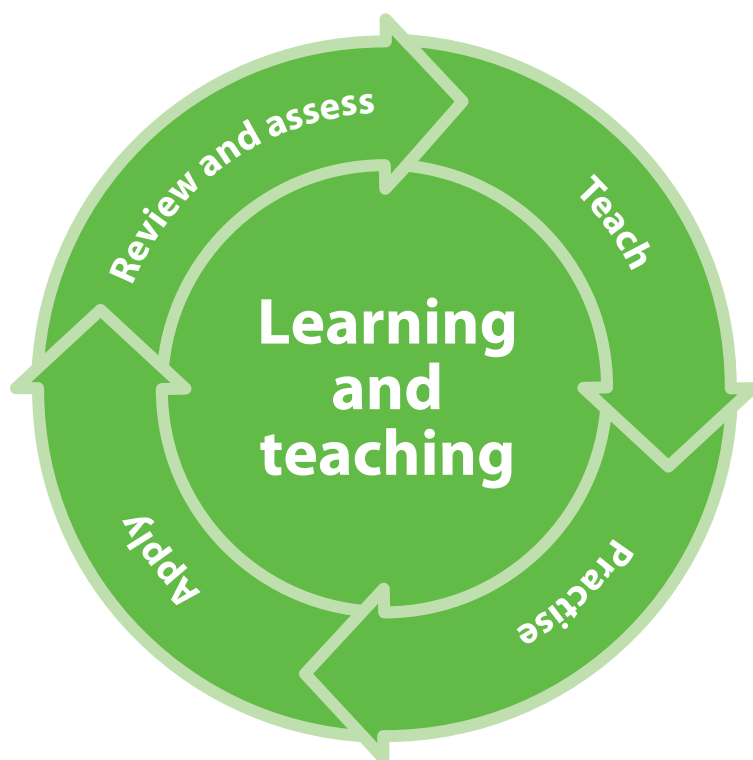


Securing Level 3 in Mathematics



Securing level 3 in Mathematics

These materials are intended to support you in ensuring that as many children as possible reach level 3 by the end of Year 4. The guidance identifies key areas of learning that children need to secure to attain level 3 in mathematics. While you will integrate the ideas from these materials into your on-going planning, they could also be used to plan targeted support for particular groups of children. There is a double-page spread for each of the six areas of mathematics:

- Understanding the number system
- Securing mental addition and subtraction
- Understanding and using multiplication and division
- Visualising and classifying shapes
- Solving problems involving money and measures
- Reading and interpreting tables and graphs

Remember:

Every day is a mental mathematics day – ensure that children engage in 10 to 15 minutes of mental work each day to practise and reinforce knowledge and skills in mathematics. *Don't expect confidence in working mentally if practice and repetition has not taken place.*

Hands-on learning is still important – provide appropriate practical equipment for children to use and manipulate, to help them to explore how and why things work and to learn to visualise, describe and represent what is in front of them. *Don't just talk about weighing scales, use one; using apparatus is better than imagining how it works.*

Seeing mathematics through models and images supports learning – help children to see how mathematics works and can be represented through a physical object, picture or diagram such as place value cards, number stick, number lines, representations of fractional parts. *Don't expect children to visualise and 'see' how something works if they have no models and images to draw on.*

Talking mathematics clarifies and refines thinking – give children the vocabulary and language of mathematics; provide activities and time for them to discuss mathematics using this language. Teach children the precision of language, for example using: 'regular', 'equals', 'factor' and how to express their reasoning using language such as 'if ... then', 'because', 'cannot be', 'never', 'sometimes', 'always'. *Don't expect children to explain or provide reasons if they have no opportunity to use, develop and refine the language to do so.*

Make mathematics interesting – share your interest in mathematics with the children. Give children mathematics that engages them in: estimating and finding out about the number of people in a sports stadium, testing out ideas such as whether the sum of three consecutive numbers is always a multiple of six, answering intriguing questions such as how many times their heart beats in 10 minutes compared with that of an elephant or a mouse. *Don't expect children to be interested in mathematics if you don't share an interest and all their mathematics is routine and dull.*

Learning from mistakes should build up children's confidence – look out for mistakes and encourage children to recognise that making mistakes is something everyone does. Show children common errors and get them to identify and correct them. Encourage children to work with a partner and share their work. *Don't just tell children something is wrong; help them to see what went right and to identify when it went wrong.*

