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Mathematics: understanding the score

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In September 2008, the Ofsted report *Mathematics: understanding the score* produced detailed evidence and analysis from inspections of mathematics teaching. The past decade has seen significant rises in standards in mathematics for pupils of all ages, but more pupils than at present should be reaching the higher GCSE grades. Strategies to improve test and examination performance, coupled with teaching that focuses heavily on preparation for the qualifications, does not equip pupils for their futures. It is vitally important to shift from a narrow emphasis on disparate skills towards a focus on pupils' mathematical understanding.

The fundamental issue for teachers is how better to develop that understanding. The essential ingredients of effective mathematics teaching are subject knowledge and understanding of the ways in which pupils learn mathematics – drawn together in the report as 'subject expertise' – together with experience of using these in the classroom. The report describes how the best teaching in both phases is enthusiastic, knowledgeable and focuses clearly on developing pupils' understanding of important concepts. Good assessment throughout each lesson enables teachers to see how pupils are thinking and to adjust teaching and learning strategies accordingly. By developing pupils' mathematical independence, teachers also equip them for success in examinations and beyond.

The report highlights many examples of good mathematics teaching and weaker practice. With this in mind Ofsted has produced this booklet to help teachers in providing the best possible opportunities for all pupils.

What are the essentials of good mathematics teaching?

The following table does not define what constitutes good or satisfactory teaching, but shows the difference between good and satisfactory features. Teaching that encompasses most of the good features may well be outstanding. Similarly, the cumulative effect of many weaker features can slow pupils' progress.

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Meeting needs and addressing misconceptions	
Teaching successfully focuses on each pupil learning.	Teaching successfully focuses on teaching some content.
Teachers monitor all pupils' understanding throughout the lesson, recognising quickly when pupils already understand the work or what their misconceptions might be, for example, circulating to check all have started correctly, to spot errors and extend thinking.	Pupils generally complete some correct work but the teacher does not recognise when some pupils are stuck, have made errors or already understand the work, for example the teacher moves on too quickly or does not circulate to check so gives answers or methods when pupils have already done the work correctly.
The teacher listens carefully and interprets pupils' comments correctly, building on pupils' contributions, questions and misconceptions to aid learning, flexibly adapting to meet needs and confidently departing from plans.	The lesson features competent questioning but the teacher is focused more on what has been asked than on the information about understanding that pupils' responses or lack of responses offers; misses opportunities to respond to needs, for example, does not build on errors or pupils' comments that they are stuck, and sticks too closely to plans.
Work challenges higher and lower attainers, as well as middle attainers, because it is informed by teachers' knowledge of pupils' learning; for example, through setting different work for different groups, or encouraging pupils capable of doing so to improve their explanations or use more efficient methods.	Pupils complete some correct work that extends or consolidates their competence but does not stretch the high attainers or support the low attainers well; for example, pupils are given challenging work only if they finish many routine questions quickly or the numbers used in a problem create barriers to the concept for lower attainers.
The plenary extends learning and meets the needs identified during the lesson.	The plenary draws the lesson to an orderly close.
Understanding concepts and explaining reasoning	
Lesson objectives involve understanding.	Lesson objectives are procedural, such as descriptions of work to be completed, or are general, such as broad topic areas.

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Lesson activities are structured around key concepts and misconceptions, so that carrying out the activities enhances understanding; for example, involving pupils in developing suitable methods to solve problems, selecting questions carefully from exercises. Pupils can explain why a method works and solve again a problem they solved a few weeks earlier.	There is a successful focus on developing skills and obtaining correct answers rather than enhancing understanding; such as providing examples which do not illustrate **why** the method works, or doing questions identical to worked examples, too many of which are similar and are not carefully selected. These skills may be short-lived so pupils cannot answer questions which they have completed correctly a few weeks earlier.
Work requires thinking and reasoning and enables pupils to compare approaches.	Methods are clearly conveyed by teachers and used accurately by pupils; pupils rely on referring to examples, formulae or rules rather than understanding or remembering them.
Practical, discussion and ICT work enhance understanding, for example, using demonstration and mental visualisation of shapes being rotated, with pairs deciding which method gives the correct answer and why.	Practical, discussion and ICT work is motivating and enables pupils to reach correct answers but is superficial and not structured well enough to enhance their understanding, such as unfocused pair work on a book exercise, group tasks where the highest attainer does all the work or free choice of hands-on ICT.
Pupils give explanations of their reasoning as well as their methods.	Questioning is clear and accurate but does not require explanation or reasoning; pupils describe the steps in their method accurately but do not explain **why** it works; for example, discussion activities enable pupils to share approaches but do not ensure they explain their reasoning.
Pupils spend enough time working to develop their understanding.	Teachers give effective exposition that enables pupils to complete work correctly but restricts the time they have to develop their understanding through their own work; for example, teachers talk for too long, pupils spend too long copying examples, notes or questions, or drawing diagrams.
Good use of subject knowledge capitalises on opportunities to extend understanding, such as through links to other subjects, more complex situations or more advanced mathematics.	Any small slips or vagueness in use of subject knowledge do not prevent pupils from making progress.
Teachers introduce new terms and symbols meaningfully, they expect and encourage correct use; pupils and teachers use mathematical vocabulary and notation fluently.	Teachers introduce new terms and symbols accurately and demonstrate correct spelling.

