Better education and care

Evaluating mathematics provision for 14–19-year-olds

Ofsted visited 26 schools, sixth form colleges and general further education colleges to determine the main factors leading to high achievement, motivation and participation in 14–19 mathematics, and the factors which act against high achievement. The findings are intended to contribute to the debate on the future of mathematics education in England, following the publication of the Smith Report, *Making mathematics count.*

This report draws principally on the outcomes of these inspections, but also on other relevant Ofsted inspection evidence. Its findings are illustrated by examples of good and of less effective practice.

Of particular interest to:

Schools, colleges, DfES, LSC, QCA, and organisations concerned with mathematics in education and society.

Age group	Published	Reference no.
14–19	May 2006	HMI 2611

Document reference number: HMI 2611

Website: www.ofsted.gov.uk

Contents

Executive summary

In summer and autumn 2005 Ofsted carried out a survey inspection to establish the factors which make the most significant contributions to high achievement, motivation and participation in 14–19 mathematics, and the factors which act against high achievement. The findings are intended to contribute to the debate on the future of mathematics education in England, following the publication of the Smith Report, *Making mathematics count.*[1]

Her Majesty's Inspectors (HMI) visited 26 schools, sixth form colleges and general further education colleges in a variety of areas. This report draws principally on the outcomes of these inspections, but also on other relevant Ofsted inspection evidence. Its findings are illustrated by examples of good and of less effective practice.

The quality of teaching was the key factor influencing students' achievement. The majority of the teaching seen was at least satisfactory in preparing students for examinations. However, in promoting a really secure understanding of mathematical ideas, in stimulating students to think for themselves and to apply their knowledge and skills in unfamiliar situations, the picture was less encouraging. In approximately half of the lessons observed, the teaching did not sufficiently encourage these important aspects of learning in mathematics.

The best teaching gave a strong sense of the coherence of mathematical ideas; it focused on understanding mathematical concepts and developed critical thinking and reasoning. Careful questioning identified misconceptions and helped to resolve them, and positive use was made of incorrect answers to develop understanding and to encourage students to contribute. Students were challenged to think for themselves, encouraged to discuss problems and to work collaboratively. Effective use was made of information and communication technology (ICT). In contrast, teaching which presented mathematics as a collection of arbitrary rules and provided a narrow range of learning activities did not motivate students and limited their achievement. Focusing heavily on examination questions enabled students to pass examinations, but did not necessarily enable them to apply their knowledge independently in different contexts.

A lack of suitably qualified specialist teachers affected achievement adversely in Key Stage 4 or on numeracy programmes in more than a third of visits. Discontinuity in learning occurred when students transferred between institutions at age 16, in areas without a common approach to GCSE provision in Key Stage 4.

---

[1] *Making mathematics count: the report of Sir Adrian Smith's inquiry into post-14 mathematics education* (937764), The Stationery Office, February 2004.

Key findings

The survey found that the factors which made the most significant contributions to high achievement in 14–19 mathematics were:

- ❒ Secure subject knowledge on the part of the teacher, underpinning an approach to mathematics in which all topics are seen as part of a coherent set of related ideas, with clear progression and links to previous and future learning.

- ❒ Teaching that focuses on developing students' understanding of mathematical concepts and enhances their critical thinking and reasoning, together with a spirit of collaborative enquiry that promotes mathematical discussion and debate.

- ❒ Assessment that informs teaching, and questioning skills that stimulate learning.

- ❒ Well paced lessons that set high expectations and challenge students to apply their own ideas, and that create a positive atmosphere to build students' confidence.

- ❒ The effective use of ICT and other high quality learning resources, including the new resources devised by the Standards Unit in the DfES to enhance learning and develop functional skills in mathematics.[2]

- ❒ Professional development for teachers which focuses on effective teaching and learning and promotes the sharing of good practice.

- ❒ A range of learning programmes that promote wider access to mathematics for students at all levels.

Factors which acted against effective achievement, motivation and participation were:

- ❒ Teaching which presents mathematics as a collection of arbitrary rules and procedures, allied to a narrow range of learning activities in lessons which do not engage students in real mathematical thinking.

- ❒ Weak assessment, including questioning, which fails to identify students' specific needs, to probe their understanding of ideas and to capitalise on incorrect responses.

[2] For details of the Standards Unit resources, see the paragraph headed 'Teaching and learning frameworks' on p. 21. (With effect from 3 April 2006 the Standards Unit is now known as the Improvement Group.)

- ❒ Insufficient subject expertise amongst some teachers of GCSE mathematics and numeracy programmes; a lack of imagination and the confidence to try new approaches amongst A level teachers.

