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Developing Primary Pre-service Teachers' Mathematical Content Knowledge During Practicum Teaching

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Developing Primary Pre-service Teachers' Mathematical Content Knowledge During Practicum Teaching.

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Abstract: While it is recognised that a teachers' mathematical content knowledge (MCK) is crucial for teaching, less is known about when different categories of MCK develop during teacher education. This paper reports on two primary pre-service teachers, whose MCK was investigated during their practicum experiences in first, second and fourth years of a four-year Bachelor of Education program. The results identify when and under what conditions pre-service teachers' developed different categories of their MCK during practicum. Factors that assisted pre-service teachers to develop their MCK included program structure providing breadth and depth of experiences; sustained engagement for learning MCK; and quality of pre-service teachers' learning experiences.

Introduction

As mathematics teacher educators we aim to assist pre-service teachers to create interest in, and passion for, learning and teaching primary mathematics (Vale & Livy, 2013). We are committed to ensuring our graduating teachers know their subject matter knowledge and are capable of demonstrating this knowledge when teaching primary mathematics. As part of the standards for teaching in Australia it is also expected that Australian teachers know the content they teach (Australian Institute for Teaching and School Leadership [AITSL], 2011; Teacher Education Ministerial Advisory Group [TEMAG], 2015). Yet there is ongoing concern within Australia that graduates are not equipped with the content knowledge they need for teaching and the integration of professional experience and theory needs to be improved (Parliament of Victoria Education and Training Committee, 2005; TEMAG, 2015). A review of teacher education suggested that professional experiences that is, teaching practicum, should be incorporated into each year of teacher education programs and pre-service teachers should practise teaching over a range of year levels (Parliament of Victoria Education and Training Committee, 2005). The recent TEMAG (2015) report recommends increasing pre-service teachers' practicum experiences with an aim of improving the quality of teacher education programs to ensure that graduate teachers are classroom ready and meet the Graduate Level of the AITSL (2011) Professional Standards. A study designed to identify what and when pre-service teachers' develop mathematical content knowledge (MCK) during their practicum experiences would be important for assisting the planning and structure of future practicum experiences that ensure pre-service teachers maximise their learning of MCK when in schools.

MCK underpins the decisions made by teachers for students' learning and is a critical attribute of mathematics teachers' knowledge (Rowland, Turner, Thwaites, & Huckstep, 2009). The literature highlights the significance of different categories of MCK used to describe MCK (Ball, Thames, & Phelps 2008; Chick, Baker, Pham, & Cheng, 2006; Ma, 1999; Rowland et al., 2009). For example in the Knowledge Quartet (Rowland et al., 2009), *foundation knowledge* focuses on what teachers know and their beliefs about mathematics; *foundation knowledge* might be used when introducing a new topic to students or when recording a mathematical expression and concerns teachers' subject matter knowledge (Rowland, et al., 2009). *Breadth* and *depth* of subject matter knowledge relates to a teacher's capacity to connect a topic with topics and to make connections with topics of greater conceptual power (Ma, 1999). Ball, et al. (2008) described the knowledge for teaching as *specialised content knowledge*, more than knowing the mathematical content of an average adult and different from *common content knowledge*. Therefore it is also important that pre-service teachers are provided with opportunities during teacher education programs to ensure they develop different categories of MCK they will rely on for teaching primary mathematics.

Recent Australian studies have reported on MCK and pedagogical content knowledge (PCK) identifying challenges when assisting pre-service teachers and practicing teachers to develop the complex knowledge required for teaching primary mathematics (Callingham, Chick, & Thornton, 2012; Frid, Goos, & Sparrow, 2009). Many studies have reported difficulties or deficiencies of teachers' and pre-service teachers' MCK but less is understood regarding how and when pre-service teachers develop MCK (Anthony, Beswick, & Elle, 2012; Ponte & Chapman, 2006). A large international study of 17 countries, Teacher Education Development Study (TEDS-M) identified that quality of opportunities to learn including practicum were important factors contributing to increased levels of MCK (Tatto, Schwille, Senk, Ingvarson et al., 2008; Tatto & Senk, 2011). Australian pre-service teachers were not part of the TEDS-M study.

Other smaller studies of pre-service teachers and teachers have highlighted the importance of teacher education program delivery and design and agree university and school partners should work together and collaborate to improve learning for all including teachers, pre-service teachers and university lecturers (Adoniou, 2013; Arnold, Edwards, Hooley, & Williams, 2011; Allen, Ambrosetti, & Turner, 2013; Kazemi, Franke, & Lampert, 2009). An example of such collaboration is a flexible integrated approach where pre-service teachers attended school-based tutorials at the school they completed their practicum, assisting development of their teacher identity (Harlow & Cobb, 2014). Turner's (2012) longitudinal study of beginning teachers in England identified that working with students and reflecting on classroom experiences assists development of MCK. At the time of commencing this study, there were no similar longitudinal studies found within Australian research.

Anthony, et al. (2012) in their review of prospective teachers of mathematics identified that there are many small-scale studies that report on pre-service teachers' MCK and coursework experiences. A longitudinal study designed to investigate the development of Australian pre-service teachers' MCK during practicum teaching will contribute to our understanding of pre-service teachers' development of different categories of their MCK during teacher education. This paper reports on part of a four-year longitudinal study of 17 primary pre-service teachers' MCK (Livy, 2014) and will consider the following research question: What factors contribute to the development of different categories of primary pre-service teachers' MCK during their practicum teaching experiences? The case of two primary pre-service teachers is presented in this paper.

Literature Review

From a research perspective, frameworks of teacher knowledge can assist with deepening our understanding of the different categories used to describe MCK as well as the MCK pre-service teachers gain during different program situations (e.g. Ball, et al., 2008; Chick, et al., 2006; Rowland et al., 2009; Shulman, 1987). Whilst developing the knowledge of mathematics includes a combination of theoretical and practical knowledge (Novotná, 2009), universities and schools are both responsible for bridging the gap between the knowledge taught during coursework and practicum (Allen et al., 2013).

