

LOGICAL AND SET THEORY MODELS  
FOR GASTROPOD LARVAE,  
NORTH AMERICAN BIRDS AND  
SEALS OF THE WORLD

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## **Abstract**

Two logically valid models are used to compare the gastropod (snail) larvae of Atlantic and Pacific equatorial oceans with birds of North America. One model is this: if there is an environment that supports many species, then there are many species that are supported by one or more environments. This model says that the many species are supported by one environment in the ocean but are supported both by one environment and each species by its own environment among birds on land. A second model is this: if one environment is suited to many species then the many species are suited (adapted) to the one environment – this of course can be reversed, if species are suited to environment then environment is suited to species; so environment and species are suited to each other. This model is applicable to gastropod larvae of the ocean and the birds of North America.

A set theory model is applied to the 32 species of seals (and sea lions) of the world. A set theory model is this: a bijective relation between each species and its environment or locale is such that there is a one-to-one correspondence between each species and its unique area or environment; whereas a surjective relation allows overlap of several species occupying the same area in a non one-to-one correspondence. There are 19 bijective seal species and 13 surjective seal species. Bijective cases are the North American birds interpreted as each being supported by and suited to its own area or environment. Surjective cases are many gastropod larvae supported by or suited to one ocean environment.

## **Introduction**

Both logically valid models and set theory models would seem to be desirable in understanding nature. Each should assist the other. Each can accomplish an ultimate aim but in quite different ways. Both are needed.

To be specific, there will be presented one logically valid model that relates one entity to many entities. This is in the equatorial Atlantic and Pacific oceans, where the environment supports many species of gastropod (snail) larvae. But this is a versatile model and, on land, will show a one-to-one correspondence when each bird species is supported by its own unique area of occurrence in North America. Then there is a second logical valid model which will develop a different approach to land and sea – the approach that the environment and its species are suited to each other.

There will be a different sort of emphasis in making as broad-scale a coverage as possible. Not only the tropical Atlantic and Pacific oceans, not just North America, but additionally the whole world will be encompassed. For the seals and seal lions of the world will be dealt with, their species and habitats throughout the world.

The seals and sea lions will usher in a set theory model, and this model will be used to reassess critically how well the logically valid models match the bird and gastropod larvae data.

## **Two Logically Valid Models**

One logically valid model has a complex if-then structure. Two if-then structures, each a part, are connected as a whole in an overall if-then structure. This is one model.

A second model has one if-then structure and the reversal of this if-then structure. An if-

then structure is an implication. So in the first model there are two implications connected as an overall implication. In the second model one implication is reversed, so there is a second implication.

Then the question is: what dictates using these two models? For the first model the relation supporting will dictate its use. The overall implication is: if there is an environment that supports many species, then these species are supported by some environment or other. We see that nature is held together by the confronting, effective relation of *supporting*. And so the environment may be viewed as *supporting, producing* many species, wherein the relation of supporting in itself is separate from the environment and the species. For the second model the attribute *being suited to, being adapted to*, will dictate the model's use – if an environment is suited to a species, then the species is suited to the environment. Here, nature only contains the benign suited to. And the environment may be viewed as having, possessing the attribute of being suited to, being adapted to, many species as a part of the composition of the environment. Thus there is a great difference between the relation supporting and the attribute being suited to.

The oceanic world will be found to be built, in part at least, on one interpretation of supporting and on one interpretation of being suited to. The terrestrial world will be seen to be built on quite different interpretations of supporting and being suited to. For the purpose of the logical models is to become the reality of nature by bending to the brute facts of nature, the brute facts of the oceanic and terrestrial worlds.

### The Relation of Supporting

The relation of supporting is the core ingredient in the following logically valid model, A.

If there is one environment that supports many species, then the many species are supported by one or more environments.

With  $(\exists x)$ , there is an  $x$  such that;  $Ex$ ,  $x$  is the same as an environment;  $(y)$ , for every  $y$ ;  $Sy$ ,  $y$  is in a species;  $Sxy$ ,  $x$  supports  $y$ ; with  $\cdot$  for *and*, and  $\supset$  between an *if* structure and a *then* structure; A is represented as follows (Quine, 1972, p. 138):

$$(\exists x) [Ex \cdot (y) (Sy \supset Sxy)] \supset (y) [Sy \supset (\exists x) (Ex \cdot Sxy)], \quad A)$$

which is at length : there is an  $x$  such that  $x$  is the same as an environment, and for every  $y$ , if  $y$  is in a species, then  $x$  supports  $y$  – if all this is the case, then for every  $y$ , if  $y$  is in a species, then there is an  $x$  such that  $x$  is the same as an environment and  $x$  is supported by  $y$ . Here a realistic approach is that the variable  $x$  is the whole of an environment, that  $x$  and environment are not two but are one. Also  $y$  is a component, a constituent, a piece of a species (of each individual of the species); thus  $y$  is in a species. Instead of the usual linguistic approach, wherein ‘ $y$  is a species’ is a linguistic identity, what the identity is driving at is an external unity – thus  $y$  is *in* a species in the real, external world.

Looking ahead, how will A be applied? The answer is that A to the left will portray gastropod larvae of the tropical ocean, wherein this environment supports many species and A to the right will portray the situation synonymously wherein these species are supported by this environment. To the left  $(\exists x)$  comes before  $(y)$ ,  $Ex$  comes before

$Sy$ , and  $x$  comes before  $y$  in  $Sxy$  so that the asymmetric  $x$  supports  $y$  is gotten in a regular manner. But to the right ( $y$ ) and  $Sy$  are before  $(\exists x)$  and  $Ex$  and so  $y$  is before  $x$ ; but  $Sxy$  has  $x$  before  $y$ , so that the asymmetry is gotten as  $x$  is supported by  $y$  (Copi, 1979, p. 119).

Looking ahead still, model A can be applied to the birds of North America. To the left the North American environment supports all bird species, but to the right there are two options that are possible. All bird species are supported by the one North American environment, or instead each bird species is supported by its own unique area of occurrence, its own environment. This is the interpretation of A, for the part  $(\exists x)$   $(Ex \cdot Sxy)$  means merely that one bird species  $y$  is supported by some environment, same or different from one bird species to the next.

This last is an important aspect of model A. In the ocean we go from one environment to many species back to one environment, a single sequence. On the land we go similarly from one environment to many species and back to one environment, a similar single sequence. But also on land we go from one environment to many species to many environments, a second unique sequence. The first sequence,  $a$ , and the second sequence,  $b$ , may be considered aspects in sets, the set  $\{a\}$  for the ocean – but the set  $\{a, b\}$  for the land. The set of these,  $\{\{a\}, \{a, b\}\}$  describes the asymmetry of an *ordered pair* – ocean first, land second :  $(a, b) = \{\{a\}, \{a, b\}\}$  (Suppes, 1972, pp. 32-33).

For proof of model A see Appendix I

**The Attribute of Being Suited To**

The attribute of being suited to is the core ingredient in the following logically valid model called equivalence (see Copi, 1979, p. 40 for basic structure) (see Appendix II for axiomatic proof):

If one environment is suited to many species, then many species are suited to the one environment; and if the many species are suited to the one environment, then the one environment is suited to the many species – equivalent to : the one environment is suited to the many species if and only if the many species are suited to the one environment.

With  $x$  the same as the one environment and  $Sxy$  as  $x$  is *suited to*  $y$  and  $(y)$  and  $Sy$  as before, B is as follows:

$$\{[(y) (Sy \supset Sxy) \supset (y) (Sy \supset Syx)] \cdot [(y) (Sy \supset Syx) \supset (y) (Sy \supset Sxy)]\} \equiv [(y) (Sy \supset Sxy) \equiv (y) (Sy \supset Syx)]. \quad \text{B)}$$

This is: for every  $y$ ,  $(y)$ , if  $y$  is in a species,  $Sy$ , then environment  $x$  is suited to  $y$ ,  $Sxy$  – that is, environment  $x$  is suited to its species. This is the first parenthesis. The first parenthesis implies the second. The second is: for every  $y$ ,  $(y)$ , if  $y$  is in a species,  $Sy$ , then  $y$  is suited to environment  $x$ ,  $Syx$  – that is, its species are suited to environment  $x$ . So far, all this is the first bracket. One is to see the reversal of  $x$  and  $y$  in  $Sxy$  to  $y$  and  $x$  in  $Syx$ , indicating that if  $x$  is suited to any species (here) then any species (here) is suited to  $x$ . In the second bracket one is to see that if any species (here) is suited to environment  $x$ , then environment  $x$  is suited to any species (here). There is the putting together of the two brackets with the dot, signifying ‘and’ which refers to an element in the objective,

external world which holds together the opposing asymmetries indicated by the first and second brackets.

In the third bracket the environment  $x$  is suited to its species if and only if they are suited to it (' $\equiv$ ' may be 'equivalent to', as in the first occurrence, or 'if and only if', as in the second occurrence). Both the environment and the species have the attribute of being suited to each other. For B should be: if one environment has the attribute of being suited to many species, then the many species have the attribute of being suited to the one environment; and if the many species have the attribute of being suited to the one environment, then the one environment has the attribute of being suited to the many species – equivalent to: the one environment has the attribute of being suited to the many species if and only if the many species have the attribute of being suited to the one environment. The asymmetry of the attribute of being suited to species is blocked by the asymmetry of the attribute of being suited to environment.

This description holds for gastropod larvae. But the birds of North America have a one-to-one correspondence between each species and its own unique environment. So C is the logically valid model for this situation and has the basic structure of B. The bare-bones basic structure for both B and C is  $[(P \supset Q) \cdot (Q \supset P)] \equiv (P \equiv Q)$ , which is proved in Appendix II. C is:

$$[(S_{xy} \supset S_{yx}) \cdot (S_{yx} \supset S_{xy})] \equiv (S_{xy} \equiv S_{yx}), \quad \text{C)}$$

where: if environment's  $x$  is suited to species'  $y$ , then species  $y$  is suited to environment's  $x$ ; and if species'  $y$  is suited to environment's  $x$ , then environment's  $x$  is suited to species'  $y$  – equivalent to: environment's  $x$  is suited to species'  $y$  if and only if species'  $y$  is suited



to environment's  $x$ . The issue of concern is that the environment, a flat, continuous far-flung piece of material, has the same attribute that the scattered, discontinuous, collected bulk material of some bird species has. The two have a common attribute. The two exemplify the common attribute of being suited to. Thus they jointly and multiply exemplify a single, abstract constituent of nature, the constituent of being suited to (Moreland, 2001, p. 74).

### **The Gastropod Larvae of the Atlantic and Pacific Oceans**

Veliger gastropod (snail) larvae are tiny snail-like forms (Sheltema, 1971; Sheltema and Williams, 1983), (Figures 1, 2 and 3). They are planktonic, existing for as long as 55-320 days in a form capable of settlement and metamorphosis if contact with land happens (Sheltema, 1971). They occur over large areas of the Atlantic Ocean or Pacific Ocean (Figures 1 and 3). They are carried passively by equatorial westward currents; there are north equatorial and south equatorial currents and a minor eastward countercurrent between them in both the Atlantic and Pacific Oceans. The larvae are produced of course by parent sublittoral snails on the shores of the Atlantic continents and on Pacific islands. The tiny larvae (1 mm. or less) occur as a number of species in about a dozen families. A number have been studied in detail; 15 are described in detail in the two studies just mentioned. But many more are present in the samples that span the equatorial oceans. Though they are produced along the shore, they are supported by the single equatorial environment of the Atlantic and by the single equatorial environment of the Pacific (Sheltema, 1995; Sheltema et al., 1996). Additionally, the

environment of the Atlantic is suited to all larvae species which are suited to the environment. Likewise for the Pacific environment.

So the relation of supporting binds together environment and all the gastropod larvae species: if there's one environment and it supports many species, then the many species are supported by this environment, (A). And so, too, the attribute of being suited to binds together environment and species in another way: if the environment is suited to all the species then all the species are suited to the single environment, .....(B).

### **The Birds of North America**

Unlike the gastropod larvae of equatorial oceans, which are not well delineated specifically in many cases, the birds of North America are in very well delineated species – at least 900 species. Unlike the larvae, which are all likely to be caught anywhere in the equatorial oceanic regions, each bird species seems to be located in its own unique area. Some bird species stay in the same area all the year round, such as the chickadee (7 species) and breed in the spring and summer (Fig. 4). Many species migrate south within the continent in winter and north to breed in spring and summer, such as the 33 species of sparrows (National Geographic, 1999) (Fig. 5). And some species migrate to South America for the winter and return to North America to breed in summer – such as the scarlet tanager, the rose-breasted grosbeak, and the golden plover (Lincoln and Hines, 1950) (Fig. 6). Thus the North American continent (including Mexico) does more than merely support these species; it produces them, in the sense that their breeding occurs there – unlike the gastropod larvae in the ocean. Thence the relation of supporting might be strengthened as the relation of producing.

In a correlated way the attribute of being suited to might be strengthened as the attribute of being adapted to. For certainly being able to reproduce in a given area shows that the species is adapted to the area – it wouldn't be there otherwise – and that its area is adapted to the species – it couldn't be there otherwise, could it.

