

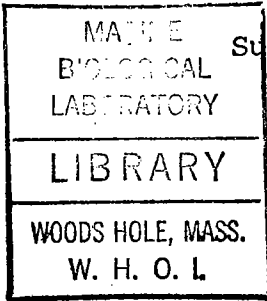
ANALYSIS OF A HIGH RESOLUTION DEEP OCEAN
ACOUSTIC NAVIGATION SYSTEM

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James Leighton Durham
Lieutenant, United States Navy

Submitted to the Department of Ocean Engineering in
partial fulfillment of the requirements for the degrees of
Ocean Engineer and Master of Science in Ocean Engineering

ABSTRACT

A high resolution acoustic navigation system for ocean use is being developed at the Woods Hole Oceanographic Institution. The system can yield navigation fixes with respect to a bottom moored reference net with accuracies (on a fix to fix basis) of a few centimeters. In order to use the system to best advantage a survey is required to determine precisely the relative positions of the net elements. Each element combines a pulse transponder with a continuous wave (CW) beacon. Accumulated phase (Doppler shift of the CW beacon) between survey points is measured as well as acoustic travel times between survey points and transponders. Non-linear regression techniques are employed to develop a maximum likelihood estimator for net element positions based on these phase and travel time measurements. An approximate error covariance matrix is generated and an optimum choice of survey points is indicated. The combined system, using these selected locations for performing the survey, can yield reference mooring coordinates with error of ± 1 meter. Improved precision appears to be limited by inaccuracies in the pulse and Doppler measuring system.

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I INTRODUCTION

There are many methods currently used to position ships, submersibles, buoys or submerged instruments in the deep ocean. Broadly speaking they can be divided into two types. The first method positions via electromagnetic transmission from a shore station or satellite whose position or orbit is precisely known. Such systems operate at long ranges but are limited in accuracy to about 100-200 meters at best. Furthermore, these systems require above surface antennas and are therefore of little use in positioning entirely submerged vehicles. The second method positions via acoustic transmission from a set of transponders or beacons, usually moored to the ocean bottom, whose relative positions are precisely known. When operated in the pulsed or transponder mode, position inaccuracies of an acoustic system can be as small as 10-20 meters in 5 km water depths. When operated in a continuous, or Doppler mode, the accuracy can approach 3-4 centimeters. The range of acoustic systems is usually severely limited by a number of factors, the most important of which are acoustic refraction and attenuation of sound in sea water due to spreading and absorption in the ocean. Current acoustic systems have ranges of about 10 km.

A navigation system utilizing acoustic beacons (Fig. 1) which operate in both a pulse and continuous wave mode is being developed. This system capitalizes on the attributes of both

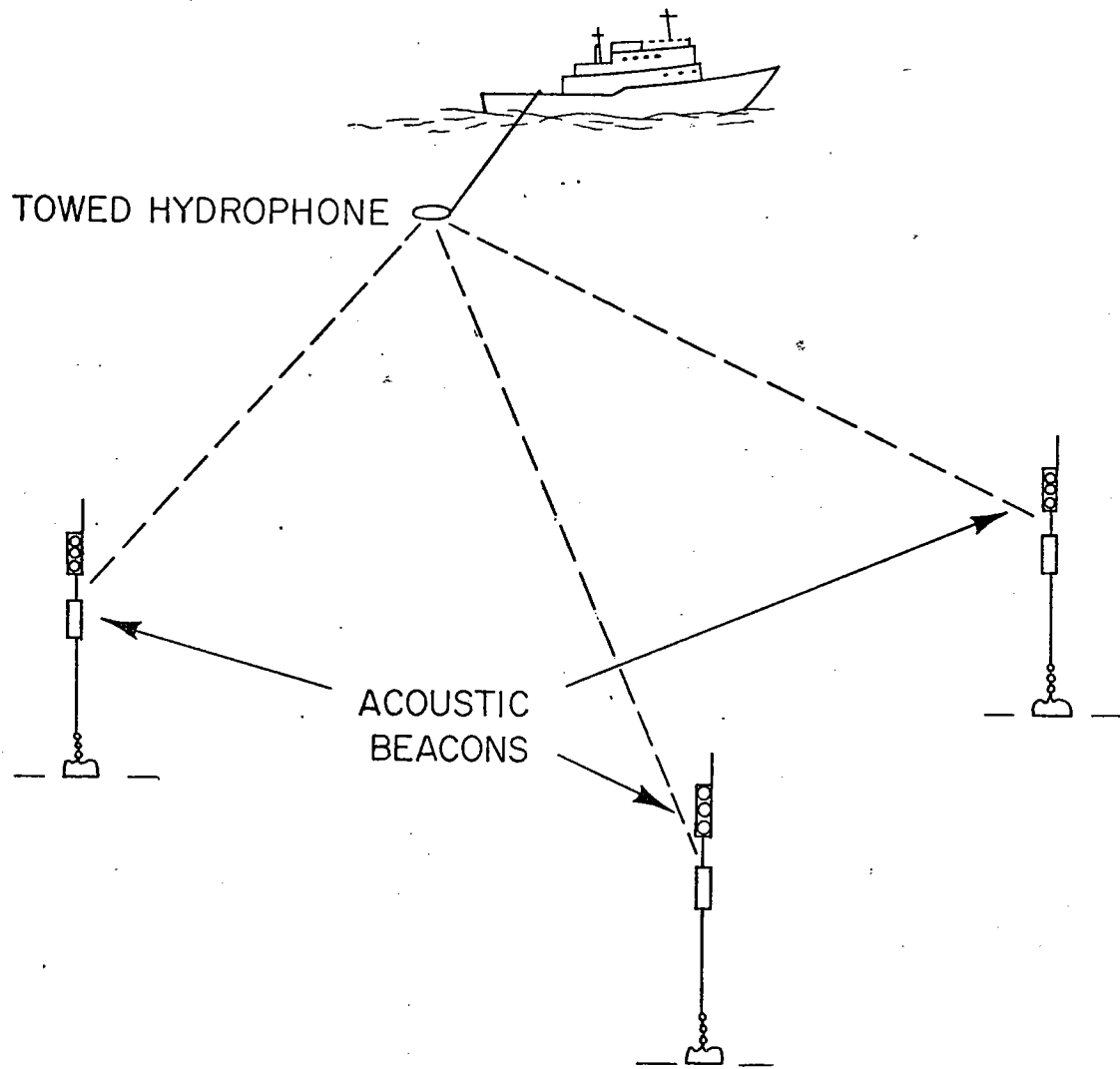


Figure 1. Acoustic Navigation Beacon Net

the pulse and Doppler modes and will be capable of positioning a platform with respect to an array of reference beacons to an accuracy of 1-2 meters and of repositioning a platform within 10 centimeters of a previous position. To make full use of the system capabilities a survey is required to accurately determine the relative positions of the reference beacons. The purpose of this investigation is to develop an accurate survey technique. Algorithms for estimating the beacon positions and the errors in those positions are derived. Performance is predicted using computer models. The estimation method used is a straightforward extension of the pulse system technique described by Hunt et al. (1974).

1) Description of System

Pulse (transponder) navigation systems measure the travel time of an acoustic pulse to estimate the slant range to a transponder. Shipboard processing equipment interrogates the bottom moored transponders and measures the time of arrival of the reply pulse from each transponder. Sound velocity corrections and corrections for fixed system delay times are applied, and travel times are converted into estimates of slant range to the transponders. The accuracy of the measurement is dependent upon the pulse bandwidth and the signal-to-noise ratio in the received pulse. Systems of this type are widely in use in

oceanographic research, submerged rescue operations and commercial enterprises in the ocean (Boegeman et al., 1972; Cestone and St. George, 1972; Van Calcar, 1969).

A continuous wave (CW) beacon system has been developed at the Woods Hole Oceanographic Institution to accurately track the position of ship-mounted and submerged hydrophones (Porter et al., 1973). The system measures the frequency change of a continuous tone due to hydrophone motion. Hydrophone velocity and displacement are derived from the beacon signal Doppler shift which is given by

$$\Delta f = f_B - f_D = -f_B(v/c),$$

where f_D is the Doppler shifted beacon frequency due to a hydrophone velocity v , f_B is the beacon frequency and c is the speed of sound in water. Doppler shift is proportional to beacon frequency. Hence the accuracy of the measurement is dependent on the beacon frequency and the signal-to-noise ratio in the narrow band that encompasses the maximum Doppler shift. Because the beacon frequency is much greater than the pulse bandwidth of transponder systems the Doppler system can measure range changes much more accurately than the pulse system assuming that both systems have the same signal-to-noise ratio. In the present implementation of the system,

accumulated phase change $\phi(t) = 2 \pi(\Delta f)t$ is measured by digital techniques and hydrophone displacements of a quarter wavelength can be resolved. A wavelength for 13 kHz (a typical beacon frequency) is $\lambda = 11.5$ centimeters, so that resolution is about $\lambda / 4 \approx 3$ cm. Phase changes due to inhomogeneities and fluctuations in the water column are negligible over the range of the system. We discuss this in Section III. 2.

2) Previous Survey Techniques

Several investigations have been conducted to develop an optimum technique for determining the coordinates of ocean bottom acoustic transponders. No satisfactory survey has been developed previously for an array of bottom-moored CW beacons.

Surveys of acoustic transponders generally fall into two categories: baseline crossing and iterative techniques. The baseline crossing technique, described by Haehnle (1967) and Hart (1967), is often referred to as the conventional or classical transponder survey method. The method requires that the depth of the transponders be previously determined from independent measurements. The baseline length, i.e., the horizontal distance between two transponders, is determined by steaming across a baseline and measuring the slant range to the two transponders during the traverse. This requires accurate ship

positioning and large amounts of ship time to yield accurate results. More recent iterative techniques are based on or are similar to that described by Vanderkulk (1961). Vanderkulk showed that for three bottom transponders and six co-planar survey positions whose depths are known, the positions and depths of the transponders can be found by solving six linear equations. A unique solution exists when the survey positions do not lie on a conic section. When more than six survey positions are used the additional equations result in an overdetermined situation for which minimum least squared error techniques have been successfully applied, (Lowenstein, 1965; McKeown, 1974; Mourad et al., 1972; Heckman and Abbott, 1973; Hunt et al., 1974).

3) Pulse-Doppler Survey Technique

The pulse-Doppler survey consists of making measurements of travel times of acoustic pulses and accumulated phase changes of CW signals at various survey points. These are converted to slant ranges and slant range differences, respectively. The pulse-Doppler survey can yield more accurate results than the pulse survey alone due to the higher resolution of the Doppler system measurement.

The slant range (SR) between a transponder and a survey point is equal to the travel time (TT) multiplied by an

effective sound speed, V_e :

$$SR = V_e \cdot TT$$

The effective sound speed is a function of the sound velocity profile, the depths of the transponder, and the survey point and varies with the amount of acoustic refraction (Vass, 1966). For m survey points there are m measurements of slant range for each transponder.

The change in slant range, DSR, between a beacon and two survey points is proportional to the change in accumulated phase, $\phi(t_2) - \phi(t_1)$ and the wavelength of the signal, $\lambda = c/f_B$:

$$DSR = [\phi(t_2) - \phi(t_1)] \lambda / 2 \pi .$$

The received signal,

$$s(t) = A(t) \exp \{ -j2 \pi f_D t \} ,$$

where $f_D = f_B + f_B v/c$ is the Doppler shifted frequency, can be written as

$$s(t) = A(t) \exp \{ -j [2 \pi f_B t + \phi(t)] \}$$

where the phase variation due to a finite receiver velocity is $\phi(t) = 2 \pi f_B t v/c$ (Porter et al., 1973). In general v is a function of time, $v = v(t)$. However, we assume that the sampling interval is short enough to consider v constant. In practice, an interval of .1 sec is used so the assumption is valid. Measurement of accumulated phase change provides

velocity and displacement information along the beacon-survey point vector, i.e., the displacement is the change in slant range. For m survey points there are $m - 1$ measurements of slant range differences for each beacon.

The measurements of travel times and phase changes may be made at the same or different survey points. Only the case where the measurements are made at the same points will be discussed in detail since this technique is more efficient in practice. The method of solution for the transponder coordinates is the same in either case but the associated errors will be different.

Non-linear regression methods as outlined by Draper and Smith (1966) are used to analyze this survey problem. Under the assumptions given in the following section the maximum likelihood estimator for the beacon positions is found. An approximate covariance matrix for the estimates and a procedure for optimizing the choice of survey points is developed. The sources of error and their effect on the estimates of the beacon coordinates are discussed.

II PARAMETER ESTIMATION

The problem of parameter estimation can be briefly described as follows: given a system from which information can be received and whose state is characterized by a set of p parameters, make some estimate of the state of the system from the information received. The information, which will generally be perturbed by noise, may be measurements, a message, or some other observations on the system.

The following definitions are used:

$$\underline{\theta}' = [\theta_1, \theta_2, \dots, \theta_p] = \text{true parameter values}$$

$$\hat{\underline{\theta}}' = [\hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_p] = \text{estimated parameter values}$$

$$\underline{F}' = \underline{F}'(\underline{\theta}) = [f_1, f_2, \dots, f_n] = \text{noiseless message}$$

$$\underline{Y}' = [y_1, y_2, \dots, y_n] = \text{message (observations perturbed by noise)}$$

where $\underline{\theta}$, $\hat{\underline{\theta}}$, \underline{F} , and \underline{Y} denote column vectors and $(\cdot)'$ denotes transpose (Manasse, 1960).

When a noiseless message $\underline{F}(\underline{\theta})$ is received, the information is sufficient to determine $\underline{\theta}$ exactly if the number of measurements n is not less than the number of parameters p . When $n = p$ the problem is called the MINIMUM DATA case. When $n > p$ it is called the REDUNDANT DATA case.

1) The Maximum Likelihood Estimate

When the observations \underline{Y} are received, some criteria must be used to determine an optimum estimate $\hat{\underline{\theta}}$ since \underline{Y} differs from \underline{F} due to the presence of noise. One often used decision rule is based on the maximum likelihood principle. This principle prescribes that the observer choose the $\hat{\underline{\theta}}$ which renders the observations \underline{Y} most likely; that is, it maximizes the conditional probability density $P(\underline{Y} | \underline{\theta})$. When expressed as a function of \underline{Y} and $\underline{\theta}$ this is called the likelihood function and is sometimes written $L(\underline{Y}; \underline{\theta})$. When the a priori probability density $P(\underline{\theta})$ can be considered to be a constant over the region of interest then the maximum likelihood estimator also maximizes $P(\underline{\theta} | \underline{Y})$. This can be shown by the use of Bayes Theorem to express the a posteriori probability density $P(\underline{\theta} | \underline{Y})$ in terms of $P(\underline{Y} | \underline{\theta})$ and $P(\underline{\theta})$:

$$P(\underline{\theta} | \underline{Y}) = \frac{P(\underline{Y} | \underline{\theta}) P(\underline{\theta})}{P(\underline{Y})} = C_1 P(\underline{Y} | \underline{\theta}) \quad (1)$$

where $P(\underline{Y}) = \int P(\underline{Y} | \underline{\theta}) P(\underline{\theta}) d\underline{\theta}$

(Note that $P(\underline{Y})$ is not a function of $\underline{\theta}$ and can be treated as a constant of proportionality).

The estimate $\hat{\underline{\theta}}$ which maximizes the likelihood function $L(\underline{Y}; \underline{\theta})$ must satisfy the equations

$$\frac{\partial L(\underline{Y}; \underline{\theta})}{\partial \theta_i} = 0 \quad i = 1, 2, \dots, p$$

for the p different parameters.

We now consider some properties of the maximum likelihood estimator and the special case of additive Gaussian noise.

2) Properties of the Maximum Likelihood Estimator

It is desirable that an estimate be both consistent, and unbiased. An estimator is consistent if the estimate, $\hat{\underline{\theta}}$, converges in probability to the true value $\underline{\theta}$. This is an asymptotic property because it concerns the behavior of an estimator as the number of measurements tends to infinity. An estimator is unbiased if the expected value of the estimate is equal to the true value, i.e.,

$$E(\hat{\underline{\theta}}) = \underline{\theta}$$

where $E(x) = \int x p(x) dx$.

In general the maximum likelihood estimator is not unbiased but it is consistent when the measurement errors are small.

The variance of an estimate,

$$V(\hat{\underline{\theta}}) = E \{ [\hat{\underline{\theta}} - E(\hat{\underline{\theta}})]^2 \} ;$$

is a measure of the fluctuation of an estimate about its mean.

Small variance is a desired property. An asymptotic measure of this property is efficiency. An estimator is said to be efficient if the asymptotic variance of the estimate is no larger than the asymptotic variance obtained using any other estimator. The maximum likelihood estimator, when it is continuous and the first and second derivatives exist and are absolutely integrable, is efficient. It can be shown that maximum likelihood estimates are asymptotically normally distributed with mean $\underline{\theta}$ and variance $1/R^2(\underline{\theta})$ where

$$R^2(\underline{\theta}) = E \left[\left(\frac{\partial \log L}{\partial \underline{\theta}} \right)^2 \right] \quad (\text{Kendall and Stuart, 1967}).$$

In the case of additive Gaussian noise the form of the likelihood function can be determined and expressions for the variance of the estimates can be developed.

3) The Maximum Likelihood Estimate in the Case of Additive Gaussian Noise

The message is assumed to be perturbed by additive Gaussian noise, \underline{e} , so that

$$\underline{Y} = \underline{F}(\underline{\theta}) + \underline{e} \quad (2)$$

where \underline{e} is characterized by an $n \times n$ moment matrix \underline{M} ;

$$\underline{M} = E \{ [\underline{Y} - \underline{F}] [\underline{Y} - \underline{F}]' \} = E(\underline{e}\underline{e}')$$

The multidimensional Gaussian distribution of the error can be expressed as

$$P(\underline{e}) = C_2 \exp \left\{ -1/2 [\underline{e}' \underline{M}^{-1} \underline{e}] \right\}$$

or

$$P(\underline{Y} - \underline{F}) = C_2 \exp \left\{ -1/2 [\underline{Y} - \underline{F}]' \underline{M}^{-1} [\underline{Y} - \underline{F}] \right\}$$

where C_2 is a positive constant (see, for example, p. 151 Davenport and Root, 1958).

From equation (2) it can be seen that the probability density of \underline{Y} given \underline{F} is equal to the probability density of the error; that is

$$P(\underline{Y}|\underline{F}) = P(\underline{e}) = P(\underline{Y} - \underline{F})$$

and since \underline{F} is a function of $\underline{\theta}$ this density is also the probability density of \underline{Y} given $\underline{\theta}$. The likelihood function, therefore, is equal to the probability density function of the error, namely a multidimensional normal distribution, that is,

$$L(\underline{Y}; \underline{\theta}) = P(\underline{Y}|\underline{\theta}) = P(\underline{Y} - \underline{F}(\underline{\theta})).$$

Thus

$$L(\underline{Y}; \underline{\theta}) = C_2 \exp \left\{ -1/2 [\underline{Y} - \underline{F}]' \underline{M}^{-1} [\underline{Y} - \underline{F}] \right\}. \quad (3)$$

The likelihood function is a maximum when the magnitude of the exponent, $[\underline{Y} - \underline{F}]' \underline{M}^{-1} [\underline{Y} - \underline{F}]$, is a minimum.

