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Australian Journal of Teacher Education

Volume 40 | Issue 12 Article 7

2015

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Recommended Citation

Kumar, R. S., & Subramaniam, K. (2015). From 'Following' to Going Beyond the Textbook: Inservice Indian Mathematics Teachers' Professional Development for Teaching Integers. *Australian Journal of Teacher Education, 40*(12).
http://dx.doi.org/10.14221/ajte.2015v40n12.7

This Journal Article is posted at Research Online.
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From 'Following' to 'Going Beyond' the Textbook: In-Service Indian Mathematics Teachers' Professional Development for Teaching Integers

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Abstract : In this paper we describe four Indian in-service middle school mathematics teachers' shifts in their roles with respect to the textbook. The shifts occurred through participation in collaborative investigation on the topic of integers in professional development meetings. Analysis of teachers' talk in these meetings indicated a shift in teachers' role from reliance on textbook to using the knowledge of integer meanings to establish the connections between contexts and representations. We claim that this change in role occurred as a result of teachers developing knowledge of important ideas and representations in the professional development setting and identifying themselves as a member of a professional learning community which values students' understanding. We argue that since roles are constitutive of teachers' professional identity, the shifts in roles indicates how teachers' identity evolved towards being an empowered mathematics teacher who design tasks and responds to students to support articulation of ideas and developing reasoning in mathematics.

Introduction

School teaching in India has a textbook-centric culture, where there is typically a single prescribed textbook in a centralised education system. A vast majority of schools follow textbooks prescribed and published by the government, which are made available to nearly all children. For a majority of children from underprivileged backgrounds, no other printed material is available apart from the prescribed textbook. A consequence is that teaching, including teaching of mathematics, is "textbook driven" (National Council of Education Research and Training [NCERT], 2006a, p.15) in the sense that resources for teaching like explanations, representations, examples, tasks, solutions and assessment used in classrooms are all largely drawn from the textbook. In some classrooms, teachers follow the textbook page by page in order to "cover" the syllabus. Thus, in the received view of teaching, teachers are not required to think explicitly about most teaching decisions as the authority is relegated to the textbook. This dependency of teachers on the textbook has been recognised as a problem in curriculum reform in India and as causing teaching to be mechanical (NCERT, 2005). One of the five basic principles underlying the National curriculum framework is "going beyond the textbook", which is thus an important goal for reforming mathematics teaching in India (NCERT, 2005, p. viii). The inclusion of this principle suggests that going beyond the textbook is part of ′good teaching′ as defined in the official discourse of curriculum reform. This however, stands in opposition to the cultural norm of teaching by strictly adhering to the textbook.

Textbook-centric teaching is not unique to India. Countries like South Africa, Korea and China also have a predominantly textbook-centric culture of teaching as reflected in research originating from these countries (Adler, Reed, Lelliott, & Setati, 2002; Li, Chen & Kulm, 2009; Kim, 1993). In the textbook-centric Asian cultures, textbooks provide a "blueprint for content coverage and instructional sequence" (Li, Chen & An, 2009, p. 813) and so the type of examples, representations and exercise problems given in the textbook may be directly used in classrooms. However, while the resources that textbooks provide may be useful, a teacher needs to have a deep understanding of connections between the important mathematical ideas in a topic and associated representations for developing conceptual understanding amongst students. Not all the resources prescribed in the textbook may be suitable for a classroom and a teacher may need to consider the level of conceptual development as well as the sociocultural background of students to select and use resources from the textbook. Thus, teachers need to develop a critical eye for evaluating textbook content for their classroom and a pedagogy based on their own informed decisions rather than relegating such decisions to the textbook.

Bringing about this change in teachers' role with respect to textbook is not simple as it does not involve merely a change in teachers' knowledge. Teachers also need to identify themselves with a style of teaching which focuses on developing students' mathematical understanding through building on students' reasoning and designing tasks to support it. Curriculum reformers often believe that such a shift in teachers' roles is possible through teachers complying with some policy suggestions without any accompanying change in thinking or any impact on their professional identity as teachers. But these shifts call for reconceptualising the meaning of teaching mathematics and reconsidering what mathematics is as well as making efforts in practice to actualise these new ideas. This impacts teachers' professional identity as it changes the way *being a mathematics teacher* is conceptualised by the teacher. Beijaard, Verloop and Vermunt (2000) have shown how teachers perceive their professional identity through roles teachers adopt in their classrooms with respect to the subject, curriculum and students. Evidences for changes in teachers' orientation and roles with respect to teaching and learning have been argued as linked to evolution of professional identity of teachers by various researchers (Gresalfi & Cobb, 2011; Stein, Silver & Smith, 1998; Chin, 2006). Teachers' participation in professional learning communities and development of shared values in the community has been considered as the reason for bringing about change in teachers' role and thus their identity (Graven, 2004; Lieberman, 2009; Gresalfi & Cobb, 2011). The identification of oneself as a member of a community which strives for developing understanding of mathematics can thus become the motivation for making efforts to change one's practice. This relationship between identity, practice and community membership is discussed by Wenger (1998) while describing how learning is situated in social contexts.

In this paper we explore the main research question: What roles were perceived initially by teachers in relation to the textbook and what was the impact of participation in professional development workshop on these perceived roles. We provide evidences of how teachers' role with respect to their use of textbook in teaching underwent a change as a result of their participation in the workshop. These roles were reflected in how teachers talked about their goals of teaching, knowledge of resources for teaching, beliefs about what is important for students' learning, classroom interaction and pedagogical decision making. We argue that since roles are constitutive of teachers' professional identity, the shifts in roles indicate teachers' evolving identities from that of textbook implementer to a designer of learning experiences. The initial identity is an institution supported identity since it is expected that the role of a teacher is to follow the textbook. The latter identity of designer is the one for which teachers' developed understanding and motivation through the professional development workshop.