So the relation of supporting, or producing, binds together environment and species: if there's an environment that supports, produces, many species, then the species as a whole are supported, produced, by the environment – and additionally each one of these many species is supported, is produced, by its own environment (A). And so the attribute of being suited to, the attribute of being adapted to, binds together environment and species in another way: if a given environment's  $x$  is suited to, is adapted to, a given species'  $y$ , then the given species'  $y$  is suited to, is adapted to, its own environment's  $x$ , and if the given species'  $y$ .....,  $[(Sxy \supset Syx) \cdot (Syx \supset Sxy)] \equiv (Sxy \equiv Syx)$ . C)

### **The Seals and Sea Lions of the World**

The gastropod larvae of the equatorial oceans are the entrained products of sublittoral land producers, their parent snails. The migrating birds that have been considered, when wintering in Central and South America, are the entrained though full grown products of the breeding birds in North America. Additionally, the environments that the bird species uniquely occupy are in a real sense producers of the birds. So there are two producers, the areas that have the food and protection for the bird species and the species themselves that produce the young. But the distinction of two sorts of producers will be altered in the next section, wherein the environment will not be merely the

tropical ocean or the North American continent but instead will be the whole world. The following account is from Nigel Bonner's Seals and Sea Lions of the World.

There are 14 species of seals and sea lions belonging to the Otariidae. They have external ears, use just their fore flippers for swimming, and on land move by short steps with their bodies held up from the ground. There are 18 species of seals belonging to the Phocidae. These have no external ears, use just their hind flippers for swimming and on land move by a humping action of their bodies against the ground.

They haul out on beaches and cobble shores and even on pack ice in spring and summer in the northern hemisphere and in winter in the southern hemisphere. They give birth and nurse their young normally for a few weeks to several months, the female returning to the water intermittently in some cases for several days to feed and thus restore her ability to produce milk. During this period they are mated by males each of which accumulates a number of females in harem-like groups. After the young are weaned, the beaches and shores are left empty as the seals spend a number of months swimming and feeding. The shore supports and produces each seal species in the sense that it is required for birth, and is a sociable and necessary place to bring up the young. Only remote shores are like this.

Only remote lonely shores compose the environment of the seals – shores free of predators but occurring anywhere from polar to tropical isles. So the lonely, remote shore environment is a single scattered entity – scattered throughout the world. Scattered but single. Its singleness and scatteredness is captured by the following description:

If there is a remote shore environment that supports (produces) the 32 seal (sea lion) species of the world, then the species are supported (produced) each by its own unique remote shore environment. A)

We can have too:

If each unique shore environment is suited (adapted) to each species, then each species is suited (adapted) to its unique shore environment; and if each species is suited (adapted) to its shore environment, then its shore environment is suited (adapted) to it; -- equivalent to: each environment is suited (adapted) to its species if and only if its species is suited to it. C)

From this we see that the issue of concern is whether or not single species and environments can be perfectly paired in fact. The factual situation is described next.

Of the nine fur seals, the northern fur seal, *Callorhinus ursinus*, had teeming, remote rookeries from Alaska to Russia, when first discovered in the mid 1700's. In spite of relentless seal hunting, large breeding populations are still present today as shown in Fig. 7, one as far south as San Miguel I. in California. Continuing southward (Fig. 7), three species have isolated island shore rookeries, *Arctocephalus townsendi* on Guadalupe I., *A. galapagoensis* on a number of Galapagos Islands, and *A. philippii* on Juan Fernandez Islands. *A. australis* is more widespread than the foregoing species but does not overlap them; it breeds on islands from Isles de Lobos around Tierra del Fuego, including the Falkland Is. and up the west coast of South America to Peru. All of these breeding colonies were reduced drastically by hunting, in some cases to dozens in the

nineteenth century. Subsequently some of the colonies regained their former numbers of thousands or 100 thousands of animals.

The remaining fur seals are found on remote islands around Antarctica and off the South African and Australian coasts (Fig. 8). Their locations for the most part seem separated from each other, though with a small amount of overlap in some places.

*A. pusillus pusillus* breeds in 23 colonies along the coast of South Africa and Namibia, the four largest not on islands but backed by deserts with no predators. *A. pusillus doriferus* breeds on islands between Tasmania, Victoria, and New South Wales off Australia.

The fur seals have an abundant underfur layer in their pelage, which is lacking in the sea lions. The five sea lion species have areas from the North Pacific to Antarctica, like the fur seals. But their areas usually do not overlap in any detailed way, except Stellar's sea-lion, *Eumatopias jubatus*, which has a distribution that closely overlaps the Fur seal *Callorhinus ursinus* from Alaska to Russia (Fig. 7). The California sea lion, *Zalophus californianus*, is found from Vancouver Island to the Tres Marias Islands off Mexico (not shown) though the breeding range does not extend so far north. *Zalophus californianus wollabaeki*, a subspecies, is found on the Galapagos Islands and thus does overlap completely *Arctocephalus galapagoensis*. But there is only small overlap of the Australian sea lion *Neophosa cinerea* found at three places in southwest and west Australia (not shown) and the fur seals *Arctocephalus pusillus doriferus* and *A. forsteri*, the last extending to many islands south of New Zealand from south and southwest Australia (shown in Fig. 8). The southern sea lion *Otaria flavescens* overlaps fairly

closely the distribution of the fur seal *Arctocephalus australis*, extending from Isles de Lobos around Tierra del Fuego and up the coast of Chile where many islands and breeding platforms exist (Fig. 7).

The seals belonging to the Phocidae, having no external ears and using the hind flippers for swimming, are unlike the Otariidae, which have external ears and swim with the fore flippers – are unlike too in having several species in the North Atlantic Ocean and Arctic Ocean. The Bearded seal, *Erignathus barbatus*, is circum arctic (Fig. 9), breeding on pack ice. The Ringed seal (*Phoca hispida*) has a circum arctic distribution too – complete overlap (Fig. 9.). These overlap partially the Harp seal (*Phoca graenlandica*, which extends northeastward to the White Sea (Fig. 10) and the Hooded seal, *Cyrtophora cristata*, which is found around Newfoundland from Svalbard to the east to the Gulf of St. Lawrence to the west and breeds as shown in Fig. 10. The Grey seal *Halichoerus gryptus* (not shown) is like the Hooded seal in its western extent but its eastern extent takes in the British Isles and the Baltic; so there is only partial overlap. The Harbor seal, *Phoca vitulina* (not shown) is circumpolar and overlaps the Atlantic areas of the last two species but extends into the North Pacific, along the west coast of U.S.A. and east rim of the Pacific as far as Hokkaido – again partial overlap. The Spotted seal (*Phoca largha*) and the Ribbon seal (*Phoca fasciata*) have duplicate distributions (complete overlap) from north of Bering Strait down along the Pacific coast to Hokkaido or Japan (Fig. 9).

Among the earless Phocidae are seven more species, in three groups. In the first group are two monk seals, one the Hawaiian monk seal (*Monachus schauinslandi*),

breeding on several islands northwest of Hawaii, and the Mediterranean monk seal (*Monachus monachus*) (not shown). These do not overlap any other seals or each other. Both are small in numbers, about 1000 for the Hawaiian and several hundred for the Mediterranean monk seals. In the second group are two elephant seals, which have proboscis-like noses. The northern one, *Mirounga angustirostris*, occupies islands from San Francisco to Baja, California (Fig. 7) and increased from about 20 animals to 125,000 animals between 1890 and the present to constitute the present range. This range overlaps completely the California sea lion (not shown). The Southern Elephant seal, *M. leonins*, has breeding areas on all the same islands around Antarctica (Fig. 7) that are the breeding areas of the fur seals *Arctocephalus gazella* and *tropicalis*, and in part of *A. forsteri* and *australis*. In the third group (Fig. 7) are the Weddel seal, the Crabeater seal, the Leopard seal, and the Ross seal, all hauling out and breeding usually on pack ice close to the Antarctic continent. They overlap only slightly the southern elephant seal and the four fur seals just mentioned. Whether they overlap each other much is hard to say.

Finally two isolated seals are a species in the Caspian Sea and a species in Lake Baikal.

From this brief account a summary of overlap of distributions is as follows:

Complete overlap: the northern fur seal and stellar's sea lion in the north Pacific; the California sea lion and the Northern Elephant seal; *Zalophus californianus wollebacki* and *Arctocephalus galapagoensis* on the Galapagos Islands; the Spotted seal and the Ribbon seal in the northwestern Pacific and Bering Straits; the Bearded seal and the



Ringed seal in the Arctic ocean; the seal lion *Otaria flavescens* and the fur seal *Arctocephalus australis* around South America. Six cases.

Partial overlap: the circum Arctic species overlap partially the three species, the Hooded seal, the Grey seal, and the Harp seal. Moderate overlap occurs among these three. Moderate overlap occurs too between the Southern Elephant seal and the four southern fur seals. Only a vague overlap occurs between all these and the Harbor seal, which extends from the North Atlantic to the North Pacific. Seven cases, approximately.

There are then 13 cases with some measure of overlap. Six of them are complete overlap, so that there is no one-to-one correspondence between species and area. Seven of these are partial overlap, so that there is no clear correspondence between species and area. The rest, 19 species, many of which are not shown, have a separate unique area for each species, so that there is a clear, one-to-one correspondence between each species and its area.

The distinction is between no overlap and one-to-one correspondence and between overlap and no one-to-one correspondence. This distinction can be reassessed through functions of set theory.

### **The Set Theory Model: Functions**

One finds these sorts of statements about functions. “Consider the function  $f(x) = x^3$ , i.e.,  $f$  assigns to each real number its cube”. “Let  $g$  assign to each country in the world its capital city” (Lipschutz, 1998, pp. 94, 95). These indicate the broad coverage of functions. More fully what the first says is: the function of being cubed, of having the property of being cubed, is to relate each number to its cube by assigning to each number

its cube. Thus there are in functions both properties and relations. More fully, what the second says is: the function of being a capital city, of having the property of being a capital city, is to relate each country to its capital city by assigning to each country its capital city. There is no property of being a capital city pure and simple. You can only have: country *a* has the property of possessing the capital city assigned to country *a* or country *a* has the capital city assigned to country *a*. Taking away property leaves the relation of country having capital city. There is still correspondence between country and capital city, between number and cube – and between seal species and its area. There are two sorts of correspondence.

One is one-to-one correspondence (no overlap) and this is called bijective – there is a bijective function of adaptedness<sup>1</sup> (the property of being adapted) that assigns to each seal species its area in the case that no other species is assigned to this area, in the case of no overlap. This bijective function is reversible – the function of adaptedness (suitedness) assigns to each area its seal species, in the case that the area does not overlap any other area. There are 19 of these species and areas. There are 19 members of the set, the class, of seals and there are 19 members of the set, the class, of areas. There are then two sets. What determines belonging to, being a member of, one of these sets is having the function of adaptedness to just one unique member of the other set. The reversible bijective function of adaptedness (suitedness) determines membership in the set of the 19 seal species via the set of their areas and the membership in the set of areas via the set of the 19 seal species.

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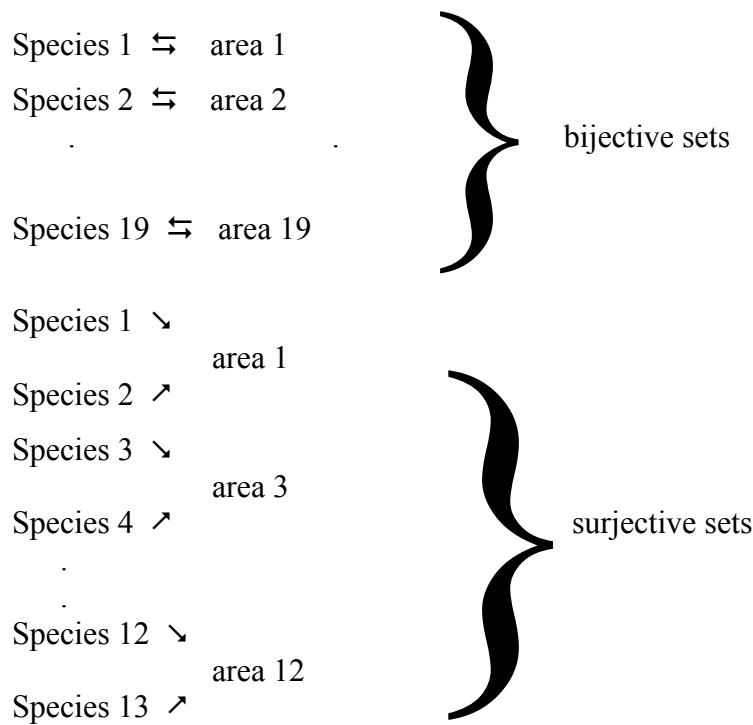
<sup>1</sup> Adaptedness = being adapted. The property of adaptedness = the property of being adapted. property = attribute.

What *does* determine being a member, being just one, of the 19 seal species set? Being adapted to its area. This is the answer to the question.

What *does* determine being a member of the area set? Being adapted to its seal species.