When the noise \underline{e} has zero mean and the error in each measurement is statistically independent of the errors in the

other measurements the moment matrix \underline{M} is diagonal

$$\underline{M} = E(\underline{ee}') = \begin{bmatrix} \sigma_1^2 & & 0 \\ & \sigma_2^2 & \\ 0 & & \ddots \\ & & & \sigma_n^2 \end{bmatrix}$$

$$\text{and } \underline{M}^{-1} = \begin{bmatrix} 1/\sigma_1^2 & & 0 \\ & 1/\sigma_2^2 & \\ & & \ddots \\ 0 & & & 1/\sigma_n^2 \end{bmatrix}$$

where σ_k^2 denotes the variance of the error in the k^{th} measurement. The moment matrix, \underline{M} , can be written in terms of weighting factors w_i and the standard error σ . The matrix of weighting factors is denoted by \underline{V}^{-1} .

$$\underline{M} = \underline{V} \sigma^2 = \begin{bmatrix} 1/w_1 & & 0 \\ & 1/w_2 & \\ & & \ddots \\ & & & 1/w_n \end{bmatrix} \sigma^2$$

and

$$\underline{V}^{-1} = \begin{bmatrix} w_1 & & 0 \\ & w_2 & \\ & & \ddots \\ 0 & & & w_n \end{bmatrix} \quad (4)$$

where $w_i = \sigma^2 / \sigma_i^2$.

The maximum likelihood estimate in this case is the weighted least squares estimate since the magnitude of the exponent in equation (3) is proportional to the weighted sum of squared errors:

$$1/2 [\underline{Y} - \underline{F}(\underline{\theta})]' \underline{M}^{-1} [\underline{Y} - \underline{F}(\underline{\theta})] = \frac{1}{2\sigma^2} \sum_{i=1}^n (y_i - f_i(\underline{\theta}))^2 w_i$$

When the variances of the errors are equal then the weighting factors, w_i , are equal to one, and the maximum likelihood estimate is the familiar least squares estimate which minimizes the sum of squared errors $SS(\underline{\theta})$:

$$SS(\underline{\theta}) = \sum_{i=1}^n (y_i - f_i(\underline{\theta}))^2.$$

Assume that a rather accurate estimate of $\underline{\theta}$ has been obtained and that $\underline{F}(\underline{\theta})$ can be approximated by a first order Taylor series

$$\underline{F}(\underline{\theta}) = \underline{F}(\underline{\hat{\theta}}) + \underline{X} \underline{\Delta\theta},$$

where $\underline{X} = \left. \frac{\partial \underline{F}}{\partial \underline{\theta}} \right|_{\underline{\hat{\theta}}}$ is an $n \times p$ matrix of derivatives with ij^{th}

element $\partial f_i / \partial \theta_j$ and the errors $\underline{\Delta\theta} = \underline{\theta} - \underline{\hat{\theta}}$ are assumed small.

When this expression for $\underline{F}(\underline{\theta})$ is substituted in equation (3), and when the a priori probability density $P(\underline{\theta})$ can be considered to be a constant over the region of interest, the probability density of $\underline{\theta}$ given \underline{Y} can be shown to be a multidimensional normal distribution with moment matrix $\underline{\Gamma}$:

$$P(\underline{\theta}|\underline{Y}) = C_3 \exp \left\{ -1/2 \underline{\Delta\theta}' \underline{\Gamma}^{-1} \underline{\Delta\theta} \right\} \quad (5)$$

where $\underline{\Gamma}^{-1} = \underline{X}'\underline{M}^{-1}\underline{X} = \frac{1}{\sigma^2} (\underline{X}'\underline{V}^{-1}\underline{X})$. (6)

The moment matrix $\underline{\Gamma}$ is known as the variance-covariance matrix since it consists of the variances (diagonal terms) and covariances (off-diagonal terms) of the estimates. It is also known as Fisher's information matrix (Van Trees, 1968). Specifically, under the conditions of uncorrelated, zero mean Gaussian noise, the variance of the estimates can be expressed as

$$\sigma_{\theta_i}^2 = \Gamma_{ii} = \sigma^2 \left[\sum_{k=1}^n \left(\frac{\partial f_k}{\partial \theta_i} \right)^2 W_i \right]^{-1}.$$

This estimate is approximately equal to the asymptotic variance $1/R^2(\underline{\theta})$ is close to $\underline{\theta}$.

4) Application of Parameter Estimation Techniques to the Survey Problem

The pulse-Doppler navigation system makes measurements which are characterized by a set of unknown parameters. The unknown parameters are the geometrical coordinates of the beacons and the ship positions at which the measurements are taken. The measurements consist of slant ranges and slant range differences.

An orthogonal XYZ coordinate system will be used in which the beacon coordinates are $(0,0,Z_1)$, $(X_2,0,Z_2)$, and (X_3,Y_3,Z_3) as shown in Figure 2. (Only the 3 beacon survey problem will be considered here; the method can easily be extended to more

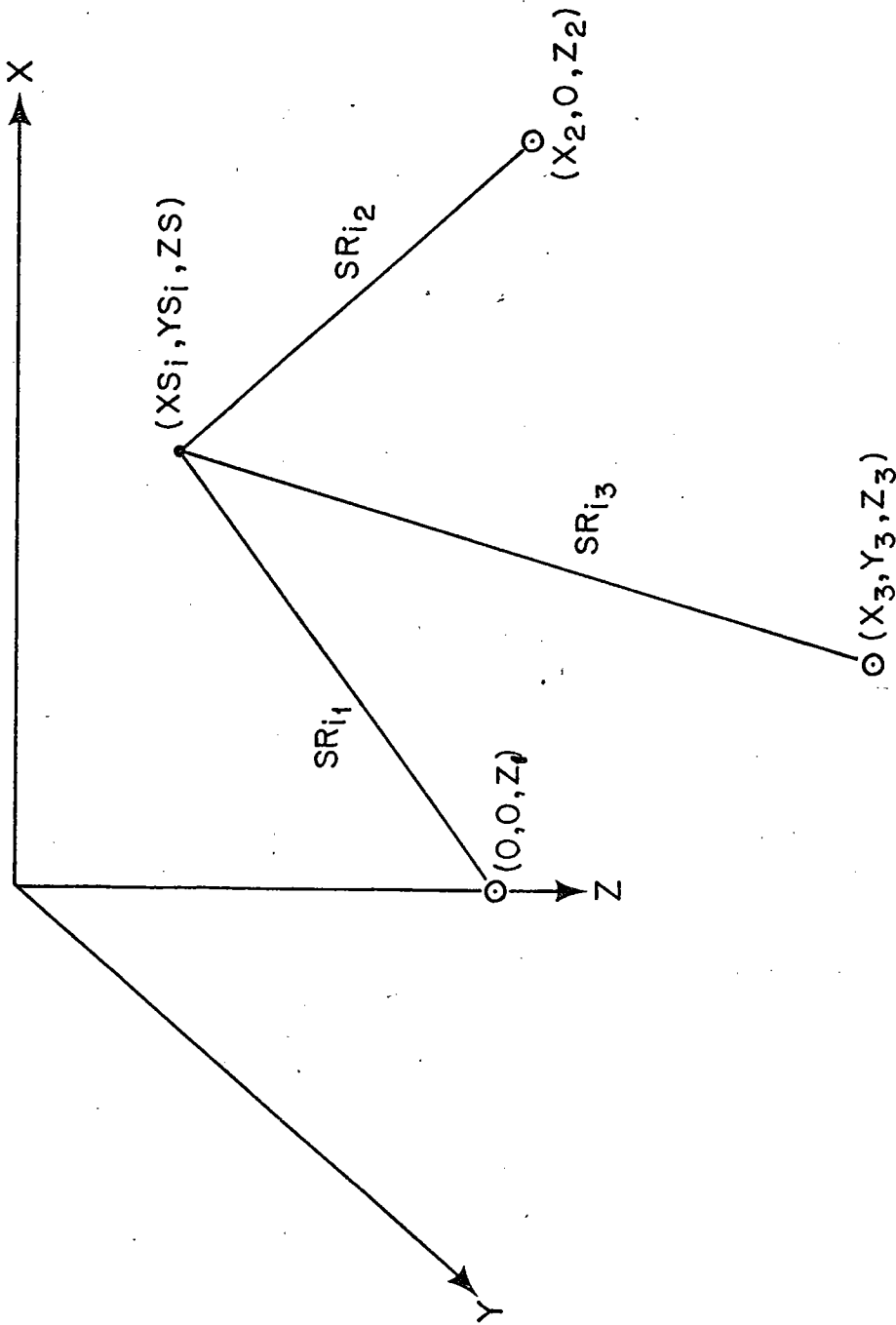


Figure 2. Survey Coordinate System

than 3 beacons). The depth of the ship hydrophone is assumed to be a known constant, Z_S . Simultaneous measurements of slant ranges and slant range differences are taken at m survey points (X_{S_i}, Y_{S_i}, Z_S) .

The unknown parameters, that is, the beacon and ship position coordinates, can be expressed as a column vector $\underline{\theta}$, with transpose

$$\begin{aligned}\underline{\theta}' &= [\theta_1, \theta_2, \dots, \theta_p] \\ &= [X_2, X_3, Y_3, Z_1, Z_2, Z_3, X_{S_1}, Y_{S_1}, \dots, X_{S_m}, Y_{S_m}]\end{aligned}$$

where $p = 2m + 6$.

The "noiseless measurements" are the exact geometrical slant ranges and slant range differences. These can be expressed as

$$SR_{ij} = SR_{ij}(\underline{\theta}) = [(X_{S_i} - X_j)^2 + (Y_{S_i} - Y_j)^2 + (Z_S - Z_j)^2]^{1/2}$$

where $X_1 = Y_1 = Z_1 = 0 \quad i = 1, 2, \dots, m$

$j = 1, 2, 3$

Similarly, the slant range differences between two survey points i and ii and the j^{th} beacon can be expressed as

$$\begin{aligned}DSR_{ij}(\underline{\theta}) &= [(X_{S_{ii}} - X_j)^2 + (Y_{S_{ii}} - Y_j)^2 + (Z_S - Z_j)^2]^{1/2} \\ &\quad - [(X_{S_i} - X_j)^2 + (Y_{S_i} - Y_j)^2 + (Z_S - Z_j)^2]^{1/2}.\end{aligned}$$

These functions can be expressed as a column vector $\underline{F}(\underline{\theta})$ with transpose,

$$\underline{F}'(\underline{\theta}) = [SR_{11}, SR_{12}, SR_{13}, SR_{21}, \dots, DSR_{11}, DSR_{12}, DSR_{13}, \dots].$$

For m survey points $\underline{F}(\underline{\theta})$ has dimensions $3m + 3(m-1) = 3(2m-1)$.

If $\underline{F}(\underline{\theta})$ could be measured without errors then some minimum

number of measurements would be sufficient to determine $\underline{\theta}$ exactly. In this case the slant range differences are linear combinations of the slant ranges and the minimum data case, as shown by Vanderkulk (1961), requires 18 slant range measurements:

$$SR_{ij}(\underline{\theta}), \quad i = 1, 2, \dots, 6, \quad j = 1, 2, 3$$

or equivalently, 3 slant range measurements and 15 slant range differences

$$SR_{1j}(\underline{\theta}), \quad j = 1, 2, 3$$

$$DSR_{ij}(\underline{\theta}), \quad i = 1, 2, \dots, 5, \quad j = 1, 2, 3$$

If only slant range differences can be measured then 24 measurements,

$$DSR_{ij}(\underline{\theta}), \quad i = 1, 2, \dots, 8, \quad j = 1, 2, 3$$

are sufficient to determine $\underline{\theta}$ exactly provided one has reasonable initial estimates of the parameters. This condition is a result of the linear approximation used to solve the non-linear equations.

The observations \underline{y} differ from $\underline{F}(\underline{\theta})$ due to the presence of noise. Slant range observations from the transponder system can be expressed as

$$S_{ij} = SR_{ij}(\underline{\theta}) + \epsilon_{ij}$$

where errors, ϵ_{ij} , are assumed to be independent identically distributed normal random variables with variance σ_ϵ^2 . Observations of slant range differences from the Doppler system can be expressed as

$$DS_{ij} = DSR_{ij}(\underline{\theta}) + \delta_{ij}$$

where the errors, δ_{ij} , are assumed to be independent identically distributed normal random variables with variance σ_δ^2 .

The errors for the complete system can be expressed as

$$\underline{e} = \begin{bmatrix} \underline{\epsilon} \\ \underline{\delta} \end{bmatrix} = [\underline{Y} - \underline{F}(\underline{\theta})] .$$

If the errors $\underline{\epsilon}$ and $\underline{\delta}$ are independent of each other, than \underline{e} is characterized by the moment matrix

$$\underline{M} = \underline{V}\sigma^2 = \begin{bmatrix} \sigma_1^2 & \sigma_2^2 & & 0 \\ & & \ddots & \\ 0 & & & \sigma_n^2 \end{bmatrix} \quad \begin{aligned} \sigma_i^2 &= \sigma_\epsilon^2 \\ i &= 1, 2, \dots, 3m \\ \sigma_{3m+i}^2 &= \sigma_\delta^2 \\ i &= 1, 2, \dots, 3(m-1) \end{aligned}$$

5) Gauss-Newton Method of Non-linear Least Squares Estimation

We have seen that the maximum likelihood estimator under conditions of additive, uncorrelated, Gaussian noise with zero mean is equivalent to the weighted least squares estimator and that this estimator is consistent and efficient. The weighted least squares estimate is the $\hat{\underline{\theta}}$ which minimizes the residual sum of squares

$$SS(\underline{\theta}) = \underline{e}' \underline{V}^{-1} \underline{e} = [\underline{Y} - \underline{F}(\underline{\theta})]' \underline{V}^{-1} [\underline{Y} - \underline{F}(\underline{\theta})] . \quad (7)$$

In general the estimate $\underline{\theta}$ is the solution of the p normal equations

$$\frac{\partial SS(\underline{\theta})}{\partial \theta_i} = 0 \quad i = 1, 2, \dots, p$$

assuming that the solution is interior to the parameter space.

When these equations are linear they can be solved directly.

When the normal equations are non-linear as they are in this case an iterative technique must be used. A number of techniques exist; for example, quasi-Newton, conjugate gradient, etc. A brief review of numerical techniques for fitting non-linear models is given by Chambers (1973).

The method used to find the estimate $\hat{\theta}$ is based on a modified Gauss-Newton procedure. The Gauss-Newton procedure consists of linearly approximating the function $F(\theta)$ and applying standard linear least squares techniques.

One linear approximation of $F(\theta)$ about an initial estimate $\hat{\theta}_0$ is a simple Taylor series

$$F(\theta) = F(\hat{\theta}_0) + \underline{X} \underline{\Delta\theta}$$

where $\underline{X} = \left. \frac{\partial F}{\partial \theta} \right|_{\hat{\theta}_0}$ is an $n \times p$ matrix of derivatives with

elements $x_{ij} = \left. \frac{\partial f_i}{\partial \theta_j} \right|_{\hat{\theta}_0}$ and $\underline{\Delta\theta} = \theta - \hat{\theta}_0$ is a column vector

of length p . Choosing the new variable $\underline{z} = \underline{y} - F(\hat{\theta}_0)$ we have the linear model

$$\underline{z} = \underline{X} \underline{\Delta\theta} + \underline{e}.$$

The linear approximation to the residual sum of squares is

$$SS(\theta) = \underline{e}' \underline{V}^{-1} \underline{e} = \left[\underline{z} - \underline{X} \underline{\Delta\theta} \right]' \underline{V}^{-1} \left[\underline{z} - \underline{X} \underline{\Delta\theta} \right] = SS(\hat{\theta}_0).$$

The Gauss-Newton step $\underline{\Delta\theta}$ is obtained by linear regression techniques on this linear residual sum of squares

(Draper and Smith, 1966):

$$\underline{\Delta\theta} = (\underline{X}'\underline{M}^{-1}\underline{X})^{-1} \underline{X}'\underline{V}^{-1}\underline{Z} \quad .$$

A new estimate $\hat{\theta}_1 = \hat{\theta}_0 + \underline{\Delta\theta}$ can be formed and the process repeated, until a given convergence criteria is reached.

6) Modifications of the Basic Gauss-Newton Procedure

Modifications of the basic Gauss-Newton procedure can be made to insure that the step $\underline{\Delta\theta}$, will reduce the residual sum of squares and avoid singularity problems in the $\underline{X}'\underline{M}^{-1}\underline{X}$ matrix. A complete discussion of the modifications used, based on stepwise regression techniques, is contained in Jennrich and Sampson (1967). A summary of that discussion is presented here.

One important modification of the basic Gauss-Newton procedure is the use of partial steps, $\eta \underline{\Delta\theta}$, in place of the full step $\underline{\Delta\theta}$ when the full step results in an increase in the residual sum of squares $SS(\hat{\theta})$. This frequently occurs when the linear approximations upon which the method is based fail. The proportion η to be used is obtained by trying the arbitrary sequence of values 1, 1/2, 1/4, 1/8, ... until the residual sum of squares is reduced. It can be shown that a sufficiently small step in the direction of $\underline{\Delta\theta}$ will always reduce the residual sum of squares unless $\hat{\theta}$ is already a minimizing value.

Another modification of the basic procedure is dictated by the possibility that the $\underline{X}'\underline{V}^{-1}\underline{X}$ matrix is, or is nearly, singular. Step-wise regression techniques are used where the parameter selected at a given step is the one which makes the greatest reduction in the residual sum of squares. If there is a singularity problem then only a subset of the parameters is used. This also provides a convenient way to handle boundary restrictions on the parameter values. A parameter is modified only if the new parameter value is within or on the parameter boundary.

7) Iterative Technique

In summary the procedure used consists of the following steps:

- 1) The value of the function $\underline{F}(\underline{\theta})$ and its derivatives \underline{X} are calculated using an initial estimate $\hat{\underline{\theta}}_0$.
- 2) The $\underline{X}'\underline{V}^{-1}\underline{X}$ matrix is formed and the residual sum of squares $SS(\hat{\underline{\theta}}_0)$ is computed.
- 3) The $\underline{X}'\underline{V}^{-1}\underline{X}$ matrix is inverted in a step-wise manner and the step size $\underline{\Delta\theta} = (\underline{X}'\underline{V}^{-1}\underline{X})^{-1} \underline{X}'\underline{V}^{-1}\underline{Z}$ is formed.
- 4) A new estimate $\hat{\underline{\theta}}_1 = \hat{\underline{\theta}}_0 + \underline{\Delta\theta}$ is formed and the new residual sum of squares $SS(\hat{\underline{\theta}}_1)$ computed. If $SS(\hat{\underline{\theta}}_1)$ is larger than $SS(\hat{\underline{\theta}}_0)$ then the step size is halved until $SS(\hat{\underline{\theta}}_1) \leq SS(\hat{\underline{\theta}}_0)$ or until $\eta = (1/2)^{10}$.