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Lesson forms clear part of a developmental sequence and pupils recognise links with earlier work, different parts of mathematics or contexts for its use.	Lesson stands alone adequately but links are superficial, for example, pupils know it is lesson two of five on a topic but not how it builds on lesson one. Contexts or applications are mentioned without indicating how the mathematics may be used in a way the pupils can understand.
Non-routine problems, open-ended tasks and investigations are used often by **all** pupils to develop the broader mathematical skills of problem solving, reasoning and generalising.	Typical lessons consist of routine exercises that develop skills and techniques adequately but pupils have few opportunities to develop reasoning, problem solving and investigatory skills, or only the higher attainers are given such opportunities.
Involving all pupils	
Pupils exude enjoyment and involvement in the lesson.	Pupils enjoy making progress in an ordered environment.
Teachers ensure all pupils participate actively in whole-class activity, such as through using mini whiteboards in ways which involve all, or partner discussions.	Questioning and whole-class activities are pitched appropriately but do not involve all pupils' actively, for example, few hands up, questions directed to few pupils, some not attempting written tasks, mini whiteboards held up whenever pupils are ready so not all give answers or some copy from others.
Respect is conveyed for pupils' contributions so that many offer right and wrong comments.	Few pupils offer responses to whole-class questions although their work is generally correct.
Pupils naturally listen to and respond to each other's comments showing engagement with them.	Pupils listen to the teacher's and pupils' contributions and respond to them when asked to.
Developing independence in learning and assessment	
Pupils develop independence by recognising when their solutions are correct and persevering to overcome difficulties because they expect to be able to solve problems; the teacher's interventions support them in estimating and checking for themselves and in raising their confidence; pupils take responsibility for following up teachers' comments on their work and seek to understand where they have gone wrong.	Pupils produce generally correct work through support that does not develop independence in solving complete problems, such as through providing answers too readily or breaking down the problem so much that pupils do not know **why** the sequence of steps was chosen; for example, pupils do not attempt hard questions and wait for answers to be read out or check them from the answer book, or focus unduly on obtaining correct answers so amend wrong answers unthinkingly when the correct ones are read out, or ask for help at each step and are given directed steps to take rather than interventions that encourage thinking and confidence that they can succeed.

Features of good mathematics teaching	Features of satisfactory mathematics teaching
Teachers and pupils have a good grasp of what **all** pupils have learnt judged against criteria that they understand, not necessarily against learning objectives or targets; this is shown through pupil discussion, reflection, oral or written summaries or explanations, and ascertained by the teacher's monitoring throughout the lesson; for example, both teacher and pupil assess whether the pupil can explain why the formula for the area of a rectangle works.	Teachers and pupils make some accurate assessment of learning; for example, the teacher correctly reflects in a plenary what many pupils have achieved, pupils make an impressionistic assessment of their learning, such as using traffic lights or against a generic lesson title like 'solving equations'.
Teachers' marking identifies errors and underlying misconceptions and helps pupils to overcome difficulties, for example, by setting clear targets to which pupils respond and teachers check against.	Accurate marking by the teacher provides pupils with feedback; important work has been marked by pupils or teacher.
Pupils are clear about what they are expected to learn in the lesson and how to show evidence of this.	Pupils complete correct work and are aware of the lesson objectives but they are not clear about which ones pertain to them, what they mean, or what they need to do to meet them, for example, when objectives are phrased in terms of 'all', 'most' and 'some' pupils without indicating which pupils; when objectives are written down but pupils do not understand their meaning by the end of the lesson; when a large quantity of questions are set and pupils do not know how they relate to the objectives; or when pupils do not have an attainable target to work towards.
Teaching assistants know the pupils well, are well briefed on the concepts and expected misconceptions, and provide support throughout the lesson that enhances thinking and independence.	Teaching assistants facilitate the production of correct work, but may not be active throughout the lesson and may provide support that leads pupils through so many small steps that independence is not encouraged.

