

**Secondary**  
*National Strategy*  
for school improvement

Mathematics subject  
leader development folder

**Guidance**

Curriculum and  
Standards

**Mathematics**  
**subject leaders**

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Mathematics subject leader  
development folder



## **Contents**

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General introduction by Sue Hackman

Mathematics introduction by Linton Waters

Handout 1.1: Core tasks

Handout 1.2: Action points

Handout 2.1: National Curriculum level descriptions

Handout 2.2: Key indicators for Level 5 in Years 7 and 8

Handout 2.3: Ma 2: Number and algebra programme of study

Handout 2.4: Key Stage 3 booster lessons

Handout 3.1: Pupils' skills as learners

# Secondary

## National Strategy

### for school improvement

The Secondary National Strategy is designed to promote the most effective practices in teaching and learning across Key Stage 3 and Key Stage 4, with a particular eye on success in the core subjects and cross-curricular skills. Our aim is to help you to create enthusiastic, independent and successful learners.

Your role as subject leader is crucial to the success of the Strategy. We are committed to providing you with up-to-date intelligence about the best practice in your subject, high-quality guidance materials, engaging resources, and opportunities to share local practice as well as new ideas. But these can only make an impact on pupils' everyday experiences through you and your teachers.

Our consultant colleagues in the LEA will be your main point of contact with the Strategy and link you to local, regional and national sources of support. Regular termly meetings are being designed to help you to lead improvements in your department. Yours is a vital, challenging and exciting role. Our aim is to add value by helping you to be a successful leader and manager, and to enjoy the support of experienced, like-minded professionals.

Subject leaders have worked hard over the last few years to secure steady improvement in Key Stage 3, once a Cinderella stage. Now we want to ensure that the same pupils hold on to the gains they made, and go even further. At first, we will extend our networks, consultancy and materials to cover both key stages. We will extend rather than expand. We don't want to overwhelm you with new initiatives; we want to make the ones you already have work well.

In future there will be opportunities to reflect on local experiences and discuss local priorities, and to feed back into the Strategy all your good ideas so that we can disseminate them to others.

This folder is designed to accommodate materials associated with the termly development days and to grow into a reference resource for you working with colleagues in school. You will be encouraged to personalise it and there is space to add your own papers.

I hope you will find the termly meetings professionally stimulating and worthwhile. In the meantime, thank you for everything you and your colleagues have done, and are doing.



*Sue Hackman*  
National Director  
Secondary National Strategy

# Secondary

## National Strategy

for school improvement

### Mathematics subject leader development folder

This folder is designed to accompany the Secondary National Strategy's mathematics subject leader development days, which your LEA will be running each term.

We will be providing guidance and resources via these meetings, which you can use with colleagues in school. The folder is designed to be large enough for you to include these resources alongside other Strategy publications or related materials from other sources.

Our aim is to keep you up-to-date with the National Strategy, to help you to make best use of the wealth of materials already published, and to support you in your crucial work of leading changes with colleagues in your department.

We aim to make the meetings topical, practical and flexible enough to reflect local priorities and expertise. A particular theme underpinning the coming terms will be securing progression from Key Stage 3 to Key Stage 4 (in terms of both the curriculum and pupils as learners). We want to help you ensure that improvements in planning, teaching and learning which are already emerging in Key Stage 3 are consolidated and extended into Key Stage 4.

The new arrangements for school inspection highlight the need for regular, sharp and accurate school-based evaluation of provision. In future terms, we will be looking at ways to assist you to monitor and evaluate key aspects of your department's work.


Now is an exciting time to be involved in teaching mathematics. Initiatives such as the National centre for excellence in mathematics teaching and planned improvements to the curriculum and assessment regime post-14 highlight the national spotlight on the subject. There is a wide consensus and determination to help more pupils enjoy mathematics, feel successful with it and be inspired to extend their mathematical studies beyond the age of 16.

We welcome feedback via your LEA mathematics team on the structure, content and timing of these meetings and materials. In the meantime, thank you for playing your part in the national drive for success in mathematics.