5) This completes an iteration. The process is repeated until the solution converges. Convergence is determined by the relative change in the residual sum of squares, i.e. the solution has converged when the relative change is less than a specified criterion, C,

$$\frac{|SS(\hat{\theta}_{n+1}) - SS(\hat{\theta}_n)|}{SS(\hat{\theta}_{n+1})} = C' < C .$$

6) In practice we require that 4 successive iterations result in $C' < C$ in order to accept the hypothesis of convergence. In some cases the algorithm may fail to converge to a minimizing solution (e.g. when it converges to a local, rather than a global, minimum). A plot of the parameter estimates and examination of the residuals will usually provide some indication of this.

8) Error Estimates

We have seen that maximum likelihood estimates are asymptotically normally distributed with known variances and that these variances are the diagonal elements of the variance-covariance matrix $\underline{\Gamma}$. For a process perturbed by uncorrelated Gaussian noise with zero mean this matrix is given by the expression

$$\underline{\Gamma} = (\underline{X}' \underline{V}^{-1} \underline{X})^{-1} \sigma^2 \text{ where } \underline{V}^{-1} = \begin{bmatrix} w_1 & & 0 \\ & w_2 & \\ 0 & & \cdot w_n \end{bmatrix} .$$

The $(\underline{X}'\underline{V}^{-1}\underline{X})$ matrix is called the correlation matrix and its inverse will be denoted by \underline{A} .

The standard error in estimating the i^{th} parameter, σ_{θ_i} is the square root of the i^{th} diagonal element of the variance-covariance matrix $\underline{\Gamma}$:

$$\sigma_{\theta_i} = \sigma \sqrt{a_i} \quad (8)$$

where a_i is the i^{th} diagonal element of the \underline{A} matrix

$$a_i = \left[\sum_{k=1}^n \left(\frac{\partial f_k}{\partial \theta_i} \mid \hat{\underline{\theta}} \right)^2 w_k \right]^{-1} \quad (9)$$

The terms on the right hand side of equation (8) are independent. The standard error, σ , depends upon the accuracy with which the slant ranges and slant range differences can be measured. An estimate of the standard error can be obtained from the residual sum of squares

$$\hat{\sigma}^2 = \frac{1}{n-p} SS(\hat{\underline{\theta}})$$

where $SS(\hat{\underline{\theta}})$ is defined in equation (7).

The other term, $\sqrt{a_i}$, is sometimes called the error magnification factor and is a function of the survey geometry. The magnitude of the error magnification can be obtained by evaluating equation (9).

The approximate errors in estimating the beacon coordinates

$$\hat{\sigma}_{\theta_i} = \hat{\sigma} \sqrt{a_i} \quad i = 1, 2, \dots, 6$$

can be reduced in two ways: 1) improvements in the measurement accuracy and 2) a suitable selection of the survey coordinates.

In the next section, we will consider in detail how this may be accomplished.

III MINIMIZATION OF ERRORS

1) Measurement errors (pulse system)

Slant ranges and slant range differences are measured by two different systems for which the magnitude and sources of error will in general be different. A discussion of these errors requires a complete description of the system operations.

The pulse system is designed to measure round trip travel time of an acoustic pulse between a ship-mounted or ship-suspended transducer and near-bottom transponders. When an acoustic pulse is transmitted from the transducer, digital counters in a shipboard receiver begin measuring elapsed time. The transponders receive and recognize the transmitted pulse and generate a "reply" pulse at a specific frequency. When a return pulse is detected by the shipboard receiver at a given frequency the corresponding counter is stopped and the elapsed time is displayed and transferred to a digital computer for processing.

There are fixed time delays associated with this process; e.g. signal recognition time, transponder turn around time and signal processing time. Accurate calibration of the system can reduce the uncertainties in each of these delay times to approximately ± 0.5 milliseconds (msec). For most applications these timing errors can collectively be considered to be normally

distributed with zero mean and standard error of 1 msec. Two of these timing errors, signal recognition time at the transponder and at the ship are functions of the signal-to-noise ratio and the pulse bandwidth. For example, the errors in recognition time increase to approximately .8 to 1 msec for a signal-to-noise ratio at the pulse receiver of 27 dB. This signal-to-noise is typical of a range of 10 km in sea state 3 for the system presently in use (source level: 189 dB, re: $1 \mu\text{Pa}$ pulse length: 10 msec). For ranges greater than about 11 km (for the present system configuration) signal recognition is erratic and the errors can be assumed infinite. For fixed bandwidth, timing errors can be reduced by increasing signal power or they may be accounted for by some appropriate weighting of the data.

Another source of error in the measurement of travel times is ship motion. The error in round trip travel time due to horizontal ship motion is given approximately by the expression

$$\epsilon = TT_m V_x \cos \theta / c$$

where TT_m = measured round trip travel time

θ = angle from the vertical between the transponder and ship transducer

V_x = horizontal component of ship speed away from the transponder

\bar{c} = average sound speed

For ship speeds of 1 knot and measured travel times of 6 to 12 seconds (typical in deep ocean applications) the round trip travel times will be in error by approximately 2 to 4 msec.

This error can be reduced by taking measurements while stopped or by reprocessing corrected data once the approximate ship-positions and speeds are known. Other ship motion such as heave, pitch and roll cannot easily be accounted for but when a large number of measurements are used their effect can be considered as an uncertainty in transducer depth equivalent to a timing error of approximately 1 msec.

A summary of these errors is present in Table 1. To achieve these errors in measured travel time of a few milliseconds it is essential to use an accurately calibrated system and make the measurements while stopped. These errors can be assumed to have zero mean for a large number of measurements.

To obtain geometric slant range from corrected travel time the velocity of sound must be known. Although the sound velocity has spatial and temporal variations, horizontal and temporal variations can be assumed negligible. Sound velocity variation over depth, however, is commonly on the order of 50 meters/sec for 5 km deep water. The structure of this variation, the sound velocity profile, can be determined from

Table 1. Sources of errors in measurement of Travel Time

| Source of Error | σ (msec) | σ (meters) |
|---------------------------------------|-----------------|-------------------|
| Recognition Time | 1.0 | 1.5 |
| Miscellaneous Timing Errors | 1.0 | 1.5 |
| Horizontal Motion ($V_x < 1/2$ knot) | 1.0 | 1.5 |
| Other Motion | 1.0 | 1.5 |
| Total = $(\sum \sigma^2)^{1/2}$ | 2.0 | 3.0 |

historical data or direct measurements. Historical data can be expected to be in error by 2.5 m/sec or more, whereas present measurement techniques have an accuracy of about .25 m/sec. For a precision navigation system the use of direct measurements is strongly preferred. When the sound velocity profile is known the travel time of an acoustic signal between two points can be determined and an effective sound speed, V_e , can be calculated.

$$V_e = SR/TT$$

When the travel time is measured using a pulse navigation system the effective sound speed is not known since it depends upon the positions of the transponders and ship transducer. The slant range, however, can be determined approximately by using a calculated range R , equal to the corrected one-way travel time TT_c multiplied by the arithmetic mean sound velocity \bar{c} :

$$SR \approx R = \bar{c}TT_c$$

$$\text{where } \bar{c} = \frac{1}{Z_i - ZS} \int_{Z_i}^{ZS} c(z) dz$$

The depth of the transducer ZS is usually known and errors of less than 50 meters in the estimate of transponder depth, Z_i , have a negligible effect on \bar{c} . The arithmetic mean sound

velocity can therefore be considered a known constant.

The ratio of slant range, SR, to calculated range $R = \overline{c}TT_c$, can be expressed as a function $K(Ve)$:

$$K(Ve) = SR/R = Ve/\overline{c}$$

For a given sound velocity profile and depths Z_S and Z_i the effective sound speed, Ve , is a monotonically increasing function of the geometric angle between the transponder and the survey point measured from the vertical (Vass, 1966) (see Fig. 3). Since the calculated range, R , is also an increasing function of the geometric angle, $K(Ve)$ can be expressed as a function of R and approximated by a second order polynomial

$$K(Ve) = K(R) \approx a_0 + a_1R + a_2R^2 .$$

The coefficients, a_1 , are found by calculating the range R and the ratio SR/R for various angles and applying a second order curve fitting subroutine. Travel times are computed using standard ray acoustic techniques (Officer, 1958). A description of program ARCOR which performs these computations is contained in Appendix A. The error in using the relationship $SR = R$ is usually less than 1.5 meters but can be as great as 3 or 4 meters for angles greater than 80° . When the relationship $SR = R(a_0 + a_1R + a_2R^2)$ is used the errors in slant range estimates are less than .1 meters. When the sound velocity profile is accurately known, errors caused by acoustic

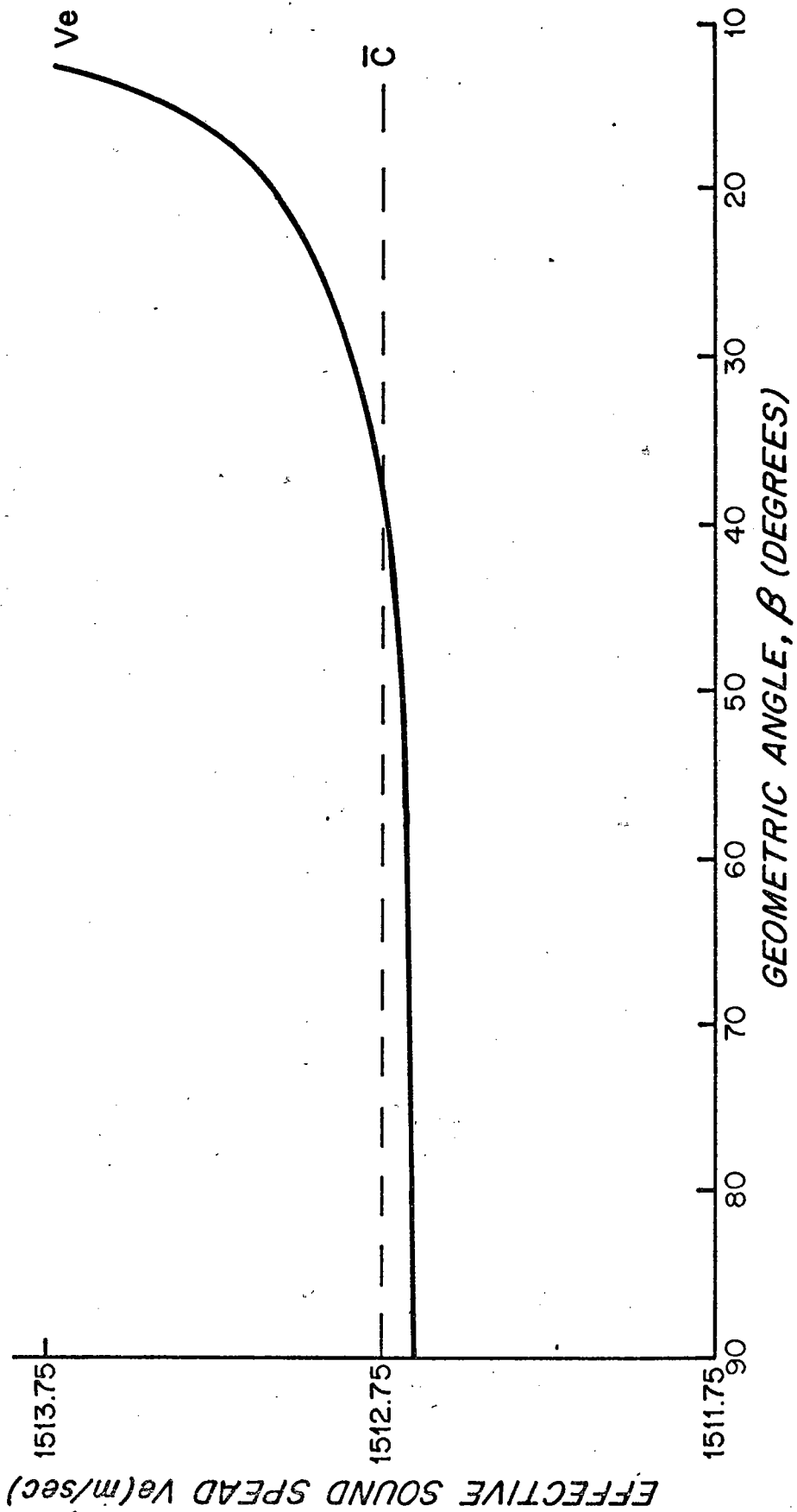


Figure 3. Typical Variation of Effective Sound Speed $V_e = SR/TT$ vs. Geometric Angle between Source and Receiver.

refraction are negligible compared to timing and transducer motion errors.

2) Measurement Errors (Doppler system)

The Doppler system in its present configuration (Fig. 4) consists of three bottom-moored CW beacons each separated by 50 Hz, a phase detector for each beacon and a real time computer processor. Each beacon contains a crystal oscillator with a frequency stability of 10^{-5} Hz and has a nominal source level of 166 dB (re ± 1 Pascal @ 1 m). The beacon signal after being processed by its phase detector is recorded on analog magnetic tape and is fed to a digital computer for real time processing. The phase detector output consists of two base-band outputs one proportional to the sine of the input phase angle, the other proportional to the cosine of the phase angle. Phase variations due to a finite receiver velocity $v(t)$, can be written as $\phi(t) = 2 \pi f_B tv(t)/c$. If the input signal is represented by

$$s(t) = A(t) \exp \left\{ -j \left[2 \pi f_B t - \phi(t) \right] \right\}$$

the phase detector outputs are given by

$$s_1(t) = A(t) \cos \phi(t)$$

$$\text{and } s_2(t) = A(t) \sin \phi(t) .$$

The instantaneous phase of the beacon signal can be determined from the ratio

$$s_2(t)/s_1(t) = \tan \phi(t) .$$

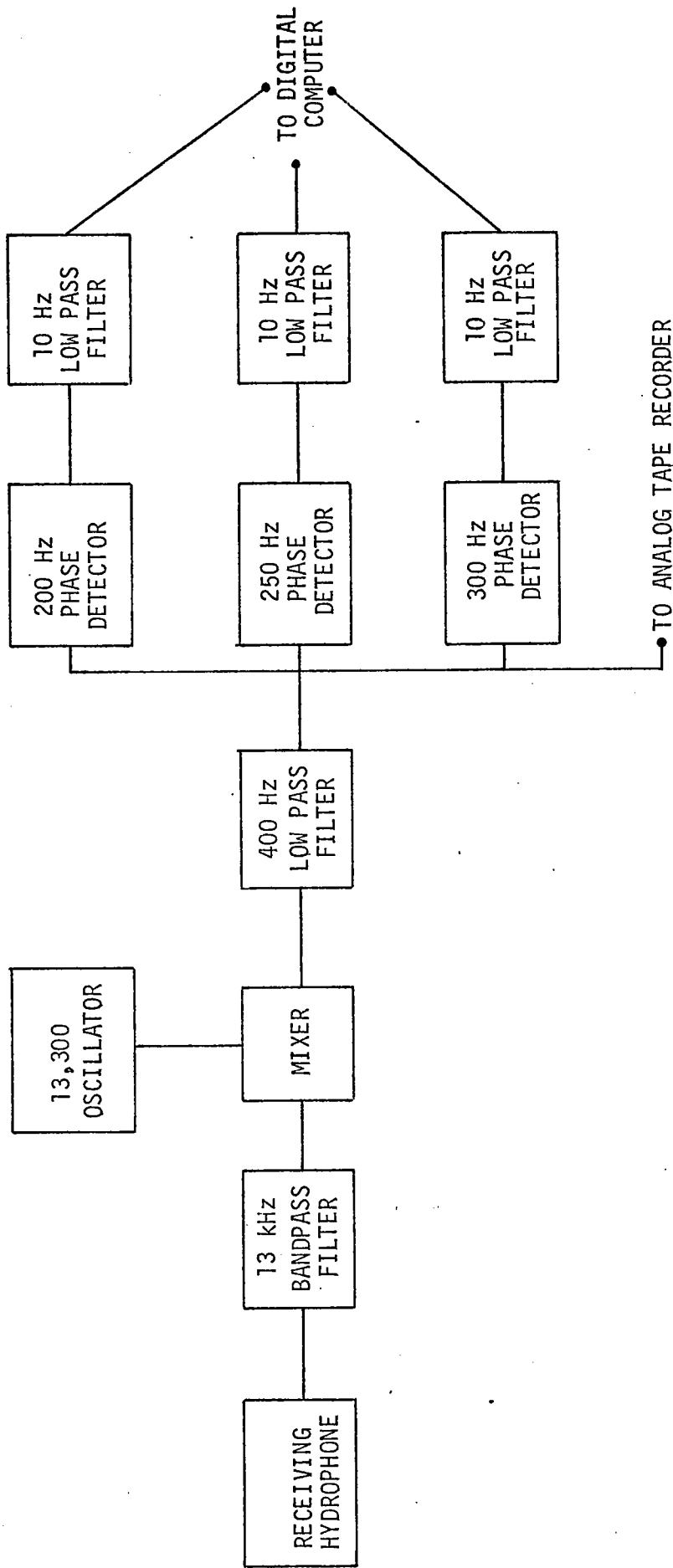


Figure 4. W.H.O.I. Doppler Receiver

In practice the quadrant into which the phase angle falls is determined simply by examining the signs of $s_1(t)$ and $s_2(t)$. Phase differences between phase detector samples are computed and accumulated in quanta of $\lambda/4 \approx 3$ cm at 13 kHz. The accumulated phase difference after T seconds is of the form

$$\phi_T = \sum_{n=1}^N [\phi(t_n) - \phi(t_{n-1})] \quad , \quad N = Tf_s$$

where f_s is the sampling frequency of the phase detector outputs (10 Hz) (Porter et al., 1973).

The distance, DSR, travelled toward or away from a beacon in time T is the accumulated phase ϕ_T multiplied by the wavelength λ of the signal:

$$DSR = \phi_T \lambda$$

The error in measurement of accumulated phase depends upon the number of samples $N = Tf_s$ and the signal detection probabilities of the receiver. Since the samples can be considered to be independent measurements the variance of the accumulated phase is the sum of the variance of the individual samples

$$\sigma_{\phi_T}^2 = \sum_{n=1}^N \sigma_{\phi_n}^2 = N \sigma_{\phi_n}^2$$

and $\sigma_{\phi_n}^2 = f(P_e)$

where P_e is the probability of error in the n^{th} sample. The probability of error is a function of the signal-to-noise

ratio at the phase detector (see Appendix B). Table 2 shows some representative values of P_e in a .1 sec interval for various signal-to-noise ratios encountered in practice. The associated errors σ_{ϕ_n} and σ_{ϕ_T} ($T = 1$ hr) are also given in both number of quadrants and meters. A quarter wavelength of 3 cm is assumed. Nominal horizontal range is calculated for a source level of 166 dB (re: $1 \mu\text{Pa}$) and a noise level of 44 dB/Hz (re: $1 \mu\text{Pa}$) assuming transmission losses are due only to spherical spreading ($20 \log r$) and attenuation (1 dB/km). Sample calculations are given in Appendix C.