Shulman's (1987) seminal work highlighted the importance of considering the different types of knowledge required for teaching and characterised teachers' content knowledge as the "amount and organisation of knowledge in the mind of the teacher..." (p.9). Building on the work of Shulman other researchers have developed frameworks as a means for understanding the complex relationship between the types of knowledge required for mathematics teaching. Each of the frameworks discussed below were developed from observation and analysis of teachers or pre-service teachers in classrooms, and therefore represent models of MCK in practice.

Ball's (1993) study of dilemmas of teaching elementary school mathematics was a turning point in her journey when she investigated her own teaching and students' learning that arose during primary mathematics lessons. Later, Hill, Ball and Schilling's (2004) video taped teachers during mathematics lessons and categorised the mathematical skills and knowledge teachers demonstrated when they: posed questions; gave explanations; chose tasks; used representations; recorded mathematics on the board; sequenced examples; analysed students' errors; appraised and mediated. Building on the scholarly work of Shulman (1987; 1998) and their own research Ball, Hill and colleagues contributed to our understanding of the categories of mathematics teaching needed to improve students' learning of mathematics (Ball & Bass, 2003; Ball, Bass, & Hill, 2004; Ball et al., 2008; Hill et al., 2004; Hill, Ball, & Schilling, 2008) by developing the Domains of Mathematical Knowledge for Teaching framework (Figure 1.)

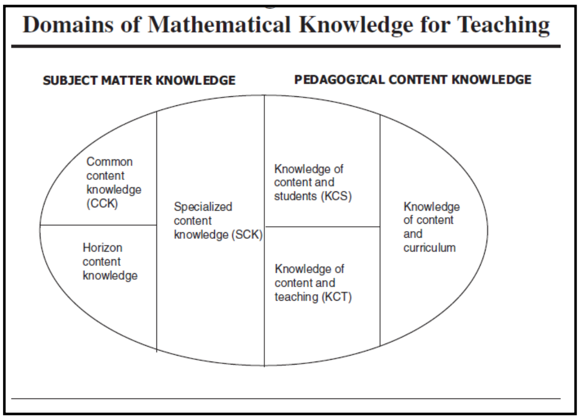

Figure 1. Domains of Mathematical Knowledge for Teaching (Ball et al., 2008, p. 403)

Ball's et al. (2008) Mathematical Knowledge for Teaching framework (Figure 1) includes two domains: subject matter knowledge and pedagogical content knowledge (PCK). Three categories describe subject matter knowledge. *Common content knowledge* enables teachers to know the mathematics they teach and *specialised content knowledge* is a unique content knowledge special to teachers. The third, *horizon content knowledge* is when a teacher demonstrates understanding of the complexities of mathematical topics, has advanced knowledge, broad understanding of mathematical ideas and connections, and links their *common content knowledge* with curriculum that their students know and will know in future years (Ball & Bass, 2009; Ball, Thames & Phelps, 2008).

Ma (1999) documented the difference between American and Chinese elementary teachers and key elements of the differences between their knowledge of mathematics. The Chinese teachers discussed mathematical problems with interconnections and demonstrated conceptual understanding wanting to know how and know why. The Chinese teachers' knowledge was described as profound understanding of fundamental mathematics (PUFM) demonstrating *breadth*, *depth* and *thoroughness* of their MCK.

> *A profound understanding of mathematics has breadth, depth, and thoroughness. Breadth of understanding is the capacity to connect a topic with topics of similar or less conceptual power. Depth of understanding is the capacity to connect a topic with those of greater conceptual power. Thoroughness is the capacity to connect all topics. (Ma, 1999, p. 124)*

Ma's study was described as a new approach and perspective on what teachers knew and how they articulated their mathematical knowledge when teaching (Even & Ball, 2003). Ma (1999) suggested that PUFM was attained during Chinese teachers' careers and built on what the teachers learnt during their own schooling. Chinese teachers learn from their colleagues, learn by doing mathematics and solving problems in several ways, they learn when teaching mathematics with their students and when studying teaching materials. These findings were significant for the professional development of teachers as it identified criteria that promoted multiple approaches for improving the quality of teachers' knowledge of the elementary mathematics curriculum.

Australian researchers Chick, Baker, et al. (2006) combined categories of PCK and MCK in their framework that was used to define teachers and pre-service teachers' knowledge when responding to interview items. The three sections of the framework assisted with identifying "subtle difference between teachers' responses, which may be attributed to differences in knowledge" (Chick & Baker, 2005, p. 256). The section, *Content Knowledge in a Pedagogical Context* (see Table 1) focused on MCK and five categories for classifying different aspects of how a teacher may demonstrate their MCK.

PCK Category	Evident when the teacher…
Profound Understanding of Fundamental Mathematics	Exhibits deep and thorough conceptual understanding of identified aspects of mathematics
Deconstructing Content to Key Components	Identifies critical mathematical components within a concept that are fundamental for understanding and applying that concept
Mathematical Structure and Connections	Makes connections between concepts and topics, including interdependence of concepts
Procedural Knowledge	Displays skills for solving mathematical problems (conceptual understanding need not be evident)
Methods of Solution	Demonstrates a method for solving a mathematical problem

Table 1: Content Knowledge in a Pedagogical Context (Chick, Baker et al., 2006 p. 299)

Many of the categories in the PCK category defined by Chick and colleagues (2006) combined or included categories of other frameworks (e.g. Ball, 2000; Ma, 1999; Shulman, 1986, 1987) and all categories are relevant to PCK. *Profound Understanding of Fundamental Mathematics* (PUFM) relates to Ma's (1999) theory of *breadth* and *depth* of mathematical topics and is evident when the teacher has deep and thorough conceptual understanding. *Deconstructing Content to Key Components* is evident when a method or estimation is used to check an answer and a teacher can identify the critical elements of the concepts. The last two categories of *procedural knowledge* and *methods of solutions* may be used by teachers and most adults in their work and could also be described as CCK. *Procedural knowledge* can be used for solving mathematical problems and *Methods of Solution* is evident when the teacher displays one method to solve the problem (Chick, Baker et al., 2006).

