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Vine copula modelling of dependence and portfolio optimization with application to mining and energy stock return series from the Australian market

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VINE COPULA MODELLING OF DEPENDENCE AND PORTFOLIO OPTIMIZATION WITH APPLICATIONS TO MINING AND ENERGY STOCK RETURN SERIES FROM THE AUSTRALIAN MARKET

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KEYWORDS

Australian resources sector, mining and energy sectors, retail and manufacturing sectors, energy and mining stocks, retail and manufacturing stocks, benchmark portfolios, pair vine copula models, r-vines, c-vines, d-vines, dependence structure, dependence risk profile, dependence concentration, copula counting technique, linear and nonlinear optimization methods, risk measures, variance, mean absolute deviation, minimizing regret, conditional Value-at-Risk, conditional Drawdown-at-Risk, portfolio allocation features, average model convergence.

ABSTRACT

This thesis models the dependence risk profile, investment risk and portfolio allocation features of seven 20-stock portfolios from the mining, energy, retail and manufacturing sectors of the Australian market in the context of the 2008-2009 global financial crisis (2008-2009 GFC) and pre-GFC, GFC, post-GFC and full sample period scenarios revolving around it. The mining and energy portfolios are the base of the study, while the retail and manufacturing are considered for benchmarking purposes. Pair vine copula models including canonical vines (*c-vines*), drawable vines (*d-vines*) and regular vines (*r-vines*) are fitted for the analysis of the portfolios' multivariate dependence and their underlying sectors' dependence risk dynamics. Besides, linear and nonlinear optimization methods threaded with the variance, mean absolute deviation (*MAD*), minimizing regret (*Minimax*), conditional Value-at-Risk (*CVaR*) and conditional Drawdown-at-Risk (*CDaR*) risk measures are implemented to examine the portfolios' investment risk and optimal portfolio allocation features.

The vine copula modelling of dependence aims at examining the dependence risk profile of the portfolios in specific market conditions; studying the changes of the portfolios' dependence structure between pairs of period scenarios; and recognizing the vine copula models that best account for the portfolios' multivariate dependence. The multiple risk measure-based portfolio optimization seeks to identify the least and most investment risky portfolios, single out the portfolio that offers the best risk-return trade-off and recognize the stocks in the portfolios that are good candidates for investment.

This thesis' main contributions stem from the "copula counting technique" and "average model convergence" perspectives proposed to handle, analyse and interpret the portfolios' dependence structure and portfolio allocation features. The copula counting technique aside from simplifying the analysis and interpretation of the assets' dependence structure, it enables an in-depth and comprehensive analysis of their underlying dependence risk dynamics in specific market conditions. The average model convergence addresses the optimal stock selection and investment confidence problems underlying any type of portfolio optimization, and faced by investors when having to select stocks from a wide array of optimal investment scenarios, in a more objective manner, through model convergence and model consensus. Both, the copula counting technique and average model convergence are new concepts that introduce new theory

to the pair vine copula and multiple risk measure-based portfolio optimization literatures.

The research findings stemming from the vine copula modelling of dependence indicate that the each of the portfolios modelled has dependence risk features consistent with specific market conditions. Out of the seven portfolios modelled the gold mining and retail benchmark portfolios are found to have the lowest dependence risk in times of financial turbulence. The iron ore-nickel mining and oil-gas energy portfolios have the highest dependence risk in similar market conditions. Out of the energy portfolios the coal-uranium is significantly less dependence risky, relative to the oil-gas. Out of the mining portfolios the iron ore-nickel is the most dependence risky, while the gold portfolio has the lowest dependence risk. The retail benchmark portfolio is significantly less dependence risky than the manufacturing benchmark portfolio in both, tranquil periods and non-tranquil periods. In terms of investment risk, the oil-gas energy portfolio is the most risky.

The "copula counting technique" is acknowledged for simplifying the analysis and interpretation of the portfolios' dependence structure and their sectors' dependence risk dynamics. The average model convergence provides an alternative avenue to identify stocks with large weight allocations and high return relative to risk. The research findings and empirical results are interesting in terms of theory and practical financial applications. Portfolio managers, risk managers, hedging practitioners, financial market analysts, systemic risk and capital requirement agents, who follow the trends of the Australian mining, energy, retail and manufacturing sectors, may find the obtained results useful to design investment risk and dependence risk-adjusted optimization algorithms, risk management frameworks and dynamic hedging strategies that best account for the downside risk the mining and energy sectors face during crisis periods.

The declaration page
is not included in this version of the thesis

PUBLICATIONS

Some sections of this thesis have already been published:

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CHAPTER 1

INTRODUCTION

This chapter consists of seven sections: introduction and background, significance of the study, purpose, research questions, assumptions, definition of terms and thesis outline.

The *introduction and background* section positions the research in the landscape of the mining, energy, retail and manufacturing sectors and in the context of the 2008-2009 global financial crisis. The size of the sectors modelled and their significance to the Australia economy is acknowledged. The problem of accurately estimating the multivariate dependence of financial variables is stated and the modelling limitations of alternative measures for dependence and correlation estimation are pointed out. The emergence of new techniques for dependence estimation and portfolio optimization is pointed out and the relevance of the multiple risk measure-based portfolio optimization approach is recognized. Reasons for selecting the 2008-2009 GFC as the context to implement the modelling framework are given, along with motivations for the selection of the mining, energy, retail and manufacturing stock portfolios. The contributions of the research conducted are also stated in this section. The *significance of the study* discusses the usefulness of the research undertaken, while the *purpose* and *research questions* sections outline the research objectives and research questions. The *assumptions* section states the assumptions upon which the research and modelling framework implemented rest. Some key concepts and ideas are explained in the *definition of terms* section.

1.1 Introduction and background

The Australian economy has grown along with the expansion of the mining and energy sectors, and in relation to the economic linkages these sectors have with the retail and manufacturing sectors (Bishop et al., 2013; KPMG Economics Group, 2013; McKay et al., 2000; Ahammad & Clements, 1999). As of December 2012 the percentages of mining (coal and uranium are included in this category) and energy (e.g. oil, gas and renewables) stocks listed and trading on the Australian Securities Exchange were approximately 39 and 9 respectively, an indication of the size of the resources sector and their relationship of dependence with the economy (Arreola & Powell, 2013).

In the last two decades Australia saw a sharp increase in the mining of precious and non-precious metals such as gold, iron ore and nickel stemming from the Asian emerging economies' increasing demand of those commodities (Bishop et al., 2013; Bingham & Perkins, 2012; Connolly & Orsmond, 2011; Gardner-Bond et al., 2008). Along with this trend of increasing demand, portfolio investors have more frequently been considering positions in the mining and energy sectors to diversify their holdings (Jennings, 2012). In 2011, gold, iron ore and nickel production placed Australia as the third, first and fourth largest exporter worldwide, respectively (Bingham & Perkins, 2012; Gardner-Bond et al., 2008). By 2014 energy production in Australia had placed the country as the ninth largest producer worldwide, with coal, uranium and natural gas accounting for 60, 20 and 13 per cent of the energy mix (BREE, 2014; DI et al., 2014).[1]

The retail and manufacturing sectors are important sectors of the Australian economy, not only because they account for 12 percent of total GDP but also because the retail sector appears to be on the rise, while the manufacturing sector has been in a declining trend and exhibits an increasing risk (Department of Industry, 2014; Kryger, 2014; Australian Bureau of Statistics, 2015). The retail sector's good performance is most likely due to the economic linkages it has with the Australian resources sector, manufacturing sector and other sectors of the economy (ARA, 2014; Savills Research, 2014; Delloite, 2013; KordaMentha, 2013; CT, 2012; Green & Roos, 2012; NAB, 2012; Mehmedovic et al., 2011; DIISR, 2010). The levels of demand, spending and

[1] The acronyms *BREE*, *DI*, *GA*, *ARA*, *CT*, *NAB* and *DIISR* used in the present chapter stand for Bureau of Resources and Energy Economics, Department of Industry, Geoscience Australia, Australian Retailers Association, Commonwealth Treasury, National Australian Bank and Department of Innovation, Industry, Science and Research.

investment in the retail sector appear to be correlated with the performance of the Australian resources sector (KPMG Economics Group, 2103).

In this context of dependence relationships and economic linkages between the Australian resources sector and Australian economy and between the Australian resources sector and Australian retail and manufacturing sectors, the accurate estimation of dependence between financial variables and their optimization is a non-trivial task that requires the use of sophisticated techniques for dependence estimation and portfolio optimization. The most promising modelling techniques to address these issues have emerged in the form of pair vine copulas and risk measures threaded with linear and nonlinear optimization methods (see e.g. Arreola & Powell, 2013; Ghalanos, 2013; Czado et al., 2012; Czado, 2010; Dissmann, 2010; Aas et al., 2009; Heinen &Valdesogo, 2009; Bedford & Cooke, 2001,2002; Cooke, 1997; Joe, 1997). In tune with that wave of financial and statistical modelling this thesis implements, in the context of the 2008-2009 GFC and pre-GFC, GFC, post-GFC and full sample period scenarios, pair regular vines (*r-vines*), pair canonical vines (*c-vines*) and pair drawable vines (*d-vines*), and linear and nonlinear optimization methods with respect to the variance, mean absolute deviation (*MAD*), minimizing regret (*Minimax*), conditional Value-at-Risk (*CVaR*) and conditional Drawdown-at-Risk risk measures to examine the dependence risk profile, investment risk and portfolio allocation features of seven 20-asset portfolios from the gold, iron ore, nickel, coal, uranium, oil, gas, retail and manufacturing sectors of the Australian stock market.

The specific objectives of the vine copula modelling of dependence undertaken are to identify the dependence risk profile of the portfolios in specific market conditions, examine the changes of the portfolios' dependence structure between pairs of period scenarios and recognize the vine copula models that best account for the portfolios' multivariate dependence. The study looks at the assets' dependence risk in times of financial turbulence characterized by low confidence in the financial stock markets, and in tranquil periods where the financial stock markets behave smoothly. The portfolios' dependence structure changes are interpreted according to standard economic theory and the price behaviour of the assets' underlying commodities across period scenarios. The multiple risk measure based portfolio optimization seeks to identify the least investment risky and most investment risky portfolios, single out the portfolio with the best risk-return trade-off and recognize the stocks that are good candidates for investment.

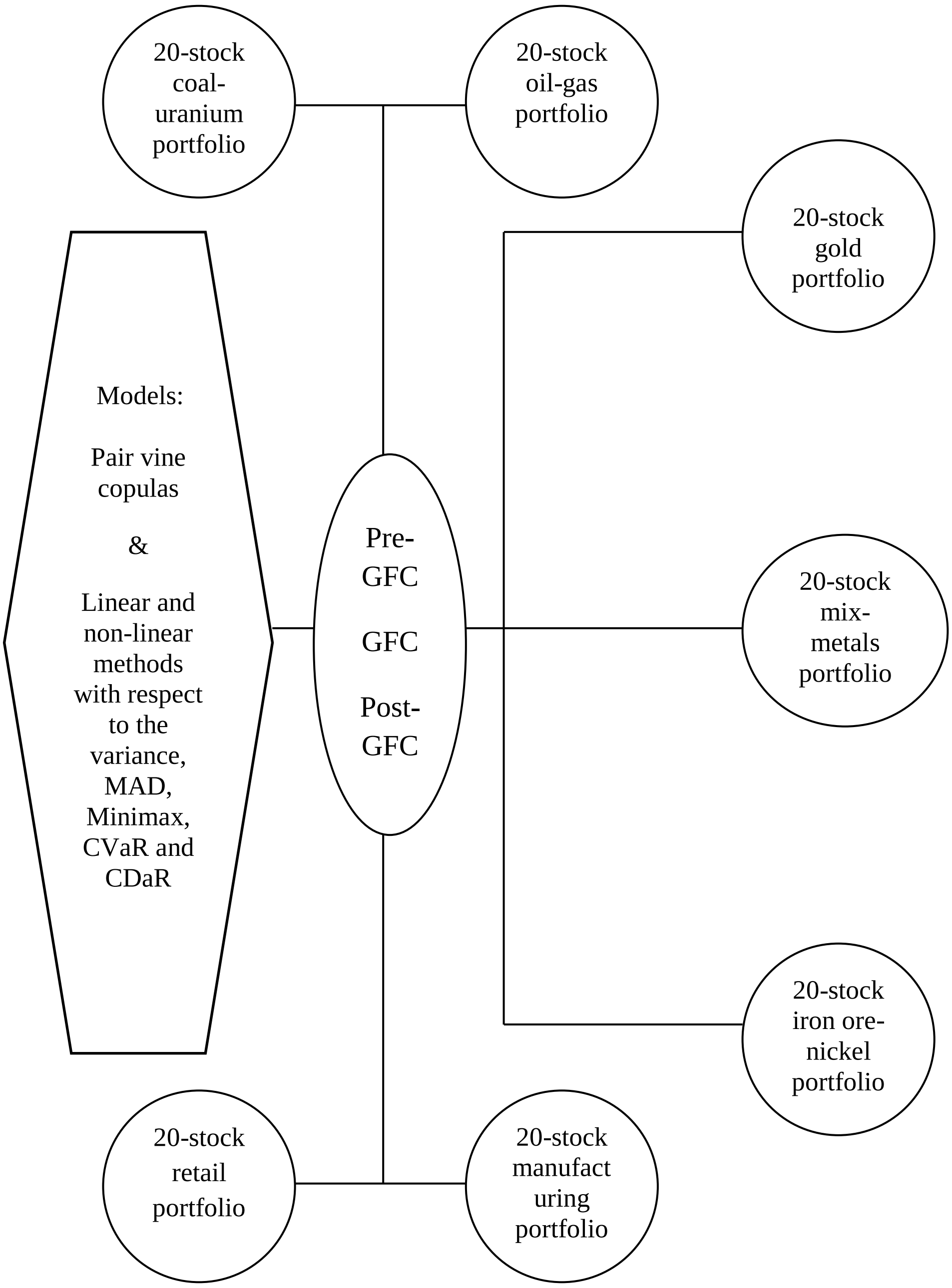

Figure 1-1: Thesis' modelling framework. This figure depicts the models fitted, the data sets modelled and the period scenarios under which the modelling framework is implemented. The pair vine copula models fitted examine the multivariate interaction and dependence risk dynamics of the assets, while the fit of the linear and nonlinear optimization methods and risk measures looks at the characteristics of the minimum risk optimal portfolios. The modelling framework is implemented under four period scenarios: pre-GFC, GFC, post-GFC and full sample.

The motivation for the selection of the pair vine copula models to account for the multivariate dependence is that they are adequate to thoroughly examine the portfolios' dependence risk dynamics in specific market conditions. Besides, the vine copula models overcome the restrictive and deterministic features of alternative measures of dependence and correlation such as the elliptical and Archimedean bivariate copulas and the Pearson, Spearman and Kendall tau (Brechmann & Czado, 2013). The portfolio optimization methods and risk measures considered are suitable because they set the ground to search for the stocks in which most of the optimization methods and risk measures assign weights which do not largely deviate from a mean of weights (i.e. the search for average model convergence). Besides, they provide a wide array of optimal investment scenarios that could cater for the investors' risk and return preferences and enable a risk comparison of the portfolios (Arreola & Powell, 2013; Eling & Tibiletti, 2010; Krokhmal et al., 2002; Cheng & Wolverton, 2001; Stone, 1973).

The motivations for the selection of the gold, iron ore-nickel, mix-metals, coal-uranium, oil-gas, retail and manufacturing portfolios are their differences in terms of structure, volatility, uses and their importance in asset investment. The retail and manufacturing benchmark portfolios are included in the mix of portfolios because of the economic linkages they have with the mining and energy sectors (KPMG Economics Group, 2103; McKay et al., 2000; Ahammad & Clements, 1999). The 2008-2009 GFC event and period scenarios revolving around it provide the market conditions to compare the volatility changes and their effect on the portfolios' dependence risk across period scenarios. Besides, the assets' price behaviour can more easily be understood when the stock markets in financial turbulence and in tranquil periods are contrasted.

This thesis fills a gap in the literature of multivariate dependence modelling with pair vine copulas and in the literature of multiple risk measure-based portfolio optimization by introducing a "copula counting technique" and an "average model convergence" perspectives. The copula counting technique is a simple procedure for the analysis and interpretation of the portfolios' multivariate interaction. The technique could be seen as an extension of unsystematic earlier attempts to dissect, organize and interpret the dependence structure of financial variables (see Allen et al., 2013; Dissmann et al., 2013; Czado et al., 2012; Heinen & Valdesogo, 2009). The average model convergence is a simple approach to handle and address in a more effective and objective manner the estimated multiple optimal weight allocations, the optimal stock selection and investment confidence problems underlying any type of portfolio optimization and faced by investors when having to select stocks from a wide array of investment scenarios.

1.2 Significance of the study

This thesis' research is significant because of the following reasons:

1) It provides a comprehensive analysis and in-depth information about the dependence structure and dependence risk dynamics of the portfolios modelled and their underlying sectors. The adequate use of this information may lead portfolio investors to reduce losses and maintain gains during crisis periods and when the financial stock markets behave smoothly (CME Group, 2011; Singh & Vyas, 2011; Heywood et al., 2003). Portfolio managers and financial market analysts, who follow the trends and performance of the Australian mining and energy sectors may also benefit from the obtained assets' dependence risk information by developing dependence risk and investment risk-adjusted portfolio management algorithms and investment strategies (Al Janabi, 2013). The results could also appeal to government agents whose responsibility is the stability of the macro economy.

2) It proposes a simple "copula counting technique" that simplifies the analysis and interpretation of the assets' dependence structure and dependence risk dynamics. The systematic aspect of the technique enables the non-specialized audience to easily access the information contained in the assets' dependence structure matrix.

3) It proposes a simple "average model convergence" perspective to address the optimal stock selection and investment confidence problems in a more objective manner through model convergence and model consensus thus, enabling the identification of the stocks in the portfolios that could be good candidates for investment.

1.3 Purpose

The purpose of the research conducted is to broaden the understanding on dependence risk in the Australian mining and energy stock portfolios modelled and their underlying sectors. It is also of interest to identify, through the use of the copula counting technique proposed, the specific market conditions under which one sector stock portfolio is riskier than others. In doing so, new insights and useful information are provided that could be used to develop dependence risk and investment risk-adjusted strategies for investment, rebalancing and hedging that more adequately account for downside risk. The portfolio optimization component of this thesis aims at examining the investment risk and resource allocation features of the asset portfolios. Another objective of the research conducted is to make the investors' stock selection process simpler and less uncertain by employing model convergence and model consensus.

1.4 Research questions

1. Are there mining portfolios with higher dependence risk than others?

2. Are there energy portfolios with higher dependence risk than others?

3. Are there mining portfolios with higher dependence risk than energy portfolios?

4. Are there mining and energy portfolios with higher dependence risk than retail and manufacturing benchmark portfolios?

5. Are the portfolios' dependence structure changes between period scenarios statistically significant?

6. Is there a pair vine copula model that best captures the multivariate dependence structure of the portfolios?

7. Is there a portfolio of stocks that offers the best risk-return trade-off?

8. Is the average model convergence of the stocks' optimal weights statistically significant?

The first research question seeks to identify the dependence risk differences between the mining portfolios: gold, iron ore-nickel and mix-metals. The second research question aims at identifying the dependence risk differences between the energy portfolios: coal-uranium and oil-gas. The third research question intends to compare the dependence risk differences between the mining and energy portfolios. The fourth research question examines the dependence risk differences between the mining and energy portfolios and the retail and manufacturing benchmark portfolios. The fifth research question wonders if the portfolios' dependence structure changes between pairs of period scenarios are statistically significant. The sixth research question recognizes the importance of identifying the pair vine copula models that best account for the multivariate dependence structure of the portfolios. The seventh research question targets the identification of the portfolio with the best risk-return trade-off. The last research question examines if the difference between the average of the optimal weights and each of the optimal weights is statistically significant.

1.5 Assumptions

1. The stock return series employed for the vine copula modelling of dependence risk and portfolio optimization reflect all the effects exerted by the price drivers of the mining, energy, retail and manufacturing stocks (Clarke et al., 2001; Jordan, 1983).
2. The mining, energy, retail and manufacturing stock portfolios are representatives of the underlying sectors.
3. Portfolio investors care about the skewed and leptokurtic features of their portfolio investments.
4. No short selling is considered in the optimization of the portfolios.

The first assumption acknowledges that the price and return series used to implement the modelling framework proposed reflect the idiosyncratic (i.e. company related) and systematic (i.e. market related) effects of the stock market. The validity of the statistical analysis rests on this assumption and implies that the stock price and return series cannot capture the effects from all existing price drivers. The second assumption is a necessary condition for the drawing of generalizations about the dependence risk profile and investment risk features of the portfolios. This assumption recognizes the difficulty to model at once all the existing stocks trading in the ASX. The third assumption, along with Xiong and Idzorek (2011), Patton (2004) and Chunhachinda et al. (1997) acknowledges the importance of considering the skewness and kurtosis of the return distribution when optimizing stock portfolios. The fourth assumption discards the selling of some stocks in the portfolios and the reinvestment of the proceeds in other stocks. The discarding of short selling in the optimization implies that negative weights are not allowed.

1.6 Definition of terms

Correlation:

There is three commonly used traditional measures of correlation: the Pearson, the Spearman and the Kendall tau. Despite their differences they all share the same restrictive and deterministic features for dependence estimation. Specifically, they are designed to be fitted in a standardized manner to diverse pairs of variables' joint distributions (Brechmann & Schepsmeier, 2013). The Pearson correlation measure is parametric, implying that it is built under the assumption of normality in the observations. The Spearman and the Kendall tau are non-parametric measures thus; do no impose any distributional constraint on the observations (Tsay, 2005; Chen & Popovich, 2002).

Cumulative distribution function:

A cumulative distribution function is defined as the probability that a random variable X takes a value which is less than or equal to x or, $F\ (x) = P\ (X \leq x)$. The behaviour of the random variable is determined by the probability distribution function employed in the modelling (Tsay, 2005). In this thesis, the cumulative distribution is represented by the stocks' return distribution.

Marginal distribution:

Let the random variables X and Y have a joint probability distribution $p\,(x, y)$. The distribution of X, or alternatively the distribution of Y, is viewed as the marginal distribution if either of them is treated separately. For instance, a data sample is considered to have a marginal distribution if it has been drawn from a larger data sample characterized by a certain probability distribution (Kijima, 2002). Although, the marginal distribution of the subsample is related to the distribution of the original data sample, it is treated as if it has its own identity. In this thesis each stock from each of the portfolios modelled represents a marginal distribution.

Normal distribution:

It is a probability distribution function with most of the observations located around the mean. The standard normal distribution function has a zero mean and a variance equal to 1. The standard normal distribution's variance keeps most of the observations around the mean and discourages extreme fluctuations. A random variable X is standard normally distributed if it satisfies:

$$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \tag{1.1}$$

where μ and σ^2 are the mean and variance parameters (Kijima, 2002). The standard normal distribution is also known as the Gaussian distribution and equation (1.1) represents the standard normal density function, which enables to observe the bell shape distribution of a random variable that satisfies the mean and variance normal conditions.

Kendall tau:

The Kendall tau correlation measure is non-parametric and as such does not impose any constraint on the distribution of the observations. The Kendall tau equation of the variables X and Y is:

$$\rho_{\tau}(X,Y) = 4\int_0^1\int_0^1 C(u,v)dC(u,v) - 1 \tag{1.2}$$

where ρ_{τ} represents the Kendall tau measure, $C(u,v)$ is the copula of the joint distribution and, d is the differential applied to $C(u,v)$.

Skewness:

It is the third central moment of a random variable and can be interpreted as "the propensity to generate negative returns with greater probability than suggested by a symmetric distribution" (Albuquerque, 2012). In this thesis the negative skewness is of concern because its effects are reflected in the left tail of the return distribution, the domain of the loss function (Kim et al., 2014; Prakash et al., 2003; Barone-Adesi, 1985; Kane, 1982; Chunhachinda et al., 1997; Lai, 1991).

$$S = \sqrt{n}\frac{\sum_{i=1}^{N}(x_i - \mu_x)^3}{\left(\sum_{i=1}^{N}(x_i - \mu_x)^2\right)^{3/2}}$$

Kurtosis:

The kurtosis is the fourth central moment of a random variable and accounts for the observations falling in the tails of the distribution. This statistical and distributional characteristic is of interest in this thesis because the stocks' asymmetric and symmetric dependence takes place in the tails of the variables' distribution (Tsay, 2005). An equation of the kurtosis is:

$$K(x) = n \frac{\sum_{i=1}^{N}(x_i - \mu_x)^4}{(\sum_{i=1}^{N}(x_i - \mu_x)^2)^2} \qquad (1.3)$$

Asymmetric dependence:

The concept of asymmetric dependence refers to the greater correlation stock return series tend to have in the tails (Hatherley, 2009; Tsafack, 2009; Alcock & Hatherley, 2008). In a macroeconomic setting, financial stock markets have been observed to display greater correlation in the negative tail when the financial stock markets lack confidence (Aloui et al., 2011; Patton, 2004; Ang & Chen, 2002; Erb et al., 1994).

The theorem of Sklar:

The theorem of Sklar (1959) shows that the multivariate distribution of a data set can be decomposed into copulas and marginal distributions. The theorem plays an important role in the statistical framework upon which the pair vine copula models are built (Brechmann & Schepsmeier, 2013). Analytically, let the random variables $X_1, \dots, X_n$ have a continuous distribution function $F_1, \dots, F_n$ and corresponding joint distribution function $F(x_1, \dots, x_n)$. It follows that a copula C exists such that,

$$F(x_1, \dots, x_n) = \mathrm{C}\big(F_1(x_1), \dots F_n(x_n)\big) \qquad (1.4)$$

for all $\boldsymbol{x} = (x_1, \dots, x_n)' \in \mathbb{R}^n$. Applying a probability integral transform on Equation (1.4) yields:

$$F\big(F_1^{-1}(x_1), \dots, F_n^{-1}(x_n)\big) = \mathrm{C}(u_1, \dots, u_n) \qquad (1.5)$$

where $F_1^{-1}(x_1)$ is the inverse distribution function (Kurowicka & Joe, 2011).

Copula counting technique:

The copula counting technique is proposed in this thesis to dissect, organize, analyse and interpret the dependence structure of the portfolios modelled. It enables an in-depth and comprehensive analysis of the assets' symmetric and asymmetric dependence risk features in specific market conditions. The technique consists of five stages: counting, recording, classification, grouping and aggregate dependence reading.

Dependence risk:

The concept of dependence risk refers to the risk stemming from the specific type of dependence relationship two variables have during times of financial turbulence and when the financial stock markets behave smoothly. The interaction between two variables during times of financial turbulence tends to be more uncertain and less predictable because of the liquidity shrinkage in the financial system. As a consequence, the dependence risk two variables have in the negative tail is higher in those market conditions. Campbell et al. (2002) find that stock securities tend to correlate more strongly when the financial stock markets are unstable. The dependence risk two stock return series have in the centre of the joint distribution is featured by mild swings in the return distribution. The dependence risk two stock return series have in the tails is characterized by large swings in the return distribution. The dependence risk of two variables could be linear, nonlinear, symmetric and asymmetric. It should also be noted that a relationship exists between dependence risk and the tail dependence coefficient, with changes in the tail dependence coefficient determining the characteristics of the dependence risk. The relationship between the tail dependence coefficient and dependence risk is reflected as stronger or weaker correlation caused by large positive or negative return variations.

Dependence concentration:

Is based on and presupposes the aggregation of bivariate copulas selected by the vine copulas to model and estimate the dependence structure of the portfolios. It refers to the location in the joint distributions where pairs of variables experience higher correlation activity, as indicated by the specific type of bivariate copulas aggregated.

Average model convergence:

The average model convergence is proposed to handle the multiple optimal weight allocations, resulting from the fit of the various portfolio optimization model specifications, and address the optimal stock selection and investment confidence problems underlying any type of portfolio optimization. The approach identifies as good candidates for investment the stocks to which most of the optimization methods and risk measures assign weights that do not largely deviate from a mean of the optimal weights.

1.7 Thesis outline

This chapter positions the research conducted in this thesis in the context of the Australian mining, energy, retail and manufacturing sectors. Motivations for the selection of the data sets and modelling framework implemented are given and this thesis' contributions and their significance are stated. Chapter 2 reviews the relevant literature in the fields of bivariate copulas, pair vine copulas and multiple risk measure-based portfolio optimization. The pair vine copula, portfolio optimization and hypothesis testing methodologies are explained in Chapter 3. Chapter 4 lays the mathematics and statistics of the pair vine copula and portfolio optimization model specifications considered. In Chapter 5 the copula counting technique is applied to examine the dependence structure of the mining portfolios. In Chapter 6 the copula counting technique is implemented to examine the dependence structure of the energy portfolios. In Chapter 7 the copula counting technique is employed to understand the dependence structure of the retail and manufacturing benchmark portfolios. In Chapter 8 linear and nonlinear optimization methods with respect to five risk measures are fitted to estimate the minimum risk optimal portfolios, identify the stocks that could be good candidates for investment and establish a risk comparison between portfolios. Chapter 9 deals with the testing of hypothesis and Chapter 10 discusses the main research findings, topics for further research and conclusions.

1.8 Summary

This chapter introduced the research conducted in this thesis and positioned it in the context of the mining, energy, retail and manufacturing sectors, and the 2008-2009 global financial crisis. The modelling framework implemented was explained and motivations for the selection of the data sets were given. The objectives and purpose of the research conducted were stated and its contributions and significance were pointed out. This thesis' modelling framework was indicated to consist of pair vine copulas, risk measures and optimization methods. The main contributions of the research are indicated to stem from the use of the copula counting technique and average model convergence perspectives. The research undertaken was recognized to be significant to portfolio managers, portfolio risk managers, investors and government agents whose responsibility is the stability of the macro economy.

CHAPTER 2

REVIEW OF THE LITERATURE

This chapter consists of four sections: graphical models, bivariate copula models, pair vine copula models, and risk measures and portfolio optimization models. The literature is surveyed chronologically, with the most recent literature discussed last.

The *graphical models* section highlights some of the key ideas and concepts underlying the path of coefficients method of Wright (1934) and their connection to the pair vine copula models fitted in this thesis. Two central concepts in this section are flexibility and branching sequential ordering. The *bivariate copula models* section looks at studies dealing with the modelling of dependence of financial variables using bivariate copulas. The central role of the bivariate copulas for the development of the pair vine copulas is recognized and the comparative advantage of the bivariate copulas relative to the traditional measures of correlation is acknowledged. The *pair vine copula models* section concentrates on the literature dealing with pair vine copula developments and applications. Studies comparing the fit of the r-vines, c-vines and d-vines are also reviewed. Key concepts in this section are pair copula constructions, multivariate density decomposition and inference of pair vine copula structures. The gap filled in the literature of dependence modelling with pair vine copulas is discussed in this section too. The *risk measures and portfolio optimization models* sections review the portfolio optimization literature dealing with applications of the variance, conditional Value-at-Risk (*CVaR*), conditional Drawdown-at-Risk (*CDaR*), minimizing regret (*Minimax*) and mean absolute deviation (*MAD*) risk measures. The gap filled by the average model convergence in the literature of multiple risk measure-based portfolio optimization is also indicated in this section.

2.1 Graphical models

Graphical models such as pair vine copulas are considered in this thesis because of their suitability to visualise and represent a problem in a simple, flexible and dissectible manner (Lauritzen, 1996). Graphical structures, in addition to that, appear to be naturally adequate to represent the interaction variables through nodes-vertices and edges thus, facilitating the estimation of dependence and the inference of causality (Guo et al., 2010).

The use of graphical models to account for the interaction between variables goes back to Wright's (1934) work where graphical path analysis, by means of the path of coefficients method, is pursued to link parent-child heritable relationship of species. The path of coefficients method is acknowledged for its flexibility to associate within a system the correlation coefficients. This flexibility aspect of Wright's path of coefficients method appeals to the pair vine copula modelling of dependence because the main strength of the pair vine copula models stems from their flexibility (Brechmann & Schepsmeier, 2013).[2] Both modelling frameworks however differ in their ability to account for nonlinearities in the joint distributions. The pair vine copulas are specifically designed to capture the nonlinear relationship between variables (Heinen & Valdesogo, 2009).

Another point of connection between Wright's path of coefficients method and the pair vine copula models relates to the branching sequential ordering of the variables. Both modelling techniques branch the variables to facilitate the estimation of dependence (see Czado et al., 2012; Dissmann, 2010). The path of coefficients method, in addition to that, estimates the correlation between two variables according to the shape of their joint distribution and the specific type of relationship each of the variable has with other variables within the system. This model feature of conditioning the correlation estimates on the type of relationship each of the variables has with other variables resembles the pair vine copula estimation of dependence. Specifically, conditional densities are used to account for the conditional dependencies (Brechmann & Schepsmeier, 2013; Czado, 2010).

[2] There is a tightly interwoven relationship between flexibility and structure in the graphical vine copula models. It is in fact the combination of flexibility and structure what leads to greater accuracy in the modelling.

Blalock (1971), Neopolitan (1990) and Cox and Wermuth (1996) have also developed and applied graph theory to model the dependence relationship between variables. The latter, in the context of large systems, uses graphs to represent the dependence and independence of variables. Neopolitan (1990) implements Bayesian networks involving paths, cycles, cliques, triangulation and belief networks to account for the relationships between variables. Blalock (1971) models the causality of variables using graphical structures.

2.2 Bivariate copula models

The copula approach in the form of bivariate copulas has been proposed for overcoming the limitations of alternative measures of correlation such as the Pearson, Spearman and Kendall tau. The elliptical and Archimedean bivariate copulas are known for providing good estimates of the underlying interaction of financial variables and for being the building blocks of the pair vine copula models (Brechmann & Czado, 2013; Low et al., 2013).

The first study to implement a copula-like modelling approach without employing the term "copula" is said to be Hull and White (1998). Their study maps the distribution of twelve currency exchange rates taking into account the changes in the market's factors driving the currency co-movements. The market factor's co-movements are estimated using suitable joint probability distribution functions. Hull and White's freedom to select adequate joint probability distribution functions in the modelling of dependence is a feature found in the pair vine copula modelling of dependence. Specifically, joint probability distribution functions such as bivariate copulas can be manually selected to build a vine structure that represents a statistical model (see Brechmann & Schepsmeier, 2013; Dissmann, 2010). Hull and White's approach to dependence estimation, just as any type of bivariate copula modelling, lacks the flexibility to accurately model high dimensional multivariate dependence structures. The bivariate copulas however, relative to the joint probability distribution functions employed by Hull and White, have the comparative advantage of splitting the joint distributions into copulas and marginals, while preserving the marginals' original distribution (Patton, 2012a).

Embrechts et al. (1999) are acknowledged in the literature of bivariate copulas for being the first to associate the concept of "copula" with measures of dependence in finance. Their study compares the fit and performance of the Student-t bivariate copula with the fit of the Pearson correlation measure. As expected, they found the Pearson correlation estimates to represent poorly the interaction between variables. The Student-t copula, on the other hand, better captures the distribution in the tails, while providing more information about the interaction between variables. Embrechts et al.'s research has in common with this thesis' research the recognition of the bivariate copulas as more accurate and adequate than the traditional measures of correlation (Brechmann et al., 2014; Brechmann & Czado, 2013).

Li (2000) employs bivariate copulas to model the correlation structure of credit risk portfolios. The study equates the survival times of credit risks (a credit risk is a fund borrowing company) with the marginals and uses the marginals to build correlation structures. The bivariate copulas are also employed to measure the credit risks' default correlation. Li's modelling framework has in common with the pair vine copula models the notion and use of correlation structures. The correlation structure concept alludes to the concept of dependence structure that is central in the pair vine copula literature.

One topic often appearing in early applications of bivariate copulas to model the interaction between financial variables is about the comparison of the Gaussian bivariate copula with the Student-t bivariate copula (e.g. Tong et al., 2013; Berg & Aas, 2009; Fischer et al., 2009; Junker & May, 2005; Malevergne & Sornetten, 2003; Embrechts et al., 1999). Despite its limitations, the Gaussian copula became a dominant model for dependence estimation due to its simplicity of application and tractability. The emergence of the Student-t copula and its symmetric dependence modelling of the tails led to the comparison of both copulas in terms of fit and performance (see e.g. Arreola et al., 2013; Malevergne & Sornetten, 2003).

Malevergne and Sornetten (2003) is one of those studies comparing the Gaussian and Student-t bivariate copulas in the context of a financial crisis event. They do so by modelling the interaction between exchange rates from six countries, six metals traded on the London Metal Exchange, and 22 large cap stocks from the New York Stock Exchange. They find the Gaussian copula to produce good estimates for normally distributed data sets in non-crisis periods, while the Student-t copula adequately captures the distribution in the tails for non-crisis periods. A point of connection between Malevergne and Sornetten's (2003) research and this thesis' research lies in the use of financial period scenarios as the context to implement their modelling framework. Both studies specifically, appear to understand the importance of considering financial crisis events to better understand the dependence risk behaviour of financial variables in stress-testing and tranquil time periods.

Junker and May (2005) establish a comparison between bivariate copulas by fitting them to a stock portfolio consisting of six assets from the German and USA markets. Their modelling framework considers the transformed Frank copula, Gaussian, Student-t, and a Clayton copula based on a linear convex combination. They model the marginals by employing the Pareto distribution that is often used in Extreme Value Theory to account for the leptokurtic features in the tails of the marginals (Këllezi & Gilli, 2000). Their findings indicate that the transformed Frank copula produces the best results. The combination of the Frank copula with the Pareto distribution captures the dependence in the tails best. Junker and May's research and this thesis' research share the concern about accounting for the asymmetric dependence in the tails during crisis periods.

An important topic in the literature of bivariate copula modelling is that of contagion across financial stock markets (e.g. Barunik & Vacha, 2013; Ozkan & Unsal, 2012; Kazi et al., 2011; Kenourgios et al., 2010; Markwat et al., 2009a; Chiang et al., 2007; Bae et al., 2003). Contagion refers to the transmission of economic conditions from one financial stock market to another (Corsetti et al., 2001). The subject of contagion has gained increasing attention since the 2008-2009 GFC took place and due to the expanding global economy (Poirson & Schmittmann, 2013).

Two pieces of research that have examined the contagion phenomenon across international stock markets are Chen and Poon (2007) and Rodriguez (2007). The former fits time-varying Student-t bivariate copulas and a dummy Student-t copula to examine the contagion effects between 28 of the largest capitalized financial stock markets (including Argentina, Chile, The Philippines and Russia) in the context of the Asian crisis of 1997. They find the European countries to have the strongest contagion effects. Their research connects to this thesis' research in the consideration of a financial crisis event as the context to implement their modelling framework. Rodriguez's (2007) piece of research, relative to that by Chen and Poon (2007), models two financial crisis events: the Asian crisis of 1997 and the Mexican crisis of 1994.

Rodriguez's (2007) approach to dependence modelling is based on "switching copula" versions of the Frank, Gumbel, Clayton and the Student-t with time-varying parameters. These copulas are indicated to capture adequately the symmetric and asymmetric dependence, and the increases and decreases of tail dependence. Rodriguez models the marginal distributions by fitting a SWARCH method that lets the variance of the variables to shift occasionally according to a Markov process. His findings indicate the presence of increased tail and asymmetric dependence in the Asian countries during the crisis period. The pair Mexico-Brazil displays symmetric dependence, while the pairs Thailand-Indonesia and Thailand-Korea display greater dependence in the centre of the joint distribution during tranquil periods. Rodriguez's research and this thesis' research have in common the consideration of crisis events as the context to implement their modelling framework. His modelling approach has the comparative advantage of using copulas with time varying parameters (Ausin & Lopes, 2010).

In the bivariate copula literature, applications assuming the parameters of the bivariate copulas to remain constant over time are seen as less complicated in terms of implementation, and less sophisticated in terms of the accuracy they provide; relative to letting the bivariate copula parameters change in time (Hautsch et al., 2013; Markwat et al., 2009b). Tong et al. (2013) and Wen et al. (2012) along with Chen and Poon (2007) and Rodriguez (2007) relax that assumption by letting the parameters of the bivariate copulas change over time thus, obtaining more accurate estimates of the multivariate dependence from energy markets.

Tong et al. (2013) model the positive and negative asymmetric tail dependence between crude oil and refined petroleum markets by fitting thirteen different copulas with time varying parameters. Among the copulas they consider are the Gaussian, Student-t, Clayton, Gumbel, Symmetrized Joe-Clayton, mixed Clayton, mixed Gumbel, asymmetric logistic model and mixed asymmetric logistic model. The mixed asymmetric logistic copula is identified to best fit the data sets, while the crude oil and refined petroleum markets are found to move in similar directions. Wen et al. (2012) specifically examine the interaction between the WTI (West Texas Intermediate), S&P500 and the Shanghai and Shenzhen composite indices.[3] Their results indicate the presence of tail and symmetric dependence between the energy, US and Chinese indices

[3] WTI is used for the modelling of energy market most likely because it is used in the Chicago Mercantile Exchange as the commodity that underlies oil futures contracts and as such it is often employed as a benchmark in the modelling of global energy markets (Vassiliou, 2009).

during the crisis period. The contagion effects between the WTI and Chinese indices are acknowledged to be weaker as compared to those between the WTI and S&P500. Their research connects to this thesis' research in the modelling of dependence in energy markets and the consideration of a financial crisis event. Unlike their research this thesis' research does not use copulas with time varying parameters because the copula models used suffice to account for the dependence concentrated at various locations of the joint distributions.

Patton (2012a, 2012b) discusses model selection, parameter estimation through maximum likelihood, model fit, and parametric and semi-parametric inference methods for model estimation. Aloui et al. (2013) estimate the conditional dependence between the Brent Crude Oil and the Central and Eastern European economies.

2.3 Pair vine copula models

In the literature of pair vine copulas Joe (1997) is seen as the starter of a series of developments. He discusses multivariate copula constructions for the design of various types of dependence structures and introduces maximum likelihood methods for the estimation of bivariate copula parameters. His research is of specific relevance to this thesis' research in that it lays some of the theoretical and statistical ground on which some of the modelling implemented in this thesis is based. Bedford and Cooke (2002, 2001) develop an equation for the construction and inference of multivariate pair copulas. Cooke (1997) employs flexible graphical vine trees or "trees of dependent random variables" to organize joint probability distributions of multiple characteristics. Berg and Aas (2009) focus on the comparison of pair vine copula constructions with nested Archimedean constructions. In their modelling of a precipitation data set and a financial data set consisting of the British Petroleum, Exxon Mobile, Deutsche Telekom and France Telecom stocks they find the pair vine copula constructions to be more flexible than the Archimedean constructions. They find the fitted Student-t pair vine copula construction to best fit the financial data set, while the Gumbel pair vine copula construction bests accounts for the dependence of the precipitation data set.

Bedford and Cooke (2002, 2001), Joe (1997) and Cooke (1997) had laid the necessary framework for the separation and inference of pair r-vine copula structures. However, no analytical models had been proposed to decompose and infer c-vine and d-vine copula structures. Aas et al. (2009) address this issue in the literature by proposing two analytical models for the decomposition of multivariate densities and the inference of c-vine and d-vine structures. Their inference and application of a Student-t pair d-vine copula to a portfolio of four financial stock return series indicates that the Student-t pair d-vine copula adequately captures the dependence of the assets and provides good estimates. The models proposed by Aas et al. (2009) have become a central theme in the relevant literature and are used in this thesis to estimate and examine the dependence structure and dependence risk dynamics of the portfolios under consideration.

There are three aspects differentiating studies implementing the Student-t copula in its bivariate and pair vine copula forms: 1) the data sets modelled, 2) the Student-t copula model variations, and 3) the copulas against which the Student-t copula is compared with (e.g. Berg & Aas, 2009; Fischer et al., 2009). In this context, Fischer et al. (2009) compare the Student-t pair vine copula with the Gaussian, Gumbel and Clayton pair vine copulas. They test the dependence modelling performance of the copulas by fitting them to a portfolio consisting of the German HVB, BMW, Allianz and Munich Re stocks. They find the Student-t pair vine copula to outperform alternative vine copula models. Their research connects to this thesis' research in the concern about identifying the vine copula model that best fits the assets' multivariate dependence structure (see also Aas, 2011; Schirmacher & Schirmacher, 2008). A possible limitation of their modelling framework lies in the use of homogeneous pair vine copulas, relative to using mixed pair vine copulas. The homogeneous pair vine copulas assume that most of the assets' dependence is concentrated in a certain area of the joint distribution (e.g. centre, negative tail, positive tail), and that the assets' dependence is either symmetric or asymmetric. This assumption may however be inadequate since the dependence variables have tends to be scattered in the centre and tails, in general. Aware of that limitation this thesis opts to fit mixed pair vine copulas. This specific type of copulas employ a wide array of bivariate copulas as the building blocks to capture the dependence scattered at various locations of the joint distributions. The homogenous pair vine copulas by contrast only use one type of bivariate copula as the building blocks of the vine structure to model the dependence scattered across all areas of the joint distributions.

Two statistical features found in multivariate distributions posing the greatest challenge to the pair vine copula modelling of dependence are the skewness and asymmetric dependence (Patton, 2004). Skewness refers to the “propensity stocks have to generate negative or positive returns with greater probability than suggested by a symmetric distribution” (Albuquerque, 2012). The asymmetric dependence refers to the greater correlation negative returns tend to have in the negative tail during crisis periods (Ammann & Süss, 2009; Okimoto, 2008; Hatherley & Alcock, 2007; Patton, 2006). Chesters (2010), Jansen and Nahuis (2003) and David (1997) have identified stock markets to have greater dependence during crisis periods.

Chollete et al. (2009) target the modelling of skewness and asymmetric dependence of an index portfolio consisting of the G5 countries, Mexico, Brazil, Chile and Argentina. They employ a mixed c-vine copula with regime switching features and a Skewed-t GARCH model to account for the distribution in the marginals. They find the Skewed-t GARCH model adequate to capture the dependence, while the fitted mixed c-vine copula provides a good estimate of the assets’ asymmetric dependence. Their research and this thesis’ research have the common feature of modelling the assets’ skewness and asymmetric dependence.

Heinen and Valdesogo’s (2009) piece of research has become in the literature of pair vine copulas a benchmark because of its clarity and contributions. Their study is significant in that it explains the relationship between the bivariate copula parameters and the Kendall tau’s correlation coefficients (see also Czado, 2010). This relationship enables one to set a bridge between the pair vine copulas’ dependence structure estimate and the standard variance-covariance estimate. They also propose a dynamic Canonical Vine Autoregressive Model that accounts for the time varying volatilities, asymmetric dependence, heteroscedasticity, leverage, skewness and kurtosis. A point of connection between their research and this thesis’ research lies in the recognition of the mixed pair vine copulas as more accurate for dependence modelling, relative to the homogeneous pair vine copulas. Both pieces of research, in addition to that, target the modelling of skewness and asymmetric dependence in the marginal and joint distributions.

The graphical aspect of the pair vine copulas while offering advantages in terms of flexibility, poses the challenge of finding the optimal graphical vine structure to be fitted and of accurately estimating the bivariate copula parameters (Brechmann & Schepsmeier, 2013). The standard method for model selection and bivariate copula parameter estimation is based on methods of maximum likelihood. Alternative models for model selection and bivariate copula parameter estimation have emerged from the field of Bayesian inference (see e.g. Smith & Vahey, 2013; Min & Czado, 2010). On this line of research Min and Czado (2010) combine Bayesian inference with portfolio optimization to examine the dependence structure and resource allocation features of an index portfolio consisting of the Norwegian BRIX bond, TOTX stock, MSCI world stock and the SSBWG hedged bond indices. They estimate the optimal vine structure, bivariate copula parameters and confidence intervals of the bivariate copula parameters. Their findings indicate that the Bayesian approach provides better estimates than alternative methods. In addition to that, conditional independence is identified to exist between the Norwegian bond index and the MSCI world stock index, conditional on the Norwegian stock index. Hofmann and Czado (2010) and Smith et al. (2010) have combined Bayesian inference with pair vine copulas to improve the model selection and bivariate copula parameter estimation. The latter models the dependence features of longitudinal data and employs the dependence structure estimates to forecast intraday electricity. This thesis opts not to use the Bayesian approach to model selection and bivariate copula parameter estimation because the focus of attention of the dependence modelling conducted is the dissection, analysis and interpretation of the estimated dependence matrix. Besides, the modelling framework implemented suffices to obtain a good grasp of the asset portfolios' dependence structure.

Czado (2010) contributes to the literature of pair vine copulas by indicating a general criterion that can be used to identify the suitability of a particular vine copula model to a data set. The c-vines, for instance, are indicated to be suitable for the modelling of data sets where among the variables involved there is one that exerts exceptional influence over the rest through high correlation values. The d-vines appear to better suit data sets where a group of variables is the most influential. She also discusses the relationship between partial, conditional and unconditional correlations within a Gaussian setting. [4]

Two main modelling trends that have emerged in the literature of pair vine copula modelling are the one focusing on the estimation of the dependence structure (e.g. Hobaek & Segers, 2012; Nikoloulopoulos et al., 2012; Panagiotelis et al., 2012; Chen & Poon, 2007; Rodriguez, 2007; Junker & May, 2005) and the one using the pair vine copula estimates of dependence to conduct portfolio optimization (e.g. Arreola & Powell, 2013; Brechmann & Czado, 2013; Low et al., 2013). In this context, Mendes et al. (2010) fit pair vine copulas to measure the strength of association between indices, treasury bonds and 100 of the largest capitalized companies from Brazil. They feed the resulting dependence structure estimate into a portfolio optimization method to find the

[4] See also Baba and Sibuya (2005) for a detailed analysis of these correlations' relationship.

optimal resource allocation. The combined modelling approach consisting of pair vine copulas and portfolio optimization is indicated to produce improved portfolio allocation estimates. Their research and this thesis' research differ in the type of model used to account for the skewness in the marginal distributions. The GARCH skewed Student-t model specification considered by them, as compared to the ARMA (1,1)-GARCH (1,1) with Student-t innovations implemented in this thesis, more suitably accounts for the skewness and asymmetric dependence in the tails. The reason for this is that the Student-t distribution accounts for the distribution in the tails symmetrically.

In the literature of pair vine copulas the number of studies dealing with the application of the r-vines is small relative to the number of applications fitting the c-vines and d-vines. As a result, the modelling properties of the r-vine models under varied conditions continue to be a subject of research. Dissmann (2010) has explored the r-vine copula models by fitting them to a portfolio of equities, fixed income securities and commodity indices. He finds these models to be more flexible than the c-vines and d-vines. The greater flexibility of the r-vines is indicated to stem from their specific shape, which reduces the number of bivariate copula parameter estimates. The r-vine copula algorithm fitted by Dissmann (2010) is distinctive in its sequential selection and estimation of the optimal vine structure and bivariate copulas.

Czado et al. (2012) also fit an algorithm that sequentially selects and estimates the c-vine structure and bivariate copulas. Their algorithm also organizes the variables in the data set according the criterion indicated in Czado (2010), while selecting the bivariate copulas in the vine tree from a catalogue of around 40 elliptical, Archimedean and rotated copulas. The variables' arrangement in the data set is indicated to influence the pair vine copula estimates (Brechmann & Schepsmeier, 2013; Dissmann et al., 2013; Czado et al., 2012). The mixed pair c-vine copula they fit to a portfolio of currency exchange rates from the US market is found to produce good estimates of dependence. This thesis' research connects to their research in the use of the sequential algorithm they propose to estimate the dependence structure of the portfolios under consideration.

Assuming the conditional distributions to be constant in a pair vine copula structure is a simplification that has advantages and disadvantages (Fan, 2010). Under this simplification, the selection of the optimal vine structure and estimation of the conditional distribution copula parameters become simpler. Letting the parameters of the bivariate copulas change over time implies that the parameters of the bivariate copulas from a certain tree are influenced by the bivariate copula parameters from previous trees and so on. As a result, the potential for error in the estimation process is higher when the parameters are allowed to change. Holding the parameters of the conditional distributions constant over time would reduce the estimation error but also poses the challenge of identifying the correct distribution function under which the vine copula model specification is valid. A wrong specification would lead to inaccurate estimates of dependence.

Stöber et al. (2012) tackle the problem of identifying appropriate distribution functions while keeping the conditional distributions constant. They find that any Archimedean copula based on the gamma Laplace transform can be simplified. In the elliptical space only the multivariate Gaussian and Student-t allow for the simplification. The simplification of pair copula constructions is indicated to be noticeably restrictive and adaptations of the pair vine copula specifications are suggested to improve the modelling of dependence. Their research is important to the pair vine copula literature because it identifies an inherent limitation and strength of the pair vine copula approach. Specifically, the use of conditional distributions and conditional densities for the estimation of interaction between variables while being an essential component of the pair vine copula algorithm, it remains as a big estimation challenge in high dimensions. The problem lies in the number of parameters to be estimated which tends to grow exponentially as the number of variables in the modelling increases (Brechmann & Schepsmeier, 2013).

An interesting pair vine copula application that combines the holding of some conditional distribution functions constant (see Stöber et al., 2012) with the letting of other bivariate copula parameters change over time (see Chen & Poon, 2007; Rodriguez, 2007) has been conducted by Almeida et al. (2012). They fit a model specification that combines a mixed pair d-vine copula and a stochastic autoregressive copula method to account for the changes in the dependence structure of 29 constituents from the German Dax 30. They find the pair d-vine copula and stochastic autoregressive copula models fitted to adequately capture the leptokurtic features in the tails of the assets' joint distributions.

Nikoloulopoulos et al. (2012) compare in the context of the 2008-2009 GFC the homogeneous Student-t c-vine and Student-t d-vine copulas with the c-vine and d-vine copulas consisting of the BB1, BB4 and BB7 rotated bivariate copulas as the building blocks (Nikoloulopoulos et al., 2012; Rodriguez, 2007; Malevergne & Sornetten, 2003).[5] Their asymmetric dependence modelling of the assets' joint distribution indicates that the vine copulas consisting of BBs rotated bivariate copulas provide the best fit to the data sets and outperform the homogeneous Student-t c-vine and Student-t d-vine copulas. Their research connects to this thesis' research in the use of the same financial crisis event as the setting to implement the modelling framework. Both studies also target the modelling of skewness and asymmetric dependence in the marginals and joint distributions; and understand that the mixed pair vine copulas are more accurate than the homogeneous. Their study, in addition to that, only considers a pre-crisis (2003-2006) and crisis (2007-2009) periods. This thesis instead considers a pre-GFC, GFC and post-GFC crisis period scenarios.

Underlying the pair vine copula models' flexibility is the optimal use of parametric distribution functions to account for the marginal distributions. The parametric distributions by providing a measurement of the observation's distribution determine the shape of the entire vine structure (Sarcia et al., 2008). If one considers that most of the existing parametric distributions are designed to model continuous data, as compared to discrete data, and that the number of existing parametric distribution functions for the modelling of discrete data (e.g. he Binomial, Probit, Hyper geometric, Multinomial, Negative binomial and Poisson) is rather small, it is natural to wonder about the performance of the pair vine copulas when fitted to discrete data sets. Panagiotelis et al. (2012) explores this issue by fitting a mixed pair d-vine copula to model the dependence of discrete microstructure and medical statistics data sets. They fit the probit, order probit, Poisson and generalized Poisson distribution functions to capture the distribution in the marginals. Their findings show that the mixed pair d-vine copula produces good results. They also recommend the use of sparsity search methods to improve the model selection and bivariate copula parameter estimation.