- ❒ A narrow focus on meeting examination requirements by 'teaching to the test', so that although students are able to pass the examinations they are not able to apply their knowledge independently to new contexts, and they are not well prepared for further study.

- ❒ Unhelpful variations in the GCSE experience of potential AS and A level students, particularly in areas where students transfer between institutions at age 16.

Recommendations

The Department for Education and Skills, the Qualifications and Curriculum Authority and awarding bodies should:

- ensure that the current revision of GCSE and A level mathematics results in examinations which encourage effective understanding and problem-solving, as well as competence in mathematical techniques.

The DfES should:

- continue to invest in the dissemination of the successful approaches to teaching and learning developed through the Standards Unit's framework for mathematics, with particular emphasis on collaborative professional development across mathematics departments
- encourage the greater use of the Standards Unit's resources and the subject coaching programme and networks in school sixth forms, as well as in the further education sector.

Schools and colleges should:

- use performance management to enhance teachers' subject knowledge and promote styles of teaching in mathematics which encourage more active participation by students and lead to greater learning
- develop more effective assessment to ensure that teaching is focused better on individual needs
- capitalise on the wide range of ICT resources to support and enhance mathematics teaching
- at Key Stage 4, focus on high levels of performance and secure understanding in GCSE mathematics, as the most effective preparation for AS and A level study, rather than pushing students too rapidly on to other programmes.

Background

1. This survey of mathematics undertaken by students aged 14–19 in schools and further education set out to answer two related questions:

   - What are the key factors contributing to high achievement, motivation and participation in mathematics?
   - What are the key factors acting against effective achievement, motivation and participation?

   It provides evidence for the debate on the future of mathematics education in England, following the publication of *Making mathematics count*.

2. Although qualifications, such as GCE A level, GCSE and key skills are the most obvious indicators of achievement, institutions of higher education and employers have indicated repeatedly that these do not guarantee that students have gained the sorts of skills and understanding needed for further study or the workplace.

3. This report therefore considers achievement more broadly, in terms of students' depth of understanding, their ability to apply their knowledge to unfamiliar situations and their preparedness for further study. It also considers the associated factors of students' motivation and participation beyond the age of 16.

4. Sir Adrian Smith's report and the recent *14–19 Education and skills implementation plan* both emphasised students' understanding of mathematical concepts and the mastery of mathematical skills as the key indicators of achievement in the subject.[3] *Making mathematics count* described mathematics as:

   > '...a major intellectual discipline in its own right, as well as providing the underpinning language for the rest of science and engineering and, increasingly, for other disciplines in the social and medical sciences. It underpins major sectors of modern business and industry, in particular, financial services and ICT. It also provides the individual citizen with empowering skills for the conduct of private and social life and with key skills required at virtually all levels of employment.
   >
   > 'An adequate supply of young people with mastery of appropriate mathematical skills at all levels is vital to the future prosperity of the UK.'

[3] *14–19 Education and skills implementation plan* (Ref: 2037-2005DCL-EN), DfES, 2005.

5. The government's *14–19 Education and skills implementation plan* defined the functional skills required in mathematics:

> 'Functional skills are those core elements of English, maths and ICT that provide an individual with the essential knowledge, skills and understanding that will enable them to operate confidently, effectively and independently in life and at work. Individuals of whatever age who possess these skills will be able to participate and progress in education, training and employment as well as develop and secure the broader range of aptitudes, attitudes and behaviours that will enable them to make a positive contribution to the communities in which they live and work...'

In the case of **mathematics** this means that:

- Each individual has sufficient understanding of a range of mathematical concepts and is able to know how and when to use them. For example, they will have the confidence and capability to use maths to solve problems embedded in increasingly complex settings and to use a range of tools, including ICT as appropriate.
- In life and work, each individual will develop the analytical and reasoning skills to draw conclusions, justify how they are reached and identify errors or inconsistencies. They will also be able to validate and interpret results, to judge the limits of their validity and use them effectively and efficiently.[4]

Teaching and learning

Factors contributing to high achievement

6. The quality of teaching was the single most important factor influencing students' achievement. Reflecting the pattern revealed by inspection more generally, the majority of teaching observed in the survey visits was at least satisfactory in preparing students for their examination objectives. Teaching was generally better on AS and A level courses than in GCSE and other level 1 and 2 programmes.[5] However, in contributing to achievement in its broader sense, and particularly in promoting a really secure understanding of mathematical ideas the picture was less encouraging. Inspectors saw some very effective teaching, examples of which are described below. However, there was also much that did not develop students' capabilities beyond the basic competences required to pass examinations.

[4] *14–19 Education and Skills Implementation Plan*, 2005.
[5] Annex 2 provides an outline of qualification levels.