In practice

The following examples give some illustration of the prime and weaker aspects of teaching of mathematics in secondary schools.

The best teaching was rooted in developing pupils' understanding of key concepts. It was inclusive in terms of ensuring that all pupils made substantial progress, no matter what their starting points. In the outstanding lessons, the teachers had high expectations of pupils' enjoyment and achievement. They made conscious efforts to foster a spirit of enquiry, developing pupils' reasoning skills through approaches that saw problem-solving and investigation as integral to learning mathematics. They checked that everyone was challenged to think hard and they adapted how they were teaching to achieve this. As a result, their classrooms were vibrant places of learning.

Many pupils describe a lack of variety in their mathematics lessons, with typical lessons concentrating on the acquisition of skills, solution of routine exercises, and preparation for tests and examinations. They seem to accept that this is what learning mathematics should be like. When asked by inspectors, most pupils recognised the difference between just getting answers right and understanding the work. Nevertheless, many of those observed in lessons were content to have the right answers in their books when they did not know how to arrive at them. They frequently replicated steps in a method without thinking and sometimes altered answers, or waited until the teacher read them out before writing them down.

Prime practice: teaching for understanding

This Year 9 lesson on the volume of cylinders enabled pupils to improve their estimation skills greatly and to understand the formula to find volume.

First, each pupil wrote an estimate for the volume of a tea candle that was on their desk. These estimates were generally far below the actual volume. The teacher then used a demonstration on the interactive whiteboard, checking very carefully that everyone could interpret the two-dimensional representation of circular layers gradually building up and could explain how the formula for the volume linked to their previous knowledge. Pupils worked in pairs with everyday objects that were well chosen for their dimensions, making measurements and calculating volumes. In doing this, they became much clearer about the size of a cubic centimetre, estimating how many would fit into an object. At the end of this very well organised lesson, pupils were much more accurate in estimating the volume of the tea candle by eye and most were very surprised that it was many more cubic centimetres than they had initially estimated.

The planning of the lesson had skilfully brought about a mismatch between the pupils' initial estimates and the actual volume. This added greatly to their learning as their surprise deepened their thinking and led to discussion about why the two amounts differed.

Weaker factors: doing well but without understanding

A pupil correctly calculated the areas of circles of radius 5cm and 7cm, by applying the standard formula $A=\pi r^2$.

When the inspector asked her whether it was reasonable that the second area was nearly twice as much as the first, she immediately assumed her answer must be wrong, as she was not used to being asked to interpret her answers. After further discussion, it became clear that she had learnt how to use the formula to calculate the area of a circle as a number, but could not say what was meant by the area of a circle. The few circles drawn in her book were all the same size. She had learnt a method to obtain answers to a problem she did not understand.

How might it be improved?

The pupil's understanding would have been better if the teacher had:

- established at the beginning how well each pupil understood the concept of area
- provided experience of finding the areas of shapes drawn to their actual size
- used pupils' previous knowledge about areas of shapes to approximate the area of a circle, for example by sandwiching it between squares and/or polygons.

Understanding would also have been strengthened if the pupil had been asked the sorts of questions that would have made her think about what she was learning and how to interpret results.

Many pupils meet mathematical ideas one at a time and therefore do not appreciate the links within mathematics at all levels. For example, pupils who can shade in ¾ of a shape often have difficulty placing ¾ on the number line; they do not think of it as a number. Good teaching ensures that these important connections are forged, but the most effective teachers enable the pupils to make the links for themselves.

Prime practice: making links within mathematics

A sixth-form further mathematics lesson in which students investigated properties of 2x2 matrices of the form: $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