Two features distinguishing several pair vine copula applications are the type of parametric distribution functions employed to account for the distribution in the marginals and the specific type of bivariate copulas employed to capture the dependence in the joint distributions (e.g. Almeida et al., 2012; Brechmann & Czado, 2012; Panagiotelis et al., 2012; Chen & Poon, 2007; Rodriguez, 2007). An alternative to using parametric distribution functions and parametric copulas is to employ empirical distribution functions and empirical copulas. These specific types of distribution

[5] The BB1, BB4 and BB7 copulas are also known as the Clayton-Gumbel, Clayton-Galambos and Joe-Clayton copulas. Each of them can be rotated by 90, 180 and 270 degrees.

functions and copulas have in theory the comparative advantage of not restricting the original distribution of the variables and as such could provide better estimates of the multivariate dependence (see Sarcia et al., 2008). In this direction, Hobaek and Segers (2012) compare a parametric pair vine copula approach with an empirical pair vine copula approach. They assume that the empirical distribution functions better capture the distribution in the marginals. The parametric distributions are seen as difficult to be specified correctly (Smith & Vahey, 2013; Patton, 2012b). Their results indicate that the empirical pair vine copula performs better than the parametric pair vine copula. This thesis' research unlike theirs only considers parametric distribution functions and parametric bivariate copulas because they have been found to adequately capture symmetries and asymmetries of dependence between pairs of variables (see e.g. Arreola et al., 2013; Brechmann & Czado, 2012; Berg & Aas, 2009; Fischer et al., 2009; Heinen & Valdesogo, 2009; Chen & Poon, 2007; Rodriguez, 2007; Junker & May, 2005; Malevergne & Sornetten, 2003).

Brechmann and Czado (2012) develop and fit a copula autoregressive model to account for the asymmetric dependence, negative skewness and nonlinearities in data sets of macroeconomic indicators (e.g. inflation and interest rates), electricity load demands and bonds. The copula autoregressive model fitted along with the skew-normal and skew-t distributions identifies nonlinear and asymmetric dependence in the data sets. The pair d-vine copula model employed by them most frequently selects the Student-t and Frank bivariate copulas.

The study by Low et al. (2013) is relevant in the literature because it identifies the optimal vine copula model with respect to the size of the portfolio. They fit a bivariate Clayton copula and a Clayton canonical vine copula. Their results indicate that the Clayton canonical vine model better accounts for the asymmetries in the joint distributions and negative skewness in the marginals. The Clayton canonical vine model considered also provides the best fit to portfolios consisting of at least 10 assets. Their research links to this thesis' research in the concern about modelling the negative skewness and asymmetric dependence. A possible limitation of the Clayton pair vine copula they implement is that it is of homogenous type and as such assumes that most of the dependence in the joint distributions is located towards the negative tail. This assumption may be inadequate since the dependence of multivariate distributions tends to be scattered across the centre, negative and positive tails. One way to address the limitation of the homogeneous Clayton pair vine copula they implement is to add other bivariate copulas such as the Gumbel, Frank, Student-t, Joe and Joe-Frank to the vine structure. By doing so, the dependence scattered at various locations of the joint distribution is accounted for.

Heinen and Valdesogo (2009) had developed a c-vine autoregressive model to estimate the dependence between stocks, stocks and the sector, and the sectors and the market. Brechmann and Czado (2013) with more or less similar objectives develop a Regular Vine Market Sector model to measure and understand the dependence structure of a data set consisting of the Euro Stoxx 50, five national indices and 46 stocks. The r-vine copula model they fit is indicated to adequately capture the asymmetries of dependence between the stocks and the sectors and between the sectors and the market. Their study has in common with this thesis' research the use of r-vines to model the assets' dependence. This thesis' research unlike theirs does not estimate the interaction between financial variables using index data. Instead, it uses stock return series to infer the portfolios' dependence risk profile. A distinctive feature of this thesis' research, relative to their research, lies in the identification of market conditions under which one sector stock portfolio is more dependence risky than others.

The consideration of a financial crisis event and period scenarios revolving around it is one of the attractive features of the modelling conducted in this thesis. In this context Allen et al. (2013) and Arreola and Powell (2013) have modelled the dependence risk dynamics of financial variables in the context of the 2008-2009 global financial crisis. Allen et al. (2013) fit r-vines to measure and understand the co-dependence of stocks in a portfolio from the Dow Jones Index. They find the r-vine model to most frequently select the Student-t copula to capture the dependence from the joint distributions. One difference between Allen et al.'s (2013) research and this thesis's research lies in the latter implementing a copula counting technique to dissect, organize and interpret the dependence structure of financial variables. Arreola and Powell (2013) examine the dependence structure of 20-stock mining and energy portfolios from the Australian market and use the resulting dependence estimates to conduct portfolio optimization with respect to multiple risk measures. This thesis' research differs from theirs in the type of vine copula models employed to account for the dependence of financial variables. Specifically, while their modelling framework considers a homogeneous Gaussian pair c-vine copula, this thesis fits mixed pair c-vine, mixed pair d-vine and mixed pair r-vine copulas (Czado et al., 2012).

Arreola et al. (2013) estimate the dependence and *CVaR* portfolio optimization of a 20-stock mix-metals leptokurtic mining portfolio from the Australian market. Their application aims at capturing the co-dependence of the assets and improving the portfolio optimization by feeding the resulting pair vine copulas' estimate of dependence into a non-convex differential evolution optimization method for risk controlled *CVaR* optimization (see Ardia et al., 2011a, 2011b). They employ a Gaussian pair c-vine copula, Student-t bivariate copula, graphical lasso and adaptive graphical lasso to estimate the interaction between variables (see Arreola & Powell, 2013; Fan et al., 2009). They find a specific variation of the Student-t copula to produce the best optimization results.

Brechmann et al. (2013) implement pair vine copulas and stress testing models to examine the dependence and contagion effects of 20 insurers and 18 banks. Their analysis of dependence is based on the modelling of spreads from credit default swaps, due to the link the credit default swaps' spreads have with systemic risk. They argue that if a systemic event occurs (e.g. a financial crisis) in the market the default expectations of banks and insurers increases, which in turn increases the default probabilities of the banks and insurers and the spreads of the credit default swaps. They find the interaction between banks and insurers to be non-elliptical and asymmetric.

Smith and Vahey (2013) combine pair vine copulas with Bayesian inference. As compared to Min and Czado (2010); Hofmann and Czado (2010) and Smith et al. (2010) who apply non-homogeneous or mixed pair vine copulas, they fit a combined modelling approach consisting of a homogeneous Gaussian pair vine copula and Bayesian inference. In order to improve the estimation of dependence they employ empirical distribution functions to account for tail asymmetric dependence in GDP growth, inflation, unemployment rate and short-term interest rate data sets (see Smith & Vahey, 2013; Patton, 2012b; Sarcia et al., 2008). Their justification for the use of empirical distribution functions, as compared to parametric distribution functions, is that the former are more accurate and easier to be specified correctly. Their motivation for the use of a homogeneous Gaussian pair vine copula in the modelling of dependence is that it provides greater parsimony in the model selection and bivariate copula parameter estimation. The Bayesian component of their modelling framework enables them to improve the vine copula structure selection and bivariate copula parameter estimation (Min & Czado, 2010).

In numerous pair vine copula applications, a trend can be observed to fit non-rotated standard elliptical and Archimedean bivariate copulas due to their easiness of implementation and their adequacy to capture symmetric and asymmetric dependence (e.g. Arreola & Powell, 2013; Aas et al., 2009; Berg & Aas, 2009; Fischer et al., 2009). Some data sets however, may have complex patterns of dependence and the dependence concentrated in areas of the joint distribution where the standard bivariate copulas have no domain. In cases like this, the 90, 180 and 270 degrees rotated versions of the elliptical and Archimedean bivariate copulas are more adequate.

Smith (2013) employs rotated Archimedean bivariate copulas as the building blocks of a mixed pair d-vine copula to account for nonlinear cross-sectional and serial dependence of daily maxima in electricity demand. He specifically models electricity spot price data sets from the Australian National Electricity Market. He fits Bayesian model average and block functionals to improve the sparsity of selected models and bivariate copula parameter estimation. He finds the modelling framework implemented to adequately account for cross-sectional and asymmetric dependence located towards the end of the tails.

Brechmann et al. (2014) fit pair vine copula models to estimate the dependence structure of operational losses and total risk capital. They source the data modelled from the Italian Database of Operational Losses that spans from January 2003 to June 2011. Their results indicate that the pair vine copula estimates of total capital requirements are significantly lower than those produced by benchmark models, an indication that the benchmark models tend to overestimate.

2.4 A gap in the literature of pair vine copulas

There are three main outputs resulting from the fit of the pair vine copulas to a data set: the Kendall tau matrix, where the correlation of the variables is displayed; the dependence structure matrix, where the vine models' bivariate copula selection is shown; and the bivariate copula parameter matrix (Brechmann & Schepsmeier, 2013). Although the Kendall tau matrix helps understand the correlation between variables, it does not inform about the location and density of the dependence concentrated in the pair of stocks' joint distributions. In addition to that, it does not provide information about changes in the dependence structure between pairs of period scenarios. The gap this thesis fills in the literature of dependence modelling with pair vine copulas refers to the analysis, processing and interpretation of the portfolios' dependence structure matrix. Specifically, a "copula counting technique" is proposed that enables an in-depth and comprehensive analysis of the portfolios' dependence structure. The technique consists of five stages: counting, recording, classification, grouping and aggregate dependence reading. [6]

In the pair vine copula literature there have been some studies (e.g. Allen et al., 2013; Czado et al., 2012; Dissmann et al., 2013; Heinen & Valdesogo, 2009) that most likely unknowingly and unintendedly have in a way engaged in one or two of the stages of the copula counting technique.[7] Allen et al. (2013) for instance, indicate that the Student-t bivariate copula is the model most frequently selected by the r-vine model they fit. However, they do not count the vine models' frequency of bivariate copula selection contained in the dependence structure matrix. As a consequence, their study does not thoroughly examine the information contained in the dependence structure matrix.

Czado et al. (2012) do identify by name the bivariate copulas most frequently selected by the implemented pair vine copulas. However, they also do not count the frequency of bivariate copula selection. In Dissmann et al. (2013) the vine models' frequency of bivariate copula selection is counted and recorded, however it is not classified and grouped. The absence of grouping the bivariate copulas in their dependence modelling approach does not allow for generalizations and a comprehensive interpretation of the assets' dependence risk. The study by Heinen and Valdesogo (2009) does count, record and classify the bivariate copulas contained in the dependence structure matrix, however, it does not group them and is unable to draw generalizations about the assets' co-movements and dependence risk profile. The copula counting technique by taking into account all five stages provides an in-depth analysis of dependence risk.

[6] In Chapter 5 a detailed explanation of the "copula counting technique" is given.
[7] What those studies lack is systematization in their processing and interpretation of the dependence structure matrix.

2.5 Risk measures and portfolio optimization models

This thesis fits the *variance, MAD*, *Minimax, CVaR* and *CDaR* risk measures to model the portfolio allocation features of mining, energy, retail and manufacturing portfolios. Research into the field of portfolio optimization has attracted significant attention from both academics and finance practitioners (e.g. Yin & Zhou, 2004; Zhou & Yin, 2003; Alexander & Baptista, 2002; Li et al., 2002; Steinbach, 2001; Yoshimoto, 1996; Kroll et al., 1984) since the seminal mean-variance quadratic optimization method of Markowitz (1952). He and Litterman (1999), Bevan and Winkelmann (1998) and Samuelson (1970) are among those highlighting its importance. Samuelson (1970) while criticizing the portfolio optimization approach, points out its usefulness in situations involving low risk. The mean-variance quadratic method is considered in this thesis' research because it is of nonlinear type and can be compared with alternative linear optimization methods.

In the context of energy markets, Chang et al. (2011) measure the correlation between Brent Crude Oil, Western Texas Intermediate and Futures securities by applying the constant conditional correlation, VARMA-GARCH, dynamic conditional correlation and BEKK models. Aside from modelling the volatility of the energy and non-energy assets they conduct portfolio optimization by feeding the resulting variance estimates into a portfolio optimization algorithm (see also Arreola & Powell, 2013; Brechmann & Czado, 2013; Low et al., 2013). Their results indicate a preference to investing in Futures securities. Their research and this thesis' research have in common the modelling of energy markets and the optimization of portfolios.

De Oliveira et al. (2011) fit a *CVaR* model to optimize a mix of energy market contracts from Brazil. Bhattacharya and Kojima (2012) optimize a portfolio of renewable energy from Japan. They find the need to increase the use of renewable energy sources in Japan. Delarue et al. (2011) implement a mean-variance quadratic method to optimize a portfolio mix of electricity generation. As compared to those studies, the portfolio optimization modelling framework implemented in this thesis is more complete because it employs multiple risk measures for portfolio optimization.

Konno and Yamazaki (1991) introduced the *MAD* risk measure as a simple, fast and non-computationally expensive approach to solving large-scale optimization problems. Their risk measure weights the observations deviating from the mean according to a linear function and does not require the estimation of a covariance matrix. The key feature of their risk measure lies in its ability to solve nonlinear optimization problems by treating them as linear optimization problems. This linearization enables the solving of large-scale problems (Konno & Shirakawa, 1994). A possible weakness of the *MAD* risk measure lies in its discarding of a covariance estimate (Simaan, 1997). The motivation for considering the *MAD* risk measure is that it is threaded with a linear optimization method and can be compared with alternative nonlinear optimization methods.

The *Minimax* risk measure was proposed by Young (1998) as a conservative approach to optimize portfolios. The risk measure seeks to minimize the risk of loss even in exchange of a zero portfolio return. It is considered in the modelling framework of this thesis because it can be compared with other risk measures such as the *CDaR*, which tends to be less conservative. Rockafellar and Uryasev (2000) introduced the *CVaR* risk measure as a means to overcome the limitations of the *VaR* measure. The *CVaR* is incorporated in the modelling framework of this thesis because it has become an important measure of downside risk in the relevant literature. Chekhlov et al. (2003) introduced the *CDaR* as an alternative to the *CVaR*. The *CDaR* has in common with the *CVaR* the modelling of observations falling below a threshold value. The *CDaR* is particularly interested in the drawdowns of return distribution (Krokhmal et al., 2002).

One topic of interest in the literature of portfolio optimization has to do with the identification of the best risk measure to be used for the optimization of portfolios. In this regard Krokhmal et al. (2002) fit the *CVaR*, *CDaR*, *MAD*, and *MaxLoss* risk measures to optimize portfolios of stocks. They find the *CVaR* to perform best for the "real" out-of-sample analysis, while the *CDaR* does best for "mixed" out of sample analysis. Despite the good performance of the *CVaR* and *CDaR* in their study, neither of them is proclaimed as the optimal risk measure to be used for the optimization of portfolios. Instead each of the risk measures fitted is indicated to allocate the resources in its own risk space. Stone (1973) addresses the same problem by fitting the *probability of an outcome worse than some disaster level*, *variance*, *semi-variance* and *MAD* risk measures. His findings indicate the importance of considering the amount to be invested, the significance investors place on small and large deviations and the expected return of the investment when selecting an appropriate risk measure. Cheng and Wolverton (2001) also deal with that problem by fitting downside risk measures and risk measures from modern portfolio theory (e.g. variance, semi-variance) to a four dimensional financial data set. Their findings indicate that some investors prefer some risk measures to others, while the risk measures produce results in their own risk space.

The above-mentioned studies appear to suggest that the selection of risk measure is ultimately dependent on the investors' preferences. Hence, in the relevant literature no satisfactory solution appears to be given to the problem of identifying the optimal risk measure to be used for the optimization of portfolios.

2.6 A gap in the literature of risk measures and portfolio optimization.

The research conducted in this thesis recognizes the difficulty former research in the field of multiple risk measure-based portfolio optimization has had to identify the best risk measure to be used for the optimization of portfolios. In order to address this issue this thesis looks at the underlying problem. First of all, this thesis understands that underlying any type of portfolio optimization approach, including that which seeks to identify the optimal risk measure to be used for the optimization of portfolios, is a problem of investment confidence, faced by investors when having to select stocks from a wide array of optimal investment scenarios. Secondly, since the investors' gains and losses are dependent on the optimal portfolio choice, it suffices to identify a non-subjective way to recognize the stocks that could be good candidates for investment to mitigate the investment confidence and optimal stock selection problems. By doing so, investors instead of selecting stocks according to a particular risk measure and specific risk and return preferences; they base their optimal stock selection on model convergence and model consensus, on the optimal weights. The focus is therefore shifted from trying to identify the optimal risk measure to be used for the optimization of portfolios to identifying the optimal stocks to invest in according to the average model convergence. This thesis, in this context, attempts to fill a gap in the literature of multiple risk measure-based portfolio optimization by introducing a simple "average model convergence" perspective that addresses the optimal stock selection and investment confidence problems in a more objective manner. The average model convergence identifies the stocks to which most of the portfolio optimization model specifications assign weights that do not largely deviate from the mean of the optimal weights.

2.7 Summary

This chapter discussed the most relevant literature in the fields of graphical models, bivariate copulas, pair vine copulas, portfolio optimization and risk measures. The literature review on graphical models revealed the importance of flexibility in modelling correlation structures. The bivariate copula literature review highlighted the flexibility aspect of the bivariate copulas and indicated the restrictive and deterministic features of traditional measures of correlation. The pair vine copula literature review acknowledged the worthiness of the pair vine copulas in multivariate dependence modelling. The pair vine copulas were indicated to overcome the restrictive and deterministic features of both, bivariate copulas and traditional measures of correlation. The most common types of pair vine copula models fitted in the literature are the mixed pair vine copulas, as compared to the homogeneous pair vine copulas. A simple copula counting technique was proposed to fill a gap in the literature of dependence modelling with pair vine copulas. The portfolio optimization literature review discussed studies using risk measures in portfolio optimization. A simple average model convergence perspective was proposed to fill a gap in the multiple risk measure-based portfolio optimization.

CHAPTER 3

METHODOLOGY

This chapter consists of three sections: introduction, model application methodology and hypotheses testing methodology.

The *introduction* section discusses the modelling framework implemented in this thesis and the data sets modelled. The *model application methodology* section explains how the pair vine copula and portfolio optimization modelling is conducted. The *hypotheses testing methodology* section states the hypotheses and briefly indicates how each of them is tested.

3.1 Introduction

The copula models for dependence estimation fitted in this thesis are the pair regular vines (*r-vines*), pair canonical vines (*c-vines*) and pair drawable vines (*d-vines*). The portfolio optimization model specifications fitted consist of linear and nonlinear optimization methods threaded with the variance, mean absolute deviation (*MAD*), minimizing regret (*Minimax*), conditional Value-at-Risk (*CVaR*) and conditional Drawdown-at-Risk (*CDaR*) risk measures. Seven 20-stock portfolios from the gold, iron ore, nickel, coal, uranium, oil, gas, retail and manufacturing sectors of the Australian market are modelled in the context of the 2008-2009 GFC and the full sample (Jan 7, 2005 -July 2, 2012), pre-GFC (Jan 7, 2005 - July 6, 2007), GFC (July 9, 2007 - Dec 31, 2009) and post-GFC (Jan 1, 2010 - July 2, 2012) period scenarios revolving around the financial crisis event. The full sample period accounts for 7.5 years and each of the sub periods accounts for 2.5 years. In selecting the size of these period scenarios we follow Baur (2012), The Bank for International Settlements (2009) and the Federal Reserve Bank of St. Louis (2009) who also use similar time periods in their analysis.

Only 20 stocks are included in each portfolio because of the high computational demands when fitting the pair vine copula models (see Brechmann & Schepsmeier, 2013; Haff, 2013; Brechmann et al., 2012). Besides, the number of stocks available in the mining and energy sectors satisfying the 7.5 years trading period is not large enough. As a consequence, some of the portfolios consist of stocks from two sectors (e.g. coal-uranium, oil-gas, iron ore-nickel). The oil and gas stocks are modelled together because several of the companies selected work with both, oil and gas. They are selected for the analysis because their representation in the Australian energy market is increasing continuously.

The coal and uranium stocks are group together because the coal and uranium commodities are used as energy sources for electricity generation thus, could share some price behaviour similarities in times of financial turbulence and when the financial stock markets behave smoothly. The coal and uranium stocks are also selected for the analysis because their representation in the energy sector of the Australian energy market is increasing. The gold, iron ore-nickel and mix-metals leptokurtic portfolios are classified as mining portfolios. Only gold stocks are included in the gold mining portfolio because the number of stocks available that satisfy the trading period sought is large enough and because gold tends to behave in peculiar ways during crisis periods (Andrew, 2012; Bingham, 2012). Thus, its price and dependence risk behaviour can be studied and analysed in those market conditions.

The iron ore and nickel stocks are grouped together because both are non-precious metals and could be used for more or less similar purposes. Stocks from the iron ore sector have been considered in the analysis of dependence and portfolio optimization because iron ore production has a special place in the mining sector of the Australian economy because of the scale of the iron ore business exports. The mix-metals leptokurtic mining portfolio is included in the mix of portfolios because it is of interest to understand the characteristics of a non-homogeneous multivariate dependence structure. By non-homogeneous it is meant that the stocks in the portfolio belong to various sections of the Australian resources sector. This portfolio, in addition to that, has been drawn out of 801 mining stocks listed and trading on the ASX by the end of 2012.

The stocks have been selected according to their kurtosis. Those with the largest kurtosis are included in the portfolio. The stocks' kurtosis is in the range (29.60, 1074). Some of the stocks from the mix-metals leptokurtic mining portfolio are also found in the mining and energy portfolios. All the stocks in the mining and energy portfolios have been selected at random.

The retail and manufacturing benchmark portfolios are considered in this thesis because their underlying market sectors figure highly in the Australian economy, each contributing roughly 5% and 6.5% of total GDP. Besides, the manufacturing sector has been in a declining trend and exhibiting decreasing risk, while the retail sector has been expanding. The manufacturing sector specifically employed around 20 percent of the Australian workforce before the 2008-2009 GFC, which dropped to 8 percent in 2014. On the other hand, the retail sector has experienced a slow but steady increase in recent years, contributing with AD 23.88 billion to the Australian economy in 2013 (Department of Industry, 2014; Kryger, 2014; Australian Bureau of Statistics, 2015). In addition to that, the retail and manufacturing benchmark portfolios' underlying sectors have economic linkages with the mining and energy sectors (Savills Research, 2014; Delloite, 2013; Mehmedovic et al., 2011) and could be used for benchmarking purposes. All the stocks in the retail and manufacturing portfolios have been selected at random.

A variety of portfolios are considered because of their differences in terms of structure, volatility, uses, and their importance in asset investment. For example, the retail stocks along with the gold stocks, which tend to be defensive in times of financial turbulence, could be used to hedge investment positions in the iron ore and nickel sectors, which have shown to be more volatile. Also, the portfolios could be used to diversify an investment position in traditional equity sectors such as the financial sector. The frequency of the stock return series is "daily" so that a sufficiently large number of observations are taken into account and the volatility changes across period scenarios are accounted. The data sets have been downloaded from DataStream International.

Table 3-1: Gold and iron ore-nickel portfolios' stocks' names and codes

Gold stocks' codes	Gold stocks' names	Iron ore-nickel stocks' codes	Iron ore-nickel stocks' names
C1:D10: SBMX	ST Barbara	C1:D12: BHPX	BHP Billiton
C2:D9: NWRX	Northwest Resources	C2:D19: GBGX	Gindalbie Metals
C3:D5: NSTX	Northern Star	C3:D14: MCRX	Mincor Resources
C4:D12: SHKX	Stone Resources of Australia	C4:D8: WSAX	Western Areas
C5:D8: DEGX	Degrey Mining	C5:D6: AGOX	Atlas Iron
C6:D13: RSGX	Resolute Mining	C6:D11: FMSX	Flinders Mines
C7:D4: AXMX	Apex Minerals	C7:D20: GRRX	Grange Resources
C8:D16: ORNX	Orion Gold	C8:D7: ARHX	Australasian Resources
C9:D11: RCFX	Redcliffe Resources	C9:D5: ARI	Arrium
C10:D6: EXMX	Excalibur Mining	C10:D2: FCNX	Falcon Minerals
C11:D1: TAMX	Tanami Gold	C11:D13:POSX	Poseidon Nickel
C12:D14:GLNX	Gleneagle Gold	C12:D9: HRRX	Heron Resources
C13:D3: MOYX	Millenium Minerals	C13:D1: MGXX	Mount Gibson Iron
C14:D20:EVNX	Evolution Mining	C14:D15:ADYX	Admiralty Resources
C15:D7: AUZX	Australian Mines	C15:D4: FMGX	Fortescue Metals
C16:D2: HEGX	Hill End Gold	C16:D17:ILUX	Iluka Resources
C17:D15:KMCX	Kalgoorlie Mining	C17:D3: IGOX	Independence group
C18:D18:IRCX	Intermin Resources	C18:D16:SHDX	Sherwin Iron
C19:D19:HAOX	Haoma Mining	C19:D10:MLMX	Metallica Minerals
C20:D17:CTOX	Citigold	C20:D18:MOLX	Moly Mines

Notes: This table displays the names and ASX codes of the gold, iron ore and nickel stocks modelled. The letters *C* and *D* with their corresponding numbers refer to the type of pair vine copula model (e.g., c-vine or d-vine) and the location of the stock return series columns in the data set. The column order of the stock return series for the c-vine modelling follows the criterion suggested by Czado et al. (2012) and Czado (2010).

Table 3-1 displays the gold and iron ore-nickel mining portfolios' stocks' names and codes. Based on the c-vine column order of the data sets ST. BARBARA (SBMX) and BHP BILLITON (BHPX) occupy the first columns in the gold and iron ore-nickel mining portfolios, respectively. SBMX started as an oil endeavour in 1969 and then refocused its operations on gold in the 2000s. BHPX calls itself the world leading diversified resources company and it is among the world's largest producers of iron ore. Based on the d-vine column order of the data set TANAMI GOLD (TAMX) and MOUNT GIBSON IRON (MGXX) occupy the first columns in the gold and iron ore-nickel mining portfolios. TAMX is engaged in gold mining operations and mineral exploration.

The c-vine column order of the coal-uranium data set indicates that PALADIN ENERGY (PDNX) is the rootstock of the entire vine structure. In the oil-gas energy portfolio the c-vine selects WOODSIDE PETROLEUM (WPLX) as the rootstock.

Table 3-2: Coal-uranium and oil-gas energy portfolios' stocks' names and codes

Coal-uranium stocks' codes	Coal-uranium stocks' names	Oil-gas stocks' codes	Oil-gas stocks' names
PDNX	Paladin Energy	WPLX	Woodside
CBQX	Coal Bank	AWEX	Awe
CLAX	Celsius Coal	BPTX	Beach Energy
LRRX	Leopard Resources	MOGX	Moby oil-gas
AQAX	Aquila Resources	NWEX	Norwest
SMMX	Summit Resources	STOX	Santos
GLLX	Galilee Energy	STXX	Strike
CPLX	Coalspurn	ACN	Acer
RESX	Resource Generation	LNGX	Liquified Ng
CNXX	Carbon Energy	CTXX	Caltex
BWDX	Blackwood	ORGX	Origin
UEQX	Uranium Equities	CUEX	Cue Energy
AGSX	Alliance Resources	BASX	Bass St. oil
EMAX	Energy Resources of Australia	ROCX	Roc oil
FYIX	Fyi Resources	MELX	Metgasco
BLZX	Blaze International	TPTX	Tangiers
NSLX	Nsl Consolidated	DLSX	Drill Search
AQCX	Aupacific Coal	APAX	Apa
BKYX	Berkeley Resources	SYSX	Syngas
WALX	Wavenet International	COEX	Cooper

Notes: This table displays the names and ASX codes of the stocks in the coal-uranium and oil-gas energy portfolios.

Table 3-3: Retail and manufacturing benchmark portfolios' stocks' names and codes

Retail stocks' codes	Retail stocks' names	Manufacturing stocks' codes	Manufacturing stocks' names
CCLX	Coca-cola	SFCX	Schaffer Corp.
HILX	Hills Hld	BLDX	Boral
GWAX	Gwa Grp.	BKWX	Brickworks
MTUX	M2 Telecom	CSRX	Csr
MTSX	Metcash	JHXX	James Hardie
WOWX	Woolworths	OLHX	Oilfield Hld.
ARPX	Arb	CKLX	Colorpak
CCVX	Cash Conv.	ANNX	Ansell
DJSX	David Jones	SDIX	Sdi
DLCX	Delecta	SOMX	Somnomed
HVNX	Harvey Norman	UCMX	USCOM
JBHX	Jb Hi-Fi	FWDX	Fleetwood
RCG	Rcg	FANX	Fantastic Hld.
SFHX	Specialty Fashion	KRSX	Kresta Hld.
SULX	Super retail	ASBX	Austal
WESX	Wesfarmers	MHIX	Merchant House
FANX	Fantastic Hld.	CSLX	Csl
GZLX	Gazal	IDTX	Idt Australia
FLTX	Flight Centre	CDAX	Codan
JETX	Jetset Travel	LGDX	Legend

Notes: This table displays the names and ASX codes of the stocks in the retail and manufacturing benchmark portfolios.

Table 3-4: Mix-metals mining portfolio's stocks' name and codes

Mix-metals leptokurtic stocks' codes	Mix-metals leptokurtic stocks' names
RIOX	Rio Tinto
BCDX	BCD Resources
CAZX	Cazaly Resources
CDUX	Cudeco
FMSX	Flinders Mines
FNTX	Frontier Resources
GLNX	Gleneagle Gold
KMCX	Kalgoorlie Mining
MAHX	McMahon Holdings
NAVX	Navigator Resources
PNAX	Panaust
PHRX	Phillips River
PDZX	Prairie Downs
RMSX	Ramelius Resources
SARX	Saracen Mineral
SIRX	Sirius Resources
AYNX	Alcyone Resources
UMLX	Unity Mining
BWDX	Blackwood
WECX	White Energy

Notes: This table displays the names and ASX codes of the stocks in the mix-metals leptokurtic portfolio.

In the mi-metals leptokurtic portfolio RIO TINTO (RIOX) is selected as the rootstock by the c-vine. RIOX is an international mining and energy company working in the extraction, processing and sale of aluminium, copper, iron ore, diamonds, coal, uranium, gold, borax, titanium dioxide and salt.

3.2 Pair vine copula methodology

The fitting of pair vine copulas begins by inspecting and cleaning the data sets. The first stage of the estimation procedure deals with the transformation of the price series to logarithmic return series. Next, the logarithmic returns are filtered to avoid convergence problems in the estimation process. Once the logarithmic returns have been filtered, their residuals and standardized residuals are estimated, and a probability integral transform is fitted to the standardize residuals to estimate the "copula data".

The distribution in the centre and tails of the marginal distributions is captured through the fit of an ARMA (1, 1)-GARCH- (1, 1) with student-t innovations to the copula data. The R package "vines" is used to estimate the order of the variables in the data set prior to the fit of the vine copula models (Fernandez & Ortiz, 2012). Once the variables have been ordered in the data set, the optimal vine structure, optimal bivariate copulas and optimal bivariate copula parameters are estimated (Brechmann & Schepsmeier, 2013).[8] Next, the dependence structure matrix resulting from the fit of the pair vine copulas to the data sets is dissected, organized and interpreted using the "copula counting technique". Finally, the counting, recording, classification, grouping and aggregate dependence reading stages of the technique are implemented. A detailed explanation of the technique is given in Chapter 5.

3.3 Portfolio optimization methodology

The portfolio optimization methodology as compared to the pair vine copula methodology is simpler. The R routine only requires logarithmic returns of the price series to be run. Once the logarithmic returns have been estimated, the constraints in the optimization problem are set, the minimum risk optimal weights are estimated for a constant level of return, and the efficient frontiers of the optimal portfolios are plotted.[9] Next, the resulting optimal weight allocations are processed and handled using the average model convergence and the stocks to which most of the optimization methods and risk measures assign weights, which do not largely deviate from a mean of the optimal weights, are identified as good candidates for investment.

3.4 Hypotheses testing methodology

There are a total of eight hypotheses tested. Each of them corresponds to one of the research questions stated in Chapter 1. The hypotheses are stated in the alternative format.

H_a 1: There are mining portfolios with higher dependence risk than others.

[8] The R packages used to fit the pair vine copulas are "vines", "CDVine" and "VineCopula".

[9] The R package used to estimate the minimum risk optimal portfolios with respect to the *variance, minimax, MAD, VaR* and *CDaR* risk measures is "parma".

Applying a two-sample two-tailed t-test for the difference of means between two mining portfolios' dependence concentrations tests the alternative hypothesis 1. The objective is to find out if the difference between two portfolios' dependence concentrations is statistically significant at the 95% confidence level. The selection of this confidence level assures with 95% probability that the difference between the means of the dependence concentrations is either significant or not significant.

H_a 2: There are energy portfolios with higher dependence risk than others

Applying the procedure used for the testing of the alternative hypothesis 1 test this alternative hypothesis.

H_a 3: There are mining portfolios with higher dependence risk than energy portfolios

Applying the procedure used for the testing of the alternative hypotheses 1 and 2 tests this alternative hypothesis.

H_a 4: There are mining and energy portfolios with higher dependence risk than retail and manufacturing benchmark portfolios.

Applying the procedure used for the testing of the alternative hypotheses 1, 2 and 3 tests this alternative hypothesis.

H_a 5: The portfolios' dependence structure changes between period scenarios are statistically significant.

The alternative hypothesis 5 can also be tested using a two-sample two-tailed t-test. The size differences of the dependence concentration between pairs of period scenarios are tested.

H_a 6: There is a pair vine copula model that best captures the multivariate dependence structure of the portfolios.

Applying the *ECP* and *ECP2* goodness-of-fit tests, which are based on the empirical copula process, tests the alternative hypothesis 6. These goodness-of-fit tests use the Cramer-von Mises (*CvM*) and Kolmogorov-Smirnov (*KS*) test statistics to check for the fit of the vine copula models to the multivariate dependence of the stocks. The goodness-of-fit tests are applied to the fit of the c-vine, d-vine and r-vine to the data sets (see Schepsmeier, 2013; Genest et al., 2009; Panchenko, 2005).

H_a 7: There is a portfolio of stocks that offers the best risk-return trade-off

Applying the non-parametric Spearman rank correlation and Kruskal-Wallis tests tests the alternative hypothesis 7. The Spearman rank correlation is fitted to account for the strength of correlation between pairs of portfolios' risk rankings, while the Kruskal-Wallis test is fitted to account for the strength of association of the entire group of portfolios' risk rankings. The tests are applied on the rankings of the portfolios' risk so that the direction of the rankings' co-movement is determined.

H_a 8: The average model convergence of the stocks' optimal weights is statistically significant.

Applying a one-sample two-tailed t-test for the difference between the average of the optimal weights and each of the optimal weights tests the alternative hypothesis 8.

3.5 Summary

This chapter discussed the methodology of model implementation. The pair vine copula, portfolio optimization and hypothesis testing methodologies were explained. The pair vine copula methodology was indicated to be the most complex since it required the transformation of the stock price series into the copula data; the ordering of the variables in the data set according to a certain criterion, and the processing of the dependence structure matrix using the copula counting technique. The portfolio optimization methodology was recognized to be simpler since it did not require the implementation of a probability integral transform to the data. The use of logarithmic returns and the implementation of the average model convergence on the resulting optimal weights were indicated to suffice.

CHAPTER 4

MODEL EXPLANATION

This chapter consists of two sections: dependence estimation and portfolio optimization.

The *dependence estimation* section explains the pair vine copula models fitted to estimate the dependence structure of the mining, energy and benchmark portfolios. The models' capabilities, structure and comparative advantages, relative to the bivariate copulas, are stated. The central role played by the bivariate copulas in the vine copula modelling of dependence risk is indicated. The flexibility feature of the pair vine copulas is highlighted and the central role of the theorem of Sklar (1959) for the development of the pair vine copulas is acknowledged. The *portfolio optimization* section explains the risk measures and the linear and nonlinear portfolio optimization methods considered.

4.1 Pair vine copulas

Copulas have been proven to be successful statistical tools for the flexible modelling of cross-sectional dependence structures of random variables (Brechmann & Czado, 2013; Low et al., 2013; Smith et al., 2010). The bivariate copulas are designed to split the marginal distribution from the joint dependence while maintaining the original distribution of the marginals (Patton, 2012b). In the bivariate copula literature a well established set of copula families exists that includes the elliptical (e.g. Gaussian and Student-t) and Archimedean (e.g. Gumbel, Frank and Clayton). Both sets of copulas have extensively been used in financial modelling due to their ability to capture symmetric and asymmetric dependence risk features from joint distributions (Louie, 2014; Hua & Joe, 2011; Joe et al., 2010; Fischer et al., 2009; Li & Peng, 2009; Aas, 2004; Frahm et al., 2003).

The elliptical bivariate copulas are often used in financial modelling because of their simplicity of implementation and interpretation and their somewhat adequate modelling of dependence in the tails (Fischer et al., 2009; Li & Peng, 2009; Frahm et al., 2003). Despite their well-accepted properties, the elliptical bivariate copulas are built to symmetrically account for the dependence in the joint distributions. As a consequence, they are unable to account for the asymmetric dependence and skewness in the joint and marginal distributions (Frahm et al., 2003). Although the Archimedean bivariate copulas can capture distributional features that the elliptical copulas cannot (Louie, 2014; Junker et al., 2006; Murray-Smith, 2002), they lack the necessary flexibility to model multivariate distributions in high dimensions (Brechmann & Schepsmeier, 2013). The pair vine copulas, as compared to the bivariate copulas, overcome the restrictive and deterministic features of the elliptical and Archimedean bivariate copulas (Brechmann & Schepsmeier, 2013; Daeyoung et al., 2013; Chollete et al., 2009).

The pair vine copulas' use of the bivariate copulas as the building blocks makes the bivariate copulas essential to the pair vine copulas' modelling of dependence risk. The Gaussian and the Frank bivariate copulas are used by the pair vine copulas to capture greater dependence in the centre of the joint distributions (Trivedi & Zimmer, 2007). Out of these two copulas, the Frank is more suitable to capture nonlinearities of dependence in the centre (McCarthy & Orlov, 2013; Junker et al., 2006). The Student-t copula is used by the pair vine copulas to symmetrically capture the tail dependence in the pair of variables' joint distribution (Arreola et al., 2013; Tong et al., 2013; Berg & Aas, 2009; Fischer et al., 2009; Junker & May, 2005; Malevergne & Sornetten, 2003; Embrechts et al., 1999). This copula, in addition to that, has been found to provide good estimates of dependence between financial variables (Smith et al., 2010). The Clayton and Gumbel copulas are used by the pair vine copulas to account for the asymmetric dependence in the negative and positive tails, respectively.

Pair vine copulas are graphical tree models that make possible the design of high dimensional multivariate distributions. Their flexibility, which is built in the theory of graphs, enables a localized modelling of stylized facts such as kurtosis, negative skewness and symmetric and asymmetric dependence through the use of bivariate copulas as the building blocks (Brechmann & Schepsmeier, 2013; Czado et al., 2012; Czado, 2010). The theorem of Sklar (1959) laid the statistical framework on which the bivariate copula and pair vine copula developments are built on (Brechmann & Schepsmeier, 2013; Aas et al., 2009). Bedford and Cooke (2002, 2001) are among the first to employ graphical r-vine models to organize, specify and fit multivariate statistical models to data sets of diverse and complex distributional features.

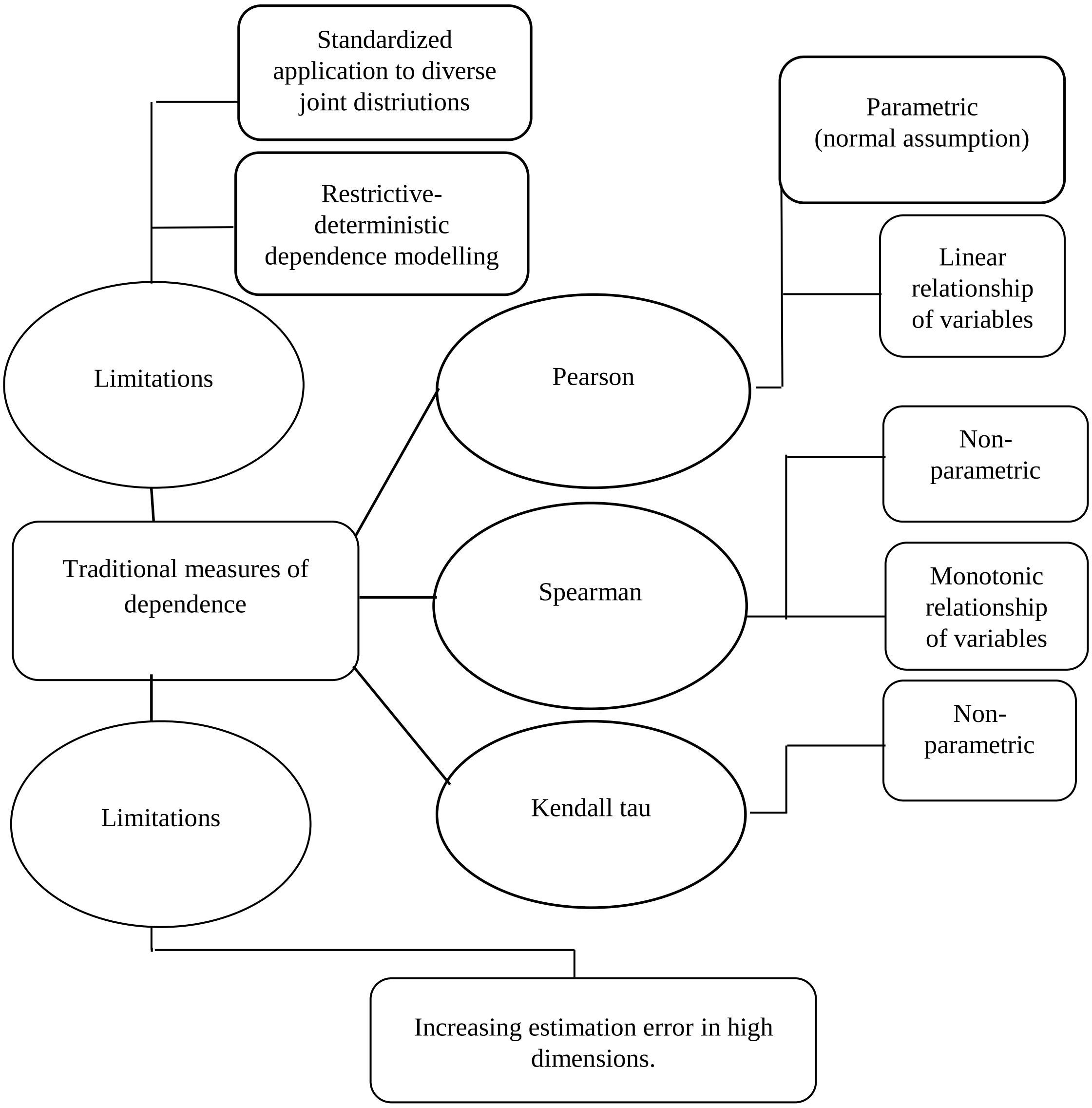

Figure 4-1: Modelling features and limitations of alternative measures of correlation. The Pearson correlation measure assumes variables relate linearly and is built to perform best under the assumption of normality (Heinen & Valdesogo, 2009). The Spearman and Kendall tau are non-parametric measures of correlation that do not constrain the distribution of the marginals to conform to a particular parametric distribution. The Spearman correlation measure assumes variables relate according to an increasing and decreasing monotonic function (Croux & Dehon, 2010; Danacica & Babucea, 2007; Chen & Popovich, 2002).

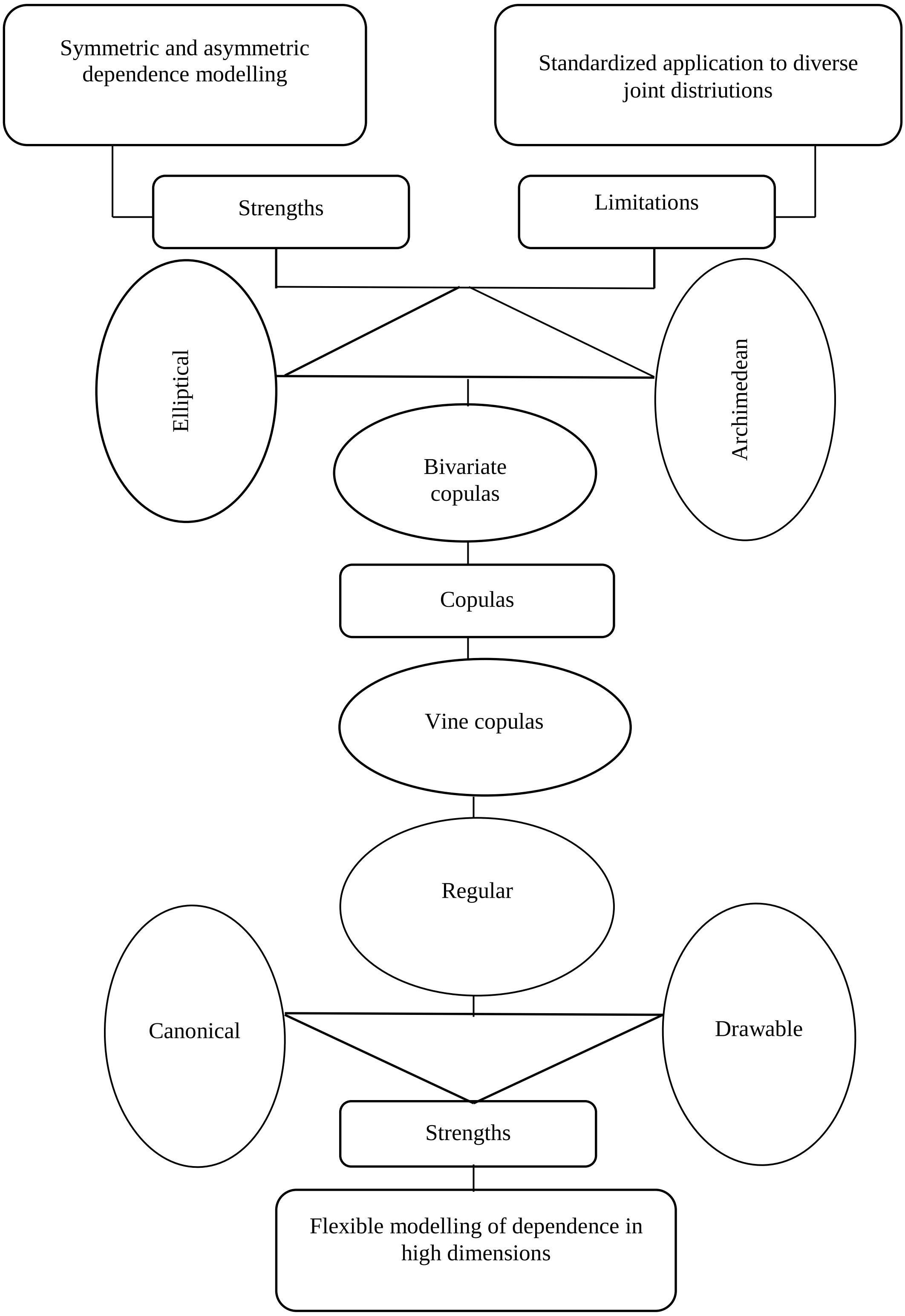

Figure 4-2: The bivariate copula and pair vine copula sets. The bivariate copulas fall into two main categories: elliptical and Archimedean. Among the elliptical copulas are the Gaussian and Student-t. Among the Archimedean copulas are the Frank, Clayton and Gumbel. The bivariate copulas' strength stems from their ability to model the symmetries and asymmetries of dependence from the joint distributions. Their major limitation stems from their inability to adequately model multivariate distributions in high dimensions due to their standardized application to joint distributions that differ in characteristics (Brechmann & Schepsmeier, 2013; Czado et al., 2012; Czado, 2010). The set of the pair vine copulas includes the regular, canonical and drawable, with the canonical and drawable being special cases of the regular. The main strength of the pair vine copulas lies in their flexible modelling of dependence (Brechmann & Schepsmeier, 2013; Czado et al., 2012; Czado, 2010).

The pair vine copulas' fit entails the identification of an adequate vine tree structure; the selection of the optimal bivariate copulas in the vine and the estimation of the bivariate copula parameters. The accuracy of the pair vine copula modelling is consequently dependent on the optimality of those three components (Alcock et al., 2013; Brechmann & Schepsmeier, 2013; Daeyoung et al., 2013).

DEFINITION 1:

A vine V is a graphical structure of n elements so that in $V = (T_1, \dots, T_{n-1})$ every tree T_i is connected with nodes $N_i = E_{i-1}$ and edge set E_i, implying that the edges of T_i are the nodes of tree T_{i+1} (Kurowicka & Cooke, 2006).

Since V is a nested set of trees with n variables such that the edges of the tree j are the nodes of the tree $j+1$, the constraint set of an edge is located on the first tree of a vine and consists of the nodes linked by an edge. However, if two edges are joined by an edge on the following tree, the conditioning set is represented by the intersection of the constraint sets. Given the constraint sets and conditioning set, the latter becomes the union of the former without the intersection, so that they represent the symmetric difference of the constraint sets. When the bivariate copulas are added to a vine structure, the conditioned and conditioning sets are replaced by the conditioned and conditioning variables (Kurowicka & Cooke, 2006).

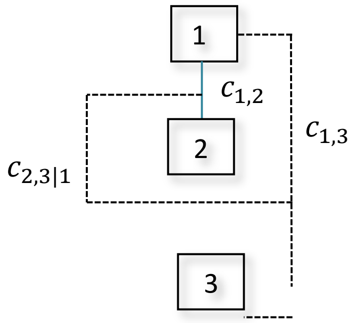

Figure 4-3: Simplified 3-dimensional pair c-vine copula.[10] **Each square in the figure represents a node. The lines connecting the squares are the edges, which represent the dependence between the nodes. The $c_{i,j}$ parameter represents the bivariate copulas and conditional bivariate copulas, used to measure the dependence between the edges.**

[10] Figure 4-3 is an adaptation of that found in Kurowicka and Joe (2011).

The connection between the theorem of Sklar (1959) and the pair vine copula models is as follows:

Let $\boldsymbol{x} = (x_1, \dots, x_n)$ for $i = 1, \dots, n$ be a sequence of random variables with continuous distribution and inverse distribution functions $F_1(x_1), \dots, F_n(x_n)$ and $F_1^{-1}(x_1), \dots, F_n^{-1}(x_n)$, respectively. Let also their probability density functions be $f_1(x_1), \dots, f_n(x_n)$ and $f_1^{-1}(x_1), \dots, f_n^{-1}(x_n)$. It follows then that their joint distribution and joint density functions are $F(\boldsymbol{x}) = F(x_1, \dots, x_n)$ and $f(\boldsymbol{x}) = f(x_1, \dots, x_n)$. If the properties of a probability integral transform are considered, a random variable $U_i \equiv F_i(X_i)$ is understood as being uniformly distributed on [0, 1] and with reverse expression $X_i = F_1^{-1}(U_i)$, for $i = 1, \dots n$. If this relationship is applied to a joint distribution the following expression is obtained:

$$P\left(X_1 \leq F_1^{-1}(u_1), \dots, X_n \leq F_n^{-1}(u_n)\right) = \mathrm{P}(\, U_1 \leq F_1(x_1), \dots, U_n \leq F_n(x_n)\ )$$
$$\equiv \mathrm{C}(x_1, \dots, x_n\,) \qquad (4.1)$$

If the property of inverse distributions $F_i\left(F_i^{-1}(X_i)\right) \geq X_i$ is employed on Equation (4.1) it follows that,

$$P(X_1 \leq x_1, \dots, X_n \leq x_n) = \mathrm{P}\left(\, F_1(X_1) \leq F_1(x_1), \dots, F_n(X_n) \leq F_n(x_n)\right)$$
$$= \mathrm{C}\left(F_1(x_1), \dots, F_n(x_n)\right) \qquad (4.2)$$

Applying the theorem of Sklar (1959) on Equation (4.2) shows that the following equality holds:

$$F(\boldsymbol{x}) = F(x_1, \dots, x_n) = \mathrm{C}\left(F_1(x_1), \dots, F_n(x_n)\right) \qquad (4.3)$$

The parameter C in Equations (4.1), (4.2) and (4.3) represents the copula of the joint distribution function for n-dimensions. Its values range in the set $[0,1]^n$, implying that the margins $F_1, \dots, F_n$ are uniformly distributed. Now, by differentiation on Equation (4.3) it is obtained:

$$f(x_1, \dots, x_n) = \frac{\partial^n \mathrm{C}(F_1(x_1), \dots F_n(x_n))}{\partial F_1(x_1), \dots \partial F_n(x_n)} \times \frac{\partial F_1(x_1)}{\partial x_1} \times \frac{\partial F_2(x_2)}{\partial x_2} \times, \dots, \times \frac{\partial F_n(x_n)}{\partial x_n}$$

$$= c_{1\dots n}(F_1(x_1), \dots, F_n(x_n)) \prod_{i=1}^{n} f_i(x_i) \qquad (4.4)$$

Equation (4.4) represents the decomposition of a density function into marginals and joint densities. Also since the joint densities can be modelled using bivariate copulas, it contains the necessary components to construct a statistical vine copula model. This transition from compact densities to decomposed densities is due to the theorem of Sklar (1959).

4.1.1 Regular vines

The set of the r-vines is large and includes the c-vines and d-vines as subsets and special cases. The r-vines, relative to the c-vine and d-vines, are indicated to be more flexible for the modelling of high dimensional dependence structures (Dissmann, 2010). An r-vine on n variables is one in which two edges in tree j are joined by an edge in tree $j+1$, only if these edges share a common node. The proximity condition governs the structural conditioning and linking of the regular vine structures. The following definition states this relationship (Kurowicka & Cooke, 2006).

DEFINITION 2:

$\mathcal{V}$ is an r-vine on n elements the Definition 1 and the following proximity condition hold: for $i = 2, \dots, n-1$, let the set $\{a, b\} \epsilon E_i$, then $\#a\Delta b = 2$, where Δ denotes the symmetric difference or union without the intersection. This means that, if a and b are nodes of T_i and are connected by an edge, then exactly one a_i equals one b_i, for $a = \{a_1, a_2\}$ and b= $\{b_1, b_2\}$ (Kurowicka & Cooke, 2006).

Definition 2 implies that one edge from two linked nodes in the tree T_i must share a common node in previous tree T_{i-1} so that the decomposition of a multivariate density follows the sequential selection and estimation of the vine tree, copulas and their parameters.

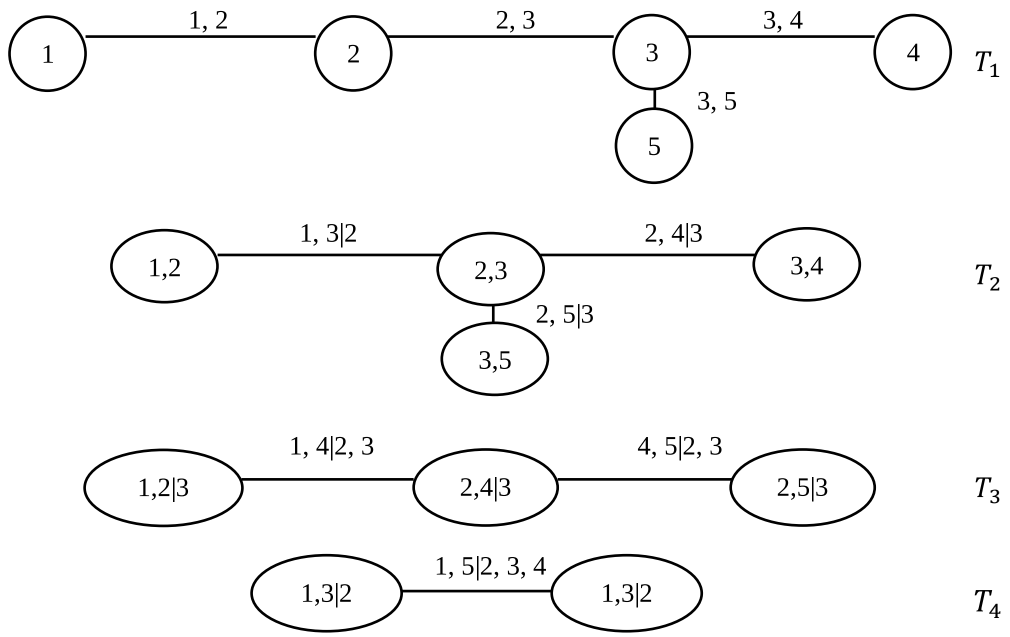

Figure 4-4: An r-vine on 5 variables.[11] The connecting lines represent the edges or, the dependencies and conditional dependencies between variables. The numbers on the connecting lines represent the bivariate copulas and conditional copulas, which are used to measure the strength of dependence. The circles of the first tree are the marginal distributions. In the second and third trees are the copulas and conditional copulas.

