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# HEAVE AND ROLL RESPONSE OF FREE FLOATING BODIES OF CYLINDRICAL SHAPE

by

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TECHNICAL REPORT

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## ABSTRACT


The following report describes a computer solution to help predict the heave and roll response of free floating bodies of cylindrical shape when excited by random seas with known spectra.

The basic concepts of harmonic analysis and statistics used in the method are first briefly reviewed. The report then presents a detailed derivation of the linear heave and roll response amplitude operators, that is the expressions of the vertical and angular displacements produced by a simple harmonic wave of one foot amplitude.

The second part of the report reviews the computation procedure and the program's logic. It gives a detailed set of instructions for the program users, reviews the program's capabilities and limitations, and presents three case studies.

The heave and roll response programs are written for use with XEROX SIGMA 7 computers. Program listings are given in the appendix.

Acknowledgments

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## 1.0 PROBLEM STATEMENT

The heave and roll motion of a cylindrical body of constant cross section when excited by a simple harmonic wave is a relatively straightforward problem. However, very few buoys can be realistically modeled as a pillbox or a telephone pole. Most spar buoys are made of circular cylinders of varying diameters (see Fig. #1). Some spar buoys extend to considerable depths below the water level. Furthermore, most seaways are not made of regular harmonic waves of single frequency and amplitude and in general irregularity and randomness of the sea surface will prevail.

The objective of this report is to present a method which can be used to compute reasonable expectations of vertical and angular displacement that a complex shape buoy will experience when free floating in a random stationary seaway.

The computer solution presented in this report was originally derived to investigate the dynamic behavior of specific spar buoys used by the Woods Hole Oceanographic Institution. This solution is here presented in a generalized form, with the hope that it becomes a constructive addition to the solutions already in the literature.

## 2.0 THEORETICAL BACKGROUND

### 2.1 Statistical Response of Floating Bodies to Ocean Waves Excitation

Readers unfamiliar with the probabilistic theory of ship and buoy dynamics should resort to References I, II, and III for a theoretical introduction to the subject.

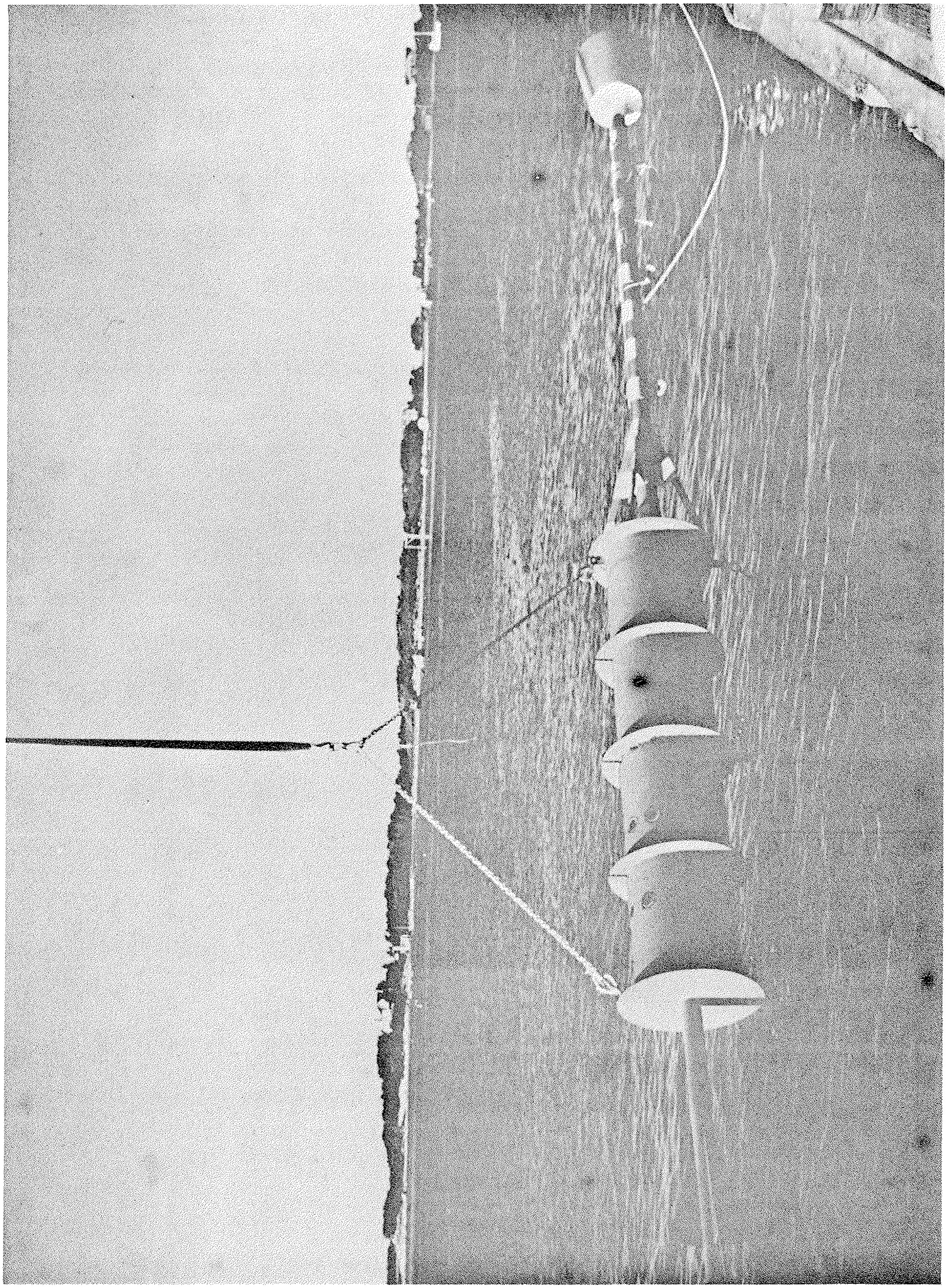

Fig. No. 1

Basic concepts borrowed from this theory and used in the formulation of the heave and roll computer programs described in this report are hereafter summarized.

If the probability density function $p(x)$ of the wave amplitudes "$x$" for a given seaway can be explicitly expressed, then the expectation of certain values of wave amplitudes can be directly computed.

For example:

- The most probable amplitude $x_m$ is the value of $x$ for which

$$\frac{d}{dx} p(x) = 0 \tag{2.1.1}$$

- The average amplitude $\bar{x}$ is given by

$$\bar{x} = \int_0^\infty x\, p(x)\, dx \tag{2.1.2}$$

- The average of a fraction $f\ (0 \leq f \leq 1)$ of wave amplitudes larger than a given amplitude $x_o$ can be obtained

$$\bar{x}_f = \frac{\int_{x_o}^\infty x\, p(x)\, dx}{\int_{x_o}^\infty p(x)\, dx} = \frac{1}{f} \int_{x_o}^\infty x\, p(x)\, dx \tag{2.1.3}$$

etc.

When certain restrictive conditions prevail, wave amplitudes have been found to follow a Rayleigh distribution given by:

$$p(x) = \frac{2x}{\overline{x^2}}\, e^{-\frac{x^2}{\overline{x^2}}} \tag{2.1.4}$$

where $\overline{x^2}$ is the mean square value of the wave amplitudes.

This probability density function has been used to compute the expectation of particular wave amplitude means and maxima. Results of these computations are found to be proportional to the root mean square $\sqrt{\overline{\chi^2}}$ of the wave amplitudes. Those retained in this study are summarized in Tables 1 and 2, "Value of expected means" and "Value of expected maxima."

Now let $S(\omega)$ be the spectral density function of the wave amplitudes of the given sea way. Assuming the seaway to be stationary, then the integral over all positive frequency ranges of $S(\omega)$ is equal to the mean square value of the wave amplitudes, i.e.

$$\overline{\chi^2} = \int_0^\infty S(\omega)\, d\omega \qquad (2.1.5)$$

This result can be used to compute $\overline{\chi^2}$ and $\sqrt{\overline{\chi^2}}$. The value of $\sqrt{\overline{\chi^2}}$ thus obtained can in turn be used to compute the expected wave amplitude means and maxima listed in Tables 1 and 2.

From the definition of the mean square value

$$\overline{\chi^2} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=0}^{i=N} \chi_i^2$$

and the result (2.1.5) it is clear that the quantity

$$\lim_{d\omega \to 0} \sqrt{S(\omega)\, d\omega} \qquad (2.1.6)$$

is proportional to the amplitude of the elementary component wave of the spectrum with frequency $\omega_n$ .

Now if $Y(\omega)$ is the expression of the linear response of a

Table 1

Wave Amplitude Means

| Fraction, $f$, of Largest Amplitudes Considered | Mean Values $\bar{r}_f \div \sqrt{\overline{r^2}}$ |
|---|---|
| 0.01 | 2.359 |
| 0.10 | 1.800 |
| 0.333 | 1.416 |
| 0.50 | 1.256 |
| 1.00 | 0.886 |

Table 2

Expected Maximum Amplitudes

| Number of Waves | Maximum Wave Amplitudes $r_{max} \div \sqrt{\overline{r^2}}$ |
|---|---|
| 50 | 2.12 |
| 100 | 2.28 |
| 500 | 2.61 |
| 1,000 | 2.78 |
| 10,000 | 3.13 |
| 100,000 | 3.47 |

free floating body to a simple harmonic wave of unit amplitude and frequency $\omega$ , then clearly the quantity

$$\lim_{d\omega \to 0} Y(\omega)\sqrt{S(\omega_m)\, d\omega} \tag{2.1.7}$$

is proportional to the amplitude of the body response to the elementary component wave of frequency $\omega_m$ .

It thus follows that

$$Y^2_{(\omega)}\, S(\omega_m)\, d\omega \tag{2.1.8}$$

is proportional to the amount of the response mean square value contained in the frequency band $d\omega$ centered at $\omega_m$ . The response mean square value $\overline{r^2}$ is therefore given by:

$$\overline{r^2} = \int_0^\infty Y^2_{(\omega)}\, S(\omega)\, d\omega \tag{2.1.9}$$

The response $Y(\omega)$ of the body being linear, the probability density function of the response will also follow the probability density function of the wave amplitudes. Thus the results tabulated in Tables 1 and 2 can be used again, together with expression (2.1.9) to compute statistical means and maxima of body response amplitude.

For example, the average of the one third highest response amplitudes will be given by

$$\overline{r_{1/3}} = 1.416\sqrt{\overline{r^2}}$$

with

$$\sqrt{\overline{r^2}} = \sqrt{\int_0^\infty Y^2_{(\omega)}\, S(\omega)\, d\omega} \tag{2.1.10}$$

Empirical formulation of wave amplitude spectra used in the computer program are:

$$S(\omega) = \frac{16.875\, e^{-\frac{9.7\times 10^4}{V^4 \omega^4}}}{\omega^5} \quad ft^2\text{- sec (Pierson Moskowitz)} \qquad 2.1.11$$

where $V$ is the wind speed (knots).

$$S(\omega) = \frac{525\, H_s^2\, e^{-\frac{1050}{T_s^4 \omega^4}}}{T_s^4 \omega^5} \quad ft^2\text{- sec (Bretschneider)} \qquad 2.1.12$$

$$S(\omega) = \frac{345\, H_s^2\, e^{-\frac{630}{T_s^4 \omega^4}}}{T_s^4 \omega^5} \quad ft^2\text{- sec (I.S.S.C.)} \qquad 2.1.13$$

In formula (2.1.12) and (2.1.13) $H_S$ is the significant wave height (feet) and $T_S$ is the significant wave period (seconds).

## 2.2 Derivation of the Heave and Roll Response Amplitude Operators (RAO)

### 2.2.1 Heave response

#### 2.2.1.1 Initial conditions

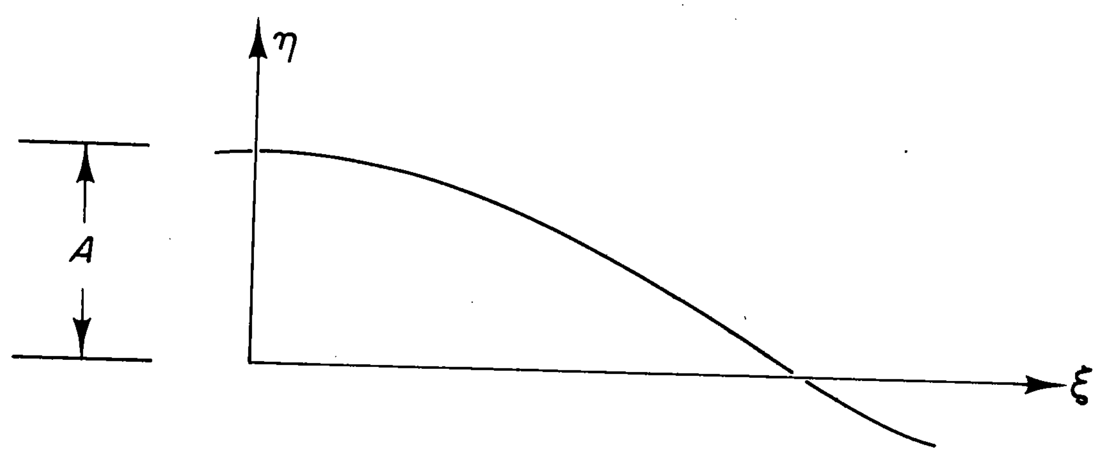


Fig. No. 2

Let us consider a simple harmonic, deep sea wave, as shown on Fig. No. 2. The coordinates of a point on the surface of this wave are given by:

$$\xi = A \sin(\omega t - K\xi) \quad (2.2.1)$$

$$\eta = A \cos(\omega t - K\xi) \quad (2.2.2)$$

If we select to observe this wave at $\xi = 0$, then the parametric equation of the water particle motion around this point become:

$$\xi = A \sin \omega t$$

$$\eta = A \cos \omega t$$

where $A$ is the wave amplitude, and $\omega$ the wave angular frequency.

We also know from the simple harmonic wave theory (Ref. I, pp. 14-27) that the parametric equations of water particles at any depth $z$ below the mean water level would then be

$$\xi = A e^{-Kz} \sin \omega t$$

$$\eta = A e^{-Kz} \cos \omega t$$

where $K$ is the wave number. $K = \frac{\omega^2}{g}$ for deep water waves, $g$ being the gravity acceleration.

The vertical components of water particle velocity and acceleration would in turn be given by:

$$\dot{\eta} = - \omega A e^{-Kz} \sin \omega t$$

$$\ddot{\eta} = - \omega^2 A e^{-Kz} \cos \omega t$$

In this case, at time $t = 0 + \varepsilon$, the amplitude of the vertical displacement of the water particles start to decrease from their maximum value, the water particles vertical velocity component starts to increase and is in the downwards direction, and the water particles vertical acceleration component starts decreasing and is also in the downwards direction.

As shown on Fig. No. 3, let $x$ be the distance from the still water surface to the buoy water line. At time $t = 0 + \varepsilon$ the buoy is assumed to move downwards, that is the distance $x$ is increasing.

2.2.1.2 General equation of heave motion

The equation of heave motion will be obtained from:

$$\sum_i F_i = (m + m')\ddot{x} = m_v \ddot{x} \tag{2.2.3}$$

where $\sum_i F_i$ = sum of the vertical forces applied to the buoy,

$m$ = mass of the buoy

$m'$ = added mass of the buoy due to the water entrained in the vertical direction

$m_v = m + m'$ = virtual mass of the buoy (in the vertical direction).

2.2.1.3 Expression of the forces applied to the buoy

The vertical forces applied to the buoy are:

- Its weight "$W$"
- The resultant "$\underline{P}$" of the pressure forces exerted by water particles on the top and the bottom plates of

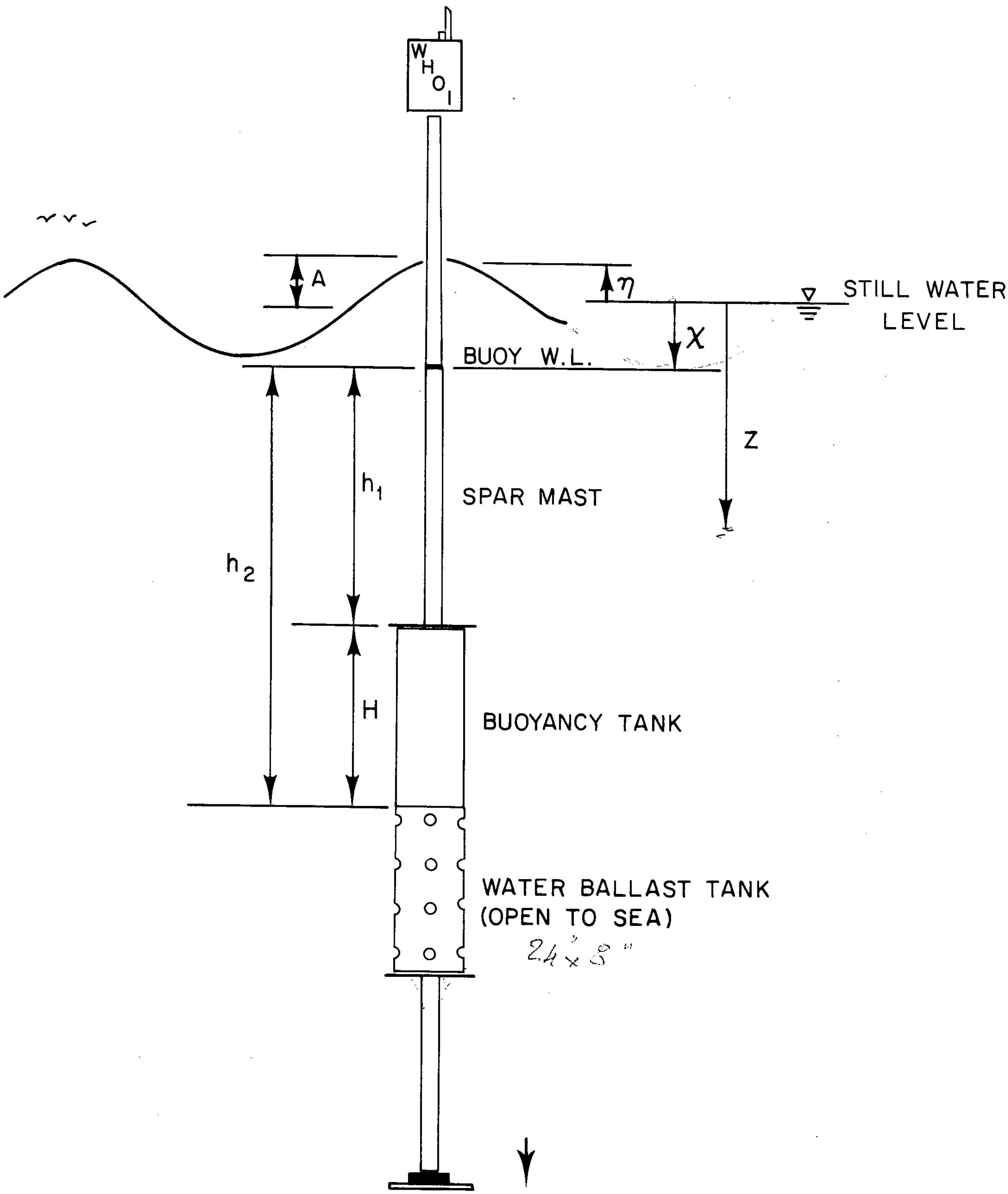
WHOI
A
η
STILL WATER LEVEL
χ
BUOY W.L.
Z
$h_1$
SPAR MAST
$h_2$
H
BUOYANCY TANK
WATER BALLAST TANK
(OPEN TO SEA)
24" x 8"

Fig. No. 3

the watertight compartments of the buoy

- The damping force "$D$" resulting from the water opposing the buoy vertical motion
- The friction force "$G$" exerted by the water particles vertical velocity on the buoy
- The inertial force "$I$" exerted by the water particles vertical acceleration on the buoy.

Forces in the direction of increasing $x$ (downwards) will be considered positive.

The expression of these forces is obtained as follows:

- Pressure force "$P$".

With the initial conditions assumed, the pressure $p$ at a depth $z$ is given by:

$$p = \rho g \left( z + A e^{-kz} \cos \omega t \right)$$

To help find a general expression for the resultant $P$, let us consider the spar buoy shown in Fig. No. 3.

At the bottom of the buoyancy tank

$$z = x + h_2$$

where $h_2$ = depth of the bottom plate below buoy water line

Assuming $x \ll h_2$, the upwards pressure force $P_B$ on the tank bottom is thus given by

$$P_B = \rho g \left\{ x + h_2 + A e^{-kh_2} \cos \omega t \right\} S_B$$

where $S_B$ = area of bottom plate subjected to water pressure (the entire area of the plate in this case).

Similarly the pressure force $P_T$ on the top plate of the buoyancy tank is given by

$$P_T = \rho g \left\{ x + h_1 + A e^{-kh_1} \cos \omega t \right\} S_T$$

where $S_T$ = area of top plate subjected to water pressure.

If the spar mast has a cross section $S_M$ and is watertight then obviously $S_T = S_B - S_M$

The resultant $\underline{P}$ will be the difference between the bottom pressure force and the top pressure force. Being in the upwards direction, $\underline{P} = -\left(P_B - P_T\right)$ i.e.

$$\underline{P} = -\rho g \left\{ (S_B - S_T)x + (h_2 S_B - h_1 S_T) - A \cos \omega t \left( S_B e^{-kh_2} - S_T e^{-kh_1} \right) \right\}$$

The constant terms in the expression of the pressure force must equal the buoy static weight $\bar{W}$. This can be easily established. Noting that

$$h_2 = h_1 + H$$

where $H$ = length of the buoyancy tank, the constant terms

$$\rho g \left( S_B h_2 - S_T h_1 \right)$$

can be written:

$$\rho g \left[ S_B (h_1 + H) - S_T h_1 \right] = \rho g \left[ (S_B - S_T) h_1 + S_B H \right]$$

which obviously is the sum of the weight of the water displaced by the immersed portion of the mast and by the buoyancy tank under equilibrium conditions, and therefore is equal to the buoy weight.

The sum of the weight force and the pressure force can then be in general expressed by:

$$\underline{P} + W = -\rho g (S_B - S_T) x + \left( \rho g \sum_i' e^{-kh_i} S_i \right) A \cos \omega t \qquad (2.2.4)$$

where $S_i$ is the surface at a depth $h_i$ subjected to the pressure. $S_i$ is positive if the pressure exerted upon it is in the upwards direction, and vice versa $S_i$ is negative if the pressure exerted upon it is in the downwards direction.

This expression can further be simplified and written:

$$P + W = -Cx + MA \cos \omega t \tag{2.2.5}$$

where

$$C = \rho g (S_B - S_T) = \rho g S_c \tag{2.2.6}$$

is the heave restoring force constant and

$$M = \rho g \sum_i S_i e^{-kh_i} = \rho g \sum_i' S_i e^{-\frac{\omega^2}{g} h_i} \tag{2.2.7}$$

- <u>Damping force "$D$"</u>.

The damping force $D_i$ exerted by the water on a buoy component "$i$" will be assumed to be directly proportional to the buoy speed $\dot{x}$. It will therefore be of the form

$$D_i = -b_i \dot{x} \tag{2.2.8}$$

where $b_i$ is the linearized coefficient of damping associated with buoy heave motion.

It can be shown (see Appendix I) that the general expression of linearized damping coefficients "$d$" for periodic motion of amplitude $X$ and frequency $\omega$ is of the form

$$d = \frac{4}{3\pi} \rho C_D S X \omega \tag{2.2.9}$$

where $\rho$ = water mass density = 2 slugs/ft$^3$

$C_D$ = conventional drag coefficient

$S$ = area normal to the flow.

In order to keep the differential equation of heave motion linear, an arbitrary constant value $\overline{X}_b$ of average heave motion must be selected to compute the linearized damping coefficients $b_i$. The value of $\overline{X}_b$ selected is left as an input for the program users. One can use, for example, a reasonable fraction of the average wave amplitude for the sea state considered in a given study.

The expression of $b_i$ then becomes

$$b_i = \frac{4}{3\pi}\rho C_{Di} S_i \overline{X}_b \omega = \omega b_i' \tag{2.2.10}$$

The total damping force is thus finally

$$D = \sum_i{}' D_i = -\omega \dot{X} \sum_i{}' \frac{4}{3\pi}\rho C_{Di} S_i \overline{X}_b$$

or simply

$$D = -B\dot{X}$$

with

$$B = \omega \sum_i{}' \frac{4}{3\pi} C_{Di} S_i \overline{X}_b = \omega \sum_i{}' b_i' \tag{2.2.11}$$

- <u>Wave induced drag force "G".</u>

The drag force $G_i$ resulting from the water particle impinging with a velocity $\dot{\eta}$ on a buoy component "$i$" is also assumed to be linearly proportional to $\dot{\eta}$. It therefore will be expressed by

$$G_i = c_i \dot{\eta} \tag{2.2.12}$$

where $c_i$ is the linearized coefficient of drag associated with water particle velocity.

Following previous reasoning the expression of $c_i$ will be given by

$$c_i = \frac{4}{3\pi} \rho\, C_{D_i} S_i \overline{X}_c\, \omega = \omega c_i' \qquad (2.2.13)$$

where $\overline{X}_c$ is now the arbitrary average value of wave amplitudes retained for the particular study.

A comment should also now be made regarding the water particle velocity $\dot{\eta}$. It will be recalled that the expression of $\dot{\eta}$ is

$$\dot{\eta} = -A\omega e^{-\kappa z} \sin \omega t$$

In the case of a plate or a cylinder of small height placed at a distance $h$ below the buoy $W.L.$ and if $x \ll h$, then $z \cong h$ and the speed of the water particles acting on this plate is well established. On the other hand, if the cylinder is one of considerable height, as for example the buoyancy tank shown in Fig. No. 3, then the speeds at the top and at the bottom must be somehow averaged and replaced by a unique equivalent speed.

For simplicity, one could consider this averaged speed to be the speed at the depth of the cylinder midpoint. For the buoyancy tank previously mentioned this speed would then be:

$$-A\omega e^{-\kappa\left(h_1 + \frac{H}{2}\right)} \sin \omega t$$

More appropriate values of equivalent depths could also be devised. With these remarks in mind, the expression of the friction force becomes

$$G = \sum_i{}' G_i = -A\omega \sin \omega t \sum_i{}' \frac{4}{3\pi} \rho\, C_{D_i} S_i \overline{X}_c\, \omega e^{-\kappa z_i}$$

$z_i$ being the true or the equivalent depth of the component "$i$".

More simply written,

$$G = -NA\omega \sin\omega t \tag{2.2.14}$$

with

$$N = \omega \sum_i{}' \frac{4}{3\pi} \rho G_i S_i \bar{X}_i e^{-kz_i} = \omega \sum_i c_i' e^{-\frac{\omega^2}{g} z_i} \tag{2.2.15}$$

- <u>Inertial force "$I$".</u>

The inertial force $I_i$ produced by the water particle acceleration $\ddot{\eta}$ on a component "$i$" of the buoy is given by

$$I_i = m_i' \ddot{\eta}$$

where $m_i'$ is the added mass of the component "$i$" and is given by $m_i' = \rho C_{m_i} V_i$

with $C_{m_i}$ = added mass coefficient of component "$i$"

$V_i$ = volume of the $i$th component (ft$^3$).

The values of $C_{m_i}$ and $V_i$ depending of course on the dimensions and shape of the component "$i$", are left as an input for the program users.

The remarks on the averaged value of the water particle speed also apply for the water particle acceleration.

The expression of the inertial force "$I$" is therefore given by

$$I = \sum_i{}' I_i = -A\omega^2 \cos\omega t \sum_i{}' m_i' e^{-kz_i} \tag{2.2.16}$$

or simply,

$$I = -QA\omega^2 \cos\omega t$$

with

$$Q = \sum_i{}' m_i' e^{-kz_i} = \rho \sum_i{}' C_{m_i} V_i e^{-\frac{\omega^2}{g} z_i} \tag{2.2.17}$$

2.2.1.4 Expression of the differential equation of heave

Using $\sum_i F_i = m_v \ddot{x}$ yields:

$$-Cx + MA\cos\omega t - B\dot{x} - NA\omega\sin\omega t - QA\omega^2\cos\omega t = m_v\ddot{x}$$

or,

$$Cx + B\dot{x} + m_v\ddot{x} = A\left\{(M - \omega^2 Q)\cos\omega t - N\omega\sin\omega t\right\}$$

This expression can be further reduced to:

$$Cx + B\dot{x} + m_v\ddot{x} = F_o\cos(\omega t + \sigma) \qquad (2.2.18)$$

where $F_o$, the exciting force is given by

$$F_o = A\sqrt{(M - Q\omega^2)^2 + (N\omega)^2} \qquad (2.2.19)$$

and $\sigma$, the phase angle between the wave and the force is given by

$$\sigma = \tan^{-1}\frac{N\omega}{M - \omega^2 Q} \qquad (2.2.20)$$

2.2.1.5 Expression of the heave response amplitude operator

Let us assume that a particular solution of the heave equation is given by:

$$x = X_o\cos(\omega t + \psi)$$

where $\psi$ is the phase angle between the exciting force and the heave response. Then,

$$x = X_o(\cos\omega t\cos\psi - \sin\omega t\sin\psi)$$
$$\dot{x} = -X_o\omega(\sin\omega t\cos\psi + \cos\omega t\sin\psi)$$
$$\ddot{x} = -X_o\omega^2(\cos\omega t\cos\psi - \sin\omega t\sin\psi)$$

Introducing these values of $x$, $\dot{x}$, $\ddot{x}$ in the equation of heave motion and ignoring for the moment the phase angle $\sigma$ yield:

$$CX_o(\cos\omega t\cos\psi - \sin\omega t\sin\psi) - BX_o\omega(\sin\omega t\cos\psi + \cos\omega t\sin\psi) - m_v X_o\omega^2(\cos\omega t\cos\psi - \sin\omega t\sin\psi) = F_o\cos\omega t$$

Thus,

$$X_o\left\{(C - m_v\omega^2)\cos\psi - B\omega\sin\psi\right\} = F_o$$

and

$$X_o\left\{(-C + m_v\omega^2)\sin\psi - B\omega\cos\psi\right\} = 0$$

From the second result,

$$\tan\psi = \frac{-\omega B}{C - m_v\omega^2} \tag{2.2.21}$$

Therefore

$$\sin\psi = \frac{-\omega B}{\sqrt{(C - m_v\omega^2)^2 + (\omega B)^2}}$$

and

$$\cos\psi = \frac{C - m_v\omega^2}{\sqrt{(C - m_v\omega^2)^2 + (\omega B)^2}}$$

Introducing these values of $\sin\psi$ and $\cos\psi$ in the first result, yields:

$$X_o = \frac{F_o}{\sqrt{(C - m_v\omega^2)^2 + (\omega B)^2}}$$

The expression of the heave response is thus finally given by

$$x = \frac{A\sqrt{(M-\omega^2 Q)^2 + (N\omega)^2}}{\sqrt{(C - m_v\omega^2)^2 + (\omega B)^2}} \cos(\omega t + \sigma + \psi) \qquad (2.2.22)$$

The response amplitude operator being the ratio of the heave amplitude $x$ by the wave amplitude $A$ is thus in turn expressed by

$$R.A.O. = \frac{x}{A} =$$

$$\frac{\sqrt{\left(\rho g \sum_i' S_i e^{-\frac{\omega^2}{g}h_i} - \omega^2 \rho \sum_i' Cm_i V_i e^{-\frac{\omega^2}{g}z_i}\right)^2 + \left(\omega^2 \sum_i' c_i' e^{-\frac{\omega^2}{g}z_i}\right)^2}}{\sqrt{\left(\rho g S_c - m_v\omega^2\right)^2 + \left(\omega^2 \sum_i' b_i'\right)^2}} \qquad (2.2.23)$$

2.2.1.6 <u>Phase relationship between heave and wave</u>

As previously established, the phase angle $\sigma$ between wave and exciting force is given by

$$\sigma = \tan^{-1} \frac{\omega^2 \sum_i' c_i' e^{-\frac{\omega^2}{g}z_i}}{\rho g \sum_i' S_i e^{-\frac{\omega^2}{g}h_i} - \omega^2 \rho \sum_i' Cm_i V_i e^{-\frac{\omega^2}{g}z_i}} \qquad (2.2.24)$$

The phase angle $\psi$ between the exciting force and the heave response is in turn given by

$$\psi = \tan^{-1} \frac{-\omega^2 \sum_i' b_i'}{\rho g S_c - m_v \omega^2} \qquad (2.2.25)$$

The phase angle $\Phi$ between wave and heave response is finally given by

$$\Phi = \sigma + \psi \qquad (2.2.26)$$

### 2.2.2 Roll Response

#### 2.2.2.1 Initial conditions

Let us consider again the simple harmonic, deep sea wave shown in Fig. No. 2.