Making links

7. In all settings, both schools and FE colleges, the most effective teachers understood how the particular aspects of mathematics they were teaching fitted into the wider development of mathematical themes and concepts. They were aware of the progression of mathematical ideas and the rich links across them. This enabled teachers to develop students' secure understanding by making links with previous and forthcoming work on the same topic and by emphasising the recurring mathematical themes and ways of thinking.

Making sense

8. Students had more positive attitudes when teachers expected them to make sense of the mathematics they were learning and enabled them to do this. Effective teachers:

   - gave clear explanations which focused on developing students' understanding of mathematical concepts
   - required students to articulate and refine their ideas, focusing on developing their critical thinking and reasoning.

   Consequently, students were helped to recognise the power of mathematical thinking; they began to gain a secure basis to develop ideas and methods in order to tackle more complex problems. For example:

AS mathematics lesson in a sixth form college

In a good lesson on trigonometric equations such as $3\cos x = 2$,

students already knew how to find one solution in the interval 0 to 360 degrees, by rearranging the equation and using the appropriate inverse trigonometric function. The teacher encouraged the students to reason out additional solutions by sketching a graph and thinking about the symmetries and periodicity found in trigonometric functions. She therefore gave them opportunities to think mathematically, to show initiative and perseverance and to reflect on their learning by discussing ideas with others.

After the lesson, the teacher was able to suggest improvements by thinking about the mathematics. The teacher and students had drawn only the 0 to 360 degree section of the sine and cosine graphs. She realised that this would not be enough to locate all the solutions for more complex equations like $5\cos(2x–60°) = 3$, and decided she would follow this idea up in the next lesson.

Building confidence

9. The best teachers consciously set out to give their students the confidence to succeed. They did this by:
   - careful assessment of the students' starting points
   - challenging students to think for themselves
   - enabling them to talk through misconceptions in a non-threatening way
   - focusing on ideas rather than just procedures.

10. The impact of such an approach was seen in Key Stage 4 GCSE classes and in General Certificate of Education (GCE) AS and A level. But it was most striking in an FE college where the rate of success for students re-taking GCSE in achieving a higher-grade pass far exceeded the national rate. The example below is from that college.

Inner-city FE college

This GCSE session was extremely well planned and resourced. The theme was simple: representing written expressions algebraically.

In the main activity, students matched pre-prepared cards and then extended this to devise their own expressions. Mini-whiteboards helped to ensure that all students contributed to the starter discussion, and the teacher was able to assess individual students' understanding. The teacher valued and used all answers and ideas from students, whether correct or not, and as a result the students were very responsive and prepared to try things for themselves, think for themselves and learn from their mistakes.

After the initial teacher-led discussion, which got the session off to a well paced and purposeful start, the students worked productively in pairs or small groups. The regular emphasis on substituting values into expressions to check equivalence reinforced the notion of letters representing variables. The students were highly motivated, talked about what they were doing with their teacher and each other, and remained fully on task throughout. They were making clear progress in their thinking and showed an understanding of key algebraic ideas well beyond the level normally seen from GCSE re-sit students.

Informal assessment and questioning skills

11. Teachers who exposed and tackled students' misconceptions through well judged questioning enhanced students' understanding. Teachers responded flexibly, judging their next intervention according to what students said or wrote. They left 'thinking time' before directing their

questions at individuals, or used mini-whiteboards to gain responses from all students, rather than taking answers only from volunteers. Crucially, they tackled incorrect answers positively, using them as an aid to learning and encouraging students to try things out, even if they might be wrong.

Structure, pace and challenge

12. Teaching which was planned effectively, lessons which were structured clearly, high expectations and a challenging pace all contributed to successful outcomes. Such teaching ensured that students remained focused and that the time available was used effectively. They were given a variety of tasks, which moved them forward in their thinking as well as allowing them to practise and consolidate standard techniques. All students were expected to contribute their own ideas and questions.

A level mechanics lesson in a sixth form college

An outstanding mechanics lesson held the full attention of the class for 75 minutes with a mix of highly effective, well focused teaching activities. The teaching was a model of clarity, rigour and pace. The students were constantly pushed for ideas and explanations, and no one was allowed to be inattentive or detached.

The teacher had prepared the lesson carefully. Presentation software was used very skilfully, with helpful annotations and commentary. Carefully designed, worked examples built students' understanding of increasingly complex situations and they were encouraged to contribute their own data to make problems more realistic. There was a strong emphasis on considering whether answers were sensible and realistic; it was clear from students' responses that they were used to doing this. A computer simulation was incorporated effectively to compare theoretical and experimental answers; this also provided good revision of how to use a spreadsheet to derive quadratic equations to fit given data. The students contributed well, and their understanding was evident in their responses to questions, in their own questions and in their suggestions for tackling problems.

