PROPAGATION AND ATTENUATION CHARACTERISTICS

OF MULTILAYERED MEDIA

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by

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of the requirements for the degrees of Master of Science and Ocean

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ABSTRACT

An algorithm is proposed to numerically integrate the inhomogeneous depth-separated wave equation using a state variable technique. The solution obtained for two simple shallow water models is shown to agree well with the known exact solutions. Integration grid density is discussed and a minimum required density specified.

The use of complex sound speeds to simulate bottom attenuation is reviewed. A numerical instability inherent to the technique that arises during the use of complex sound speeds is investigated.

The algorithm is also applied to a deep ocean profile, and the solution characteristics discussed. Sensitivity problems that arise when modelling the seafloor as a layered elastic medium are analyzed.

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INTRODUCTION

Since Pekeris's study of shallow water sound propagation(1), considerable effort has been directed toward developing a better understanding of the interaction of oceanic acoustic waves with the seabed. This effort has occurred in two disciplines, seismology and ocean acoustics.

In seismology, primary interest in wave propagation below the oceanseabed interface. Such information as layer depths and crustal structure is desired, with the goal being to confirm seismic models through the generation of synthetic seismograms. Mathematical extensions of ray theory are usually used to construct the solution to the inhomogeneous wave equation(2,3,4,6,7). Ocean acoustics, on the other hand, is concerned with the seabed as a lossy and dispersive medium which influences the structure of the acoustic pressure field in the ocean. Construction of the solution via an expansion of the eigenfunctions associated with the homogeneous equation is most common(9,10, 12), although several recent methods obtain the Green's function solution to the inhomogeneous equation directly(13,14). The advantages and disadvantages of the most important analysis techniques that have evolved in each field will be discussed in the following.

SEISMOLOGY

The most basic approach to elastic wave propagation in the seismic literature is the Thomson-Haskell method(2,3). It considers a medium of parallel, isotropic layers which have constant elastic parameters and sound speeds. A plane elastic wave at specified wave number is propagated at oblique incidence through the stratified media. Matrices which express the boundary conditions for each interface are written. The properties of the wave in the Nth layer are obtained in terms of the same properties in the

 1^{st} layer via a recurrence relation which involves the product of N matrices(2).

This matrix product is a major problem with the Thomson-Haskell method and prevents its more widespread use. Accurate modeling of the medium often requires a large number of layers(due to the restriction that each layer must have constant physical properties) and computation of the product of a large number of matrices is both time-consuming and inaccurate. The solution contains both positive and negative exponential terms and may suffer from numerical instability. This instability results from the loss of that arises as the exponentially growing term increases through the profile.

As an example, the Thomson-Haskell method has been applied to the special case of low-frequency sound propagation in the Arctic Ocean(22,23). In this application the upward-refracting sound speed profile required the division of the fluid into layers of constant speed overlying an infinite half-space. The half-space was taken to be either a high-speed fluid(22) or a solid(23). The calculation of the transmission matrices required the use of double precision arithmetic and limited depths to approximately 1 kilometer. These restrictions prevent the application of the method to more general deep ocean models.

The method of characteristics(4,5) can be used to solve the wave equation since it is a second order hyperbolic differential equation. The characteristics are the natural coordinates of the differential equation and are roughly analogous to the natural coordinates used in multidegree of freedom vibration systems. In application of the method to wave propagation problems, an initial wavefront is assumed on which the velocity potential and all first order derivatives are known. The characteristic equations, which represent

propagation paths forward of the initial wavefront, can be derived from the inhomogeneous form of the original differential equation. A second set of equations, the conditionals, are then derived from the inhomogeneous equation. These two sets of equations are equivalent to the boundary value problem.

Numerical calculation of the solution is straightforward. The characteristics form a grid forward of the initial wavefront. By using the known initial conditions to begin integration of the conditionals, the solution is sequentially determined throughout the region.

Since the conditionals are usually integrated numerically, an advantage of this technique is that all elastic properties and sound speeds can vary. Discontinuities in the initial conditions are also acceptable and will propagate along the grid.

The main problem with the method of characteristics is similar to the instability problem of the Thomson-Haskell method. The propagation paths represent increasing and decreasing amplitude terms. As the solution is computed, the increasing term tends to swamp the contribution of the decreasing term, causing a loss of precision and numerical stability.

Finite difference techniques are most commonly implemented in both the integration and the approximation of the appropriate boundary conditions. The errors due to these procedures require high grid density and a resulting increase in computation time. The sequential nature of the solution is also a direct cause of long computation times.

Two Fourier transform techniques that have become important in the siesmic literature in recent years are the ray-theoretical and reflectivity methods. Both methods consider an earth model consisting of an oceanic

liquid layer overlying a solid medium of plane, homogeneous, isotropic, elastic layers and an elastic half-space.

The ray-theoretical method requires the calculation of the step function response of the earth model for a point source and receiver in the ocean layer. The step-function response involves using generalized reflection and transmission coefficients to obtain an integral representation of the acoustic field. Next, a source transfer function is written which describes the nature of the pressure field due to the underwater explosion that acts as the acoustic source. The synthetic seismogram is then obtained by convolving the step response with the time derivative of the source transfer function(6). The initial step of the reflectivity method is the numerical integration in the horizontal wave number domain of the plane wave reflection coefficient(reflectivity). This procedure is repeated for each horizontal phase velocity and the seismogram is obtained by the inverse transformation of the product of the source spectrum and the reflectivity(7).

Since it includes multiple reflections and converted waves, the reflectivity method provides highly accurate synthetic seismograms. However, it suffers from long computation times if the reflection response has a long duration(7). The ray-theoretical method does not have a dependence on response duration but instead has a computation time which is directly related to the number of multiple reflections and conversions that are included(6). For any particular application, the choice must be made between the error resulting from omission of these waves and the computation time resulting from their inclusion. Common procedure is to use the ray-theoretical method to obtain an overall seismic picture , and the reflectivity method to study specific details of the profile under consideration(7).

OCEAN ACOUSTICS

In the ocean acoustic literature, emphasis has been on the application of various numerical techniques to the integration in the water column of the depth-separated wave equation. In addition, various computer programs have been developed which obtain solutions for those ocean models in which the sound speed can be written in terms of specified mathematical functions. Finally, Fourier transform theory has been used to study acoustic propagation for given source and receiver configurations.

The Naval Research Laboratory(NRL) programs FLUID and SOLID are examples of the direct application of finite difference techniques to the solution of the homogeneous wave equation(10). The primary goal in each is the determination of the eigenvalues and eigenfunctions of the normal mode form of the solution. The physical model considered is a fluid layer with a specified sound speed profile that overlies a half-space which is either fluid of solid. The elastic parameters of the half space are constants. The iteration procedure employed involves integration to obtain a trial eigenfunction for an estimated eigenvalue and adjustment of the estimate depending on the error obtained between the value of the eigenfunction at the surface and the surface boundary condition. This procedure is repeated until the error at the surface is within a prespecified bound.

Expression of the differential equation and the boundary conditions in finite difference form is straightforward. For multilayered work the particular form of the boundary condition approximation is important both with regard to the accuracy of the solution and with regard to the nature of the compressional-shear coupling in the elastic medium. The elastic multilayered model would require consideration of this topic. It is likely that the

extension to the more complex model would also cause a substantial increase in computation time.

The Applied Research Laboratory at the University of Texas has developed a normal mode program which also obtains the eigenvalues and eigenfunctions of the homogeneous equation(12). It employs numerical integration as well as a parallel shooting technique for efficient computation of each mode and such of its properties as group velocity and attentuation. Parallel shooting involves parameterizing the velocity potential by introducing an independent variable, and dividing the depth coordinate intosubintervals(31). The differential equation is then written as a first order system where each member corresponds to a particular subinterval and the coupling of the equations is expressed by the appropriate continuity conditions. Using an assumed eigenvalue, the system is integrated towards the sound channel. The secant method is used to improve the estimate and reduce the error between the upward and downward integrated terms to within the desired bounde. Although reasonably efficient in its present application, the technique suffers from the same drawbacks that influenced the NRL programs, when applied to the more complex model.

The parabolic wave equation method(13) is a technique of solving the inhomogeneous wave equation that is particularly useful for studying long range propagation in the ocean. It allows both depth and range-dependent sound speed profiles. The two-dimensional parabolic wave equation is obtained from the elliptic wave equation by replacing the velocity potential with the product of a Hankel function and an envelope function, and assuming that:1) the receiver is in the far field, 2) only small angles to the horizontal are to be considered, and 3) motions are uncoupled in azimuthal directions. The

integration is then performed by using a finite difference Fourier algorithm.

Since the parabolic equation is valid only in the far field, its solution must be matched with a near field solution, which is known for simple sources. The upper boundary condition is specified through the use of an image source out of phase with the actual source. The bottom outgoing wave boundary condition is obtained by introducing a large artificial absorption term which prevents the backscattering of waves into the ocean. The validity of this assumption is still in question(13).

Inaccuracies arise from the small angle approximation and at discontinuities in sound speed and density. Characteristically, the technique has long computation times, although recent versions have seen substantial improvement(13).

The Fast Field Program (14) is an application of Fourier transform techniques to the solution of the inhomogeneous wave equation. The acoustic potential is considered the output of a linear system and is the result of the transform of the product of the transfer function of the medium and the transform of the source waveform. The transfer function of the medium is itself a Fourier-Bessel transform and contains all information about the environment. These integrals are evaluated using the fast Fourier transform.

Computation time is the major problem with the FFP. The Green's function solution must be obtained either by numerical integration or by restricting the sound speed profile to situations for which known solutions are available. In the latter case, recurrence relations must be evaluated to obtain the Green's function. These recurrence relations suffer from numerical instability and are time-consuming to evaluate.

Green's functions obtained by numerical integration are usually sensitive

to round-off errors and also substantial computer time. They are constructed by integrating the homogeneous equation for two solutions that independently satisfy the surface and bottom boundary conditions(a similar procedure is used in section 2.1 to obtain an exact solution for a simple shallow water ocean model). The emphasis of this thesis will be on obtaining the Green's function through a state variable representation of the wave equation. This solution technique will be shown to be numerically superior toothesestwo procedures. In addition, when combined with a new technique of evaluating the Hankel transform(15), a substantial improvement in computational efficiency is expected.

A variety of other normal-mode related programs exist which do not fall in any of the above classifications. These include the works of Bucker(9), Stickler(16), and Mckisic and Hamm(17). The first two authors specify the sound speed profileas mathematical functions for which the solution is known. Mckisic and Hamm have applied a new method of solution of eigenvalue problems to the depth-dependent wave equation. The method assumes exponential solutions where the functions in the exponents are specified by Riccati equations. The exponent functions can be obtained by an iteration procedure on the Riccati equations. The additional complexity of the elastic multilayered model is such that the use of any of these techniques in a computer program would be time-consuming and inefficient.

This thesis will study a new technique for evaluating the Green's function solution to the inhomogeneous depth-separated wave equation. The technique involves the use of a state variable representation of the original differential equation that was originally proposed by Baggeroer(18) to obtain the eigenvalues of the homogeneous or normal mode program. Emphasis will be on

the study of the technique and its potential in the solution of acoustic propagation problems. The numerical advantages of the representation will be apparent for the oceanic model that includes shear wave propagation in the bottom.

Chapter I will present the mechanics of the state variable system and introduce the continuity conditions required at the boundaries of each layer.

Chapter II will compare the state variable solution with known solutions for two simple shallow water models.

Chapter III will discuss the use of complex sound speeds to model bottom attentuation and investigate an associated numerical instability. It will conclude with a discussion of a deep ocean profile application and a sensitivity problem that arises when the bottom is modelled as a layered elastic medium.

1. DERIVATION OF STATE VARIABLE ALGORITHM

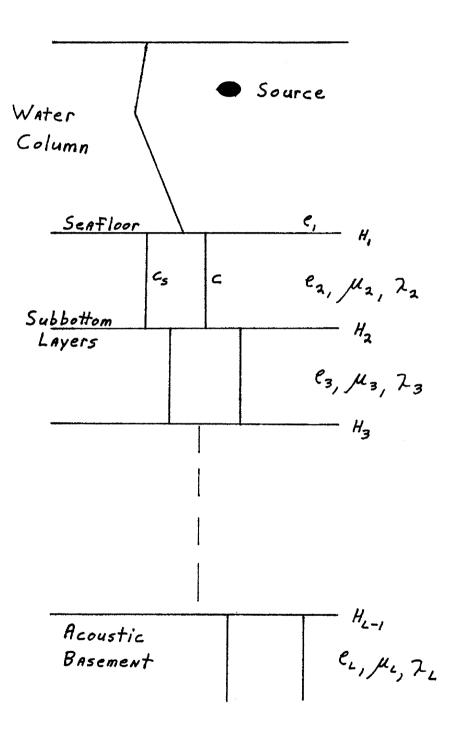
Certain assumptions and definitions are required to simplify the ocean model. These are presented in Section 1. The depth-separated wave equations are then derived and state variable theory used to obtain the state representations. Surface and basement boundary conditions are specified in Section 3, along with the continuity conditions required at the interface between any two layers. Finally, Section 4 introduces a magnitude and phase representation of the state equations that is numerically superior to the linear state equations.

1.1 SPECIFICATION OF THE OCEAN MODEL

For the purpose of this discussion, the ocean is assumed to be an compressible fluid media of constant depth. The seabed is modeled as an elastic medium consisting of a specified number of horizontal, homogeneous isotropic layers in which both compressional and shear waves can propagate. The sound speed of the ocean can vary in the vertical direction. All elastic layers are restricted to have constant sound speeds and elastic properties. Compressional and shear speeds C and C_s are defined in terms of Lame's constant λ , rigidity μ , and density ρ as follows:

$$2 + 2\mu = e^{2}$$
$$\mu = e^{2}$$

(all variables are defined in Appendix I). It should be noted that the technique to be described requires constant elastic parameters only in that the standard derivation of the wave equation for elastic media makes this assumption (19). The final requirement of the model is that coupling



The General Ocean Model

Figure 1.1.1

between compressional and shear waves occurs only at the interface between layers. This coupling is specified by the continuity of vertical and horizontal velocity and normal and shear stress. Figure 1.1,1 depicts the ocean-seabed model.

For convenience, a Cartesian coordinate system is employed. The depth-separated equation that results is identical to that resulting from the use of a cylindrical coordinate system (32). Therefore, the potential of the technique for use with the FFP or with the Hankel transform algorithm of reference (15) can still be evaluated. In addition, the problems associated with expressing the continuity conditions between elastic layers in cylindrical coordinates can be avoided.

1.2 DERIVATION OF THE STATE EQUATIONS

Consider the general form of the acoustic wave equation for a compressional velocity potential $\underline{\mathcal{F}}(X, Y, \overline{z}, t)$ $\nabla^2 \underline{\mathcal{F}} - \frac{1}{c^2(\overline{z})} \frac{\partial^2 \underline{\mathcal{F}}}{\partial t^2} = \mathcal{F}(\overline{r} - \overline{r_o}) \overset{-iwt}{e} 1.2.1$

Assuming harmonic time dependence and horizontal plane wave propagation (the far field) the substitution

$$\overline{\phi}(x,\overline{z},t) = e \qquad \phi(\overline{z}) \qquad 1.2.2$$

reduces 1.2.1 to

$$\frac{d^2\phi(z)}{dz^2} + \left[\frac{(2\pi f)^2}{c^2(z)} - (2\pi v)^2\right]\phi(z) = \delta(z-z)_{1,2,3}$$

Following the same procedure for a shear velocity potential gives the result

$$\frac{d^2 \Psi(z)}{dz^2} + \left[\frac{(2\pi f)^2}{c_s^2(z)} - (2\pi V_s)^2\right] \Psi(z) = 0$$
1.2.4

It is useful to define normalized sound speed and horizontal wave number functions as well as a depth parameter that is normalized by the longest wavelength of the sound speed profile (18). The resulting forms of the differential equations are

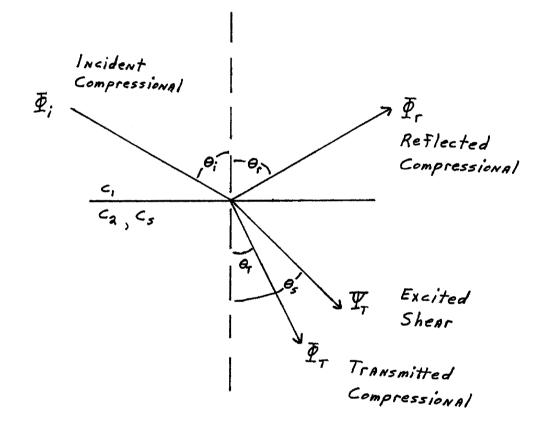
Note that the depth parameter has been normalized by two different wavelengths corresponding to the compressional and shear maximum wavelengths, respectively. This is necessary because the normalization process requires that

$$\left(\frac{2\pi f}{c(i)}\right)^{2} - (2\pi V)^{2} = -(g^{2}(i) - V^{2})$$

$$\left(\frac{2\pi f}{c_{s}(i)}\right) - (2\pi V_{s})^{2} = -(g^{2}(i) - V_{s}^{2})$$
1.2.7

Had a single normalized depth parameter been used, both of equations 1.2.7 could not have been satisfied. This becomes clear upon the substitution of 1.2.5 c & d and 1.2.6 c & d into 1.2.7. Since it is preferable to scale 1.2.3 and 1.2.4 into a form which retains their mathematical similarity (i.e., 1.2.5a and 1.2.6a), the independent depth parameters are introduced. Further considerations regarding the use of the two depth parameters will be discussed in Section 1.4.

At this point, it is important to state the relationship between the horizontal wave numbers V and V_s. Consider the case of a plane compressional wave incident from a fluid onto an elastic solid as is depicted in Figure 12.1. Potential expressions for the various waves are $\overline{P}_i(X, \overline{z}) = e_X p \left[i A_i(X 5in \theta_i + \overline{z} \cos \theta_i) \right]$ $\overline{P}_r(X, \overline{z}) = e_X p \left[i A_i(X 5in \theta_i - \overline{z} \cos \theta_i) \right]$ $\overline{P}_r(X, \overline{z}) = e_X p \left[i A_i(X 5in \theta_i - \overline{z} \cos \theta_i) \right]$ $\overline{P}_r(X, \overline{z}) = e_X p \left[i A_i(X 5in \theta_i + \overline{z} \cos \theta_i) \right]$ $\overline{P}_r(X, \overline{z}) = e_X p \left[i A_i(X 5in \theta_i + \overline{z} \cos \theta_i) \right]$ By Snell's law, the wave number parallel to the boundary must be the same for all waves. Hence,



Snell's Law Relationships

Figure 1.2.1

Since

$$2\pi V = k, 5in \Theta;$$

$$2\pi V_s = k, 5in \Theta_s$$

it follows that $V = V_s$.

Representation of systems in state form is common in the control theory literature (20,21). The general objective is to model the particular process by a system of first order ordinary differential equations of the form

$$\frac{d}{di} \begin{bmatrix} X_{i} & Ci \\ ... \\ ... \\ ... \\ X_{n} & Ci \end{bmatrix} = \begin{bmatrix} A_{ii} & Ci \\ ... \\ ... \\ A_{ini} & (i) - - A_{ini} & Ci \end{bmatrix} \begin{bmatrix} X_{i} & Ci \\ ... \\ ... \\ X_{n} & Ci \end{bmatrix} + \begin{bmatrix} U_{i} & Ci \\ ... \\ ... \\ U_{ni} & Ci \end{bmatrix}$$
1.2.8

where \underline{X} (f) are the states of the process and \underline{U} (f) the control inputs. Since control processes of this form have been well studied, representation of the acoustic wave equation in state form is straightforward.

The first step in the derivation is to define two state variables. The velocity potential, $\phi(\varsigma)$, being of prime importance here, is specified as the first variable. The second variable, $P(\varsigma)$, has no direct acoustic significance. The state system is

$$\begin{bmatrix} \dot{\phi}(\mathfrak{s}) \\ \dot{P}(\mathfrak{s}) \end{bmatrix} = \begin{bmatrix} A_{11}(\mathfrak{s}) & A_{12}(\mathfrak{s}) \\ A_{21}(\mathfrak{s}) & A_{22}(\mathfrak{s}) \end{bmatrix} \begin{bmatrix} \phi(\mathfrak{s}) \\ \phi(\mathfrak{s}) \\ \phi(\mathfrak{s}) \end{bmatrix} + \begin{bmatrix} U_{1}(\mathfrak{s}) \\ U_{2}(\mathfrak{s}) \end{bmatrix}^{-1.2.9}$$

where the dot refers to the derivative with respect to ${\circleftillef$

The coefficient matrix <u>A</u> is determined by matching 1.2.9 to the original depth-separated wave equation 1.2.5. This is done by taking the derivative of 1.2.9a with respect to f and substituting in 1.2.9

for all first order derivatives. The second order equation that results is

$$\begin{split} \ddot{\phi}(\vec{s}) &= \left[\dot{A}_{11}(\vec{s}) + \dot{A}_{12}^{2}(\vec{s}) + \dot{A}_{12}(\vec{s}) \dot{A}_{21}(\vec{s}) \right] \dot{\phi}(\vec{s}) \\ &+ \left[A_{11}(\vec{s}) A_{21}(\vec{s}) + \dot{A}_{21}(\vec{s}) A_{22}(\vec{s}) + \dot{A}_{22}(\vec{s}) \right] P(\vec{s})_{1.2.10} \\ &+ \dot{A}_{12}(\vec{s}) \dot{U}_{2}(\vec{s}) + \dot{U}_{1}(\vec{s}) \end{split}$$

Matching the coefficients of 1.2.10 with those of 1.2.5a, three equations are obtained for the six unknowns of 1.2.9.

$$\dot{A}_{11}(i) + A_{11}^{2}(i) + A_{12}(i)A_{21}(i) = g^{2}(i) - r^{2}$$

$$A_{11}(i)A_{21}(i) + A_{21}(i)A_{22}(i) + A_{22}(i) = 0$$

$$A_{12}(i)U_{2}(i) + \dot{U}_{1}(i) = 7_{0}f(i - i_{0})$$

Exact values of the coefficients $A_{ii}(\xi)$ and $U_i(\xi)$ can be chosen in any manner that satisfies all of equations 1.2.11. Particularly convenient is $A_{12}(\xi) = 1$ and $U_1(\xi) = 0$, which reduces 1.2.11 to

$$\dot{A}_{11}(s) + A_{11}^{2}(s) + A_{21}(s) = g^{2}(s) - r^{2}$$

$$A_{11}(s) = -A_{22}(s)$$

$$U_{2}(s) = 7_{0} \int (s - s_{0})$$
1.2.12

The well-known Riccati equation can be obtained from 1.2.12a by the substitution $f(\xi) = -A_{11}(\xi)$.

$$\dot{f}(\hat{s}) = f^{2}(\hat{s}) + A_{2}(\hat{s}) - g^{2}(\hat{s}) + \Gamma^{2} \qquad 1.2.13$$

The above derivation is a repetition in greater detail of reference (18), which also discusses the choice of $A_{21}(\zeta)$. That choice is

$$A_{21}(\xi) = -\nabla^2$$
 1.2.14

The compressional state system is

$$\begin{bmatrix} \dot{\phi}(\hat{s}) \\ \dot{\rho}(\hat{s}) \end{bmatrix} = \begin{bmatrix} -f(\hat{s}) & i \\ -\nabla^2 & f(\hat{s}) \end{bmatrix} \begin{bmatrix} \phi(\hat{s}) \\ P(\hat{s}) \end{bmatrix} + \begin{bmatrix} 0 \\ 7_0 \end{bmatrix} \delta(\hat{s} - \hat{s}_0)$$

$$1.2.15$$

$$f(s) = f^{2}(s) - g^{2}(s)$$
 1.2.16

Notice that the Riccati equation is a function of the sound speed profile only and is independent of wavenumber. Riccati equations have been well studied and 1.2.16 can be numerically integrated. The coefficient matrix in 1.2.15 is determined by the Riccati solution and the equations can then be integrated to obtain the velocity potential.

The shear state system is

$$\begin{bmatrix} \dot{\psi}(\tilde{s}_{s}) \\ \dot{P}_{s}(\tilde{s}_{s}) \end{bmatrix} = \begin{bmatrix} -\bar{F}_{s}(\tilde{s}_{s}) & I \\ -\bar{F}_{s}(\tilde{s}_{s}) \end{bmatrix} \begin{bmatrix} \psi(\tilde{s}_{s}) \\ P_{s}(\tilde{s}_{s}) \end{bmatrix} \begin{bmatrix} -\bar{F}_{s}^{2} & \bar{F}_{s}(\tilde{s}_{s}) \end{bmatrix} \begin{bmatrix} P_{s}(\tilde{s}_{s}) \\ P_{s}(\tilde{s}_{s}) \end{bmatrix}$$

$$i.2.17$$

$$\dot{F}(\tilde{s}_{s}) = \bar{F}_{s}^{2}(\tilde{s}_{s}) - \bar{\theta}_{s}^{2}(\tilde{s}_{s})$$

At this point, the advantages of the state representation are probably not obvious. They become more apparent as the state equations are implemented in a computer program. The simplification from a second to a first order system allows the use of numerical integration techniques which are easier to apply than those for the second order system and for which approximation errors are less crucial. The state systems also allow the analysis of sound speed profiles which are more representative of the real ocean. Finally, the analogy to the FFP is apparent, with the extension to long-range propagation applications theoretically straightforward.

1.3 CONTINUITY CONDITIONS

In general, continuity of nine quantities is required at the interface between two elastic layers. Velocity components comprise three of the quantities and stress components the remaining six. The velocity terms follow from the definition of compressional and shear velocity potentials, which is

$$\overline{V} = (u, v, w) = \nabla \underline{\mathcal{P}} + \nabla \times \underline{\mathcal{P}}$$
 1.3.1

For a homogeneous media in a Cartesian coordinate system with wave propagation occurring only in two dimensions, the "V" velocity and all Y terms can be neglected. The remaining horizontal and vertical velocity

terms, neglecting the exponential of equation 1.2.2, are

$$\begin{aligned}
\mathcal{U} &= \frac{\partial \overline{\Psi}}{\partial \chi} - \frac{\partial \overline{\Psi}}{\partial \overline{z}} = i2\pi V \phi(i) - \frac{1}{7_{os}} \frac{d \Psi(i)}{d i_{s}} \\
w &= \frac{\partial \overline{\Psi}}{\partial \overline{z}} + \frac{\partial \overline{\Psi}}{\partial \chi} = \frac{1}{7_{o}} \frac{d \phi(i)}{d i_{s}} + i2\pi V \Psi(i_{s})^{1.3.2}
\end{aligned}$$

Since equations 1.2.5 and 1.2.6 were scaled using different normalized depth variables, notice that ϕ and ψ are functions of ξ and ξ_s , respectively. The integration size for ϕ and ψ will in general be different. However, the coupling at each interface requires that they

be expressed in the continuity conditions with respect to the same coordinate. Since

$$\frac{\partial \phi}{\partial z} = \frac{\partial \phi}{\partial \xi} \frac{\partial \xi}{\partial z} = \frac{1}{7_o} \frac{\partial \phi}{\partial \xi}$$
$$\frac{\partial \psi}{\partial z} = \frac{\partial \psi}{\partial \xi} \frac{\partial \xi_s}{\partial z} = \frac{1}{7_os} \frac{\partial \psi}{\partial \xi_s}$$

this has in fact been done in 1.3.2.

For an isotropic media, the six stress components can be written in terms of two elastic parameters: the rigidity μ and the Lamé constant λ . The stress expressions, written in terms of the velocity components and the dilation $\boldsymbol{\Theta}$, are

$$\begin{aligned}
\nabla_{XX} &= 7 \theta + 2\mu \frac{du}{dX} \\
\nabla_{YY} &= 7 \theta + 2\mu \frac{dv}{dY} \\
\nabla_{ZZ} &= 7 \theta + 2\mu \frac{dw}{dZ} \\
\nabla_{XY} &= \mu \left(\frac{du}{dY} + \frac{d'v}{dX}\right) \\
\nabla_{YZ} &= \mu \left(\frac{du}{dZ} + \frac{dw}{dY}\right) \\
\nabla_{ZX} &= \mu \left(\frac{dw}{dX} + \frac{dw}{dZ}\right) \\
\theta &= \frac{du}{dX} + \frac{dv}{dY} + \frac{dw}{dZ}
\end{aligned}$$
1.3.3

The assumptions of the model reduce the above six expressions to the normal stress ∇_{zz} and the tangential stress ∇_{zx} . Expressed in potential form, they are $\nabla_{zz} = 7 (i 2\pi v)^2 \phi(s) + \frac{7 + 2u}{7_o^2} \frac{d^2 \phi(s)}{ds^2} + \frac{u i 4\pi v}{7_o s} \frac{d' \psi}{ds}$ $\nabla_{zx} = \mu \left[\frac{i 4\pi v}{7_o} \frac{d \phi(s)}{ds} + (i 2\pi v)^2 \psi(s) - \frac{1}{7_o^2} \frac{d'^2 \psi(s)}{ds} \right]^{1.3.4}$

It is important to note that the reduction of 1.3.1 to 1.3.2 and 1.3.3 to 1.3.4 is possible because of the use of an X, Y, Z geometry and the

homogeneous, isotropic media assumption (33). Extension to a cylindrical geometry, as may be desirable for future applications, will require reconsideration of the velocity and stress equations.

The result of these simplifications is that there are four quantities that must be continuous across the interface between any two elastic layers. The formal expression of these continuity conditions, as shown in Figure 1.3.1, is

$$\begin{split} \mathcal{U}_{i}(H_{i}) &= \mathcal{U}_{i+i}(H_{i}) \\ \mathcal{W}_{i}(H_{i}) &= \mathcal{W}_{i+i}(H_{i}) \\ \nabla_{\mathbb{Z}^{2}i}(H_{i}) &= \nabla_{\mathbb{Z}^{2}i+i}(H_{i}) \\ \nabla_{\mathbb{Z}^{X}_{i}}(H_{i}) &= \nabla_{\mathbb{Z}^{X}_{i+i}}(H_{i}) \end{split}$$
1.3.5

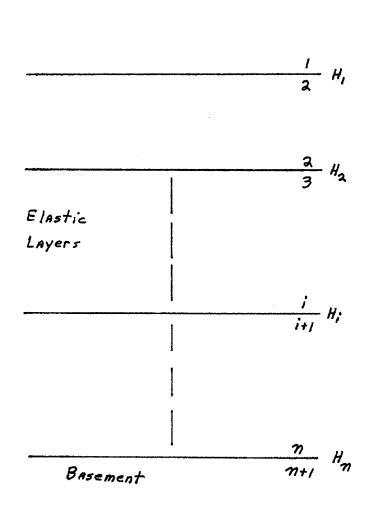
At the interface between the ocean and the uppermost elastic layer, a simplification of these conditions is in order. First, since the fluid is not allowed to hold shear, the tangential stress in the solid must vanish. Second, continuity of horizontal velocity is not required since slippage can occur. The formal expressions are

$$\begin{aligned} \mathcal{U}_{i}(H_{i}) &= \mathcal{U}_{2}(H_{i}) \\ \nabla_{zz_{i}}(H_{i}) &= \nabla_{zz_{2}}(H_{i}) \\ \mathcal{O} &= \nabla_{zx_{2}}(H_{i}) \end{aligned}$$
 1.3.6

The boundary condition at the ocean's surface is that the normal stress must vanish. From 1.3.4a and 1.2.5a, the expression for this condition is

$$\phi(o) = 0 \qquad 1.3.7$$

The bottom boundary condition can take either of two forms. For an N^{th} layer bounded by a rigid surface, both velocity components must vanish.



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Continuity Conditions

Figure 1.3.1

$$i2\pi V \phi \begin{pmatrix} H_{m}/Z_{o} \end{pmatrix} - \frac{1}{7_{os}} \frac{\partial \Psi (H_{m}/Z_{os})}{\partial S_{s}} = 0$$

$$\frac{1}{7_{o}} \frac{\partial \Phi (H_{m}/Z_{o})}{\partial S_{s}} + i2\pi V \Psi (\frac{H_{m}}{7_{os}}) = 0$$
1.3.8

For an Nth layer bounded by a propagating, infinite elastic half space, constrained motions are required.

$$|\psi(s)| < N < \infty$$
 $\xi \rightarrow \infty$
 $|\psi(s_{3})| < N < \infty$ $\xi \rightarrow \infty$ ^{1.3.9}

The boundary value problem has now been completely specified except for the exact nature of the solution. The homogeneous shear state equations are excited at the ocean bottom by the coupling of $\phi(i)$ and $\Psi(i)$, expressed by 1.3.6, and the shear solution consists solely of the integration of 1.2.17 satisfying 1.3.6 and either 1.3.8 or 1.3.9. The compressional state equations are inhomogeneous and their solution must consist of some combination of homogeneous and particular terms which satisfy 1.3.7 and either 1.3.8 or 1.3.9. Formally, the solutions are

$$\phi(\mathfrak{s}) = \phi_{p}(\mathfrak{s}) + \phi_{H}(\mathfrak{s})$$

$$P(\mathfrak{s}) = P_{p}(\mathfrak{s}) + P_{H}(\mathfrak{s})$$

1.3.10

Shear:

Compressional:

$$\Psi(\varsigma) = \Psi_{H}(\varsigma_{s})$$

$$P_{s}(\varsigma) = P_{sH}(\varsigma_{s})$$

The boundary conditions for the particular solution depend on the integration direction. Baggeroer (18) discusses the convenience of integrating up the profile, and the resulting choice of

$$\oint_{P} \left(\frac{H_{m}}{Z_{o}} \right) = 0$$

$$P_{P} \left(\frac{H_{m}}{Z_{o}} \right) = 0$$

$$1.3.12$$

These initial conditions result in a particular solution that is identically zero below the source.

Equations 1.3.12 allow expression of the bottom boundary conditions solely in terms of the homogeneous potentials. For the rigid bottom case $i 2\pi V \phi_{H} \begin{pmatrix} H_{\eta} \\ 7_{o} \end{pmatrix} - \frac{1}{7_{os}} \frac{\partial \Psi_{H} \begin{pmatrix} H_{\eta} \\ 7_{os} \end{pmatrix}}{\partial \xi_{s}} = 0$ $\frac{1}{7_{o}} \frac{\partial \phi_{H} \begin{pmatrix} H_{\eta} \\ 7_{o} \end{pmatrix}}{\partial \xi} + i 2\pi V \Psi_{H} \begin{pmatrix} H_{\eta} \\ 7_{os} \end{pmatrix} = 0$ 1.3.13

For the propagating basement model, the assumption of constant sound speed allows an analytic solution to be obtained. That solution is

$$\begin{split} \psi_{H}\left(\mathfrak{f}^{2}\frac{H_{m}}{2_{o}}\right) &= \frac{1}{2}\left[\psi_{H}\left(\frac{H_{m}}{2_{o}}\right) - \frac{1}{r}\frac{\partial\psi_{H}\left(\frac{H_{m}}{2_{o}}\right)}{\partial\mathfrak{f}^{2}}\right]e^{-r\left(\mathfrak{f}-\frac{H_{m}}{2_{o}}\right)} \\ &+ \frac{1}{2}\left[\psi_{H}\left(\frac{H_{m}}{2_{o}}\right) + \frac{1}{r}\frac{\partial\psi_{H}\left(\frac{H_{m}}{2_{o}}\right)}{\partial\mathfrak{f}^{2}}\right]e^{-r\left(\mathfrak{f}-\frac{H_{m}}{2_{o}}\right)} \\ \psi_{H}\left(\mathfrak{f}_{s}^{2}\frac{H_{m}}{2_{os}}\right) &= \frac{1}{2}\left[\psi_{H}\left(\frac{H_{m}}{2_{os}}\right) - \frac{1}{r}\frac{\partial\psi_{H}\left(\frac{H_{m}}{2_{os}}\right)}{\partial\mathfrak{f}_{s}}\right]e^{-r\left(\mathfrak{f}_{s}-\frac{H_{m}}{2_{os}}\right)} \\ &+ \frac{1}{2}\left[\psi_{H}\left(\frac{H_{m}}{2_{os}} + \frac{1}{r}\frac{\partial\psi_{H}\left(\frac{H_{m}}{2_{os}}\right)}{\partial\mathfrak{f}_{s}}\right]e^{-r\left(\mathfrak{f}_{s}-\frac$$

$$\psi_{H}\left(\frac{H_{m}}{T_{o}}\right) + \frac{1}{\Gamma} \frac{\partial \psi_{H}\left(\frac{H_{m}}{T_{o}}\right)}{\partial \varsigma} = 0$$

$$\psi_{H}\left(\frac{H_{m}}{T_{os}}\right) + \frac{1}{\Gamma_{s}} \frac{\partial \psi_{H}\left(\frac{H_{m}}{T_{os}}\right)}{\partial \varsigma_{s}} = 0$$

$$1.3.15$$

Both 1.3.13 and 1.3.15 contain four unknowns and therefore have two apparently arbitrary constants. As integration proceeds upward, however, these constants are specified by other requirements of the model. In particular, equation 1.3.6c, which must be satisfied at the ocean-sediment interface, determines the third unknown by expressing the required relationship between the compressional and shear fields. The final unknown is specified by the surface boundary condition. Recall that the particular solution can be integrated using 1.3.12. Therefore, from 1.3.7,

$$\phi_{\mu}(o) = -\phi_{\mu}(o)$$
 1.3.16

and the boundary value problem is completely specified.