The slope of this wave is given by:

$$\frac{\partial \eta}{\partial \xi} = KA \sin (\omega t - K\xi)$$

If we again select to observe this wave at $\xi = 0$, the expression of the slope $\Phi_s$ becomes

$$\Phi_s = KA \sin \omega t$$

The horizontal components of the velocity and acceleration of a water particle at a depth $z$ will in turn be given by:

$$\dot{\xi} = \omega A e^{-Kz} \cos \omega t$$
$$\ddot{\xi} = -\omega^2 A e^{-Kz} \sin \omega t$$

At the time $t = 0 + \varepsilon$ the magnitude of the horizontal particle velocity is maximum and is positive (i.e., in the direction of increasing $\xi$ ), the magnitude of the horizontal particle acceleration is minimum and in the opposite direction, and the magnitude of the slope is minimum and starts to increase.

Let $\Theta$ be the angle of roll, measured from the vertical in a clockwise direction. At time $t = 0 + \varepsilon$, the buoy will be assumed to roll in this direction, i.e., the angle of roll is increasing.

These initial conditions are depicted in Fig. No. 4. Rotation of the buoy is assumed to take place around the buoy center

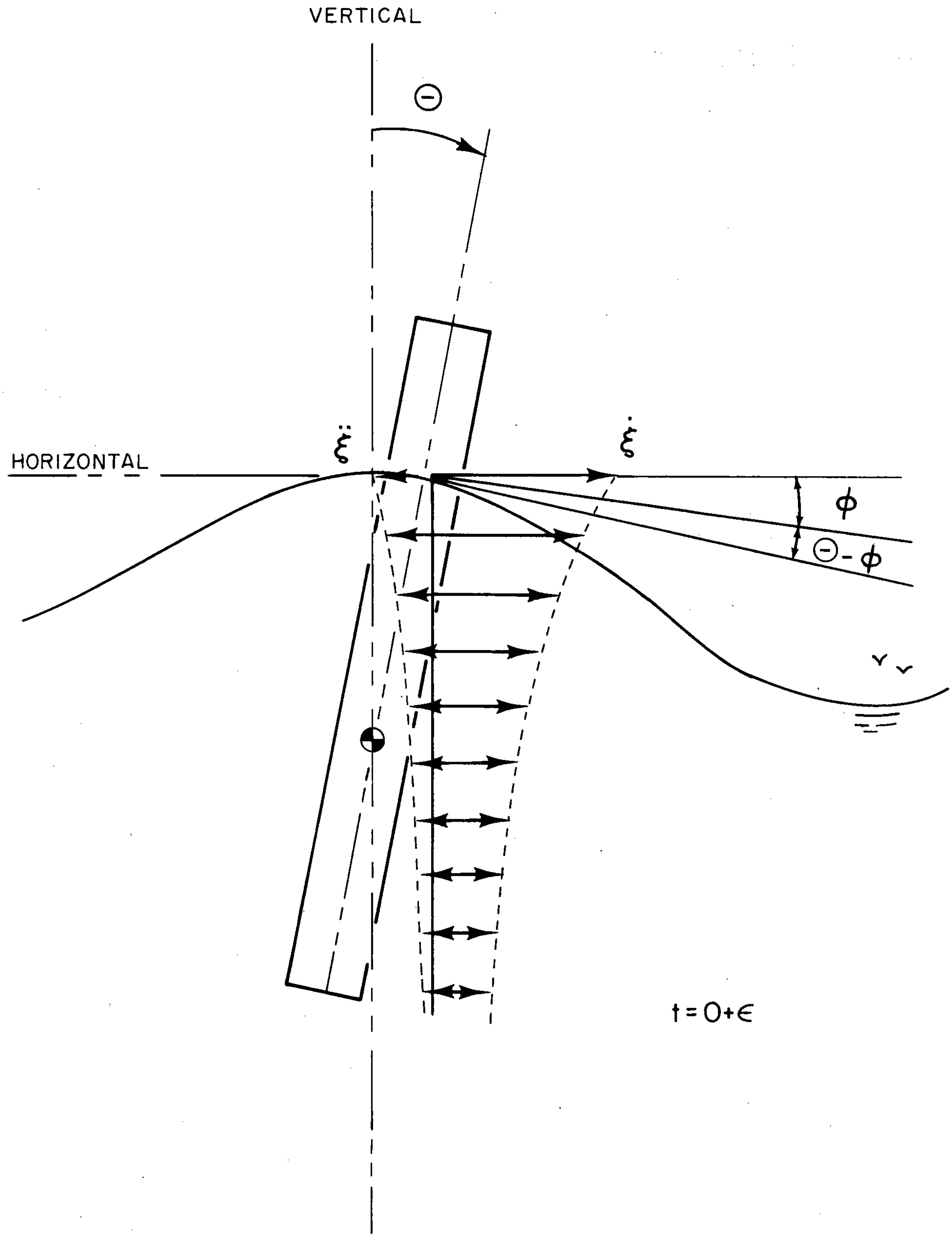
VERTICAL
Θ
HORIZONTAL
$\ddot{\xi}$
$\dot{\xi}$
$\phi$
$\Theta - \phi$
$t = 0 + \epsilon$

Fig. No. 4

of gravity.

2.2.2.2 General equation of roll motion

The equation of roll motion is given by:

$$\sum_i \mathcal{M}_i = (I + I_F)\ddot{\theta} = I_V \ddot{\theta} \qquad (2.2.27)$$

where $\sum_i \mathcal{M}_i$ = sum of the moments applied to the buoy

$I$ = moment of inertial with respect to c.g. of buoy

$I_F$ = added moment of inertia due to entrained water, also with respect to buoy c.g.

$I_V$ = virtual moment of inertia = $I + I_F$

2.2.2.3 Expression of the moments applied to the buoy

Moments applied to the buoy are:

- righting moment caused by displacement of center of buoyancy, $\mathcal{M}_R$.
- damping moment due to buoy motion in the water $\mathcal{M}_D$.
- friction moment due to drag forces induced on the buoy by horizontal water particle velocity $\mathcal{M}_F$.
- inertia moment due to inertia forces induced on the buoy by horizontal water particle acceleration $\mathcal{M}_I$.

Clockwise capsizing moments will be considered positive, and vice versa counterclockwise righting moments will be considered negative.

The expression of these moments can be derived as follows:

- Righting moment - $\mathcal{M}_R$

The righting moment opposes buoy motion. Its value is given by:

$$\mathcal{M}_R = - W\overline{gm}\,(\theta - \Phi) = - W\overline{gm}\,(\theta - KA\sin\omega t) \qquad (2.2.28)$$

where $\overline{W}$ = buoy weight

$\overline{gm}$ = distance from buoy center of gravity to buoy metacenter.

- Damping moment - $\mathcal{M}_D$

The drag forces due to buoy motion alone oppose the roll both above and below the buoy center of gravity. Therefore, the damping moment is negative. Its expression is derived as follows:

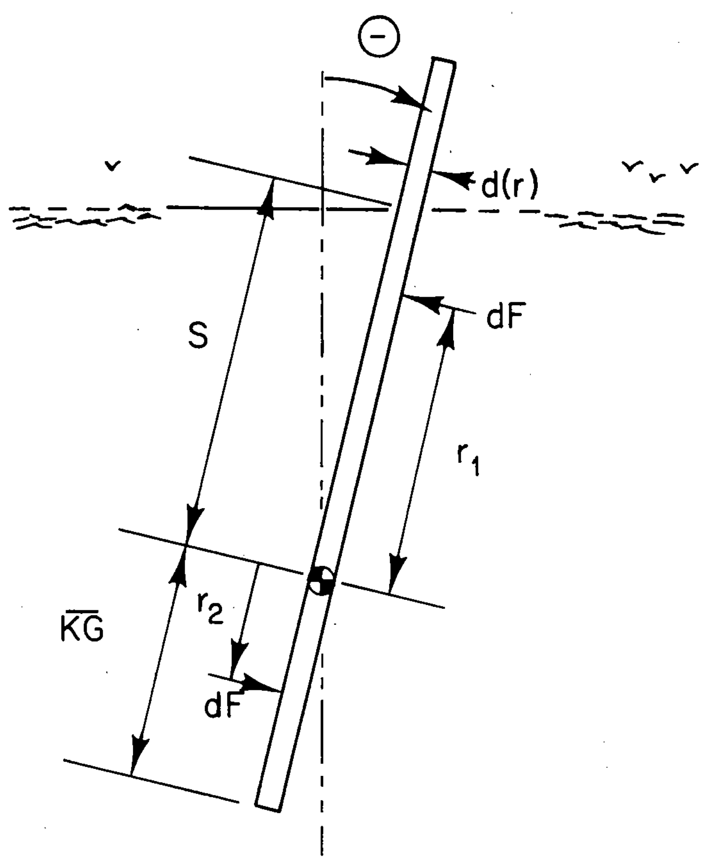


Fig. No. 5

Let us consider an elementary buoy section at a distance $r$ from the buoy c.g. (See Fig. No. 5)

The elementary damping force on this element will be assumed to be of the form:

$$dF_D = b(r)\, r\, \dot{\theta} \cos\theta$$

or, for small angles of roll $\theta$

$$dF_D = b(r)\, r\, \dot{\theta}$$

where $b(r)$ is a linearized damping coefficient again given by:

$$b(r) = \frac{4}{3\pi} \rho\, C_D\, S_{(r)}\, X_{(r)}\, \omega$$

with

$\rho$ = fluid density = 2 slugs/ft$^3$

$C_D$ = drag coefficient for cylinders, normal flow

$S(r)$ = area across the flow = $d(r)\, dr$ with $d(r)$ the cylinder diameter at distance $r$

$X(r)$ = amplitude of cyclic motion at distance $r$

$X(r)$ = $r\theta$ (in order to keep the equation of motion linear an arbitrary constant value of $\theta$ must be selected, say $\theta = \bar{\theta}$ ).

$\omega$ = angular frequency of cyclic motion, which under steady state conditions should equal the frequency of the exciting wave.

The expression of the damping force thus becomes:

$$dF_D = \left( \frac{4}{3\pi} \rho\, C_D\, \bar{\theta}\, \omega\, d(r)\, r\, dr \right) r\, \dot{\theta}$$

or

$$dF_D = \alpha \omega \, d(r) \, r^2 dr \, \dot{\theta}$$

where

$$\alpha = \frac{4}{3\pi} \rho \, C_D \, \bar{\Theta} \tag{2.2.29}$$

The moment of this elementary force is:

$$d\mathcal{M}_D = -r F_D = -\alpha \omega \, \dot{\theta} \, d(r) \, r^3 dr$$

and the total damping moment is found from

$$\mathcal{M}_D = -\dot{\Theta} \alpha \omega \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1^3 dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2^3 dr_2 \right\}$$

or

$$\mathcal{M}_D = -B \dot{\Theta} \tag{2.2.30}$$

where

$$B = \alpha \omega \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1^3 dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2^3 dr_2 \right\} \tag{2.2.31}$$

Appendix II outlines a method for computing these integrals.

- Wave drag moment - $\mathcal{M}_F$

Drag forces due to water particle velocity will tend to capsize the buoy or to upright it depending on their point of application with respect to the c.g.

The resulting moment will thus be positive above the c.g. and negative below the c.g.

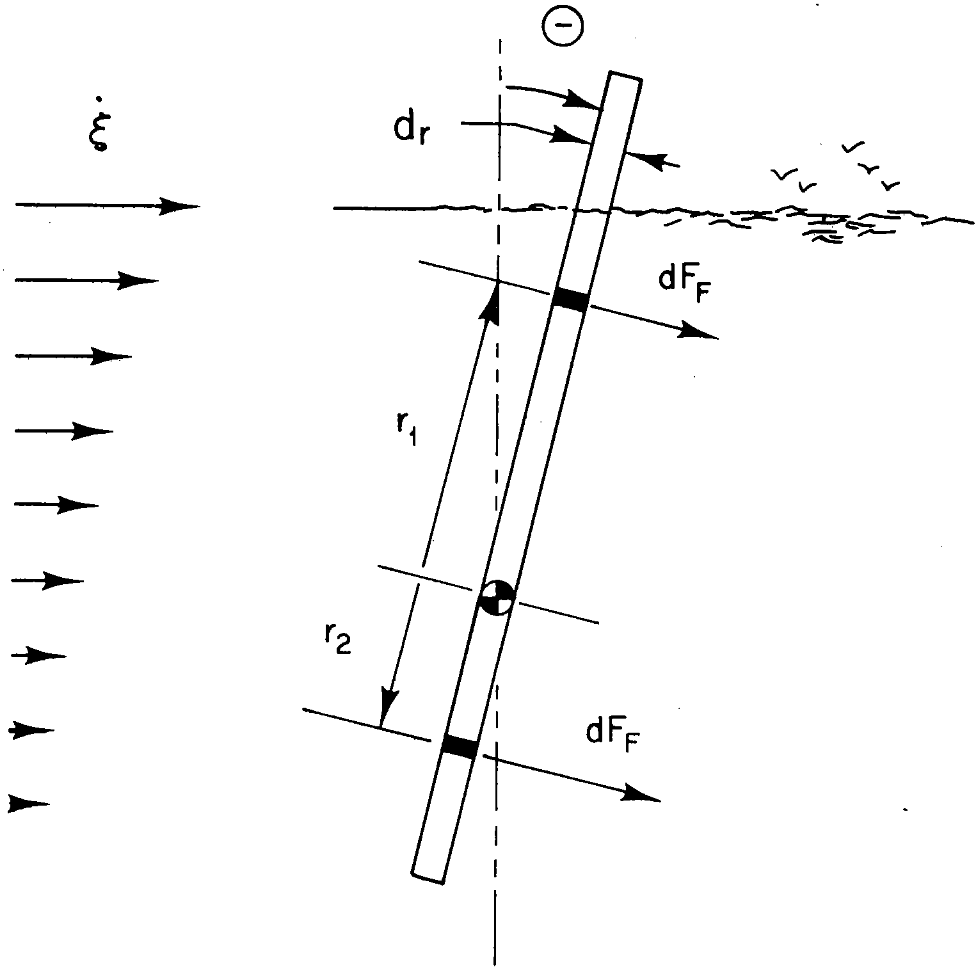


Fig. No. 6

Consider again an elementary buoy section of area $d(r)\,dr$ at a distance $r$ from the buoy center of gravity (Fig. No. 6). The elementary drag force due to the water horizontal velocity on this elementary section will be assumed to be of the form:

$$dF_F = c(r)\,\dot{\xi}$$

where $c(r)$ the linearized damping coefficient, will be expressed by:

$$c(r) = \frac{4}{3\pi}\rho\, C_D\, S(r)\, X(r)\, \omega$$

$X(r)$ in this case is the amplitude of the water particle cyclic motion and is therefore given by

$$X(r) = A e^{-Kz}$$

Here again, in order to maintain linearity in the expression of the roll RAO, an arbitrary constant amplitude $\bar{A}_F$ must be selected. One could, for example, select the average amplitude $\bar{A}$ of the waves in the particular sea state.

With these remarks in mind, the expression of $C(r)$ can be written:

$$c(r) = \frac{4}{3\pi} \rho C_D d(r) dr \bar{A}_F \omega e^{-Kz}$$

or

$$c(r) = \omega \beta d(r) e^{-Kz} dr$$

where

$$\beta = \omega \frac{4}{3\pi} \rho C_D \bar{A}_F \tag{2.2.32}$$

The expression of the elementary drag force $dF_F$ thus becomes

$$dF_F = \beta A \omega \cos\omega t \, e^{-2Kz} d(r) dr$$

The moment of this elementary force is in turn:

$$dM_F = r dF_F = \beta A \omega \cos \omega t \, d(r) r e^{-2Kz} dr$$

Noting that the drag forces have a tendency to capsize the buoy when applied above the buoy c.g., and to upright the buoy when applied below the c.g., the expression of the wave drag moment becomes:

$$\mathcal{M}_F = \beta A \cos\omega t \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1 e^{-2Kz} dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2 e^{-2Kz} dr_2 \right\}$$

or,

$$\mathcal{M}_F = D A \omega \cos\omega t \qquad (2.2.33)$$

where

$$D = \beta \left\{ \int_{r_1=0}^{r_1=S} d(r_1) r_1 e^{-2Kz} dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2) r_2 e^{-2Kz} dr_2 \right\} \qquad (2.2.34)$$

A method for the evaluation of the coefficient $D$ is outlined in Appendix III.

- Wave inertia moment - $\mathcal{M}_I$

The elementary inertia force $dI$ due to the water particle horizontal acceleration acting on an elementary buoy section of volume

$$dV = \frac{\pi}{4} d(r)^2 \, dr$$

is of the form:

$$dI = C_m \rho \, dV \, \ddot{\xi} \cos\theta$$

or, for small angles $\theta$,

$$dI = C_m \rho \frac{\pi}{4} d(r)^2 \ddot{\xi} \, dr$$

where $C_m$ = coefficient of added mass for cylinders. This elementary force can be more simply written:

$$dI = \gamma \, \ddot{\xi} \, d(r)^2 dr$$

with $$\gamma = \frac{\pi}{4}\rho C_m \qquad (2.2.35)$$

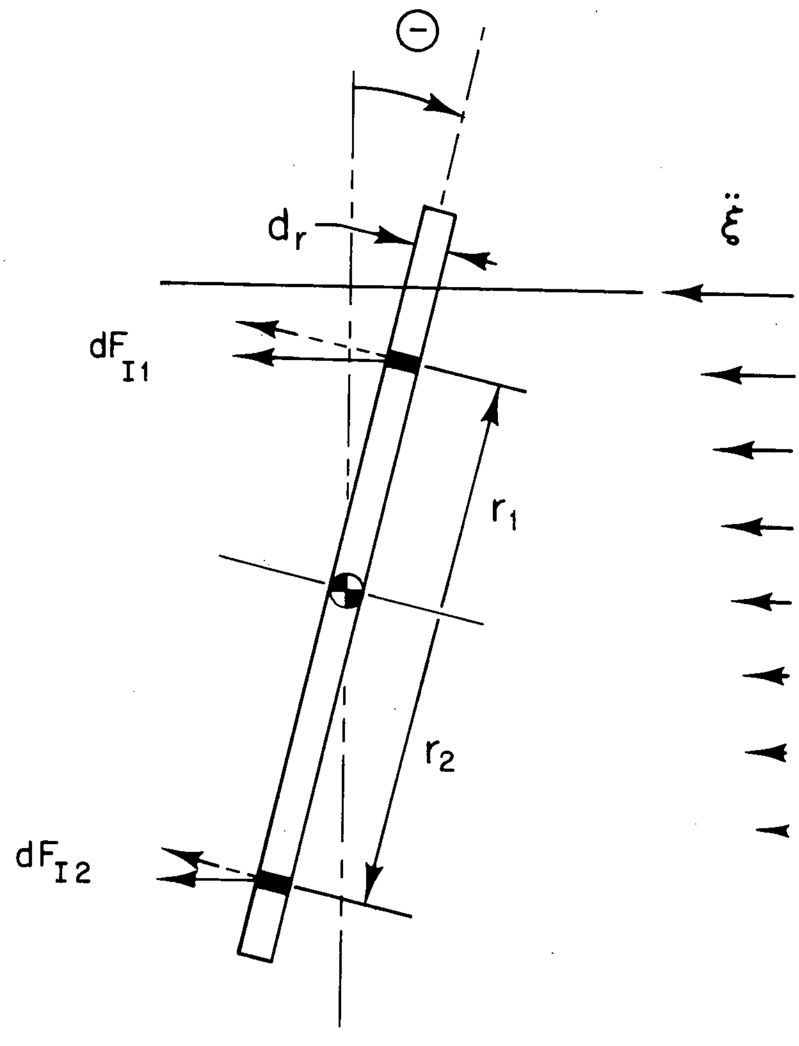


Fig. No. 7

The moment with respect to the c.g. of this elementary force is in turn given by

$$dM_I = \gamma \, d(r)^2 \, r \, \ddot{\xi} \, dr$$
$$= \gamma \omega^2 A \sin \omega t \, d(r)^2 \, r e^{-kz} \, dr$$

Nothing again (Fig. No. 7) that inertia forces due to the wave action have a tendency to upright the buoy above the c.g. and to capsize it below the c.g., the expression of the total moment will be given by:

$$\mathcal{M}_I = \rho\omega^2 A \sin\omega t \left\{ \int_{r_1=0}^{r_1=S} d(r_1)^2 r_1 e^{-Kz} dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)^2 r_2 e^{-Kz} dr_2 \right\} \quad (2.2.36)$$

or,

$$\mathcal{M}_I = \mathcal{P} A \omega^2 \sin\omega t \quad (2.2.37)$$

A method for the evaluation of the coefficient $\mathcal{P}$ is outlined in Appendix IV.

2.2.2.4 Expression of the added moment of inertia $I_F$

The added mass of an elementary buoy section of volume $dV$, located at a distance $r$ from the buoy c.g. is given by

$$dm' = C_m \rho_F \, dV$$

$C_m$ = added mass coefficient = 1 for cylinders.

The moment of inertia of this elementary mass with respect to the c.g. is:

$$dI_F = r^2 dm' = r^2 \rho_F \, dV$$

and the total moment of inertia is

$$I_F = \int dI_F = \iiint \rho_F \, r^2 dV \quad (2.2.38)$$

Thus

$\bar{I}_F$ = moment of inertia of the water displaced by the buoy with respect to the buoy c.g.

$\bar{I}_F$ can be evaluated following the method outlined in Appendix V.

2.2.2.5 Expression of the differential equation of roll

Summing the moments and applying the angular form of Newton's law yield:

$$-W\bar{g}\bar{m}\left(\theta-\frac{\omega^2}{g}A\sin\omega t\right)-B\dot{\theta}+DA\cos\omega t+PA\omega^2\sin\omega t=(I+\bar{I}_F)\ddot{\theta}$$

The resulting equation of motion is then:

$$I_V\ddot{\theta}+B\dot{\theta}+C\theta=A\left\{\left(\frac{\omega^2}{g}+P\omega^2\right)\sin\omega t+D\omega\cos\omega t\right\}$$

where $C=W\bar{g}\bar{m}$ is the roll restoring constant.

The equation of roll motion can be further reduced to:

$$C\theta+B\dot{\theta}+I_V\ddot{\theta}=M\cos(\omega t+\nu) \qquad (2.2.39)$$

where $M$, the exciting torque due to wave action, is given by:

$$M=A\sqrt{\left(\frac{C\omega^2}{g}+P\omega^2\right)^2+(D\omega)^2} \qquad (2.2.40)$$

and $\nu$ the phase angle between wave and resulting torque is in turn given by:

$$\nu=-\tan^{-1}\frac{\frac{C\omega^2}{g}+P\omega^2}{D\omega} \qquad (2.2.41)$$

### 2.2.2.6 Expression of the roll response amplitude operator

Assuming again that a particular solution of the differential equation of roll is given by:

$$\Theta = \Theta_0 \cos(\omega t + \psi)$$

where $\psi$ is the phase angle between roll and the external torque $M$ and introducing this value of $\Theta$ and the values of its first and second derivatives $\dot{\Theta}$ and $\ddot{\Theta}$ in the equation of motion, will yield

$$\Theta = \frac{M}{\sqrt{(C - I_v \omega^2)^2 + (\omega B)^2}}$$

and

$$\psi = \tan^{-1} \frac{-\omega B}{C - I_v \omega^2} \tag{2.2.42}$$

The expression of the roll response will then be given by:

$$\Theta = \frac{A \sqrt{\left(\frac{C\omega^2}{g} - P\omega^2\right)^2 + (D\omega)^2}}{\sqrt{(C - I_v \omega^2)^2 + (\omega B)^2}} \tag{2.2.43}$$

The response amplitude operator being the ratio of the roll amplitude by the wave amplitude $A$, will thus be given by:

$$R.A.O. = \sqrt{\frac{\left(\frac{C\omega^2}{g} + P\omega^2\right)^2 + (D\omega)^2}{(C - I_v \omega^2)^2 + (\omega B)^2}} \tag{2.2.44}$$

2.2.2.7 Phase relationship between roll and wave

As previously established, the phase angle $\sigma$ between wave and exciting torque is given by:

$$\sigma = \tan^{-1}\left[-\frac{\frac{C\omega^2}{g} + P\omega^2}{D\omega}\right] \qquad (2.2.45)$$

The phase angle $\psi$ between the exciting torque and the roll response is in turn given by:

$$\psi = \tan^{-1}\left[-\frac{\omega B}{C - I_V\omega^2}\right] \qquad (2.2.46)$$

Finally the phase angle $\Phi$ between wave and roll response will be the sum of the two, i.e.,

$$\Phi = \sigma + \psi \qquad (2.2.47)$$

# 3.0 COMPUTER PROGRAMS

## 3.1 Heave Computer Program. (HERAO)

### 3.1.1 Program logic

The operations performed by the heave computer program can be summarized as follows:

- It computes the heave Response Amplitude Operator, using formula (2.2.23) for decreasing values of the wave angular frequency $\omega$ .

The recurrence formula used to change the value of the angular frequency between two consecutive computations of the RAO is

$$\omega_n = \frac{2\pi}{T_{n-1} + \Delta T}$$

where $\Delta T$, the change in wave period is constant. The value of $\Delta T$ to use for a particular set of computations is left as a program input, and so is the range of variation of wave periods to be considered.

- It computes the phase angles between force and wave, heave and force, and heave and wave using formulas (2.2.24), (2.2.25), and (2.2.26), for the same set of angular frequencies $\{\omega_n\}$.
- It computes the wave amplitudes spectral density using one of the spectral density formulas (2.1.11), (2.1.12), or (2.1.13) for the same set $\{\omega_n\}$. The choice of spectral density formula is left as an input.
- It computes the heave response spectral density $R(\omega)$ using formula (2.1.8) and the computed values of the RAO for the selected set $\{\omega_n\}$.
- It computes the root mean square values of the wave amplitudes and of the heave response amplitudes by taking, as suggested by formulas (2.1.5) and (2.1.9), the square root of the area under the wave and heave amplitudes spectral density curves established for the set $\{\omega_n\}$.

- It uses the statistical results of Tables I and II and the two root mean square values of wave amplitudes and heave response to compute the corresponding expectations of wave and heave means and maxima.

### 3.1.2 Program input

The program is designed to handle any number of cases in consecutive order. All input is format free. Values for any parameters are entered in a continuous string, separated by commas. The program is designed to run either in the batch mode or interactively from an on-line remote terminal. The method of input is the same for either case. As the interactive mode is also self-explanatory and types user prompts, the input will be discussed for the batch mode. All depths are considered as positive downwards. An equivalent depth is an average, or more accurately an effective, depth at which a body or surface is located.

Input data must be provided on the following cards:

Card 1 -- Number of pressure surfaces.

NP an integer value, starting in column 1, used to specify the number of horizontal pressure surfaces.

Card 2 -- Pressure depth, surface area.

There will be as many card 2's as specified on card 1. Each will contain the following information.

DEPTHP a value specifying the "equivalent depth" of the ith pressure surface, in feet.

AREA a value specifying the area of the pressure surface, in square feet. A negative value is entered for a surface that has exerted upon it a downward force. A positive value is entered for surface subjected to an upward force.

Card 3 -- Number of inertial components.

NE an integer value, starting in column 1, specifying the number of inertial components which comprise the buoy.

Card 4 -- Depth, added mass coefficient, volume.

There will be as many card 4's as specified on card 3. Each card will contain the following information.

DEPTHI a value specifying the equivalent depth of the ith inertial component, in feet.

ADDMSC a value specifying the added mass coefficient for the ith inertial component.

VOLUME a value specifying the volume of the ith inertial component, in cubic feet.

Card 5 -- Number of drag surfaces.

ND an integer value, starting in column 1, specifying the number of drag surfaces of the buoy body.

Card 6 -- Drag depth, linearized damping coefficient, linearized wave drag coefficient.

There will be as many card 6's as there are drag surfaces specified on card 5. Each card will contain the following information.

DEPTHD a value specifying the equivalent depth, in feet, of the ith drag surface.

DAMPC a value specifying the linearized damping coefficient of the ith drag surface, in lbs force/(ft/sec)/(rad/sec).

WDRAGC a value specifying the linearized wave drag coefficient of the ith drag surface, in lbs force/(ft/sec)/(rad/sec).

Card 7 -- Cross sectional area at water surface.

CAREAWL a value, starting in column 1, specifying the cross sectional area at the water line, in square feet. For the purposes of this analysis this area is assumed to be constant over the range of motion at the water line.

Card 8 -- Virtual mass.

VIRTMASS a value specifying the virtual mass of the body, in slugs.

Card 9 (3F.0) -- Starting, ending, increments of wave periods.

TIME1 a value specifying the lowest wave period to be studied, in seconds.

TIME2 a value specifying the highest wave period to be studied, in seconds. This time should be an integral multiple of the incremental time (TIMEDEL) greater than TIME1.

TIMEDEL a value specifying the incremental wave period, in seconds, used in the analysis from TIME1 to TIME2.

Card 10 (I,F.0,F.0) -- Amplitude spectrum selection, parameters.

This card has a general form as follows.

ISEASEL, PARAMA, PARAMB

The necessity and meaning of the parameters will depend on the amplitude spectrum (ISEASEL) selected. In reality these are double height formulas which are converted internally to give the amplitude spectrum. The following options are available.

ISEASEL = 1 Pierson-Moskowitz formula.

PARAMA = wind speed, in knots

no PARAMB

ISEASEL = 2 Bretschneider formula.

PARAMA = significant wave height, in feet

PARAMB = significant wave period, in seconds

ISEASEL = 3 International Ship Structure Congress

PARAMA = significant wave height, in feet

PARAMB = significant wave period, in feet

Card 11 (A1, 1X, A1, 1X, 2E.0) -- Selection of listing, line printer plot, plot scale minimum, plot scale maximum.