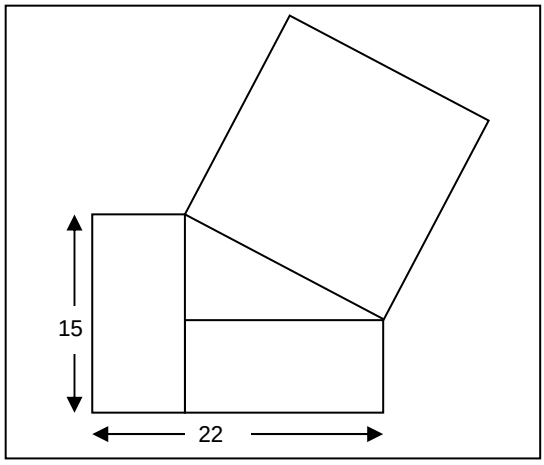
13. The survey showed that students particularly need the opportunity to tackle challenging multi-step problems. Such problems require careful consideration of a solution strategy, and the selection of appropriate techniques. To find the solution, students need to be able to recall facts and apply basic techniques accurately and speedily, as this example shows:

Year 11 GCSE lesson

Higher level GCSE students were challenged to find the area of the square in the diagram shown, in which the two rectangles are congruent, and to determine if it was greater or less than the combined area of the other three shapes.

Students had to recognise the need to find the height and base of the triangle, and to recall and use Pythagoras' Theorem to find the area of the square. They also needed to recall and use the area formulae for rectangles and triangles and accurately carry out the necessary calculations.

A challenging extension problem asked students to find a rectangle for which the area of the square was equal to the combined area of the other three shapes. This required them to form and solve a quadratic equation, selecting appropriate unknowns.



Learning through collaboration and discussion

14. Most students liked to work in groups and appreciated the opportunity to discuss their solutions and ideas with other learners. They were able to learn very effectively from one another, were prepared to 'chance their arm' in a non-threatening situation and developed their reasoning and use of mathematical language. Problem-solving carried out jointly was usually more productive than that done individually.

AS and A level lessons in a sixth form college

The teaching of one of the teachers was consistently good. He grouped students around tables and set tasks which encouraged them to discuss and to try out ideas. He introduced topics by means of problems, which were sufficiently open-ended to involve all the students, regardless of ability. He then skilfully developed the mathematical ideas through their responses. Students were

encouraged to present their solutions to the rest of the class. They were made to think for themselves, and all were fully involved in the work. The progress that they were making was very evident.

Using information and communication technology

15. Where information and communication technology (ICT) was used effectively, it had a marked positive impact on learning in mathematics. Presentation software allowed teaching material to be presented efficiently; high quality graphs and diagrams frequently had much more impact than hand-drawn sketches. Specialist software enabled topics and ideas to be developed quickly and effectively.

A level and further mathematics lessons in a sixth form college

Examples of the use of specialist mathematics software included:

- demonstrating graphically the convergence of the Newton-Raphson method, and the effect of changing the starting values
- illustrating the accuracy of the trapezium rule for the area under a curve
- a simulation to investigate the principle of conservation of momentum in collisions.

This kind of visual approach to studying mechanics considerably aided the students' understanding of underlying principles and enabled theory to be developed from practical numerical examples.

16. Where they had the opportunity to do so, students were increasingly confident in using graph-plotting software, and some used graphical calculators very effectively. In AS and A level lessons where the teaching was good, teachers made sure that students appreciated the limitations of such tools, and encouraged students to discuss their findings with one another.

AS mathematics lesson in an 11–18 school

In an effective lesson on numerical techniques for solving equations, students made good use of a spreadsheet and a graph-plotting package to compare the efficiency of a range of methods. Because the teachers had emphasised the underlying geometric interpretation, students were able to invent situations in which the methods might fail. In one case, students were expected to attempt an initial example before the teacher demonstrated the solution. This encouraged them to discuss their work with each other and to reflect on the notes they had just taken on the underlying theory.

Relevant contexts

17. The strongest and most inspirational teaching in numeracy programmes:
    - conveyed enthusiasm for mathematics
    - stemmed from robust subject knowledge
    - showed excellent classroom skills
    - embedded the work in compelling and relevant vocational contexts.

    The following examples illustrate these features vividly:

Session on the application of number in a general FE college

A stimulating session with hairdressing students struck just the right balance between engaging the learners and keeping their mathematics moving forward. The excellent teaching was informed by the tutor's own very secure capability in mathematics and a high degree of relevance to the vocational situation (enhanced by the presence of the vocational as well as the numeracy tutor in the session). Level 1 and level 2 work was under way, with the two groups of students largely unaware of the extent of the differentiation. In a quick-fire role-play, students had to make creative and viable changes to the timing of appointments. The session overall had a good variety of creative and challenging activities.