Since the boundary conditions 1.3.6c, 1.3.7, and 1.3.13 or 1.3.15 are split (that is, evaluated at different points in the depth profile), it is useful to consider the general step-by-step procedure to be employed in solving the boundary value problem. First, the particular solution is obtained. For the initial conditions of 1.3.12, $\phi_{p}(i)$ and $P_{p}(i)$ will be non-zero from the source point to the surface. Second, two unknowns are chosen from the four in 1.3.15 (for a propagating basement) and given unit amplitude. For example,

For simplicity in satisfying 1.3.6c, the third step is to integrate upward using only the compressional field in the basement as the initial condition (assuming for the moment zero shear $a + H_n$). Compressional and shear fields will be excited in all elastic layers and integration will proceed to the uppermost interface of the profile where 1.3.6c will not in general be satisfied. Next, the shear field is used as the initial condition at H_n (with a zero compressional field) and integration again will proceed upward to the uppermost interface.

At this point, two independent solutions will have been obtained for the elastic layers, neither of which satisfies 1.3.6c. The linearity of 1.2.15 and 1.2.17 allow superposition of these solutions in such a manner as to satisfy 1.3.6c. This superposition amounts to a scaling of the shear solution to insure that the tangential stress at H_1 vanishes. This procedure specifies the third constant as was discussed earlier.

Equations 1.3.6 a & b are used to obtain the appropriate compressional field quantities in the ocean and integration proceeds to the surface to obtain $\phi_{\mu}(c)$. The superposition used earlier determined the relationship between the compressional and shear fields but not the absolute magnitude of either (the fourth constant) and in general 1.3.16 will not be satisfied. Scaling of the entire compressional and shear solutions by the constant

$$C = - \frac{\phi_{p}(o)}{\phi_{H}(o)}$$
 1.3.18

will insure that the surface boundary condition is satisfied.

The compressional and shear solutions are

$$\phi(s) = \phi_{p}(s) - \frac{\phi_{p}(o)}{\phi_{H}(o)} \phi_{H}(s)$$
1.3.19

$$\Psi(\varsigma) = - \frac{\phi_{p}(\sigma)}{\phi_{\mu}(\sigma)} \Psi_{\mu}(\varsigma_{s})$$
 1.3.20

This completes the specification of the boundary value problem. The equations can now be programmed and solutions obtained for any ocean and seabed parameters. It has been found, however, that a representation independent of absolute magnitude is more convenient numerically. This representation will be presented in Section 1.4.

1.4 PHASE PLANE REPRESENTATION

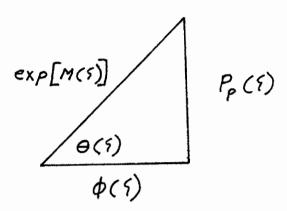
The state equations of Section 1.3 can be integrated to obtain the solution for any given ocean model. However, the exponential nature of the solution causes large amplitude terms to arise, especially in deep ocean examples. Baggeroer (18) introduced a magnitude and phase representation of the state equations which is numerically preferable. This representation allows integration of differential equations which are independent of the absolute magnitude of $p(\zeta)$ and $\psi(\zeta_{s})$. As shown in Figure 1.4.1, the respective variables are defined

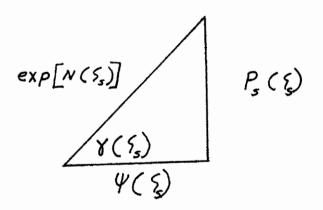
$$\Theta(i) = TAN^{-1} \left(\frac{P(i)}{\phi(i)} \right)$$
1.4.1

$$M(\mathfrak{f}) = \frac{1}{2} l_{N} \left[\phi^{2}(\mathfrak{f}) + P^{2}(\mathfrak{f}) \right]$$

$$1.4.2$$

$$\chi(\tilde{s}) = T_{AN} \left(\frac{F_{3}(\tilde{s}_{3})}{\psi(\tilde{s}_{3})} \right)$$
 1.4.3





Phase Plane Relationships Figure 1.4.1

$$N(\xi_{5}) = \frac{1}{2} l_{N} \left[\Psi^{2}(\xi_{5}) + P_{5}^{2}(\xi_{5}) \right]$$
1.4.4

Under this transformation, the state equations can be derived to be

$$\dot{\Theta}(\hat{s}) = 2f(\hat{s})\cos\Theta(\hat{s})\sin\Theta(\hat{s}) - \nabla^2\cos^2\Theta(\hat{s}) - 5in^2\Theta(\hat{s}) = 1.4.5$$

$$\dot{M}(\vec{s}) = f(\vec{s}) \begin{bmatrix} 5in^2 \Theta(\vec{s}) - \cos^2 \Theta(\vec{s}) \end{bmatrix}$$

$$+ (1 - \sigma^2) 5in \Theta(\vec{s}) \cos \Theta(\vec{s}) \\ \dot{\gamma}(\vec{s}_s) = 2f_s(\vec{s}_s) \cos \gamma(\vec{s}_s) 5in \gamma(\vec{s}_s)$$

$$- r_s^2 \cos^2 \gamma(\vec{s}_s) - 5in^2 \gamma(\vec{s}_s) \\ \dot{N}(\vec{s}_s) = f_s(\vec{s}_s) \begin{bmatrix} 5in^2 \gamma(\vec{s}_s) - \cos^2 \gamma(\vec{s}_s) \end{bmatrix}$$

$$\dot{N}(\vec{s}_s) = f_s(\vec{s}_s) \begin{bmatrix} 5in^2 \gamma(\vec{s}_s) - \cos^2 \gamma(\vec{s}_s) \end{bmatrix}$$

$$\dot{N}(\vec{s}_s) = f_s(\vec{s}_s) \begin{bmatrix} 5in^2 \gamma(\vec{s}_s) - \cos^2 \gamma(\vec{s}_s) \end{bmatrix}$$

$$\dot{N}(\vec{s}_s) = f_s(\vec{s}_s) \begin{bmatrix} 5in^2 \gamma(\vec{s}_s) - \cos^2 \gamma(\vec{s}_s) \end{bmatrix}$$

$$\dot{N}(\vec{s}_s) = f_s(\vec{s}_s) \begin{bmatrix} 5in^2 \gamma(\vec{s}_s) - \cos^2 \gamma(\vec{s}_s) \end{bmatrix}$$

The surface and propagating basement boundary conditions become

$$\Theta(o) = \frac{T}{2}$$

$$\theta(\frac{H_{m}}{R_{o}}) = T_{AN} \left[f(\frac{H_{m}}{R_{o}}) - f^{2}(\frac{H_{m}}{R_{o}}) - \nabla^{2} \right]_{1.4.10}$$

$$\delta(\frac{H_{m}}{R_{o}s}) = T_{AN} \left[f_{s}(\frac{H_{m}}{R_{o}s}) - f^{2}(\frac{H_{m}}{R_{o}s}) - \nabla^{2} \right]_{1.4.11}$$

The rigid bottom boundary conditions 1.3.13 and 1.3.5 and 1.3.6 do not transform into convenient magnitude and phase expressions. When they are required, the simplest procedure is to return to the linear plane, obtain the desired result and transform back to the phase plane. Implementation of these equations is straightforward. The particular solution is integrated in the linear plane. The homogeneous solutions are obtained in the phase plane and the total solution is given by

$$\psi(\tilde{s}_{3}) = e^{N(\tilde{s}_{3})} \cos \chi(\tilde{s}_{3})$$

$$(\tilde{s}) = \psi_{p}(\tilde{s}) - \frac{\psi_{p}(o)}{e^{N(o)}\cos\theta(o)}} \cos \theta(\tilde{s})$$

$$(1.4.12)$$

$$(\tilde{s}) = \psi_{p}(\tilde{s}) - \frac{\psi_{p}(o)}{e^{N(o)}\cos\theta(o)}} \cos \theta(\tilde{s})$$

$$(1.4.13)$$

Notice that the scaling constant of 1.3.18 has been included in 1.4.13.

In the following chapter, these equations will be integrated for two specific ocean-seabed models and compared with known Green's function solutions.

2. CONFIRMATION OF STATE VARIABLE ALGORITHM

At any given wavenumber, the Green's function solution to the wave equation is equivalent to the superposition of an interfering set of up and down travelling plane waves. Construction of the solution by an appropriate combination of plane waves will provide an accurate test of the theory of Chapter I. In Chapter II an endpoint method of constructing the Green's function will be used to obtain the exact solution of the wave equation for two simple shallow water oceanic models. The state variable solution will then be compared to the exact solution.

2.1 SINGLE LAYER OCEAN EXACT SOLUTION

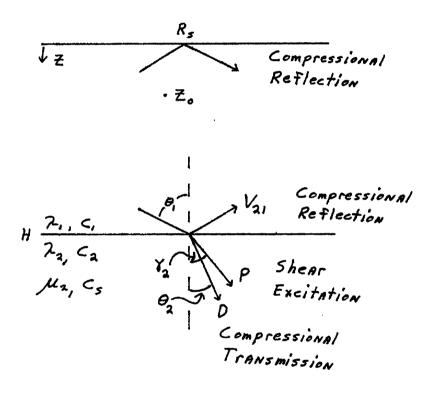
An exact solution of the inhomogeneous depth-separated wave equation can be written for the simple model of a constant sound speed fluid layer overlying an infinite elastic half space. Figure 2.1.1 depicts this model, which is commonly referred to as the generalized Pekeris waveguide.

The assumption of a pressure release surface at z = 0 reduces R_s , the surface reflection coefficient, to the value -1. The remaining three coefficients do not in general reduce to such a convenient value, and must be written in terms of the appropriate elastic parameters and wave numbers. For a fluid with $K_1 = \frac{2}{c_1} \frac{\gamma r}{c_1} f$, the horizontal and vertical wave numbers are defined in terms of the angle of incidence Θ , as follows:

$$2\pi F = R, 5 \approx \theta,$$

$$K_{z_1} = K, Cos \theta,$$

The corresponding expressions in the bottom are



Single Layer Ocean Description

Figure 2.1.1

$$2\pi V = K_2 \quad 5iv \quad \theta_2 \qquad \qquad 2\pi V = K_{25}$$
$$K_{22} = K_2 \quad \cos \theta_2 \qquad \qquad K_{225} = K_{25} \quad \cos \delta_2$$

The angles θ_{1} , θ_{2} , and δ_{2} are measured from the z-axis to the respective propagation vectors as shown in Figure 2.1.1. Brekhovskikh(25) writes the reflection and transmission coefficients in terms of the impedances z_{1} of the media. His expressions are:

$$V_{21} = \frac{Z_2 \cos^2 2 \delta_2 + Z_{25} \sin^2 2 \delta_2 - Z_1}{Z_2 \cos^2 2 \delta_2 + Z_{25} \sin^2 2 \delta_2 + Z_2} \quad 2.1.1$$

$$D = \frac{\ell_1}{\ell_2} \frac{2 Z_2 \cos 2\chi_2}{Z_2 \cos^2 2\chi_2 + Z_{25} \sin^2 2\chi_2 + Z_2} 2.1.2$$

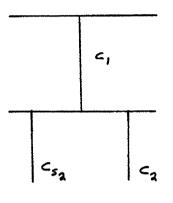
$$P = -\frac{l_1}{l_2} \frac{2 Z_{52} S_{12} Z_{2}}{Z_2 C_{05}^2 2 Z_2 + Z_{25} S_{12}^2 Z_2 Z_2 + Z_2} 2.1.3$$

$$Z_{1} = \frac{e_{1}c_{1}}{cos \theta_{1}}$$
 $Z_{2} = \frac{e_{2}c_{2}}{cos \theta_{2}}$ $Z_{52} = \frac{e_{2}c_{52}^{2}}{cos \delta_{2}^{2}}$

The common oceanic model has $C_2 > C_1$. The magnitude of the reflection coefficient V_{21} can be characterized by considering the various wave number domains that occur for $C_s < C_1$ and for $C_s > C_1 (C_s < C_2)$.

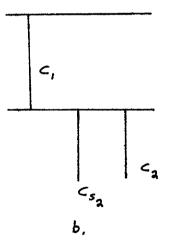
For the case $C_{s_2} < C_1^2 < C_2$, Figure 2.1.2a, the magnitude of V_{21} is less than 1.0 for all real incident angles e_1 , since from Snell's law

 Y_2 is also real and the resulting shear wave in the bottom is propagating. The presence of a propagating wave, compressional or shear, in the bottom signifies energy lost from the water column and hence $|V_{21}| < 1.0$. When $\theta_1 = \theta_2 = 5in^{-1} \frac{\zeta_1}{\zeta_2}$, the compressional wave in the bottom is



.

a,



Sound Speed Relationships

Figure 2.1.2

inhomogeneous and the shear wave propagating. For $\Theta_1 < \Theta_c$ both waves are propagating.

For the case $C_1 < C_2 < C_2$, Figure 2.1.2b, total internal reflection $|V_{21}| = 1$ occurs for $\Theta_1 > 5_{12} - \frac{C_1}{C_{52}}$. Both the compressional and the shear waves in the bottom are inhomogeneous. When $5_{12} - \frac{C_1}{C_{52}} > \Theta_1 > 5_{12} - \frac{C_1}{C_2}$, $|V_{21}| < 1$ and shear propagates in the bottom. For $\Theta_1 < 5_{12} - \frac{C_1}{C_2}$ both waves again propagate and $|V_{21}| < 1$. Typical plots of the reflection and transmission coefficients are given in (25) and (33).

In general, for the differential equation

$$\frac{d^{-G}}{dz^{2}} + k_{z}^{2}G = f(z-z_{o})$$

$$0 \le z \le H$$
2.1.5

the solution obtained via the endpoint method (24,30) has the form

$$G(\overline{z},\overline{z}_{0}) = \begin{cases} G_{a}(\overline{z},\overline{z}_{0}) = W' \mathcal{U}_{a}(\overline{z}) \mathcal{U}_{b}(\overline{z}_{0}) & 0 \leq \overline{z} \leq \overline{z}_{0} \\ \\ G_{b}(\overline{z},\overline{z}_{0}) = W' \mathcal{U}_{a}(\overline{z}_{0}) \mathcal{U}_{b}(\overline{z}) & \overline{z}_{0} \leq \overline{z} \leq \overline{H} \end{cases}$$

where W is the Wronskian

$$W(z_o) = \mathcal{U}_a(z_o) \frac{d\mathcal{U}_b(z_o)}{dz} - \frac{d\mathcal{U}_a(z_o)}{dz} \mathcal{U}_b(z_o)^{2.1.7}$$

 $U_a(z)$ and $U_b(z)$, which satisfy the upper and lower boundary conditions respectively, are linearly independent solutions of the homogeneous equation . The compressional and shear solutions in the lower half

space satisfy the equations

Compressional:
$$\frac{d^2 G_2}{d z^2} + k_{z_2}^2 G_2 = 0$$
 2.1.8
 $z = H$

Shear:

$$\frac{d^2 G_{25}}{dz^2} + k_{z_{25}}^2 G_{25} = 0 \qquad 2.1.9$$

$$z = H$$

and can be written as

$$G_{2}(\overline{z},\overline{z}) = W' \mathcal{U}_{a}(\overline{z}) \mathcal{U}_{2}(\overline{z})$$

$$Z^{2}H^{2.1.10}$$

$$G_{2s}(z, z_{o}) = W^{-1} U_{a}(z_{o}) U_{2s}(z)$$

 $z \ge H$

 $U_2(z)$ and $U_{2s}(z)$ are solutions of 2.1.8 and 2.1.9 that satisfy the boundary condition at z = H and the radiation condition as $z \rightarrow \infty$. Note that the solutions 2.1.10 and 2.1.11 are not, strictly speaking, Green's functions. The notation G_2 and G_{2s} has been employed for convenience. For the single layered ocean of Figure 2.1.1, the various expressions are

$$U_a(z) = e^{-ik_{z_1}z_1 + ik_{z_1}z_2} = 0 \le z \le z_0^{-2.1.12}$$

$$\begin{aligned} \mathcal{U}_{b}(z) &= e^{ik_{z_{1}}z} + V_{z_{1}}e^{2ik_{z_{1}}H - ik_{z_{1}}z} \\ z_{s} \leq z \leq H \quad 2.1.13 \end{aligned}$$

$$\begin{aligned} \mathcal{U}_{a}(z) &= De^{ik_{z_{2}}z} \\ H \leq z \quad 2.1.14 \end{aligned}$$

$$\begin{aligned} \mathcal{U}_{a}(z) &= De^{ik_{z_{2}}z} \\ H \leq z \quad 2.1.14 \end{aligned}$$

$$\begin{aligned} \mathcal{U}_{a}(z) &= Pe^{ik_{z_{2}}z} \\ H \leq z \quad 2.1.15 \end{aligned}$$

$$\begin{aligned} \mathcal{G}_{a}(z, z_{s}) &= -2i5ik(k_{z_{1}}z) \left[e^{ik_{z_{1}}z} + V_{z_{1}}e^{2ik_{z_{1}}H} - ik_{z_{2}}z\right] \\ \mathcal{W}^{-1}(z, 1, 15) \end{aligned}$$

$$\begin{aligned} \mathcal{G}_{a}(z, z_{s}) &= -2i5ik(k_{z_{1}}z) \left[e^{ik_{z_{1}}z} + V_{z_{1}}e^{2ik_{z_{1}}H} - ik_{z_{1}}z\right] \\ \mathcal{W}^{-1}(z, 1, 17) \\ z_{s} \leq z \leq H \end{aligned}$$

$$\begin{aligned} \mathcal{G}_{a}(z, z_{s}) &= -2i5ik(k_{z_{1}}z) \left[DW^{-1}e^{ik_{z_{1}}H} - ik_{z_{2}}(z, -H)\right] \\ \mathcal{G}_{a}(z, z_{s}) &= -2i5ik(k_{z_{1}}z) \left[DW^{-1}e^{ik_{z_{1}}H} - ik_{z_{2}}(z, -H)\right] \\ z = H \end{aligned}$$

$$\begin{aligned} \mathcal{W}^{-1} &= 2k_{z_{1}} \left[5ik_{z_{1}}z_{s}(z) \left[PW^{-1}e^{ik_{z_{1}}H} - ik_{z_{2}}z(z-H)\right] \\ z = H \end{aligned}$$

+
$$i \cos k_{z_1} z_{z_2} (e^{i k_{z_1} z_{z_2}} + V_{z_1} e^{2i k_{z_1} H} - i k_{z_1} z)$$

Equations 2.1.16 through 2.1.20 comprise the exact solution for the model of Figure 2.1.1. As was discussed earlier, the compressional and shear waves in the bottom are either propagating or exponentially decaying (inhomogeneous) depending on the incident angle \bullet_1 . It should also be noted that the Wronskian is identically zero at discrete mode wavenumbers and therefore the Green's function solution diverges in those cases. Perfectly trapped discrete modes occur when $C_1 < S_2 < C_2$ and are limited to the wavenumber domain where $|V_{21}| = 1.0$. Section 2.2 will compare the above solution with the state variable solution.

2.2 SINGLE LAYER OCEAN SOLUTION COMPARISONS

Realistic values of sound speeds and elastic parameters for Figure 2.1.1 can be obtained from the geophysical literature. Hamilton (28,29) suggests this model as common in continental shelf areas. The table below details the information to be used in this section. These parameters correspond to an ocean bottom of sand or silty-sand of unknown depth.

Source Depth 30 m C_1 1500 m/s λ_1 2.25 $\cdot 10^{-5}$ dynes/cm ² C_2 1675 m/s λ_2 4.80125 $\cdot 10^{-5}$ dynes/cm ² S_2 450 m/s	Frequency	100 hz
λ_{1} $2.25 \cdot 10^{-5} \text{ dynes/cm}^{2}$ C_{2} 1675 m/s $4.80125 \cdot 10^{-5} \text{ dynes/cm}^{2}$	Source Depth	30 m
$\begin{array}{c} 1 \\ C_2 \\ \lambda_2 \end{array} \qquad \begin{array}{c} 1675 \text{ m/s} \\ 4.80125 \cdot 10^{-5} \text{ dynes/cm}^2 \end{array}$	c ₁	1500 m/s
$\lambda_{2}^{2} \qquad 4.80125 \cdot 10^{-5} \text{ dynes/cm}^{2}$	λ _l	$2.25 \cdot 10^{-5} \text{ dynes/cm}^2$
2	C ₂	1675 m/s
s ₂ 450 m/s	λ2	$4.80125 \cdot 10^{-5} \text{ dynes/cm}^2$
	s ₂	450 m/s

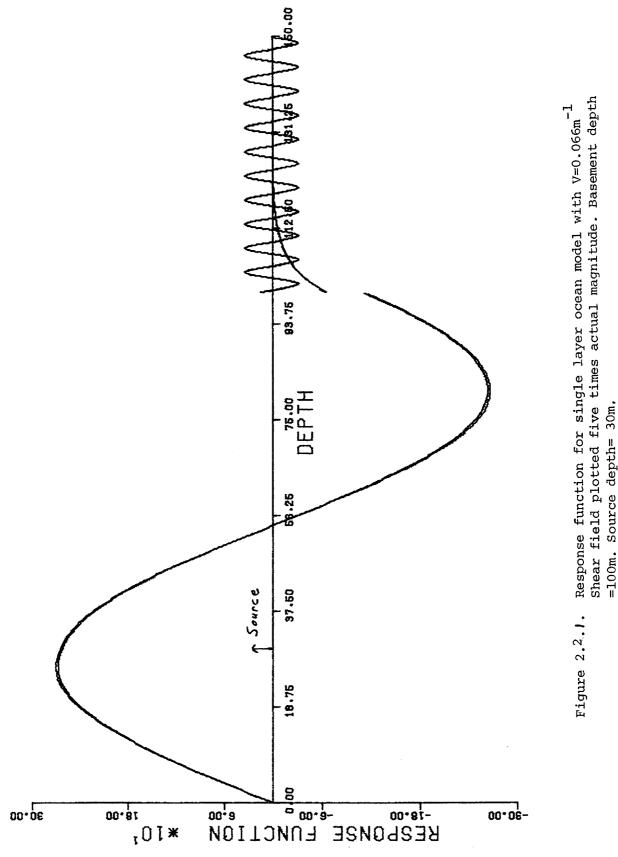
The water and bottom wave numbers for this example are

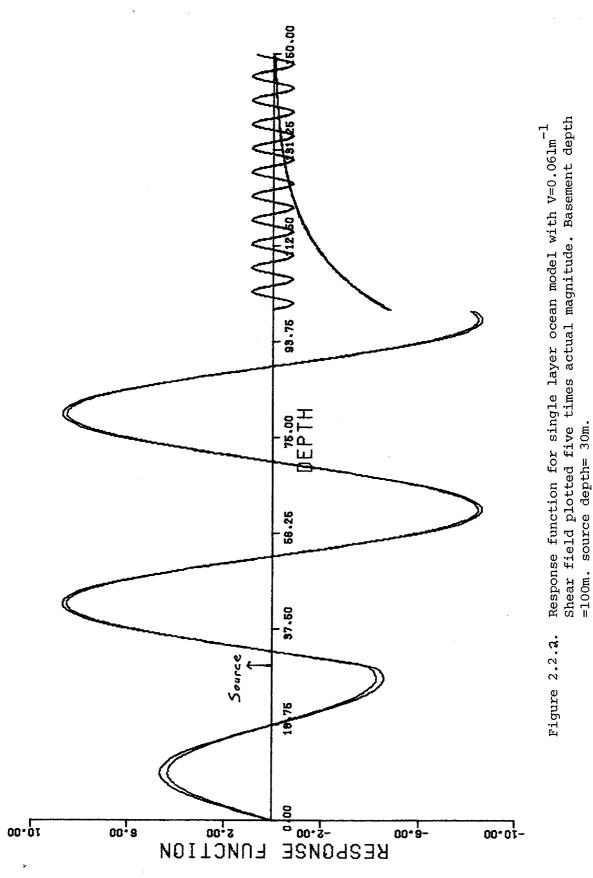
$$V_W = 0.0667 \text{ m}^{-1}$$

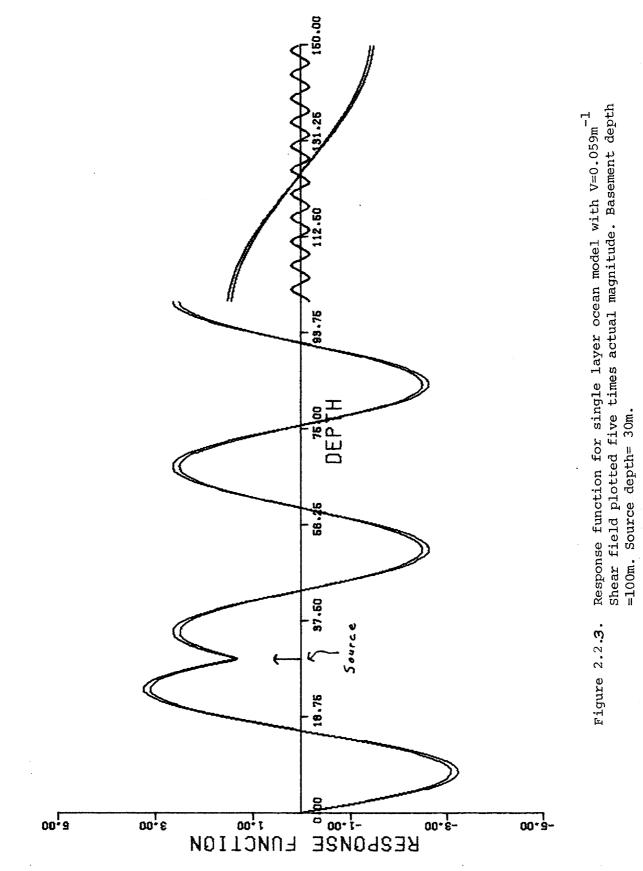
 $V_B = 0.0597 \text{ m}^{-1}$
 $V_{BS} = 0.2222 \text{ m}^{-1}$

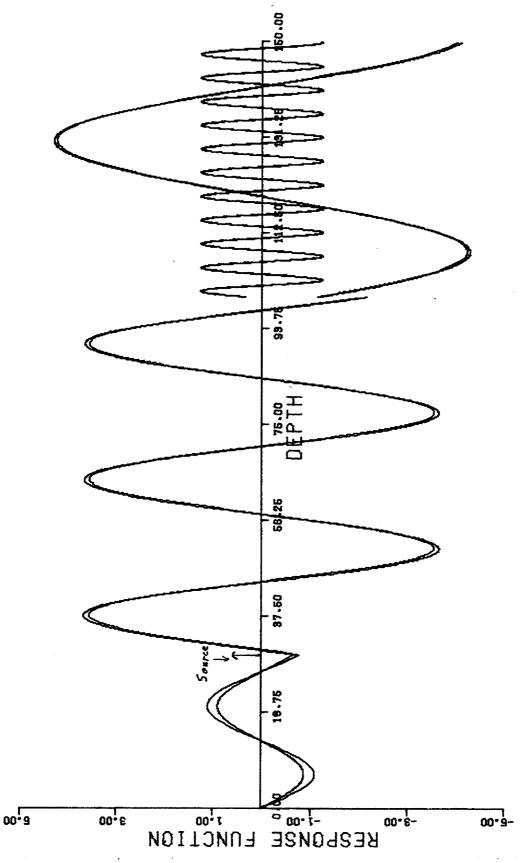
Solutions to 1.2.5 for wave numbers V such that $V_W > B > V_B$ will consist of sinusoids in the fluid layer and decaying exponentials in the bottom. As discussed earlier, there are no perfectly trapped discrete modes for this model, since the parameters and sound speeds correspond to Figure 2.1.2c. The shear contribution is so small, however, that the response functions, when computed at the modal wavenumbers of the equivalent two fluid case, will appear unchanged from the mode shapes. A discontinuity in the slope of the response function will occur at the source for all other wave numbers. For wave numbers V < V_B a propagating compressional wave will be excited in the bottom. Since $V_{BS} > V_{H_20}$, a propagating shear wave will be generated in the bottom.

Figures 2.2.1, 2.2.2, 2.2.3, and 2.2.4 display the response functions obtained from the two techniques for the horizontal wave numbers V = 0.066, 0.061, 0.059, and 0.055 (the term response function is defined as the response of the given ocean model at the horizontal wavenumber V to a source at depth z_0 and is synonymous with the term Green's function.) Agreement between the two solution techniques is quite good in all cases. Notice a slight decrease in accuracy as V decreases. This phenomenon









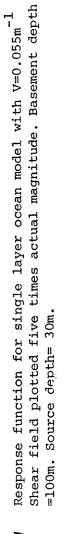


Figure 2.2.4

is coupled with the integration grid density and will be discussed in the next section.

2.3 INTEGRATION GRID DENSITY REQUIREMENTS

Section 2.2 showed that the state variable technique does produce accurate results. To determine the efficiency of the algorithm, the integration grid size that is required to produce these solutions must be investigated. Minimum computer time will occur when the number of integration points is minimized.

The Nyquist sampling theorem of communication theory states the relationship between the sampling period of a process and the minimum period present in the data to be sampled that is required to enable exact reconstruction of the data. A comparable relationship for this application would be one which specifies the number of integration points required per spatial wavelength to insure solution accuracy. Defining the vertical spatial wavelength λ_s as

$$\mathcal{F}_{s} = (2\pi)' \left(\frac{f^{2}}{c_{s}^{2}} - V^{2}\right)^{-1/2}$$
 2.3.1

the number of integration points per wavelength is defined as

$$N_{Z} = \frac{\overline{\mathcal{P}_{s} N_{z}}}{H_{N}}$$
 2.3.2

where N is the total number of integration points and H the basement depth.

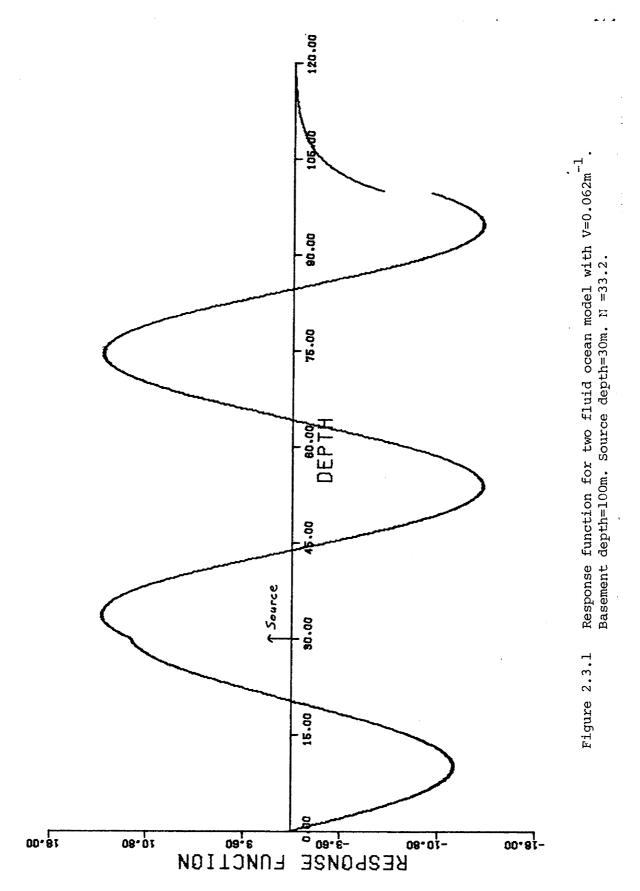
A simplified version of Figure 2.1.1 will be used to study the

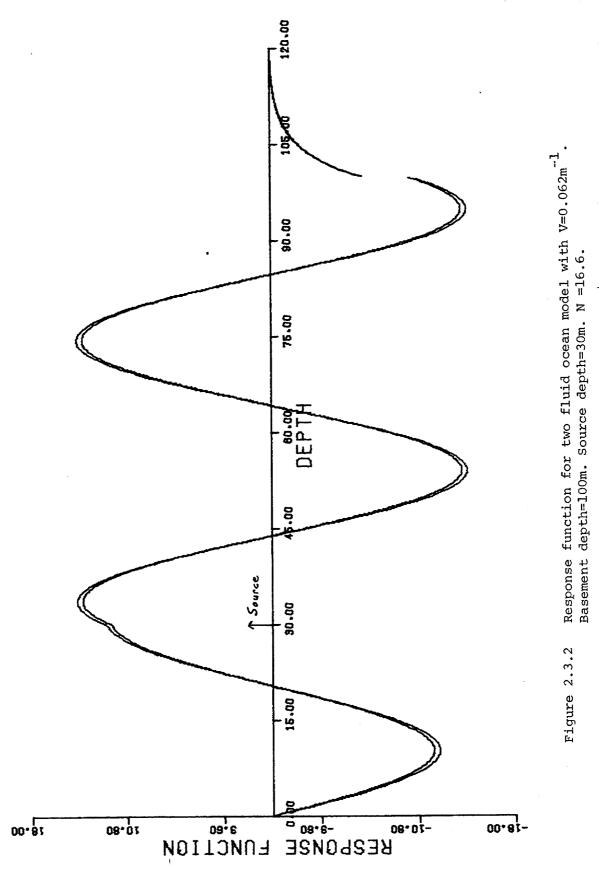
variation of solution accuracy with grid size. The ocean bottom will be assumed fluid with a sound speed of 2000 m/s and Lamé constant of $6.0 \cdot 10^{-5}$ dynes/cm². Source depth and horizontal wave number will be held constant at 30 m and 0.062 m⁻¹. Brief comments will be made shortly on the effect on solution accuracy of varying these parameters.