*Linton Waters*  
Mathematics Strand Director  
Secondary National Strategy

## Core tasks



<p><b>1a</b></p> <p>Lead the department in discussion about priorities for mathematics</p>	<p><b>2a</b></p> <p>Agree targets for raising pupils' attainment in the context of whole-school targets</p>
<p><b>3a</b></p> <p>Develop a strategy for the improvement of mathematics</p>	<p><b>4a</b></p> <p>Lead the improvement of teaching quality</p>
<p><b>5a</b></p> <p>Lead the review, the construction and resourcing of the curriculum</p>	<p><b>6a</b></p> <p>Embed the improvement in the department's systems and practices</p>

<p><b>2b</b></p> <ul style="list-style-type: none"> <li>• Establish targets for improvement for cohorts, groups of pupils and individual pupils, using the outcomes of review and monitoring</li> <li>• Link targets for pupils' attainment to targets for pupils' learning</li> <li>• Cross-refer targets for each year group with the scheme of work and teaching plans</li> </ul>	<p><b>1b</b></p> <ul style="list-style-type: none"> <li>• Allocate regular department meeting / INSET time to discussion of teaching and learning</li> <li>• Embed agreed values and approaches in schemes of work</li> <li>• Make explicit links between national, school and departmental strategies for raising standards and embed these within the action plan</li> </ul>
<p><b>4b</b></p> <ul style="list-style-type: none"> <li>• Devise and implement an effective CPD strategy</li> <li>• Share, extend and improve effective teaching</li> <li>• Challenge and support the improvement of ineffective teaching</li> </ul>	<p><b>3b</b></p> <ul style="list-style-type: none"> <li>• Audit to establish the capacity for improvement and to identify curricular / staff / resource implications</li> <li>• Establish curricular targets and staff development priorities with clear success criteria linked to raised pupil attainment and clear monitoring and evaluation procedures</li> <li>• Allocate time, resources, status and responsibilities, and relate precisely to time, finance and personnel available</li> </ul>
<p><b>6b</b></p> <ul style="list-style-type: none"> <li>• Make the improvement of teaching and learning and the sharing of good practice the focus of each departmental meeting</li> <li>• Build new ideas and approaches from development work and training into schemes of work</li> </ul>	<p><b>5b</b></p> <ul style="list-style-type: none"> <li>• Take action, as required, following the regular review of the scheme of work, ensuring suitable differentiation and progression for pupils</li> <li>• Build commitment to a set of agreed standards across the subject</li> </ul>

<p><b>7a</b></p> <p>Create time for staff to learn together</p>	<p><b>8a</b></p> <p>Involve collaboration with other organisations</p>
<p><b>9a</b></p> <p>Focus systematically on teaching and learning</p>	<p><b>10a</b></p> <p>Discuss work, progress and attitudes with sample groups of pupils</p>
<p><b>11a</b></p> <p>Review with teachers their assessments of progress against targets for classes, identified groups and individuals</p>	<p><b>12a</b></p> <p>Evaluate schemes of work to ensure they focus on effective teaching and learning</p>



<p><b>8b</b></p> <ul style="list-style-type: none"> <li>• Take up opportunities to join networks</li> <li>• Take up opportunities to work with leading departments</li> <li>• Access NCSL and higher education programmes to support developments in mathematics</li> </ul>	<p><b>7b</b></p> <ul style="list-style-type: none"> <li>• Identify opportunities for relevant teachers to attend National Strategy training and to disseminate key information to colleagues</li> <li>• Promote collaborative working and support colleagues developing work in planning, teaching and evaluating</li> <li>• Manage meetings to ensure collaborative time for work on teaching and learning</li> <li>• Contribute to CPD meetings</li> <li>• Participate in a coaching programme to develop and share expertise</li> <li>• Provide training and support for new, supply, second-subject and returning teachers</li> </ul>
<p><b>10b</b></p> <ul style="list-style-type: none"> <li>• Agree with colleagues the sample and how the outcomes will be used</li> <li>• Arrange meetings with a suitable sample of pupils, taking into account the ability range and year groups</li> <li>• Make judgements about pupils' attitudes, engagement and confidence in their learning</li> <li>• Use the outcomes of the sample to discuss with colleagues areas of effective practice, development issues and groups of pupils for intervention</li> </ul>	<p><b>9b</b></p> <ul style="list-style-type: none"> <li>• Establish the link between improvements in teaching and learning and higher standards</li> <li>• Carry out an audit with the whole department to identify development priorities which contribute to the SIP and SEF</li> </ul>
<p><b>12b</b></p> <ul style="list-style-type: none"> <li>• Ensure that the scheme of work: focuses on teaching objectives; includes cross-curricular themes; and promotes a range of appropriate teaching and learning styles</li> <li>• Confirm full understanding and use of the agreed scheme of work by all teachers of mathematics</li> <li>• Ensure that assessment opportunities built into the scheme of work support improved learning and progress</li> </ul>	<p><b>11b</b></p> <ul style="list-style-type: none"> <li>• With colleagues, regularly review progress related to key curricular targets, underperforming groups and individuals</li> <li>• Use the outcome of the reviews to: share effective practice, design support, and agree ways of tackling pupils' underperformance</li> </ul>