Theoretical Framework: Relation Between Perceived Role of The Teacher, The Textbook and Teachers' Professional Identity

The textbook is a central artefact of teaching in certain educational cultures and structures the teaching learning process. The textbook is assumed to have a position of authority with respect to the teacher in the education system in India. This cultural norm is in direct opposition to the agenda of new curriculum framework (NCERT, 2005) which encourages teachers to go beyond the textbook. Thus cultural norm and new policy initiatives like curricular reform and professional development initiatives exist as oppositional forces acting on the way teachers perceive their roles.

Teachers and students assume roles with regard to the textbook, and in the course of practice, these roles acquire stability and are constitutive for teacher identities as reflected in actual teaching practice. Hence, it is important to describe and analyse such roles and the way in which they constitute teacher identities. Further, the textbook also functions as a locus of authority, as an embodiment of the expectations and directives of the school system. Thus, a teacher may assume the role of a follower of textbook strictly adhering to the tasks, representations and explanations given in the textbook while not considering which of these resources might be more accessible or relevant for children's conceptual development. In doing this, the teacher perceives her role as passive and her own knowledge of mathematics and students are relegated to a position inferior to that of the textbook, which is considered as authority. On the other hand, when the teacher perceives her role as that of an active decision maker and considers the role of the textbook as a tool for teaching, she uses her own knowledge and experience to make decisions about tasks, use of representations and explanations of procedures. The roles that teachers perceive for themselves are governed by the beliefs that they have about their own self efficacy and their social position in the education system and society as a whole. They may also be influenced by the level of knowledge about mathematics and its teaching. A teacher with low self-efficacy and limited knowledge of mathematics teaching is more likely to consider the textbook as an authority. Teachers' interactions with others in their social contexts may also influence what roles teachers perceive for themselves. While directives in institution may affirm teachers' role as a textbook follower, professional development may encourage teachers to take a more active role in their pedagogical decision making.

We believe that textbooks are artefacts of practice that can serve as an entry point to re-negotiate meanings of the resources of teaching given in it like examples, contexts, representations, and exercises. Contexts refers to the different situations where a mathematical idea can be used. Representations refers to the modes in which these situations are represented, which can be as a description of context, as a model e.g. number line, or as a symbolic expression. The connection between contexts and representations is through meanings that convey different aspects of a mathematical concept. In this paper, we claim that teachers' understanding of meanings of integers, helped to build the connection between contexts and representation. This understanding of meanings occurred as a result of negotiations that occurred during the professional development meetings.

In this paper, we show how negotiation of meanings can pave the way for negotiation of teachers' roles with respect to textbook, eventually having an impact on teacher identities. Identity has been defined as how one gets recognised as "a kind of person" in a social interaction (Gee, 2000, p.100). One can either accept, contest or negotiate these interpretations made by others which can be based on one's nature (e.g. gender), position (e.g. Professor) or interest group (e.g. Rock music fan). Thus identities can be ascribed by others or achieved. However, it has been argued that people may have multiple identities in different social situations and identities keep on evolving throughout life with experience (Gee, 2000). In this paper, we explore how teachers' participation in a professional

development initiative impacts the way these teachers perceive themselves to be a certain kind of mathematics teacher and how it gets reflected in the way they talk about their role in the teaching and learning of mathematics.

The Study

This study is part of a larger study aimed at developing a model for the professional development of in-service teachers in workshop and school based settings. In this study, we illustrate the dynamic relationship between goals, beliefs, knowledge and practices of participating teachers, who are situated in a textbook centred culture, while they collaborated in a professional development initiative. The collaboration aimed to develop resources for teaching, and included reflection on teaching experiences using those resources. Teachers' professional development is illustrated through shifts in the teachers' goals, beliefs, knowledge and practice. These shifts amount to a change of role from being a 'follower' of the textbook to being a designer of learning experiences using resources like explanations, examples and tasks, in planning as well as in teaching.

In this study, four middle school in-service teachers (3 female: Swati, Anita and Rajni; 1 Male: Ajay, all pseudonyms) engaged in collaborative investigation (Smith & Bill, 2004) on the Grade 6 topic of *integers* in six one-day meetings over a period of five months (July-November 2010). The teachers were selected by the principals as effective teachers. The teachers had many years of experience of teaching ranging from 17 to 23 years and were between the age range of 42 to 54 years. They all had bachelor degrees in Mathematics and in Education, while two also had a master's degree in mathematics. During the course of these meetings, the first author observed classroom teaching of two teachers Swati and Anita, and held post lesson discussion on classroom interactions. Swati had a master's degree and Anita a bachelor's degree in mathematics. These two teachers had expressed an intent to change their classroom practice and had shown a more active engagement in the meeting as compared to the other two teachers and thus were provided support for classroom teaching. In this report, we will draw mainly on data from the professional development workshop, where the teachers reported their classroom experiences instead of using classroom observation data, as it was assumed that teachers would report experiences which were important to them and would justify their importance in collaborative settings. Classroom observations by the first author were used in a supportive manner, to corroborate teachers' statements.

During the six meetings, teachers shared their resources for teaching integers, designed tasks, prepared lesson plans and reflected on their teaching. They also jointly planned and conducted a session for their teacher peers based on their experiences in the collaboration. A research-based framework, to interpret the meanings associated with integers and with the integer operations of addition and subtraction, was shared with the participating teachers. In real contexts, integers may designate a *state* (an attribute), a *change* in state, or a *static relation* between two states (Vergnaud, 2009). State refers to the use of integers to represent the magnitude of attributes of an object like height, depth and temperature in relation to a standard reference point taken as zero. Change refers to the change in state of the object and the use of integers to represent the change, for e.g., while going from an altitude of 100m to 50m, the change in altitude can be represented by –50m. Static relation involves the use of integers to indicate the relation of a state (or change) to a reference point of interest, which may be non-zero, for e.g., the relative altitudes of other planes in relation to a plane in the air. Teachers worked with a range of contexts, many of which they themselves suggested, identifying whether integers as applied to the context denoted state, change or relation. Similarly they interpreted addition and subtraction operations as corresponding to the actions of *combine*, *change* and *compare* across a variety of contexts. For more details see Kumar,

Subramaniam & Naik (Accepted, under revision). The focus of the discussion and tasks during the six one day meetings are given in Fig. 1.