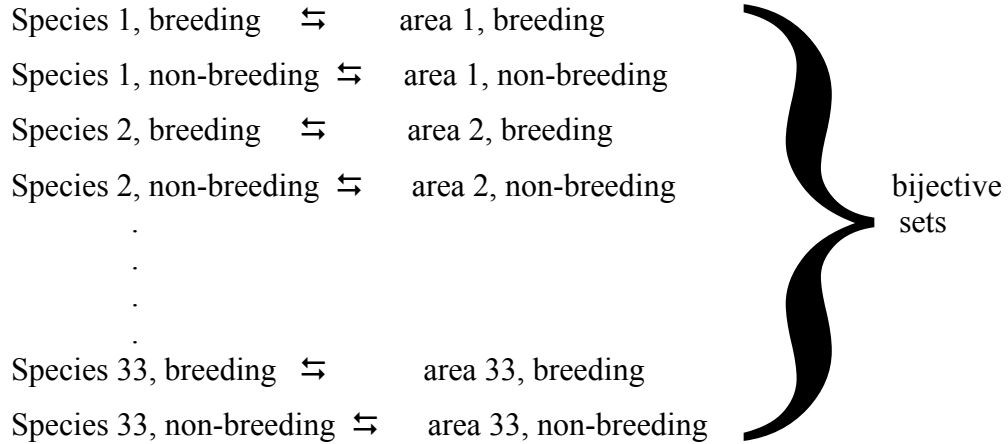
Next, the 13 species with overlapping areas do not have a one-to-one correspondence. They have a surjective function of adaptedness that assigns two or more of them to the same or partially the same area. What determines being a member of this set of species is, as before, having an area of its own, but not having a unique area – because another species has this area, at least in part, as its own too.

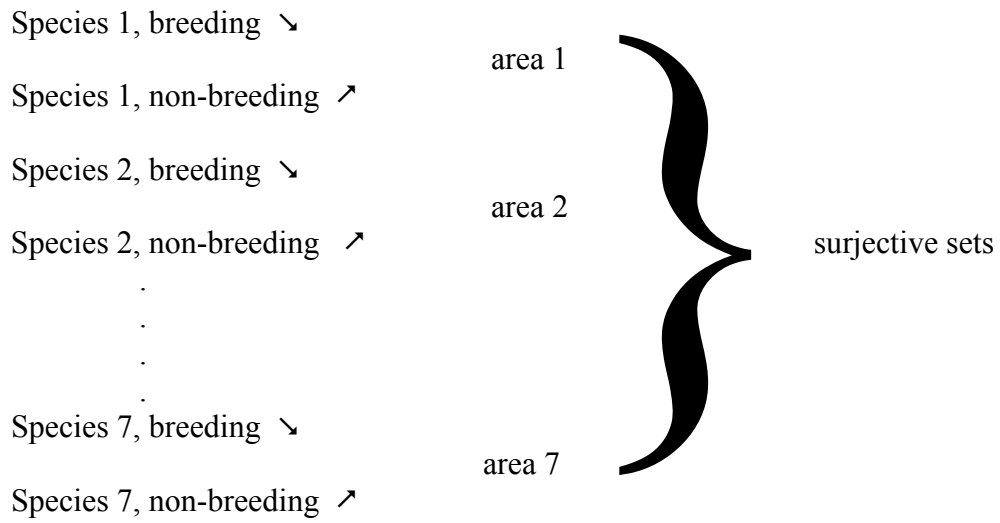
Having an area as its own, being adapted or suited to its area, can be shown for the bijective and surjective species as follows (see Hulburt, 2004; Lipschutz, 1998, p. 99).



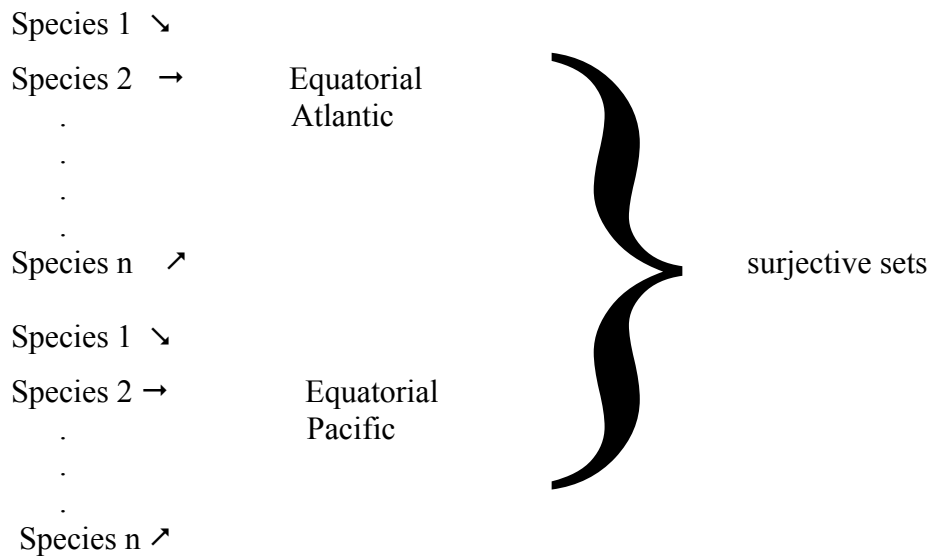
The 19 non-overlapping seals make bijective sets. The 13 overlapping seals make surjective sets.

Among birds, sparrows and the three selected species are examples of migrators, a large bijective set with one-to-one, reversible assignments of its members to members of the area set. But there are four sets now, the breeding northern pairs of sets and the non-breeding southern pairs of sets. Then chickadees, that don't migrate, compose three sets. All this is shown as follows for 33 sparrow species and 7 chickadee species. It is important to note that one-to-one correspondence in the case of seals means no overlap of areas of species residence but in the case of birds means no overlap of winter and summer areas of each species.





Among gastropod larvae all the species of the Atlantic equatorial environment form two sets, one with many species and the other with the single membered equatorial environment set. Same for the Pacific. Both of these pairs of sets are pictured in the following way.



What would set theory say about the relation of supporting (producing) as compared to the property or attribute of being suited to – the property of being adapted to, the property of adaptedness to? Model A left says that the one environment supports the many larval species, that one environment supports many bird species, that one scattered environment supports 32 seal species. This is surjective with the set of species to the left and the one-membered set environment to the right (as just pictured). Model A right says many species are supported by one environment or many species are supported (produced) each by its own environment. The first option is surjective; the second option is bijective, except for the surjective non-migratory birds and the overlapping seals which are surjective. The situation is mixed. But the bijective, reversible, one-to-one corresponding pattern seems to emerge as a somewhat dominating feature in these samples from land.

Model C endorses the ecological suited-to attribute and is applicable only in reversible bijective cases.

### **The Perfect World Model**

What would an ideal, perfect world model in the context of the samples of nature so far presented be? Should there be one-to-one correspondences in reversible bijective sets of *ordered pairs* of entities that are described actively by structural elements that produce and animate the ecological spectrum and that are also described passively by the integration of species traits and externality, not leaving out an overriding salience, an overriding assertive *asymmetry* that dictates, one would think, the changing panorama

that confronts us. For we cannot get away from asymmetry. *Support* is asymmetric.

*Produce* is asymmetric. *Suited to* and *adapted to* are likewise asymmetric.

It is asymmetric that the snail larvae are supported by the equatorial ocean, that the bird species of North America are produced each by its own area, that the seal species of the world are partially produced each by its fragment of lonely, remote shores of northern, tropical, and southern coasts. Suppose there were only this babel of assertive producings and supportings. But such a chaotic world is not what we deduce.

What we deduce is the blocking of assertive asymmetries. An initial step is the switch from support to suited to, from produce to adapted to. But there is no structural guarantee that these more benign asymmetries will get us the real world. The asymmetries must be blocked. If implication is part of nature, then the asymmetry of implication can be blocked by reverse implication. Next are shown parts of models B and C, which have reverse implications. It will be recalled that  $Sxy$  is *x is suited to y* and that  $Syx$  is *y is suited to x* and that  $Sy$  is *y is in a species*. We have:

$$[(y) (Sy \supset Sxy) \supset (y) (Sy \supset Syx)] \cdot [(y) (Sy \supset Syx) \supset (y) (Sy \supset Sxy)] \quad \text{B)}$$

$$(Sxy \supset Syx) \cdot (Syx \supset Sxy) \quad \text{C)}$$

$$(P \supset Q) \cdot (Q \supset P) \quad \text{D)}$$

Asymmetry is blocked by reversing the first implication to get the second. The bare-bones structure is given by D. What is vital is joining implication and reverse implication by the dot, meaning the linguistic ‘and’. In the real external world there must be a linkage for the word ‘and’ to refer to. Such a linkage is abstract, just as sets

(species) are abstract, just as symbolic variables are abstract. Abstract but real. For the models we are presenting are not models of the world; they are the world.

The annulment of asymmetry is, of course, a forlorn enterprise in a world that departs from a perfect world model. The crucial and basic flaw of the enterprise comes from set theory. For the entities of the two sets that are bijective (one-to-one) or surjective (not one-to-one) are ordered pairs. Ordered pairs are such that  $a$  comes before  $b$ . And this order is asymmetrical. One larval species must be to the left and its environment to the right, one bird species left and its area right, seal species left and its area to the right, species and areas taken in pairs. But these are small ordered pairs.

Two very different ordered pairs are from model A: in the ocean the environment supports many species, so the many species are supported by the environment – this sequence is interpreted as a sole aspect in set  $\{a\}$ . On land model A has two aspects where sequence  $a$  means the land environment supports many species, so the many species are supported by the single land environment, North America or whole world – or where sequence  $b$  means the land environment supports many species, so the many species are supported each by its own environment, about 900 bird species and their areas or 32 seal species and their areas.

Thus  $a$  is a single aspect in the ocean set  $\{a\}$ . Thus  $a$  and  $b$  are two aspects, a unified aspect from a many aspect in the land set  $\{a, b\}$ . The ordered pair is the set of these,  $(a, b) = \{\{a\}, \{a, b\}\}$ . This is irreversible, left precedes right, the littler set first, the bigger set second, the way 1 comes before 2 (see section on relations; see Appendix III). There are two of these pairs, the larvae-bird pair and the larvae-seal pair. These



are two vast pairs. They bestride the natural world in a way undreamt of by the usual presentations of set theory.

Larvae

Birds

One aspect,  $\{a\}$

Two aspects,  $\{a, b\}$

Larvae

Seals

One aspect,  $\{a\}$

Two aspects,  $\{a, b\}$

**Appendix I**

The proof of A is as follows.

1.	$(\exists x) [Ex \cdot (y) (Sy \supset Sxy)]$	Assum.
2.	$Sy$	Assum.
3.	$Ex \cdot (y) (Sy \supset Sxy)$	1, EI
4.	$Ex$	3, Simp.
5.	$(y) (Sy \supset Sxy)$	3, Simp.
6.	$Sy \supset Sxy$	5. UI
7.	$Sxy$	6, 2, MP
8.	$Ex \cdot Sxy$	4, 7, Conj.
9.	$(\exists x) (Ex \cdot Sxy)$	8, EG
10.	$Sy \supset (\exists x) (Ex \cdot Sxy)$	2-9, CP
11.	$(y) [Sy \supset (\exists x) (Ex \cdot Sxy)]$	10, UG
12.		
	$(\exists x) [Ex \cdot (y) (Sy \supset Sxy)] \supset (y) [Sy \supset (\exists x) (Ex \cdot Sxy)]$	1-12, CP

Line 1 is assumed, that is, *if* is placed before it and so it is left dangling until line 12.

Line 2 is similarly assumed and its dangling status is resolved in line 10. In both cases the *if* part of lines 1 or 2 is followed by a *then* part in lines 12 or 10; this is called conditional proof, CP. Line 3 shows that just one environment suffices for the proof and (  $x$  ) of line 1 is dropped – a process called existential instantiation, EI. Line 4 shows that from the and-connected parts of line 3 one part,  $Ex$ , may be deduced; line 5 shows that the other part,  $(y) (Sy \supset Sxy)$ , may be deduced too – this is simplification, Simp.. Line 6 is universal instantiation, UI, wherein, if  $Sy \supset Sxy$  of line 5 is true of every  $y$ ,  $(y)$ , then  $(y)$

is dropped and just one  $y$  and its species,  $Sy$ , suffices for the proof. In line 7  $Sxy$  is deduced by the argument: if  $Sy$  then  $Sxy$  of line 6, given  $Sy$  of line 2, therefore  $Sxy$  of line 7 – modus ponens, MP. Line 8 shows that the putting together of  $Ex$  and  $Sxy$  is deduced from  $Ex$  of line 4 and  $Sxy$  of line 7 – conjunction, Conj. Line 9 just says that since  $x$  is an environment,  $Ex$ , and environment  $x$  supports  $y$ ,  $Sxy$ , then there is one such  $x$  – existential generalization, EG. And line 11, universal generalization, UG, makes a contrasting claim that for every  $y$  if  $y$  is in a species then .....

The issue of deduction occurs three times. The first is simplification. If both parts of line 3 are true the whole is true. If one or both are false, then the whole is false. The outcome from this situation is line 4 and line 5. And the outcome is arresting, because you can't get false out of line 3 going to line 4 or to line 5. The reason is that if the antecedent parts of line 3 are both true so are the consequent parts (4 and 5) and the whole works is true. But if the antecedent parts are singly false then each yields a false consequent (4 or 5) – and *false* antecedent yielding *false* consequent is considered *true* as a whole.