# Action points

Priorities for action	Timescale / notes

# National Curriculum level descriptions

## Ma1 Using and applying

### Level 5 description

In order to carry through tasks and solve mathematical problems, pupils identify and obtain necessary information. They check their results, considering whether these are sensible. Pupils show understanding of situations by describing them mathematically using symbols, words and diagrams. They draw simple conclusions of their own and give an explanation of their reasoning.

## Ma2 Number and algebra

### Level 5 description

Pupils use their understanding of place value to multiply and divide whole numbers and decimals by 10, 100 and 1000. They order, add and subtract negative numbers in context. They use all four operations with decimals to two places. They reduce a fraction to its simplest form by cancelling common factors and solve simple problems involving ratio and direct proportion. They calculate fractional or percentage parts of quantities and measurements, using a calculator where appropriate. Pupils understand and use an appropriate non-calculator method for solving problems that involve multiplying and dividing any three-digit number by any two-digit number. They check their solutions by applying inverse operations or estimating using approximations. They construct, express in symbolic form, and use simple formulae involving one or two operations. They use brackets appropriately. Pupils use and interpret coordinates in all four quadrants.

## Key indicators for level 5 in Years 7 and 8

These key indicators are from the Framework yearly teaching programmes and are significant for tracking pupils' progress towards level 5.

Year 7	Year 8
<p><b>Using and applying mathematics to solve problems</b></p> <ul style="list-style-type: none"> <li>• Break a complex calculation into simpler steps, choosing and using appropriate and efficient operations and methods.</li> <li>• Solve word problems and investigate in a range of contexts, explaining and justifying methods and conclusions.</li> </ul> <p><b>Numbers and the number system</b></p> <ul style="list-style-type: none"> <li>• Understand and use decimal notation and place value; multiply and divide integers and decimals by 10, 100, 1000 and explain the effect.</li> <li>• Simplify fractions by cancelling all common factors; identify equivalent fractions.</li> <li>• Recognise the equivalence of percentages, fractions and decimals; calculate simple percentages and use percentages to compare simple proportions.</li> </ul>	<p><b>Using and applying mathematics to solve problems</b></p> <ul style="list-style-type: none"> <li>• Identify the necessary information to solve a problem; represent problems and interpret solutions in algebraic, geometrical or graphical form.</li> </ul> <p><b>Numbers and the number system</b></p> <ul style="list-style-type: none"> <li>• Add, subtract, multiply and divide integers.</li> </ul>

## Key indicators for level 5 in Years 7 and 8

Year 7	Year 8
<p><b>Calculations</b></p> <ul style="list-style-type: none"> <li>• Extend mental methods of calculation to include decimals, fractions and percentages.</li> <li>• Multiply and divide three-digit by two-digit whole numbers; extend to multiplying and dividing decimals with one or two places by single-digit whole numbers.</li> <li>• Check a result by considering whether it is of the right order of magnitude and by working the problem backwards.</li> </ul> <p><b>Algebra</b></p> <ul style="list-style-type: none"> <li>• Use letter symbols to represent unknown numbers or variables.</li> <li>• Know and use the order of operations and understand that algebraic operations follow the same conventions and order as arithmetic operations.</li> <li>• Plot the graphs of simple linear functions.</li> </ul>	<p><b>Calculations</b></p> <ul style="list-style-type: none"> <li>• Use standard column procedures for multiplication and division of integers and decimals, including by decimals such as 0.6 and 0.06; understand where to position the point by considering equivalent calculations.</li> </ul> <p><b>Algebra</b></p> <ul style="list-style-type: none"> <li>• Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket.</li> <li>• Substitute integers into simple formulae.</li> </ul>

