

# Available potential energy in the world's oceans

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## ABSTRACT

It is shown that in the case with bottom topography, the available gravitational potential energy cannot be represented by the available pressure potential energy. Thus, a suitable quantity for the study of large-scale circulation is the total available potential energy which is defined as the sum of available gravitational potential energy and available internal energy.

A simple computational algorithm for calculating the available potential energy in the world's oceans is proposed and tested. This program includes the compressibility of seawater and realistic topography. It is estimated that the world's oceans available gravitational potential energy density is about  $1474 \text{ J/m}^3$  and the available internal energy density is  $-850 \text{ J/m}^3$ ; thus, the net available potential energy density is  $624 \text{ J/m}^3$ , and the total amount of available potential energy is  $805 \times 10^{18} \text{ J}$ .

## 1. Introduction

The oceanic general circulation takes place in the environments of gravity; thus, balance of gravitational potential energy (GPE) plays a vital role in the energetics of the oceanic circulation (Huang, 2004). Using the mean depth of the world's oceans,  $z = 3750 \text{ m}$ , and the reference level, the total amount of GPE in the world's oceans is estimated as  $2.09 \times 10^{25} \text{ J}$  (Oort *et al.*, 1994). However, most of such a huge energy is dynamically inert and only a very small fraction, the so-called available potential energy (APE), is dynamically active.

The concept of APE was first introduced by Margules (1905). However, the application of this concept in studying the dynamics of the atmosphere appeared primarily after Lorenz (1955) introduced an approximate definition for APE.

Concepts related to APE also appeared in classical papers in oceanography. For example, Sandstrom and Helland-Hansen (1903) introduced the dynamic height  $\Delta D$ , which is the potential energy per unit mass relative to a reference ocean with temperature  $0^\circ\text{C}$  and salinity 35 psu; Fofonoff (1962a,b) discussed the concept of the anomaly of potential energy, which is defined as the potential energy per unit area found by integrating  $\Delta D$  over the water column from surface to a given pressure,  $p$ .

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Different forms of APE have been proposed for ocean study. For an incompressible ocean, the available gravitational potential energy (AGPE) is defined as

$$\chi = g \int (z - Z)dm \quad (1)$$

where  $g$  is the gravitational acceleration;  $z$  and  $Z$  are the geopotential height of a mass element in the physical and reference states. The reference state is defined as a state with minimal GPE.

Due to the nonlinear equation of state of the seawater, searching such a state of minimal GPE in the ocean is complicated; thus, in previous studies the horizontal mean density profile has been used as the reference state as a compromise. For example, Wright (1972) discussed the deep circulation of the Atlantic using such a reference state to infer the rate of the release of APE in the Atlantic. Oort *et al.* (1989, 1994) studied APE in the world's oceans using a similar definition. However, seawater is nearly incompressible and we omit the compressibility of seawater; such a reference state of minimal GPE can be found through a sorting program (Huang, 1998).

There are also other forms of APE proposed for the study of oceanic circulation. For example, Bray and Fofonoff (1981) used the following definition

$$\Pi_g = \frac{1}{g} \int_{p_s}^{p_b} (v - v_r) p dp dx dy \quad (2)$$

where  $p_s$  and  $p_b$  represent the pressure at the sea surface and bottom;  $v$  and  $v_r$  are the specific volume of the water parcels in the physical and reference states. It is readily seen that under the hydrostatic approximation this definition provides the AGPE, assuming that the contribution due to bottom pressure is negligible. Since this definition is based on a thermodynamic variable, it is convenient to apply *in-situ* data, as discussed by Bray and Fofonoff (1981). However, there are several potential problems in the application of this definition. First, as shown in Appendix A, this definition can introduce errors for the case with bottom topography. Second, this definition does not include a contribution due to the compressibility of seawater. In addition, the reference state might be difficult to define for the world's oceans due to the complicated nature of the equation of state. A modified definition was introduced by Reid *et al.* (1981)

$$\Pi = \frac{1}{g} \int_{p_s}^{p_b} (h - h_r) p dp dx dy \quad (3)$$

where  $h$  and  $h_r$  are the enthalpy of the water parcels in the physical and reference state. This definition includes the contribution due to changes in the internal energy. A rather peculiar feature associated with this definition is that the component associated with the internal energy is negative (Reid *et al.*, 1981), and this is due to the fact that cold water is

more compressible than warm water. As a result, during the adjustment the total volume of seawater is reduced and the total amount of internal energy is increased. Therefore, a noticeable part of the gravitational potential energy released is used to compress water parcels, so the net amount of APE is smaller than the AGPE.

An important and difficult question is how to find the reference state that is used in the definitions (2) and (3). Due to the nonlinear nature of the equation of state for seawater, a reference state with a minimal enthalpy is difficult to define. In a slightly different approach, one can use the reference state with minimal gravitational potential energy; however, due to the nonlinear nature of the equation of state such a reference state is also difficult to find. Thus, in most studies for the basin-scale oceanic circulation APE is calculated according to a third definition which is based on the quasi-geostrophic (QG) approximation (Pedlosky, 1987; Oort *et al.*, 1989, 1994; Reid *et al.*, 1981)

$$\Pi_{QG}^a = -g \iiint_V \frac{[\rho - \bar{\rho}(z)]^2}{2\bar{\rho}_z^\theta} dx dy dz \quad (4)$$

where the reference state  $\bar{\rho}(z)$  is defined by horizontal averaging of the density field, and  $\bar{\rho}_z^\theta$  is the vertical gradient of the horizontal-mean potential density.

Although this definition is simple and easy to use, it has some problems as well. First, this definition is based on the quasi-geostrophic approximation. Although it is a sound approximation in the study of meso-scale eddies and baroclinic instability, it is inaccurate in the study of thermohaline circulation. It is found that the application of this traditional definition of APE and its sources can lead to substantial errors, as shown in the case of an incompressible ocean by Huang (1998). Second, the contribution due to internal energy is not clearly defined in this formulation. Third, according to the APE definition (4), horizontal adjustment of the density field is required during the adjustment from the physical state to the reference state; thus, this definition is not based on a truly adiabatic process. Furthermore, such horizontal shifting of water mass does not change the total GPE. For the details the reader is referred to Appendix B.