The second case of deduction is modus ponens, wherein antecedent,  $Sy$  and consequent,  $Sxy$ , are laid out:

$Sy$	$\supset$	$Sxy$	
True	$\supset$	True	True
True	$\supset$	False	False
False	$\supset$	True	True
False	$\supset$	False	True

The first has both parts true, so it is true as a whole. The second is flatly the opposite of the first, so it is false as a whole. What shall we do about the third and fourth? They are certainly not the flat opposites of the first, so they are counted as true. The second in conjunction with the antecedent  $Sy$  would yield false as a whole, and this conjunction implies the consequent  $Sxy$  (true = T, false =  $\perp$ ):

$$Sy \supset Sxy \cdot Sy : \supset Sxy$$

$$T \supset \perp \cdot T : \supset \perp$$

$$\perp \cdot T : \supset \perp$$

$$\perp \supset \perp$$

$$T$$

The other three in conjunction with the antecedent are:

$$Sy \supset Sxy \cdot Sy : \supset Sxy$$

$$1^{\text{st}} \quad T \supset T \cdot T : \supset T$$

$$T$$

$$3^{\text{rd}} \quad \perp \supset T \cdot \perp : \supset T$$

$$T \cdot \perp : \supset T$$

$$\perp \supset T$$

$$T$$

$$4^{\text{th}} \quad \perp \supset \perp \cdot \perp : \supset \perp$$

$$T \cdot \perp : \supset \perp$$

$$\perp \supset \perp$$

$$T$$

Thus the analysis of modus ponens is very important, because modus ponens is everywhere. There is no such thing as stimulus – response; instead we have: if stimulus then response, given stimulus; therefore response. There is no such thing as cogito ergo sum; instead we have: if cogito then sum, given cogito; ergo sum.

The third deduction is conjunction: if  $Ex$  (line 4), then if  $Sxy$  (line 7), then both  $Ex$  and  $Sxy$  (line 8). Instead of doing a true-false analysis as just done, an elaborate analysis in Appendix II step 37 is given.

The three deductions, simplifications, modus ponens, and conjunction compose the structure of all nature. Their use in the analysis lines 1-12 is part of nature.

### Appendix II. Equivalence

The following derivation of  $[(P \supset Q) \cdot (Q \supset P)] \equiv (P \equiv Q)$  is from Hilbert and Ackerman (1950, p. 27-39), as developed by Copi (1979, p. 266-268). The derivation is from Hilbert and Ackerman's four axioms and takes 44 steps. A different derivation is from Rosser's three axioms and takes 89 steps (Hulburt, 2002).

The method of derivation is by substitution. Where one expression is put for another expression, the one expression is substituted for the other. At step 7.  $P$  is put for  $Q$ , and so on.

There is one rule of inference, R.1 (where  $\therefore$  = therefore):

$P \supset Q$

$P$

$\therefore Q$

There are two definitions (where  $\cdot$  is *and*):

$$P \supset Q = (\text{def.}) \sim P \vee Q, \text{ step 11.}$$

$$\sim P \vee [\sim Q \vee \sim (\sim P \vee \sim Q)] = (\text{def.}) P \supset [Q \supset (P \cdot Q)], \text{ step 37.}$$

There are several assumed expressions. These are premises.

The method follows fairly closely Copi’s presentation of Hilbert and Ackerman, except for rule 1, which is taken from Rosser (1953), p. 65, Theorem 4.20.

The Hilbert and Ackerman axiomatic presentation, like that of Rosser, derives all the major principles of logic – contraposition, conjunction, simplification, association, commutation, distribution, double negation, Dr. Morgan’s laws, equivalence.

In following the steps of each proof, sometimes a step initiates something wholly new, as in step 17, and sometimes each step depends on the last as in 24.-27., and in 28.-33. Just the bracketed part of axiom 4 is used.

Axioms

$$(P \vee P) \supset P \qquad \text{A1}$$

$$P \supset (P \vee Q) \qquad \text{A2}$$

$$(P \vee Q) \supset (Q \vee P) \qquad \text{A3}$$

$$(P \vee Q) \supset [(R \vee P) \supset (R \vee Q)] \qquad \text{A4}$$

Rule 1

$$P \supset P_1$$

$$P_1 \supset P_2$$

$$P_2 \supset P_3$$

⋮

⋮

⋮

$$P \supset P_n$$

Proof of  $P \supset (Q \vee P)$ , Theorem 1

1.  $P \supset (P \vee Q)$                       A2
2.  $(P \vee Q) \supset (Q \vee P)$               A3
3.  $P \supset (Q \vee P)$                       Rule 1, from 1 and 2

From  $(P \vee Q)$  derive  $(Q \vee P)$ , Theorem 2

4.  $(P \vee Q) \supset (Q \vee P)$               A3
5.  $P \vee Q$                                   Premiss
6.  $Q \vee P$                                   R.1, from 4 and 5

Proof of  $P \supset P$ , Theorem 3

7.  $P \supset (P \vee P)$                       A2;  $P$  for  $Q$
8.  $(P \vee P) \supset P$                       A1
9.  $P \supset P$                                   Rule 1, from 7 and 8

Proof of  $P \vee \sim P$ , Theorem 4

10.  $P \supset P$                                   Theorem 3
11.  $\sim P \vee P$                               Def.;  $P$  for  $Q$ , from 10
12.  $P \vee \sim P$                               Theorem 2;  $(\sim P \vee P)$  for  $(P \vee Q)$ ,  $(P \vee \sim P)$  for  $(Q \vee P)$ ; from 11

Proof of  $[P \vee (Q \vee R)] \supset [Q \vee (P \vee R)]$ , Theorem 5

13.  $R \supset (P \vee R)$                       Theorem 1;  $R$  for  $P$ ,  $P$  for  $Q$
14.  $(Q \vee R) \supset [Q \vee (P \vee R)]$       A4;  $Q$  for  $R$ ,  $R$  for  $P$ , and  $(P \vee R)$  for  $Q$ ; from 13
15.  $[P \vee (Q \vee R)] \supset \{P \vee [Q \vee (P \vee R)]\}$       A4;  $P$  for  $R$ ,  $(Q \vee R)$  for  $P$ ,  
 $[Q \vee (P \vee R)]$  for  $Q$ ; from 14
16.  $\{P \vee [Q \vee (P \vee R)]\} \supset \{[Q \vee (P \vee R)] \vee P\}$       A3; obvious substitutions; from 15

17.  $P \supset (P \vee R)$  A2;  $R$  for  $Q$
18.  $(P \vee R) \supset [Q \vee (P \vee R)]$  Theorem 1;  $(P \vee R)$  for  $P$
19.  $P \supset [Q \vee (P \vee R)]$  Rule 1; 17, 18, 19
20.  $\{[Q \vee (P \vee R)] \vee P\} \supset \{[Q \vee (P \vee R)] \vee [Q \vee (P \vee R)]\}$  From 19 by A4,  
 $[Q \vee (P \vee R)]$  for  $R, Q$
21.  $\{[Q \vee (P \vee R)] \vee P\} \supset [Q \vee (P \vee R)]$  A1; on right second  $[Q \vee (P \vee R)]$  eliminated  
 in 20
22.  $[P \vee (Q \vee R)] \supset [Q \vee (P \vee R)]$  Rule 1; 15, 16, 21, 22
- Proof of  $[P \vee (Q \vee R)] \supset [(P \vee Q) \vee R]$  Theorem 6
23.  $(Q \vee R) \supset (R \vee Q)$  A3,  $Q$  for  $P, R$  for  $Q$
24.  $[P \vee (Q \vee R)] \supset [P \vee (R \vee Q)]$  A4,  $P$  for  $R, (Q \vee R)$  for  $P, (R \vee Q)$  for  $Q$ ; from 23
25.  $[P \vee (R \vee Q)] \supset [R \vee (P \vee Q)]$  Theorem 5; from 24
26.  $[R \vee (P \vee Q)] \supset [(P \vee Q) \vee R]$  A3;  $R$  for  $P, (P \vee Q)$  for  $Q$ ; from 25
27.  $[P \vee (Q \vee R)] \supset [(P \vee Q) \vee R]$  Rule 1; 24, 25, 26, 27
- Proof of  $[(P \vee Q) \vee R] \supset [P \vee (Q \vee R)]$ , Theorem 7
28.  $[(P \vee Q) \vee R] \supset [R \vee (P \vee Q)]$  A3, obvious substitutions
29.  $[R \vee (P \vee Q)] \supset [P \vee (R \vee Q)]$  Theorem 5; from 28
30.  $[P \vee (R \vee Q)] \supset [(P \vee R) \vee Q]$  Theorem 6; from 29
31.  $[(P \vee R) \vee Q] \supset [Q \vee (P \vee R)]$  A3; from 30
32.  $[Q \vee (P \vee R)] \supset [P \vee (Q \vee R)]$  Theorem 5; from 31
33.  $[(P \vee Q) \vee R] \supset [P \vee (Q \vee R)]$  Rule 1; 28, 29, 30, 31, 32, 33



Proof of  $P \supset [Q \supset (P \cdot Q)]$

Theorem 8

34.  $(\sim P \vee \sim Q) \vee \sim(\sim P \vee \sim Q)$  Theorem 4;  $\sim P \vee \sim Q$  for  $P$ ,  $\sim(\sim P \vee \sim Q)$  for  $\sim P$
35.  $[(\sim P \vee \sim Q) \vee \sim(\sim P \vee \sim Q)] \supset$   
 $\{\sim P \vee [\sim Q \vee \sim(\sim P \vee \sim Q)]\}$  Theorem 7;  $\sim P$  for  $P$ ,  $\sim Q$  for  $Q$ ,  $\sim(\sim P \vee \sim Q)$  for  $R$ ; from 34
36.  $\sim P \vee [\sim Q \vee \sim(\sim P \vee \sim Q)]$  R.1; from 35 and 34
37.  $P \supset [Q \supset (P \cdot Q)]$  Def.; from 36

Proof of  $(P \supset Q) \cdot (Q \supset P)$

38.  $(P \supset Q) \supset [(Q \supset P) \supset [(P \supset Q) \cdot (Q \supset P)]]$  37.,  $(P \supset Q)$  for  $P$ ,  $(Q \supset P)$  for  $Q$
39.  $P \supset Q$  Premiss
40.  $(Q \supset P) \supset [(P \supset Q) \cdot (Q \supset P)]$  R.1; from 38 and 39
41.  $Q \supset P$  Premiss
42.  $(P \supset Q) \cdot (Q \supset P)$  R.1; from 40 and 41

If  $P$  then  $Q$  is:  $Q$  if  $P$ . Also, if  $Q$  then  $P$  can be:  $Q$  only if  $P$ . So  $(P \supset Q)$  and  $(Q \supset P)$  condense to  $(Q$  if  $P)$  and  $(Q$  only if  $P)$ , which condenses to  $(Q$  if and only if  $P)$ , which is shown as  $(Q \equiv P)$ , which can be switched to  $(P \equiv Q)$ . So if you have 42. then you get  $(P \equiv Q)$ , and if you have  $(P \equiv Q)$  then you get 42. – just like 42. - and putting these in conjunction you get:

43.  $\{[(P \supset Q) \cdot (Q \supset P)] \supset (P \equiv Q)\} \cdot \{(P \equiv Q) \supset [(P \supset Q) \cdot (Q \supset P)]\}$
44.  $[(P \supset Q) \cdot (Q \supset P)] \equiv (P \equiv Q)$

44. is the basic structure of models B and C, as derived from the Hilbert-Ackerman axioms, substitution, definitions, R.1, rule 1, and premisses.

Steps 36. to 37. are very important. The part  $\sim(\sim P \vee \sim Q)$  says that the denial of  $P$  denied or  $Q$  denied gives you both  $P$  and  $Q$ , as shown in 37. The denial of the initial  $P$  or  $Q$  is a different matter. One has always the choice of  $\sim P$ . So you don't get  $P$  is  $\sim P$ . And if you don't don't get  $P$  is  $\sim \sim P$ , which is  $P$ . So instead of  $\sim P$  you have  $P$ . Same for  $Q$ . So you get from  $\sim P \vee (\sim Q \dots)$  to  $P \supset [Q \supset \dots]$ . This is what is happening between steps 10. and 11. earlier. And this is presaged by the first definition.

**Appendix III. Set of Sets**

A collection of sets, a set of sets, may be assembled from the following structures:

the set of	$\{\dots\}$
all members $x$ such that	$x$ :
$x$ belongs to set $A$	$x \in A$
a set of sets	$S$

which together are:

$$\{x : x \in A \text{ for some } A \text{ belonging to } S\}$$

which is : the set of all members  $x$  such that  $x$  belongs to set  $A$  for some  $A$  belonging to  $S$ .

This is related to the uniting or union,  $U$ , of all the sets  $A$  belonging to  $S$ :

$$U\{A : A \in S\}.$$

What does this mean? Let us say  $S = \{\{a, b\}, \{a\}, \{a, c\}\}$ . Each set in inner braces is an  $A$ , and  $S$  equals the set in outer braces of these inner sets. There are three  $A$ 's, two with two members,  $a, b$  and  $a, c$ , and one with a single member,  $a$  – these members corresponding to  $x$  in  $x \in A$ . But  $U\{A : A \in S\}$  has a different meaning from the three sets  $A$  in  $S$ , for  $U\{A : A \in S\}$  means the set  $\{a, b, c\}$  and unifies the set of sets that is  $S$ . Thus the set of sets,  $S$ , brings together directly its sets  $A$ .  $U$  brings together these sets in another way – by having the common members without repeats (from Lipschutz, 1998, p. 117, and Milewsky, 1989, p. 16).

On the other hand, the power set expands the number of sets. The power set is : the set of all subsets of a set. The power set,  $P(A)$ , of a set having two members,  $a$  and  $b$ , has four subsets:

$$P(A) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$$

$\emptyset$  is the empty set defined by  $\{x \in A : x \neq x\}$  (Halmos, 1974, p. 8) which has no members. The other three sets are nested in the set of all of them. Thus they are subsets. Two have only single members  $a$  in  $\{a\}$  and  $b$  in  $\{b\}$ . Only one subset has both  $a$  and  $b$ ; this subset is  $\{a, b\}$ .