Understanding the number system

Level 3 standards to be achieved:

Count in equal steps including counting over boundaries

Compare and order whole numbers to 1000 using knowledge of the value of each digit

Multiply and divide numbers by 10 understanding the effect

Round whole numbers to the nearest 10 and 100 and use this to make approximations

Use sense of number to make approximations and compare numbers and amounts

Understand and use negative numbers and numbers with one or two decimal places in contexts involving measures or money

Read, write and recognise fractions

For children to attain level 3, they need to:

- read and write numbers that contain zero as a place holder, understanding its role
- count in ones and tens backwards and forwards over boundaries e.g. 187, 197, 207...
- identify significant digits to compare and order two or more numbers
- position numbers approximately on partially marked number lines
- round a number by identifying the multiple of 10/100 that it is closest to
- compare and order negative and positive numbers using a number line
- identify the value of each digit in decimal measures such as money
- understand the role of the numerator and denominator of a fraction
- identify, read and write fractions to describe a proportion of a shape or amount e.g. appreciate that, since there are 100cm in 1m, 1cm is equal to 1/100 of a metre

Make sure that:

you regularly use partially-marked or blank number lines as part of oral and mental work

you use models such as place-value cards, beadstrings and base ten apparatus to help children develop a secure understanding of place value

children regularly count in equal steps as part of oral and mental work – include counting in decimal numbers and measures, for example: 25cm, 50cm, 75cm, 1m, 1m 25cm...

children position fractions on number lines and count in fractions e.g. 1/5, 2/5, 3/5...

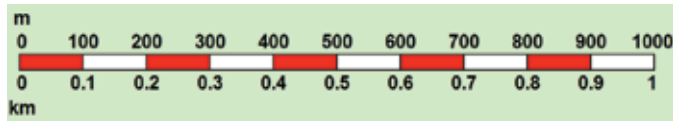
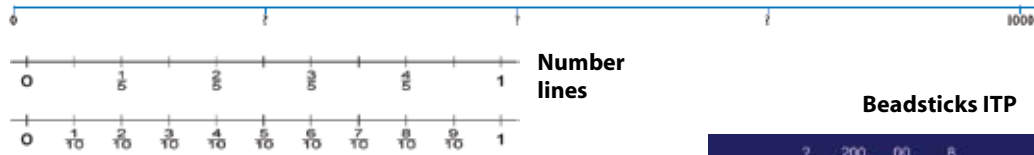
you use calculators or place value grids to demonstrate the effect of multiplying and dividing numbers by 10 – help children appreciate that moving each digit one column to the left makes the number 10 times greater e.g. 50p is 10 times greater than 5p

children experience situations or problems where rounding or approximating are needed

children regularly have opportunities to explain how they used their understanding of the number system to help them to calculate efficiently

Understanding the number system

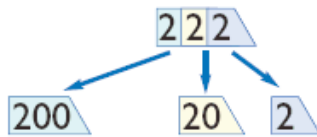
Teaching and learning resources



Beadsticks ITP



Place value cards



Intervention materials

Springboard 4

•Units 1, 4 and 7

Overcoming barriers in mathematics: level 2 to 3

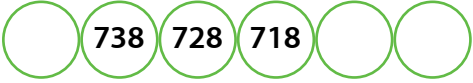

- Can I read, write and partition whole numbers to 1000?
- Can I order two-digit and three-digit numbers and position these on a number line?
- Can I round whole numbers up to 1000 to the nearest 10 or 100?
- Can I multiply one-digit and two-digit numbers by 10 and 100?
- Can I read and write fractions and explain their meaning?

Wave 3 materials

Fractions ITP



Assessment checklist

I can statements	Assessment examples
<i>I can find a missing number in a sequence</i>	Find the next two numbers: 189, 192, 195, __, __ Find the missing numbers: 
<i>I understand what each digit in a three-digit number is worth and can explain how I know</i>	Max puts these numbers in order from smallest to largest. What would be the third number? 835, 535, 538, 388, 508 How many three-digit numbers is it possible to write that have 6 in the tens column?
<i>I can multiply/divide a number by 10</i>	Ann says that 38×10 is 308. Explain how you know she is wrong. How many £10 notes are needed to make £470?
<i>I can round numbers to find approximate answers to calculations or problems</i>	Which of these numbers is closest to the answer of $342 - 119$: 200 220 230 250 300
<i>I can order negative and positive numbers</i>	The temperature at noon on Monday is -2°C and on Tuesday is -6°C . Which day was warmer at noon? Explain how you know.
<i>I can solve problems that involve decimal numbers as money or measures</i>	How many 10p pieces do you need to make £2.30? Samir walks 0.8 km. How many metres is this?
<i>I can recognise and write a fraction of a shape</i>	Would a chocolate lover rather have $\frac{1}{2}$ or $\frac{1}{5}$ of this bar of chocolate? Explain your answer. 