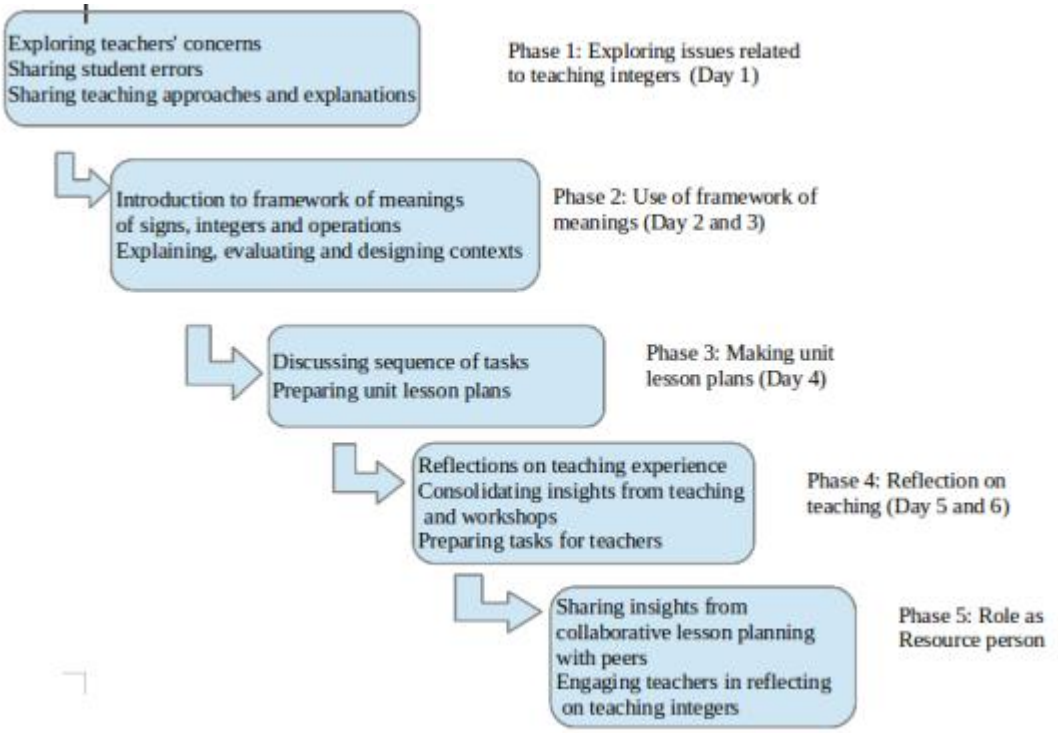

Figure 1: Study design showing phases of collaborative lesson planning workshops and tasks engaged by teachers

Data collection and analysis

The data during these meetings were collected in form of audio and video records of the discussions which amounted to approximately 40 hours of data. The first author reviewed the records several times, as well as fully transcribed them for analysis. The notes of the meetings made who were participant observers were also used. Tasks, worksheets, lesson plans and presentations designed by the teachers in the meetings were included for analysis. The textbook chapter was analysed for the types of resources that it offered for teaching. Transcripts of discussions in the meetings were analysed to identify resources developed by teachers collaboratively and incorporated in lesson plans, and resources that teachers reported using in their teaching. Analysis was done to identify in what ways these resources were different from those given in the textbook and how teachers went beyond the textbooks in their description of teaching. Using transcripts, critical events during professional development were identified that illustrated expressions of teachers' goals, beliefs, knowledge, reported practices and basis of decision making. Shifts in these dimensions were identified by comparing these events and identifying changes in discourse of teachers. Analytical memos were written to illustrate how these different dimensions reflected change in teachers' role with respect to the textbook.

Results

In this section, we describe findings from our analysis, which includes a comparison of the resources for teaching the topic of integers drawn from the textbook and the resources developed in workshops and used by teachers. Textbook resources were limited in terms of

the variety of contexts used, the absence of clear and detailed justifications for procedures using models and in the paucity of opportunities to generate, interpret and reason with mathematical ideas. Teachers' initial discourse in the professional development workshop indicated their strong belief in following the textbook while teaching. This was reflected in the teachers' description of their teaching approaches and assessment questions. The teachers laid emphasis on teaching rules and on students attaining fluency in solving number problems, goals that were closely aligned with the textbook. We argue, citing teachers' reflections, that this was a major factor in the skewed student participation and limited opportunities for learning during classroom interaction. Thus, initially teachers' role was that of a textbook follower which constrained teachers' thinking and practice for teaching mathematics.

In the collaborative workshops, teachers engaged in tasks of explaining, evaluating and designing resources for teaching based on considering a variety of meanings that can be attributed to signs, integers and their operations. We describe how teachers' engagement made it possible for them to go beyond the textbook and assume the more active role of a designer by modifying their goals for teaching, use of representations, classroom interaction practices, their beliefs about what was important to learn and assess, and pedagogical decision making. Teachers reported increased student participation in classroom interaction and opportunities for student learning following these changes.

Textbook Resources for Teaching – Paucity of Reasoning Opportunities

The Grade 6 chapter on integers in the textbook (NCERT, 2006b) was analysed by the researcher to identify the nature of resources for teaching and the opportunities they offer for meaning making and reasoning with mathematics. The textbook included very few examples of the use of integers in contexts and did not contain a discussion of the variety of meanings associated with integers and integer operation. It did not allow for exploration of different aspects of contexts that can be represented by integers or connections between various representation of integers.