In the study of thermohaline circulation (or meridional overturning cells) in the world's oceans, it is desirable to return to the original concept introduced by Margules (1905). In order to do so, we will re-examine the total APE in the compressible oceans. A major technical obstacle in the study of the APE is how to find the reference state with minimal gravitational potential energy. This problem is solved by a simple computational algorithm discussed in Section 2. The application of this algorithm to the world's oceans provides interesting insight to the dynamic structure of the stratification in the world's oceans, as discussed in Section 3. Finally, conclusions are presented in Section 4.

## 2. Available potential energy in the compressible ocean

### a. The available potential energy

The available potential energy,  $\Pi^a$ , can be defined as

$$\Pi^a = g \int (z - Z)dm + \int (e - e^r)dm + p_s(V - V^r) \quad (5)$$

where  $z$  is the vertical position in the physical state, with  $z = 0$  defined as the deepest point on the bottom;  $Z$  is the vertical position in the reference state;  $e$  is the internal energy, superscript  $r$  indicates the reference state,  $dm = \rho dv$  is the mass of each water parcel, and  $V$  and  $V^r$  are the total volume of the ocean in the physical state and the reference state, respectively. Note that the ocean is an open system which interacts with the atmosphere by exchanging force, mass, heat, and freshwater fluxes. For this study we will omit the exchange of mass, heat and freshwater fluxes; however, the pressure force between atmosphere and oceans must be included. For example, if the total volume of the oceans changes after adjustment, atmospheric pressure does work on the ocean. Assuming the atmospheric work remains unchanged, this amount of work is represented by the last term in Eq. (5).

Thus, the total amount of APE consists of three parts; i.e., the available gravitational potential energy (AGPE), the available internal energy (AIE), and the pressure work (PW). Note that although the total amount of gravitational potential energy depends on the choice of the reference level, the total amount of AGPE is independent of the choice of reference level, if the mass of each water parcel is conserved during the adjustment from the physical to reference states.

The UNESCO equation of state is used for the density calculation (Fofonoff and Millard, 1983). Internal energy is defined in terms of the Gibbs function

$$e = G - T \left( \frac{\partial G}{\partial T} \right)_{s,p} - pv \quad (6)$$

where  $G$  is the Gibbs function and  $v$  is the specific volume of seawater. The basic idea of using the Gibbs function to unify the thermodynamics of seawater was first proposed by Fofonoff (1962a). After 30 years, this idea was finally put into practice (Feistel and Hagen, 1995; Feistel, 2003).

### b. Finding the reference state

There have been theoretical studies in meteorology in which the APE is examined using a variational approach, e.g., Dutton and Johnson (1967), Wiin-Nielsen and Chen (1993). Although a variational approach can lead to a well-defined problem mathematically, no practical algorithms have been discussed that can be used to find a state of minimum potential energy.

The reference state is defined as a state which can be reached adiabatically and

reversibly; i.e., there is no mixing involved during the adjustment process, and this state should be the state in which the total potential energy of the system is a global minimum. In this study we used an iterative computer program to search the reference state that is stably stratified and has a global minimal gravitational potential energy, as discussed in Appendix C. However, we are unable to show that such a state is also a state of global minimum of total potential energy.

The first step is to calculate the potential density of each grid box, using a large value as the reference pressure. Since the maximal depth defined in the Levitus data is 5750 m, we will use 5880 db as the highest reference pressure to begin the iteration process. One can sort the density in all grid boxes according to their density, with the heaviest water parcel sitting on the bottom. In this process the bottom topography enters the calculation of the thickness of the individual layer  $dz$ :

$$dz = \frac{dm_{ijk}/\rho_{ijk}}{S(z)}. \quad (7)$$

where  $dm_{ijk}$  and  $\rho_{ijk}$  are the mass and density of the grid box  $ijk$ , and  $S(z)$  is the horizontal area at level  $z$ . To improve the stability of the water column at all depths, repeat the sorting process as follows. All water parcels lying above the pressure of 5800 db will be resorted by using a smaller reference pressure of 5750 db. A new array of density is formed, including the new density calculated from the new reference pressure (5750 db) and the old density calculated from the old reference pressure (5880 db). This new density array is sorted according to the density. Since this new reference pressure is smaller than the previous one, density calculated using this reference pressure is smaller than that calculated based on the previous reference pressure. Thus, the order of the water parcels below the 5800 db pressure level will not be alternated in sorting afterward.

Similarly, re-sorting using reference pressures of 5650 db, 5550 db, . . . will guarantee the stability of the whole water column at any given depth. A simple computer program can be written to sort out the stratification for any given number of reference pressure levels. Thus, the reference state, which is stably stratified at any given level, can be constructed to any reasonably desired degree of accuracy.

### 3. APE in the world's oceans

The search program discussed above is applied to the world's oceans, with realistic topography. The annual-mean climatology of temperature and salinity, with "realistic" topography, from the Levitus (1998) dataset is used. Since the deepest two levels of grid in temperature and salinity of this dataset are at 5000 m and 5500 m, the bottom of the last grid is assumed to be at 5750 m.

In order to simplify the calculation we assume that water parcels can be stacked up from the deepest places in the world's oceans; i.e., we ignore the blocking and separation of basins at different levels due to the existence of bottom topography. Thus, the world's

Table 1. APE dependence on the number of level of reference pressure, in units of  $J/m^3$ .

Number of ref. level	AGPE	AIE	APE
5	844.7	-829.7	15.0
50	1468.3	-852.4	615.9
500	1474.6	-850.7	623.9
2000	1475.4	-850.4	625.0
5800	1474.5	-850.3	624.2

oceans are treated as a single basin, with the horizontal area equivalent to the world's oceans at the corresponding levels. This kind of calculation is, of course, an idealization in order to avoid cumbersome manipulations. It is fair to say that the minimal state of GPE obtained from such a searching program may not be reachable. Nevertheless, the AGPE and AIE calculated in this way may provide a useful upper bound of APE in the oceans.