Since the number of existing regular vine structures is large and diverse no exact analytical expression has been proposed in the literature of pair vine copulas to decompose and infer regular vine structures. Kurowicka and Cooke (2006) proposed an equation to approximate regular vine structures:

Let $\mathbb{N} = \{N_1, \dots, N_{n-1}\}$ and $\mathcal{E} = \{E_1, \dots, E_{n-1}\}$ be the set of nodes and set of edges corresponding to an r-vine structure. Next, let $j(e)$ and $k(e)$ be the conditioned nodes and $D(e)$ the conditioning set. It follows that every edge $e = j(e), k(e) |\, D(e)$ is an element of $\mathcal{E}$ conditioned by $D(e)$ and can be modelled by a conditional bivariate copula density of the form $c_e = c_{j(e),k(e)|\,D(e)}$. Now let $\boldsymbol{X} = (X_1, \dots, X_n)$ be a vector of variables so that if $\boldsymbol{X}$ is conditioned by $D(e)$ it becomes $\boldsymbol{X}_{D(e)}$. Putting all parameters together yields the following expression:

$$f(x_1, \dots, x_n) =$$
$$\left[\prod_{k=1}^{n} f_k(x_k)\right] \times \left[\prod_{i=1}^{n-1} \prod_{e \in E_i} c_{j(e),k(e)|D(e)} \left(\mathrm{F}\left(x_{j(e)} \middle| x_{D(e)}\right), \mathrm{F}\left(x_{k(e)} \middle| x_{D(e)}\right)\right)\right] \qquad (4.5)$$

[11] Figure 4-4 is an adaptation of that found in Czado et al. (2013).

The model represented by Equation (4.5) is uniquely determined, implying that each inferred r-vine structure is unique (Kurowicka & Cooke, 2006). The uniqueness of determination of each r-vine tree structure makes the task of selecting the optimal r-vine tree more complicated since it requires the storage of the optimal bivariate copulas in the search process.

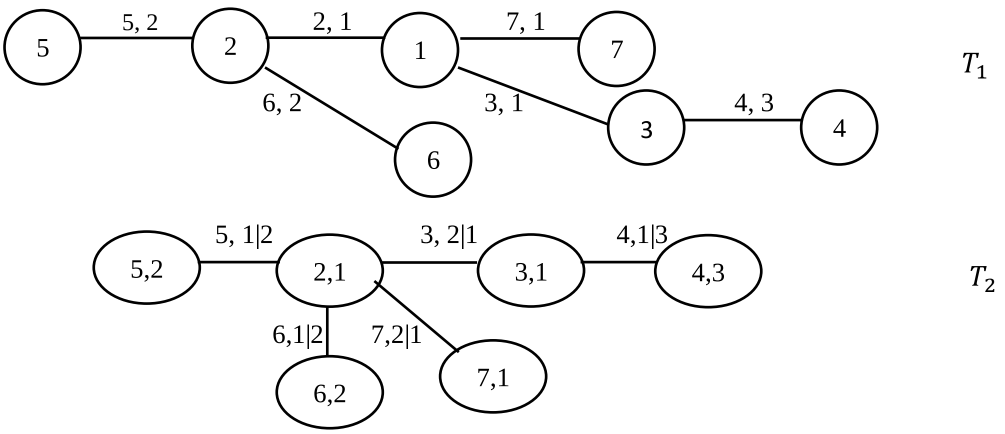

Figure 4-5: First two trees of an r-vine on 7 variables.[12] **The connecting lines represent the edges or, the dependencies and conditional dependencies between variables. The numbers in the connecting lines represent the bivariate copulas and conditional copulas used to measure the strength of dependence between the nodes of the r-vine structure. The circles of the first tree represent the marginal distributions of the vine. In the second tree are located the copulas and conditional copulas.**

$$M = \left| \begin{array}{ccccccc} 7 & & & & & & \\ 4 & 6 & & & & & \\ 3 & 4 & 5 & & & & \\ 6 & 3 & 4 & 4 & & & \\ 5 & 5 & 3 & 2 & 3 & & \\ 2 & 1 & 1 & 1 & 2 & 2 & \\ 1 & 2 & 2 & 3 & 1 & 1 & 1 \end{array} \right.$$

Figure 4-6: Diagonal matrix of an r-vine on 7 variables. This figure contains the components of the seven-variable r-vine displayed in Figure 4-5. The diagonal elements of the matrix *M* represent the original nodes-variables in the first tree of the r-vine. An edge in the first tree of the vine is formed by a diagonal component and a component from the base row (i.e. first row from the bottom up). The conditional edges (i.e. the

[12] Figure 4-5 is an adaptation of that found in Brechmann et al. (2012).

dependence between two nodes, given the dependence relationship each of the nodes has with another node from previous trees) start to appear from the second vine tree onwards. A conditional edge on the second vine tree consists of one component from the diagonal, one component from the second row (from the bottom up), and a component from the base row (this is the conditioning component of the conditional edge), and so on.

4.1.2 Canonical vines

Canonical vines have a star like tree structure and for every tree T_i, $i \in \{1, \ldots, n-1\}$ a root node is selected. The criterion for the selection of a root node in a c-vine tree structure requires from the root node to have the strongest correlation with the rest of the nodes in the tree. The c-vines are indicated to best fit data sets that have a dominant variable (Czado et al., 2013).

DEFINITION 3:

An r-vine is called a canonical vine if Definitions 1 and 2 hold, and each tree T_i has a unique node of degree $n-i$. The node with maximal degree in tree T_1 is identified as the root node of the entire vine tree structure. Aas et al. (2009) proposed the following model for the separation of multivariate densities and the inference of pair c-vine copula structures:

$$f(\boldsymbol{x}) = \prod_{k=1}^{n} f_k(x_k)$$
$$\cdot \prod_{i=1}^{n-1} \prod_{j=1}^{n-i} c_{i,i+j|1:(i-1)} \left(\mathrm{F}(x_i|x_1, \ldots x_{i-1}), \mathrm{F}\left(x_{i+j}\middle|x_1, \ldots, x_{i-1}\right) \middle| \boldsymbol{\theta}_{i,i+j|\,1:(i-1)} \right) \quad (4.6)$$

In Equation (4.6) the index i identifies the trees and index j runs over the edges in each tree. An example of a 4-dimensional c-vine density decomposition and its corresponding graph is:

$$\mathrm{f}(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4) = \mathrm{c}_{1,2}\big(\mathrm{F}_1(\mathrm{x}_1), \mathrm{F}_2(\mathrm{x}_2)\big) \cdot \mathrm{c}_{1,3}\big(\mathrm{F}_1(\mathrm{x}_1), \mathrm{F}_3(\mathrm{x}_3)\big) \cdot \mathrm{c}_{1,4}\big(\mathrm{F}_1(\mathrm{x}_1), \mathrm{F}_4(\mathrm{x}_4)\big)$$
$$\cdot\, \mathrm{c}_{2,3|1}\left(\mathrm{F}_{2|1}(\mathrm{x}_2|\mathrm{x}_1), \mathrm{F}_{3|1}(\mathrm{x}_3|\mathrm{x}_1)\right) \cdot \mathrm{c}_{2,4|1}\left(\mathrm{F}_{2|1}(\mathrm{x}_2|\mathrm{x}_1), \mathrm{F}_{4|1}(\mathrm{x}_4|\mathrm{x}_1)\right)$$
$$\cdot\, \mathrm{c}_{3,4|1,2}\left(\mathrm{F}_{3|1,2}(\mathrm{x}_3|\mathrm{x}_1, \mathrm{x}_2), \mathrm{F}_{4|1,2}(\mathrm{x}_4|\mathrm{x}_1, \mathrm{x}_2)\right)$$
$$\cdot\, \mathrm{f}_1(x_1) \cdot \mathrm{f}_2(\mathrm{x}_2) \cdot \mathrm{f}_3(\mathrm{x}_3) \cdot \mathrm{f}_4(\mathrm{x}_4) \quad (4.7)$$

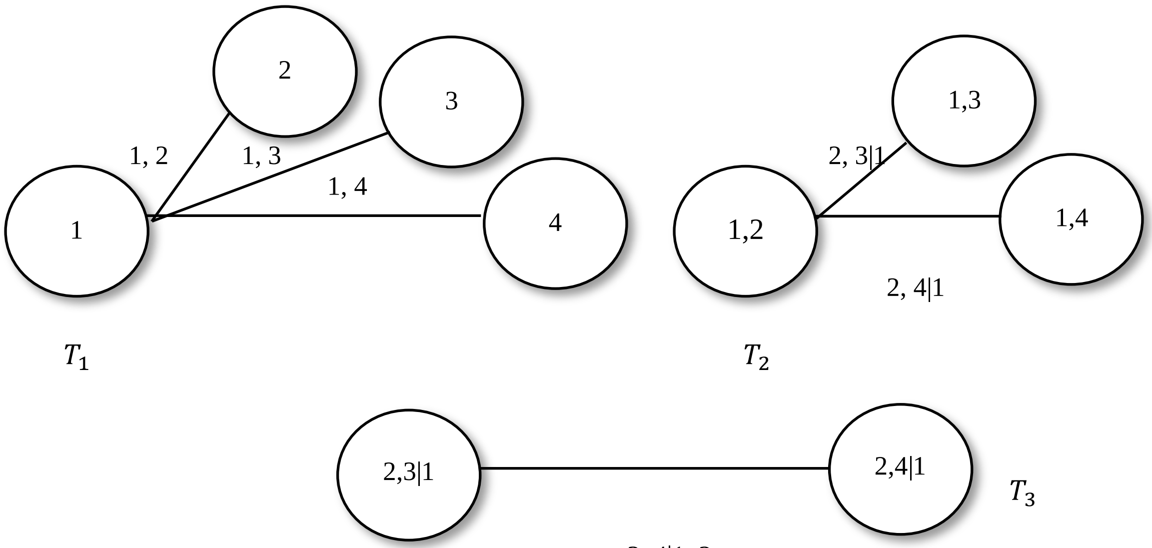

Figure 4-7: 4-dimensional c-vine structure with 3 trees and 6 edges.[13] The connecting lines represent the edges or the dependencies and conditional dependencies between variables. The numbers on the connecting lines represent the bivariate copulas and conditional copulas used to measure the strength of dependence between nodes. The circles of the first tree represent the marginal distributions. In the second and third trees are the copulas and conditional copulas.

In Equation (4.7) the unconditional copulas $c_{1,2}$, $c_{1,3}$ and $c_{1,4}$ of the first tree T_1 (see Figure 4-7) model the edges 1, 2, 1, 3 and 1, 4. The conditional copulas $c_{2,3|1}$, $c_{2,4|1}$ and $c_{3,4|1,2}$ from the T_2 model the conditional edges 2, 3|1 and 2, 4|1. The marginal densities of the nodes in tree T_1 are represented by the functions f_1,f_2 f_3 and f_4. The first node of the tree T_1 represents the root node of the entire vine structure.

4.1.3 Drawable vines

Drawable vines are represented through line trees and every node of any tree T_i cannot be linked to more than two edges. In the d-vine tree structures the first tree of the vine plays a central role in the definition of subsequent trees. Hence, the most influential variables, in terms of correlation, are found in the first tree. The d-vines are indicated to best fit the data sets where instead of a single variable being the dominant, a group of

[13] Figures 4-7 and 4-8 are an adaptation of those found in Min and Czado (2010).

variables exerts the most influence over the rest through large correlation values (Czado, 2010; Min & Czado, 2010).

DEFINITION 4:

A regular vine is called a drawable vine if each of its nodes in T_i has a degree of at most 2.

Aas et al. (2009) proposed the following model for the separation of multivariate densities and the inference of pair d-vine copulas:

$$f(\boldsymbol{x}) = \prod_{k=1}^{n} f_k(x_k)$$
$$\cdot \prod_{i=1}^{n-1} \prod_{j=1}^{n-i} c_{j,j+i|(j+1):(j+i-1)} \left(\mathrm{F}\left(x_j \middle| x_{j+1}, \ldots x_{j+i-1}\right), \mathrm{F}\left(x_{j+i} \middle| x_{j+1}, \ldots, x_{j+i-1}\right) \middle| \boldsymbol{\theta}_{j,j+i|\,(j+1):(j+i-1)}\right) \quad (4.8)$$

In Equation (4.8) the index i identifies the trees and index j runs over the edges in each tree. An example of a 5-dimensional d-vine density decomposition and its corresponding graph is:

$$\begin{aligned} \mathrm{f}(\mathrm{x}_1, \mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4, \mathrm{x}_5) = {} & \mathrm{c}_{1,2}\big(\mathrm{F}_1(\mathrm{x}_1), \mathrm{F}_2(\mathrm{x}_2)\big) \cdot \mathrm{c}_{2,3}\big(\mathrm{F}_2(\mathrm{x}_2), \mathrm{F}_3(\mathrm{x}_3)\big) \cdot \mathrm{c}_{3,4}\big(\mathrm{F}_3(\mathrm{x}_3), \mathrm{F}_4(\mathrm{x}_4)\big) \\ & \cdot \mathrm{c}_{4,5}\big(\mathrm{F}_4(\mathrm{x}_4), \mathrm{F}_5(\mathrm{x}_5)\big) \cdot \mathrm{c}_{1,3|2}\left(\mathrm{F}_{1|2}(\mathrm{x}_1|\mathrm{x}_2), \mathrm{F}_{3|2}(\mathrm{x}_3|\mathrm{x}_2)\right) \\ & \cdot \mathrm{c}_{2,4|3}\left(\mathrm{F}_{2|3}(\mathrm{x}_2|\mathrm{x}_3), \mathrm{F}_{4|3}(\mathrm{x}_4|\mathrm{x}_3)\right) \cdot \mathrm{c}_{3,5|4}\left(\mathrm{F}_{3|4}(\mathrm{x}_3|\mathrm{x}_4), \mathrm{F}_{5|4}(\mathrm{x}_5|\mathrm{x}_4)\right) \\ & \cdot \mathrm{c}_{2,4|3}\left(\mathrm{F}_{2|3}(\mathrm{x}_2|\mathrm{x}_3), \mathrm{F}_{4|3}(\mathrm{x}_4|\mathrm{x}_3)\right) \cdot \mathrm{c}_{3,5|4}\left(\mathrm{F}_{3|4}(\mathrm{x}_3|\mathrm{x}_4), \mathrm{F}_{5|4}(\mathrm{x}_5|\mathrm{x}_4)\right) \\ & \cdot \mathrm{c}_{1,4|2,3}\left(\mathrm{F}_{1|2,3}(\mathrm{x}_1|\mathrm{x}_2, \mathrm{x}_3), \mathrm{F}_{4|2,3}(\mathrm{x}_4|\mathrm{x}_1, \mathrm{x}_2)\right) \\ & \cdot \mathrm{c}_{2,5|3,4}\left(\mathrm{F}_{2|3,4}(\mathrm{x}_2|\mathrm{x}_3, \mathrm{x}_4), \mathrm{F}_{5|3,4}(\mathrm{x}_5|\mathrm{x}_3, \mathrm{x}_4)\right) \\ & \cdot \mathrm{c}_{1,5|2,3,4}\left(\mathrm{F}_{1|2,3,4}(\mathrm{x}_1|\mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4), \mathrm{F}_{5|2,3,4}(\mathrm{x}_5|\mathrm{x}_2, \mathrm{x}_3, \mathrm{x}_4)\right) \\ & \cdot \mathrm{f}_1(x_1) \cdot \mathrm{f}_2(\mathrm{x}_2) \cdot \mathrm{f}_3(\mathrm{x}_3) \cdot \mathrm{f}_4(\mathrm{x}_4) \cdot \mathrm{f}_5(\mathrm{x}_5) \qquad (4.9) \end{aligned}$$

In Equation (4.9) the unconditional copulas $\mathrm{c}_{1,2}$, $\mathrm{c}_{2,3}$ and $\mathrm{c}_{3,4}$ and $\mathrm{c}_{4,5}$ from the first d-vine tree T_1 (see Figure 4-8 below) model the edges 1,2; 2,3; 3,4 and 4,5. The conditional copulas $\mathrm{c}_{1,3|2}$, $\mathrm{c}_{2,4|3}$ and $\mathrm{c}_{3,5|4}$ from the second d-vine tree T_2 model the conditional edges 1, 3|2; 2, 4|3 and 3, 5|4, and so on with the rest of the trees. Some equivalent expressions of the conditional factors from Equations (4.7) and (4.9) are:

$$f_{2|1}(x_2|x_1) = \frac{f_{12}(x_1,x_2)}{f_1(x_1)} = c_{1,2}(\mathrm{F}_1(\mathrm{x}_1), \mathrm{F}_2(\mathrm{x}_2)) \cdot f_2(x_2) \tag{4.10}$$

$$\begin{aligned} f_{3|1,2}(x_3|x_1,x_2) = \frac{f_{1,3|2}(x_1,x_3|x_2)}{f_{1|2}(x_1|x_2)} &= c_{1,3|2}(\mathrm{F}_{1|2}(\mathrm{x}_1|\mathrm{x}_2), \mathrm{F}_{3|2}(\mathrm{x}_3|\mathrm{x}_2)) \cdot f_{3|2}(x_3|x_2) \\ &= c_{1,3|2}(\mathrm{F}_{1|2}(\mathrm{x}_1|\mathrm{x}_2), \mathrm{F}_{3|2}(\mathrm{x}_3|\mathrm{x}_2)) \\ &\quad \cdot c_{2,3}(\mathrm{F}_1(\mathrm{x}_1), \mathrm{F}_2(\mathrm{x}_2)) \cdot f_3(x_3) \end{aligned} \tag{4.11}$$

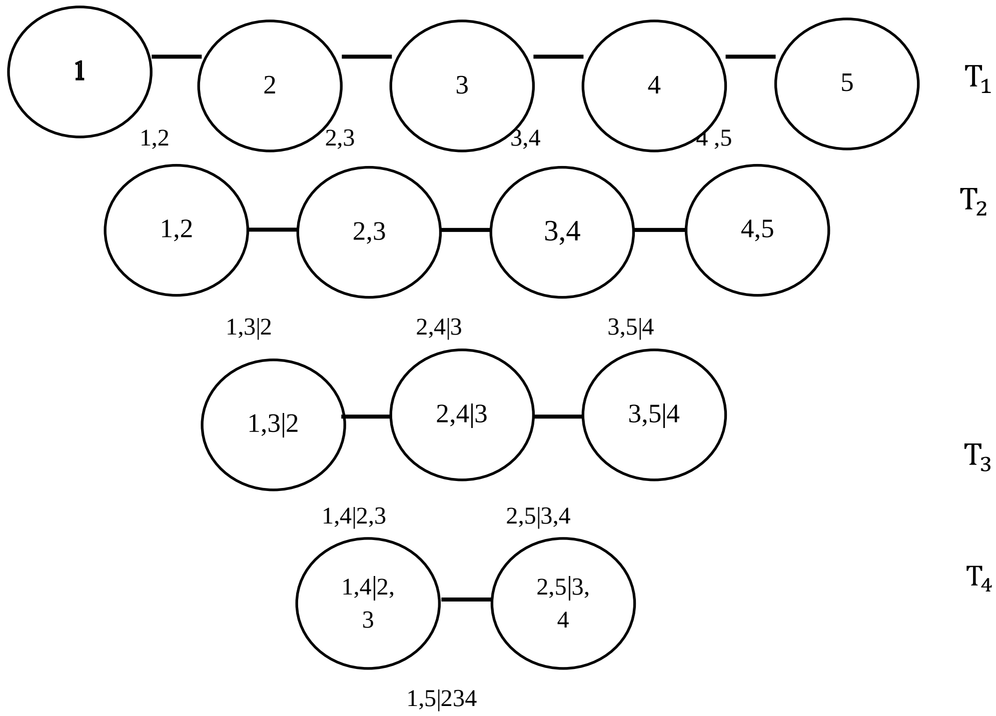

Figure 4-8: 5-dimensional d-vine structure with 4 trees and 10 edges.[14] **The connecting lines represent the edges or the dependencies and conditional dependencies between variables. The numbers on the connecting lines represent the bivariate copulas and conditional copulas used to measure the strength of dependence between nodes. The circles of the first tree represent the nodes of the vine or marginal distributions. In the second and third trees are the copulas and conditional copulas.**

The left hand side of Equation (4.10) represents the density function of x_2 conditional on the values of x_1. It is equal to the ratio between the bivariate density function of x_1 and x_2 and the marginal density function of x_1. By the theorem of Sklar (1959) the right hand side of the same equation is expressed as the product of the bivariate copula density of x_1 and x_2 and the marginal density of x_2. The left hand side of Equation (4.11) is the density function of x_3 conditional on the values of x_1 and x_2. It is equal to the ratio between the conditional bivariate density function of x_1 and x_3 given the values of x_2 and the conditional density of x_1 given the values of x_2. The right hand side of Equation (4.11) is expressed as the product of the conditional bivariate copula density $c_{1,3|2}$, the bivariate copula density $c_{2,3}$, and the marginal density $f_3(x_3)$.

[14] Figures 4-7 and 4-8 are an adaptation of those in Min and Czado (2010).

4.2 Risk measures and optimization models

The optimization methods and risk measures discussed in this chapter are fitted to estimate the minimum risk optimal portfolios. The risk measures considered are the *variance*, *MAD*, *Minimax*, *CVaR* and *CDaR*. Each of them has interesting theoretical properties that enable the optimization of portfolios from a specific angle (see e.g. Gao et al., 2014; Arreola & Powell, 2013; Chang et al., 2009). Thus, a comparison can be established between them in terms of resource allocation and investment risk.

4.2.1 The variance

The *variance* risk measure threaded with the nonlinear mean-variance quadratic (*QP*) portfolio optimization problem (4.12)-(4.15) assumes the return distribution to be normal. Investors' preferences are represented by a quadratic utility function (Brooks & Kat, 2002; Pratt, 1964). The convexity and symmetry of the quadratic utility function causes the observations deviating from the mean to be penalized with an escalating rate (Ghalanos, 2013; Markowitz, 1959,1952). The nonlinear portfolio optimization problem to be solved is:

$$\min_{w} \quad \frac{1}{n}\sum_{i=1}^{n}\left(\sum_{j=1}^{m} w_j\left(r_{i,j}-\mu_j\right)\right)^2 \tag{4.12}$$

Subject to:

$$\sum_{j=1}^{m} w_j\,\mu_j = \mu_P\,; \tag{4.13}$$

$$\sum_{j=1}^{m} w_j = 1 \tag{4.14}$$

$$w_j \geq 0, \text{ for } j = 1, \ldots, m \tag{4.15}$$

Equations (4.13), (4.14) and (4.15) appear in all subsequent portfolio optimization model specifications. They represent respectively the portfolio's target return, the constraint on the sum of the optimal weights to be equal to 1, and the constraint on each of the optimal weights to be positive semi definite (at least zero). This implies that no short selling is considered in the optimization of the portfolios. The parameter μ_j represents the average of the returns or expected return value, while $r_{i,j}$ represents the return of the security i at time j.

4.2.2 The mean absolute deviation

The *MAD* risk measure was introduced by Konno and Yamazaki (1991) as a simpler and none computationally expensive measure. The risk measure solves nonlinear optimization problems in their linearized form thus; simplifying the solution of large-scale optimization problems. Under this risk measure deviations from the mean are weighted according to a linear function, while a covariance estimate is not required (Konno & Shirakawa, 1994). Since the risk measure does not penalize leptokurtic observations as heavily as the *variance* does, it may be seen as more robust. The linear portfolio optimization problem to be solved is:

$$\min_{w,d} \frac{1}{n} \sum_{i=1}^{n} d_i \tag{4.16}$$

Subject to:

$$\sum_{j=1}^{m} (r_{i,j} - \mu_j) w_j \leq y_i, \quad \forall_i \in \{1, \dots, n\} \tag{4.17}$$

$$\sum_{j=1}^{m} w_j \, \mu_j = \mu_P \; ; \tag{4.18}$$

$$\sum_{j=1}^{m} w_j = 1 \tag{4.19}$$

$$\sum_{j=1}^{m} (r_{i,j} - \mu_j) w_j \geq -y_i, \; \forall_i \in \{1, \dots, n\} \tag{4.20}$$

$$w_j \geq 0, \quad \forall_j \in \{1, \dots, m\} \tag{4.21}$$

where the parameter d_i accounts for absolute deviations from the forecast mean. Equations (4.17) and (4.20) delineate the lower and upper bounds of y_i, respectively.

4.2.3 The minimizing regret

Young (1998) introduces the *Minimax* risk measure as a conservative approach to minimize the risk of portfolios. In problem (4.22)-(4.26) for instance, the constraint (4.23) states that the difference between the maximum loss of the portfolio M_p and the forecast return of the portfolio is less or equal to zero. The portfolio optimization problem to be solved is:

$$\min_{M_p,w} M_p \tag{4.22}$$

Subject to:

$$M_p - \sum_{j=1}^{m} w_j r_{ij} \leq 0, \forall_i \in \{1, \dots, n\} \tag{4.23}$$

$$\sum_{\mathrm{j}=1}^{m} w_j \, \mu_j = \mu_{\mathrm{P}} \tag{4.24}$$

$$\sum_{\mathrm{j}=1}^{m} w_j = 1 \tag{4.25}$$

$$w_j \geq 0, \forall_j \in \{1, \dots, m\} \tag{4.26}$$

4.2.4 The conditional Value-at-Risk

Rockafellar and Uryasev (2000) introduced the *CVaR* measure as a way to compensate for the inadequacies of the *VaR* measure. As compared to the *VaR* it does fulfill the subadditivity property. Also, by being a spectral risk measure it weights the average of the loss distribution according to a probability and is more in tune with the loss function of the tails' distribution (Szego, 2002; Uryasev, 2000). The linear portfolio optimization problem to be solved is:

$$\min_{w,d,v} \frac{1}{na} \sum_{i=1}^{n} d_i + v \tag{4.27}$$

Subject to:

$$\sum_{j=1}^{m} w_j r_{i,j} + v \geq -d_i, \forall_i \in \{1, \dots, n\} \tag{4.28}$$

$$\sum_{\mathrm{j}=1}^{m} w_j \, \mu_j = \mu_{\mathrm{P}} \tag{4.29}$$

$$\sum_{\mathrm{j}=1}^{m} w_j = 1 \tag{4.30}$$

$$w_j \geq 0, \forall_j \in \{1, \dots, m\}; \tag{4.31}$$

$$d_i \geq 0, \forall_j \in \{1, \ldots, n\} \qquad (4.32)$$

where μ_P represents the target return of the portfolio, v is the *VaR* at the a-coverage rate and d_i accounts for the deviation values below the *VaR*.

4.2.5 The conditional Drawdown-at-Risk

Chekhlov et al. (2003) proposed the *CDaR* as an alternative to the *CVaR* measure. A common feature the *CDaR* and *CVaR* have is the modelling of observations in the negative tail. The *CDaR* is concerned with the drawdowns in the asset distribution. It records and averages the drawdowns ending below a threshold value (Ghalanos, 2013). The linear portfolio optimization problem to be solved is:

$$\min_{w,u,v,z} v + \frac{1}{na} \sum_{i=1}^{n} z_i \qquad (4.33)$$

Subject to:

$$z_i - u_i + v \geq 0, \forall_i \in \{1, \ldots, n\} \qquad (4.34)$$

$$\sum_{j=1}^{m} w_j r_{i,j} + u_i - u_{i-1} \geq 0, \ \ u_0 = 0, \forall_i \in \{1, \ldots, n\} \qquad (4.35)$$

$$z_i \geq 0, \ \ u_i \geq 0, \forall_i \in \{1, \ldots, n\}$$

$$\sum_{j=1}^{m} w_j \mu_j = \mu_P ; \qquad (4.36)$$

$$\sum_{j=1}^{m} w_j = 1 ; \ w_j \geq 0, \forall_j \in \{1, \ldots, m\} \qquad (4.37)$$

Where the parameters z and u are auxiliary vectors. The parameter v accounts for the *CDaR* at the a quantile level.

5. Summary

This chapter explained the pair vine copula, risk measure and portfolio optimization models implemented in this thesis to examine the dependence risk profile and portfolio allocation features of the mining, energy and retail and manufacturing benchmark portfolios under consideration. The pair vine copulas were acknowledged for their flexible modelling of dependence in high dimensions. The r-vines were recognized to be the largest set of vine structures, while the c-vines and d-vines were acknowledged to be special cases of them. The central role of the theorem of Sklar (1959) for the development of the pair vine copulas was indicated and the linear and nonlinear portfolio optimization model specifications with respect to the *variance*, *Minimax*, *MAD*, *CVaR*, *CDaR* were explained. The *CVaR* and *CDaR* were identified as threshold and downside risk measures, while the *variance*, *Minimax*, *MAD* were identified as risk measures from modern portfolio theory.

CHAPTER 5

DEPENDENCE STRUCTURE ESTIMATION: MINING PORTFOLIOS

This chapter consists of three sections: introduction, copula counting technique and dependence structure estimation

The *introduction* section provides an overview of the gold, iron ore and nickel commodities that underlie the Australian mining stock portfolios modelled. The *copula counting technique* section briefly contextualizes in the relevant literature the "copula counting technique" proposed, states the stages of the technique and its usefulness. The *dependence estimation* section implements the copula counting technique to dissect, organize, analyse and interpret the mining portfolios' dependence structure.

5.1 Introduction

In the last two decades Australia saw a sharp increase in the mining of precious and non-precious metals such as gold, iron ore and nickel stemming from the Asian emerging economies' increasing demand of those commodities (Bishop et al., 2013; Bingham & Perkins, 2012; Connolly & Orsmond, 2011; Gardner-Bond et al., 2008). In 2011 gold, iron ore and nickel production placed Australia as the third, first and fourth largest exporter worldwide, respectively (Bingham & Perkins, 2012; Gardner-Bond et al., 2008). During the 2008-2009 GFC gold prices, contrary to iron ore and nickel prices, rose to historical levels and investors saw gold as a "relatively secure defensive investment and storage of wealth" as the confidence in the financial stock markets eroded (BREE, 2014; Collins, 2013; DRET & BREE, 2013; Silvennoinen & Thorp, 2013; Bingham & Perkins, 2012; WGC, 2012; Connolly & Orsmond, 2011)[15]. Iron ore prices suffered a sharp decline (e.g., a 48% from US$138 per ton to US$71 per ton) in the period from Oct-2008 to Dec-2009 and displayed a strong negative correlation with financial stock market uncertainty (Bingham & Perkins, 2012; Connolly & Osmond, 2011). Nickel prices, relative to iron ore prices, undergo a more severe price decline from May 2007 (e.g. at US$51,783 per metric ton) to the second half of 2008, when they reached their lowest price (e.g. US$10,000 per metric ton). Similarly to the iron-ore prices, nickel prices show to be negatively correlated to financial stock market uncertainty.

This chapter's objectives are to examine the dependence risk profile of the mining portfolios in specific market conditions; account for the portfolios' dependence structure changes between pairs of period scenarios; and recognize the pair vine copula models that best capture the dependence structure of the portfolios. The copula counting technique is used for this purpose.

5.2 The "copula counting technique"

The fit of the pair vine copula models to a data set produces three outputs: the Kendall tau correlation matrix, representing the correlation between pairs of variables; the dependence structure matrix, where the vine copula models' bivariate copula selection is contained; and the matrix of bivariate copula parameters (see Brechmann & Schepsmeier, 2013). The copula counting technique focuses on the dissection, organization, analysis and interpretation of the dependence structure matrix. The reason for this is that the information about the assets' dependence risk is contained in the dependence structure matrix. The copula counting technique consists of five stages: 1) counting, 2) recording, 3) classification, 4) grouping and, 5) aggregate dependence reading. In the literature of pair vine copula modelling there have been studies (e.g. Allen et al., 2013; Dissmann et al., 2013; Czado et al., 2012; Heinen & Valdesogo, 2009) that most likely unintendedly have engaged in one or two of the bivariate copula counting technique's stages. Hence, the technique could be seen as an extension of those earlier attempts aimed at examining

[15] The acronyms *WGC*, *DRET* and *BREE* used in this chaper stand for World Gold Council, Department of Resources, Energy and Tourism, and Bureau of Resources and Energy Economics.

the dependence structure and dependence risk profile of stock portfolios. In what follows each of the techniques' stages is described in detail.

1) Counting

The bivariate copulas selected by the vine models and contained in the diagonal dependence structure matrices presented in the next section are counted to know how often a certain copula is selected for the estimation of the stocks interaction. Knowing the frequency of the selection is essential because aggregation is used to draw generalizations and inferences about the portfolios' dependence risk profile. The aggregation of the bivariate copulas is crucial to the analysis because single bivariate copulas considered in isolation do not provide sufficient information about the dependence risk in high dimensional dependence structures.

2) Recording:

The counted bivariate copulas are organized in tables so that the patterns of dependence concentration are easily recognized. The recording of the frequency of bivariate copula selection also facilitates the identification of dependence concentration shifts across financial period scenarios or changes in the dependence structure across time. [16]

3) Classification:

The bivariate copulas selected by the vine copula models are distinguished on the basis of the type of dependence modelling they perform. This process of differentiation needs not be recorded; however, it does require from the modeller to understand the dependence modelling properties of each bivariate copula so that they are adequately classified. The adequate classification of the bivariate copulas lays in turn a reliable ground to accurately interpret the dependence structure of financial variables and the dependence risk profile of stock portfolios.

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[16] The term "dependence concentration" is based on and presupposes the aggregation of bivariate copulas selected by the vine copulas to model and estimate the dependence structure of the portfolios. It refers to the location in the joint distributions where pairs of variables experience higher correlation activity, as indicated by the specific type of bivariate copulas aggregated.

4) Grouping:

The selected bivariate copulas are grouped (on the tables where they were recorded) according to the type of dependence modelling they perform and the location (e.g. centre, positive tail and negative tails) of the dependence they model.

5) Aggregate dependence reading

This stage deals with the identification of symmetric and asymmetric patterns of dependence and the recognition of the size and location of the dependence and its concentration in the joint distributions. The shifts of dependence concentration between pairs of period scenarios are also identified and interpreted and the vine copula models that best account for the dependence structure of the portfolios are acknowledged. The risk profile of the portfolios is explained in detail by looking at the actual behaviour of the underlying commodities and using standard economic theory.

5.3 Dependence structure estimation

This section deals with the implementation of the copula counting technique to the mining portfolios: gold, iron ore-nickel and mix-metals leptokurtic. The counting stage of the copula technique is only implemented to the full sample period scenario of each portfolio since the counting of the bivariate copulas for the rest of the period scenarios is summarized and recorded in subsequent tables as part of the recording stage of the copula counting technique. In addition to that, the counting, recording and classification stages are summarized in those tables together with the grouping stage. Only the Kendall tau and dependence structure matrices corresponding to one period scenario from each portfolio is displayed in this section. The remaining matrices have been placed in Appendix A.

The bivariate copulas found in Table 5-1 belong to the Archimedean and elliptical families. The 90, 180 and 270 degrees rotated versions of them are also considered to account for distributional characteristics that the standard version of the Archimedean and elliptical cannot (Brechmann & Schepsmeier, 2013; Smith, 2013; Nikoloulopoulos et al., 2012; Chollete et al., 2009). The Gumbel, Joe and Clayton 180 copulas are designed to model greater concentration of asymmetric dependence at various locations of the positive tail. The Student-t copula models the dependence in the tails symmetrically. All copulas listed in Table 5-1 are also used to model the dependence of the energy, retail and manufacturing portfolios in Chapters 6 and 7. Table 5-1 is omitted in those chapters to avoid repetition.

Table 5-1: Set of the bivariate copula families employed by the vine copula models

One Par	Archimedean 2Par	90 Rotated	180 Rotated	270 Rotated
Gaussian (1)	Clayton-Gumbel(BB1) (7)	Clayton (23)	Clayton (13)	Clayton (33)
Student-t (2)	Joe-Gumbel(BB6) (8)	Gumbel (24)	Gumbel (14)	Gumbel (34)
Clayton (3)	Joe-Clayton(BB7) (9)	Joe (26)	Joe (16)	Joe (36)
Gumbel (4)	Joe-Frank(BB8) (10)	Clayton-Gumbel(BB1) (7)	Clayton-Gumbel (BB1) (17)	Clayton-Gumbel(BB1) (37)
Frank (5)		Joe-Gumbel(BB6) (28)	Joe-Gumbel(BB6) (18)	Joe-Gumbel(BB6) (38)
Joe (6)		Joe-Clayton(BB7) (29)	Joe-Clayton(BB7) (19)	Joe-Clayton(BB7) (39)
		Joe-Frank(BB8) (30)	Joe-Frank(BB8) (20)	Joe-Frank(BB8) (40)

Notes: the table lists the bivariate copulas employed by the c-vine, d-vine and r-vine copula models and their corresponding conventional numbers. The top row of the table classifies the bivariate copulas according to the number of parameters they use and their degree of rotation. Each of the bivariate copulas in the table is assigned one number to make the pair vine copula estimation of dependence less complex, while also simplifying the interpretation of the dependence structure. The number 1 is used to represent the Gaussian bivariate copula, number 2 to represent the Student-t copula, and so on. These numbers appear in the diagonal dependence structure matrices of subsequent sections.

Fitting a two-sample two-tailed t-test for the difference of means between two portfolios' dependence concentrations enables one to identify the dependence risk differences between the mining portfolios. The two-sample two-tailed t-test fitted at the 95% confidence level is:

$$t = \frac{\text{The difference between smaple means}}{\text{Estimated standard error of difference between means}} \quad (5.1)$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S_{\bar{x}_1 - \bar{x}_2}} \quad (5.2)$$

where $$S_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \quad (5.3)$$

In Equation (5.3) the variables s_1^2 and s_2^2 represent the variances of the populations, and n_1 and n_2 account for the number of observations in each population. The degrees of freedom are estimated as follows:

$$df = (n_1 - 1) + (n_2 - 1) \quad (5.4)$$

The dependence concentrated at a certain location of stocks' joint distribution is considered to be significantly larger or significantly smaller if the resulting t-test values are larger or smaller than the critical values. If the resulting t-test values are neither larger nor smaller than the critical values, one portfolio’s dependence concentration is neither significantly larger nor significantly smaller than that of other portfolio. Also, while the concentration of dependence in the portfolios is measured by counting the frequency of bivariate copula selection, as indicated for example in Table 5-2 below, the t-statistics are estimated using the same frequency of bivariate copula selection at some location of the joint distributions between pairs of portfolios. Specifically, they are obtained using the difference between frequencies of a given copula selected by the vine models for different sector portfolios

5.3.1 Gold portfolio

The dependence structure matrices of the gold mining portfolio displayed in Panel (a) of Figure 5-1 contain the information about the concentration of dependence in the centre and in the tails of the pairs of gold stocks. The Kendall tau matrix displayed in Panel (b), as a measure of correlation, represents the strength of association between pairs of stocks. The assets' co-movements can more easily be interpreted using the copula counting technique.

20																		
20	2																	
2	2	20																
3	2	20	10															
2	5	14	1	5														
1	2	5	5	20	1													
20	10	5	2	5	10	10												
20	2	5	20	5	5	2	5											
10	10	6	20	5	1	4	20	10										
1	20	13	1	5	20	13	5	14	3									
5	10	5	5	5	5	4	10	5	4	1								
14	2	5	5	2	1	5	5	5	5	10	1							
20	2	20	1	10	2	10	4	13	23	20	5	10						
20	10	5	5	5	4	20	20	5	4	20	4	5	4					
14	14	14	20	2	1	2	33	14	5	14	5	4	10	5				
20	10	5	5	5	5	5	3	10	3	34	14	1	24	4	3			
14	2	5	5	5	23	20	5	5	10	23	1	2	5	5	1	5		
5	5	5	3	10	4	5	14	14	4	1	24	20	5	16	23	4	5	
2	14	1	3	20	20	20	2	4	5	14	4	14	2	14	16	1	33	5

14																		
14	20																	
2	2	10																
2	2	3	5															
30	1	5	5	5														
4	10	20	5	3	20													
10	10	2	20	2	2	5												
2	5	14	20	5	14	14	3											
20	3	20	10	20	1	20	14	5										
20	2	5	5	10	5	2	20	14	14									
2	2	2	5	5	20	3	5	5	14	1								
14	5	2	5	14	4	1	20	1	1	1	20							
5	5	5	1	5	4	4	4	5	4	1	5	1						
14	20	5	10	23	20	5	4	36	6	4	3	5	23					
20	10	5	2	5	20	10	5	20	5	5	10	5	23	5				
20	20	4	20	3	4	10	13	16	14	5	4	20	16	16	14			
14	5	10	1	6	2	20	16	2	20	20	3	5	2	2	1	23		
5	5	20	4	16	13	1	5	13	5	6	16	4	5	1	5	23	1	
5	2	2	5	5	1	20	2	14	2	16	10	4	1	3	10	10	10	16

Panel (a)

0.100																		
0.133	0.518																	
0.140	0.164	0.078																
0.073	0.480	0.232	0.051															
0.247	0.119	0.046	0.060	0.033														
0.139	0.349	0.143	0.054	0.082	0.046													
0.112	0.299	0.158	0.026	0.155	0.047	0.055												
0.083	0.313	0.144	0.089	0.129	0.031	0.057	0.037											
0.044	0.087	0.001	0.068	0.048	0.010	0.001	0.033	0.058										
0.166	0.142	0.036	0.049	0.040	0.115	0.009	0.049	0.035	0.003									
0.051	0.164	0.060	0.040	0.051	0.067	0.003	0.111	0.060	0.003	-0.002								
0.128	0.218	0.077	0.092	0.041	0.049	0.051	0.063	0.034	0.025	0.039	-0.011							
0.207	0.121	0.095	0.048	0.082	0.087	0.046	0.007	0.027	-0.003	0.042	-0.010	0.040						
0.072	0.228	0.063	0.023	0.069	0.002	0.031	0.051	0.021	0.005	0.031	0.003	0.022	0.012					
0.108	0.070	0.008	0.079	0.028	0.043	0.017	-0.006	0.016	0.054	0.050	0.030	0.007	0.041	-0.011				
0.020	0.284	0.073	0.042	0.082	0.031	0.061	0.017	0.065	0.007	-0.006	0.003	0.017	-0.003	0.003	0.002			
0.075	0.574	0.298	0.052	0.222	-0.012	0.098	0.104	0.072	0.034	-0.021	-0.023	0.024	-0.018	0.041	-0.019	0.077		
0.027	0.273	0.108	0.003	0.071	0.002	0.056	0.004	0.002	0.005	0.009	-0.007	0.019	0.005	0.005	-0.006	0.003	0.093	
0.140	0.086	0.049	0.060	0.034	0.084	0.055	0.028	0.013	0.034	0.032	0.004	0.017	0.022	0.004	0.026	0.008	-0.009	-0.026

Panel (b)

Figure 5-1: Dependence structure and Kendall tau matrices of the gold mining portfolio. Panel (a) displays the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the portfolio. Panel (b) displays the c-vine Kendall tau correlation matrix of the portfolio based on the full sample period. Each of the diagonal matrices consists of 192 components. The numbers in the diagonal dependence structure matrices represent the bivariate copulas listed and numbered in Table 5-1.

Counting (gold portfolio):

According to Figure 5-1 and Table 5-2 the bivariate copulas more frequently selected by the c-vine, d-vine and r-vine models to measure the dependence from the pairs of gold stocks' joint distributions are: the Frank 54, 46 and 54 times for the c-vine, d-vine and r-vine models, respectively; the Joe-Frank rotated 180 degrees 26, 28 and 19 times; the Student-t 20, 23 and 21 times; the Gaussian 17,17 and 15 times; the Gumbel 180 degrees rotated 16, 16 and 15 times; the Joe-Frank 15, 16 and 20 times; the Gumbel 15, 14 and 11 times; the Clayton 6, 8 and 11 times; the Joe 180 degrees rotated 1, 8 and 8 times and; the Clayton 90 degrees rotated 4, 5 and 0 times. Table 5-2 summarizes the counting, recording, classification and grouping stages for all financial period scenarios of the gold mining portfolio.

Recording, classification and grouping (gold portfolio):

Table 5-2: C-vine, d-vine and r-vine models' bivariate copula selection for the gold mining portfolio

Bivariate Copula	**Full sample**			**Pre-GFC**			**GFC**			**Post-GFC**		
Vine model	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**
						Negative Tail						
Clayton	6	8	11	12	18	19	9	11	12	15	12	18
Gumbel180	16	16	15	22	14	14	14	15	12	9	12	11
Studen-t	20	23	21	14	14	17	16	19	21	19	17	19
Joe 180	1	8	8	15	15	10	3	7	6	0	0	8
Joe-Frank 180	26	28	19	0	0	8	8	8	11	0	0	6
Clayton 270	0	0	0	5	8	0	0	0	0	5	7	0
						Centre						
Frank	54	46	54	48	49	51	85	69	72	58	59	53
Gaussian	17	17	15	27	25	22	17	21	18	30	26	28
						Positive Tail						
Gumbel	15	14	11	13	4	10	0	0	3	9	11	9
Clayton 180	0	0	6	11	18	14	8	6	13	10	11	9
Clayton 90	4	5	0	4	4	0	0	0	0	7	8	0
Studen-t	20	23	21	14	14	17	16	19	21	19	17	19
Joe	0	0	3	0	0	3	0	0	5	0	0	6
Joe-Frank	15	16	20	7	3	2	7	8	4	0	0	4

Notes: the top row of the table displays the four financial period scenarios under consideration and the type of pair vine copulas fitted. The first column lists the bivariate copulas most frequently selected by the vine copula models to measure the dependence between the pairs of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The Student-t copula has been grouped with the copulas for positive and negative tail dependence because it measures the dependence in both tails symmetrically. The letters *C*, *D* and *R* stand for canonical, drawable and regular. The dependence structure located in the centre, negative tail and positive tail of the portfolio has been dissected, organized, counted, classified and grouped.

Aggregate dependence reading (gold portfolio):

In the modelling of the gold mining portfolio, the c-vine, d-vine and r-vine models select the Frank and Gaussian bivariate copulas the most under each of the four financial period scenarios to model the dependence of the gold stocks. This implies that most of the dependence in the gold mining portfolio is concentrated in the centre of the joint distributions. This information in turn indicates that the gold stocks have high dependence risk when the financial stock markets are stable and low dependence risk when they lack investors' confidence. This specific type of dependence risk feature is found to be coherent with the price behaviour of gold during the 2008-2009 global financial crisis. Gold stocks during the GFC and part of the post-GFC period scenarios displayed an exceptionally strong negative correlation with financial stock market confidence. They reached historical levels and were perceived by investors as a "relatively secure defensive investment and storage of wealth" (Collins, 2013; Andrew, 2012; Bingham, 2012). The high concentration of dependence the mining portfolio has in the centre also implies that its return values are liable to change more frequently when the stock markets are tranquil and less frequently when they are unstable. Gold stocks could therefore be used to hedge an investment position in other mining and energy assets that have high dependence risk during financial crisis periods (Baur & McDermott, 2010; Baur & Lucey, 2010).

The Frank copula is observed to have its largest presence in the GFC, indicating that it is the most suitable copula to capture the nonlinear and linear dependence in the centre of the joint distribution. The Gaussian bivariate copula has its largest presence in the post-GFC and pre-GFC period scenarios, suggesting that most of the dependence relationships during the GFC period are of nonlinear nature, while those during the pre-GFC and post-GFC are mainly of linear type. In general, the level of complexity in the gold stocks' interaction appears to decrease as the financial stock market confidence increases. The noticeable decrease of the copulas for the modelling of asymmetric dependence in the negative tail confirms the immunity of gold to financial crisis periods' effects. With regard to model selection, the r-vine model is observed to most frequently select the Frank copula under most of the period scenarios considered. Consequently, the r-vine is discerned to be the model that best captures the multivariate dependence structure of the gold mining portfolio.

The significance testing of the gold mining portfolio's relative comparison of dependence concentration displayed in Table 5-3 indicates that its overall dependence in the centre is at the 95% confidence level significantly larger than those of the iron ore-nickel, oil-gas and retail, and neither significantly larger nor significantly smaller than those of the coal-uranium, mix-metals and manufacturing. In the negative tail it has significantly smaller dependence concentration than the iron ore-nickel coal-uranium, oil-gas and mix-metals portfolios. The gold stocks are therefore significantly less dependence risky than the iron ore and nickel stocks during crisis periods and, as a consequence, could be used to hedge and diversify an investment position with high concentration in the iron ore and nickel sectors.

The asymmetric dependence concentration in the negative tail of the gold mining portfolio is significantly smaller than those of the iron ore-nickel, coal-uranium, oil-gas and mix-metals. This information is an indication of the gold stocks' high propensity to yield positively skewed returns in times of financial turbulence and negatively skewed returns when the stock markets are tranquil. These findings are consistent with the behaviour of gold prices during 2008-2009 global financial crisis, with gold price increases being followed by subsequent price increases. On the other hand, once gold prices reached their peak in the post-GFC period (e.g. around the third quarter of 2011), a negatively skewed behaviour is observed to dominate them (Baur & McDermott, 2010; Baur & Lucey, 2010). The comparison of the gold mining portfolio's symmetric dependence concentration indicates that its dependence concentration is significantly larger than those of the iron ore-nickel, coal-uranium and mix-metals, and significantly smaller than those of the retail and manufacturing benchmark portfolios. Figure 5-2 depicts the significance testing of symmetric dependence concentration.

Table 5-3: Significance testing of the gold mining portfolio's relative comparison of dependence

Significance testing of dependence	Iron ore-nickel	Coal-uranium	Oil-gas	Mix-metals	Retail	Manufacturing
			Overall dependence (centre)			
Frank T-test	6.44	1.07	3.40	0.84	3.02	0.25
Statistical significance	Sig. larger	Neither	Sig. larger	Neither	Sig. larger	Neither
			Overall dependence (negative tail)			
Clayton T-test	-4.59	-4.33	-2.43	-2.18	-0.84	-1.08
Gumbel 180 T-test	-4.16	-0.11	-2.49	1.73	2.87	3.59
Joe 180 T-test	-4.26	-2.28	-0.69	-1.65	2.00	-0.60
Joe-Frank 180 T-test	-0.78	-0.93	-1.42	-2.08	1.35	1.04
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller	Neither	Neither
			Overall dependence (positive tail)			
Gumbel T-test	3.55	2.15	1.20	2.39	-0.05	1.07
Clayton 180 T-test	0.80	1.16	0.52	-0.84	-3.14	-2.36
Joe T-test	-0.81	-0.17	-4.40	-1.19	-4.85	-3.00
Joe-Frank T-test	3.16	1.52	3.54	-1.44	1.86	1.46
Statistical significance	Sig. larger	Neither	Neither	Neither	Sig. smaller	Sig. smaller
			Symmetric dependence (tails)			
Student-t T-test	-0.11	5.26	1.11	6.54	-2.56	0.10
Statistical significance	Neither	Sig. larger	Neither	Sig. larger	Sig. smaller	Neither
			Asymmetric dependence (negative tail)			
Clayton T-test	-4.59	-4.33	-2.43	-2.18	-0.84	-1.08
Gumbel 180 T-test	-4.16	-0.11	-2.49	1.73	2.87	3.59
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller	Sig. larger	Sig. larger
			Asymmetric dependence (positive tail)			
Gumbel T-test	3.55	2.15	1.20	2.39	-0.05	1.07
Clayton 180 T-test	0.80	1.16	0.52	-0.84	-3.14	-2.36
Statistical significance	Sig. larger	Sig. larger	Neither	Sig. larger	Sig. smaller	Sig. smaller
			Critical value= $t_{(0.05,22)}=\pm 2.07$			

Notes: The table displays the significance testing of the gold mining portfolio's relative comparison of dependence concentration. The top row displays the names of the portfolios against which the gold mining portfolio is compared with. The first column from left to right shows the copulas to which the t-test is implemented and the statistical significance category. The rest of the columns display the resulting t-test values, the type of dependence being tested and its location, and the significance testing results. The bottom row states the critical value used to determine the existence or not existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine statistical significance, the t-value of at least one copula is required to be larger or smaller than the critical value.

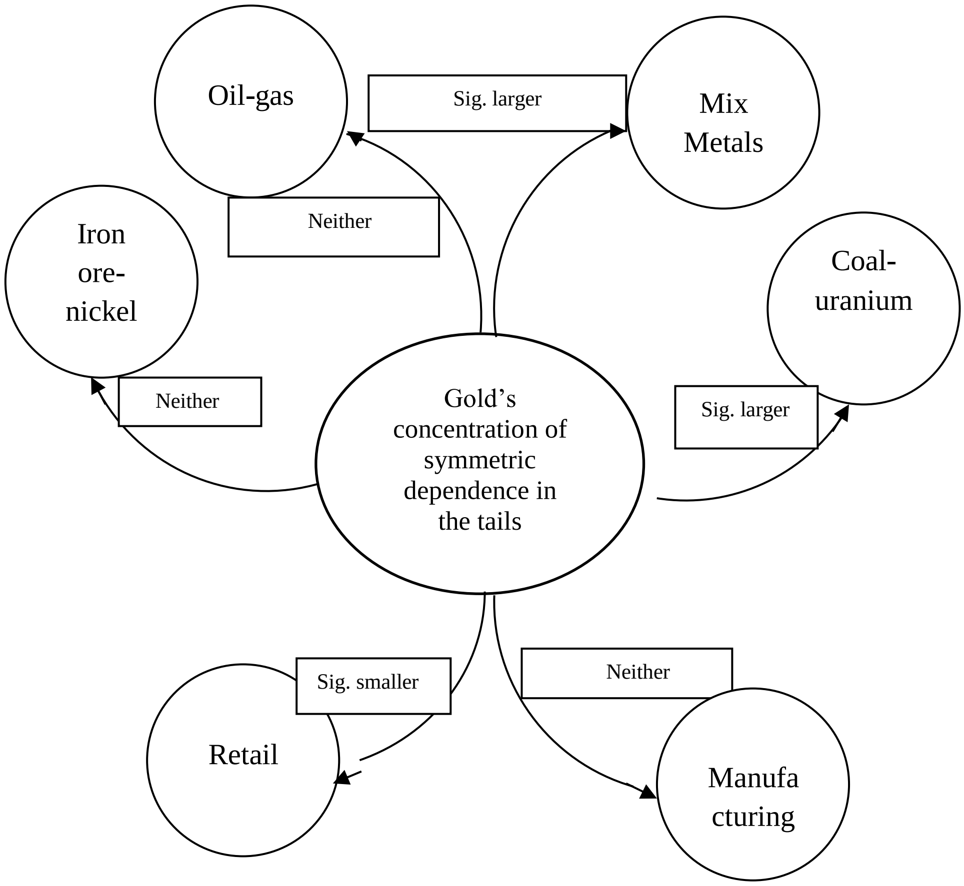

Figure 5-2: Gold mining portfolio's symmetric dependence concentration in the tails. The estimate of symmetric dependence is based on an average of all four period scenarios.

5.3.2 Iron ore-nickel portfolio

The diagonal matrices of the iron ore-nickel mining portfolio displayed in Panel (a) of Figure 5-3 differ from those of the gold mining portfolio (see Panel (a) from Figure 5-1) in the specific type of numbers they contain. Those from Figure 5-3 have the bivariate copula families 3 and 14 in more occasions. These numbers represent the Clayton and 180 rotated Gumbel copulas that model greater concentration of dependence in the negative tail. In the diagonal dependence structure matrices of the iron ore mining portfolio the bivariate copula family number 5 appears more frequently than any other copula. That number represents the Frank copula, which is designed to capture greater concentration of dependence in the centre of the joint distributions.