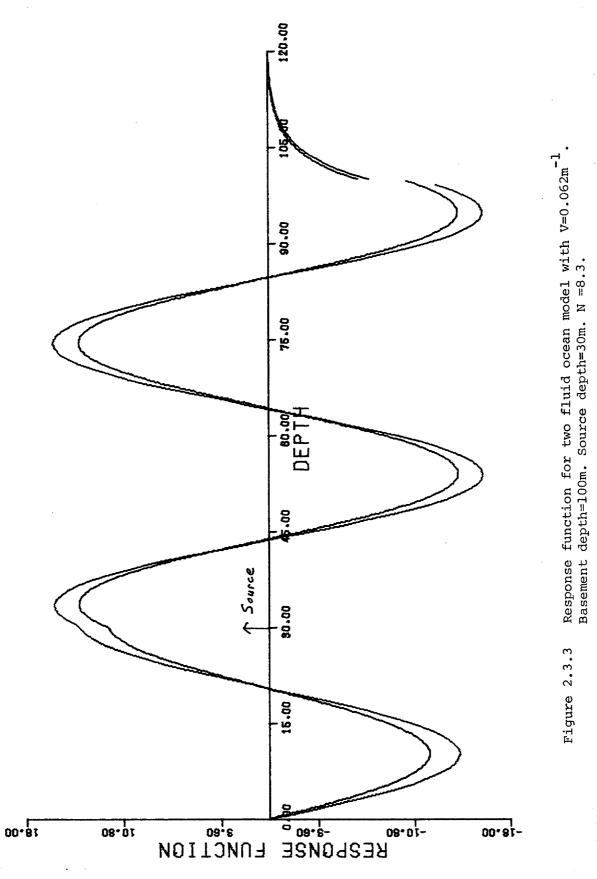
Figures 2.3.1 through 2.3.5 are the response functions obtained by integrating with the values of N_{λ} ranging from 33.25 down to 2.1, as is noted below each figure. Notice that excellent results are obtained for $N_{\lambda} = 33.28$ and only slight degradation in accuracy visible for $N_{\lambda} = 16.6$. More substantial error is noticed for $N_{\lambda} = 8.4$ and entirely unacceptable solutions obtained with $N_{\lambda} = 4.2$ and $N_{\lambda} = 2.1$.

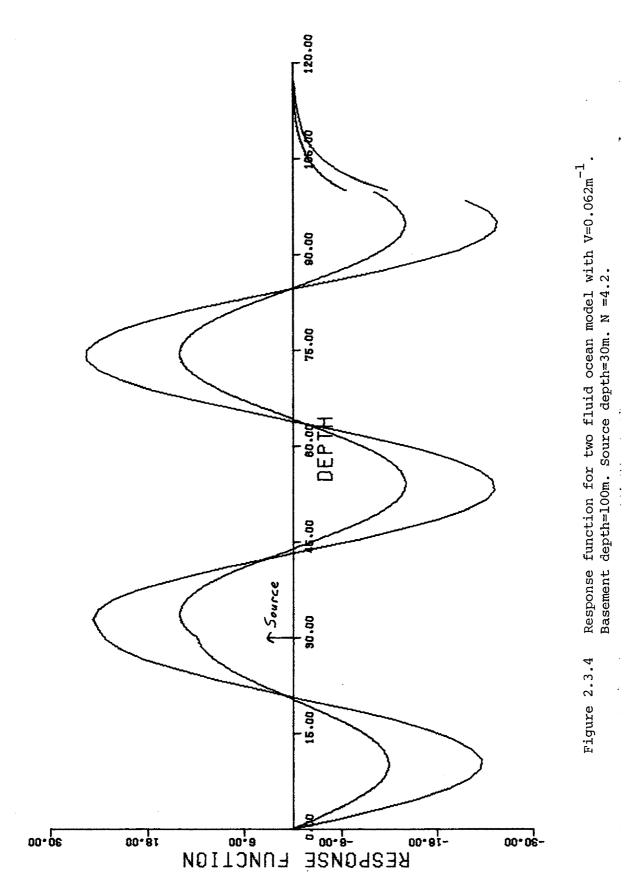
Repeated computation for a variety of bottom parameters and source depths has shown that the value of $N_{\lambda} = 16$ is the minimum for which accurate solutions can be expected. In a few individual cases, values as low as 12 or 13 produced acceptable results but these cases were not common. Nor did any unique characteristic exist which would enable the a priori knowledge that $N_{\lambda} = 12$ was acceptable. Similarly, increasing N_{λ} above 16.0 did not uniformly increase solution accuracy except as a function of horizontal wave number, which will be discussed below. Therefore, the value of $N_{\lambda} = 16.0$ appears to be most appropriate.

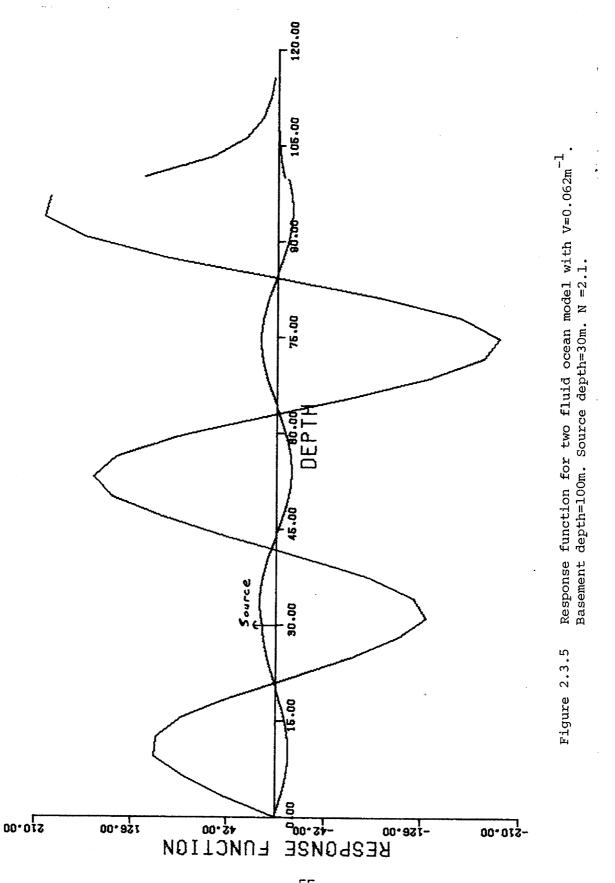
Decreasing the horizontal wave number V, which decreases the spatial wavelength λ_s , does have an influence on solution accuracy. Consider Figures 2.3.6, 2.3.7, and 2.3.8. The ocean model is identical to that used above except that V = 0.058 m⁻¹. The values of N_{λ} are 17, 34, and 51. The error for N_{λ} = 17 is greater in Figure 2.3.6 for V = 0.058 m⁻¹ than in Figure 2.3.2 for V = 0.062 m⁻¹. Also,

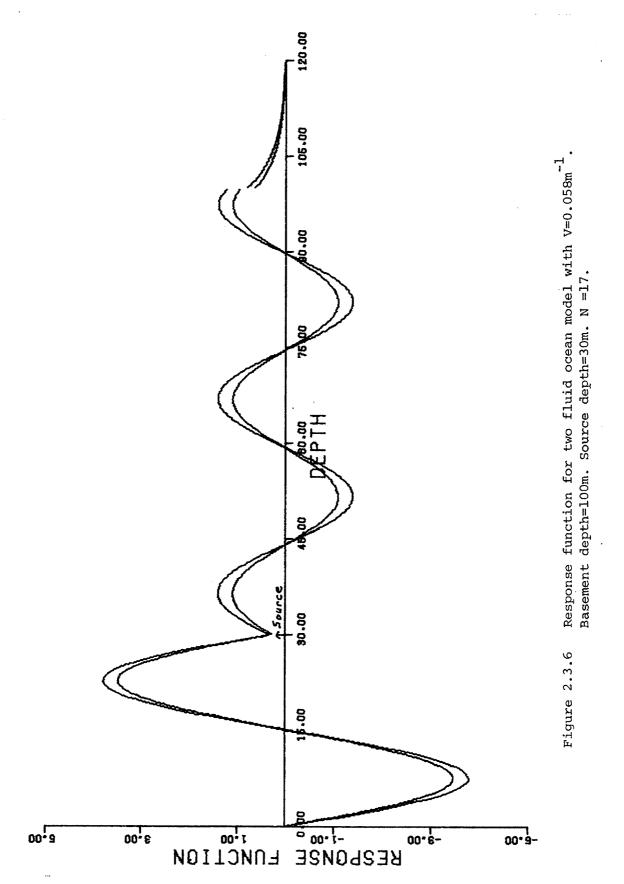












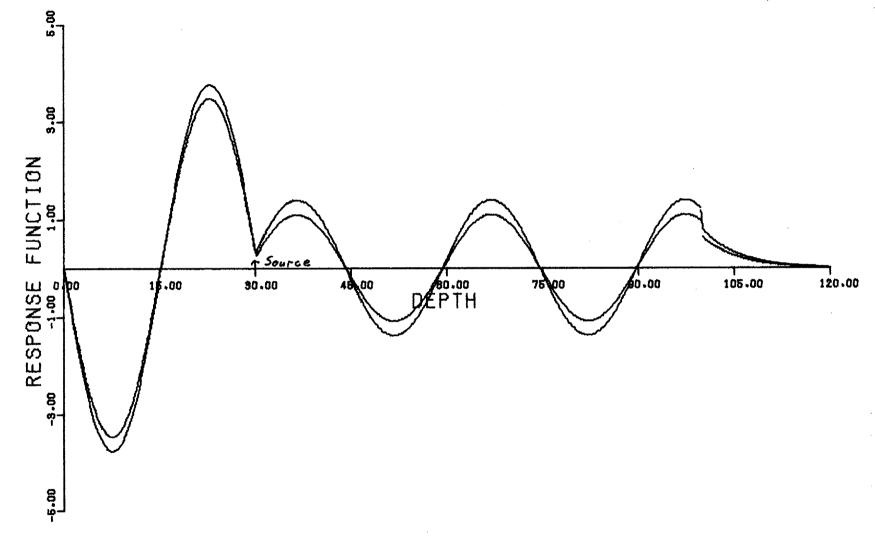
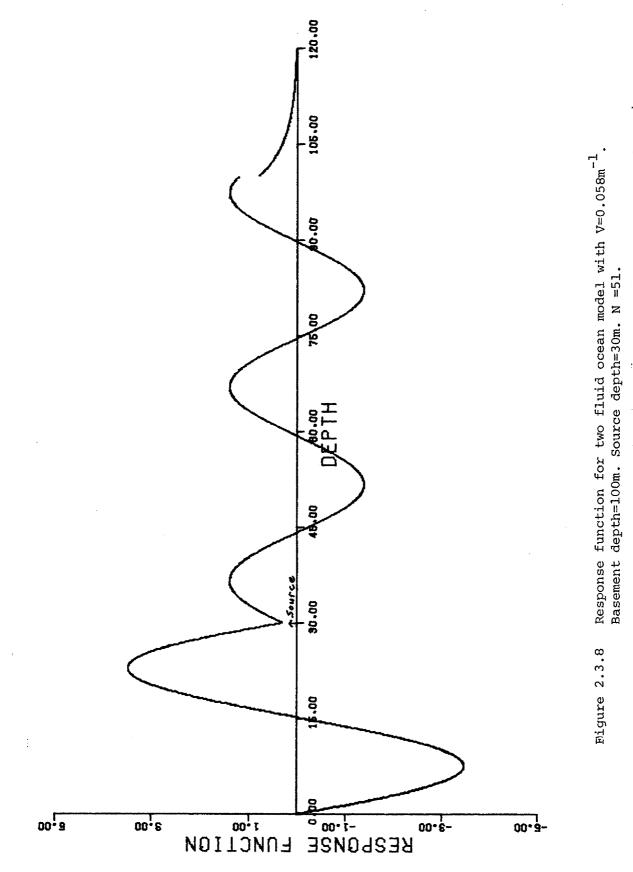


Figure 2.3.7 Response function for two fluid ocean model with V=0.058m⁻¹. Basement depth=100m. Source depth=30m. N =34.



there is little difference between 2.3.7 and 2.3.8, as there was between 2.3.1 and 2.3.2 for the equivalent change in N_{λ}. In fact, convergence to the exact solution does not occur for V = 0.058 m⁻¹ until N_{λ} = 51, which is a very high grid density solution (1056 integration points in a 100 meter channel). Continued use of the algorithm has shown that in some situations gradual convergence to the solution occurs as N_{λ} increases, and in other situations there is a region of variation in N_{λ} where the error remains constant, as was the case above. No apparent difference exists between these two cases which enables prediction of the type of convergence to expect as V decreases.

To conclude, equation 2.3.2 provides a valid means of selecting the optimal integration grid size. For the wave number domain corresponding to the higher wave numbers (lower order modes), $N_{\lambda} = 16$ is sufficient. For the lower wave number region (higher order and continuous modes) it appears that increasing error can be expected and N_{λ} should probably be increased accordingly.

2.4 TWO LAYER OCEAN EXACT SOLUTION AND COMPARISONS

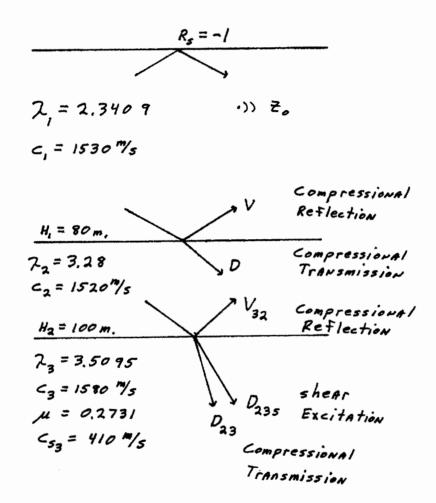
Application of the endpoint method to more complicated models of the ocean is straightforward, providing the required reflection coefficients can be written. As a second test of the state variable technique, the exact solution for the two layered ocean of Figure 2.4.1 will be written. This model, also proposed by Hamilton (28), is common in continental shelf areas where deposition of silts occurred as the sea level rose. For convenience, the second layer will be assumed a fluid. This is realistic, as silts generally have very low shear speeds.

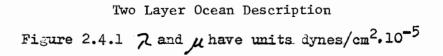
Brekhovskikh's reflection coefficient expressions are again most convenient. The reflection, transmission, and shear conversion coefficients at the lower interface have the same form as Equations 2.1.1, 2.1.2, and 2.1.3. The reflection and transmission coefficients for the upper interface are compound expressions, taking into account the influence of both of the lower media. They are:

$$V = \frac{V_{21} + V_{32} \exp(2ik_{22}(H_2 - H_1))}{1 + V_{21}V_{32}\exp(2ik_{22}(H_2 - H_1))}$$
2.4.1

$$D = \frac{1 + V_{21}}{1 + V_{21} V_{32} \exp(2ik_{22}(H_2 - H_1))} 2.4.2$$

$$V_{21} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$
 2.4.3





$$G_{a}(\overline{z}, \overline{z}_{o}) = W^{-1} \mathcal{U}_{a}(\overline{z}) \mathcal{U}_{b}(\overline{z}_{o}) \quad o \in \overline{z} \leq \overline{z}_{o} 2.4.4$$

$$G_{b}(\overline{z}, \overline{z}_{o}) = W^{-1} \mathcal{U}_{a}(\overline{z}_{o}) \mathcal{U}_{b}(\overline{z}) \quad \overline{z}_{o} \leq \overline{z} \leq H, 2.4.5$$

$$G_{a}(\overline{z}, \overline{z}_{o}) = W^{-1} \mathcal{U}_{a}(\overline{z}_{o}) \mathcal{U}_{a}(\overline{z}) \quad H, \leq \overline{z} \leq H_{a} 2.4.6$$

$$G_{3}(\overline{z}, \overline{z}_{o}) = W^{-1} \mathcal{U}_{a}(\overline{z}_{o}) \mathcal{U}_{3}(\overline{z}) \quad H_{a} \leq \overline{z} 2.4.7$$

$$G_{35}(\overline{z}, \overline{z}_{o}) = W^{-1} \mathcal{U}_{a}(\overline{z}_{o}) \mathcal{U}_{35}(\overline{z}) \quad H_{a} \leq \overline{z} 2.4.8$$

where W is the Wronskian defined in 2.1.7.

The linearly independent solutions to the homogeneous equation are .

$$\mathcal{U}_{a}(z) = e^{-ik_{z_{1}}z} - e^{ik_{z_{1}}z} \quad 0 \le z \le z_{o} \ _{2.4.9}$$

$$U_{b}(z) = e^{ik_{z_{1}}z} + Ve^{2ik_{z_{1}}H_{i}} - ik_{z_{1}}z = z_{0} \leq z \leq H_{i}^{2.4.10}$$

$$\begin{aligned} \mathcal{U}_{2}(z) &= De^{-ik_{22}H_{1}} \left[e^{ik_{22}Z} + V_{32}e^{2ik_{22}H_{2} - ik_{22}Z} \right]_{2.4.11} \\ &\quad H_{1} \leq z \leq H_{2} \\ \mathcal{U}_{3}(z) &= DD_{23}e^{ik_{23}Z} \\ H_{2} \leq Z \\ H_{2} \leq Z \\ \mathcal{U}_{3s}(z) &= DD_{23}e^{ik_{23}Z} \\ H_{2} \leq Z \\ H_{2} \leq Z \\ \mathcal{U}_{4.12} \end{aligned}$$

.

The resulting solution is

$$\begin{aligned} G_{a}(\overline{z},\overline{z}_{o}) &= -2i Sin(k_{\overline{z}_{1}}\overline{z}) \left[e^{ik_{\overline{z}_{1}}\overline{z}_{o}} + Ve^{2ik_{\overline{z}_{1}}H - ik_{\overline{z}_{1}}\overline{z}_{o}} \right] V_{2.4.14}^{-1} \\ G_{b}(\overline{z},\overline{z}_{o}) &= -2i Sin(k_{\overline{z}_{1}}\overline{z}_{o}) \left[e^{ik_{\overline{z}_{1}}\overline{z}_{+}} + Ve^{2ik_{\overline{z}_{1}}H} - ik_{\overline{z}_{1}}\overline{z} \right] V_{2.4.15}^{-1} \\ G_{a}(\overline{z},\overline{z}_{o}) &= -2i Sin(k_{\overline{z}_{1}}\overline{z}_{o}) \left[e^{ik_{\overline{z}_{2}}\overline{z}_{+}} + V_{\overline{z}_{2}} e^{2ik_{\overline{z}_{2}}H_{2}} - ik_{\overline{z}_{2}}\overline{z} \right] \\ \cdot D W^{-i} e^{iH_{i}(k_{\overline{z}_{1}} - k_{\overline{z}_{2}})} \right] \end{aligned}$$

$$G_{3}(\overline{z},\overline{z}_{0}) = -2i Sin(k_{\overline{z}_{1}} \overline{z}_{0}) D D_{23} W^{-1} e^{i k_{\overline{z}_{1}} H_{1}} .$$

$$e^{i k_{\overline{z}_{2}}(H_{2} - H_{1})} e^{i k_{\overline{z}_{3}}(\overline{z} - H_{2})}$$

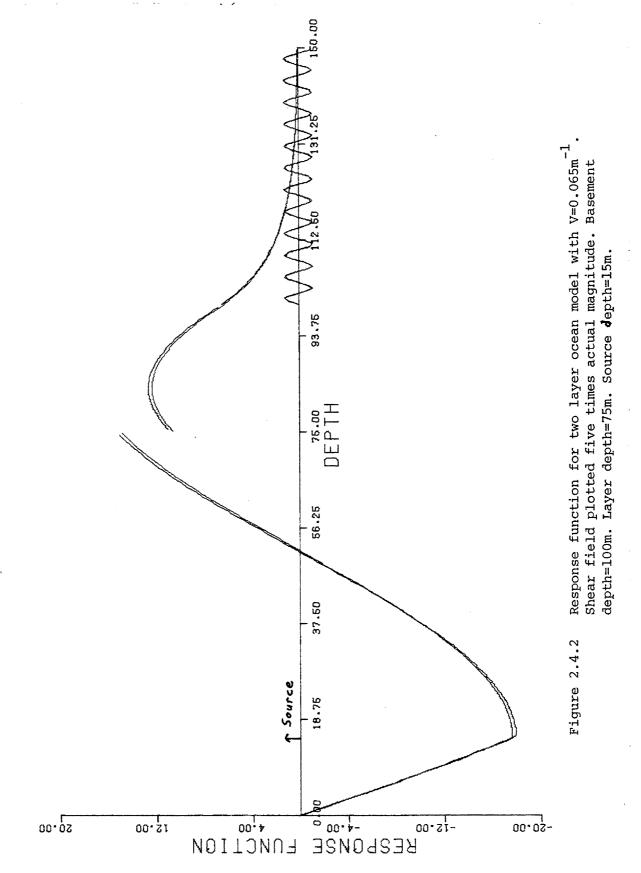
$$G_{35}(\bar{z},\bar{z}_{0}) = -2i Sin(k_{\bar{z}_{1}}\bar{z}_{0}) DD_{23}W^{-1}e^{ik_{\bar{z}_{1}}H_{1}} = 2.4.18$$

$$e^{ik_{\bar{z}_{2}}(H_{2}-H_{1})}e^{ik_{\bar{z}_{35}}(\bar{z}-H_{2})}$$

$$W^{-1} = 2k_{\bar{z}_{1}}\left[Sin(k_{\bar{z}_{1}}\bar{z}_{0})(e^{ik_{\bar{z}_{1}}\bar{z}_{0}} - Ve^{2ik_{\bar{z}_{1}}H_{-ik_{\bar{z}_{1}}\bar{z}_{0}})^{2.4.19} + i(\cos(k_{\bar{z}_{1}}\bar{z}_{0})(e^{ik_{\bar{z}_{1}}\bar{z}_{0}} + Ve^{2ik_{\bar{z}_{1}}H_{-ik_{\bar{z}_{1}}\bar{z}_{0}})\right]$$

Figures 2.4.2 through 2.4.5 are the solutions obtained from Equations 2.4.14 through 2.4.19 and from the state variable algorithm. Integration step size was chosen to satisfy 2.3.2 for $N_{\lambda} \geq 16$. The respective horizontal wavenumbers are V = 0.065, 0.064, 0.061, and 0.058 m⁻¹. As in the single layered ocean results, these figures show that accurate results are again obtained by the state variable algorithm. Further study of this two layered ocean proves that, as in the single layer case, solution accuracy is not a function of any oceanic parameters other than vertical wavenumber. The same characteristics of the solution were found for the two layer ocean as were found for the single layer ocean with regard to the value of N_{λ} in 2.3.2.

The state variable technique can be used to obtain accurate solutions of the depth-separated wave equation. The solutions are stable with regard to all oceanic parameters. Required grid density has been specified by 2.3.2. In Chapter 3 the technique will be used to study the effects of including complex sound speeds to model attenuation.



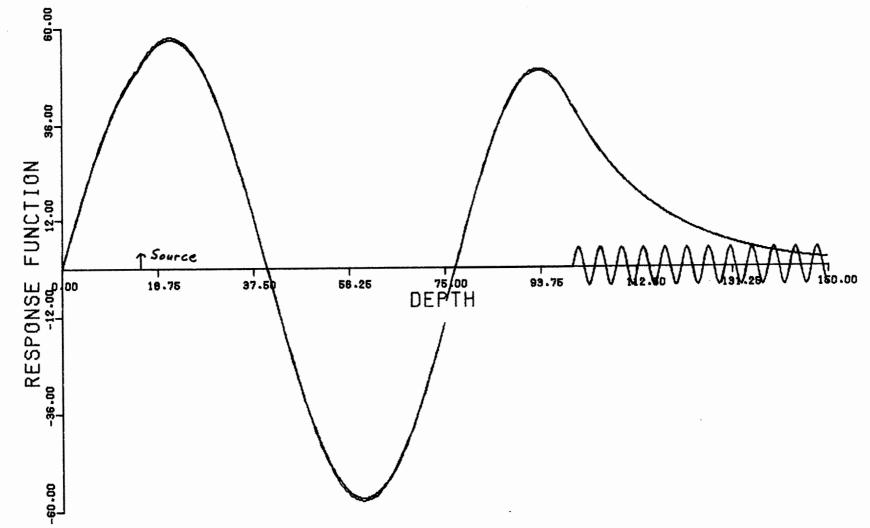
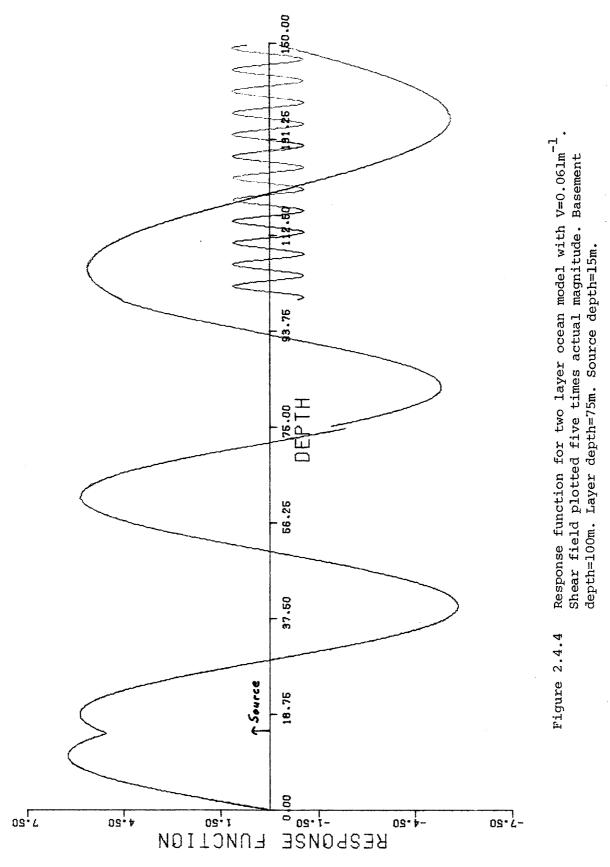
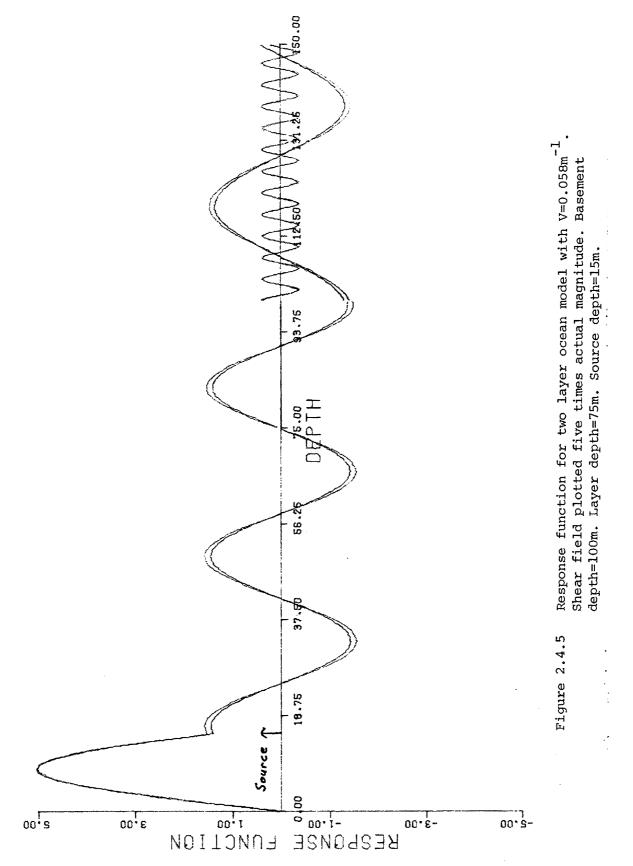


Figure 2.4.3 Response function for two layer ocean model with V=0.064m⁻¹. Shear field plotted five times actual magnitude. Basement depth=100m. Layer depth=75m. Source depth=15m.





3. ATTENUATION AND DEEP OCEAN MODEL

Chapters 1 and 2 introduced the state variable technique for obtaining solutions to the inhomogeneous depth separated wave equation. By incorporating complex sound speeds, the technique can also be used to study the influence of bottom attenuation on the solution. Section 3.1 will consider the single layer ocean of Chapter 2 with bottom attenuation. Section 3.2 will discuss an instability that arises during the integration process when complex sound speeds are included and the influence of that instability on the resulting response function's accuracy. Section 3.3 will conclude with a general discussion of the use of the algorithm on a deep ocean model. The complex sound speeds will, in general, cause the response functions to have real and imaginary components. Consequently, both the magnitude and phase of the complex response function will be discussed. The phase state variable will also be presented with and without attenuation.

3.1 SINGLE LAYER OCEAN BOTTOM ATTENUATION

Shear and compressional wave attenuation in marine sediments has been reviewed by Hamilton (34, 35). Experimental data indicates that compressional attenuation coefficients have a dependence on the first power of frequency, i.e., $a_c = kf^{\dagger}$ (34). The coefficient k is largest for sands, smallest for silty-sands, and intermediate for silt-clay muds. Very little data is available on shear wave attenuation. Relating the shear attenuation coefficient a_s to the compressional attenuation coefficient a_s is most conveniently done by defining the logarithmic Δ

$$\Delta_{\rm S} = \frac{a_{\rm s} \, c_{\rm s}}{F} \qquad \Delta_{\rm c} = \frac{a_{\rm c} \, c_{\rm c}}{F}$$

where c_s and c_c are bottom shear and compressional wave speeds and f the frequency. The ratio of Δ_c to Δ_s has been found to be 0.3 for sands and 0.1 for silt-clays (35). Until more thorough experimental data is available, the best method of obtaining a_s is to determine a_c from reference 34 and use

$$\frac{\Delta_{c}}{\Delta_{s}} = \frac{a_{c}C_{c}}{a_{s}C_{s}} = \begin{cases} 0.3 & \text{sand} \\ 0.1 & \text{silt-clay} \end{cases}$$
3.1.1

to compute the best estimate of shear attenuation.

When attenuation mechanisms are present, the horizontal wavenumber becomes complex, i.e., V +i δ . Horizontally propagating plane waves then consist of a product of two exponentials.

$$e^{i2\pi(V+i\delta)X} = e^{i2\pi VX} - 2\pi \delta X$$

$$= e^{3.1.2}$$

The second exponential provides the decay with distance due to attenuation.

When bottom attenuation is the only loss mechanism to be considered, a slightly more convenient method of including attenuation uses complex sound speeds in the propagation constant $K_c = 2\pi f/c_c$ for the bottom. Recall from Section 2.1,

$$2\pi V = \frac{2\pi f}{c_c} Sin \theta,$$

as

Writing the bottom sound speed as $c_c - ic_c'$, K_c becomes

$$K_{c} = \frac{2\pi f}{c_{c}^{-i}c_{c}'} = \frac{2\pi f c_{c}}{c_{c}^{2} + c_{c}'^{2}} + i \frac{2\pi f c_{c}'}{c_{c}^{2} + c_{c}'^{2}}$$

Similarly for $c_s - ic_s'$ $K_s = \frac{2\pi F}{c_s - ic_s'} = \frac{2\pi F c_s}{c_s^2 + c_s'^2} + i \frac{2\pi F c_s'}{c_s^2 + c_s'^2}$

Therefore,

$$a_{e} = \frac{2\pi f c_{e}'}{c_{e}^{2} + c_{e}'^{2}}$$

$$a_{s} = \frac{2\pi f c_{s}'}{c_{s}^{2} + c_{s}'^{2}}$$
3.1.3

Notice that the imaginary sound speed term has a negative sign. This insures that Equation 3.1.2 does in fact have a decaying exponential.

Representative values of the attenuation coefficients can be obtained from the literature (34, 35). At 100 Hz, for example, experimental data gives the following limits on a_c and a_s

$$7 \cdot 10^{-4} < a_{c} < 1 \cdot 10^{-2}$$

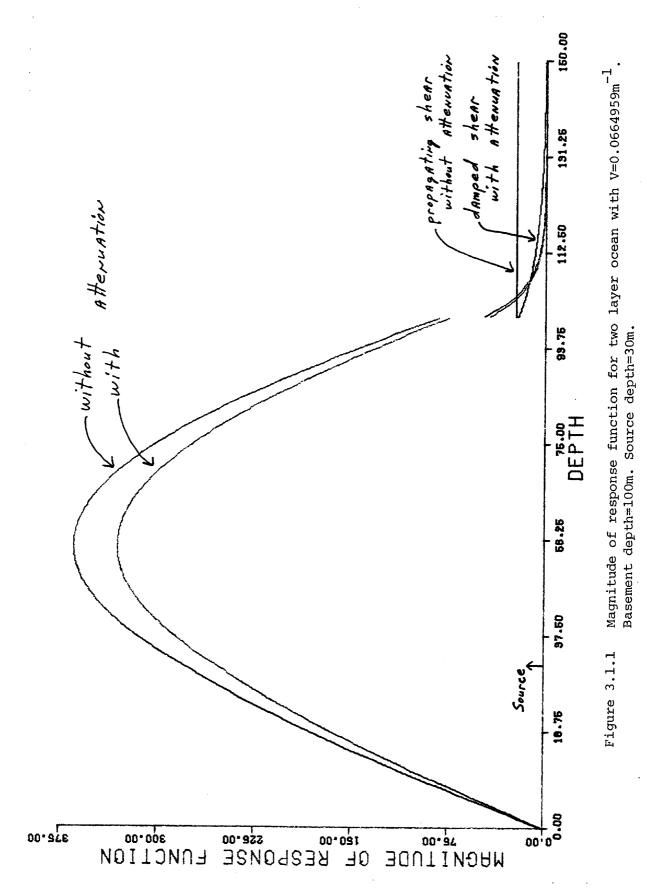
 $4 \cdot 10^{-2} < a_{s} < 2 \cdot 10^{-1}$

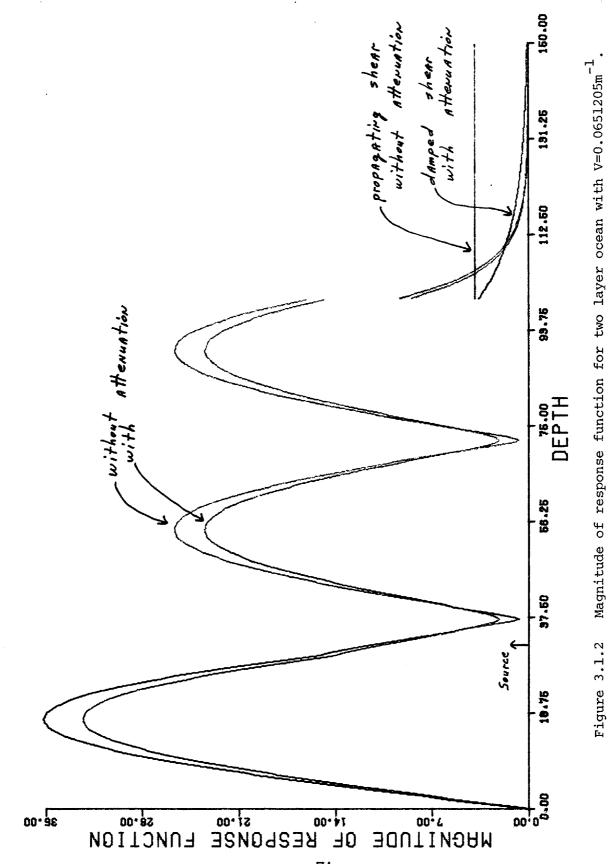
For sand, a typical value from reference 35 is a $_{\rm C}$ = 0.0056. Using 3.1.1 and 3.1.3, the sound speeds that result are

Figures 3.1.1 through 3.1.7 display various facets of the solution. On each of the first five, the magnitude of the response function evaluated both with and without bottom attenuation is plotted. Figures 3.1.1, 3.1.2, and 3.1.3 correspond to the first, third, and fifth modes of the comparable Pekeris waveguide. For the Pekeris case without bottom attenuation, infinite response would result at modal wavenumbers. In this case, the shear acts as a loss mechanism and the response function is finite. The fourth and fifth are for wavenumbers on either side of the critical angle, which occurs at V = 0.0597. Figure 3.1.6 plots the phase of the complex potential for the fifth mode. Figure 3.1.7 is an example of the variation of the state variable $\Theta(s)$ with depth for V = 0.0651205.