ILIST enter a Y (for yes) in column 1 if you desire a listing of the various output parameters. Any other character in column 1 will not produce a listing.

IPLOT enter a Y (for yes) in column 3 if you desire a line printer plot of the RAO. Any other character in column 3 will not produce a plot. The line printer will plot a point at each selected wave period. A check is made on the length of the plot for the following criteria.

$$N = \frac{T2 - T1 + TIMEDEL}{TIMEDEL} \leq 250$$

Note: This limitation is computer dependent.

RAOMIN if a plot is desired, you may enter, beginning in column 5, the minimum value for the RAO scale. If left blank, RAOMIN = 0.

RAOMAX if a plot was selected, the maximum value of the RAO scale may be entered following RAOMIN (separated by a comma). Under the current version RAOMAX $\geq$ 5. For best results RAOMAX-RAOMIN should be an integral multiple of 5. If it is not, the program adjusts it to be so. The current default is RAOMAX = 5.

Card 12 (A1) -- Another case?

IEND If you wish to run another case, enter a Y or YES beginning in column 1. Any character other than a Y in column 1 will cause the program to terminate.

The sequence of card types 1 through 12 is repeated for each additional case desired.

There is a special entry mode for additional cases. Because the buoy configuration may be quite complex, it is undesireable to enter all the descriptive parameters if all that is changing is the wind speed for the sea state. Another alternative is that all the parameters may remain constant except the inertia terms. A special input code of -1 will allow the user to keep in effect the values last entered for any of the parameters. This input code may be used for any of the following input cards.

Card 1

NI = -1

Use the pressure parameters from the previous case.

Do not input any type 2 cards.

Card 3

NI = -1

Use the inertia parameters from the previous case.

Do not input any type 4 cards.

Card 5

ND = -1

Use the drag parameters from the previous case.

Do not input any type 6 cards.

Card 7

CAREAWL = -1

Use the previously entered value of the cross sectional area.

Card 8

VIRTMASS = -1

Use the previously entered value of the virtual mass.

Card 9

TIME 1 = -1

Use the previously entered time range and increments.

Card 10

ISEASEL = -1

Use the previously entered amplitude spectrum for the sea state.

Usage modes

As previously mentioned, the program can be used in either a batch or an interactive mode.

The control card sequence necessary to compile, load, and run the HERAO program in a batch mode is as follows:

```
!JOB aaa,uuu
!LIMIT (TIME,3),(CORE,10)
!FORTRAN LS,GO
```

FORTRAN source deck of program HERAO

```
!METASYM SI,LO,GO
```

source deck for subprogram IAMTERM

```
!LOAD (GO),(UNSAT),(3)),(MAP),(LDEF),(LMN,HERAOR),
  (PERM)
!RUN (LMN,HERAOR)
!DATA
```

data cards for each case

```
!EOD
```

To run a subsequent job utilizing the existing load module, only the following cards need to be submitted:

```
!JOB aaa,uuu
!LIMIT (TIME,2),(CORE,10)
!RUN (LMN,HERAOR)
!DATA
```

data cards for each case

```
!EOD
```

To use the run module in an interactive mode from a terminal, simply log on and enter, to a ! prompt, the following:

START HERAOR

where HERAOR is the name of a previously created load module.

From this point, the operator simply responds to the program prompts as if you were punching up the cards. The only difference is that the operator does not need to start in column 1, but should start as though the head were already positioned correctly. It is.

## Restrictions

The user should not enter a wave period of zero (0.0) or less. While the program will still run, the integral of the wave amplitudes from an angular frequency of $\infty$ will be unreasonable. This in turn will cause the resulting wave statistics to be in error. All other parameters computed should be satisfactory. The method of integration used is that of a trapezoidal approximation. The user must therefore exercise some care in selecting the time period increment. Too large an increment may cause the peak of the RAO or wave amplitude spectrums to be "smoother", resulting in lower values for the integrals of the heave response and amplitude spectrum.

## Subprograms required

IAMTERM — a metasymbol subprogram which checks to see if the program is being run in batch or from an on-line terminal.

SEASPEC — computes the double height density spectrum for the sea state according to one of several empirical formulas; internal.

PLOTINIT — initializes the line printer plot routine; internal.

PLOTHEAV — executes the line printer plot routine; internal.

LPPLOT (PLOT1, PLOT2, PLOT3, PLOT4, PLOT5, PLOT7)

a subprogram which helps create and list a line printer plot; fromW.H.O.I. account 3 library.

### 3.1.3 Program output

The output of the program is comprised of four basic parts. These are:

1. Summary of input parameters.
2. Summary of RAO, phases, and amplitude spectrum, all given as a function of time and frequency.
3. Tabular summaries of wave and heave response statistical properties.
4. Line printer plot of the RAO.

The summary of the input parameters is only given for a run made in the batch mode. For an on-line hard copy terminal, the users entries constitute the input summary.

The listing of the RAO, phases, and other information is optional, as specified in column 1 of input card type 11. The list has the same format whether in the batch or on-line mode. Note that the amplitude spectrum is output for the sea state.

Tabular summaries of the wave and heave response statistical properties are always output and are the same regardless of the mode of operation.

The line printer plot is also optional, as specified in column 3 of input card type 11.

Typical program outputs for the batch and terminal modes are shown under "Case Studies", Section 4.

Errors and diagnostics

***NUMBER OF ENTRIES IS GREATER THAN ARRAY
SIZE ALLOWS

$nn_1$ $nn_2$

THE PROGRAM TERMINATES

The input for the number of components describing the buoy configuration exceeds the array size allocated. Currently $nn_2 = 20$.

***THE PLOT BUFFER IS NOT LARGE ENGOUGH FOR
THE PERIOD RANGE SPECIFIED
THE PLOT IS SUPPRESSED

The number of wave periods analyzed must meet constraints described in the input section, card 11.

Timing

The program's execution time is a function of the buoy configuration and the number of wave periods analyzed. In any case, the execution time normally is negligible, being about 3 seconds (0.05 minutes) per case.

## 3.2 Roll Computer Program (ROLLRAO)

### 3.2.1 Program logic

The operations performed by the roll computer program are similar to those performed by the heave program. They include:

- Computation of the roll RAO, using formula (2.2.44) over the set $\{\omega_n\}$ previously defined. The roll RAO is expressed in units of degrees of roll per foot of wave amplitude. The recurrence formula to change the value of the angular frequency between two consecutive computations of the RAO is again

$$\omega_n = \frac{2\pi}{T_{n-1} + \Delta T}$$

where $\Delta T$, the change in the wave period is a constant set by the user.

- Computation of the phase angles between external torque and wave, roll response and torque, and roll and wave using formulas (2.2.45, 46, 47) for the same set $\{\omega_n\}$.
- Computation of the wave amplitude and roll response spectral densities $S(\omega)$ and $R(\omega)$ and of the root mean square values of wave amplitudes and roll amplitudes. The choice of spectral density formula is left as a program input.
- Finally, computations of expectations of means and maxima of wave and roll amplitudes with the help of the statistical parameters shown in Tables I and II.

3.2.2 Program input

The program is designed to handle any number of cases. Almost all input is format free. Values for any parameter are entered in a continuous string, separated by commas or blanks. The program is designed to run either in the batch mode or interactively from an on-line remote terminal. As the interactive mode is also self-explanatory and types user prompts, the input will be discussed for the batch mode. All depths are considered as positive downwards.

Card 1 (3F.0) -- Period range of time.

TIME1 a value specifying the lowest wave period to be studied, in seconds. As the wave velocity expression contains an exponent with wave frequency, the user is cautioned against using a starting period of less than 1.0 seconds.

TIME2 a value specifying the highest wave period to be studied, in seconds. This value of time should be an integral multiple of the incremental time (TIMEDEL) greater than the value of TIME1.

TIMEDEL a value specifying the incremental wave period, in seconds, used in the analysis from TIME1 to TIME2.

Card 2 (I,F.0,F.0) -- Amplitude spectrum selection, parameters.

This card has a general form as follows.

ISEASEL, PARAMA, PARAMB

The necessity and meaning of the parameters will depend on the amplitude spectrum (ISEASEL) selected. In reality, these are double height formulas which are converted internally to produce the amplitude spectrum. the following options are available.

ISEASEL = 1 Pierson-Moskowitz formula

PARAMA wind speed, in knots.

no PARAMB

ISEASEL = 2 Bretschneider formula.

PARAMA significant wave height, in feet.

PARAMB significant wave period, in seconds.

ISEASEL = 3 International Ship Structure Congress

PARAMA significant wave height, in feet.

PARAMB significant wave period, in seconds.

Card 3 (F.0) -- Radius of buoy at water surface plane.

RWL a value specifying the outer radius of the buoy at the surface (still water assumed), in feet.

Card 4 (F.0) -- Depth to keel.

DEPTHK a value specifying the depth to the keel (bottom) of the buoy, in feet.

Card 5 (F.0) -- Average wave amplitude.

AVERGAMP a value specifying the average expected wave amplitude, in feet.

Card 6 (F.0) -- Average roll constant.

THETABAR a value specifying the average expected roll, in degrees.

Card 7 (I) -- Number of buoy components.

NP a value specifying the number of buoy components. This may be set to zero in subsequent cases.

<u>Cards 8 ... (7+NP)</u> (I, 6F.0, 3A4, A2) -- Component specification.

| | |
|---|---|
| ISHAPE | an integer value used as a code to specify the component shape.<br>1) hollow cylinder<br>2) solid cylinder<br>3) solid disc<br>4) right triangular plate |
| WIDTH | a value specifying the width (diameter or base) of the component, in feet. |
| HEIGHT | a value specifying the height of the component, in feet. |
| THICK | a value specifying the thickness of the component, in inches. Entering THICK = -1 for ISHAPE = 1 will generate a solid (THICK = WIDTH/2.0). For ISHAPE = 2 or 3, also enter a -1 as it is ignored. |
| DENSITY | a value specifying the density of the component, in pounds mass per cubic foot ($lbsm/ft^3$). |
| DISTCG | a value specifying the vertical distance from the buoy keel to the component center of gravity, in feet. The component c.g. is vertical vector only. |

FRACNORM a value specifying the fractional proportion of the component area normal to the roll motion. For cylinders this entry = 1.0. For triangular plates it will vary from 0.0 (oriented in line with roll) to 1.0 (area normal to roll motion).

ICOMMENT a character string used to describe the component.

Note: For the purpose of visually inspecting data cards used in batch input, it may be desirable to "format" the data. The recommended format is (I5, 6F10.0, 1X, 3A4, A2).

Card 9 (I) -- Redefined part code.

N an integer value specifying the number (index) of the component to be redefined. This allows the user to change the dimensions of a component(s) or to add new components. To add a new component, the specified value of N must be one greater than the current maximum number of parts defined for the buoy. There may be as many redefinition pairs of cards as desired. To terminate the sequence enter a value for N = -1.

Card 10 (I, 6F.0, 3A4, A2) -- Redefinition specification.

This card has the same format as card 7, the component specification. Any number of pairs of this and the previous card may be entered as desired.

Card 11 (A1, 1X, A1, 1X, 2F.0) -- Selection of listing, line printer plot, plot scale minimum and maximum.

ILIST — enter a Y (for yes) in column 1 if you desire a listing of the various output parameters. Any other character in column 1 will cause the listing to be suppressed.

IPLOT — enter a Y (for yes) in column 3 if you desire a line printer plot of the roll RAO. Any other character in column 3 will cause the line printer plot to be suppressed. The line printer plots a point at each selected wave period. A check is made on the length of the plot for the following criteria.

$$\frac{\text{T2} - \text{T1} + \text{TIMEDEL}}{\text{TIMEDEL}} \leq 100$$

RAOMIN — if a plot is desired, you may enter, beginning in column 5, the minimum value for the RAO scale. If left blank RAOMIN = 0.0.

RAOMAX — if a plot was selected, the maximum value of the RAO scale may be entered following the RAOMIN value (separated by a blank or comma). For the best results RAOMAX -

RAOMIN should be an integral multiple of 5. If it is not, the program adjusts it to be so. The current default is RAOMAX = 10.0.

Card 12 (A1) -- Another case?

IEND — If you wish to run another case, enter a Y or YES beginning in column 1. Any character other than a Y in column 1 will cause the program to terminate.

The sequence of card types 1 through 12 is repeated for each additional case desired.

There is a special entry mode for additional cases. Because the buoy configuration may be quite complex, it is undesirable to enter all the descriptive parameters to investigate the effect of a different wind speed or sea state. Another alternative is that the design of the buoy remains the same except for the size of the counterweight. A special input code of -1 and the component redefinition options allow the user to keep in effect the values last entered for any of the parameters or components.

Cards 1-7 — A -1 will maintain those values previously entered. For card type 7, this simply uses the same buoy and no new components are defined (no type 8 cards).

Cards 9-10 — As many of these pairs as desired may be entered to define a new buoy configuration. The entry sequence is terminated by a -1 for card type 9 and no card type 10.

Usage

As previously mentioned, this program is meant to be used in either a batch or an interactive mode. The following control card sequence is a complete list of steps necessary to compile, load, and execute the program ROLLRAO in the batch mode.

```
!JOB aaa,uuuu
!LIMIT (TIME,3),(CORE,20)
!FORTRAN LS,GO
   Fortran source deck of program ROLLRAO
!METASYM SI,LO,GO
   source deck of subprogram IAMTERM
!LOAD (GO),(UNSAT,(3)),(MAP),(LDEF),(LMN,ROLLR),(PERM)
!RUN (LMN,ROLLR)
!DATA
   data cards for all cases
!EOD
```

To run a subsequent job utilizing the existing load module, only the following cards need to be submitted:

```
!JOB aaa,uuuu
!LIMIT (TIME,2),(CORE,20)
!RUN (LMN,ROLLR)
!DATA
   data cards for all cases
!EOD
```

To run the load module from a terminal, simply log on and enter, to a ! prompt, the following:

```
START ROLLR
```

From this point on you simply respond to the program prompts as if you were punching the data cards. The only difference is that you do not need to start in column 1, but should start as if you are already there. (There is also a slight difference in the manner in which the buoy components are described and entered. This is explained to the user on line.)

Restrictions

The user should exercise care in entering wave periods of less than 1.0 seconds. The water velocity terms contain a natural exponent of the wave number times displacement from the center of gravity and this computation may exceed the machine computational capability.

The method of integration used is that of a trapezoidal approximation over frequency. The user must therefore exercise additional care in selecting the time period increment. Too large an increment may cause the peak of the RAO or wave amplitude spectrums to be "smoothed", resulting in lower values for the integrals of the amplitude spectrum and roll response. At the other extreme, the time period of 0.0 will cause an "infinite" angular frequency and the resulting wave statistics will be in error.

Also note that the entire program is executed in single precision. For normal buoy configurations this mode is adequate. However, in certain cases such as the case of a small flat cylinder (case study no. 1, Section 4.1), the response amplitude operator exhibited signs of instability in the numerical computation. This can be overcome by using double precision computations.

Subprograms Required

| | |
|---|---|
| IAMTERM | a Metasymbol subprogram which checks to see if the program is being run in batch or from an on-line terminal. |
| SEASPEC | computes the amplitude density spectrum for the sea state according to one of several empirical formulae; internal. |
| PLOTINIT | initializes the line printer plot routine; internal. |
| PLOTROLL | executes the line printer plot routine; internal. |
| TINPUT | prompts and inputs data from a user on-line. |
| BINPUT | reads and summarizes data entered in batch mode. |
| BODYVOL | computes the volume of a component. |
| BODYMI | computes the shape dependent contribution of a component's moment of inertia. |
| DISPLACE | computes those parameters associated with the displacement of the buoy's components. |
| BUOYDAMP (WATERDAMP) | computes the moments of damping for the buoy and water drag forces. |
| WATRINRT | computes the inertia moment contribution from the water wave particle acceleration. |

LPPLOT (PLOT1, PLOT2, PLOT3, PLOT4, PLOT5, PLOT7)

a subprogram which creates and tests a line printer plot; W.H.O.I. account 3 library.

3.2.3 Program output

The output of the program is comprised of four basic parts. These are:

1. Summary of input parameters and physical properties of the buoy.
2. Summary of RAO, phases, and amplitude spectrum, all given as a function of time frequency.
3. Tabular summaries of wave and roll response statistical properties.
4. Line printer plot of the roll RAO.

The summary of the input parameters is only given for a run made in the batch mode. For an on-line hard copy terminal, the user entries constitute the input summary.

The listing of the RAO, phases and other information is optional, as specified in column 1 on input card type 11. The list has the same format whether in batch or on-line mode. Note that the amplitude spectrum is output for the sea state. This is derived directly from the selected double height spectrum.

Tabular summaries of the wave and roll response statistical properties are always output and are the same regardless of the mode of operation.

The line printer plot is also optional, as specified in column 3 of the input card 11.

Timing

The program's execution time is a function of the buoy configuration and the number of wave periods analyzed. Normally the execution time is much less than a minute per case.

Errors and Diagnostics

A number of checks are made on the input parameters and the stability of the buoy. The messages indicate the nature of the error and take appropriate action depending on the execution mode.

## 4.0 CASE STUDIES

To illustrate the use of the computer solutions the following three case studies are presented.

a. Heave and roll response of a flat cylinder of small dimensions.

b. Heave and roll response of a ballasted "telephone pole."

c. Heave and roll response of a complex shape W.H.O.I. spar buoy.

### 4.1 Heave and Roll Response of a Small Flat Cylinder

This case study has relatively little practical value. Every one knows that a thin slice of pulpwood, if thrown in the sea, will essentially follow the heave and slope of the waves, both in magnitude and phase. It is included here mainly for the purpose of program result verification.

The small cylinder considered has a diameter of one foot. The ratio of its height to its diameter is 1:3. Its density is 3/4 that of sea water. The heave and roll motion of this cylinder will be studied using a Pierson-Moskowitz spectrum with a 20 knots wind.

4.1.1 Program input

Buoy draft. The buoy draft is obviously 0.25'.

Pressure surface. The pressure force is exerted exclusively on the cylinder lower face. The pressure surface is therefore at 0.25' from the surface. Its area $S$ is $\pi/4 = 0.785$ sq. ft. The pressure force being upwards $S$ is positive.

Inertial component. The added mass effect will be con-considered to take place at the lower face of the cylinder, i.e. at 0.25' from the surface. It will be assumed to take place only half of the time, during the downwards part of the heave cycle. It will be estimated to be the same as the one produced by a flat plate of same radius as the cylinder. Thus the averaged added mass will be:

$$m' = \frac{1}{2}\left(\frac{8}{3}\rho a^3\right) = \frac{1}{2}\left(C_m \rho \, Vol\right) = 0.33 \text{ slug.}$$

Assuming the added mass coefficient to be equal to one, the averaged added mass coefficient and volume will be

$$C_m' = \frac{1}{2} C_m = 0.50$$

$$V_{bL} = \frac{8}{3} a^3 = 0.33 \text{ cu-ft.}$$

Damping/drag surface. Damping and wave drag will also be considered to take place at the lower face of the cylinder, i.e. at 0.25' from the surface. These effects will again be assumed to take place only half the time.

A choice must now be made for the arbitrary values of average heave $\overline{X_b}$ and average wave amplitude $\overline{X_c}$. Assuming that the heave equals the wave amplitude, a fair assumption in this case, and selecting the average wave amplitude for winds of 20 knots to be 3' will yield $\overline{X_b} = \overline{X_c} = 3'$.

Using this value of $\overline{X_b}$ and $\overline{X_c}$ in expressions (2.2.10) and (2.2.13), and a drag coefficient $C_D = 0.9$, the value of the linearized damping and wave drag coefficients $b'$ and $c'$ is found to be:

$$b_1' = c_1' = 1.8 \quad \frac{\text{lb-f}}{\text{ft-sec}} \bigg/ \frac{\text{rad.}}{\text{sec}}$$

To account for the time average only half of these coefficients value, i.e. 0.9, should be used as program input.

Cross sectional area at surface. This area equals $\pi/4$ or 0.785 sq. ft.

Buoy virtual mass $m_v$

$$m_v = m + m'$$

$m$ = buoy mass = $\frac{\pi}{4} \times \frac{1}{3} \times \frac{3}{4} \times \frac{64}{32}$ = 0.392 slug.

$m'$ = added mass = 0.333 slug

Thus, buoy virtual mass $m_v$ = 0.725 slug

In addition to these parameters, the user must also specify the arbitrary roll constant $\bar{\Theta}$ and wave amplitude constant $\bar{A}$. In this case the average angle roll $\bar{\Theta}$ will be assumed to equal the slope of the average wave in 20 knots wind. Assuming an average amplitude $\bar{A}$ of 3' and an average wave length of 120', the average wave slope $\bar{\Phi}_s$ is found to be:

$$\bar{\Phi}_s = \bar{\Theta} = \frac{2\pi \bar{A}}{L} = \frac{\pi}{20} \qquad = 0.157 \text{ radians}$$

A summary of the data input is shown in the Data Coding Form Fig. No. 8 and Fig. No. 9.

4.1.2 <u>Program output</u>

Heave and roll response amplitude operators are depicted in Fig. Nos. 10 and 11.

Computed expectations of average and maximum values of wave amplitudes and of heave and roll motions are also obtained with the help of the computer programs. Corresponding values of wave slopes are calculated independently using

$$\bar{\Phi} = \alpha \sqrt{\int \Phi(\omega)\, d\omega}$$

where $\alpha$ is the applicable Raleigh constant

and $\Phi(\omega) = k^2 S(\omega)$,

$k$ being the wave number = $\omega^2/g$ and

$S(\omega)$ being the wave amplitude spectral density.

These results are summarized in Table Nos. 3 and 4.

The heave RAO, with a value of one over most of the

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# DATA CODING FORM

DATA FOR THE HEAVE MOTION OF A SMALL, FLAT CYLINDER (CASE #1)

| | 1–80 |
|---|---|
| 1 | 1 |
| 2 | 0.25, 0.785 |
| 3 | 1 |
| 4 | 0.25, 0.50, 0.33 |
| 5 | 1 |
| 6 | 0.25, 1.80, 1.80 |
| 7 | 0.785 |
| 8 | 0.725 |
| 9 | 1.0, 50.0, 1.0 |
| 10 | 1, 20.0 |
| 11 | Y, Y |
| 12 | N |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |
| 19 | |
| 20 | |

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Fig. No. 8

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# DATA CODING FORM



```
DATA FOR THE ROLL MOTION OF A SMALL, FLAT CYLINDER  (CASE #1)
1.0, 50.0, 1.0
1, 20.0
0.50
0.25
3.0
9.0
1
2, 1.0, 0.333, -1.0, 48.0, 0.1667, 1.0, FLAT DISC
-1
Y, Y
N
```

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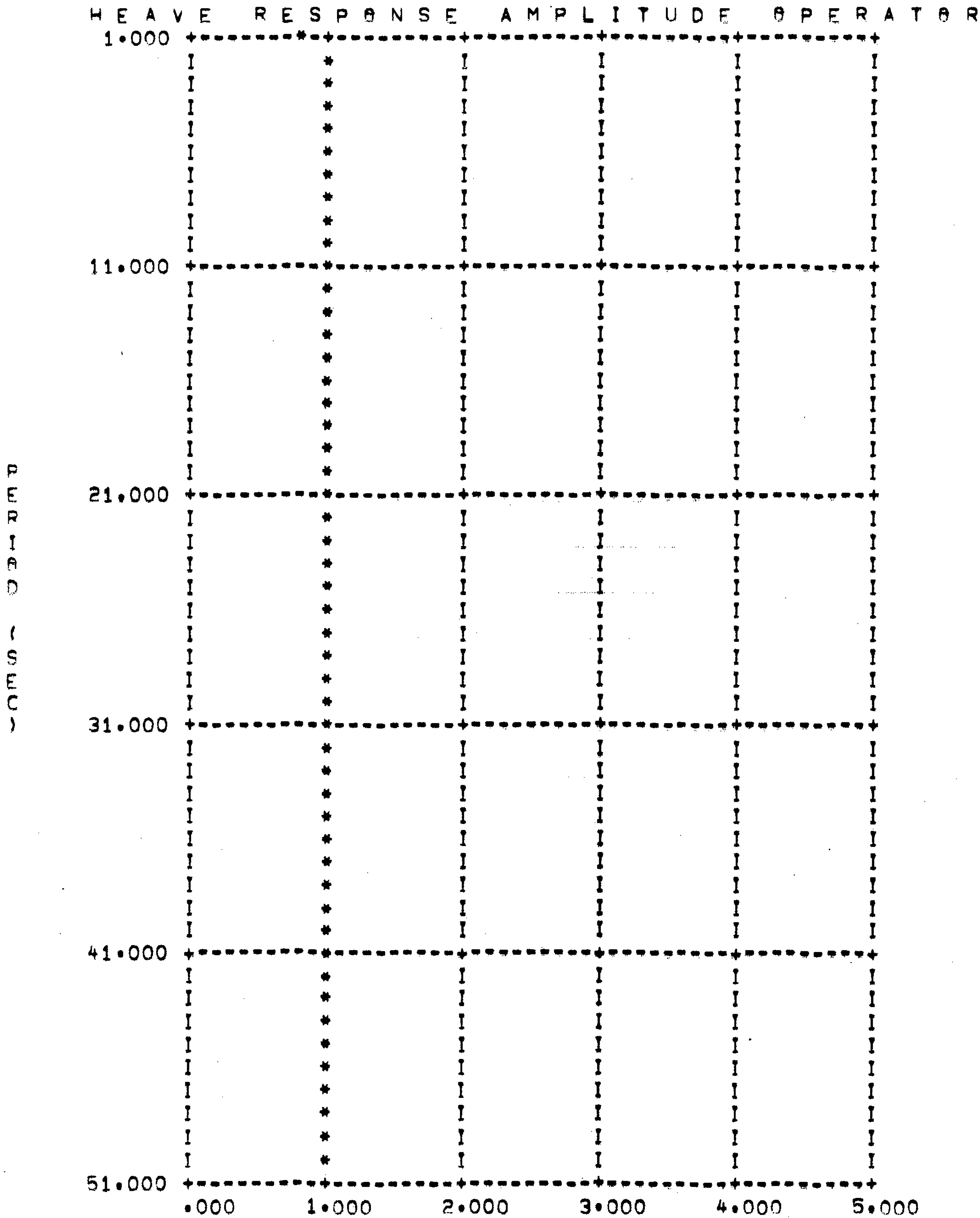
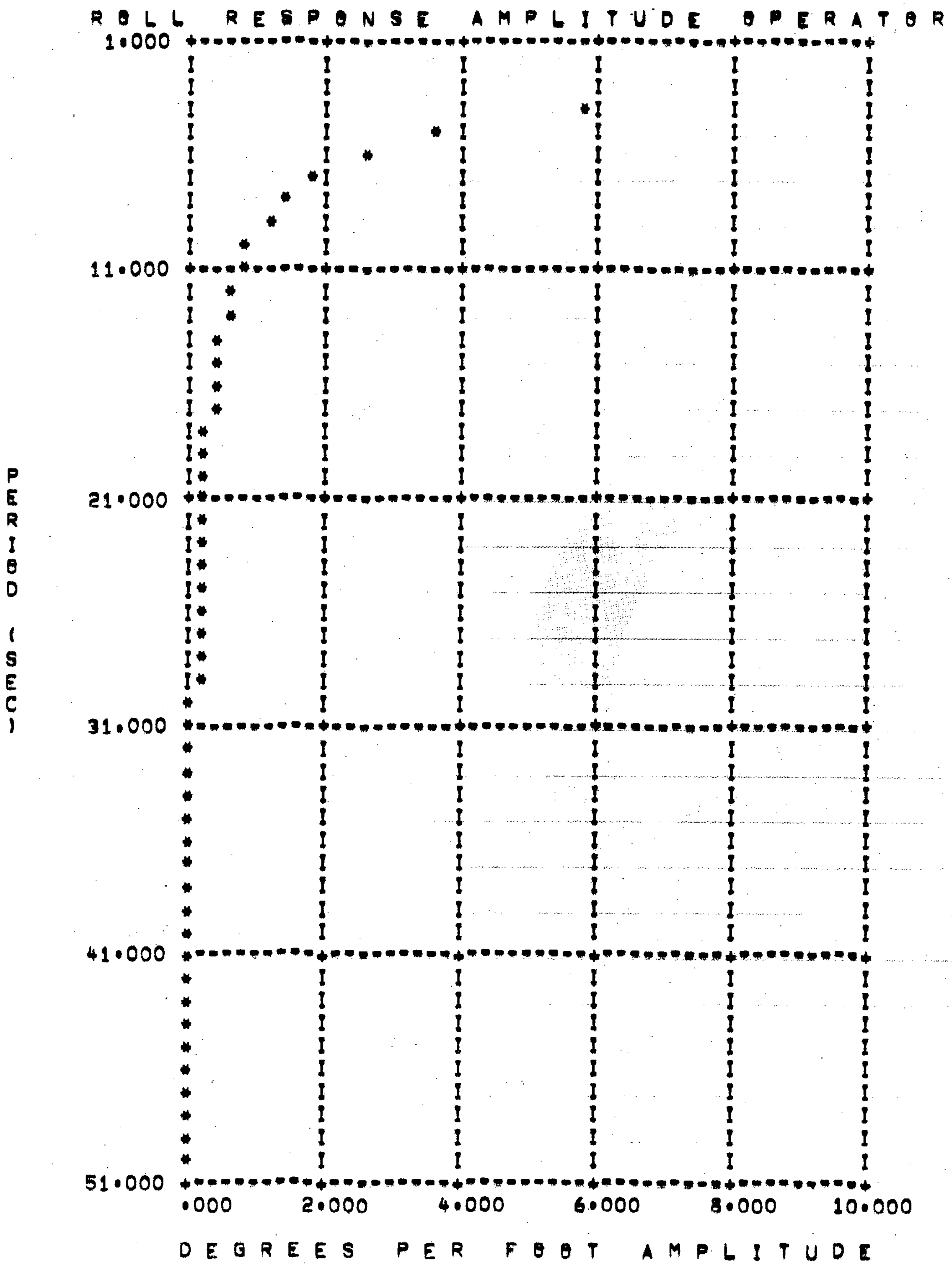
Fig. No. 9




*STOP* 0

Fig. No. 10




*STOP* 0

Fig. No. 11

RMS OF WAVE SPECTRUM = 2.683 FEET

PROBABLE AMPLITUDE
OF WAVE = 1.897 FEET

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE WAVE AMPLITUDE | NUMBER OF WAVES | EXPECTED WAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 6.330 | 50 | 5.689 |
| .100 | 4.830 | 100 | 6.118 |
| .333 | 3.799 | 500 | 7.003 |
| .500 | 3.370 | 1000 | 7.459 |
| 1.000 | 2.377 | 10000 | 8.399 |
| | | 100000 | 9.311 |

RMS OF RESPONSE SPECTRUM = 2.684 FEET

PROBABLE AMPLITUDE
OF HEAVE RESPONSE = 1.897 FEET

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE HEAVE AMPLITUDE RESPONSE | NUMBER OF WAVES | EXPECTED HEAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 6.331 | 50 | 5.690 |
| .100 | 4.831 | 100 | 6.119 |
| .333 | 3.800 | 500 | 7.005 |
| .500 | 3.371 | 1000 | 7.461 |
| 1.000 | 2.378 | 10000 | 8.400 |
| | | 100000 | 9.313 |

Table No. 3 Heave Response of Flat Small Cylinder

Amplitudes of Roll are in Degrees

| Fraction of Largest Amplitudes Considered | Average Roll Amplitude Response | Number of Waves | Expected Roll Maximum Amplitude |
|---|---|---|---|
| .010 | 25.070 | 50 | 22.530 |
| .100 | 19.129 | 100 | 24.230 |
| .333 | 15.048 | 500 | 27.737 |
| .500 | 13.348 | 1000 | 29.544 |
| 1.000 | 9.416 | 10000 | 33.264 |
| | | 100000 | 36.877 |

Amplitudes of Wave Slope are in Degrees

| Fraction of Largest Amplitudes Considered | Average Slope | Number of Waves | Expected Maximum Slope |
|---|---|---|---|
| .010 | 23.905 | 50 | 21.483 |
| .100 | 18.240 | 100 | 23.104 |
| .333 | 14.349 | 500 | 26.448 |
| .500 | 12.728 | 1000 | 28.171 |
| 1.000 | 8.978 | 10000 | 31.718 |
| | | 100000 | 35.163 |

Table No. 4 Roll Response of Small Flat Cylinder wave periods considered, clearly indicates that the small cylinder is a perfect wave follower. Averages and expected maximum values of heave when compared to corresponding values of wave amplitude confirm this expected result.