The main topics covered in the textbook chapter were introduction to integers, comparing integers, integer addition and integer subtraction. The chapter contained examples, tasks and a summary of main ideas at the end of the chapter. Examples included descriptions of contexts for illustrating a mathematical idea as well as worked out examples of different types of problems. Tasks refer to individual questions posed to students within the text after introduction of an idea to explore it as well as questions in chapter exercises which are the main focus of teaching. Table 1 below gives the number of examples and tasks given in the textbook chapter based on context, models or symbolic expressions along with a few instances. There were three sets of exercises in the chapter. Each exercise consisted of a set questions, with each question containing sub questions of the same type. In the table, the number of questions is given for each category and the number of sub questions is given in brackets.

	Type of examples/tasks	Introduction of integers	Assigning integer sign/position	Integer comparison	Addition of integers	Subtraction of integers
1.	**Context based**	Debit-credit	Height above-below sea level	Height of steps of a well	Representing movement on stairs	
	Examples	2	2	0	1	0
	Tasks	1 (6)	3 (13))	1 (6)	1 (7)	0
2	**Model based**	Extending beyond zero in a number line based game to reach negative numbers	Represent -5 on number line	Write four negative numbers greater than -20	Find the solution using number line : (-2) + 6	What would we do for 6 -(-2) ?
	Examples	1	2	5	13(3)	6
	Tasks		7 (32)	1 (4)	6 (24)	0
3	**Symbolic expression based**				Add without using number line 11+ (-7)	Find 23 - (-12)
	Examples	0	0	0	3	2
	Tasks	0	2 (10)	3 (16)	4 (16)	4 (19)

Table 1: Examples and tasks (including exercises) given in the integer chapter in the sixth grade

The textbook introduced integers under a topic titled “need of integers”, where the context of borrowing money was described. Negative integers were needed, the textbook pointed out, to differentiate the amount borrowed from the amount possessed. Explanations for addition and subtraction of integers were based on models: neutralisation model (explained below) for addition and number line model for addition and subtraction. There was no clear justification for procedures or actions performed on the models corresponding to the operations of addition and subtraction. The textbook explained the procedure for solving different types of integer addition and subtraction problems using models by presenting examples. For the neutralisation model, students were asked to do an activity with black and white buttons representing negative and positive integers. The textbook stated that “A pair of one white button (+1) and one black button (-1) will denote zero i.e. [1+ (-1) = 0]” without explaining why they sum to zero. After solving a few examples using the neutralisation model, the textbook states “You add when you have two positive numbers like (+3) +(+2) =+5[=3+2]. You also add when you have two negative numbers, but the answer will take a minus sign like (-2) +(-1)= -(2+1) = -3”. No additional explanation is offered for the rules. This pattern is followed when the number line model is introduced. After asking students to solve a few addition and subtraction problems using the number line, students were expected to move quickly to finding the answer “without use of number line”, presumably using rules, although this was not mentioned explicitly. The summary at the end of the chapter also emphasised rules which were listed as observations like “We observe that when we have the same sign, add and put the same sign...”. The dominant representation used in the chapter, besides symbolic numerical expressions, was the horizontal number line. The only exceptions were the two examples described briefly in the text, where the vertical number line was used along with a context (water level in a well; going up and down the stairs). The tasks neither required students to explore the connection between a context and a model nor was the connection explicitly made in the textbook. Tasks expecting students to use representations

like the number line were procedural in nature since students were expected to follow the worked out examples given in the textbook. All the exercise tasks of the chapter were closed ended having a single correct answer while only two tasks expected students to engage in reasoning by evaluating statements or generalisations about integers and giving examples to support their reasoning.

Teachers' Initial Discourse: Relation Between Textbook Use and Goals and Beliefs of Teachers

The teachers' discourse on the first day of the workshop indicated that they did not feel the need to develop a detailed understanding of contexts where integers can be used and the meanings attributed to integers in different contexts. This could be because they focused largely on teaching rules and procedures. All the teachers agreed that they teach rules for addition and subtraction of integers. While some considered teaching rules as the main goal of the chapter on integers since "ultimately students need to know rules", others were sceptical about the effectiveness of teaching rules, since students may not "remember the rules" thus resulting in their making errors. Anita proposed that one must allow students to themselves construct rules of addition of integers from the activity with two coloured buttons given in the textbook. She illustrated how addition of integers could be demonstrated as "neutralization" of equal numbers of positive (black) and negative (red) buttons. However, she did not know how subtraction could be illustrated using the two-coloured buttons since it was not given in the textbook.

The above example illustrates how the use of representations by teachers was limited to what was given in the textbook and how the teachers' goal of focusing on rules followed the textbook's focus on the same. More evidence of this emerged in the teachers' discussion in the meetings. For example, the textbook used the number line as the dominant representation to explain addition and subtraction of integers, which the teachers discussed most during the first two days of workshop. The teachers used the number line as a device to find the answer to integer addition and subtraction problems by focusing on a procedure or action to be performed on the number line to get the answer rather than focusing on the meaning. For e.g., Rajni shared that "we are telling them that add means [moving towards] right and subtract means [moving towards] left" without any explanation as to why one should move in a particular direction since it was not in the textbook. Further discussions indicated that explanations, representations, tasks, the types of solutions expected and assessment of learning were all determined by the textbook. Although the teacher educators discussed alternative approaches and representations, in the initial sessions of the workshop teachers felt constrained in adopting them since these were not part of the textbook and the end of the term exams required students to solve questions based on methods given in textbook.

Anita: ... but we cannot give [different method] if others [i.e., other teachers in the school] will be following the same syllabus... using the number line add... like that question will come [in the exam]. (session 1.2, 186)

This assessment practice reinforced the teachers' feeling that they did not have the agency to select and explore alternative approaches in the classroom. Thus, the teachers' knowledge was limited by what was given in the textbook as the teachers perceived their role as only to engage with the school mathematics in the textbook.