In addition, it may be conceptually difficult to assume that water in some of the semi-closed marginal seas, such as the Mediterranean Sea, the Black Sea and the Caspian Sea, can be redistributed over the bottom of the world's oceans; thus, in most of the discussion hereafter marginal seas are excluded from our calculation.

Since the search program is based on iterations, a natural question is how many reference levels are needed in order to obtain a reliable estimate of APE in the oceans? First, we performed a series of runs with different numbers of reference pressure levels (Table 1). It is clear that when the number of reference pressure levels is larger than 500, the amount of AGPE and AIE calculated from the search program converges. For the following discussion, the results are based on 5800 reference levels; i.e., the reference pressure increment is 1 db. The computation in the case with 5800 reference levels took less than two hours to complete on a personal computer; thus, such a calculation is considered to be computationally inexpensive. Since the difference between the reference pressure and the *in-situ* pressure is less than 1 db, errors in the *in-situ* density and the vertical positions of each water layer is negligible.

AGPE calculation has been carried out for the world's oceans and three major basins separately. The boundary between the Atlantic and Indian oceans is set along 20E, the boundary between the Indian and Pacific oceans is set along 156E, and the boundary between the Atlantic and Pacific oceans is set along 70W. The Arctic Ocean is separated from the North Pacific Ocean along the Arctic circle and AGPE is calculated for the Atlantic and the Arctic together. The results are summarized in Table 2. Note that after adjustment, the sea level drops 4.2 cm; thus, the sea-surface atmospheric pressure does work on the oceans, as shown in the last term in Eq. 5. For the world's oceans, the total work is about  $1.43 \times 10^{16}$  J, and it corresponds to an energy source of  $0.01 J/m^3$  which is negligible. In comparison, AGPE density in the Atlantic Ocean is the highest, and in the Pacific Ocean it is the lowest.

Previously, Vulis and Monin (1975) made an estimate of the AGPE density in the

Table 2. APE for different basins in the world's oceans (including or excluding the Mediterranean Sea), in units of  $\text{J/m}^3$ .

Basin		AGPE	AIE	APE
Atlantic	With Medi.	2316.4	-1608.4	708.0
	No Medi.	2338.1	-1699.3	638.8
Pacific		970.7	-489.0	481.7
Indian		1235.4	-762.7	472.7
World Oceans	With Medi.	1463.8	-799.4	664.4
	No Medi.	1474.5	-850.3	624.2

Atlantic. Neglecting the contribution due to salinity and using a linearized approximation of the thermodynamic equations, their estimate of the AGPE for the Atlantic is  $700 \text{ J/m}^3$ .

The AGPE calculated from (5) represents a theoretical upper limit of energy that can be converted into kinetic energy if all the forcing of the system is withdrawn. Under such an idealized situation, water parcels with large densities sink to the bottom and push the relatively lighter water parcels upward. This process is illustrated by the potential contribution of GPE through adiabatic adjustment (Figs. 1 and 2). It is readily seen that dense water in the Southern Oceans associated with steep isopycnal surfaces sustained by a strong wind stress forcing is the primary contributor to AGPE. On the other hand, deep

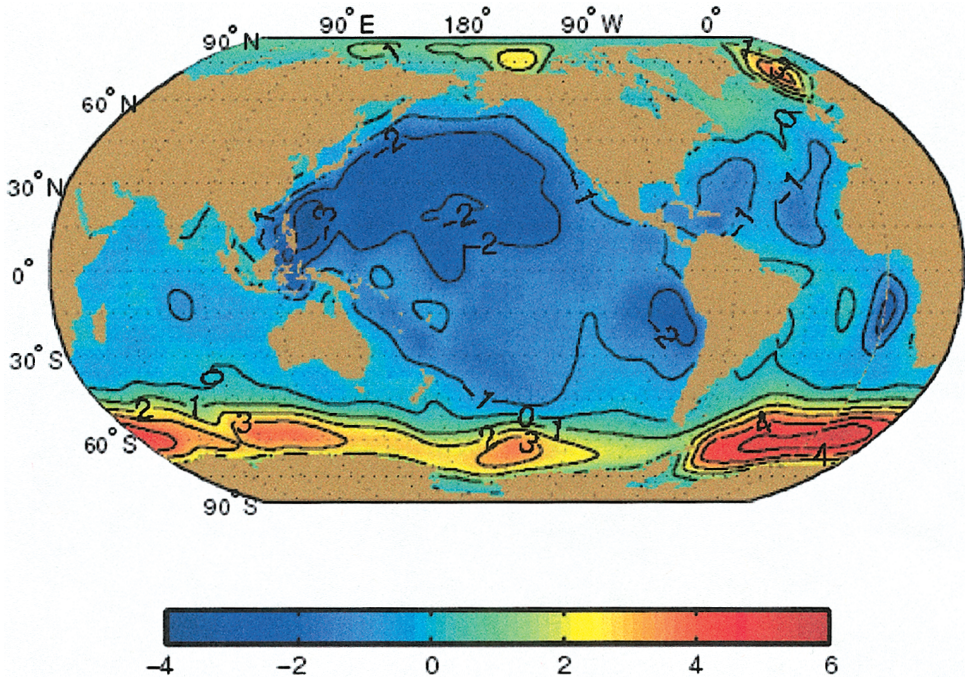


Figure 1. Contribution to AGPE through adiabatic adjustment, in units of  $10^{11} \text{ J/m}^2$ .