14																		
7	2																	
17	20	5																
17	20	2	20															
3	20	5	16	20														
20	5	14	14	20	1													
20	20	20	3	14	20	5												
2	5	5	10	20	13	14	14											
1	3	3	3	10	2	20	20	2										
20	14	14	16	2	14	2	20	2	2									
3	14	13	16	14	16	14	20	2	2	2								
14	2	10	2	2	2	1	3	5	1	4	36							
20	16	14	16	13	16	3	20	4	20	4	13	4						
2	2	2	2	5	1	13	5	2	33	6	5	10	6					
2	20	20	20	3	3	3	5	20	20	1	33	5	6	14				
7	2	2	14	5	5	16	5	4	6	13	16	10	33	14	14			
3	20	5	3	20	2	5	2	3	2	2	16	23	2	5	23	4		
3	2	10	4	16	2	20	5	6	3	1	3	5	13	5	24	6	2	
20	20	20	2	16	5	14	13	2	20	3	1	2	10	2	13	16	13	26

14																		
1	7																	
7	1	2																
14	20	4	5															
14	5	5	3	2														
20	20	5	5	20	3													
20	14	5	5	20	3	2												
16	20	3	2	1	2	2	4											
14	1	5	16	6	5	4	20	5										
2	16	20	20	20	13	1	16	20	2									
3	3	3	17	5	10	2	2	2	5	20								
20	16	1	2	20	20	2	2	1	13	2	13							
14	17	5	10	13	10	20	2	2	2	2	3	10						
14	14	1	16	2	13	5	5	13	4	6	1	20	13					
16	14	5	1	2	3	16	5	10	20	1	5	2	2	23				
20	5	20	1	5	3	2	16	20	1	3	5	2	16	20	5			
20	16	14	20	3	20	5	3	1	2	13	2	2	2	3	5	2		
20	14	14	20	2	5	5	20	10	5	20	4	2	5	2	2	5	2	
14	20	20	5	3	20	5	5	5	20	14	3	5	3	14	13	16	13	1

Panel (a)

0.272																		
0.303	0.107																	
0.339	0.148	0.127																
0.286	0.214	0.092	0.084															
0.111	0.123	0.074	0.041	0.083														
0.172	0.105	0.05	0.057	0.063	0.036													
0.153	0.081	0.054	0.033	0.076	0.1	0.039												
0.344	0.107	0.094	0.065	0.065	0.033	0.033	0.03											
0.137	0.057	0.047	0.051	0.068	0.101	0.046	0.108	0.018										
0.164	0.074	0.051	0.062	0.043	0.09	0.023	0.054	0.028	0.054									
0.086	0.053	0.036	0.055	0.024	0.058	0.048	0.065	-0.003	0.048	0.028								
0.336	0.164	0.111	0.076	0.075	0.026	0.048	0.014	0.065	0.01	0.018	-0.002							
0.136	0.063	0.063	0.049	0.034	0.098	0.051	0.117	0.023	0.133	0.029	0.031	0.01						
0.36	0.137	0.05	0.091	0.088	0.075	0.021	0.051	0.045	-0.011	0.007	-0.011	0.072	0.009					
0.265	0.083	0.065	0.074	0.041	0.026	0.032	0.015	0.077	0.047	0.008	-0.024	0.021	0.009	0.017				
0.305	0.113	0.162	0.097	0.04	0.009	0.011	0.046	0.027	0.029	0.017	0.026	0.036	-0.023	0.024	0.009			
0.074	0.064	0.042	0.025	0.092	0.078	0.033	0.043	0.022	0.051	0.025	0.039	-0.006	0.024	-0.01	-0.012	0.007		
0.09	0.072	0.056	0.03	0.03	0.061	0.076	0.018	0.015	0.027	0.041	0.048	0.031	0.037	0.012	-0.014	0.005	0.003	
0.169	0.092	0.111	0.073	0.039	0.059	0.049	0.024	0.002	0.053	0.067	0.055	0.019	0.03	-0.002	0.014	0.013	0.011	-0.007

Panel (b)

Figure 5-3: Dependence structure and Kendall tau matrices of the iron ore-nickel mining portfolio. Panel (a) displays the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the portfolio. Panel (b) displays the c-vine Kendall tau correlation matrix of the portfolio based on the full sample period. Each diagonal matrix consists of 192 components. The numbers in the diagonal dependence structure matrices of Panel (a) represent the bivariate copulas listed and numbered in Table 5-1.

Counting (Iron ore-nickel portfolio):

According to Figure 5-3 and Table 5-6 for the iron ore-nickel mining portfolio, the bivariate copulas more frequently selected by the c-vine, d-vine and r-vine models to measure the dependence in the joint distributions are: the Frank 22, 33 and 17 times for the c-vine, d-vine and r-vine models, respectively; the Joe-Frank 180 degrees rotated 32, 34 and 31 times; the student-t 36, 35 and 31times; the Gaussian 6, 13 and 11 times; the Gumbel rotated 180 degrees 16, 15 and 23 times; the Clayton 18, 17 and 23 times; Joe 180 degrees rotated 13, 12 and 9 times and; the Clayton 180 degrees rotated 0,0 and 13 times. Table 5-4 summarizes the counting, recording, classification and grouping stages of the bivariate copula counting technique.

Recording, classification and grouping (Iron ore-nickel portfolio):

Table 5-4: C-vine, d-vine and r-vine models' bivariate copula selection for the iron ore-nickel portfolio

Bivariate Copula	Full sample			Pre-GFC			GFC			Post-GFC		
Vine model	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**
Negative Tail												
Clayton	18	17	23	24	22	20	30	22	30	19	14	14
Gumbel 180	16	15	23	23	17	20	27	28	30	22	20	14
Studen-t	36	35	31	10	11	12	8	11	10	20	16	24
Joe 180	13	12	9	22	22	23	13	10	14	19	9	19
Joe-Frank 180	32	34	31	0	0	6	12	8	3	7	13	9
Clayton 270	0	0	3	0	0	6	0	0	5	0	0	4
Centre												
Frank	22	33	17	32	38	37	34	36	30	37	47	37
Gaussian	6	13	11	15	16	16	16	25	23	14	25	16
Positive Tail												
Gumbel	0	0	9	0	0	6	0	0	6	0	0	5
Clayton 180	0	0	13	17	13	16	0	0	11	0	0	12
Clayton 90	0	0	2	0	0	5	0	0	4	0	0	7
Studen-t	36	35	31	10	11	12	8	11	10	20	16	24
Joe	0	0	5	0	0	11	0	0	8	0	0	5
Joe-Frank	0	0	8	0	0	0	0	0	3	0	0	2

Notes: the top row of the table displays the four financial period scenarios under consideration and the type of pair vine copulas fitted. The first column lists the bivariate copulas most frequently selected by the vine copula models to measure the dependence between the pairs of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The Student-t copula has been grouped with the copulas for positive and negative tail dependence because it measures the dependence in both tails symmetrically. The letters *C*, *D* and *R* stand for canonical, drawable and regular. The dependence structure located in the centre, negative tail and positive tail of the portfolio has been dissected, organized, counted, classified and grouped.

Aggregate dependence reading (Iron ore-nickel portfolio):

In the iron ore-nickel mining portfolio, the c-vine, d-vine and r-vine models also select the Frank copula the most under each of the four financial period scenarios considered to model the dependence from the joint distributions. Nevertheless, despite the Frank copula being the most predominant in each of the period scenarios, most of the dependence in the iron ore-nickel mining portfolio is located in the negative tail. This is verified by aggregating the Clayton, 180 Gumbel and 180 Joe copulas. The dependence concentration in the negative tail of the portfolio is clearly larger than that in the centre, implying that the portfolio has high dependence risk in non-tranquil stock market conditions and low dependence risk when the financial stock markets behave smoothly.

A look into the 2008-2009 GFC shows that the price of the iron ore and nickel commodities did experience a severe decline during the crisis period. Iron ore prices specifically fell 48 per cent (from US$138 per tonne to US$71 per tonne) from Oct-2008 to Dec-2009 (Bingham & Perkins, 2012). Nickel prices relative to iron ore prices undergo a more drastic decline from May 2007 (e.g. at US$51,783 per metric tonne) to the second half of 2008 (e.g. US$10,000 per metric ton) (Bingham, 2012). Nickel prices, moreover, appear to react more rapidly to changes in financial stock market confidence. For instance, while iron ore prices were still on the rise from the middle of 2006 to the end of 2007, nickel prices were already in decline starting from the end of 2006 to the fourth quarter of 2008. A possible reason for this is that nickel prices do not seem to have the same strength of positive association the iron ore prices have with steel demand. Steel-based products are perhaps more indispensable than nickel-based products during crisis periods (Bingham, 2012).

The high concentration of dependence the iron ore-nickel mining portfolio has in the negative tail also makes its returns values liable to change less frequently in tranquil stock market conditions, while having a high probability of being extreme in those market conditions. As compared to the gold mining portfolio, the iron ore-nickel mining portfolio is more dependence risky in crisis periods due to the high concentration of dependence it has in the negative tail. The reason for this is that greater losses can be incurred in times of financial turbulence, relative to tranquil periods. The decrease of the Frank copula and the increase of the Clayton and 180 Gumbel copulas during the GFC period scenario represents a shift of the dependence structure from the pre-GFC to the GFC (refer to Table 5-4). This shift of dependence concentration indicates that the iron ore and nickel stocks tend to correlate more strongly when the financial stock markets lack confidence, and are markedly riskier than the gold stocks in those market conditions (Connolly & Orsmond, 2011).

Table 5-5: Significance testing of the iron ore-nickel portfolio's relative comparison of dependence

Significance testing of dependence	Gold	Coal-uranium	Oil-gas	Mix-metals	Retail	Manufacturing
			Overall dependence (centre)			
Frank T-test	-6.44	-6.08	-3.68	-7.12	-5.42	-7.26
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller
			Overall dependence (negative tail)			
Clayton T-test	4.59	0.66	0.88	2.05	2.57	3.48
Gumbel 180 T-test	4.16	3.59	1.83	4.86	6.52	6.89
Joe 180 T-test	4.26	1.59	2.71	3.19	7.74	3.28
Joe-Frank 180 T-test	0.78	0.11	-0.39	-1.18	2.07	1.78
Statistical significance	Sig. larger	Neither	Neither	Sig. larger	Sig. larger	Sig. larger
			Overall dependence (positive tail)			
Gumbel T-test	-3.55	-2.19	-4.08	-1.44	-5.74	-3.63
Clayton 180 T-test	-0.80	0.32	-0.32	-1.57	-3.39	-2.74
Joe T-test	0.81	0.63	-2.21	0.00	-2.54	-1.29
Joe-Frank T-test	-3.16	-2.37	0.57	-4.16	-3.08	-2.61
Statistical significance	Sig. smaller	Neither	Sig. smaller	Neither	Sig. smaller	Sig. smaller
			Symmetric dependence (tails)			
Student-t T-test	0.11	2.39	0.64	2.50	-1.84	0.16
Statistical significance	Neither	Sig. larger	Neither	Sig. larger	Neither	Neither
			Asymmetric dependence (negative tail)			
Clayton T-test	4.59	0.66	0.88	2.05	2.57	3.48
Gumbel 180 T-test	4.16	3.59	1.83	4.86	6.52	6.89
Statistical significance	Sig. larger	Sig. larger	Neither	Sig. larger	Sig. larger	Sig. larger
			Asymmetric dependence (positive tail)			
Gumbel T-test	-3.55	-2.19	-4.08	-1.44	-5.74	-3.63
Clayton 180 T-test	-0.80	0.32	-0.32	-1.57	-3.39	-2.74
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller	Neither	Sig. smaller	Sig. smaller
			Critical value= $t_{(0.05,22)}$=±2.07			

Notes: The table displays the significance testing of the iron ore-nickel mining portfolio's relative comparison of dependence concentration. The top row displays the names of the portfolios against which the iron ore-nickel mining portfolio is compared with. The first column from left to right shows the copulas to which the t-test is implemented and the statistical significance category. The rest of the columns display the resulting t-test values, the type of dependence being tested and its location, and the significance testing results. The bottom row states the critical value used to determine the existence or not existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine statistical significance the t-value of at least one copula is required to be larger or smaller than the critical value.

Table 5-4 also indicates that from the GFC to the post-GFC period scenarios the dependence structure shifts from the negative tail to the centre, indicating post-GFC increases in financial stock market confidence, greater financial stability in the stock markets and a higher propensity of the iron ore and nickel stocks to yield positively skewed returns. The gold stocks during the post-GFC period scenario contrary to the iron ore and nickel stocks are characterized for having a high propensity to yield negatively skewed returns. Research conducted by Baur and McDermott (2010) and Baur and Lucey (2010) indicates that gold prices did display a negatively skewed behaviour from the middle of the post-GFC onwards. With respect to model selection, the c-vine is observed to select under each of the four financial period scenarios considered the copulas for negative tail dependence modelling more frequently than the r-vine and d-vine do. As a result, the c-vine is acknowledged for best capturing the dependence structure of the iron ore and nickel stocks.

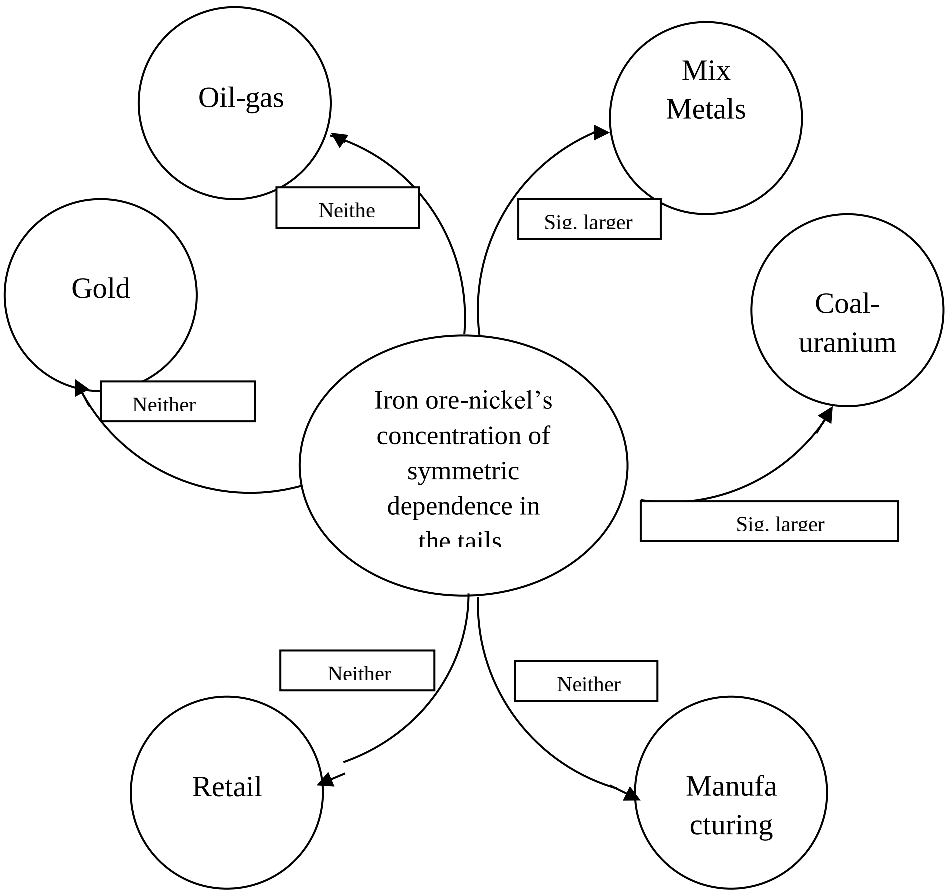

Figure 5-4: Iron ore-nickel portfolio's symmetric dependence concentration in the tails. The estimate of symmetric dependence is based on an average of all four period scenarios.

The significance testing of the iron ore-nickel mining portfolio's relative comparison of dependence concentration displayed in Table 5-5 indicates that its overall dependence concentration in the negative tail is at the 95% confidence level significantly larger than those of the gold, mix-metals, retail and manufacturing. This information confirms the higher dependence riskiness of the iron ore and nickel stocks relative to the gold, retail and manufacturing stocks in non-tranquil periods. With respect to the oil-gas energy portfolio, it has neither significantly larger nor significantly smaller dependence concentration in the negative tail. The same applies to its asymmetric dependence in the negative tail. However, in the centre and positive tail it has significantly smaller dependence concentration relative to the oil-gas energy portfolio. As a consequence, it is more dependence risky than the oil-gas energy portfolio. Mining portfolio investors could therefore use retail and manufacturing stocks to diversify an investment position heavily concentrated in the iron ore and nickel sectors, in tranquil stock market conditions.

The iron ore-nickel mining portfolio's asymmetric dependence concentration in the negative tail is, with exception of that of the oil-gas, significantly larger than that of any other portfolio. This information confirms the high propensity of the iron ore and nickel stocks to yield negatively skewed returns in times of financial turbulence. A look into the 2008-2009 GFC shows that the price of the iron ore and nickel commodities did behave according to a negatively skewed function. In the period 2008-2009 for instance, iron ore prices decline 48 per cent of their value, from US$138 per tonne to US$71 per tonne (Bingham & Perkins, 2012). The iron ore-nickel mining portfolio's symmetric dependence concentration is significantly larger than those of the mix-metals and coal-uranium, and neither significantly larger nor significantly smaller than those of the rest of the portfolios.

5.3.3 Mix-metals leptokurtic portfolio

The diagonal dependence structure matrices of the mix-metals leptokurtic mining portfolio displayed in Figure 5-5 differ from those of the iron ore-nickel mining portfolio in the number of times the copulas 5 and 3 appear. In the mix-metals leptokurtic mining portfolio specifically the copula number 5 appears in more occasions, indicating that relative to the iron ore-nickel mining portfolio is less dependence risky when the financial stock markets lack confidence. As compared to the gold mining portfolio, the copula number 5 appears in fewer occasions, indicating that is still safer to invest in the gold mining portfolio as opposed to the mix-metals leptokurtic mining portfolio during crisis periods.

5																		
20	10																	
20	10	20																
3	10	16	20															
5	5	2	20	14														
5	5	10	4	10	10													
5	20	5	16	10	5	16												
20	5	20	20	5	26	1	5											
1	10	2	20	20	20	5	1	20										
14	20	20	1	20	5	13	1	2	13									
5	10	20	20	2	20	14	10	13	2	5								
1	10	20	5	20	20	13	5	20	5	1	5							
2	5	16	20	5	16	5	33	3	20	20	13	5						
20	10	20	5	20	20	1	10	5	5	10	20	14	10					
5	10	2	20	2	20	10	10	3	5	4	10	5	5	5				
20	10	2	20	20	20	10	10	24	10	5	10	10	5	20	5			
20	10	16	5	20	16	10	33	20	4	20	5	1	1	13	13	1		
1	10	20	10	2	20	5	10	5	5	3	10	5	33	10	5	5	23	
20	20	14	20	20	2	6	10	1	5	5	5	5	5	5	5	5	3	5

10																		
2	5																	
20	20	10																
20	5	5	1															
20	3	5	3	1														
14	5	1	5	1	5													
20	1	5	20	20	10	16												
20	2	20	3	16	5	3	3											
5	20	1	5	5	20	5	10	10										
5	14	20	2	20	10	16	3	5	33									
20	20	5	20	20	5	24	20	5	10	5								
5	20	20	1	20	34	5	13	2	5	5	5							
2	5	33	5	16	20	13	23	13	14	20	5	5						
20	5	5	5	14	5	5	3	1	20	3	5	5	1					
2	14	10	5	4	5	14	10	1	1	20	5	3	3	1				
10	10	10	20	5	10	10	4	5	23	5	4	1	10	5	20			
10	2	20	14	20	5	20	5	3	5	3	13	20	5	3	20	5		
2	10	2	20	5	20	1	13	10	5	14	1	5	1	10	5	5	5	
10	5	5	5	13	5	14	14	10	14	5	4	3	13	5	10	26	3	16

Panel (a)

0.052																		
0.126	0.073																	
0.237	0.061	0.076																
0.122	0.042	0.112	0.112															
0.125	0.317	0.139	0.107	0.112														
0.049	0.013	0.054	0.005	0.047	0.041													
0.063	0.031	0.022	0.003	0.032	0.056	0.002												
0.217	0.038	0.073	0.092	0.075	-0.021	-0.006	-0.022											
0.122	0.083	0.103	0.090	0.126	0.177	0.062	-0.008	0.051										
0.302	0.057	0.054	0.087	0.068	0.012	0.007	0.012	0.078	0.019									
0.105	0.283	0.205	0.114	0.134	0.430	0.002	0.043	0.017	0.074	0.031								
0.132	0.062	0.115	0.101	0.107	0.187	0.005	0.069	0.040	0.106	0.040	0.121							
0.136	0.055	0.059	0.087	0.024	0.039	-0.009	-0.003	0.024	0.072	0.055	0.026	0.046						
0.145	0.125	0.065	0.108	0.074	0.109	0.011	0.038	0.059	0.113	0.083	0.084	0.007	0.073					
0.119	0.117	0.174	0.098	0.127	0.336	0.045	0.069	0.013	0.130	0.010	0.221	0.089	-0.017	0.06				
0.153	0.092	0.106	0.096	0.078	0.151	0.042	0.045	-0.008	0.107	0.059	0.108	0.056	0.078	0.069	0.101			
0.160	0.079	0.077	0.060	0.057	0.024	0.056	-0.009	0.048	0.036	0.060	0.052	0.022	0.063	0.039	0.018	0.058		
0.088	0.117	0.162	0.069	0.063	0.252	0.038	0.064	0.057	0.115	0.021	0.218	0.109	-0.029	0.052	0.199	0.066	-0.009	
0.174	0.120	0.134	0.111	0.135	0.185	0.001	0.062	0.051	0.089	0.075	0.118	0.049	0.022	0.058	0.094	0.037	0.024	0.095

Panel (b)

Figure 5-5: Dependence structure and Kendall tau matrices of the mix-metals leptokurtic mining portfolio. Panel (a) displays the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the portfolio. Panel (b) displays the c-vine Kendall tau correlation matrix of the portfolio based on the full sample period. Each diagonal matrix consists of 192 components. The numbers in the diagonal dependence structure matrices of Panel (a) represent the bivariate copulas listed and numbered in Table 5-1.

Counting (mix-metals leptokurtic portfolio):

According to Figure 5-5 and Table 5-6 for the mix-metals leptokurtic portfolio, the bivariate copulas more frequently selected by the c-vine, d-vine and r-vine models to measure the dependence in the joint distributions are: the Frank 51, 59 and 73 times for c-vine, d-vine and r-vine models respectively; the Joe-Frank 180 degrees rotated 48, 35 and 30 times; the Joe-Frank 33, 22 and 17 times; the Gaussian 13, 16 and 10 times; the Clayton 5, 15 and 13 times; the Student-t 11, 9 and 15 times; the Gumbel 180 degrees rotated 5, 11 and 7 times; the Clayton 180 degrees rotated 7,7 and 9 times each; the Joe 180 degrees rotated 7, 5 and 2 times and; the Gumbel 3, 4 and 6 times. Table 5-6 summarizes the counting, recording, classification and grouping stages of the bivariate copula counting technique.

Recording, classification and grouping (mix-metals leptokurtic portfolio):

Table 5-6: C-vine, d-vine and r-vine models' bivariate copula selection for the mix-metals leptokurtic portfolio

Bivariate Copula	**Full sample**			**Pre-GFC**			**GFC**			**Post-GFC**		
Vine model	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**
Negative Tail												
Clayton	5	15	13	24	18	19	13	17	13	19	19	26
Gumbel 180	5	11	7	8	15	8	20	18	18	7	12	5
Studen-t	11	9	15	6	11	11	14	13	15	11	8	11
Joe 180	7	5	2	12	11	12	12	10	15	14	7	9
Joe-Frank 180	48	35	30	8	13	9	15	9	10	17	16	14
Clayton 270	0	0	2	0	0	6	6	5	10	8	6	14
Centre												
Frank	51	59	73	67	49	68	54	58	51	64	66	61
Gaussian	13	16	10	20	20	19	15	19	21	20	19	20
Positive Tail												
Gumbel	3	4	6	10	8	5	0	0	7	0	0	6
Clayton 180	7	7	9	10	14	14	10	10	5	11	13	13
Clayton 90	0	0	3	0	0	6	5	9	4	0	0	7
Studen-t	11	9	15	6	11	11	14	13	15	11	8	11
Joe	0	0	3	4	6	3	0	0	2	5	3	3
Joe-Frank	33	22	17	6	7	5	12	8	13	5	6	4

Notes: the top row of the table displays the four financial period scenarios under consideration and the type of pair vine copulas fitted. The first column lists the bivariate copulas most frequently selected by the vine copula models to measure the dependence between the pairs of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The Student-t copula has been grouped with the copulas for positive and negative tail dependence because it measures the dependence in both tails symmetrically. The letters *C*, *D* and *R* stand for canonical, drawable and regular. The dependence structure located in the centre, negative tail and positive tail of the portfolio has been dissected, organized, counted, classified and grouped.

Aggregate dependence reading (mix-metals leptokurtic portfolio):

In the mix-metals leptokurtic mining portfolio, the c-vine, d-vine and r-vine models also select the Frank copula the most under each of the four financial period scenarios considered to capture the dependence from the pair of stocks' joint distributions. As a result, most of the dependence in the portfolio is concentrated in the centre. This implies that the stocks in the mix-metals leptokurtic mining portfolio have high dependence risk in tranquil stock market conditions and low dependence risk in non-tranquil stock market conditions. Another implication stemming from the large concentration of dependence the mix-metals leptokurtic mining portfolio has in the centre is that its return values are liable to change more frequently in times of financial turbulence and have a low probability of being extreme in those market conditions. Given the above-mentioned dependence risk profile of the mix-metals leptokurtic mining portfolio, some of its stocks could be used to hedge, diversify and minimize the risk of an investment position in the iron ore and nickel sectors during crisis periods. As compared to the gold mining portfolio, the mix-metals leptokurtic mining portfolio is less preferable in terms of dependence risk during crisis periods.

The mix-metals leptokurtic mining portfolio's dependence structure located in the centre and positive tail of the joint distributions changes significantly in size from the pre-GFC to the GFC period scenarios. Specifically, the number of copulas for the modelling of asymmetric dependence in the negative tail increases significantly, indicating that the dependence during the GFC period is of asymmetric type. It follows that stocks with higher concentration of dependence in the negative tail tend to correlate more strongly during the GFC period scenario. The c-vine copula model, relative to the r-vine and d-vine, is observed to select the Frank copula more frequently under each of the four financial period scenarios considered. As a consequence, the c-vine is the model that best captures the multivariate dependence structure of the mix-metals leptokurtic mining portfolio.

The significance testing of the mix-metals leptokurtic mining portfolio's relative comparison of dependence concentration displayed in Table 5-7 indicates that its overall dependence concentration in the centre is at the 95% confidence level significantly larger than those of the iron ore-nickel, oil-gas and retail. In the negative tail it has it significantly smaller than that of the iron ore-nickel, and significantly larger than those of the gold mining and retail benchmark portfolios. The portfolios' asymmetric dependence in the negative tail is significantly larger than those of the gold mining and retail benchmark portfolios.

Table 5-7: Significance testing of the mix-metals portfolio's relative comparison of dependence

Significance testing of dependence	Gold	Iron ore-nickel	Coal-uranium	Oil-gas	Retail	Manufacturing
			Overall dependence (centre)			
Frank T-test	-0.84	7.12	0.35	3.23	2.84	-0.69
Statistical significance	Neither	Sig. larger	Neither	Sig. larger	Sig. larger	Neither
			Overall dependence (negative tail)			
Clayton T-test	2.18	-2.05	-1.57	-1.48	0.44	1.18
Gumbel 180 T-test	-1.73	-4.86	-1.62	-3.52	0.00	0.79
Joe 180 T-test	1.65	-3.19	-1.07	0.50	5.27	0.79
Joe-Frank 180 T-test	2.08	1.18	1.61	1.02	3.18	3.30
Statistical significance	Sig. larger	Sig. smaller	Neither	Neither	Sig. larger	Neither
			Overall dependence (positive tail)			
Gumbel T-test	-2.39	1.44	-0.55	-2.15	-3.81	-1.96
Clayton 180 T-test	0.84	1.57	1.98	1.37	-3.12	-2.21
Joe T-test	1.19	0.00	0.86	-3.34	-3.82	-1.91
Joe-Frank T-test	1.44	4.16	2.85	4.45	3.17	2.82
Statistical significance	Neither	Neither	Neither	Sig. smaller	Sig. smaller	Neither
			Symmetric dependence (tails)			
Student-t T-test	-6.54	-2.50	-0.06	-3.63	-4.83	-4.29
Statistical significance	Sig. smaller	Sig. smaller	Neither	Sig. smaller	Sig. smaller	Sig. smaller
			Asymmetric dependence (negative tail)			
Clayton T-test	2.18	-2.05	-1.57	-1.48	0.44	1.18
Gumbel 180 T-test	-1.73	-4.86	-1.62	-3.52	0.00	0.79
Statistical significance	Sig. larger	Sig. smaller	Neither	Sig. smaller	Sig. larger	Neither
			Asymmetric dependence (positive tail)			
Gumbel T-test	-2.39	1.44	-0.55	-2.15	-3.81	-1.96
Clayton 180 T-test	0.84	1.57	1.98	1.37	-3.12	-2.21
Statistical significance	Sig. smaller	Neither	Neither	Sig. smaller	Sig. smaller	Sig. smaller
			Critical value= $t_{(0.05,22)}=\pm 2.07$			

Notes: The table displays the significance testing of the mix-metals leptokurtic mining portfolio's relative comparison of dependence concentration. The top row displays the names of the portfolios against which the mix-metals leptokurtic mining portfolio is compared with. The first column from left to right shows the copulas to which the t-test is implemented and the statistical significance category. The rest of the columns display the resulting t-test values, the type of dependence being tested and its location, and the significance testing results. The bottom row shows the critical value used to determine the existence or not existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine statistical significance, the t-value of at least one copula is required to be larger or smaller than the critical value.

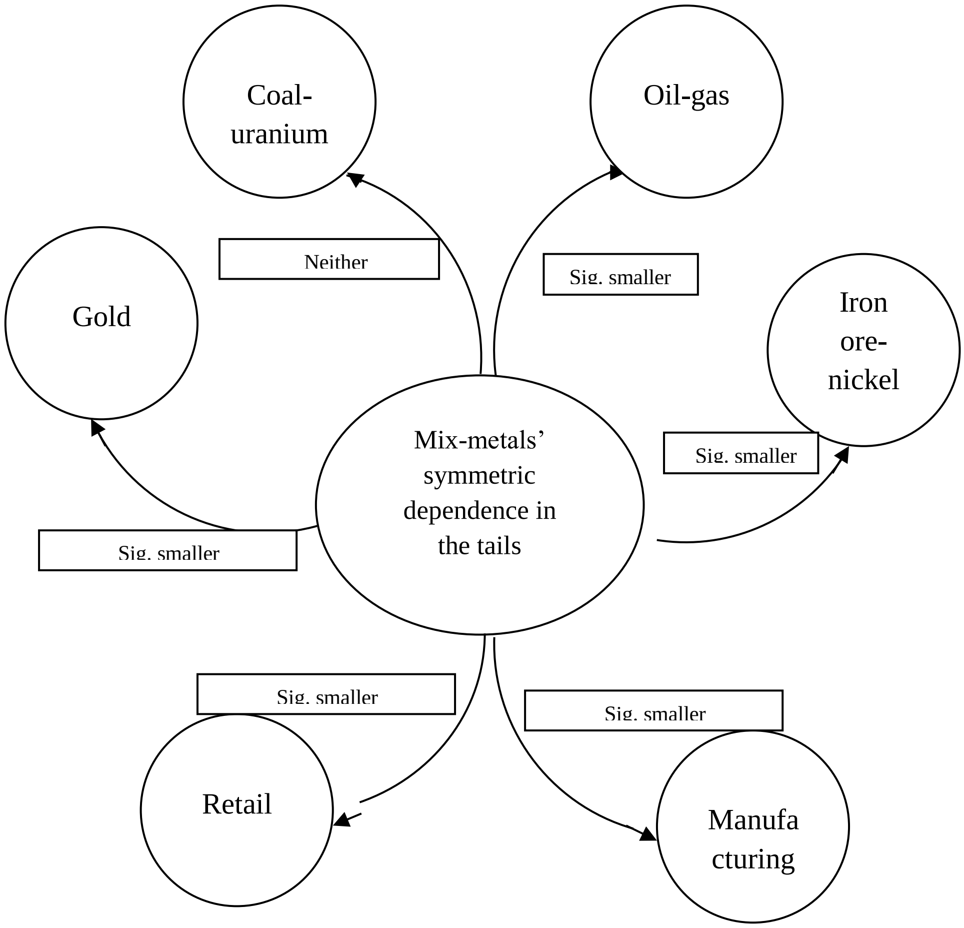

Figure 5-6: Mix-metals leptokurtic mining portfolio's symmetric dependence concentration in the tails. The estimate of symmetric dependence is based on an average of all four period scenarios.

5.4 Discussion of results

The identification of the gold mining portfolio as having low dependence risk in times of financial turbulence is consistent with the results from previous studies looking at the price behaviour of gold in those market conditions. For instance, Morales and Andreosso-O'Callaghan (2011) acknowledge gold markets for not being affected by crisis periods; Dee et al. (2013) point out the low risk aspect of gold in the long run when investing in stocks; Baur and Lucey (2010) recognize the short-period safe haven features of gold in extreme market conditions; Courdert and Raymond (2010) in their modelling of stocks from the US and the G7 also acknowledge the safe heaven characteristics of gold during bear markets; and Faff and Chan (1998) identify a relationship between the performance of Australian gold stocks and gold prices.

The suitability of the r-vines to best account for the multivariate dependence structure of the gold stocks could imply, along with Dissmann (2010) and Dissmann et al. (2013), that their multivariate dependence is more complex relative to the dependence structure of the iron ore-nickel and mix-metals leptokurtic mining portfolios. It could also mean that the gold mining portfolio does not have a stock that has high correlation values with the rest of the stocks in the portfolio. The higher dependence risk the mix-metals leptokurtic mining portfolio has during crisis periods relative to the gold mining portfolio is most likely due to the wide variety of stocks it consists of. Specifically, some of its stocks belong to the iron ore and nickel sectors, identified in this chapter as significantly more dependence risky than the gold stocks. With respect to the adequacy of the c-vines to best account for the multivariate dependence structure of the iron ore-nickel and mix-metals leptokurtic mining portfolios, the presence of a stock in each of the portfolios heavily influencing the interaction between stocks appears to be the reason why. In this respect, while the c-vine identifies BHP BILLITON (BHPX) as the rootstock of the iron ore-nickel mining portfolio, RIO TINTO (RIOX) is recognized as the rootstock in the mix-metals leptokurtic mining portfolio. Other studies where the c-vines have been found to adequately model the multivariate interaction of financial assets are Czado et al. (2012), Chollete et al. (2009) and Heinen and Valdesogo (2009).

The modelling of gold stocks conducted in this chapter, relative to modelling of gold stocks undertaken by Baur and Lucey (2010), Dee et al. (2013), Courdert and Raymond (2010) and Morales and Andreosso-O'Callaghan (2011) has the distinctive feature of identifying the stocks' symmetric and asymmetric dependence risk characteristics in specific market conditions, as well as their negatively and positively skewed price and return behaviour n both, tranquil and non-tranquil periods. The dependence risk analysis of the iron ore and nickel stocks undertaken in this chapter could be the first that thoroughly examines the stock portfolio's underlying sector's dependence risk behaviour in stressed and non-stressed stock market conditions, within an Australian macro economic context. This chapter's modelling of dependence, as compared to the dependence risk modelling of Low et al. (2013), Allen et al. (2013), Arreola and Powell (2013) and Brechmann et al. (2014) has the comparative advantage of scrutinizing the dependence concentration at various locations in the joint distribution. The difference lies in this thesis' use of the copula counting technique to interpret the portfolios' dependence structure and dependence risk profile.

5.5 Summary

This chapter implemented c-vines, d-vines and r-vines to estimate the dependence structure of the gold, iron ore-nickel and mix-metals leptokurtic mining portfolios. The implementation of the copula counting technique indicated that the gold mining portfolio has most of the dependence concentrated in the centre of the joint distributions, due to the predominance of the Frank copula across period scenarios. This information was interpreted as the gold stocks having low dependence risk in times of financial turbulence and high dependence risk in non-crisis periods. The dependence risk dynamics of gold stocks were confirmed by the price behaviour of gold during the 2008-2009 global financial crisis. The mix-metals portfolio was also found to have most of the dependence concentrated in the centre of the joint distributions, making it less dependence risky than the iron ore-nickel during crisis periods and more dependence risky than the gold mining portfolio in similar market conditions.

The iron ore-nickel mining portfolio, despite the large presence of the Frank copula in each of the four financial period scenarios considered, was found to have most of the dependence concentrated in the negative tail. This dependence risk feature makes it high dependence risky during crisis periods and low dependence risky in tranquil stock market conditions. A look into the 2008-2009 GFC confirmed the dependence risk dynamics of the mix-metals leptokurtic portfolio. The significance testing of the portfolios' relative comparison of dependence concentration indicated that the gold mining portfolio is significantly less dependence risky than the iron ore-nickel and mix-metals in times of financial turbulence characterized by low confidence in the financial stock markets. In similar market conditions the iron ore-nickel mining portfolio is found to be significantly more dependence risky than the gold and mix-metals mining portfolios. The r-vine was found to best account for the multivariate dependence and dependence risk dynamics of the gold mining portfolio, while the c-vine was identified to best capture the dependence structure of the iron ore-nickel and mix-metals leptokurtic mining portfolios.

CHAPTER 6

DEPENDENCE STRUCTURE ESTIMATION: ENERGY PORTFOLIOS

This chapter consists of two sections: introduction and dependence structure estimation

The *introduction* section provides an overview of the coal, uranium, oil and gas commodities that underlie the Australian energy stock portfolios modelled. The *dependence estimation* section deals with the dissection, analysis and interpretation of the energy portfolios' dependence structure and dependence risk profile, using the copula counting technique explained in Chapter 5.

6.1 Introduction

According to the Department of Industry, Geoscience Australia and The Buerau of Resources and Energy Economics in 2014 Australia was the ninth largest producer of energy worldwide, accounting for 2.4 per cent of the world's energy. In the period 2011-2012 it exported roughly 80 per cent of the energy it produced, with coal, uranium and gas accounting for 60, 20 and 13 per cent of the local energy production, respectively. Besides, roughly 64 and 20 per cent of the electricity produced within the country stemmed from the burning of coal and gas, respectively. Around the same time period, Australia occupied the third place in uranium production worldwide, contributing with 11 per cent of total global production (BREE, 2014).[17]

This chapter's objectives are to examine the dependence risk profile of the energy portfolios in specific market conditions; account for the portfolios' dependence structure changes between pairs of period scenarios; and recognize the pair vine copula models that

[17] The acronyms *BREE*, *DI* and DRET used in the present chapter stand for Bureau of Resources and Energy Economics, Department of Industry, and Department of Resources, Energy and Tourism.

best capture the multivariate dependence structure and dependence risk dynamics of the portfolios. The copula counting technique is used for this purpose.

6.2 Dependence structure estimation

6.2.1 Coal-uranium portfolio

The dependence structure matrices of the coal-uranium energy portfolio displayed in Panel (a) of Figure 6-1 differ from those of the iron ore-nickel mining portfolio in the number of times the copula number 5 appears. The Frank copula specifically is more predominant in the dependence structure matrices of the coal-uranium energy portfolio, relative to the mix-metals and gold mining portfolios.

1																		
20	5																	
3	2	5																
14	5	23	3															
5	5	20	5	1														
3	2	20	20	3	5													
20	5	33	1	5	13	13												
20	5	5	20	20	5	5	14											
14	20	14	5	5	5	14	5	5										
20	5	20	5	3	5	5	23	5	5									
14	2	5	5	14	4	1	1	3	2	5								
14	16	1	5	20	1	3	1	5	2	3	13							
2	5	20	5	5	3	3	5	3	5	34	14	5						
5	10	5	10	33	5	5	6	3	36	5	1	33	1					
3	5	5	5	14	5	5	3	5	36	5	34	1	13	10				
20	10	5	5	20	13	5	4	20	1	4	5	1	23	14	3			
3	5	5	20	6	10	5	20	5	13	5	26	33	33	14	5	13		
14	20	5	20	3	3	5	1	5	10	4	1	3	2	1	5	23	3	
3	5	14	5	5	1	1	24	16	33	14	13	33	3	14	16	4	5	1

5																		
2	20																	
20	2	5																
14	2	3	4															
20	16	5	14	3														
5	2	20	5	5	5													
5	20	14	20	36	33	23												
2	3	14	14	2	13	3	13											
20	20	3	20	16	3	36	3	2										
20	5	20	5	4	5	1	1	33	3									
20	13	16	14	10	10	3	20	20	20	5								
5	1	5	3	3	5	5	3	13	10	5	5							
5	20	20	20	5	5	5	20	3	5	2	5	5						
20	5	1	5	14	14	5	1	5	2	1	5	24	26					
14	20	3	3	1	20	1	14	3	13	5	3	5	2	3				
16	14	2	5	20	1	16	36	13	2	5	16	1	2	4	6			
10	1	1	5	5	14	6	1	10	10	5	14	5	1	5	1	13		
14	1	1	5	26	5	13	16	5	1	1	5	4	13	1	5	1	5	
16	10	20	20	20	5	5	5	23	1	5	3	5	5	26	13	10	2	6

Panel (a)

0.101																		
0.116	0.590																	
0.076	0.502	0.166																
0.196	0.105	-0.004	0.007															
0.242	0.462	0.178	0.100	0.013														
0.103	0.501	0.158	0.075	0.061	0.046													
0.120	0.033	-0.006	0.01	0.135	0.002	0.003												
0.164	0.483	0.131	0.057	0.095	0.064	0.060	0.004											
0.160	0.218	0.022	0.066	0.060	0.042	0.039	0.022	0.088										
0.055	0.553	0.251	0.141	0.031	0.057	0.064	-0.005	0.064	0.011									
0.130	0.132	0.031	0.054	0.039	0.038	0.028	0.008	0.022	0.026	-0.015								
0.188	0.143	0.025	0.056	0.069	0.034	0.020	0.028	0.027	0.055	0.010	0.037							
0.294	0.106	0.038	0.031	0.115	0.023	0.010	0.030	0.023	0.049	-0.006	0.021	0.055						
0.035	0.297	0.117	0.038	-0.002	0.042	0.038	0.004	0.009	-0.001	0.017	-0.010	-0.008	0.001					
0.046	0.547	0.201	0.144	0.013	0.069	0.067	0.005	0.075	-0.003	0.125	-0.008	0.016	0.010	0.055				
0.052	0.209	0.017	0.072	0.039	0.007	0.017	0.005	0.039	0.022	0.002	0.072	0.005	-0.004	0.004	0.003			
0.065	0.521	0.154	0.115	0.016	0.080	0.063	0.055	0.083	0.011	0.107	-0.008	-0.007	-0.015	0.005	0.103	0.006		
0.145	0.256	0.063	0.085	0.043	0.016	0.015	-0.004	0.061	0.053	0.016	0.069	0.035	0.026	0.017	0.019	-0.007	0.014	
0.006	0.240	0.002	0.056	0.011	-0.005	-0.003	-0.005	0.00	-0.003	0.003	0.004	-0.002	0.004	0.005	0.001	0.005	0.061	-0.004

Panel (b)

Figure 6-1: Dependence structure and Kendall tau matrices of the coal-uranium energy portfolio. Panel (a) displays the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the portfolio. Panel (b) shows the c-vine Kendall tau correlation matrix of the portfolio. All matrices consist of 192 components. The numbers in the diagonal dependence structure matrices of Panel (a) represent the bivariate copulas listed and numbered in Table 5-1.

Counting (coal-uranium portfolio):

According to the diagonal matrices displayed in Panel (a) of Figure 6-1 and in Table 6-1 the bivariate copulas more frequently selected by the c-vine, d-vine and r-vine models under the full sample period scenario to measure the dependence between the coal and uranium stocks are: the Frank 65, 51 and 53 times for the c-vine, d-vine and r-vine models, respectively; the Joe-Frank 180 degrees rotated 20, 26 and 33 times; the student-t 7, 14 and 18 times; the Gaussian 19, 22 and 20 times; the Gumbel rotated 180 degrees 16, 14 and 17 times; the Clayton 22, 19 and 14 times; Joe 180 degrees rotated 3, 8 and 8 times and; the Clayton 180 degrees rotated 0,0 and 5 times. Table 6-1 summarizes the counting, recording, classification and grouping stages of the copula counting technique.

Recording, classification and grouping (coal-uranium portfolio):

Table 6-1: C-vine, d-vine and r-vine models' bivariate copula selection for the coal-uranium portfolio

Bivariate Copula	**Full sample**			**Pre-GFC**			**GFC**			**Post-GFC**		
Vine model	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**
						Negative Tail						
Clayton	22	19	14	15	18	18	16	22	30	23	23	18
Gumbel 180	16	14	17	14	15	12	12	13	27	9	10	13
Studen-t	7	14	18	13	10	11	12	17	10	4	11	9
Joe 180	3	8	8	26	15	16	4	13	14	12	8	15
Joe-Frank 180	20	26	8	8	5	16	8	8	14	11	11	15
						Centre						
Frank	65	51	53	50	48	58	59	51	30	55	61	64
Gaussian	19	22	20	29	28	14	32	26	23	27	25	17
						Positive Tail						
Gumbel	5	4	3	3	3	2	11	2	6	6	4	8
Clayton 180	0	0	5	11	21	16	0	0	11	0	0	7
Clayton 90	6	2	0	3	11	11	6	6	4	10	4	3
Studen-t	7	14	18	13	10	11	12	17	10	4	11	9
Joe	0	0	1	0	0	4	0	0	8	0	0	6
Joe-Frank	6	8	12	0	0	3	5	6	3	0	0	5

Notes: the top row of the table displays the four financial period scenarios under consideration and the type of pair vine copulas fitted. The first column lists the bivariate copulas most frequently selected by the vine copula models to measure the dependence between the pairs of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The Student-t copula has been grouped with the copulas for positive and negative tail dependence because it measures the dependence in both tails symmetrically. The letters *C, D* and *R* stand for canonical, drawable and regular. The dependence structure located in the centre, negative tail and positive tail of the portfolio has been dissected, organized, counted, classified and grouped.

Aggregate dependence reading (coal-uranium portfolio):

The pair vine copula modelling of dependence of the coal-uranium energy portfolio indicates that the Frank copula is the most frequently selected under each of the four financial period scenarios considered, indicating that most of the dependence in the portfolio is concentrated in the centre of the joint distributions (refer to Table 6-1). This implies that coal and uranium stocks have high dependence risk in non-crisis periods and low dependence risk in times of financial turbulence. A look into the 2008-2009 GFC indicates that coal prices did not suffer the severe decline oil, iron ore and nickel prices did. Coal prices overall remain robust during the crisis period (BREE, 2014; DRET & BREE, 2013; Bingham & Perkins, 2012). One explanation for this is that the demand for electricity tends to remain more or less constant even when the financial stock markets lack confidence. Besides, coal in Australia is still a major energy source for electricity generation.

Uranium prices also enjoyed a relative stability during the 2008-2009 GFC most likely because some of its price drivers are not directly linked to the traditional macroeconomic fundamentals. Some drivers of uranium prices are global concerns about greenhouse gas emissions and clean energy; price increases in fossil fuel, and nuclear power plant events such as the Fukushima and Chernobyl. Uranium prices, in addition to that, appear to be strongly correlated with electricity demand and the levels of nuclear power plant operation (DI et al., 2014). Another important implication from the high concentration of dependence the coal-uranium energy portfolio has in the centre is that its return values are liable to change more frequently in tranquil stock market conditions and have a low probability of being extreme in those market conditions. Energy investors could therefore benefit from the relative safeness of coal, gas and uranium stocks in times of financial turbulence by using them to diversify and hedge an investment position with high concentration in the oil, iron ore and nickel sectors.

A noticeable shift of dependence concentration in the coal-uranium energy portfolio takes place from the positive tail in the pre-GFC to the centre and negative tail in the GFC. This information reflects the high volatility of the financial stock markets during the most critical period and the low probability of coal and uranium stocks to realize positive returns in those market conditions. Table 6-1 also indicates that the largest concentration of asymmetric dependence in the negative tail of the coal-uranium energy portfolio occurs during the GFC, reflecting the propensity of the coal and uranium stocks to yield negatively skewed returns in non-tranquil stock market conditions.

Table 6-2: Significance testing of the coal-uranium energy portfolio's relative comparison of dependence

Significance testing of dependence	Gold	Iron ore-nickel	Oil-gas	Mix-metals	Retail	Manufacturing
			Overall dependence (centre)			
Frank T-test	-1.07	6.08	2.60	-0.35	2.10	-0.95
Statistical significance	Neither	Sig. larger	Sig. larger	Neither	Sig. larger	Neither
			Overall dependence (negative tail)			
Clayton T-test	4.33	-0.66	0.21	1.57	2.12	3.10
Gumbel 180 T-test	0.11	-3.59	-2.02	1.62	2.29	2.99
Joe 180 T-test	2.28	-1.59	1.23	1.07	4.56	1.55
Joe-Frank 180 T-test	0.93	-0.11	-0.72	-1.61	3.74	2.78
Statistical significance	Sig. larger	Neither	Neither	Neither	Sig. larger	Sig. larger
			Overall dependence (positive tail)			
Gumbel T-test	-2.15	2.19	-1.88	0.55	-3.88	-1.65
Clayton 180 T-test	-1.16	-0.32	-0.66	-1.98	-3.76	-3.15
Joe T-test	0.17	-0.63	-3.66	-0.86	-4.06	-2.46
Joe-Frank T-test	-1.52	2.37	3.06	-2.85	0.29	-0.12
Statistical significance	Neither	Sig. larger	Neither	Neither	Sig. smaller	Sig. smaller
			Symmetric dependence (tails)			
Student-t T-test	-5.26	-2.39	-3.17	0.06	-4.66	-3.82
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller	Neither	Sig. smaller	Sig. smaller
			Asymmetric dependence (negative tail)			
Clayton T-test	4.33	-0.66	0.21	1.57	2.12	3.10
Gumbel 180 T-test	0.11	-3.59	-2.02	1.62	2.29	2.99
Statistical significance	Sig. larger	Sig. smaller	Neither	Neither	Sig. larger	Sig. larger
			Asymmetric dependence (positive tail)			
Gumbel T-test	-2.15	2.19	-1.88	0.55	-3.88	-1.65
Clayton 180 T-test	-1.16	-0.32	-0.66	-1.98	-3.76	-3.15
Statistical significance	Sig. smaller	Sig. larger	Neither	Neither	Sig. smaller	Sig. smaller
			Critical value= $t_{(0.05,22)}=\pm 2.07$			

Notes: The table displays the significance testing of the coal-uranium energy portfolio's relative comparison of dependence concentration. The top row displays the names of the portfolios against which the coal-uranium energy portfolio is compared with. The first column from left to right shows the copulas to which the t-test is implemented and the statistical significance category. The rest of the columns display the resulting t-test values, the type of dependence being tested and its location, and the significance testing results. The bottom row states the critical value used to determine the existence or not existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine statistical significance, the t-value of at least one copula is required to be larger or smaller than the critical value.

Since the c-vine copula model relative to the r-vine and d-vine selects the Frank copula more frequently under each of the four period scenarios, it is acknowledged for better capturing the multivariate dependence structure of the coal-uranium energy portfolio.

The significance testing of the coal-uranium energy portfolio's relative comparison of dependence concentration displayed in Table 6-2 indicates that its overall dependence concentration in the centre is at the 95% confidence level significantly larger than those of the iron ore-nickel, oil-gas and retail portfolios. This implies that the coal and uranium stocks are less dependence risky than the oil, iron ore and nickel stocks in times of financial turbulence. The portfolios' asymmetric dependence concentration in the positive tail is significantly smaller than those of the gold, iron ore-nickel, retail and manufacturing and significantly larger than those of the gold, retail and manufacturing, in the negative tail. This information indicates that the coal and uranium stocks relative to the retail and manufacturing benchamark are less propense to yield positively skewed returns in tranquil stock market conditions. The portfolio's symmetric dependence concentration in the tails is at the 95% confidence level significantly smaller than those of the gold, iron ore-nickel, oil-gas, retail and manufacturing.

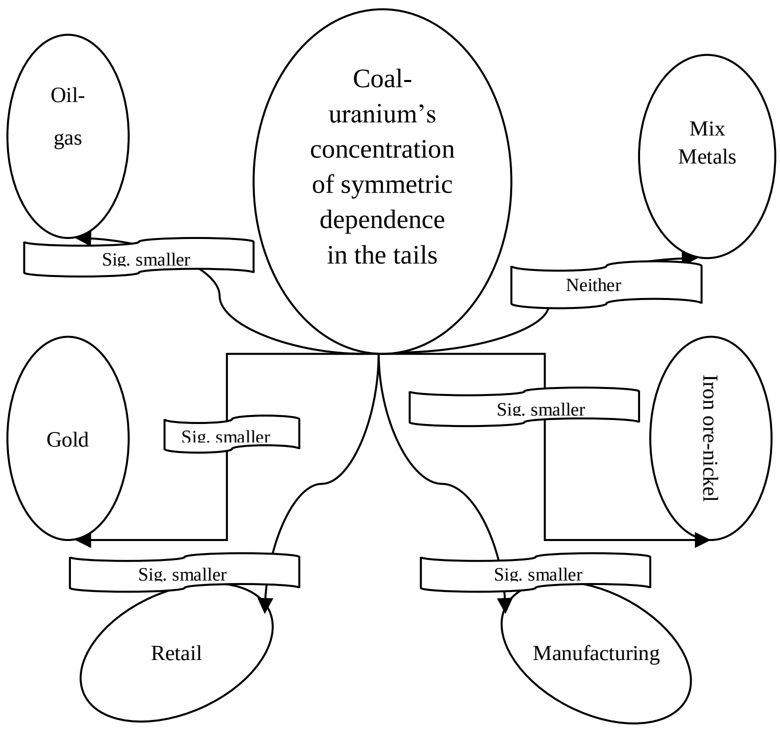

Figure 6-2: Coal-uranium portfolio's symmetric dependence concentration in the tails. The estimate of symmetric dependence is based on an average of all four period scenarios.

6.2.2 Oil-gas portfolio

The diagonal dependence structure matrices of the oil-gas energy portfolio displayed in Figure 6-3 differ from those of the coal-uranium energy portfolio in the number of times the copulas 3 and 14 appear. The oil-gas energy portfolio due to the large concentration of dependence it has in the negative tail requires the use of the Clayton and 180 degrees rotated Gumbel copulas more often. This feature is also found in the dependence structure matrices of the iron ore-nickel mining portfolio, while it is absent in the dependence structure matrices of the gold, coal-uranium and mix-metals leptokurtic portfolios.