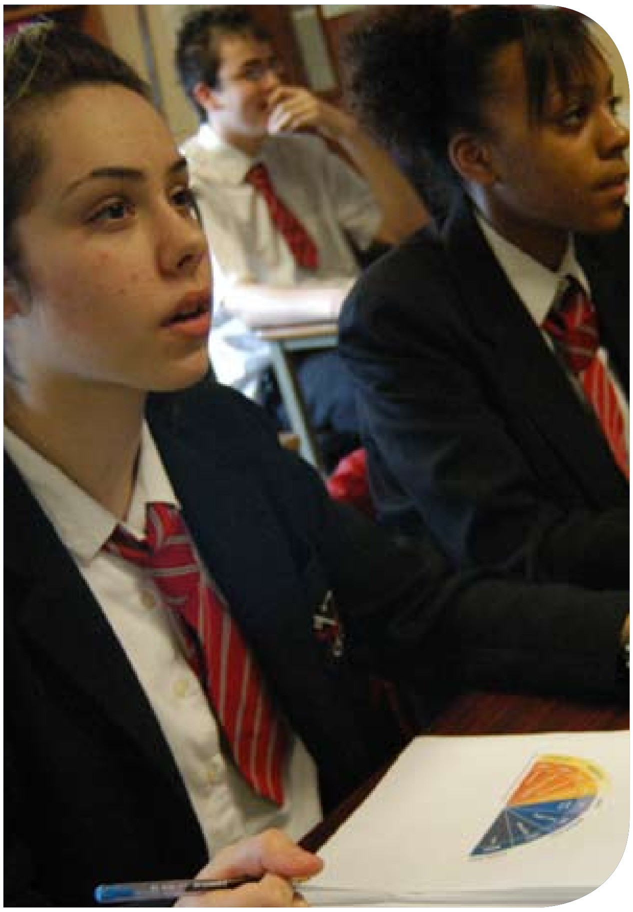
Guided by the teacher, the students found that each matrix represented an enlargement of scale factor $r = \sqrt{a^2 + b^2}$ with rotation by θ about the origin, where $r \cos \theta = a$, $r \sin \theta = b$. They established that the matrices had the same properties as complex numbers of form $a+ib$, and that the set formed a group and a ring. They therefore found links across the three topics of matrices, complex numbers and algebraic structure.

Teachers who have effective subject expertise know how to structure learning in ways that allow pupils to connect apparently different topics, and build on their earlier learning.

Prime practice: the mathematical progression of ideas

A Year 7 lesson introducing the calculation of probabilities.

A teacher, realising that probability is a difficult idea for many pupils, had made sure that pupils were used to marking fractions and decimals on a number line before they met the idea of the probability scale. He emphasised the need to consider equally likely outcomes in calculating probabilities through groups of three pupils playing a game which was based on the number of heads obtained from spinning two coins. At each turn, the player whose number came up scored a point. The pupils quickly learnt that this game was 'not fair'. They realised that there were four equally likely outcomes (tail-tail, tail-head, head-tail, head-head) rather than three (0, 1, 2 heads) and that this was why 1 head was more likely than either 0 or 2 heads.



More typically, pupils complete questions on calculating probabilities, for example, 'There are five red and three white balls in a bag. What is the probability of obtaining a red ball?', but do not connect this to work on marking probability estimates on a number line.

In the secondary lessons observed, the most prevalent style was one where the teacher demonstrated a new mathematical method which pupils then practised. When this approach was used well, teachers developed pupils' understanding of why the method worked through explanations and activities. They selected a suitable range of questions so that pupils developed the necessary breadth of skills and understanding of the applicability of the method. In weaker lessons, pupils were expected to memorise and apply the method.

Weaker factors: learning without understanding

In a Year 9 lesson, pupils learnt how to plot straight-line graphs but without appreciating the relationship between coordinate pairs and the equation of the graph, and with little idea how to interpret the gradient in terms of the slope.

The teacher showed the pupils how to substitute three values for x in an equation such as $y = 2x - 3$ to obtain three pairs of coordinates. Pupils plotted the three points and joined them with a straight line. They rarely extended the line beyond these three points. A few pupils had difficulty because they did not realise that the numbers on the axes needed to be regularly spaced, and this led to dog-leg graphs rather than perfectly straight lines. The teacher had not checked quickly all pupils' work to ensure they had scaled their axes appropriately or to point out the problem resulting from not doing so.

While pupils drew a selection of such graphs, the inspector asked some of them which other points were on the line. Most recognised only those where the line segment they had drawn passed through a point on the grid. They did not appreciate that the straight line consists of all points with coordinates (x,y) that fit the equation and no others, a principle that underpins much future graph-related work.

Some pupils could identify the gradient in the formula because they had been told it was the coefficient of x, but not by looking at the graph. These pupils had no concept of what gradient meant in terms of slope. They could usually determine the intercept from memory as the constant term in the formula, but could not explain why it was necessary to put $x = 0$ into the equation to find the intercept on the y axis.

How might it be improved?

Inaccuracies in pupils' work could have been spotted quite easily, if the teacher had checked the work throughout the lesson, looking especially at axes to pick up on errors in spacing and at the line segments drawn to check for position, straightness and length.

For better learning, the teacher might:

- pose questions to check understanding, for example, whether points such as (5.5, 8), (-10, -17), (3, 2), (100, 197) lie on the line with equation $y = 2x - 3$
- focus more on the meaning of 'gradient' and how it might be read directly from the graph as the increment in y for unit increment in x.

Learning could be extended by asking pupils what lines might be parallel to $y = 2x - 3$, or how could they use it to draw the lines with equations $y = 4x - 3$ and $y = -2x - 3$?