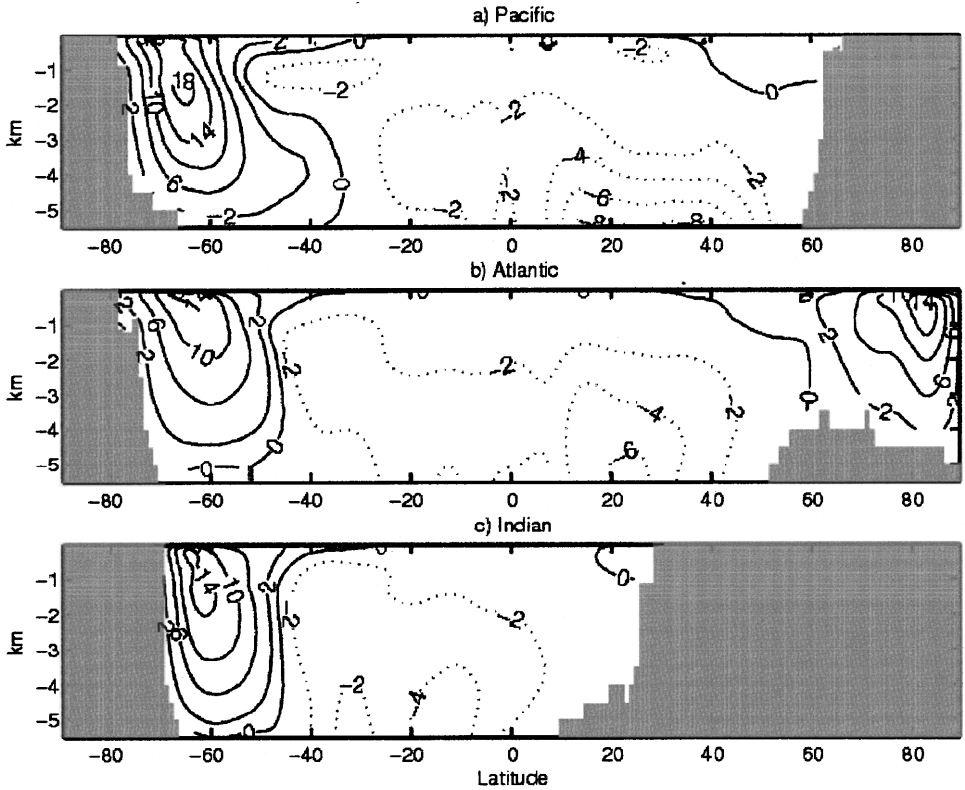


Figure 2. Contribution to AGPE through adiabatic adjustment, in units of  $10^{13} \text{ J/m}^2$ .

water at middle latitude basins would be pushed upward during the adjustment, and thus contribute to AGPE negatively.

In order to illustrate the effect of nonlinearity in the equation of state on APE, we discuss a simple case, with two  $1^\circ \times 1^\circ$  boxes sitting side by side. The center of the first box is at the equator, and the second box is at  $1\text{N}$ . Temperature and salinity of the first (second) box are  $20^\circ\text{C}$  ( $10^\circ\text{C}$ ) and 35 (33). The thickness of the boxes is equal and it varies from 100 m to 1000 m. In addition, there can be two layers of water, each of the same thickness. The corresponding density of AGPE and AIE can be calculated. It is readily seen that the magnitude of both AGPE and AIE increases quickly as the depth of the boxes increases. What contributes to the rapid increase of AGPE? There are two major contributors—the nonlinearity of the equation of state and the geometric factor. To illustrate the geometric factor we first analyze the generation of AGPE and AIE with a linear equation of state:

$$\rho = \rho_0 - \alpha(T - T_0) + \beta(S - S_0) + \gamma P, \quad (8)$$

where  $\rho_0 = 1.0369 \text{ g/cm}^3$ ,  $\alpha = 0.1523 \times 10^{-3}/^\circ\text{C}$ ,  $\beta = 0.7808 \times 10^{-3}$ ,  $\gamma = 4.462 \times 10^{-6}/\text{db}$ .



Table 3. APE dependence on the pressure effect, in units of  $J/m^3$ .

Equation of state	Thickness of each layer (m)	UNESCO (1981), Feistel and Hagen (1995)			Linear in TS	Linear in TSP
		AGPE	AIE	APE	AGPE	AGPE
One layers	100	106	5.3	112	4.59	4.73
	250	376	-41.1	335	11.5	11.8
	500	1116	-261	855	22.9	23.7
	1000	3671	-1219	2452	45.9	47.4
Two Layer	100	381	22.3	403	9.44	225
	250	1801	-8.1	1793	23.6	1389
	500	6407	-220	6187	47.2	5540
	1000	23876	-1200	22675	94.4	22163

If the equation of state is linear, it is readily seen that the total GPE in this box model increases proportionally to the square of the thickness of the box, as does the total amount of AGPE. Since the volume increases linearly with the depth of the bottom, the density of AGPE is linearly proportional to the depth, as shown in the sixth column in Table 3. Comparison of the third column with the sixth and seventh columns indicates that the nonlinearity of the equation of state is a major source of AGPE, and for a model with a linear equation of state, the corresponding AGPE would be greatly reduced.

In the case of density linearly depending on pressure, the AGPE with a single layer increases linearly with layer thickness; however, it increases quadratically with two layers. It is interesting to note that a single-layer model AGPE diagnosed from a model with linear dependence on  $(T, S)$  or  $(T, S, P)$  is much smaller than that from the UNESCO equation. However, a two-layer model AGPE diagnosed from a model with the equation of state linearly depending on  $(T, S)$  is doubled, but is still much smaller than that diagnosed from a model based on the UNESCO equation of state. On the other hand, for a model based on an equation of state linearly depending on  $(T, S, P)$  this difference is greatly reduced, as shown in the last column of Table 3.

It is clear that the effect of nonlinearity of the equation of state is very important in the calculation of AGPE. This fact can be further illustrated by processing the same climatological dataset but with different equations of state. For example, if we use a linear equation of state to calculate, the results are different (Table 4). It is of interest to note that

Table 4. AGPE dependence on the equation of state, in units of  $J/m^3$ .

Equation of state	UNESCO	Linear in $T, S$	Linear in $T, S, P$	Bouss. approx.
AGPE	1474.5	793.4	1316.0	904.6

an equation of state, with a linear dependence on pressure can substantially improve the calculation of the AGPE.

Note that our calculation above is based on mass coordinates, thus the mass of each grid box is conserved during the adjustment. A model based on the traditional Boussinesq approximation does not conserve mass, so the meaning of APE inferred from such a model is questionable. As an example, APE based on the Boussinesq approximation can be calculated and it is noticeably smaller than the truly compressible model (Table 4). (This calculation was done as follows: density is calculated from the UNESCO equation of state; the volume of an individual water parcel remains unchanged after adjustment, even if its density is adjusted to the new pressure.) It is speculated that a smaller AGPE may affect the model's behavior during the transient state; however, this is left for further study.