The Knowledge Quartet framework was developed by analysing and identifying primary teachers' MCK in action (Rowland et al., 2009; Thwaites, Huckstep, & Rowland, 2005; Turner, 2008). The Knowledge Quartet (see Table 2) has four categories *foundation knowledge*, *transformation*, *connection* and *contingency* (Rowland et al., 2009).

Category	Code
Foundation	Adheres to textbook Awareness of purpose Concentration on procedures Identifying errors Overt subject knowledge Theoretical underpinning Use of terminology
Transformation	Choice of examples Choice of representation Demonstration
Connection	Anticipation of complexity Decisions about sequencing Making connections between concepts Making connections between procedures Recognition of conceptual appropriateness
Contingency	Deviation from agenda Responding to children's ideas Use of opportunities

Table 2: The four categories and codes of the Knowledge Quartet framework (Rowland et al., 2009, p. 29)

The first category of the Knowledge Quartet (Table 2) , *foundation knowledge* assists teachers to make decisions for mathematics teaching and lists codes concerning subject knowledge that is evident when planning and teaching. In addition *foundation knowledge* underpins the other three dimensions of the *knowledge quartet* framework relating to knowledge in action (Rowland et al., 2009). Unlike the previous frameworks *foundation knowledge* includes beliefs about mathematics such as a clear awareness of the purpose of the mathematics education (Thwaites et al., 2005).

Transformation identifies how the teacher is required to use what they know when presenting ideas to their students, such as teacher choice of examples, procedures or choice of student activities. Appropriate representations will assist students' learning (Rowland et al., 2009). *Connection* relates to the "coherence of the planning or teaching across an episode, lesson or series of lessons" (Rowland et al., 2009, p. 31) and relates to Ma's (1999) description of *breadth* and *depth*. *Contingency* is when a teacher is presented with an unexpected teaching event during their lesson, and they have to decide how they will respond (Thwaites et al., 2005). This has also been described as a teachable moment (Clarke, Cheeseman, Gervasoni, Gronn et al., 2002) and requires the teacher to draw on their MCK and PCK that will determine the quality of the response (Rowland et al., 2009).

The origin and structure of these frameworks indicate that pre-service and in-service teachers continue to develop MCK in practice. Therefore practicum is not simply an opportunity to put CCK and *foundation knowledge* into practice, but ought to provide an opportunity for pre-service teachers to further develop MCK, including *specialised content knowledge, transformation, connections*, *contingencies, breadth* and *depth* of MCK. Since difficulties in pre-service teachers' MCK have been reported in a number of studies across the world (Ponte & Chapman, 2008), teaching practicum during pre-service teacher education also needs to provide an opportunity to develop or extend *common content knowledge* and *foundation knowledge* depending on the amount of knowledge pre-service teachers commence their program demonstrating.

Other studies have identified different factors that may influence the development of MCK such as teacher identity, teacher beliefs and program structure. Pre-service teachers' practicum experiences are important for providing realistic understanding of what it means to be a teacher and for developing their teacher identity and beliefs (Harlow & Cobb, 2014; Philipp, Ambrose, Lamb, Sowder et al., 2007). Philipp et al. (2007) reported that early practicum experiences where pre-service teachers identify and then debrief about children's mathematical thinking could improve their MCK. Others agree that pre-service teachers as early as first-year of their program need to develop professional knowledge, both for practice and in practice (Adoniou, 2013; Harlow & Cobb, 2014). Butterfield (2012) suggests programs designed to immerse pre-service teachers in practicum experiences that engage them in activities, focusing on the mathematics and areas of their mathematical difficulties will assist with developing their MCK. Research has also highlighted difficulties pre-service teachers experience during practicum. Huntley's (2013) study of pre-service teachers' mathematics lesson plans identified that they lacked structure and the ability to choose tasks that scaffolded level of difficulty. Similarly Livy (2010) reported a second-year pre-service teacher having difficulty choosing appropriate examples when teaching a subtraction lesson to Year 3 students.

As mentioned earlier, combining theory and practice is important and some studies have reported on such experiences. A Praxis Inquiry approach allows for teachers to reflect and make connections with teaching experiences during tutorials at university (Arnold et al., 2011). Kazemi's et al. (2009) guided public rehearsals during tutorials aimed to assist pre-service teachers to develop their knowledge of mathematics for practicum teaching. Others agree a collaboration of university and school based teaching experiences provides varied opportunities for pre-service teachers to learn how to teach, plan and reflect on their teaching with their lecturers and mentor teachers (Adoniou, 2013; Arnold et al., 2011; McDonough & Sexton, 2011).

The literature on MCK demonstrates the complexities and different categories of knowledge required for primary mathematics teaching. Therefore identifying when different categories of MCK develop during teacher education is important for enhancing pre-service teachers' understanding of the mathematics they will rely on when teaching. Furthermore as the expectation to increase the role of teaching experience in schools is mounting, we need evidence that these opportunities are capable of promoting pre-service teachers' MCK.

Method

The study reported in this paper is part of a mixed-methods, four-year longitudinal study designed to extend understanding of the MCK that 17 Australian pre-service teachers developed during their teacher education program. The participants were enrolled in a combined primary and secondary (Preparatory[1]-Year 12) Bachelor of Education. The larger study identified factors that enhanced their MCK for teaching primary mathematics (Livy, 2014; Livy & Herbert, 2013; Livy & Vale, 2011; Muir & Livy, 2012). The study was judged as valid, as reported in the larger study (Livy, 2014).