The roll RAO, being in degree of roll per foot of wave rather than per degree of slope does not immediately correlate roll and wave slope. It shows simply that roll is large at small periods, and tends to zero as the wave period increases, which is of course precisely what the slope of the wave does. The computed statistical averages however do confirm that for practical purposes the roll angle is strongly correlated to wave slope. It thus appears that this first study case is a good test of the program validity.

## 4.2 Heave and Roll Response of a Ballasted "Telephone Pole"

We next consider a cylindrical body made of two cylinders of same diameter, but of different lengths and densities, as shown on Fig. No. 12.

The density of the large cylinder is 16 lbs/cu. ft. The density of the small cylinder is 384 lbs/cu. ft. As in the previous case, the heave and roll response of this body will be studied using a Pierson-Moskowitz spectrum with a 20 knots wind.

### 4.2.1 Program input

In order to provide the necessary input data to the heave and roll programs, the following computations must be first performed.

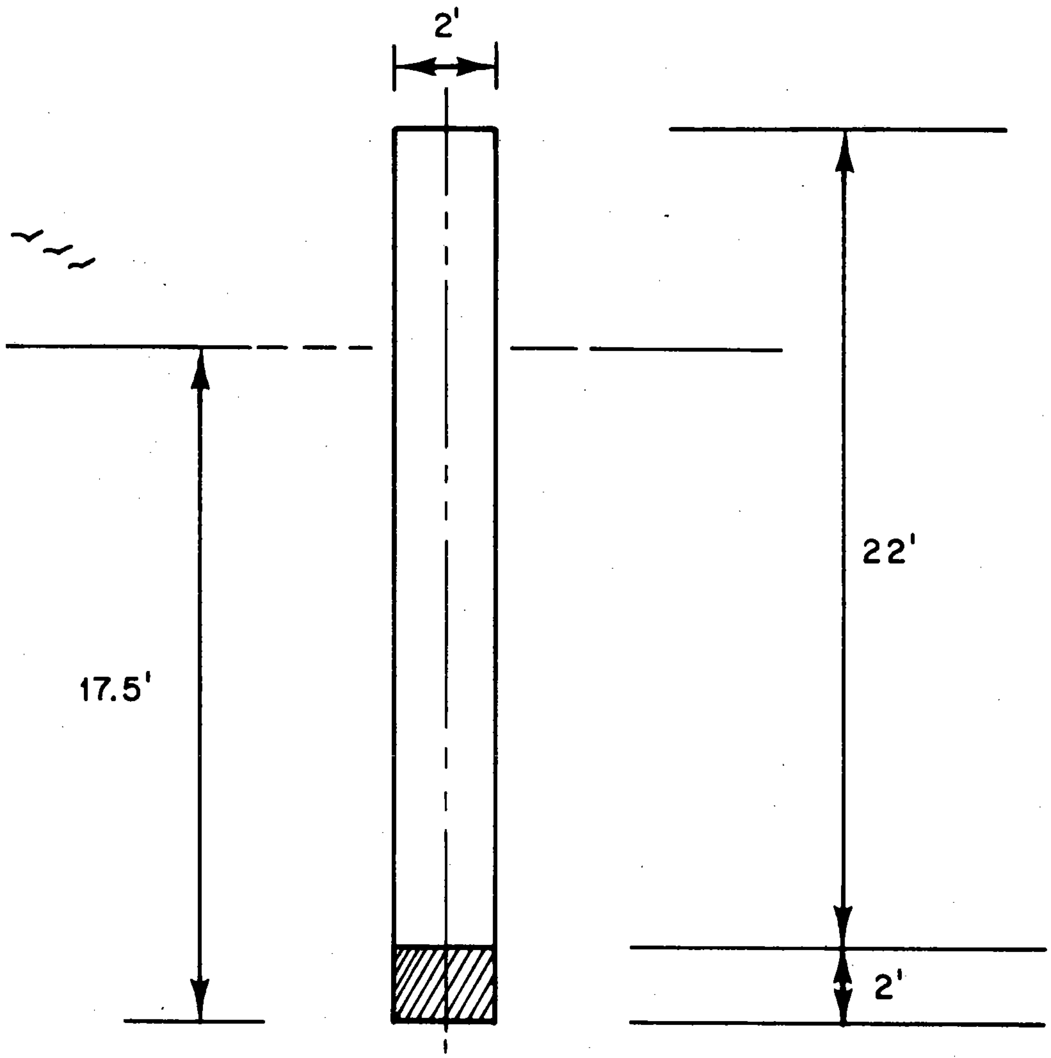



Fig. No. 12

Buoy draft.

weight of small cylinder = $\pi$ x1x1x2x384 = 2413 lbs

weight of long cylinder = $\pi$ x1x1x22x16 = 1105 lbs

Total Weight = 3518 lbs

$$\text{Draft} = \frac{3518}{\pi \times 1 \times 1 \times 64} = 17.5'$$

Number, depth, and area of pressure surfaces. In this simple case there is again only one pressure surface, namely the pole lower face. It is located 17.5' from the surface. Its area "$S$" is

$$S = \pi \times 1 \times 1, \quad S = 3.14 \text{ sq. ft.}$$

The pressure force being upwards, $S$ is positive.

Number, depth, added mass coefficient and volume of inertial components. In heave motion, the added mass effect will be considered to be essentially produced by the pole lower end. There will thus be only one inertial component, acting at 17.5' from the surface. The added mass effect will be estimated to be the same as the one produced by a sphere of same radius as the pole (long cylinder approximation), but acting only half of the time. Thus the averaged added mass will be expressed by

$$m' = \frac{1}{2} C_m \rho \, Vol = \frac{1}{2}\left(\frac{1}{2}\frac{64}{32.2} \times \frac{4\pi}{3}\right) = 2.08 \text{ slugs}$$

The corresponding added mass coefficient and volume are therefore

$$C_m' = \frac{1}{2} C_m = 0.250$$
$$Vol = \frac{4}{3}\pi \times 1 = 4.19 \text{ cu. ft.}$$

Number, depth, damping and wave drag coefficients of drag surfaces. Damping and wave drag effects are assumed to be also essentially produced by the pole lower face. Thus there will be only one drag surface located at 17.5'. These effects will be assumed to take place only half of the time.

The average heave $\overline{X}_b$ will be assumed to be half of the average wave amplitude $\overline{X}_c$ and the later will be assumed to be 3'. Using these values of $\overline{X}_b$ and $\overline{X}_c$ in expressions (2.2.10) and (2.2.13) together with $C_D$ = 0.9, yield

$$b_1' = 3.6 \quad \frac{lb\text{-}f}{ft\text{-}sec} \Big/ \frac{rad.}{sec}$$
$$c_1' = 7.2 \quad \text{"} \quad \text{"}$$

Again, to account for the fact that damping effects are assumed to occur only half of the time, only half of the coefficients values, i.e. 1.8 and 3.6 respectively, should be used as program input.

Cross sectional area at surface.

$$\pi R^2 = \pi \times 1 \times 1 = 3.14 \text{ sq-ft}.$$

Buoy virtual mass $m_v$.

$$m = \frac{3518}{32.2} = 109 \text{ slugs}$$
$$m' = 2.08 \text{ slugs}$$
$$m_v = m + m' = 111.08 \text{ slugs}$$

In addition to these computed parameters, the program user must select the arbitrary average roll constant $\overline{\Theta}$ and wave amplitude $\overline{A}$. For this example $\overline{\Theta}$ is set equal to 5° and $\overline{A}$ equal to 3.0 feet.

Obviously the number of buoy parts is two. Their characteristics are summarized as follows:

| Part No. | Name | Shape | Width (ft) | Height (ft) | Thick (ft) | Density (lbs/cu.ft) | C.G. Above Keel (ft) |
|---|---|---|---|---|---|---|---|
| 1 | Upper Cyl. | Solid Cyl. | 2 | 22 | | 16 | 13 |
| 2 | Lower | Solid Cyl. | 2 | 2 | | 384 | 1 |

All pertinent input data are listed in the data coding form shown in Fig. Nos. 13 and 14.

4.2.2 Program output

Values of the heave and roll response amplitude operators of the telephone pole for the prescribed period interval (50 seconds) and increment (1 second) are presented in the typical computer printouts shown in Fig. No. 15 and Fig. No. 16. The response amplitude operators are also graphically represented in Fig. Nos. 17 and 18. Expected average and maximum values of wave amplitudes and of heave and roll responses are as tabulated in Tables 5 and 6.

4.3 Heave and Roll Response of a Complex Shape W.H.O.I. Spar Buoy

Fig. No. 19 shows the dimension and shape of the spar buoy to be studied next.

Heave and roll response will be again established using a

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# DATA CODING FORM

DATA FOR THE HEAVE MOTION OF A BALLASTED POLE (CASE #2)

| | 1–80 |
|---|---|
| 1 | 1 |
| 2 | 17.5, 3.14 |
| 3 | 1 |
| 4 | 17.5, 0.25, 4.19 |
| 5 | 1 |
| 6 | 17.5, 1.8, 3.6 |
| 7 | 3.14 |
| 8 | 111.08 |
| 9 | 1.0, 50.0, 1.0 |
| 10 | 1, 20.0 |
| 11 | Y, Y |
| 12 | N |
| 13 | |
| 14 | |
| 15 | |
| 16 | |
| 17 | |
| 18 | |
| 19 | |
| 20 | |

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Fig. No. 13

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## DATA CODING FORM

```
DATA FOR THE ROLL MOTION OF A BALLASTED POLE  (CASE #2)
1.0, 50.0, 1.0
1, 20.0
1.0
17.5
3.0
5.0
2
2, 2.0, 22.0, -1.0, 16.0, 13.0, 1.0, LIGHT TOP
2, 2.0, 2.0, -1.0, 384.0, 1.0, 1.0, HEAVY BOTTOM
-1
Y, Y, 0.0, 15.0
N
```

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Fig. No. 14

| PERIOD | ANG FREQ | RAO | W-F PHASE | F-H PHASE | W-H PHASE | AMP SPEC |
|---|---|---|---|---|---|---|
| 1.000 | .628E 01 | .000 | 50.115 | -179.027 | -128.912 | .002 |
| 2.000 | .314E 01 | .001 | 11.136 | -178.863 | -167.727 | .055 |
| 3.000 | .209E 01 | .062 | 4.703 | -178.420 | -173.716 | .406 |
| 4.000 | .157E 01 | .701 | 2.596 | -176.519 | -173.923 | 1.597 |
| 5.000 | .126E 01 | 3.246 | 1.646 | -6.319 | -4.673 | 4.223 |
| 6.000 | .105E 01 | 1.381 | 1.138 | -1.427 | -.289 | 8.094 |
| 7.000 | .898E 00 | 1.153 | .833 | -.745 | .089 | 11.382 |
| 8.000 | .785E 00 | 1.078 | .637 | -.480 | .157 | 11.477 |
| 9.000 | .698E 00 | 1.044 | .502 | -.342 | .160 | 7.928 |
| 10.000 | .628E 00 | 1.028 | .407 | -.259 | .148 | 3.524 |
| 11.000 | .571E 00 | 1.018 | .336 | -.204 | .132 | .933 |
| 12.000 | .524E 00 | 1.012 | .282 | -.166 | .116 | .135 |
| 13.000 | .483E 00 | 1.009 | .240 | -.138 | .103 | .010 |
| 14.000 | .449E 00 | 1.006 | .207 | -.116 | .091 | .000 |
| 15.000 | .419E 00 | 1.005 | .180 | -.100 | .081 | .000 |
| 16.000 | .393E 00 | 1.004 | .158 | -.086 | .072 | .000 |
| 17.000 | .370E 00 | 1.003 | .140 | -.076 | .065 | .000 |
| 18.000 | .349E 00 | 1.002 | .125 | -.067 | .058 | .000 |
| 19.000 | .331E 00 | 1.002 | .112 | -.060 | .053 | .000 |
| 20.000 | .314E 00 | 1.001 | .101 | -.054 | .048 | .000 |
| 21.000 | .299E 00 | 1.001 | .092 | -.048 | .044 | .000 |
| 22.000 | .286E 00 | 1.001 | .084 | -.044 | .040 | .000 |
| 23.000 | .273E 00 | 1.001 | .077 | -.040 | .037 | .000 |
| 24.000 | .262E 00 | 1.001 | .070 | -.037 | .034 | .000 |
| 25.000 | .251E 00 | 1.001 | .065 | -.034 | .031 | .000 |
| 26.000 | .242E 00 | 1.000 | .060 | -.031 | .029 | .000 |
| 27.000 | .233E 00 | 1.000 | .056 | -.029 | .027 | .000 |
| 28.000 | .224E 00 | 1.000 | .052 | -.027 | .025 | .000 |
| 29.000 | .217E 00 | 1.000 | .048 | -.025 | .023 | .000 |
| 30.000 | .209E 00 | 1.000 | .045 | -.023 | .022 | .000 |
| 31.000 | .203E 00 | 1.000 | .042 | -.022 | .021 | .000 |
| 32.000 | .196E 00 | 1.000 | .040 | -.020 | .019 | .000 |
| 33.000 | .190E 00 | 1.000 | .037 | -.019 | .018 | .000 |
| 34.000 | .185E 00 | 1.000 | .035 | -.018 | .017 | .000 |
| 35.000 | .180E 00 | 1.000 | .033 | -.017 | .016 | .000 |
| 36.000 | .175E 00 | 1.000 | .031 | -.016 | .015 | .000 |
| 37.000 | .170E 00 | 1.000 | .030 | -.015 | .015 | .000 |
| 38.000 | .165E 00 | 1.000 | .028 | -.014 | .014 | .000 |
| 39.000 | .161E 00 | 1.000 | .027 | -.014 | .013 | .000 |
| 40.000 | .157E 00 | 1.000 | .025 | -.013 | .012 | .000 |
| 41.000 | .153E 00 | 1.000 | .024 | -.012 | .012 | .000 |
| 42.000 | .150E 00 | 1.000 | .023 | -.012 | .011 | .000 |
| 43.000 | .146E 00 | 1.000 | .022 | -.011 | .011 | .000 |
| 44.000 | .143E 00 | 1.000 | .021 | -.011 | .010 | .000 |
| 45.000 | .140E 00 | 1.000 | .020 | -.010 | .010 | .000 |
| 46.000 | .137E 00 | 1.000 | .019 | -.010 | .009 | .000 |
| 47.000 | .134E 00 | 1.000 | .018 | -.009 | .009 | .000 |
| 48.000 | .131E 00 | 1.000 | .018 | -.009 | .009 | .000 |
| 49.000 | .128E 00 | 1.000 | .017 | -.009 | .008 | .000 |
| 50.000 | .126E 00 | 1.000 | .016 | -.008 | .008 | .000 |

Fig. No. 15

Heave Response Amplitude Operator
"Telephone Pole"

RAO IS IN DEGREES/FOOT OF WAVE AMPLITUDE

| PERIOD | ANG FREQ | RAO | W-T PHASE | T-R PHASE | W-R PHASE | AMP SPEC |
|---|---|---|---|---|---|---|
| 1.000 | .628E 01 | 2.373 | 94.644 | -172.529 | -77.886 | .002 |
| 2.000 | .314E 01 | 1.898 | 114.370 | -171.541 | -57.171 | .055 |
| 3.000 | .209E 01 | 2.241 | 145.846 | -169.158 | -23.312 | .406 |
| 4.000 | .157E 01 | 4.270 | 163.392 | -162.214 | 1.178 | 1.597 |
| 5.000 | .126E 01 | 14.685 | 170.591 | -112.391 | 58.200 | 4.223 |
| 6.000 | .105E 01 | 5.609 | 173.937 | -19.060 | 154.877 | 8.094 |
| 7.000 | .898E 00 | 2.622 | 175.732 | -8.363 | 167.369 | 11.382 |
| 8.000 | .785E 00 | 1.638 | 176.806 | -5.052 | 171.753 | 11.477 |
| 9.000 | .698E 00 | 1.155 | 177.500 | -3.485 | 174.015 | 7.928 |
| 10.000 | .628E 00 | .871 | 177.978 | -2.587 | 175.391 | 3.524 |
| 11.000 | .571E 00 | .685 | 178.329 | -2.014 | 176.315 | .933 |
| 12.000 | .524E 00 | .556 | 178.578 | -1.620 | 176.958 | .135 |
| 13.000 | .483E 00 | .461 | 178.773 | -1.336 | 177.437 | .010 |
| 14.000 | .449E 00 | .390 | 178.930 | -1.124 | 177.806 | .000 |
| 15.000 | .419E 00 | .334 | 179.054 | -.960 | 178.094 | .000 |
| 16.000 | .393E 00 | .290 | 179.126 | -.830 | 178.296 | .000 |
| 17.000 | .370E 00 | .254 | 179.243 | -.726 | 178.517 | .000 |
| 18.000 | .349E 00 | .225 | 179.268 | -.640 | 178.628 | .000 |
| 19.000 | .331E 00 | .201 | 179.315 | -.570 | 178.746 | .000 |
| 20.000 | .314E 00 | .180 | 179.357 | -.510 | 178.847 | .000 |
| 21.000 | .299E 00 | .162 | 179.393 | -.460 | 178.934 | .000 |
| 22.000 | .286E 00 | .147 | 179.523 | -.417 | 179.106 | .000 |
| 23.000 | .273E 00 | .134 | 179.610 | -.379 | 179.230 | .000 |
| 24.000 | .262E 00 | .123 | 179.542 | -.347 | 179.195 | .000 |
| 25.000 | .251E 00 | .113 | 179.497 | -.318 | 179.179 | .000 |
| 26.000 | .242E 00 | .104 | 179.542 | -.293 | 179.249 | .000 |
| 27.000 | .233E 00 | .096 | 179.629 | -.271 | 179.358 | .000 |
| 28.000 | .224E 00 | .089 | 179.601 | -.252 | 179.349 | .000 |
| 29.000 | .217E 00 | .083 | 179.396 | -.234 | 179.162 | .000 |
| 30.000 | .209E 00 | .078 | 179.762 | -.218 | 179.543 | .000 |
| 31.000 | .203E 00 | .073 | 179.897 | -.204 | 179.693 | .000 |
| 32.000 | .196E 00 | .068 | 179.926 | -.191 | 179.735 | .000 |
| 33.000 | .190E 00 | .064 | 179.963 | -.179 | 179.784 | .000 |
| 34.000 | .185E 00 | .060 | -179.955 | -.169 | -180.123 | .000 |
| 35.000 | .180E 00 | .057 | 179.621 | -.159 | 179.462 | .000 |
| 36.000 | .175E 00 | .053 | 179.778 | -.150 | 179.628 | .000 |
| 37.000 | .170E 00 | .051 | 179.438 | -.142 | 179.296 | .000 |
| 38.000 | .165E 00 | .048 | -179.805 | -.134 | -179.939 | .000 |
| 39.000 | .161E 00 | .046 | -179.875 | -.128 | -180.002 | .000 |
| 40.000 | .157E 00 | .043 | 179.417 | -.121 | 179.296 | .000 |
| 41.000 | .153E 00 | .041 | -179.352 | -.115 | -179.467 | .000 |
| 42.000 | .150E 00 | .039 | 179.696 | -.110 | 179.586 | .000 |
| 43.000 | .146E 00 | .037 | -179.358 | -.105 | -179.462 | .000 |
| 44.000 | .143E 00 | .036 | -179.559 | -.100 | -179.658 | .000 |
| 45.000 | .140E 00 | .034 | 179.657 | -.095 | 179.562 | .000 |
| 46.000 | .137E 00 | .033 | 179.571 | -.091 | 179.479 | .000 |
| 47.000 | .134E 00 | .031 | -178.071 | -.087 | -178.158 | .000 |
| 48.000 | .131E 00 | .030 | 178.501 | -.084 | 178.417 | .000 |
| 49.000 | .128E 00 | .029 | 177.126 | -.080 | 177.046 | .000 |
| 50.000 | .126E 00 | .028 | 178.549 | -.077 | 178.472 | .000 |

Fig. No. 16

Roll Response Amplitude Operator
"Telephone Pole"

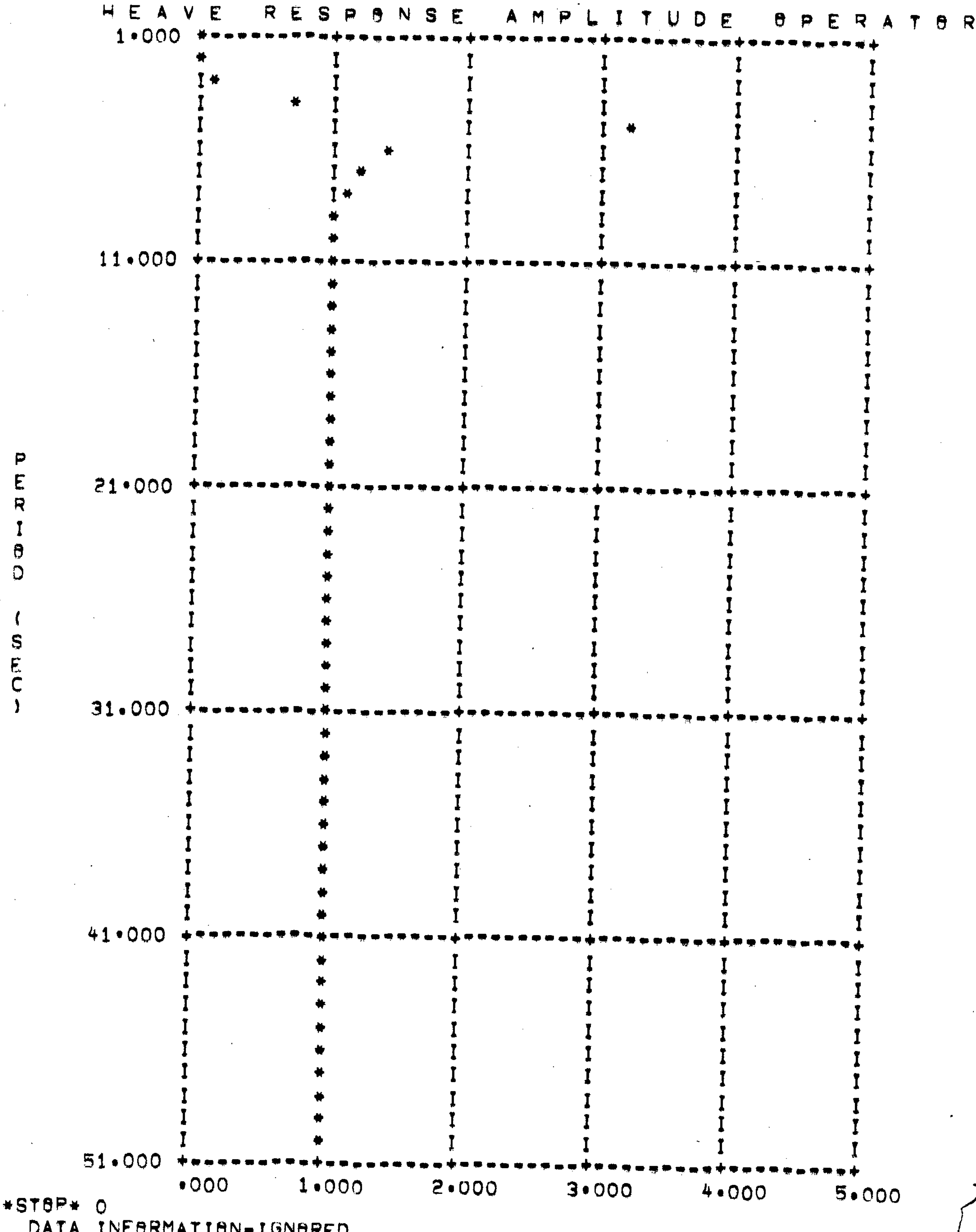

HEAVE RESPONSE AMPLITUDE OPERATOR
1.000
11.000
21.000
31.000
41.000
51.000
PERIOD (SEC)
.000
1.000
2.000
3.000
4.000
5.000
*STOP* 0
DATA INFORMATION=IGNORED

Fig. No. 17

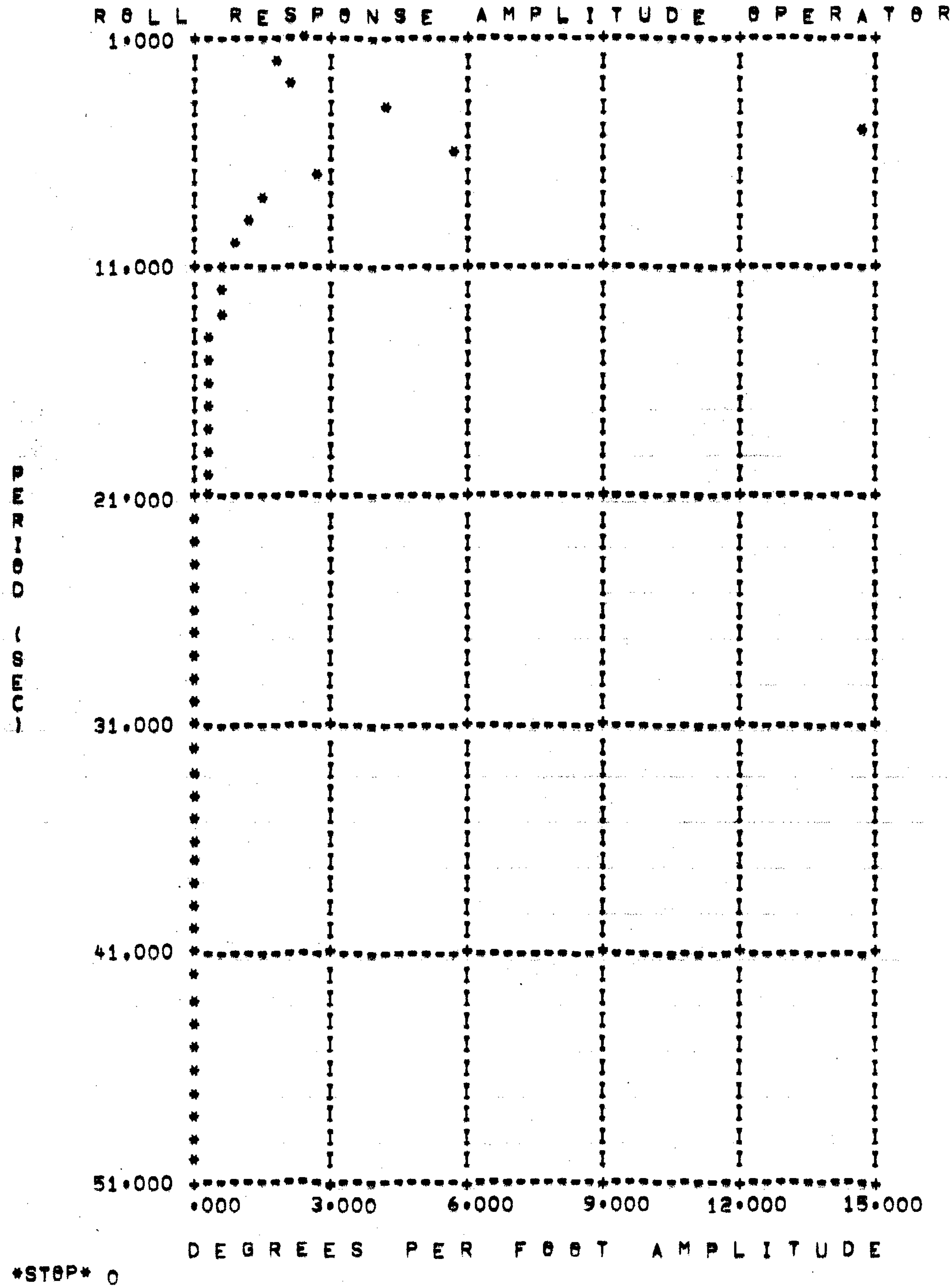

ROLL RESPONSE AMPLITUDE OPERATOR
1.000
11.000
21.000
31.000
41.000
51.000
PERIOD (SEC)
.000
3.000
6.000
9.000
12.000
15.000
DEGREES PER FOOT AMPLITUDE
*STOP* 0

Fig. No. 18

RMS OF WAVE SPECTRUM = 2.683 FEET

PROBABLE AMPLITUDE OF WAVE = 1.897 FEET

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE WAVE AMPLITUDE | NUMBER OF WAVES | EXPECTED WAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 6.330 | 50 | 5.689 |
| .100 | 4.830 | 100 | 6.118 |
| .333 | 3.799 | 500 | 7.003 |
| .500 | 3.370 | 1000 | 7.459 |
| 1.000 | 2.377 | 10000 | 8.399 |
| | | 100000 | 9.311 |

RMS OF RESPONSE SPECTRUM = 4.362 FEET

PROBABLE AMPLITUDE OF HEAVE RESPONSE = 3.084 FEET

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE HEAVE AMPLITUDE RESPONSE | NUMBER OF WAVES | EXPECTED HEAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 10.291 | 50 | 9.248 |
| .100 | 7.852 | 100 | 9.946 |
| .333 | 6.177 | 500 | 11.386 |
| .500 | 5.479 | 1000 | 12.128 |
| 1.000 | 3.865 | 10000 | 13.654 |
| | | 100000 | 15.138 |

Table No. 5 Heave Response of Telephone Pole

RMS ØF WAVE SPECTRUM = 2.683 FEET

PRØBABLE AMPLITUDE
ØF WAVE = 1.897 FEET

| FRACTIØN ØF LARGEST AMPLITUDES CØNSIDERED | AVERAGE WAVE AMPLITUDE | NUMBER ØF WAVES | EXPECTED WAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 6.330 | 50 | 5.689 |
| .100 | 4.830 | 100 | 6.118 |
| .333 | 3.799 | 500 | 7.003 |
| .500 | 3.370 | 1000 | 7.459 |
| 1.000 | 2.377 | 10000 | 8.399 |
| | | 100000 | 9.311 |