An interesting and potentially very important point is that available internal energy in the world's oceans is negative. For reversible adiabatic and isohaline processes, changes in internal energy obey  $de = -pdv$ . Since cold water is more compressible than warm water, during the exchange of water parcels internal energy increase associated with the cold water parcel is larger than the internal energy decline associated with the warm water parcel. Thus, available internal energy associated with the adjustment to the reference state is negative. A similar phenomenon was discussed previously by Reid *et al.* (1981), who demonstrated analytically that the net amount of total APE in a compressible ocean is less than the total amount of available gravitational potential energy.

To illustrate this point, we can examine changes in internal energy in the following idealized experiment: there are two water masses: water parcel  $m_1$  sits on the sea surface (so, the pressure difference in the constant atmospheric pressure is  $p_1 = 0$ ) and the temperature is  $1^\circ\text{C}$ ; water parcel  $m_2$  sits at a pressure  $p_2$  and the temperature is  $10^\circ\text{C}$ ; both have salinity  $S = 35$ . If  $m_1$  were moved down to pressure level  $p_2$  adiabatically, its volume is reduced, as indicated by the heavy line in Figure 3a, and its internal energy is increased, as indicated by the heavy line in Figure 3b. On the other hand, if water parcel  $m_2$  is moved to the surface adiabatically, its volume expands, as indicated by the thin line in Figure 3a, and its internal energy declines by the amount indicated by the thin line in Figure 3b.

In comparison, warm air is more compressible than cold air; thus, the corresponding available internal energy in the atmosphere is positive. Since most studies about available potential energy have been based on the quasi-geostrophic approximation, this issue remains to be studied, using a similar algorithm for the available potential energy based on the original definition like (5). For the comparison of thermodynamics of seawater and air the reader is referred to a new textbook by Curry and Webster (1999).

#### 4. Conclusion

A simple computational program for calculating AGPE and AIE in the world's oceans is discussed. This program is applied to the world's oceans with realistic topography and

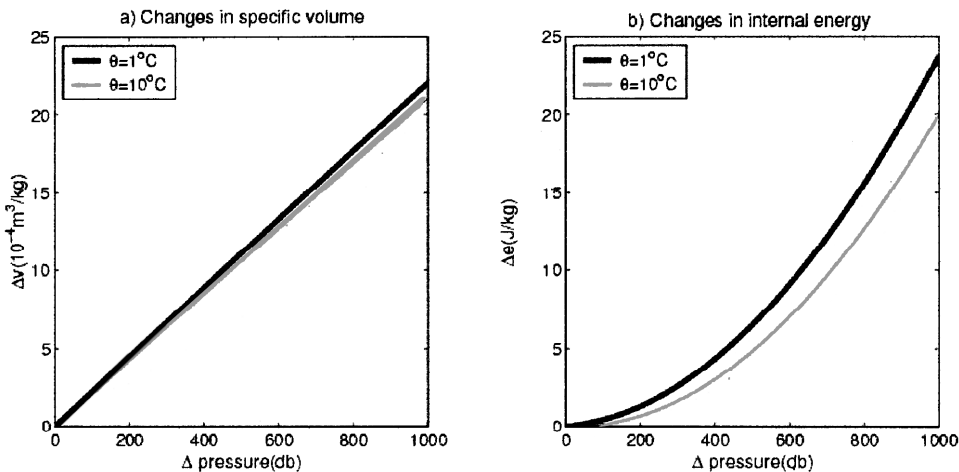


Figure 3. Changes in the specific volume and internal energy as a function of potential temperature and pressure.

temperature and salinity climatology. The distribution of available potential energy in the world's oceans reveals the fundamental role of strong fronts and currents in the Southern Ocean. In a word, these strong currents hold up the dense water around the Antarctic continent and thus facilitate the storage of AGPE and APE in the world's oceans. In the event of a decline of wind stress over the Southern Ocean, the relaxation of the currents will give rise to a slump of density fronts. As a result, a large amount of mechanical energy will be released that can drive a very energetic adjustment in the world's oceans.

*Acknowledgment.* RXH was supported by the National Science Foundation through grant OCE-0094807 and the National Aerospace Administration through contract no. 1229833 (NRA-00-OES-05) to the Woods Hole Oceanographic Institution. This is Woods Hole contribution no. 11209.

## APPENDIX A

### Potential energy in an ocean with topography

Total potential energy in the oceans consists of two parts, the gravitational potential energy and internal energy. In some earlier studies it was suggested that enthalpy could be used as a state variable for the study of total potential energy; e.g., Bray and Fofonoff (1981) and Reid *et al.* (1981).

Using enthalpy is rather convenient because it is a state variable; however, as will be shown shortly, defining the available potential energy using enthalpy can introduce major errors in the case of bottom topography.

The total potential energy per unit of mass is defined as

$$\Pi = e + gz, \quad (\text{A1})$$

where  $e$  is the internal energy, while the enthalpy is defined as

$$H = e + pv, \quad (\text{A2})$$

where  $p$  and  $v$  are pressure and specific volume. The total amount of enthalpy is obtained by integrating (A2) over all the mass elements  $dm = \rho dx dy dz$ .

For simplicity, our discussion in this appendix is based on the approximation assuming water is incompressible; thus, the potential contribution due to atmospheric pressure, last term in Eq. (5), can be omitted. Using the hydrostatic relation and integration by parts, the last term in (A2) is reduced to

$$\iiint pdz dx dy = \iint (p_s Z_s - p_b Z_b) dx dy + g \iiint \rho z dx dy dz \quad (\text{A3})$$

where  $p_s$  and  $p_b$  are pressure at the sea surface and bottom,  $Z_s$  and  $Z_b$  are the geometric height of the sea surface and bottom.