Random phase fluctuations can be caused by multipath interference and forward scattering. The root mean square phase fluctuations from these sources has been estimated to be about .13 quadrants (Porter et al., 1973). When compared with the errors due to ambient sea noise shown in Table 2 they have little effect (about 5-15% increase in σ_{ϕ_T}) and can be considered negligible.

Slant range difference is obtained from accumulated phase change by multiplication by a scaling factor: the wavelength of the signal. Errors in wavelength $\lambda = c/f_B$ are caused primarily by errors in sound velocity (the instability of the beacon frequency is negligible). When the arithmetic mean

TABLE 2. Doppler Errors vs Signal-to-Noise Ratio

| 10 log S/N (dB) | P_e | σ_{ϕ_n} (quad) | σ_{ϕ_T} (1 hr) | | X (km) |
|--------------------|-------|-----------------------------|--------------------------|----------|-----------|
| | | | (quad) | (meters) | |
| 33 | .023 | .217 | 41 | 1.2 | 0 |
| 30 | .031 | .253 | 48 | 1.4 | 3.7 |
| 27 | .042 | .293 | 56 | 1.7 | 5.5 |
| 24 | .055 | .341 | 65 | 1.9 | 7.5 |
| 21 | .072 | .393 | 75 | 2.2 | 9.2 |
| 18 | .095 | .456 | 87 | 2.6 | 11.1 |

sound velocity of an accurately measured sound velocity profile is used, the standard error in wavelength will usually be less than 1 meter. This can be reduced further by using the average sound,

$$\dot{S} = \text{length of travel path/travel time,}$$

once the approximate survey positions are known.

In summary, when operating above the pulse system threshold, slant ranges can be measured by the pulse system with an accuracy of 3 to 5 meters. Under the same conditions slant range differences can be measured with an accuracy of 1.2 to 2.5 meters for measuring intervals of 1 hour.

3) Selection of Survey Coordinates

The error in estimating beacon coordinates also depends on the error magnification terms of the beacon coordinates,

$$M_i = \sqrt{a_i}, \quad i = 1, 2, \dots, 6$$

where the a_i are the i^{th} diagonal elements of the $\underline{A} = (\underline{X} \underline{V}^{-1} \underline{X})^{-1}$ matrix. These terms are dependent upon the number and relative locations of the survey points. In general the error magnifications will vary inversely with the degrees of freedom which increase with the number of survey points above a certain minimum. For example the slant range difference equations require at least 9 survey points for a solution to exist, and the degrees of freedom for this set of equations is equal to

the number of additional survey points. Comparison of survey techniques should be based on the same number of degrees of freedom. For example a pulse survey of 7 points (1 degree of freedom) should be compared to a Doppler survey of 10 points, and a pulse survey of 10 points should be compared to a pulse-Doppler survey of 10 points.

A measure of "good" survey locations is the trace of the beacon coordinate covariance sub-matrix, that is, the sum of the squares of the error magnification factors of the beacon coordinates:

$$\text{TRACE} = \sum_{i=1}^6 M_i^2 = \sum_{i=1}^6 a_i.$$

The trace is a function of the beacon and survey location coordinates, i.e. the parameter vector $\underline{\theta}$, and will be denoted by $\text{TR}(\underline{\theta})$. For a given number of survey locations there will be a set or sets of survey locations for which the trace is a minimum. A typical deep ocean deployment of the pulse-Doppler navigation system will be discussed for illustration.

The most commonly used 3-beacon acoustic navigation net is ideally an equilateral triangle. The baseline length, i.e. the length of the sides of the triangle, are typically on the order of the beacon depths: about 5 km. For this configuration of the net the covariance matrix was evaluated for several sets of survey locations, each consisting of 10 survey points.

A few examples are given in Figures 5A through 5E. The value of the trace of the beacon covariance matrix for each arrangement is listed in Table 3. The trace for the corresponding survey with only pulse data is given for comparison. A more rigorous technique for determining "optimum" survey locations was also used. Powell's algorithm to find a (local) minimum of a function of several variables was applied to a function $G(\underline{\theta}) = TR(\underline{\theta}) + C(\underline{\theta})$, where $C(\underline{\theta})$ is an arbitrary function added to the trace to constrain the survey locations within the maximum range of the system (Powell, 1965). No significant improvement to the starting value of $G(\underline{\theta}) \approx 3.6$ was realized although several survey geometries were tried.

4) Computer Simulation

A computer program called PDSRV was developed to solve the survey problem using the parameter estimation techniques discussed. It is a modification of a Biomedical Computer Program (BMD 07R) (see Dixon, 1973). The program requires pulse data from at least six survey points and/or Doppler data from at least ten survey points. In addition to the data, the user must input the average sound velocity and the acoustic refraction coefficients found by program ARCOR. The beacon frequencies, the depth(s) of the transducer and initial estimates of the beacon positions must also be provided.