Mixed ability creative arts group in a general FE college

The numerical work in a performance session was outstanding. The session had a brisk pace, a rich variety of relevant and intrinsically interesting activities and excellent rapport between the tutor and the students. At one stage, students developed a dance routine to measure a studio by arm span and stride.

The detailed scheme of work showed that such relevant activities were the norm with this group: for example, in a later session probability was used to predict developments in the plot of a drama sequence.

Level 1 programme in a general FE college

A bricklaying group was working on Excel spreadsheets at a much higher level of demand than is usually seen with such a group. In handling data on accidents in the building trade, they used mean and median as measures of risk. They were very well motivated and gave full attention and effort to the work. They responded extremely well to the badinage, high expectations and learning culture that the tutor had generated.

Factors acting against high achievement

18. Many of the factors which militated against students achieving as well as they might have done reflected the absence of the positive features already described. At least some of these factors were present in about half of the work observed in the survey. In many of the lessons seen, the teaching had some strengths, but lacked sufficient flair, imagination and challenge to get the best from students.

Limited teaching strategies

19. Many teachers had a good personal knowledge of mathematical techniques but a restricted range of teaching strategies: demonstration, followed by practice of standard procedures, predominated. The teachers were effective in showing students what to do but mathematics became an apparently endless series of algorithms for them rather than a coherent and interconnected body of knowledge. The result was that lessons did not develop sufficiently the students' ability to reason and discover solutions for themselves.

> **Year 10 GCSE intermediate level mathematics lesson**
>
> In a lesson on simultaneous equations, the teacher had adapted her approach to respond to students' difficulty in previous lessons. By carefully controlling the difficulty of the problems, she was able build students' confidence in the basic technique before reintroducing questions that required manipulation of two equations. She showed the students how to set out the equations and, for each new type, she guided them through one or two examples before they tried a few themselves. They were largely successful at learning the method, but without a clear sense of why it worked. Earlier, students had covered the intersections of lines, but the teacher did not make the obvious link, so the students did not appreciate why two equations were needed to reach a solution. The teacher avoided cases that required operations with negative numbers.

20. Teachers frequently talked for too long without allowing students to engage with mathematics and find out the difficulties for themselves. Lessons often had too little practical or group work to stimulate discussion and mathematical thinking amongst students.

> **Year 10 GCSE intermediate level mathematics lesson**
>
> In a lesson on the circumference of a circle, the teacher showed the students how to calculate the circumference given its diameter, and vice versa. She demonstrated a worked example of each type, but the students were not particularly attentive. She then set an

exercise. After a few minutes some students had calculated the circumference of a circle, with a diameter of 5 cm, to be 1.6 cm, seeing nothing wrong. It became clear that the students could not confidently picture, draw or make with their hands, a circle of diameter 5cm, nor did they visualise the circumference of the circle as the distance around the circle. Students had been taught to use a formula but not to understand its meaning or purpose. Their lesson had shown them how to use a formula (and a calculator) to get an answer, but not the importance of selecting the right formula in the first place.

Ineffective assessment for learning

21. A common barrier to success in many GCSE re-sit courses was the lack of planning to provide work which reflected the students' strengths and weaknesses. Instead of using assessment information to identify the most important topics, most courses attempted to repeat the entire syllabus.

22. In-class assessment and monitoring of students' understanding were often weaknesses in the mathematics lessons seen in the survey. Questioning frequently sought correct answers rather than probing whether students understood the ideas. Teachers made too little use of incorrect answers to identify and deal with students' errors and misconceptions.

GCSE re-sit class in a sixth form college

A suitable starter activity, on long multiplication, was less effective than it could have been since the teacher did not pick up on the students' errors: once a student had answered correctly the teacher moved on to the next question without checking what others had done.

The main topic, on ratios, summarised the ideas students had covered in their high schools without attempting to investigate their prior knowledge and understanding. Some of the examples were impractical, and others, for example scaling up recipes, did not need the use of ratios. Explanations were clear but, beyond a few closed questions, the students were not much involved. There was little evidence of progress.

Subject knowledge

23. Almost always, teachers of AS and A level mathematics had sufficient subject knowledge to teach the syllabuses. However, they did not

always show the confidence and real depth of understanding to teach in the most creative or imaginative ways.

24. On post-16 vocational programmes, tutors' lack of mathematical qualifications or experience in the vocational sector sometimes hindered students' learning. This affected half of the colleges visited.