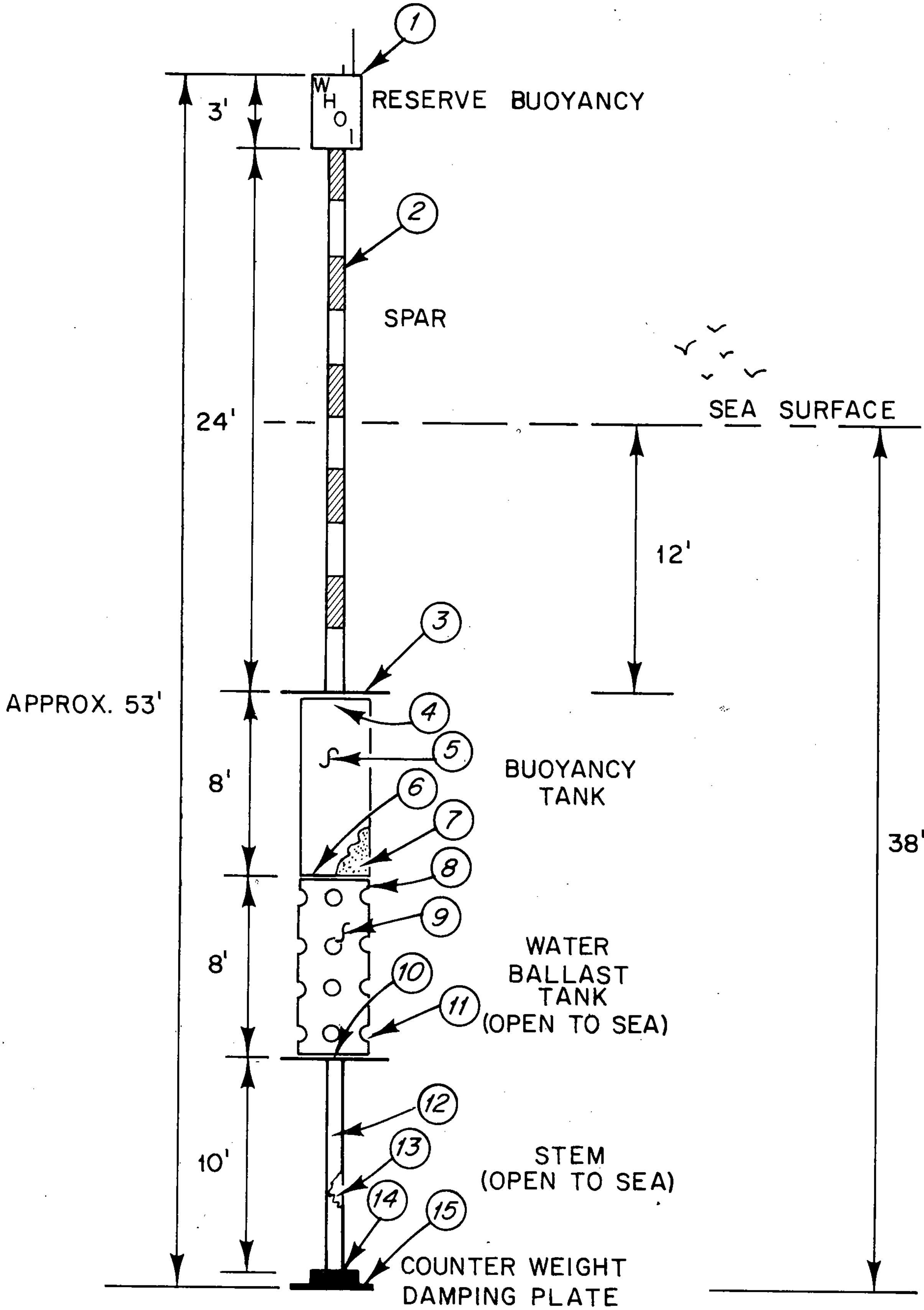
RMS ØF RESPØNSE SPECTRUM = 17.684 DEG

PRØBABLE AMPLITUDE
ØF RØLL RESPØNSE = 12.502 DEG

AMPLITUDES ØF RØLL ARE IN DEGREES

| FRACTIØN ØF LARGEST AMPLITUDES CØNSIDERED | AVERAGE RØLL AMPLITUDE RESPØNSE | NUMBER ØF WAVES | EXPECTED RØLL MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 41.716 | 50 | 37.489 |
| .100 | 31.831 | 100 | 40.319 |
| .333 | 25.040 | 500 | 46.154 |
| .500 | 22.211 | 1000 | 49.160 |
| 1.000 | 15.668 | 10000 | 55.350 |
| | | 100000 | 61.362 |

Table No. 6 Roll Response of Telephone Pole



1
W H O I
RESERVE BUOYANCY
3'
2
SPAR
24'
SEA SURFACE
12'
APPROX. 53'
3
4
5
BUOYANCY TANK
8'
6
7
8
38'
9
WATER BALLAST TANK (OPEN TO SEA)
8'
10
11
12
13
STEM (OPEN TO SEA)
10'
14
15
COUNTER WEIGHT DAMPING PLATE


Fig. No. 19

Pierson-Moskowitz spectrum with a wind of 20 knots.

This relatively complex shape buoy is made of the following parts:

- A reserve buoyancy cylinder 3 ft. high, 2.5 ft. in diameter, made of 2 lbs/cu.ft. polyurethane foam.
- A spar 24 ft. long made of 8" O.D by 1/4" thick wall aluminum tubing.
- A spar base plate, made of 4 ft. diameter by 1/2" thick aluminum plate.
- A 3'.0" diameter by 8'.0" long buoyancy tank made of 3/16" steel plates. The buoyancy tank is filled with 4 lbs/cu.ft. foam.
- A 3'.0" diameter by 8'.0" water ballast tank made of 1/8" steel plates. The tank is filled with sea water. The bottom plate is 4'.0" in diameter.
- A 10'.0" long stem made of 6 5/8" O.D. schedule 40 steel pipe filled with sea water.
- A 4'.0" diameter by 1/2" thick damping plate.
- A counterweight cylinder 2.5 ft. in diameter by 0.848 ft. high, made of cast iron.

The physical parameters of the buoy main components are summarized in Table No. 7.

Table No. 7

**Table of Spar Buoy Components Parameters**

| Part No. | Name | Shape | Width (ft) (diameter) | Height (ft) | Thick (ft) (wall) | Density (lbs/ft$^3$) | Weight (lbs) | C.G. Above Keel |
|---|---|---|---|---|---|---|---|---|
| 1 | Reserve Buoyancy | Solid Cyl. | 2.5 | 3.0 | | 2.0 | 29.45 | 52.383 |
| 2 | Spar | Hollow Cyl. | 0.666 | 24.0 | 0.0208 | 160.0 | 167.11 | 38.889 |
| 3 | Plate Al. | Solid Cyl. | 4.0 | 0.416 | | 160.0 | 83.64 | 26.909 |
| 4 | Plate St. | Solid Cyl. | 3.0 | 0.0156 | | 490.0 | 54.03 | 26.889 |
| 5 | Plate St. | Hollow Cyl. | 3.0 | 8.0 | 0.0156 | 490.0 | 573.72 | 22.889 |
| 6 | Plate St. | Solid Cyl. | 3.0 | 0.0156 | | 490.0 | 54.03 | 18.900 |
| 7 | Foam | Solid Cyl. | 3.0 | 8.0 | | 4.0 | 226.19 | 22.889 |
| 8 | Plate St. | Solid Cyl. | 3.0 | 0.0104 | | 490.0 | 36.02 | 18.8844 |
| 9 | Plate St. | Hollow Cyl. | 3.0 | 8.0 | 0.0104 | 490.0 | 384.22 | 14.8896 |
| 10 | Plate St. | Solid Cyl. | 4.0 | 0.0104 | | 490.0 | 64.03 | 10.8948 |
| 11 | Water | Solid Cyl. | 3.0 | 8.0 | | 64.0 | 3619.11 | 14.8896 |
| 12 | Stem | Hollow Cyl. | 0.552 | 10.0 | 0.0233 | 490.0 | 197.98 | 5.8896 |
| 13 | Water | Solid Cyl. | 0.552 | 10.0 | | 64.0 | 153.16 | 5.8896 |
| 14 | Counter-weight | Solid Cyl. | 2.5 | 0.848 | | 450.0 | 1875.00 | 0.4656 |
| 15 | Damping Plate | Solid Cyl. | 4.0 | 0.0416 | | 490.0 | 256.15 | 0.0208 |
| | | | | | | Total = | 7773.84 | |

### 4.3.1 Program input

Computations and considerations in the support of program input are as follows.

Buoy draft. The weight of the water displaced by the buoy equals the weight of the buoy.

Weight of water displaced by the stem

$$\frac{\pi}{4}(0.552)^2 \times 10 \times 64 = 153.16$$

Weight of water displaced by the ballast tank

$$\frac{\pi}{4} \times 3 \times 3 \times 8 \times 64 = 3619.11$$

Weight of water displaced by the buoyancy tank

$$\frac{\pi}{4} \times 3 \times 3 \times 8 \times 64 = 3619.11$$

Weight of water displaced by the immersed portion of the spar of length "$h$"

$$\frac{\pi}{4} \times (0.666)^2 h \times 64 = 22.295\,h$$

Solving for $h$

$$h = \frac{7773.84 - (153.16 + 2 \times 3{,}619.11)}{22.295} \cong 12.0'$$

The buoy draft is therefore 38'.

Number, depth and area of pressure surfaces. There are two pressure surfaces to consider, namely the top and the bottom of the foam filled buoyancy tank. The area of the first pressure surface $S_1$ is given by

$$S_1 = \pi\left(R_1^2 - R_2^2\right) \quad \text{where}$$

$R_1$ is the radius of the tank, $R_1 = 1.5'$

$R_2$ is the radius of the spar, $R_2 = 0.33'$

Thus $S_1 = \pi(1.5^2 - 0.33^2) = 6.71$ sq. ft.

$S_1$ is located 12' below the surface. The pressure force acting on $S_1$ being downwards, $S_1$ is negative. The area of the second pressure surface $S_2$ is in turn given by

$$S_2 = \pi R_1^2 = 7.06 \text{ sq. ft.}$$

It is located 20' below the surface. The pressure force is upwards and thus $S_2$ is positive.

<u>Number, depth, added mass coefficients, and volume of inertial components.</u> For computing the heave response, two distinct added mass effects must be accounted for: the added mass due to the water entrained by the top and bottom plates of the buoyancy and water ballast tanks, and the added mass due to the water entrained by the damping plate.

The first added mass effect will be assumed to be the same as the one produced by a sphere with a diameter equal to the diameter (4') of the plates located at the top of the buoyancy tank and the bottom of the water ballast tank. The equivalent depth will be selected midway between the two plates, i.e. 20' below the surface. The added mass coefficient for a sphere is $\frac{1}{2}$. The volume $VOL_1$ of this first inertial component is

$$VOL_1 = \frac{4}{3}\pi (2)^3 = 33.51 \text{ cu. ft.}$$

The second added mass effect will be considered to take place at the buoy keel, i.e. at 38' below the surface. The formula for the added mass of a circular plate of radius "$a$" being

$$m' = \frac{8}{3}\rho a^3 = C_m \rho \, VOL$$

An arbitrary added mass coefficient of 1 will yield a volume $VOL_2$

$$VOL_2 = \frac{8}{3} a^3 = \frac{8}{3}(2)^3 = 21.33 \text{ cu-ft}$$

Both effects in this case are happening all the time.

Number, depth, damping and wave drag coefficients of drag surfaces. Damping and wave drag will be assumed to be produced mainly by or on the upper and lower faces of the buoyancy tank and water ballast tank and by the damping plate at the buoy lower end. There will thus be two drag surfaces, one assumed to be located half way between the two ends of the tanks at an equivalent depth of 20', and the other at the buoy keel 38' below the surface. Assuming $\overline{X}_b = \frac{1}{2}\overline{X}_c$ and $\overline{X}_c$ = 3' the corresponding damping and wave drag coefficients given by expressions (2.2.10) and (2.2.13) are found to be

average heave amplitude.

$$b_1' = \frac{4}{3\pi} \times 2 \times 0.9 \times \pi (2)^2 \times 1.5 = 14.40$$

$$b_2' = \frac{4}{3\pi} \times 2 \times 1.2 \times \pi (2)^2 \times 1.5 = 19.20$$

$$c_1' = \frac{4}{3\pi} \times 2 \times 0.9 \times \pi (2)^2 \times 3 = 28.80$$

$$c_2' = \frac{4}{3\pi} \times 2 \times 1.2 \times \pi (2)^2 \times 3 = 38.40$$

Cross sectional area at surface.

$$\pi R^2 = \pi \times 0.33^2 = 0.342 \text{ sq-ft}$$

Buoy virtual mass.

Added mass of first inertial component

$$m_1' = \frac{1}{2}\times\frac{64}{32.2}\times\frac{4}{3}\times(2)^3\pi \qquad = \quad 33.3 \text{ slugs}$$

added mass of second inertial component

$$m_2' = \frac{8}{3}\times\frac{64}{32.2}\times(2)^3 \qquad = \quad 42.4 \text{ slugs}$$

Buoy mass "$m$"

$$m = \frac{7773.84}{32.2} \qquad = \quad \underline{237.85} \text{ slugs}$$

Virtual mass $m_v = m + m_1' + m_2'$ = 313.55 slugs

As in the preceding case study, the program user must also provide an arbitrary value of average buoy roll $\bar{\Theta}$ and wave amplitude $\bar{A}$. In this case $\bar{\Theta}$ and $\bar{A}$ are selected to be 5° and 2.5 feet respectively.

All pertinent data are listed in the data coding form shown in Fig. Nos. 20 and 21.

4.3.2 Program output

The heave and roll response amplitude operators are graphically represented in Fig. No. 22 and Fig. No. 23. The expected average and maximum values of wave amplitudes and of heave and roll motion are summarized in Table Nos. 8 and 9.

As a point of interest, Table No. 10 presents a succinct performance comparison of the three buoy types when submitted to the same random excitation.

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WOODS HOLE OCEANOGRAPHIC INSTITUTION
WOODS HOLE, MASSACHUSETTS

# DATA CODING FORM

DATA FOR THE HEAVE MOTION OF A WHOI BUOY (CASE #3)

| | 1–80 |
|---|---|
| 1 | 2 |
| 2 | 12.0, -6.71 |
| 3 | 20.0, 7.06 |
| 4 | 2 |
| 5 | 20.0, 0.5, 33.51 |
| 6 | 38.0, 1.0, 21.33 |
| 7 | 2 |
| 8 | 20.0, 14.4, 28.8 |
| 9 | 38.0, 19.2, 38.4 |
| 10 | 0.342 |
| 11 | 313.55 |
| 12 | 1.0, 50.0, 1.0 |
| 13 | 1, 20.0 |
| 14 | Y, Y |
| 15 | N |
| 16 | |
| 17 | |
| 18 | |
| 19 | |
| 20 | |

PROGRAM

PROGRAMMER | PROGRAM NUMBER | DATE | PAGE OF

7/64 IPC 301-1



Fig. No. 20

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WOODS HOLE, MASSACHUSETTS

# DATA CODING FORM

DATA FOR THE ROLL MOTION OF THE WHOI BUOY (CASE #3)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1.0, 50.0, 1.0 | | | | | | | |
| 1, 20.0 | | | | | | | |
| 0.333 | | | | | | | |
| 38.0 | | | | | | | |
| 2.5 | | | | | | | |
| 5.0 | | | | | | | |
| 15 | | | | | | | |
| 2 | 2.500 | 3.0 | -1.0 | 2.0 | 52.389 | 1.0 | RESERVE BUOYAN |
| 1 | 0.666 | 24.0 | 0.250 | 160.0 | 38.889 | 1.0 | SPAR |
| 2 | 4.000 | 0.0416 | -1.0 | 160.0 | 26.309 | 1.0 | PLATE AL. |
| 2 | 3.000 | 0.0156 | -1.0 | 490.0 | 26.889 | 1.0 | PLATE ST. |
| 1 | 3.000 | 8.0 | 0.187 | 490.0 | 22.889 | 1.0 | PLATE ST. |
| 2 | 3.000 | 0.0156 | -1.0 | 490.0 | 18.900 | 1.0 | PLATE ST. |
| 2 | 3.000 | 8.0 | -1.0 | 4.0 | 22.889 | 1.0 | FOAM |
| 2 | 3.000 | 0.0104 | -1.0 | 490.0 | 18.884 | 1.0 | PLATE ST. |
| 1 | 3.000 | 8.0 | 0.125 | 490.0 | 14.889 | 1.0 | PLATE ST. |
| 2 | 4.000 | 0.0104 | -1.0 | 490.0 | 10.895 | 1.0 | PLATE ST. |
| 2 | 3.000 | 8.0 | -1.0 | 64.0 | 14.889 | 1.0 | WATER |
| 1 | 0.552 | 10.0 | 0.280 | 490.0 | 5.889 | 1.0 | STEM |
| 2 | 0.552 | 10.0 | -1.0 | 64.0 | 5.889 | 1.0 | WATER |
| 2 | 2.500 | 0.848 | -1.0 | 450.0 | 0.466 | 1.0 | COUNTERWEIGHT |
| 2 | 4.000 | 0.0416 | -1.0 | 490.0 | 0.021 | 1.0 | DAMPING PLATE |
| -1 | | | | | | | |
| Y, Y | | | | | | | |
| N | | | | | | | |

PROGRAM

PROGRAMMER | PROGRAM NUMBER | DATE | PAGE OF

7/64 IPC 301-1

Fig. No. 21

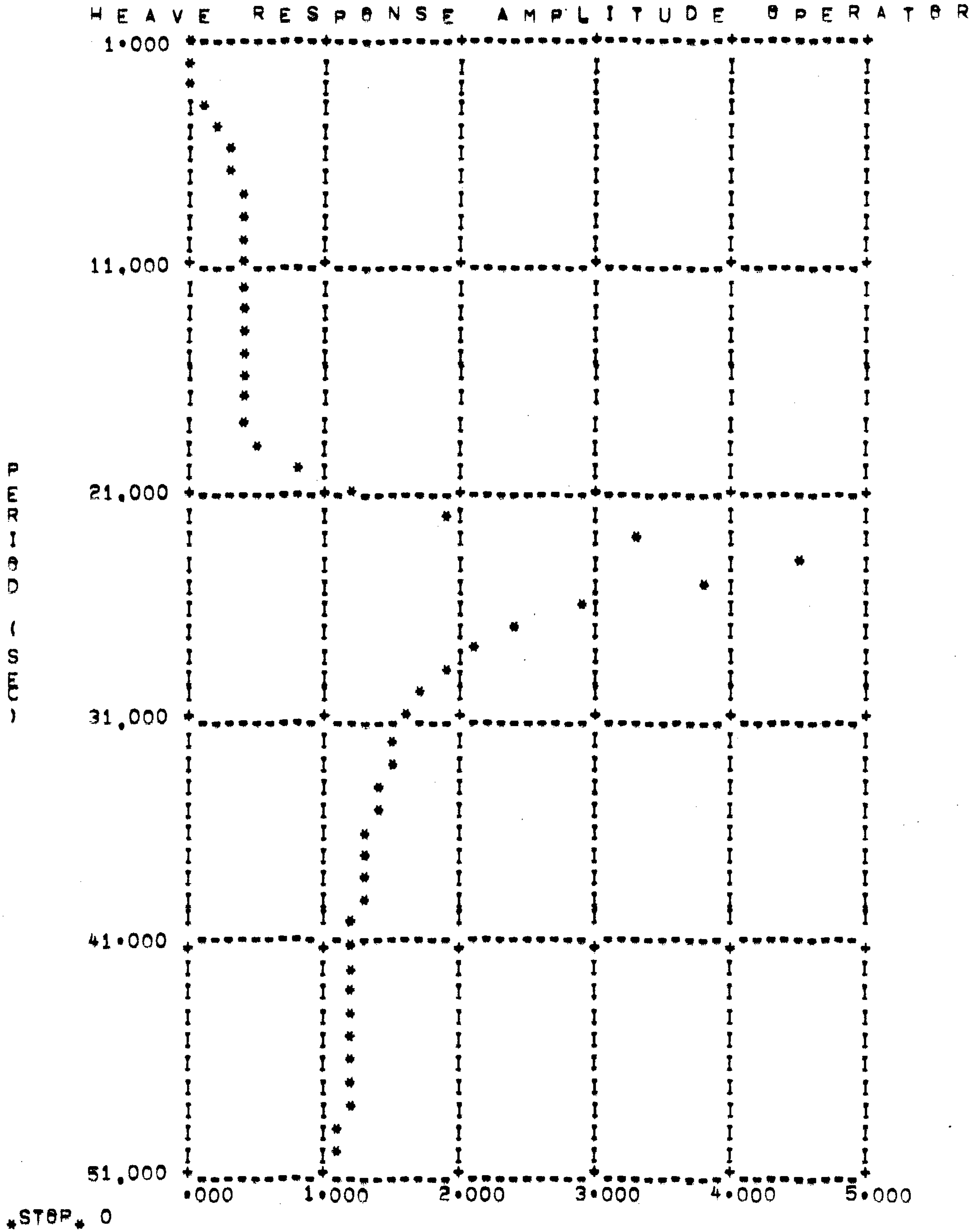
HEAVE RESPONSE AMPLITUDE OPERATOR
1.000
11,000
21,000
31,000
41.000
51,000
PERIOD (SEC)
.000
1.000
2.000
3.000
4.000
5.000
*STOP* 0

Fig. No. 22

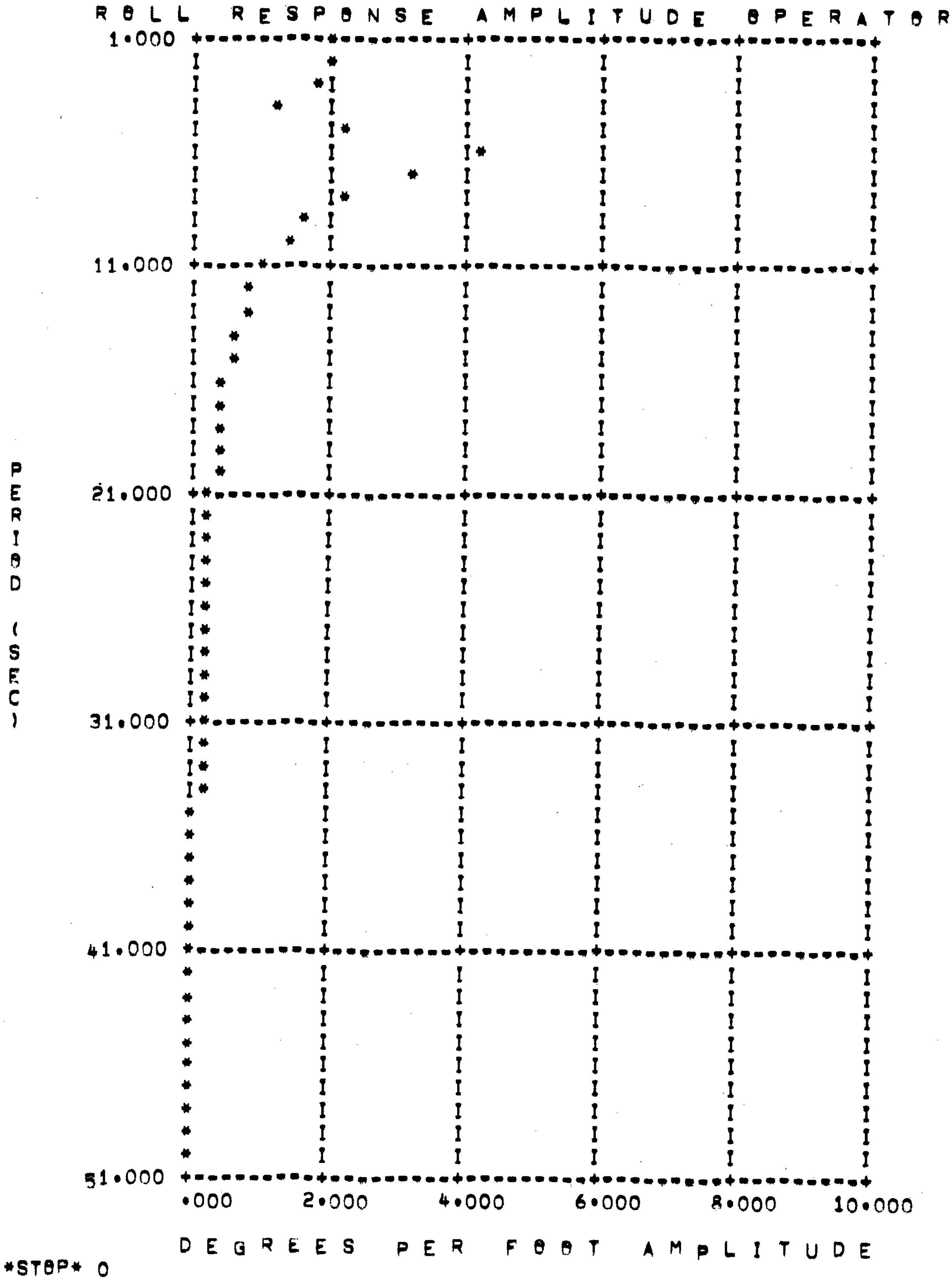
ROLL RESPONSE AMPLITUDE OPERATOR
1.000
11.000
21.000
31.000
41.000
51.000
PERIOD (SEC)
.000
2.000
4.000
6.000
8.000
10.000
DEGREES PER FOOT AMPLITUDE
*STOP* 0

Fig. No. 23

RMS OF WAVE SPECTRUM = 2.683 FEET

PROBABLE AMPLITUDE OF WAVE = 1.897 FEET

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE WAVE AMPLITUDE | NUMBER OF WAVES | EXPECTED WAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 6.330 | 50 | 5.689 |
| .100 | 4.830 | 100 | 6.118 |
| .333 | 3.799 | 500 | 7.003 |
| .500 | 3.370 | 1000 | 7.459 |
| 1.000 | 2.377 | 10000 | 8.399 |
| | | 100000 | 9.311 |

RMS OF RESPONSE SPECTRUM = .809 FEET

PROBABLE AMPLITUDE OF HEAVE RESPONSE = .572 FEET

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE HEAVE AMPLITUDE RESPONSE | NUMBER OF WAVES | EXPECTED HEAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 1.908 | 50 | 1.714 |
| .100 | 1.456 | 100 | 1.844 |
| .333 | 1.145 | 500 | 2.111 |
| .500 | 1.016 | 1000 | 2.248 |
| 1.000 | .716 | 10000 | 2.531 |
| | | 100000 | 2.806 |

Table No. 8 Heave Response of Complex Shape Buoy

RMS OF WAVE SPECTRUM = 2.683 FEET

PROBABLE AMPLITUDE
OF WAVE = 1.897 FEET

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE WAVE AMPLITUDE | NUMBER OF WAVES | EXPECTED WAVE MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 6.330 | 50 | 5.689 |
| .100 | 4.830 | 100 | 6.118 |
| .333 | 3.799 | 500 | 7.003 |
| .500 | 3.370 | 1000 | 7.459 |
| 1.000 | 2.377 | 10000 | 8.399 |
| | | 100000 | 9.311 |

RMS OF RESPONSE SPECTRUM = 7.489 DEG

PROBABLE AMPLITUDE
OF ROLL RESPONSE = 5.295 DEG

AMPLITUDES OF ROLL ARE IN DEGREES

| FRACTION OF LARGEST AMPLITUDES CONSIDERED | AVERAGE ROLL AMPLITUDE RESPONSE | NUMBER OF WAVES | EXPECTED ROLL MAXIMUM AMPLITUDE |
|---|---|---|---|
| .010 | 17.667 | 50 | 15.877 |
| .100 | 13.481 | 100 | 17.076 |
| .333 | 10.605 | 500 | 19.547 |
| .500 | 9.407 | 1000 | 20.820 |
| 1.000 | 6.636 | 10000 | 23.442 |
| | | 100000 | 25.988 |

Table No. 9 Roll Response of Complex Shape Buoy

Table No. 10

Performance Comparison

| Buoy Type | Average Heave / Average Wave Amplitude | Significant Heave / Average Wave Amplitude |
|---|---|---|
| Flat Cylinder | 1.000 | 3.961 |
| Telephone Pole | 1.625 | 6.591 |
| Spar Buoy | 0.301 | 2.791 |

## 5.0 CONCLUSIONS AND LIMITATIONS

The theoretical introduction and the case studies presented point out the positive aspects of the computer solution as well as some of its limitations. In the formulation of the equations of heave and roll motion, an attempt has been made to account for the effects of the water particle velocity and acceleration. The depth dependence of these effects has been included. The model thus obtained is more realistic than simpler models which consider only buoy displacement and wave slope as the predominant exciting forces.

As illustrated in case study number 3, the response of buoys of relatively complex shape can be easily studied. The heave and roll response amplitude operators can be used to compute the response of the buoys to waves of known or specified amplitude and frequencies. In addition to this time domain approach, specified spectral densities can be used to derive certain statistical expectancies of buoy heave and roll amplitudes. Parametric studies of buoy performance can thus readily be made.

On the other hand, to satisfy the condition of linearity, certain assumptions are made which introduce in the solution a degree of arbitrariness difficult to evaluate. Certainly the initial choice of the average values of wave amplitude, buoy heave, and buoy roll angle used to compute the linearized coefficients of drag and inertia will reflect on the accuracy of the solution. To improve this accuracy an iterative procedure can be followed which replaces the initial assumed values by computed ones until sufficient agreement is achieved.

Experimentally verified values of linear equivalents of inertia and viscous effects would greatly help validate or improve the computer solution described in this report. The assumption of small roll angles, also required by the condition of linearity, further limits the use of this program.

Energy dissipation by wave radiation is not considered. This factor could be important in large disk buoys. Finally the effects that mooring lines and tether lines would have on the buoy response have not been included, thus restricting this solution to free floating buoys.

Despite these limitations, the rationale used in the derivation of the solution and the program input flexibility make the computer solution useful as well as practical.

## 6.0 REFERENCES

1. Berteaux, H. O., Buoy Engineering, John Wiley & Sons, Inc., 1976.

2. Marks, W., "The Application of Spectral Analysis and Statistics to Seakeeping", The Society of Naval Architects and Marine Engineers , No. 1-24, September, 1963.

3. Price, W. G. and R.E.D. Bishop, Probabilistic Theory of Ship Dynamics, Chapman and Hall, 1974.

# 7.0 APPENDICES

## APPENDIX I

### Expression of Linearized Damping Coefficient

When the drag force on a body moving with a velocity $V$ is assumed to be linearly proportional to the velocity, the expression of the force is simply

$$D_1 = d V$$

In most cases, however, drag forces are expressed in terms of $V^2$, using the familiar formula

$$D_2 = \frac{1}{2} \rho C_D A V^2$$

where $\rho$ = water mass density

$C_D$ = drag coefficient

$A$ = body area across the flow (blunt bodies).