In the most effective lessons, teachers often presented new topics by challenging pupils to apply their mathematics to solve problems, drawing ideas from them and using probing questions to gauge their initial understanding and develop it. They sequenced learning carefully, helping pupils to make links to related areas of mathematics. The teachers listened to pupils carefully and observed their work throughout the lesson, aiming to identify any misconceptions or barriers to understanding.

Prime practice: an interesting approach to a new topic

This lesson was a challenging introduction to three-dimensional applications of Pythagoras' theorem for a top set of Year 10 pupils. The approach enabled pupils to see how their existing knowledge of two-dimensional Pythagoras' theorem might be extended to the new three-dimensional context.

The teacher provided models of a cuboid and a square-based pyramid made from straws. She asked the pupils to find the length of the diagonal of the cuboid and the height of the pyramid. After briefly inviting questions, she let the pupils get on with the task, circulating around the classroom to ensure they were all on a fruitful track. She intervened only if pupils appeared stuck when, by asking questions, she ascertained their thinking and moved it on. She did not steer pupils, at any stage, towards a particular method. This was a successful approach with alternative methods arising, which she discussed with the whole class later in the lesson.

In such circumstances, pupils become confident learners as they develop skills in articulating their thinking about mathematics. They learn to make sense of ideas, and reason and justify their methods and solutions because discussion is a regular feature. Learning is therefore active and cumulative; they make good progress because they make connections with their existing knowledge and understanding.

Prime practice: pupils persevering

This was a lesson on constructing triangles for low-attaining Year 10 pupils, who discovered for themselves why pairs of compasses are needed for constructing some triangles: it became a meaningful problem. (More usually, pupils are guided through the standard construction. As a result, they do not necessarily realise that to draw the triangle accurately without compasses is not possible.)

The pupils were asked to draw triangles of given dimensions for the three sides. They had access to rulers, pencils, protractors and pairs of compasses. They tried to carry out the task; the teacher gave them no extra advice or support at that stage. After 10 minutes, the pupils were concerned that they could draw two sides with the correct length but not the third. In essence, they had discovered the problem with the construction. One pupil used the compasses to draw some arcs but could not see how he could complete the triangle. The teacher used this pupil's ideas, demonstrating to the class what he had done, and asking them to think how it could help them. Again they worked in groups and, gradually, pupils were able to use the compasses effectively to draw the triangles. The fact that they had persevered with the task until they found the method, and realised the reason for it, gave them a very good understanding of how to draw triangles when the lengths of the three sides were given.

Most teachers establish clear routines and pupils pay attention to their explanations. A common shortcoming, however, is that teachers do not give sufficient attention to whether all the pupils have understood the work. Errors or misconceptions are not always exposed: some pupils get the answers from their peers, others alter their answers to the stated correct ones, and some do not progress far through the exercise. Some teachers refer frequently to how the work relates to examination requirements, which can be helpful for the pupils, but they do not monitor the quality of the learning or assess whether the work makes sense to the pupils.

Weaker factors: learning without understanding

A Year 10 lesson on finding the fraction of an amount. The teacher had a clear view of the types of question pupils needed to be able to do to be successful in the foundation tier of the GCSE examinations.

The teacher showed the pupils how to calculate ¾ of £10.80 by dividing by 4 and multiplying by 3. He did not explain why. One pupil called out, offering her own method. The teacher discouraged her, but she insisted on telling him, even though he didn't want to know that 'You halve it, then halve it again, and add the two halves together.' She probably meant that she would add the first ½ to the halved half, ¼, to make ¾ but the teacher did not pick up on this clue. Instead he repeated his method of dividing by the denominator and multiplying by the numerator, all as one calculation.

When pupils tackled similar questions, many of them reached the right answers, but none of the pupils to whom the inspector spoke could explain why they were dividing by the denominator and multiplying by the numerator. Some pupils could understand why dividing by the denominator gave one part and, coaxed for the answer, why they would then multiply by the numerator.

The teacher moved around the classroom while pupils worked steadily through the exercise, helping those who were stuck by demonstrating the same method again.

Discussing the lesson afterwards with the inspector, the teacher could see how he had emphasised the technique without any reference to understanding. He commented to the inspector the next day that the discussion had made him reflect critically on the methods he often used in his teaching.

How might it be improved?

A different starting point would have been to use an easy example that pupils could do in their heads, say ¾ of £20 or £10, and then probe how they worked it out. Listening to their responses could provide insights into their thinking, and the teacher could use their explanations as the starting point for developing a method.

Learning would have been better if pupils had been enabled to make the connection between finding the fraction, ¼, of something and dividing it by 4 to give four equal parts. Practical equipment might help, although most pupils find money easy to understand.