Settings and Program Structure

As part of the program pre-service teachers completed coursework at university and practicum experiences in primary and secondary schools. Graduates were qualified to teach in both primary and secondary schools, including the teaching of primary mathematics and their discipline specialisation in secondary schools. The program structure is summarised in Table 3 listing when pre-service teachers undertook three core primary mathematics units (Units 1A, 2A and 2B) and the number of practicum days completed in primary schools during first, second and fourth years.

Year	Core Mathematics Education Units	Elective Mathematics Education Units	Practicum Experience
1	Unit 1A	Unit 1B Secondary discipline specialisation units	Primary school (20 days)
2	Unit 2A Unit 2B	Unit 1B[2] Secondary discipline specialisation units	Primary school (32 days)
3	None	Unit 1B[2] Secondary discipline teaching units	Secondary school – discipline specialisation (42 days)
4	None		Primary school (50 days)

Table 3: Course structure for mathematics teacher education

Satisfactory completion of a Mathematical Competency Skills and Knowledge (MCSK) test was a requirement for Unit 2B. The MCSK test included 49 items ranging in difficulty up to Year 8 mathematics (ACARA, 2015). Items were closed question types and required short answers using words or symbols (numbers). The items assessed pre-service teachers' knowledge of number, geometry, measurement, statistics and probability. All MCSK tests were completed under exam conditions, with working out encouraged, no calculators were permitted and pre-service teachers were given 180 minutes to respond to all MCK items. At the time of the study many pre-service teachers completed Unit 1B because they required additional knowledge for teaching primary mathematics and had difficulty passing the MSK test during Year 2.

[1] Preparatory is the first year of schooling and is now called Foundation Level throughout Australia

[2] Compulsory for pre-service teachers who did not pass the MCSK test during Year 2

Practicum, called Project Partnership at this university, was a collaborative partnership between the university and schools. Pre-service teachers were usually assigned different schools for each year of the program, attending regularly on Tuesday as well as undertaking full week placements. During practicum, pre-service teachers taught collaboratively with their mentor teachers (classroom teacher), to enhance their knowledge of teaching including primary mathematics.

The Participants

Two of the 17 pre-service teachers participating in the larger study were selected to analyse and report on the factors influencing the development of MCK during practicum experiences. Kerri and Esther[3] were chosen for this study because, unlike some participants in the larger study, they were able to demonstrate MCK by passing a Mathematical Skills and Knowledge (MCSK) test during second-year of their pre-service teacher education program and longitudinal study. Therefore it might be expected that they would continue to extend their MCK and demonstrate other categories of MCK as described in the review of literature during their practicum experiences. Participants in the larger longitudinal study were enrolled in a combined primary and secondary teacher education program. Esther chose Drama and English and Kerri chose Drama and Studies of Society and Education (SOSE) as their secondary discipline specialisations; neither chose or completed a secondary mathematics discipline specialisation.

Prior to university Kerri had completed Year 12 Further Mathematics and Esther had completed Year 11 Mathematical Methods (VCAA, 2010). Further Mathematics consisted of study in data analysis and then a selection of three of six modules: number patterns; geometry and trigonometry; graphs and relations; business mathematics; networks and decision mathematics; and matrices (Victorian Curriculum and Assessment Authority (VCAA), 2010). Esther completed Year 12 but had not taken mathematics past Year 11. She completed Mathematics Methods (CAS)[4] in Year 11 consisting of study in functions and graphs, algebra, rates of change and calculus and probability. Esther also had two years between completing secondary school and commencing her study, some of this time she worked as a nanny and travelled overseas.

Although Kerri had completed a higher level of mathematics at secondary school compared to Esther they both responded correctly to most items in the MCSK test in second-year, confirming they could demonstrate an accurate understanding of mathematical ideas or concepts, a category of *foundation knowledge*. Therefore their practicum experiences could be compared to identify opportunities that developed categories of MCK.

Data Collection and Analysis

Data were collected at different times and in different situations throughout the longitudinal study and included quantitative and qualitative methods. An ethnographical design was chosen and included four methods of data collections (McMillan, 2004) (Table 4).

[3] Pseudonyms used throughout

[4] Computer Algebra System

Method	Data gathered	Analysis
Qualitative	Artefacts: Primary mathematics lesson plans (Years 2 and 4 lesson observation)	Data were uploaded into NVivo and coded using the four categories of the Knowledge Quartet (Rowland et al., 2009) to identify categories of MCK
Qualitative	Responses to questionnaire (Year 2)	Descriptive analysis to determine demographic factors and open coding to identify prior learning, beliefs and attitudes
Quantitative	All pre-service teachers' MCSK test short answer item responses (Year 2 or as completed)	Ranking MCSK test items by percentage of correct responses to indicate level of difficulty by content domains (see Livy, 2014)
Qualitative	Lesson observations (Years 2 and 4)	Interviews and lesson observations were transcribed and data were uploaded into NVivo and coded using the four categories of the Knowledge Quartet (Rowland et al., 2009) to identify evidence of MCK and coded for breadth and depth using Ma (1999) and open coding to identify opportunities to develop MCK
Qualitative	Interview responses (Years 2, 3 and 4)	Open coding to identify opportunities to develop MCK or constraints on developing MCK

Table 4: Data gathering and analysis

A situated perspective was used for identifying how pre-service teachers interacted during practicum experiences, to identify when and under what conditions they developed their MCK. A situated perspective is when a study occurs in multiple contexts including the physical and social systems (Peressini, Borko, Romagnano, Knuth, et al., 2004). Data collection, management and analysis occurred simultaneously and included content analysis, reducing the data to identify when and under what conditions pre-service teachers demonstrated categories of MCK identified within the review of literature (Ball et al., 2008; Chick, Baker et al., 2006; Ma, 1999; Rowland et al., 2009). The data were read several times to identify factors and recurring themes. Three themes emerged when considering contributing factors and the findings were organised according to: program structure including breadth and depth of MCK, sustained engagement, and quality of pre-service teachers' learning experiences during practicum.