19																		
7	1																	
1	5	5																
14	5	5	2															
2	19	5	5	1														
14	3	4	3	20	13													
3	5	5	20	5	13	5												
1	5	5	1	5	5	5	5											
14	1	3	5	16	5	4	1	5										
7	14	14	14	20	20	5	36	16	1									
14	1	16	20	6	2	5	14	5	3	14								
1	5	3	2	20	23	1	5	3	3	16	4							
1	20	5	14	20	5	5	10	3	1	2	5	33						
14	1	14	20	20	13	3	14	5	3	13	5	3	1					
23	13	1	16	5	5	3	36	13	33	1	13	1	23	1				
2	5	1	20	20	1	5	20	3	5	5	5	3	1	10	5			
7	20	14	13	2	6	6	5	3	1	20	5	26	5	23	1	5		
13	13	1	2	20	5	16	5	5	6	1	1	5	26	5	5	20	23	
9	14	5	20	1	5	3	5	3	5	6	10	14	5	14	10	16	2	5

14																		
3	3																	
14	16	5																
14	1	20	5															
5	20	5	5	1														
1	3	5	5	1	5													
20	5	14	10	5	13	1												
5	20	13	3	5	6	23	5											
1	1	5	16	5	1	4	1	33										
1	4	20	13	33	5	5	6	1	5									
2	5	4	10	4	1	13	3	5	3	1								
19	19	5	20	2	5	5	13	3	3	1	14							
14	1	5	3	14	5	13	5	16	1	6	2	14						
14	14	5	3	1	20	2	1	14	13	23	5	13	2					
5	14	13	5	5	16	4	14	1	2	13	5	14	6	14				
20	14	10	2	2	5	5	5	3	16	20	5	5	24	13	16			
20	2	16	1	5	1	23	3	1	23	3	5	13	5	1	3	16		
2	20	5	33	1	26	24	5	6	5	26	16	20	5	5	5	5	5	
5	5	3	1	23	1	5	33	5	33	5	23	1	5	1	1	33	5	13

Panel (a)

0.132																		
0.182	0.068																	
0.175	0.137	0.068																
0.204	0.081	0.091	0.094															
0.215	0.174	0.033	0.053	0.081														
0.184	0.121	0.134	0.064	0.099	0.066													
0.209	0.045	0.123	0.118	0.127	0.047	0.082												
0.195	0.085	0.053	0.032	0.063	0.033	-0.011	0.071											
0.189	0.179	0.085	0.011	0.062	0.074	0.052	0.052	-0.009										
0.264	0.056	0.229	0.068	-0.006	0.097	0.036	0.014	0.092	0.093									
0.412	0.132	0.087	0.153	0.030	0.009	0.049	0.031	0.072	0.020	0.064								
0.310	0.105	0.105	0.100	0.116	0.015	-0.006	0.043	0.008	0.052	0.070	0.049							
0.250	0.210	0.125	0.041	0.038	0.113	0.018	0.047	0.017	0.052	0.013	0.010	0.035						
0.198	0.178	0.115	0.108	0.063	0.121	0.052	0.035	0.023	0.028	-0.015	0.022	0.014	0.004					
0.249	0.081	0.054	0.086	-0.021	0.055	0.040	0.081	0.098	0.190	0.040	0.154	0.040	0.013	0.021				
0.242	0.114	0.091	-0.007	0.006	-0.019	0.034	0.023	0.022	0.017	0.091	0.065	0.015	-0.027	0.037	0.033			
0.255	0.241	0.034	0.013	-0.027	0.076	-0.038	0.030	-0.012	-0.034	0.045	0.40	0.028	0.082	0.032	0.019	0.054		
0.469	0.145	0.142	-0.011	0.010	-0.009	-0.046	0.020	0.034	-0.054	-0.036	0.027	0.221	0.078	0.079	-0.031	-0.050	-0.041	
0.315	0.186	0.004	-0.010	-0.002	0.006	-0.012	-0.003	0.045	-0.006	0.009	-0.003	-0.008	0.124	0.008	-0.008	-0.002	0.014	0.002

Panel (b)

Figure 6-3: Dependence structure and Kendall tau matrices of the oil-gas energy portfolio. Panel (a) displays the GFC c-vine (on the left) and d-vine (on the right) dependence structure matrices of the portfolio. Panel (b) shows the d-vine Kendall tau correlation matrix of the portfolio. All matrices consist of 192 components. The numbers in the diagonal dependence structure matrices of Panel (a) represent the bivariate copulas listed and numbered in Table 5-1.

Counting (oil-gas portfolio):

According to Panel (a) from Figure 6-3 and Table 6-3 the bivariate copulas more frequently selected under the full sample period by the c-vine, d-vine and r-vine models to measure the dependence of the oil and gas stocks are: the Frank 47, 41 and 46 times for the c-vine, d-vine and r-vine models, respectively; the Joe-Frank 180 degrees rotated 26, 30 and 22 times; the student-t 23, 20 and 22 times; the Gaussian 17, 19 and 16 times; the Gumbel rotated 180 degrees 22, 23 and 23 times; the Clayton 16, 18 and 23 times; Joe 180 degrees rotated 0, 0 and 2 times and; the Clayton 180 degrees rotated 8, 7 and 10 times. Table 6-3 summarizes the counting, recording, classification and grouping stages of the bivariate copula counting technique.

Recording, classification and grouping (oil-gas portfolio):

Table 6-3: C-vine, d-vine and r-vine models' bivariate copula selection for the oil-gas portfolio

Bivariate Copula	**Full sample**			**Pre-GFC**			**GFC**			**Post-GFC**		
Vine model	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**
					Negative Tail							
Clayton	16	18	23	21	25	24	18	15	16	14	21	23
Gumbel 180	22	23	23	17	14	12	17	16	23	11	18	18
Studen-t	23	20	22	20	14	17	9	10	11	18	18	18
Joe 180	0	0	2	15	17	17	0	0	10	16	9	16
Joe-Frank 180	26	30	22	4	8	6	18	12	10	13	13	12
					Centre							
Frank	47	41	46	40	35	30	54	55	51	54	48	37
Gaussian	17	19	16	21	19	25	28	29	25	21	16	20
					Positive Tail							
Gumbel	9	7	8	8	8	7	3	5	6	6	4	6
Clayton 180	8	7	10	14	11	13	0	0	14	0	0	15
Clayton 90	3	8	4	6	9	10	5	6	5	3	6	4
Studen-t	23	20	22	20	14	17	9	10	11	18	18	18
Joe	2	6	3	6	9	7	5	5	4	3	7	4
Joe-Frank	0	0	3	0	0	1	0	0	3	0	0	1

Notes: the top row of the table displays the four financial period scenarios under consideration and the type of pair vine copulas fitted. The first column lists the bivariate copulas most frequently selected by the vine copula models to measure the dependence between the pairs of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The Student-t copula has been grouped with the copulas for positive and negative tail dependence because it measures the dependence in both tails symmetrically. The letters *C*, *D* and *R* stand for canonical, drawable and regular. The dependence structure located in the centre, negative tail and positive tail of the portfolio has been dissected, organized, counted, classified and grouped.

Aggregate dependence reading (oil-gas portfolio):

In the oil-gas energy portfolio, despite the large presence of the Franck copula most of the dependence between the stocks in the portfolio is concentrated in the negative tail. This concentration of dependence in the negative tail is however not noticeably larger than that in the centre. The oil-gas energy portfolio, consequently, has a high dependence risky in non-tranquil stock market conditions relative to tranquil periods. A look into the 2008-2009 GFC shows that oil prices did fall sharply during the crisis period, so much so that by the end of 2008 they have reached levels seen in the 2000's (DI et al., 2014). Other factors known to adversely impact oil prices in the short run are the cyclical and seasonal demand for oil, supply disruptions triggered by political instability in oil producing countries, monopoly power, currency exchange rate changes and oil stock market speculation. In the long run, the marginal cost of oil production tends to have the greatest impact. Gas prices during the crisis period did not decline as much as the oil prices did. One reason for this is that the demand for electricity tends to remain more or less constant even during crisis periods, due to gas still being used in Australia as a source for electricity generation (DI et al., 2014).

Other implications from the high concentration of dependence the oil-gas energy portfolio has in the negative tail is that its oil stocks'returns are liable to change less frequently in times of financial turbulence and have a high probability of being extreme in those market conditions. The inverse applies to gas stocks, which have most of the dependence concentrated in the centre of the joint distributions. Energy investors could therefore be better off by avoiding oil stock investments during crisis periods characterized by low confidence in the financial stock markets and instead investing in gas stocks in those market conditions. With respect to model selection, since the c-vine model relative to the r-vine and d-vine most frequently selects copulas for the modelling of negative tail dependence under each of the four financial period scenarios considered, the c-vine is discerned to best capture the multivariate interaction and dependence structure of the oil-gas energy portfolio.

Unlike in the pre-GFC and post-GFC period scenarios, the c-vine, d-vine and r-vine copula models in the GFC select the Student-t copula in fewer occasions, an indication that most of the stocks' dependence relationships in the tails are of asymmetric and nonlinear type. This information implies that the oil stocks tend to correlate more strongly in stock market conditions with low investors' confidence and have a high propensity to yield negatively skewed returns in those market conditions. In the pre-GFC period scenario the oil-gas energy portfolio has most of the dependence concentrated towards the end of the positive tail, indicating that the oil stocks tend to generate positively skewed returns in market conditions similar to those found in the pre-GFC period scenario.

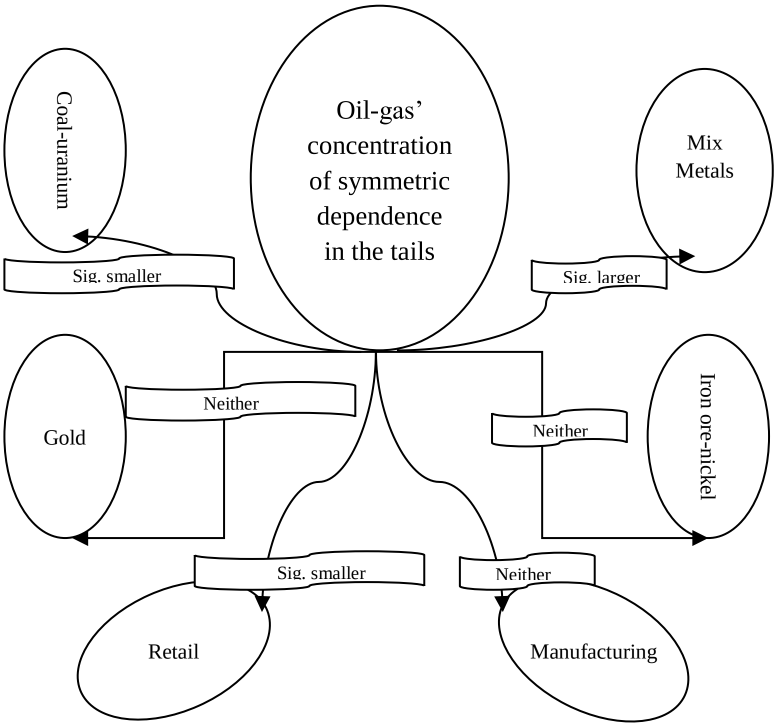

Figure 6-4: Oil-gas energy portfolio's symmetric dependence concentration in the tails. The estimate of symmetric dependence is based on an average of all four period scenarios.

The significance testing of the oil-gas energy portfolio's relative comparison of dependence concentration displayed in Table 6-4 indicates that its overall dependence in the negative tail is at the 95% confidence level significantly larger than those of the gold, retail and manufacturing and neither significantly larger or smaller than that of the iron ore-nickel mining portfolio. In the centre and positive tail however it has significantly larger concentration of dependence than the iron ore-nickel, making it less dependence risky than the iron ore-nickel mining portfolio. This information indicates that the oil stocks are significantly more dependence risky than the gold, retail and manufacturing when the financial stock markets lack investors' confidence.

Table 6-4: Significance testing of the oil-gas energy portfolio's relative comparison of dependence

Significance testing of dependence	Gold	Iron ore-nickel	Coal-uranium	Mix-metals	Retail	Manufacturing
			Overall dependence (centre)			
Frank T-test	-3.40	3.68	-2.60	-3.23	-0.98	-3.67
Statistical significance	Sig. smaller	Sig. larger	Sig. smaller	Sig. smaller	Neither	Sig. smaller
			Overall dependence (negative tail)			
Clayton T-test	4.45	-0.88	-0.21	1.48	2.06	3.12
Gumbel 180 T-test	2.49	-1.83	2.02	3.52	5.17	5.62
Joe 180 T-test	0.67	-2.61	-1.19	-0.48	2.09	0.15
Joe-Frank 180 T-test	1.42	0.39	0.72	-1.02	3.79	3.09
Statistical significance	Sig. larger	Neither	Neither	Neither	Sig. larger	Sig. larger
			Overall dependence (positive tail)			
Gumbel T-test	-1.20	4.08	1.88	2.15	-2.58	-0.09
Clayton 180 T-test	-0.52	0.32	0.66	-1.37	-3.81	-2.73
Joe T-test	4.40	2.21	3.66	3.34	-0.52	1.32
Joe-Frank T-test	-3.54	-0.57	-3.06	-4.45	-4.93	-3.40
Statistical significance	Neither	Sig. larger	Neither	Sig. larger	Sig. smaller	Sig. smaller
			Symmetric dependence (tails)			
Student-t T-test	-1.11	-0.64	3.17	3.63	-2.94	-0.78
Statistical significance	Neither	Neither	Sig. larger	Sig. larger	Sig. smaller	Neither
			Asymmetric dependence (negative tail)			
Clayton T-test	4.45	-0.88	-0.21	1.48	2.06	3.12
Gumbel 180 T-test	2.49	-1.83	2.02	3.52	5.17	5.62
Statistical significance	Sig. larger	Neither	Neither	Sig. larger	Sig. larger	Sig. larger
			Asymmetric dependence (positive tail)			
Gumbel T-test	-1.20	4.08	1.88	2.15	-2.58	-0.09
Clayton 180 T-test	-0.52	0.32	0.66	-1.37	-3.81	-2.73
Statistical significance	Neither	Sig. larger	Neither	Sig. larger	Sig. smaller	Sig. smaller
			Critical value= $t_{(0.05,22)}$=±2.07			

Notes: The table displays the significance testing of the oil-gas energy portfolio's relative comparison of dependence concentration. The top row displays the names of the portfolios against which the oil-gas energy portfolio is compared with. The first column from left to the right shows the copulas to which the t-test is implemented and the statistical significance category. The rest of the columns display the resulting t-test values, the type of dependence being tested and its location and the significance testing results. The bottom row states the critical value used to determine the existence or not existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine statistical significance, the t-value of at least one copula is required to be larger or smaller than the critical value.

The portfolio's asymmetric dependence concentration in the negative tail is significantly larger than those of the gold, mix-metals, retail and manufacturing, confirming the higher dependence riskiness of the oil stocks when the financial stock market are unstable. The oil-gas energy portfolio's symmetric dependence concentration in the tails as indicated by Figure 6-4 is significantly smaller than those of the coal-uranium and retail, and significantly larger than that of the mix-metals.

6.3 Discussion of results

The identification of the coal-uranium energy portfolio as having low dependence risk in times of financial turbulence alludes to the relative stability the coal and uranium commodities displayed during the 2008-2009 global financial crisis. The Bureau of Resources and Energy Economics (2014), The Department of Resources, Energy and Tourism (2013) and Bingham and Perkins (2012) have, among many others, pointed out the relative mild price fluctuations of those commodities during the period. The predominance of the Frank copula in the dependence structure matrices of the coal-uranium energy portfolio is a feature shared with the gold and mix-metals portfolios. As compared to those portfolios, the coal-uranium energy portfolio was found to be significantly more dependence risky than the gold and significantly different from the mix-metals leptokurtic in terms of dependence risk. The recognition of the c-vine as the model that best accounts for the multivariate dependence structure of the coal-uranium energy portfolio suggests that one stock in the portfolio exerts significant influence over the rest through large correlation values. This stock appears to be PALADIN ENERGY (PDNX), which is selected by the c-vine as the rootstock of the portfolio.

The identification of the oil-gas energy portfolio as having high dependence risk in market conditions characterized by low confidence in the financial stock markets is in line with the literature modelling oil markets. The Department of Industry, Geoscience Australia and the Bureau of Resources and Energy Economics (2014) have documented the high risk and negatively skewed price behaviour of oil during the 2008-2009 global financial crisis. Du et al. (2012) identify volatility increases in a stock portfolio as a result of increases in oil prices; Killian and Park (2009) estimate that around 22 per cent of the long run fluctuation in the US stock market is due to the supply and demand shocks experienced by the crude oil prices; Park and Ratti (2008) recognize the statistical significance of impact oil prices have on real stock returns from the US and

13 European countries; and Basher and Sadorsky (2006) acknowledge the presence of oil price risk in stock markets from emerging economies.

The predominance of the Clayton and 180 Gumbel copulas in the dependence structure of the oil-gas energy portfolio is a feature shared with the iron ore-nickel mining portfolio. The iron ore-nickel mining portfolio is however more dependence risky due to the demand and supply dynamics of the iron ore and nickel commodities which tend to be more heavily skewed towards the negative tail. The identification of the c-vine as the model that best accounts for the multivariate dependence structure of the oil and gas stocks is also an indication that a stock in the portfolio has strong correlation values with the rest. This stock is WOODSIDE (WPLX), recognized by the c-vine as the rootstock of the portfolio. Czado et al. (2012), Chollete et al. (2009) and Heinen and Valdesogo (2009) have noted the suitability of the c-vines to adequately capture the multivariate dependence of financial assets.

This chapter's research, as compared to the dependence risk modelling of Brechmann and Schepsmeier (2013), Min and Czado (2010), Czado et al. (2012) and Brechmann and Czado (2012), has comparative advantages. Firstly, it provides a detailed and comprehensive account of the assets' dependence risk features in specific market conditions. Secondly, it proposes a systematic approach, in the form of the copula counting technique, to examine the assets' dependence concentration and dependence at various locations of the joint distributions. In addition to that, this chapter's analysis of dependence risk appears to be the first to model the risk of oil stocks from the Australian market using pair vine copulas and the copula counting technique.

6.4 Summary

This chapter implemented the copula counting technique to dissect, analyse and interpret the dependence structure of the energy portfolios. The coal-uranium energy portfolio was found to have most of the dependence concentrated in the centre of the pairs of stocks' joint distributions, as indicated by the predominance of the Frank copula in each of the four period scenarios. As a result, the coal-uranium energy portfolio was acknowledged to have high dependence risk in tranquil stock market conditions and low dependence risk in times of financial turbulence. These findings were confirmed by actual price behaviour of the coal and uranium commodities during the 2008-2009 global financial crisis. Out of the three vine copula models fitted to the energy portfolio the c-vine was acknowledged to best account for the multivariate dependence structure of the coal-uranium energy portfolio.

The oil-gas energy portfolio, contrary to the coal-uranium portfolio, was found to have most of the dependence concentrated in the negative tail. As a result the portfolio has high dependence risk in stock market conditions characterized by low investors' confidence and low dependence risk in stock market conditions with restored confidence. These findings were also confirmed by actual price behaviour of the oil during the 2008-2009 global financial crisis. The c-vine was identified to best capture the multivariate dependence structure of the oil-gas energy portfolio.

CHAPTER 7

DEPENDENCE STRUCTURE ESTIMATION: RETAIL AND MANUFACTURING PORTFOLIOS

This chapter consists of two sections: introduction and dependence estimation

The *introduction* section provides an overview of the Australian retail and manufacturing sectors underlying the retail and manufacturing stock benchmark portfolios modelled. The *dependence estimation* section dissects, analyses and interprets the dependence structure of the retail and manufacturing benchmark portfolios using the copula counting technique explained in Chapter 5.

7.1 Introduction

The retail and manufacturing sectors are two important sectors of the Australian economy because they together account for about 12 per cent of total GDP. Besides, the manufacturing sector has been in a declining trend and exhibiting decreasing risk, while the retail sector has been expanding. The manufacturing sector specifically employed around 20 percent of the Australian workforce before the 2008-2009 GFC, which dropped to 8 percent in 2014. On the other hand, the retail sector has experienced a slow but steady increase in recent years, contributing about AD 23.88 billion to the Australian economy in 2013 (Department of Industry, 2014; Kryger, 2014; Australian Bureau of Statistics, 2015). Both sectors, in addition to that, can be easily identified for having a strong relationship of dependence and economic linkages with the Australian resources sector: the mining and energy sectors. The performance of the Australian resources sector could therefore in this sense be thought as directly impacting the levels of demand, spending and investment within the retail and manufacturing sectors. Evidence of this relationship of dependence and multiplier effects the manufacturing sector has with the Australian resources sector is that the mining sector in 2011 supplied 20 per cent of the raw material used by the manufacturing sector, while the mining sector demanded 5 per cent of the goods produced by the manufacturing sector (ARA, 2014; Savills Research, 2014; Delloite, 2013; Mehmedovic et al., 2011).[18] The Australian manufacturing sector is the second largest exporter of goods, next to the mining sector.

Although one of the primary performance drivers of the retail sector is the Australian resources sector, all other sectors of the economy also impact its performance through spill over effects. Other performance drivers of the manufacturing and retail sectors are the financial stock market confidence, business cycles, interest rate fluctuations, exchange rate changes and currency fluctuations, advances in manufacturing technology, the increasing world of the web and communications and the expansion of the digital economy (ARA, 2014; Savills Research, 2014; Delloite, 2013; Mehmedovic et al., 2011).

This chapter's objectives are to examine the dependence risk profile of the retail and manufacturing benchmark portfolios in specific market conditions; account for the portfolios' dependence structure changes between pairs of period scenarios; and recognize the pair vine copula models that best capture the multivariate dependence of the portfolios. The copula counting technique is used for this purpose.

7.2 Dependence structure estimation

7.2.1 Retail portfolio

The diagonal dependence structure matrices of the retail benchmark portfolio displayed in Panel (a) of Figure 7-1 share the common feature with those of the gold, coal-uranium and mix-metals of having a large presence of the Frank copula (i.e. copula number 5). This copula is designed to capture greater concentration of dependence in the centre of the joint distributions. On the other hand, they have a reduced presence of the copulas for negative tail dependence modelling.

[18] The acronyms *ARA*, *AGPC*, *PC*, *NAB*, *DIISR*, *CT* and *DI* used in the present chapter stand for stands for The Australian Retailers Association, Australian Government Productivity Commission, Productivity Commission, National Australian Bank, Department of Industry, Innovation, Sceince and Research, Commonwealth Treasury and Department of Industry.

2																		
14	5																	
14	5	5																
10	5	5	1															
3	5	5	5	20														
33	6	2	5	4	36													
5	5	2	2	5	1	26												
33	23	13	20	2	33	6	4											
2	1	1	6	3	1	13	5	23										
2	24	5	5	33	14	5	23	13	13									
33	5	13	3	2	2	13	13	13	13	20								
2	5	2	3	5	5	5	2	5	23	20	5							
5	13	1	5	13	4	4	2	5	33	5	5	16						
3	4	5	5	14	5	2	5	2	4	14	5	13	1					
5	5	1	5	2	14	13	2	2	23	23	2	2	2	16				
16	2	1	5	4	13	2	2	2	1	33	10	14	1	14	13			
5	3	2	6	10	5	5	5	13	5	34	4	5	5	2	3	1		
20	2	1	1	2	2	1	5	1	14	1	2	20	2	5	5	2	5	
14	2	2	14	14	19	2	2	1	5	5	2	7	2	2	2	5	20	20

26																		
2	16																	
2	23	2																
5	13	1	10															
3	23	3	16	10														
2	5	2	16	5	2													
14	14	2	5	2	4	20												
20	13	2	1	33	5	1	13											
13	2	4	13	14	5	13	3	24										
5	14	1	6	5	5	6	5	4	20									
5	14	10	3	16	5	5	2	4	13	1								
2	4	2	20	5	20	3	1	2	13	2	13							
2	13	1	5	2	13	5	20	13	5	6	13	5						
14	14	5	14	20	2	2	5	10	40	4	14	13	3					
6	5	4	5	2	33	10	10	36	10	13	3	20	13	5				
2	1	1	5	2	4	2	2	1	5	5	1	2	3	2	2			
5	5	5	23	1	5	14	5	2	14	14	2	2	2	2	2	2		
3	2	2	23	16	2	4	5	5	4	2	5	1	5	13	1	5	13	
5	2	2	3	3	2	14	2	1	5	2	7	1	2	14	1	20	2	1

Panel (a)

0.03																		
0.00	0.20																	
0.00	0.19	0.25																
0.03	0.04	0.14	0.03															
0.02	0.30	0.09	0.03	0.06														
-0.02	0.02	0.28	0.03	0.00	-0.01													
-0.01	-0.02	0.04	0.09	0.04	0.01	-0.01												
0.00	-0.02	0.02	0.10	0.05	0.00	0.01	0.01											
0.05	0.01	0.01	0.02	0.02	0.04	0.00	0.02	-0.01										
-0.02	-0.02	-0.01	0.02	-0.01	0.01	-0.01	0.00	0.03	0.01									
-0.02	0.03	0.04	0.03	0.03	0.02	0.01	0.02	0.01	0.01	0.02								
0.04	0.03	0.05	0.04	-0.02	0.02	0.05	0.02	0.18	-0.01	0.04	-0.03							
0.03	0.02	0.07	0.07	0.02	0.02	0.04	0.01	0.07	-0.01	0.01	0.03	0.02						
0.04	0.02	-0.02	0.02	0.03	0.00	0.03	0.04	0.32	0.01	0.00	0.04	0.02	0.06					
0.05	0.02	0.04	0.04	0.05	0.05	0.04	0.02	0.09	-0.01	0.00	0.05	0.06	0.03	0.03				
0.02	0.03	0.07	0.06	0.03	0.04	0.10	0.08	0.08	-0.01	0.00	0.07	0.06	0.08	0.02	0.03			
0.04	0.03	0.09	0.03	0.03	0.08	0.11	0.08	0.03	0.25	0.00	0.09	0.09	0.09	0.11	0.03	0.06		
0.07	0.36	0.07	0.06	0.06	0.13	0.13	0.11	0.04	0.01	-0.01	0.12	0.17	0.14	0.13	0.14	0.12	0.10	
0.09	0.09	0.15	0.10	0.10	0.20	0.22	0.21	0.12	0.08	0.19	0.26	0.26	0.28	0.20	0.21	0.22	0.15	0.21

Panel (b)

Figure 7-1: Dependence structure and Kendall tau matrices of the retail benchmark portfolio. Panel (a) displays the full sample period r-vine (on the left) and c-vine (on the right) dependence structure matrices of the portfolio. Panel (b) displays the full sample period r-vine Kendall tau correlation matrix of the portfolio. All matrices consist of 192 components. The numbers in the diagonal dependence structure matrices of Panel (a) represent the bivariate copulas listed and numbered in Table 5-1.

Counting (retail portfolio):

According to Figure 7-1 and Table 7-1 the bivariate copulas more frequently selected by the c-vine, d-vine and r-vine models under the full sample period scenario to measure the dependence from pairs of retail stocks' joint distributions are: the Frank 38, 44 and 51 times for c-vine, d-vine and r-vine models respectively; the Joe-Frank 180 degrees rotated

9, 6 and 8 times; the Joe-Frank 7, 5 and 3 times; the Gaussian 18, 16 and 18 times; the Clayton 11, 9 and 7 times; the Student-t 45, 43 and 41 times; the Gumbel 180 degrees rotated 14, 13 and 12 times; the Clayton 180 degrees rotated 18, 15 and 16 times each; the Joe 180 degrees rotated 5, 3 and 3 times and; the Gumbel 10, 7 and 8 times. Table 7-1 summarizes the counting, recording, classification and grouping stages of the bivariate copula counting technique.

Recording, classification and grouping (retail portfolio):

Table 7-1: C-vine, d-vine and r-vine models' bivariate copula selection for the retail portfolio

Bivariate Copula	Full sample			Pre-GFC			GFC			Post-GFC		
Vine model	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**
Negative Tail												
Clayton	11	9	7	18	17	18	25	21	20	13	15	16
Gumbel 180	14	13	12	12	8	9	13	10	10	13	11	9
Studen-t	45	43	41	17	15	16	23	25	31	20	19	21
Joe 180	5	3	3	2	3	5	3	4	6	4	3	5
Joe-Frank 180	9	6	7	9	7	10	4	3	4	4	2	3
Centre												
Frank	38	44	51	52	53	58	46	44	49	47	43	45
Gaussian	18	16	18	28	24	22	21	19	21	31	27	25
Positive Tail												
Gumbel	10	7	8	5	7	10	9	7	6	10	9	12
Clayton 180	18	15	16	9	7	10	15	13	15	20	19	21
Clayton 90	4	5	6	10	8	10	7	5	3	7	5	4
Studen-t	45	43	41	17	14	16	23	25	31	20	19	21
Joe	4	3	4	7	5	2	5	6	8	7	6	9
Joe-Frank	7	5	3	6	5	4	4	3	1	2	1	3

Notes: the top row of the table displays the four financial period scenarios under consideration and the type of pair vine copulas fitted. The first column lists the bivariate copulas most frequently selected by the vine copula models to measure the dependence between the pairs of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The Student-t copula has been grouped with the copulas for positive and negative tail dependence because it measures the dependence in both tails symmetrically. The letters *C*, *D* and *R* stand for canonical, drawable and regular. The dependence structure located in the centre, negative tail and positive tail of the portfolio has been dissected, organized, counted, classified and grouped.

Aggregate dependence reading (retail portfolio):

According to Table 7-1 in the retail benchmark portfolio, the c-vine, d-vine and r-vine models select the Frank copula the most under each of the four period scenarios considered, indicating that most of the dependence in the portfolio is concentrated in the centre of the joint distributions. This information implies that the retail stocks have high dependence risk in non-crisis periods and low dependence risk when the financial stock markets lack confidence. The high concentration of dependence located in the centre of the portfolio’s joint distribution also implies that its returns are liable to change more frequently in tranquil stock market conditions and have a low probability of being extreme in similar market conditions.

A look into the 2008-2009 GFC shows that stock investments in the Australian retail sector were exposed to lower risk as compared to investments in the United States retail sector (AGPC, 2011). A plausible explanation for this is that the Australian economy has a strong resource-based economy and the Australian retail sector has important economic linkages with the performance of the Australian resources sector. The retail sector, moreover, went through moderate economic shocks during the financial crisis mainly because the Australian resources sector overall manoeuvre the financial crisis’ effects fairly well. Evidence of this is that the gold mining sector had its best historical performance during the crisis period period (Connolly & Orsmond, 2011).

The most significant shift of dependence concentration in the retail benchmark portfolio occurred from the pre-GFC to the GFC period scenarios. Specifically, the portfolio’s dependence structure, as indicated by the decrease of the Frank, Joe-Frank and the 180 Joe-Frank copulas, and the increase of the Clayton and 180 Gumbel copulas (the latter two copulas are designed to capture asymmetric dependence in the negative tail), moves from the centre of the portfolio’s joint distribution towards the end of the tails. This dependence structure shift between pairs of period scenarios reflects the highly volatile market conditions during the GFC period scenario and the propensity of some retail stocks to yield negatively skewed returns in those market conditions. The second largest concentration of dependence in the retail benchmark portfolio is located in the negative tail.

In the post-GFC period scenario, the copulas for the modelling of positive tail dependence have their largest presence, suggesting a recovery of the financial stock markets; an increased probability for the retail stocks to realize positive returns; and the propensity of the retail stocks to yield positively skewed returns in those market conditions. The shift of dependence concentration from the GFC to the post-GFC suggests that the Australian retail sector had a relatively slow recovery during the post-crisis period. This assertion is consistent with alternative research indicating that the retail sector began to recover as the confidence in the financial stock markets increased; as the price of the mining and energy commodities recovered; and as the Australian dollar depreciated (AGPC, 2011; PC, 2011). With respect to model selection, the r-vine relative to the c-vine and d-vine is observed to select the Frank copula more frequently under each of the four financial period scenarios considered. Thus, the r-vine is discerned to be the model that best accounts for the multivariate dependence structure of the retail benchmark portfolio.

The significance testing of the retail benchmark portfolio's relative comparison of dependence concentration displayed in Table 7-2 indicates that its overall dependence concentration in the centre is at the 95% confidence level significantly smaller than those of the gold, coal-uranium, mix-metals and manufacturing, and significantly larger than that of the iron ore-nickel mining portfolio. This information implies that the returns of the retail benchmark portfolio are liable to change less frequently than those of the gold, coal-uranium, mix-metals and manufacturing when the confidence in the financial stock markets is low and have a high probability of being extreme in those market conditions

The retail benchmark portfolio's asymmetric dependence in the positive tail is, with the exception of that of the manufacturing, significantly larger than those of most portfolios at the 95% confidence level. In the negative tail it has significantly smaller dependence concentration than the mining and energy portfolios. The symmetric dependence concentration in the tails of the retail benchmark portfolio is, with exception of that of the iron ore-nickel, significantly larger than those of most portfolios. The relative comparison of the retail and manufacturing benchmark portfolios indicates that the former is less dependence risky than the latter during crisis periods. This information appears to be consistent with the performance of the retail and manufacturing sectors during the 2008-2009 global financial crisis. A possible explanation is that a large percentage of the money in circulation during the crisis period was spent and invested for the acquisition of basic household and livelihood goods instead of durables that require larger investment and capital.

Table 7-2: Significance testing of the retail benchmark portfolio's relative comparison of dependence

Significance testing of dependence	Gold	Iron ore-nickel	Coal-uranium	Oil-gas	Mix-metals	Manufacturing
			Overall dependence (centre)			
Frank T-test	-3.02	5.42	-2.10	0.98	-2.84	-3.34
Statistical significance	Sig. smaller	Sig. larger	Sig. smaller	Neither	Sig. smaller	Sig. smaller
			Overall dependence (negative tail)			
Clayton T-test	1.77	-2.57	-2.12	-2.06	-0.44	0.75
Gumbel 180 T-test	-2.87	-6.52	-2.29	-5.17	0.00	1.38
Joe 180 T-test	-2.00	-7.74	-4.56	-2.20	-5.27	-2.46
Joe-Frank 180 T-test	-1.35	-2.07	-3.74	-3.79	-3.69	-0.35
Statistical significance	Neither	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller	Neither
			Overall dependence (positive tail)			
Gumbel T-test	0.05	5.74	3.88	2.58	3.81	1.95
Clayton 180 T-test	3.14	3.39	3.76	3.43	3.12	1.35
Joe T-test	4.85	2.54	4.06	0.52	3.82	1.81
Joe-Frank T-test	-1.86	3.08	-0.29	4.02	-3.17	-0.45
Statistical significance	Sig. larger	Sig. larger	Sig. larger	Sig. larger	Sig. larger	Neither
			Symmetric dependence (tails)			
Student-t T-test	2.56	1.84	4.66	2.94	4.83	2.44
Statistical significance	Sig. larger	Neither	Sig. larger	Sig. larger	Sig. larger	Sig. larger
			Asymmetric dependence (negative tail)			
Clayton T-test	1.77	-2.57	-2.12	-2.06	-0.44	0.75
Gumbel 180 T-test	-2.87	-6.52	-2.29	-5.17	0.00	1.38
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller	Neither	Neither
			Asymmetric dependence (positive tail)			
Gumbel T-test	0.05	5.74	3.88	2.58	3.81	1.95
Clayton 180 T-test	3.14	3.39	3.76	3.43	3.12	1.35
Statistical significance	Sig. larger	Sig. larger	Sig. larger	Sig. larger	Sig. larger	Neither
			Critical value= $t_{(0.05,22)}$=±2.07			

Notes: The table displays the significance testing of the retail benchmark portfolio's relative comparison of dependence concentration. The top row displays the names of the portfolios against which the retail benchmark portfolio is compared with. The first column from left to right shows the copulas to which the t-test is implemented and the statistical significance category. The rest of the columns display the resulting t-test values, the type of dependence being tested and its location and, the significance testing results. The bottom row states the critical value used to determine the existence or not existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine statistical significance, the t-value of at least one copula is required to be larger or smaller than the critical value.

7.2.2 Manufacturing portfolio

The diagonal dependence structure matrices of the manufacturing benchmark portfolio displayed in Panel (a) of Figure 7-2 differ from those of the retail benchmark portfolio in the reduced presence of the Frank copula (i.e. copula number 5). As compared to the dependence structure matrices of the iron ore-nickel mining and oil-gas energy portfolios, those of the manufacturing benchmark portfolio have a significantly larger number of copulas for the modelling of dependence in the centre of the joint distributions.

5																		
5	5																	
33	1	3																
16	3	30	1															
36	4	2	14	1														
26	33	5	14	1	1													
5	10	3	3	1	23	13												
13	26	10	13	13	1	1	5											
13	5	2	5	20	24	14	2	13										
3	20	4	1	5	3	5	36	14	5									
5	26	5	5	5	34	1	5	33	13	30								
5	2	5	5	1	10	5	5	2	36	10	30							
1	20	5	1	1	3	2	16	5	2	4	2	4						
23	33	5	4	34	5	33	5	1	14	10	5	1	30					
1	1	5	1	24	1	13	5	13	4	10	20	2	5	5				
14	5	5	10	10	5	33	5	13	36	5	5	3	5	5	5			
1	13	2	1	3	5	1	2	4	5	5	20	2	2	2	9	2		
2	1	5	6	4	14	1	5	5	3	5	5	33	5	5	5	5	5	
13	7	2	13	20	5	2	1	20	19	14	5	20	1	5	1	1	1	3

5																		
6	40																	
26	13	24																
5	33	1	40															
1	5	1	5	1														
14	5	16	13	24	24													
4	20	13	5	2	5	36												
5	5	1	5	5	33	3	5											
4	23	5	5	5	2	13	20	36										
5	23	1	5	5	5	5	24	5	1									
1	23	3	4	20	20	5	5	40	5	5								
5	10	23	13	13	5	5	5	5	3	26	13							
13	4	5	13	23	13	2	5	13	2	16	2	1						
13	5	5	10	5	1	5	2	23	1	5	3	3	13					
5	5	4	20	20	33	3	3	23	3	1	2	5	14	5				
10	5	4	10	1	5	1	5	14	5	5	5	1	36	2	10			
2	1	5	5	5	1	2	20	1	14	2	2	20	3	36	20	5		
1	2	13	2	2	1	14	5	5	5	20	5	2	20	3	2	2	5	
2	10	10	14	1	5	2	5	3	3	19	20	7	2	14	1	1	5	5

Panel (a)

-0.03																		
0.02	-0.063																	
-0.014	0.016	-0.006																
0.057	-0.005	0.006	-0.041															
-0.005	0.059	0.009	0.025	0.019														
0.01	0.063	0.001	0.019	-0.02	-0.002													
0.042	0.103	0.006	0.084	0.022	-0.011	-0.003												
0.063	0.157	-0.011	0.082	0.08	-0.001	0.007	0.027											
0.017	-0.013	0.047	0.105	0.079	0.015	0.026	0.053	-0.001										
0.056	-0.016	0.014	0.192	0.145	0.11	0.082	-0.013	-0.057	0.005									
0.021	-0.006	0.01	0.015	0.2	0.213	0.063	0.1	-0.115	0.037	-0.01								
0.024	0.069	-0.002	0.026	0.023	0.302	0.146	0.047	-0.251	0.019	-0.015	0.002							
0.028	0.016	0.067	0.015	-0.013	0.002	0.282	0.164	0.011	0.038	0.019	0.018	0.003						
0.009	0.133	0.031	0.063	-0.007	0.002	-0.02	0.336	-0.006	0.043	0.021	0.023	0.014	0.002					
0.183	0.232	0.005	0.024	0.093	-0.001	0.011	0.032	-0.006	0.028	0.103	0.072	-0.018	0.017	-0.011				
0.209	0.286	0.001	0.163	0.025	0.128	0.021	0.04	0.002	0.02	0.053	0.063	0.065	-0.011	0.015	0.213			
0.359	0.059	-0.016	0.155	0.259	0.011	0.096	0.069	-0.003	0.06	0.059	0.043	0.062	0.02	-0.013	0.252	0.361		
-0.003	0.21	0.005	0.149	0.296	0.002	0.078	0.114	0.056	0.064	0.081	0.104	0.098	0.088	0.042	0.649	0.665	-0.021	
0.644	0.108	0.114	0.049	0.091	0.429	0.097	0.14	0.014	0.096	0.16	0.176	0.254	0.308	0.164	0.086	0.125	0.099	0.539

Panel (b)

Figure 7-2: Dependence structure and Kendall tau matrices of the manufacturing benchmark portfolio. Panel (a) displays the full sample period r-vine (on the left) and c-vine (on the right) dependence structure matrices of the portfolio. Panel (b) displays the full sample period r-vine Kendall tau correlation matrix of the portfolio. All matrices consist of 192 components. The numbers in the diagonal dependence structure matrices of Panel (a) represent the bivariate copulas listed and numbered in Table 5-1.

Counting (manufacturing portfolio):

According to Figure 7-2 and Table 7-3 the copulas most frequently selected by the c-vine, d-vine and r-vine models, under the full sample period scenario, to account for the dependence in the pairs of manufacturing stocks' joint distributions are: the Frank 56, 68 and 60 times for c-vine, d-vine and r-vine models respectively; the Joe-Frank 180 degrees rotated 8, 9 and 12 times; the Joe-Frank 8, 12 and 7 times; the Gaussian 30, 15 and 23 times; the Clayton 11, 9 and 12 times; the Student-t 17, 24 and 21 times; the Gumbel 180 degrees rotated 8, 8 and 7 times; the Clayton 180 degrees rotated 13, 9 and 14 times each; the Joe 180 degrees rotated 2, 1 and 2 times and; the Gumbel 8,6 and 6 times. Table 7-3 summarizes the recording, classification and grouping stages of the bivariate copula counting technique for the manufacturing bencportfolio.

Recording, classification and grouping (manufacturing portfolio):

Table 7-3: C-vine, d-vine and r-vine models' bivariate copula selection for the manufacturing portfolio

Bivariate Copula	**Full sample**			**Pre-GFC**			**GFC**			**Post-GFC**		
Vine model	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**	**C vine**	**D vine**	**R vine**
						Negative Tail						
Clayton	11	9	12	24	15	14	20	14	14	10	19	11
Gumbel 180	8	8	7	11	15	8	8	13	5	12	12	11
Studen-t	17	24	21	18	23	28	11	15	12	13	17	19
Joe 180	2	1	21	15	11	14	5	7	9	3	3	6
Joe-Frank 180	8	9	12	2	0	4	4	4	5	5	20	2
						Centre						
Frank	56	68	60	45	42	48	65	64	69	61	51	57
Gaussian	30	15	23	22	24	23	25	23	21	25	26	32
						Positive Tail						
Gumbel	8	6	6	5	5	3	9	11	10	8	5	2
Clayton 180	13	9	14	13	14	10	10	10	11	14	17	19
Clayton 90	2	6	7	4	12	11	0	8	3	7	4	5
Studen-t	17	24	21	18	23	28	11	15	12	13	17	19
Joe	1	3	1	7	3	3	7	3	3	7	5	5
Joe-Frank	8	12	7	4	1	3	1	2	7	1	2	2

Notes: the top row of the table displays the four financial period scenarios under consideration and the type of pair vine copulas fitted. The first column lists the bivariate copulas most frequently selected by the vine copula models to measure the dependence between the pairs of stocks. Each number in the table represents the number of times a certain bivariate copula has been selected by a certain vine copula model. The Student-t copula has been grouped with the copulas for positive and negative tail dependence because it measures the dependence in both tails symmetrically. The letters *C*, *D* and *R* stand for canonical, drawable and regular. The dependence structure located in the centre, negative tail and positive tail of the portfolio has been dissected, organized, counted, classified and grouped.

Aggregate dependence reading (manufacturing portfolio):

In the manufacturing benchmark portfolio most of the dependence is also concentrated in the centre of the pairs of stocks' joint distributions, as indicated by the predominance of the Frank copula in each of the four period scenarios considered. However, despite the manufacturing and retail benchmark portfolios having most of the dependence concentrated in the centre of the joint distributions, the manufacturing benchmark portfolio has it significantly smaller. This makes the manufacturing benchmark portfolio more dependence risky than the retail in times of financial turbulence. Another implication stemming from the high concentration of dependence the manufacturing benchmark portfolio has in the centre is that its returns are liable to change less frequently than those of the retail in tranquil stock market conditions and have a high probability of being extreme in non-tranquil periods.

A look into the Australian economy indicates that the higher riskiness of the manufacturing benchmark portfolio stems from the specific type of interdependence and multiplier effects it has with the Australian resources sector (ARA, 2014; Savills Research, 2014; Delloite, 2013; Mehmedovic et al., 2011), an important driver of the Australian economy. Specifically, the spill over effects the resources sector has on the Australian manufacturing sector differ from those spill over effects the Australian resources sector has on the retail sector. The spills over effects on the manufacturing sector are more volatile and deal with higher levels of uncertainty and risk aversion on behalf of investors. A possible explanation for this is that spending and investment in the manufacturing sector tends to require more capital.

The predominance of the Frank copula in the GFC period scenario suggests that most of the dependence relationships in that period are of nonlinear type. Besides, the returns of the manufacturing benchmark portfolio appear to be driven by complex relationships of dependence in the centre. The reduced presence of the Frank and increased presence of the Gaussian during the post-GFC suggests that the dependence relationships of the manufacturing stocks in that period are more of linear type. It also reflects the reduced volatility in the financial stock markets during the post-crisis period and a less chaotic world of dependence relationships.

Unlike in the retail benchmark portfolio, the Student-t copula in the manufacturing benchmark portfolio has its smallest presence in the GFC and its largest in the pre-GFC, indicating that most of the dependence during the GFC period scenario is of nonlinear and asymmetric type. This in turn, supports the idea about the manufacturing stocks being riskier than the retail stocks, and their propensity to yield negatively skewed returns when the financial stock markets lack the investors' confidence.

From the GFC to the post-GFC period scenarios the dependence structure is observed to change slightly, with minor increases in the number of stocks displaying positive tail dependence. This information suggests that the Australian manufacturing sector lagged behind the effects of the 2008-2009 GFC until the end of 2012. The recovery rate of the retail sector during the 2008-2009 GFC was observed to be higher than that of the manufacturing sector. Research conducted by the Australian Department of Innovation, Industry, Science and Research indicates that the manufacturing sector indeed recover at a slower pace during the post-crisis period (see also KordaMentha, 2013; CT, 2012; Green & Roos, 2012; NAB, 2012; DIISR, 2010). With respect to model selection, the d-vine copula model is observed to most frequently select the Frank copula under each of the four period scenarios considered. The d-vine is consequently the best model to account for the multivariate dependence structure of the manufacturing benchmark portfolio.

The significance testing of the manufacturing benchmark portfolio's relative comparison of dependence concentration displayed in Table 7-4 indicates that its overall dependence concentration in the centre is at the 95% confidence level significantly larger than those of the iron ore-nickel and oil-gas, and significantly smaller than that of the retail. This information confirms the lower dependence riskiness of the manufacturing stocks relative to the iron ore, nickel and oil stocks during crisis periods. It also confirms the higher dependence riskiness of the manufacturing stocks relative to the retail stocks in similar market conditions. The asymmetric dependence in the negative tail of the manufacturing stocks is significantly smaller than those of the iron ore, nickel, coal, uranium, oil, gas and mix-metals stocks. The portfolio's symmetric dependence concentration in the tails is significantly larger than those of the coal-uranium and mix-metals, and significantly smaller than that of the retail benchmark portfolio.

Table 7-4: Significance testing of the manufacturing benchmark portfolio's relative comparison of dependence

Significance testing of dependence	Gold	Iron ore-nickel	Coal-uranium	Oil-gas	Mix-metals	Retail
			Overall dependence (centre)			
Frank T-test	-0.24	7.26	0.95	3.67	0.69	3.34
Statistical significance	Neither	Sig. larger	Neither	Sig. larger	Neither	Sig. smaller
			Overall dependence (negative tail)			
Clayton T-test	1.08	-3.48	-3.10	-3.12	-1.18	-0.75
Gumbel 180 T-test	-3.59	-6.89	-2.99	-5.62	-0.79	-1.38
Joe 180 T-test	0.60	-3.28	-1.55	-0.15	-0.79	2.46
Joe-Frank 180 T-test	-1.04	-1.78	-2.78	-3.09	-3.30	0.35
Statistical significance	Neither	Sig. smaller	Sig. smaller	Sig. smaller	Neither	Neither
			Overall dependence (positive tail)			
Gumbel T-test	-1.07	3.63	1.65	0.09	1.96	-1.95
Clayton 180 T-test	2.36	2.74	3.15	2.73	2.21	-1.35
Joe T-test	3.00	1.29	2.46	-1.32	1.91	-1.81
Joe-Frank T-test	-1.46	2.61	0.12	3.40	-2.82	0.45
Statistical significance	Sig. larger	Sig. larger	Sig. larger	Sig. larger	Neither	Neither
			Symmetric dependence (tails)			
Student-t T-test	-0.10	-0.16	3.82	0.78	4.29	-2.44
Statistical significance	Neither	Neither	Sig. larger	Neither	Sig. larger	Sig. smaller
			Asymmetric dependence (negative tail)			
Clayton T-test	1.08	-3.48	-3.10	-3.12	-1.18	-0.75
Gumbel 180 T-test	-3.59	-6.89	-2.99	-5.62	-0.79	-1.38
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller	Sig. smaller	Neither	Neither
			Asymmetric dependence (positive tail)			
Gumbel T-test	-1.07	3.63	1.65	0.09	1.96	-1.95
Clayton 180 T-test	2.36	2.74	3.15	2.73	2.21	-1.35
Statistical significance	Sig. larger	Sig. larger	Sig. larger	Sig. larger	Sig. larger	Neither
			Critical value= $t_{(0.05,22)}$=±2.07			

Notes: The table displays the significance testing of the manufacturing benchmark portfolio's relative comparison of dependence concentration. The top row displays the names of the portfolios against which the manufacturing benchmark portfolio is compared with. The first column from left to right shows the copulas to which the t-test is implemented and the statistical significance category. The rest of the columns display the resulting t-test values, the type of dependence being tested and its location, and the significance testing results. The bottom row states the critical value used to determine the existence or not existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine statistical significance, the t-value of at least one copula is required to be larger or smaller than the critical value.

7.3 Discussion of results

The identification of the retail benchmark portfolio as being less dependence risky than the manufacturing in times of financial turbulence and in tranquil periods has to do with the specific type of economic linkages and relationship of dependence each of the stock portfolios' underlying sectors has with the Australian resources sector: the mining and energy sectors. In this regard, The Australian Retailers Association (2014), Savills Research (2014), Delloite (2013) and Mehmedovic et al. (2011) have identified a relationship of dependence between the Australian benchmark manufacturing sector and the resources sector. The predominance of the Frank copula in the dependence structure matrices of the retail and manufacturing benchmark portfolios is a feature shared with the gold, coal-uranium and mix-metals. The large presence of the Frank copula in the portfolios' dependence structure matrices indicates that the portfolios have low dependence risk in market conditions characterized by low confidence in the financial stock markets. This specific type of dependence concentration also indicates that the Frank copula is the most suitable model to account for the linear and nonlinear dependence relationships in the centre.

The recognition of the r-vine copula model as the most suitable to account for the multivariate dependence structure of the retail benchmark portfolio could, along with Dissmann (2010) and Dissmann et al. (2013), indicate that the retail benchmark portfolio's dependence structure is more complex than that of the manufacturing thus, requiring a vine copula model with greater flexibility. The vine copula modelling of dependence undertaken in this chapter, relative to the dependence risk modelling conducted by Fischer et al. (2009), Berg and Aas (2009), Aas et al. (2009), Chollete et al. (2009) and Heinen and Valdesogo (2009), has the comparative advantage of using a five-stage copula counting technique to dissect, organize, analyse and interpret the dependence structure of the assets modelled. As a result, a more comprehensive analysis of dependence risk is conducted. This chapter's modelling of the Australian retail and manufacturing sectors may also be the first that thoroughly examines their dependence risk dynamics using pair vine copulas.

7.4 Summary

This chapter implemented the copula counting technique to dissect, analyse and interpret the dependence structure and dependence risk dynamics of the retail and manufacturing benchmark portfolios. The retail benchmark portfolio was found to have most of the dependence concentrated in the centre of the joint distributions, an indication that the retail stocks have low dependence risk in non-tranquil stock market conditions and high dependence risk when the financial stock markets behave smoothly. These findings were confirmed by the actual performance of the retail sector during the 2008-2009 global financial crisis.

The manufacturing sector while having most of the dependence concentrated in the centre of the joint distributions, was acknowledged to have significantly smaller concentration of dependence in the centre, relative to the retail benchmark portfolio. As a result, the manufacturing benchmark portfolio is identified, at the 95% confidence level, to be significantly more dependence risky than the retail when the stock markets are unstable and when they are stable. The specific type of relationship and economic linkages the Australian resources sector has with the retail sector were identified to be important determinants of the retail sector's outperformance over the manufacturing sector.

CHAPTER 8

PORTFOLIO OPTIMIZATION

This chapter consists of three sections: introduction, average model convergence, and portfolio optimization

The *introduction* section states the motivation for the selection of multiple risk measures threaded with linear and nonlinear optimization methods to estimate the minimum risk optimal portfolios. The *average model convergence* section explains the average model convergence perspective proposed in this thesis to identify the stocks that could be good candidates for investment and to address the investment confidence problem underlying any type of portfolio optimization. The *portfolio optimization* section handles the estimated multiple risk measure-based optimal portfolios' weight allocations using the average model convergence perspective. The portfolios are compared in terms of their riskiness, with the most investment risky and least investment risky portfolios being identified.

8.1 Introduction

The multiple risk measure-based portfolio optimization conducted in this chapter minimizes the risk of the portfolios subject to a constant target return across portfolios. Some of the risk measures employed penalize the return distribution taking as a reference point a threshold value. These risk measures are known as downside risk measures and are characterized for being asymmetric in their dealing with the left tail or loss function of the return distribution (Morton et al., 2006; Chekhlov et al., 2003; Krokhmal et al., 2002; Rockafellar & Uryasev, 2000; Grootveld & Hallerbach, 1999; Nawrocki, 1999; Young, 1998; Konno & Shirakawa, 1994; Sortino & Price, 1994; Konno & Yamazaki, 1991). The risk measures identified as risk measures from modern portfolio theory scale the observations deviating from the measure of central tendency according to convex and linear functions (Ghalanos, 2013; Markowitz, 1959, 1952). Multiple risk measures and optimization methods are chosen for the optimization of the mining, energy, retail and manufacturing portfolios because they provide a wide array of investment scenarios that may cater for the investor's risk and return preferences, while setting the ground for the use of the "average model convergence" perspective.

This chapter's objectives are to identify the stocks that could be good candidates for investment, the least and most investment risky portfolios and the portfolio that offers the best risk-return trade-off.

8.2 The "average model convergence"

The average model convergence perspective proposed in this thesis is a simple approach to handling the multiple optimal weight allocations resulting from the fit of various optimization methods and risk measures in the form of portfolio optimization model specifications. The average model convergence represents a shift of perspective in the sense that it switches the focus of attention from the search of the best optimization method and risk measure to be used for the optimization of portfolios to the search for the stocks in which most of the model specifications' optimal weight allocations converge, on average. Model convergence and model consensus in the optimal weights is sought.

The proposed approach by shifting the focus of attention it attempts to address in a more objective manner the optimal stock selection and investment confidence problems underlying any type of portfolio optimization and faced by investors when having to select stocks from a wide array of optimal investment scenarios. The multiple risk measure-based portfolio optimization that does not consider the average model convergence tends to adopt a subjective solution to the optimal stock selection and investment confidence problems.

In identifying the stocks that are good candidates for investment, the average of the stocks' optimal weights resulting from the fit of the various model specifications is compared with each of the optimal weights. Thus, the stocks whose optimal weights do not largely deviate from the average of the optimal weights satisfy the average model convergence and are discerned as good candidates for investment. In Chapter 9 the average model convergence on the selected stocks is tested for statistical significance at the 95 and 99 per cent confidence levels. Also, it will be noticed that several of the stocks selected by the average model convergence have the investment features of being allocated large weights by most of the portfolio optimization model specifications and of having a high return relative to risk.

8.3 Portfolio optimization

The five risk measures fitted to the mining, energy, retail and manufacturing data sets to estimate the minimum risk optimal portfolios are the *variance*, *MAD*, *Minimax*, *CVaR* and *CDaR*. With the exception of the variance risk measure, which is threaded with a nonlinear quadratic optimization method, all others are threaded with linear model specifications. The *CVaR* risk measure uses probabilities to model the negative tail of the return distribution and forecast the portfolio's tail loss. Thus, it provides a probabilistic approximation of the loss that will exceed the *VaR*, with the *VaR* being used as the portfolio's threshold risk value in the negative tail (Krokhal et al., 2003). The estimation of the *CVaR* considers a 1-day time horizon and a 95% confidence level. The *CVaR* estimate indicates that with 95% probability the portfolio's loss will not exceed the 1-day *VaR*.