The power set of two members has  $2^2 = 4$  subsets. The power set of three members has  $2^3 = 8$  subsets. This power set is thus the set of eight subsets :  $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ . What we see from the array of subsets is the prevalence of sets having a single member. It is perfectly all right for a set to have a single member, because it makes no sense for a member to belong to itself but does make sense for it to belong to a set even though the set has only itself. The prevalence of sets of one member plus the sets of two members makes possible the set of any pair of these sets. Thus the structure of a set of subsets is the valuable outcome paving the way for the set of subsets that is the ordered pair,  $(a, b) = \{\{a\}, \{a, b\}\}$  (Halmos, 1974, pp. 22-24), Suppes, 1972, pp. 32-33).

The content of a member in a single membered set can be anything. The content of *member* has a vastly greater range in this study than in standard set theory texts.

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Figure 1. Gastropod (snail) larvae in the Pacific Ocean.

Left: A, *Heliacus trochoides*; B, *Heliacus variegatus*;  
C, *Architectonica perspectiva*. Right: distribution of larvae in the Pacific;  
1 and 2 = unknown species, 3 = *Heliacus variegatus*, 4 = *Architectonica perspectiva*, 5 = *Philippia oxytropis*, 6 = *Philippia radiata*, 7 = *Heliacus trochoides*. (from Sheltema, R.S., and I.S. Williams, 1983, fig. 3 and fig. 4)

Figure 2. Larval shells, opercula, and protoconchs of some species belonging to the families Muricidae, Ovulidae, Architectonicidae and Neritidae:  
 (a) *Thais* (?) *rustica* veliger larva, Eastern Atlantic off West Africa;  
 (b) protoconch of *Thais rustica*, Port Royal, Jamaica; (c) protoconch of *Pedicularia sicula decussate*, off coast of Georgia, Western Atlantic; (d) *Thais haemastoma* larval shell, Western Atlantic east of Bahamas; (e) protoconch of *Thais haemastoma*, Corpus Christi Bay, Texas; (f) larval shell of *Pedicularia sicula*, Western Atlantic, Gulf Stream; (g) larval shell of *Pedicularis sicula*, Eastern Atlantic, off Azores; (h) larval shell of *Thais haemastoma*, Western Atlantic east of Bahamas; (i) larval shell of *Thais haemastoma*, Mid-equatorial Atlantic, South Equatorial Current; (j) larval shell of *Thais haemastoma*, Eastern Atlantic off West Africa; (k) *Philippia krebsii* larval shell, fully developed, Western North Atlantic; (l) *Philippia krebsii* larval shell, same as (k); (m) operculum of *Philippia krebsii*; (n) three larval shells of *Smaragdia viridis*, northern end of Gulf Stream; (o) operculum of *Smaragdia viridis*, northern end of Gulf Stream; (p) larval shells of *Smaragdia viridis*, off West Africa; (q) protoconch of *Smaragdia viridis viridemaris*, Castle Harbor, Bermuda. Scale – 1mm. (Sheltema, 1971. p. 292)



Figure 3. Upper figure: tropical Atlantic Ocean. Filled circles are locations where larvae of the family Architectonicidae were found. Divided circles are locations where both Architectonicidae and Ranellidae were found in the same sample. Open circles are locations where larvae of Ranellidae were found. Triangles are locations of other families of larvae. Small open circles no larvae. (From Shetlema, R.S., 1995, fig. 4)

Lower figure: tropical Pacific Ocean. Distribution of larvae belonging to the family Architectonicidae. (from Shetlema, et al., 1996, fig. 2)

Figure 4. Distribution of chickadees. Top panel: left, Mexican Chickadee (*Poecile sclateri*); right, Chestnut-Backed Chickadee (*Poecile rufescens*). Next to top panel: left, Mountain Chickadee (*Poecile gambeli*); right, Gray-Headed Chickadee (*Poecile cinctus*). Next to bottom panel: left, Black-Capped Chickadee (*Poecile atricapillus*); right, Boreal Chickadee (*Poecile hudsonicus*). Bottom panel: Carolina Chickadee (*Poecile carolinensis*). (National Geographic, 1999, pp. 328, 336)

Figure 5. Distribution of sixteen North American sparrows. Spring-summer breeding areas in the north shown in black. Wintering areas in the south shown in gray. (National Geographic, 1999, pp. 402, 404, 408, 412, 414, 416)

Figure 6. Migration routes and northern breeding areas and southern wintering areas of the Scarlet Tanager (*Piranga olivacea*), upper left, of the Rose-Breasted Grosbeak (*Pheucticus ludovicianus*), upper right, and of the Golden Plover (*Pluvialis apricaria*). (Lincoln and Hines, 1950, pp. 44, 46, 54)

Figure 7. Distribution of four Fur seals (1), lower left. Distribution of the Southern sea lion, lower right. Locations of the Northern Elephant seal, upper left. The Northern Fur seal and Stellar's sea lion, upper right. (Bonner, 1999, pp. 41, 66, 126, 52, 68)

Figure 8. The distribution of the southern fur seals, upper left. The distribution of the four Antarctic seals, upper right. The distribution of the Southern Elephant seal, bottom. (Bonner, 1999, pp. 45, 139, 124)

Figure 9. Distribution of the Ribbon seal and the Spotted seal, to the left, and of the Bearded seal and the Ringed seal, to the right. (Bonner, 1999, pp. 102, 111, 88, 104)

Figure 10. Distribution of the Hooded seal and the Harp seal. (Bonner, 1999, pp. 90, 108)



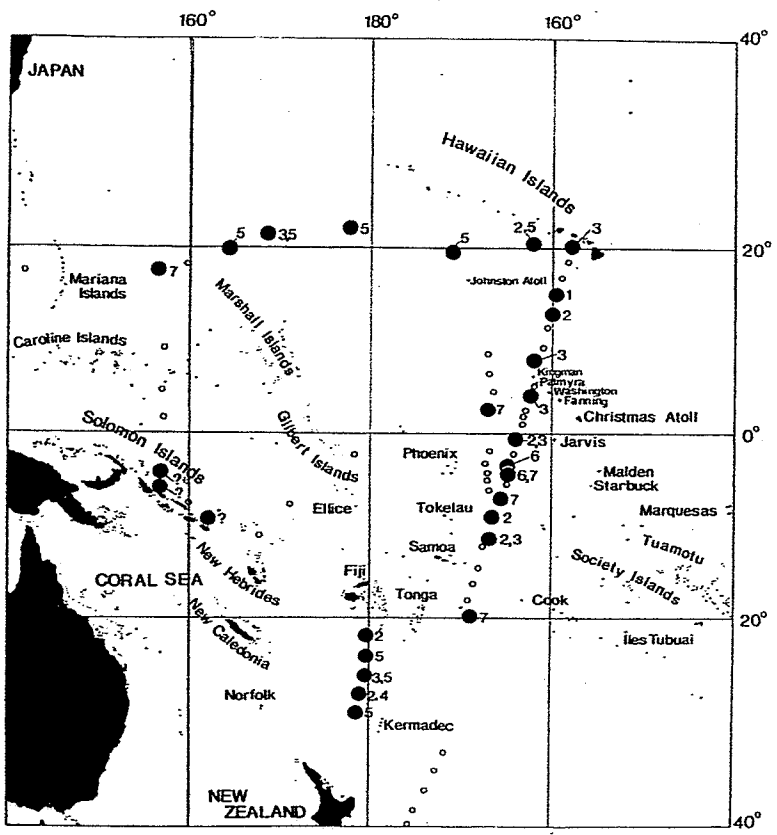
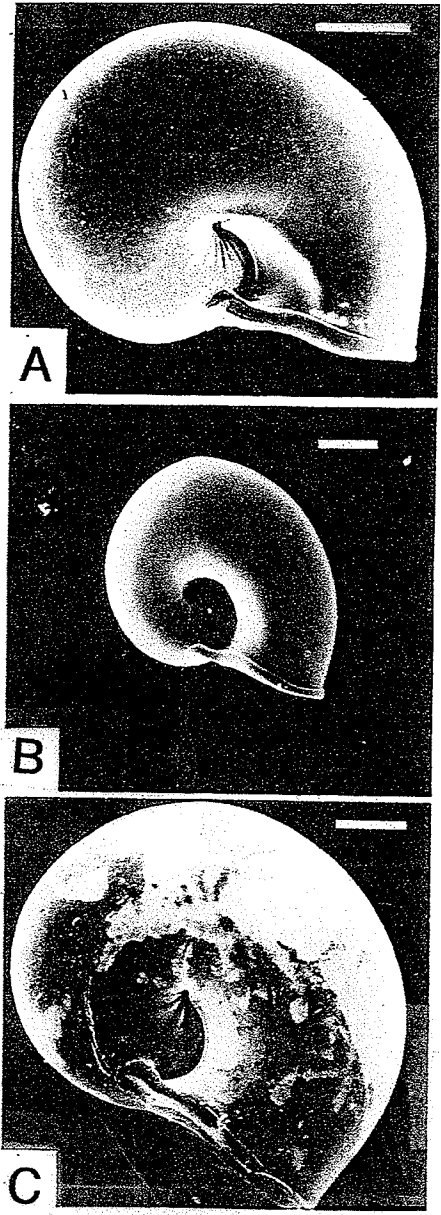