Securing mental addition and subtraction

Level 3 standards to be achieved:

Use number facts and place value to add and subtract two-digit numbers efficiently

Use informal jottings, including number lines, to record stages in mental calculations

Understand that addition and subtraction are inverse operations

Select from a range of mental methods depending on the numbers in a calculation

Use appropriate calculation methods to solve problems involving addition/subtraction

Draw on experience of mental methods to explain steps in written methods for addition and subtraction

For children to attain level 3, they need to:

- use known facts to work out related ones, for example use $7 + 8 = 15$ to work out $37 + 8 = 45$
- partition two-digit numbers to support efficient calculation
- draw their own number lines to show steps in a calculation
- use the inverse operation to check answers, particularly for subtraction
- identify the appropriate calculation(s) needed to solve a problem
- consider the numbers involved in a particular calculation to make appropriate decisions on which mental method to choose
- work out subtraction by counting backwards and by counting forwards and decide which is the more efficient method for particular calculations
- use correct mathematical vocabulary to describe/explain their calculation methods

Make sure that:

children rehearse addition and subtraction facts regularly through daily oral and mental work

children handle, make and draw shapes rather than simply look at drawings of shapes

you move children from counting in ones to using known facts to calculate efficiently

children are able to add and subtract multiples of 10 and 100 rapidly using known facts

you pick up on common errors such as subtracting the wrong digit, for example saying that $92 - 38 = 66$ because $90 - 30 = 60$ and $8 - 2 = 6$

children can find missing numbers in calculations such as $82 - \square = 39$

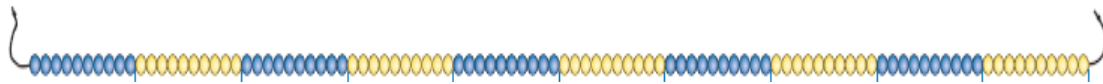
children understand and use appropriate vocabulary including the term 'difference'

children have regular opportunities to explain and compare calculation methods

you use children's understanding of mental methods such as partitioning as the basis for the development of written methods, initially using expanded methods

Securing mental addition and subtraction

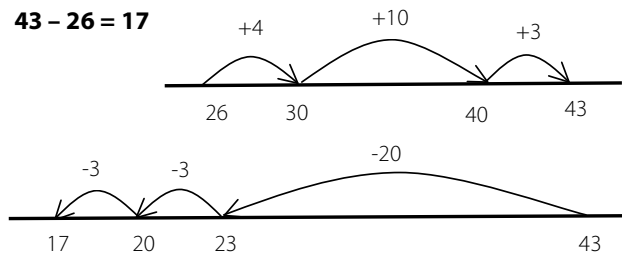
Teaching and learning resources



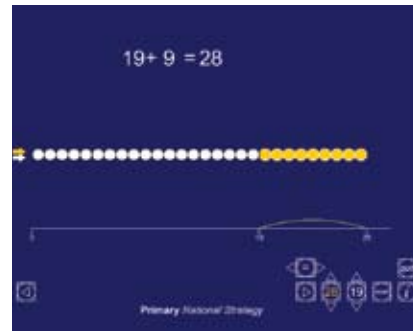
Beadstrings

Number lines

$$43 - 26 = 17$$



Difference ITP



Addition and subtraction facts spreadsheet



Intervention materials

Springboard 4

- Units 2, 3 and 9

Overcoming barriers in mathematics: level 2 to 3

- Can I recall and use addition and subtraction facts for numbers to 20?
- Can I find pairs of numbers that total 100?
- Can I subtract mentally combinations of one-digit and two-digit numbers?
- Can I say a subtraction fact that is the inverse of an addition fact and vice versa?