In the optimization of the portfolios with respect to the *CDaR*, the drawdowns of the historical return distribution are model to forecast the portfolio's loss. A portfolio's drawdown on a sample path is understood as the "the drop of the portfolio's value relative to the maximal value attained in the previous path's returns" (Krokhmal et al., 2003). The time horizon and confidence level used in the portfolio optimization with respect to the *CDaR* is similar to that used in the optimization with respect to the *CVaR*. The *CDaR* estimate indicates that with 95% probability the portfolio's drop in value will not exceed a certain percentage. The *Minimax* risk measure is mainly concerned with wealth preservation even if the return of the portfolio is zero (Ortobelli et al., 2005). This specific type of portfolio optimization is indicated to adjust quickly to structural shocks (Schaarschmidt & Schanbacher, 2012). The risk of the portfolio under the *Minimax* risk measure is interpreted in percentage values, just as the risk of the portfolios resulting from the fit of the *variance* and *MAD*. While the optimization with respect to the *MAD* penalizes the absolute deviations from the mean, the variance penalizes the deviation from the mean without considering their absolute value (Konno et al., 1993).

Each of the portfolios considered is optimized using the logarithmic return series corresponding to the pre-GFC, GFC, post-GFC and full sample period scenarios. In this chapter only the minimum risk optimal portfolios based on the full sample period are stated, analysed and discussed. The portfolio optimization results based on the logarithmic return series corresponding to the pre-GFC, GFC, post-GFC have been placed in Appendix D. Those results are not discussed because the analysis conducted in this chapter suffices to show how to use the average model convergence to optimally select stocks. The target return for the optimization of the portfolios is 4.2%. This specific target return value is reasonable relative to the 3.7% average annual return offered by cash deposits in Australia (Russell Investments & ASX, 2014). It should be noticed that the minimum risk portfolio optimization proposed is time invariant and the resulting weights are, as a consequence, determined by the historical distribution of the stock returns.

8.3.1 Mining portfolios

The application of the average model convergence to the gold mining portfolio's optimal weight allocations displayed in Table 8-1 indicates that most of the optimization methods and risk measures converge on average in the ST. BARBARA (SBMX) stock, when the portfolio optimization with respect to the *CDaR* is ignored. If the model specifications with respect to the *CDaR* and *Minimax* are discarded, the remaining models' optimal weights converge on average in NORTHWEST RESOURCES (NWRX) and RESOLUTE MINING (RSGS). This type of model convergence could be discerned by gold portfolio investors as model consensus and be used to select those stocks as good candidates for investment. In addition to that, the average model convergence is able to identify two gold stocks that have two of the best mean returns relative to risk in the portfolio. ST. BARBARA (SBMX) has in fact the best risk-return trade-off in the entire portfolio and is allocated large weights by most of the portfolio optimization model specifications.

Table 8-1: Optimal weights of the gold mining portfolio

Codes	Portfolio optimization					Weights' average			Stocks' descriptive statistics			
Gold codes	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR	μ	σ^2	K	SK
SBMX	30.01	44.28	24.25	24.93	29.23	30.54	27.11	28.06	0.07	0.18	4.56	-0.05
NWRX	3.53	0	0	4.18	4.53	2.45	3.06	4.08	-0.02	0.44	26.64	-1.10
NSTX	19.62	6.39	31.72	23.75	19.95	20.29	23.76	21.11	0.11	0.37	10.66	0.16
SHKX	0	0	0	0	0	0.00	0.00	0.00	-0.17	0.30	4.29	0.47
DEGX	0	0	0	0	0	0.00	0.00	0.00	-0.18	0.32	11.40	1.08
RSGX	13.54	0	0	14.15	13.28	8.19	10.24	13.66	0.01	0.15	5.75	-0.23
AXMX	0	0	0	0	0	0.00	0.00	0.00	-0.22	0.44	16.79	-0.15
ORNX	0	0	0	0	0	0.00	0.00	0.00	-0.16	0.36	6.61	-0.03
RCFX	0	0	0	0	0	0.00	0.00	0.00	-0.14	0.61	5.67	0.65
EXMX	0	0	0	0	0	0.00	0.00	0.00	-0.17	1.78	13.85	0.02
TAMX	0	0	0	1	0	0.20	0.25	0.33	-0.05	0.26	17.94	0.85
GLNX	0	0	0	0	0	0.00	0.00	0.00	-0.41	1.14	563.41	-17.93
MOYX	0	0	0	0	0	0.00	0.00	0.00	-0.15	0.45	22.31	0.11
EVNX	6.91	14.28	0	4.21	5.98	6.28	4.28	5.70	0.00	0.32	10.79	0.74
AUZX	0	0	0	0	0	0.00	0.00	0.00	-0.14	2.15	16.55	-0.00
HEGX	0	0	0	0	0	0.00	0.00	0.00	-0.09	0.29	3.09	0.45
KMCX	0	0	0	0	0	0.00	0.00	0.00	-0.21	0.53	45.01	-2.27
IRCX	13.66	35.05	0	13.9	15.63	15.65	10.80	14.40	0.01	0.28	10.24	0.70
HAOX	6.97	0	0	5.24	3.59	3.16	3.95	5.27	-0.02	0.67	18.06	1.85
CTOX	5.77	0	44.03	8.66	7.8	13.25	16.57	7.41	-0.02	0.19	27.91	2.05
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA	NA	NA	NA	NA
P-Risk	5.55	103.02	15.63	1.80	0.062	NA	NA	NA	NA	NA	NA	NA

Notes: This table reports the minimum risk optimal weights of the gold sector portfolio in percentage. The abbreviations LP, QP, MW and Var stand for linear programing, mean-variance quadratic programming, mean of weights and variance. The names and codes of the stocks are provided in Table 3-1. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively. The μ, σ^2, K and SK stand for mean, variance, kurtosis and skewness.

Although the descriptive statistics could be used to identify the stocks that could be good candidates for investment, their identification by means of the average model convergence provides investment confidence that is based on model convergence and model consensus. The most extreme weight allocations are produced by the model specifications with respect to the *CDaR* and *Minimax*. This weight allocation pattern is encountered in each of the portfolios considered. Also, notice that most likely the reason why several stocks are allocated zero weights is because the risk measures and optimization models identify those stocks as high risk.

Out of the gold stocks with the largest kurtosis CITIGOLD (CTOX) offers the best risk-return trade-off since it has a small negative mean return, a relatively small variance and a large positive skewness. Out of the gold stocks with the largest skewness CITIGOLD (CTOX) has the less adverse mean return relative to risk. The gold stocks' descriptive statistics also indicate that ST. BARBARA (SBMX), NORTHERN STAR (NSTX), RESOLUTE MINING (RSGS) and INTERMIN RESOURCES (IRCX) have the largest mean return relative to risk. Despite INTERMIN RESOURCES (IRCX) and NORTHERN STAR (NSTX) having two of the best risk-return trade-offs in the portfolio, the average model convergence does not identify them as good candidates for investment. The risk comparison of the portfolio with the rest of the portfolios indicates that it is more investment risky than the iron ore-nickel, coal-uranium, mix-metals, retail and manufacturing and less investment risky than the oil.

Table 8-2: Optimal weights of the iron ore-nickel mining portfolio

Codes	**Portfolio optimization**					**Weights' average**			**Stocks' descriptive statistics**			
Ore-nickel codes	**CVaR (LP)**	**CDaR (LP)**	**Minimax (LP)**	**MAD (LP)**	**Var (QP)**	**MW**	**MW ex. CDaR**	**MW ex. Minimax & CDaR**	**μ**	**σ^2**	**K**	**SK**
BHPX	46.72	53.15	39.52	39.38	39.62	43.68	41.31	41.91	0.04	0.05	4.10	-0.23
GBGX	0.00	0.00	0.00	2.27	0.62	0.58	0.72	0.96	0.09	0.20	5.25	0.32
MCRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.13	3.91	0.43
WSAX	1.70	0.00	0.00	5.50	2.74	1.99	2.49	3.31	0.05	0.10	4.62	0.08
AGOX	1.83	0.00	4.19	2.10	4.04	2.43	3.04	2.66	0.11	0.21	6.70	0.64
FMSX	1.48	0.86	0.00	3.15	2.21	1.54	1.71	2.28	0.08	0.64	283.20	10.78
GRRX	0.00	0.00	0.00	1.62	2.20	0.76	0.96	1.27	-0.01	0.20	10.65	0.39
ARHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.13	0.33	8.81	1.01
ARI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05	0.08	6.26	-0.18
FCNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.18	0.37	12.14	0.72
POSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05	0.38	22.34	1.94
HRRX	0.63	0.00	0.00	4.12	2.59	1.47	1.84	2.45	-0.03	0.22	13.26	1.37
MGXX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.16	6.59	0.12
ADYX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11	0.42	13.36	1.46
FMGX	6.81	4.32	0.55	5.35	5.41	4.49	4.53	5.86	0.15	0.19	10.82	0.44
ILUX	27.35	41.66	46.59	22.64	27.38	33.12	30.99	25.79	0.04	0.07	3.30	0.10
IGOX	1.44	0.00	9.14	5.88	3.84	4.06	5.08	3.72	0.06	0.12	3.31	0.22
SHDX	3.32	0.00	0.00	2.70	2.46	1.70	2.12	2.83	-0.05	0.29	10.49	0.50
MLMX	8.71	0.00	0.00	5.27	6.87	4.17	5.21	6.95	0.02	0.22	2.91	0.36
MOLX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.12	0.29	5.61	0.67
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA	NA	NA	NA	NA
P-Risk	4.39	40.91	7.94	1.35	0.035	NA	NA	NA	NA	NA	NA	NA

Notes: This table reports the minimum risk optimal weights of the iron ore-nickel sector portfolio in percentage. The abbreviations LP, QP, MW and Var stand for linear programing, mean-variance quadratic programming, mean of weights and variance. The names and codes of the stocks are provided in Table 3-1. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively. The μ, σ^2, K and SK stand for mean, variance, kurtosis and skewness.

The optimal weight allocations of the iron ore-nickel mining portfolio displayed in Table 8-2 indicate that most of the optimization methods and risk measures converge on average in the BHP BILLITON (BHPX), when the optimal weight allocations with respect to the *CDaR* and *CVaR* are ignored. This stock, in addition to that, is allocated large weights by most of the portfolio optimization model specifications and, according to the stocks' descriptive statistics, it has the largest return relative to risk in the entire portfolio. The stock is thus a good candidate for investment not only because of the large weights it is allocated and the large positive return it has relative to risk but primarily because it is backed or supported by model convergence or model consensus. Notice also that there are cases where most of the portfolio optimization model specifications allocate zero weights to stocks with high return. As compared to other stocks with high return and low cvariance that are allocated large weights, those stocks while having a high return also have a large variance. Consequently, this is the reason why they are allocated a zero weight, and occurs in other portfolios too.

Out of the iron ore and nickel stocks with the largest kurtosis and skewness FLINDERS MINES (FMSX) offers the best risk-return trade-off. BHP BILLITON (BHPX), ILUKA RESOURCES (ILUX) and FORTESCUE METALS (FMGX) offer the highest return relative to risk. Out of these stocks only BHP BILLITON (BHPX) is backed by the average model convergence. The risk comparison of the portfolio with the rest of the portfolios indicates that it is less dependence risky than the gold, mix-metals coal-uranium and oil-gas and more risky than the retail and manufacturing benchmark portfolios.

Table 8-3: Optimal weights of the mix-metals leptokurtic portfolio

Codes	Portfolio optimization					Weights' average			Stocks' descriptive statistics			
Mix-metals codes	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR	μ	σ^2	K	SK
RIOX	31.22	0.00	25.39	33.03	32.88	24.50	30.63	32.38	0.03	0.07	33.29	-2.25
BCDX	0.18	0.00	0.00	0.00	0	0.04	0.05	0.06	-0.21	0.31	85.38	-3.95
CAZX	2.22	0.03	0.00	1.01	1.6	0.97	1.21	1.61	-0.01	0.47	48.68	-2.00
CDUX	7.97	34.73	0.00	7.36	8.32	11.68	5.91	7.88	0.09	0.32	35.59	-0.34
FMSX	3.38	12.14	0.00	4.00	4.03	4.71	2.85	3.80	0.08	0.64	283.20	10.78
FNTX	0.16	0.00	0.00	0.69	1.27	0.42	0.53	0.71	-0.04	0.68	49.29	2.91
GLNX	0.00	0.00	0.00	0.00	0	0.00	0.00	0.00	-0.41	1.14	563.41	-17.94
KMCX	0.00	0.00	0.00	0.00	0	0.00	0.00	0.00	-0.21	0.53	45.01	-2.27
MAHX	6.17	0.00	18.85	11.22	13.67	9.98	12.48	10.35	0.01	0.13	29.61	-2.04
NAVX	0.00	0.00	0.00	0.00	0	0.00	0.00	0.00	-0.13	0.38	14.66	-0.66
PNAX	1.53	0.00	0.00	3.54	4.49	1.91	2.39	3.19	0.07	0.21	33.56	1.86
PHRX	2.03	0.00	5.92	0.08	0.06	1.62	2.02	0.72	-0.12	0.78	304.36	-0.24
PDZX	0.00	0.00	0.00	0.00	0	0.00	0.00	0.00	-0.23	1.11	868.23	-24.02
RMSX	16.10	26.64	7.71	8.82	10.86	14.03	10.87	11.93	0.07	0.25	68.89	3.86
SARX	17.67	0.00	38.10	17.67	13.18	17.32	21.66	16.17	0.10	0.26	142.80	6.49
SIRX	0.00	0.00	0.00	0.00	0	0.00	0.00	0.00	-0.42	1.80	1074.1	-28.03
AYNX	0.00	0.00	0.00	0.00	0	0.00	0.00	0.00	-0.21	0.65	888.76	-24.45
UMLX	0.00	12.36	4.03	0.00	0	3.28	1.01	0.00	-0.10	0.20	85.65	-3.42
BWDX	8.06	0.00	0.00	8.53	5.07	4.33	5.42	7.22	-0.05	0.32	480.33	13.98
WECX	3.31	14.09	0.00	4.05	4.55	5.20	2.98	3.97	-0.03	0.24	150.40	-6.72
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA	NA	NA	NA	NA
P-Risk	4.77	88.52	10.83	1.44	0.043	NA	NA	NA	NA	NA	NA	NA

Notes: This table reports the minimum risk optimal weights of the mix-metals leptokurtic sector portfolio in percentage. The abbreviations LP, QP, MW and Var stand for linear programing, mean-variance quadratic programming, mean of weights and variance. The names and codes of the stocks are provided in Table 3-4. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively. The μ, σ^2, K and SK stand for mean, variance, kurtosis and skewness.

The optimal weight allocations of the mix-metals leptokurtic portfolio displayed in Table 8-3 indicate that the portfolio optimization model specifications converge on average in RIO TIONTO (RIOX) and CUDECO (CUX), when the model specifications with respect to the *CDaR* and *Minimax* are ignored. This stock, just as ST. BARBARA (SBMX) from the gold mining portfolio and BHP BILLITON (BHPX) from the iron ore-nickel mining portfolio, is allocated large weights by most of the model specifications and has the best risk-return trade-off in the entire portfolio. Hence, the good investment characteristics of the stock, as indicated by its large mean return relative to risk and the large weights it is allocated, are further supported by the model convergence.

The descriptive statistics of the stocks in the mix-metals portfolio indicate that out of the stocks with largest kurtosis and skewness FLINDERS MINES (FMSX) offers the best risk-return trade-off. In terms of mean return and variance, RIO TIONTO (RIXO), SARACEN MINERALS (SARX) and PANAUST (PNAX) offer the best risk-return trade-off. Out of these stocks only RIO TIONTO (RIXO) is backed by the average model convergence. The risk comparison of the portfolio with the rest of the portfolios indicates that it is less investment risky than the gold and oil-gas and more risky than the iron ore-nickel, coal-uranium, retail and manufacturing.

8.3.2 Energy portfolios

The optimal weight allocations of the coal-uranium energy portfolio displayed in Table 8-4 indicate that most of the portfolio optimization model specifications converge on average in COAL BANK (CBQX), AQUILA RESOURCES (AQAX) and COALSPURN (CPLX), if the portfolio optimizations with respect to the *CDaR* and *Minimax* are ignored. While COAL BANK (CBQX) has a negative return, AQUILA RESOURCES (AQAX) and COALSPURN (CPLX) have two of the best risk-return trade-offs in the portfolio and are allocated relatively large weights. Out of the coal and uranium stocks with the largest kurtosis BLACKWOOD (BWDX) has the largest positive skewness and the least adverse risk-return trade-off.

The stocks with the largest return relative to risk in the portfolio are PALADIN ENERGY (PDNX), AQUILA RESOURCES (AQAX), SUMMIT RESOURCES (SMMX), COALSPURN (CPLX), ALLIANCE RESOURCES (AGSX) and BERKELEY RESOURCES (BKYX). Out of these stocks, only AQUILA RESOURCES (AQAX) and COALSPURN (CPLX) are backed by the average model convergence. AQUILA RESOURCES (AQAX), in addition to that, has the largest return relative to risk in the entire portfolio. The risk comparison of the coal-uranium energy portfolio indicates that it is less investment risky than the mix-metals and gold, and more investment risky than the iron ore-nickel mining portfolio.

Table 8-4: Optimal weights of the coal-uranium energy portfolio

Codes	**Portfolio optimization**					**Weights' average**			**Stocks' descriptive statistics**			
Coal-uranium Codes	**CVaR (LP)**	**CDaR (LP)**	**Minimax (LP)**	**MAD (LP)**	**Var (QP)**	**MW**	**MW ex. CDaR**	**MW ex. Minimax & CDaR**	**μ**	**σ^2**	**K**	**SK**
PDNX	0.00	0.00	0.00	2.17	0.00	0.43	0.54	0.72	0.05	0.17	3.65	-0.02
CBQX	3.97	4.36	1.47	3.70	4.16	3.53	3.33	3.94	-0.03	0.46	13.79	0.15
CLAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.31	1.48	1558.4	-37.34
LRRX	0.00	0.00	0.00	0.10	0.00	0.02	0.03	0.03	-0.21	1.09	9.84	0.26
AQAX	17.45	0.00	1.41	17.63	16.70	10.64	13.30	17.26	0.10	0.15	7.06	0.59
SMMX	26.06	59.72	32.32	17.31	19.94	31.07	23.91	21.10	0.11	0.17	4.82	0.43
GLLX	3.57	14.26	0.00	3.34	4.40	5.11	2.83	3.77	-0.08	0.34	8.34	0.50
CPLX	12.60	2.40	19.41	12.26	13.58	12.05	14.46	12.81	0.13	0.25	13.43	0.02
RESX	0.00	13.14	0.00	0.08	0.00	2.64	0.02	0.03	-0.12	0.57	377.64	-12.75
CNXX	0.84	6.13	0.00	3.50	2.59	2.61	1.73	2.31	0.01	0.36	6.62	0.37
BWDX	11.49	0.00	0.00	14.86	7.99	6.87	8.59	11.45	-0.05	0.32	480.33	13.98
UEQX	0.00	0.00	0.00	0.35	0.40	0.15	0.19	0.25	-0.08	0.33	2.86	0.18
AGSX	1.88	0.00	0.00	4.90	3.84	2.12	2.66	3.54	0.09	0.32	10.80	1.25
EMAX	5.92	0.00	0.00	5.65	8.36	3.99	4.98	6.64	-0.05	0.11	5.23	-0.45
FYIX	4.56	0.00	6.83	1.02	2.17	2.92	3.65	2.58	-0.23	0.39	17.55	0.26
BLZX	0.00	0.00	0.65	0.30	0.69	0.33	0.41	0.33	-0.22	1.08	16.50	0.05
NSLX	0.00	0.00	0.00	0.60	0.25	0.17	0.21	0.28	-0.17	0.73	57.46	-3.08
AQCX	0.62	0.00	8.48	0.89	0.81	2.16	2.70	0.77	-0.08	0.69	9.75	0.18
BKYX	5.51	0.00	14.62	6.88	8.04	7.01	8.76	6.81	0.07	0.29	10.77	-0.17
WALX	5.51	0.00	14.80	4.47	6.08	7.72	7.72	5.35	-0.04	0.41	28.08	1.04
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA	NA	NA	NA	NA
P-Risk	4.81	83.68	9.21	1.44	0.042	NA	NA	NA	NA	NA	NA	NA

Notes: This table reports the minimum risk optimal weights of the coal-uranium sector portfolio in percentage. The abbreviations LP, QP, MW and Var stand for linear programing, mean-variance quadratic programming, mean of weights and variance. The names and codes of the stocks are provided in Table 3-2. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively. The μ, σ^2, K and SK stand for mean, variance, kurtosis and skewness.

The minimum risk optimal weight allocations of the oil-gas energy portfolio displayed in Table 8-5 indicate that most of the optimization methods and risk measures converge on average in BEACH ENERGY (BPTX). This stock, in addition to that, is allocated extremely large weights by each of the model specifications and has one of the best risk-return trade-offs in the portfolio. This stock could therefore be seen as a good candidate for investment because of the large weights it is allocated, the large mean return relative to risk it offers and the backing it receives from the model convergence and model consensus. Also, with the exception of the model specifications with respect to the *MAD* and *variance* risk measures, the remaining models assign weights to ORIGIN ENERGY

(ORGX) that do not vary much from each other. This stock also has a relatively high return relative to risk.

The oil-gas stocks' descriptive statistics indicate that out of the oil and gas stocks with the largest kurtosis only ORIGIN ENERGY (ORGX) and COOPER (COEX) have a positive mean return. Out of the stocks with the largest skewness ORIGIN (ORGX) has the best risk-return trade-off. In terms of mean and variance, WOODSIDE (WPLX), ORIGIN (ORGX), APA (APAX), BEACH ENERGY (BPTX), SANTOS (STOX) and CALTEX (CTXX) have the best risk-return trade-offs in the portfolio. The average model convergence only supports the selection of two of these stocks. The risk comparison of the portfolio with the rest of the portfolios indicates that it is more risky than any other portfolio.

Table 8-5: Optimal weights of the oil-gas energy portfolio

Codes	**Portfolio optimization**					**Weights' average**			**Stocks' descriptive statistics**			
Oil-gas Codes	**CVaR (LP)**	**CDaR (LP)**	**Minimax (LP)**	**MAD (LP)**	**Var (QP)**	**MW**	**MW ex. CDaR**	**MW ex. Minimax & CDaR**	**μ**	**σ^2**	**K**	**SK**
WPLX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	3.73	-0.11
AWEX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.07	2.40	-0.21
BPTX	94.94	95.13	95.13	93.87	93.87	94.59	94.45	94.23	0.04	0.09	2.35	0.07
MOGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.11	0.58	109.85	2.31
NWEX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04	0.40	19.03	1.41
STOX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.05	4.14	-0.20
STXX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.20	3.79	0.55
ACN	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08	0.34	145.90	6.48
LNGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.22	2.83	0.47
CTXX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.06	1.83	-0.18
ORGX	4.29	4.87	4.87	0.00	0.00	2.81	2.29	1.43	0.03	0.03	36.52	2.24
CUEX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	0.14	3.74	0.37
BASX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.16	0.37	29.47	-1.70
ROCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08	0.12	9.70	-0.52
MELX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.22	8.23	1.16
TPTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.51	20.88	-0.86
DLSX	0.77	0.00	0.00	0.00	0.00	0.15	0.19	0.26	0.03	0.24	9.63	-0.11
APAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.04	6.42	0.05
SYSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.19	0.57	23.41	0.96
COEX	0.00	0.00	0.00	6.13	6.13	2.45	3.07	4.09	0.03	0.12	21.40	0.98
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA	NA	NA	NA	NA
P-Risk	6.16	88.00	15.69	2.04	0.078	NA	NA	NA	NA	NA	NA	NA

Notes: This table reports the minimum risk optimal weights of the oil-gas sector portfolio in percentage. The abbreviations LP, QP, MW and Var stand for linear programing, mean-variance quadratic programming, mean of weights and variance. The names and codes of the stocks are provided in Table 3-2. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively. The μ, σ^2, K and SK stand for mean, variance, kurtosis and skewness.

8.3.3 Retail and manufacturing portfolios

The multiple optimal weight allocations of the retail benchmark portfolio displayed in Table 8-6 indicate that most of the portfolio optimization model specifications converge on average in M2 TELECOM (MTUX), WOOLWORTHS (WOWX) and ARB (ARPX), when the optimizations with respect to the *CDaR* and *Minimax* are ignored. All three stocks have some of the best risk-return trade-offs in the entire portfolio and have been allocated large weights. The stocks are therefore desirable for investment not only because of their large return relative to risk and the large weights they have been allocated, but also because they are backed by the average model convergence.

Table 8-6: Optimal weights of the retail benchmark portfolio

Codes	**Portfolio optimization**					**Weights' average**			**Stocks' descriptive statistics**			
Retail Codes	**CVaR (LP)**	**CDaR (LP)**	**Minimax (LP)**	**MAD (LP)**	**Var (QP)**	**MW**	**MW ex. CDaR**	**MW ex. Minimax & CDaR**	**μ**	**σ^2**	**K**	**SK**
CCLX	16.83	15.02	30.47	12.78	13.95	17.81	18.51	14.52	0.03	0.03	5.15	-0.17
HILX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07	0.06	6.78	0.14
GWAX	0.00	0.00	1.97	0.00	0.00	0.39	0.49	0.00	-0.02	0.05	2.77	0.14
MTUX	12.82	22.62	14.13	11.08	11.34	14.40	12.34	11.75	0.12	0.10	4.69	0.50
MTSX	10.10	33.90	1.77	7.71	7.55	12.21	6.78	8.45	0.01	0.02	4.38	-0.14
WOWX	29.52	28.46	1.84	27.45	25.60	22.57	21.10	27.52	0.03	0.02	5.57	-0.33
ARPX	19.57	0.00	31.72	22.11	22.90	19.26	24.08	21.53	0.05	0.03	5.47	0.10
CCVX	3.51	0.00	0.00	5.21	4.29	2.60	3.25	4.34	0.04	0.10	5.38	-0.21
DJSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.05	6.22	-0.26
DLCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07	0.60	9.17	0.33
HVNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02	0.05	4.05	0.16
JBHX	0.00	0.00	0.00	2.54	2.93	1.09	1.37	1.82	0.04	0.06	4.72	-0.11
RCG	1.85	0.00	0.00	0.28	0.44	0.51	0.64	0.86	0.00	0.21	8.64	0.20
SFHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04	0.10	5.17	0.48
SULX	5.38	0.00	10.78	8.75	8.56	6.69	8.37	7.56	0.05	0.06	6.68	-0.25
WESX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.03	8.31	-0.39
FANX	0.00	0.00	0.00	0.20	0.00	0.04	0.05	0.07	-0.03	0.06	9.59	-0.44
GZLX	0.41	0.00	7.32	1.71	2.42	2.37	2.97	1.51	-0.03	0.05	17.29	-0.80
FLTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.07	9.55	0.07
JETX	0.00	0.00	0.00	0.16	0.00	0.03	0.04	0.05	-0.02	0.10	5.78	0.12
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA	NA	NA	NA	NA
P-Risk	2.09	25.94	3.65	0.669	0.008	NA	NA	NA	NA	NA	NA	NA

Notes: This table reports the minimum risk optimal weights of the retail sector portfolio in percentage. The abbreviations LP, QP, MW and Var stand for linear programing, mean-variance quadratic programming, mean of weights and variance. The names and codes of the stocks are provided in Table 3-3. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively. The μ, σ^2, K and SK stand for mean, variance, kurtosis and skewness.

According to the stocks' descriptive statistics, out of the retail stocks with the largest kurtosis FLIGHT CENTER (FLTX) has the best risk-return trade-off. Out of the retail stocks with the largest skewness M2 TELECOM (MTUX) and WOOLWORTHS (WOWX) offer the best risk-return trade-off. In terms of mean and variance, COCA-COLA (CCLX), M2 TELECOM (MTUX), WOOLWORTHS (WOWX) and ARB (ARPX) offer the best risk-return trade-off. The average model convergence supports the selection of most of these stocks. The risk comparison of the portfolio with the rest of the portfolios indicates that it is less risky than any other portfolio.

The multiple optimal weight allocations of the manufacturing benchmark portfolio displayed in Table 8-7 indicate that most of the portfolio optimization model specifications converge on average in CSL (CSLX), BRICKWORKS (BKWX) and ANSELL (ANNX), when the model specifications with respect to the *CDaR* and *Minimax* are ignored. These stocks' descriptive statistics indicate that CSL (CSLX) and ANSELL (ANNX) have two of the largest mean returns relative to risk in the portfolio and are allocated large weights by most of the optimization methods and risk measures. These two investment features together with the average model convergence make those stocks to be good candidates for investment.

Table 8-7: Optimal weights of the manufacturing benchmark portfolio

Codes	Portfolio optimization					Weights' average			Stocks' descriptive statistics			
Manufacturing codes	**CVaR (LP)**	**CDaR (LP)**	**Minimax (LP)**	**MAD (LP)**	**Var (QP)**	**MW**	**MW ex. CDaR**	**MW ex. Minimax & CDaR**	**μ**	**σ^2**	**K**	**SK**
SFCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06	0.04	16.39	-1.38
BLDX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04	0.05	3.78	-0.14
BKWX	13.03	7.33	5.33	12.10	10.18	9.59	10.16	11.77	-0.01	0.03	7.57	0.26
CSRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07	0.05	7.69	-0.68
JHXX	0.00	8.47	0.00	0.00	0.00	1.69	0.00	0.00	0.01	0.06	4.83	0.42
OLHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.13	0.15	23.73	-1.04
CKLX	0.00	0.76	0.00	1.38	1.57	0.74	0.74	0.98	-0.01	0.06	5.08	0.10
ANNX	17.32	13.03	35.14	15.81	18.35	19.93	21.66	17.16	0.02	0.03	2.50	0.33
SDIX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.13	0.17	12.64	0.45
SOMX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08	0.37	10.52	0.18
UCMX	0.00	2.79	0.00	0.00	0.00	0.56	0.00	0.00	-0.17	0.25	17.43	-0.60
FWDX	13.79	0.00	10.24	9.55	12.22	9.16	11.45	11.85	0.02	0.04	6.85	0.04
FANX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03	0.06	9.59	-0.44
KRSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.08	0.15	11.24	-0.43
ASBX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.06	8.43	0.46
MHIX	0.66	0.00	0.00	3.78	2.27	1.34	1.68	2.24	0.00	0.13	23.90	-0.03
CSLX	54.70	65.88	49.29	56.52	54.82	56.24	53.83	55.35	0.07	0.03	2.73	0.04
IDTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.12	0.10	11.57	-0.13
CDAX	0.50	0.00	0.00	0.87	0.58	0.39	0.49	0.65	-0.01	0.08	11.39	0.69
LGDX	0.00	1.75	0.00	0.00	0.00	0.35	0.00	0.00	-0.03	0.18	90.45	-4.44
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA	NA	NA	NA	NA
P-Risk	2.72	24.49	5.14	0.901	0.015	NA	NA	NA	NA	NA	NA	NA

Notes: This table reports the minimum risk optimal weights of the manufacturing sector portfolio in percentage. The abbreviations LP, QP, MW and Var stand for linear programing, mean-variance quadratic programming, mean of weights and variance. The names and codes of the stocks are provided in Table 3-3. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* refer to the mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively. The μ, σ^2, K and SK stand for mean, variance, kurtosis and skewness.

Out of the stocks with the largest kurtosis MERCHANT HOUSE (MHIX) offers the best risk-return trade-off. Out of the stocks with the largest skewness CODAN (CDAX) has the least adverse risk-return trade-off. In terms of mean and variance, CSL (CSLX), FLEETWOOD (FWDX), ANSELL (ANNX) and JAMES HARDIE (JHXX) have the best risk-return trade-offs in the portfolio. The average model convergence only supports the selection of two of these stocks. The risk comparison of the manufacturing benchmark portfolio with the rest of the portfolios indicates that it is more risky than the retail and less risky than any other portfolio.

According to Figure 8-1, where the full sample period efficient frontiers with respect to the *CVaR* are depicted, the efficient frontier of the oil-gas energy portfolio moves towards the right at a higher risk-return ratio. Hence, under this particular risk measure the oil-gas portfolio is the most risky. The efficient frontiers of the portfolios for each of the four period scenarios and risk measures considered have been placed in Appendix C.

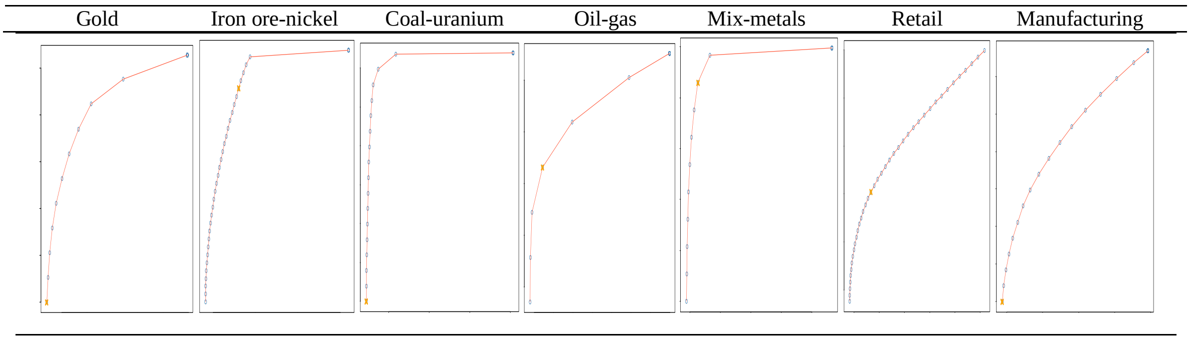

Figure 8-1: Full sample period efficient frontiers of the portfolios under the *CVaR* measure. From left to right the efficient frontiers of the gold, iron ore-nickel, coal-uranium, oil-gas, mix-metals, retail and manufacturing portfolios are displayed.

The portfolios' risk analysis for each of the four risk measures and period scenarios considered displayed in Table 8-8 indicates that under the full sample period the oil-gas energy portfolio is the most risky, followed by the gold mining portfolio. By contrast, the retail benchmark portfolio is the least risky, followed by the manufacturing benchmark portfolio. Under the pre-GFC and GFC period scenarios the gold mining portfolio is the most risky, while the retail remains as the least risky. In the post-GFC period scenario the coal-uranium energy portfolio is the most risky, while the retail remains as the least risky. Considering that the full sample period accounts for the dependence structure and volatility changes between period scenarios and that the number of observations in the full sample period is the largest, a higher weight is given to the measure of the portfolios' risk under this period. Thus, overall the oil-gas energy portfolio is the most risky. Since each of the portfolios has been optimized using the same target return, and the least risky portfolio is the retail, this portfolio offers the best risk-return trade-off.

An analysis of the changes in the risk of the portfolios between pairs of period scenarios indicates that the portfolios' risk for each of the risk measures considered is lower during the pre-GFC; an indication that the 2008-2009 GFC had not yet unfolded. On the contrary, during the GFC period scenarios the portfolios display the highest risk exposure, reflecting the exceptionally high volatility in the stock markets. In the post-GFC the portfolios display lower risk relative to the GFC. The portfolio's risk that fluctuates the most between period scenarios is the one estimated according to the *CDaR*. The largest *CDaR* values appear in the GFC and full sample period scenarios. The portfolio's risk that fluctuates the least is the one estimated according to the *MAD* risk measure.

Table 8-8: Four-period scenario portfolios' risk comparison for all risk measures

Portfolios	**Gold**	**Iron ore-nickel**	**Coal-uranium**	**Oil-gas**	**Mix-metals**	**Retail**	**Manufacturing**
Target portfolio return =0.042							
Pre-GFC							
CVaR	3.181	2.971	3.059	2.128	2.318	1.309	1.429
CDaR	19.839	15.674	17.108	8.287	15.983	7.089	10.776
MinMax	4.256	5.147	4.028	2.615	2.771	1.659	1.821
MAD	1.117	0.965	0.994	0.731	0.782	0.448	0.519
Var	0.023	0.014	0.018	0.009	0.011	0.004	0.005
GFC							
CVaR	5.962	5.428	5.751	3.556	5.864	2.58	4.675
CDaR	116.086	39.559	55.146	26.033	93.664	26.864	23.576
MinMax	11.71	10.387	8.561	6.997	12.762	3.537	7.427
MAD	1.969	1.756	1.698	1.182	1.829	0.879	1.38
Var	0.080	0.063	0.057	0.029	0.072	0.014	0.045
Post-GFC							
CVaR	4.12	4.003	4.172	2.656	3.914	1.561	1.992
CDaR	20.167	31.116	38.512	10.704	24.233	8.178	9.323
MinMax	5.983	5.898	8.305	4.166	6.441	2.344	3.028
MAD	1.41	1.32	1.303	0.889	1.261	0.518	0.68
Var	0.037	0.031	0.049	0.014	0.04	0.005	0.009
Full sample period							
CVaR	5.55	4.39	4.81	6.16	4.77	2.09	2.72
CDaR	103.02	40.91	83.68	88.00	88.52	25.94	24.49
MinMax	15.63	7.94	9.21	15.69	10.83	3.65	5.14
MAD	1.80	1.35	1.44	2.04	1.44	0.669	0.901
Var	0.062	0.035	0.042	0.078	0.043	0.008	0.015

Notes: This table displays the risk of the portfolios resulting from the fit of the various optimization methods and risk measures. The risk of the portfolios is estimated for each of the four financial period scenarios considered. The first column from left to right defines the risk of the portfolios for each of the risk measures considered. The target portfolio return used on each of the portfolio optimization model specifications to estimate the risk of the portfolios is 4.2%. The same target portfolio return is used for the estimation of the portfolio's risk under the pre-GFC, GFC, post-GFC and full sample period scenarios.

A summary of the results indicates that the most risky portfolio is the oil-gas energy portfolio, while the least risky is the retail. The retail benchmark portfolio offers the best risk-return trade-off because it has the lowest risk subject to a constant target return across portfolios. These findings are in its majority consistent with the results from Chapters 5, 6 and 7, where the oil-gas energy portfolio is identified to be the second most risky during crisis periods, while the retail is the second least dependence risky in similar market conditions, and the least dependence risky in tranquil stock market conditions. In general, the average model convergence is observed to select stocks that are allocated large weights and have a high return relative to risk. The approach proposed appears to address in a more objective manner the optimal stock selection and investment confidence problems.

8.4 Discussion of results

The identification of the oil-gas energy portfolio as the most investment risky is to a large extent consistent with the results from Chapters 5 and 6, where the oil-gas is recognized to be the second most dependence risky. This finding is also in line with the literature examining the risk of oil stock assets. Faff and Brailsford (1999), for instance, find oil prices to exert some influence on the Australian stock markets. Du et al. (2012), Killian and Park (2009), Park and Ratti (2008) and Basher and Sadorsky (2006) also identify the risk in oil markets in various contexts and conditions. The identification of the retail portfolio as the least risky and, consequently, as the one offering the best risk-return trade-off is in congruence with the results from Chapter 7, where the retail benchmark portfolio outstands as the second least dependence risky during crisis periods and the least dependence risky in tranquil periods.

Relative to the studies of the retail and manufacturing sectors conducted by ARA (2014), Savills Research, (2014), Delloite (2013), KordaMentha (2013), CT (2012), Green and Roos (2012), NAB (2012), Connolly and Orsmond (2011), AGPC (2011), PC (2011), Mehmedovic et al. (2011) and DIISR (2010), this chapter's research examines thoroughly and comprehensively the dependence risk of the asset's modelled in specific market conditions. The comparison of the retail and manufacturing benchmark portfolios with the mining and energy portfolios shows that the benchmark portfolios are overall less dependence risky and less investment risky. The specific economic linkages each of the sectors has with the Australian resources sector and the diversity of commodity assets driving the performance of the Australian resources sector are identified to be the main reason for their dependence risk differences.

The suitableness of the average model convergence perspective to identify stocks with high return relative to risk and with large weight allocations suggests that the approach proposed is practical, easy to implement and useful. The model convergence and model consensus used to address the optimal stock selection and investment confidence problems appears to be a distinctive feature in the multiple risk measure-based portfolio optimization literature. It is however puzzling that the average model convergence does not identify INTERMIN RESOURCES (IRCX) and NORTHERN STAR (NSTX) in the gold mining portfolio, SARACEN MINERALS (SARX) and PANAUST (PNAX) in the iron ore-nickel mining portfolio, PALADIN ENERGY (PDNX), SUMMIT RESOURCES (SMMX), ALLIANCE RESOURCES (AGSX) and BERKELEY RESOURCES (BKYX) in the coal-uranium energy portfolio, WOODSIDE (WPLX), APA (APAX), SANTOS (STOX) and CALTEX (CTXX) in the oil-gas energy portfolio, COCA-COLA (CCLX) in the retail benchmark portfolio and JAMES HARDIE (JHXX) in the manufacturing benchmark portfolio as good candidates for investment despite having some of largest returns relative to risk.

The multiple risk measure-based portfolio optimization conducted in this chapter relative to single risk measure optimization by Markowitz (1952), Zhou (2004), Zhou and Yin (2003), Alexander and Baptista (2002), Li et al. (2002), Steinbach (2001), Yoshimoto (1996), Kroll et al. (1984), He and Litterman (1999), Bevan and Winkelmann (1998), Samuelson (1970), Chang et al. (2011) and De Oliveira et al. (2011) is in the least more informative. Those studies lack the multi-angle portfolio optimization perspective that could cater for the specific risk and return preferences of investors. Relative to the multiple risk measure-based portfolio optimization by Krokhmal et al. (2002), Stone (1973) and Cheng and Wolverton (2001), the implemented portfolio optimization framework addresses more effectively and objectively the optimal stock selection and investment confidence problems.

8.5 Summary

This chapter fitted linear and nonlinear model specifications with respect to the *variance*, *MAD*, *CVaR*, *Minimax* and *CDaR* risk measures to estimate the minimum risk optimal mining, energy, retail and manufacturing portfolios. The average model convergence perspective proposed in this thesis for the optimal stock selection was implemented. In the gold mining portfolio the average model convergence selected ST. BARBARA (SBMX), NORTHWEST RESOURCES (NWRX) and RESOLUTE MINING (RSGS) as good candidates for investment. In the iron ore-nickel mining portfolio, despite the descriptive statistics indicating that ILUKA RESOURCES (ILUX) and FORTESCUE METALS (FMGX) have two of the largest returns relative to risk, the average model convergence only identified BHP BILLITON (BHPX) as good candidate for investment.

In the mix-metals portfolio RIO TIONTO (RIOX) and CUDECO (CUX) drew the attention of the average model convergence. In the coal-uranium energy portfolio, COAL BANK (CBQX), AQUILA RESOURCES (AQAX) and COALSPURN (CPLX) were identified as good investment choices. In the oil-gas energy portfolio BEACH ENERGY (BPTX) and ORIGIN ENERGY (ORGX) were spotlighted by the average model convergence as good investment choices. In the retail benchmark portfolio M2 TELECOM (MTUX), WOOLWORTHS (WOWX) and ARB (ARPX) caught the attention of the proposed approach to selecting stocks. In the manufacturing portfolio, CSL (CSLX), BRICKWORKS (BKWX) and ANSELL (ANNX) were selected by the average model convergence as good candidates for investment.

It is noticed that most of the stocks selected by the average model convergence have some of the highest mean returns relative to risk and are allocated large weights by most of the portfolio optimization model specifications. Those stocks, in addition to that, were backed by the average model convergence and model consensus. It was also noticed that some stocks despite having a high return relative to risk were not selected by the average model convergence as good candidates for investment. The oil-gas energy portfolio was identified as the most risky, while the retail benchmark portfolio was the least risky. The retail benchmark portfolio offered the best risk-return trade-off.

CHAPTER 9

HYPOTHESIS TESTING

This chapter consists of two sections: pair vine copula hypothesis testing and portfolio optimization hypothesis testing.

The *pair vine copula hypothesis testing* section deals with the testing of the hypotheses that arise from the fit of the pair vine copulas to the portfolios' data sets. The portfolios' dependence risk differences at various locations of their joint distributions are tested. The *portfolio optimization hypothesis testing* section deals with the testing of the hypothesis stemming from the fit of the various optimization methods and risk measures to the portfolios modelled. The statistical significance of the average model convergence on the stocks is tested and the stock portfolio with the best risk-return trade-off is identified.

9.1 Pair vine copulas hypothesis testing

The number of alternative hypotheses tested in this chapter is eight. Each of these alternative hypotheses has been stated in Chapter 3. The number of alternative hypotheses tested with respect to the fit of the pair vine copula models is six. Those hypotheses compare the portfolios' dependence risk and the portfolios' dependence structure changes between pairs of period scenarios. The objective is to test for the statistical significance of the portfolios' dependence risk differences and dependence structure changes. The portfolios' dependence risk differences are identified to stem from their dependence structure differences which in turn are determined by the dependence concentration differences at various locations in the pairs of stocks joint distributions.

The alternative hypotheses 1 to 4 are tested at the 95% confidence level using a two-sample two-tailed t-test for the difference of means between dependence concentrations. The concentration of dependence in the centre and in the tails of the pairs of stocks' joint distributions is tested. The selection of the 95% confidence level assures with 95%

probability that the difference between the means of the dependence concentrations is either significant or not significant. Each of the two samples used for the testing of the alternative hypotheses 1 to 4 consists of 12 observations, corresponding to 3 vine copula models and 4 period scenarios. The degrees of freedom for 12 observations are 22. The difference between two portfolios' dependence concentration is acknowledged to be statistically significant if the resulting t-test value is larger or smaller than the critical value.

The testing of the alternative hypothesis 5 follows the same procedure as in the testing of the alternative hypotheses 1 to 4. The only differences lie in the number of observations used in each of the samples and the number of period scenarios used for the testing. The alternative hypothesis 6 is tested using goodness-fit-tests. The pair vine copula models that best fit the multivariate dependence structure of the portfolios are sought. Estimating the rankings of the optimal weights and applying non-parametric tests to measure the strength of association between the portfolios' risk rankings test the alternative hypothesis 7. Employing a one-sample two-tailed t-test for the difference between the average of the optimal weights and each of the optimal weights tests the alternative hypothesis 8.

As to the alternative hypotheses 1 to 5, when testing for the dependence risk differences in the centre of two portfolios' joint distributions the two-tailed t-test is fitted to the vine models' frequency of selection of the Frank copula. The reason for this is that this copula is designed to capture greater concentration of dependence in the centre of the joint distributions. When testing for dependence risk differences in the negative tail the t-test is fitted to the vine models' frequency of selection of the Clayton and 180 degrees rotated Gumbel, Joe and Joe-Frank copulas. These copulas are designed to best capture the dependence scattered at various locations in the negative tail of the joint distributions. When testing for dependence risk differences in the positive tail the t-test is fitted to the vine models' frequency of selection of the Gumbel, Joe, Joe-Frank and 180 degrees rotated Clayton copulas. These copulas are suitable to capture the dependence concentrated in the positive tail.

When testing for asymmetric dependence risk differences in the negative and positive tails the t-test is fitted to the vine models' frequency of selection of the Clayton, 180 degrees rotated Gumbel, and the Gumbel and180 degrees rotated Clayton, respectively. These copulas are designed to better account for the dependence concentrated in the negative and positive tails. The testing of the portfolios' symmetric dependence risk differences uses the vine models' frequency of selection of the Student-t copula. The Student-t copula accounts for the dependence in the tails symmetrically.

This chapter's objective is to verify and validate the statistical significance of the findings from Chapters 5, 6, 7 and 8.

9.1.1 Hypothesis 1

H_a 1: There are mining portfolios with higher dependence risk than others.

Since the dependence risk of the portfolios is determined by the specific characteristics of their dependence concentration in the joint distributions, implementing a two-sample two-tailed t-test for the difference of means between two portfolios' dependence concentration tests the alternative hypothesis 1. The significance testing of the mining portfolios displayed in Table 9-1 indicates that the gold mining portfolio's dependence concentration in the centre is at the 95% confidence level significantly larger than that of the iron ore-nickel mining portfolio. In the negative tail however, the iron ore-nickel has it significantly larger than the gold mining portfolio. This information confirms the results from Chapter 5 where the iron ore-nickel mining portfolio is identified to be more dependence risky than the gold in times of financial turbulence characterized by low confidence in the financial stock markets. The gold mining portfolio is observed to be less dependence risky in similar market conditions.

The significance testing of dependence concentration of the iron ore-nickel mining portfolio relative to the mix-metals indicates that the iron ore-nickel has significantly larger concentration of dependence in the negative tail, implying that it is significantly more dependence risky than the mix-metals in non-tranquil stock market conditions. In the centre however, the mix-metals has significantly larger concentration of dependence than the iron ore-nickel, making it more dependence risky during crisis periods. The significance testing between the iron ore-nickel and mix-metals mining portfolios also confirms the results from Chapter 5. The significance testing of the mix-metals leptokurtic portfolio relative to the gold shows that the mix-metals has significantly larger concentration of dependence in the negative tail, making it less dependence risky than the gold mining portfolio in non-tranquil periods. This information also confirms the results from Chapter 5. The statistically significant dependence risk differences between the mining portfolios lead to the acceptance of the alternative hypothesis 1.

Table 9-1: Significance testing of dependence concentration for the mining portfolios

Significance testing of dependence	Gold relative to the iron ore-nickel	Iron ore-nickel relative to the mix-metals	Mix-metals relative to the gold
Overall dependence (negative tail)			
Clayton T-test	-4.59	2.05	2.18
Gumbel 180 T-test	-4.16	4.86	-1.73
Joe 180T-test	-4.26	3.19	1.65
Joe-Frank 180 T-test	-0.78	-1.18	2.08
Statistical significance	Sig. smaller	Sig. larger	Sig. larger
Overall dependence (centre)			
Frank T-test	6.44	-7.12	-0.84
Statistical significance	Sig. larger	Sig. smaller	Neither
Overall dependence (positive tail)			
Gumbel T-test	3.55	-1.44	-2.39
Clayton 180 T-test	0.80	-1.57	0.84
Joe T-test	-0.81	0.00	1.19
Joe-Frank T-test	3.16	-4.16	1.44
Statistical significance	Sig. larger	Neither	Neither
Asymmetric dependence (positive tail)			
Gumbel T-test	3.55	-1.44	-2.39
Clayton 180 T-test	0.80	-1.57	0.84
Statistical significance	Sig. larger	Neither	Sig. smaller
Asymmetric dependence (negative tail)			
Clayton T-test	-4.59	2.05	2.18
Gumbel 180 T-test	-4.16	4.86	-1.73
Statistical significance	Sig. smaller	Sig. larger	Sig. larger
Symmetric dependence (tails)			
Student-t T-test	-0.11	2.50	-6.54
Statistical significance	Neither	Sig. larger	Sig. smaller
Critical value	$t_{(0.05,22)} = \pm 2.074$		

Notes: On the top row are displayed the names of the portfolios being compared in terms of dependence concentration and dependence risk. On the first column from left to right are displayed the copulas to which the t-test is fitted, and the statistical significance category. The second and third columns from left to right display the resulting t-test values, the statistical significance, the type of dependence and its location. The bottom row displays the critical value used to determine the existence or non-existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio, or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

9.1.2 Hypothesis 2

H_a 2: There are energy portfolios with higher dependence risk than others

The energy portfolios' significance testing of dependence concentration displayed in Table 9-2 indicates that the coal-uranium energy portfolio, relative to the oil-gas, has at the 95% confidence level significantly larger concentration of dependence in the centre. This makes the coal-uranium energy portfolio more dependence risky than the oil-gas when the financial stock markets are tranquil. The oil-gas energy portfolio, on the other hand, has neither significantly larger nor significantly smaller dependence concentration in the negative tail, relative to the coal-uranium energy portfolio. This information confirms the results from Chapter 6 regarding the dependence risk differences between the energy portfolios. The statistically significant dependence risk differences between the energy portfolios lead to the acceptance of the alternative hypothesis 2.

Table 9-2: Significance testing of dependence concentration for the energy portfolios

Significance testing of dependence	Coal-uranium relative to the oil-gas	Oil-gas relative to the coal-uranium
	Overall dependence (negative tail)	
Clayton T-test	0.21	-0.21
Gumbel 180 T-test	-2.02	2.02
Joe 180T-test	1.23	-1.19
Joe-Frank 180 T-test	-0.72	0.72
Statistical significance	Neither	Neither
	Overall dependence (centre)	
Frank T-test	2.60	-2.60
Statistical significance	Sig. larger	Sig. smaller
	Overall dependence (positive tail)	
Gumbel T-test	-1.88	1.88
Clayton 180 T-test	-0.66	0.66
Joe T-test	-3.66	3.66
Joe-Frank T-test	3.06	-3.06
Statistical significance	Neither	Neither
	Asymmetric dependence (positive tail)	
Gumbel T-test	-1.88	1.88
Clayton 180 T-test	-0.66	0.66
Statistical significance	Neither	Neither
	Asymmetric dependence (negative tail)	
Clayton T-test	0.21	-0.21
Gumbel 180 T-test	-2.02	2.02
Statistical significance	Neither	Neither
	Symmetric dependence (tails)	
Student-t T-test	-3.17	3.17
Statistical significance	Sig. smaller	Sig. larger
Critical value	$t_{(0.05,22)}=\pm 2.074$	

Notes: On the top row are displayed the names of the portfolios being compared in terms of dependence concentration and dependence risk. On the first column from left to right are displayed the copulas to which the t-test is fitted, and the statistical significance category. The second and third columns from the left to right display the resulting t-test values, the statistical significance, the type of dependence and its location. The bottom row displays the critical value used to determine the existence or non-existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio, or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

9.1.3 Hypothesis 3

H_a3: There are mining portfolios with higher dependence risk than energy portfolios

The significance testing of dependence concentration of the mining and energy portfolios displayed in Table 9-3 indicates that in the negative tail the iron ore-nickel mining portfolio has at the 95% confidence level neither significantly larger nor significantly smaller concentration of dependence than the coal-uranium energy portfolio. However, its asymmetric dependence concentration is significantly larger than that of the coal-uranium in the negative tail; indicating that the iron ore and nickel stocks tend to yield negatively skewed returns in stock market conditions characterized by low confidence in the financial stock markets. The significance testing of the iron ore-nickel mining portfolio relative to the oil-gas energy portfolio shows that its dependence concentration in the negative tail is neither significantly larger nor significantly smaller than that of the oil-gas energy portfolio. Its asymmetric dependence in the negative tail is also neither significantly larger nor significantly smaller. It however has in the centre and positive tail significantly smaller concentration of dependence than the oil-gas energy portfolio. As a consequence, the iron ore-nickel mining portfolio is more dependence risky than the oil-gas during crisis periods. The significance testing of the mix-metals relative to the oil-gas indicates that the latter has higher dependence risk than the mix-metals when the financial stock markets are unstable. The statistically significant dependence risk differences between the mining and energy portfolios lead to the acceptance of the alternative hypothesis 3.

Table 9-3: Significance testing of dependence concentration for the mining and energy portfolios

Significance testing of dependence	Iron ore-nickel relative to the coal-uranium	Iron ore-nickel relative to the oil-gas	Mix-metals relative to the oil-gas
	Overall dependence (negative tail)		
Clayton T-test	0.66	0.88	-1.48
Gumbel 180 T-test	3.59	1.83	-3.52
Joe 180T-test	1.59	2.71	0.48
Joe-Frank 180 T-test	0.11	-0.39	1.02
Statistical significance	Neither	Neither	Neither
	Overall dependence (centre)		
Frank T-test	-6.08	-3.68	3.23
Statistical significance	Sig. smaller	Sig. smaller	Sig. larger
	Overall dependence (positive tail)		
Gumbel T-test	-2.19	-4.08	-2.15
Clayton 180 T-test	0.32	-0.32	1.37
Joe T-test	0.63	-2.21	-3.34
Joe-Frank T-test	-2.37	0.57	4.45
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller
	Asymmetric dependence (positive tail)		
Gumbel T-test	-2.19	-4.08	-2.15
Clayton 180 T-test	0.32	-0.32	1.37
Statistical significance	Sig. smaller	Sig. smaller	Sig. smaller
	Asymmetric dependence (negative tail)		
Clayton T-test	0.66	0.88	-1.48
Gumbel 180 T-test	3.59	1.83	-3.52
Statistical significance	Sig. larger	Neither	Sig. smaller
	Symmetric dependence (tails)		
Student-t T-test	2.39	0.64	-3.63
Statistical significance	Sig. larger	Neither	Sig. smaller
Critical value	$t_{(0.05,22)} = \pm 2.074$		

Notes: On the top row are displayed the names of the portfolios being compared in terms of dependence concentration and dependence risk. On the first column from left to right are displayed the copulas to which the t-test is fitted, and the statistical significance category. The second and third columns from the left to right display the resulting t-test values, the statistical significance, the type of dependence and its location. The bottom row displays the critical value used to determine the existence or non-existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio, or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

9.1.4 Hypothesis 4

H_a4: There are mining and energy portfolios with higher dependence risk than retail and manufacturing benchmark portfolios.