Teachers' Engagement in Workshops to Develop Resources for Teaching

The teachers engaged in the workshop on tasks involving explaining, evaluating and designing contexts and models; they identified and generated examples of different meanings of integers and their operations, and designed tasks based on the meanings and contexts discussed. We argue that this had an impact on teachers' perceived roles while using textbooks for teaching. This was reflected in their discourse about selecting and evaluating resources for teaching, their reflections on classroom experiences of teaching integers during the project, and in their discourse and choice of tasks while designing a workshop session for another group of teachers.

The impact was observed along five dimensions, namely, goals of teaching, knowledge of resources drawn on or constructed by teachers, beliefs about what was important for student learning, practices related to classroom interaction and focus on students' thinking as a basis for pedagogical decisions. The teachers' professional growth across these dimensions led to a change in the way the teachers used resources for teaching from "following" the textbook to "designing" learning opportunities, thereby assuming a more active role in selecting resources for teaching. Teachers' growth across these dimensions is illustrated in Table 2.

Dimensions of teachers' thinking and practice	Characteristics in teachers' initial discourse	Shifts indicated through teachers' discourse in later workshop meetings
Goals of teaching	Focus on telling rules and developing fluency in computation	Exploration of constructing rules meaningfully using different contexts
Teachers' knowledge of resources of teaching	Use of representations and teaching approaches given in the textbook and resisting alternative approaches not given in the textbook	Exploring alternative representations and approaches and using a mix of textbook resources and representations developed in the professional development meetings
Teachers' beliefs about what is important for student learning	Solving textbook questions and getting correct answers	Articulating thinking and reasoning about mathematical ideas
Teachers' practices of classroom interaction	Classroom interaction dominated by students who have already done the chapter in 'tuition' classes; Focus on procedures and getting correct answers in interaction	Classroom interaction more equitable because of use of tasks and contexts not in textbook; Teachers' sensitivity to listening and encouraging students' thinking
Teachers' pedagogical decision making	Classroom and assessment tasks based on textbook exercises	Use of classroom and assessment tasks designed during meetings to elicit students' thinking and reasoning

Table 2: Shifts indicated in teachers' discourse in workshops across five dimensions of teachers' thinking and practice

Shifts in Goals of Teaching

As discussed above, the teachers' initial goal was to teach rules. The textbook too indicated that students were expected to become fluent in solving integer problems given in textbook exercises using rules. Rules were stated as observations after working out example problems in the textbook. While exploring contexts and models in the workshop, the teachers were able to explore how rules could be meaningfully constructed using the neutralisation

model or through an exploration of using integers in contexts. The teachers discussed how students could possibly generalise that while adding a positive and a negative integer, if the positive integer is larger in absolute magnitude than the negative integer, then it would result in the answer being positive by relating this to the movement of an elevator in a building. The addition would correspond to combining a larger upward movement with a downward movement leading to a net upward movement (with upward signifying the positive direction).

Another shift in the goal was from having students avoid making mistakes to understanding the thinking behind student errors. The teachers' initial talk in the workshop indicated that they tried to have students avoid making mistakes by giving clear explanations. Teachers were familiar with common mistakes that students generally make in integer addition or subtraction and shared them on the first day. For e.g., Rajni shared that when doing problems like "subtract 7 from 3", students frequently made a mistake by writing this as "7–3". She said that her response would be to tell students to remember that the second number would come first while writing the symbolic statement. According to her, the error was because students forget to "keep in mind" that they have to reverse the order of numbers while writing in numerical form (3 – 7). Here the teacher is attributing a central role to learning through memorization in addressing errors, indicating a certain belief about mathematics learning. Also, the focus is on telling students what to do to get the right answer given a particular type of problem rather than exploring what it means to subtract one number from another.

On the first day, the teachers discussed problems that involved only calculation. Even while discussing the number line, the teachers' focus was on how to find the answer to a problem. The meaning of integers and operations was not focused or considered as important for students to understand. However, during the course of the workshop meetings, the teachers developed an understanding of meanings of integers and operations and designed several tasks and contexts based on meanings. They also tried to frame questions to encourage students to reason using contexts indicating the setting of a new goal of developing students' understanding of the meaning of integer and integer operations. Examples of this would be discussed in a later subsection on pedagogical decision making.

Covering the textbook and the exercises given in it seemed to be an important goal for the teachers initially as they resisted using approaches not discussed in the textbook. When the teachers designed tasks and contexts on their own in the meetings, it motivated them to go beyond the textbooks and use these tasks in classroom. The teachers' reflections on classroom experiences discussed in a later subsection indicate that they used these tasks to engage students in thinking and reasoning rather than having them solve problems mechanically. Thus the goal of covering of textbook made way for the use of tasks to engage students in thinking.

Knowledge of Resources for Teaching

Initially, the teachers showed resistance to using alternative approaches, but later they were open to exploring alternative approaches by designing resources for teaching such as representations, explanations and contexts for teaching integers. This occurred during the discussion in the collaborative workshop where different approaches were evaluated by teachers and teacher educators from various perspectives. These included judgements about whether the approach was accessible to students and whether it made sense mathematically. For example, subtraction of integers is dealt in the textbook using the idea of additive inverse – the textbook shows through examples how subtracting a number is the same as adding its additive inverse. In the course of a workshop discussion, Rajni reported that she taught students how all subtraction problems can be converted into addition by changing the sign of

subtraction to addition and writing the additive inverse in place of the number being subtracted. Anita responded that it would not make sense to students as to why one should convert the subtraction problem into an addition problem. Ajay challenged the approach in a different way – he asked why in the expression "–5 – 3" only the minus sign of the second number is changed to a plus sign and not that of the first. Here Anita used her knowledge of students' thinking to evaluate the approach while Ajay used his knowledge about mathematics to challenge what appeared to be a logical inconsistency. Further discussion led to acknowledging how students find it challenging to associate the negative sign with an integer when they know from previous experience that it denotes the subtraction operation. This led the teachers to recognise that it was important to distinguish the two meanings of the minus sign as indicating a negative integer and as indicating the subtraction operation. With regard to changing subtraction to addition of the additive inverse, the teachers had identified a challenge in conveying the justification for a familiar procedure, which the textbook clearly did not offer. This created a need for the teachers to know justifications of procedures carried out with the representations and contexts used for teaching integers, thereby making them sensitive to this criterion in selecting representations and tasks.