If the motion of the body is periodic, with amplitude $X_0$ and frequency $\omega = \frac{2\pi}{T}$, i.e. if for example

$$x = X_0 \sin \omega t$$

then the amount of energy dissipated per cycle by the damping force is given by

$$E = \int_0^T D_2\, dl = \int_0^T D_2 V\, dt = \frac{1}{2} \rho C_D A \int_0^T \omega^3 X_0^3 (\cos \omega t)^3\, dt$$

or

$$E = \frac{4}{3} \rho C_D A X_0^3 \omega^2$$

The amount of energy dissipated per cycle by the linear damping force is in turn given by

$$E = \int_0^T D_1\, dl = \int_0^T D_1 V\, dt = d \int_0^T \omega^2 X_0^2 (\cos \omega t)^2\, dt$$

or

$$E = \pi d X_0^2 \omega$$

Assuming both forces to dissipate the same amount of energy will yield the expression of the linearized damping coefficient $d$ , namely

$$d = \frac{4}{3\pi} \rho C_D A X_o \omega$$

## APPENDIX II

<u>Evaluation of the Coefficient "B" of Damping Moment</u>

"B" has been previously defined as:

$$B = \alpha\omega \left\{ \int_{r_1=0}^{r_1=S} d(r_1)\, r_1^3\, dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)\, r_2^3\, dr_2 \right\}$$

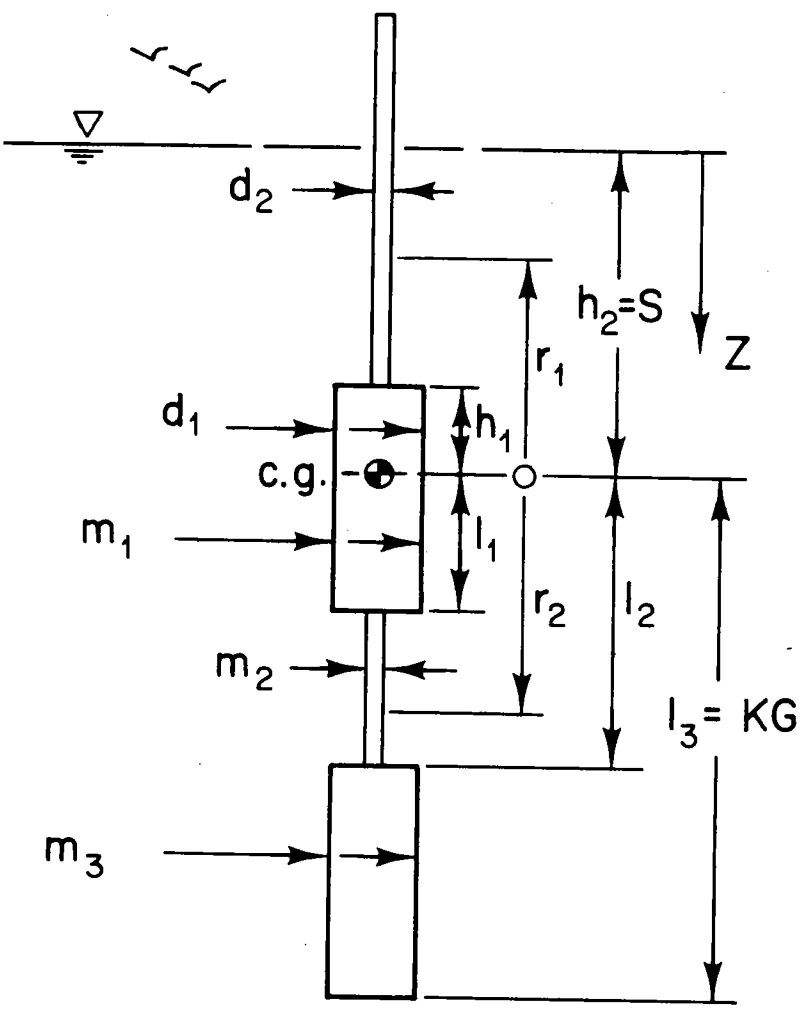


Fig. No. 24

Let us consider a buoy made of different cylinders as shown on the sketch. (Fig. No. 24) Let $d_1$, $d_2$, be the values of $d(r_1)$ and $h_1$ and $h_2$ be the corresponding limits of the variable $r_1$. The integration of the first integral

$$\int_{r_1=0}^{r_1=S} r_1^3\, d(r_1)\, dr_1$$

yields:

$$d_1 \int_{r_1=0}^{r_1=h_1} r_1^3 dr_1 + d_2 \int_{r_1=h_1}^{r_1=h_2} r_1^3 dr_1 = \frac{d_1 h_1^4}{4} + \frac{d_2 h_2^4}{4} - \frac{d_2 h_1^4}{4}$$

This result obviously leads to the recurrence formula

$$\int_{r_1=0}^{r_1=S} r_1^3 d(r_1)\, dr_1 = \frac{1}{4} \sum_i h_i^4 (d_i - d_{i+1}) \qquad i = 1,2$$

Let now $m_1$, $m_2$, $m_3$ be the values of $d(r_2)$ and $\ell_1$, $\ell_2$, $\ell_3$ the corresponding limits of the variable $r_2$. The second integral can then be evaluated as follows:

$$\int_{r_2=0}^{r_2=\overline{KG}} r_2^3 d(r_2) dr_2 = m_1 \int_{r_2=0}^{r_2=\ell_1} r_2^3 dr_2 + m_2 \int_{r_2=\ell_1}^{r_2=\ell_2} r_2^3 dr_2 + m_3 \int_{r_2=\ell_2}^{r_2=\ell_3} r_2^3 dr_2$$

$$= \frac{1}{4} \left\{ \ell_1^4 (m_1 - m_2) + \ell_2^4 (m_2 - m_3) + \ell_3^4 m_3 \right\}$$

This result in turn yields to the recurrence formula

$$\int_{r_2=0}^{r_2=\overline{KG}} d(r_2)\, r_2^3 dr_2 = \frac{1}{4} \sum_j{}' \ell_j^4 (m_j - m_{j+1}) \qquad j = 1,2,3$$

The expression of the coefficient "B" is therefore

$$B = \frac{\gamma \omega}{4} \left\{ \sum_i h_i^4 (s_i - s_{i+1}) + \sum_j \ell_j^4 (m_j - m_{j+1}) \right\}$$

where

$$\gamma = \frac{4}{3\pi} \rho C_D \bar{\Theta}$$

APPENDIX III

Evaluation of the Coefficient "D" of Wave Drag Moment

The coefficient "D" has been previously defined as

$$D = \beta \left\{ \int_{r_1=0}^{r_1=S} d(r_1)\, r_1\, e^{-2Kz}\, dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)\, r_2\, e^{-2Kz}\, dr_2 \right.$$

The integrals can be readily evaluated with the help of the following argument. For small angles $\Theta$, the projection of $r$ on the vertical is approximately equal to $r$.

Thus, from $r_1 = 0$ to $r_1 = S$, $z \cong S - r_1$

and from $r_2 = 0$ to $r_2 = \overline{KG}$, $z \cong S + r_2$

Introducing these values of $z$ in the integrals yields

$$D = \beta e^{-2KS} \left\{ \int_{r_1=0}^{r_1=S} d(r_1)\, e^{2Kr_1}\, r_1\, dr_1 - \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)\, r_2\, e^{-2Kr_2}\, dr_2 \right\}$$

Considering the same buoy geometry as in Appendix II, and evaluating the first integral over the domain of variation of $r_1$ yield:

$$\frac{1}{4K^2} \left\{ d_1 \left[ e^{2Kh_1}(2Kh_1 - 1) + 1 \right] + d_2 \left[ e^{2Kh_2}(2Kh_2 - 1) - e^{2Kh_1}(2Kh_1 - 1) \right] \right\}$$

This result leads to the recurrence formula:

$$\int_{r_1=0}^{r_1=S} d(r_1)\, r_1\, e^{2Kr_1}\, dr_1 = \frac{1}{4K^2} \sum d_i \left[ e^{2Kh_i}(2Kh_i - 1) - e^{2Kh_{i-1}}(2Kh_{i-1} - 1) \right]$$

$i = 1, 2$

Similarly, the evaluation of the second integral over the domain of variation of $r_2$ yields:

$$\frac{m_1}{4K^2}\left[e^{-2Kl_1}(2Kl_1+1)-1\right]$$
$$+\frac{m_2}{4K^2}\left[e^{-2Kl_2}(2Kl_2+1)-e^{-2Kl_1}(2Kl_1+1)\right]$$
$$+\frac{m_3}{4K^2}\left[e^{-2Kl_3}(2Kl_3+1)-e^{-2Kl_2}(2Kl_2+1)\right]$$

The recurrence formula thus is

$$-\int_{r_2=0}^{r_2=\overline{KG}} d(r_2)\, r_2\, e^{-2Kr_2}\, dr_2 = \frac{1}{4K^2}\sum_j m_j\left[e^{-2Kl_j}(2Kl_j+1)-e^{-2Kl_{j-1}}(2Kl_{j-1}+1)\right] \quad j=1,2,3$$

Thus, the expression of the coefficient "D" is finally

$$D=\frac{\beta e^{-2Ks}}{4K^2}\left\{\sum_i d_i\left[e^{2Kh_i}(2Kh_i-1)-e^{2Kh_{i-1}}(2Kh_{i-1}-1)\right]\right.$$
$$\left.+\sum_j m_j\left[e^{-2Kl_j}(2Kl_j+1)-e^{-2Kl_{j-1}}(2Kl_{j-1}+1)\right]\right\}$$

with $K=\frac{\omega^2}{g}$

and $\beta=\frac{4}{3\pi}\rho\, C_D\, \overline{A_F}\, \omega$

# APPENDIX IV

## Evaluation of the Coefficient "P" of Wave Inertia Moment

"P" has been previously defined as

$$P = \rho \left\{ -\int_{r_1=0}^{r_1=S} d(r_1)^2 r_1 e^{-Kz} dr_1 + \int_{r_2=0}^{r_2=\overline{KG}} d(r_2)^2 r_2 e^{-Kz} dr_2 \right\}$$

Using the change of variables previously discussed, the integrals in the

bracket can be written:

$$-\int_0^S d(r_1)^2 r_1 e^{-K(S-r_1)} dr_1 + \int_0^{\overline{KG}} d(r_2)^2 r_2 e^{-K(S+r_2)} dr_2$$

or

$$e^{-KS} \left\{ -\int_0^S d(r_1)^2 r_1 e^{Kr_1} dr_1 + \int_0^{\overline{KG}} d(r_2)^2 r_2 e^{-Kr_2} dr_2 \right\}$$

Noting that

$$-\int_a^b r_1 e^{Kr_1} dr_1 = \frac{e^{Kr_1}}{K^2} (Kr_1 - 1) \Big/_b^a$$

Then, over the intervals

$$0 \le r_1 \le h_1 \quad \text{with } d(r_1) = d_1$$
$$h_1 \le r_1 \le h_2 \qquad d(r_2) = d_2$$

the evaluation of the integrals yield:

1) over the first interval

$$-\frac{d_1^2}{K^2} \left[ 1 + e^{Kh_1} (Kh_1 - 1) \right]$$

2) over the second interval,

$$\frac{d_2^{\,2}}{K^2}\left[e^{-Kh_1}(Kh_1-1) - e^{Kh_2}(Kh_2-1)\right]$$

It thus appears that

$$-\int_{r_1=0}^{r_1=S} d(r_1)^2\, r_1 e^{-KZ}\, dr_1 = \frac{e^{-KS}}{K^2}\sum_i d_i^{\,2}\left[e^{-Kh_{i-1}}(Kh_{i-1}-1) - e^{Kh_i}(Kh_i-1)\right]$$

$$i = 1, 2$$

As far as the second integral is concerned, noting

$$\int r_2 e^{-Kr_2}\, dr_2 = \frac{e^{-Kr_2}}{K^2}(-Kr_2-1)\Big|_a^b$$

$$= \frac{e^{-Kr_2}}{K^2}(Kr_2+1)\Big|_b^a$$

and evaluating over the range $\ell_1 \leq r_2 \leq \ell_2$ , $(d(r_2) = m_2)$

yield:

$$\frac{m_2^{\,2}}{K^2}\left\{e^{-K\ell_1}(K\ell_1+1) - e^{-K\ell_2}(K\ell_2+1)\right\}$$

which shows that

$$\int_0^{\overline{KG}} d(r_2)^2\, r_2 e^{-KZ}\, dr_2 = \frac{e^{-KS}}{K^2}\sum_j m_j^{\,2}\left[e^{-K\ell_{j-1}}(K\ell_{j-1}+1) - e^{-K\ell_j}(K\ell_j+1)\right]$$

$$j = 1, 2, 3$$

The expression of the coefficient "P" is therefore

$$P = \frac{\gamma e^{-Ks}}{K^2} \left\{ \sum_i d_i^2 \left[ e^{-Kh_{i-1}} (Kh_{i-1} - 1) - e^{Kh_i} (Kh_i - 1) \right] + \sum_j m_j^2 \left[ e^{-Kg_{j-1}} (Kg_{j-1} + 1) - e^{-Kg_j} (Kg_j + 1) \right] \right\}$$

with

$$K = \omega^2 / g$$

$$\gamma = \frac{\pi C_m}{4} \rho$$

APPENDIX V

Evaluation of the Coefficient "$I_F$" of Added Moment of Inertia

The moment of inertia of the water displaced by the buoy with respect to the buoy c.g. is evaluated with the help of the parallel axis theorem, and is given by

$$I_F = \sum_i I(\bar{x})_i + \sum_i M_i \left(\bar{x}_i - \overline{KG}\right)^2$$

where $i = 1, 2, 3 \ldots n$

$I(\bar{x})_i$ = moment of inertia of cylinder "$i$" with respect to its own c.g.

$$I(\bar{x})_i = \frac{M_i}{4}\left(R_i^2 + \frac{H_i^2}{3}\right)$$

with $M_i$ = mass of water displaced by cylinder "$i$"

$R_i$ = radius of cylinder "$i$"

$H_i$ = height of cylinder "$i$"

$\bar{X}_i$ = distance of c.g. of cylinder "$i$" to keel

$\overline{KG}$ = distance of buoy c.g. to keel.

# APPENDIX VI

## Computation Method for Coefficients "B", "D" and "P"

The actual computation of the roll response amplitude operator in computer program ROLLRAO is performed using different forms of the expressions for some of the moments. This was done because the buoy configuration is input as geometrical "solids" rather than surfaces. It was, therefore, more straightforward to implement the computation of the damping forces using an iterative procedure on the components. The following expressions give the form of the equations used.

For the buoy damping moment, "B"

$$B = \frac{\alpha\omega}{4} \sum_{i=1}^{i=n} d_i \, sign(z - z_{cg}) * (z - z_{cg})^4 \Big|_{z=z_{T_i}}^{z=z_{B_i}}$$

where $\alpha = \dfrac{4\rho C_D \bar{\Theta}}{3\pi}$ as defined in Appendix II

$d_i$ = diameter of the ith buoy component

$z_{cg}$ = depth to the buoy center of gravity

$z_{B_i}$ = depth to the bottom surface of the ith buoy component

$z_{T_i}$ = depth to the top surface of the ith buoy component

$$sign(z - z_{cg}) = \frac{z - z_{cg}}{|z - z_{cg}|}$$

$n$ = number of buoy components

For the water damping moment, "D"

$$D = \frac{\beta e^{-2kz_{cg}}}{4k^2} \sum_{i=1}^{i=n} d_i \left[ \left( -2k(z - z_{cg}) - 1.0 \right) e^{-2k(z - z_{cg})} + 1.0 \right] \Big|_{z=z_{T_i}}^{z=z_{B_i}}$$

where $\beta = \dfrac{\omega 4 \rho \bar{A} C_D}{3\pi}$

and $K = \omega^2 / g$ as defined in Appendix III.

For the water particle acceleration inertia moment "P"

$$P = -\frac{\gamma e^{-KZ_{cg}}}{K^2} \sum_{i=1}^{i=m} d_i^2 \left[ \left( -K(Z - Z_{cg}) - 1.0 \right) e^{-K(Z - Z_{cg})} + 1.0 \right] \Bigg|_{Z=Z_{T_i}}^{Z=Z_{B_i}}$$

where $\gamma = \dfrac{\pi \rho C_m}{4}$ as defined in Appendix IV.

# APPENDIX VII

## Heave Program Listing

```
C                             PROGRAM HERA0
C
C                 VERSION 2.0    JAN, 1977             R. GOLDSMITH
C                 VERSION 1.1    JUNE, 1976            R. GOLDSMITH
C
C                 THIS PROGRAM IS USED TO COMPUTE THE HEAVE RESPONSE
C                   AMPLITUDE OPERATOR, AND ASSOCIATED PHASE ANGLES, FOR
C                   SPAR TYPE BUOY SYSTEMS
C
C                 VERSION 2.0 - MODIFIED TO INCORPORATE A WAVE DRAG COEFF
C
C
      LOGICAL IAMTERM
C
      DIMENSION DEPTHP(25),AREA(25)
      DIMENSION DEPTHI(25),ADDMSC(25),VOLUME(25)
      DIMENSION DEPTHD(25),DAMPC(25),WDRAGC(25)
      DIMENSION MAXWAVNO(6),WVMAXCOF(6),HEVMAXHT(6)
      DIMENSION FRACAMPS(5),AVRCOEFF(5),AVRESPNS(5)
C
      DATA NCR,NLP/105,108/
      DATA NMAX/25/
      DATA PI/3.141592/,RTOD/57.2958/
      DATA RHO,G/1.99035, 32.174/
C
      DATA FRACAMPS /0.01,0.10,0.333,0.50,1.0 /
      DATA AVRCOEFF /2.359,1.800,1.416,1.256,0.886 /
      DATA MAXWAVNO /50,100,500,1000,10000,100000 /
      DATA WVMAXCOF / 2.12,2.28,2.61,2.78,3.13,3.47 /
C                 ***********************************
C
C                 INITIALIZATION
C
      NP = 0
      NI = 0
      ND = 0
      CAREAWL = 0.0
      VIRTMASS = 0.0
      ISEASEL = 0
      TIME1 = 0.200
      TIME2 = 50.0
      TIMEDEL = 0.200
      WINDV = 0.0
      PI2 = PI*2.0
      RHOG = RHO*G
C
C                 CHECK FOR ON-LINE
      IONFLAG = 0
      IF (IAMTERM (IDUM) )  IONFLAG = 1
C
C                 INPUT DATA
C
  100 CONTINUE
      WRITE (NLP,9400)
C
C                 INPUT NUMBER OF PRESSURE SURFACES
      IF (IONFLAG .EQ. 1)  WRITE (NLP,9410)
      INPUT NTEST
      IF (NTEST .GT. NMAX)  WRITE (NLP,9700) NTEST,NMAX ;  STOP 100
```

```
      IF (NTEST .LT. 0)  GØ TØ 175
      IF (NTEST .GE. 0)  NP = NTEST
      IF (NP .EQ. 0)  GØ TØ 200
C                INPUT PRESSURE TERMS
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9420)
      DØ 150  I = 1,NP
        INPUT DEPTHP(I),AREA(I)
  150 CØNTINUE
C                ØUTPUT TERMS
  175 CØNTINUE
      IF (IØNFLAG .EQ. 1)  GØ TØ 200
      WRITE (NLP,9430) NP
      IF (NP .GT. 0)  WRITE (NLP,9440)  (DEPTHP(I),AREA(I), I=1,NP)
C
C                INPUT NUMBER ØF INERTIAL CØMPØNENTS
  200 CØNTINUE
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9450)
      INPUT NTEST
      IF (NTEST .GT. NMAX)  WRITE (NLP,9700) NTEST,NMAX ;  STØP 200
      IF (NTEST .LT. 0)  GØ TØ 275
      IF (NTEST .GE. 0)  NI = NTEST
      IF (NI .EQ. 0)  GØ TØ 300
C                INPUT INERTIAL TERMS
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9460)
      DØ 250  I = 1,NI
        INPUT DEPTHI(I),ADDMSC(I),VØLUME(I)
  250 CØNTINUE
C                     ØUTPUT TERMS
  275 CØNTINUE
      IF (IØNFLAG .EQ. 1)  GØ TØ 300
      WRITE (NLP,9470)  NI
      IF (NI .GT. 0)  WRITE (NLP,9480)  (DEPTHI(I),ADDMSC(I),VØLUME(I),
     +                                                   I=1,NI)
C
C                INPUT NUMBER ØF DRAG SURFACES
  300 CØNTINUE
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9490)
      INPUT NTEST
      IF (NTEST .GT. NMAX)  WRITE (NLP,9700)  NTEST,NMAX ;  STØP 300
      IF (NTEST .LT. 0)  GØ TØ 375
      IF (NTEST .GE. 0)  ND = NTEST
      IF (ND .EQ. 0)  GØ TØ 400
C                INPUT DRAG CØMPØNENTS
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9500)
C                     SUM DRAG SURFACE CØEFF F(DEPTH=0)
      SUMDC0 = 0.0
      DØ 350  I = 1,ND
        INPUT DEPTHD(I),DAMPC(I),WDRAGC(I)
        SUMDC0 = SUMDC0 + DAMPC(I)
  350 CØNTINUE
C                     ØUTPUT TERMS
  375 CØNTINUE
      IF (IØNFLAG .EQ. 1)  GØ TØ 400
      WRITE (NLP,9510)  ND
      IF (ND .GT. 0)  WRITE (NLP,9520)  (DEPTHD(I),DAMPC(I),WDRAGC(I),
     +                                   I = 1,ND)
C
C                INPUT WATER LEVEL CRØSS SECTIØN AREA
C                     (TØ SIMPLIFY THE CØMPUTATIØN THIS IS ASSUMED
C                      CØNSTANT ØVER THE RANGE ØF VERTICAL MØTIØN AT
```

```
C                        THE WATER LINE)
  400 CØNTINUE
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9530)
      INPUT CSATEST
      IF (CSATEST .LT. 0.0)  GØ TØ 450
      CAREAWL = CSATEST
  450 CØNTINUE
      IF (IØNFLAG .NE. 1)  WRITE (NLP,9540)  CAREAWL
      RF = CAREAWL*RHØG
C
C                  INPUT VIRTUAL MASS
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9550)
      INPUT VMTEST
      IF (VMTEST .LT. 0.0)  GØ TØ 550
      VIRTMASS = VMTEST
  550 CØNTINUE
      IF (IØNFLAG .NE. 1)  WRITE (NLP,9560)  VIRTMASS
C
C                  INPUT TIME RANGE
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9570)
      READ (NCR,9025)  T1,T2,T3
      IF (T1 .LT. 0.0)  GØ TØ 675
      TIME1 = T1
      TIME2 = T2
      IF (TIME2 .LT. TIME1)  TIME2 = TIME1
      IF (T3 .LE. 0.0)  T3 = TIMEDEL
      TIMEDEL = T3
  675 CØNTINUE
      IF (IØNFLAG .NE. 1)  WRITE (NLP,9580)  TIME1,TIME2,TIMEDEL
C
C                  INPUT WIND VELØCITY FØR SEA STATE
  700 CØNTINUE
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9590)
      READ (NCR,9020)  ISTEST,WAVEHT,WAVEPER
      IF (ISTEST .GT. 3)  ISTEST = -1
      IF (ISTEST .LT. 0)  GØ TØ 775
      ISEASEL = ISTEST
      IF (ISEASEL .EQ. 1)  WINDV = WAVEHT
      WINDVP4 = WINDV**4
      WAVEHTP2 = WAVEHT*WAVEHT
      WAVPERP4 = WAVEPER**4
  775 CØNTINUE
      IF (IØNFLAG .EQ. 1)  GØ TØ 800
      IF (ISEASEL .EQ. 0)  WRITE (NLP,9600)
      IF (ISEASEL .EQ. 1)  WRITE (NLP,9601)  WINDV
      IF (ISEASEL .EQ. 2)  WRITE (NLP,9602)  WAVEHT,WAVEPER
      IF (ISEASEL .EQ. 3)  WRITE (NLP,9603)  WAVEHT,WAVEPER
C
C                  CHECK ØUTPUT ØPTIØNS
  800 CØNTINUE
      IF (IØNFLAG .EQ. 1)  WRITE (NLP,9605)
      READ (NCR,9000)  IL,IP,RMIN,RMAX
      ILIST = 0
      IF (IL .EQ. 1HY)  ILIST = 1
      IPLØT = 0
      IF (IP .EQ. 1HY)  IPLØT =1
      IF (IPLØT .EQ. 1)  CALL PLØTINIT
C
C                  CØMPUTE RESPØNSE AND PHASE CØMPØNENT ØN TIME ITERATIØN
C
```

```
      900 CØNTINUE
          IF (ILIST .EQ. 1)  WRITE (NLP,9610)
          RRSINTG = 0.0
          SINTG = 0.
          DØ 2000  TIME = TIME1,TIME2,TIMEDEL
            FREQ = PI2*10000000000.0
            IF (TIME .NE. 0.0)  FREQ = PI2/TIME
            FREQP4 = FREQ**4
            FREQP5 = FREQ*FREQP4
            EXPTERM = -FREQ*FREQ/G
C
C                           SUM PRESSURE CØMPØNENTS
            SUMP = 0.0
            REPEAT 1100, FØR I= (1,NP)
              SUMP = SUMP + RHØG*AREA(I)*EXP(EXPTERM*DEPTHP(I))
     1100   CØNTINUE
C                           SUM INERTIAL CØMPØNENTS
            SUMI = 0.0
            REPEAT 1200, FØR I = (1,NI)
              SUMI = SUMI + ADDMSC(I)*VØLUME(I)*EXP(EXPTERM*DEPTHI(I))
     1200   CØNTINUE
            SUMI = RHØ*FREQ*FREQ*SUMI
C
C                           SUM DRAG CØMPØNENTS
            SUMD = 0.0
            REPEAT 1300, FØR I = (1,ND)
              SUMD = SUMD + WDRAGC(I)*EXP(EXPTERM*DEPTHD(I))
     1300   CØNTINUE
            SUMD = FREQ*FREQ*SUMD
            SUMDC = FREQ*FREQ*SUMDCO
C
C                           CØMPUTE RESPØNSE AMPLITUDE ØPERATØR
            RNUM = (SUMP - SUMI)**2 + SUMD**2
            RDENØM = (RF - VIRTMASS*FREQ*FREQ)**2 + SUMDC**2
            RAØ = SQRT(RNUM/RDENØM)
C
C                           CØMPUTE PHASE BETWEEN FØRCE AND HEAVE
            PHI = RTØD*ATAN2(-SUMDC,(RF - VIRTMASS*FREQ*FREQ) )
C
C                           CØMPUTE PHASE BETWEEN FØRCE AND WAVE
            SIGMA = RTØD*ATAN2(SUMD,(SUMP - SUMI) )
C
C                           CØMPUTE PHASE BETWEEN WAVE AND HEAVE
            THETA = SIGMA + PHI
C
C                           GET SEA SPECTRA
            CALL SEASPEC
C
C                           CØMPUTE RESPØNSE AND INTEGRATE
            RRS = RAØ*RAØ*S
            IF (TIME .LE. TIME1)  GØ TØ 1400
            DELF = FREQLAST - FREQ
            RRSINTG = (RRS + RRSLAST)*0.50*DELF + RRSINTG
            SINTG = (S + SLAST)*0.50*DELF + SINTG
     1400   CØNTINUE
            RRSLAST = RRS
            SLAST = S
            FREQLAST = FREQ
C
C                           ØUTPUT LIST IF IT WAS SELECTED
```

```
            IF (ILIST .LE. 0)  GO TO 1500
            WRITE (NLP,9615)  TIME,FREQ,RAO,SIGMA,PHI,THETA,S
C
C                          CHECK FOR PLOT
 1500       CONTINUE
            IF (IPLOT .LE. 0)  GO TO 2000
            CALL PLOT3 ('*',RAO,TIME,1)
 2000 CONTINUE
C
C                    GET STATISTICS
C
C                          COMPUTE ROOT MEAN SQUARE OF WAVE
      RMS = SQRT(SINTG)
      PROBAMP = 0.707*RMS
C                          COMPUTE AVERAGE WAVE HEIGHT
      DO 2300 I = 1,5
        AVRESPNS(I) = AVRCOEFF(I)*RMS
 2300 CONTINUE
C                          COMPUTE MAXIMUM WAVE AMPLITUDES
      DO 2400 I = 1,6
        HEVMAXHT(I) = WVMAXCOF(I)*RMS
 2400 CONTINUE
      WRITE (NLP,9635)  RMS
      WRITE (NLP,9640)  PROBAMP
      WRITE (NLP,9645)  (FRACAMPS(I),AVRESPNS(I),
     +                    MAXWAVNO(I),HEVMAXHT(I), I=1,5),
     +                    MAXWAVNO(6),HEVMAXHT(6)
C
C                          COMPUTE ROOT MEAN SQUARE OF RESPONSE
      RMS = SQRT(RRSINTG)
      PROBAMP = 0.707*RMS
C                          COMPUTE AVERAGE RESPONSE OF HEAVE
      DO 2500 I = 1,5
        AVRESPNS(I) = AVRCOEFF(I)*RMS
 2500 CONTINUE
C                          COMPUTE MAXIMUM AMPLITUDES OF HEAVE
      DO 2600 I = 1,6
        HEVMAXHT(I) = WVMAXCOF(I)*RMS
 2600 CONTINUE
      WRITE (NLP,9620)  RMS
      WRITE (NLP,9625)  PROBAMP
      WRITE (NLP,9630)  (FRACAMPS(I),AVRESPNS(I),
     +                    MAXWAVNO(I),HEVMAXHT(I), I=1,5),
     +                    MAXWAVNO(6),HEVMAXHT(6)
C
C                    CHECK FOR PLOT
C
      IF (IPLOT .LE. 0)  GO TO 3000
      CALL PLOTHEAV
 3000 CONTINUE
      IF (IONFLAG .EQ. 1)  WRITE (NLP,9655)
      READ (NCR,9015,END=8000)  IEND
      IF (IEND .EQ. 1HY)  GO TO 100
C
C
C
 8000 CONTINUE
      STOP
C
 9000 FORMAT (A1,1X,A1,1X,2E.0)
```

```
      9015 FORMAT (A1)
      9020 FORMAT (I,F.0,F.0)
      9025 FORMAT (3F.0)
C
      9400 FORMAT (1H1,'   HEAVE RESPONSE ANALYSIS PROGRAM'/
     +            '         ALL DEPTHS ARE POSITIVE'/)
      9410 FORMAT (/' INPUT THE NUMBER OF PRESSURE SURFACES 0')
      9420 FORMAT (/' ENTER FOR EACH SURFACE'/
     +            '  DEPTH (FT), AREA (SQ FT) (- AREA FOR DOWNWARD FORCE)')
      9430 FORMAT (/' NUMBER OF PRESSURE SURFACES =',I5)
      9440 FORMAT (/' DEPTH (FT), AREA (SQ FT) (- AREA FOR DOWNWARD FORCE)'/
     +            (F10.3,2X,F8.2) )
      9450 FORMAT (/' INPUT THE NUMBER OF INERTIAL COMPONENTS 0')
      9460 FORMAT (/' ENTER FOR EACH COMPONENT'/
     +            '  DEPTH (FT), ADDED MASS COEFF, VOLUME (CU FT)' )
      9470 FORMAT (/' NUMBER OF INERTIAL COMPONENTS =',I5)
      9480 FORMAT (/'  DEPTH (FT)  ADDED MASS COEFF   VOLUME (CU FT)'/
     +          (F10.3,6X,F10.3,7X,F12.3) )
      9490 FORMAT (/' INPUT THE NUMBER OF DRAG SURFACES 0')
      9500 FORMAT (/' ENTER FOR EACH SURFACE'/
     +            '  DEPTH (FT), DAMPING COEFF, WAVE DRAG COEFF'
     +            ' (LBF/(FT/SEC)/(RAD/SEC))' )
      9510 FORMAT (/' NUMBER OF DRAG SURFACES =',I5)
      9520 FORMAT (/'  DEPTH (FT)  DAMPING COEFF  WAVE DRAG COEFF'
     +            ' (LBF/(FT/SEC)/(RAD/SEC))' /
     +          (F10.3,3X,F11.3, 5X,F11.3) )
      9530 FORMAT (/'  ENTER CROSS SECTION AREA AT SURFACE (SQ FT) 0')
      9540 FORMAT (/'  CROSS SECTIONAL AREA AT SURFACE =',F10.4,' SQ FEET')
      9550 FORMAT (/'  ENTER VIRTUAL MASS (SLUGS) 0')
      9560 FORMAT (/'  VIRTUAL MASS =', F11.2, ' SLUGS')
      9570 FORMAT (/'  ENTER START, END, INCREMENT OF PERIOD RANGE (SEC) 0')
      9580 FORMAT (/'  PERIOD RANGE, IN SECONDS   START     END    DELTA'/
     +            26X,F8.3,F8.3,2X,F8.3)
      9590 FORMAT (/'  ENTER SEA SPECTRUM TYPE AND PARAMETERS'
     +           /'      = 1.0                  0 '
     +           /'      PIERSON-MOSKOWITZ 1, WIND SPEED (KNOTS)'
     +           /'      BRETSCHNEIDER      2, SIGNIF WAVE HT (FT), SIGNIF'
     +           ' WAVE PERIOD (SEC)'
     +           /'      I.S.S.C.            3, SIGNIF WAVE HT (FT), SIGNIF'
     +           ' WAVE PERIOD (SEC)'         )
      9600 FORMAT (/'  SEA SPECTRUM = 1.0')
      9601 FORMAT (/'  PIERSON-MOSKOWITZ SEA SPECTRUM '
     +           /'      WIND SPEED = ',F10.3,' KNOTS')
      9602 FORMAT (/'  BRETSCHNEIDER SEA SPECTRUM'
     +           /'      SIGNIFICANT WAVE HT = ',F10.3,' FEET'
     +           /'      SIGNIFICANT WAVE PERIOD = ',F10.3,' SEC')
      9603 FORMAT (/'  I.S.S.C. SEA SPECTRUM'
     +           /'      SIGNIFICANT WAVE HT = ',F10.3,' FEET'
     +           /'      SIGNIFICANT WAVE PERIOD = ',F10.3,' SEC')
      9605 FORMAT (/'  ENTER Y OR N FOR A LISTING, PLOT OF RESPONSE   '/
     +           '     FOR PLOT, YOU MAY ALSO ENTER RAO MIN,MAX 0')
      9610 FORMAT (1H1/'   PERIOD    ANG FREQ      RAO    W-F PHASE F-H PHASE '
     +           'W-H PHASE  AMP SPEC ')
      9615 FORMAT (F10.3,E10.3,F10.3,F10.3,F10.3,F10.3,F10.3)
      9620 FORMAT (///////'  RMS OF RESPONSE SPECTRUM =',F10.3,' FEET')
      9625 FORMAT (/'  PROBABLE AMPLITUDE '/
     +           '              OF HEAVE RESPONSE =',F10.3,' FEET')
      9630 FORMAT (//
     +        /' FRACTION OF        AVERAGE  I                    EXPECTED  '
     +         /'   LARGEST          HEAVE   I                     HEAVE    '
```

```
     +        /' AMPLITUDES        AMPLITUDE I    NUMBER     MAXIMUM    '
     +        /' CONSIDERED        RESPONSE  I   OF WAVES   AMPLITUDE   '
     +        /' ----------        --------- I   --------   ---------   '
     +       5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
     +        /                    T28,'I', T33,I6, T43,F9.3 //)
 9635 FORMAT (1H1////'  RMS OF WAVE SPECTRUM =',F10.3,' FEET')
 9640 FORMAT (/'  PROBABLE AMPLITUDE '/
     +         '       OF WAVE             =',F10.3,' FEET')
 9645 FORMAT (//
     +        /' FRACTION OF                       I                 EXPECTED   '
     +        /'    LARGEST        AVERAGE   I                  WAVE     '
     +        /' AMPLITUDES          WAVE    I    NUMBER     MAXIMUM    '
     +        /' CONSIDERED        AMPLITUDE I   OF WAVES   AMPLITUDE   '
     +        /' ----------        --------- I   --------   ---------   '
     +       5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
     +        /                    T28,'I', T33,I6, T43,F9.3    )
 9655 FORMAT (/' DO YOU WANT ANOTHER CASE ?')
C
 9700 FORMAT (' *** NUMBER OF ENTRIES IS GREATER THAN ARRAY SIZE ALLOWS'
     +        /14X,I8,20X,I7,
     +        /'       THE PROGRAM TERMINATES' )
C
C              ********************************************************
C
      SUBROUTINE SEASPEC
      IF (ISEASEL .EQ. 0)  S = 1.0 ;  RETURN
      GO TO  (4100,4200,4300), ISEASEL
C              PIERSON - MOSKOWITZ
 4100 CONTINUE
      S = 135.0/FREQP5*EXP(-97000.0/(FREQP4*WINDVP4))
C                   CORRECT FOR DOUBLE HEIGHT SPECTRUM
      S = S/8.0
      RETURN
C              BRETSCHNEIDER
 4200 CONTINUE
      S = 4200.0*WAVEHTP2/(WAVPERP4*FREQP5)*
     +     EXP(-1050.0/(WAVPERP4*FREQP4))
C                   CORRECT FOR DOUBLE HEIGHT SPECTRUM
      S = S/8.0
      RETURN
C              I.S.S.C.
 4300 CONTINUE
      S = 2760.0*WAVEHTP2/(WAVPERP4*FREQP5)*
     +     EXP(-630.0/(WAVPERP4*FREQP4))
C                   CORRECT FOR DOUBLE HEIGHT SPECTRUM
      S = S/8.0
      RETURN
C
C
C              ********************************************************
C
      SUBROUTINE PLOTINIT
C              THIS SUBROUTINE IS USED TO INITIALIZE A LINE PRINTER
C                PLOT OF THE HEAVE RESPONSE.
C
      DIMENSION IPLOTBUF(3300)
C
      DATA IBUFSIZE /3300/
C
C
```

```
          IF (RMAX .GT. RMIN)  GO TO 1490
          RMAX = 100*RMIN
          IF (RMAX .EQ. 0.0)  RMAX = 5.0
 1490     CONTINUE
          RDEL = RMAX - RMIN
          RMAX = IFIX(RDEL/5.0 + 0.999)*5.0 + RMIN
C
          NLINES = (TIME2 - TIME1)/TIMEDEL
          NBARS = (NLINES + 9.1)/10.0
          TMAX = TIME1 + NBARS*10.0*TIMEDEL
          NLINES = (TMAX - TIME1)/TIMEDEL + 1.0
          IF (NLINES .LT. IBUFSIZE/13)  GO TO 5000
          IPLOT = 0
          WRITE (NLP,9710)
          RETURN
C
 5000     CONTINUE
          CALL PLOT1 (NBARS,10,5,10)
          CALL PLOT2 (IPLOTBUF,RMIN,RMAX,TMAX,TIME1)
          RETURN
C
 9710     FORMAT (/' ***** THE PLOT BUFFER IS NOT LARGE ENOUGH '
     +            /'           FOR THE PERIOD RANGE SPECIFIED '
     +            /'        THE PLOT IS SUPPRESSED')
C
C
C                   **************************************************************
C
          SUBROUTINE PLOTHEAV
C
C                   THIS SUBROUTINE IS USED TO OUTPUT THE LINE PRINTER
C                     PLOT OF THE HEAVE RESPONSE.
C
          CALL PLOT5 (3,33,'HEAVE RESPONSE AMPLITUDE OPERATOR')
          CALL PLOT4 (14,' PERIOD (SEC) ')
          CALL PLOT7 (10)
          RETURN
          END
```