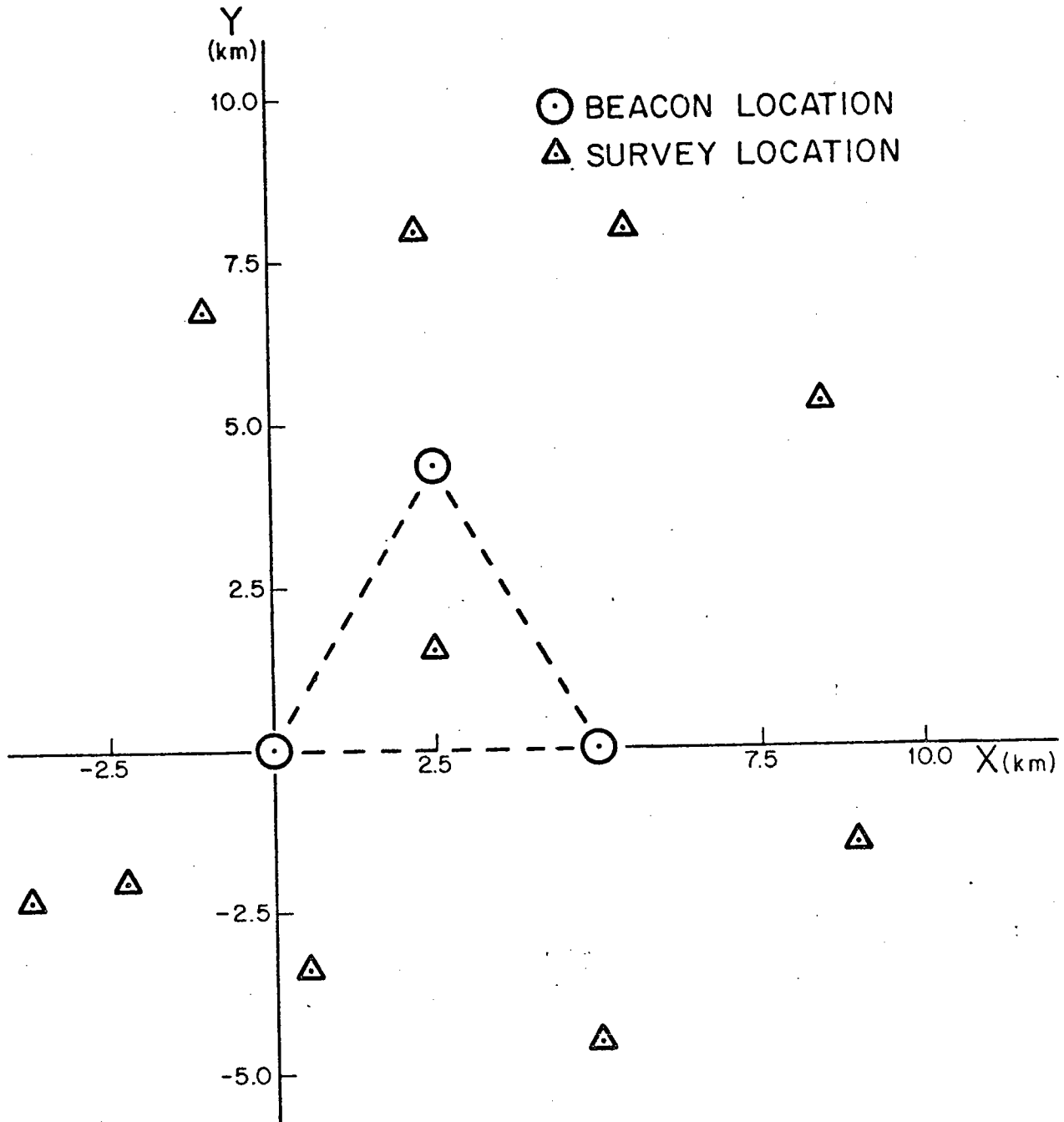


FIGURE 5A, TRACE = 3.6

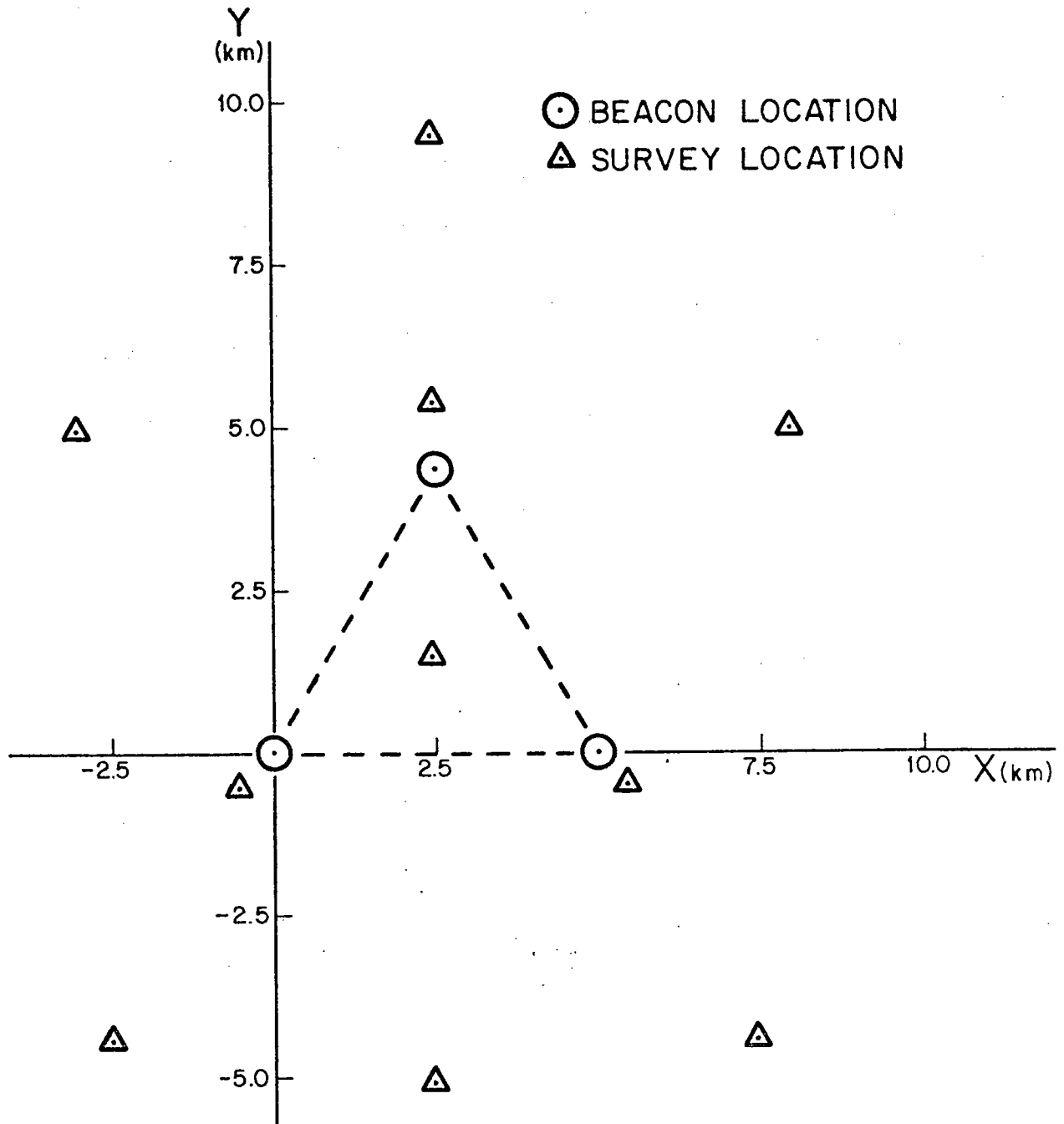


FIGURE 5B, TRACE = 4.0

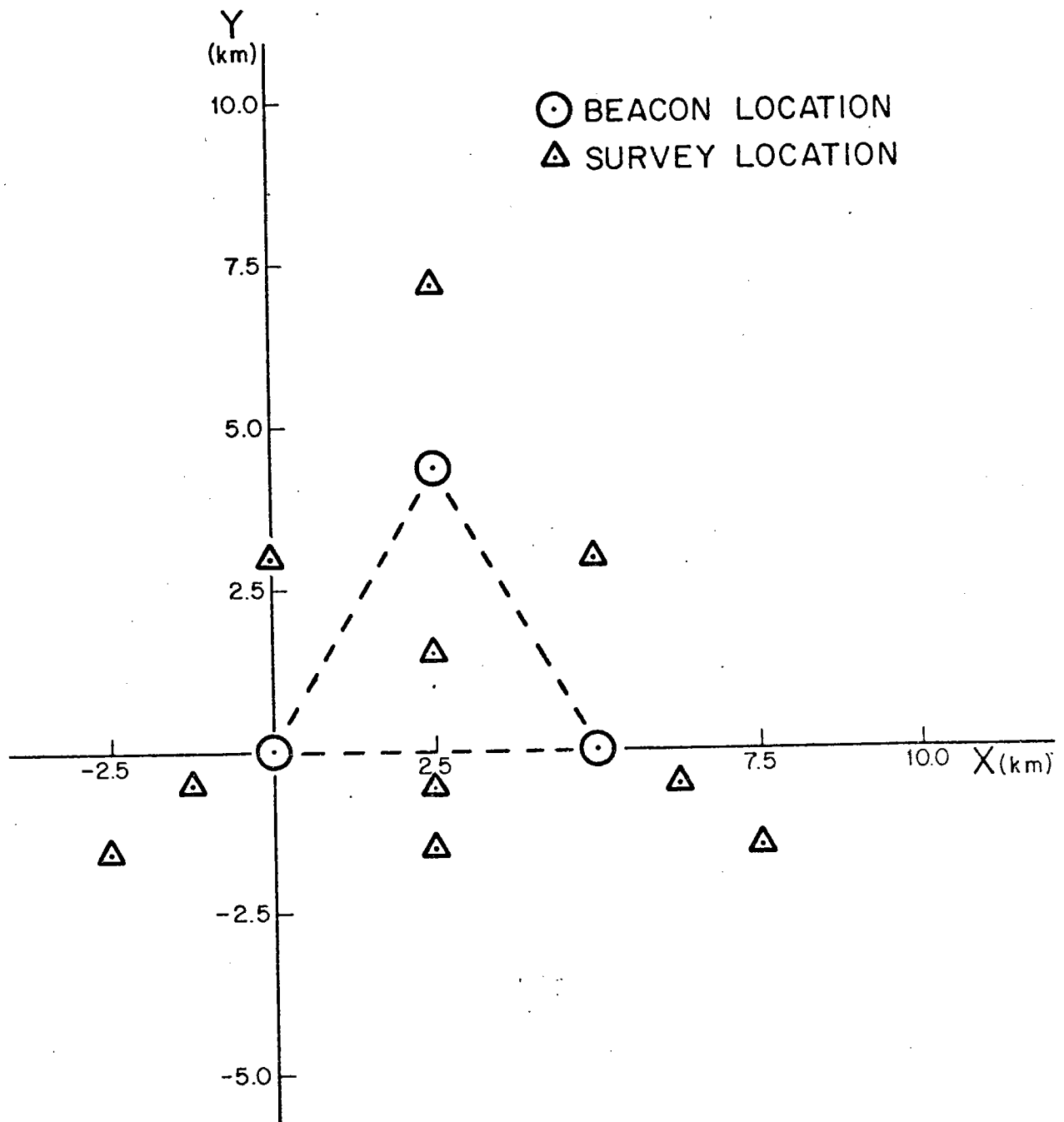


FIGURE 5C, TRACE = 4.3

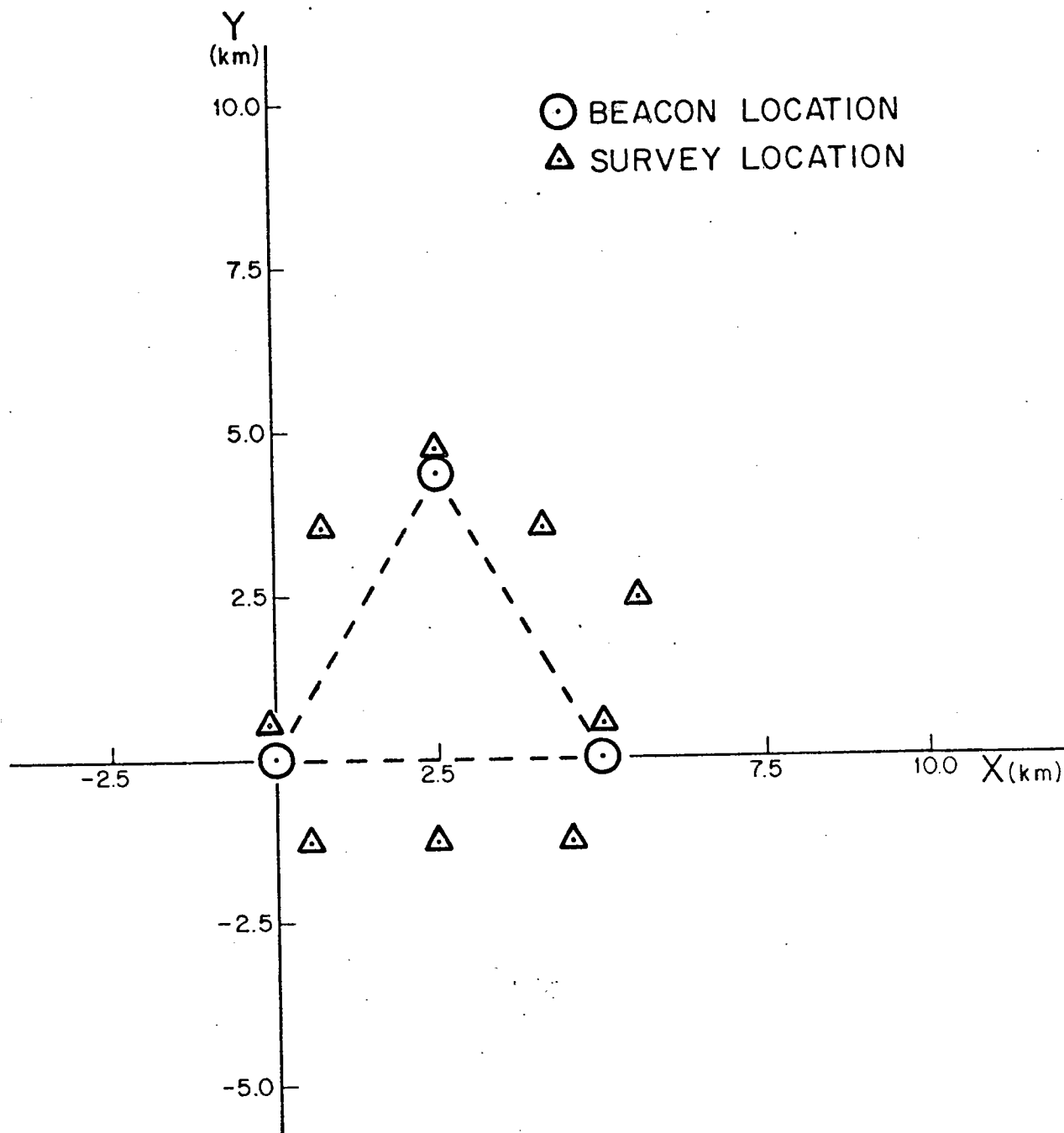


FIGURE 5D, TRACE = 17.0

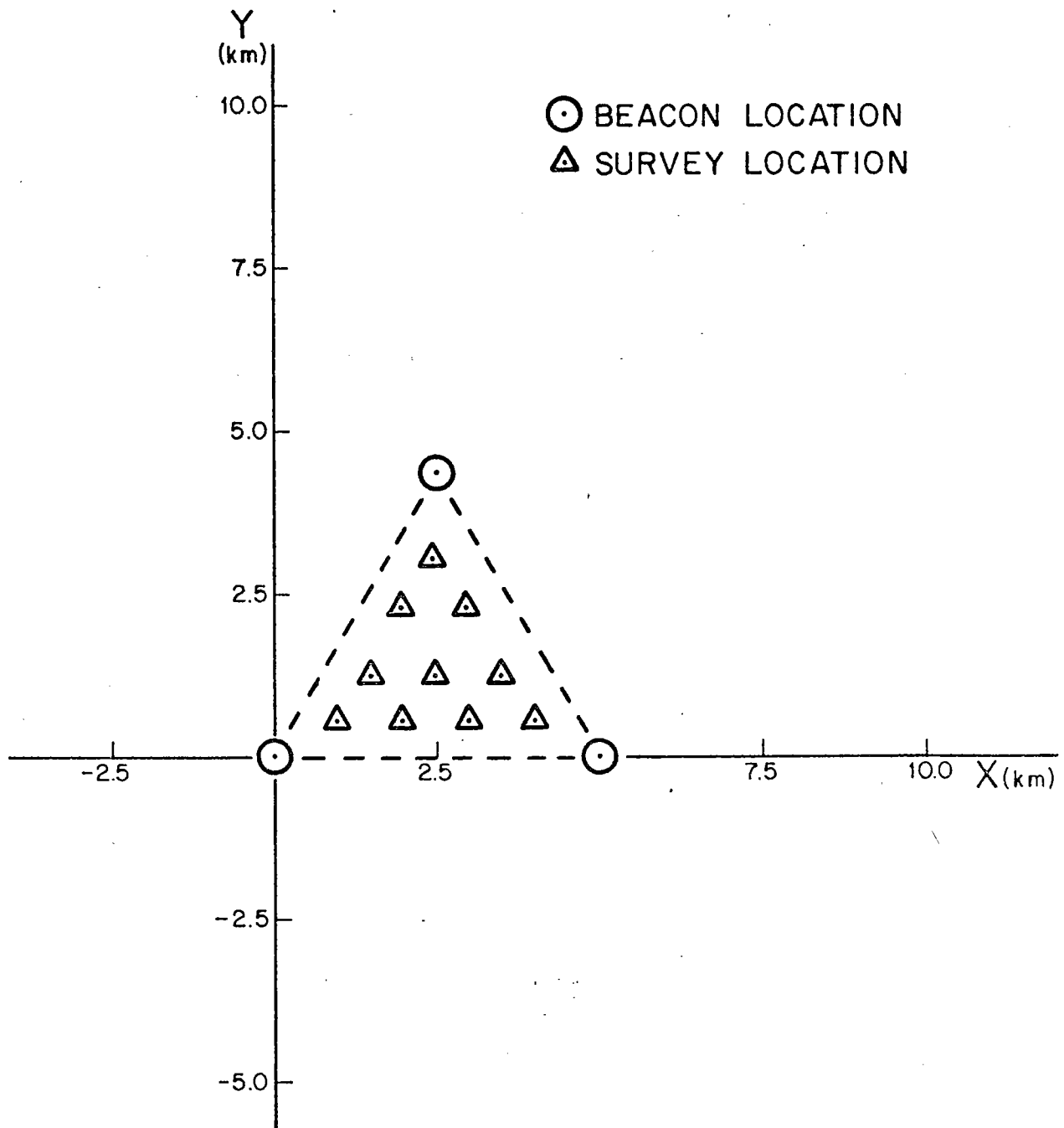


FIGURE 5E, TRACE = 138

TABLE 3. Comparison of the Trace of the Beacon Covariance Matrix
for Pulse/Doppler and Pulse Surveys

| Figure # | Trace Pulse & Doppler | Trace Pulse Only |
|----------|--------------------------|---------------------|
| 5A | 3.6 | 6.8 |
| 5B | 4.0 | 8.3 |
| 5C | 4.3 | 10.4 |
| 5D | 17.0 | 58.2 |
| 5E | 138 | 325 |

Maximum and minimum values for the beacon coordinates are constrained to provide more rapid convergence. The constraints are set as follows:

$$X_j = X_j \pm 2000 \text{ m}$$

$$Y_j = Y_j \pm 2000 \text{ m}$$

$$Z_j = Z_j \pm 1000 \text{ m}$$

where (X_j, Y_j, Z_j) are the initial estimates of the j^{th} beacon coordinates. Other parameter bounds may be specified if desired. Although it is unlikely, even with the most rudimentary primary navigation system, that initial estimate errors will exceed those specified above.

The program consists of three main sections: (1) initialization and data input; (2) minimization of weighted squared error; (3) determination of the approximate error covariance matrix of the beacon positions. For convenience the program is divided into 7 subroutines as illustrated in Figure 6. A description of each subroutine is given in Table 4.

Two additional subroutines were used to simulate survey problems. Subroutine READ calculates data for a given set of survey locations and beacon coordinates. Subroutine NORMA generates Gaussian distributed numbers with a specified mean and standard deviations. These are used to simulate data with errors.

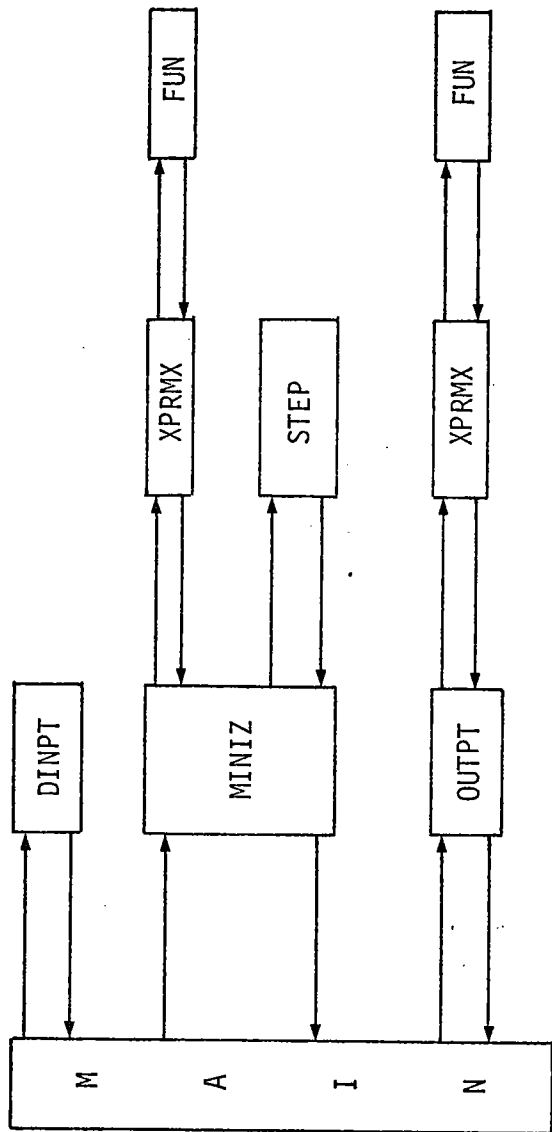


Figure 6. Program PDSRV.

TABLE 4. Description of Program PDSRV

| Name of Subroutine | Description |
|--------------------|--|
| MAIN | <p>Reads the following user inputs:</p> <ol style="list-style-type: none"> 1. Transducer depths 2. Convergence criteria 3. Bounds for beacon parameters 4. Beacon coordinate estimates 5. Survey position estimates (optional) 6. Bounds of Survey position (optional) |
| DINPT | <ol style="list-style-type: none"> 1. Reads input data from specified device(s) 2. Calculates slant ranges (SR) from travel times 3. Calculates changes in slant range (DSR) from accumulated phase change |
| MINIZ | <ol style="list-style-type: none"> 1. Computes the mean squared error (EE) from the weighted sum of squared errors: $EE = (1/DOF) SS(\hat{\theta})$ 2. Checks that EE is less than previous value; if not the step size is halved 3. Checks convergence criteria |
| XPRMX | <ol style="list-style-type: none"> 1. Computes the elements of the $\underline{X} \underline{V}^{-1} \underline{X}$ matrix 2. Calculates the weighted sum of square errors: $SS(\underline{\theta}) = \sum_{i=1}^n (y_i - f_i) w_i$ |

TABLE 4. Continued

| Name of Subroutine | Description |
|--------------------|---|
| FUN | <p>1. Calculates values of the slant range and slant range differences (f_i) using latest estimates of beacon and ship positions</p> <p>2. Calculates the elements of the \underline{X} matrix, i.e. the derivatives of the slant ranges and slant range differences:</p> $x_{ij} = \left. \frac{\partial f_i}{\partial \theta_j} \right _{\hat{\theta}}$ |
| STEP | <p>1. Inverts the $\underline{X} \underline{V}^{-1} \underline{X}$ matrix in a step-wise fashion</p> <p>2. Computes the step sizes $\underline{\Delta \theta}$</p> |
| OUTPT | <p>Computes and outputs the following</p> <ol style="list-style-type: none"> 1. Final estimates 2. Correlation matrix for Beacon Positions 3. Root mean square error (standard error, σ) 4. Error magnification terms of the covariance matrix 5. Residuals and standard errors of each measurement (optional) |

Typical output of a simulated survey problem is shown in Tables 5a through 5d. Table 5a lists the beacon and survey coordinates used by subroutine READ and the errors in data generated by subroutine NORMA. Table 5b lists the simulated data which is input to program PDSRV. The final estimates, correlation matrix, error magnification factors and standard errors of the beacon coordinates are also shown in Table 5b. Similar information for the survey locations is listed as illustrated in Table 5c. The residuals,

$$SR(\text{observed}) - SR(\text{estimated}) \text{ and}$$

$$DSR(\text{observed}) - DSR(\text{estimated})$$

can be compared with the estimates of standard error of each measurement. This output, labeled SURVEY ERRORS, is optional and is shown in Table 5c.

After each iteration of the minimization process the mean squared error and latest estimates of the beacon positions can be output as shown in Table 5d. The number of the iteration and the number of times the step size was halved is also included.

Survey problems with measurement errors of more than 750 meters have been simulated. The minimization process converged in every case and the beacon coordinate estimates were nearly normally distributed about the true values with the standard

| BEACON POSITIONS | | | |
|------------------|-----------|----------|------------------------|
| X | Y | Z | |
| .000 | .000 | 5000.000 | Beacon Position 1 |
| 5000.000 | .000 | 5000.000 | Beacon Position 2 |
| 2500.000 | 4330.000 | 5000.000 | Beacon Position 3 |
| SURVEY POSITIONS | | | |
| X | Y | Z | |
| 2500.000 | 1500.000 | .000 | Survey Position 1 |
| 2500.000 | 5300.000 | .000 | Survey Position 2 |
| -3000.000 | 5000.000 | .000 | Survey Position 3, etc |
| -700.000 | -700.000 | .000 | |
| -2500.000 | -4300.000 | .000 | |
| 2500.000 | 5000.000 | .000 | |
| 7500.000 | -4300.000 | .000 | |
| 5700.000 | -700.000 | .000 | |
| 8000.000 | 5000.000 | .000 | |
| 2500.000 | 10000.000 | .000 | |

ERROR IN PULSE DATA

| | | | |
|---------|---------|-----------------|--|
| -2.000 | -11.598 | .239 | |
| -7.301 | -4.215 | 19.437 | |
| -6.168 | -1.207 | -5.621 | |
| -9.410 | -12.574 | 15.113 | |
| 2.973 | -8.316 | 11.020 | Errors in the slant range measurements to beacons 1,2, and 3, made at each survey point. |
| .981 | -8.433 | 12.778 | |
| -5.387 | 7.075 | -9.840 | |
| -6.129 | 0.200 | 16.032 | |
| -11.246 | -15.035 | -18.199 | |
| 9.262 | -2.652 | 6.059 | |
| MEAN= | .000 | STD.DEV.= 10.00 | WT= 1.000 |

ERROR IN DOPPLER DATA

| | | | |
|---------|---------|-----------------|---|
| -16.027 | 11.101 | 1.064 | |
| 2.822 | -3.546 | 11.963 | |
| 9.345 | -11.397 | -.264 | |
| -7.257 | -12.374 | -15.616 | |
| 13.017 | 3.525 | -14.093 | Errors in the slant range difference measurements between survey points 1&2, 2&3, 3&4, etc. |
| .165 | -3.702 | 4.306 | |
| -5.811 | 5.947 | -.421 | |
| -4.913 | -7.530 | -8.272 | |
| -7.140 | -14.132 | .751 | |
| MEAN= | .000 | STD.DEV.= 10.00 | WT= 1.000 |

Table 5a. List of beacon and survey coordinates used for survey simulation. Random normal errors generated by Subroutine NORMA.

| PULSE SYSTEM DATA | | Slant Ranges to Beacons | | |
|-------------------|--------------|-------------------------|----------|----------|
| NR | TIME (H*M*S) | 1 | 2 | 3 |
| SLR 1 | 71*71*71 | 5785.11 | 5776.32 | 5745.58 |
| SLR 2 | 142*142*142 | 7695.95 | 7699.83 | 5112.72 |
| SLR 3 | 213*213*213 | 7674.98 | 10675.87 | 7457.55 |
| SLR 4 | 284*284*284 | 5687.65 | 7601.89 | 7765.69 |
| SLR 5 | 355*355*355 | 7855.63 | 9978.68 | 11167.84 |
| SLR 6 | 426*426*426 | 7500.98 | 7491.57 | 18598.89 |
| SLR 7 | 497*497*497 | 9981.61 | 7859.73 | 11147.88 |
| SLR 8 | 568*568*568 | 7688.33 | 5105.27 | 7763.97 |
| SLR 9 | 639*639*639 | 10665.83 | 7666.11 | 7444.97 |
| SLR 10 | 710*710*710 | 11465.70 | 11453.79 | 7565.75 |

(1)

| DOPPLER SYSTEM DATA | | Slant Range Differences with respect to Beacons | | |
|---------------------|--------------|---|----------|----------|
| NR | TIME (H*M*S) | 1 | 2 | 3 |
| DSL R 1 | 71*71*71 | .00 | .00 | .00 |
| DSL R 2 | 142*142*142 | 1898.50 | 1926.51 | -651.85 |
| DSL R 3 | 213*213*213 | -19.28 | 2970.29 | 2381.91 |
| DSL R 4 | 284*284*284 | -2574.74 | -3874.82 | 317.37 |
| DSL R 5 | 355*355*355 | 1948.34 | 2368.16 | 3368.58 |
| DSL R 6 | 426*426*426 | 468.36 | -2483.47 | -585.78 |
| DSL R 7 | 497*497*497 | 2487.16 | -451.84 | 575.91 |
| DSL R 8 | 568*568*568 | -2378.34 | -1949.65 | -3376.54 |
| DSL R 9 | 639*639*639 | 3857.71 | 2576.56 | -325.91 |
| DSL R 10 | 710*710*710 | 772.22 | 3761.16 | 97.27 |

(1)

| FINAL ESTIMATES | | |
|-----------------|---------|---------|
| X | Y | Z |
| .00 | .00 | 4995.80 |
| 5008.89 | .00 | 5063.22 |
| 2514.16 | 4325.92 | 5083.16 |

(2)

| CORRELATION MATRIX FOR BEACON POSITIONS | | | | | | |
|---|--------|--------|--------|--------|--------|--------|
| | X2 | X3 | Y3 | Z1 | Z2 | Z3 |
| | 1.0000 | .4986 | -.1951 | -.1038 | -.1242 | .1263 |
| | .4986 | 1.0000 | -.8442 | -.8884 | -.8872 | .8978 |
| | -.1951 | -.8442 | 1.0000 | .1299 | .1287 | .8339 |
| | -.1038 | -.8884 | .1299 | 1.0000 | .8189 | .1588 |
| | -.1242 | -.8872 | .1287 | .8189 | 1.0000 | .1313 |
| | .1263 | .8978 | .8339 | .1588 | .1313 | 1.0000 |

(3)

| | | | | | | |
|---------------------------------|---------|---------|--------|--------|--------|--------|
| ERR. MAG. | 1.8947 | 1.1326 | .6714 | .5998 | .5912 | .5850 |
| STD. ERR. | 10.9884 | 11.3896 | 6.7401 | 6.0128 | 5.9348 | 5.8725 |
| TRACE | = | 3.983 | | | | |
| STANDARD ERROR IN DOPPLER DATA= | | | 10.838 | | | |

(4)

Table 5b. (1) Simulated Pulse & Doppler measurements. (2) Final estimates of beacon positions. (3) Correlation matrix for beacon positions. (4) Error magnification factors and estimated standard errors for each beacon position.