By practising only one method at a time, pupils do not gain the confidence and intellectual flexibility they need. This can fragment the subject, because it is presented as a collection of apparently arbitrary rules for memorising. The rules can be incomplete or confusing.

Weaker factors: unhelpful rules

Teachers usually introduce rules to help pupils remember results or steps in methods. However, few are always true and many are never fully developed so that pupils understand the context of a rule. Here are three examples.

(a) 'To multiply by 10 you add a nought' but $3.4 \times 10 \neq 3.40$

Discussion about place value is the most powerful way of tackling multiplying by 10.

(b) 'Always measure from the end of the ruler' but this doesn't always work, and is a common mistake young pupils make when learning to measure. Another error is that they measure from 1 on the scale.

The emphasis should be placed on measuring from 0, which is often at the end of a tape measure but the scale on most rulers starts a little way in from the end of the ruler.

(c) 'Two minuses make a plus' $-5 \times -3 = +15$ but $-5 + -3 \neq +8$.

This rule is an inaccurate simplification of a generalisation. Incorrectly applied 'rules' on signs and operations are the source of many errors for secondary pupils in work on number and algebra, usually because the 'rule' is learned without understanding and they do not take into account the different contexts of the operations of multiplication and addition, and the positive and negative states.

How might it be improved?

Where it is considered that rules might be useful, they should be unambiguous and developed with the pupils. The unthinking use of rules should be discouraged.

The lack of development of 'using and applying mathematics' is a prime reason why pupils' understanding of mathematics lags behind their proficiency in executing techniques and recalling facts. Some mathematics departments are introducing approaches that focus more on pupils' learning; for example, starting lessons with tasks or problems that make pupils think, and encouraging discussion. More generally, teachers need support and guidance in planning, teaching and assessing 'using and applying mathematics' and, thereby, in teaching for understanding.

Prime practice: teaching mathematical thinking

The context of this Year 9 problem-solving lesson was a series of questions about the number of permutations of letters in different names, such as LUCY, ALI or WAYNE.

Rather than show pupils the standard formula, the teacher provided them with an opportunity to find their own solutions. This was not as haphazard as it might seem, because he also had a very clear idea about which kinds of thinking he wanted to encourage and the point he wanted pupils to move towards. This type of problem solving might be characterised as 'open in the middle' rather than open-ended.

The lesson objectives were 'Pupils will learn: the value of working systematically to solve problems; to refine their understanding of the methods they develop; to refine their oral and written explanations of their methods; and the value of reducing a problem to a simpler case.'

For much of the lesson, the teacher's role was to listen to pupils explaining their ideas, to encourage and nurture any systematic thinking, and to intervene with additional problems when appropriate. Mini-plenaries were used as appropriate to encourage pupils to share their ideas with the class, draw out key ideas that emerged and stimulate further thought about variations on the original problem. By the end of the lesson, most pupils had worked out that the number of permutations of n distinct letters would be $n! = 1 \times 2 \times 3 \times \ldots \times n$. More importantly, they understood the importance of making systematic lists and therefore understood in a concrete sense the recursive nature of the solution: that a five-letter word could begin with any of the five letters, followed by any of the 24 permutations of the other four letters, giving $5 \times 24 = 120$, and that 24 arose as 4 (starting letters) × 6 (ways of arranging the other three letters), and 6 as 3×2, and so on.

Variations of the problem were held in reserve, such as EMMA, ANN, GEMMA and DONALD, leading to the generalised problem of counting permutations when some letters repeat. Many pupils recognised that having two letters the same halved the number of possibilities and that having three letters the same reduced the number further, but realised that this needed more thought.

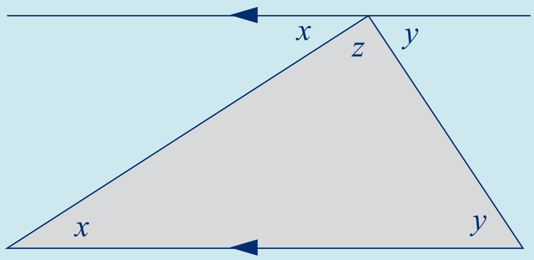
The following example illustrates how good expertise enables mathematical correctness to underpin explanations without making the ideas inaccessible. It pays attention to detail and is precise.

Prime practice: mathematical correctness

A Year 7 lesson on the sum of the angles in a triangle.

The teacher started the lesson by rehearsing what pupils knew about the angle properties of intersecting and parallel lines. Pupils were expected to recall facts about vertically opposite, corresponding, alternate and supplementary angles. Pupils could explain that vertically opposite angles had to be equal because they were both supplementary to the same angle (totalling 180 degrees together).