Development of MCK during Practicum

The following results include discussion of Kerri and Esther's distribution of practicum experiences. Next, first, second and fourth year practicum experiences are reported describing the MCK that Kerri and Esther may have or could have developed during practicum experiences. Finally a discussion is included reflecting on the responsibilities of the mentor teachers who supervised Kerri and Esther's practicum experiences. Additionally during their coursework at university Kerri and Esther were provided with opportunities that extended their *foundation knowledge*, *transformation* and *connections* when participating in lectures, tutorials and responding to assignments during their practicum experiences (Livy, 2014). The scope of this paper does not report on artefacts such as assignments or coursework experiences.

Distribution of Practicum Experiences

Not all pre-service teachers experienced different year levels during practicum experiences. It was by chance where the pre-service teachers were placed in first, second and fourth year and schools were responsible for allocating pre-service teachers to different mentor teachers rather than the university or pre-service teachers requesting their preferences for year levels. Although there was no evidence in this study, pre-service teachers with less MCK may opt to teach the lower levels so they do not have to teach older students mathematics avoiding an opportunity to extend their MCK and *depth* of MCK. *Depth* of MCK could be developed when pre-service teachers practised their teaching across different year levels.

Table 5 lists the distribution by year level of Kerri and Esther's experiences teaching primary mathematics during first, second and fourth years and is evidence of their *breadth* and *depth* of primary teaching experiences. For example, when pre-service teachers planned and taught a series of lessons they had the opportunity to demonstrate their *breadth* of mathematical knowledge building on from one lesson to the next or from one topic to the next, also assisting them to make *connections* of the topics they were teaching.

Name	First-year (20 days)	Second-year (32 days)	Fourth-year (50 days)
Esther	Year 2/3	Year 1	Foundation
Kerri	Year 1	Year 3/4	Year 5/6

Table 5: Distribution of practicum teaching during the Teacher Education Program (n=2)

Esther's primary teaching experiences occurred in lower primary year levels. In contrast Kerri's practicum experiences were distributed across lower, middle and upper primary year levels. Kerri's practicum provided conditions for her to extend her *depth* of MCK because she had the opportunity to watch, participate and teach students ranging from Year 1 to Year 6. As a comparison Esther's practicum was limited to lower year levels, including Preparatory to Year 3.

Pre-service teachers who do not experience upper year levels during their practicum experience may lack confidence to teach Year 6 students once graduated. During Esther's interview in fourth year she explained that she had not completed practicum with Year 6 students:

> *My school visits have been at the lower levels and I don't have any problems with mathematics so it hasn't [the maths] been a problem. [Also] if you told me to teach area and volume in Grade 6 level with a couple of days in advance I would have no problems coming up and teaching it. [I would] just do a bit of reading on it and understanding it is not a problem.*

Esther believed that she could rely on her MCK with revision but agreed that having the opportunity to teach Year 6 mathematics during the program may have helped her to learn more difficult mathematical concepts.

Kerri and Esther completed 102 days of practicum in primary schools throughout first, second and fourth year. The total number of days in schools (144 days) was far in excess of the teacher education program accreditation minimum of 45 days (Victorian Institute of Teaching, 2011) and during their program pre-service teachers at this university experienced more days in schools when compared to other Australian universities or counties (Tatto et al., 2012; Victorian Institute of Teaching, 2011). Therefore, providing pre-service teachers with many opportunities to participate and teach primary mathematics lessons.

MCK Developed During First-Year Primary School Practicum

As identified by Philipp et al. (2007) pre-services teachers can develop their MCK when observing and thinking about students' responses to mathematical problems. Kerri and Esther were not observed teaching during first-year of their practicum but some of the primary mathematical experiences may have developed or revised their *foundation knowledge*. For example they may have extended their knowledge of mathematical terms when listening to their mentor teacher during mathematics lessons or observed teaching strategies used to promote student understanding such as the choice of materials used to *transform* and represent the concepts being taught.

In first-year Kerri assisted in a Year 1 classroom and Esther a Year 2 and Year 3 composite classroom where they mainly worked with small groups of students, helping their mentor during different lessons. They reported that they gained minimal experience in planning and teaching mathematics lessons because their coursework expectations were connected to primary literacy teaching. Kerri and Esther had limited opportunities during first-year to develop their MCK during practicum when planning, teaching and reflecting on their mathematics' teaching. As a result, program structure and expectations of other core primary units of study may have distracted pre-service teachers from focusing on developing their MCK during first-year practicum.

MCK Developed During Second-Year Primary School Practicum

In second-year as part of their program requirements and coursework pre-service teachers were expected to teach at least 20 primary mathematics lessons. In addition to practicum experiences further opportunities to extend their MCK occurred when they completed two core second-year subjects during second semester and prepared and passed their MCSK test. The program structure of second-year provided pre-service teachers with increased opportunities to make *connections* with their MCK including theory and practice that would have fostered development of knowledge for teaching primary mathematics.

Kerri

The first author observed Kerri teaching a measurement lesson to a small group of Year 3 and 4 students. All pre-service teachers as part of their practicum guidelines were expected to prepare a lesson plan before teaching. The aim of the lesson, as listed in her lesson plan, was to re-cap on o'clock, half past and how many minutes each number on the clock represented. Kerri played a Time Bingo game with the students, holding up cards with different o'clock times and the students covered the time on a card with six different clock faces. The use of the bingo game suggested that Kerri believes that mathematics should be enjoyable and was evidence of her *foundation knowledge*. Next the students were given a worksheet and recorded different time periods on analogue clock faces. Kerri assisted different students, demonstrating how she could rely on her basic *foundation knowledge* when checking their responses and explaining the different times to the students.