SURVEY POSITIONS

| NR | X | ERR. MAG | STD. ERR | Y | ERR. MAG | STD. ERR | Z |
|----|----------|----------|----------|----------|----------|----------|-----|
| 1 | 2526.56 | 1.53 | 15.39279 | 1492.20 | 1.37 | 13.71078 | .00 |
| 2 | 2495.33 | 1.57 | 15.74754 | 5292.56 | .68 | 6.86014 | .00 |
| 3 | -2989.48 | 1.45 | 14.55447 | 4997.56 | 1.36 | 13.66243 | .00 |
| 4 | -676.73 | 1.37 | 13.72096 | -711.37 | 1.47 | 14.72249 | .00 |
| 5 | -2463.60 | 1.96 | 19.70095 | -4317.24 | 1.56 | 15.65258 | .00 |
| 6 | 2522.26 | 1.55 | 15.53885 | -4997.62 | .68 | 6.80046 | .00 |
| 7 | 7502.79 | 1.50 | 15.01551 | -4299.39 | 1.55 | 15.56735 | .00 |
| 8 | 5697.97 | 1.05 | 10.55979 | -698.21 | 1.46 | 14.67308 | .00 |
| 9 | 7995.83 | 1.25 | 12.57012 | 4996.21 | 1.36 | 13.62897 | .00 |
| 10 | 2531.10 | 2.68 | 26.86143 | 9991.16 | .62 | 6.19868 | .00 |

(1)

SURVEY ERRORS

| PT NR | RESIDUALS (OBS - EST) | | | STD. ERROR OF ESTIMATES | | NORM. SE. | |
|----------|-----------------------|--------|---------|-------------------------|-------|-----------|--------|
| | BEACON | | | BEACON | | | |
| | 1 | 2 | 3 | 1 | 2 | 3 | |
| 1 | -8.699 | -4.408 | -4.363 | 7.282 | 7.287 | 7.548 | 1.0643 |
| 2 | 2.057 | -5.346 | 16.997 | 6.713 | 6.719 | 5.768 | 1.7868 |
| 3 | -2.254 | .246 | -10.566 | 6.637 | 6.273 | 6.467 | 1.0766 |
| 4 | -3.725 | -4.394 | -10.124 | 5.924 | 6.680 | 6.672 | 1.8945 |
| 5 | -8.223 | -3.855 | -9.355 | 6.178 | 6.676 | 6.185 | 1.2989 |
| 6 | -2.087 | -4.283 | 16.977 | 6.384 | 6.356 | 5.984 | 1.7565 |
| 7 | -5.121 | 7.038 | -2.545 | 6.676 | 6.113 | 6.072 | .9034 |
| 8 | -1.690 | 6.673 | -8.429 | 6.691 | 5.895 | 6.625 | 1.0041 |
| 9 | -4.381 | -9.884 | -6.802 | 6.360 | 6.752 | 6.415 | 1.2751 |
| 10 | 11.980 | 8.660 | 7.521 | 7.545 | 7.570 | 6.138 | 1.6523 |
| 2 | -1.579 | 2.861 | 3.165 | 7.707 | 7.675 | 6.947 | .4532 |
| 3 | 1.806 | -9.961 | 9.518 | 7.418 | 7.189 | 6.792 | .9714 |
| 4 | -6.689 | -4.670 | 1.669 | 6.612 | 7.247 | 7.365 | .8231 |
| 5 | -7.692 | -8.383 | -14.270 | 5.169 | 7.409 | 6.892 | 1.8180 |
| 6 | 4.701 | -4.496 | -8.230 | 6.641 | 7.318 | 6.327 | 1.0450 |
| 7 | 3.498 | -7.890 | 7.402 | 7.330 | 6.644 | 6.365 | 1.1327 |
| 8 | -1.639 | -4.449 | .607 | 7.419 | 5.031 | 6.951 | .4772 |
| 9 | -2.486 | -.845 | -5.327 | 7.274 | 6.710 | 7.406 | .5916 |
| 10 | -11.286 | 7.978 | -9.134 | 7.689 | 7.754 | 7.093 | 1.6500 |

(2)

Table 5c. (1) Estimates of survey positions (X,Y) with their associated error magnification factors and estimated standard errors.
 (2) Residuals of slant range measurements (1-10) and slant range difference measurements (2-10).

| ITERATION | ERROR | MEAN | SQUARE | | | | | |
|-----------|-------|-------------|---------|---------|---------|---------|---------|---------|
| 0 | 0 | 1.19000E+07 | 4900.00 | 2000.00 | 4000.00 | 5200.00 | 4700.00 | 4600.00 |
| 1 | 0 | 5.77917E+06 | 4900.00 | 2000.00 | 4000.00 | 6200.00 | 5162.38 | 4690.23 |
| 2 | 0 | 4.61724E+06 | 5700.32 | 3579.39 | 3175.10 | 6200.00 | 3700.00 | 4725.52 |
| 3 | 0 | 2.70681E+06 | 4793.68 | 1637.97 | 3506.94 | 6200.00 | 3700.00 | 5600.00 |
| 4 | 0 | 1.40231E+06 | 3757.42 | 414.57 | 3093.15 | 6200.00 | 3700.00 | 4000.79 |
| 5 | 1 | 1.37630E+06 | 3404.73 | 207.20 | 4007.40 | 6200.00 | 4307.20 | 4903.93 |
| 6 | 0 | 1.36017E+06 | 3001.65 | .00 | 4037.62 | 6200.00 | 4097.46 | 5053.59 |
| 7 | 0 | 1.30778E+06 | 2900.00 | .00 | 4120.63 | 6200.00 | 5105.11 | 5040.62 |
| 8 | 0 | 1.05873E+06 | 2900.00 | 710.03 | 4624.55 | 6200.00 | 4900.54 | 4031.00 |
| 9 | 0 | 9.62426E+05 | 2900.00 | 1436.23 | 5121.00 | 6106.78 | 5203.44 | 4941.72 |
| 10 | 1 | 9.21551E+05 | 3260.94 | 2572.45 | 4757.03 | 6154.50 | 5172.11 | 5201.27 |
| 11 | 0 | 8.03565E+05 | 4015.20 | 4000.00 | 4497.50 | 6066.09 | 5116.21 | 5079.12 |
| 12 | 0 | 6.21777E+05 | 4675.70 | 4000.00 | 3039.10 | 5770.43 | 5047.44 | 5326.55 |
| 13 | 0 | 5.10541E+05 | 4741.90 | 3565.09 | 3025.33 | 5554.60 | 5175.16 | 5600.00 |
| 14 | 1 | 2.93913E+05 | 4761.22 | 2024.30 | 3004.77 | 5221.38 | 5245.07 | 5600.00 |
| 15 | 1 | 1.15640E+05 | 4626.09 | 2337.39 | 4104.07 | 5072.07 | 5194.47 | 5357.01 |
| 16 | 0 | 4.06715E+05 | 4977.57 | 2460.02 | 4331.46 | 4995.64 | 5022.17 | 5023.19 |
| 17 | 0 | 1.01663E+02 | 5006.91 | 2512.76 | 4326.17 | 4995.69 | 5003.65 | 5003.33 |
| 18 | 0 | 1.00767E+02 | 5000.00 | 2514.16 | 4325.92 | 4995.00 | 5003.22 | 5003.16 |

Table 5d. Beacon coordinate estimates (X2,X3,X3,Z1,Z2,Z3) at each iteration.

deviation approximately the same as that predicted.

Single simulations with measurement errors of 5 to 800 meters were made to determine the stability of the estimates of standard error, $\hat{\sigma}$, and the approximate error covariance matrix. Table 6 lists some examples showing the simulated measurement error, σ ; the estimated standard error, $\hat{\sigma}$, and the estimate of the TRACE of the beacon error covariance matrix. The stability of the trace indicates the linearity of the model in the region around $\underline{\theta}$.

The stability of the trace has another important implication: suppose some "optimum" set of survey points have been determined for a particular navigation net; the actual survey points used might differ from the optimum by hundreds of meters due to navigation errors while conducting the survey; the resulting errors in the estimates of beacon coordinates, however, would not be very different. The primary geometric factor which affects the errors of the estimates is the distribution of the survey points not their exact location.

TABLE 6.

| σ | $\hat{\sigma}$ | Trace |
|----------|----------------|-------|
| 5 | 48 | 3.96 |
| 50 | 478 | 3.95 |
| 500 | 513 | 4.76 |
| 600 | 640 | 3.92 |
| 700 | 751 | 3.98 |
| 800 | 820 | 4.35 |

IV DISCUSSION

Using the parameter estimation technique and the procedures described, the relative positions of acoustic navigation beacons can be determined with an accuracy of ± 1 meter.

A non-linear model is developed for a combined pulse-Doppler navigation system whose measurements of travel time and accumulated phase are perturbed by additive zero-mean Gaussian noise. The maximum likelihood estimation for the beacon coordinate is found. The numerical technique used to solve the non-linear estimation problem is a modified Gauss-Newton method. This method consists of approximating the non-linear model with a linear expansion and iteratively solving the linear model using standard least squares estimation techniques. An approximate error covariance matrix is found from which errors in the beacon coordinate can be estimated. Computer simulation has shown this technique to be stable for measurement errors within (and beyond) the range of errors encountered in practice.

Many sources of error are present in the survey problem. The preceding sections detail the several steps to be taken in order to achieve an accuracy of ± 1 meter in the beacon estimates. These can be generalized as follows:

1. Sound velocity should be determined with a precision instrument, for example, a Conductivity-Temperature-Depth (CTD)

probe (see Gregg and Cox, 1971). Errors in the measurement of sound velocity will usually bias the estimates but will not affect their variance.

2. Corrections should be applied to the measurements to account for acoustic refraction. When these corrections are applied, the errors due to acoustic refraction can be considered negligible.

3. Travel time measurements should be made while the surveying platform is dead-in-the-water, or nearly so, to eliminate errors due to horizontal platform motion. This is a potential source of large errors in the pulse system, although it does not affect the accuracy of the Doppler system measurement.

4. The time interval between selected Doppler survey points should be less than 1 hour since errors in the measurement of accumulated phase increase with time. Intervals of approximately 1/2 hour each are recommended.

5. Survey locations which will minimize the error magnification factors of the beacon coordinate estimates should be used. The choice of survey locations is critical to obtaining accurate estimates and can also affect the ability of the estimation technique to converge to a global rather than a local minimum. Several examples of good survey point selections are given in Section III. 3 above.

6. Pulse and Doppler measurements should be made simultaneously at no less than 10 survey locations. The use of more than 10 survey locations will, in general, reduce the errors in the beacon estimates. For example, the use of 40 survey locations can reduce these errors by a factor of 1/2. In most cases the capacity of the processing computer will limit the maximum number of survey locations which can be used practically.

APPENDIX A

DESCRIPTION OF PROGRAM ARCOR

Pulse navigation systems calculate the distance between two points from the measured time of travel of an acoustic pulse between those two points. This distance can be approximated by multiplying the travel time by an average sound velocity. This is an approximation since the sound travels in a refracted path rather than a straight line. The purpose of Program ARCOR is to improve the approximation by accounting for the refraction of sound in water. For a given sound velocity profile and the depths of a sound source and receiver, this program uses standard ray-tracing techniques (Officer, 1958) to find the time of travel, TT , of an acoustic pulse between the two points is the travel time TT multiplied by the arithmetic mean sound velocity \bar{c} :

$$R = \bar{c}TT .$$

The actual distance, SR is given by the expression

$$SR = (X^2 + Z^2)^{1/2}$$

where X is the horizontal distance traveled and Z is the vertical distance between source and receiver. The ratio between the actual distance and the approximate distance, $K_i = SR_i/R_i$, is calculated for several travel times. The set of approximate distances and their ratios (R_i, K_i) , $i = 1, N$, are fit to a

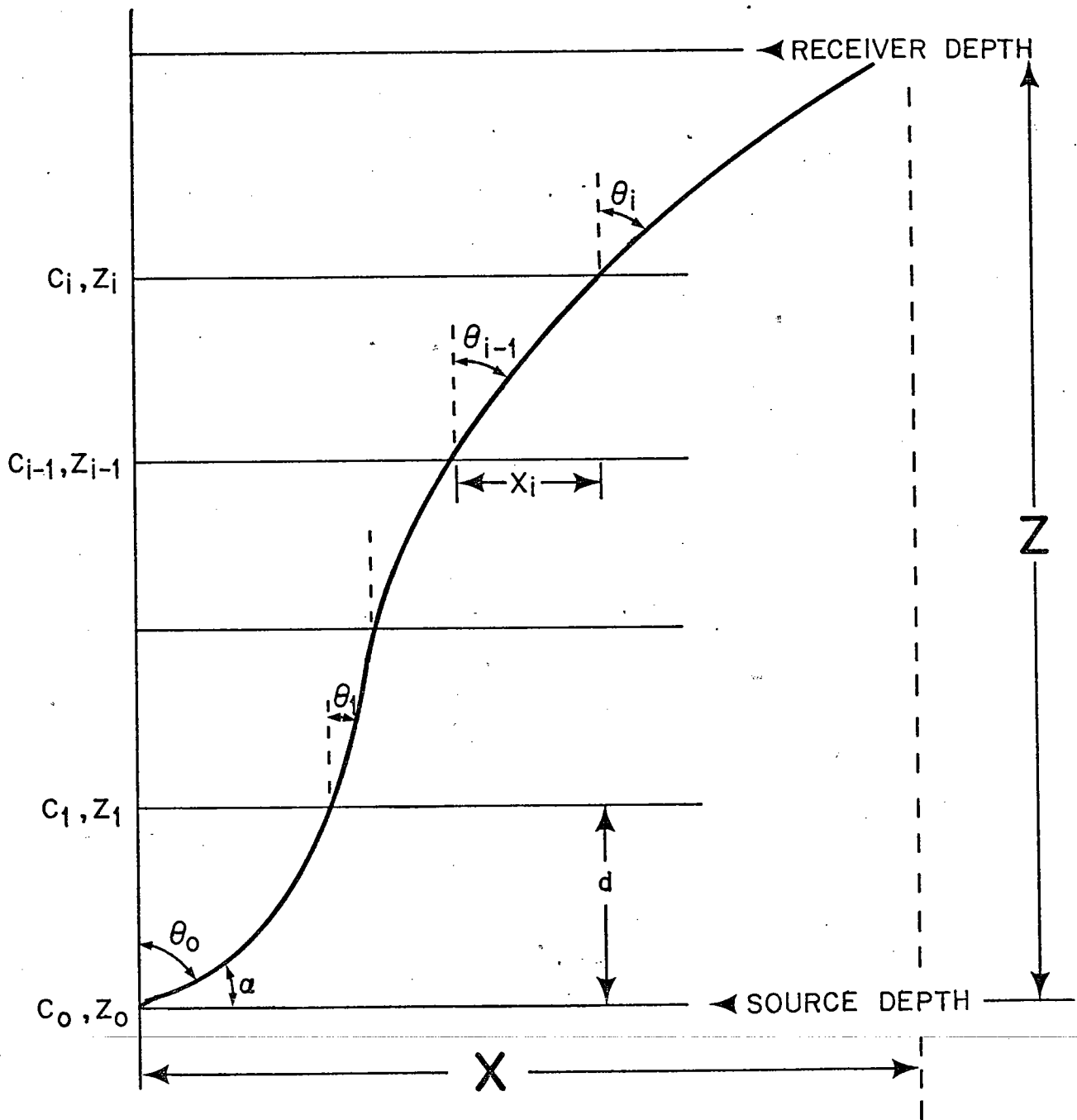


Figure 7. Notation used for Program ARCOR.

second degree polynomial using standard least squares regression:

$$K = \frac{SR}{R} = a_0 + a_1R + a_2R^2$$

where the a_i are the acoustic refraction coefficients. The actual distance SR can therefore be calculated when the approximate distance $R = \overline{CTT}$ is known:

$$SR = R(a_0 + a_1R + a_2R^2)$$

The set of approximate distances and their associated ratios are generated by calculating the necessary parameters for various angles θ_0 at which sound leaves the source (see Fig. 7). For each angle θ_0 there is a constant given by Snell's law

$$\frac{\sin\theta_0}{c_0} = p(\text{constant}) = \frac{\sin\theta_i}{c_i}$$

where c_0 = sound velocity at the source and c_i = sound velocity at interface i . This constant and the sound velocity gradients

$$g_i = \frac{c_i - c_{i-1}}{z_i - z_{i-1}},$$

determine the travel time and the distance an acoustic signal must travel to reach the receiver depth, assuming constant gradient in each layer. The angle at which sound leaves a layer is

$$\theta_i = \sin^{-1} (pc)$$

The travel time for each layer is

$$t = \frac{1}{g} \ln \left(\frac{\tan \theta_i/2}{\tan \theta_{i-1}/2} \right).$$

The horizontal distance traveled is

$$x_i = \frac{1}{pg} (\cos \theta_{i-1} - \cos \theta_i) \text{ (Officer, 1958).}$$

For each starting angle θ_0 the total travel time and horizontal distance traveled are found by summing the above quantities

for each layer. We then have

$$\text{Total horizontal distance: } X = \sum x_i$$

$$\text{Vertical distance: } Z = \text{source depth-receiver depth}$$

$$\text{Total travel time: } TT = \sum t$$

$$\text{Distance between source and receiver: } SR = (X^2 + Z^2)^{1/2}$$

$$\text{Approximate distance: } R = \overline{c}TT$$

$$\text{Ratio: } K = SR/R$$

The program computes the first starting angle by defining an angle, $\alpha = 90^\circ$ and setting the departure angles equal to $90^\circ - \alpha$. Each successive α is reduced by a factor, for example .95, and a new departure angle is computed. The values of K and R are fitted with a second degree polynomial to produce the coefficients, a_i . A typical output is shown on page 73. A listing of the program begins on page 74. The program was originated by W.M. Marquet of the Woods Hole Oceanographic

Institution and includes calculations of coefficients for the case of a receiver at the same depth of the source. This particular computation is not used in the survey problem and is not discussed here. The original version of this program was called Program SETUP and written by Mary Hunt and Roger Goldsmith of the Woods Hole Oceanographic Institution.