Pupils cut out triangles and tore off the corners, but each pupil had a different triangle, and all were pasted onto a class poster. The teacher elicited from them a proof that the angles of a triangle are supplementary by drawing a line through a vertex parallel to the opposite side of a triangle and encouraging them to apply their existing knowledge.



Too many secondary pupils expect to find learning mathematics difficult and seem to accept that this is so. They know the difference between being proficient at carrying out techniques and understanding the underlying mathematical ideas. They recognise that they often learn methods by following teachers' illustrative examples and working through many exercises, obtaining correct answers without really understanding why. Some pupils quite like the security of being given rules and structured methods, but tend to become dependent on them and, in turn, on their teachers. Many pupils refer frequently to prompts provided by the teacher about how to carry out a technique, but such methods, memorised without understanding, often later become confused or forgotten, and subsequent learning becomes insecure. Moreover, such an approach fragments the mathematics curriculum.

Weaker factors: right answers but insecure learning

A Year 8 lesson in which pupils learnt a method for solving simple equations of the form $2x + 5 = 13$ and $5x - 7 = 8$ but with superficial understanding. Although the technique was initially demonstrated correctly, pupils' thinking was not developed in a way that would support further learning.

The teacher demonstrated correctly the technique of adding to or subtracting from each side of the equation to create a simpler equation, such as $2x = 8$ and $5x = 15$, and then dividing by the coefficient of x. Pupils were set an exercise with around 20 similar questions. The teacher gave help as needed until most had answered several questions. The answers were read out and pupils gave themselves a mark out of 20, with many scoring full marks.

Noticing that every question had the same format, and that several pupils had omitted their working, the inspector tried out some variations with a few pupils. These pupils tackled $3 + 18x = 42$ with confidence. When asked to explain how they arrived at their (incorrect) answer of $x = 8$, they said they had subtracted 18 and divided by 3. Their choices were based on the position of the numbers 3 and 18 in the equation, and not their meaning.

By setting all questions in the same format, pupils took a short cut to the answers and did not think about the method they had originally been taught. Critically, the teacher gained a false impression of pupils' learning, believing they could now solve simple equations, whereas this was in fact restricted to a particular subset of such equations. Pupils could not extend their approach to any other equations.

How might it be improved?

To improve learning in this lesson the teacher, when first demonstrating the method, could have checked that pupils understood each step by selecting examples in which the positions of the numbers within the equations varied. Following this by independent work that included a range of equations would allow any misconceptions to be exposed. Insisting on good presentation of solutions would help reinforce the need for logical thinking.

Sometimes, teachers do not assess the extent of pupils' difficulty accurately. Typically, the teacher asks a question, very few hands go up, a selected pupil answers it well, and the teacher assumes that all the class knows and understands. Actually, the pupils' books and discussions indicate that many are unclear.

Weaker factors: teachers not circulating

A lesson starter in which the teacher was unaware that pupils' progress was very variable.

A low-attaining Year 7 class was given a worksheet as a quick lesson starter. It contained several questions of the form 400 + 300 = 600 +…

The teacher did not circulate to check anyone's work so did not realise that some pupils had written 1,300 and attempted the remaining questions incorrectly as additions. While some pupils finished very quickly, others had managed only a few questions. The speed of responses showed that the pupils who already knew how to do this work were not extended and those who did not know gained little benefit.

How might it be improved?

If the teacher had moved around the class quickly checking pupils' first answers, or used mini whiteboards for the starter activity, he would have identified those pupils who were making the mistake of adding the three numbers. Continuing to circulate as pupils worked would show the teacher who was struggling and who was not challenged by the task.

Learning may have been better if the questions had been tailored to pupils' prior attainment, perhaps through two or three worksheets at different levels of challenge.

At some stage, most teachers are asked questions by pupils about the usefulness of what is being taught. Many feel uncomfortable with these, especially with more abstract concepts, often resorting to answering, 'It's on the syllabus'. Few talk about specific applications or explain the power of being able to think mathematically.

Prime practice: applications of mathematics

A teacher's response to 'Why do we have to learn algebra? What use will it be?'

The teacher reminded the pupils that algebra is important in science because formulae are needed to express the laws of science; spreadsheets use algebraic formulae and are a very powerful tool used by thousands of businesses; and computer graphics require complicated algebraic methods to make sure that objects are portrayed correctly. He also pointed out the power of algebraic notation as a means of communicating within mathematics.