Fig 1

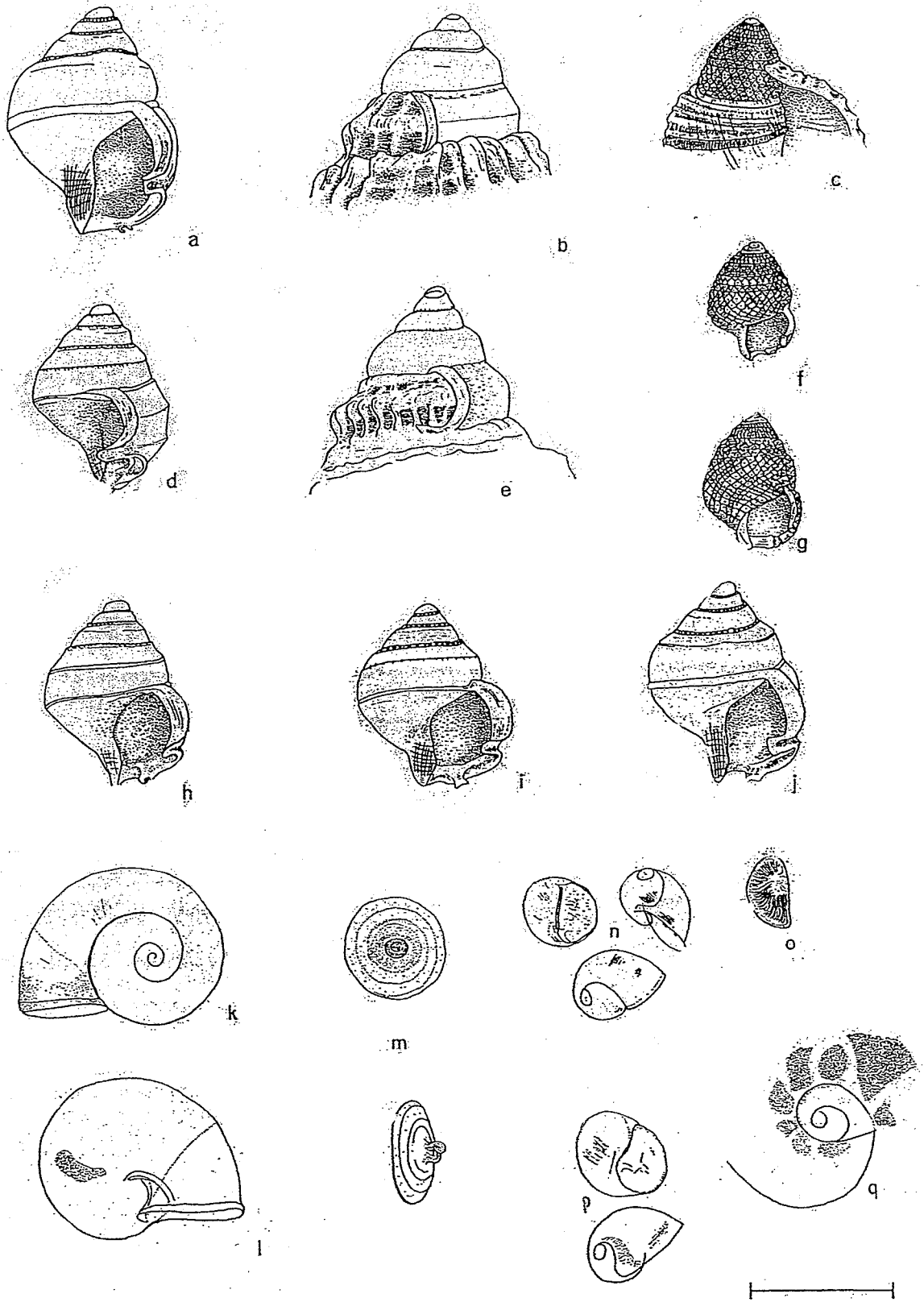


FIG. 2

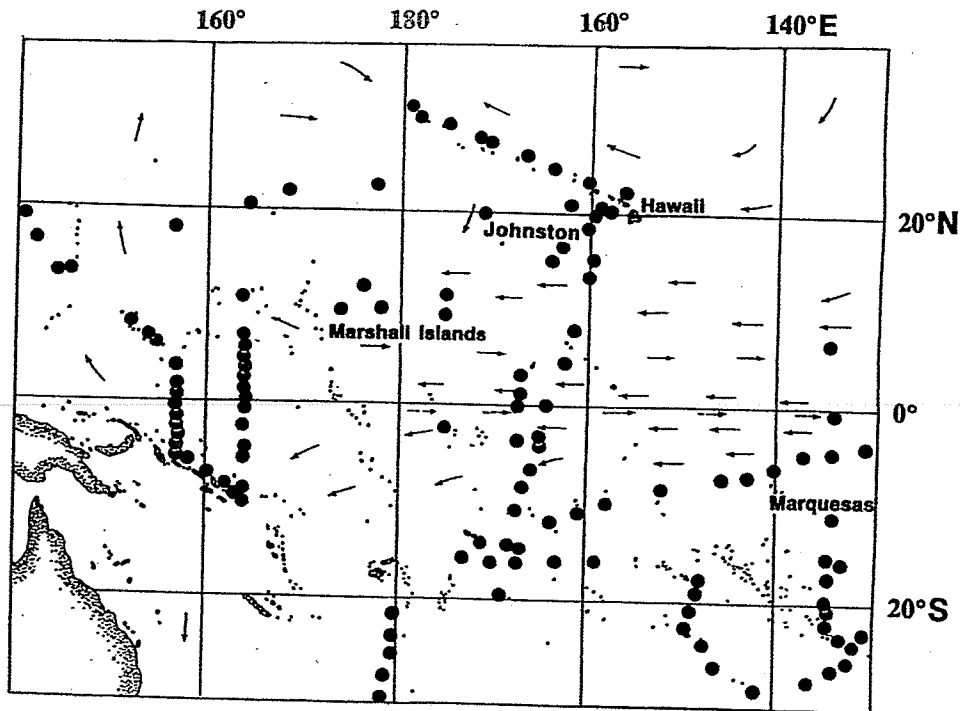
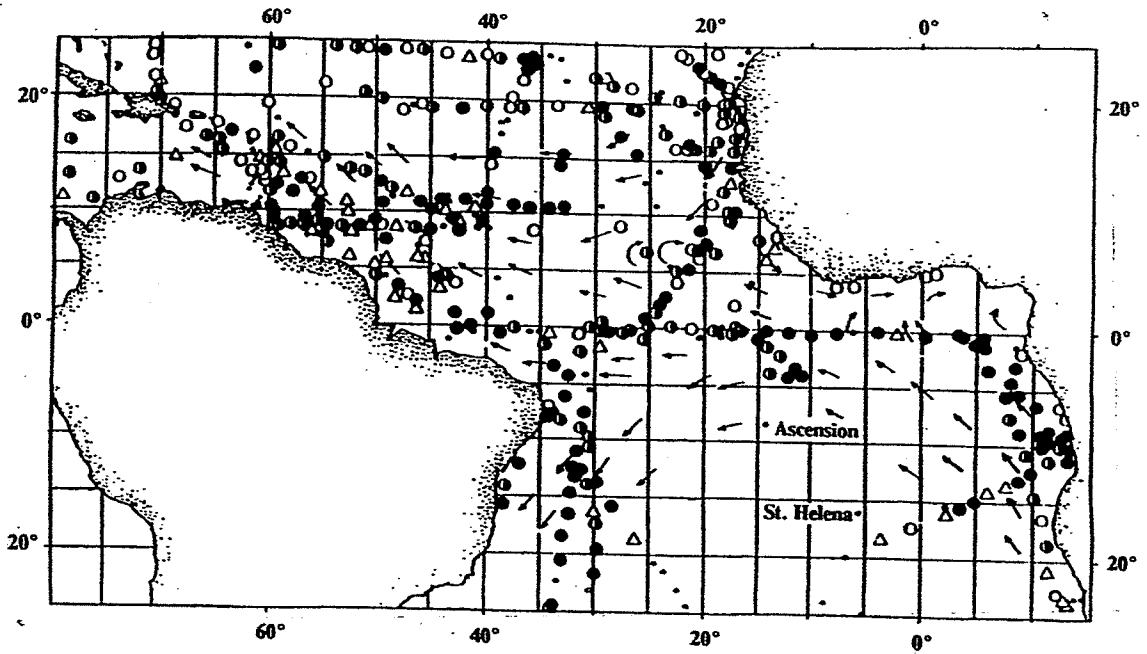


Fig. 3

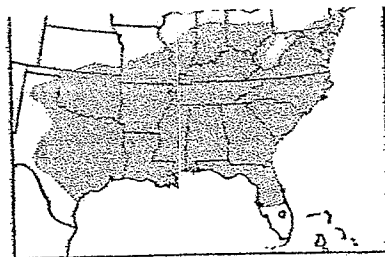
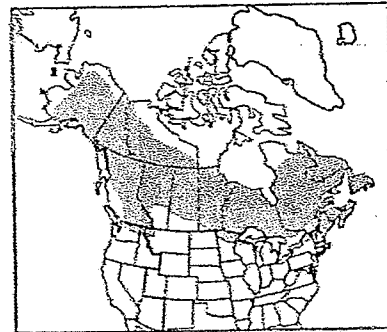
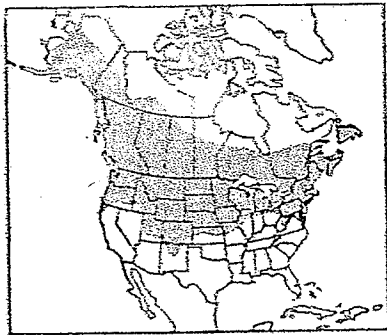
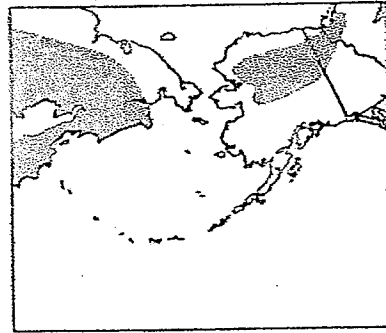
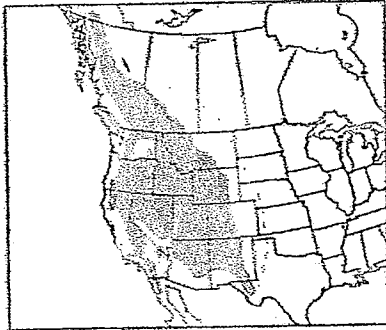
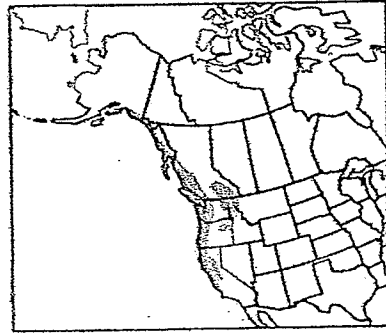
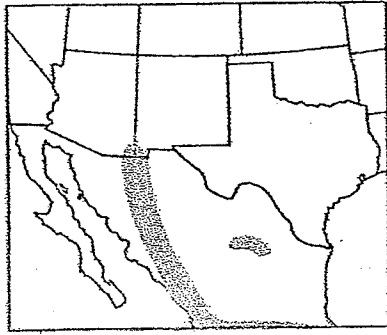
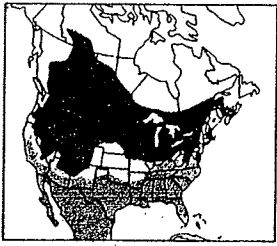
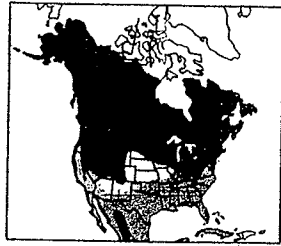