# APPENDIX VIII

## Roll Program Listing

```
C                                   PROGRAM ROLLRAO
C
C               VERSION 1.0         SEP, 1976           R. GOLDSMITH
C
C               THIS PROGRAM IS USED TO COMPUTE THE ROLL RESPONSE
C                 AMPLITUDE OPERATOR, AND ASSOCIATED PHASE ANGLES, FOR
C                 SPAR TYPE BUOY SYSTEMS.
C
C                 CURRENT VERSION RESTRICTS DESIGN TO CYLINDRICAL AND
C                 TRIANGULAR BODIES ON END, AND RECTANGULAR PLATES.
C
C
      LOGICAL IAMTERM
C
      DIMENSION FRACAMPS (5), AVRCOEFF(5), AVRESPNS(5)
      DIMENSION MAXWAVNO(6), WVMAXCOF(6), ROLLMAX(6)
C
      COMMON / IODEV / NCR,NLP
      COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
      COMMON / SEASTATE / ISEASEL,WINDV,WAVEHT,WAVEPER,
     +                    WINDVP4,WAVEHTP2,WAVPERP4
      COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
     +                DENSITY(50),DISTCGK(50),FRACNORM(50)
      COMMON / BOUTS / VOLUME(50),WEIGHT(50)
      COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
     +                    DEPTHB(50),DEPTHT(50)
      COMMON / CONSTANT / PI,RTOD,RHO,G
      COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
     +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
      COMMON / COEFS / DRAG(5),COEFM(5)
      COMMON / MOMENTS / BUOYMI,ADDMI,VIRTINRT,WATERIM,BUOYMR,
     +                   BUOYMDT,BUOYMD,WATERMD,DAMPM
      COMMON / OUTPUTS / ILIST,IPLOT,RMIN,RMAX
C
C
      DATA NCR,NLP /105,108/
      DATA PI,RTOD /3.141592,57.2958/
      DATA RHO,G / 1.99035, 32.174/
C             MAXIMUM ARRAY SIZES
      DATA NPMAX /50/
C             DRAG COEFFICIENTS FOR CYLINDER AND PLATE
      DATA DRAG / 1.2, 1.2, 1.5, 0.0, 0.0 /
C             ADDED MASS COEFFICIENTS FOR CYLINDER AND PLATE
      DATA COEFM / 1.0, 1.0, 1.0, 0.0, 0.0 /
C             STATISTICAL COEFFICIENTS
      DATA FRACAMPS /0.01,0.10,0.3333,0.50,1.0/
      DATA AVRCOEFF /2.359,1.800,1.416,1.256,0.886/
      DATA MAXWAVNO /50,100,500,1000,10000,100000/
      DATA WVMAXCOF /2.12,2.28,2.61,2.78,3.13,3.47/
C
C             ***********************************
C
C             INITIALIZATION
C
      PI2 = PI*2.0
      RHOG = RHO*G
      TIME1 = 0.2
      TIME2 = 50.0
      TIMEDEL = 0.200
```

```
           ISEASEL = 0
           WINDV = 0.0
           WAVEHT = 0.0
           WAVEPER = 0.0
           RWL = 0.0
           AVERGAMP = 3.0
           THETABAR = 11.5/RTØD
           NPARTS = 0
      C
       100 CØNTINUE
      C
      C                    CHECK FØR ØN LINE AND INPUT MØDE
           IØNFLAG = 0
       150 CØNTINUE
           WRITE (NLP,9400)
           IF (IAMTERM (IDUM) )  IØNFLAG = 1 ;  CALL TINPUT
           IF (IØNFLAG .EQ. 0)  CALL BINPUT
           IF (IPLØT .EQ. 1)  CALL PLØTINIT
      C
      C                    CØMPUTE TØTAL BUØY WEIGHT,     DISTANCE FRØM KEEL TØ
      C                      CENTER ØF GRAVITY,     DEPTH ØF CG
      C
           BUØYWGT = 0.0
           SUMT = 0.0
           DØ 400  I = 1,NPARTS
             BUØYWGT = BUØYWGT + WEIGHT(I)
             SUMT = SUMT + WEIGHT(I)*DISTCGK(I)
       400 CØNTINUE
           BUØYCGK = SUMT/BUØYWGT
           DEPTHCG = DEPTHK - BUØYCGK
      C
      C                    CØMPUTE THE PART BASIC MØMENT ØF INERTIA CØNTRIBUTIØN
      C
           SUMT = 0.0
           DØ 500  I = 1,NPARTS
             CALL BØDYMI (ISHAPE(I),HEIGHT(I),WIDTH(I),THICK(I),PINERT)
      C                    FØR THE BØDY ABØUT ITS ØWN AXIS
             PMI = WEIGHT(I)*PINERT/G
      C                    ABØUT THE CG
             BMICØMP = (WEIGHT(I)/G)*(DISTCGK(I) - BUØYCGK)**2
             SUMT = SUMT + PMI + BMICØMP
       500 CØNTINUE
           BUØYMI = SUMT
      C
      C                    GET DISPLACEMENT CØNTRIBUTIØNS
      C
           CALL DISPLACE
      C
           BUØYCGCB = BUØYCBK - BUØYCGK
      C
           WRITE (NLP,9405)  BUØYWGT,WDISPLAC
      C                    CØMPUTE DISTANCE TØ METACENTER FRØM CB
      C
           SURFINRT = PI*RWL**4/4.0
           BUØYCBM = SURFINRT*RHØG/WDISPLAC
      C
      C                    CØMPUTE RIGHTING ARM, GM
      C
           BUØYCGM = BUØYCGCB + BUØYCBM
      C
```

```
      C              CHECK FOR STABILITY
            IF (BUOYCGM .LT. 0.0)  WRITE (NLP,9700)  BUOYCBK+BUOYCBM,
           +                                         BUOYCGK ;
           +                         STOP 550
      C
      C          ************************************
      C
      C          COMPUTE RIGHTING MOMENT TERM
      C
            BUOYMR = BUOYWGT*BUOYCGM
            BUOYMRG = -BUOYMR/G
      C
      C          COMPUTE NATURAL PERIOD OF ROLL
      C              FIRST GET VIRTUAL MOMENT
            VIRTINRT = BUOYMI + ADDMI
            PERIODO = 2.0*PI*SQRT (VIRTINRT/BUOYMR)
            WRITE (NLP,9410)  PERIODO
      C
      C          MAKE ASSUMPTION OF UNIT AMPLITUDE
      C          AND START FREQUENCY ANALYSIS
      C
            IF (ILIST .EQ. 1)  WRITE (NLP,9450)
            RRSINTG = 0.0
            SINTG = 0.0
      X     OUTPUT BUOYMI,BUOYMR,BUOYMRG
      C            ***********************************************
      C          NOTE: THE CALL TO BUOYDAMP IS PLACED HERE TO
      C                SIMPLIFY THE COMPUTATION. THE CONSTANT TERM
      C                IS COMPUTED HERE AND THE FREQ IS MULTIPLIED
      C                IN AT THE BEGINNING OF THE FREQUENCY ITERATION.
      C            ***********************************************
            CALL BUOYDAMP
            DO 2000  TIME = TIME1,TIME2,TIMEDEL
              FREQ = PI2*1.0E+10
              IF (TIME .NE. 0.0)  FREQ = PI2/TIME
              WAVEN = FREQ*FREQ/G
      C
      C         *** COMPUTE DAMPING MOMENTS
      C
      C                  SEE NOTE ABOVE
              BUOYMD = BUOYMDT*FREQ
              CALL WATERDAMP
      C
      C         *** COMPUTE WAVE INERTIA MOMENTS
      C
              CALL WATRINRT
      X       OUTPUT BUOYMD,WATERMD,WATERIM
      C
      C
              ETEM = BUOYMRG + WATERIM
              FTEM = WATERMD
      C
      C            SET MAIN COMPONENT
              E = ETEM*FREQ*FREQ
              F = FTEM*FREQ
      C                  PHASE BETWEEN WAVE AND TORQUE
              SIGMA = RTOD*ATAN2 (-E,F)
      C
      C                  EXCITING TORQUE
              T = SQRT (E*E + F*F)
```

```
C
C                    PHASE ANGLE BETWEEN TORQUE AND ROLL
           ATEM = BUOYMD*FREQ
           BTEM = BUOYMR - VIRTINRT*FREQ*FREQ
           PHI = RTOD*ATAN2 (-ATEM,BTEM)
C
C                    ROLL RAO
C
           ROLLRAO = T/SQRT (ATEM*ATEM + BTEM*BTEM)
C
C                    PHASE ANGLE BETWEEN WAVE AND ROLL
C
           THETA = SIGMA + PHI
C
C                    GET SEA SPECTRA
           CALL SEASPEC
C
C                    COMPUTE RESPONSE AND INTEGRATE
           RRS = ROLLRAO*ROLLRAO*S
           IF (TIME .LE. TIME1)  GO TO 1400
           DELF = FREQLAST - FREQ
           RRSINTG = (RRS + RRSLAST)*0.50*DELF + RRSINTG
           SINTG = (S + SLAST)*0.50*DELF + SINTG
  1400     CONTINUE
           RRSLAST = RRS
           SLAST = S
           FREQLAST = FREQ
C
C                    OUTPUT LIST IF IT WAS SELECTED
           IF (ILIST .LE. 0)  GO TO 1500
           WRITE (NLP,9455)  TIME,FREQ,ROLLRAO*RTOD,SIGMA,PHI,THETA,S
C
C                    CHECK FOR PLOT
  1500     CONTINUE
           IF (IPLOT .LE. 0)  GO TO 2000
           CALL PLOT3 ('*',ROLLRAO*RTOD,TIME,1)
C
 2000 CONTINUE
C
C              GET STATISTICS
C
C                    COMPUTE ROOT MEAN SQUARE OF WAVE
      RMS = SQRT(SINTG)
      PROBAMP = 0.707*RMS
C                    COMPUTE AVERAGE WAVE HEIGHT
      DO 2300 I = 1,5
        AVRESPNS(I) = AVRCOEFF(I)*RMS
 2300 CONTINUE
C                    COMPUTE MAXIMUM WAVE AMPLITUDES
      DO 2400 I = 1,6
        ROLLMAX(I) = WVMAXCOF(I)*RMS
 2400 CONTINUE
      WRITE (NLP,9500)  RMS
      WRITE (NLP,9505)  PROBAMP
      WRITE (NLP,9510)  (FRACAMPS(I),AVRESPNS(I),
     +                    MAXWAVNO(I),ROLLMAX(I), I=1,5),
     +                    MAXWAVNO(6),ROLLMAX(6)
C
C                    COMPUTE ROOT MEAN SQUARE OF RESPONSE
      RMS = SQRT(RRSINTG)
```

```
            PRØBAMP = 0.707*RMS
      C                   CØMPUTE AVERAGE RESPØNSE ØF RØLL
            DØ 2500 I = 1,5
              AVRESPNS(I) = AVRCØEFF(I)*RMS*RTØD
       2500 CØNTINUE
      C                   CØMPUTE MAXIMUM AMPLITUDES ØF RØLL
            DØ 2600 I = 1,6
              RØLLMAX(I) = WVMAXCØF(I)*RMS*RTØD
       2600 CØNTINUE
            WRITE (NLP,9515)  RMS*RTØD
            WRITE (NLP,9520)  PRØBAMP*RTØD
            WRITE (NLP,9525)  (FRACAMPS(I),AVRESPNS(I),
           +                   MAXWAVNØ(I),RØLLMAX(I), I=1,5),
           +                   MAXWAVNØ(6),RØLLMAX(6)
      C
      C               CHECK FØR PLØT
      C
            IF (IPLØT .LE. 0)  GØ TØ 3000
            CALL PLØTRØLL
       3000 CØNTINUE
      C
            IF (IØNFLAG .EQ. 1)  WRITE (NLP,9485)
            READ (NCR,9085,END=8000)  IEND
            IF (IEND .EQ. 1HY)  GØ TØ 150
      C
       8000 CØNTINUE
            STØP
      C
       9085 FØRMAT (A1)
      C
       9400 FØRMAT (1H1,'        RØLL RESPØNSE ANNALYSIS PRØGRAM'/
           +        '                ALL DEPTHS ARE PØSITIVE '/)
       9405 FØRMAT (/'  CHECK FØR BUØYANCY      BUØY WEIGHT = ',F10.1,' LBSM'/
           +        '                         WATER DISPLACED = ',F10.1,' LBSM')
       9410 FØRMAT (/'  NATURAL PERIØD =',F8.3,' SECØNDS')
       9450 FØRMAT (1H1/'       RAØ IS IN DEGREES/FØØT ØF WAVE AMPLITUDE'/
           +        '    PERIØD   ANG FREQ      RAØ    W-T PHASE T-R PHASE '
           +        'W-R PHASE  AMP SPEC ')
       9455 FØRMAT (F10.3,E10.3,F10.3,F10.3,F10.3,F10.3,F10.3)
       9485 FØRMAT (/' DØ YØU WANT ANØTHER CASE Ø')
       9500 FØRMAT (1H1////'  RMS ØF WAVE SPECTRUM =',F10.3,' FEET')
       9505 FØRMAT (/'  PRØBABLE AMPLITUDE '/
           +        '         ØF WAVE            =',F10.3,' FEET')
       9510 FØRMAT (//
           +      /' FRACTIØN ØF                   I                    EXPECTED  '
           +      /'    LARGEST         AVERAGE    I                      WAVE    '
           +      /' AMPLITUDES          WAVE      I    NUMBER          MAXIMUM   '
           +      /' CØNSIDERED        AMPLITUDE   I   ØF WAVES        AMPLITUDE  '
           +      /' -----------       ---------   I   --------        ---------  '
           +     5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
           +      /                    T28,'I', T33,I6, T43,F9.3   )
       9515 FØRMAT (///////'  RMS ØF RESPØNSE SPECTRUM =',F10.3,' DEG    ')
       9520 FØRMAT (/'  PRØBABLE AMPLITUDE '/
           +        '           ØF RØLL RESPØNSE  =',F10.3,' DEG    ')
       9525 FØRMAT (//
           +      /' AMPLITUDES ØF RØLL ARE IN DEGREES'/
           +      /' FRACTIØN ØF       AVERAGE    I                    EXPECTED  '
           +      /'    LARGEST          RØLL     I                      RØLL    '
           +      /' AMPLITUDES        AMPLITUDE  I    NUMBER          MAXIMUM   '
           +      /' CØNSIDERED        RESPØNSE   I   ØF WAVES        AMPLITUDE  '
```

```
     +        /' -----------      --------- I   --------   ---------  '
     +      5(/T4,F5.3, T16,F9.3, T28,'I', T33,I6, T43,F9.3)
     +       /                    T28,'I', T33,I6, T43,F9.3 //)
C
 9700 FORMAT (//'    *** STOP EVERYTHING - THIS BUOY WILL ROLL OVER'/
     +          '         THE CENTER OF GRAVITY IS ABOVE THE META'/
     +          '         - CENTER.     MC ABOVE KEEL = ',F6.2,' FEET'/
     +          '                       CG ABOVE KEEL = ',F6.2,' FEET')
C
C
C                 *********************************************************
C
      SUBROUTINE SEASPEC
      IF (ISEASEL .EQ. 0)  S = 1.0 ;  RETURN
      FREQP4 = FREQ**4
      FREQP5 = FREQP4*FREQ
      GO TO  (4100,4200,4300), ISEASEL
C                PIERSON - MOSKOWITZ
 4100 CONTINUE
      S = 135.0/FREQP5*EXP(-97000.0/(FREQP4*WINDVP4))
C                     CORRECT FOR DOUBLE HEIGHT SPECTRUM
      S = S/8.0
      RETURN
C                BRETSCHNEIDER
 4200 CONTINUE
      S = 4200.0*WAVEHTP2/(WAVPERP4*FREQP5)*
     +        EXP(-1050.0/(WAVPERP4*FREQP4))
C                     CORRECT FOR DOUBLE HEIGHT SPECTRUM
      S = S/8.0
      RETURN
C                I.S.S.C.
 4300 CONTINUE
      S = 2760.0*WAVEHTP2/(WAVPERP4*FREQP5)*
     +        EXP(-630.0/(WAVPERP4*FREQP4))
C                     CORRECT FOR DOUBLE HEIGHT SPECTRUM
      S = S/8.0
      RETURN
C
C
C                 *********************************************************
C
      SUBROUTINE PLOTINIT
C              THIS SUBROUTINE IS USED TO INITIALIZE A LINE PRINTER
C                PLOT OF THE ROLL RESPONSE.
C                        SIZE LIMITED FOR ON-LINE USE ONLY
C
      DIMENSION IPLOTBUF(1300)
C
      DATA IBUFSIZE / 1300/
C
C
      IF (RMAX .GT. RMIN)  GO TO 1490
      RMAX = 100*RMIN
      IF (RMAX .EQ. 0.0)  RMAX = 10.0
 1490 CONTINUE
      RDEL = RMAX - RMIN
      RMAX = IFIX(RDEL/5.0 + 0.999)*5.0 + RMIN
C
      NLINES = (TIME2 - TIME1)/TIMEDEL
      NBARS = (NLINES + 9.1)/10.0
```

```
      TMAX = TIME1 + NBARS*10.0*TIMEDEL
      NLINES = (TMAX - TIME1)/TIMEDEL + 1.0
      IF (NLINES .LT. IBUFSIZE/13)  GO TO 5000
      IPLOT = 0
      WRITE (NLP,9710)
      RETURN
C
 5000 CONTINUE
      CALL PLOT1 (NBARS,10,5,10)
      CALL PLOT2 (IPLOTBUF,RMIN,RMAX,TMAX,TIME1)
      RETURN
C
 9710 FORMAT (/' ***** THE PLOT BUFFER IS NOT LARGE ENOUGH '
     +        /'           FOR THE PERIOD RANGE SPECIFIED '
     +        /'        THE PLOT IS SUPPRESSED')
C
C
C                 ***********************************************************
C
      SUBROUTINE PLOTROLL
C
C                 THIS SUBROUTINE IS USED TO OUTPUT THE LINE PRINTER
C                   PLOT OF THE ROLL RESPONSE.
C
      CALL PLOT5 (3,33,'ROLL RESPONSE AMPLITUDE OPERATOR')
      CALL PLOT4 (14,' PERIOD (SEC) ')
      CALL PLOT5 (2,28,' DEGREES PER FOOT AMPLITUDE ')
      CALL PLOT7 (10)
      RETURN
C
      END
```

```
      SUBROUTINE TINPUT
C
C              VERSION 1.0         SEP, 1976            R. GOLDSMITH
C
C              THIS ROUTINE INPUTS DATA FOR THE ROLL RAO IN ON-LINE MODE
C
C
      COMMON / IODEV / NCR,NLP
      COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
      COMMON / SEASTATE / ISEASEL,WINDV,WAVEHT,WAVEPER,
     +                   WINDVP4,WAVEHTP2,WAVPERP4
      COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
     +                DENSITY(50),DISTCGK(50),FRACNORM(50)
      COMMON / BOUTS / VOLUME(50),WEIGHT(50)
      COMMON / CONSTANT / PI,RTOD,RHO,G
      COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
     +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
      COMMON / OUTPUTS / ILIST,IPLOT,RMIN,RMAX
C
C
C              INPUT TIME AND RANGE
      WRITE (NLP,9400)
      READ (NCR,9000)  T1,T2,T3
      IF (T1 .LT. 0.0) GO TO 200
      TIME1 = T1
      TIME2 = T2
      IF (TIME2 .LT. TIME1)  TIME2 = TIME1
      IF (T3 .LE. 0.0)  T3 = TIMEDEL
      TIMEDEL = T3
C
C              SELECT SEA STATE PARAMETERS
  200 CONTINUE
      WRITE (NLP,9405)
      READ (NCR,9005)  ISTEST, WAVEHT, WAVEPER
      IF (ISTEST .GT. 3)  ISTEST = -1
      IF (ISTEST .LT. 0)  GO TO 300
      ISEASEL = ISTEST
      IF (ISEASEL .EQ. 1)  WINDV = WAVEHT
      WINDVP4 = WINDV**4
      WAVEHTP2 = WAVEHT*WAVEHT
      WAVPERP4 = WAVEPER**4
C
C              ENTER WATER PLANE RADIUS
  300 CONTINUE
      WRITE (NLP,9410)
      READ (NCR,9010)  RWLTEST
      IF (RWLTEST .LT. 0.0)  GO TO 400
      RWL = RWLTEST
C
C              INPUT DEPTH TO KEEL
  400 CONTINUE
      WRITE (NLP,9415)
      READ (NCR,9015)  ZKTEST
      IF (ZKTEST .LT. 0.0)  GO TO 500
      DEPTHK = ZKTEST
C
C              ENTER ESTIMATED AVERAGE AMPLITUDE
  500 CONTINUE
      WRITE (NLP,9420)
```

```
            READ (NCR,9020)  AMPTEM
            IF (AMPTEM .LT. 0.0)  GO TO 600
            AVERGAMP = AMPTEM
C
C                     ENTER ESTIMATED AVERAGE ROLL
        600 CONTINUE
            WRITE (NLP,9425)
            READ (NCR,9025)  THETATEM
            IF (THETATEM .LT. 0.0)  GO TO 700
            THETABAR = THETATEM/RTOD
C
C
        700 CONTINUE
C
C                     DEFINE BUOY
C
            N = 0
       1000 CONTINUE
            WRITE (NLP,9455)
            READ (NCR,9055)  NPTEM
            IF (NPTEM .LE. 0)  GO TO 2000
C
            WRITE (NLP,9460)
            N = 1
C
C                     LOOP ON ENTRY
       1050 CONTINUE
            IF (N .GT. NPMAX)  WRITE (NLP,9725)  NPMAX ;  GO TO 200
            WRITE (NLP,9465)  N
            READ (NCR,9065)  IDUM
            WRITE (NLP,9470)
            READ (NCR,9070)  K,W,H,T,D,X,F
C
C                     SET INPUTS FOR CORRECT SHAPE
            IF ((K .GT. 5) .OR. (K .LT. 1))  OUTPUT K ;  WRITE (NLP,9700) ;
           +                                 GO TO 1050
            IF (W .LE. 0.0)  WRITE (NLP,9705) ;  OUTPUT W ;  GO TO 1050
            WIDTH(N) = W
            IF (H .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT H ;  GO TO 1050
            HEIGHT(N) = H
            IF (D .LT. 0.0)  WRITE (NLP,9710) ;  OUTPUT D ;  GO TO 1050
            DENSITY(N) = D
            IF (X .LT. 0.0)  WRITE (NLP,9715) ;  OUTPUT X ;  GO TO 1050
            DISTCGK(N) = X
            GO TO (1100,1100,1100,1400,1500), K
C
C                     CYLINDERS
       1100 CONTINUE
            IF (T .EQ. -1.0)  T = W*6.0
            IF ((K .EQ. 2) .OR. (K .EQ. 3) )  T = W*6.0
            IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  GO TO 1050
            THICK(N) = T/12.0
            FRACNORM(N) = 1.0
            ISHAPE(N) = 1
            GO TO 1800
C
C                     TRIANGLE
       1400 CONTINUE
            IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  GO TO 1050
            THICK(N) = T/12.0
```

```
          IF ((F .GT. 1.0) .ØR. (F .LT. 0.0))  WRITE (NLP,9720) ;  ØUTPUT F
         +                                     ;  GØ TØ 1050
          FRACNØRM(N) = F
          ISHAPE(N) = 2
          GØ TØ 1800
C
C                   RECTANGLE
 1500 CØNTINUE
          IF (T .LT. 0.0)  WRITE (NLP,9705) ;  ØUTPUT T ;  GØ TØ 1050
          THICK(N) = T/12.0
          IF ((F .GT. 1.0) .ØR. (F .LT. 0.0))  WRITE (NLP,9720) ;  ØUTPUT F
         +                                     ;  GØ TØ 1050
          FRACNØRM(N) = F
          ISHAPE(N) = 3
          GØ TØ 1800
C
C                   CØMPUTE VØLUME AND WEIGHT
 1800 CØNTINUE
          CALL BØDYVØL (ISHAPE(N),H,W,THICK(N),V)
          WEIGHT(N) = V*D
          VØLUME(N) = V
C
C                   CHECK NUMBER ØF ENTRIES
          IF (N .GE. NPTEM)  GØ TØ 2000
          N = N + 1
          GØ TØ 1050
C
C                   PART CHANGE
C
 2000 CØNTINUE
          IF (NPTEM .GT. 0)  NPARTS = NPTEM
          NPTEM = 0
          NPARTS = MAX (NPARTS,N)
          WRITE (NLP,9475)
          READ (NCR,9075)  N
          IF ((N .GT. 0) .AND. (N .LE. NPARTS + 1))  GØ TØ 1050
C
 3000 CØNTINUE
          WRITE (NLP,9480)
          READ (NCR,9080)  IL,IP,RMIN,RMAX
          ILIST = 0
          IF (IL .EQ. 1HY)  ILIST = 1
          IPLØT = 0
          IF (IP .EQ. 1HY)  IPLØT = 1
C
          RETURN
C
 9000 FØRMAT (3F.0)
 9005 FØRMAT (I,F.0,F.0)
 9010 FØRMAT (F.0)
 9015 FØRMAT (F.0)
 9020 FØRMAT (F.0)
 9025 FØRMAT (F.0)
 9055 FØRMAT (I)
 9065 FØRMAT (A)
 9070 FØRMAT (I,6F.0)
 9075 FØRMAT (I)
 9080 FØRMAT (A1,1X,A1,1X,2F.0)
C
 9400 FØRMAT (/'  ENTER START, END, INCREMENT ØF PERIØD RANGE (SEC) Ø')
```

```
      9405 FØRMAT (/'  ENTER SEA SPECTRUM TYPE AND PARAMETERS'
     +           /'      = 1.0                 0 '
     +           /'      PIERSØN-MØSKØWITZ 1, WIND SPEED (KNØTS)'
     +           /'      BRETSCHNEIDER      2, SIGNIF WAVE HT (FT), SIGNIF'
     +           ' WAVE PERIØD (SEC)'
     +           /'      I.S.S.C.           3, SIGNIF WAVE HT (FT), SIGNIF'
     +           ' WAVE PERIØD (SEC)'       )
      9410 FØRMAT (/'  ENTER WATER PLANE RADIUS AT SURFACE (FT) Ø')
      9415 FØRMAT (/'  ENTER DEPTH TØ KEEL (FT) Ø')
      9420 FØRMAT (/'  ENTER EXPECTED AVERAGE AMPLITUDE (FT) Ø').
      9425 FØRMAT (/'  ENTER EXPECTED AVERAGE RØLL (DEG) Ø')
      9455 FØRMAT (/'  ENTER NUMBER ØF BUØY PARTS Ø')
      9460 FØRMAT (/'  *** FØR EACH PART NUMBER YØU MUST ENTER '/
     +           '       SØME IDENTIFIER BEFØRE YØU RETURN.'/
     +           '       THEN ENTER  K,W,H,T,D,X,F                  '/
     +           '          K = SHAPE CØDE  1 - HØLLØW CYLINDER      '/
     +           '                          2 - SØLID CYLINDER       '/
     +           '                          3 - DISC                 '/
     +           '                          4 - TRIANGULAR (RT) PLATE '/
     +           '                          5 - RECTANGULAR PLATE'/