ARITHMETIC MEAN SOUND VELOCITY = 1512.7344

| X | SR | TT | THETA | K | R |
|----------|----------|--------|-------|---------|----------|
| .00 | 4645.00 | 3.0709 | .00 | .99991 | 4645.43 |
| 287.43 | 4653.88 | 3.0768 | 3.60 | .99991 | 4654.32 |
| 565.98 | 4679.28 | 3.0935 | 7.06 | .99991 | 4679.72 |
| 825.94 | 4719.62 | 3.1202 | 10.37 | .99991 | 4720.05 |
| 1100.83 | 4773.66 | 3.1559 | 13.56 | .99991 | 4774.10 |
| 1361.53 | 4840.43 | 3.2001 | 16.62 | .99991 | 4840.87 |
| 1619.50 | 4919.16 | 3.2521 | 19.55 | .99991 | 4919.59 |
| 1875.24 | 5009.25 | 3.3117 | 22.37 | .99991 | 5009.68 |
| 2100.01 | 5110.21 | 3.3784 | 25.07 | .99992 | 5110.64 |
| 2385.35 | 5221.68 | 3.4521 | 27.67 | .99992 | 5222.10 |
| 2641.14 | 5343.37 | 3.5325 | 30.17 | .99992 | 5343.79 |
| 2898.26 | 5475.08 | 3.6196 | 32.56 | .99992 | 5475.49 |
| 3157.62 | 5616.64 | 3.7132 | 34.86 | .99993 | 5617.04 |
| 3419.52 | 5767.94 | 3.8132 | 37.06 | .99993 | 5768.33 |
| 3684.56 | 5928.91 | 3.9196 | 39.18 | .99994 | 5929.29 |
| 3953.23 | 6099.51 | 4.0324 | 41.21 | .99994 | 6099.80 |
| 4225.98 | 6279.73 | 4.1515 | 43.16 | .99995 | 6280.07 |
| 4503.24 | 6469.56 | 4.2769 | 45.04 | .99995 | 6469.80 |
| 4785.07 | 6669.02 | 4.4088 | 46.84 | .99996 | 6669.31 |
| 5072.75 | 6878.14 | 4.5470 | 48.56 | .99996 | 6878.40 |
| 5365.69 | 7096.95 | 4.6916 | 50.22 | .99997 | 7097.17 |
| 5664.21 | 7325.48 | 4.8427 | 51.81 | .99998 | 7325.66 |
| 5969.47 | 7563.77 | 5.0001 | 53.34 | .99998 | 7563.89 |
| 6280.04 | 7811.85 | 5.1641 | 54.81 | .99999 | 7811.91 |
| 6598.02 | 8069.72 | 5.3349 | 56.21 | 1.00000 | 8069.71 |
| 6923.61 | 8337.41 | 5.5114 | 57.56 | 1.00001 | 8337.32 |
| 7255.07 | 8614.90 | 5.6948 | 58.86 | 1.00002 | 8614.72 |
| 7594.25 | 8902.17 | 5.8846 | 60.11 | 1.00003 | 8901.89 |
| 7940.03 | 9199.17 | 6.0809 | 61.30 | 1.00004 | 9198.70 |
| 8293.60 | 9505.85 | 6.2835 | 62.45 | 1.00006 | 9505.32 |
| 8654.04 | 9822.10 | 6.4925 | 63.55 | 1.00007 | 9821.42 |
| 9022.00 | 10147.81 | 6.7077 | 64.61 | 1.00008 | 10146.90 |
| 9397.03 | 10482.02 | 6.9290 | 65.63 | 1.00010 | 10481.79 |
| 9779.93 | 10826.96 | 7.1564 | 66.60 | 1.00011 | 10825.72 |
| 10169.45 | 11180.31 | 7.3896 | 67.54 | 1.00013 | 11178.54 |

DEEP TO SHALLOW (4650.00 TO 5.00)

ASK(1) = .9998616E+00

ASK(2) = -.5962590E-09

ASK(3) = .2213737E-11

Output of Program ARCOR. X=Horizontal distance between source and receiver; SR=Slant range; TT=Travel time; THETA=Angle of departure from source; K=SR/R; R=cTT=approximate slant range.

```

0001 FTN,L
0002 PROGRAM ARCOR
0003 C DOUBLE PRECISION VERSION FOR HP
0004 C HP DOS VERSION BY MARY HUNT & JAMES DURHAM AUG 74
0005 C FINDS RAY BENDING COEFFICIENTS
0006 DOUBLE PRECISION PI,XI,XF,XD,XMAX,XC,SK,DK,TH0,TH
0007 DOUBLE PRECISION TH02,TH2,TT,ARG,ALPH,ALPHT,RFAC
0008 DOUBLE PRECISION XS,TS,CS,AL2,RT,ASN,XT,RACT,P,ALS
0009 DIMENSION R(50),DX(50)
0010 DIMENSION DTDC(3),DTSC(3)
0011 DIMENSION Z(50),C(50)
0012 DIMENSION G(50),CB(50)
0013 DIMENSION COEF(3,3)
0014 DIMENSION SAVE(3)
0015 EQUIVALENCE (COEF(1,1),DTDC),(COEF(1,2),DTSC),(COEF(1,3),G/VE)
0016 PI = 3.14159265358979
0017 C SET UNIT DEVICE CODES
0018 NI = 1
0019 NO = 1
0020 C
0021 C ** PAUSE FOR SENSE SWITCHES
0022 C
0023 WRITE(NO,1002)
0024 1002 FORMAT(" PROGRAM ARCOR, HP VERSION"/" SET SENSE SWITCHES"/
0025 1" SSW 1 ON - S.VEL PROFILE LISTED ON OUTPUT DEVICE"/
0026 1" SSW 3 ON - DEEP TO DEEP CALCULATION SKIPPED"/
0027 1" SSW 7 ON - RAY BENDING CALCULATION SKIPPED"/
0028 1" SSW 14 ON - S.VEL PROFILE AND COEFFS OUTPUT ON PAPER 30 1")
0029 WRITE(NO,1003)
0030 1003 FORMAT("/" ALL INPUTS ARE FREE FIELD"/" READY? -")
0031 READ(NI,1004) IANS
0032 1004 FORMAT(/)
0033 WRITE(NO,1014)
0034 1014 FORMAT("/" LIST DEVICE? -")
0035 READ(NI,*) LP
0036 C
0037 DO 2 I = 1,3
0038 COEF(1,I) = 1.0
0039 COEF(2,I) = 0.0
0040 COEF(3,I) = 0.0
0041 2 CONTINUE
0042 C
0043 C ** INPUT SOUND VELOCITY PROFILE
0044 C
0045 WRITE(NO,1005)
0046 1005 FORMAT(" S. VEL. INPUT DEVICE? -")
0047 READ(IJ,*) ISVC
0048 WRITE(NO,1006)
0049 1006 FORMAT(" INPUT PROFILE"/" DEPTH(I),S.VEL(I/Z)"/

```

```

0000      1" ENTER NEGATIVE NUMBER WHEN DONE!"
0001      I=0
0002      5 CONTINUE
0003      I = I + 1
0004      READ(15VC,*) Z(I),C(I)
0005      IF(Z(I)) 12,8,8
0006      8 CONTINUE
0007 C
0008 C      NO MORE THAN 50 ALLOWED
0009 C
0010      IF(I - 50) 5,30,30
0011      12 CONTINUE
0012      I = I - 1
0013 C
0014 C
0015 C ** INITIALIZE VARIABLES
0016 C      SET X INITIAL, TERMINAL AND INCREMENTAL VALUES
0017 C
0018      30 CONTINUE
0019      NSV = I
0020      IF(ISSW(1)) 31,32,32
0021      31 WRITE(LP,3000)
0022      3000 FORMAT(/' I      Z      C"/)
0023      WRITE(LP,3001) (I,Z(I),C(I),I=1,NSV)
0024      3001 FORMAT(I5,2F10.4)
0025      32 IF(ISSW(7)) 2014,34,34
0026      34 CONTINUE
0027      XI = 0.000
0028      XF = 2000.000
0029      XD = 2000.000
0030      MORD = 2
0031      MORD1 = MORD + 1
0032      50 CONTINUE
0033      ITER = -1
0034 C      INPUT TYPICAL DEEP DEPTH
0035      WRITE(NO,4000)
0036      4000 FORMAT(" BEACON DEPTH? ")
0037      READ(NI,*) Z0
0038 C      INPUT SHALLOW RECEIVER DEPTH
0039      WRITE(NO,4010)
0040      4010 FORMAT(" DEPTH OF RECEIVER 1? ")
0041      READ(NI,*) ZR
0042 C
0043 C      *** START DEEP TO DEEP PROCESSING
0044 C
0045 C      FIND AVERAGE VELOCITY AT TYPICAL DEEP DEPTH
0046      70 CONTINUE
0047      IF(ISSW(3)) 1000,75,75
0048      75 CONTINUE

```

```

0079      CBAR = YP(Z0,Z,C,NSV)
0100 C      FIND THE DEEP DEPTH STARTING LAYER
0101      DO 110 II = 1,NSV
0102      IF (Z0 - Z(II)) 115,115,110
0103      110 CONTINUE
0104      II = NSV
0105      115 CONTINUE
0106 C
0107 C      FIND GRADIENT OF SOUND VELOCITY AT DEEP DEPTH
0108      G(II) = (C(II) - C(II-1))/(Z(II) - Z(II-1))
0109 C
0110 C      IF GRADIENT IS NEGATIVE, DEEP WATER ANALYSIS IS SKIPPED
0111      IF (G(II)) 125,130,135
0112      125 WRITE (NO,9102)
0113      9102 FORMAT(53H NEGATIVE GRADIENT - DEEP TO DEEP PROCESSING BYPASSED)
0114      GO TO 1000
0115      130 WRITE (NO,9101)
0116      9101 FORMAT(42H GRADIENT = 0 FOR DEEP TO DEEP PROCESDING /
0117      +      20H K = 1.0 FOR ALL X )
0118      GO TO 1000
0119 C
0120 C      CALCULATE DO LOOP PARAMETERS
0121 C
0122      135 CONTINUE
0123      IE = (XF - XI)/XD + 0.00100
0124 C      SET DO LOOP TO VARY X FOR DEEP TO DEEP
0125      WRITE (LP,9105)
0126      9105 FORMAT(///5X,"X",10X,"K",10X,"K-1")
0127      DO 400 I = 1,IE
0128      FI = I
0129      XC = XI + FI*XD
0130      IF (XC) 175,150,175
0131 C      SET THETAC FOR X = 0
0132      150 CONTINUE
0133      DK = -1.000
0134      R(I) = 0.0
0135      GO TO 350
0136 C      CALCULATE THETA0
0137      175 CONTINUE
0138      TH0 = 2.0*CBAR/(XC*G(II))
0139      TH0 = DATAN(TH0)
0140 C      CALCULATE ANGLE TH
0141      200 CONTINUE
0142      TH = PI - TH0
0143 C      FIND HALF ANGLES
0144      TH2 = TH0/2.000
0145      TH2 = TH/2.000
0146 C      CALCULATE TIME T
0147      TT = !DSIN(TH2)/ECOS(TH2) / !DSIN(TH0)/LCOS(TH0)

```

```

0148      TT = DLOG(TT)/G(I1)
0149 C      CALCULATE APPARENT DISTANCE TRAVELLED
0150      RACT = CBAR*TT
0151      R(I) = RACT
0152 C      CALCULATE RATIO K
0153      DK = XC/RACT
0154      350 CONTINUE
0155      DX(I) = DK - 1.0DR
0156      WRITE (LP,9106) XC,DK,DX(I)
0157      400 CONTINUE
0158 C      FIT POLYNOMIAL WITH X AND K TO GET COEFFICIENTS
0159      CALL HCRFT(R,DX,DTDC,IE,MORD)
0160 C      OUTPUT CURVE FIT COEFFICIENTS
0161      DTDC(1) = DTDC(1) + 1.0
0162      WRITE (LP,9103) Z0,(I,DTDC(I),I = 1,MORD1)
0163 C
0164 C      ** DEEP TO SHALLOW CALCULATIONS
0165 C
0166      1000 CONTINUE
0167      CS = R.0DR
0168 C      FIND STARTING DEPTH AND LAYER
0169 C      FIND LAYER GRADIENT AND AVERAGE SOUND VELOCITY
0170      DO 1150 J = 1,NSV
0171      IF (Z0 - Z(J)) 1150,1180,1100
0172      1100 CONTINUE
0173      J = NSV + 1
0174      1150 CONTINUE
0175      K = J
0176      ZJ1 = Z(J-1)
0177      CPAR = YP(ZR,Z,C,NSV)
0178      ZJS = Z(J)
0179      CJS = C(J)
0180      JS = J
0181      Z(J) = Z0
0182      C(J) = CPAR
0183      G(J) = (CBAR - C(J-1))/(Z(J) - ZJ1)
0184      CP(J) = (CBAR + C(J-1))/2.0
0185      GO TO 1250
0186 C      FIND LAYER GRADIENT AND AVERAGE VELOCITY
0187      1180 JS = J
0188      1200 ZJ1 = Z(J-1)
0189      1205 CONTINUE
0190      K = J
0191      IF (Z(J) - ZR) 1300,1300,1200
0192      1200 IF (ZJ1 - ZR) 1210,1230,1250
0193      1210 CONTINUE
0194      CPAR = YP(ZR,Z,C,NSV)
0195      G(J) = (C(J) - CPAR)/(Z(J) - ZR)
0196      CP(J) = (CBAR + C(J))/2.0

```

```

0197      K = 1
0198      ZJ1 = ZR
0199      GO TO 1250
0200 1230  G(J) = (C(J) - C(J-1))/(Z(J) - ZJ1)
0201      CB(J) = (C(J) + C(J-1))/2.0
0202 1250  CONTINUE
0203      CS = CS + CB(J)*(Z(J) - ZJ1)
0204      K = K-1
0205      J = K
0206      IF(J-1) 1300,1275,1200
0207 1275  ZJ1 = -0.00001
0208      GO TO 1200
0209 1300  CS = CS/(Z0 - ZR)
0210      WRITE(LP,9109) CS
0211 1320  CONTINUE
0212      ITER = ITER + 1
0213 C      SET STARTING ANGLE = 0 DEG
0214      ALPH = 0.000
0215      ALPHT = PI/2.000
0216 C      SET ARRAY INDEX
0217      I = 1
0218      RFAC = .9500
0219      WRITE(NO,9201)
0220 9201  FORMAT(" ENTER DECREMENT FACTOR BETWEEN 0 AND 1 -")
0221      READ(NI,*) RFAC
0222      WRITE(NO,9202)
0223 9202  FORMAT(" ENTER MAX HORIZONTAL RANGE -")
0224      XMAX = 1000.00
0225      READ(NI,*) XMAX
0226      WRITE(LP,9107)
0227 C      BEGIN RAY TRACE
0228 1400  CONTINUE
0229 C      INITIALIZE SUMS
0230      XS = 0.000
0231      TS = 0.000
0232      S = 0.000
0233      P=DSIN(ALPH/C(JS))
0234      THETA = ALPH+180.000/PI
0235      J = JS
0236      K = J
0237 1420  ZJ1 = Z(J-1)
0238 1425  CONTINUE
0239      CJ1 = C(J-1)
0240      IF(Z(J) - ZR) 1500,1500,1428
0241 1428  IF(ZJ1 - ZR) 1430,1400,1400
0242 1430  CONTINUE
0243      K = 1
0244      ZJ1 = ZR
0245      CJ1 = C(AR)

```

```

0246 1440 CONTINUE
0247 C      FIND ANGLE RAY LEAVES LAYER
0248      ASN = CJI*P
0249      IF (ASN - 1.0D0) 1450,1450,1600
0250 1450 CONTINUE
0251 C      TH = ASIN(ASN)
0252      ARG = ASN/DSORT(1.0D0 - ASN*ASN)
0253      TH = DATAN(ARG)
0254      IF (G(J)) 1460,1480,1460
0255 1460 CONTINUE
0256 C      FIND TIME
0257      TH2 = TH/2.0D0
0258      AL2 = ALPH/2.0D0
0259      IF (I-1) 1470,1480,1470
0260 1470 CONTINUE
0261 C      TT = DTAN(TH2)/DTAN(AL2)
0262      TT = (DSIN(TH2)/DCOS(TH2)) / (DSIN(AL2)/DCOS(AL2))
0263      TT = (1-DLOG(TT))/G(J)
0264      RT = (-1.0D0)/P+G(J)
0265      XT = (DCOS(ALPH)-DCOS(TH))*RT
0266      AL = (TH-ALPH)*RT
0267      GO TO 1490
0268 C      CALCULATE LAYER VALUES FOR STRAIGHT UP OR G = 0 CASES
0269 1480 CONTINUE
0270 C      XT = (Z(J)- ZJ1)*DTAN(ALPH)
0271      XT = (Z(J)- ZJ1)*(DSIN(ALPH)/DCOS(ALPH))
0272      AL = XT
0273      TT = DSORT ((Z(J)- ZJ1)*(Z(J)- ZJ1) + XT*XT)/CJ(J)
0274 C
0275 1490 CONTINUE
0276 C      SUM X DISTANCE
0277      XS = XS + DABS(XT)
0278 C      SUM TRAVEL TIME
0279      TS = TS + DABS(TT)
0280 C      SUM ARC LENGTH
0281      S = S + AL
0282 C      RESET ENTRY RAY FOR NEXT LAYER
0283      ALPH = TH
0284 C      RESET INDEX AND CHECK
0285      K = K-1
0286      J = K
0287      IF (J - 1) 1500,1495,1420
0288 1495 ZJ1 = -0.00001
0289      GO TO 1425
0290 C      STORE TOTALS
0291 1500 CONTINUE
0292      RACT = XS*XS + (Z0-ZR)*(Z0-ZR)
0293      RACT = DSORT(RACT)
0294      R(I) = CS+TS

```

```

0295      SK = RACT/R(I)
0296      WRITE(LP,9108) XS,RACT,TS,THETA,SK,R(I)
0297  9108  FORMAT(F10.2,F10.2,F10.4,F10.2,F10.5,F10.2)
0298      DX(I) = SK - 1.000
0299  C      REINITIALIZE ANGLE AND INDEX
0300  C      I = I + 1
0301      ALPHT = RFAC*ALPHT
0302      ALPH = PI/2.000 - ALPHT
0303      IF(XS - XMAX) 1550,1600,1600
0304  1550  CONTINUE
0305  C      CHECK THAT INDEX DOES NOT BECOME TOO GREAT
0306      IF (I - 48) 1400,1400,1600
0307  1600  I = I - 1
0308  C      FIT POLYNOMIAL WITH X AND K COEFFICIENTS
0309  1650  CONTINUE
0310      CALL HCRET(R,DX,DTSC,I,MORD)
0311  C      OUTPUT CURVE FIT COEFFICIENTS
0312      DTSC(1) = DTSC(1) + 1.0
0313      WRITE (LP,9104) Z,ZR,(II,DTSC(II),II=1,MORD)
0314      IF(ITER) 50,1700,2000
0315  1700  CONTINUE
0316      DO 25 I=1,3
0317      SAVE(I) = DTSC(I)
0318      25  CONTINUE
0319      WRITE(NO,4020)
0320  4020  FORMAT(/" DEPTH OF RECEIVER 2"/
0321      1" (ENTER NEGATIVE NUMBER IF ANOTHER SET OF"
0322      1" COEFFS IS NOT DESIRED) -")
0323      READ(NI,1) ZR
0324      IF(ZR) 1950,1700,1000
0325  1950  CONTINUE
0326      DTSC(1) = 1.0
0327      DTSC(2) = 0.0
0328      DTSC(3) = 0.0
0329  2000  CONTINUE
0330      IF(JSSW(14)) 2010,7000,7000
0331  2010  CONTINUE
0332      IF(ZB - Z(JS)) 2014,2010,2014
0333  2012  CONTINUE
0334      Z(JS) = ZJS
0335      C(JS) = CJS
0336  2014  CONTINUE
0337      NPT = 4
0338      WRITE(NPT,5000) NSV
0339  5000  FORMAT(I5)
0340      DO 45 I=1,NSV
0341      C(I) = C(I)/1000.
0342      WRITE(NPT,5040) Z(I),C(I)
0343  5040  FORMAT(2F10.4)