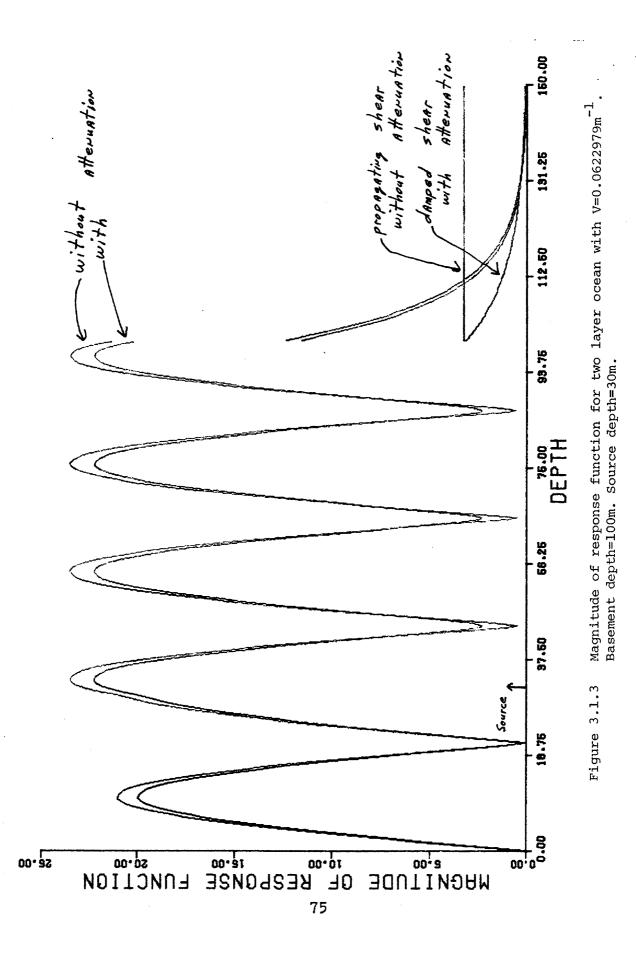
The first three figures exhibit characteristics that are predictable. The magnitudes for the attenuated cases are all smoothed compared to the unattenuated cases; that is, the maxima are smaller and the minima larger. The propagating shear wave, formerly of constant magnitude, now exhibits the expected exponential damping. The inhomogeneous compressional wave in the bottom shows a slight decrease in amplitude from the unattenuated case. Finally, increased coupling to the bottom as wave number decreases is apparent via the increasing proportion of shear wave generation.

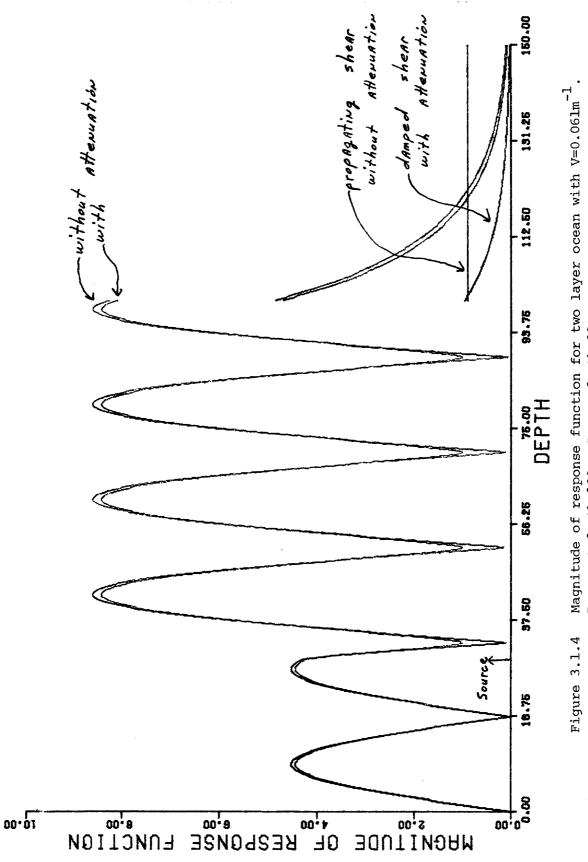
Decreasing the horizontal wave number further to V = 0.061 and V = 0.059 produces an unexpected effect on the response function. On the first three figures, all lobes of the magnitude were uniformly attenuated. This characteristic is not evident as V decreases. In particular in 3.1.4, the upper two lobes are only very slightly smaller, while the lower four are more noticeably attenuated, as was the case





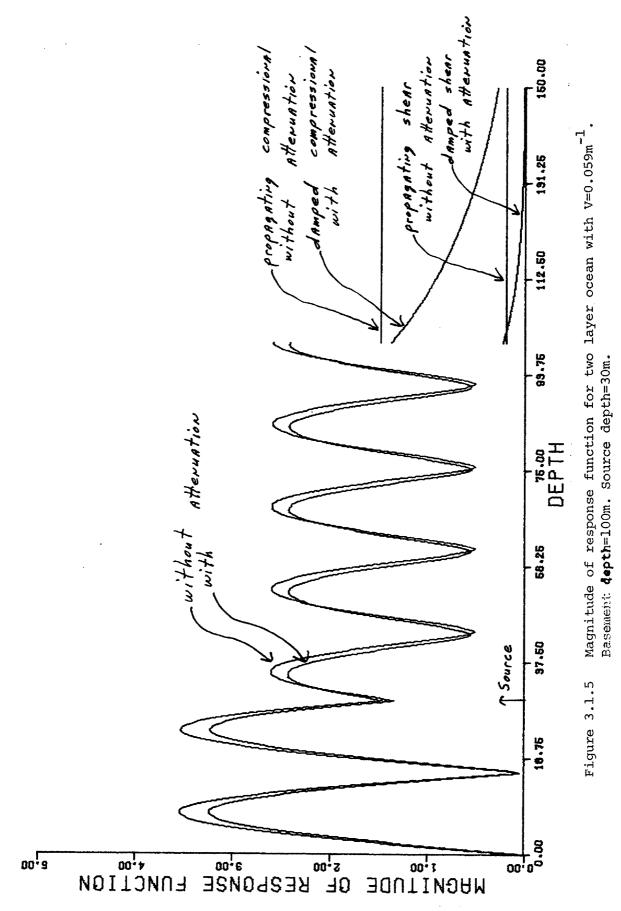
Magnitude of response function for two layer ocean with V=0.0651205m⁻¹. Basement depth=100m. Source depth=30m.

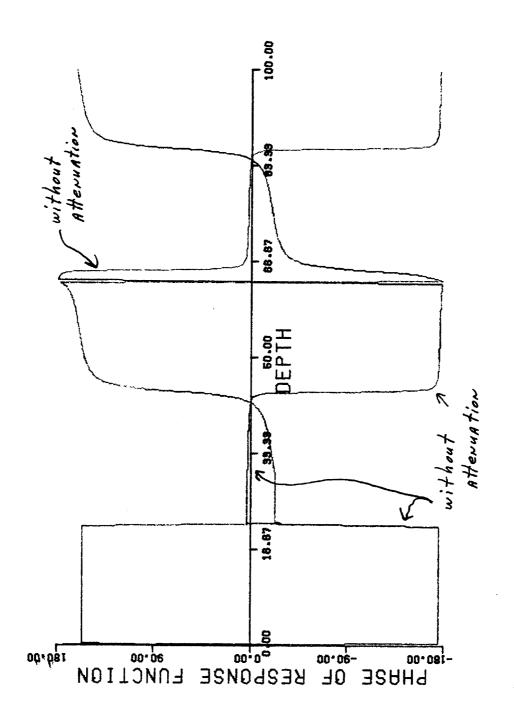




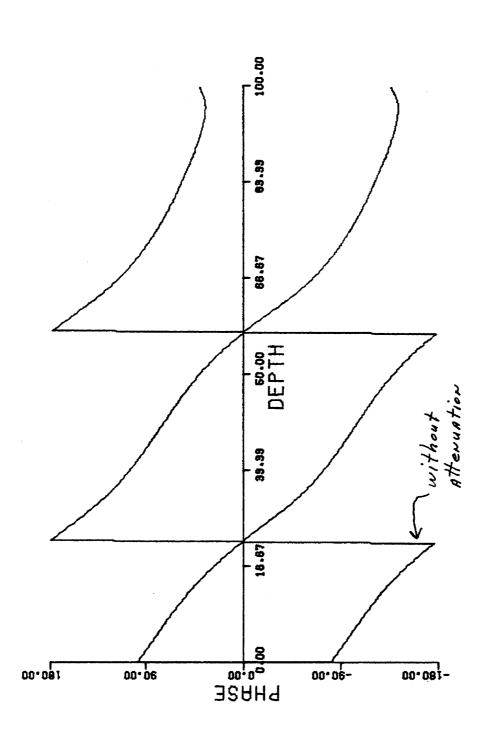
Basement depth=100m. Source depth=30m.







Phase of response function for two layer ocean with V=0.0622979m⁻¹ Basement depth=100m. Source depth=30m. Figure 3.1.6





earlier. In Figure 3.15. the upper two lobes have substantially larger magnitude in the attenuated case than in the unattenuated case. The remaining lobes continue the uniformly reduced amplitude trend.

Note that the source depth for this case is 30 meters, and it becomes clear that the larger magnitude region is above the source. This tendency was investigated for various source depths and held true in all cases. The region above the source is slightly amplified and that below slightly attenuated as the horizontal wave number V decreases below critical. Unfortunately, the numerical instability discussed in the following section became apparent for V below 0.059 and this phenomenon could not be investigated further. It is, however, believed to be an accurate description of the influence of attenuation on the response function and not a product of the numerical technique.

A typical plot of the phase of the complex potential is given in Figure 3.1.6. The only effect of attenuation is to shift the phase by approximately 20° . Without attenuation, the phase is initially -175° and remains constant until 22 m depth, where it jumps to about 5° . It then decreases to nearly 540° . All values are plotted on the primary branch of -180° to $+180^{\circ}$ so the phase at 63 m jumps up to $+180^{\circ}$. With attenuation, the phase is initially -195° (plotted at $+165^{\circ}$) and tracks the unattenuated phase throughout the profile. No variation in the phase occurred as a function of wave number, as was noticed in the magnitude figures.

The phase state variable $\Theta(i)$ for V = 0.059 is shown in Figure 3.1.7. The complex sound speed shifts $\Theta(i)$ by nearly 180° and, as with the phase of the complex potential, has no other influence. The plots

are otherwise exactly identical. This characteristic was consistent throughout the wave number domain.

Stability problems with the technique arose in certain profiles and wave number domains. The source of these problems will be discussed in the next section.

3.2 COMPLEX SOUND SPEED NUMERICAL INSTABILITY

Reference 18 includes a thorough discussion of the numerical stability of the solution of the Riccati and phase differential equations. While investigating the use of complex sound speeds to model attenuation, instability during the integration of the phase differential equation occasionally occurred which prevented a solution from being obtained. Although a specific expression defining the limits within which the solution, with attenuation, converges was not found, a discussion of the cause of the problem is in order.

Consider the phase differential equation:

$$\Theta(i) = f(i) Sin 2\Theta(i) - r^2 Cos^2 \Theta(i) - Sin^2 \Theta(i)$$
 3.2.1

Given an ocean model with pure real sound speeds, the solution of 3.2.1 can be obtained. Including complex sound speeds in the basement will alter initial conditions for upward integration of $\dot{\mathcal{O}}(\varsigma)$. To study the change in $\mathcal{O}(\varsigma)$, it is convenient to use a perturbation method. First write

$$\Theta(i) = \Theta(i) + \epsilon \int \Theta(i) \\
= F_{i}(i) + \epsilon \int F(i) \\
^{3.2.3}$$

where the first term on the right hand side of each equation is the

unperturbed, without attenuation, solution. The second term is a first-order variation on that solution due to the inclusion of attenuation. To obtain the first order linearized equation for $f \theta(\varsigma)$, expand the right hand side of 3.2.1 about $\theta_{\sigma}(\varsigma)$ by a Taylor series.

In general, if

$$\dot{\Theta}_{s}(s) = F(\Theta_{s}(s), s, f(s), r^{2})$$

$$3.2.4$$

then

$$\dot{\Theta}(\mathfrak{l}) + \mathcal{J}\dot{\Theta}(\mathfrak{l}) = F(\Theta_{0}(\mathfrak{l}) + \mathcal{J}\Theta(\mathfrak{l}), \mathfrak{l}, \mathfrak{F}_{0}(\mathfrak{l}) + \mathcal{J}\mathfrak{F}(\mathfrak{l}), \nabla^{2}) \\ = F(\Theta_{0}(\mathfrak{l}), \mathfrak{l}, \mathfrak{F}_{0}(\mathfrak{l}), \nabla^{2}) + \frac{\partial F}{\partial \Theta} / \mathcal{J}\Theta(\mathfrak{l}) \\ + \frac{\partial F}{\partial \mathfrak{F}} / \mathcal{J}\mathfrak{F}(\mathfrak{l}) + higher \text{ order terms}^{3.2.5} \\ \text{The equation of first variation is}$$

$$\delta \dot{\theta}(\hat{s}) = \frac{\partial F}{\partial \theta} \left| \delta \theta(\hat{s}) + \frac{\partial F}{\partial F} \right| \delta F(\hat{s}) \qquad 3.2.6$$

where the partial derivatives are evaluated using unattenuated values of all parameters. Notice that the variation in $\dot{\phi}(\xi)$ is a function of the variation in both $\Theta(\xi)$ and $f(\xi)$. Therefore an equation of variation for the Riccati equation must be studied before continuing with the analysis of equation 3.2.6.

Equation 1.3.16 is of the form

$$\dot{F}(i) = g(F(i), g(i))$$
 3.2.7

The equation of first variation is obtained as follows: $\dot{F}_{o}(s) + \delta \dot{F}(s) = g\left(F_{o}(s), g_{o}(s)\right) + \frac{\delta g}{\delta F} \int \delta F(s) + \frac{\delta g}{\delta g} \int \delta g(s)$ $\mathcal{J}\dot{F}(\xi) = \frac{\partial 2}{\partial F} \int \mathcal{J}F(\xi) + \frac{\partial 2$

$$ff(i) = 2f_{i}(i) ff(i) - 2g_{i}(i) fg(i)$$
 3.2.8

Equation 3.2.8 was obtained via a locally stationary analysis. That is, the solution to 3.2.8 is valid only in a particular neighborhood of $f(\xi)$. The neighborhood is specified by that region of ξ in which $f(\xi)$ is approximately constant. The solution to 3.2.8 is

$$\delta \mp (\hat{i}) = e^{2 \frac{1}{5} (\hat{i}) \Delta \hat{i}} + \frac{\Re_0 (\hat{i}) \delta \Re (\hat{i})}{\frac{1}{5} (\hat{i})} 3.2.9$$

Equation 3.2.9 describes the changes in the Riccati solution that result from a change in the initial conditions. Notice that for integration upward through the water column $\Delta \xi \subset 0.0$ and therefore 3.2.9 has a stable solution. The conclusion is that the second term on the right hand side of 3.2.6 is not the source of the numerical problem and the first term must now be considered.

The equation of first variation for 3.2.3 is

$$\delta \dot{\theta}(\mathbf{i}) = \left[(1 - r^2) 5 i \times 2\theta_0(\mathbf{i}) - 2 \overline{f_0(\mathbf{i})} \cos 2\theta_0(\mathbf{i}) \right] \delta \theta(\mathbf{i})$$

$$+ 5 i \times 2\theta_0(\mathbf{i}) \delta \overline{f(\mathbf{i})}$$
3.2.10

To study the instability which arose during the integration of the phase differential equation, the feedback term of equation 3.2.10 (the coefficient of $\mathcal{FO}(\mathcal{F})$) must be studied in conjunction with equation 3.2.1.

Before examining 3.2.10 in detail, it is useful to recall a result of Laplace transform theory(37). For the differential equation

$$\dot{X}(s) = AX(s)$$
 $X(o) = 1$

with transform

$$\chi(s) = \frac{1}{s-A}$$

the stability of the solution can be analyzed by considering the location of the pole at A. Stable solutions, in the bounded-input bounded-output sense, are obtainable for A < 0 and integration down the profile(pole in the left half plane) or for A > 0 and integration up the profile(pole in the right half plane). In both cases, the solution is a decaying exponential.

Now reconsider 3.2.10, which has a pole at

$$A = (1 - r^{2}) 5 in 28(1) - 2f_{0}(1) \cos 28(1) \qquad 3.2.11$$

Note again this is a locally stationary analysis. The solution of 3.2.10 will be stable for integration up the profile if A>0. Therefore, the local requirement is that

$$T_{AN} 2 \theta_{o}(5) \ge \frac{2 f_{o}(5)}{1 - r^{2}}$$
 3.2.9

If stable solutions of 3.2.1 can be obtained such that 3.2.9 is satisfied, then the use of complex sound speeds to model attenuation would be acceptable. If not, however, then the numerical instability will result. A discussion of the critical points of 3.2.1 is now in order.

Baggeroer(18) discussed the importance of the equilibrium points of equation 3.2.1. In general, the larger of the two roots of 3.2.1, given by

$$T_{AN} \ \Theta(S) = F_g^2(S) \pm \sqrt{F_g^2(S) - V_o^2} \qquad F_g^2(S) = V_o^2$$

is stable for upward integration. Modes form when, at some point,

$$v_{0}^{2} = f_{g}^{2}(1)$$

and the equilibrium points disappear. In this case $\dot{\Theta}(\tilde{f}) < 0.0$ and $\Theta(\tilde{f})$ is continually decreasing up the profile. Note, for example, figure 3.1.7 where $\Theta(\tilde{f})$ has been plotted modulo 360.

The source of the instability is now clear. As ∇_{σ}^{2} increases, the lower boundary on the region for stable solutions of 3.2.10, as given by 3.2.12, increases(the denominator becomes more negative, therefore $\mathcal{G}_{\sigma}(\varsigma)$ must approach zero from below the origin). However, the solution of 3.2.1 tracks $\mathcal{G}_{\sigma}(\varsigma)$ in the opposite direction- by continually decreasing. At some point, 3.2.12 is no longer satisfied and A becomes less than zero. It is important to note that the solution to 3.2.10 is unstable in this case at the same time that the solution to 3.2.1 is stable. If the solution to 3.2.1 were not stable the entire analysis would be meaningless.

This unstable solution is not solely the cause of the breakdown of the numerical technique. It means simply that the perturbed solution diverges from the unperturbed solution in these regions of the depth profile. As A becomes increasingly more negative, the solution becomes relatively more unstable. In other words, $\Theta(\varsigma)$ in equation 3.2.2 diverges from $\Theta_{\sigma}(\varsigma)$.

The conclusion is that the region of convergence of the solution with attenuation is controlled by the sound speed profile through $f_{\sigma}(\varsigma)$, the horizontal wave number through Γ^2 , and the amount of attenuation through $\delta \theta(\varsigma)$. The ability to study the attenuation characteristics of any given ocean model must be determined to a large extent by trial and error since the technique has this inherent instability.

3.3 Deep Ocean Model Considerations

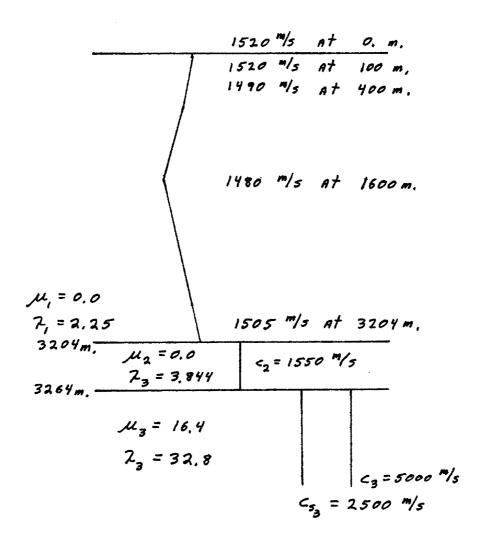
The thesis to this point has dealt entirely with shallow water ocean models. This section will begin with a discussion of a simple deep ocean model that consists of a fluid layer and a thin sediment layer assumed to act as a fluid. The basement is allowed to be elastic. It will conclude

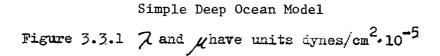
with an investigation of the technique for ocean models in which the bottom is a multilayered elastic medium.

A simple deep ocean model is depicted in figure 3.3.1. The seafloor, characterized as young oceanic crust, consists of a thin sediment layer that separates the ocean from the elastic basement. The sediment is assumed fluid since typical shear speeds are generally small and resulting shear waves of negligible amplitude. The shallow water models of chapter 2 are proof of this assumption. In both of those cases the basement shear field had substantially smaller magnitude than the compressional field.

The sampling density requirement that $N_{2}=16$ was derived from the study of a simple constant sound speed shallow water model. A question that arises is whether or not this density is valid for the more complex model. Since the exact solution cannot be written, another method of verification must be found. Reconsideration of figures 2.3.1 through 2.3.4 suggests the method. Notice that as the sampling density is increased, the state solution approaches the exact solution in steps of decreasing size. For example, in figure 2.3.4, the state solution has approximately one-half the magnitude of the exact solution. In figure 2.3.3, the ratio is about nine-tenths and in figures 2.3.2 and 2.3.1, the difference is almost negligible. By varying the sampling density and computing the response function for figure 3.3.1, this asymptotic tendency can be used to study the validity of $N_{2}=16$ for the deep ocean model.

The result of this study is that $N_2=16$ is valid in general. Accurate solutions can be expected for all profiles in which this criterion is satisfied. As with the shallow water study, values below 16 appeared occasionally to be acceptable. These occurrences were infrequent and unpredictable however.



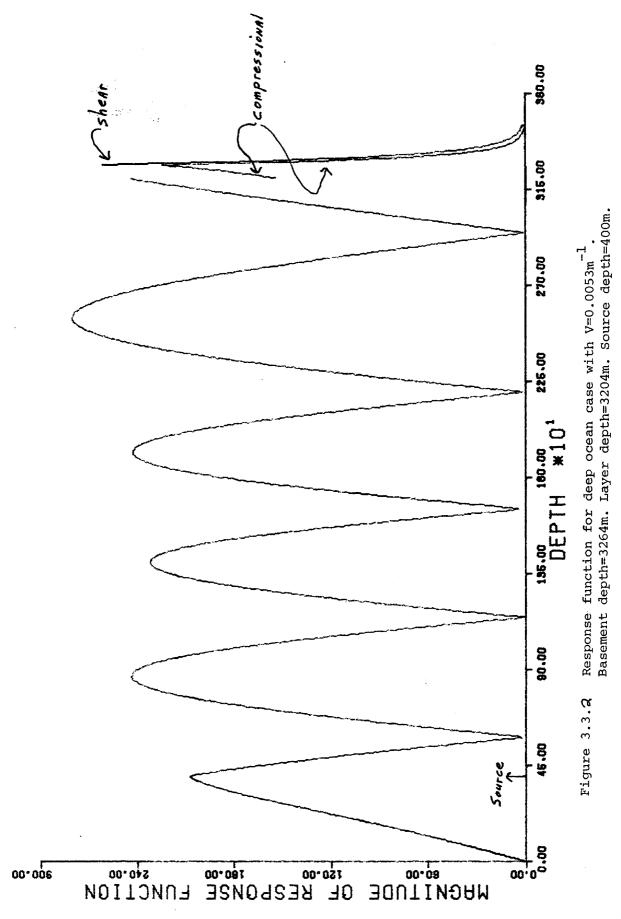


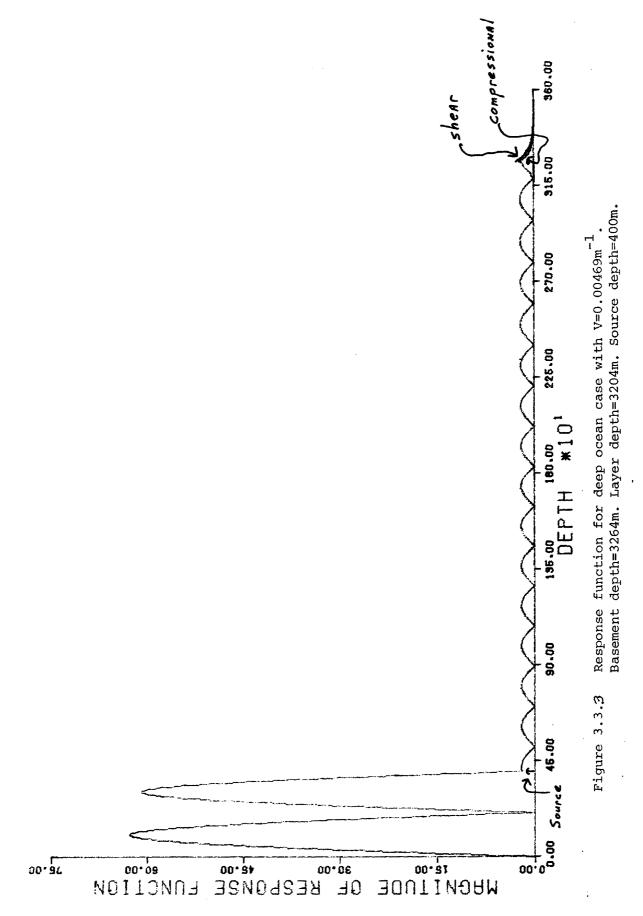
Study of the deep ocean profile for decreasing wavenumber V gave the same results as was obtained earlier for the shallow water cases. N₂ should be increased as V is decreased, especially in the continuous mode region. Good procedure would involve selecting several typical horizontal wave numbers and computing the response of each for N₂=12, N₂=16, and N₂=20. Study of the results should then aid in selecting the optimal sampling density.

Figures 3.3.2, 3.3.3, and 3.3.4 are typical response functions for figure 3.3.1 where an eight hertz source is at 400 meters. The horizontal wave numbers V=0.0053 and V=0.00469 are near the fifth and seventeenth modes of the 18 mode profile and V=0.003 is in the continuous region.

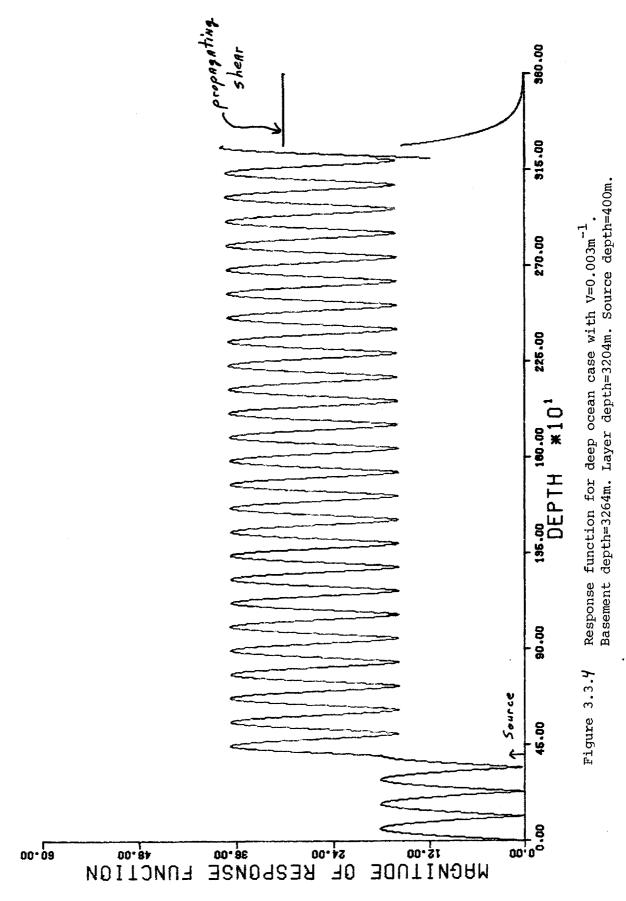
Notice that the basement shear waves, which are plotted at full value, are on the same order of magnitude as the compressional waves. In all cases shear has the larger magnitude, substantially so for V=0.003. This is to be expected for the dense, high speed basement. The excitation of shear is a much more important attenuation mechanism than in any of the models considered earlier. In addition, the propagating shear wave present in the basement for V=0.003 appears to have a strong influence on the compressional wave response function below the source. This is a good example of the type of phenomenon that can be easily investigated using the state variable algorithm.

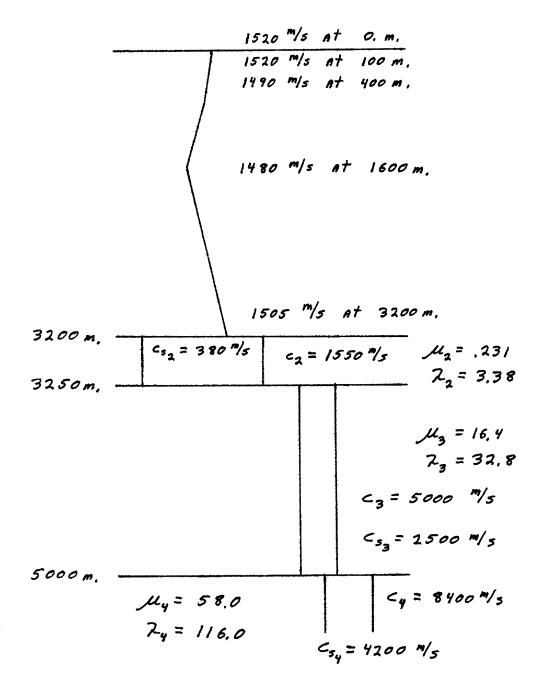
A more complex model of the same ocean region is shown in figure 3.3.5. Shear propagation is allowed in the sediment layers and the basement is divided into two elastic media. Most of the listed parameters and sound speeds are assumed from laboratory experiments and field data(33,38). The remainder of the section will discuss the application of the algorithm to this model.



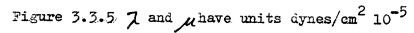








Multilayered Deep Ocean Model

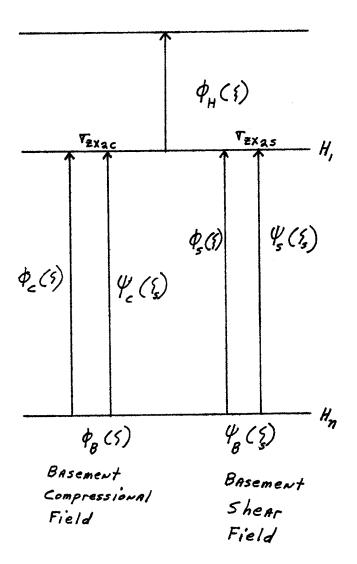


From section 1.3 and figure 3.3.6 recall the method of solution for this model. Two independent integrations of the state equations will occur. The first will use $\phi_{g}(\zeta)$ as the initial condition and the second $\mathscr{W}_{g}(\zeta)$. The principle of superposition will then be used to combine the two solutions in a manner that satisfies equation 1.4.6c. This superposition amounts to a scaling of $\phi_{g}(\zeta)$ and $\mathscr{Y}_{g}(\zeta)$ such that the total tangential stress at H is zero. In particular

$$\nabla_{z_{x_{2r}}} = 0.0 = \nabla_{z_{x_{2c}}} + A, \nabla_{z_{x_{2s}}} = 3.3.1$$

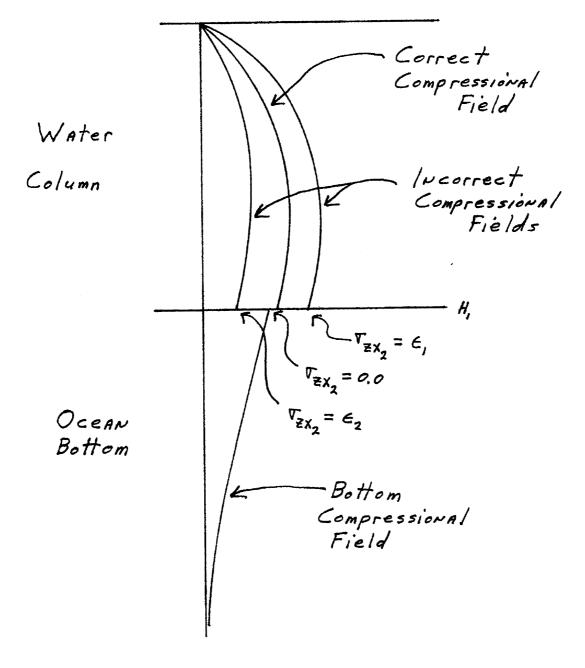
where A_i is the appropriate scaling factor. This procedure determines the relative magnitude of the total shear field to the total compressional field and specifies one of the unknown constants of equation 1.3.15. Equation 1.3.6a and 1.3.6b are then used to obtain $P_i(H_i)$ and integration proceeds to the ocean surface. The remaining constant of 1.3.15 is then determined and the entire solution scaled to satisfy equation 1.3.7.

This theory breaks down upon implementation however. Enough precision cannot be retained during the computation to insure that $V_{2X_{2T}}$ does in fact equal zero. Referring again to figure 3.3.6, the apparent cause of the problem lies in the relative magnitude of $\oint_{c} \langle \zeta \rangle$, $\oint_{c} \langle \zeta \rangle$, and $\bigvee_{s} \langle \zeta \rangle$. $\bigvee_{s} \langle \zeta \rangle$ is generally half an order of magnitude smaller than $\bigvee_{c} \langle \zeta \rangle$. $\bigvee_{c} \langle \zeta \rangle$ in turn is generally at least two orders of magnitude smaller than $\oint_{c} \langle \zeta \rangle$ and $\oint_{s} \langle \zeta \rangle$. The result is that the constant A₁ in equation 3.3.1 is strongly dependent on $\oint_{c} \langle \zeta \rangle$ and $\oint_{s} \langle \zeta \rangle$ and weakly dependent on $\bigvee_{c} \langle \zeta \rangle$. Difficulty then arises in keeping $V_{2X_{2T}} = 0.0$. Figure 3.3.7 depicts this sensitivity problem. For convenience, the shear field and the bottom elastic layers are not drawn. As was discussed earlier, a total compressional field is computed for the ocean bottom. The correct solution requires that the tangential stress at H₁ equal zero. In this case, the appropriate continuity



Multilayered Model Integration

Figure 3.3.6



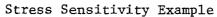


Figure 3.3.7

conditions are used to calculate the compressional field in the water column, and integration continues to the ocean surface. However, the relative magnitudes of the potentials cause difficulty in controlling the exact value of the tangential stress, and in general the stress has a small nonzero value, depicted as $\boldsymbol{\epsilon}_{,}$ and $\boldsymbol{\epsilon}_{2}$ in figure 3.3.7. Notice that although the bottom compressional field is not influenced by the stress error, the water column compressional field is extremely sensitive to the error. As a result, the correct water column compressional field could not be computed. No other criterion exists for determining the correct magnitude of the compressional field above H. The possibility of using double precision arithmetic was precluded by the requirement of complex variables. The conclusion is that further analysis of the state variable algorithm is required before being useful in the study of oceanic models of multilayered media.

To conclude, an accurate and efficient technique for computing the Green's function solution to the depth-separated wave equation has been presented. The technique has no inherent limitations when sound speeds are real quantities. The use of complex sound speeds to simulate attenuation is limited. In most cases attenuation is acceptable and solutions for wavenumbers corresponding to the discrete mode region. Study of the continuous mode region is limited.

The technique can be applied to both shallow and deep ocean sound speed profiles. The basement can be modelled as elastic and one or more sediment layers assumed to act as fluids are acceptable. Sensitivity limitations do not allow the modelling of the basement as a multilayered elastic medium.