However, Kerri had difficulty *transforming* her *foundation knowledge* and making *connections* when questioning a student during the lesson. She could not rely on her *foundation knowledge* when choosing questions that might assist the student's understanding for reading half past nine, instead she provided prompts that assisted the student to say the correct time:

> *Kerri: The minutes hand is half way past the twelve, the hours hand is half way past the nine, what time is it?*
> *Student: Ten past six*
> *Kerri: Not ten past six*
> *Student: Oh six past ten*
> *Kerri: No, no don't worry about the sixes, do you know how many minutes it represents? So it is 30 minutes past*
> *Student: Past six*
> *Kerri: It is not past six it is past the hour hand, what is that hour? What is that number? [points to nine].*
> *Student: nine*
> *Kerri: That is it what is another way to say that?*
> *Student: Half past nine*
> *Kerri: Good girl*

Rather than using a single representation of the concept being taught Kerri could have extended her choice of representations. She could have chosen to use a clock face with moving hands to assist the student's understanding by demonstrating the movement of the big and little hands as the time approaches half past nine. A range of representations would assist the student's understanding and demonstrate Kerri's knowledge of *transformation*.

During her interview with the first author Kerri explained that the time lesson was part of a series of five lessons she was completing as part of the requirements for her practicum. Each week Kerri had been teaching one lesson, focusing on the concept of time and this was the fifth lesson. When asked about how she planned these lessons she said,

> *[I] just look at last week's lesson and picking up on the strengths and weaknesses and what we still need to work on and things like that. That is what I did today because I needed to see whether they actually understood what I have been teaching them. My mentor writes me an evaluation every week on my lesson. She doesn't get a chance to come in and watch [Kerri took the lesson in a space next to where the mentor was teaching] but she can see me a bit so she just writes for me and gives me advise on what their [the students'] strengths and weaknesses and this will form the bases of the next lesson. I chose the activities today and I am free to do what I want. My mentor gets a copy of my lesson plan and any worksheets and stuff so she knows what we have been doing. [Before the lesson] she got the box out of the resource room with the clocks and things and said this is what you can do and she gave me a sheet of really small clock faces and they are the ones she uses and they are just blank and you have to fill in the hands and stuff. They were too small for my kids so I made up some myself. She hasn't shown me any other resources do with maths. I found a website myself to use…*

Kerri's mentor teacher could have offered further guidance before the lesson. Rather than only providing resources, explaining how to use the clocks and worksheet or discussion of how to help students who might have difficulties would most likely afford further opportunity to extend Kerri's choice of representations. Therefore when teaching Kerri may have demonstrated better *connections* within the lesson when responding to students' needs or *contingencies*. The mentor could also suggest a teacher resource book or website that might assist Kerri to extend her knowledge of the key concepts of the topic that would assist Kerri when planning.

Furthermore whilst Kerri was teaching the small group of students, (as expected in second-year) the mentor teacher was responsible for teaching the remainder of the class and was unable to notice difficulties Kerri might have with her MCK during the lesson. Also the mentor teacher's lesson feedback provided after the lesson suggests that the mentor teacher did not report on Kerri's MCK or PCK, strengths or weaknesses of the lesson. Instead, the mentor's feedback focused on the students' knowledge of mathematics rather than the skills of the pre-service teacher. In this situation the mentor teacher provided Kerri with limited supported when planning, during and after the lesson limiting Kerri's opportunities to develop her MCK.

Esther

In second-year Esther also taught a lesson focusing on how to tell the time and was observed teaching a group of 16 Year 1 students. This lesson was taught toward the end of second semester and Esther had nearly completed both of her core second year primary mathematics units at university. This was the first occasion that Esther had taught time. Before the lesson, her mentor teacher had discussed some ideas and explained that students would have most difficulty telling the time on an analogue clock compared to telling the time using a digital clock. Esther's mentor teacher provided an opportunity for Esther to reflect on how the students might respond to the mathematical concepts before the lesson and this could also assist Esther when planning including the choice of tasks she might use when teaching.

Esther prepared her lesson plan after meeting with her mentor teacher. The aim of the lesson as recorded in the lesson plan was for the students to revise time including half past, one-quarter past, o'clock and one-quarter to. The lesson plan provided evidence of *foundation knowledge* such as choice of mathematical language, big hand, small hand and o'clock.

Esther also listed four rotational tasks she had chosen for the students to complete. When planning the lesson Esther chose activities related to her topic, providing evidence of making *connections*. She also considered the materials she would use to help the students learn, for example she had laminated cards with clock faces, digital times and clock times (Figure 2). This was evidence of *transformation* by representing the mathematics to the students.

Figure 2. Esther's cards used for matching analogue and digital time

Esther commenced the lesson with the students sitting on the floor. She asked questions demonstrating how she was relying on her *foundation knowledge* assisting

> students to focus on the purpose of the lesson and also chose to use a large clock face with moving hands to promote student understanding.
> *The big hand is at the 12 and the little hand is at the four. What time do you think that might be…. If the big hand is on the three and the little hand is on the nine… Can someone come and show me what a quarter two eleven might be…*

Esther chose closed question types when teaching. Open questions would have provided an opportunity for students to respond with a range of answers, promoting opportunities for *contingencies*.

Next the students rotated in groups and completed different tasks designed to extend their understanding for telling the time. After the lesson Esther provided a comment about her lesson.

> *I think I should have broken up the lesson, into two lessons. One doing o'clock and half past, the next day doing quarter past and quarter two again.*

After the lesson, Esther reflected on the concepts she had chosen to teach during her lesson considering how students might learn and the sequence for learning to tell the time. Planning and teaching mathematics lessons provided an opportunity for Esther to use her *foundation knoweldge* to make *connections.*

MCK Developed During Fourth Year Primary School Practicum

In fourth year Kerri had experience of teaching a Year 5 and 6 class and Esther a Preparatory Year class. They reported that as part of their coursework they had not completed any primary mathematics teacher education since second-year and the focus of third year was on their secondary disciplines. Therefore they were both concerned that they had forgotten a lot of the mathematics they had learnt when revising for their MCSK test. The program did not provide them with sustained opportunities to revise and develop their MCK for each year of the program.