The significance testing of dependence concentration of the mining and energy portfolios and the retail and manufacturing benchmark portfolios displayed in Table 9-4 indicates that the iron ore-nickel mining portfolio relative to the retail and manufacturing benchmark portfolios has at the 95% confidence level significantly larger concentration of dependence in the negative tail. As a result, the iron ore-nickel mining portfolio has higher dependence risk than the retail and manufacturing benchmark portfolios in times of financial turbulence. The significance testing of the oil-gas energy portfolio relative to the retail and manufacturing benchmark portfolios shows that the oil-gas energy portfolio has significantly larger concentration of dependence in the negative tail, an indication of higher dependence risk in the gold mining portfolio, relative to the retail and manufacturing benchmark portfolios during non-tranquil periods. The statistically significant dependence risk differences between the mining and energy portfolios and the retail and manufacturing benchmark portfolios lead to the acceptance of the alternative hypothesis 4.

Table 9-4: Significance testing of dependence concentration for the mining and energy portfolios and the retail and manufacturing benchmark portfolios

Significance testing of dependence	Iron ore-nickel relative to the retail	Iron ore-nickel relative to the manufacturing	Oil-gas relative to the retail	Oil-gas relative to manufacturing
		Overall dependence (negative tail)		
Clayton T-test	2.57	3.48	2.06	3.12
Gumbel 180 T-test	6.52	6.89	5.17	5.62
Joe 180T-test	7.74	3.28	2.20	0.15
Joe-Frank 180 T-test	2.07	1.78	3.79	3.09
Statistical significance	Sig. larger	Sig. larger	Sig. larger	Sig. larger
		Overall dependence (centre)		
Frank T-test	-5.42	-7.26	-0.98	-3.67
Statistical significance	Sig. smaller	Sig. smaller	Neither	Sig. smaller
		Overall dependence (positive tail)		
Gumbel T-test	-5.74	3.63	-2.58	-0.09
Clayton 180 T-test	-3.39	2.74	-3.43	-2.73
Joe T-test	-2.54	1.29	-0.52	1.32
Joe-Frank T-test	2.07	2.61	3.79	-3.40
Statistical significance	Sig. smaller	Sig. larger	Sig. smaller	Sig. smaller
		Asymmetric dependence (positive tail)		
Gumbel T-test	-5.74	3.63	-2.58	-0.09
Clayton 180 T-test	-3.39	2.74	-3.43	-2.73
Statistical significance	Sig. smaller	Sig. larger	Sig. smaller	Neither
		Asymmetric dependence (negative tail)		
Clayton T-test	2.57	-3.48	2.06	3.12
Gumbel 180 T-test	6.52	-6.89	5.17	5.62
Statistical significance	Sig. larger	Sig. smaller	Sig. larger	Sig. larger
		Symmetric dependence (tails)		
Student-t T-test	-1.84	-0.16	-2.94	-0.78
Statistical significance	Neither	Neither	Sig. smaller	Neither
Critical value		$t_{(0.05,22)}=\pm 2.074$		

Notes: On the top row are displayed the names of the portfolios being compared in terms of dependence concentration and dependence risk. On the first column from left to right are displayed the copulas to which the t-test is fitted, and the statistical significance category. The second and third columns from the left to right display the resulting t-test values, the statistical significance, the type of dependence and its location. The bottom row displays the critical value used to determine the existence, or non-existence of statistical significance. The dependence concentration of a portfolio could be significantly smaller or significantly larger than that of other portfolio or neither. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

9.1.5 Hypothesis 5

H_a5: The portfolios' dependence structure changes between period scenarios are statistically significant

Applying a two-sample two-tailed t-test for the difference of means between two portfolios dependence concentration tests the alternative hypothesis 5. The number of observations used for the testing of this hypothesis is 3. Each observation corresponds to one of the vine copula models fitted to the data sets. It should be noticed that using 3 observations to fit the t-test could be considered to be a limitation of the analysis since it could lead to questions regarding the reliability of the results. The degrees of freedom used are 4 and the confidence level on the t-test is 95%. A dependence structure change is acknowledged to be statistically significant if the resulting t-test value is larger or smaller than the critical value, and not statistically significant otherwise. If more than two copulas are used to account for the dependence concentration at a specific location of the pairs of stocks' joint distributions, the t-test values of at least two copulas are required to be larger or smaller than the critical value to determine the existence of statistical significance. The statistical significance of the dependence structure changes is tested in the pre-GFC-GFC, GFC-post-GFC, and pre-GFC-post-GFC pairs of period scenarios.

Table 9-5: Gold portfolio's significant testing of dependence structure changes

Significance testing of dependence structure changes	Pre-GFC to GFC	Pre-GFC to post-GFC	GFC to Post-GFC
	Overall dependence structure changes (negative tail)		
Clayton T-test	2.94	0.59	-2.73
Gumbel 180 T-test	1.31	2.62	2.95
Joe 180T-test	4.77	4.15	1.12
Joe-Frank 180 T-test	-2.72	0.24	3.83
Statistical Significance	Significant	Not significant	Significant
	Overall dependence structure changes (centre)		
Frank T-test	-6.38	-4.37	4.36
Statistical Significance	Significant	Significant	Significant
	Overall dependence structure changes (positive tail)		
Gumbel T-test	3.46	-0.30	-8.83
Clayton 180 T-test	2.25	2.52	-0.57
Joe T-test	-0.42	-0.55	-0.16
Joe-Frank T-test	-1.47	1.46	3.02
Statistical Significance	Not significant	Not significant	Not significant
	Asymmetric dependence structure changes (positive tail)		
Gumbel T-test	3.46	-0.30	-8.83
Clayton 180T-test	2.25	2.52	-0.57
Statistical Significance	Not significant	Not significant	Not significant
	Asymmetric dependence structure changes (negative tail)		
Clayton T-test	2.94	0.59	-2.73
Gumbel 180T-test	1.31	2.62	2.95
Statistical Significance	Not significant	Not significant	Not significant
	Symmetric dependence structure changes (tails)		
Student-t T-test	-1.09	-1.45	0.13
Statistical Significance	Not significant	Not significant	Not significant
	Critical value=$t_{(0.05,4)}$= ± 2.776		

Notes: The top row states the pairs of period scenarios in which the dependence structure changes are tested. The first column from the left to the right shows the bivariate copulas used to identify the location of the dependence and its concentration. The overall, symmetric and asymmetric dependence in the centre and in the tails is tested. The row in the bottom displays the critical value. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

The significance testing of the gold mining portfolio's dependence structure changes displayed in Table 9-5 indicates that in the centre its dependence structure changes significantly from the pre-GFC to the GFC, from the GFC to the post-GFC and from the pre-GFC to the post-GFC. In the negative tail its dependence structure changes significantly from the pre-GFC to the GFC and from the pre-GFC to the post-GFC.

Table 9-6: Iron ore-nickel portfolio's significant testing of dependence structure changes

Significance testing of dependence structure changes	Pre-GFC to GFC	Pre-GFC to post-GFC	GFC to Post-GFC
Overall dependence structure changes (negative tail)			
Clayton T-test	-2.25	3.83	4.54
Gumbel 180 T-test	-5.00	0.55	4.49
Joe 180T-test	9.82	2.44	-1.15
Joe-Frank 180 T-test	-2.11	-3.52	-0.78
Statistical Significance	Not significant	Not significant	Significant
Overall dependence structure changes (centre)			
Frank T-test	1.12	-1.50	-2.27
Statistical Significance	Not significant	Not significant	Not significant
Overall dependence structure changes (positive tail)			
Gumbel T-test	0.00	0.16	0.16
Clayton 180 T-test	3.62	3.26	-0.08
Joe T-test	0.27	0.61	0.39
Joe-Frank T-test	-1.22	-1.22	0.34
Statistical Significance	Not significant	Not significant	Not significant
Asymmetric dependence structure changes (positive tail)			
Gumbel T-test	0.00	0.16	0.16
Clayton 180T-test	3.62	3.26	-0.08
Statistical Significance	Significant	Significant	Not significant
Asymmetric dependence structure changes (negative tail)			
Clayton T-test	-2.25	3.83	4.54
Gumbel 180T-test	-5.00	0.55	4.49
Statistical Significance	Significant	Significant	Significant
Symmetric dependence structure changes (tails)			
Student-t T-test	1.55	-4.63	-5.12
Statistical Significance	Not significant	Significant	Significant
Critical value= $t_{(0.05,4)}$ = ± 2.776			

Notes: The top row states the pairs of period scenarios in which the dependence structure changes are tested. The first column from left to the right shows the bivariate copulas used to identify the location of the dependence and its concentration. The overall, symmetric and asymmetric dependence in the centre and in the tails is tested. The row in the bottom displays the critical value. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

The significance testing of the iron ore-nickel mining portfolio's dependence structure changes displayed in Table 9-6 indicates that in the negative tail the dependence structure changes are significant from the GFC to the post-GFC. Its asymmetric dependence structure at the same location also changes significantly from the pre-GFC to the GFC, from the pre-GFC to the post-GFC and from the GFC to the post-GFC. The portfolio's symmetric dependence structure in the positive tail changes significantly from the pre-GFC to the post-GFC and from the GFC to the post-GFC.

Table 9-7: Coal-uranium portfolio's significant testing of dependence structure changes

Significance testing of dependence structure changes	Pre-GFC to GFC	Pre-GFC to post-GFC	GFC to Post-GFC
Overall dependence structure changes (negative tail)			
Clayton T-test	-1.66	-2.73	0.37
Gumbel 180 T-test	-0.91	2.46	1.64
Joe 180T-test	2.24	2.21	-0.43
Joe-Frank 180 T-test	-0.11	-0.92	-1.19
Statistical Significance	Not significant	Not significant	Not significant
Overall dependence structure changes (centre)			
Frank T-test	0.71	-2.42	-1.81
Statistical Significance	Not significant	Not significant	Not significant
Overall dependence structure changes (positive tail)			
Gumbel T-test	-1.71	-3.40	-1.43
Clayton 180 T-test	3.24	4.51	0.38
Joe T-test	-0.55	-0.34	0.24
Joe-Frank T-test	-3.37	-0.42	1.95
Statistical Significance	Not significant	Not significant	Not significant
Asymmetric dependence structure changes (positive tail)			
Gumbel T-test	-1.71	-3.40	-1.43
Clayton 180T-test	3.24	4.51	0.38
Statistical Significance	Significant	Significant	Not significant
Asymmetric dependence structure changes (negative tail)			
Clayton T-test	-1.66	-2.73	0.37
Gumbel 180T-test	-0.91	2.46	1.64
Statistical Significance	Not significant	Not significant	Not significant
Symmetric dependence structure changes (tails)			
Student-t T-test	-0.90	1.81	2.08
Statistical Significance	Not significant	Significant	Significant
Critical value=$t_{(0.05,4)}= \pm 2.776$			

Notes: The top row states the pairs of period scenarios in which the dependence structure changes are tested. The first column from left to the right shows the bivariate copulas used to identify the location of the dependence and its concentration. The overall, symmetric and asymmetric dependence in the centre and in the tails is tested. The row in the bottom displays the critical value. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

The significance testing of the coal-uranium energy portfolio's dependence structure changes displayed in Table 9-7 indicates that its asymmetric dependence concentration changes significantly from the pre-GFC to the GFC and from the pre-GFC to the post-GFC. Its symmetric dependence concentration also changes significantly from the pre-GFC to the post-GFC and from the GFC to the post-GFC.

Table 9-8: Oil-gas energy portfolio's significant testing of dependence structure changes

Significance testing of dependence structure changes	Pre-GFC to GFC	Pre-GFC to post-GFC	GFC to Post-GFC
Overall dependence structure changes (negative tail)			
Clayton T-test	5.75	1.64	-1.28
Gumbel 180 T-test	-2.02	-0.59	1.15
Joe 180T-test	4.68	1.35	-3.11
Joe-Frank 180 T-test	-3.37	-6.79	0.34
Statistical Significance	Significant	Not significant	Not significant
Overall dependence structure changes (centre)			
Frank T-test	-7.18	-2.41	1.67
Statistical Significance	Significant	Not significant	Not significant
Overall dependence structure changes (positive tail)			
Gumbel T-test	3.90	3.83	-0.74
Clayton 180 T-test	2.06	1.85	-0.06
Joe T-test	3.46	2.19	0.00
Joe-Frank T-test	-0.77	0.00	0.77
Statistical Significance	Significant	Not significant	Not significant
Asymmetric dependence structure changes (positive tail)			
Gumbel T-test	3.90	3.83	-0.74
Clayton 180T-test	2.06	1.85	-0.06
Statistical Significance	Not significant	Not significant	Not significant
Asymmetric dependence structure changes (negative tail)			
Clayton T-test	5.75	1.64	-1.28
Gumbel 180T-test	-2.02	-0.59	1.15
Statistical Significance	Significant	Not significant	Not significant
Symmetric dependence structure changes (tails)			
Student-t T-test	4.70	-0.71	-16.97
Statistical Significance	Significant	Not significant	Significant
Critical value= $t_{(0.05,4)}$ = ± 2.776			

Notes: The top row states the pairs of period scenarios in which the dependence structure changes are tested. The first column from left to the right shows the bivariate copulas used to identify the location of the dependence and its concentration. The overall, symmetric and asymmetric dependence in the centre and in the tails is tested. The row in the bottom displays the critical value. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

The significance testing of the oil-gas energy portfolio's dependence structure changes displayed in Table 9-8 indicates that in the centre and in the tails its dependence structure changes significantly from the pre-GFC to the GFC. Its symmetric dependence structure changes significantly from the pre-GFC to the post-GFC and from the GFC to the post-GFC.

Table 9-9: Mix-metals portfolio's significant testing of dependence structure changes

Significance testing of dependence structure changes	Pre-GFC to GFC	Pre-GFC to post-GFC	GFC to Post-GFC
Overall dependence structure changes (negative tail)			
Clayton T-test	3.22	-0.41	-3.19
Gumbel 180 T-test	-4.21	0.91	5.98
Joe 180T-test	-0.55	0.97	1.13
Joe-Frank 180 T-test	-0.68	-3.93	-2.58
Statistical Significance	Not significant	Not significant	Not significant
Overall dependence structure changes (centre)			
Frank T-test	1.32	-0.45	-4.58
Statistical Significance	Not significant	Not significant	Significant
Overall dependence structure changes (positive tail)			
Gumbel T-test	2.38	2.81	0.13
Clayton 180 T-test	1.67	0.27	-1.65
Joe T-test	4.06	0.74	-3.90
Joe-Frank T-test	-3.75	1.50	4.50
Statistical Significance	Significant	Not significant	Not significant
Asymmetric dependence structure changes (positive tail)			
Gumbel T-test	2.38	2.81	0.13
Clayton 180T-test	1.67	0.27	-1.65
Statistical Significance	Not significant	Significant	Not significant
Asymmetric dependence structure changes (negative tail)			
Clayton T-test	3.22	-0.41	-3.19
Gumbel 180T-test	-4.21	0.91	5.98
Statistical Significance	Significant	Not significant	Significant
Symmetric dependence structure changes (tails)			
Student-t T-test	-3.24	-0.42	4.24
Statistical Significance	Significant	Not significant	Significant
Critical value= $t_{(0.05,4)}$ = ± 2.776			

Notes: The top row states the pairs of period scenarios in which the dependence structure changes are tested. The first column from left to right shows the bivariate copulas used to identify the location of the dependence and its concentration. The overall, symmetric and asymmetric dependence in the centre and in the tails is tested. The row in the bottom displays the critical value. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

The significance testing of the mix-metals portfolio's dependence structure changes displayed in Table 9-9 indicates that in the centre its dependence structure changes significantly from the GFC to the post-GFC. Its symmetric dependence structure in the tails changes significantly from the pre-GFC to the GFC and from the GFC to the post-GFC.

Table 9-10: Retail portfolio's significant testing of dependence structure changes

Significance testing of dependence structure changes	Pre-GFC to GFC	Pre-GFC to post-GFC	GFC to Post-GFC
Overall dependence structure changes (negative tail)			
Clayton T-test	-3.39	3.90	5.09
Gumbel 180 T-test	-1.04	-0.98	0.00
Joe 180T-test	-0.98	-0.77	0.39
Joe-Frank 180 T-test	6.50	6.58	1.22
Statistical Significance	Not significant	Significant	Not significant
Overall dependence structure changes (centre)			
Frank T-test	4.16	5.23	0.88
Statistical Significance	Significant	Significant	Not significant
Overall dependence structure changes (positive tail)			
Gumbel T-test	0.00	1.93	1.04
Clayton 180 T-test	-6.28	-13.17	-7.87
Joe T-test	-1.20	-1.92	-0.98
Joe-Frank T-test	2.71	4.50	0.77
Statistical Significance	Not significant	Not significant	Not significant
Asymmetric dependence structure changes (positive tail)			
Gumbel T-test	0.00	1.93	1.04
Clayton 180T-test	-6.28	-13.17	-7.87
Statistical Significance	Not significant	Not significant	Not significant
Asymmetric dependence structure changes (negative tail)			
Clayton T-test	-3.39	3.90	5.09
Gumbel 180T-test	-1.04	-0.98	0.00
Statistical Significance	Not significant	Not significant	Not significant
Symmetric dependence structure changes (tails)			
Student-t T-test	-5.12	-6.00	3.14
Statistical Significance	Significant	Significant	Significant
Critical value= $t_{(0.05,4)}$ = ± 2.776			

Notes: The top row states the pairs of period scenarios in which the dependence structure changes are tested. The first column from the left to the right shows the bivariate copulas used to identify the location of the dependence and its concentration. The overall, symmetric and asymmetric dependence in the centre and in the tails is tested. The row in the bottom displays the critical value. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

The significance testing of the retail benchmark portfolio's dependence structure changes displayed in Table 9-10 indicates that in the centre its dependence structure changes significantly from the pre-GFC to the GFC and from the pre-GFC to the post-GFC. Its dependence structure in the negative tail changes significantly from the pre-

GFC to the post-GFC. The significance testing of the manufacturing benchmark portfolio's dependence structure changes displayed in Table 9-11 indicates that in the centre its dependence structure changes significantly from the pre-GFC to the GFC, from the GFC to the post-GFC and from the pre-GFC to the post-GFC. The portfolios' asymmetric dependence concentration also changes significantly from the pre-GFC to the GFC, from the pre-GFC to the post-GFC and from the GFC to the post-GFC.

Table 9-11: Manufacturing benchmark portfolio's significant testing of dependence structure changes

Significance testing of dependence structure changes	**Pre-GFC to GFC**	**Pre-GFC to post-GFC**	**GFC to Post-GFC**
	Overall dependence structure changes (negative tail)		
Clayton T-test	9.41	12.15	8.07
Gumbel 180 T-test	6.37	2.81	3.70
Joe 180T-test	4.65	7.31	2.41
Joe-Frank 180 T-test	-2.38	-1.51	-1.02
Statistical Significance	Significant	Significant	Significant
	Overall dependence structure changes (centre)		
Frank T-test	-11.14	-4.10	3.61
Statistical Significance	Significant	Significant	Significant
	Overall dependence structure changes (positive tail)		
Gumbel T-test	-7.87	-0.44	3.35
Clayton 180 T-test	1.96	-2.81	-5.20
Joe T-test	0.00	-1.10	-1.10
Joe-Frank T-test	-0.40	5.63	1.08
Statistical Significance	Not significant	Not significant	Not significant
	Asymmetric dependence structure changes (positive tail)		
Gumbel T-test	-7.87	-0.44	3.35
Clayton 180T-test	1.96	-2.81	-5.20
Statistical Significance	Significant	Not significant	Significant
	Asymmetric dependence structure changes (negative tail)		
Clayton T-test	9.41	12.15	8.07
Gumbel 180T-test	6.37	2.81	3.70
Statistical Significance	Significant	Significant	Significant
	Symmetric dependence structure changes (tails)		
Student-t T-test	4.05	2.41	-2.10
Statistical Significance	Significant	Not significant	Not significant
	Critical value= $t_{(0.05,4)}$ = ± 2.776		

Notes: The top row states the pairs of period scenarios in which the dependence structure changes are tested. The first column from the left to the right shows the bivariate copulas used to identify the location of the dependence and its concentration. The overall, symmetric and asymmetric dependence in the centre and in the tails is tested. The row in the bottom displays the critical value. When 4 copulas are used to determine the statistical significance it is required that the t-values of at least 2 copulas are larger or smaller than the critical value. If only two copulas are used to determine the statistical significance it is required that the t-value of at least one copula is larger or smaller than the critical value.

The statistically significant dependence structure changes identified to take place in most of the portfolios between periods scenarios lead to the acceptance of the alternative hypothesis 5.

9.1.6 Hypothesis 6

H_a 6: There is a pair vine copula model that best captures the multivariate dependence structure of the portfolios

Applying the *ECP* and *ECP2* goodness-of-fit tests, which are based on the empirical copula processes, tests the alternative hypothesis 6. The tests are used to identify the pair vine copula model that best fits the multivariate dependence structure of the portfolios. The tests are non-parametric and are based on the Cramer-von Mises (*CvM*) and Kolmogorov-Smirnov (*KS*) test statistics, which use a 95% confidence level. Relative to the Akaike and Bayesian Information Criteria they are more reliable sources of information regarding the goodness-of-fit of the pair vine copula models fitted (Schepsmeier, 2013; Genest et al., 2009; Panchenko, 2005).

The *ECP* and *ECP2* are implemented on the c-vine, d-vine and r-vine copula modelling of the portfolios' data sets under the pre-GFC, GFC, post-GFC and full sample period scenarios. The identification of the vine copula model that most adequately fits the multivariate dependence structure of the portfolios is based on the p-values resulting from the goodness-of-fit testing. The smaller the p-values are, the larger the distance between the fitted parametric vine copula model and the empirical distribution of the multivariate dependence, and vice versa. When evaluating the goodness-of-fit testing results across period scenarios, a higher weight is given to the p-values resulting from the full sample period goodness-of fit-testing because that period accounts for the volatility and dependence structure changes across the three sub periods under consideration.

According to Table 9-12, where the goodness of fit testing for the c-vine, d-vine and r-vine modelling of the gold, iron ore-nickel and coal-uranium portfolios under the four period scenarios is displayed, the r-vine, relative to the c-vine and d-vine, is the model that best accounts for the multivariate dependence structure of the gold mining portfolio. The p-values for the fit of the r-vine are specifically larger than those for the c-vine under the pre-GFC, GFC and full sample periods. The p-values for the fit of the r-vine are also larger than those resulting from the goodness-of-fit testing for the d-vine modelling under most of the period scenarios.

Table 9-12: Goodness-of-fit testing for the gold, iron ore-nickel and coal-uranium energy portfolios

Portfolio	**Gold**			**Iron ore-nickel**			**Coal-uranium**		
Vine copula	**C-vine**	**D-vine**	**R-vine**	**C-vine**	**D-vine**	**R-vine**	**C-vine**	**D-vine**	**R-vine**
				Full sample period					
ECP(*CvM*)	*ts*=0.016 *p*=0.44	*ts*=0.003 *p* =0.975	*ts*=0.004 *p*=0.98	*ts*=0.023 *p*=0.71	*ts*=0.039 *p*=0.70	*ts*=0.02 *p*=0.51	*ts*=0.062 *p*=0.29	*ts*=0.008 *p* =0.205	*ts*=0.005 *p*=0.81
ECP2(*CvM*)	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00
ECP(*CvM*)	*ts*=1.825 *p*=0.23	*ts*=0.952 *p*=0.425	*ts*=1.339 *p*=0.04	*ts*=2.028 *p*=0.73	*ts*=3.018 *p*=0.04	*ts*=2.072 *p*=0.64	*ts*=2.617 *p*=0.37	*ts*=0.849 *p*=0.555	*ts*=1.337 *p*=0.03
ECP2(*CvM*)	*ts*=0.022 *p*=1.00	*ts*=0.022 *p*=1.00	*ts*=0.022 *p*=1.00	*ts*=0.055 *p*=1.00	*ts*=0.066 *p*=1.00	*ts*=0.047 *p*=1.00	*ts*=0.022 *p*=1.00	*ts*=0.022 *p*=1.00	*ts*=0.022 *p*=1.00
				Pre-GFC					
ECP(*CvM*)	*ts*=0.003 *p*=1.00	*ts*=0.003 *p* =1.00	*ts*=0.003 *p*=1.00	*ts*=0.008 *p*=0.96	*ts*=0.008 *p*=0.98	*ts*=0.009 *p*=0.95	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=0.607 *p*=0.28	*ts*=0.849 *p*=0.27	*ts*=0.824 *p*=0.34	*ts*=0.438 *p*=0.53	*ts*=0.354 *p*=0.78	*ts*=0.336 *p*=0.80	*ts*=0.195 *p*=0.99	*ts*=0.144 *p*=1.00	*ts*=0.195 *p*=0.98
ECP2(*CvM*)	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.079 *p*=1.00	*ts*=0.078 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00
				GFC					
ECP(*CvM*)	*ts*=0.012 *p*=0.77	*ts*=0.004 *p*=1.00	*ts*=0.003 *p*=1.00	*ts*=0.015 *p*=0.88	*ts*=0.016 *p*=0.96	*ts*=0.018 *p*=0.78	*ts*=0.009 *p*=0.995	*ts*=0.008 *p*=0.99	*ts*=0.011 *p*=0.97
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=1.010 *p*=0.395	*ts*=0.770 *p*=0.22	*ts*=0.367 *p*=0.78	*ts*=1.308 *p*=0.575	*ts*=1.580 *p*=0.2	*ts*=1.553 *p*=0.32	*ts*=1.815 *p*=0.065	*ts*=1.231 *p*=0.35	*ts*=0.950 *p*=0.79
ECP2(*CvM*)	*ts*=0.077 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.076 *p*=1.00	*ts*=0.062 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.078 *p*=1.00	*ts*=0.039 *p*=1.00
				Post-GFC					
ECP(*CvM*)	*ts*=0.002 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.002 *p*=1.00	*ts*=0.013 *p*=0.945	*ts*=0.013 *p*=0.97	*ts*=0.010 *p*=1.00	*ts*=0.002 *p*=1.00	*ts*=0.003 *p*=1.00	*ts*=0.002 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=0.431 *p*=0.055	*ts*=0.131 *p*=1.00	*ts*=0.304 *p*=0.43	*ts*=1.053 *p*=0.425	*ts*=0.983 *p*=0.29	*ts*=0.849 *p*=0.62	*ts*=0.200 *p*=0.965	*ts*=0.404 *p*=1.00	*ts*=0.287 *p*=0.53
ECP2(*CvM*)	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.078 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00

Notes: The first column from left to righ displays the ECP and ECP2 goodness-of-fit tests and the underlying *CvM* and *KS* test statistics employed by the goodness of fit tests. *ECP* and *ECP2* stand for empirical copula process number 1 and empirical copula process number 2. The abbreviations *ts* and *p* stand for test statistic and p-value. *CvM* and *KS* stand for Cramer-von Mises and Kolmogorov-Smirnov tests. The fit of the c-vine, d-vine and r-vine pair vine copulas for the full sample period, pre-GFC, GFC and post-GFC are tested.

The goodness-of-fit testing of the iron ore-nickel mining portfolio indicates that the c-vine is the model that best captures its multivariate dependence structure. Specifically, despite the r-vine best fitting its multivariate dependence structure in the pre-GFC and post-GFC, the c-vine does it better in the GFC and full sample period scenarios. In addition to that, the p-values resulting from the goodness-of-fit testing for the d-vine modelling under most of the period scenarios are smaller than those for the fit of the c-vine. In the coal-uranium energy portfolio, the c-vine, relative to the r-vine and d-vine, also provides the best fit under most of the period scenarios.

The goodness of fit testing for the c-vine, d-vine and r-vine modelling of the oil-gas, mix-metals and retail portfolios under the four period scenarios displayed in Table 9-13 indicates that the c-vine model, relative to the r-vine and d-vine, best accounts for the multivariate dependence structure of the coal-uranium energy portfolio. The p-values for the fit of the c-vine under the pre-GFC, GFC and post-GFC are larger than those for the fit of the r-vine. Besides, the p-values for the fit of the d-vine are also smaller than those for the fit of the c-vine in most period scenarios.

Table 9-13: Goodness-of-fit testing for the oil-gas, mix-metals and retail benchmark portfolios

Portfolio	**Oil-gas**			**Mix-metals leptokurtic**			**Retail**		
Vine copula	**C-vine**	**D-vine**	**R-vine**	**C-vine**	**D-vine**	**R-vine**	**C-vine**	**D-vine**	**R-vine**
Full sample									
ECP(*CvM*)	*ts*=0.012 *p*=0.80	*ts*=0.012 *p*=0.725	*ts*=0.016 *p*=0.53	*ts*=0.024 *p* =0.85	*ts*=0.004 *p*=0.925	*ts*=0.011 *p*=0.555	*ts*=0.011 *p* =0.65	*ts*=0.0093 *p* =0.87	*ts*=0.00 *p*=0.96
ECP2(*CvM*)	*ts*=0.012 *p*=0.80	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p* =1.00	*ts*=0.000 *p* =1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p*=1.00	*ts*=0.000 *p* =1.00	*ts*=0.000 *p*=1.00
ECP(*CvM*)	*ts*=1.566 *p*=0.65	*ts*=2.890 *p*=0.12	*ts*=2.758 *p*=0.30	*ts*=1.701 *p*=0.615	*ts*=0.944 *p* =0.26	*ts*=1.789 *p*=0.045	*ts*=1.200 *p*=0.505	*ts*=1.597 *p* =0.14	*ts*=1.275 *p*=0.61
ECP2(*CvM*)	*ts*=0.044 *p*=1.00	*ts*=0.022 *p*=1.00	*ts*=0.022 *p*=1.00	*ts*=0.022 *p*=1.00	*ts*=0.022 *p* =1.00	*ts*=0.022 *p* =1.00	*ts*=0.022 *p*=1.00	*ts*=0.045 *p* =1.00	*ts*=0.022 *p*=1.00
Pre-GFC									
ECP(*CvM*)	*ts*=0.002 *p*=1.00	*ts*=0.002 *p*=1.00	*ts*=0.003 *p*=1.00	*ts*=0.004 *p*=1.00	*ts*=0.002 *p* =1.00	*ts*=0.006 *p*=0.93	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=0.274 *p*=0.81	*ts*=0.233 *p*=0.75	*ts*=0.274 *p*=0.74	*ts*=0.644 *p*=0.155	*ts*=0.444 *p* =0.43	*ts*=0.444 *p*=0.575	*ts*=0.391 *p*=0.24	*ts*=0.290 *p* =0.12	*ts*=0.315 *p*=0.24
ECP2(*CvM*)	*ts*=0.039 *p*=1.00	*ts*=0.078 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00
GFC									
ECP(*CvM*)	*ts*=0.013 *p*=0.915	*ts*=0.016 *p*=0.935	*ts*=0.012 *p*=0.93	*ts*=0.004 *p*=1.00	*ts*=0.004 *p* =1.00	*ts*=0.004 *p*=1.00	*ts*=0.007 *p*=1.00	*ts*=0.004 *p* =1.00	*ts*=0.004 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=1.218 *p*=0.195	*ts*=2.079 *p*=0.15	*ts*=1.520 *p*=0.015	*ts*=0.578 *p*=0.38	*ts*=0.680 *p*=0.175	*ts*=0.883 *p*=0.50	*ts*=0.754 *p*=0.53	*ts*=0.435 *p* =0.66	*ts*=0.550 *p*=0.40
ECP2(*CvM*)	*ts*=0.039 *p*=1.00	*ts*=0.039 p=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00
Post-GFC									
ECP(*CvM*)	*ts*=0.016 *p*=0.815	*ts*=0.010 *p*=0.97	*ts*=0.012 *p*=0.945	*ts*=0.002 *p*=1.00	*ts*=0.003 *p* =1.00	*ts*=0.003 *p*=1.00	*ts*=0.003 *p*=1.00	*ts*=0.004 *p* =1.00	*ts*=0.005 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=1.078 *p* =0.555	*ts*=0.681 *p*=0.37	*ts*=1.001 *p*=0.435	*ts*=0.332 *p*=0.69	*ts*=0.394 *p*=0.445	*ts*=0.613 *p*=0.125	*ts*=0.394 *p*=0.305	*ts*=0.397 *p* =0.57	*ts*=0.468 *p*=0.245
ECP2(*CvM*)	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00

Notes: The first column from left to righ displays the ECP and ECP2 goodness-of-fit tests and the underlying *CvM* and *KS* test statistics employed by the goodness of fit tests. *ECP* and *ECP2* stand for empirical copula process number 1 and empirical copula process number 2. The abbreviations *ts* and *p* stand for test statistic and p-value. *CvM* and *KS* stand for Cramer-von Mises and Kolmogorov-Smirnov tests. The fit of the c-vine, d-vine and r-vine pair vine copulas for the full sample period, pre-GFC, GFC and post-GFC are tested.

In the mix-metals portfolio the c-vine also captures best its multivariate dependence structure in most of the period scenarios. For instance, the p-values for the fit of the c-vine under the pre-GFC, post-GFC and full sample period scenarios are larger than those from the goodness of fit testing for the r-vine. Besides, the p-values for the fit of the d-vine are not larger than those for the fit of the c-vine under most period scenarios. In the retail benchmark portfolio the r-vine most adequately accounts for its multivariate dependence structure. Specifically, the p-values for the fit of the r-vine under the full sample and pre-GFC periods are larger than those for the fit of the c-vine, while the p-values for the fit of the d-vine are smaller than those for the fit of the r-vine.

Table 9-14: Goodness-of-fit testing for the manufacturing benchmark portfolio

Portfolio	**Manufacturing**		
Vine copula	**C-vine**	**D-vine**	**R-vine**
		Full sample	
ECP(*CvM*)	*ts*=0.023 *p*=0.19	*ts*=0.0033 *p* =0.98	*ts*=0.021 *p*=0.67
ECP2(*CvM*)	*ts*=0.000 *p*=1.00	*ts*=0.000 *p* =1.00	*ts*=0.000 *p*=1.00
ECP(*CvM*)	*ts*=1.498 *p*=0.38	*ts*=1.089 *p* =0.21	*ts*=2.293 *p*=0.15
ECP2(*CvM*)	*ts*=0.022 *p*=1.00	*ts*=0.022 *p* =1.00	*ts*=0.022 *p*=1.00
		Pre-GFC	
ECP(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=0.117 *p*=1.00	*ts*=0.117 *p* =1.00	*ts*=0.117 *p*=1.00
ECP2(*CvM*)	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00
		GFC	
ECP(*CvM*)	*ts*=0.004 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.005 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=0.902 *p*=0.07	*ts*=0.195 *p* =0.99	*ts*=0.851 *p*=0.07
ECP2(*CvM*)	*ts*=0.902 *p*=0.07	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00
		Post-GFC	
ECP(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.002 *p* =1.00	*ts*=0.001 *p*=1.00
ECP2(*CvM*)	*ts*=0.001 *p*=1.00	*ts*=0.001 *p* =1.00	*ts*=0.001 *p*=1.00
ECP(*CvM*)	*ts*=0.117 *p*=1.00	*ts*=0.470 *p* =0.21	*ts*=0.139 *p*=1.00
ECP2(*CvM*)	*ts*=0.039 *p*=1.00	*ts*=0.039 *p* =1.00	*ts*=0.039 *p*=1.00

Notes: The first column from left to righ displays the ECP and ECP2 goodness-of-fit tests and the underlying *CvM* and *KS* test statistics employed by the goodness of fit tests. *ECP* and *ECP2* stand for empirical copula process number 1 and empirical copula process number 2. The abbreviations *ts* and *p* stand for test statistic and p-value. *CvM* and *KS* stand for Cramer-von Mises and Kolmogorov-Smirnov tests. The fit of the c-vine, d-vine and r-vine pair vine copulas for the full sample period, pre-GFC, GFC and post-GFC are tested.

As indicated by the p-values displayed in Table 9-14, the d-vine is the model that best accounts for the multivariate dependence structure of the manufacturing benchmark portfolio. Although the p-values for the fit of the r-vine are larger than those for the fit of the c-vine, the p-values for the fit of the d-vine are larger than those for the fit of the r-vine in most period scenarios.

The summary of the results displayed in Table 9-15 indicates that the c-vine model is adequate to best account for the multivariate dependence structure of the iron ore-nickel, coal-uranium, oil-gas and mix-metals leptokurtic portfolios. The r-vine most closely approximates the interaction between the gold and retail stocks, while the d-vine better captures the co-movements of the manufacturing stocks. These findings are in line with the dependence structure modelling results from Chapters 5, 6 and 7. The identification of specific vine copula models as best accounting for the multivariate dependence structure of the portfolios leads to the acceptance of the alternative hypothesis 6.

Table 9-15: Goodness-of-fit testing summary

Portfolios	R-vine	C-Vine	D-Vine
Gold	✓	✗	✗
Iron Ore-Nickel	✗	✓	✗
Coal-Uranium	✗	✓	✗
Oil-Gas	✗	✓	✗
Mix-metals	✗	✓	✗
Retail	✓	✗	✗
Manufacturing	✗	✗	✓

Notes: This table displays a summary of the goodness-of-fit testing for the r-vine, c-vine and d-vine modelling of dependence of the gold, iron ore-nickel, mix-metals leptokurtic, coal-uranium, oil-gas, retail and manufacturing portfolios.

9.2 Portfolio optimization hypothesis testing

9.2.1 Hypothesis 7

H_a: There is a portfolio of stocks that offers the best risk-return trade-off

The testing of the alternative hypotheses 7 requires two steps. First, the analysis of the portfolios' risk rankings displayed in Table 9-17 so that the least risky portfolio is identified. Next, by testing the statistical significance of the strength of association between the risk measures and portfolios' risk rankings identifying the least risky portfolio, it is possible to test the alternative hypothesis 7 indirectly or implicitly. That is, if each of the risk measures converges on a certain portfolio as the least risky, it sufficies to show that their co-movements are in the same direction and that this co-movements are statistically significant. For this purpose the non-parametric Spearman rank correlation and Kruskal-Wallis tests are fitted. The Spearman rank correlation test is used to measure the strength of association between pairs of portfolios' risk rankings, and to know if their association is statistical significant. The Kruskal-Wallis test is used to measure the significance of the strength of association of the entire group of portfolios' risk rankings. The confidence level specified in the tests is 95%. Also, since all portfolios have been optimized using the same target return, the portfolio with the lowest risk offers the best risk-return trade-off.

Table 9-16 displays the risk of the portfolios for each of the five risk measures and period scenarios considered. The portfolios' rankings and significance testing results displayed in Table 9-17 indicate that all the risk measures identify the retail benchmark portfolio as the least risky. In addition to that, the strength of association between the risk measures and portfolios' risk rankings that identify the retail benchmark portfolio as the least risky is close to 1, positively correlated and statistically significant. The Kruskal-Wallis results show that the strength of association of the entire group of portfolios' risk rankings is statistically significant. As a result, the retail benchmark portfolio is indeed the least risky and since each of the portfolios under consideration has been optimized using the same target portfolio return, the retail benchmark portfolio offers the best risk return trade-off. These findings lead to the acceptance of the alternative hypothesis 7.

Table 9-16: Portfolios' risk for the full sample, pre-GFC, GFC and post-GFC

Portfolios' Risk	CVaR	CDaR	Minimax	MAD	Var
		Full Sample Period			
Gold	5.55	103.02	15.63	1.8	0.062
Iron ore-nickel	4.39	40.91	7.94	1.35	0.035
Coal-uranium	4.81	83.68	9.21	1.44	0.042
Oil-gas	6.16	88	15.69	2.04	0.078
Mix-metals	4.77	88.52	10.83	1.44	0.043
Retail	2.09	25.94	3.65	0.669	0.008
Manufacturing	2.72	24.49	5.14	0.901	0.015
		Pre-GFC			
Gold	3.181	19.839	4.256	1.117	0.023
Iron ore-nickel	2.971	15.674	5.147	0.965	0.014
Coal-uranium	3.059	17.108	4.028	0.994	0.018
Oil-gas	2.128	8.287	2.615	0.731	0.009
Mix-metals	2.318	15.983	2.771	0.782	0.011
Retail	1.309	7.089	1.659	0.448	0.004
Manufacturing	1.429	10.776	1.821	0.519	0.005
		GFC			
Gold	5.962	116.09	11.71	1.969	0.08
Iron ore-nickel	5.428	39.56	10.39	1.756	0.063
Coal-uranium	5.751	55.15	8.56	1.698	0.057
Oil-gas	3.556	26.03	7.00	1.182	0.029
Mix-metals	5.864	93.66	12.76	1.829	0.072
Retail	2.58	26.86	3.54	0.879	0.014
Manufacturing	4.675	23.58	7.43	1.38	0.045
		Post-GFC			
Gold	4.12	20.167	5.983	1.41	0.037
Iron ore-nickel	4.003	31.116	5.898	1.32	0.031
Coal-uranium	4.172	38.512	8.305	1.303	0.049
Oil-gas	2.656	10.704	4.166	0.889	0.014
Mix-metals	3.914	24.233	6.441	1.261	0.04
Retail	1.561	8.178	2.344	0.518	0.005
Manufacturing	1.992	9.323	3.028	0.68	0.009

Notes: The table displays the risk of the portfolios for each of the risk measures and period scenarios considered.

Table 9-17: Full sample period portfolios' risk rankings

Portfolios' risk Rankings	CVaR	CDaR	Minimax	MAD	Var
Portfolios' risk rankings (full sample period)					
Gold	2	1	2	2	2
Iron ore-nickel	5	5	5	5	5
Coal-uranium	3	4	4	3.5	4
Oil-gas	1	3	1	1	1
Mix-metals	4	2	3	3.5	3
Retail	7	6	7	7	7
Manufacturing	6	7	6	6	6
CVaR-CDaR ρ=0.785 pvalue = 0.036**	CVaR-Minimax ρ =0.964 pvalue = 0.00***	CVaR-MAD ρ=0.991 pvalue=0.00***	CVaR-Var ρ=0.964 pvalue= 0.00***		CDaR-Minimax ρ =0.857 pvalue = 0.013**
CDaR- Var ρ=0.857 pvalue =0.013**	CDaR-MAD ρ=0.828 pvalue =0.021**	Minimax-MAD ρ=0.991 pvalue=0.00***	Minimax-Var ρ=1.00 pvalue =0.00***		MAD-Var ρ=0.991 pvalue =0.00***
Kruskal-Wallis Test: Chi-squared = 0.000, df = 4, p-value = 1					
Portfolios' risk rankings (Pre-GFC)					
Gold	1	1	2	1	1
Iron ore-nickel	3	4	1	3	3
Coal-uranium	2	2	3	2	2
Oil-gas	5	6	5	5	5
Mix-metals	4	3	4	4	4
Retail	7	7	7	7	7
Manufacturing	6	5	6	6	6
CVaR-CDaR ρ =0.928 pvalue = 0.00***	CVaR-Minimax ρ =0.892 pvalue = 0.00***	CVaR-MAD ρ =1.00 pvalue =0.00***	CVaR-Var ρ =1.00 pvalue = 0.00***		CDaR-Minimax ρ =0.75 pvalue = 0.052
CDaR-MAD ρ =0.92 pvalue = 0.00***	CDaR-Var ρ=0.928 pvalue = 0.00***	Minimax-MAD ρ=0.892 pvalue=0.00***	Minimax-Var ρ=0.892 pvalue = 0.00***		MAD-Var ρ=1.00 pvalue = 0.00***
Kruskal-Wallis Test: Chi-squared = 0, p-value = 1					
Portfolios' risk rankings (GFC)					
Gold	1	1	2	1	1
Iron ore-nickel	4	4	3	3	3
Coal-uranium	3	3	4	4	4
Oil-gas	6	6	6	6	6
Mix-metals	2	2	1	2	2
Retail	7	5	7	7	7
Manufacturing	5	7	5	5	5
CVaR- CDaR ρ =0.857 pvalue = 0.013**	CVaR-Minimax ρ = 0.928 pvalue = 0.00***	CVaR-MAD ρ = 0.964 pvalue=0.00***	CVaR- Var ρ = 0.964 pvalue = 0.00***		CDaR- Minimax ρ = 0.785 pvalue = 0.036**
CDaR- MAD ρ = 0.821 pvalue = 0.023**	CDaR- Var ρ = 0.821 pvalue = 0.023**	Minimax- MAD ρ = 0.964 pvalue=0.00***	Minimax- Var ρ = 0.964 pvalue = 0.00***		MAD- Var ρ = 1.00 p-value =0.00***
Kruskal-Wallis Test: Chi-squared = 0, p-value = 1					
Portfolios' risk rankings (Post-GFC)					
Gold	2	4	3	1	3
Iron ore-nickel	3	2	4	2	4
Coal-uranium	1	1	1	3	1
Oil-gas	5	5	5	5	5
Mix-metals	4	3	2	4	2
Retail	7	7	7	7	7
Manufacturing	6	6	6	6	6
CVaR- CDaR ρ = 0.892 pvalue = 0.00***	CVaR- Minimax ρ = 0.892 pvalue = 0.00***	CVaR-MAD ρ = 0.892 pvalue=0.00***	CVaR- Var ρ = 0.892 pvalue = 0.00***		CDaR- Minimax ρ = 0.892 pvalue =0.00***
CDaR- MAD ρ = 0.75 pvalue = 0.052	CDaR- Var ρ = 0.892 pvalue = .006***	Minimax- MAD ρ = 0.714 pvalue = 0.07	Minimax- Var ρ = 1.00 pvalue = 0.00***		MAD- Var ρ = 0.714 pvalue = 0.07
Kruskal-Wallis Test: Chi-squared = 0, pvalue = 1					

Notes: This table displays the rankings of the portfolios' risk estimates. The portfolios' risk estimates are displayed in Table 9-16. The results from the fit of the nonparametric Spearman rank correlation and Kruskal-Wallis tests are also displayed. The parameter ρ represents the strength of association between the rankings of the portfolios' risk estimates. Each column of rankings has a mean value of 4. The ** correspond to p-values < .05 and *** for p-values < .01. The pairs of risk measures CVaR-CDaR, CVaR-Minimax, CVaR-MAD and the remaining ones represent the pairs of portfolios' risk rankings.

9.2.2 Hypothesis 8

H_a: The average model convergence of the stocks' optimal weights is statistically significant.

Applying a one-sample two-tailed t-test for the difference between each of the optimal weights and the average of the optimal weights tests the alternative hypothesis 8. The average model convergence of the stocks' optimal weights is determined to be statistically significant if the difference between the average of the optimal weights and each of the optimal weights is not statistically significant. If the resulting t-test value is neither larger nor smaller than the critical value, the distance between two values is determined not to be statistically significant. The one-sample two-tailed t-test test employed is:

$$t = \frac{\bar{x} - \Delta}{s / \sqrt{n}} \tag{9.1}$$

The parameter $\bar{x}$ represents the sample mean or the mean of the optimal weights. The parameter Δ accounts for each of the optimal weights, while s represents the standard deviation of the sample of optimal weights. The parameter n stands for the size of the data sample. The degrees of freedom are estimated as follows:

$$df = (n_1 - 1) \tag{9.2}$$

The degrees of freedom and critical values across portfolios vary since the number of observations or optimal weights vary when fitting the t-test. The reason for this is that in some portfolios the weight allocations stemming from some of the fitted risk measures are ignored when searching for the average model convergence. Table 9-18 displays the degrees of freedom and critical values corresponding to 3, 4 and 5 observations.

Table 9-18: Degrees of freedom and critical values of observations

Number of observations	Degrees of freedom	Confidence level	Critical value
3 Observations	2	95%	±4.30
4 Observations	3	95%	±3.18
5 Observations	4	95%	±2.77
3 Observations	2	99%	±9.92
4 Observations	3	99%	±5.84
5 Observations	4	99%	±4.60

Notes: The table displays the degrees of freedom, confidence levels and critical values corresponding to 3, 4 and 5 observations. The parameters are considered for the fitting of the one sample two-tailed t-test.

According to Chapter 8 in the gold mining portfolio the optimal weights converge on average in ST. BARBARA (SBMX), if the model specification with respect to the *CDaR* is ignored. They also converge on average in NORTHWEST RESOURCES (NWRX) and RESOLUTE MINING (RSGS), if the model specifications with respect to the *CDaR* and *Minimax* are discarded. In the iron ore-nickel mining portfolio the optimal weights converge in BHP BILLITON (BHPX), if the model specifications with respect to the *CDaR* and *CVaR* are ignored. In the coal-uranium energy portfolio they converge in COAL BANK (CBQX), AQUILA RESOURCES (AQAX) and COALSPURN (CPLX), when the model specifications with respect to the *CDaR* and *Minimax* are ignored.

In the oil-gas energy portfolio the optimal weights converge on average in BEACH ENERGY (BPTX), and in ORIGIN ENERGY (ORGX) when the model specifications with respect to the *MAD* and *variance* risk are discarded. In the mix-metals portfolio they converge in RIO TIONTO (RIOX) and CUDECO (CUX), when the model specifications with respect to the *CDaR* and *Minimax* are ignored. In the retail benchmark portfolio they converge on average in the M2 TELECOM (MTUX), WOOLWORTHS (WOWX) and ARB (ARPX) stocks when the model specifications with respect to the *CDaR* and *Minimax* are ignored. In the manufacturing benchmark portfolio they converge in CSL (CSLX), BRICKWORKS (BKWX) and ANSELL (ANNX) if the model specifications with respect to the *CDaR* and *Minimax* are ignored.

Table 9-19 displays the results of the significant testing on the stocks selected by the average model convergence. It is observed that none of the resulting t-test values is larger or smaller than the critical values displayed in Table 9-18. As a result, the difference between the average of the optimal weights and each of the optimal weights is not statistically significant at the 95% and 99% confidence levels. This in turn implies that the average model convergence of the stocks' optimal weights is a statistically significant. This information leads to the acceptance of the alternative hypothesis 8.

Table 9-19: Significance t-testing of the portfolios' optimal weights

Stocks/ Risk Measures	CVaR	CDaR	Minimax	MAD	Var	MW	MW ex. CDaR	MW ex. CDaR & Minimax	MW ex. CVaR & CDaR	MW ex. MAD & Variance
					Gold portfolio					
SBMX	30.01 t=-1.98	44.28 Discarded	24.25 t=1.95	24.93 t=1.48	29.23 t=-1.45	30.54	**27.11**	28.06	26.14	32.85
NWRX	3.53 t=1.88	0 Discarded	0 Discarded	4.18 t=-0.34	4.53 t=-1.54	2.45	3.06	**4.08**	2.90	1.18
RSGX	13.54 t=0.45	0 Discarded	0 Discarded	14.15 t=-1.91	13.28 t=1.46	8.19	10.24	**13.66**	9.14	4.51
					Iron ore-nickel portfolio					
BHPX	46.72 Discarded	53.15 Discarded	39.52 t=-0.19	39.38 t=1.82	39.62 t=-1.63	43.68	41.31	41.91	**39.51**	4.46
					Coal-uranium portfolio					
CBQX	3.97 t=-0.20	4.36 Discarded	1.47 Discarded	3.70 t=1.82	4.16 t=-1.62	3.53	3.33	**3.94**	3.11	3.27
AQAX	17.45 t=-0.67	0.00 Discarded	1.41 Discarded	17.63 t=-1.30	16.70 t=1.97	10.64	13.30	**17.26**	11.91	6.29
CPLX	12.60 t=0.54	2.40 Discarded	19.41 Discarded	12.26 t=1.40	13.58 t=-1.94	12.05	14.46	**12.81**	15.08	11.47
					Oil-gas portfolio					
BPTX	94.94 t=-1.19	95.13 t=-1.84	95.13 t=-1.84	93.87 t=2.43	93.87 t=2.43	**94.59**	94.45	94.23	94.29	95.07
ORGX	4.29 t=2.00	4.87 t=-1.00	4.87 t=-1.00	0.00 Discarded	0.00 Discarded	2.81	2.29	1.43	1.623	**4.677**
					Mix-metals portfolio					
RIOX	31.22 t=1.99	0.00 Discarded	25.39 Discarded	33.03 t=-1.13	32.88 t=-0.87	24.50	30.63	**32.38**	30.43	18.87
CDUX	7.97 t=-0.31	34.73 Discarded	0.00 Discarded	7.36 t=1.87	8.32 t=-1.56	11.68	5.91	**7.88**	5.23	14.23
					Retail portfolio					
MTUX	12.82 t=-1.98	22.62 Discarded	14.13 Discarded	11.08 t=1.23	11.34 t=0.75	14.40	12.34	**11.75**	12.18	16.52
WOWX	29.52 t=-1.76	28.46 Discarded	1.84 Discarded	27.45 t=0.06	25.60 t=1.70	22.57	21.10	**27.52**	18.30	19.94
ARPX	19.57 t=1.95	0.00 Discarded	31.72 Discarded	22.11 t=-0.58	22.90 t=-1.37	19.26	24.08	**21.53**	25.58	17.10
					Manufacturing portfolio					
CSLX	54.70 t=1.10	65.88 Discarded	49.29 Discarded	56.52 t=-2.00	54.82 t=0.90	56.24	53.83	**55.35**	53.54	56.62
BKWX	13.03 t=-1.50	7.33 Discarded	5.33 Discarded	12.10 t=-0.39	10.18 t=1.89	9.59	10.16	**11.77**	9.20	8.56
ANNX	17.32 t=-0.22	13.03 Discarded	35.14 Discarded	15.81 t=1.83	18.35 t=-1.61	19.93	21.66	**17.16**	23.10	21.83

Notes: This table displays the stocks from the mining, energy, retail and manufacturing portfolios in which the optimal weights from the various portfolio optimization model specifications converge on average. The abbreviations MW, MW ex. *CDaR* and MW ex. *Minimax* and *CDaR* stand for mean of the optimal weights, mean of the optimal weights excluding the weights from the optimization with respect to the *CDaR* measure, and so on with the rest. The letter *t* represents the t-test value resulting from the fit of the one-sample two-tailed t-test. The average values in bold are used to test for the statistical significance.

Table 9-20: Hypothesis testing results

Alternative Hypotheses	Hypothesis Statement	Acceptance/Rejection
H_a 1:	There are mining portfolios with higher dependence risk than others.	Accepted
H_a 2:	There are energy portfolios with higher dependence risk than others	Accepted
H_a 3:	There are mining portfolios with higher dependence risk than energy portfolios	Accepted
H_a 4:	There are mining and energy portfolios with higher dependence risk than retail and manufacturing benchmark portfolios.	Accepted
H_a5:	The portfolios' dependence structure changes between period scenarios are statistically significant	Accepted
H_a 6:	There is a pair vine copula model that best captures the multivariate dependence structure of the portfolios	Accepted
H_a 7:	There is a portfolio of stocks that offers the best risk-return trade-off	Accepted
H_a 8:	The average model convergence of the stocks' optimal weights is statistically significant.	Accepted

Notes: This table shows the alternative hypotheses tested and their acceptance. The number of hypotheses tested is eight. Six of them stem from the pair vine copula modelling of dependence, while the remaining two are based on the portfolio optimization component of this thesis.

A summary of the hypothesis testing indicates that each of the alternative hypotheses formulated is accepted. The alternative hypotheses 1 to 4 are accepted because dependence risk differences are found to exist between the mining portfolios, energy portfolios, mining and the energy portfolios, and between the mining and energy portfolios and the retail and manufacturing benchmark portfolios. The alternative hypothesis 5 is accepted because statistically significant dependence structure changes are observed to take place between pairs of period scenarios. The alternative hypothesis 6 is accepted because specific vine copula models are identified to best suit the multivariate dependence structure of each of the portfolios. The alternative hypothesis 7 is accepted because one portfolio is identified to have the lowest risk and offer the best risk-return trade-off. Finding the distance between the average of the optimal weights and each of the optimal weights not to be statistically significant leads to the acceptance of the alternative hypothesis 8.