Wave 3 materials

- +/- Year 4 booklet 3

Assessment checklist

I can statements	Assessment examples
<i>I can add two-digit numbers, choosing an efficient method</i>	What number is 27 more than 45? What number is 19 more than 45? Explain how you worked out these two calculations. Work out the missing digits: $3 \square + \square 2 = 85$
<i>I can subtract two-digit numbers, choosing an efficient method</i>	Work out these two subtraction calculations: $72 - 5$ $72 - 68$ Did you use the same method for each calculation? If not, why not? Explain your methods to a friend and compare your methods with theirs.
<i>I can record the steps of my addition/subtraction methods</i>	Work out $47 + 38$. Record how you work this out and explain what you have written.
<i>I can check my answer to a calculation</i>	Paul says $72 - 15 = 63$. Write down an addition calculation that you could do to check this. Paul's working is: $70 - 10 = 60$ and $5 - 2 = 3$ so $72 - 15 = 63$ Can you identify where Paul has gone wrong?
<i>I can use addition and subtraction to solve problems</i>	I have 45p in my money bank and 28p in my purse. How much more money do I need to buy a comic that costs £1?

Understanding and using multiplication and division

Level 3 standards to be achieved:

Use multiplication/division facts and place value to solve problems involving two-digit numbers

Use informal jottings to record steps of mental calculations

Understand that multiplication and division are inverse operations

Understand that division can involve both equal sharing and equal grouping

Use division and multiplication to find fractions of amounts

Use understanding of mental methods and place value to start to develop secure written methods for multiplication and division

For children to attain level 3, they need to:

- appreciate that situations involving repeated addition are more efficiently represented using multiplication
- use known facts to work out related ones e.g. use $3 \times 4 = 12$ to answer $30 \times 4 = 120$
- represent arrays using multiplication and multiplication calculations using arrays
- use partitioning to multiply a two-digit number by a single digit and record steps
- interpret division as the inverse of multiplication, for example understanding that $24 \div 4$ can be found by working out 'How many 4s make 24?'
- divide a two-digit number by a single digit number through splitting it into sensible 'chunks'
- find and interpret remainders in division, rounding up or down where appropriate
- find a unit fraction (e.g. $\frac{1}{5}$) of an amount using division, then multiply the answer to find non-unit fractions (e.g. $\frac{2}{5}$, $\frac{3}{5}$...)

Make sure that:

children rehearse multiplication and division facts regularly through daily oral and mental work

you help children move on to using multiplication if they continue to use repeated addition when it has become inefficient

children can use known facts to multiply or divide multiples of 10, working out for example 30×5 or $280 \div 4$

children are able to recognise when a word problem involves multiplication or division

children recognise 'special case' calculations, for example dividing by 4 through halving and halving again

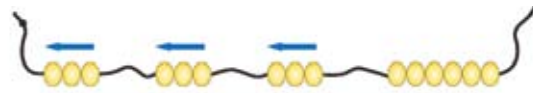
children can find missing numbers in calculations such as $\square \div 5 = 12$

children have regular opportunities to explain and compare calculation methods

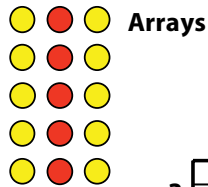
you use children's understanding of mental methods as the basis for the development of written methods such as the grid method and chunking

Understanding and using multiplication and division

Teaching and learning resources

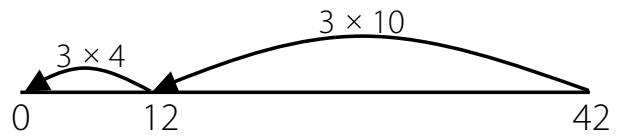


Division by grouping using beadstrings

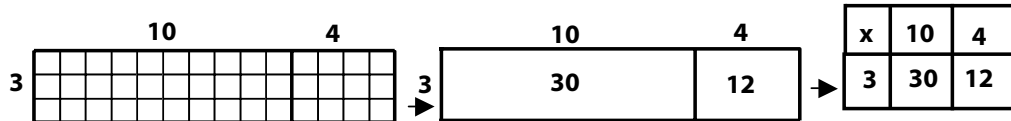


Arrays

Division by chunking on a number line $42 \div 3$



Multiplication: progression from arrays to the grid method



Intervention materials

Springboard 4

- Units 5, 9 and 10

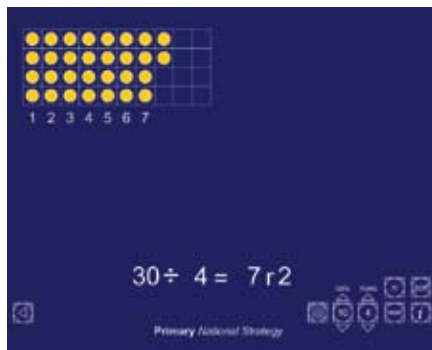
Overcoming barriers in mathematics: level 2 to 3