The range of pupils' errors and misconceptions when they learn algebra means that their written work on algebraic topics is an important source of clues to their thinking. The best teachers focus on pupils' errors as a learning point. They spot the significant misconceptions which are illuminated by pupils' mistakes. Skilful teachers select a range of questions for the pupils to tackle, making sure that all pupils are challenged and each is exposed to potential misconceptions.

Prime practice: the need for diagnostic marking

A teacher's views on the importance of marking pupils' work on 'collecting like terms' in algebra.

In the lesson, the teacher developed an activity for pupils adapted from a National Strategy training pack. She provided plenty of graduated practice that gradually introduced complications such as negative terms. Her explanation to the pupils emphasised that terms could be added and subtracted in any order, provided that 'positive terms stay positive and negatives stay negative'. She explained that many pupils had trouble with this topic, and that she used a specific range of questions to enable the different types of error to be revealed. Because they would need this skill often in later work, she would mark the work herself to identify any misconceptions.

Although most secondary teachers recognise the importance of pedagogic skills in mathematics, they often comment on the pressures of external assessments on them and their pupils. Feeling constrained by these pressures and by time, many concentrate on approaches they believe prepare pupils for tests and examinations, in effect, 'teaching to the test'. This practice is widespread and is a significant barrier to improvement.

Weaker factors: poor use of subject expertise

A Year 8 lesson following homework on 'collecting like terms' in algebra. Although the teacher realised that the pupils had had difficulty with the homework, the teacher's subsequent approach was unhelpful because it was not mathematically precise and compounded existing misconceptions.

In the starter activity, pupils took turns to go to the interactive whiteboard to match equivalent algebraic expressions by collecting like terms. The examples involved positive terms only. Pupils then marked their homework on the same topic, the teacher reading out answers. When it became apparent that several had not completed the homework, the teacher amended his lesson to explain the topic again, using the imagery of counting apples, bananas, and so on. When one question involved both c (cats) and c^2, the teacher stretched the imagery, saying 'c^2 is different to c. It is like a cat with two black ears'. Despite the bizarre imagery, pupils were eventually able to complete the homework. However, the idea that algebraic terms represent objects is unhelpful; indeed such a method reinforces this misconception.

How might it be improved?

The teacher might have found it useful to have marked this homework himself.

An approach to collecting like terms that generalises arithmetic would be more powerful mathematically; for instance two 7s added to three 7s makes five 7s might help with $2c + 3c = 5c$, and 7^2 is clearly different from 7.

The interactive whiteboard features in many, but not all, secondary classrooms, bringing positives and negatives to teaching and learning. Good practice includes the use of high-quality diagrams and relevant software to support learning through, for example, construction of graphs or visualisation of transformations. However, too often teachers use whiteboards simply for PowerPoint presentations with no interaction by the pupils.

Prime practice: ICT

Use of the interactive whiteboard and internet to scale a picture from very tiny to extremely large.

A Year 7 class, working on scales, was shown a website using the interactive whiteboard where a picture was scaled from 10^{-16} metres to 10^{16} metres, that is, 10,000,000,000,000,000 metres. The pupils were amazed; they became animated and excited, discussing the effect of scaling by powers of 10. The teacher posed questions, asking pupils, in pairs, to describe and explain their thinking. Some presented this from the front of the class with their peers critically appraising it in a lively discussion.

Some of the report's examples of primary mathematics lessons are pertinent to secondary mathematics teaching. In both phases, good curricular planning provides pupils with opportunities to apply mathematics to a variety of interesting tasks, enabling them to choose approaches and reason and refine their thinking in the light of their solutions. Teachers encourage pupils to discuss mathematical problems in depth and this helps to build their confidence. In a primary school where developing pupils' understanding was promoted effectively, pupils were confident in 'thinking aloud' and were not afraid to have their mistakes used to help others.

Prime practice: discussion

An interesting approach to ratio and proportion with Year 6 pupils with lots of discussion.

The teacher engaged pupils throughout the lesson by incorporating many activities and encouraging discussion and argument in pairs until an answer was agreed. A reverse approach to solving problems was effective in getting pupils to think about clarity of expression. The teacher put one cup of fruit juice and two cups of water in a jug and one cup of fruit juice and three cups of water into another jug. The contents of both jugs were poured into a bowl, which, by then, contained 2,800ml of the mixture.

The teacher posed the question: how many millilitres of fruit juice are in the bowl? Pupils worked in pairs with jottings on mini whiteboards. Many struggled at first, argued with each other, but eventually worked out that $\frac{2}{7}$ of the mixture would be juice. Pupils were then asked to write a question, in words not just numbers, to match the problem they had just solved. As the lesson went on, middle-attaining pupils in the group completed more, similar questions and higher-attaining pupils were given some that required much deeper thinking.

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