APPENDIX I

C(z) = Compressional Sound Speed = Compressional Sound Speed Minimum c_ $C_{c}(z)$ = Shear Sound Speed C_{s0} = Shear Sound Speed Minimum = Transmission Coefficient From 1st to 2nd Layer D $\delta(\vec{r}-\vec{r}_0)e^{-i\omega t} = \delta(x-x_0) \delta(y-y_0) \delta(z-z_0) e^{-i\omega t}$ Harmonic Sound Source At (x₀, y₀, z₀) = Integration Step Size Δξ = Frequency f f(ξ) = Riccatti Parameter $G_{u}(z,z_{0})$ = Green's Function Solution Above Source at z_{0} $G_{T}(z_{1}z_{0}) =$ Green's Function Solution Below Source at z_{0} = Depth of ith Layer н_і = Radian Wave Number $2\pi f/c$ k k_{s} = Shear Radian Wave Number $2\pi f/c_{s}$ = Vertical Radian Wave Number in ith Layer k zi = Vertical Shear Radian Wave Number in ith Layer k zsi = Lamé's Constant; ith Layer λ_i λo = Maximum Compressional Wavelength $\lambda_0 = f/c_0$ = Maximum Shear Wavelength $\lambda_{0s} = f/c_{0s}$ λ_{0s} = Magnitude of Compressional Potential In Phase Plane Μ(ξ) N(ξ) = Magnitude of Shear Potential In Phase Plane = Rigidity of ith Layer μ

P	= Transmission Coefficient of Shear Wave in 2nd Layer Excited by Compressional Wave in 1st Layer
Pij	= Stress Component, Convention Related to 3-D Cube, i Referring to Direction of Face On Which Stress Is Acting and j Being Direction On Which Stress Is Acting
Ρ(ξ)	= Compressional State Variable
Ρ _s (ξ)	= Shear State Variable
φ(z)	= Compressional Velocity Potential
q(ξ)	= Normalized Compressional Sound Speed Parameter
q _s (ξ)	= Normalized Shear Sound Speed Parameter
Rs	= Surface Compressional Reflection Coefficient
ρ _i	= Density i th Layer
ψ(z)	= Shear Velocity Potential
σ^2	= Normalized Compressional Vertical Wave Number
σ ² s	= Normalized Shear Vertical Wave Number
θ(ξ)	= Phase of Compressional Potential in Phase Plane
γ (ξ)	= Phase of Shear Potential in Phase Plane
θ _i	= Direction of Plane Wave Propagation in i th Layer
Υ _i	= Direction of Shear Propagation in i th Layer
θ _d	$= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \text{dilatation}$
v	= (u, v, w) Velocity Vector
v	= Horizontal Wave Number
V _{ij}	= Reflection Coefficient for Wave in j th Layer Off Of i th Interface
^z i	= Acoustic Impedance in i th Layer
z _s i	= Shear impedance in i Layer

<u>9</u>8

 ξ = Normalized Compressional Depth Parameter ξ_s = Normalized Shear Depth Parameter

APPENDIX II

The implementation of the algorithm of Chapter I for the HP 2100 computer consists of a main program "SMAIN" and four subroutines "SWBC", "RNSV", "SCSPH", and "SSBC". "SMAIN" asks a series of questions which completely specify the ocean model to be studied. Table AII-1 lists these quantities. This profile is then displayed on the console for review by the operator before calculation begins.

The four subroutines are called at various times during calculation of the response function. "SWBC" implements equations 1.4.6, given the potentials in the second layer. "SSBC" implements equations 1.4.5, given the potentials in the lower layer. "RNSV" computes the transformation of equations 1.5.1,1.5.2,1.5.5, and 1.5.6. "SCSPH" integrates the differential equations 1.5.3,1.5.4,1.5.7, and 1.5.8 upwards in any layer.

The calculated compressional and shear potentials are stored in disc files for later use. In addition, printed output of most important quantities is provided. Table AII-2 lists those quantities.

Figure AII-1 is a flowchart of program "SMAIN". It can be divided into several general calculation sections. The first section contains the input sequence as well as the computation of the normalized sound speed parameters $q(\xi)$ and $q(\xi)$ and Riccati parameters f(f) and $f(\xi)$. It also contains the integration of the linear state equations for the particular solution. Initialization of the potentials as discussed in f1.4 then occurs.

The decision block titled "interface number" requires explanation. Each interface is numbered, with the fluid-sediment interface(the uppermost interface) number 1. Integration starts at the lowest interface. Therefore, for a one layer model, the interface number is one and the program jumps to "SWBC". For two or more layers the superposition principle of $\oint 1.4$ is required and "SMAIN" continues to "SSBC", after setting $\bigvee_{H} = \bigvee_{H} = 0.0$ as discussed earlier. The trio of subroutines then computes the field in the layer up to the next interface. If this interface number is greater than one, the procedure is repeated. If not, the program continues with the calculation of the shear excited field in a completely analogous manner.

Once the fields due to compressional and shear excitation have both been calculated, the correction factor of equation 3.3.2 can be computed and the total field in the elastic layers obtained by superposition. The next sequence of subroutines then calculates the homogeneous compressional solution in the water column, which is combined with the particular solution in the following step. Finally, the basement solution is computed.

The numerical integration technique used throughout "SMAIN" is a thirdorder Adams-Bashforth method(27). This method provides the same accuracy as the more common Runge-Kutta techniques, and is more straightforward to use in the desired application.

Notice that this program has been organized to accomodate any desired ocean model, including those which were found not solvable in Chapter 3. This allows further study of the problems which these models have presented.

"SMAIN" stores all information in fifteen binary data disc files, and all computations are done in groups of 32 integration steps, which is the maximum number of complex quantities that can be stored on one sector of the disc. Consequently, the disc file length in sectors should be

$$5 = \frac{H_{N+1}}{\Delta z} \cdot \frac{1}{32}$$

where Δz is the unnormalized integration step size and H_{N+1} the depth in the basement to which the solution is to be calculated.

TABLE AII-1

FRQ	= Complex Sound Speed
ZBS	= Basement Depth
CMN	= Minimum Value of Compressional Sound Speed(real number)
SCMN	= Minimum Value of Shear Sound Speed(real number)
NDZ	= Number of Integration Points
KBT	= Number of Sound Speed Transition Points
CSSP(i)	= Complex Compressional Sound Speed at Transition Point i
SSSP(i)	= Complex Shear Sound Speed at Transition Point i
ZBT(i)	= Depth at Transition Point i
NLAY S	= Number of Layers
JPB	= Basement Type; =1 if propagating, =0 if rigid
MU(j)	= Rigidity in Layer j
LAM(j)	= Lame's Constant in Layer j
LDEP(j)	= Depth of Layer j
CCBS	= Complex Compressional Basement Speed
SCBS	= Complex Shear Basement Speed
VP	= Complex Horizontal Wave Number
S	= Depth of Source
SS	= Strength of Source

TABLE AII-2

The following data is printed out only when the appropriate sense switch is on.

Switch 1	Sound Speed Profile Data
Switch 2	Riccati Data
Switch 3	Particular Solution
Switch 4	RNSV Data
Switch 5	SCSPH Data
Switch 6	Superposed Total Solution

The following data is printed out only when the appropriate sense switch is off.

Switch	7	Total	Solution	Above	Basement

Switch 8 Total Solution In Basement

When switches 7 and 8 are off and switch 10 is on, data for each integration step is printed out. If 10 is off , a number of steps are skipped between each step that is printed out; the interval equal to NDZ/32.

Switch 11 must be off to use the batch input mode.

Switch 12, when on, prevents calculation of the basement solution.

Switch 13, when on, prevents calculation of sound speed and Riccati information and is useful when information stored in disc files from previous computation will remain unchanged.

Switch 14, when on, prevents calculation of particular solution and is useful when information stored in disc file from previous computation will remain unchanged.

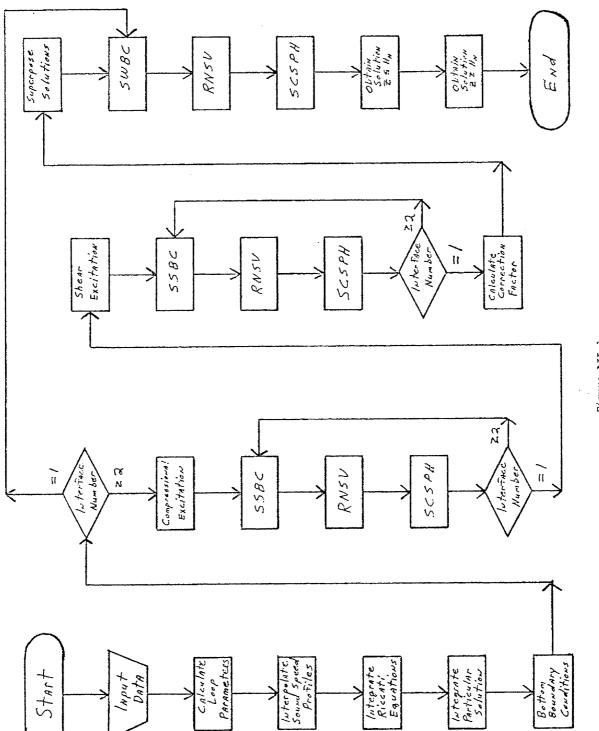


Figure AII-1.



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	6626		COMPLEX R. M. E. G. F. H. C. D. Y1, Y2, Y3, Y4, D5, D6, D7, D8, D1, D2, D3, D4
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$\left(\right)$	6623		DIMENSION W8(32), YP(32), YBF(32), ZBF(32), XBF(32), ZBT(25), CSSP
	8829		DIMENSION 555P(25), MU(15), CXR(32), SXR(32), SFRX(32), CFRX(32)
	6630		DIMENSION ALAM(15), LDEP(15), IF1(3), IF2(3), IF3(3), IF4(3), IF5(
7	6031		DIMENSION IF6(3), IF7(3), IF8(3), IF9(3), IF18(3), IF11(3), IF12(3
	6632		DIMENSION IF13(3), IF14(3), IF15(3), DH(15)
	8833		REAL MU, LDEP, MPG, HAGS, MAGB
(8834		DATA PL, JAY, IN/3, 1415926, (0, 0, 1, 0), 1/
	6935		IF1(1)=2HDN
~	8936		IF1(2)=2H1
C	86 37		IF2(1)=2HDW
	6633		IF2(2)=2H2
1	683 9		IF3(1)=2HDN
(8848		IF3(2)=2H3
	6641		IF4(1)=2HDN
1	6642		IF4(2)=244
(6643		IF5(1)=2HDW
	6644		IF5(2)=2H5
i	8845		IF6(1)=2HDW
Ć	0046		IF6(2)=2H6
	0047		IF7(1)=2HDW
(6648		IF7(2)=2H7
C	8849		IF8(1)=2HDH
	8658		IF8(2)=248
(6651		1F9(1)=2HDY
	8852		IF9(2)=2HP
	9953		IF18(1)=2HDY
(6654		IF18(2)=2H9F
	6855		IF11(1)=2HDZ
	0056		IF11(2)=2HFF
(8857		IF12(1)=2HDC
`	8858		IF12(2)=2HXQ
	9059		IF13(1)=2HD5
L.	8868		IF13(2)=24X0
` .	8861		IF14(1)=2HDC
	0000		1044/23-2000

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-	UUUL.	15 TAVE 1-218 (C	
	8863	IF14(3)=2HX	
	0064	IF15(1)=2HD5	
	6665	IF15(2)=2HFQ	
C	0366	IF15(3)=24X	
Ċ.	8667	IF1(3)=2H	
	6663	IF2(3)=2H	
(0869	IF3(3)=2H	
`	6979	IF4(3)=2H	
	8871	IF5(3)=2H	
(8972	IF6(3)=2H	
	8973	IF7(3)=2H	
	0074 0075	IF9(3)=2H	
(<i>0</i> 075	IF9(3)=2H	
	8876 8877	TPI=PI*2. ♥ IF19(3)=2H	
	0071 0078	IF10(3)=2H	
Ċ	6679	IF12(3)=2H	
	8888	IF13(3)=2H	
	6681	PI2=P1**2	
Ċ	8682	IF(I55H(11)) 99,98	
	0083	98 IN=22	
	8884	99 CONTINUE	
C	6685	TPISQ=TPI++2	
	0086		
~	99 87	C INPUT	
C	8988	C	
	8889	5 WRITE(1, 198)	
1	8898	100 FORMAT("TYPE COMPLEX FREQUENCY, NUMBER OF GRID POINTS *	
C	0891	1" AND NUMBER OF SOUND SPEED TRANSITIONS")	
	8892	READ(IN. *) FRQ. NDZ. KBT	
1	8893	CNN=10099.8	
(88 94	SCH1=19988. 8	
	8895	00 162 J=1. KBT	
\langle	6996	WRITE(1.103) J	
~	8897	183 FORMAT("TYPE COMPLEX C AND 5 SPEEDS AND DEPTH AT TRANSITION"	
	6668	READ(IN.*) CSSP(J), SSSP(J), ZBT(J) ~	
C	8899	IF(CHN GT. REAL(CSSP(J))) CHN=REAL(CSSP(J))	
× •	6189 64.04	IF (SCHN, GT, REAL (SSSP(J)), AND, SSSP(J), NE. 8, 8) SCHN-REAL (SSSP(
	9191 9192	182 CONTINUE	
(8182 9193	IF(SCIN, EQ. 12909.0) SCIN=CIN HRITE(1, 105)	
	8185 8184	165 FORMAT("TYPE # OF LAYERS AND 1 OR 8 IF PROPAGATING OR RIGID"	
	8185	READ(IN.*) HEAVS, JPB	
Ć	8185	に12000-7-1 12-12月15-1	
	8187	00 196 J=L.NL	
	0108	HRITE(1.187) J	
\subset	8109	187 FORMAT("TYPE VISCOSITY, LAME AND DEPTH OF LAYER ", 13)	
	8118	186 READ(IN. *) MU(J), ALAM(J), LDEP(J)	
~	0111	ZBS=LDEP(NL)	
C	8112	IF(JPB. EQ. 8) 60 TO 153	
	811 3	WRITE(1, 104)	
~	0 114	184 FORMAT("TYPE COMPLEX BASEMENT C AND S SPEEDS, COMPUTATION "	
C	8115	1"DEPTH VISCOSITY AND LAME">	
	0116		
(0 117		
C	6118		
	0119		
Ċ	8128		
C	8121		
		C DISPLAY PROFILE PRRAMETERS	
(8123		
	8124		
	0125 0125		
i,	0126 0127		
	0127 0400		

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CAA OT DO	10 10	
t≕an 8¢Þ	2670)
IF(155W(10)) 440, 441	8435	
0 10=56M	161 8	
	96199 COTO	\mathcal{L}
CZZZS NH=TLIX(2<(ZB2\LEGDIU(0ZC))+T)	6878	
IF(45, EQ, CMP1,40, 0, 0, 8, 8)) 51G5=CMP1,X(0, 0, 0, 0)	88 70 28 70	
IE/A2 ED CAAL A(0 0 0 3/) 2102-CAAD A(0 0 0 3/) 2102=1bi20*(6E182-A23) ×471225	2010 9810)
Starsheiter - Star	5878	
ILE(3CB2) E0 CHATX(0' 8' 9' 0) / A2=CHATX(0' 8' 9' 0' 0)	91816 91816	
	2878) L
21Gt=11F15Gt+(8E1R-VP2)*%LP3	2819	
4h*4h=Z4h	T 8 T 8	Ŋ
<pre>(WTX*<zgt)1t6t3) 58z="Z0</pre"></zgt)1t6t3)></pre>	9879)
(<2@i)LH0~LH×SH7X)/S8Z=SZO	62.178	
BETRAFEROZ/CARG	8278	С
BETHS-FRR2/SCTN2	2210	2
1405+1405=27405	9278	
FROM FROM	SZT18)
CARG=CARHAGAN	+2.70 ε2.70	-
20=¥5=V(XTV)*€25 XTV3=2XTV*+5	Z2148 Z2149	
X11X=C3111X X11X5≤=X11X=×5	7278	Э
BH-MUS-SNIK	8270	
A RECONLED	6978	2
368 CONTINUE	8918	С
Э	29748	
C CATCONDALE ANSIONE CONCLUME AND INTERVATION AUGMETERS.	9978	\mathbf{i}
3	9165	1
395 EOSWBH(#SIEID BOLLOW#)	1978	
362 MILE(T6) 385)	5978)
802 01 09	2918	-
MBITE(LP, 46L) COBS, SOBS	1918	
369 EOSABIL("PROPAGATING EOTION")	09 1 0)
H&ILE(T5'396) IE(1168'E0'3) 20:10:392	6570 8578	
16/166/16/16/2010/2626(1)/281(1)	2510	
10/10/10/10/10/10/10/10/10/10/10/10/10/1	9510	.)
IECIDS EGT T) PRILECTD (CS) MICHTHAR) HIGH HAR)	9722	
356 MELLE(LP. 165) 11 MI(1); GTEA(1)	VSTƏ	1
D0 350 1=1°/1T	5578) J
MALLE(LP, 160) ERG, NDZ, VP, S	6125	
trep(tr/+) LP	tsto	\rightarrow
TTO LOUGHL(.LALE FIRE 661RLEK FO #.)	otes	
HALLE(T' 170)	6743	
IL(1'E6'T) 00 10 2	8148)
KEUD(IN**) 1 VC2 LOSMAU(«IO CHARGE BUCKARELES IABE J OINESMICE 8«)	247 0 9778	
TEZ EUDMOT(*IC UMMUE DODOWLIEB INDE T UIHLBWICE 8") 378 MENIE(T TEZ)	94 42 97 42	
365 17111an 812	5778 5778)
672 01 05	5410	
461 FORMER "BREEREN" SETE 4/ /ISX SETE 4/	6442	
HEILE(T' 46T) CCB2' 2CB2	T+T8	<i></i>
MSILE(T) 389)	01-TH	
IE(168' E0' 0' 0) C0 10 383	6210	4
ST4X0EBLH., 3GX E18 4)	8578)
TSETO + 1/14X .ZHEHK ZORNO ZEED. / 3X SETO + 1/1	2578	
Tes Fooder("Terminon", 13, 14, 14, Conformation States States "	9278)
(1) TSS (1) T (222b(1)) 282(1) 281(1)	SETH	5
00 759 1=7 K81	1218	
T-FURE CORFLEMENTED STORE S	8133	\mathcal{I}
462 EGENEL("BEEERELL'VBX" «MISCOSTIA") ETS' 5' VJSX	22118	F
IE(TEBE EBT) MELLE(T'46S) MO(MTUAR)/ETEMOMTUAR) IT6X: JT6XE COMPLEAL "LETB'S' VIGX: USELH "LETB'S)	6737 6736	
48X ••• 94E UNALBAL •* E48 2* 346X •//EDIA •* E48 2/	01.30 0758	-
202 PRATIC ALL DOLD ## (127407) #C107120 #C42 5 7	0710	

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	112.PF	UK 10 1772.	
	8195	441 NP=NDZ/32	
	81.96	442 CONTINUE	
	6197	DO 640 J=1. NLRVS	
~	81 98	640 DN(J)=LDEP(J)+10Z/(ZE5+32)	
$\langle \cdot \rangle$	8199	WRITE(LP, 641) (DN(J), J=1, NLRYS)	
	8288	641 FORMAT (6F10, 5)	
Ć	8281	WDEP=LDEP(1)	
`	8282	IF (MU(2), EQ. 0. 8) WDEP=LDEP(2)	
	8283	COR=CMPLX(1.0.0.0)	
C	0204	HRITE(LP, 210) XLM, XLMS, SIGP, SIGS, DZ, DZS	
<u>C</u>	8285	220 FORMAT("XLM=", 2F10, 5, " XLM5=", 2F10, 5, /	
	8286	1"SIGP=", 2F10. 5, " SIG5=", 2F10. 5, /	
	6207	2"DZ=", 2F10. 5, " DZ5=", 2F10. 5)	
$\left(\right)$	8288	K0=1	
	8289	CCL=CSSP(1)	
(8218	SCL=SSSP(1)	
`	8211	ZL=0. 6	
	8212	L=LMAX	
ŕ	9213	IF(I554(13)) 191 198	
i	8214	C	
		C CALCULATE SOUND SPEED PROFILE NORMALIZED FUNCTION	
		C STORE ON DISC FILES DSXQ AND DCXQ	
<u></u>			
	8217		
	8218	190 I=0	
\mathcal{C}	8219	L=8	
ч.,	8228	DO 50 J=1.NDZ	
	8221	I=I+1	
	8222	CS=CMPLX(0, 0, 0, 0)	
C	6223	SS=CMPLX(0, 0, 0, 0)	
	8224	CXR(1)=CHPLX(0, 0, 0, 0)	
(8225	SXQ(I)=CMPLX(0, 0, 0, 0)	
	8226	Z=FLOAT(J-1)*ZBS/FLOAT(NDZ)	
	8227	D0 48 K-KR, KBT	
	8228	IF(ZBT(K), GE, Z) GO TO 41	
$\langle -$	8229	Z1=ZBT(K)	
	8238	CCL=CSSP(K)	
	8231	SQL=SSSP(K)	
(
	8232	40 CONTINUE	
	0233	CCU=CCBS	
C	8234	SCU=SCBS	
C .	8235	ZU=2B5	
	8236	GO TO 42	
<i>p</i>	6237	41 CONTINUE	
C	8238	CCU=CSSP(K)	
	8239	SCU=SSSP(K)	
Ċ	8248	ZU=ZBT(K)	
~	8241	42 CONTINUE	
	8242	CS=CCL+((CCU-CCL)/(ZU-ZL))*(Z-ZL)	
(8243	SS=SQL+((SQL-SQL)/(ZU-ZL))+(Z-ZL)	
,	8244	HL(I)=05	
	8245	W2(I)=55	
<i></i>	8246	CXQ(I)=TPISQ+FRQ2+(1,/CM+2-(1,/C5)++2)+XLM2	
(6247	5XQ(1)=TPISQ#FRQ2*(1, /SCH12-(1, /SS)**2)*XLM52	
	8248	XBF(I)=Z	
	8249	IF(55, E9, 0, 0) 5%9(I)=0, 0	
0		··· ···· ··· ··· ··· ··· ··· ··· ··· ·	
•.	8258	KA=K	
	8251	52 JF(I.LT. 32) GO TO 50	
1	6252	CALL EXEC(15, 1038, CX0, 128, JF12, L)	
C .	8253	CRLL EXEC(15, 1039, SXR, 128, IF13, L)	
	8254	I=0	
	8255	l=1+1	
(8256	IF(ISSN(1)) 51,58	
	8257	51 D0 55 JX=1, 32 MP	
ł	8258	55 WRITE(LP, 154) XBF(JX), WL(JX), W2(JX), CXR(JX), SXR(JX)	
C.	8259		
	9769		

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	8261	191 CONTINUE	()
	8262	I≓1	
	8263	LNRX=L	
~	8264	CS=CSSP(KBT)	C
(6265	SS=SSSP(KBT)	· ·
	8266	CX9(1)=TPISQ+FR92+(1_/C19)2-(1_/C5)++2)+XL192	}
-	8267	5x9(1)=TPI5R+FR92+(1_/501H2-(1_/55)++2)+3LH52	
(8268	IF(SS, EQ, 0, 0) SA(2(1)=0, 0	C
	6269	IF(ISSW(1)) 583,582	
C	8270	583 WRITE(LP, 154) ZBS, CS, SS, CXR(1), SXR(1)	
	6271		İ
	6272	5XQ(2)=TPIS0+FRQ2+(1, /5CPN2-(1, /5CPS)++2)+XLH52	
	8273	B1=5X8(2)	$c \mid$
Ę	8274	IF(SCES.EQ.CHPLX(0.6),0,0)) SXR(2)=CHPLX(0.6),0,0)	
	8275	Z=ZES+1_0	
	8276		
<	0277	565 [F(JPB, Eg. 1) JRTE(LP, 154) Z. CCBS, 5CPS, CXR(2), 5XR(2)	
	6278		:
			i
(6279		()
`	8288		
		C RICCATI PROPAGATING BASEMENT INITIALIZATION	
1	6282	-	$\langle $
(8283	IF(JPB. EQ. 1) GO TO 17	
	8284	CFQ2H=TP1+FR0+C5QRT(1_/C11N2-(1_/C55P(KBT))++2)+XLM	
~	8285	5F08H=TP1+FR2+C5QR1(1, /SC1N2-(1, /SS5P(KBT))++2)+XLM5	
(8286		
	8287		ļ
	8298		
Ć			
	8289		1
	8298		:
6	8291		
(6292	SFQ=SFQ8H	- ` i
	6293	HF=1A	
	6294	K5=1	,
	6295		
	8296		
	9297		
(
	8298		
		C RICCATI RIGID BOTTOM INITIALIZATION	
Ċ	0300		
<u> </u>	9391		` .
	8382		
6	8383	IF (SCBS, EQ, CHP1.X(0, 0, 0, 0)) SF9BS=CHP1.X(0, 0, 0, 0)	(
C	0304	CFRX(2)=CFQBS	
	0395	SFRX(2)=5FBB5	-
,	0306	6 WF=LH+1	1
(0307		C.
	8388		
	8389		
Ċ			(
	6318		
	8311		
Č	8312		(
·-	0313		× .
	8314		
6	8315	C RICCATI INTEGRATION	
C	0316	C STORE ON DISC FILES DOFOX AND DSFOX	ζ.
	8317		
	0318		
Ć	8319		N.
•			
	8328		
<u>,</u>	8321		(
ς.	0322		~
	6323		
	0324	IF(ISSN(13)) 192,193	4
ί.	8325		7
	A776		

	UNCU	LIT 10 V
	8327	CALL EXEC (14, 1038, CXQ, 128, IF12, LN)
	0328	CALL EXEC (14, 1038, SXR, 128, IF13, LN)
	8329	DO 68 I=NS, 32
C	8338	NX=33-1
`	8331	CFRX(NX)=CMPLX(0, 0, 0, 0)
	6332	SFQX(NX)=CMPLX(0, 0, 0, 0)
(0333	CFQP=CFQ-DZ+CDFQ
•	9334	SF0P=SF0-DZS+SDF0
	0335 0336	CDFQP=CFQP++2-CXQ(NX)
Ċ	9337 9337	SDF&P=SF&P*+2-SXQ(HX) CF&=CF&-, 5+D2+(CDF&+CDF&P)
	9338	SFR=SFR-, 5+DZS+(SDFR+SDFR)
	9339	CDFG=CFR++2-CXR(NX)
	0348	SDFR=SFR++2-SXR(NX)
	8341	CFCX(NX)=CFQ
	8342	SF0X(NX)=SF0
$\langle \cdot \rangle$	8343	Z=FLOAT(NX-1+LN+32)*2E5/FLOAT(NDZ)
	0344	IF (Z. LT. HDEP) SFRX(NX)=CTPLX(0, 0, 0, 0)
	8345	IF(NX NEQ 1) GO TO 68
	8346	CALL EXEC (15, 103B, CF0X, 123, IF14, LN)
	8347	CALL EXEC (15, 1938, SFRX, 128, IF15, LN)
C	9348	474 CALL EXEC(15, 1038, YP, 128, IF9, LN)
C	8349	NS1=NS
	9358	NS=1
C	8351	473 JF(155W(2)) 61, 68
Ν.	8352	61 00 65 JY=N51, 32 NP
	8353	JX=33-JY
C	0354	Z=FLOAT(JX-1+LN+32)+ZB5/FLOAT(ND2)
×	8355	65 WRITE(LP, 152) Z. CFOX(JX), SFOX(JX)
	0356	152 FORMAT(F10, 3, 4(F10, 6, 2X))
í.	8357	68 CONTINUE
	8358 8359	192 CONTINUE
	0.507	
		C THEFEODTE DODITION OF COLUCTION NETWOR I THEOD CONDITIONS
Ç	0368	C INTEGRATE PARTICULAR SOLUTION USING LINEAR EQUATIONS
Ç	0368 0361	C STORE ON DISC FILE DYP
-	0368 0361 0362	C STORE ON DISC FILE DYP C
((0368 0361 0362 0363	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195
-	0368 0361 0362	C STORE ON DISC FILE DYP C
Ċ	0368 0361 0362 0363 0364	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE
-	0368 0361 0362 0363 0364 0365	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE PHII=CMPLX(0. 0, 0. 0)
Ċ	0368 0361 0362 0363 0364 0365 0366	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE PHII=CMPLX(0. 0, 0. 0) PRII=CMPLX(0. 0, 0. 0)
C (0368 0361 0362 0363 0364 0365 0366 0367 0368 0369	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE PHII=CMPLX(0. 0, 0. 0) PRII=CMPLX(0. 0, 0. 0) DPHII=CMPLX(0. 0, 0. 0)
Ċ	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0369 0370	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE PHII=CHPLX(0, 0, 0, 0) PRII=CHPLX(0, 0, 0, 0) DPHII=CHPLX(0, 0, 0, 0) DPHII=CHPLX(0, 0, 0, 0) DPHII=CHPLX(0, 0, 0, 0) DPHI=CHPLX(0, 0, 0, 0)
C (0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0370 0371	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINE PHII=CMPLX(0, 0, 0, 0) PRII=CMPLX(0, 0, 0, 0) DPRII=CMPLX(0, 0, 0, 0) DPRII=CMPLX(0, 0, 0, 0) DPRI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0)
C C C	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0370 0371 0372	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINE PHII=CPPLX(0.0.0.0) PRII=CPPLX(0.0.0.0) DPRII=CPPLX(0.0.0.0) DPRII=CPPLX(0.0.0.0) DPRI=CPPLX(0.0.0.0) PHI=CPPLX(0.0.0.0) PRI=CPPLX(0.0.0.0)
C (0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0370 0371 0372 0373	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE PHII=CTPLX(0.0.0.0) PRII=CTPLX(0.0.0.0) DPHII=CTPLX(0.0.0.0) DPHII=CTPLX(0.0.0.0) DPHI=CTPLX(0.0.0.0) PHI=CTPLX(0.0.0.0) PHI=CTPLX(0.0.0.0) PRI=CTPLX(0.0.0.0) PRI=CTPLX(0.0.0.0) PRI=CTPLX(0.0.0.0)
C C C	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0373 0374	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CHPLX(8, 0, 0, 0) PRII=CHPLX(8, 0, 0, 0) DPHII=CHPLX(8, 0, 0, 0) DPHI=CHPLX(8, 0, 0, 0) DPHI=CHPLX(8, 0, 0, 0) PHI=CHPLX(8, 0, 0, 0) PHI=CHPLX(8, 0, 0, 0) LS=NH-72 NX=LMFX+1
	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0373 0374 0375	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CHPLX(0, 0, 0, 0) PRII=CHPLX(0, 0, 0, 0) DPHII=CHPLX(0, 0, 0, 0) DPHI=CHPLX(0, 0, 0, 0) DPHI=CHPLX(0, 0, 0, 0) PHI=CHPLX(0, 0, 0, 0) PHI=CHPLX(0, 0, 0, 0) ILS=MA/32 NX=LMFX+1 IF(ISSN(3)) 501, 500
C C C	0368 9361 9362 9363 8364 9365 9366 9367 9366 9369 9379 9374 9373 9374 9374 9375 9376	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CMPLX(0, 0, 0, 0) PRII=CMPLX(0, 0, 0, 0) DPHI=CMPLX(0, 0, 0, 0) DPRII=CMPLX(0, 0, 0, 0) DPRI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) CSSN(3)) 501, 500 501 MRITE(LP, 62)
	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0371 0372 0374 0375 0374 0376 0377	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194,195 195 CONTINE PHII=CMPLX(0,0,0,0) PRII=CMPLX(0,0,0,0) DPHI=CMPLX(0,0,0,0) DPHI=CMPLX(0,0,0,0) DPHI=CMPLX(0,0,0,0) PHI=CMPLX(0,0,0,0) PHI=CMPLX(0,0,0,0) PHI=CMPLX(0,0,0,0) ES=NN/32 NX=LMXX+1 IF(ISSN(3)) 501,500 501 MRITE(LP,62) 62 FORMAT(/"PRTICULAR SOLUTION")
	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0379 0374 0372 0374 0375 0374 0375 0377 0378	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194,195 195 CONTINUE PHII=CMPLX(0.0.0.0) PRII=CMPLX(0.0.0.0) DPHII=CMPLX(0.0.0.0) DPHI=CMPLX(0.0.0.0) DPHI=CMPLX(0.0.0.0) DPHI=CMPLX(0.0.0.0) PHI=CMPLX(0.0.0.0) PHI=CMPLX(0.0.0.0) PHI=CMPLX(0.0.0.0) CSN(3) 501,500 S01 MRITE(LP.62) 62 FORMAT(/*PARTICULAR SOLUTION*) S00 NA=(LS+1)*32-NA
	0368 0361 0362 0363 0364 0365 0366 0367 0379 0371 0372 0374 0375 0374 0376 0377 0378 0377 0378 0377	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE PHII=CMPLX(0, 0, 0, 0) PRII=CMPLX(0, 0, 0, 0) DPHII=CMPLX(0, 0, 0, 0) DPHI=CMPLX(0, 0, 0, 0) DPHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) ESSM0.72 NX=LMFX+1 IF(ISSW(3)) 501,500 501 MRITE(LP, 62) 62 FORMAT(/"PRRTICULAR SOLUTION") 500 NA=(L5+1)*32-NA DO 13 J=1,NA
	0368 0361 0362 0364 0365 0366 0367 0368 0369 0371 0372 0374 0374 0375 0374 0376 0377 0378 0378 0378 0378 0378 0378	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINE PHII=CMPLX(0, 0, 0, 0) PRII=CMPLX(0, 0, 0, 0) DPRII=CMPLX(0, 0, 0, 0) DPRI=CMPLX(0, 0, 0, 0) DPRI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) CSMN:72 NX=LMFX+1 IF(ISSW(3)) 501,500 501 MRITE(LP, 62) 62 FORMMT(/*PRETICULAR SOLUTION*) 500 NA=(LS+1)*32-NA DO 13 J=1,NA 13 YP(33-J)=CMPLX(0, 0, 0, 0)
	0368 0361 0362 0363 0364 0365 0366 0367 0379 0371 0372 0374 0375 0374 0376 0377 0378 0377 0378 0377	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINUE PHII=CMPLX(0, 0, 0, 0) PRII=CMPLX(0, 0, 0, 0) DPHII=CMPLX(0, 0, 0, 0) DPHI=CMPLX(0, 0, 0, 0) DPHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) ESSM0.72 NX=LMFX+1 IF(ISSW(3)) 501,500 501 MRITE(LP, 62) 62 FORMAT(/"PRRTICULAR SOLUTION") 500 NA=(L5+1)*32-NA DO 13 J=1,NA
	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0374 0375 0374 0375 0374 0375 0379 0379 0380 0381	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194,195 195 CONTINUE PHII=CHPLX(0.0.0.0) PRII=CHPLX(0.0.0.0) DPHI=CHPLX(0.0.0.0) DPHI=CHPLX(0.0.0.0) DPHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) LS=NH/72 NX=LMFX+1 IF(ISSN(3)) 501,500 501 MRITE(LP.62) 62 FORMMT(/*PRRIICULAR SOLUTION*) 509 NA=(LS+1)*32-NA DO 13 J=1.NA 13 YP(33-J)=CHPLX(0.0.0) PRII=-0.5*DZ*SD*XLM
	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0373 0374 0375 0376 0377 0378 0377 0379 0388 0381 0382	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194,195 195 CONTINUE PHII=CHPLX(0.0.0.0) PRII=CHPLX(0.0.0.0) DPHI=CHPLX(0.0.0.0) DPHI=CHPLX(0.0.0.0) DPRI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) ES=NA/72 NX=LHFX+1 IF(ISSN(3)) 501,500 501 MRITE(LP.62) 62 FORMMT(/*PRRTICULAR SOLUTION*) 500 NA=(LS+1)*32-NA D0 13 J=1.NA 13 YP(33-J)=CHPLX(0.0.0) PRII=-0.5*DZ*SD*XLM DPRI=SD*XLM
	0368 0361 0362 0363 0366 0366 0367 0368 0369 0370 0371 0372 0373 0374 0375 0376 0377 0378 0377 0378 0377 0378 0379 0381 0381 0381 0381	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194,195 195 CONTINUE PHII=CHPLX(0.0.0.0) PRII=CHPLX(0.0.0.0) DPHII=CHPLX(0.0.0.0) DPHI=CHPLX(0.0.0.0) DPRI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHI=CHPLX(0.0.0.0) PHII=CHPLX(0.0.
	 0368 0361 0362 0363 0364 0365 0366 0367 0368 0379 0373 0374 0375 0376 0377 0378 0379 0388 0381 0382 0384 0385 0386 	C STORE ON DISC FILE DYP C IF(ISSW(14)) 194, 195 195 CONTINE PHII=CMPLX(0, 0, 0, 0) PRII=CMPLX(0, 0, 0, 0) DPHI=CMPLX(0, 0, 0, 0) DPHI=CMPLX(0, 0, 0, 0) PHI=CMPLX(0, 0, 0, 0) PRI=CMPLX(0, 0, 0, 0)
	 0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0374 0375 0376 0377 0378 0384 0384 0386 0387 	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CHPLX(0, 0, 0, 0) PRII=CHPLX(0, 0, 0, 0) DPHI=CHPLX(0, 0, 0, 0) DPHI=CHPLX(0, 0, 0, 0) PHI=CHPLX(0, 0, 0, 0) PHI=CHPLX(0, 0, 0, 0) LS=NH-72 NX=LMFX+1 IF(ISSN(3)) 501, 500 501 MRITE(LP, 62) 62 FORMMT(/*PRRITCULAR SOLUTION*) 500 NA=(LS+1)*32=NA D0 13 J=1, NA 13 YP(33-J)=CHPLX(0, 0, 0, 0) PRII=-0, S+D2*SD*XLM DPRI=SD*XLM LS=LS+1 NB=ND+1 D0 12 I=1,LS LT=LS-I CALL EXEC(14, 1038, CF0X, 128, IF14, LT)
	 0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0372 0374 0375 0374 0375 0376 0377 0378 0384 0385 0384 0385 0388 	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CHPLX(8, 0, 0, 0) PRII=CHPLX(8, 0, 0, 0) DPHI=CHPLX(8, 0, 0, 0) DPHI=CHPLX(8, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) LS=NH/72 N%=LHFX+1 IF(ISSN(3)) 501, 500 501 MRITE(LP, 62) 62 FORMHT(/*PRRIICLAR SOLUTION*) 509 NA=(LS+1)*32=NA DO 13 J=1,NA 13 YP(33-J)=CHPLX(0, 0, 0, 0) PRII=0, S+0Z*SD=XLM DPRI=D+XLM LS=LS+1 NB=NB+1 DO 12 J=1,LS LT=LS-1 CALL EXEC(14, 1038, CF0X, 128, IF14, LT) DO 12 J=NA, 32
	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0373 0374 0375 0376 0377 0378 0377 0378 0377 0378 0378 0378	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CHPLX(0, 0, 0, 0) PRII=CHPLX(0, 0, 0, 0) DPRII=CHPLX(0, 0, 0, 0) DPRI=CHPLX(0, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) IS=NA/72 NX=LHFX+1 IF(ISSN(3)) 501, 500 501 MRITE(LP, 62) 62 FORMMT(/*PRETICULAR SOLUTION*) 500 NA=(LS+1)*32=NA D0 13 J=1, NA 13 YP(33-J)=CHEX(0, 0, 0, 0) PRII=-0, S+DZ*SD=XLM DPRI=SD=XLM LS=LS+1 NA=NA+1 D0 12 I=1_LS LI=LS=I CALL EXEC(14, 1038, CF0X, 123, IF14, LT) D0 12 J=NA, 32 NY=33-J
	0366 0361 0362 0363 0366 0366 0367 0368 0369 0371 0372 0373 0374 0375 0376 0377 0378 0377 0378 0377 0378 0377 0378 0379 0383 0384 0381 0384 0385 0384 0385 0387 0388 0389 0389 0399	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CMPLX(0, 0, 0, 0) PRII=CMPLX(0, 0, 0, 0) DPRII=CMPLX(0, 0, 0, 0) DPRI=CMPLX(0, 0, 0, 0) PRI=CMPLX(0, 0, 0, 0, 0, 0) PRI=CMPLX(0, 0, 0, 0, 0) PRI=
	0368 0361 0362 0363 0364 0365 0366 0367 0368 0369 0371 0372 0373 0374 0375 0376 0377 0378 0377 0378 0377 0378 0378 0378	C STORE ON DISC FILE DYP C IF(ISSN(14)) 194, 195 195 CONTINUE PHII=CHPLX(0, 0, 0, 0) PRII=CHPLX(0, 0, 0, 0) DPRII=CHPLX(0, 0, 0, 0) DPRI=CHPLX(0, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) PRI=CHPLX(0, 0, 0, 0) IS=NA/72 NX=LHFX+1 IF(ISSN(3)) 501, 500 501 MRITE(LP, 62) 62 FORMMT(/*PRETICULAR SOLUTION*) 500 NA=(LS+1)*32=NA D0 13 J=1, NA 13 YP(33-J)=CHEX(0, 0, 0, 0) PRII=-0, S+DZ*SD=XLM DPRI=SD=XLM LS=LS+1 NA=NA+1 D0 12 I=1_LS LI=LS=I CALL EXEC(14, 1038, CF0X, 123, IF14, LT) D0 12 J=NA, 32 NY=33-J