If we use the sea surface as the reference level, the surface pressure term vanishes. From Eq. (A3), the difference between the physical and reference states is

$$\Delta P + \Delta P Z_b = \Delta \chi \quad (\text{A4})$$

where  $\Delta P = \Delta \iiint p dx dy dz$  is the difference in the volume integration of pressure and will be called the available pressure potential energy (APPE),  $\Delta P Z_b = \Delta \iint p_b Z_b dx dy$  is the difference in the area integral of the bottom pressure multiplied by the bottom height and will be called the available bottom pressure potential energy (ABPPE), and  $\Delta \chi = g \Delta \iiint \rho z dx dy dz$  is the available gravitational potential energy (AGPE).

Since we are interested in the difference of GPE and bottom pressure energy before and after the adjustment, the results calculated from a mass-conserved model are independent of the choice of the reference level used for the geopotential. (Note that for models that do not conserve mass, an artificial source of GPE may be introduced, so results from such models may depend on the choice of the reference level.) Using the lowest level of the bottom as the reference level, the calculation of the available bottom pressure energy is greatly simplified.

For an ocean with a flat bottom, ABPPE is zero if we choose the bottom as the reference level for GPE. However, this term is nonzero in the instance of bottom topography. Since both  $\Pi$  and  $H$  contain internal energy, we will only discuss the second part in their definitions; i.e., we will show that in general  $\Delta \chi$  cannot be represented in terms of  $\Delta P$ .

To illustrate the basic idea we will discuss several simple two-dimensional cases. First, we discuss four box models shown in Figure A1. In each case, the initial and reference states are shown in the upper and lower rows. By definition, the reference state is a state with a stable stratification and minimum GPE. In all cases the energy terms involve simple back-of-the-envelope calculations, so the reader can check them out with ease.

Since the bottom is flat in Case a, ABPPE is zero. As a result, AGPE and APPE are the

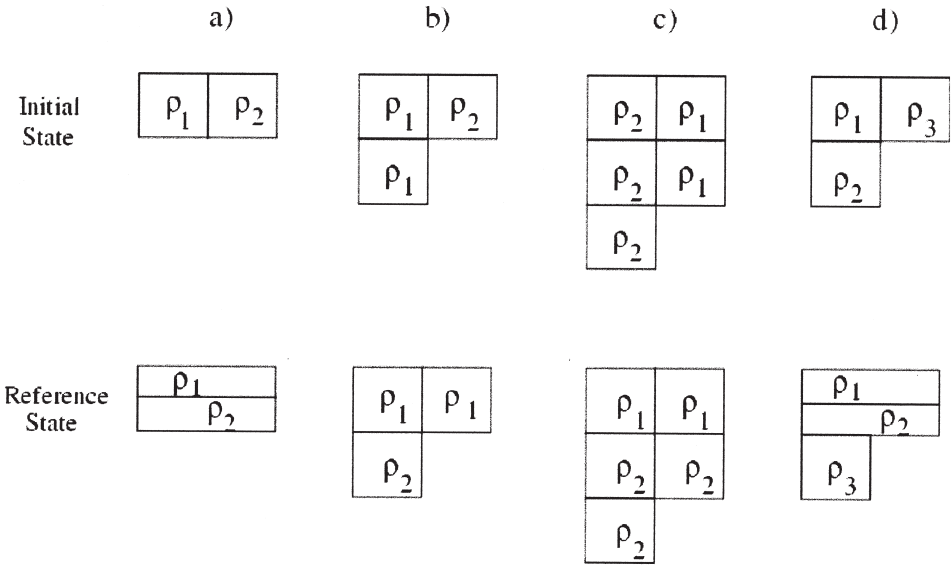


Figure A1. Sketch of box models: density distribution in the initial and reference states. The width and height of each box is  $d$ .

same. However, in all cases with bottom topography, ABPPE energy is nonzero. As shown in the last column of Table A1, ABPPE can be positive or negative. In Case b, ABPPE is equal to AGPE, and APPE is zero. In Case c, ABPPE is negative. As a result, the amount of APPE is twice as much as the AGPE, so that it can offset this imbalance. Finally, it is interesting to note that it may require at least three water masses to produce a case with a negative APPE.

As an example of the case of continuous stratification, we will discuss a case for an incompressible ocean with a sloping bottom, as shown in the nondimensional coordinates in Figure A2a. The density distribution has a simple form

$$\rho = \rho_0[1 + a(x - z)], \quad 0 < x < 1, 0 < z < 1 \tag{A5}$$

where both  $x = X/D$  and  $z = Z/D$  are nondimensional coordinates. The reference state with minimum gravitational potential energy is a state where all isopycnal surfaces are level, with density increasing downward monotonically (Fig. A2b).

Table A1. Comparison of different terms during adjustment from the initial state to the reference state, including AGPE, available pressure energy, and bottom pressure energy terms.

Case	$\Delta\chi$	$\Delta P$	$\Delta PH_0$
a	$\frac{1}{4}gd^3(\rho_2 - \rho_1)$	$\frac{1}{4}gd^3(\rho_2 - \rho_1)$	0
b	$gd^3(\rho_2 - \rho_1)$	0	$gd^3(\rho_2 - \rho_1)$
c	$gd^3(\rho_2 - \rho_1)$	$2gd^3(\rho_2 - \rho_1)$	$-gd^3(\rho_2 - \rho_1)$
d	$\frac{1}{4}gd^3(4\rho_3 - 3\rho_2 - \rho_1)$	$\frac{1}{4}gd^3(\rho_1 - \rho_2) < 0$	$\frac{1}{4}gd^3(4\rho_3 - 2\rho_2 - 2\rho_1)$

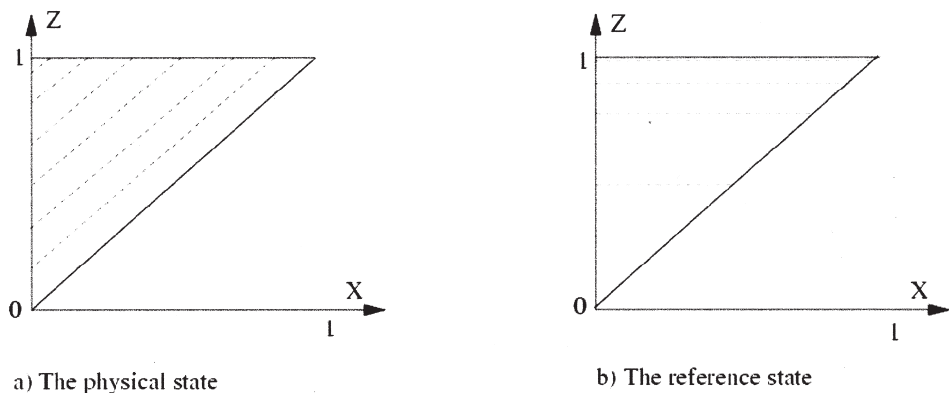


Figure A2. The physical and reference states for an ocean with a strong current and topography, with isopycnal surfaces depicted by dashed lines.