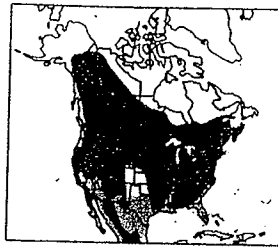
FIG. 4



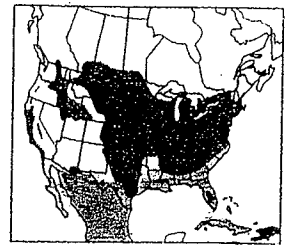
*Poecetes gramineus*



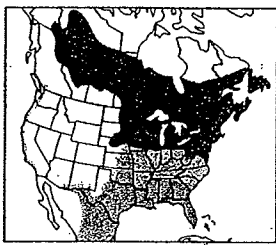
*Passerculus sandwichensis*



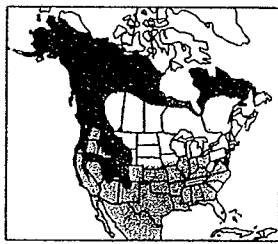
*Spizella passerina*



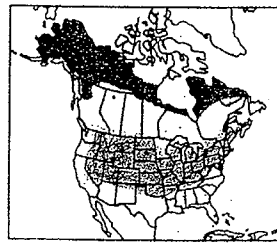
*Ammodramus savannarum*



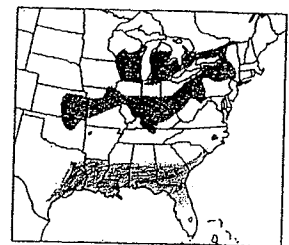
*Melospiza georgiana*



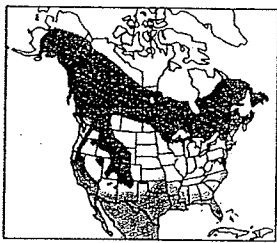
*Zonotrichia leucophrys*



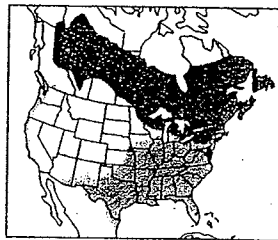
*Spizella arborea*



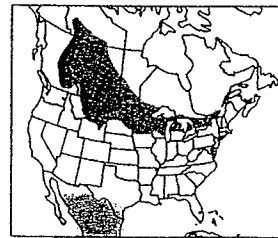
*Ammodramus henslowii*



*Melospiza lincolnii*



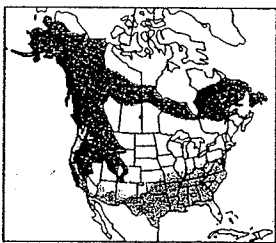
*Zonotrichia albicollis*



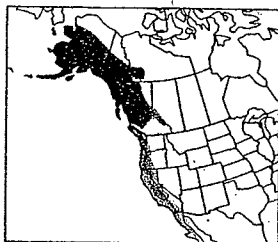
*Spizella pallida*



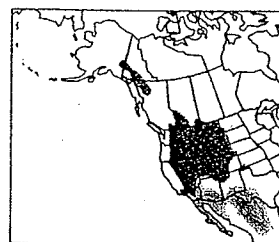
*Ammodramus bairdii*



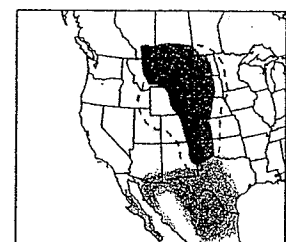
*Passerella iliaca*



*Zonotrichia atricapilla*



*Spizella breweri*



*Calamospiza melanocorys*

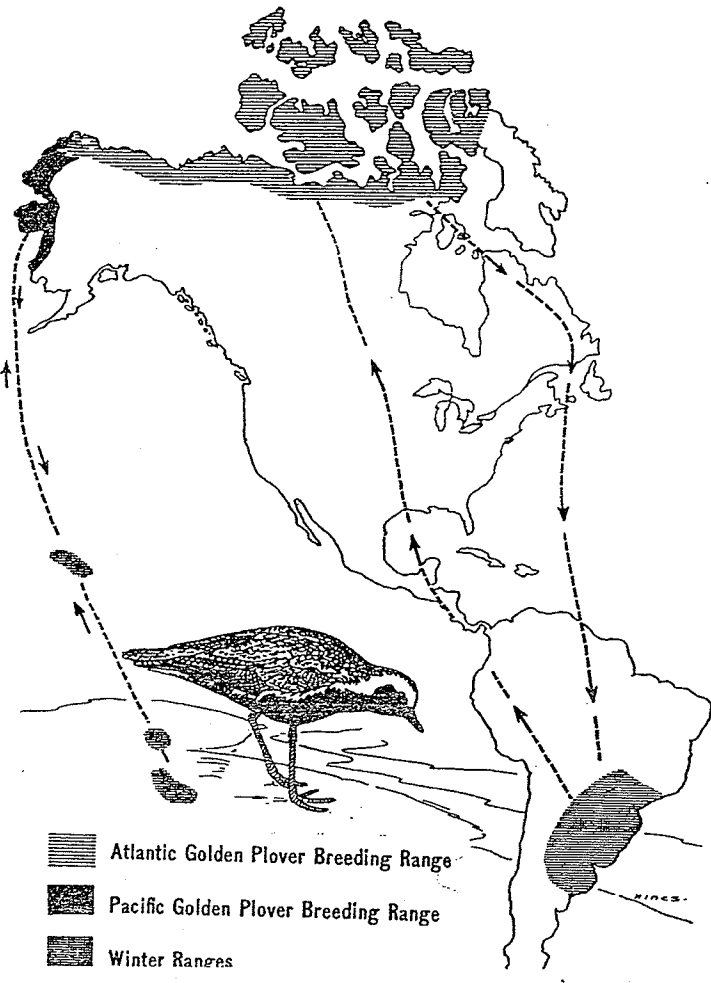
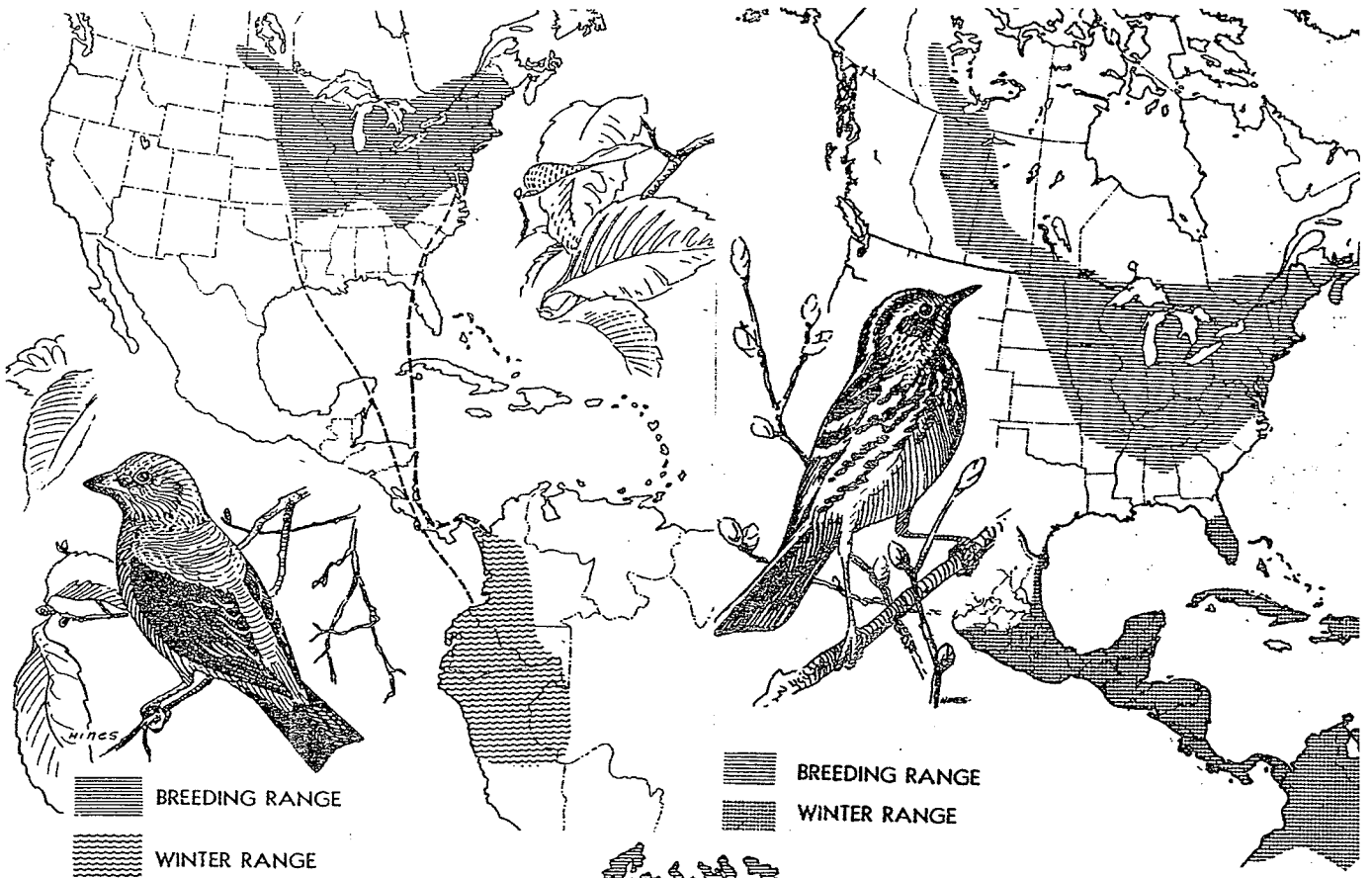
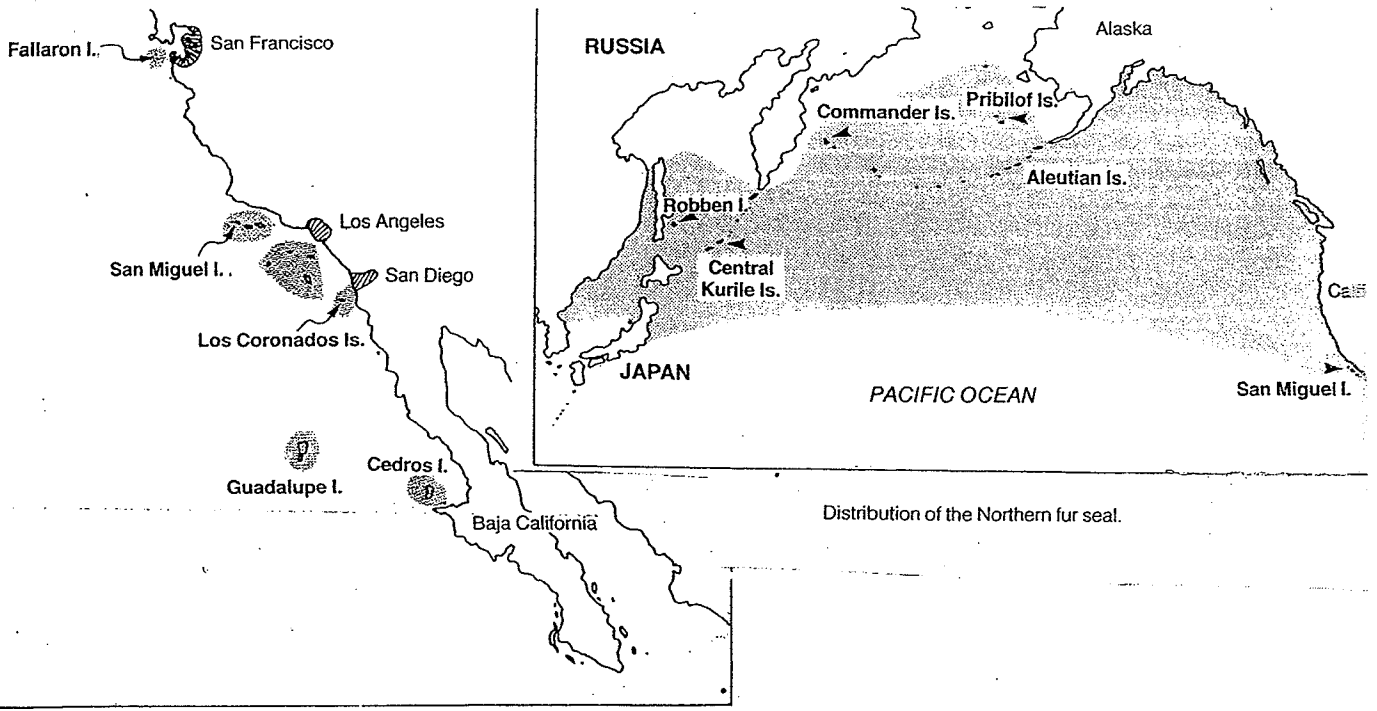
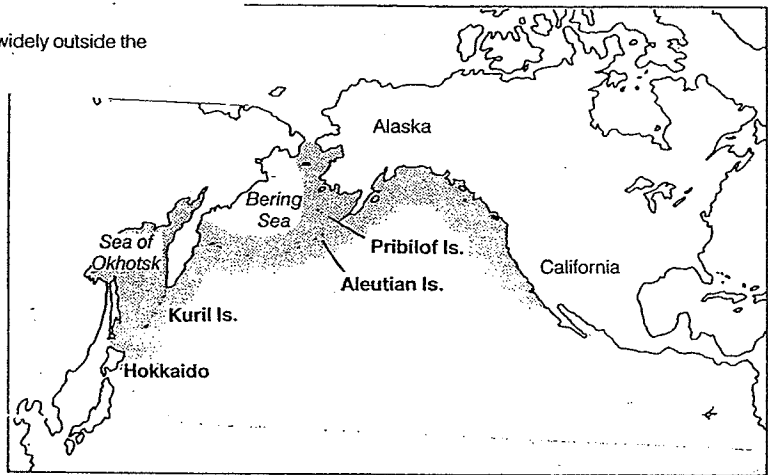
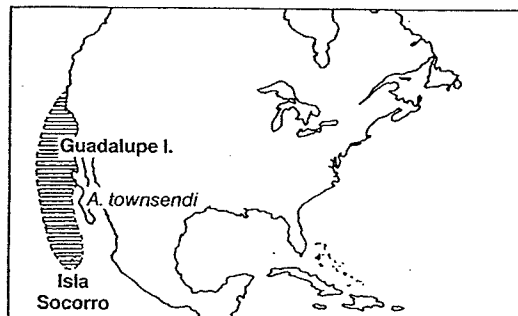


FIG. 6



Feeding distribution of Northern elephant seals. Animals disperse widely outside the breeding season.



Distribution of Steller's sea lion.

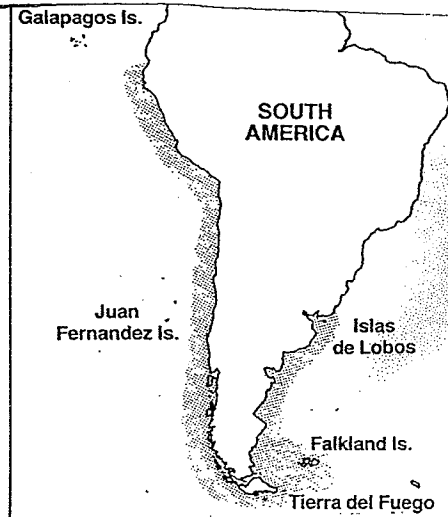
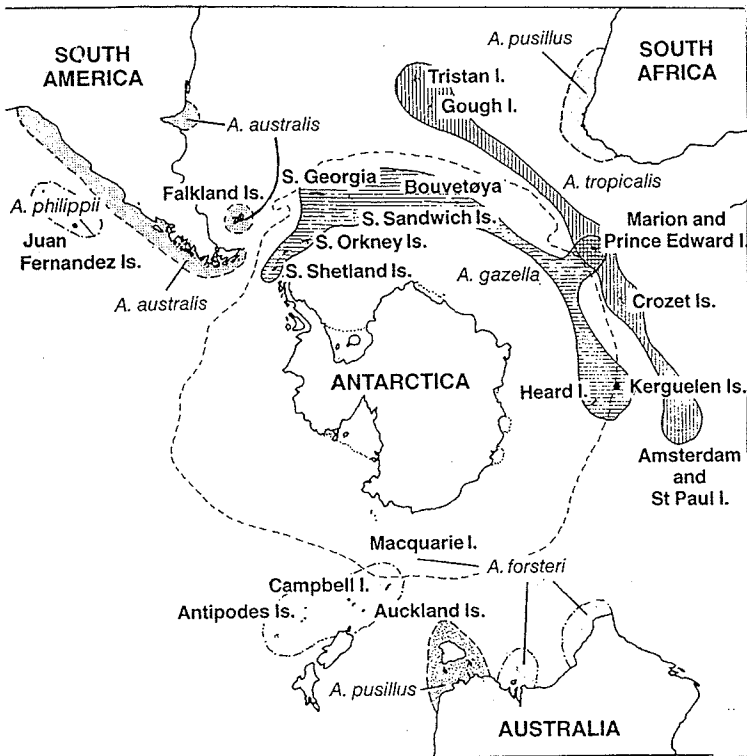
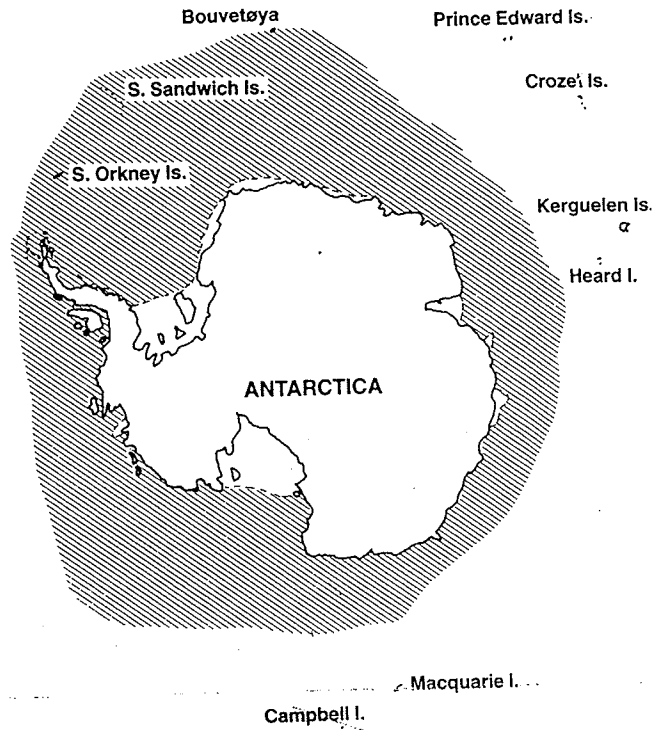


FIG. 7



Distribution of southern fur seal



The area occupied by the Antarctic seals

Breeding distribution of Southern elephant seals. Non-breeding animals may stray far to the north and south of the indicated areas.