An important step taken by the teachers that facilitated going beyond the textbooks, was to create tasks different from the textbooks by considering meanings that can be attributed to signs, integers and their operations. Discussions with the teachers had indicated a variety of attributes that they associated with integers like position, change in quantity, direction, movement, and in general, attributes containing opposite senses. However, the teachers mostly discussed these notions as unconnected examples and did not use them as an organising principle for selecting, evaluating or designing tasks. These notions later served as resources for the teachers to build their knowledge when they engaged in activities like identifying and exploring contexts where it made sense to use integers as representing state, change and relation in different contexts. They realised that they mostly used examples in which integers are used to represent state as in temperature, height/depth below sea level and profit-loss. It was an interesting insight for the teachers that change in state or comparison of two states can also be represented using integers. The idea that integers can represent change is considered important for learning integers by Thompson & Dreyfus, (1988). For e.g., the increase and decrease in a baby's weight over weeks can be represented using integers corresponding to the "change" meaning (see Fig. 2). Further, the textbook analysis done with teachers revealed that the questions given in the textbook were based on an interpretation of integers as representing "state". The discussion on meanings of integers not only helped teachers in understanding how these examples are different and related but also helped in making teachers aware of the significance of considering a variety of examples of contexts and meanings of integers to develop a rich and connected understanding of integers.

A NEW BORN BABY'S WEEKLY WEIGHT CHART

WEIGHT OF THE BABY AT THE TIME OF BIRTH 3kg.(or 3000 gm.)

Week	Baby's wt. (in gms)	Loss / Gain	Total Loss/Gain
1st week	2800		
2nd week	2750		
3rd week	2900		
4th week	3100		
5th week	3400		
6th week	3500		

Questions based on baby weight context : What could be the reason for 50 gm. decrease in first week? What could be the reason for 300 gm. increase in 5th week?

Figure 2: Task based on baby weight prepared by teachers in the workshop

The textbook used the neutralisation model to illustrate addition of integers (using buttons with two colours to represent positive and negative integers). During the discussion, Swati asked how one might explain why the pair of +1 and –1 sums to zero as she had observed that students start counting all the buttons instead of considering a black and white pair as cancelling one another yielding zero. A suggestion made by the teacher educator was that one might think of +1 as representing increase in quantity, and –1 as representing decrease in quantity and thus both together represents no change. The idea suggested to teachers that integers may represent change in quantity, which led them to use this central idea in other contexts and representations. For e.g., Anita designed a context in which stones removed and added from a bowl containing an unknown quantity of stones can be represented as integers. She used this context in her teaching. Similarly, Rajni designed and used a task in which the same amount of movement of an elevator up and down the floors of building can be represented with addition of integers of equal positive and negative integers thus resulting in no change or zero change. Thus developing justification through meaning in one context helped the teachers to justify actions/procedures on representations in other contexts.

Although the textbook was dominated by the number line and numerical expressions, the teachers developed a variety of representations in the workshops and attempted connections between different representations. The textbook chapter on integers had used 8 different contexts in the text and exercises, while the teachers explored more than 38 different contexts in their discussions in the workshop. The eachers and researchers developed a context of a multi-storey building with a number of basement floors, where integers were used to represent not only floors above and below the ground level, but also the up and down movement of the elevator. The teachers gradually connected the building representation to the vertical number line as students were able to use the idea of representing movement between floors using integers, going beyond the floors shown in the drawing of the building. Here the teachers were able to connect context, model and numerical expressions together by relating them to the meaning of integers.

Shifts in Beliefs About What Was Important For Student Learning

As noted earlier, the teachers' resistance to alternative approaches was also determined by the assessment practice of giving only textbook questions in exams. Classroom observations also indicated that the teachers used mostly questions that were given in textbook exercises and rarely discussed or gave any questions different from the type given in the textbook. The teachers' engagement in designing tasks different from the ones given in the textbook based on the meanings of integers led teachers to look critically at their assessment practice. Many students used to cover the textbook ahead of lessons in coaching classes ("tuition classes"), which resulted in them knowing the procedures and answers to most questions discussed from the textbook. They quickly solved the textbook problems and were considered as bright students by teachers. Swati realised that this identification of students as "bright" based on their quick response to textbook questions is an artefact of the practice of using only textbook questions rather than indication of real learning by students. She spoke about this common problem faced by many teachers across the education system. She said

> Students whom we call bright are not really bright because it's just that they have already done the chapter and thus know the answers. But if we twist the question they are not able to answer. They don't know the basics but they will solve it. (Workshop meeting excerpt, 8-09-10)

She discussed how as a result of using tasks designed in workshop she was able to engage students in thinking rather than mechanically giving answers by using rules. .. Some children had already done[the problem]. They knew the answer but when I asked them to explain they were not able to explain. So taking such an activity made them also think. When I asked the reasoning they told madam we don't know the reason but we know this is the answer.... Because they have learnt the rules... they directly learnt it.... Learning rules is easier and those bright children you know they are able to learn the rules very fast. What is the disadvantage [is that] they don't want to know the reason…. So that is advantage of having something different in the class which is not there in the textbook. (Workshop meeting excerpt, 26-11-10)

This focus on reasoning indicated her change of focus in assessment from evaluating whether the answer was correct or incorrect to evaluating whether the student is able to explain her/his answer.