General FE college

One group working on the application of number was taught by a tutor whose own highest mathematics qualification was a grade D at GCSE level. Another tutor, with a bricklaying group, had previous experience of teaching, mostly in primary schools, and had little knowledge of the construction industry. In accountancy and engineering, tutors' subject knowledge was often excellent, but they were not skilled at putting across mathematical ideas such as direct proportion.

25. Most, but not all, teachers in Key Stage 4 were suitably qualified to teach GCSE, but only half of the schools had a full complement of specialist mathematics teachers. In a few of the lessons seen, teachers gave students incorrect, incomplete, inappropriate or misleading information, as in the examples here:

GCSE lessons in various schools

- Students were told that 1/6 = 0.1666666666... is not a recurring decimal because the first digit is different to the others.
- Students were told to copy a definition of a regular polygon as one with equal sides (neglecting to mention the requirement for equal angles).
- Students were told that the best outcomes from a dice-rolling experiment would be those where the observed frequencies were closest to the expected frequencies (and that it was therefore a good idea for the number of repetitions of the experiment to be a multiple of six).

26. In areas where recruitment and retention of mathematics teachers were a problem, schools sometimes had to rely on unsuitable staff. Some of the students attending colleges in these areas complained to inspectors that they had been taught by several different temporary teachers during Key Stage 4. At best, non-specialist, temporary teachers were given clear instructions about what to do; at worst, they simply set students exercises, with little guidance.

Covering the syllabus

27. Teachers' attention to detail and the discipline needed to cover the syllabus were major factors in students' success in GCSE, AS and A level examinations. However, this was sometimes at the expense of developing students' ability to apply their mathematical knowledge, since a good proportion of time was spent teaching them how to approach and solve examination questions. Some centres focused single-mindedly on the examination requirements, often supported by intensive 'drill and practice' revision activities, and their students achieved good examination results. Some practice is necessary, but the danger of using such an approach exclusively is that it gives a temporary boost that is not sustained at the next examination level. In particular, it does not prepare GCSE students well for advanced level study in mathematics.

28. Because of what teachers saw as the pressure of time and the compartmentalised nature of some syllabuses, they often felt that they could not afford to make links with other mathematical topics, or felt constrained to tackle topics in a fixed order. As a result, students did not build the essential links for a secure understanding of mathematics as a whole.

AS further mathematics lesson in a sixth form college

In a lesson on simultaneous equations in three unknowns, the teacher concentrated on the standard technique of elimination and back substitution, including how to handle under-specification and inconsistent equations. Where solutions were lines, they were given in parametric form. Though the text book and the teacher both referred to diagrams of intersecting planes, the link between parametric equations and vector equations was not made. This was because the teacher was reluctant to bring forward A2 mathematics work on vectors for the AS further mathematics students, even though it would have enhanced their geometric understanding considerably.

Support for teaching and learning

Factors contributing to high achievement

Professional development and good practice

29. Frequently, examples of good and weak teaching and learning were observed in the same school or college. However, in departments with a culture of disseminating good practice and which were open to innovation, the quality of learning and achievement was more consistent. Their departmental meetings focused on professional development as well as dealing with necessary administrative matters.

30. The focus for the professional development of mathematics teachers was often on preparing to teach new courses, for example GCSE or AS/A2 mathematics units. Such development was recognised as necessary and worthwhile by participants.

31. However, the professional development opportunities which had the most impact on learning and achievement helped teachers to plan programmes that developed mathematical understanding coherently over time and improved methodology by encouraging reflection on the effectiveness of teaching approaches.

Standards Unit teaching and learning framework

32. Resources from the teaching and learning framework for mathematics produced by the DfES Standards Unit were being used successfully in a number of colleges. These materials encouraged teachers to be more reflective and offered strategies to encourage students to think more independently. They encouraged discussion and active learning in AS, A level and GCSE lessons.

33. While some colleges were just dipping into the resources, a few had used the full package to transform teaching and learning across an entire mathematics team. The new approach had enabled one of the colleges visited to accept students on to its AS courses with GCSE grade C or grade B at the intermediate tier, with the expectation that they would be successful. Success rates at all levels were impressive in this college.

34. The materials helped to expose students' common misconceptions and to develop their skills and interest successfully. In addition, students' individual needs were identified and appropriate activities selected for them. These colleges gave students sufficient time to develop secure understanding of key concepts and helped to achieve good progress.

Additional support for students

35. Most schools and colleges provided formal or informal support beyond the taught sessions. This was given by teachers in their free time or through a mathematics workshop or homework club where duty teachers were available to help at scheduled times during the week. This was sometimes an important factor in helping students who were struggling to keep up with course requirements. It was also seen as an effective means of motivating students to study the subject. Some of this provision made imaginative use of outside resources or encouraged the students to help one another in practical ways.