Kerri

When observed teaching in fourth year Kerri was assisting students to prepare for a National Assessment Program Literacy and Numeracy (NAPLAN) (Australian Curriculum Assessment and Reporting Authority (ACARA), 2010). Kerri had not prepared a lesson plan and said that her mentor teacher did not expect her to write one. For the lesson the students sat together on the floor while Kerri asked them to explain how they worked out their responses to items from a practice NAPLAN mathematics test. Kerri and the students discussed and shared their responses and thinking to 15 multiple-choice problems.

This lesson provided an opportunity for Kerri to make *connections* across different domains of the curriculum. For a geometry problem that required identifying the number of rectangles needed to make a hexagonal prism, Kerri drew a net of the prism to help the students to visualise the shape and count the six rectangles. Later during her interview with the researcher she also commented that using a net or other materials during her lesson would have helped students to visualise the mathematics and that some of the difficult problems could be followed up in the following lesson as further revision. This comment was evidence of her developing *specialised content knowledge* by making *connections* between lessons and concepts and thinking about the complexity of the mathematical topics.

For another problem the students were asked to divide 2515 by five, and the class agreed that the correct response was 503. One student said, "I got 53 because I did five times five is 25 and three times five is 15."

Kerri replied, "You just mucked it up," and did not consider or understand the student's misconception. The student most likely divided 25 by 5 and 15 by 5 rather than thinking 2500 divided by 5 is 500.

Instead she continued the lesson and did not act on the *contingency*, but rather completed a short division on the whiteboard demonstrating the correct response. This was the last problem the students discussed before packing up.

During the lesson Kerri was able to rely on her *foundation knowledge* and make *connections* with different student responses to a range of problems. She dealt with student comments and answers leading the discussion and demonstrating how she could rely on her *foundation knowledge*. She modelled calculations correctly recording some of the students' responses and *method of solutions* they described on a whiteboard. She was also able to *transform* her MCK by using appropriate mathematical language, teaching strategies or representations demonstrating her *breadth* of MCK throughout the lesson.

Kerri may have developed greater connections with the students' responses if she had completed the responses herself before the lesson, and discussed the correct solutions with her mentor teacher. Multiple choice questions, such as NAPLAN questions are designed with one response illustrating a misconception. Kerri could have also discussed the likely errors with her mentor and/or students providing an opportunity to extend her depth of mathematical knowledge. Lessons similar to this provide an opportunity for pre-service teachers to also deal with *contingencies* as students discuss the strategies they used to solve the range of questions.

Esther

In fourth-year Esther was observed teaching 16 Preparatory students and teaching teen numbers. The students were focussing on teen numbers because Esther's mentor teacher had explained that students often had difficulty saying and writing teen numbers and it was important to teach these numbers over a series of lessons. The lesson was the third lesson in a series of five lessons that Esther taught with guidance from her mentor teacher. Esther had prepared a lesson plan before the lesson, listing her choice of tasks for teaching teen numbers. The lesson plan provided evidence of her *foundation* knowledge and *transformation* by mathematical terminology that assisted students to develop efficient counting skills and understanding of teen numbers and choice of appropriate activities. Esther planned the mathematics lessons herself but always discussed her ideas with her mentor before teaching and any necessary changes were made before teaching.

> *I got to do the lesson myself… I was given the topic and my mentor said to come up with some lessons. I came up with four lessons and I put them to my mentor and said what do you think. She said, yes, yes, yes, she said this might work and you might want to think about this… she wanted them to do some acitivities with tens frames … I tweaked them then we found something that we were both happy with…. I used VELS and Nelson maths… VELS was least helpful and tells you the knowledge the kids should have like knoweldge of numbers one to twenty…the teacher resource [Nelson books] both helped [the most] it had language to use and examples of activities which is really what a new teacher needs… it is how to teach that is the issue [what I need to find out] because I know my maths.*

Esther's mentor teacher was in her final year of teaching having taught for many years and was an experienced classroom teacher who shared her resources and knowledge with Esther. She also provided Esther with guidance of what to teach and references that had suggestions for teaching. Esther also embraced the passion of her mentor and was demonstrating her own teacher identity. Esther's mentor had the skills to guide and assist Esther to *transform* her *foundation knoweldge* and make *connections* when teaching. Esther's development of MCK most likely occurred because she could rely on her *foundation knoweldge* when teaching.

For the lesson introduction Esther gave each of the students a card with a numeral from 1 to 16. They were asked to line up in order from the smallest to biggest number: "We have the smallest down here and biggest here... line up in the right order. Smallest to biggest…What comes after four? Who has five?"

One student who had the numeral 13 said, "I have thirty-three."

Esther asked him to try again and other students correctly answered, "Thirteen."

This situation could be coded, as a *contingency* and Esther should follow up with the student to assist them with reading teen numbers correctly as part of the lesson.

Throughout the lesson Esther relied on her foundation and did not have any difficulty with the mathematical content. She chose suitable mathematical language for teaching this year level:

> *What comes after six…which order do these numbers come in? [held up three cards 19, 18 and 20]. And identified an incorrect response when a student had confusion with thirteen. Next Esther used the interactive white board and asked different students to make teen numbers on tens frames. Then they recorded the digits making connections with the model and symbol. One student recorded the digit 9 back-to-front and Emily assisted him to write the digit correctly. "You need to draw a stick then a circle on the other side... well done.*

For the remainder of the lesson the students rotated through four activities: an ordering task, involving cutting pictures of people with numbers 1 to 20 on their clothing and pasting them in number order; a reading task, where students caught paper fish with a magnet on a string and read the number on the fish; and a tens frame counting activity on the computer with numbers less than 20. The mentor teacher assisted the students on the computer and other groups while Esther worked with a teacher focus group on the fourth rotating activity. Esther's choice of activities demonstrated how she could make *connections* by choosing a range of activities that assisted students to develop their understanding of one and two digit numbers to 20 and included tasks for ordering, naming, recording and counting.