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	0202	DPRII=-SIGP*PHI+CFQX(NY)*FRI
	0393	
	0394	PHII=PHII-, 5+DZ+(DPHI+DPHII)
	8395	PRII=PRII 5*DZ*(DPRI+DPRII)
Ċ	8396	DPHI=-CFQX(NY)*PHII+PRII
C	8397	DPRI=-SIGP*PHII+CF0X(NY)*PRII
	0398	IF(155H(3)) 176,175
	8399	176 Z=(LT+32+NY-1)+ZB5/FLOAT(NDZ)
$\left(\right)$	6400	WRITE (LP, 152) Z, PHII
	9491	175 YP(NY)=PHII
C	6462	IF(J.1.T. 22) 60 TO 12
•	6483	CALL EXEC (15, 1038, YP, 123, IF9, LT)
	8484	NA=1
	8495	12 CENTINUE
- Ç	8486	194 CONTINUE
C		C PROPAGATING PRSEMENT INITIALIZATION
`	8489	C
	8410	PS=CMPLX(0.0901_0.0)
	8411	IF (VS, EQ, CMPLX(0, 0, 0, 0)) PS=CMPLX(0, 0, 0, 0)
	8412	682 PP=CMPLX(0, 0001, 0, 0)
	6413	RP=CSQRT(CFQBS*+2-SIGP)
Ć	8414	RS=CSQRT(SFQBS*+2-SIGS)
V.	8415	RP1=CFQBS-RP
	8416	RS1=SF 885-FS
	8417	PH=PP/RP1
(8413	PHDOT=-CFQ85+PH+PP
	8419	IF (VS. EQ. CMPLX(8. 0, 8. 0)) GO TO 681.
C	8428	SI=P5/RS1
\	0421	SMPR=(XLM+(B1-5IG5)/(4, *PI*XLM52)+PI*/52*XLM)*JAY/VP
	8422	NX=NLAYS-1
	6423	IF (MU(NX), NEQ. 0, 0) GO TO 601
(8424	NX=NLRYS-1
	9425	IF (MU(NX), NEQ. 0. B) GO TO 601
ſ	8426	PhDOT=-SYPR+SI
	8427	PH=PHDOT/(RP1-CFQBS)
	9423	PP=PH*RP1
~	8429	581 CONTINUE
(6430	IF(JPB, EQ. 8) GOTO 814
	8431	SID0T=-SF0BS*SI+PS
	6432	WRITE(LP, 990)
C	6433	990 FORMAT(//"PROPAGATING BASEMENT"/)
-		
	0434	218 FORMAT(/4F15.6)
Ċ	0435	624 FORMAT (//4F12. 6, //)
Υ.	6436	PHE=PH
	8437	SIB=SI
	6438	PPB=PP
Ċ	8439	
	6448	
Ċ	8441	
	8442	
	0443	WRITE(LP, 218) PH, PHDOT, SL, SIDOT
~	8444	814 IF (JPB, EQ. 1) GO TO 201
Ç	0445	C
		C RIGID POTTON INITIALIZATION
	6447	
(
	6448	
	8449	SI=CHPLX(8.0,0.0)
	0456	PP=CFQBS
Ć	8451	PH=CMPLX(1. 0. 0. 0)
	8452	
	8453	
(Ĩ		
	8454	
	6455	
,	6456	5 WRITE(LP, 210) PH, PHDOT
1	6457	? PHB=PH
	P459	ς αn ⊕punnt .

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••	· · · · ·	July (AV)	
	8459	SIB=SI	(
	0460	CCU=SIP0T	· · · ·
	8461	JK≓NLAY5	1
	8162	W=0	~
Ć	8463	NX=1	(
	8464	CALL EXEC(14, 103B, CFQX, 4, IF14, LMPX)	-
(0465	CALL EXEC (14, 103B, SFQX, 4, IF15, LMBX)	(
N .	8466	5F3=5F0X(1)	
	9467	CF3=CF0X(1)	f
r	6468	JK=JK-1	
C	8469	IF(JK, EQ. 1) 60 TO 303	
	047 9	GD TO 282	
	8471		i
4			(
		C CONFRESSIONAL POTENTIAL EXCITATION	
		C USE INTERFACE EQUATIONS TO OBTAIN POTENTIAL ADOVE INTERFACE	
(C Integrate Magnitude and Phase equations upward	1
I.	8475	C	,
	8476	281_JK=NLRY5	
,	6477	5I=CHPLX(0, 0, 0, 0)	(
	8478	SID0T=CMPLX(0, 0, 0, 0)	Ç
	8479	P5=C1PLX(0, 0, 0, 0)	
	0488		
(0FLL EXEC(14, 1838, CXB, 8, 1F12, L16X)	(
`	8481		
	0482	CALL EXEC(14, 103B, SXQ, S, IF13, LHAX)	
C	8483	SF1=SXQ(1)	(
Ç	8434	SF2=SX2(2)	
	8485	CF1=CXQ(1)	
	8486	CF2=CX9(2)	1
(6437	CRLL EXEC(14, 1938, CF6%, 4, IF14, LMR%)	:
	8438	CHL EXEC(14, 103B, FF0X, 4, IF15, LMRX)	
(8489	SF3=SF0X(1)	
	8498	CF3=CF0X(1)	
	8491	L=LMPX-1	
<i>(</i> -	8492	N5=1	Ć
Ç	8493	M9G≈0. 9	·.
	8494	HRITE(LP, 707)	
	8495	787 FORMAT(/*COMPRESSIONAL EXCITATION PH PHDOT, SI, SIDOT*/)	
ζ	8496	200 JK=JK-1	C .
	8497	IF(JK EQ. 1) GD TO 388	
(8498		(
<u> </u>	8499	NEXK+1	
	8588	CALL SSBC (ML VS, VP, SF1, SF2, CF1, CF2, MLN, NU, ALAAN, STG5, STG9,	
···.	8581	1Ph, Phoot, SI, Sidot, LP, An, XLAS)	(
1	8582	282 CONTINUE	
	8503	CALL, RXSV(SF3, CF3, LP, SIGS, SIGP, PH, PHDOT, SI, SIDOT, TH, THDOT,	
	8584	111 NEOT, N. NEOT, GA. GADOT)	1
(8585	IF(FU(ML), NED, 0, 8) GO TO 27	(
	8586	GADOT=()#PLX(8, 6, 6, 8)	
	8597	NDOT=CMPLX(0, 6, 6, 0)	
C			. (
	8598	GR=CMPLX(0, 6, 6, 0)	•
	6589	SIGS=CMPLX(0, 0, 0)	
1	8518	SI=CMPLX(0, 0, 0, 0)	(
	8511	SID0T=CHPLX(0, 0, 0, 0)	
	6512	N=CT\$LX(9, 0, 0, 0)	
,	8513	27 CONTINUE	
(8514	IF(JK NER (NLAYS-1)) 60 TO 413	ъ.
	8515	M1(1)=TH	
	8516	₩2(1)=TH-7	
(ς.
•	8517	42(1)=6A	-
	6518	144(1)=N-7	
(6519	D1=PH	(
ι.	6528	02=51	×.,
	8521	D3=PHDOT	
	8522	D4=51D0T	,
	8523	CALL EXEC(15, 1038, M1, 4, IF1, LMRX)	C
	9524	ppi Everya A30 U. A TEA HARV	

	44C.T	CHEL LALOYAN JOST MATH AT LENKY	
·	0525	CALL EXEC(15, 1038, M3, 4, IF3, LMAX)	(
	8526	CALL EXEC(15, 1838, W4, 4, IF4, LMRX)	
	8527	418 CONTINUE	
C	8523	FN=DN(JK)-DN(JK-1)	C
	8529 8529	OPLI SCSPH(SFOX, CFOX, LP, NS, NF, ZRS, DZ, NOZ, SIGP, SIGS, MJ, M2, M3,	
	6530 6534	1TH, THEOT, GR. GREDT, M. MEDT, N. NEDT, PH. PHEDT, PP. PPECT, SL. SIDOT,	
(6531 6532	2PS, PSD0T, D2S, FN, L, IF1, IF2, IF3, IF4, IF14, IF15, NAG)	C
•	6533	L1=L+1 IF(NF.NE.32) L1=L	
	8534	CRLL EXEC(14, 1038, CX9, 128, IF12, L1)	
C	8535	CALL EXEC(14, 1038, 5%) 123, IF13, L1)	
	8536	CALL EXECT (14, 103B, CF0X, 128, IF14, L1)	
	6537	CALL EXEC(14, 1038, SF0X, 128, IF15, L1)	r
C	6538	SF1=SX8(32+N5f1)	C
	8539	G2 = 500 (32 + 15+2)	
	8549	CT1=CX8(32-H5+1)	1
Ĉ	8541	CF2=CXQ(32-H5+2)	:
	8542	SF3=SF0X(32-H5+1)	
,	8543	CF3=CF0X(32-H6+1)	1
$\langle \cdot \rangle$	8544	IF (NS, NE, 1) NS=NS+1	(
	8545	66 GO TO 200	
<i>r</i> .	8546	380 CONTINUE	(
Ċ	8547	IF(NLRYS, EQ. 2) GO TO 301	ζ.
	6548	IF(MU(NLRYS), EQ. 8, 0) SI=CMPLX(0, 0, 0, 0)	
~	8549	C WRITE(LP, 210) MRG	(
Ç	6556	IF(MU(MLRYS), EQ. 0, 9) 60 TO 782	Ċ.
	8551	c	
C	8552	C SHEAR POTENTIAL EXCITATION	Ć
(C INTEGRATE MAGNITUDE AND PHASE EQUATIONS UPWARD	•.
	8554		
C	6555	WRITE(LP, 796)	(
•	8556	786 FORMAT(/*SHEAR EXCITATION-PH, PHDOT, SL SIDOT*/)	
	8557	SMPR=((SF2-5165)*XL1/(4. +P1*XL1/52)+P1*Y52*XL1)*(JRY/YP)	
(8558	CCR=PHDOT+SNPR+SI	(
	8559		
	8568	701 JK-HA RYS	
(8561	PH=CMPLX(0, 6, 0, 0)	ξ.
``	8562	PHD0T=C#PLX(0, 0, 0, 0)	
	8563	PP=C#PLX(0. 0. 0. 0)	
Ċ	8564 8565	SI=SIP SID0T=CCU	4
	8566	P2=P28	
	8567	r⊃-r⊃t ₩=:32	
(6563	CALL EXEC(14, 1938, CX0, 8, IF12, LMAX)	_ (
	6569	(RLL EXEC(14, 1838, SAR, S, IF13, LNFX)	
	6578	SF1=5XQ(1)	r
C	8571	SF2=5XQ(2)	C
	8572	(F1-CARI)	
,	0573	CF2=CXR(2)	ŕ
(8574	CALL EXEC(14, 1038, CF0X, 4, IF14, LMRX)	Ċ.
	8575	CPLL EXEC(14, 1038, SF0X 4, IF15, LM9X)	
<i>,</i> •	9575	SF3-SF0X(1)	(
Ç	8577	CF3=CF0X(1)	N.,
	6578	L=LMX-1.	
ť	8573	NS=1	<u> </u>
i	8588		`
	6581	IF(JPR. EQ. 0) GO TO 783	
ſ	8582		Ú,
(6583		•
	8584		
ć :	6585		C
N.	9586		
	6587		
ć	6588		Ć.
` .	9589 9599		

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022 H=N 025 TCL ENSIGNED_CLEP. SUBS. SUP. PM. PMOT. SL. SIDD. TA TMOT. 026 TCL ENSIGNED_CLEP. SUBS. SUP. PM. PMOT. SL. SIDD. TA TMOT. 027 DESTIGNED_CLEP. SUBS. SUP. PM. PMOT. SL. SIDD. TA TMOT. 027 DESTIGNED_CLEP. SUBS. SUP. PM. PMOT. SL. SIDD. TA TMOT. 027 DESTIGNED_CLEP. SUBS. SUP. PMOT. SL. SUP. SUBS. SUBS. SUP. SUBS. SUP. SUBS.				,	
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9532 PB-51071 9534 ORL, EPEC(15, 1928, M5, 4, 1F5, LH90) 9635 ORL, EPEC(15, 1928, M6, 4, 1F5, LH90) 9636 ORL, EPEC(15, 1928, M6, 4, 1F5, LH90) 9637 CRUE, EPEC(15, 1928, M6, 4, 1F5, LH90) 9638 784 ONTIME 9639 FH-FML(0, H0, K-T, LH90, DE, M5, M5, M5, M5, M7, OR, LS, M6, M0, THE, M0, M0, H0, DE, PP, M0, LS, S100, LS, M5, M5, M7, OR, EG, M0, LH0, MB, M4, MP, M0, PP, PPOT, S1, S100, DE, FM, LH5, LF5, LF5, H7, LF3, LF14, LF5, FMDS) 9632 LG+H4 9633 LG+H4 9634 IFOF ME, 20, L14, LF5, LF5, LF7, LF3, LF14, LF5, FMDS) 9632 CH, L DEC(14, 1928, S00, L23, LF13, L1) 9633 GRL, DEC(14, 1928, S00, L23, LF13, L1) 9634 IFOF ME, 20, L14, L15, L15, L10 9635 S1+500(24-M54) 9636 GRL, DEC(14, 1928, S00, L23, LF13, L1) 9637 GRL, DEC(14, 1928, S00, L23, LF14, L15) 9638 S2+500(22-M54) 9631 G2+500(22-M54) 9633 S2+500(22-M54) 9633 S2+500(22-M54) 9633 S2+500(22-M54) 9635 COMPUTE PALODIAL PED (MB-MTHE, L15, L15, L15, L15, L15, L15, L15, L15				í.	
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6616 OPLL DEC(45.1928, H0, 4, 1F2, LMRO) 9617 OPL DEC(45.1928, H0, 4, 1F2, LMRO) 8619 704 DONTINE 9621 IN-MALIO-TALLE, HS, INF, ZES, DZ, NDZ, S159, 5105, HS, HK, TZ 9621 11H, THOU, CH, MCN, LP, NS, NF, ZES, DZ, NDZ, S159, 5105, HS, HK, TZ 9622 11H, THOU, DS, PNL, LHS, INF, ZES, DZ, NDZ, S159, 5105, HS, INF, TS 9623 11H, THOUT, DS, PNL, LHS, INF, ZES, DZ, NDZ, S159, 5105, HS, JK, HK, TZ 9624 IFORF, ME, 32) L14, 9625 DELL DEC(14, 1038, CND, 128, IF12, L13) 9626 DELL DEC(14, 1038, CND, 128, IF14, L1) 9627 DELL DEC(14, 1038, CND, 128, IF14, L1) 9628 DELL DEC(14, 1038, CND, 128, IF14, L1) 9629 GEL DEC(14, 1038, CND, 128, IF14, L1) 9629 GEL DEC(14, 1038, CND, 128, IF14, L1) 9629 GEL SEC(14, 1038, CND, 128, IF14, L1) 9629 GEL SEC(14, 1028, CND, 128, IF14, L1) 9629 GEL SEC(14, 1038, CND, 128, IF14, L1) 9629 GEL SEC(14, 1038, CND, 128, IF14, L1) 9621 GEL SEC(14, 1028, CND, 128, IF14, L1) 9623 GEL SEC(14, 1028, CND, 128, IF14, L1) 9633 GEL SEC(14, 1028, CND, 128, IF14, L1) 9633 <td></td> <td></td> <td>CRL EXEC(15, 1038, NG, 4, IF6, LN9X)</td> <td>C</td> <td>l</td>			CRL EXEC(15, 1038, NG, 4, IF6, LN9X)	C	l
8618 784 CONTINUE 9621 PHENCIKO-TWO, CTWO, LP, MS, MF, 285, D2, MD2, SIGS, MS, MS, M7, 9621 11%, THOT, B0, GROTT, M, TEOT, M, MOT, PM, PHEOT, SL, SLUDT, 9622 List, M 9623 List, M, List, STF, IF7, IF7, IF8, IF44, IF25, IM65) 9624 IFOM, MS, 201, L25, THE, IF7, IF8, IF44, IF25, IM65) 9625 List, M, List, STF, IF7, IF8, IF44, IF25, IM65) 9626 CRL, DECCI44, 1028, CM0, 128, IF14, L10 9627 CRL, DECCI44, 1028, CM0, 128, IF14, L10 9628 CRL, DECCI44, 1028, CM0, 128, IF14, L10 9629 SF1-SR0(22-H54) 9621 CF4-CM2(22-H54) 9623 SF2-SF00(22-H54) 9623 SF2-SF00(22-H54) 9623 SF2-SF00(22-H54) 9623 SF2-SF00(22-H54) 9623 SF2-SF00(22-H54) 9623 SF2-SF00(22-H54) 9625 COMPINE ONLOWERT TO SATISFY TAWGENTIAL 9636 COMPINE ONLOWERT TO SATISFY TAWGENTIAL 9637 COMPINE ONLOWERT TO SATISFY TAWGENTIAL 9638 COMPINE ONLOWERT TO SATISFY TAWGENTIAL 9633 COMPINE ONLOWERT TO SATISFY TAWGENTIAL 9644 COMP	\mathcal{C}		CRLL EXEC(15,1032,47,4, IF7, LM8X)		
SEGS FMEMICIC-MORC-10 BEGS COLL SCEPH(SF02, CF02, LP, MS, NF, ZBS, D2, ND2, ST09, ST05, MS, MA, M7, BEGS COLL SCEPH(SF02, CF02, LP, MS, NF, ZBS, D2, ND2, ST09, ST05, MS, MA, M7, BEGS COLL SCEPH(SF02, CF02, LP, ML, IPS, IFS, IF7, IPA, IP14, IP15, MBS) BEGS COLL, DEC014, DBB, CM0, L20, IF12, L13) BEGS COLL SEC014, DBB, CM0, L20, IF12, L13) BEGS COLL, SEC014, DBB, CM0, L20, IF12, L13) BEGS CF1-CM2, C2+C5+C13) CE14, DBB, CM0, L20, L20, L20, L20, L20, L20, L20, L2		9617	CRLL EXEC(15, 1032, H8, 4, IF8, LNRX)		
9639 PH-PMC/02-PMC/CH-19, NS, NF, 25S, 02, N02, STEP, STES, NS, NS, NS, NS, NS, NS, NS, NS, NS, N		8618	704 CONTINUE	(
9621 11% THOT, BE, GHOT, M. 1907, M. 1907, FP. PP00T, SL SLDDT. 9622 25%, PD01, D25, PML IFS, IFG. IF7, IF8, IF14, IF15, MRSD 9624 IF06, NE, 32 L34. 9625 CH1, PE004, 1038, COL 128, IF12, L13 9626 CH1, DE004, 1038, COL 128, IF12, L13 9627 CH1, DE004, 1038, CF08, 128, IF13, L13 9628 CF1, DE004, 1038, CF08, 128, IF14, L13 9629 SF1-S00(2+664) 9620 CF1-D0002+664) 9621 CF2-S00(2+664) 9623 SF2-S00(2+664) 9633 SF2-S00(2+664) 9634 C72-CF00(2+664) 9635 SF2-S00(2+664) 9636 C72-CF00(2+664) 9637 722 C001TNE 9638 C72-CF00(2+664) 9639 C140001-2070400 9630 C140001-2070400 9643 C07+C0001-2070400 9644 C <td>(</td> <td>8619</td> <td>FN=DN(JK)-DN(JK-1)</td> <td></td> <td></td>	(8619	FN=DN(JK)-DN(JK-1)		
622 25, PSOL DZ, PKL L FS, IF6, IF7, IF8, IF14, IF15, IF85) 6623 List, H 6624 IFOR, NG, 32) L14, 6625 CRL DECC(4, 1038, CND, 128, IF13, L1) 6626 CRL DECC(4, 1038, CRD, 128, IF13, L1) 6627 CRL DECC(4, 1038, CRD, 128, IF13, L1) 6628 SF1-SOU(2+K1) 6629 SF1-SOU(2+K1) 6620 CF2-COR(2+K1) 6621 CF2-COR(2+K1) 6623 SF2-SOU(2+K1) 6633 SF2-SOU(2+K2) 6633 SF2-SOU(2+K54) 6633 C 6634 C COMPINE CRULINES DOUTIONS TO SATISFY TRADENTIAL 6635 IF (MURINFS-1) E 200 CONDITION 6646 KETTE(LP, 220 OR 6647 IF (MURINFS-1) E 0. B () OR COMPINAL 0. 8. 0 () 6648 MS-MERMULA-2-NK(1) 0. 0 () 6649 MS-MERMULA-2-NK(8628	CALL SCSPH(SF0%, CF0%, LP, NS, NF, ZBS, DZ, NDZ, SIGP, SIGS, MS, We, W7,		
6622 List4 6624 IF(NF, NE, S2) List. 6625 CRL EXEC(14, 1038, CN0, 123, IF12, L1) 6626 CRL EXEC(14, 1038, CN0, 123, IF12, L1) 6627 CRL EXEC(14, 1038, CN0, 123, IF13, L1) 6627 CRL EXEC(14, 1038, CN0, 123, IF13, L1) 6628 SF1-SN(12-4541) 6629 SF1-SN(12-4541) 6620 SF2-SN(12-4542) 6631 CF2-CN(12-4542) 6632 SF2-SN(12-4542) 6633 CF2-SN(12-4542) 6634 CF3-CFDX(12-4542) 6635 IF(NS, NE, 1), NS-MSC41 6636 GF1 NS-SNE44 6637 C22 CONTINE 6638 C 6637 C2 CONTINE 6638 C 6639 C 6630 C 6631 C120-HS-HMOHT-SIENE CONDITIONS TO SATISFY TANGENTIAL 6640 C 6641 C 6642 METECHEN-SED NEW TH = ZERD CONDITION 6643 COMPENDATISKA & 0 6644 IF(MICHANS-SED R. 0 6645 IF(MICHANS-SED R. 0 <		9621	1TH, THDOT, GR, GRDOT, M, MDOT, N, MDOT, PH, PHDOT, PP, PPDOT, SL, SIDOT,	< c	
6624 IFOF.NE 32) L34. (6625 CML EXEC(34, 1008, CMD, 128, IF12, L3) (6627 CPL EXEC(34, 1008, CPM, 128, IF13, L3) (6627 CPL EXEC(34, 1008, CPM, 128, IF13, L3) (6628 CPL EXEC(34, 1008, SPE, 128, IF13, L3) (6629 SF1-SPM(124-154) (6621 CF1-CM(124-1542) (6633 SF2-SPM(124-1542) (6634 CF3-CPM(124-1542) (6635 IF (NK, NE 1) NS-MS41 (6636 GF3-SPM(124-1542) (6637 CF3-CPM(124-1542) (6638 CF3-CPM(124-1542) (6637 CF3-CPM(124-1542) (6636 CF3-CPM(124-1542) (6637 CB CONTINUE (6638 C CONTINUE (6637 CB CONTINUE (6638 C CONTINUE (6639 C CONTINUE (6631 C CONTINUE (6632 C CONTINUE (6633 <td< td=""><td>(</td><td>8522</td><td>295, PSD0T, D25, FN, L. 1F5, 1F6, 1F7, 1F8, 1F14, 1F15, MPGS)</td><td></td><td></td></td<>	(8522	295, PSD0T, D25, FN, L. 1F5, 1F6, 1F7, 1F8, 1F14, 1F15, MPGS)		
9625 CR1_EPEC(4, 1038, CS0, 123, IF12, L1) 9626 CR1_EPEC(4, 1038, CS0, 123, IF13, L1) 9627 CR1_EPEC(4, 1038, CF0, 123, IF13, L1) 9628 CR1_EPEC(4, 1038, CF0, 123, IF13, L1) 9629 SF1-SR(12+K41) 9631 CF1-CR(12+K41) 9652 CF2-SR(12+K41) 9653 SF2-SR(12+K41) 9654 CF3-CF0R(12+K41) 9655 IF(0K, NE.1) NS=K41 9656 GD 10 769 9657 CR2 CMTINE 9658 C 9656 GD 10 769 9657 CR2 CMTINE 9658 C 9659 COMPLIE CRULATED SOLUTIONS TO SATISFY INNERVIAL 9657 CR2 CMTINE 9658 C 9659 COMPLIE CRULATED SOLUTIONS TO SATISFY INNERVIAL 9640 C STRESS RT WHTER-SEDIMENT = ZERO CONDITION 9641 C 9642 C 9644 IF(MUNLAMS-1): EQ.8, 0: DOR=CMPLX(0, 8, 8, 0) 9645 IF(MUNLAMS): EQ.8, 0: DOR=CMPLX(0, 8, 8, 0) 9646 METEUP.20: DOR 9647 MED=MEGA7		9 623			
0025 CH1 DEC(14, 1038, S00, 123, IF13, L1) 0627 CH1 DEC(14, 1038, S700, 123, IF14, L1) 0628 CP1 DEC(14, 1038, S700, 123, IF14, L1) 0629 SF1-SN(12-N541) 0630 0621 CF2-CN(12-N541) 0631 0622 CF2-CN(12-N541) 0631 0633 SF3-SF0X(12-N541) 074 0634 CF2-CN(12-N541) 074 0635 IF(NS NE 1) NS=NS41 074 0636 C COMPUNE 0010 NS 0637 722 CONTINE 074 0638 C COMPUNE 0010 NS 0633 C 0010 768 074 0635 IF(NS NE 1) NS=NS41 075 0760 0636 C COMPUNE 0010 NS 0100 NS 0643 C COMPUNE 0000 NS 0000 NS 0644 C MSTESS AT WHITE-SED IMENT = ZERO CONDITION 080 08643 002-COMPUNE NS 0644 IF(NUUNAWS-3): ED, 0, 0) 002-COMPUNE NS 0000 0000 0000 0644 IF(NUUNAWS-3): ED, 0, 0)<		8624		(
0627 CPLL DEC(14, 1038, CF0X, 128, IF14, 11) 0628 CPLL DEC(14, 1038, CF0X, 128, IF15, 11) 0629 SF1-S0X(12+15+1) 0629 SF1-S0X(12+15+1) 0621 CF2-C0X(12+15+2) 0633 SF2-SF0X(12+15+1) 0634 CF2-CFX(12+15+1) 0635 IF10KS, NE1) NS=NS+1 0636 CF2-CFX(12+15+1) 0637 T22-CFX(12+15+1) 0638 CF2-CFX(12+15+1) 0637 T22-CFX(12+15+1) 0638 CF2-CFX(12+15+1) 0637 T22-CFX(12+15+1) 0638 C 0637 T22-CCNTINE 0638 C 0637 C22-CCNTINE 0638 C 0637 C22-CCNTINE 0648 C 0649 C STRESS AT WHTER-SEDIMENT = ZERO CONDITION 0644 C 0645 C 0646 C NETTER-SEDIMENT = ZERO CONDITION 0647 D22-CONC/(PHD0T+STR=SEDMENDS) 0648 MG=MEND(1495, BE) 0649 MG=MEND(1495, BE) 0644	(8625	CRLL EXEC(14, 1038, CXQ, 128, IF12, 1.1)		
8528 CPLL EXEC(14.1028.5F0%.123.1F15.L1) 8629 SF1=SR0(22+16+2) 6638 SF2=SR0(22+16+2) 6631 CF1=CR0(22+16+2) 8633 SF3=SF0%(22+16+1) 8653 CF2=CR0(22+16+2) 8653 SF3=SF0%(22+16+1) 8653 CF2=CR0(22+16+1) 8653 CF2=CR0(22+16+1) 8653 CF3=CF0%(22+16+1) 8653 CF3=CF0%(22+16+1) 8653 CF3=CF0%(22+16+1) 8653 C 8654 C 8655 L1000, PMP DMPT, SL, SIDDT, SMPR, COR 8644 TF(MUNAPS-1): ED 8, 0) COR=CMPLX(0, 0, 0, 0) 8645 IF(MUNAPS-1): ED 9, 0) COR=CMPLX(0, 0, 0, 0) 8646 MSG=MENCHSC 8647 212 FORMAL PS-1, 50 8648 MSG=MENCHSC 8649 MSG=MENCHSC 8653 J=000000000000000000000000000000000000		8626	CALL EXEC(14, 103B, 5X0, 129, IF13, L1)		÷
BC28 SF1-SXR(2+R54) BC38 SF2-SXR(2+R54) BC31 CF2-CXR(22+R54) BC32 CF2-CXR(22+R54) BC33 SF3-SF0X(32+R54) BC34 CF3-CFRX(22+R54) BC35 IF (KK, NE, 1) KS-N54 BC36 GD T0 789 BC37 722 CONTINUE BC38 C BC37 722 CONTINUE BC38 C BC37 722 CONTINUE BC38 C BC37 C2 CONTINUE BC38 C BC39 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL BC48 C BC39 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL BC49 C BC41 C BC41 C BC42 C BC43 COR-CORV(PHOT-SATEX-DOR BC44 IF (MU(MAPS-SE DE 0, 0) COR=CATEX(1, 0, 0, 0) BC44 IF (MU(MAPS-SE DE 0, 0) COR=CATEX(1, 0, 0, 0) BC44 IF (MU(MAPS-SE DE 0, 0) COR=CATEX(1, 0, 0, 0) BC44 IF (MU(MAPS-SE DE 0, 0) COR=CATEX(1, 0, 0, 0)	r	6627			
 	Ç	8628			
6531 CF1=CX8(32-H541) 6672 CF2=CX8(32-H542) 6633 SF3=SF0X(32-H541) 6634 CF2=CF0X(32-H541) 6635 IF(NS, NE, 1) NS=NS41 6636 G9 T0 700 6637 722 CONTINUE 6638 C 6639 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL 6639 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL 6640 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL 6641 C 6642 C MRITE(P, 210) PH, PHOOT, SL SIDOT, SMPR COR 6643 COR=COR/(PHODT=SIPR+SI)=PEP/MRG=MADS) 6644 IF (MU(NLAYS-1): EQ & 0) OBC=COFFLX(0 & 0. 0) 6645 IF (MU(NLAYS-1): EQ & 0) OBC=COFFLX(0 & 0. 0) 6646 WRITE(P, 212) COR 6647 212 FORMAT(2F15 & 0) 6648 MT0=MAU(1) +2=DR(1) 6659 NI=DN(NL)+2=DR(1) 6651 F(RZ, ED, 0, 0; NL=HL-1) 6652 P2=DN(1)-JR 6653 IF (RZ, ED, 0, 0; NL=HL-1) 6655 I=LHPX					
0031 012000(22005) 0633 SF3=SF0X(32+05+1) 0633 SF3=SF0X(32+05+1) 0633 SF3=SF0X(32+05+1) 0635 IF(NS, NE, 1) NS=NS+1 0636 GD TO 709 0637 702 CONTINUE 0638 C 0637 702 CONTINUE 0638 C 0639 C COMPLIXE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL 0640 C STRESS AT WATER-SEDIMENT = ZERO CONDITION 0641 C 0642 C 0644 C COMPLIXE TO PH. PHOOT, SI, SIDOT, SMPR. COR 0644 IF(MU(NLAYS-1), EQ. 0.0) COR=CMPLX(1.0.0.0.0) 0644 IF(MU(NLAYS-1), EQ. 0.0) COR=CMPLX(1.0.0.0.0.0) 0645 IF(MU(NLAYS-1), EQ. 0.0) COR=CMPLX(1.0.0.0.0.0) 0646 WRITE(1P, 212) COR 0647 212 FORENT(2F15.0.0) 0648 MS=DN(NL)+2-DN(1) 0649 MS=DN(NL)+2-DN(1) 0641 MS=DN(NL)+2-DN(1) 0652 P2=DN(1)-JR 0653 IF(AZ ED.0.0) NL=HL-1 0654 R2=1.0-AZ 0655 I=L=MSX	C				
0633 SF3=SFDX(32=NS+1) 0634 CF3=CFDX(32=NS+1) 0635 IF(NS, NE, 1) NS=NS+1 0636 G0 T0 760 0637 762 CONTINUE 0638 C 0639 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL 0638 C 0649 C STRESS AT WATER-SEDIMENT = ZERO CONDITION 06410 C 0642 C 0643 COR=-CORX(HDDT-SIDENPERSD) 0644 IF(MUNUARS-1), EQ. 0.00R=CMPLX(1.0, 0.0) 0644 IF(MUNUARS-1), EQ. 0.00R=CMPLX(1.0, 0.0) 0645 IF(MUNUARS-1), EQ. 0.00R=CMPLX(1.0, 0.0) 0646 WEITE(IP, 212) COR 0647 212 FORMAT(2F15.8) 0648 MS=MMEXI(MED, HMOS) 0649 MS=MMEXI(MED, HMOS) 0651 JR=MI(1) 0652 62-0N(1)-JR 0653 IF(RZ ED.0, 0.0) NL=NL=1 0653 IF(RZ ED.0, 0.0) NL=NL=1 0655 I=(JMX)	C				
0634 CF3=CF0X(72=4541) 0635 IFOXS NE 1) NS=NS+4 0636 G0 T0 760 0637 722 CONTINUE 0638 C 0639 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL 0640 C STRESS AT WATER-SEDIMENT = ZERO CONDITION 0644 C 9642 C 9643 COR=-COR/(PHODT-STPRAST) MEXP(MBC-MBGS) 9644 IF(MU(NLAYS-LE 0, 0) COR=CMPLX(1, 0, 0, 0) 9644 IF(MU(NLAYS-LE 0, 0) COR=CMPLX(0, 0, 0) 9644 IF(MU(NLAYS-LE 0, 0) COR=CMPLX(0, 0, 0, 0) 9645 IF(MU(NLAYS-LE 0, 0) COR=CMPLX(0, 0, 0, 0) 9646 WRITE(IP, 212) COR 9647 212 FORMAT(2F15, 8) 9648 MPG=MPR0-7 9659 N=DN(UL)+2-DN(L) 9651 JR=DN(L) 9653 IF(AZ, ED, 0, 0) NL=HL-1 9653 IF(AZ, ED, 0, 0) NL=HL-1 9653 IF(AZ, ED, 0, 0) NL=HL-1 9654 AZ=L 0:AZ 9655 IL=MPX					
0031 0131 0131 0131 0035 0131 0131 0131 0036 0131 0131 0131 0037 782 0011104 0131 0037 782 0011104 0131 0037 782 0011104 0131 0037 782 0011104 0131 0037 782 0011104 0131 0037 782 0011104 0131 0037 782 0011104 0131 0037 782 0011104 0131 0041 01114 0141 0141 0042 011104 0141 0101 0044 011100 0101 0101 0045 0111 0101 0101 0046 WRITE(1P, 212) 008 0101 <td>C</td> <td></td> <td></td> <td>C</td> <td></td>	C			C	
0633 06 TO 700 (0637 722 CONTINUE (0638 C (0639 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL (0640 C STRESS AT WATER-SEDIMENT = ZERO CONDITION (0641 C (0642 C NRITE(LP, 210) PH, PHDOT, SL SIDOT, SNPR, COR 0643 COR=-CDR/(PHDOT+SIPR*SI)*EXP (MHG-MRGS) 0644 IF (MUKNLAYS-1), EQ. 8, 0) COR=CMPLX(1, 0, 0, 0) 0644 IF (MUKNLAYS-1), EQ. 8, 0) COR=CMPLX(2, 0, 0, 0) 0645 IF (MUKNLAYS-1), EQ. 8, 0) COR=CMPLX(2, 0, 0, 0) 0646 WRITE(LP, 212) COR 0647 212 FORMAT(2F15, 8) 0648 MHG=MMMCALMED, MHOS) 0649 MHG=MMMCALMED, MHOS) 0651 JR=MA(1) 0652 PZ=DM(1)-JR 0653 IF (AZ, ED. 0, 0) NL=ML=1 0653 IF (AZ, ED. 0, 0) NL=ML=1 0655 L=LMX	(
6637 782 CONTINUE 0638 C 0639 C 0639 C 0640 C 0641 C 0642 C 0643 CORF-COR/CPHOOT, SL, SIDOT, SNPR, COR 0644 C 0645 CORF-COR/CPHOOT+SLYEENP(MBC-MBOS) 0644 IF(MU(NLAYS-1), EQ, 0, 0) CORFCMPLX(1, 0, 0, 0) 0644 IF(MU(NLAYS-1), EQ, 0, 0) CORFCMPLX(1, 0, 0, 0) 0645 IF(MU(NLAYS), EQ, 0, 0) CORFCMPLX(1, 0, 0, 0) 0646 WRITE(LP, 212) COR 0647 212 FORMAT(2F15, 8) 0648 MBG=MBQL(MEG, MBOS) 0649 MBG=MBQL(MEG, MBOS) 0649 MBG=MBQL(MEG, MBOS) 0651 JR=DN(1) 0652 R2=DN(1)-JR 0653 IF(MZ, EQ, 0, 0) NL=H1-1 0655 L=LMBX					1
0633 C 0633 C 0634 C 0635 C 0640 C 0541 C 0642 C 0643 COR=-COR/(PHDOT+SIPR+SI)+EXP(PHO-PHAGS) 0644 IF (PMU(NLAYS-1), EQ, 0, 0) COR=CMPLX(1, 0, 0, 0) 0645 IF (PMU(NLAYS), EQ, 0, 0) COR=CMPLX(1, 0, 0, 0) 0646 WRITE(I.P. 212) COR 0647 212 FORMAT(2F15, 8) 0648 MPG=MPMX(MPG-PHAGS) 0649 MPG=MPMX(MPG-PHAGS) 0651 JR=DN(1) 0652 P2=DN(1)-JR 0653 IF (PLZ ER, 0, 0) NL=MI-1 0655 L=LPRX	r			(
0639 C COMPINE CALCULATED SOLUTIONS TO SATISFY TANGENTIAL. 0648 C STRESS AT WATER-SEDIMENT = ZERO CONDITION 0641 C 0642 C WRITE(LP, 210) PH, PHDOT, SI, SIDOT, SMPR, COR 0643 COR=-COR/(PHDOT+SMPR*SI)*EXP(MAG-MAGS) 0644 IF (MU(NLAYS-1), EQ, 0, 0) COR=CMPLX(1, 0, 0, 0) 0645 IF (MU(NLAYS-1), EQ, 0, 0) COR=CMPLX(1, 0, 0, 0) 0646 WRITE(LP, 212) COR 0647 212 FORMAT(2F15, 8) 0648 MAGE-MAGS) 0649 MAGE-MAGS) 0651 JR=DN(1) 0652 P2=DN(1)-JR 0653 IF (R2, EQ, 0, 0) N1=M1=1 0654 R2=L 0=R2 0655 L=LMRX	`				
0640 C STRESS AT WATER-SEDIMENT = ZERO CONDITION 0641 C 0642 C WRITE(LP, 210) PH, PHDOT, SI, SIDOT, SNPR, COR 0643 COR=-COR/(PHDOT+SNPR*SI)*EXP(MAG-MAGS) 0644 IF (MU(NLAYS), EQ, 0, 0) COR=CMPLX(1, 0, 0, 0) 0645 IF (MU(NLAYS), EQ, 0, 0) COR=CMPLX(0, 0, 0, 0) 0646 WRITE(LP, 212) COR 0647 212 FORMAT(2F15, 8) 0648 MAGE=MAMSI-7 0659 NI=DN(1) 0651 JR=DN(1) 0652 P2=DN(1)-JR 0653 IF (RZ, EQ, 0, 0) NI=MI-1 0654 R2=L 0-RZ 0655 L=LMRX			U		
9641 C 9642 C WRITE(LP, 210) PH, PHDOT, SL, SIDDT, SNPR, COR 9643 COR=-COR/(PHDOT+STMPR*SI)*EXP(MHC-THGOS) 9644 IF (MU(NLAYS-1), EQ, 0, 0) COR=CMPLX(1, 0, 0, 0) 9645 IF (MU(NLAYS, EQ, 0, 0) COR=CMPLX(0, 0, 0, 0) 9646 WRITE(LP, 212) COR 9647 212 FORMATICPHS, R005) 9648 M96=MMM2(MMG, MHOS) 9649 M96=MMM2(MMG, MHOS) 9649 M96=MMM2(MHS, MHOS) 9651 JR=DN(1) 9652 MI=DN(1)-JR 9653 IF (RZ, EQ, 0, 0) NI=MI-1 9655 L=LMPX	(8639	C CUTENE CALUCIANED SOLUTIONS TO SATISFY PROCEDURE.	. `	
9642 C WRITE(LP, 210) PH, PHDOT, SJ, SIDOT, SMPR, COR 9643 COR=-COR/(PHDOT+SMPR+SI)>HEXP(MHG-HPGGS) 9644 IF (MU(N,AYS-1), EQ, 0, 0) COR=CMPLX(1, 0, 0, 0) 9645 IF (MU(N,AYS), EQ, 0, 0) COR=CMPLX(2, 0, 0, 0) 9646 WRITE(LP, 212) COR 9647 212 FORMATICE/NS, 80 9648 MHG=MMM2(MHG, MHGS) 9649 MHS=MM21(MHG, MHGS) 9649 MHS=MM21(MHG, MHGS) 9651 JR=DN(1) 9652 P2=DN(1)-JR 9653 IF (RZ, EQ, 0, 0) 9655 L=LMX					
9643 COR=-COR/(PHDOT+STMPR*SI)*EXP(MRC-MRGS) 9644 IF(MU(NLAYS-1), EQ. 0, 0) COR=CMPLX(1, 0, 0, 0) 9645 IF(MU(NLAYS), EQ. 0, 0) COR=CMPLX(0, 0, 0, 0) 9646 WRITE(LP, 212) COR 9647 212 FORMATICENS, 8) 9648 MAG=AMMOX(MAG, MAGS) 9649 MAG=AMMOX(MAG, MAGS) 9649 MAG=AMMOX(MAG, MAGS) 9651 JAH-DN(1) 9652 P2=DN(1)-JR 9653 IF(RZ, EQ. 0, 0) NI=N1=1 9654 R2=1.0-HZ 9655 L=1.MRX					
9644 IF (MU(NLAYS-1), EQ. 0, 0) COR=CMPLX(1, 0, 0, 0) 9645 IF (MU(NLAYS), EQ. 0, 0) COR=CMPLX(0, 0, 0, 0) 9646 WRITE(LP, 212) COR 9647 212 FORMAT(2F15, 8) 9648 MAG=RMPX1(MAG, MAGS) 9649 MAG=RMPX1(MAG, MAGS) 9649 MAG=RMPX1(MAG, MAGS) 9651 JR=DN(NL)+2-DN(1) 9652 P2=DN(1)-JR 9653 IF (RZ, EQ. 0, 0) NI=N1=1 9654 R2=L 0=HZ 9655 L=LMPX	6			`	
#645 IF (MU(MLAYS): EQ. 9, 8) COR=CMPLX(0, 0, 0, 0) #646 WRITE(LP, 212) COR #647 212 FORMAT(2FA5, 8) #648 MAGENMAX(MAG, MADS) #649 MAGENMAX(MAG, MADS) #649 MAGENMAX(MAG, MADS) #649 MAGENMAX(MAG, MADS) #649 MAGENMAX-7 #653 NI=DN(ML)+2-DN(1) #651 JR=DN(1) #652 P2=DN(1)-JR #653 IF (RZ, EQ, 0, 0) MI=M1=1 #6554 RZ=1, 0-RZ #6555 L=1.MRX			UKE-UUKY(TRUTTSTRYS) THEN (TRUTTSTRYS)		ļ
B646 WRITE(LP, 212) COR B647 212 FORMAT(2F15.8) B648 MAGE-AMPX1(MAG. MAGES) B649 MAGE-AMPX1(MAG. MAGES) B659 M1=0N(NL)+2-DH(1) B651 JR=DN(1) B652 P2=DH(1)-JR B653 IF(R2, EQ, 0, 0) N1=H1=1 B655 L=L MPX					
6647 212 FORMAT (2F15. 8) 0648 MASS-AMPX2 (1496. HADS) 0649 MASS-AMPX2 (1496. HADS) 0659 MASSA 0651 JAR-DN(1) 0652 A2-DN(1)-JR 0653 IF (RZ, ED, 0, 0) N1=H1-1 0655 L-1. MAX	(· ·	1
0648 M96=81901(N96, M865) 0649 M99=N93-7 0659 N1=0N(NL)+2=DN(1) 0651 JR=0N(1) 0652 P2=0N(1)-JR 0653 IF(RZ, 50, 0, 0) NL=NL-1 0655 L=LIPX	``				
0643 M99-M99-7 0553 N1=0N(NL)+2-DH(1) 0651 JR=DH(1) 0652 P2=DH(1)-JR 0653 IF(RZ_ER_0_0) N1=N1=1 0654 R2=1.0-AZ 0655 L=1.M9X					
0013 01-01(1) 0653 N1-0N(NL)+2-0H(1) 0651 JR=0H(1) 0652 R2=0H(1)-JR 0653 IF(RZ_ER_0.0) N1=H1=1 06554 R2=1.0=HZ 06555 L=1.09X	((
0651 JR=DM(1) (0652 62=0N(1)-JR (0653 IF(6Z, EQ, 0, 0) N1=N1=1 (0654 62=1, 0=42 (0655 L=1.199X (~				ł
0 6652 62=0N(1)-JR 6653 IF(AZ_ER_0_0) N1=N1=1 06534 A2=1_0-A2 06555 L=1.199X				(
6653 IF(AZ_EQ_0_0) N1=N1=1 6654 AZ=1_0=AZ 6655 L=1_MAX	(ζ.	
θ6554 f2=1. θ−f2 () θ6555 L-1.1#9X ()	-				
96555 L-1.199X				(
				<u> </u>	