Inner-city FE college

The college has a well established system of peer tutoring, and there are three twilight homework clubs each week at which the duty lecturer is assisted by undergraduates from the local university. The latter provide good role models for the college students. The sessions are well attended and highly valued by the students.

Factors acting against effective achievement

Teaching resources

36. Textbooks written to match specific GCSE syllabuses or individual AS or A2 units, often with awarding body approval, were popular with teachers and students. The sharp focus on the examination, with specimen questions and practice papers, meant they were an effective tool for revision and preparation for examinations. However, this strength was also a potential weakness, as only the best texts made mathematical connections beyond the targeted unit and promoted mathematical enquiry and understanding. Effective teachers compensated for this, but in less confident hands the subject was reduced to techniques for passing examinations.

37. The teaching of some key topics, particularly in level one and two courses, was hindered by a lack of resources to provide concrete experience. For example, students learnt about converting between kilograms and pounds, but had no idea of what a kilogram or a pound mass weighed. Many students learnt formulas for calculating areas, without appreciating that their answer told them how many unit squares were needed to cover the shape. Resources did not always meet the needs identified by initial diagnostic testing in sixth form colleges and general further education colleges.

Courses and examinations

Factors contributing to achievement and participation

Bridging programmes

38. Some colleges made effective use of preliminary courses, such as application of number key skills, basic skills numeracy and level 1 free-standing mathematics qualifications (FSMQ), to meet the particular needs of students who were not yet ready to achieve GCSE grade C. Similarly, for students with a relatively weak GCSE background, a few colleges successfully provided a two year course for AS mathematics. More commonly, colleges offered AS use of mathematics and a range of appropriate FSMQ units as a bridging course prior to studying AS or A level mathematics. Relatively few students took these qualifications, but

those who did usually found them interesting and worthwhile. They learnt to use mathematical methods in more vocational contexts and, where a course was well prepared and effectively marketed, they achieved good results.

A range of options

39. Large colleges and bigger sixth forms were able to provide sufficient mathematics AS and A2 courses to allow students to mix and match with almost any combination of subjects. There was also the possibility of allowing students to choose between optional units, for example, mechanics, statistics or decision mathematics. In these circumstances, participation levels could be relatively high.

40. Further mathematics, at AS and A level, was normally available in the larger institutions. In some places, particularly areas of high social disadvantage, further mathematics provision was not readily available, although an increasing number of further mathematics centres were being set up to tackle the problem.

Accelerated examination entry

41. A response of some departments to the Gifted and Talented initiative was to enter students early for GCSE. Early entry can be successful in circumstances where the students all have the ability to achieve high grades and there is a carefully planned learning pathway available which ensures an appropriate depth of understanding of mathematics beyond GCSE.

> **A selective girls' school**
>
> In this school the entire top set was entered for GCSE mathematics in Year 10. This proved successful, with all entrants securing A* grades. Students then progressed to complete the AS and A2 mathematics courses in Years 11 and 12, with a number completing further mathematics in Year 13.

42. Inspection evidence suggests, however, that only a small minority of schools are in a position to use such a policy effectively.

Factors acting against effective achievement

Examination demands and the criteria for assessment

43. Teachers naturally wish their students to be successful and conscientiously prepare them for examinations, which are the main public measures of achievement. However, there is considerable evidence, not only from this survey, that many students are learning to

pass mathematics examinations without necessarily gaining the mathematical skills and understanding to apply their knowledge accurately and independently.

44. The assessment criteria for key skills application of number portfolios and the data-handling component of GCSE coursework are sometimes interpreted by teachers in a way that does not always reward appropriate mathematical thinking. Students are advised that extra marks can be obtained if they demonstrate particular techniques, even though they might be tokenistic or irrelevant. The survey showed that higher-achieving students were sometimes taught to use more advanced statistical techniques specifically to access the highest marks, even though they might have been statistically inappropriate.

Upper ability Year 11 GCSE group

In a sequence of lessons on data handling, students were using paired data from a widely available data set to test out hypotheses of their own choice. They were encouraged to select samples of size 5 from their data sets of size 40 and to calculate Spearman's rank correlation coefficient for the sample, something they had been taught the previous week. If the correlation coefficient was near to 1 or 1, the students plotted a scatter diagram for the full data set and drew a line of best fit (by eye). The process, which is of dubious statistical or educational validity, was justified by the need to meet certain assessment criteria.