## Key indicators for level 5 in Years 7 and 8

Year 7	Year 8
<p><b>Shape, space and measures</b></p> <ul style="list-style-type: none"> <li>• Identify parallel and perpendicular lines; know the sum of angles at a point, on a straight line and in a triangle.</li> <li>• Use a ruler and protractor to measure and draw lines to the nearest millimetre and angles, including reflex angles, to the nearest degree.</li> <li>• Recognise and visualise the transformation and symmetry of a 2-D shape:               <ul style="list-style-type: none"> <li>– reflection in given mirror lines and line symmetry;</li> <li>– rotation about a given point and rotational symmetry.</li> </ul> </li> <li>• Convert one metric unit to another (e.g. grams to kilograms); read and interpret scales on a range of measuring instruments.</li> <li>• Know and use the formula for the area of a rectangle.</li> </ul>	<p><b>Shape, space and measures</b></p> <ul style="list-style-type: none"> <li>• Transform 2-D shapes by simple combinations of rotations, reflections and translations, on paper and using ICT; identify all the symmetries of 2-D shapes.</li> <li>• Use units of measurement to estimate, calculate and solve problems in everyday contexts involving length, area, volume, capacity, mass, time, angle and bearings; know rough metric equivalents of imperial measures in daily use (feet, miles, pounds, pints, gallons).</li> </ul>

## Key indicators for level 5 in Years 7 and 8

Year 7	Year 8
<p><b>Handling data</b></p> <ul style="list-style-type: none"> <li>• Compare two simple distributions using the range and one of the mode, median or mean.</li> <li>• Interpret diagrams and graphs (including pie charts), and draw simple conclusions based on the shape of graphs and simple statistics for a single distribution.</li> <li>• Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts.</li> </ul>	<p><b>Handling data</b></p> <ul style="list-style-type: none"> <li>• Estimate probabilities from experimental data; understand that:               <ul style="list-style-type: none"> <li>– if an experiment is repeated there may be, and usually will be, different outcomes;</li> <li>– increasing the number of times an experiment is repeated generally leads to better estimates of probability.</li> </ul> </li> </ul>

# Ma 2: Number and algebra programme of study

## Mathematics: Higher

### Using and applying number and algebra

1 Pupils should be taught to:

#### Problem solving

- a select and use appropriate and efficient techniques and strategies to solve problems of increasing complexity, involving numerical and algebraic manipulation
- b identify what further information may be required in order to pursue a particular line of enquiry and give reasons for following or rejecting particular approaches
- c break down a complex calculation into simpler steps before attempting a solution and justify their choice of methods
- d make mental estimates of the answers to calculations; present answers to sensible levels of accuracy; understand how errors are compounded in certain calculations

#### Communicating

- e discuss their work and explain their reasoning using an increasing range of mathematical language and notation
- f use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem
- g present and interpret solutions in the context of the original problem
- h use notation and symbols correctly and consistently within a given problem
- i examine critically, improve, then justify their choice of mathematical presentation; present a concise, reasoned argument

#### Reasoning

- j explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether a particular case may be generalised further and understand the importance of a counter-example; identify exceptional cases when solving problems
- k understand the difference between a practical demonstration and a proof
- l show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning
- m recognise the significance of stating constraints and assumptions when deducing results; recognise the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem.