The calculation of AGPE and other terms is elementary, but rather tedious; thus, it is not included here. Accordingly, changes in the energy balance (A4) are

$$\Delta P + \Delta PZ_b = \Delta \chi \quad (\text{A6})$$

where

$$\Delta P = g\rho_0 a D^3 \left( \frac{1}{12} - \frac{\pi}{32} \right) = -0.015 g\rho_0 a D^3;$$

$$\Delta PZ_b = g\rho_0 a D^3 \left( \frac{3}{24} - \frac{\pi}{32} \right) = 0.027 g\rho_0 a D^3;$$

$$\Delta \chi = g\rho_0 a D^3 \left( \frac{5}{24} - \frac{\pi}{16} \right) = 0.012 g\rho_0 a D^3.$$

Although the AGPE is positive, the APPE is negative. In reality, there is a large bottom pressure contribution  $\Delta PZ_b$  that offsets the errors induced by the APPE term. Thus, using APPE as the energy available is troublesome in the present case.

Therefore, examples discussed above clearly demonstrate that using enthalpy to replace total potential energy in the study of APE for the world's oceans can lead to serious errors. Thus, the available potential energy in the ocean with bottom topography should be based on the sum of the internal energy and gravitational potential energy.

## APPENDIX B

### APE in QG approximation

For simplicity, all variables are nondimensionalized, and dimensional variables are labeled with \*. Assume a simple stratification

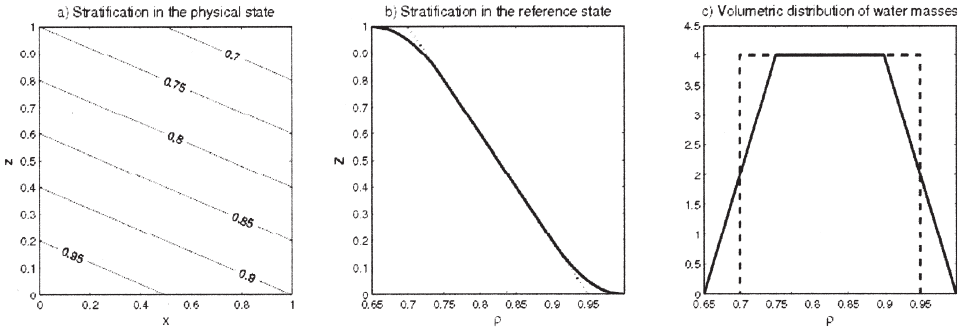


Figure B1. Sketch of a simple model ocean, in nondimensional units: (a) stratification in the physical space; (b) stratification in the reference space, the dashed line for the QG definition and the solid line for the exact definition; (c) volumetric distribution of water mass in the density coordinate, the solid line for the physical space and the dashed line for the reference space under the QG approximation.

$$\rho = \frac{\rho^*}{\rho_0} = 1 - ax - bz, \quad 0 \leq x = \frac{x^*}{L} \leq 1, \quad 0 \leq z = \frac{z^*}{H} \leq 1.$$

A special case with  $a = 0.1$ ,  $b = 0.25$  is shown in Figure B1. By definition, stratification in the reference state used in the QG approximation is the horizontal mean density  $\bar{\rho} = \bar{\rho}^x(z)$ , and it is depicted by the dashed line in the middle panel. In comparison, stratification in the reference state according to the exact definition can be calculated and it is depicted as the solid line.

From Figures B1(b) and B1(c), it is clear that water with density lighter than 0.7 and heavier than 0.95 does not exist in the reference state under the QG approximation. Thus, the transition from the physical to reference states must invoke horizontal mixing; i.e., the reference state under the QG approximation cannot be reached adiabatically.

GPE is nondimensionalized as  $\chi = \chi^*/g\rho_0 H^2 L$ . Under the QG approximation it is readily seen that

$$\chi_0 = \iiint \rho z dx dz = \int \bar{\rho}^x dz = \chi_{ref}$$

Therefore, under the QG approximation the total amount of GPE in the physical state and the reference state is exactly the same. In fact, the adjustment process implied in the QG definition involves only horizontal shifting and mixing of water masses, so the total amount of GPE should not change.

On the other hand, the amount of GPE in the system under both the QG approximation and the exact definition can be calculated from the definitions. The calculation is straightforward but tedious, so it is not included here. For the case with  $a = 0.1$  and  $b = 0.25$ , we obtained  $\Pi_{QG}^a = 0.00167$  and  $\Pi_{real}^a = 0.00153$ ; thus, APE from the QG

definition is about 10% larger than that from the exact calculation. Therefore, there is apparently a conflict between the real change of GPE in the system and the APE calculated from the QG definition. These puzzles associated with QG definition applied to the basin-scale circulation problems need to be interpolated properly.

### APPENDIX C

#### An iterative algorithm for searching the reference state with minimal AGPE

For the world's oceans, we use an  $N$ -loop search algorithm as follows. First we define two arrays:

$$P_{cut}(n) = p_B \frac{n}{N}$$

$$p_{ref}(n) = p_B \frac{n + 0.5}{N}$$

where  $p_B$  is the maximum bottom pressure used in our search;  $p_{ref}$  is used as the reference pressure for the  $n$ th iteration, and  $p_{cut}$  is the pressure level where the list of water parcel density will be cut off. Since the maximum depth in the Levitus data is 5,750 m, we set  $p_B = 5800$  db. As explained below, water parcels under *in-situ* pressure smaller than  $p_{cut}(n)$  will be subject to the next loop of sorting, using a reference pressure  $p_{ref}(n - 1)$ , which is smaller than  $p_{cut}(n)$ .