- Can I recall multiplication and division facts for the 2, 3, 4, 5 and 10 times tables?
- Can I use understanding of multiplication and division to solve problems?
- Can I multiply a two-digit number by a one-digit number?
- Can I divide a two-digit number by a one-digit number and explain any remainder?
- Can I find a unit fraction of a shape, number or quantity by splitting it into the correct number of equal parts?

Wave 3 materials

- \times/\div Year 4 booklets 1, 2, 4 and 5

Remainders after division ITP



Assessment checklist

I can statements	Assessment examples
<i>I can give the multiplication sentence that is linked to a division sentence and vice versa</i>	What multiplication could you work out to check $32 \div 4 = 8$? What is the missing number: $35 \div \square = 5$ How do you know?
<i>I can multiply a two-digit by a one-digit number and record the steps I take</i>	What is 20×3 ? Use your answer to work out 21×3 . Explain how you did this. What is the total value of seventeen 5p pieces? Record your working.
<i>I can divide a two-digit by a one-digit number and record the steps I take</i>	How can you use the fact that $60 \div 3 = 20$ to help you find $72 \div 3$? Divide 75 by 5, recording your working.
<i>I can solve problems that involve multiplication or division</i>	My dad does 25 minutes of exercise every day. How much exercise does he do in a week? 36 children need to sit on benches. Five children can sit on one bench. How many benches are needed?
<i>I can find fractions of amounts</i>	There are 28 children in the class. $\frac{3}{4}$ of them are girls. How many girls is this?

Visualising and classifying shapes

Level 3 standards to be achieved:

Name 2-D and 3-D shapes and recognise them in different orientations

Recognise 3-D shapes from 2-D drawings

Describe properties of 2-D and 3-D shapes using appropriate vocabulary

Use properties to sort and classify sets of shapes

Recognise symmetry in 2-D shapes

Visualise shapes from descriptions or partial drawings

For children to attain level 3, they need to:

- describe and sort 2-D shapes using a range of properties including number of sides, equal sides and number of right angles
- describe and sort 3-D shapes using number and shape of faces, number of edges and vertices
- compare shapes by describing what is the same and what is different about them
- use shape vocabulary accurately including: names of common shapes, 2-D, side, vertex, polygon, regular, irregular, 3-D, face, edge, net, prism
- understand that, in regular shapes, all sides are equal and all angles are equal
- classify shapes and sort them onto diagrams such as Venn and Carroll diagrams
- draw on their practical experience of 2-D and 3-D shapes to visualise shapes and patterns

Make sure that:

shape and space activities are included in oral and mental starters

children handle, draw and make shapes rather than simply looking at representations of them

children have opportunities to draw and manipulate shapes using squared and isometric paper and ICT as well as art straws, construction kits or pinboards

you model and prompt accurate and precise use of shape vocabulary and display key terms

activities involve structured opportunities for children to use shape vocabulary

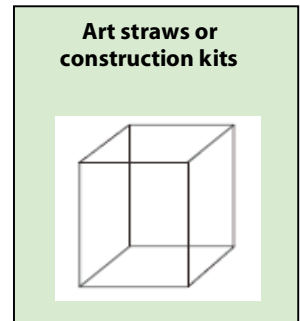
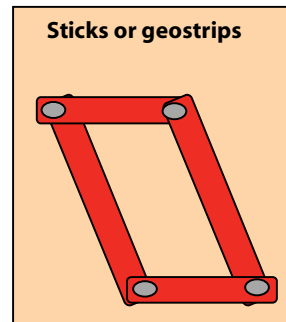
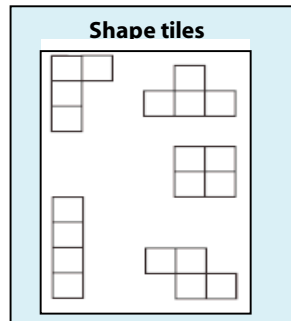
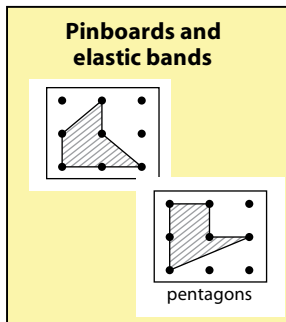
children use shapes to solve problems, for example investigating and describing the different shapes that can be created by placing two given shapes together side by side