```



```
0344 45 CONTINUE
0345 5060 FORMAT(3E20.10)
0346 WRITE(NPT,5060) (SAVE(J),J=1,3)
0347 WRITE(NPT,5060) (DTSC(J),J=1,3)
0348 WRITE(NPT,5060) (DTDC(J),J=1,3)
0349 7000 CONTINUE
0350 STOP
0351 9103 FORMAT(///20H DEEP TO DEEP DEPTH = ,F10.2/
0352 + (5H ADK(,I2,3H) =,E14.7))
0353 9104 FORMAT(///21H DEEP TO SHALLOW (,F10.2,3H TO,F10.2,2H)/
0354 + (5H ASK(,I2,3H) =,E14.7))
0355 9106 FORMAT(3E12.5)
0356 9107 FORMAT(///5X,1HX,9X,2HSR,9X,1HT,9X,5HTHETA,5X,1HK,9X,1HR)
0357 9109 FORMAT( /" ARITHMETIC MEAN SOUND VELOCITY = ", F12.4)
0358 END
```

```
* * NO ERRORS*
```

```

0001 C CURVE-FITTING SUBROUTINE F.K.KEYTE (W.H.O.I.) 12TH MAY 1964
0002 SUBROUTINE HCRET(X,Y,C,N,MCURV)
0003 C VERSION FOR HEWLETT - PACKARD
0004 C THIS IS A CUT-DOWN VERSION OF SUBROUTINE CRVET
0005 C DONE TO SAVE ROOM
0006 C DONE BY M. HUNT MARCH 1974
0007 DIMENSION X(1),Y(1),C(1)
0008 DIMENSION A(5,6),AP(5,6)
0009 C M IS COEFF INDCTR, L=1 IS SPCL ROUTE INDCTR FOR Y=C(1) ONLY
0010 M = 3
0011 MC = M + 1
0012 C M IS ORDER OF COEFFICIENT MATRIX
0013 C MC IS J NUMBER OF Y COLUMN
0014 C ZERO ALL ARRAYS
0015 DO 2 I=1,M
0016 C(I)=0.0
0017 DO 2 J=1,MC
0018 A(I,J)=0.0
0019 2 AP(I,J)=0.0
0020 A(1,1) = N
0021 C A IS COEFFICIENT MATRIX
0022 C AP (A PRIME) IS AUXILIARY MATRIX
0023 C C(J),J=1,M ARE COEFFICIENTS OF A(I,J), AND OF THE INDEP. VARIABLE
0024 17 INCR = -1
0025 I=1
0026 DO 18 J=2,M
0027 IXP = J+INCR
0028 DO 18 JJ=1,N
0029 18 A(I,J)=A(I,J)+X(JJ)**IXP
0030 C CANNOT INCLUDE J=1 SINCE ANY ZERO TO THE ZERO NOT ALLOWED
0031 DO 19 I=2,M
0032 INCR=INCR+1
0033 DO 19 J=1,M
0034 IXP=J+INCR
0035 DO 19 JJ=1,N
0036 19 A(I,J)=A(I,J)+X(JJ)**IXP
0037 DO 20 JJ=1,N
0038 20 A(1,MC)=A(1,MC)+Y(JJ)
0039 DO 21 I=2,M
0040 IXP=I-1
0041 DO 21 JJ=1,N
0042 21 A(I,MC)=A(I,MC)+Y(JJ)*X(JJ)**IXP
0043 C THIS FILLS ARRAY. NOW SOLVE FOR C(J) BY CROUT REDUCTION
0044 DO 22 I=1,M
0045 22 AP(I,1)=A(I,1)
0046 DO 23 J=2,MC
0047 23 AP(1,J) = A(1,J)/A(1,1)

```

```
0048      DO 28 J=2,MC
0049      DO 28 I=2,M
0050      S=0.0
0051      IF (I-J) 26,24,24
0052      24 MA = J-1
0053      DO 25 K=1,MA
0054      25 S=S+AP(I,K)*AP(K,J)
0055      AP(I,J) = A(I,J) - S
0056      GO TO 28
0057      26 MA = I-1
0058      DO 27 K=1,MA
0059      27 S=S+AP(I,K)*AP(K,J)
0060      AP(I,J) = (A(I,J) -S)/AP(I,I)
0061      28 CONTINUE
0062      C(M)=AP(M,MC)
0063      DO 30 I=2,M
0064      S=0.0
0065      MA=I-I+2
0066      MB=MA-1
0067      DO 29 K=MA,1
0068      29 S=S+AP(MB,K)*C(K)
0069      30 C(MB)= AP(MB,MC) - S
0070      RETURN
0071      END
```

** NO ERRORS*

```
0001 C      YP=1
0002 C      INTERPOLATION FUNCTION
0003 C      WRITTEN BY HANK PERKINS
0004      FUNCTION YP(XP,X,Y,N)
0005      DIMENSION X(1),Y(1)
0006      YQ(XQ,X1,Y1,X2,Y2) = (Y2*(XQ-X1)-Y1*(XQ-X2))/(X2-X1)
0007      IF(N-1) 10,10,20
0008      10 CONTINUE
0009      YP = Y(1)
0010      RETURN
0011      20 CONTINUE
0012      IF(XP-X(1)) 30,30,40
0013      30 CONTINUE
0014      YP = YQ(XP,X(1),Y(1),X(2),Y(2))
0015      RETURN
0016      40 CONTINUE
0017      IF(XP-X(N)) 60,50,50
0018      50 CONTINUE
0019      YP = YQ(XP,X(N-1),Y(N-1),X(N),Y(N))
0020      RETURN
0021      60 CONTINUE
0022      DO 70 I=1,N
0023      IF(XP-X(I)) 80,80,70
0024      70 CONTINUE
0025      80 CONTINUE
0026      J = I
0027      YP = YQ(XP,X(J-1),Y(J-1),X(J),Y(J))
0028      RETURN
0029      END
```

```
*** NO ERRORS***
```

APPENDIX B

DETECTION PROBABILITIES OF WOODS HOLE OCEANOGRAPHIC INSTITUTION

CYCLE COUNTER

The detection probabilities of the Woods Hole Oceanographic Institution cycle counter can be calculated by considering the problem of detecting a sine wave in the presence of Gaussian noise. This treatment is sufficient because the estimate of phase is not based on the optimum estimator

$$\varnothing = \tan^{-1} \frac{s_2(t)}{s_1(t)}$$

where $s_1(t)$ and $s_2(t)$ are the quadrature components of the input to the phase detector; the estimate of phase is based on the signs of $s_1(t)$ and $s_2(t)$. The detection probabilities of the sine component will be developed for illustration. The corresponding probabilities of the cosine component can be developed in a similar manner with identical results.

Consider a sine wave in the absence of noise

$$s(t) = A(t) \sin \varnothing(t)$$

The amplitude $A(t)$ varies slowly compared with the phase $\varnothing(t)$ and can be considered a constant. The phase can be assumed uniformly distributed

$$p(\varnothing) = 1/2\pi, |\varnothing| < \pi$$

The probability density of the signal $p(s)$ is therefore given by the expression:

$$p(s) = \frac{1}{\pi A} \frac{1}{\left[1 - \left(\frac{s}{A}\right)^2\right]^{1/2}} \left| \frac{s}{A} \right| < 1 .$$

The phase detector output in the presence of noise will be denoted by $z(t) = s(t) + n(t)$ where the noise is normally distributed:

$$p(n) = \frac{1}{\sqrt{2\pi}\sigma_n} e^{-n^2/2\sigma_n^2} \quad \sigma_n = \text{r.m.s. noise voltage.}$$

The probability density $p(z)$ may be found by the convolution of $p(s)$ and $p(n)$ assuming statistical independence of the two processes. This leads to the following integral for $p(z)$:

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma_n} \int e^{-n^2/2\sigma_n^2} \frac{1}{\pi A} \frac{1}{\left[1 - \frac{(z-n)^2}{A^2}\right]^{1/2}} dn$$

This integral has several representations (see Ol'shevskii, 1967) but is approximately a constant near $z = 0$ for $A/\sigma_n \gg 1$. Figure 8a illustrates the shape of this probability density as a function of the parameter $q^2 = \frac{A^2}{2\sigma_n^2}$.

The probability of error is a weighted sum of the probabilities of false alarm $P(\text{FA})$ and false dismissal $P(\text{FD})$. The cycle counter decides that the signal is positive when the voltage exceeds some threshold $V_T > 0$. Similarly the decision

that the signal is negative occurs when the voltage is less than some negative threshold $-V_T < 0$. A "false alarm" occurs when the signal is below the threshold and the signal plus noise exceeds the thresholds:

$$|z(t)| > V_T \text{ given } |s(t)| < V_T .$$

The probability of this event, $P(\text{FA})$, is approximately the same as the probability that the noise will exceed the thresholds in the absence of signal. This is represented by the shaded area in Figure 8b and is given by the expression

$$P(\text{FA}) \approx 2 \int_{V_T}^{\infty} p(n) dn = 2 \int_{V_T/\sigma_n}^{\infty} \frac{1}{\sqrt{2\pi}} \exp(-x^2/2) dx .$$

A false dismissal occurs when the signal is above the thresholds but the signal plus noise is below the thresholds:

$|z(t)| < V_T$ given $|s(t)| > V_T$. The probability of this event $P(\text{FD})$, is approximately the same as the probability that the signal plus noise is below the thresholds for any signal. This is represented by the shaded area in Figure 8c and given by the expression

$$P(\text{FD}) \approx \int_{-V_T}^{V_T} p(z) dz .$$

In the region of interest $p(z)$ can be approximated by a constant

$$p(z) \approx \frac{1}{\pi A} , A/\sigma_n \gg 1, V_T/A \ll 1 .$$

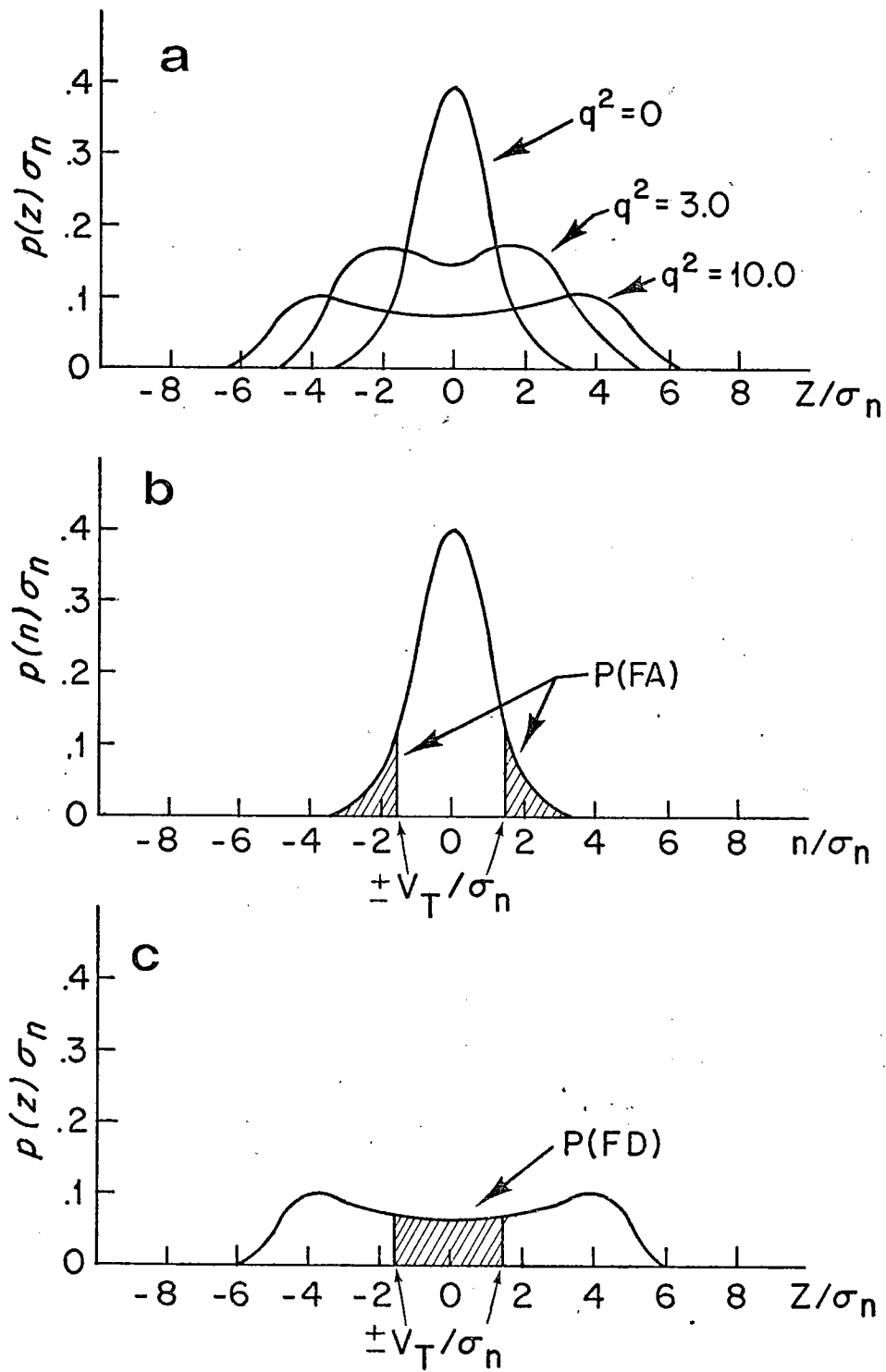


Figure 8. Probability Densities of Signal Plus Noise $p(z)$ and noise $p(n)$.

The probability of false dismissal can then be found

$$P(\text{FD}) \approx \frac{2V_T/\pi A}{\pi (A/\sigma_n)} = \frac{2 (V_T/\sigma_n)}{\pi (A/\sigma_n)}$$

It can be seen that an increase in the threshold voltage-to-noise ratio V_T/σ_n will increase the probability of false dismissal and decrease the probability of false alarm for a given signal-to-noise ratio. Since the losses assigned to these errors are approximately equal, the optimum threshold setting for a given signal-to-noise ratio will satisfy the following equation:

$$\frac{P(\text{FA})}{P(\text{FD})} = 1$$

or equivalently

$$\int_{V_T/\sigma_n}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx = \frac{V_T/\sigma_n}{\pi (A/\sigma_n)} \quad (10)$$

The expression on the left hand side of equation (10) is a tabulated function of V_T/σ_n and can be solved by trial-and-error for various values of the signal power-to-noise ratio $S/N = A^2/2\sigma_n^2$ (see Table 7).

The probability of error P_e in determining the sign of the signal is given by the following expression for the optimum threshold setting:

$$P_e = 1/2 [P(\text{FD}) + P(\text{FA})] = P(\text{FD}) \quad (11)$$

TABLE 7

| $10 \log S/N$ | V_T/σ_n | P_e | $\sigma_{\phi_n}^2$ |
|---------------|----------------|-------|---------------------|
| 12 | 1.40 | .158 | .366 |
| 18 | 1.67 | .095 | .208 |
| 24 | 1.92 | .055 | .116 |
| 30 | 2.16 | .031 | .064 |

An error in the cycle count, i.e. the number of quadrants of phase change between two successive cycles can be caused by an error in either or both of the quadrature components. When an error (false alarm or false dismissal) occurs in only one component the cycle count will be in error by ± 1 quadrant. When an error occurs in both components the cycle count will be in error by ± 2 quadrants. The following probability of cycle count error can be calculated from the probability of error P_e previously determined by equation (11):

$$P(e=0) = (1-P_e)^2$$

$$P(e=\pm 1) = 2P_e(1-P_e)$$

$$P(e=\pm 2) = P_e^2$$

The variance in the n^{th} sample can be calculated as follows

$$\begin{aligned} \sigma_{\phi_n}^2 &= P(e=0)(0)^2 + P(e=\pm 1)(1)^2 + P(e=\pm 2)(2)^2 \\ &= 2P_e(1-P_e) + 4P_e^2 = 2P_e(1+P_e). \end{aligned}$$

The variance of error in the measurement of accumulated phase change ϕ_T after T seconds is

$$\sigma_{\phi_T}^2 = \sum_{n=1}^N \sigma_{\phi_n}^2 = N \sigma_{\phi_n}^2 \quad N = Tf_s$$

where f_s is the sampling frequency. For a sampling frequency of 10 Hz and an interval of 1 hour = 3600 seconds the standard error in the measurement of ϕ_T is $\sigma_{\phi_T} \approx 190 \sigma_{\phi_n}$.

Table 7 lists the values of P_e , $\sigma_{\theta_n}^2$ and the optimum threshold-to-noise ratios for various signal-to-noise ratios typically encountered in practice. The variance is calculated assuming that the bias is zero or negligible. This assumption is valid when the velocity relative to a beacon is small.

APPENDIX C

SAMPLE CALCULATION OF SIGNAL-TO-NOISE RATIO

In general the signal-to-noise level will decrease as the range between a sound source (e.g. CW beacon) and a receiver (e.g. ship transducer) increases. A sample calculation is made to demonstrate the relationship between signal-to-noise ratio at the source $(S/N)_s$, the range between source and receiver R , and signal-to-noise ratio at the receiver, $(S/N)_r$. The signal level of the CW beacons in the present configuration of the pulse-Doppler system is 166 dB (re: 1μ Pa). A typical ambient sea noise level in sea state 3 is 44 dB/Hz (re: 1μ Pa) or 54 dB/10 Hz (see Urick, 1967). The signal-to-noise level in a 10 Hz band is therefore 112 dB. Transmission losses are assumed due to spherical spreading and attenuation. The loss due to spherical spreading is $20 \log R$ where R is the range in meters. The attenuation of a 13 kHz signal is approximately 1 dB/km. The signal-to-noise level at the receiver $(S/N)_r$ is therefore: $10 \log(S/N)_r = 112 \text{ dB} - 20 \log R - R \times 10^{-3}$. For a horizontal range of 10 km and water depth of 5 km the range between a bottom-moored source and a surface receiver is approximately

$$R = (5^2 + 10^2)^{1/2} \times 10^3 = 11.2 \times 10^3 \text{ m}$$

The $(S/N)_r$ in decibels is therefore

$$10 \log(S/N)_r = 112 - 81.0 - 11.2 = 19.8 \text{ dB.}$$

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