Shifts in Classroom Interaction Practices

Teachers also reported shifts in their classroom practice and interaction as a result of engagement in the professional development workshop, where they developed sensitivity to probing students' thinking and encouraging reasoning among students using tasks that they themselves designed. Anita acknowledged that "usually we follow the textbook method.... This was entirely different as I used this button activity and lift context wherever required". She described how students were able to explain their answers and give reasons using these contexts. Swati talked about how it made children *think* instead of solving problems mechanically, especially when tuition classes drilled the mechanical solving of problems. She said

...So when you are taking a new example which is not there... they have not gone through that example, it catches their attention also and we can involve the whole class with such an activity....And actually that is what happened when I took the lift case [a task designed in the professional development workshop based on the context of a multi-storey building as described earlier] in the class.... There were some very bright children also who used to... you know when I used to ask them 2 – 3... five or six of them were ready with the answer... but when I took the lift problem they also tried to do and actually find the answer. They were also taking time to find the answer and this also gave opportunity to other children who participated. And we also framed questions and exercises which were not there in the book.... The benefit of having different exercises... different examples is that they are motivated to think and answer... they are not ready with the answer. [Workshop meeting excerpt, Swati, 26-11-2010]

Thus both teachers acknowledged that the tasks designed by the teachers helped in engaging students in thinking and reasoning with mathematics. They both also shared students' errors and unexpected interpretations that they came across in their teaching. Their reflections indicated that they had begun to accept that while exploring models and contexts, students may interpret mathematical expressions differently and may over-generalise certain observations. For e.g., in Anita's class a student interpreted the expression '4 – 3' as moving 4 floors up starting from 5th floor and then coming down 3 floors. Although, this was different from the movement that she intended the expression to convey (starting from the fourth floor and moving three floors down), she responded to the student by representing her answer as 5 + 4 – 3 where she discussed the resultant expression 5 +1=6 and in the context also discussed the net movement in terms of the expression 4 – 3 being one. While reporting this episode, she pointed out how she had changed her "track according to the student's answer" and

shared how "this makes us also feel good in being able to interpret their answers.... That is why it is a learning experience.... Instead of saying their answer is wrong it is one way of thinking".

Shifts in Pedagogical Decision Making

The most significant way in which teachers went beyond the textbook was in the way they used students' thinking as a basis to make pedagogical decisions rather than mechanically following the textbook. At the end of the study, the teachers led a session in a workshop for a group of teacher colleagues from the same educational system. The themes that teachers chose for discussion with other teachers indicated the sensitivity that they had developed for students' thinking and how this influenced their pedagogical decisions in classroom. Swati reflected,

Actually we did it in so much detail here so I could... I was more aware.... I realised that the students need clearer understanding of integer... otherwise we would clearly say 'no, not like this- do like this'... this is how we used to deal … so that is the change in us I could observe. [Workshop meeting excerpt, Swati, 26-11-2010]

This reflection indicated how Swati has started valuing giving opportunities to students to think on their own rather than telling them procedures to solve problems. Anita gave examples of 'eliciting questions' that can be asked of students like "How to represent the loss or gain of weight through integers?" and in this way "go beyond the textbook as such questions are not there in the textbook". She justified that it helps students in "relating to their daily life situation" and allows students opportunities to speak and "not just sit and listen... they speak up... they tell their mind". In the final meeting of the workshop, where the teachers took on the role of workshop leaders, Ajay discussed the difficulty that students face in understanding that 5 – 3 cannot be equal to 3 – 5 since they have not developed the meaning of negative sign as sign of integer and thus will say that "larger number cannot be subtracted from [smaller] integer". The fact that he chose to talk about challenges faced by students, rather than how rules should be told to students (which was the focus of his talk in the initial meetings), indicates the sensitivity that he had developed to seeing the mathematics from the students' perspective. Rajni, who also initially focused on procedural explanations and rules, constructed her explanation based on what students find challenging and provided justification of the procedure in her explanation. She first talked about issues related to minus sign and its meaning which students find confusing like finding answer of expressions "5 – 7" or "5 + –7". While her earlier explanation in workshops did not take meaning into account and focused on how to manipulate numbers and signs, she shared in the workshop that "we cannot say you put whatever sign you want [in 5 – +7 or 5+ –7]... everything should have some meaning". She said that when a teacher tells the student that one should convert the subtraction problem into addition, students find this hard to accept. She said that students need to understand the concept of additive inverse to do so. She chose to introduce the idea of opposite or inverse through movement on number line where movement in two opposite direction in equal amounts leads to coming back to the starting same position thus establishing that 2 numbers are additive inverses of each other if their sum is zero. This idea is an important part of justifying why subtracting a number is the same as adding its additive inverse. The idea that subtracting a number is equivalent to adding the additive inverse is one of the big ideas associated with the topic of integers.

Thus in all the teachers' talk there is indication of considering students' thinking for making pedagogical decisions like selecting tasks, questions and explanations which would address the challenges faced by students and build on students' thinking.

Discussion

The teachers' knowledge of resources for teaching like representations, explanations and examples were found initially to be limited by what was given in the textbook. This was consistent with initial teaching goals which were closely aligned with the textbook in terms of emphasising rules and computational problems, and consistent with the teachers' role as followers of the textbook governed by cultural and institutional norms. Initially, the teachers deferred to the authority that the textbooks represented. The education system reinforced their role in following the textbook through the norm of giving textbook based questions for assessment. The education system also communicated through its reform policies the expectation that teachers aid the development of understanding of mathematics among students. However, teachers resisted alternative approaches and avoided students making mistakes by telling rules and procedures to be followed clearly. Thus, the choice of resources was influenced by teachers' beliefs and systemic norms apart from teachers' knowledge.