Using potential temperature  $\theta$  and salinity  $S$  for each water parcel and any given reference pressure, we generate an initial density array  $\rho_{sort}(m)$ . With this initial density array, we can enter the following outer loop, with  $n$  decreasing from  $N$  to 1.

- (A) Using a sorting program to rearrange the density list  $\rho_{sort}(m)$  to the new order so that the density increases with  $m$ .
- (B) Inner loop 1, for  $m = M$  to 1. This loop starts from the deepest point in the world's oceans and moves upward. The geometric height increment of each parcel  $dz(m)$  is calculated using *in-situ* density  $\rho(m)$  and the total horizontal area  $A(z)$  at a given depth in the world's oceans:

$$dz(m) = dm(m)/\rho(m)/A(z).$$

- (C) Inner loop 2, for  $m = 1$  to  $M$ . This loop starts from the sea surface and moves downward. Using the geometric height information obtained from the previous inner loop, the *in-situ* pressure and density are calculated. In addition, the density under the reference pressure  $p_{ref}(n)$  is calculated. From the list of *in-situ* pressure, the position to cut the list of density is determined as follows.

$$p(m) = p(m - 1) + g\rho(m)dz(m)$$

$$\rho(m) = \rho(\theta, S, p(m))$$

$$\rho_{ref}(m) = \rho(\theta, S, p_{ref}(n))$$

Set  $m_{cut} = m$  when  $p(m) \geq p_{cut}(n)$  is satisfied for the first time.



(D) Updating the density array for the next run of sorting

$$\rho_{\text{sort}}(m) = \begin{cases} \rho_{\text{ref}}(m), & \text{for } m = [1, m_{\text{cut}}] \\ \text{unchanged}, & \text{for } m > m_{\text{cut}} \end{cases}$$

End of the outer loop and return to step A.

When the outer loop is finished, a reference state is obtained which has a stable stratification at any given level. Note that the *in-situ* properties such as pressure and density are accurately calculated. The errors associated with the calculation can be improved to any desired accuracy, if finer vertical resolution in pressure is chosen.

#### REFERENCES

- Bray, N. A. and N. P. Fofonoff. 1981. Available potential energy for MODE eddies. *J. Phys. Oceanogr.*, *11*, 30–47.
- Curry, J. A. and P. J. Webster. 1999. *Thermodynamics of Atmospheres and Oceans*, Academic Press, London, UK, 471 pp.
- Dutton, J. A. and D. R. Johnson. 1967. The theory of available potential energy and a variational approach to atmospheric energetics. *Adv. Geophys.*, *12*, 333–436.
- Feistel, R. 2003. A new extended Gibbs thermodynamic potential of seawater. *Prog. Oceanogr.*, *58*, 43–114.
- Feistel, R. and E. Hagen. 1995. On the Gibbs thermodynamic potential of seawater. *Prog. Oceanogr.*, *36*, 249–327.
- Fofonoff, N. P. 1962a. Physical properties of seawater, *in* *The Sea: Ideas and Observations*, Interscience, 3–30.
- 1962b. Dynamics of ocean currents, *in* *The Sea, Ideas and Observations*, Interscience, 323–395.
- Fofonoff, N. P. and R. C. Millard. 1983. Algorithms for the computation of fundamental properties of seawater. UNESCO Technical Papers in Marine Sciences, *44*, 53 pp.
- Huang, R. X. 1998. On available potential energy in a Boussinesq ocean. *J. Phys. Oceanogr.*, *28*, 669–678.
- 2004. Ocean, energy flow, *in* *Encyclopedia of Energy*, C. J. Cleveland, ed., *4*, Elsevier, 497–509.
- Levitus, S., T. Boyer, M. E. Conkright, T. O'Brien, J. Antonov, C. Stephens, L. Stathoplos, D. Johnson and R. Gelfeld. 1998. World ocean database 1998, Vol. 1: Introduction. NOAA Atlas NESDIS 18, Nat'l. Oceanic and Atmos. Admin., Dept. Commerce, Washington, DC, 346 pp.
- Lorenz, E. N. 1955. Available potential energy and the maintenance of the general circulation. *Tellus*, *7*, 157–167.
- Margules, M. 1905. *Über die energie der sturme*, Wein: K. K. Hof-und. Staatsdruckerei, 26 pp.
- Oort, A. H., L. A. Anderson and J. P. Peixoto. 1994. Estimates of the energy cycle of the oceans. *J. Geophys. Res.*, *99*, 7665–7688.
- Oort, A. H., S. C. Ascher, S. Levitus and J. P. Peixoto. 1989. New estimates of the available potential energy in the world ocean. *J. Geophys. Res.*, *94*, 3187–3200.
- Pedlosky, J. 1987. *Geophysical Fluid Dynamics*, Springer-Verlag, NY, 710 pp.
- Reid, R. O., B. A. Elliott and D. B. Olson. 1981. Available potential energy: A clarification. *J. Phys. Oceanogr.*, *11*, 15–29.
- Sanstrom, J. W. and B. Helland-Hanson. 1903. *Ueber die Berechnung von Meerestromungen*, Rep. Norweg. Fish Invest., *2*, 43 pp.
- UNESCO. 1981. Background papers and supporting data on the International Equation of State of Seawater, 1980. UNESCO Tech. Pap. Mar. Sci., *38*, 192 pp.

- Vulis, I. L. and A. S. Monin. 1975. The available potential energy of the ocean. *Doklady, Akademii Nauk SSSR*, 221, 597–600.
- Wiin-Nielsen, A. and T.-C. Chen. 1993. *Fundamentals of Atmospheric Energetics*, Oxford Univ. Press, NY, 376 pp.
- Wright, W. R. 1972. Northern sources of energy for the deep Atlantic. *Deep-Sea Res.*, 19, 865–877.

Received: 1 June, 2004; revised: 1 September, 2004.