children are given opportunities to develop visualisation skills, for example being asked to complete a partially-drawn shape from its description

Visualising and classifying shapes

Teaching and learning resources

Practical equipment to make shapes



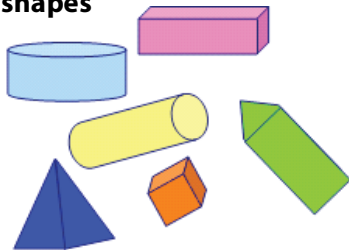
Isometric grid ITP



Fixing points ITP



Sets of plastic or wooden shapes

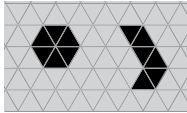

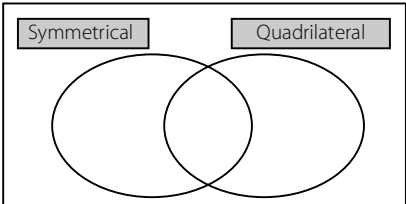



Intervention materials

Overcoming barriers in mathematics: level 2 to 3

- Can I make, name and describe 2-D and 3-D shapes?
- Can I sort shapes choosing my own criteria?

Assessment checklist

I can statements	Assessment examples
<i>I can recognise 2-D and 3-D shapes and describe their properties</i>	Describe some ways in which these two shapes are the same and some ways in which they are different. Which shape is regular? Describe how you know. 
<i>I can sort shapes describing how I have classified them</i>	Place the shapes below in the correct place in the Venn diagram. Make one shape of your own to add to each section of the diagram 
<i>I can identify whether shapes are symmetrical</i>	
<i>I can visualise shapes</i>	On the grid join dots to make a triangle which does not have a right angle. Use a ruler. 

Solving problems involving money and measures

Level 3 standards to be achieved:

Identify the appropriate operation(s) needed to solve a problem

Record working for each calculation needed to solve a problem

Understand and use £.p notation

Understand how to use subtraction to find change

Recognise units of measurement and make simple conversions when needed to solve problems involving measures

Work out simple time differences by counting on from one time to another

For children to attain level 3, they need to:

- identify important information in a word problem and use this to select the appropriate operation(s)
- recognise when a problem involves more than one step, identify the steps, and record working for each step
- explain their method and compare methods with others
- check that their answer to a problem is of a reasonable size and answers the original question
- include units of measure in the answer where appropriate
- draw timelines to find time differences

Make sure that:

children understand that problems involving equal groups, measures or amounts can be solved using multiplication or division

you spend time in lessons discussing the strategies involved in solving a word problem – it is more beneficial to unpick one or two problems in depth than to look at several superficially

children have opportunities to solve real problems, for example doubling the ingredients of a recipe to cook double the amount

you give children opportunities to write, try out and discuss their own word problems and those written by their peers

children appreciate why one pound and five pence should be written as £1.05 not £1.5

children use appropriate methods to work out change, for example using a number line to find the difference between the cost of goods bought and money given

children know the relationships between units of metric measure and units of time and can make simple conversions

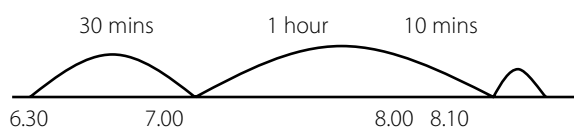
Solving problems involving money and measures

Teaching and learning resources



Coins

Timelines



BANKSLEY POOL
Swimming prices
 Adult swim – £2.50
 Child swim – £1.20
Opening hours
 8 a.m. – 7 p.m. daily

Price lists and catalogues



Clocks

**Measuring scales and
Measuring cylinder ITPs**



Intervention materials

Springboard 4

- Unit 6
- Unit 10

Overcoming barriers in mathematics: level 2 to 3

- Can I identify the calculation needed to solve a problem?
- Can I identify the stages in a 2-step problem?
- Can I explain and record my methods and solutions
- Can I explain the relationships between kilometres and metres, metres and centimetres, kilograms and grams, litres and millilitres?
- Can I work out how long it is between two times?