     +           '          W = WIDTH ØR ØUTSIDE DIAMETER (FT)       '/
     +           '          H = HEIGHT (FT)                          '/
     +           '          T = THICKNESS (IN)                       '/
     +           '              A -1 ENTERED FØR CASE K=1 WILL ASSUME '/
     +           '                A SØLID (T = W/2/12)               '/
     +           '              FØR CASES K=2,3 ENTER ANYTHING       '/
     +           '          D = DENSITY (LBM/FT**3)                  '/
     +           '          X = DISTANCE FRØM KEEL TØ PART CG (FEET)  '/
     +           '          F = FØR PLATES ØNLY, FRACTIØNAL AREA'/
     +           '                        ØF THE PLATE NØRMAL TØ MØTIØN'/
     +           '              ENTER 1 FØR CYLINDERS              '////)
      9465 FØRMAT (/ '  PART NØ. ', I3,'Ø')
      9470 FØRMAT ( '  ENTER K,W,H,T,D,X,F')
      9475 FØRMAT (/'  ENTER PART NUMBER TØ CHANGE (-1 TØ STØP) Ø')
      9480 FØRMAT (/'  ENTER Y ØR N FØR LIST AND PLØT ØPTIØNS'/
     +           '      FØR PLØT YØU MAY ALSØ ENTER RAØ MIN AND MAX Ø')
C
      9700 FØRMAT (/' *** YØURE KIDDING - THE CØDES ØNLY GØ FRØM 1 TØ 5 '/
     +           '     TRY AGAIN '/)
      9705 FØRMAT (/' *** WHAT KIND ØF SHAPE IS THIS Ø')
      9710 FØRMAT (/' *** WHAT DØ YØU HAVE IN THERE Ø')
      9715 FØRMAT (/' *** WHERE IS IT Ø')
      9720 FØRMAT (/' *** RANGE ØF F = 0.0 TØ 1.0 Ø')
      9725 FØRMAT (/' *** ØNLY ',I3,' CØMPØNENTS ARE ALLØWED '/
     +           '       BUØY DEFINITIØN TERMINATES '/
     +           '       BUØY CHANGES WILL PRØCEED  '/)
C
           END
```

```
      SUBROUTINE BINPUT
C
C           VERSION 1.0         SEP, 1976            R. GOLDSMITH
C
C           THIS ROUTINE INPUTS DATA FOR THE ROLL RAO IN BATCH MODE
C
C
C
C
      DIMENSION ICOMMENT(4), ID(10)
C
      COMMON / IODEV / NCR,NLP
      COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
      COMMON / SEASTATE / ISEASEL,WINDV,WAVEHT,WAVEPER,
     +                    WINDVP4,WAVEHTP2,WAVPERP4
      COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
     +                DENSITY(50),DISTCGK(50),FRACNORM(50)
      COMMON / BOUTS / VOLUME(50),WEIGHT(50)
      COMMON / CONSTANT / PI,RTOD,RHO,G
      COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
     +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
      COMMON / OUTPUTS / ILIST,IPLOT,RMIN,RMAX
C
C
      DATA ID /'  H CYL ','  S CYL ','   DISC ',' TRI PLT',' RCT PLT'/
C
C           INPUT TIME AND RANGE
      READ (NCR,9005)  T1,T2,T3
      IF (T1 .LT. 0.0) GO TO 175
      TIME1 = T1
      TIME2 = T2
      IF (TIME2 .LT. TIME1)  TIME2 = TIME1
      IF (T3 .LE. 0.0)  T3 = TIMEDEL
      TIMEDEL = T3
  175 CONTINUE
      WRITE (NLP,9405)  TIME1,TIME2,TIMEDEL
C
C           SELECT SEA STATE PARAMETERS
  200 CONTINUE
      READ (NCR,9010)  ISTEST, WAVEHT, WAVEPER
      IF (ISTEST .GT. 3)  ISTEST = -1
      IF (ISTEST .LT. 0)  GO TO 275
      ISEASEL = ISTEST
      IF (ISEASEL .EQ. 1)  WINDV = WAVEHT
      WINDVP4 = WINDV**4
      WAVEHTP2 = WAVEHT*WAVEHT
      WAVPERP4 = WAVEPER**4
  275 CONTINUE
      IF (ISEASEL .EQ. 0)  WRITE (NLP,9410)
      IF (ISEASEL .EQ. 1)  WRITE (NLP,9411)  WINDV
      IF (ISEASEL .EQ. 2)  WRITE (NLP,9412)  WAVEHT,WAVEPER
      IF (ISEASEL .EQ. 3)  WRITE (NLP,9413)  WAVEHT,WAVEPER
C
C           ENTER WATER PLANE RADIUS
  300 CONTINUE
      READ (NCR,9015)  RWLTEST
      IF (RWLTEST .LT. 0.0)  GO TO 375
      RWL = RWLTEST
  375 CONTINUE
```

```
          WRITE (NLP,9415)  RWL
C
C                   INPUT DEPTH TO KEEL
    400 CONTINUE
        READ (NCR,9020)  ZKTEST
        IF (ZKTEST .LT. 0.0)  GO TO 475
        DEPTHK = ZKTEST
    475 CONTINUE
        WRITE (NLP,9420)  DEPTHK
C
C                   ENTER ESTIMATED AVERAGE AMPLITUDE
    500 CONTINUE
        READ (NCR,9025)  AMPTEM
        IF (AMPTEM .LT. 0.0)  GO TO 575
        AVERGAMP = AMPTEM
    575 CONTINUE
        WRITE (NLP,9425)  AVERGAMP
C
C                   ENTER ESTIMATED AVERAGE ROLL
    600 CONTINUE
        READ (NCR,9030)  THETATEM
        IF (THETATEM .LT. 0.0)  GO TO 675
        THETABAR = THETATEM/RTOD
    675 CONTINUE
        WRITE (NLP,9430)  THETABAR*RTOD
C
C
    700 CONTINUE
C
C                   DEFINE BUOY
C
        N = 0
   1000 CONTINUE
        READ (NCR,9055)  NPTEM
        IF (NPTEM .LE. 0)  GO TO 2000
C
        WRITE (NLP,9455)  NPTEM
        WRITE (NLP,9460)
        N = 1
C
C                   LOOP ON ENTRY
   1050 CONTINUE
        IF (N .GT. NPMAX)  WRITE (NLP,9725)  NPMAX ;  STOP 1050
        READ (NCR,9070)  K,W,H,T,D,X,F,ICOMMENT
C
C                   SET INPUTS FOR CORRECT SHAPE
        IF ((K .GT. 5) .OR. (K .LT. 1))  OUTPUT K ;  WRITE (NLP,9700) ;
       +                                  STOP 1050
        IF (W .LE. 0.0)  WRITE (NLP,9705) ;  OUTPUT W ;  STOP 1050
        WIDTH(N) = W
        IF (H .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT H ;  STOP 1050
        HEIGHT(N) = H
        IF (D .LT. 0.0)  WRITE (NLP,9710) ;  OUTPUT D ;  STOP 1050
        DENSITY(N) = D
        IF (X .LT. 0.0)  WRITE (NLP,9715) ;  OUTPUT X ;  STOP 1050
        DISTCGK(N) = X
        GO TO (1100,1100,1100,1400,1500), K
C
C                   CYLINDERS
   1100 CONTINUE
```

```
          IF (T .EQ. -1.0)  T = W*6.0
          IF ((K .EQ. 2) .OR. (K .EQ. 3) )  T = W*6.0
          IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  STOP 1100
          THICK(N) = T/12.0
          FRACNORM(N) = 1.0
          ISHAPE(N) = 1
          GO TO 1800
C
C                    TRIANGLE
   1400 CONTINUE
          IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  STOP 1400
          THICK(N) = T/12.0
          IF ((F .GT. 1.0) .OR. (F .LT. 0.0))  WRITE (NLP,9720) ;  OUTPUT F
       +                                        ;  STOP 1400
          FRACNORM(N) = F
          ISHAPE(N) = 2
          GO TO 1800
C
C                    RECTANGLE
   1500 CONTINUE
          IF (T .LT. 0.0)  WRITE (NLP,9705) ;  OUTPUT T ;  STOP 1500
          THICK(N) = T/12.0
          IF ((F .GT. 1.0) .OR. (F .LT. 0.0))  WRITE (NLP,9720) ;  OUTPUT F
       +                                        ;  STOP 1500
          FRACNORM(N) = F
          ISHAPE(N) = 3
          GO TO 1800
C
C                    COMPUTE VOLUME AND WEIGHT
   1800 CONTINUE
          CALL BODYVOL (ISHAPE(N),H,W,THICK(N),V)
          WEIGHT(N) = V*D
          VOLUME(N) = V
C
          WRITE (NLP,9465)  N,ID(2*K-1),ID(2*K),W,H,THICK(N),D,X,F
       +                    ,ICOMMENT
C                    CHECK NUMBER OF ENTRIES
          IF (N .GE. NPTEM)  GO TO 2000
          N = N + 1
          GO TO 1050
C
C                    PART CHANGE
C
   2000 CONTINUE
          IF (NPTEM .GT. 0)  NPARTS = NPTEM
          NPTEM = 0
          NPARTS = MAX (NPARTS,N)
          READ (NCR,9075)  N
          IF ((N .GT. 0) .AND. (N .LE. NPARTS + 1))  WRITE (NLP,9470) ;
       +                                               GO TO 1050
C
   3000 CONTINUE
          READ (NCR,9080)  IL,IP,RMIN,RMAX
          ILIST = 0
          IF (IL .EQ. 1HY)  ILIST = 1
          IPLOT = 0
          IF (IP .EQ. 1HY)  IPLOT = 1
C
          RETURN
C
```

```
      9005 FØRMAT (3F.0)
      9010 FØRMAT (I,F.0,F.0)
      9015 FØRMAT (F.0)
      9020 FØRMAT (F.0)
      9025 FØRMAT (F.0)
      9030 FØRMAT (F.0)
      9055 FØRMAT (I)
      9070 FØRMAT (I,6F.0,3A4,A2)
      9075 FØRMAT (I)
      9080 FØRMAT (A1,1X,A1,1X,2F.0)
     C
      9405 FØRMAT (/'  PERIØD RANGE, IN SECØNDS   START     END     DELTA'/
         +            26X,F8.3,F8.3,2X,F8.3)
      9410 FØRMAT (/'  SEA SPECTRUM = 1.0')
      9411 FØRMAT (/'  PIERSØN-MØSKØWITZ SEA SPECTRUM '
         +         /'     WIND SPEED = ',F10.3,' KNØTS')
      9412 FØRMAT (/'  BRETSCHNEIDER SEA SPECTRUM'
         +         /'     SIGNIFICANT WAVE HT = ',F10.3,' FEET'
         +         /'     SIGNIFICANT WAVE PERIØD = ',F10.3,' SEC')
      9413 FØRMAT (/'  I.S.S.C. SEA SPECTRUM'
         +         /'     SIGNIFICANT WAVE HT = ',F10.3,' FEET'
         +         /'     SIGNIFICANT WAVE PERIØD = ',F10.3,' SEC')
      9415 FØRMAT (/'  WATER PLANE RADIUS AT SURFACE = ',F6.2,' FT')
      9420 FØRMAT (/'  DEPTH TØ THE KEEL = ',F6.2,' FT')
      9425 FØRMAT (/'  ESTIMATED AVERAGE AMPLITUDE = ',F6.2,' FT')
      9430 FØRMAT (/'  ESTIMATED AVERAGE RØLL = ',F6.2,' DEG')
      9455 FØRMAT (/'  NUMBER ØF PARTS = ',I3)
      9460 FØRMAT (//'                                                    ',
         +         '  C.G. '/
         +         '                                               DENSITY',
         +         ' ABØVE'/
         +         ' PART            WIDTH    HEIGHT    THICK    (LBSM/ ',
         +         '  KEEL  FRACT'/
         +         ' NØ    SHAPE     (FT)     (FT)      (FT)      FT**3)',
         +         '  (FT)   NØRM  CØMMENTS'/
         +         ' ---- -------- -------  -------  --------  ---------',
         +         ' -----  -----  ---------------'/)
      9465 FØRMAT (I5, 2X,2A4, F8.2, F8.2, F11.4, F10.1, F7.2, F7.2, 2X,4A4)
      9470 FØRMAT (/)
     C
      9700 FØRMAT (/' *** YØURE KIDDING - THE CØDES ØNLY GØ FRØM 1 TØ 5 '/)
      9705 FØRMAT (/' *** WHAT KIND ØF SHAPE IS THIS Ø ')
      9710 FØRMAT (/' *** WHAT DØ YØU HAVE IN THERE Ø ')
      9715 FØRMAT (/' *** WHERE IS IT Ø ')
      9720 FØRMAT (/' *** RANGE ØF F = 0.0 TØ 1.0 Ø ')
      9725 FØRMAT (/' *** ØNLY ',I3,' CØMPØNENTS ARE ALLØWED '/
         +        '          THE PRØGRAM TERMINATES '/)
     C
           END
```

```
      SUBRØUTINE BØDYMI (IS,H,W,T,PMI)
C
C               VERSIØN 1.0         SEP, 1976          R. GØLDSMITH
C
C               THIS RØUTINE CØMPUTES THE BASIC SHAPE MØMENT ØF INERTIA
C                 ASSUMING UNIFØRM DENSITY
C
      CØMMØN / IØDEV / NCR,NLP
C
C
      IF ((IS .GT. 3) .ØR. (IS .LT. 1))  WRITE (NLP,9700)  IS ;
     +                                                      STØP 10
C
      GØ TØ (100,200,300), IS
C
C               CYLINDER
  100 CØNTINUE
      WT = W - 2.0*T
      IF (WT .LT. 0.0)  WT = 0.0
      PMI = (W*W + WT*WT)/16.0 + H*H/12.0
      RETURN
C
C               TRIANGLES
  200 CØNTINUE
C ****MØST PLATES ARE SMALL SØ THE INERTIA IS IGNØRED FØR NØW
      PMI = 0.0
      RETURN
C
C               RECTANGULAR PLATE
  300 CØNTINUE
      PMI = H*H/12.0
      RETURN
C
C
 9700 FØRMAT (//'    *** WHAT KIND ØF SHAPE IS CØDE ', I6/
     +      '           PRØGRAM STØPS IN RØUTINE BØDYMI')
C
      END
```

```
      SUBRØUTINE BØDYVØL (IS,H,W,T,V)
C
C                 VERSIØN 1.0          SEP, 1976            R. GØLDSMITH
C
C                 CØMPUTE THE BØDY VØLUME
C
      CØMMØN / IØDEV / NCR,NLP
      CØMMØN / CØNSTANT / PI,RTØD,RHØ,G
C
C
      IF ((IS .GT. 3) .ØR. (IS .LT. 1))  WRITE (NLP,9700)  IS ;
     +                                                       STØP 10
      GØ TØ (100,200,300), IS
C
C
C                 CYLINDERS
  100 CØNTINUE
      WT = W - 2.0*T
      IF (WT .LT. 0.0)  WT = 0.0
      V = H*PI*(W*W - WT*WT)/4.0
      RETURN
C
C                 TRIANGLES
  200 CØNTINUE
      V = 0.5*H*W*T
      RETURN
C
C                 RECTANGULAR PLATE
  300 CØNTINUE
      V = H*W*T
      RETURN
C
C
 9700 FØRMAT (//'    *** WHAT IN THE WØRLD IS SHAPE CØDE ', I6/
     +       '               PRØGRAM STØPS IN RØUTINE BØDYVØL')
C
      END
```

```
      SUBROUTINE DISPLACE
C
C                 VERSION 1.0         SEP, 1976           R. GOLDSMITH
C
C                 THIS ROUTINE IS USED TO COMPUTE THOSE PARAMETERS
C                    ASSOCIATED WITH THE BUOY DISPLACEMENT
C
      COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
     +                DENSITY(50),DISTCGK(50),FRACNORM(50)
      COMMON / BOUTS / VOLUME(50),WEIGHT(50)
      COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
     +                  DEPTHB(50),DEPTHT(50)
      COMMON / CONSTANT / PI,RTOD,RHO,G
      COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
     +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
      COMMON / MOMENTS / BUOYMI,ADDMI,VIRTINRT,WATERIM,BUOYMR,
     +                  BUOYMDT,BUOYMD,WATERMD,DAMPM
C
C
      RHOG = RHO*G
      WDISPLAC = 0.0
      SUMCBK = 0.0
      ADDMI = 0.0
C
C                 BEGIN LOOP ON EACH PART
C
      DO 1000  I = 1,NPARTS
        IS = ISHAPE(I)
        H = HEIGHT(I)
        W = WIDTH(I)
        T = THICK(I)
        V = VOLUME(I)
        X = DISTCGK(I)
        F = WEIGHT(I)
        WD(I) = W
        HD(I) = H
        XD(I) = X
C
C                 COMPUTE DEPTH OF PART BOTTOM AND TOP
C
        DCG = DEPTHK - X
C                       CHECK SHAPE
        GO TO (100,200,100), IS
C                             CYLINDER AND RECTANGULAR PLATE
  100   CONTINUE
        TEM = H/2.0
        DB = DCG + TEM
        DT = DCG - TEM
        GO TO 300
C                             TRIANGLE
  200   CONTINUE
        DB = DCG + H/3.0
        DT = DCG - H*0.6667
C   ***           THE ABOVE IS ONLY A GUESS - CORRECT
        GO TO 300
C
C                 IF PART TOTALLY OUT OF WATER IGNORE
  300   CONTINUE
        IF (DB .LE. 0.0)  VCOR = V ;  GO TO 750
```

```
C                   IF PART TOP IS IN WATER OK
           IF (DT .GE. 0.0)  VCOR = 0.0 ;  GO TO 750
C
C                   COMPUTE VOLUME CORRECTION FOR OUT OF WATER
           IF ((IS .NE. 1) .AND. (IS .NE. 4))  GO TO 350
C                        CYLINDER AND RECTANGULAR PLATE MOD
           XD(I) = DEPTHK - DB/2.0
           HD(I) = DB
           GO TO 600
C                        TRIANGLE MOD
      350  CONTINUE
           IF (IS .NE. 2)  GO TO 400
C ****     XD(I) SHOULD ALSO CHANGE IF MI IS TO BE CORRECT ALSO HD
           WD(I) = H*W/ABS(DT)
           GO TO 600
      400  CONTINUE
C
      600  CONTINUE
           CALL BODYVOL (IS,ABS(DT),WD(I),T,VCOR)
           DT = 0.0
C
C                        VOLUME IN WATER
      750  CONTINUE
           VD(I) = V - VCOR
           DEPTHB(I) = DB
           DEPTHT(I) = DT
C
C                        WEIGHT OF WATER
           WGTW = VD(I)*RHOG
           WDISPLAC = WDISPLAC + WGTW
           SUMCBK = SUMCBK + WGTW*XD(I)
           FD(I) = WGTW
C
C                        COMPUTE THE ADDED MI OF THE WATER BODY
C                        ABOUT ITS OWN AXIS
           CALL BODYMI (IS,HD(I),WD(I),T,WBODYMI)
           ADWMI = VD(I)*RHO* (XD(I) - BUOYCGK)**2
           ADDMI = ADDMI + WGTW*WBODYMI/G + ADWMI
C
     1000 CONTINUE
C
C                   COMPUTE THE CENTER OF BUOYANCY
C
          BUOYCBK = SUMCBK/WDISPLAC
          DEPTHCB = DEPTHK - BUOYCBK
C
          RETURN
C
          END
```

```
      SUBROUTINE BUOYDAMP
C
C               VERSION 1.0         SEP, 1976           R. GOLDSMITH
C
C               THIS ROUTINE COMPUTES THE BUOY DAMPING MOMENT
C                  ASSUMPTIONS :- THAT THE ANGLE OF ROLL IS 'SMALL'
C                                 SO THAT THE HORIZONTAL COMPONENT OF
C                                 BUOY MOTION AND WATER VELOCITY HAVE
C                                 PERPENDICULAR EFFECT.
C                               - DAMPING FORCE IS LINEAR AND PROPORTION
C                                 TO SPEED
C
C
      COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
      COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
     +                DENSITY(50),DISTCGK(50),FRACNORM(50)
      COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
     +                DEPTHB(50),DEPTHT(50)
      COMMON / CONSTANT / PI,RTOD,RHO,G
      COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
     +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
      COMMON / COEFS / DRAG(5),COEFM(5)
      COMMON / MOMENTS / BUOYMI,ADDMI,VIRTINRT,WATERIM,BUOYMR,
     +                BUOYMDT,BUOYMD,WATERMD,DAMPM
C
C
      ALPHAT= 4.0*RHO*THETABAR/(3.0*PI)
C
C               THIS SECTION COMPUTES JUST THE BUOY DAMPING
C
      BUOYMDT = 0.0
      DO 50 I = 1,NPARTS
C               CHECK IF ITS OUT OF WATER
        IF (VD(I) .LE. 0.0)  GO TO 50
C               CHECK SHAPE
        GO TO  (20,30,30), ISHAPE(I)
C                    CYLINDER
   20   CONTINUE
        XB = DEPTHB(I) - DEPTHCG
        XT = DEPTHT(I) - DEPTHCG
        PMD = 0.25*WIDTH(I)*(SIGN (XB**4,XB) - SIGN (XT**4,XT) )
        GO TO 40
C                    TRIANGULAR RECTANGULAR PLATES
   30   CONTINUE
        PAREA = VD(I)/THICK(I)
        XC = BUOYCGK - XD(I)
        PMD = PAREA*XC*XC*FRACNORM(I)
        GO TO 40
   40   CONTINUE
        ALPHA = ALPHAT*DRAG(ISHAPE(I))
        BUOYMDT = BUOYMDT + ALPHA*PMD
X       OUTPUT ALPHA*PMD
   50 CONTINUE
C              ***************************************************
C              NOTE: TO SIMPLIFY THE COMPUTATION THIS TERM HAS BEEN
C                    COMPUTED AS A CONSTANT.  THE FREQUENCY
C                    CONTRIBUTION IS MULTIPLIED IN AT THE BEGINNING
C                    OF THE MAIN FREQUENCY ITERATION IN THE MAIN
C                    PROGRAM.
```

```
C               ***********************************************
      RETURN
C
C               ********************************
C
      ENTRY WATERDAMP
C
C                      WATER MOMENT OF DAMPING
C
      BETAT = 4.0*RHO*AVERGAMP*FREQ/(3.0*PI)
      WATERMD = 0.0
C
C                 COMPUTE DAMPING
      DO 1000  I = 1,NPARTS
C                      CHECK IF ITS OUT OF WATER
        IF (VD(I) .LE. 0.0)  GO TO 1000
C                      CHECK SHAPE
        GO TO (100,300,300), ISHAPE(I)
C                      CYLINDERS
  100   CONTINUE
        XB = DEPTHB(I) - DEPTHCG
        XT = DEPTHT(I) - DEPTHCG
        WAVEXB = WAVEN*XB*2.0
        WAVEXT = WAVEN*XT*2.0
        WAVENP2 = WAVEN*WAVEN
        TERMB = ((-WAVEXB - 1.0)*EXP (-WAVEXB) + 1.0)
        TERMT = ((-WAVEXT - 1.0)*EXP (-WAVEXT) + 1.0)
        WMD =  (TERMB - TERMT)*WIDTH(I)*EXP (-2.0*WAVEN*DEPTHCG)
     +           /(4.0*WAVENP2)
        GO TO 900
C
C                      TRIANGULAR AND RECTANGULAR PLATES
  300   CONTINUE
        PAREA = VD(I)/THICK(I)
        XC = BUOYCGK - XD(I)
        ZC = DEPTHK - XD(I)
C                      WATER MOMENT
        WMD = PAREA*XC*FRACNORM(I)*EXP (-WAVEN*ZC)
        GO TO 900
  900   CONTINUE
C
C                 GET TOTAL DAMPING
        BETA = BETAT*DRAG(ISHAPE(I))
        WMD = BETA*WMD
        WATERMD = WATERMD + WMD
 1000 CONTINUE
C
C                 TOTAL DAMPING MOMENT
C
      DAMPM = BUOYMD + WATERMD
C
      RETURN
C
      END
```

```
      SUBROUTINE WATRINRT
C
C           VERSION 1.0          SEP, 1976          R. GOLDSMITH
C
C           THIS ROUTINE COMPUTES THE COEFFICIENT FOR THE INERTIA
C              MOMENT DUE TO WATER PARTICLE ACCELERATION.
C
C
      COMMON / TP / TIME1,TIME2,TIMEDEL,FREQ,WAVEN
      COMMON / BINS / NPARTS,ISHAPE(50),WIDTH(50),HEIGHT(50),THICK(50),
     +                DENSITY(50),DISTCGK(50),FRACNORM(50)
      COMMON / WATERDIS / WD(50),HD(50),XD(50),VD(50),FD(50),
     +                    DEPTHB(50),DEPTHT(50)
      COMMON / CONSTANT / PI,RTOD,RHO,G
      COMMON / BUOY / NPMAX,RWL,AVERGAMP,THETABAR,PERIODO,
     +                DEPTHK,BUOYCGK,DEPTHCG,BUOYCBK,DEPTHCB,WDISPLAC
      COMMON / COEFS / DRAG(5),COEFM(5)
      COMMON / MOMENTS / BUOYMI,ADDMI,VIRTINRT,WATERIM,BUOYMR,
     +                   BUOYMDT,BUOYMD,WATERMD,DAMPM
C
      BETAT = PI*RHO/4.0
      CONST = BETAT/(WAVEN*WAVEN)*EXP (-WAVEN*DEPTHCG)
C
      WATERIM = 0.0
C
C           COMPUTE WATER PARTICLE INERTIA MOMENT
      DO 1000  I = 1,NPARTS
C                CHECK IF ITS OUT OF WATER
        IF (VD(I) .LE. 0.0)  GO TO 1000
C                CHECK SHAPE
        GO TO (100,300,300),  ISHAPE(I)
C
C                CYLINDERS
  100   CONTINUE
        XB = DEPTHB(I) - DEPTHCG
        XT = DEPTHT(I) - DEPTHCG
        WAVENXB = WAVEN*XB
        WAVENXT = WAVEN*XT
        TERM1B = (-WAVENXB - 1.0)*EXP (-WAVENXB)
        TERM1B = + TERM1B + 1.0
        TERM1T = (-WAVENXT - 1.0)*EXP (-WAVENXT)
        TERM1T = + TERM1T + 1.0
        WMI = CONST*(WIDTH(I)*WIDTH(I) )*(TERM1B - TERM1T)
        GO TO 900
C
C                TRIANGULAR AND RECTANGULAR PLATES
  300   CONTINUE
C ****CURRENTLY SET TO ZERO
        WMI = 0.0
        GO TO 900
C
  900   CONTINUE
        WMI = -WMI*COEFM(ISHAPE(I))
        WATERIM = WATERIM + WMI
C
 1000 CONTINUE
C
      RETURN
C
      END
```

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>The following report describes a computer solution to help predict the heave and roll response of free floating bodies of cylindrical shape when excited by random seas with known spectra.<br>The basic concepts of harmonic analysis and statistics used in the method are first briefly reviewed. The report then presents a detailed derivation of the linear heave and roll response amplitude operators (Cont.) | | |

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The heave and roll response programs are written for use with XEROX SIGMA 7 computers. Program listings are given in the appendix.

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