One group used a stamp pad to print pictures and then counted the number of pictures. When Esther was helping one boy she noticed that he was recording his pictures in rows and said, "I like the way you did rows so you could keep track of the number of pictures you were doing."

This was an example of a *contingency* because Esther had not planned for the students to place their pictures in a certain order. She had most likely considered during the lesson that making rows would assist students to count their pictures with greater accuracy rather than randomly stamping all over the page and hence highlighted one student's response to other students in the class. Esther was also making *connections* with students' thinking whilst promoting her knowledge needed for teaching this topic.

During this lesson Esther used her *foundation knowledge* to choose appropriate teaching strategies that promoted the required mathematical understanding of teen numbers and numbers less than twenty. She demonstrated evidence of *transformation* because of the appropriate examples, materials and resources she used during the lesson. Esther also made *connections* that were evident in her planning and teaching and chose tasks that were different but focused on the same mathematical topic. Throughout the lesson Esther was able to question and respond to children's answers, questions and comments while teaching. This was most likely because Esther had planned the lesson herself with the support and guidance of a mentor who had the skills to assist her to extend the *foundation knowledge* she brought to the lesson.

As part of data collection, Esther provided a lesson plan for each of the lessons observed for this study as well as other lesson plans she had prepared when teaching with her mentor teachers. Consequently, Esther's development of her MCK may have been extended because she rehearsed her lessons by planning and considered different categories of her MCK when preparing to teach. For example, when planning Esther considered activities, choice of materials, choice of appropriate mathematical language she would use and questions she might pose to assist student learning. Esther relied on different categories of her MCK when planning and teaching during fourth-year.

Factors Contributing to Enhancement of MCK during Practicum

Both of the pre-service were chosen for this study because in second-year they could demonstrate correct responses to a MCSK test and therefore could rely on their foundation knowledge. The university provided practicum experiences in primary school during first, second and fourth years that provided opportunities for Kerri and Esther to *transform* their *foundation knowledge* when teaching, make *connections* when planning lessons, including a sequence of lessons on one topic and provided experiences of *breadth* and *depth* of MCK. Overall the results identified contributing factors that assisted or in some instances limited development of MCK during practicum experiences in primary schools.

Program Structure Including Breadth and Depth

The program provided many days in primary school settings and opportunities for pre-service teachers to experience mathematics lessons and develop their *breadth* and *depth* of MCK. Pre-service teachers were able to develop *breadth* of experience when teaching a series of lessons with the same year level. However, opportunities to learn and develop *depth* of MCK were restricted when pre-service teachers did not teach a range of levels. Pre-service teachers who might not experience upper year levels may have difficulty relying on their MCK of more advanced mathematical knowledge. Future studies may consider examining different combinations of *depth* of program experiences that also include teaching in lower secondary mathematics classrooms.

Sustained Engagement of MCK throughout the Program

The larger study reported on coursework and practicum experiences and sustained engagement of MCK throughout the program was a factor that contributed to the development of pre-service teachers' MCK (Livy, 2014). The scope of this paper reported on practicum experiences. However, it was likely that completing two core units of study related to primary mathematics whilst also completing practicum and assignments related to primary mathematics teaching most likely assisted pre-service teachers to make connections with theory and practise and their identity as a primary mathematics teacher. In contrast an emphasis on other areas of the curriculum, including coursework assignments during practicum distracted pre-service teachers from sustained engagement with their MCK during first year.

The pre-service teachers in this study were completing a primary and secondary teacher education program. As a result the secondary discipline experience in secondary schools (not mathematics) during third year was problematic as pre-service teachers did not sustain engagement with mathematics for each year of their program. Many Australian universities offer a Bachelor of Education program in primary teacher education but this primary and secondary structure is less common. The findings of this study could be compared with other programs that focus on straight primary teacher education.

Quality of Pre-service Teachers' Learning Experiences

The mentor teacher played an important role in assisting the quality of pre-service teachers' learning experiences and development of MCK for teaching. All mentor teachers should facilitate learning by guiding pre-service teachers when they are planning lessons; observing pre-service teachers teaching; providing feedback after the lesson; as well as modelling good practice when teaching primary mathematics lessons.

Pre-service teachers should also be expected to prepare detailed lesson plans bearing in mind the knowledge they need for teaching different topics at various year levels, seeking assistance from their mentor and reflecting on their own teaching and categories of MCK. Lesson plans are important for assisting pre-service teachers to consider the tasks and examples they will use with their students (Huntley, 2013). Kazami et al. (2009) identified guided rehearsals as important for assisting pre-service teachers to learn. Future studies could identify in greater detail the types of feedback the mentor teachers provided the pre-service teachers. Limited data were collected as part of this study; mentor teachers were not interviewed and their written practicum assessment reports were not collected.

Conclusion

While limited conclusions can be drawn from two cases this study highlights how practicum experiences were important for providing opportunities for pre-service teachers to develop different categories of MCK. Both pre-service teachers were provided with opportunities that extended their MCK including *foundation* knowledge, *transformation*, *connections*, *contingencies*, *breadth* and *depth* of MCK or *specialised* MCK for teaching during practicum. The contributing factors that assisted pre-service teachers to develop MCK during practicum teaching included program structure providing *breadth* and *depth* of experiences; sustained engagement for learning MCK; and quality of pre-service teachers' learning experiences. The findings of this smaller study were consistent with the study reporting on the larger cohort of participants (Livy, 2014).

Ensuring future pre-service teachers are given and can seek ongoing opportunities to enhance the different categories of MCK during different program situations will aim to improve the quality of teacher education programs. These findings will be useful when considering the recent TEMAG report (2015) and improving the quality of graduates and preparation of pre-service teachers to be classroom ready to teach mathematics.

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