Assessment checklist

I can statements	Assessment examples
<i>I can identify what operation(s) I need to do to solve a problem</i>	Ben and Jess are answering this problem: Mary has collected 61 key rings, Jo has 45. How many more keyrings does Mary have than Jo? Ben does the calculation $61 + 45$. Jess does the calculation $61 - 45$. Who is correct? Explain how you know.
<i>I can jot down the steps to show how I worked out a problem</i>	Josh buys one coconut and half a kilogram of bananas. What does he pay? Show your working.
<i>I can explain how I solved a problem</i>	Explain your method to a friend.
<i>I can solve problems involving money, including finding change</i>	Holly has these coins. She wants to buy a notebook costing £1.50 How much more money does she need? I pay for a coach trip costing £7.80 with a £10 note. How much change should I get?
<i>I can solve problems that involve measures</i>	A jug holds 2 litres of juice. How many 200ml cups of juice can be filled from the jug?
<i>I can solve problems that involve time</i>	A film starts at 6.30 p.m. and ends at 8.10 p.m. How long is it?



Reading and interpreting tables and graphs

Level 3 standards to be achieved:

Understand how different types of graphs, charts and sorting diagrams are presented

Read titles and labels to identify what information is shown on a particular graph

Work out the size of each interval to read non-unit scales accurately

Make sensible estimates for data that falls between two marked intervals on a scale

Identify the information that needs to be read from a graph to answer a question

Identify the appropriate calculations that need to be carried out to solve a data problem

For children to attain level 3, teach them how to:

- recognise key features of tables, tally charts, frequency charts, pictograms, bar charts, Venn and Carroll diagrams
- use all of the information given in a graph or table, including the title and labels, to interpret the data it represents
- identify the appropriate column, row or cell of a table to find required information
- work out the value of each interval on a scale, count along the scale to check and write in unmarked amounts
- use their understanding of proportion to make sensible estimates for measures that fall between two intervals
- find and note down all of the information needed to solve a problem
- identify and carry out the appropriate calculations needed to solve a problem involving data, including questions such as 'How many more...?' and 'How many... altogether?'

Make sure that:

you build regular opportunities into oral and mental starters to rehearse the skills needed to read scales accurately

children are given opportunities to interpret a wide range of tables, graphs and charts

children have experience of vertical and horizontal bar charts

children regularly interpret real data, for example timetables, tables from magazines or graphs from the internet

children annotate graphs and charts where possible to help them read data accurately

children discuss and compare their methods for solving data handling problems

children record their working when answering problems that involve calculating with data

information presented in tables and graphs is used as a basis for practice of mental and written calculation

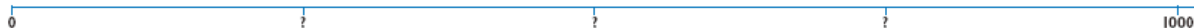
Reading and interpreting tables and graphs

Teaching and learning resources

Counting stick



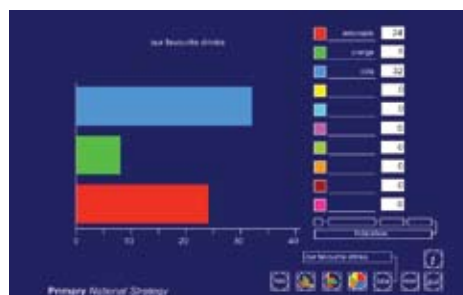
Number lines



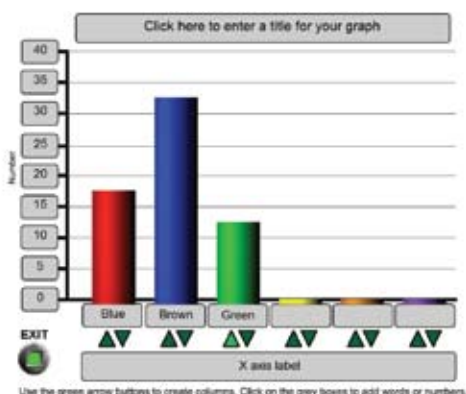
Real data

Edinburgh	–	09:35	–	–	13:35	–	–
Glasgow	09:15	–	11:15	13:15	–	13:45	15:15
Stirling	09:57	–	11:57	13:57	–	14:29	15:57
Perth	10:34	10:51	12:34	14:34	14:50	15:15	16:35
Inverness	–	13:10	–	–	17:05	–	–