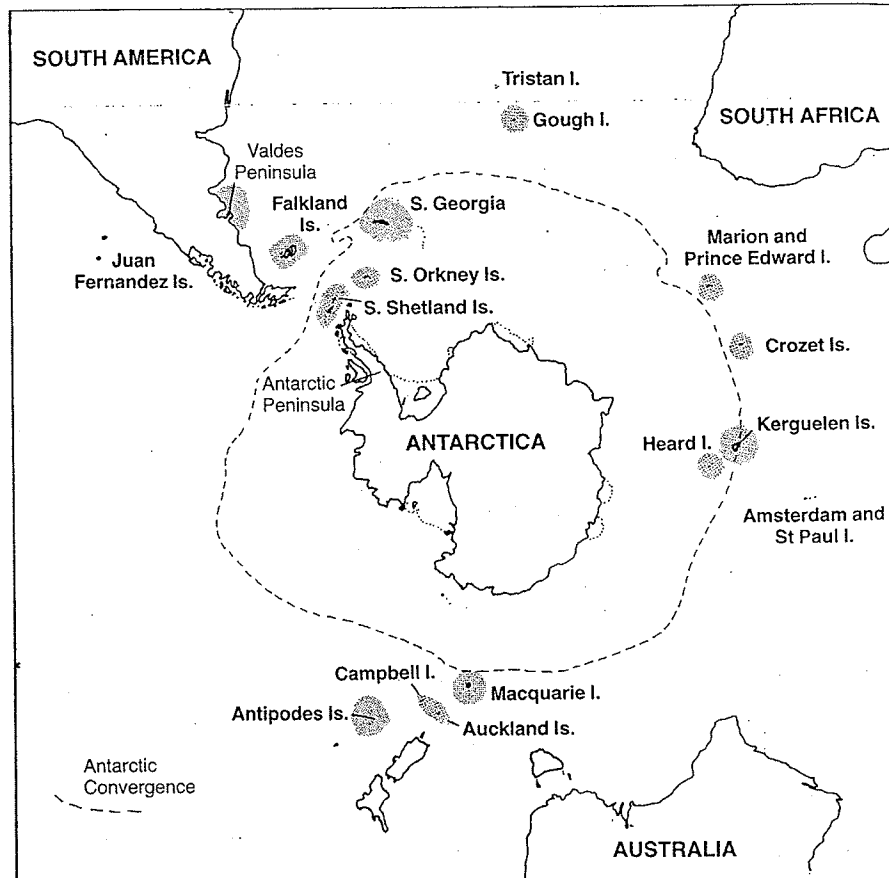
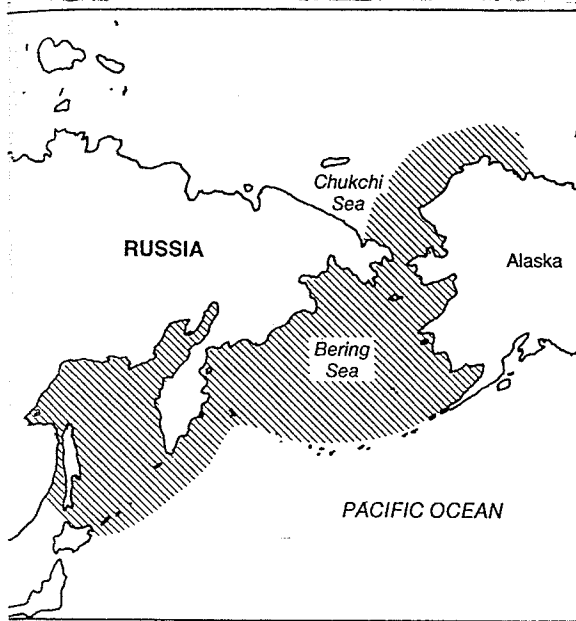
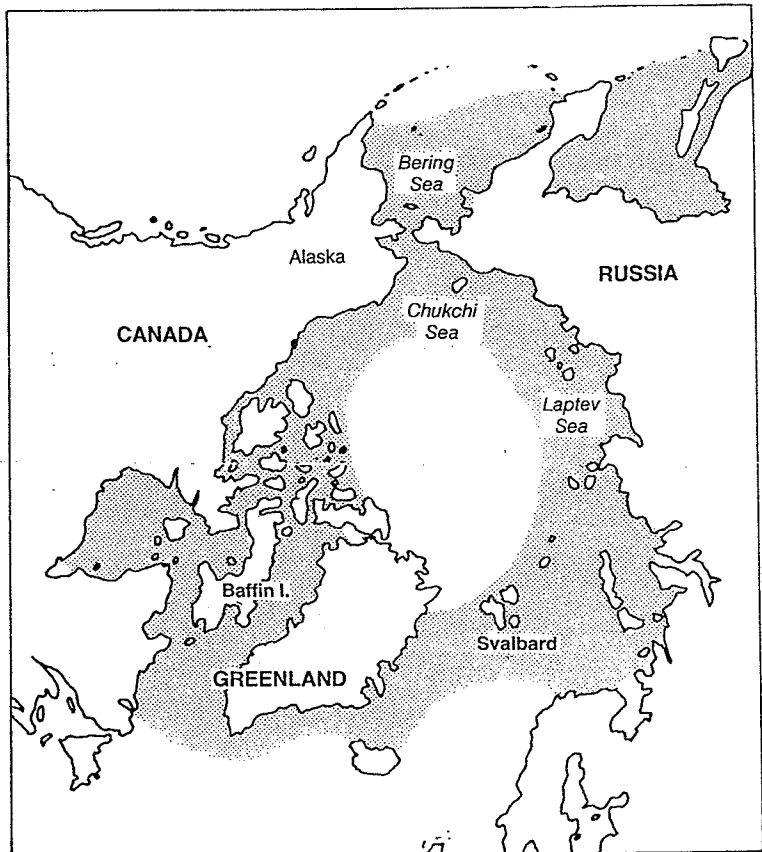


FIG. 8

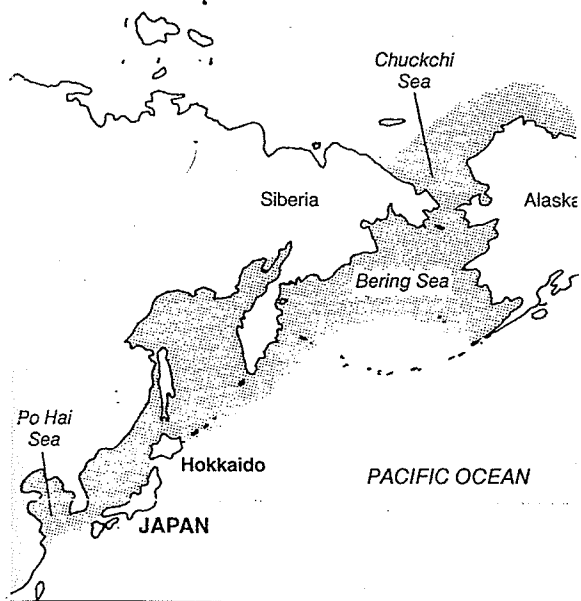




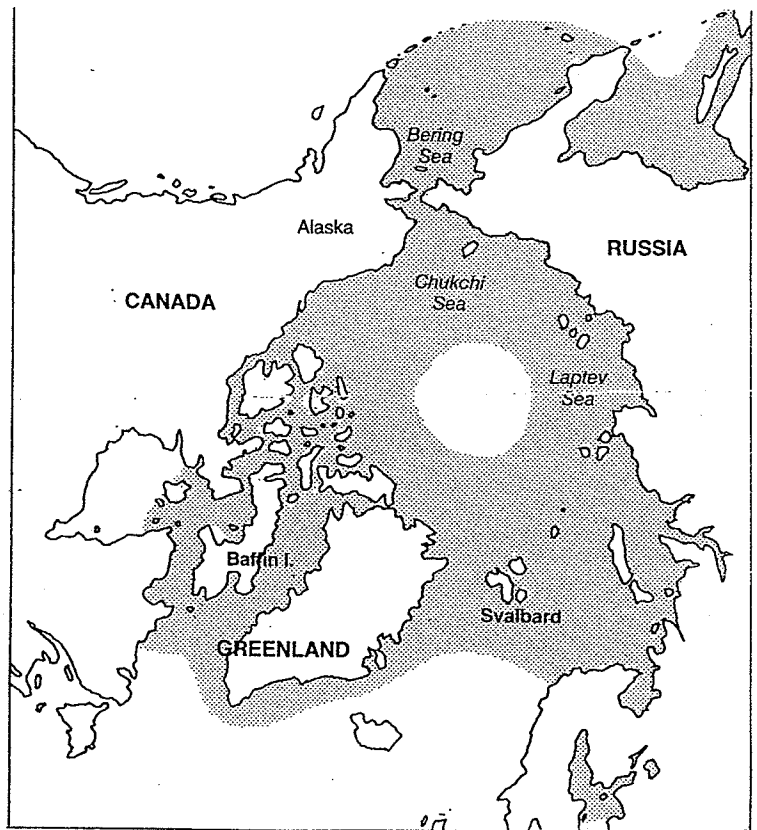
Distribution of the Ribbon seal.



Distribution of the Bearded seal.

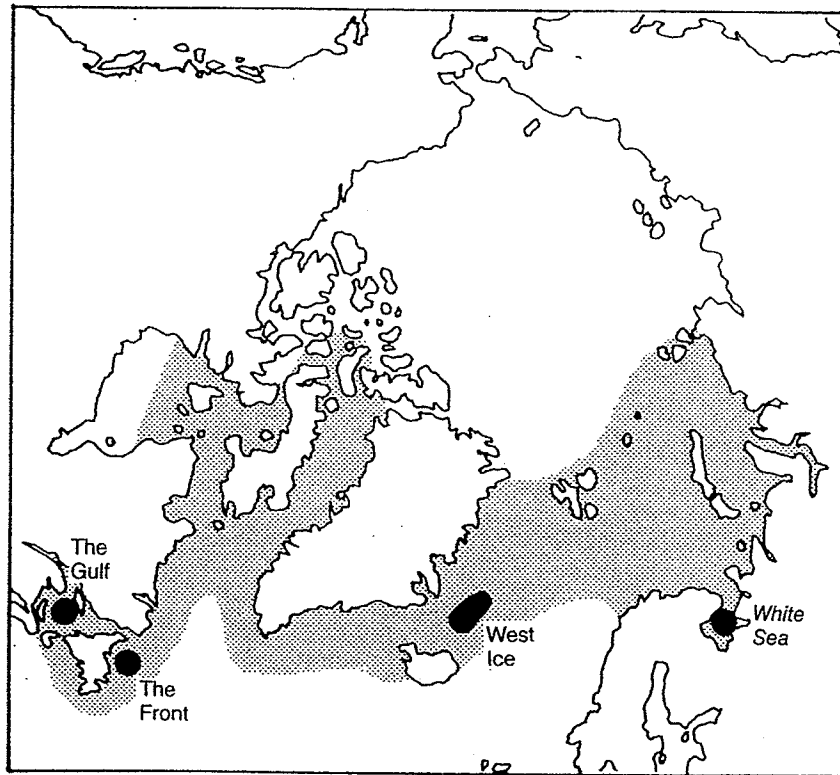
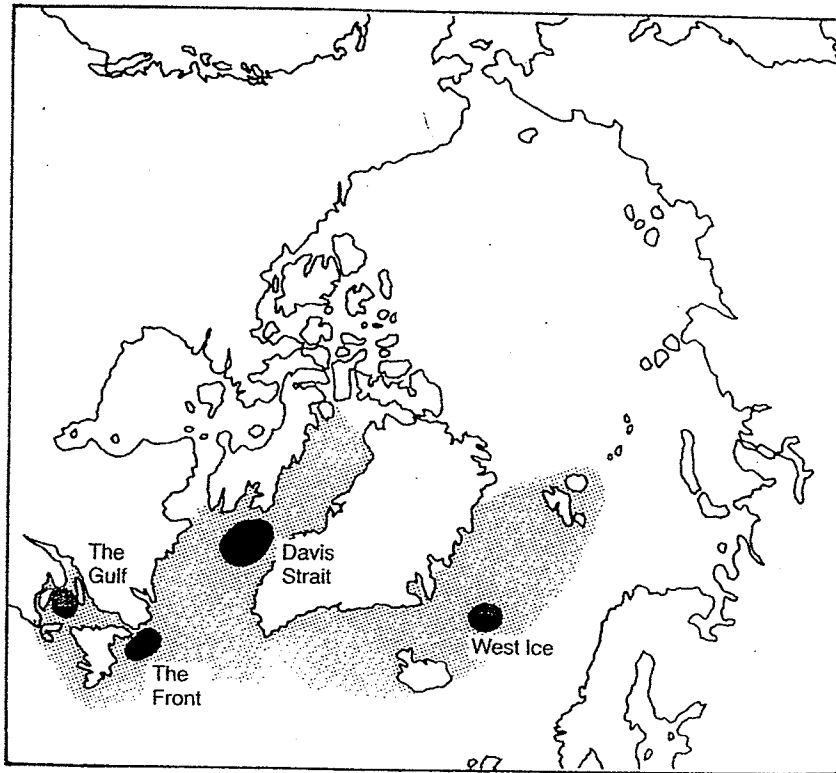


Distribution of the Spotted seal.



Distribution of the Ringed seal.

Distribution of the Hooded seal. The four breeding areas are indicated in black.



Distribution of the Harp seal. The four breeding areas are indicated in black.

FIG. 10