**Numbers and the number system**

2 Pupils should be taught to:

**Integers**

- a use their previous understanding of integers and place value to deal with arbitrarily large positive numbers and round them to a given power of 10; understand and use negative integers both as positions and translations on a number line; order integers; use the concepts and vocabulary of factor (divisor), multiple, common factor, highest common factor, least common multiple, prime number and prime factor decomposition

**Powers and roots**

- b use the terms square, positive square root, negative square root, cube and cube root; use index notation [for example,  $8^2$ ,  $8^{-\frac{2}{3}}$ ] and index laws for multiplication and division of integer powers; use standard index form, expressed in conventional notation and on a calculator display

**Fractions**

- c understand equivalent fractions, simplifying a fraction by cancelling all common factors; order fractions by rewriting them with a common denominator

**Decimals**

- d recognise that each terminating decimal is a fraction [for example,  $0.137 = \frac{137}{1000}$ ]; recognise that recurring decimals are exact fractions, and that some exact fractions are recurring decimals [for example,  $\frac{1}{7} = 0.142857142857\dots$ ]; order decimals

**Percentages**

- e understand that 'percentage' means 'number of parts per 100', and interpret percentage as the operator 'so many hundredths of' [for example, 10% means 10 parts per 100 and 15% of Y means  $\frac{15}{100} \times Y$ ]

**Ratio**

- f use ratio notation, including reduction to its simplest form and its various links to fraction notation.

**Calculations**

3 Pupils should be taught to:

**Number operations and the relationships between them**

- a multiply or divide any number by powers of 10, and any positive number by a number between 0 and 1; find the prime factor decomposition of positive integers; understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal, because division by zero is not defined); multiply and divide by a negative number; use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer, fractional and negative powers; use inverse operations, understanding that the inverse operation of raising a positive number to power  $n$  is raising the result of this operation to power  $\frac{1}{n}$
- b use brackets and the hierarchy of operations

- c calculate a given fraction of a given quantity, expressing the answer as a fraction; express a given number as a fraction of another; add and subtract fractions by writing them with a common denominator; perform short division to convert a simple fraction to a decimal; distinguish between fractions with denominators that have only prime factors of 2 and 5 (which are represented by terminating decimals), and other fractions (which are represented by recurring decimals); convert a recurring decimal to a fraction [for example,  $0.142857142857\dots = \frac{1}{7}$ ]
- d understand and use unit fractions as multiplicative inverses [for example, by thinking of multiplication by  $\frac{1}{5}$  as division by 5, or multiplication by  $\frac{6}{7}$  as multiplication by 6 followed by division by 7 (or vice versa)]; multiply and divide a given fraction by an integer, by a unit fraction and by a general fraction
- e convert simple fractions of a whole to percentages of the whole and vice versa; then understand the multiplicative nature of percentages as operators [for example, a 15% increase in value Y, followed by a 15% decrease is calculated as  $1.15 \times 0.85 \times Y$ ]; calculate an original amount when given the transformed amount after a percentage change; reverse percentage problems [for example, given that a meal in a restaurant costs £36 with VAT at 17.5%, its price before VAT is calculated as  $\pounds \frac{36}{1.175}$ ]
- f divide a quantity in a given ratio

### Mental methods

- g recall integer squares from  $2 \times 2$  to  $15 \times 15$  and the corresponding square roots, the cubes of 2, 3, 4, 5 and 10, the fact that  $n^0 = 1$  and  $n^{-1} = \frac{1}{n}$  for positive integers  $n$  [for example,  $10^0 = 1$ ;  $9^{-1} = \frac{1}{9}$ ], the corresponding rule for negative numbers [for example,  $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ ],  $n^{\frac{1}{2}} = \sqrt{n}$  and  $n^{\frac{1}{3}} = \sqrt[3]{n}$  for any positive number  $n$  [for example,  $25^{\frac{1}{2}} = 5$  and  $64^{\frac{1}{3}} = 4$ ]
- h round to a given number of significant figures; develop a range of strategies for mental calculation; derive unknown facts from those they know; convert between ordinary and standard index form representations [for example,  $0.1234 = 1.234 \times 10^{-1}$ ], converting to standard index form to make sensible estimates for calculations involving multiplication and/or division

### Written methods

- i use efficient methods to calculate with fractions, including cancelling common factors before carrying out the calculation, recognising that in many cases only a fraction can express the exact answer
- j solve percentage problems, including percentage increase and decrease [for example, simple interest, VAT, annual rate of inflation]; and reverse percentages
- k. represent repeated proportional change using a multiplier raised to a power [for example, compound interest]
- l calculate an unknown quantity from quantities that vary in direct or inverse proportion
- m calculate with standard index form [for example,  $2.4 \times 10^7 \times 5 \times 10^3 = 12 \times 10^{10} = 1.2 \times 10^{11}$ ,  $(2.4 \times 10^7) \div (5 \times 10^3) = 4.8 \times 10^3$ ]
- n use surds and  $\pi$  in exact calculations, without a calculator; rationalise a denominator such as  $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