The findings described in this paper indicate that the shifts in teachers' thinking and practice with respect to their goals, beliefs, knowledge, practice and decision making were intricately connected since these shifts indicated change in the way the teachers used the textbook in teaching. In all the dimensions there was a shift from the teachers' reliance on the textbook to using the knowledge and insights from professional development experience. In the teachers' goals, the shift was from focusing on rules and computational fluency to developing understanding of integers through use of contexts and focus on reasoning. These shifts were connected to the knowledge of resources that the teachers developed during professional development and the roles that the teachers exercised in their teaching practice. The shifts were visible in the teachers' role of being a task selector, facilitator of learning and evaluator of learning. As task selectors, the teachers made decisions for selecting tasks for teaching and assessment, as facilitators they interacted and responded to students and as evaluators they indicated what they valued as learning of mathematics. As a result of the professional development experience, the teachers in the role of task selector made decisions to use certain tasks based on the underlying mathematical idea rather than based on whether it is given in the textbook. The increase in variety of resources that the teachers had access to in terms of representations, explanations, tasks, examples and contexts also helped the teachers to exercise their agency in pedagogical decision making. The teachers' interaction with students also changed when they started focusing on eliciting student thinking and reasoning. While initially the teachers shared student errors, they shared students' interpretation of contexts and their thinking in later meetings. This indicated development of sensitivity among the teachers towards listening to students' thinking and responding rather than focusing on evaluation of their answers as right or wrong. The teachers also reported that they included opportunities for students to think on their own rather than providing answers. We believe that the shifts in roles described above would not have been possible without supporting the teachers' development of knowledge of meanings of signs, integers and operations and how they are embedded in contexts and models. It not only led to developing deeper understanding of the resources of teaching given in the textbook but also helped the teachers in addressing their pedagogical concerns. We claim that, the teachers' empowerment in terms of knowledge of resources is the driving factor for change in relationship between the teachers and the textbook and the teachers' role with respect to textbook.

Participating in the collaborative professional development context allowed teachers the opportunity to exercise their agency in several ways, thus developing, on the one level, deeper understanding of the resources for teaching and, on the other, becoming empowered with deeper understanding of integers to design and use resources from and beyond the textbook. Teachers' agency was restricted when they directly used the textbook resources without reflecting on meanings held by students and critically appraising representations.

During the collaborative professional development, the teachers engaged in elucidating the meaning of integers and the operations of integer addition and subtraction, designing tasks and contexts, and evaluating and contesting approaches for teaching integers. These discussions provided teacher educators with a good idea about how teachers think and thus to think of ways to challenge beliefs and support knowledge development. While the teachers' beliefs of using rules and avoiding errors supported their role of being textbook followers, the professional development meetings provided them with the space to use their knowledge of resources from textbooks as an entry point to extend their knowledge. They were able to expand their example spaces by discussing various representations and contexts for teaching integers and integer addition and subtraction. Engaging the teachers in the role of evaluating and designing resources of teaching helped in bringing the social and conceptual conflicts to the surface like the use of alternative approaches other than those given in the textbook or the differences in the meaning of the minus sign as representing integers or the operation of subtraction.

While engaging in workshop discussions, the teachers exercised their agency and drew on their mathematical knowledge and situated knowledge of teaching and of students to evaluate approaches instead of conceding the authority of the textbook. The differences that existed among teachers' knowledge challenged them to move towards deeper explanations considering students' thinking and mathematical consistency. Engaging in discussions situated in one's own teaching experience thus provided the teachers with the opportunity to reflect on their beliefs and norms and develop their knowledge in a community.

We believe that shifts in the teachers' roles are the result of interaction of teachers' beliefs, goals, knowledge and practices. These roles contribute to shaping the teachers' identity as a mathematics teacher by influencing what aspects of mathematics are considered important by the teacher to discuss in the classroom and what type of knowledge about representations and students are drawn upon in mathematics teaching. The teachers, who focused initially on rules and procedures, started valuing the role of contexts in mathematics learning in the course of the workshop meetings. This indicates that considering meanings and reasoning related to contexts and representing contexts through mathematics also gained the status of being a part of mathematics from the teachers' viewpoint. On the other hand, students' ideas were found to be valued by the teachers indicating a shift in their theory of learning mathematics from transmission of knowledge towards understanding how children construct knowledge by making sense of ideas around them. These deep changes in the way the teachers' viewed mathematics and mathematical learning contributed to the teachers' evolving identity of a mathematics teacher who focuses on developing students' understanding by designing tasks and contexts rather than mechanically following the textbook. The identity of the textbook follower was an identity which was affirmed by the education system and thus represented a form of Institutional identity (Gee, 2000) where teachers are expected to follow the norms of the education system. In the course of professional development, however, teachers developed understanding and affinity for the image of a teacher who evaluates the tasks critically for student accessibility and listens closely to student ideas thereby leading to development of the identity of a designer of learning experiences. The latter identity is thus a form of affinity identity (Gee, 2000), which the teachers adopted by their own choice because of identifying themselves as a group member of a professional learning community which focuses on students' understanding. The reason for the teachers' affinity identity being more prominent than the institutional identity in this study could be the empowerment that teachers had as a result of their development of knowledge of resources of teaching and its validation in the professional learning community. This finding contributes to the research body on teachers' identity about how professional development can contribute to the development of identities based on affinity groups and overcome the institutional norms and expectations (Gresalfi & Cobb, 2011; Lieberman 2009).

This study illustrates how in the social setting of professional development, space is provided for teachers to renegotiate their relationship with the textbook by exploring and designing resources for teaching. The textbook is an artefact which is central to professional practice in some educational cultures. This is because practices around artefacts distribute roles that acquire stability over time and are constitutive for identities. Adopting these socially defined roles, teachers internalise the beliefs, knowledge and practices associated with their roles with respect to textbook. Negotiation of the these roles is illustrated in this study through the development of knowledge of resources for use in teaching in a professional development context, which led to concomitant change in the teacher's beliefs and practice, indicating how teachers can work towards developing an identity of an empowered mathematics teacher.

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