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2	8657	NF=32	(
	0658	DO 759 I=1. N1	(
	8659	CALL EXEC(14, 183B) MJ 128, IF1, L)	
$\boldsymbol{\zeta}$	8668	CRLL EXEC(14, 1838, W2, 128, IF2, L)	C
ι,	8661	CRLL EXEC(14, 1038) W3, 128, IF3, L)	,
	0662	CALL EXEC(14, 183B, W4, 128, IF4, L)	
r	9663	CALL EXEC(14, 103B, W5, 128, IF5, L)	
(8664	CALL EXEC(14, 1838, N5, 128, IF5, L)	
	8665	CPLL EXEC(14, 103B, N7, 128, IF7, L)	
~	8666	CALL EXEC(14, 103B, WS, 128, IF8, L)	
(6667	CALL EXEC(14, 1938, CFRX, 128, IF14, L)	•
	0668	CRL1_EXEC(14, 1038, SF0X, 129, IF15, L)	
	8669	D0 705 J-NS, NE	(
ť	6678	NX=33-J	(
	8671	XRF(NX)=FLORT(NX-1+L+32)+ZBS/FLORT(NDZ)	
	6672	B1=CC05(H1 (N2))	<i>r</i> -
-	8673	B2+CSIN(W1(NX))	C
	8674	B3=ECOS(HE(HEQ))	
	8675	IF(SF(X(1X), EQ, CMPLX(0, 0, 0, 0)) B3=CMPLX(0, 0, 0, 0)	-
(8676	BI-CSIN(H3(NN))	(
	6677	YE (N)=81+0EXP(H2(NX)=#86)	
	8673	ZEF (HX)=R3*CEXP(W4(NX)=M9G)	
(8679	L2(10)=(-CF0)(10)+R1+R2)+CE2P(112(10)-1496)	(
	8688	H4(NX)=(-SF(X)(NX)+R3+B4)*(CEXP(H4(NX)-HRG)	
	8681	IF (MICNLRYS), EQ. 8, 8) 60 TO 755	_
C	8682	B5=CC05(45(NX))	(
	8683	BS=CSIN(VS(NX))	
Ċ	8684	87=CC05(47(NX))	ĺ.
	8685	IF (SFRX(NX), ER, CMPLX(0, 0, 0, 0)) B7=CMPLX(0, 0, 0, 0)	
	8686	88=CSIN(47(NX))	
(0637	YBF(NX)=YRF(NX)+CCR+85+CEXP(N6(NX)-MRG)	ξ.
•	6638	2BF(1EX)=2BF(1EX)+C0R+B7+CEXP(48(1EX)-HB6)	
	6689	W2(NX)=W2(NX)+(-CF2X(NX)+85+86)+C0R*CEXP(W6(NX)-M96)	
(6698	M4(NX)=M4(NX)+(-SF0X(NX)+87+B8)+C0R+CEXP(M8(NX)-MBG)	(
٠.	8691	755 CONTINUE	
	8692	IF(L NEQ LIMAX) 60 TO 756	
(8693	YBF(1)=(D1+CQR+D5)*EYP(-H9G-7)	(.
` .	9694	D1=YEF(1)	
	8695	ZBF(1)=(D2+CD2+CD5)+EXP(-MAG-7)	
(8696	D2=ZPF(1)	
(6697	W3(1)=(D3+COR+D7)+EXP(-MAG-7)	
	8698	D3=#3(1)	
C	8699	H4(1)=(D4+C0R+D8)+EXP(-HAG-7)	(
Ç	8799	D4=W4(1)	`
	8701	756 CONTINUE	
Ċ	8782	IF(NX. NE. 1) GO TO 705	C
(0703	CRL1_EXEC(15,1038, YEF, 128, IF18, L)	`
	8784	CALL EXEC(15, 1838, ZBF, 128, IF11, L)	
C	8785		ſ
C	8786	751 (F(153H(6)) 760,790	ν.
	0707	768 00 789 JY=NS, NF, NP	
1	6769	JX=33~JY	(
(, j	6789	789 URITE(LP, 715) MBF(JX), VBF(JX), ZBF(JX)	
	6718	715 FORMAT(F5. 1, 4F15. 8)	
C	8711	729 NS=1	
C.	0712	桥=32	ι,
	8713	IF(I.EQ. (NL-1), RND, RZ, NE, 1, 0) HF=R2*32	
	8714	795 CONTINE	
Ç	8715	IF(NF, LE, 0) GO TO 752	
	0716	750 CONTINUE	
1.	8717	752 CONTINUE	
(<u> </u>	0718	JF(ISSW(6)) 589, 583	۲.
	8719	589 IF(NF, LT, 32) NRITE(LP, 715)(XBF(33-JY), YBF(33-JY),	
	0728	128F(33-JY), JY=16, NF, NP)	, I
•	0721	568 PH-78F (NX)	Ų,
	A722	CI=7PC(MY)	

	9723		í
	8724		
	8725	Y2=MU(2)*(PHEOT+SMPR+SI)	
(6726) KRITE(LP, 238) PH, PHDOT, SI, SIDOT, Y2, SNPR NS=NF+2	(
•	8727 9728	10=10=12 19=32	
	0728 0729	:er=s∠ NX=4X=1	
(0738	788 FORMAT(F4, 6, 2%, 8F8, 4)	
•	0738	IF (AZ, NE, C, O) CALL EXEC(15, 1038, Y8F, 128, IF18, L)	
	0732	IF (AZ, NE, O, O) CALL, EXEC(15, 1039, 785, 120, IF 10, E)	
{	0733	IF (NS. EQ. 34) NS=2	
	0734		
		. Integrate magnitude and phase equations in the hater column	
Ċ	0736	4.	C
	8737	301 CONTINUE	
-	0738	IF (16, EQ, 0) 18-1	
()	8739	L=DN(1)-1	
	0740	N5=N5=1	
,	0741	IF (K5, NE, 1) L=L+1	(
(0742	IF(N5, NE, 1) NX=NX+1	(
	0743	Ni=DN(1)	
~	8744	IF (NLRYS, NEQ. 2) GO TO 721	(
(8745	NS=1	C
	8746		
6	0747	N%=1	C
(0748	NL=LMPX	C
	8749	L-LBAX-1.	
1	8750	SI=5IB	
(8751	SIDOT=CCU	
	8752	721 CALL EXEC(14, 103B, C30, 128, IF12, N1)	
C	8753	CALL EXEC(14, 1038, SX0, 128, IF13, NL)	
,	8754	ORLL EXEC(14, 1838, CF0%, 128, IF14, NL)	
	0755	CALL EXEC(14, 1038, SF2X, 128, IF15, NL)	
Ć	8756	728 SF2=Sx8(Nx+1)	(
	8757	CF1=CXQ(NX)	`
	8758		
Ċ	8759 0760	CALL SNEC (VS. VP. SF2, CF1, CF2, XLN, MU, ALAN, LP, SIGP,	Ę.
	8768 8764	15105, PH, PHDOT, SL, SIDOT, XLMS)	
	0761 0762	383 CONTINUE	
(8763	SF3=SF8X(NX) CF3=CF8X(NX)	
	8764	SI=CMPLX(0, 0, 0)	
	0765	SID01=(1)PLX(0, 0, 0, 0)	
Ċ	0766	PS=(NPLX(0, 0, 0, 0)	(
	8757	PSD0T=CMP1X(0, 6, 6, 0)	
~	0768	IF(ULAYS NEC 2) 60 TO 453	ŕ
Ċ	0769	DI=PH+EXP(-7, 0)	(
	0770	D2=SI	
<i>,</i> -	8771	D3=PHD0T+EXP(-7, 0)	
(8772	D4=SIDOT	• •
	0773	453 CALL RNSV(SF3, CF3, LP, SIGS, SIGP, PH, PHD0T, SI, SIDOT, TH, THD0T,	
ç	8774	1/M, MDOT, NJ, NDOT, GA, GADOT)	
•	0775	GADOT=C1PLX(0, 0, 0, 0)	
	0776	GA=CMPLX(0, 0, 0, 0)	
ć	8777	454 CONTINUE	
í.	6778	IF (NLAYS, GT. 2) M=M-MAG+7	
	6779	IF (NLAYS GT. 2) N=N-MAG+7	
(0780	MAGE=0.0	
ς.	6731	MACE=MAC+7	
	6782	JR=DN(1)	
(0783		
`	0784 0705	SIGS=CMPLX(0, 0, 0, 0)	
	8785 0700		
	0736 0727	CALL SCSPH(SF02), CF02, LP, NS, NF, 205, DZ, NDZ, SIGP, SIGS, NL, NZ, NZ, 17H, TH20T, GR, GROOT, N, MDOT, N, MDOT, PH, PHDOT, PP, PPDOT, SI, SIDOT,	, N
	8787 9709	115 1500 / 05 0500 / 10 120 / 10 120 120 120 120 120 120 120 120 120	

	0100	SUDAL POOLS ASSAUDED TO TE TO TE SUITE SUITE SUITE TO TRAVELOUS
	0789	SI=CMPLX(0,0,0,0)
	8790	CPLL. EXEC(14, 103B, YP, 128, IF9, 0)
	8791	CCL=YP(1)/(CCO5(M1(1))*CEXP(W2(1)))
~	8792	WRITE(LP, 212) CCL
(8793	211 FORMAT (/F15, 2, 2% F15, 2)
	6794	4RITE(LP, 800)
	0795	800 FORMAT(/"TOTAL SOLUTION-PH SI"/)
(8795	
	0797	C OBTAIN TOTAL SOLUTION
	9798	C C C C C C C C C C C C C C C C C C C
C	8799	
		DO 468 I=1 LMAX
	0990	
(0801	CALL EXEC(14, 1838, YP, 128, IF9, M1)
	6862	CFLL EXEC(14, 1038, YRF, 128, IF18, M1)
	6803	CRLL EXEC(14, 1038, ZRF, 128, IF11, N1)
(8884	00 70 J=1,32
	8885	YEF(J)=YP(J)-CCL*YEF(J)
	3896	XPF(J)=FLOAT(J-1+N1+32)+ZB5/FLOAT(NDZ)
	6867	ZBF(J) = -CCL * ZBF(J)
	8888	YEF(J)=YEF(J)*NDZ/ZES
	8869	ZRF(J)=ZEF(J)+40Z/ZE5
1	6810	155 FORMAT(F6, 2, 4F15, 5)
Ç	6811	70 CONTINUE
	0212	CHLL EXEC(15, 1038, YBF, 128, IF10, N1)
~	6813	CALL EXEC(15, 103B, ZBF, 128, IF11, N1)
Ć.	0814	191-D11(1)-1.0
	6815	NH=DN(1)+4,8
	0316	NP=NDZ/32
C	0817	IF(I, GE, NM, AND, I, LE, NN) NP=1
	8818	IF(ISSW(7)) 460, 173
	9819	
<u>_</u>	9820	
	8821	460 CONTINUE
$\left(\right)$	0822	IF(JPB.EQ.0) GO TO 1000
	6823	
	6824	
Ć	0825	C
	8826	LB=LM9X
	6827	IF(ISEN(12)) 314, 315
Ç	6823	
•	6829	
	0830	PH=-CCL**10Z/ZB5*D1
(0831	
C	9832	
	6833	
1.	9834	HRITE(LP, 218) PH, PHDOT, SL, SIDOT
C	9835	IF (NLRYS, EQ. 2) MRGB=7, 0
	9836	WRITE(LP, 990)
r	6837	PH=PHB+(NDZ/ZES)+(CCL)+EXP(MAGB)
(8838	PHDOT=SCL*(NDZ/ZBS)*(-CCL)*EXP(-19908)
	8839	IF (NU(NLRYS-1), EQ. 0, 0) COR=CNPLX(1, 0, 0, 0)
,	6840	SI=SIB*NDZ/ZRS*(-CCL)*EXP(-MAGB)*COR
4	0841	SIDOT=CCU+HDZ/ZBS+(-CCL)+EXP(-HAGB)+COR
	6842	WRITE (1.P. 210) PH. PHDOT, SI, SIDOT
	8843	YEF(1)=PH
C	6844	
	8845	
	8846	
(8847	
	6848	
	6849	
(
	885 9 9954	
	8851	
(0852	
• .	6623	
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		NO JES L'ALINER LA
	0255	po 26 J=1, 31
	0856	N1=J+1
	3357	X2F(N1)=FLOAT(N1-1+L+32)+ZBS/FLOAT(NDZ)
	AS22	YBF(H1)=PH+CEXP(-RP+FLORT(J+H5)+DZ)
\boldsymbol{c}	8859	ZEF(N1)=51*CEXP(-RS*FLOAT(J+N5)*DZ5)
	8868	571 FORMAT(14, 2X, 8F8, 5)
	8861	26 CONTINUE
(9862	IF(1554(8)) 415, 416
	9863	415 DO 415 J=1. 32. NP
	8864	WRITE(1.P. 155) XBF(J), YBF(J), ZBF(J)
C	8865	415 CONTINUE
	0866	CALL EXEC(15, 1038, YBF, 128, IF10, L)
	0867	CALL EXEC(15, 103B, ZBF, 128, IF11, L)
C	9862	1988 CONTINUE
	0869	PH=YP=(32)
	0005 0879	51=ZBF(32)
Ċ	8371	YEF(1)=PH+CEXP(-RP+DZ)
	6872	ZRF(1)=51*CEXP(-R5*DZ5)
	8873	YEF(1)=FLOAT(N1+L+32)+ZES/FLOAT(ND2)
Ç	8874	NS=1
•	0875	313 CONTINUE
	0876	314 CONTINUE
(6877	549(1)=CMPLX((FLOAT(MLAYS)), 0. 0)
	0378	SXQ(2)=CMPLX((FLOAT(NDZ)), 0. 8)
	6879	SXR(3)=CPPLX(Z25, 0, 0)
Ċ	9889	SXR(4)=HER
	6881	SXR(5)=SHER
	8882	DO 129 J=L NLRY5
C	6883	5XQ(J+5)=CMPLX(LDEP(J), 0. 0)
·.	9384	128 CONTINUE
	8885	CPLL_EXEC(15, 1038, SAD, 128, IF8, 0)
Ć	8585	
	9287 0000	C THIS SUPROUTINE CALCULATES POTENTIALS ABOVE WATER-SEDIMENT
\langle	0000	C INTERFACE GIVEN POTENTIALS RELOW INTERFACE
	8898	
	6891	SUBROUTINE SNBC (VS, VP, SF2, CF1, CF2, XLM, MU, ALBM, LP, STGP,
Ċ	6892	15165, PH. PHDOT, SI, SIDOT, MLMS)
	6893	COMPLEX X1, X3, Y5, VP, PH, PHDOT, SI, SIDOT, SIGP, SIGS
	66353 6894	COMPLEX XLM CF1. CF2. SF2. VP2. VS2. XLM2. XLMS2, JRY
(8895	COMPLEX AND G D G D G D G D G D G D G D G D G D G
	6896	COMPLEX, YJ, Y3, Y4, SIPR
	0897	REAL MU
Ć	6898	DIMENSION MU(1), ALAM(1)
	9899	
	9998	
(0981	VP2=VP++2
	6982	Y52=Y5**2
	0903	X1 M2=X1 M**2
(8964	X1.1152=X1.115*X1.115
	8985	Y1=CMPLX(0. 0. 0. 0)
	8986	
C	8987	
	8968	
	8989	
	8910	E=CMPIX(0,0,0,0)
	0910 0911	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0)
C	8918 8911 8912	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SHPR-CHPLX(0, 0, 0, 0)
C	0910 0911 0912 0913	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SMPR-CHPLX(0, 0, 0, 0) F=CHPLX(0, 0, 0, 0)
	8918 8911 8912 8913 8913	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SHPR=CHPLX(0, 0, 0, 0) F=CHPLX(0, 0, 0, 0) H=CHPLX(0, 0, 0, 0)
	8918 8911 8912 8913 8914 8914	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SHPR=CHPLX(0, 0, 0, 0) F=CHPLX(0, 0, 0, 0) H=CHPLX(0, 0, 0, 0) C=CHPLX(0, 0, 0, 0)
	8910 8911 8912 8913 8914 8914 8916	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SHPR=CHPLX(0, 0, 0, 0) F=CHPLX(0, 0, 0, 0) H=CHPLX(0, 0, 0, 0) C=CHPLX(0, 0, 0, 0) D=CHPLX(0, 0, 0, 0)
	8910 8911 8912 8913 8914 8914 8915 8916 8916	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SHPR=CHPLX(0, 0, 0, 0) F=CHPLX(0, 0, 0, 0) H=CHPLX(0, 0, 0, 0) C=CHPLX(0, 0, 0, 0) D=CHPLX(0, 0, 0, 0) R=JHY+2, *PI*YS*XLM
	8910 8911 8912 8913 8914 8914 8916 8916 8917 8918	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SHRF: THPLX(0, 0, 0, 0) H=CHPLX(0, 0, 0, 0) C=CHPLX(0, 0, 0, 0) D=CHPLX(0, 0, 0, 0) P=JAY*2, *PI*V5*XLM N=JAY*2, *PI*V5*XLM N=JAY*2, *PI*V5*XLM
	8910 8911 8912 8913 8914 8914 8915 8916 8916	E=CHPLX(0, 0, 0, 0) G=CHPLX(0, 0, 0, 0) SHPR=CHPLX(0, 0, 0, 0) H=CHPLX(0, 0, 0, 0) C=CHPLX(0, 0, 0, 0) C=CHPLX(0, 0, 0, 0) B=SHY42, *PI*V5*XLM M=JRY42, *PI*V5*XLM M=JRY42, *PI*V7*XLM5 E=(FLAM(1)+2, *MU(1))/(-4, *PI2*VP2*XLM2)