**Calculator methods**

- o use calculators effectively and efficiently, knowing how to enter complex calculations; use an extended range of function keys, including trigonometrical and statistical functions relevant across this programme of study
- p understand the calculator display, knowing when to interpret the display, when the display has been rounded by the calculator, and not to round during the intermediate steps of a calculation
- q use calculators, or written methods, to calculate the upper and lower bounds of calculations, particularly when working with measurements
- r use standard index form display and how to enter numbers in standard index form
- s use calculators for reverse percentage calculations by doing an appropriate division
- t use calculators to explore exponential growth and decay [for example, in science or geography], using a multiplier and the power key.

**Solving numerical problems**

- 4 Pupils should be taught to:
- a draw on their knowledge of operations and inverse operations (including powers and roots), and of methods of simplification (including factorisation and the use of the commutative, associative and distributive laws of addition, multiplication and factorisation) in order to select and use suitable strategies and techniques to solve problems and word problems, including those involving ratio and proportion, repeated proportional change, fractions, percentages and reverse percentages, inverse proportion, surds, measures and conversion between measures, and compound measures defined within a particular situation
  - b check and estimate answers to problems; select and justify appropriate degrees of accuracy for answers to problems; recognise limitations on the accuracy of data and measurements.

**Equations, formulae and identities**

- 5 Pupils should be taught to:

**Use of symbols**

- a distinguish the different roles played by letter symbols in algebra, using the correct notational conventions for multiplying or dividing by a given number, and knowing that letter symbols represent definite unknown numbers in equations [for example,  $x^2 + 1 = 82$ ], defined quantities or variables in formula [for example,  $V = \pi R^2$ ], general, unspecified and independent numbers in identities [for example,  $(x + 1)^2 = x^2 + 2x + 1$  for all  $x$ ], and in functions they define new expressions or quantities by referring to known quantities [for example,  $y = 2 - 7x$ ;  $f(x) = x^3$ ;  $y = \frac{1}{x}$  with  $x \neq 0$ ]

- b understand that the transformation of algebraic entities obeys and generalises the well-defined rules of generalised arithmetic [for example,  $a(b + c) = ab + ac$ ]; expand the product of two linear expressions [for example,  $(x + 1)(x + 2) = x^2 + 3x + 2$ ]; manipulate algebraic expressions by collecting like terms, multiplying a single term over a bracket, taking out common factors [for example,  $9x - 3 = 3(3x - 1)$ ], factorising quadratic expressions including the difference of two squares [for example,  $x^2 - 9 = (x + 3)(x - 3)$ ] and cancelling common factors in rational expressions [for example,  $2(x + 1)^2 / (x + 1) = 2(x + 1)$ ]
- c know the meaning of and use the words 'equation', 'formula', 'identity' and 'expression'

### Index notation

- d use index notation for simple integer powers, and simple instances of index laws [for example,  $x^3 \times x^2 = x^5$ ;  $x^2 / x^3 = x^{-1}$ ;  $(x^2)^3 = x^6$ ]; substitute positive and negative numbers into expressions such as  $3x^2 + 4$  and  $2x^3$

### Equations

- e set up simple equations [for example, find the angle  $a$  in a triangle with angles  $a$ ,  $a + 10$ ,  $a + 20$ ]; solve simple equations [for example,  $5x = 7$ ;  $11 - 4x = 2$ ;  $3(2x + 1) = 8$ ;  $2(1 - x) = 6(2 + x)$ ;  $4x^2 = 49$ ;  $3 = \frac{12}{x}$ ] by using inverse operations or by transforming both sides in the same way

### Linear equations

- f solve linear equations in one unknown, with integer or fractional coefficients, in which the unknown appears on either side or on both sides of the equation; solve linear equations that require prior simplification of brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution

### Formulae

- g use formulae from mathematics and other subjects [for example, for area of a triangle or a parallelogram, area enclosed by a circle, volume of a prism, volume of a cone]; substitute numbers into a formula; change the subject of a formula, including cases where the subject occurs twice, or where a power of the subject appears [for example, find  $r$  given that  $A = \pi r^2$ , find  $x$  given  $y = mx + c$ ]; generate a formula [for example, find the perimeter of a rectangle given its area  $A$  and the length  $l$  of one side]

### Direct and inverse proportion

- h set up and use equations to solve word and other problems involving direct proportion or inverse proportion [for example,  $y \propto x$ ,  $y \propto x^2$ ,  $y \propto \frac{1}{x}$ ,  $y \propto \frac{1}{x^2}$ ] and relate algebraic solutions to graphical representation of the equations

### Simultaneous linear equations

- i find the exact solution of two simultaneous equations in two unknowns by eliminating a variable, and interpret the equations as lines and their common solution as the point of intersection
- j solve simple linear inequalities in one variable, and represent the solution set on a number line; solve several linear inequalities in two variables and find the solution set

**Quadratic equations**

- k solve quadratic equations by factorisation, completing the square and using the quadratic formula

**Simultaneous linear and quadratic equations**

- l solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns, one of which is linear in each unknown, and the other is linear in one unknown and quadratic in the other [for example, solve the simultaneous equations  $y = 11x - 2$  and  $y = 5x^2$ ], or where the second is of the form  $x^2 + y^2 = r^2$

**Numerical methods**

- m use systematic trial and improvement to find approximate solutions of equations where there is no simple analytical method of solving them [for example,  $x^3 - x = 900$ ].

**Sequences, functions and graphs**

- 6 Pupils should be taught to:

**Sequences**

- a generate common integer sequences (including sequences of odd or even integers, squared integers, powers of 2, powers of 10, triangular numbers); generate terms of a sequence using term-to-term and position-to-term definitions of the sequence; use linear expressions to describe the  $n$ th term of an arithmetic sequence, justifying its form by reference to the activity or context from which it was generated

**Graphs of linear functions**

- b use conventions for coordinates in the plane; plot points in all four quadrants; recognise (when values are given for  $m$  and  $c$ ) that equations of the form  $y = mx + c$  correspond to straight-line graphs in the coordinate plane; plot graphs of functions in which  $y$  is given explicitly in terms of  $x$  (as in  $y = 2x + 3$ ), or implicitly (as in  $x + y = 7$ )
- c find the gradient of lines given by equations of the form  $y = mx + c$  (when values are given for  $m$  and  $c$ ); understand that the form  $y = mx + c$  represents a straight line and that  $m$  is the gradient of the line, and  $c$  is the value of the  $y$ -intercept; explore the gradients of parallel lines and lines perpendicular to these lines [for example, know that the lines represented by the equations  $y = -5x$  and  $y = 3 - 5x$  are parallel, each having gradient  $(-5)$  and that the line with equation  $y = \frac{x}{5}$  is perpendicular to these lines and has gradient one-fifth]

**Interpreting graphical information**

- d construct linear functions and plot the corresponding graphs arising from real-life problems; discuss and interpret graphs modelling real situations [for example, distance-time graph for a particle moving with constant speed, the depth of water in a container as it empties, the velocity-time graph for a particle moving with constant acceleration]

**Quadratic functions**

- e generate points and plot graphs of simple quadratic functions [for example,  $y = x^2$ ;  $y = 3x^2 + 4$ ], then more general quadratic functions [for example,  $y = x^2 - 2x + 1$ ]; find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function; find the intersection points of the graphs of a linear and quadratic function, knowing that these are the approximate solutions of the corresponding simultaneous equations representing the linear and quadratic functions

**Other functions**

- f plot graphs of: simple cubic functions [for example,  $y = x^3$ ], the reciprocal function  $y = \frac{1}{x}$  with  $x \neq 0$ , the exponential function  $y = k^x$  for integer values of  $x$  and simple positive values of  $k$  [for example,  $y = 2^x$ ;  $y = (\frac{1}{2})^x$ ], the circular functions  $y = \sin x$  and  $y = \cos x$ , using a spreadsheet or graph plotter as well as pencil and paper; recognise the characteristic shapes of all these functions

**Transformation of functions**

- g apply to the graph of  $y = f(x)$  the transformations  $y = f(x) + a$ ,  $y = f(ax)$ ,  $y = f(x + a)$ ,  $y = a f(x)$  for linear, quadratic, sine and cosine functions  $f(x)$

**Loci**

- h construct the graphs of simple loci, including the circle  $x^2 + y^2 = r^2$  for a circle of radius  $r$  centred at the origin of coordinates; find graphically the intersection points of a given straight line with this circle and know that this corresponds to solving the two simultaneous equations representing the line and the circle.