Progression within mathematics

45. In some areas where students transferred between institutions at age 16, post-16 achievement and participation were affected adversely by variations among schools in the preparation of potential AS and A level students. A number of factors contributed to this, including specialist school status, Gifted and Talented programmes, and the availability of the GCSE statistics qualification, which allows many higher-attaining mathematics students to gain an extra GCSE with no extra teaching time being required.

46. There were usually liaison arrangements between sixth form colleges and 11–16 schools, but rarely an entitlement for students to be taught the full higher level GCSE mathematics in Key Stage 4. The variations made it harder for colleges to meet the needs of all students wishing to study mathematics at level 3 and beyond, and some students became demotivated when they felt less well prepared than their peers.

Interviews with staff and students in a sixth form college

Students, teachers and senior managers in one sixth form college all commented on the difficulties that arose because of incomplete syllabus coverage and lack of specialist teaching in local schools. They reported that some schools had not covered higher level GCSE in full, but others had entered their most able students for GCSE in year 10. Early entry took two forms: higher level GCSE progressing to AS units in year 11, and intermediate level GCSE followed by GCSE statistics. The latter group was usually weak on algebra. The accelerated students in the first group were often left to teach themselves for part of their programme, usually from standard textbooks, with limited success. They did not necessarily follow the same examination board as the college. Some were put off by their first taste of level 3 mathematics and had consequently refused to consider taking further mathematics.

Notes

The survey was carried out to establish the factors which make the most significant contributions to high achievement, motivation and participation in 14–19 mathematics, and the factors which act against high achievement.

The findings are intended to contribute to the debate on the future of mathematics education in England, following the publication of the Smith Report, *Making mathematics count.*

In the summer and autumn 2005, Her Majesty's Inspectors visited 26 schools, sixth form colleges and general further education colleges in a variety of areas. They were selected to be broadly representative of the range of provision for 14–19 mathematics and post-16 numeracy. The institutions involved are listed in Annex 1.

Specialist inspectors conducted two-day visits. They observed lessons, scrutinised students' work and other documentation, and held discussions with teachers and students. This report draws principally on these inspections, but also on additional evidence from other mathematics subject visits and Secondary National Strategy visits to schools, carried out by Ofsted during 2005.

Further information

Making mathematics count: the report of Sir Adrian Smith's inquiry into post-14 mathematics education (937764), The Stationery Office, February 2004.
Available at: www.mathsinquiry.org.uk/report/

14–19 Education and skills implementation plan (Ref: 2037-2005DCL-EN), DfES, 2005.
Available at: www.dfes.gov.uk/publications/14–19implementationplan/

Teaching and learning frameworks

From April 2003 the DfES Standards Unit (now known as the Improvement Group) worked with colleges, school sixth forms and others to develop teaching and learning frameworks in priority curriculum areas. In mathematics the framework includes teaching resources, a professional training programme for subject coaches and regional mathematics subject-coaching networks. Each framework has a guidance book for teachers, a range of resources for learners, teacher-training resources including training videos and DVDs (where appropriate), and guides and a CD-ROM illustrating the approaches and containing supporting materials and session plans. The framework for mathematics was launched in September 2005. Details of the teaching and learning frameworks are available at:
www.successforall.gov.uk/index.cfm?pg=84

Improving learning in mathematics: challenges and strategies. This speech, given on 6 October 2005 by Bill Rammell MP, Minister of State for Lifelong Learning, Further and Higher Education, is also available through the above link.

Annex 1

Institutions visited for the survey

Colleges

Boston College

Braintree College

Calderdale College, Halifax

Dewsbury College

East Norfolk Sixth Form College, Gorleston

Epping Forest College, Loughton

Furness College, Barrow-in-Furness

Havering Sixth Form College

Huddersfield Technical College

Joseph Priestley College, Leeds

Kendal College

Lakes College West Cumbria, Workington

Newham Sixth Form College

Palmer's College, Grays

Paston College, North Walsham

Stockport College

Tower Hamlets College

Xavarian College, Manchester

Schools

Local authority shown in brackets.

Ilkley Grammar School (Bradford)

Malton School (North Yorkshire)

Moulton School (Northamptonshire)

Northampton School for Boys (Northamptonshire)

Rivington and Blackrod High School (Bolton)

Saints Peter and Paul Catholic High School, Widnes (Halton)

Sponne School Technology College (Northamptonshire)

The Catholic High School, Chester – a Specialist Science College (Cheshire)

Annex 2

Mathematics and numeracy programmes available for 14–19 students in the institutions visited

Level 3: AS mathematics
AS use of mathematics
AS further mathematics
A level mathematics
A level further mathematics

Level 2: GCSE mathematics
Key skills application of number
Free-standing mathematics qualifications

Level 1: GCSE mathematics
Key skills application of number
Free-standing mathematics qualifications