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	0.420	STANDARISEZ TE TRACEZZES AND LETTERTARIES AND A	
	8921	F=(2 *MU(1))/(-4 *PI2*VP2*XLM*XLM5)	
t (8922	H=(2, *****(2))/(-4, *PI2**/P2**1.M**/1.M5)	
	6923	C=VS/(2, *VP)	
C	0924	D=:XLM/(JAY+4, *VP+PI+XLMS2)	
(8925	X1=CMPLX(0, 0, 0, 0)	
	8926	X3=CMPLX(0, 6, 0, 0)	
(8927	X1=ALAM(2)+6*(CF2-5IGP)	
`	0 928	X3=ALAM(1)+E*(CF1-SIGP)	
	0929	5199R=A*C-D*(5F2-5IG5)	
	0930	Y1=PHDOT+R+SI	
~			
	0931	Y3=MU(2)*(PHDOI+SMPR*SI)	
	8932	Y4=X1*PH+A*H*SIDOT	
,	6933	WRITE(LP, 2)	
$\langle \rangle$	8934	2 FORMAT (/"PH, PHDOT, SI, SIDOT-BELOW INTERFACE #1"/)	
	8935	NRITE(LP, 1) PH. PHDOT, SI, SIDOT	
C	8 936	C URITE(LP, 1) YL Y3, Y4, SYPR SF2	
×	8937	1 FORMAT(4F15, 7/)	
	6938	C	
	8939		
(*			
``	8948		
	8941	PHDOT=PHDOT+fi+SI	
	8942	PH=(X1+PH+R+H+SID0T)/X3	
C	8943	SI=C#PLX(0.9,0.0)	
	0944	SIDOT=CMPLX(0. 0, 0. 0)	
C	0945	C	
C	6946	C	
	6947		
C	8948	WRITE(LP, 346)	
 	8949	346 FORMAT(/"PH, PHDOT-ABOVE INTERFACE #1"/)	
	0950	Y1≈PHDOT+A*SI	
	0951	Y4=X3+PH+P+F+SIDOT	
Ċ	0952		
		WRITE(LP, 1) PH, PHDOT	
	6953	C WRITE(LP, 1) YL YA	
~	8954	RETURN	
Ć.	8955	END	
	8956		
C	8957	C THIS SUPROUTINE CALCULATES POTENTIALS AROVE INTERFACE	
Ľ,		C GIVEN POTENTIALS BELOW INTERFACE FOR INTERFACE SEPARATING	
	8958	6 UTEN FOILNING DELAN INTERFICE FOR INTERFICE PERMITING	
	8959	C THO ELASTIC LAVERS.	
C	8959 8968	C THO ELASTIC LAVERS. C	
Ç	8959 8968 8961	C THO ELASTIC LAVERS. C SUBROUTINE SSPC(ML, VS, VP, SF1, SF2, CF1, CF2, XLM, ML, ALAM, SIGS, SI	
Ç	8959 8968	C THO ELASTIC LAVERS. C	
	8959 8968 8961	C THO ELASTIC LAVERS. C SUBROUTINE SSPC(ML, VS, VP, SF1, SF2, CF1, CF2, XLM, ML, ALAM, SIGS, SI	
C	8959 8968 8961 8962 8963	C THO ELASTIC LAVERS. C SUBROUTINE SSPC(ML, VS, VP, SF1, SF2, CF1, CF2, XLM, MU, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHDOT, SI, SIDOT, R, M, E, G, F, H, C, D, XLMS, XLMS	
	8959 8968 8961 8962 8963 8963	C THO ELASTIC LAVERS. C SUBROUTINE SSPC(ML, VS, VP, SFL SF2, CFL CF2, XLM, MJ, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHDOT, SI, SIDOT, A. M. E. G. F. H. C. D. XLMS, XLMS COMPLEX, V52, VP2, SIGP, SIGS, XLM, SFL SF2, YL, Y2, V3, V4, JAY, CFL CF	
	8959 8968 8961 8962 8963 8963 8964 8965	C THO ELASTIC LAVERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, XLM, MJ, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHDOT, SI, SIDOT, A. N. 5, G, F, H. C, D, XLMS, XLMS COMPLEX, V52, VP2, SIGP, SIGS, XLM, SFL SF2, YL, Y2, Y3, Y4, JAY, CFL CF COMPLEX, XLM2	
C	8959 8968 8961 8962 8963 8964 8965 8965	C THO ELASTIC LAVERS. C SUBROUTINE SSPC(ML, YS, YP, SFL SF2, CFL CF2, XLM ML, ALAM, SIGS, SI 1PH, PHOT, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHOT, SI, SIDOT, R, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX, Y2, VP2, SIGP, SIGS, XLM, SFL SF2, YL, Y2, Y3, V4, JAY, CFL CF COMPLEX, XLM2 DIMENSION, MU(1), RLAM(1)	
	8959 8968 8961 8962 8963 8963 8964 8965	C THO ELASTIC LAVERS. C SUBROUTINE SSEC(ML, VS, VP, SF1, SF2, CF1, CF2, XLM, MJ, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHDOT, SI, SIDOT, A, M, 5, G, F, H, C, D, XLMS, XLMS COMPLEX, V52, VP2, SIGP, SIGS, XLM, SF1, SF2, Y1, Y2, Y3, Y4, JAY, CF1, CF COMPLEX, X1M2	
C	8959 8968 8961 8962 8963 8964 8965 8966 8966	C THO ELASTIC LAVERS. C SUBROUTINE SSPC(ML, VS, VP, SFL SF2, CFL CF2, XLM, ML, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHDOT, SI, SIDOT, A. M. E. G. F. H. C. D. XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, VL, V2, V3, V4, JAY, CFL CF COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, VL, V2, V3, V4, JAY, CFL CF COMPLEX, MM2 DIMENSION, MU(1), ALAM(1) REAL, MU	
C	8959 8968 8961 8962 8963 8964 8965 8965 8965 8966 8967 8968	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX X7, VS, VP, PH, PHDOT, SL SIDOT, A. M. E. G. F. H. C. D. XLMS, XLMS COMPLEX X7, VS, VP, PH, PHDOT, SL SIDOT, A. M. E. G. F. H. C. D. XLMS, XLMS COMPLEX VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL V2, V3, V4, JAY, CFL CF COMPLEX MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/	
C	8959 8968 8961 8962 8963 8964 8965 8966 8967 8968 8969	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOOT, SL, SIDOT, LP, PM, MLMS) COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, MLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, MLMS, MLMS COMPLEX X7, VS, VP, SIGP, SIGS, MLM, SFL SF2, YL, Y2, V3, V4, JAY, CFL CF COMPLEX MIM2 DIMENSION MU(1), ALAM(1) REAL MM DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0)	
C	8959 8968 8961 8962 8963 8964 8965 8966 8967 8966 8967 8968 8969 8969 8978	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOOT, SL, SIDOT, LP, PM, MLNS) COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A. M. E. G, F, H. C. D. XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A. M. E. G, F, H. C. D. XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A. M. E. G, F, H. C. D. XLMS, MLMS COMPLEX X7, VS, VP, SIGP, SIGS, MLM SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0)	
C	8959 8968 8961 8962 8963 8964 8965 8966 8967 8968 8969	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOOT, SL, SIDOT, LP, PM, MLMS) COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, MLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, MLMS, MLMS COMPLEX X7, VS, VP, SIGP, SIGS, MLM, SFL SF2, YL, Y2, V3, V4, JAY, CFL CF COMPLEX MIM2 DIMENSION MU(1), ALAM(1) REAL MM DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0)	
C C	8959 8968 8961 8962 8963 8964 8965 8966 8967 8966 8967 8968 8969 8979 8979	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOOT, SL, SIDOT, LP, PM, MLNS) COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A. M. E. G, F, H. C. D. XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A. M. E. G, F, H. C. D. XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A. M. E. G, F, H. C. D. XLMS, MLMS COMPLEX X7, VS, VP, SIGP, SIGS, MLM SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0)	
C	8959 8968 8961 8962 8963 8964 8965 8966 8967 9968 8967 9968 8969 8979 8970 8971 8972	C THO ELASTIC LAWERS. C SUBROUTINE SSPC(ML, VS, VP, SF1, SF2, OF1, CF2, XLM, ML, ALAM, SIGS, SI 1PM, PHDOT, SL, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHDOT, SL, SIDOT, R, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SF1, SF2, VL, V2, V3, V4, JAY, OF1, CF COMPLEX, X1M2 DIMENSION MU(1), ALAM(1) REAL MU DATA, JAY, PJ/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0)	
C C	8959 8968 8961 8962 8963 8964 8965 8966 8967 8968 8969 8978 8971 8972 8973	C TWO ELASTIC LAWERS. C SUBROUTINE SSEC(ML, VS. VP. SFL SF2, CFL CF2, XLM ML, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX X7, VS. VP, PH, PHDOT, SI, SIDOT, R, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V102 DIMENSION MU(1), RLAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0)	
C C	8959 8968 8961 8962 8963 8964 8965 8966 8967 8968 8969 8978 8971 8972 8973 8974	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS. VP. SFL SF2, CFL CF2, XLM ML, ALAM, SIGS, SI 1PH, PHODT, SI, SIDOT, LP, MM, XLMS) COMPLEX X7, VS. VP, PH, PHODT, SI, SIDOT, A. M. E. G. F. H. C. D. XLMS, XLMS COMPLEX VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL V2, V3, V4, JAY, CFL CF COMPLEX VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL V2, V3, V4, JAY, CFL CF COMPLEX MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) H=CMPLX(0, 0, 0, 0)	
с с с	8959 8968 8961 8962 8963 8964 8965 8966 8967 8968 8967 89578 89579 89772 89773 8974 89773	C TWO ELASTIC LAWERS. C SUBROUTINE SSEC(ML, VS. VP. SFL SF2, CFL CF2, XLM ML, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX X7, VS. VP, PH, PHDOT, SI, SIDOT, R, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V72, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JAY, CFL CF COMPLEX V102 DIMENSION MU(1), RLAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0)	
C C	8959 8968 8961 8962 8963 8964 8965 8966 8967 8968 8969 8978 8971 8972 8973 8974	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, XLM, ML, ALAM, SIGS, SI 1PH, PHDOT, SI, SIDOT, LP, MM, XLMS) COMPLEX X7, VS, VP, PH, PHDOT, SI, SIDOT, A. M. E, G, F, H. C, D, XLMS, XLMS COMPLEX VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL V2, V3, V4, JAY, CFL CF COMPLEX VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL V2, V3, V4, JAY, CFL CF COMPLEX MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) H=CMPLX(0, 0, 0, 0)	
с с с	8959 8968 8961 8962 8963 8964 8965 8966 8967 8968 8969 8971 8972 8973 8974 8974 8975 8976	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHODT, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHOOT, SI, SIDOT, A. M. E. G. F. H. C. D. XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL, V2, V3, V4, JAY, CFL CF COMPLEX, MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) G=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) KARAGE AND AND AND AND AND AND AND AND AND AND	
с с с	8959 8968 8961 8962 8963 8966 8966 8966 8967 8968 8978 8979 8979	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI JPH, PHOOT, SL, SIDOT, LP, PM, MLMS) COMPLEX, X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, MLMS, MLMS COMPLEX, VS2, VP2, SIGP, SIGS, MLM, SFL SF2, YL, V2, V3, V4, JAY, CFL CF COMPLEX, MLM2 DIMENSION MU(1), ALAM(1) REAL, MM DATA, JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0)	
с С с с	8959 8968 8961 8962 8963 8964 8965 8965 8965 8965 8963 8969 8970 8971 8972 8973 8973 8975 8976 8977 8975	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOOT, SL, SIDOT, LP, PM, MLMS) COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X1M2 DIMENSION MU(1), PLAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) X7=CTPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) C=C	
с с с	8959 8968 8961 9962 8963 8965 8965 8966 8967 8976 8977 8974 8977 8974 8977 8977 8977 8977	C TWO ELASTIC LAWERS. C SUBROUTINE SSEC(ML, YS, YP, SFL SF2, CFL CF2, XLM ML, ALAM, SIGS, SI 1PH, PHOT, SI, SIDOT, LP, MM, XLNS) COMPLEX X7, VS, VP, PH, PHOT, SL, SIDOT, R, M, E, G, F, H, C, D, XLNS, XLMS COMPLEX X7, VS, VP, SIGP, SIGS, XLM, SFL SF2, YL Y2, V3, V4, JAY, CFL CF COMPLEX X1M2 DIMENSION MU(1), RLAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) D=CMPLX(0, 0, 0, 0) D=CMPLX(0, 0, 0, 0) D=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0)	
с С с с	8959 8968 8961 8962 8963 8964 8965 8965 8965 8965 8963 8969 8970 8971 8972 8973 8973 8975 8976 8977 8975	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOOT, SL, SIDOT, LP, PM, MLMS) COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, MLMS COMPLEX X1M2 DIMENSION MU(1), PLAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ R=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) X7=CTPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) C=C	
C C C C C C C	8959 8968 8961 9962 8963 8965 8966 8967 8966 8967 8976 8977 8977 8977	C TWO ELASTIC LAWERS. C SUBROUTINE SSEC(ML, YS, YP, SFL SF2, CFL CF2, XLM, ML, ALAM, SIGS, SI 1PH, PHOD, SL, SIDDT, LP, MM, XLMS) COMPLEX X7, VS, VP, PH, PHODT, SL, SIDDT, R, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX Y2, VP2, SIGP, SIGS, XLM, SFL SF2, VL V2, V3, V4, JMY, CFL CF COMPLEX MM2 DIMENSION MU(1), RLAM(1) REAL MU DATA JMY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) D=CMPLX(0, 0, 0, 0) V1=TMPLX(0, 0, 0, 0) V1=TMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0)	
с С с с	8959 8968 8961 9962 8963 8964 8965 8966 8967 8970 8970 8977 8977 8977 8977 8977 897	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, XLM, ML, ALAM, SIGS, SI 1PH, PHOD, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHODT, SI, SIDOT, A. M. E, G, F, H. C, D, XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, VL, V2, V3, V4, JAY, CFL CF COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, VL, V2, V3, V4, JAY, CFL CF COMPLEX, MM2 DIMENSION, MU(1), ALAM(1) REAL, MI DATA, JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0, 0)	
C C C C C C C	8959 8968 8961 9962 8963 8965 8965 8966 8967 8976 8977 8977 8977 8977 8977	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOD, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHOOT, SI, SIDOT, A. M. E, G, F, H. C, D, XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL, V2, V3, V4, JAY, CFL CF COMPLEX, MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0, 0) V4=C	
C C C C C C C	8959 8968 8961 9962 8963 8965 8965 8965 8966 8967 9968 8977 8977 8977 8977 8977	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOD, SI, SIDOT, LP, PM, XLMS) COMPLEX, X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL, V2, V3, V4, JAY, CFL CF COMPLEX, MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) H=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0)	
	8959 8968 8961 9962 8963 8965 8965 8966 8967 8976 8977 8977 8977 8977 8977	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOD, SI, SIDOT, LP, MM, XLMS) COMPLEX, X7, VS, VP, PH, PHOOT, SI, SIDOT, A. M. E, G, F, H. C, D, XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL, V2, V3, V4, JAY, CFL CF COMPLEX, MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) E=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0, 0) V4=C	
C C C C C C C	8959 8968 8961 9962 8963 8965 8965 8965 8966 8967 9968 8977 8977 8977 8977 8977	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHODT, SI, SIDOT, LP, PM, XLMS) COMPLEX, X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX, X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX, X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX, X7, VS, VP, PH, PHOOT, SL, SIDOT, A, M, E, G, F, H, C, D, XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL, SF2, YL, V2, V3, V4, JRY, CFL, CF COMPLEX, MM2 DIMENSION MU(1), RLAM(1) REAL, ML DATA, JRY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) H=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0, 0) V5:W5W5	
	8959 8968 8961 9962 8963 8964 8965 8966 8967 8976 8977 8977 8977 8977 8977	C THO ELASTIC LAMERS. C SUBROUTINE SSEC(ML, VS, VP, SFL SF2, CFL CF2, MLM, ML, ALAM, SIGS, SI 1PH, PHOD, SI, SIDOT, LP, PM, XLMS) COMPLEX, X7, VS, VP, PH, PHOOT, SL, SIDOT, A. M. E, G, F, H. C, D, XLMS, XLMS COMPLEX, VS2, VP2, SIGP, SIGS, XLM, SFL SF2, YL, V2, V3, V4, JAY, CFL CF COMPLEX, MM2 DIMENSION MU(1), ALAM(1) REAL MU DATA JAY, PI/(0, 0, 1, 0), 3, 1415926/ A=CMPLX(0, 0, 0, 0) H=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) F=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) C=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V1=CMPLX(0, 0, 0, 0) V2=CMPLX(0, 0, 0, 0) V3=CMPLX(0, 0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0) V4=CMPLX(0, 0, 0)	

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		0938	С	
		0989		A+JAY+2, +PI+Y5+XLM
		6990		M=J9Y+2 +PI+VP+XLMS
I	(
	,	6991		E=(ALAM(ML)+2, *MU(ML))/(-4, *PT2*VP2*YLM2)
		0992		G=(ALAM(AM)+2, *MU(AM))/(-4, *PI2*VP2*XLM2)
	1	8993		F=(2, *(1)(11_))/(-4, *PI2*VP2*XL1*XL15)
	(8994		H=(2.*MJ(M))/(-4.*PI2*VP2*XLM*XLMS)
		0995		C=VS/(2 *VP)
	(8996		D=X1.M2(J9Y+4, +VP+P1+X1MS2)
		0997		Y1=PHDOT+A*SI
		0998		Y2=-SID0T+M*PH
	r'	6999		Y3=mJ(1m1)*(PHD0T+A+C+SI-D*(SF2-51G5)*51)
	(1000		Y4=(ALAN(MN)+G*(CF2-SIGP))*PH+A*H*SIDOT
		1801		WRITE(LP, 2) ML
	Ć	1082		2 FORMAT(/"PH, PHDOT, SI, SIDOT-BELCH INTERFACE #", I3)
		1003		WRITE(LP, 1) PH, PHDOT, SL, SIDOT
		1004	С	IRITE(LP, 1) YL, Y2, Y3, Y4
		1005		1. FORMAT(/4F15. ?)
	4,	1996	£	
		1007		
		1003	v	X7=(ALAM(MM)+G*(CF2-SIGP)+A*F*M)*PH
	C			
	`	1009		X7=X7+(H-F)+A+SIDOT
		1010		X7=X7/(11.91)(11.)+E*(CF1=SIGP)+f*f*f)
	1	1011		SID0T=SID0T-M*(PH-X7)
	Ċ	1812		IF (SIGS, EQ, CMPLX(0, 0, 0, 0)) SIDOT=CMPLX(0, 0, 0, 0)
		1013		PH=X?
		1014		X7=ChPl X(0, 0, 0, 0)
	(AF-GELAGE DE D
		1015	-	
		1016	C	
	1	1017		X?=-MU(M_)*A+MU(MM)*(-D*(5F2-5I05)+A+C)
	$\langle \cdot \rangle$	1818		X7=X7+SI+PHDOT*(NU(NU)-NU(NL))
		1019		X7=X2/(%)(%_)*(-A+A+A+C-D*(SF1~SI6S)))
		1028		IF (SIGE EQ CMPLX(0, 0, 0, 0)) X7=CMPLX(0, 0, 0, 0)
	$\langle \cdot \cdot \cdot \rangle$	1021		IF (m)(mL), EQ. 9. 9) X7=CMPLX(8. 8, 9. 9)
		1022		PHDOT=PHDOT+PH*(SI-X7)
	Ċ	1823		SI=X7
	C	1024		IF(HU(ML), EQ. 0, 0) SIDOT=CMPLX(0, 0, 0, 0)
		1025		IF(MU(ML), EQ. 0. 0) SI=CMPLX(0. 0, 0. 0)
		1026	С	
	(1827	3	
		1028		WRITE(LP. 913) ML
		1.029		913 FORMAT(/"PH, PHDOT, SI, SIDOT-ABOYE INTERFACE # , ", 13)
	C			
	`	1030		WRITE(LP, 1) PH, PHDOT, SI, SIDOT
		1031		Y1=CMPLX(8, 8, 6, 6)
	1	1032		Y2=C19LX(0, 0, 0, 0)
	C	1033		Y3=C7#LX(0, 0, 0, 0)
		1034		Y4=CMPLX(0, 0, 0, 0)
		1935		Y=PHD0T+P#5I
	Ċ			
		1036		Y2=-51001+114PH
		1937		Y3=#U(#_)*(PHDOT+A*C*SI-(\$F1-5105)*D*5I)
	<i>,-</i>	1038		Y4=(PLAN(ML)+E+(CF1-SIGP))+PH+A+F+SIDOT
	$\langle -$	1839	C	WRITE(LP, 1) YL Y2, Y3, Y4
		1649		RETURN
		1041		END
	í			4 ₀ 1.97
	·	1042		
				THIS SUBROUTINE TRANSFORMS FROM THE LINEAR TO THE MAGNITUDE AND
	r			Phase plane.
	C	1045	C	
		1845		SUPROUTINE RNSY (SF, CF, LP, SIGS, SIGP, PH, PHOOT, SI, SIDOT, TH, THOO
		1847		1M. HOOT, N. NDOT, GA. GADOT)
	0	1048		COMPLEX PH, SI, PHDOT, SIDOT, PP, PS, PSDOT, PPDOT, GR, M, N, TH, SIGP, S
				COMPLEX MOOT, NDOT, THOOT, GADOT, SF, CF, A. CSZ1, CSZ2, SSZ2
		1049		
	•	1258		DINENSION X(18)
		1851		DATR P1/3, 1415925/
		4052		τμ Υμοίναρια),

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		571 YO IR LEWICE VEG. U.Z	
	1853	H=CMPLX(0, 0, 0, 0) -	
(1854	GR=CMPLX(0, 6, 0, 0)	(i
	1055		
		N=CMPLX(0, 0, 0, 0)	
C	1056	THEOT=CHPLX(8, 8, 8, 8)	$\left(\right)$
	1957	GREAT-CHIPLX(0, 0, 0, 0)	
	1858		
(1859	PP=CMPLX(0, 0, 0, 0)	C
``	1968	ndot=chpi x(e. g, g, d)	
	1861	PPD0T=C19PLX(0, 0, 0, 0)	
C	1862	PS=CTPLX(0, 6, 0, 0)	
•	1063	PEDDT=CHPLX(0, 0, 0, 0)	
	1064	PP=PHD0T+PH+CF	
C	1065	PPD0T=-SIGP+PH+CF+PP	(
•	1866	P5=5ID07+5I*5F	`
	1867	PSD0T=-SIQS*SI+SF*PS	
15	1963	E=0. 9	
C	1069	IF (AIMARG(PP), EQ. Ø, Ø, AND, AIMARG(PH), EQ. Ø, Ø) GO TO 5	1
	1078	60 T0 6	
	1071	5 IF (REAL (PP), LT. 0. 0, AND, REAL (PH), LT. 0. 0) E=3, 141592654	C
4	1872	6 R=PH++2+PP++2	(,)
	1073	7=0.5*CLCG(R)+(0.0.1.0)*E	
	1074	D=1_0	
- (°	1075	IF(AIMAG(R), EQ. 0, 0, RND, REAL(A), LT. 0, 0) GO TO 9	(
	1076	GO TO 18	
	1077	9 IF (AINAG(PP), LT. 8, 8, AND, AIMAG(PH), LT. 8, 8) D=-1, 8	
0	1078		(
	1879	M=CFPLX(REAL(N), D)	
	1079		
(IF(REAL(PH), LT. 0, 0, ADD, AIMAG(PH), GT. 0, 0)	1
`	1991	11=11+(0. 0. 3. 1415926)	
	1982		
C	1883	IF(RIMAG(SI), EQ. 0, RAD, RIMAG(PS), EQ. 0, 0) GO TO 7	
	1934	60 TO 8	
	1025	7 IF(REAL(PS), LT, 0, 0, AND, REAL(SI), LT, 0, 0) E=3, 141532654	
C	1986	8 R-PS++2+5I++2	6
×	1987	N=8.5*CLCG(A)+(0.8,1.8)*E	× .
	1888	D=1. 0 ~~	
~	1089	JF (ALMAG(A), EQ. 9. 6. AND, REFL (A), LT, 6. 9) GD TO 11	i
Ć	1698	GD TO 12	Ň
	1891	11. IF(AIMAG(PS), LT. 0, 0, AND, AIMAG(SI), LT. 0, 0) D=-1, 0	
/	1892	12 D=D#AIMAG(N)	,
Ç	1093	N=CIPLX(RERL(N),D)	•
	1894	IF(REAL(SI), LT. 0, 0, AND, AIMAG(SI), GT. 0, 0)	
,	1095	1N=N+(0. 0, 3, 1415926)	1
(1095	C521=51#+2+P5##2	C
	1097	IF(C521, E9, CHPLX(0, 9, 0, 0)) N=CHPLX(0, 0, 0, 0)	
	1098	TH=(0, - 0, 5)*(1,06(((0, 1,)-PP/PH)/((0, 1,)+PP/PH))	
C .	1099	(SZ)=PS/SI	(
	1199	IF(3). EQ. CHPLX(0. 0, 0. 0)) CSZ1=CHPLX(0. 0, 0. 0)	
	1191	GP=(0, ,-0, 5)*CLOG(((0, ,1,)-CSZ1)/((0, ,1,)+CSZ1))	
(1192	CSZI=C005(TH)	(
	1193	SSZ1=CSIN(TH)	
	1184	C572=C005(GA)	
C .	1105	SSZ2=CSIN(CA)	i.
	1185	7572-05174CH7 MD0T=CF*(SSZ1+SSZ1-CSZ1)+(1, -STGP)+SSZ1+CSZ1	
	1107	ND0T=5F*(5522+6522+6522+(1, -5165)*5522+6522	
(ί,
-	1198	THEOT=2_+CF+C521+5521+5521+5521+5521+5521+5521 CONTE-1_+CF+C521+519+C521+C521+C521+5521	
	1109	GRD0T=2, +5F+C5Z2+S5Z2-S165+C5Z2+C5Z2+S5Z2 1F415F214433, 4, 2	
(1118	IF(ISSW(4)) 4,3	(
~	1111	4 WRITE(LP, 2)	`
	1112	2 Format(/"Transform to phase plane-"	
í.	1113	1"TH: N. GR. N. THDOT, MDOT, GROOT, NDOT"/)	(
`.	1114	WRITE (LP, 1) TH, M. GR, N. THDOT, MDOT, GHOOT, NDOT	×.
	1115	WRITE(LP, 1) PP, PPD07, PS, PSD07	
;	1116	3 X(1)=ATAN2(AIMAG(PH), REAL(PH))	÷
1	1117	X(2)=ATAN2(ATMAG(PP), REAL(PP))	`.
	441Q		

																					÷																
																						uabrai si	5	, 5105, ML 0, PPD01, 5	(jj	T PDOT, 19900		2) FOX(L), OF									
										- 8	đ		205									TURNAL SADITIAN TATEGODIES MAGANITIAN AND PLACE SALAMANAN AND AND AND AND AND AND AND AND AND		SURROTTINE SCSPH(SFRX, CFRX, LP, NS, NF, ZPS, DZ, NDZ, STOP, STOS, NL, NUZ, NZ, M3, TH, TROOT, G9, GADOT, N, NDOT, N, NDOT, PH, PHDOT, PP, PPDOT, S	251001, PS, PSD01, 025, FN, LJ 1F1, 1F2, 1F3, 1F4, 1F14, 1F15, M96)	complex dz. sigp. sigs. Th. GA. R. N. Throf. Moot. Groot. Noot complex the gap. NP. (CSZ, CSZ, SSZ, SSZ) Throat. GAPOOL NP00	CONPLEX NPDOT, PH. PP. SI, PS, PHDOT, PPDOT, SIDOT, PSDOT	CONPLEX MJ. NO. MA. SFUX CFUX VZ5. NZ6. YPF (Z2), ZPF (Z2) DIMENSION 1F10(3), 1F11(3), W(1), W(1), W(1), M(1), SFUX(1), CF									
	IERM									JF(44. GT, 8. 00001. 02. 46. GT, 0. 00001.) GP=GA+41. JF(47. 1.T. A ADARA RAD, 44. 1.T. A DARAM) GD TO 22	JF (Y5, L. I, 8), 2525012, HND, Y6, L.I, 14, 14235013, 143 - 113 - 22 Continue		IF (25542 (TH), 61, 62, 2832185367) TH=TH-6, 2832185397 IF (25542 (GA), 61, 6, 2832185337) TH=TH-6, 2832183397		rrbul ≈-slor*rrhtur*rr IF(SF, FQ, Chrb.X(0, 0, 0, 0)) SI=Chrb.X(0, 0, 0, 0)					-		Saura musa :		. NF, ZRS, DZ 5, N, NDOT, P	IF3, IF4, IF	#001, MD01, \$052, 5552,	PPD01, SIDO	5 분명, 11년(1) 11년(1)									
L(PS))	CT PHAGE .			(, (PH))	((1))	£ (PS))			1. 0. 99901	1.0.00000 1.1 9.00000	L.I. N. 1993	1	11-HL (200					1001S	Inter-	FORMAT(4(F12, 3, 2%), /, 4(F12, 8, 2%))		NU LL ENCODIM		FOX, LP, NS DOT, N, NDO	, IF1, IF2,	GB R. N. T CS7, CS57,	PS, PHDOT,	X, CHUX, DZ 3), ML(L),									
and the state of the second of the second second second second second second second second second second second	ure corre		<u>.</u>	PS=CSIN(GR)*CE/2P(N) X(S)=61692(61096(PH), REAL(PH))	X(6)=RTAN2 (AINAG(PP), REAL (PP)) X(7)=AITAN2 (AINAG(SI), REAL (SI))	X(8)-ATAN2(ATMAC(PS), REAL(PS))	(I)		([+3)) 4. 69. 45. 6	11.02.46.0 11.02.46.0	0. HWD. 76.		6. 283185 6. 283185		0.6.8.9)	2	រុំ ភូមិ	URITE(LP, 1) PH, PHDOT, SI, SIDOT	WALE (LP 1) PD PRIVUD PS PSUA MATE (LP 1) TM GR M N	23), 7, 4(1		TERPOTES		94(SFR% C 201, G9, G9	DZS, FN, L	() S165, 11 2, 115, 115, 11	PP, SI,	43, 144, 5FU (3), 1F11(48, 441		
RUZ(RING	k to ins	l=1 D0 20 J=1,3 Du-frocrutureryDrM1	PP=CSIN(TH)*CEXP(N) SI=CCOS(GR)*CEXP(N)	P5=CSIN(GA)*CE2P(N) X(5)=ATAP2(A1M96(PH	FIND (BITTER	FINE (FILME	₩3=885(X(1+4)-X(1)) ₩3=885(X(1+4)-X(1))	44=885(X(1+5)-X(1+2))) ₩5=885(X(1+5)-X(1+2))	46=ABS(X(1+7)-X(1+3)) JF(Y3, GT, 0, 60604, 09, 4	31, 8, 8086 T 8 8986		ן ש	L (TH). GT. L (GA). GT.	PHD0T=-CF*PH+PP		Sd+IS+35-=100IS	15(1554(4)) 26, 25	LP. 1) PH.	WIELP 1) PP PHULF	(4(F12.3,		MITNE 1		TINE SCS MA, TH, TH	PS, P5D0T	X 02.510 X THP.GA	X NPDOT,	X HIL HZ	BG 2000	F18(1)=2HD? F18(2)=2HBF	F11(1)=2HDZ	F11(2)=2HBF F10(3)=2H	년 문 문		IF(ISSN(10)) 448, 441	442	2 0
8(4)-AI	Doublecheck to Insure correct phase term	1=1 00 20 J=1,3 04-00000700	SI=CSIO	PS=CSI) X(S)=81	X(6)=R1 X(7)=R1	X(8)-HTPL)S89=54	12-682(16-465 1F (Y3. (JF(Y4.1	22 IFCY5.LT. 33 CINTINE		IF (REA	=100Hd	E INGT	=1001S	EIUUCH	25 HRITE(HATEC HATEC	1 FORMAT		חטוב כווסם		Suppose Suppose	ZSID01,		COMPLE	DIMENS	REAL MAG	1F18(2	F11(1	IF11(2)=2H IF10(3)=2H	IF11(3)=2H	2-0004-2-2		448 NWP=1 SD TD 442	345 01 00 777 00 00 00
			1126	1128 1128	871	ន្ម	1 Å	11 36 11 36	1137 1138	51 13 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 15 14 14 15 14 14 14 14 14 14 14 14 14 14 14 14 14	1142		1145 1145	1147	1149 1149	851	1152					1159 C	ას	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	100	11 15 15 15 15 15 15 15 15 15 15 15 15 1	7782	89 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	8277	227	127	5 FI FI FI FI FI FI FI FI FI FI FI FI FI F	1176	1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2		60 11 18 1	

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(1185	GRP==(MPLX(0, 8, 8, 8)	(
;	1186	MPDOT=CMPLX(8, 6, 9, 8)	
	1137	NPDOT=CMPLX(0, 0, 0, 0)	
\mathbf{C}	1188	MP≔CMPLX(0, 0, 0, 0)	(
(1189	NP=CMPLX(0.0,0.0)	
	1190	THPDOT=C11PLX(0, 0, 0, 0)	
C	1191	GAPDOT=CMPLX(0, 0, 0, 0)	Ċ
(1192	CALL EXEC(14, 1838, Y8F, 128, IF18, L)	
	1193	CALL EXEC(14, 1038, ZEF, 128, IF11, L)	
C	1194	N1=F1+1	
1	1195	JA=FN	
	1196	A=FN-JA	
(1197	IF(NS. GE. 32) NS=1	(
•	1198	1F(A, EQ. 8, 8, AND, N5, EQ. 1) N1=FN	
	1199	IF(ISSN(5)) 4,9	
Ć	1209	4 CCSZ=TH+57, 295779	7
(1291	5552=6 8 +57. 295779	
	1282	5 WRITE(LP, 2)	
(1203	2 Format(/"Integrate in phase place-thum gain"/)	Ç
1	1294	XN1=ZB5/FLORT(NDZ)+FLORT(L+32+33-N5)	`
	1285	WRITE(LP, 10) XNN CCSZ, M, SSSZ, N	
C	1206	9 DO 51 I=1.M1	(
C	1297	CR11_EMEC(14, 103B, CF0X, 128, IF14, L)	
	1208	CRLL EXEC(14, 103B, SF0X, 128, IF15, L)	
<i>c</i>	1209	1F(1, EQ, N1, RND, A, NE, 0, 0) NF=R*32	(
(1218	IF(NS. NE. 1) NF=NS+NF	
	1211	IF(NF. GT. 32) NF=32	
C	1212	12 00 50 J⇒KS, NF	ć.
١.	1213	NX=33-J	
	1214	WL(NX)=CMPLX(0, 8, 8, 8)	
C	1215	µ2(NX)≈CNPLX(0, 0, 0, 0)	
<u></u>	1216	N3(NX)=CMPLX(0, 8, 8, 0)	
	1217	H4(NX)=CMPLX(0, 0, 0, 0)	
	1218	TH₽=TH-DZ+THDOT	C
ί.	1219	GRP=GR-DZS+GRDOT	×.
	1228	MP=M-D2+MDOT	
Ċ	1221	NP=N-DZSHNDOT	1
C	1222	(CSZ=CCOS(THP)	
	1223	CSSZ=CSIN(THP)	
(1224	5C5Z=C005(GPP)	
Υ.	1225	SSSZ=CSIN(GPP)	
	1226	THPDOT=-SIGP+CCSZ+CCSZ+CSSZ+2.+CFGX(NX)+CCSZ+CSSZ	
1	1227	GPPD0T=-5105*5C5Z*5C5Z-555Z*555Z+2. *5F0X(NX)*5C5Z*555Z	C
(1228	MPD0T=CFQX(NX)*(CSSZ*CSSZ-CCSZ*CCSZ)+(1, -51GP)*CSSZ*CCSZ	C
	1229	NPDOT=SFQX(NX)*(SSSZ*SSSZ-SCSZ*SCSZ)+(1, -SIGS)*SSSZ*SCSZ	
(.	1238	TH=TH-, 5+DZ+(THDOT+THPDOT)	$\langle \cdot \rangle$
(1231	M=M-, 5+DZ+(NDOT+NPDOT)	
	1272	GA=GA-, 5+025+(GADOT+GAPDOT)	
6	1233	N=N-, 5+DZ5+(NDOT+NPDOT)	C.
Ċ.	1234	CCSZ=CCOS(TH)	N
	1235	CSSZ=CSIN(TH)	
(1236	SCSZ=CCOS(GR)	í.
•	1237	SSSZ=CSIN(GR)	``
	1238	THEOT=-516P+CC5Z+CC5Z-C55Z+C55Z+2, *CF0X(NX)*CC5Z*C55Z	
(1239	GADOT=-5105+5052+5052-5552+5552+2, +5F0X(NX)+5052+5552	
(1248	MDOT=CFRX(NX)+(CSSZ+CSSZ=CCSZ+CCSZ)+(1SIGP)+CSSZ+CCSZ	``
	1241	NPDOT=SFQX(NX)*(S55Z+S5SZ-SCSZ+SCSZ)+(1, -SIG5)*S5SZ+SCSZ	
ć	1242	VL(HX)=TH	C
(1243	1/2(NX)=11	N.
	1244	NCKNX)=GA	
C	1245	144(NX)=N	ć
(1246	YBF(NX)=CCSZ*CEXP(M)	
	1247	ZBF(NX)=CMPLX(0, 0, 0, 0)	
	1248	50 IF (NX NE 1) 60 TO 58	(
÷.	1249	CALL EXEC(15, 1038, W1, 128, IF1, L)	<u>х</u>
	1258	COLL EVEC(15, 1020 H2 122 L)	

		فالمعالية وتعاقبه وتحافظ وتعاويه والكادرية والمحادث المالية
	1251	CALL EXEC(15, 1038, W3, 128, 1F3, L)
-	1252	CALL EXEC(15, 1938, W4, 128, IF4, L)
	1253	CALL EXEC(15, 1038, VSF, 123, IF10, L)
~	1254	CRL1_EXEC(15, 103B, ZBF, 128, IF11, L)
(1255	L1=L
	1256	L=L-1
,•	1257	151=15
C.	1253	115=1
	1259	28 IF(ISS4(5)) 7,50
	1260	7 DO 3 JX=151 NF, NP
$\langle \cdot \rangle$	1261	JY=33-JX
	1262	Z=FLOAT(JY-1+(L+1)*32)*ZB5/FLOAT(NDZ)
	1263	CC5Z-W1(JY)*57. 295779
6	1264	555Z=4B(JY)*57. 295779
•	1265	3 WRITE(LP, 10) Z. CCSZ. H2(JY), SSSZ. H4(JY)
	1266	10 FORMAT(F7, 2, 2%, 8(F6, 3, 1%))
$\langle \cdot \rangle$	1267	70 FORMAT(/16/)
	1268	50 CONTINUE
	1269	51 CONTINUE
1	1270	IF (NF. EQ. 32) GO TO 6
	1271	CALL EXEC(15, 1938, W1, 128, IF1, L)
	1272	CRLL EXEC (15, 1038, N2, 128, IF2, L)
Ċ	1273	CHLL EXEC(15, 1838, W3, 128, IF3, L)
	1274	CALL EXEC (15, 1038, W4, 128, IF4, L)
	1275	IF(ISSN(5)) 11.6
C	1276	11 DO 40 JX=15, NF, NP
	1277	JY=33-JX
	1278	2=FL0AT(JY-1+L*32)*ZB5/FL0AT(NDZ)
(1279	CCS2=JU(JY)+57. 295779
	1280	SSSZ=102(JY)+57.295779
,	1281	40 WRITE(LP, 10) Z, CCSZ, W2(JY), SSSZ, W4(JY)
Ę.	1282	6 NX=33-NF
	1283	MRG=AMAX1(REAL(M), REAL(N))
/	1284	M=M-M9G+7
(1285	N=N-1470+7
	1286	PH=CCOS(TH)*CEXP(M)
,	1287	PP=CSIN(TH)+CEXP(M)
(1283	SI=CCOS(GR)*CEXP(N)
	1289	PS=CSIN(GR)*CEXP(N)
<u></u>	1298	IF (SF0X(NX), EQ. CMPLX(0, 0, 0, 0)) SI=CMPLX(0, 0, 0, 0)
Ċ	1291	PHDOT=-CFQX(IX)*PH+PP
	1292	PPDOT=-SIGP*PH+CFRX(NX)*PP
	1293	SIDOT=-SFRX(NX)*SI+PS
Ĉ	1294	PSDOT=-SIGS*SI+SFQX(NX)*PS
	1295	3 NS=1₽
,	1296	RETURN
- C	1297	END
	1238	END\$
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