**Breadth of study**

- 1 During the Key Stage, pupils should be taught the Knowledge, skills and understanding through:
  - a activities that ensure they become familiar with and confident using standard procedures for the range of calculations appropriate to this level of study
  - b solving familiar and unfamiliar problems in a range of numerical, algebraic and graphical contexts and in open-ended and closed form
  - c using standard notations for decimals, fractions, percentages, ratio and indices
  - d activities that show how algebra, as an extension of number using symbols, gives precise form to mathematical relationships and calculations
  - e activities in which they progress from using definitions and short chains of reasoning to understanding and formulating proofs in algebra and geometry
  - f a sequence of practical activities that address increasingly demanding statistical problems in which they draw inferences from data and consider the uses of statistics in society
  - g choosing appropriate ICT tools and using these to solve numerical and graphical problems, to represent and manipulate geometrical configurations and to present and analyse data.

# Key Stage 3 Booster lessons

- 1 Place value
- 2 Fractions, decimals and percentages 1
- 3 Fractions, decimals and percentages 2
- 4 Using a calculator
- 5 Ratio and proportion
- 6 Algebraic expressions
- 7 Sequences
- 8 Lines and angles
- 9 Area
- 10 Circles
- 11 Probability
- 12 Solving word problems
- 13 Algebraic equations
- 14 Handling data
- 15 Ratio and proportion 2
- 16 Problem solving

These are contained in the *Year 9 booster kit – mathematics* available at [www.standards.dfes.gov.uk/keystage3](http://www.standards.dfes.gov.uk/keystage3)

## Pupils' skills as learners

Although pupils are acquiring mathematical knowledge and skills at a reasonable rate, their skills as learners are not sufficiently well developed.

In particular, pupils are often unaware of the standard at which they are currently working and what is required to achieve higher levels. They are not able to reflect on their relative strengths and weaknesses in mathematics and know little about what they need to do to improve in the subject. In an attempt to address these issues, some schools have adopted a range of strategies, as in these examples from the Ofsted subject report 2002/3:

*On the school intranet, the mathematics staff developed part of the site for pupils in Years 7 and 8. It included revision notes and practice material for each of the major topics. Following the half-term assessments, pupils were helped to identify aspects of mathematics about which they were still not confident and directed to use the related materials on the intranet. Many pupils did so during lunchtimes or in the homework club sessions. Some pupils emailed work to their homes for completion in their own time.*

*In a Year 8 class, the pupils knew they were working on National Curriculum level 5 material, for the most part, and had detailed descriptions of mathematics at level 5 in the back of their exercise books.*

*The school library contained a range of mathematics books and multimedia materials. For some units of work, pupils were set the task of researching some aspects ahead of studying them in lessons. For example, Year 9 pupils were asked to find out about Pythagoras and his theorem.*

*At the start of each term, all groups in Year 10 were provided with a detailed outline of the term's work in mathematics. It included samples of examination questions that they ought to be able to complete when they had mastered the work.*

*National Curriculum attainment targets and level descriptions for Key Stage 3 were rewritten for pupils and displayed as posters in the mathematics corridor. For each level, a selection of 'model' test questions was attached to help pupils understand the characteristics of a complete and correct response. This display was changed as the Year 9 units of work changed.*

More schools need to employ strategies such as these to enable pupils to be more effective learners, for example to:

- become more autonomous and independent learners
- develop research and enquiry skills
- decide where to apply effort in studying
- choose to spend additional time on particular aspects of mathematics
- consolidate their understanding and skills at their own pace and with others if they choose
- extend their learning where they are interested or think it is necessary
- take more responsibility for their own progress and achievement.

*Mathematics in secondary schools, Ofsted subject reports 2002/03;  
HMI 1978; February 2004*



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