Data handling ITP



Handygraph ITP



Intervention materials

Springboard 3

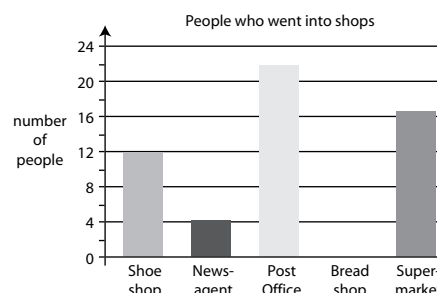
•Unit 10

Overcoming barriers in mathematics: level 2 to 3

- Can I create and interpret a pictogram where the symbol represents more than 1 object?
- Can I create and interpret bar charts?
- Can I make and use lists and tables to organise and interpret information to solve a problem?
- Can I use data to answer a question?

Assessment checklist

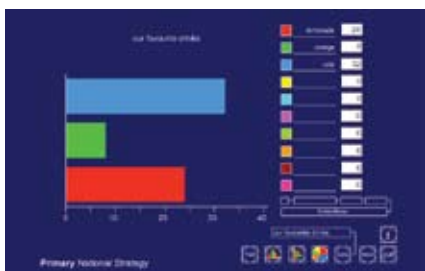
I can statements	Assessment examples
<i>I can read information accurately from different sorts of graphs and charts</i>	Chris did a survey of the number of people who went into shops in one hour.
<i>I can interpret the scales along the axes of a graph to read data accurately</i>	How many people went into the Supermarket in the hour? 13 people went into the Bread shop. Draw this on the graph.
<i>I can identify what calculations need to be done to answer a data handling problem</i>	This table shows the numbers of children who went walking, sailing or climbing at an outdoor centre.
<i>I can identify what calculations need to be done to answer a data handling problem</i>	How many children went sailing in May, June and July altogether? In June, how many more children went walking than climbing?



	May	June	July
walking	25	80	75
sailing	15	42	50
climbing	18	27	23

Where can I find the resources?

ICT resources



Data handling ITP

The image shows a screenshot of an 'Addition and subtraction facts spreadsheet'. It displays a grid of numbers. The top row contains the numbers 15, 59, and 62. The left column contains the numbers 31, 37, and 100. The number 106 is in the middle of the grid, and 27 is in the bottom left corner. The spreadsheet is titled 'Addition and subtraction facts spreadsheet'.

Addition and subtraction facts spreadsheet

ITPs (Interactive Teaching Programs)

These programs can be downloaded from www.standards.dcsf.gov.uk/nationalstrategies/primary. Navigate to the Mathematics Framework area and then to the Mathematics resource library. Refine the results by filtering down to the Interactive Teaching Programs.

Spreadsheets

These programs can be downloaded from www.standards.dcsf.gov.uk/nationalstrategies/primary. Search for spreadsheets and refine your search to display those relevant to Key Stage 2.

Intervention materials

Springboard 3 and Springboard 4

These materials provide lessons to support intervention for those children who are working just below level 3. They can be downloaded from www.standards.dcsf.gov.uk/nationalstrategies/primary. Navigate to the Mathematics Framework area and then to the Mathematics resource library. Refine the results by filtering down to the initiative Springboard.

Overcoming barriers in mathematics – helping children move from level 2 to level 3

This is a booklet with a CD-based set of materials designed to help teachers move children from level 2 into level 3. The materials can be ordered online on the teachernet publications website:

<http://publications.teachernet.gov.uk/default.aspx?PageFunction=productdetails&PageMode=publication&ProductId=DCSF-00149-2008&>

Supporting children with gaps in their mathematical understanding – Wave 3

The Wave 3 mathematics pack aims to help teachers identify and address gaps in learning for children who are working significantly below age-appropriate levels. They can be downloaded from www.standards.dcsf.gov.uk/nationalstrategies/primary. Navigate to the Mathematics Framework area and then to the Mathematics resource library. Refine the results by filtering down to Mathematics Wave 3.



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