9.3 Discussion of results

The acceptance of the alternative hypotheses 1 to 4 is not surprising since in Chapters 5, 6 and 7, through the use of the counting stage of the copula counting technique, it was noticed that the portfolios' dependence concentrations differ in size and structure. The acceptance of the alternative hypothesis 6 is an important result because in Chapters 5, 6 and 7 the identification of specific vine copula models, through the use of the counting stage of the copula counting technique, to best account for the multivariate dependence structure of the portfolios may have not sufficed to show that those models were indeed the most suitable. The acceptance of the alternative hypothesis 6 helped verify and validate the fit of the vine copulas in Chapters 5, 6 and 7. The acceptance of the alternative hypothesis 7 supports the results from Chapters 7 and 8, where the retail benchmark portfolio is recognized to be the second least dependence risky and the least investment risky, respectively. The acceptance of the alternative hypothesis 8 contributes to verify that the distance between the average of the optimal weights and each of the optimal weights, assign on the stocks identified as good candidate for investment, is not statistically significant.

9.3 Summary

This chapter tested the alternative hypotheses corresponding to the research questions posed in Chapter 1. The testing of the alternative hypotheses 1 to 5 was conducted by fitting a two-sample two-tailed t-test for the difference of means between two portfolios' dependence concentrations at various locations of the joint distributions. The alternative hypotheses 1 to 4 were accepted because statistically significant dependence risk differences were found between the mining portfolios, energy portfolios, mining and energy portfolios, and between mining and energy and retail and manufacturing benchmark portfolios. Some portfolios were found to have higher dependence risk than

others in specific market conditions. The alternative hypothesis 5 was accepted because statistically significant dependence structure changes were observed to take place between pairs of period scenarios. The alternative hypothesis 6 was accepted because specific vine copula models were identified to best fit the multivariate dependence structure of the portfolios. The alternative hypothesis 7 was accepted because one portfolio was recognized to have the lowest investment risk and offer the best risk-return trade-off. The
statistical significance of the average model convergence led to the acceptance of the alternative hypothesis 8.

The c-vine copula model was identified to best account for the multivariate dependence structure of the iron ore-nickel, coal-uranium, oil-gas and mix-metals portfolios. The r-vine copula model was acknowledged for best capturing the multivariate dependence of the gold mining and retail benchmark portfolios. The d-vine copula model, on the other hand, best fits the dependence structure of the manufacturing benchmark portfolio. Each of the alternative hypotheses formulated was accepted.

CHAPTER 10

CONCLUDING REMARKS

This chapter consists of three sections: results-discussion and contributions, limitations, and suggestions for further research.

The *results-discussion and contributions* section briefly states and discusses this thesis' contributions and main results. The *limitations* section states and discusses the main limitations of the study. The *suggestion for further research section* proposes some topics that could be worth exploring in subsequent studies and applications.

10.1 Results-discussion and contributions

This thesis implements pair vine copula models including c-vines, d-vines and r-vines, along with linear and nonlinear optimization methods with respect to the *variance*, *MAD*, *Minimax*, *CVaR* and *CDaR* risk measures, to thoroughly and comprehensively examine the dependence risk, investment risk and portfolio allocation features of seven 20-asset portfolios from the mining, energy, retail and manufacturing sectors of the Australian stock market in the context of the 2008-2009 GFC and pre-GFC, GFC, post-GFC and full sample period scenarios. In Chapters 5, 6 and 7 the analysis of the portfolios' dependence risk is based on the analysis of the dependence concentration in the centre and tails of the joint distributions. The analysis of the portfolios' investment risk and portfolio allocation features conducted in Chapter 8 stems from the examination of the portfolios' overall risk, optimal weights and model convergence in some stocks.

This thesis contributes to the literature on pair vine copula modelling of dependence and multiple risk measure-based portfolio optimization by introducing a "copula counting technique" and "average model convergence" perspectives. The copula counting technique has enabled an in-depth and comprehensive analysis of the portfolios' dependence structure and dependence risk characteristics in specific market conditions. The copula counting technique aside from being an alternative avenue for the interpretation of multivariate dependence structures, it introduces new concepts and theory to the pair vine copula literature. Overall it has made possible a broader understanding of the portfolios' underlying sectors' dependence risk dynamics. The average model convergence has offered an alternative way to address the optimal stock selection and investment confidence problems underlying any type of portfolio optimization and faced by investors when having to select stocks from a wide array of optimal investment scenarios. The approach represents a shift of perspective in the multiple risk measured-based portfolio optimization literature in the sense that it identifies stocks that could be good candidates for investment, through model convergence and model consensus.

A wide variety of portfolios is considered because of their differences in terms of structure, volatility, uses, and their importance in asset investment. For example, the retail stocks along with the gold stocks, which tend to be defensive in times of financial turbulence, could be used to hedge investment positions in the iron ore and nickel sectors, which have shown to be more volatile. Also, the portfolios could be used to diversify an investment position in traditional equity sectors such as the financial sector. Oil and gas stocks have been selected for the analysis because their representation in the Australian energy market is increasing continuously. The same is true for the coal and uranium stocks, which may share some similarities arising from their common use for electricity generation. Stocks from the iron ore sector are considered in the analysis of dependence and portfolio optimization because iron ore production has a special place in the mining sector of the Australian economy due to the large scale of the iron ore business exports. A mix-metals leptokurtic mining portfolio is included in the mix of portfolios because it is of interest to understand the characteristics of a non-homogeneous multivariate dependence structure.

The empirical results stemming from the fit of the pair vine copulas and the use of the copula counting technique indicated that the c-vines are overall the most suitable models to account for the multivariate dependence structure of the mining and energy portfolios. Also, while the iron ore-nickel mining and oil-gas energy portfolios are identified to be the most dependence risky, the gold and retail are overall the least dependence risky. The suitability of the c-vines to best account for the multivariate dependence structure of the mining and energy portfolios appears to be influenced by the presence of a rootstock in each of the portfolios having high correlation values with the rest of the stocks in the portfolios. In the iron ore-nickel mining portfolio the c-vine identifies BHP BILLITON

(BHPX) as the rootstock. RIO TINTO (RIOX) is identified in the mix-metals, PALADIN ENERGY (PDNX) in the coal-uranium, and WOODSIDE (WPLX) in the oil-gas energy portfolio. The empirical results stemming from the fit of the multiple risk measure-based portfolio optimization model specifications indicates that the portfolio with the lowest investment risk is the retail thus, offering the best risk-return trade-off out of the seven portfolios considered. The most dependence risky portfolios are the iron ore-nickel mining and oil-gas energy portfolios. The most investment risky portfolio is the oil-gas energy portfolio. Out the mining portfolios the gold portfolio is the least dependence risky. Out of the energy portfolios, the coal-uranium portfolio is less dependence risky than the oil-gas.

The pair vine copula modelling of dependence undertaken in the Chapters 5, 6 and 7 indicates that each of the portfolios modelled has dependence risk features consistent with specific market conditions. Out of the mining portfolios the gold and mix-metals have low dependence risk in times of financial turbulence. The iron ore-nickel mining portfolio has the highest dependence risk in similar market conditions, as indicated by the large concentration of dependence it has in the negative tail. Stocks from the gold and mix-metals could consequently be used to hedge an investment position with high concentration in the iron ore and nickel sectors.

With respect to the energy portfolios, the coal-uranium is identified to have low dependence risk in times of financial turbulence, while the oil-gas has high dependence risk in similar market conditions. Coal and uranium stocks could therefore be used to reduce the risk of an investment position in the oil sector. Although both benchmark portfolios have low dependence risk in market conditions characterized by low confidence in the financial stock markets, the retail is significantly less dependence risky than the manufacturing benchmark portfolio. Investments in the retail sector are therefore preferred to investments in the manufacturing sector during crisis and non-crisis periods. The identification of the gold mining portfolio as low dependence risky in times of financial turbulence is in congruence with the literature's research findings (see e.g. Baur & Lucey, 2010; Dee et al., 2013; Courdert & Raymond, 2010; Morales & Andreosso-O'Callaghan, 2011; Faff & Chan, 1998). As compared to those studies which model the risk characteristics of gold, this thesis modelling of gold markets is more complete because it identifies the symmetric and asymmetric dependence risk characteristics of the assets in specific market conditions. Besides, it examines their negatively and positively skewed price and return behaviour in different market conditions.

The mix-metals portfolio's high dependence risk relative to the gold stems from the wide variety of stocks it consists of. Specifically, some of its stocks belong to the iron ore and nickel sectors, identified to be significantly more dependence risky than the gold sector. Bingham and Perkins (2012) have pointed out the high-risk features of the iron ore and nickel sector in market conditions characterized by low confidence in the financial stock markets. The low dependence risk identified in the coal-uranium energy portfolio in similar market conditions is found to stem from the relative stability the coal and uranium commodities displayed during the 2008-2009 global financial crisis. The Bureau of Resources and Energy Economics (2014), The Department of Resources Energy and Tourism (2013) and Bingham and Perkins (2012) have pointed out the price and implied risk behavior of the coal and uranium sectors during the crisis period.

The oil-gas energy portfolio's high dependence risk in non-tranquil stock market conditions is in line with the literature modelling the risk in oil markets (e.g. Du et al., 2012; Killian & Park, 2009; Park & Ratti, 2008; Basher & Sadorsky, 2006). This thesis modelling of energy stock markets relative to the modelling of energy markets undertaken by Tong et al. (2013), Wen et al. (2012) and Chang et al. (2011) has the comparative advantage of scrutinizing the assets' dependence scattered at various locations in the joint distributions. Those studies by not considering a systematic approach, such as the copula counting technique, in their analysis of dependence are unable to thoroughly and comprehensively examine the multivariate dependence risk dynamics of the energy assets modelled. In fact, this thesis' modelling of the Australian energy stock markets appears to be the first to use pair vine copulas to model their dependence risk behavior.

The identification of the retail benchmark portfolio as less dependence risky than the manufacturing benchmark portfolio in tranquil periods and non-tranquil periods has to do with the specific type of economic linkages and relationship of dependence each of the portfolios' underlying sectors has with the Australian resources sector (i.e. mining and energy sectors), as pointed out by The Australian Retailers Association (2014), Savills Research (2014), Delloite (2013) and Mehmedovic et al. (2011). This thesis' research and examination of the Australia retail and manufacturing sectors, relative to the research conducted by The Australian Retailers Association (2014), Savills Research, (2014), Delloite (2013), KordaMentha (2013), Commonwealth Treasury (2012), Green and Roos (2012), The National Australian Bank (2012), The Productivity Commission (2011) and Mehmedovic et al. (2011), is more complete because aside from looking at the sectors' performance and price behaviour in varied market conditions, it examines thoroughly their dependence risk in specific market conditions. The research findings resulting from the fit of the pair vine copulas are validated by comparing them with the actual price behaviour of the stock portfolios‘ underlying sectors.

The multiple risk measure-based portfolio optimization implemented in Chapter 8 shows that the retail benchmark portfolio has the lowest investment risk, while the oil-gas is the most investment risky. Out of the two energy portfolios modelled the oil-gas is more investment risky. Out of the benchmark portfolios the retail is less investment risky. Out of the seven portfolios modelled the retail offers the best risk-return trade-off. In the gold mining portfolio the average model convergence proposed in this thesis identifies ST. BARBARA (SBMX), NORTHWEST RESOURCES (NWRX) and RESOLUTE MINING (RSGS) as good candidates for investment; BHP BILLITON (BHPX) is the best choice in the iron ore-nickel mining portfolio; RIO TIONTO (RIOX) and CUDECO (CUX) in the mix-metals; AQUILA RESOURCES (AQAX) and COAL SPURN (CPLX) in the coal-uranium; BEACH ENERGY (BPTX) and ORIGIN ENERGY (ORGX) in the oil-gas; M2 TELECOM (MTUX), WOOLWORTHS (WOWX) and ARB (ARPX) in the retail; and CSL (CSLX), BRICKWORKS (BKWX) and ANSELL (ANNX) in the manufacturing benchmark portfolio. Each of the stocks selected by the average model convergence have the distinctive features of having a high return relative to risk and of having being allocated large weights by most of the portfolio optimization model specifications. It is also noticed that several stocks with high return relative to risk are not allocated large weights and are not spotted by the average model convergence as good candidates for investment.

The identification of the oil-gas energy portfolio as the most *investment* risky is to a large extent consistent with the results from Chapters 5 and 6, where the oil-gas energy portfolio is recognized to be the second most dependence risky, next to the iron ore-nickel mining portfolio. This high-risk feature of the oil gas sector is in line with the literature (e.g. Faff & Brailsford, 1999; Basher & Sadorsky, 2006; Killian & Park, 2009; Park & Ratti, 2008; Du et al., 2012). The identification of the retail benchmark portfolio as the least investment risky and as the one offering the best risk-return trade-off is also in congruence with the results from Chapter 7, where the retail benchmark portfolio is identified to be the second least dependence risky in crisis and non-crisis periods. The ability of the average model convergence approach to identify stocks with high return relative to risk and with large weight allocations suggests that the approach is useful and worth considering for the optimization of portfolios with respect to multiple risk measures.

The multiple risk measure-based portfolio optimization implemented in this thesis, relative to the *single* risk measure portfolio optimization by Chang et al. (2011) and De Oliveira et al. (2011), Zhou (2004), Zhou and Yin (2003), Alexander and Baptista (2002), Li et al. (2002), Steinbach (2001), Yoshimoto (1996), He and Litterman (1999), Bevan and Winkelmann (1998), Kroll et al. (1984), Samuelson (1970) and Markowitz (1952) is in the least more informative. Those studies, specifically, lack the multi-angle portfolio optimization perspective that could cater for the investors' diverse specific risk and return preferences. Relative to the *multiple* risk measure-based portfolio optimization undertaken by Krokhmal et al. (2002), Cheng and Wolverton (2001) and Stone (1973), this thesis' modelling of the portfolios' investment risk addresses more effectively and objectively the optimal stock selection and investment confidence problems underlying any type of portfolio optimization and faced by investors when having to select stocks form a wide array of optimal investment scenarios.

The fitted pair vine copula models, along with the use of the copula counting technique, show to be worthy of consideration for the modelling of stock portfolios' dependence risk. The implemented multiple risk measure-based portfolio optimization model specifications, along with the average model convergence perspective, prove to be an attractive alternative way to address the optimal stock selection and investment confidence problems.

The hypothesis testing conducted in Chapter 9 shows that each of the alternative hypotheses formulated is accepted. The identification of statistically significant dependence risk differences between portfolios, and of statistically significant dependence structure changes between pairs of period scenarios led to the acceptance of the alternative hypotheses 1 to 5. The presence of statistically significant dependence structure changes between pairs of period scenarios is discerned to reflect the levels of confidence and volatility changes in the financial stock markets across period scenarios. The statistical significance of the strength of association and same-direction co-movements of the portfolios' risk rankings and risk measures, that identify the retail benchmark portfolio as the least risky, led to the acceptance of the alternative hypothesis 6. The identification of specific vine copula models to best fit the multivariate dependence of the portfolios led to the acceptance of the alternative hypothesis 7. The alternative hypothesis 8 is accepted because the difference between the average of the optimal weights and each of the optimal weights is found not to be statistically significant. The research findings from Chapters 5, 6, 7 and 8 are recognized to be in line with the hypothesis testing results from Chapter 9.

The acceptance of the alternative hypotheses 1 to 4 is found to be coherent with the analysis conducted in Chapters 5, 6 and 7, where through the use of the counting stage of the copula counting technique, the size of the portfolios' dependence concentration is noticed to be different in terms of size and structure. The acceptance of the alternative hypothesis 6 is an important result because the identification of specific vine copula models as best fitting the multivariate dependence structure of the portfolios in Chapters 5, 6 and 7 may have put into question the reliability of the copula counting technique for the identification of the most suitable vine copula models. The acceptance of the alternative hypothesis 7 supports the findings from Chapters 7 and 8 where the retail benchmark portfolio is identified to be the least *investment* risky and the second least *dependence* risky. The acceptance of the alternative hypothesis 8 verifies that the convergence of the optimal weight allocations in some stocks is statistically significant.

Portfolio managers, risk managers, hedging practitioners, financial market analysts, systemic risk and capital requirement agents, who follow the trends of the Australian mining, energy, retail and manufacturing sectors, may find the obtained empirical results useful to design investment risk and dependence risk-adjusted optimization algorithms, risk management frameworks and dynamic hedging strategies. For those end users, it is of interest to know what the inherent dependence risk characteristics of those sectors are like. The same is true for risk managers in performing stress-testing and robustness checks, which are particularly important in times of financial turbulence where extreme downside risk events tend to occur (Al Janabi, 2013).

10.2 Limitations

A possible limitation of this thesis' research lies in the size of the data samples used. Although the employed 7.5 years price series' length is long enough to account for the volatility and dependence structure changes across period scenarios, a larger number of stocks in each of the portfolios under consideration could have provided additional value to the portfolio optimization and dependence risk modelling. In spite of the number of stocks in each portfolio being 20, it was possible to draw generalizations and insights about the dependence risk profile and portfolio allocation features of the stock portfolios and their underlying sectors.

Another limitation of the modelling framework implemented in this thesis stems from the nature of the copula counting technique proposed to dissect, organize, analyse and interpret the asset portfolios' multivariate dependence structure. Specifically, the technique while enabling a comprehensive analysis of the portfolios dependence risk dynamics in specific market conditions, it does not provide an exact estimate of dependence risk in the negative tail. This limitation is in turn conditioned by the analytical design of each bivariate copula employed in the statistical vine copula structures or models. That is, although each of the bivariate copulas employed best captures the dependence from a certain location of the joint distribution, they simultaneously, and to a lesser degree, capture the dependence scattered in all locations of the joint distributions. An exact estimate of the dependence concentrated in the negative tail could lead to more accurate estimates of downside risk.

Another limitation arises from the specific type of market conditions under which the portfolios' dependence risk is modelled: crisis periods and non-crisis periods. While in theory it may be simple to define and conceive those notions, in practice it is difficult to distinguish between crisis and non-crisis periods, times of financial turbulence and tranquil periods, market conditions characterized by low confidence in the financial stock markets and market conditions with restored stock market confidence, tranquil stock market conditions and non-tranquil stock market conditions. One more limitation stems from the difficulty to compare each of the portfolio's optimal weight allocations across different risk measures. That is, since each of the risk measures produces an estimate of risk in its own space it becomes troublesome to grasp an overall risk estimate when comparing them. In spite of this difficulty it is possible to identify the least risk and most risky portfolios by comparing them on the same risk measure.

10.3 Suggestions for further research

One line of research within the pair vine copula field worth exploring relates to the development of criteria to fit regular vines. There are no clear criteria about the types of data sets suitable to be modelled by r-vines (i.e. the set of vines that excludes the c-vines and d-vines). There are, however, more or less clear criteria about the suitability of the c-vines and d-vines to model data sets of specific characteristics (see Czado, 2010). More applications of pair vine copulas to model the mining, energy, retail and manufacturing markets could lead to results that could impact policy and decision making related to investment in the mining and energy sectors modelled. Applications of pair vine copulas to model credit risk, market risk, liquidity risk, investment risk and dependence risk in the financial sector could provide insights that could impact policy concerned with financial and macro economic stability. The fields of behavioral finance and high frequency modelling could also benefit from the implementation of pair vine copulas.

BIBLIOGRAPHY

Aas, K. (2011). Pair-copula constructions: even more flexible than copulas, 1-39.

Aas, K. (2004). Modelling the dependence structure of financial assets: a survey of four copulas. *Norwegian Computing Centre*, 1-21.

Aas, K., Czado, C., Frigessi, A., & Bakken, H. (2009). Pair-copula constructions of multiple dependence. *Insurance: Mathematics & Economics Journal*, *44*(2), 182–198.

Ahammad, H., & Clements, K. W. (1999). What does minerals growth mean to Western Australia? *Resources Policy*, *25*(1), 1-14.

Albuquerque, R. (2012). Skewness in stock returns: reconciling the evidence on firm versus aggregate returns. *Review of Financial Studies*, *25*(5), 1630-1673.

Alcock, J., & Hatherley, A. (2008). The price of asymmetric dependence. In *21st Australasian Finance & Banking Conference*, 1-46.

Alcock, J., Brailsford, T., Faff, R., & Rand-Kwong, Y.L. (2013). Canonical vine copulas in the context of modern portfolio management: are they worth it? *Journal of Banking & Finance, 37*(8), 3085-3089.

Allen, D.E., Ashraf, M.A., McAleer, M., Powell, R.J., & Singh, A.K. (2013). Financial dependence analysis: applications of vine copulae. *Statistica Neerlandica, 67*(4), 403-435.

Alexander, G. J., & Baptista, A. M. (2002). Economic implications of using a mean-VaR model for portfolio selection: a comparison with mean-variance analysis. *Journal of Economic Dynamics & Control*, *26*(7), 1159-1193.

Al Janabi, M. A. (2013). Tactical risk analysis in emerging markets in the wake of the credit crunch and ensuing sub-prime financial crisis. *Emerging Markets & the Global Economy: A Handbook*, 413.

Almeida, C., Czado, C., & Manner, H. (2012). Modelling high dimensional time-varying dependence using d-vine SCAR models. *arXiv preprint arXiv:1202.2008,* 1-29.

Aloui, R., Hammoudeh, S., & Nguyen, D.K. (2013). A time-varying copula approach to oil and stock market dependence: the case of transition economies. *Energy Economics*, *39*, 208–221.

Ammann, M., & Süss, S. (2009). Asymmetric dependence patterns in financial time series. *The European Journal of Finance*, *15*(7-8), 703-719.

Ang, A., & Chen, J. (2002). Asymmetric correlations of equity portfolios. *Journal of Financial Economics, 63*(3), 443-494.

Andrew, J.A. (2012). Australia's trade performance 1990-91 to 2010-11. *Department of Foreign Affairs and Trade: Economic Advocacy & Analysis Branch*, 1-9.

Ardia, D., Boudt, K., Carl, P., Mullen, K., & Peterson, B. (2011a). Differential evolution with DEoptim: an application to non-convex portfolio optimization. *The R Journal, 3* (1), 27-34.

Ardia, D., Mullen, K., Ulrich, J., & Peterson, B. (2011b). DEoptim: an R package for global optimization by differential evolution. *Journal of Statistical Software, 40*(6), 1-26

Arreola, J., & Powell, R. (2013). Optimal risk minimization of Australian energy and mining portfolios of stocks under multiple risk measures. Proceedings of the *20th International Congress on Modelling & Simulation*, Adelaide, Australia, 1-7.

Arreola, J., Allen, D., & Powell, R. (2013). Dependence estimation and controlled CVaR portfolio optimization of a highly kurtotic Australian mining sample of stocks. Proceedings of the *20th International Congress on Modelling and Simulation*, Adelaide, Australia, 1-7.

Ausin, M. C., & Lopes, H.F. (2010). Time-varying joint distribution through copulas. *Computational Statistics & Data Analysis*, *54*(11), 2383-2399.

Australian Government Productivity Commission (2011). Economic structure and performance of the Australian retail industry. *Porductivity Commision Draft Report, Canberra*, 1-14.

Australian Retailers Association (2014). Financial system inquiry. *Melbourne*, *August*,1-10.

Australian Bureau of Statistics (2015). Retail trade, Australia. *Canberra*, *January*.

Baba, K., & Sibuya, M. (2005) Equivalence of partial and conditional correlation coefficients. *Journal of Japanese Statistical Society*, *35*(1), 1–19.

Bae, K.H., Karolyi, G.A., & Stulz, R.M. (2003). A new approach to measuring financial contagion. *Review of Financial Studies*, *16*(3), 717-763.

Bank for International Settlements (2009). The international financial crisis: timeline, impact and policy responses in Asia and the Pacific. *BIS papers*, 52, 21-82.

Barone-Adesi, G. (1985). Arbitrage equilibrium with skewed asset returns. *Journal of Financial & Quantitative Analysis*, *20*(03), 299-313.

Barunik, J., & Vacha, L. (2013). Contagion among central and eastern European stock markets during the financial crisis. *arXiv preprint arXiv:1309.0491*, 1-17.

Basher, S.A., & Sadorsky, P. (2006). Oil price risk and emerging stock markets. *Global Finance Journal*, *17* (2), 224-251.

Baur, D.G. (2012). Financial contagion and the real economy. *Journal of Banking and Finance*, *36* (10), 2680-2692.

Baur, D.G., & McDermott, T.K. (2009). Is gold a safe haven? International evidence. *Journal of Banking & Finance*, *34*(8), 1886-1898.

Baur, D. G., & Lucey, B. M. (2010). Is gold a hedge or a safe haven? An analysis of stocks, bonds and gold. *Financial Review*, *45*(2), 217-229.

Bedford, T., & Cooke, R.M. (2001). Probability density decomposition for conditionally dependent random variables modelled by vines. *Annals of Mathematics & Artificial Intelligence*, *32* (1-4), 245–268.

Bedford, T., & Cook, R. M. (2002). Vines: a new graphical model for dependent random variables. *Annals of Statistics*, *30*, 1031–1068.

Berg, D., & Aas, K. (2009). Models for construction of higher-dimensional dependence: a comparison study. *European Journal of Finance*, *15*, 639–659.

Bevan, A., & Winkelmann, K. (1998). Using the Black-Litterman global asset allocation model: three years of practical experience. *Fixed Income Research*, 1-19.

Bhattacharya, A., & Kojima, S. (2012). Power sector investment risk and renewable energy: a Japanese case study using portfolio risk optimization method. *Energy Policy*, *40*, 69-80.

Bingham, F. (2012). Australia's resources exports 2001 to 2011. *Department of Foreign Affairs and Trade: Economic Advocacy & Analysis Branch*, 1-4.

Bingham, F., & Perkins, B. (2012). Australia's coal and iron ore exports 2001 to 2011. *Department of Foreign Affairs and Trade: Economic Advocacy & Analysis Branch*, 1-8.

Bishop, J., Kent, C., Plumb, M., & Rayner, V. (2013). The resources boom and the Australian economy: a sectoral analysis. *RBA Bulletin*, 39-50.

Blalock, H.M. (Ed.). (1985). *Causal models in the social sciences*. Transaction Publishers.

Brechmann, E.C., & Schepsmeier, U. (2013). Modelling dependence with c and d vine copulas: The R-package C-D vine. *Journal of Statistical Software*, *52* (3), 1-26.

Brechmann, E.C., Czado, C., & Aas, K. (2012). Truncated regular vines in high dimensions with applications to financial data. *Canadian Journal of Statistics*, *40* (1), 68–85.

Brechmann, E.C., & Czado, C. (2012). COPAR- multivariate time series modelling using the copula autoregressive model. *arXiv:1203.3328*, 1-32.

Brechmann, E.C., & Czado, C. (2013). Risk management with high-dimensional vine copulas: an analysis of the Euro Stoxx 50. *Statistics & Risk Modelling*, *30*(4), 307-342.

Brechmann, E.C., Hendrich, K., & Czado, C. (2013). Conditional copula simulation for systemic risk stress testing. *Insurance: Mathematics & Economics*, *53*, 722-732.

Brechmann, E.C., Czado, C., & Paterlini, S. (2014). Flexible dependence modelling of operational risk losses and its impact on total capital requirements. *Journal of Banking & Finance*, *40*, 271-285.

Brooks, C., & Kat, H. (2002). The statistical properties of hedge fund index returns and their implications for investors. *Journal of Alternative Investments*, *5*(2), 26-44.

Bureau of Resources and Energy Economics (2014). Australian energy update. *Canberra*, *July*, 1-24.

Campbell, R., Koedijk, K., & Kofman, P. (2002). Increased correlation in bear markets. *Financial Analysts Journal*, *58* (1), 87-94.

Chang, C.L., McAleer, M., & Tansuchat, R. (2011). Crude oil hedging strategies using dynamic multivariate GARCH. *Energy Economics*, *33*(5), 912-923.

Chang, T. J., Yang, S. C., & Chang, K. J. (2009). Portfolio optimization problems in different risk measures using genetic algorithm. *Expert Systems with Applications*, *36*(7), 10529-10537.

Chen, S., & Poon, S. (2007). Modelling international stock market contagion using copula and risk appetite. *Capital Markets: Asset Pricing & Valuation eJournal*, 1-48.

Chen, P. Y., & Popovich, P. M. (2002). *Correlation: parametric and nonparametric measures* (No. 137-139). Sage.

Cheng, P., & Wolverton, M.L. (2001). MPT and the downside risk framework: a comment on two recent studies. *Journal of Real Estate Portfolio Management*, *7*(2), 125-131.

Chekhlov, A., Uryasev, S., & Zabarankin, M. (2003). *Portfolio optimization with drawdown constraints, asset and liability management tools*. Risk Books, London.

Chesters, J. (2010). The global financial crisis in Australia. *Proceedings of the Conference: Social Causes, Private Lives.* Australian Sociological Association. The University of Queensland, Australia.

Chiang, T. C., Jeon, B. N., & Li, H. (2007). Dynamic correlation analysis of financial contagion: evidence from Asian markets. *Journal of International Money & Finance*, *26*(7), 1206-1228.

Chollete, L., Heinen, A., & Valdesogo, A. (2009). Modelling international financial returns with a multivariate regime switching copulas. *Journal of Financial Econometrics*, *7*, 437-480.

Chunhachinda, P., Dandapani, K., Hamid, S., & Prakash, A. J. (1997). Portfolio selection and skewness: evidence from international stock markets. *Journal of Banking & Finance*, *21*(2), 143-167.

Clarke, J., Jandik, T., & Mandelker, G. (2001). The efficient markets hypothesis. *Expert Financial Planning: Advice from Industry Leaders*, 126-141.

CME Group (2011). Risk management for equity asset managers. *Chicago*, 1-20.

Collins, T. (2013). Australia's gold industry: trade, production and outlook. *Department of Foreign Affairs and Trade: Economic Advocacy & Analysis Branch*, 1-5.

Commonwealth Treasury (2012). The Australian dollar and the manufacturing sector. *Canberra, November*, 1-23.

Connolly, E., & Orsmond, D. (2011). The mining industry: from bust to boom. *Economic Analysis Department*, Reserve Bank of Australia, 1-59.

Cooke, R.M. (1997). Markov and entropy properties of tree- and vine-dependent variables. *Proceedings of the ASA Section on Bayesian Statistical Science,* 1-10.

Coudert, V., & Raymond, H. (2011). Gold and financial assets: are there any safe havens in bear markets. *Economics Bulletin*, *31* (2), 1613-1622.

Corsetti, G., Pericoli, M., & Sbracia, M. (2001). Correlation analysis of financial contagion: what one should know before running a test. *Yale Economic Growth Centre Discussion Paper*, (822).

Cox, D.R., & Wermuth, N. (1996). *Multivariate dependencies*. London. Chapman & Hall.

Croux, C., & Dehon, C. (2010). Influence functions of the Spearman and Kendall correlation measures. *Statistical Methods & Applications*, *19*(4), 497-515.

Czado, C. (2010). Pair-copula constructions of multivariate copulas. In *Copula theory and its applications*. Springer Berlin Heidelberg, 93-109.

Czado, C., Schepsmeier, U., & Min, A. (2012). Maximum likelihood estimation of mixed c-vines with application to exchange rates. *Statistical Modelling*, *12* (3), 229–255.

Czado, C., Brechmann, C., & Gruber, L. (2013). Selection of vine copulas. In *Copulae in mathematical & quantitative finance* by Jaworski, P., Durante, F., & Härdle, W.K. Springer, 1-20.

Daeyoung, K., Jong-Min, K., Shu-Min, L., & Yoon-Sung, J. (2013). Mixture of d-vine copulas for modelling dependence. *Computational Statistics & Data Analysis*, *64*, 1-19.

Danacica, D. E., & Babucea, A. G. (2007). Methodological aspects in using Pearson coefficient in analysing social & economical phenomena. *European Research Studies Journal*, *10*(3-4), 89-98.

David, A. (1997). Fluctuating confidence in stock markets: implications for returns and volatility. *Journal of Financial & Quantitative Analysis*, *32*(04), 427-462.

Delarue, E., De-Jonghe, C., Belmans, R., & D'haeseleer, W. (2011). Applying portfolio theory to the electricity sector: energy versus power. *Energy Economics*, *33*(1), 12-23.

Dee, J., Li, L., & Zheng, Z. (2013). Is gold a hedge or safe haven? Evidence from inflation and stock market. *International Journal of Development and Sustainability*, *2* (1), 12-27.

Delloite, L.L.P. (2013). Global powers of retailing: retail and beyond. *New York, January,* 1-44.

De Oliveira, F.A., De Paiva, A.P., Lima, J.W.M., Balestrassi, P.P., & Mendes, R.R.A. (2011). Portfolio optimization using mixture design of experiments: scheduling trades within electricity markets. *Energy Economics*, *33*(1), 24-32.

Department of Industry (2014). Australian Industry Report. *Canberra,* 1-220

Department of Innovation, Industry, Science and Research (2010). Manufacturing sector: overview of structural change- industry brief 2008-2009. *Canberra, July*, 1-80.

Department of Industry, Geoscience Australia and Bureau of Resources and Energy Economics (2014). Australian energy resource assessment. *Canberra*, *August*, 1-364.

Department of Resources, Energy and Tourism and Bureau of Resources and Energy Economics (2013). Energy in Australia. *Canberra, May,* 1-140.

Dissmann, J., Brechmann, E.C., Czado, C., & Kurowicka, D. (2013). Selecting and estimating regular vine copulae and application to financial returns. *Computational Statistics & Data Analysis*, *59*, 52-69.

Dissmann, J.F. (2010). Statistical inference for regular vines and application. *Diploma Thesis,* Technische Universitat Munchen.

Du, K., Platen, E., & Rendek, R. (2012). Modelling of oil prices (No. 321).

Eling, M., & Tibiletti, L. (2010). Internal vs. external risk measures: how capital requirements differ in practice. *Operations Research Letters*, *38*(5), 482-488.

Embrechts, P., McNeil, A., & Straumann, D. (1999). Correlation: pitfalls and alternatives. *Risk*, *12*(5), 69-71.

Erb, C., Harvey, C., & Viskanta, T. (1994). Forecasting international equity correlations. *Financial Analysts Journal*, *50* (6), 32–45.

Faff, R., & Chan, H. (1998). A multifactor model of gold industry stock returns: evidence from the Australian equity market. *Applied Financial Economics*, *8* (1), 21-28.

Faff, R., W., & Brailsford, T.J. (1999). Oil price risk and the Australian stock market. *Journal of Energy Finance and Development*, *4* (1), 69-87

Fan, Y. (2010). Copulas in econometrics. *Encyclopedia of Quantitative Finance.*

Fan, J., Feng, Y., & Wu Y. (2009). Network exploration via the adaptive lasso and scad penalties. *The Annals of Applied Statistics*, *3*(2), 521-541.

Federal Reserve Bank of St. Louis (2009). The Financial Crisis: A Timeline of Events and Policy Actions. http://timeline.stlouisfed.org.

Fernandez, Y.G., & Ortiz, M.R.S. (2012). Multivariate dependence modelling with vines: R package 'vines', 1-20.

Fischer, M., Kock, C., Schluter, S., & Weigert, F. (2009). An empirical analysis of multivariate copula models. *Quantitative Finance*, *9*(7), 839–854.

Frahm, G., Junker, M., & Szimayer, A. (2003). Elliptical copulas: applicability and limitations. *Statistics & Probability Letters*, *63*(3), 275-286.

Gao, J., Xiong, Y., & Duan, L. (2014). Dynamic mean-risk portfolio selection with multiple risk measures in continuous-time, 1-36.

Gardner-Bond, S., Spring, S., Wilkes, G., Williams, R., & Slack, W. (2008). Iron Ore sector review: leveraging exposure to iron ore through Australian pure plays. *Ocean Equities*, 1-38.

Genest, C., Remillard, B., & Beaudoin, D. (2009). Goodness-of-fit tests for copulas: a review and a power study. *Insurance: Mathematics & Economics*, *44*(2), 199-213.

Ghalanos, A. (2013). Portfolio optimization in parma (Version 1.5-0), 1-23

Green, R. and Roos, G. (2012). Australia's Manufacturing Future. *April, Sydney*, 1-111.

Grootveld, H., & Hallerbach, W. (1999). Variance vs downside risk: is there really that much difference? *European Journal of Operational Research*, *114*(2), 304-319.

Guo, J., Levina, E., Michailidis, G., & Zhu, J. (2011). Joint estimation of multiple graphical models. *Biometrika, 98* (1), 1-15.

Haff, I.H. (2013). Parameter estimation for pair-copula constructions. *Bernoulli*, *19*(2), 462-491.

Hatherley, A. (2009). Asymmetric dependence structures. *PhD Thesis*. The University of Queensland.

Hatherley, A., & Alcock, J. (2007). Portfolio construction incorporating asymmetric dependence structures: a user's guide. *Accounting & Finance, 47*(3), 447-472.

Hautsch, N., Okhrin, O., & Ristig, A. (2013). Modelling time-varying dependencies between positive-valued high-frequency time series. In *Copulae in Mathematical and Quantitative Finance*. Springer Berlin Heidelberg, 115-127

Heinen, A., & Valdesogo, A. (2009). Asymmetric CAPM dependence for large dimensions: the canonical vine autoregressive model. CORE, 1-40.

He, G., & Litterman, R. (1999). The intuition behind the Black-Litterman model portfolios. *Goldman Sachs Investment Management Series*, 1-27.

Heywood, G.C., Marsland, J.R., & Morrison, G.M. (2003). Practical risk management for equity portfolio managers. *British Actuarial Journal*, *9* (05), 1061-1123.

Hobaek Haff, I., & Segers, J. (2012). Nonparametric estimation of pair-copula constructions with the empirical pair-copula. *arXiv:1201.5133*, 1-23.

Hofmann, M., & Czado, C. (2010). Assessing the VaR of a portfolio using d-vine copula based multivariate GARCH models. *Submitted for publication*, 1-36.

Hua, L., & Joe, H. (2011). Tail order and intermediate tail dependence of multivariate copulas. *Journal of Multivariate Analysis, 102* (10), 1454-1471.

Hull, J., & White, A. (1998). Value at risk when daily changes in market variables are not normally distributed. *The Journal of Derivatives, 5* (3), 9–19.

Jansen, W. J., & Nahuis, N.J. (2003). The stock market and consumer confidence: European evidence. *Economics Letters*, *79*(1), 89-98.

Jennings, W.W. (2012). Energy stocks as a separate portfolio allocation. *The Journal of Wealth Management*, *14*(4), 70-86.

Joe, H. (1997). *Multivariate models and dependence concepts*. Chapman & Hall, London.

Joe, H., Li, H., & Nikoloulopoulos, A. K. (2010). Tail dependence functions and vine copulas. *Journal of Multivariate Analysis*, *101*(1), 252-270.

Jordan, J.S. (1983). On the efficient markets hypothesis. *Econometrica: Journal of the Econometric Society*, 1325-1343.

Junker, M., & May, A. (2005). Measurement of aggregate risk with copulas. *The Econometrics Journal, 8*, 428–454.

Junker M., Szimayer, A., & Wagner, N. (2006) Nonlinear term structure dependence: copula functions, empirics, and risk implications. *Journal of Banking & Finance, 30*(4), 1171-1199.

Kane, A. (1982). Skewness preference and portfolio choice. *Journal of Financial & Quantitative Analysis*, *17*(01), 15-25.

Kazi, I. A., Guesmi, K., & Kaabia, O. (2011). Contagion effect of financial crisis on OECD stock markets. *Economics,* 1-20.

Këllezi, E., & Gilli, M. (2000). *Extreme value theory for tail-related risk measures*. FAME.

Kenourgios, D., Samitas, A., & Paltalidis, N. (2010). Financial crises and stock market contagion in a multivariate time-varying asymmetric framework. *Journal of International Financial Markets Institutions & Money*, *21*(1), 92-106.

Kijima, M. (2002). *Stochastic processes with application to finance.* Chapman & Hall.

Killian, L., & Park, C. (2009). The impact of oil price shocks on the US stock market. *International Economic Review, 50* (4), 1267-1287.

Kim, W.C., Fabozzi, F.J., Cheridito, P., & Fox, C. (2014). Controlling portfolio skewness and kurtosis without directly optimizing third and fourth moments. *Economics Letters*, *122*(2), 154-158.

Konno, H., & Yamazaki, H. (1991). Mean-absolute deviation portfolio optimization model and its applications to Tokyo stock market. *Management Science*, *37*(5), 519-531.

Konno, H., Shirakawa, H., & Yamazaki, H. (1993). A mean-absolute deviation-skewness portfolio optimization models. *Annals of Operations Research*, *45*(1), 205-220.

Konno, H., & Shirakawa, H. (1994). Equilibrium relation in the capital market: a mean-absolute deviation case. *Financial Engineering & the Japanese Market*, *1*(1), 21-35.

KordaMentha (2013). Australian manufacturing: redefining manufacturing. *Sydney, August*, 1-39.

KPMG Economics Group (2103). Economic reach of the Western Australian resources sector, *Perth, June,* 1-47.

Krokhmal, P., Uryasev, S., & Zrazhevsky, G. (2002). Risk management for hedge fund portfolios: a comparative analysis of linear portfolio rebalancing strategies. *Journal of Alternative Investments, 5* (1), 10–29.

Krokhmal, P., Uryasev, S., & Zrazhevsky, G. (2003). Numerical comparison of *CVaR* and *CDaR* approaches: application to hedge funds. *The Stochastic Programming Approach to Asset-Liability and Wealth Management. AIMR/Blackwell.*

Kroll, Y., Levy, H., & Markowitz, H. M. (1984). Mean-variance versus direct utility maximization. *The Journal of Finance*, *39*(1), 47-61.

Kryger, A. (2014). Performance of the manufacturing industry: a quick guide. Parliament of Australia: Department of Parliamentary Services, *Canberra*, *November*, 1-5.

Kurowicka, D., & Cooke, R. M. (2006). *Uncertainty analysis with high dimensional dependence modelling*. John Wiley & Sons.

Kurowicka, D., & Joe, H. (2011). *Dependence modelling: vine copula handbook*. World Scientific.

Lai, T.Y. (1991). Portfolio selection with skewness: a multiple-objective approach. *Review of Quantitative Finance & Accounting*, *1*(3), 293-305.

Lauritzen, S.L. (1996). *Graphical models*. Oxford University Press.

Li, D. (2000). On default correlation: a copula approach. *Journal of Fixed Income, 9*(4), 43–54.

Li, X., Zhou, X.Y., & Lim, A. E. (2002). Dynamic mean-variance portfolio selection with no-shorting constraints. *SIAM Journal on Control & Optimization*, *40*(5), 1540-1555.

Li, D., & Peng, L. (2009). Goodness-of-fit test for tail copulas modelled by elliptical copulas. *Statistics & Probability Letters*, *79*(8), 1097-1104.

Louie, H. (2014). Evaluation of bivariate Archimedean and elliptical copulas to model wind power dependency structures. *Wind Energy, 17*(2), 225-240.

Low, R.K.Y., Alcock, J., Faff, R., & Brailsford, T. (2013). Canonical vine copulas in the context of modern portfolio management: are they worth it? *Journal of Banking & Finance, 37*(8), 3085-3099.

Malevergne, Y., & Sornette, D. (2003). Testing the Gaussian copula hypothesis for financial assets dependence. *Quantitative Finance, 3*(4), 231–250.

Markowitz, H. M. (1952). Portfolio selection. *Journal of Finance*, *7,* 77–91.

Markowitz, H.M. (1959). *Portfolio selection: efficient diversification of investments*. Yale University Press.

Markwat, T., Kole, E., & Van Dijk, D. (2009a). Contagion as a domino effect in global stock markets. *Journal of Banking & Finance*, *33*(11), 1996-2012.

Markwat, T. D., Kole, E., & Van Dijk, D. J. (2009b). Time variation in asset return dependence: strength or structure? In *22nd Australasian Finance & Banking Conference,*1-54.

McCarthy, J., & Orlov, A.G. (2013). Copula-based extreme market co-movements in the EU. *International Atlantic Economic Society*, 1-23.

McKay, B., Lambert, I., & Miyazaki, S. (2000). The Australian mining industry: from settlement to 2000. *Australian Mining Industry,* 8410.

Mendes, B., Semeraro, M., & Leal, R. (2010). Pair-copulas modelling in finance. *Financial Markets & Portfolio Management*, *24*(2), 193–213.

Mehmedovic, S. Pallant J., & Pacini E. (2011). Australian retail snapshot. *The Australia Centre for Retail Studies,* 1-8.

Min, A., & Czado, C. (2010). Bayesian inference for multivariate copulas using pair-copula constructions. *Journal of Financial Econometrics, 8* (4), 511-546.

Morales, L., & Andreosso-O'Callaghan, B. (2011). Comparative analysis on the effects of the Asian and global financial crises on precious metal markets. *Research in International Business and Finance*, *25* (2), 203-227

Morton, D.P., Popova, E., & Popova, I. (2006). Efficient fund of hedge funds construction under downside risk measures. *Journal of Banking & Finance*, *30*(2), 503-518.

Murray-Smith, D. (2002). Modelling sample selection using Archimedean copulas. *Econometrics Journal, 6*(1), 99-123.

National Australia Bank (2012). Sector insights: manufacturing. *Canberra, October,* 1-26.

Nawrocki, D.N. (1999). A brief history of downside risk measures. *The Journal of Investing*, *8*(3), 9-25.

Neapolitan, R. (2012). *Probabilistic reasoning in expert systems: theory and algorithms*, CreateSpace Independent Publishing Platform.

Nikoloulopoulos, A.K., Joe, H., & Li, H. (2012). Vine copulas with asymmetric tail dependence and applications to financial return data. *Computational Statistics & Data Analysis*, *56*(11), 3659-3673.

Okimoto, T. (2008). New evidence of asymmetric dependence structures in international equity markets. *Journal of Financial & Quantitative Analysis*, *43*(03), 787-815.

Ortobelli, S., Rachev, S.T., Stoyanov, S., Fabozzi, F.J., & Biglova, A. (2005). The proper use of risk measures in portfolio theory. *International Journal of Theoretical & Applied Finance, 8*(08), 1107-1133.

Ozkan, M. F. G., & Unsal, D. F. (2012). Global financial crisis, financial contagion, and emerging markets. *International Monetary Fund*, 1-58.

Panagiotelis, A., Czado, C., & Joe, H. (2012). Pair copula constructions for multivariate discrete data. *Journal of the American Statistical Assocciation*, *107* (499), 1063-1072.

Panchenko, V. (2005). Goodness-of-fit testing for vine copulas. *Physica A: Statistical Mechanics & Its Applications*, *355*(1), 176-182.

Park, J., & Ratti, R.A. (2008). Oil price shocks and stock markets in the US and 13 European countries. *Energy Economcis*, *30* (5), 2587-2608.

Patton, A.J. (2004). On the out-of-sample importance of skewness and asymmetric dependence for asset allocation. *Journal of Financial Econometrics*, *2*(1), 130–168.

Patton, A. J. (2006). Modelling asymmetric exchange rate dependence. *International Economic Review*, *47*(2), 527-556

Patton, A.J. (2012a). Copula methods for forecasting multivariate time series. *Handbook of Economic Forecasting, 2,* 1-77.

Patton, A.J. (2012b). A review of copula models for economic time series. *Journal of Multivariate Analysis*, *110*, 4-18.

Poirson, H., & Schmittmann, J. (2013). Risk exposures and financial spill overs in tranquil and crisis times: bank-level evidence. *International Monetary Fund*, 1-46.

Prakash, A.J., Chang, C.H., & Pactwa, T.E. (2003). Selecting a portfolio with skewness: recent evidence from US, European, and Latin American equity markets. *Journal of Banking & Finance*, *27*(7), 1375-1390.

Pratt, J. (1964). Risk aversion in the small and in the large. *Econometrica*: *Journal of the Econometric Society*, *32,* 122-136.

Productivity Commission (2011). The structure and performance of the Australian retail industry. *Canberra, November, 56* (4), 1-563.

Rockafellar, R.T., & Uryasev, S. (2000). Optimization of conditional Value-at-Risk. *Journal of Risk*, *2*, 21-42.

Rodriguez, J. (2007). Measuring financial contagion: a copula approach. *Journal of Empirical Finance, 41*(3), 401–423.

Russell Investments & ASX (2014). 2014 Long term investment report: taking a long-term view of the historical investment landscape. *Sydney*, *June*, 1-16.

Samuelson, P.A. (1970). The fundamental approximation theorem of portfolio analysis in terms of means, variances and higher moments. *Review of Economic Studies*, *37*(4), 537-542.

Sarcia, S.A., Basili, V.R., & Cantone, G. (2008). An approach to improving parametric estimation models in the case of violation of assumptions based upon risk analysis. Technical Report, 1-49

Savills Research (2014). Insight Australian retail market. *Perth, Febreuary*,1-24.

Schaarschmidt, S., & Schanbacher, P. (2014). Minimax: portfolio choice based on pessimistic decision making. *International Journal of Economics & Finance*, *6*(8), 1-18.

Schirmacher, D., & Schirmacher, E. (2008). Multivariate dependence modelling using pair-copulas. *Technical report*, 1-52.

Schepsmeier, U., (2013). A goodness-of-fit test for regular vine copula models. Http://arXiv preprint arXiv:.org/abs/1306.0818.

Simaan, Y. (1997). Estimation risk in portfolio selection: the mean variance model versus the mean absolute deviation model. *Management Science*, *43*(10), 1437-1446.

Singh, M., & Vyas, R.K. (2011). The impact of portfolio risk on performance of scheduled commercial banks in India. *The IUP Journal of Bank Management*, *10*(3), 34-48.

Silvennoinen, A., & Thorp, S. (2013). Financialization, crisis and commodity correlation dynamics. *Journal of International Financial Markets, Institutions & Money*, *24*, 42-65.

Sklar, A. (1959). Functions de repartition and dimensions et leurs marges. *Universite de Paris 8*, 229–231.

Smith, M., Min, A., Czado, C., & Almeida, C. (2010). Modelling longitudinal data using a pair-copula decomposition of serial dependence. *Journal of the American Statistical Association, 105* (492), 1467–1479.

Smith, M.S. (2013). Bayesian approaches to copula modelling. *arXiv:1112.4204 ,* 1-33.

Smith, M., & Vahey, S. (2013). Asymmetric density forecasting of U.S. macroeconomic variables using a Gaussian copula model of cross-sectional and serial dependence. *Journal of Business & Economic Statistics*, 1-44.

Smith, M. (2013). Copula modelling of dependence in multivariate time series. *Melbourne Business School*, 1-46.

Sortino, F.A., & Price, L.N. (1994). Performance measurement in a downside risk framework. *The Journal of Investing, 3*(3), 59-64.

Steinbach, M.C. (2001). Markowitz revisited: mean-variance models in financial portfolio analysis. *SIAM review*, *43*(1), 31-85.

Stöber, J., Joe, H., & Czado, C. (2012). Simplified pair copula constructions—limits and extensions. *Journal of Multivariate Analysis, 119*, 101-118.

Stone, B. (1973) A general class of three-parameter risk measures. *The Journal of Finance*, *28* (3), 675-685.

Szego, G. (2002). Measures of risk. *Journal of Banking & Finance*, *26*(7), 1253-1272.

Tong, B., Wu, C., & Zhou, C. (2013). Modelling the co-movements between crude oil and refined petroleum markets. *Energy Economics, 40*, 882–897.

Trivedi, P.K., & Zimmer, D.M. (2007). *Copula modelling: an introduction for practitioners.* Now Publishers Inc.

Tsay, R.S. (2005). *Analysis of financial time series.* John Wiley & Sons, 2nd ed.

Tsafack, G. (2009). Asymmetric dependence implications for extreme risk management. *Journal of Derivatives*, *17*(1), 1-7.

Uryasev, S. (2000). Conditional value-at-risk: optimization algorithms and applications. In *Computational Intelligence for Financial Engineering, 2000. Proceedings of the IEEE/IAFE/INFORMS 2000 Conference*, 49-57.

Vassiliou, M. (2009). *Historical dictionary of the petroleum industry*. Lanham, MD: Scarecrow Press.

Wen, X., Wei, Y., & Huang, D. (2012). Measuring contagion between energy markets and stock markets during financial crisis: a copula approach. *Energy Economics*, *34*(5), 1435–1446.

World Gold Council (2012). Gold as a strategic asset for UK investors: portfolio risk management and capital preservation, 1-36.

Wright, S. (1934). The method of path coefficients. *Annals of Mathematical Statistics*, *5*(3), 161–215.

Xiong, J., & Idzorek, T. (2011). The impact of skewness and fat tails on the asset allocation decision. *Financial Analyst Journal*, *67*, 23-25.

Yin, G., & Zhou, X.Y. (2004). Markowitz's mean-variance portfolio selection with regime switching: from discrete-time models to their continuous-time limits. *SIAM Journal on Control and Optimization*, *49*(3), 349-360.

Yoshimoto, A. (1996). The mean-variance approach to portfolio optimization subject to transaction costs. *Journal of the Operations Research Society of Japan*, *39*(1), 99-117.

Young, M.R. (1998). A minimax portfolio selection rule with linear programming solution. *Management Science*, *44*(5), 673-683.

Zhou, X.Y., & Yin, G. (2003). Markowitz's mean-variance portfolio selection with regime switching: a continuous-time model. *SIAM Journal on Control and Optimization*, *42*(4), 1466-1482.

APPENDIX A: Dependence structure diagonal matrices

2																		
14	16																	
5	3	14																
16	16	3	13															
14	3	14	5	4														
1	20	5	5	5	1													
5	16	2	5	2	16	1												
3	3	16	4	5	1	5	16											
1	1	1	5	4	5	24	20	33										
2	20	4	5	1	1	24	13	2	1									
5	13	13	13	5	3	20	4	5	33	3								
1	14	5	5	4	2	1	16	1	5	5	5							
14	14	14	20	2	5	1	4	4	1	3	3	13						
14	14	2	2	14	16	20	2	14	1	1	1	5	5					
14	3	4	14	14	2	4	2	5	20	14	5	6	1	5				
14	26	2	40	14	4	5	2	13	5	33	5	23	34	13	10			
14	16	5	5	5	16	1	9	13	1	5	33	5	5	14	33	5		
23	23	5	5	3	13	4	5	34	24	1	30	5	1	1	1	5	5	
14	14	34	4	3	16	5	16	16	13	1	5	5	16	5	5	5	23	1

4																		
1	3																	
2	3	23																
3	1	5	24															
1	1	5	16	1														
20	16	14	5	4	1													
14	30	5	5	5	5	40												
2	16	13	16	5	16	1	1											
14	3	14	5	3	5	5	30	1										
2	5	5	14	23	16	13	13	1	1									
2	20	14	13	5	1	16	3	3	13	33								
5	14	13	16	10	2	20	16	4	14	1	5							
3	3	5	1	16	1	13	5	1	6	2	1	10						
14	2	2	13	16	16	3	5	1	6	2	13	33	23					
2	3	2	16	24	33	5	3	5	1	14	14	13	3	1				
2	5	13	5	5	14	13	5	5	1	33	5	3	5	13	5			
5	7	5	26	5	26	5	5	33	16	33	1	13	5	5	23	1		
5	13	5	3	14	1	3	6	5	4	26	14	5	1	5	3	5	34	
40	5	5	2	33	6	2	24	13	16	5	3	13	5	13	5	5	5	33

Panel (a)

14																		
33	5																	
2	2	5																
5	2	5	13															
5	5	16	5	3														
1	2	5	13	5	5													
14	5	5	5	20	13	5												
20	2	10	5	5	20	2	3											
2	14	23	20	5	5	16	5	4										
1	20	5	3	5	5	13	5	5	2									
5	10	16	5	5	5	13	10	5	14	5								
3	2	4	3	5	5	2	5	5	1	14	5							
1	20	5	14	5	5	5	5	14	5	14	5	5						
5	10	14	5	5	6	5	5	5	5	20	5	5	5					
19	3	3	14	5	5	1	33	6	5	3	1	24	13	23				
14	5	2	1	5	20	33	5	1	5	33	5	5	23	4	34			
5	2	10	14	5	5	5	5	5	10	3	5	5	16	5	23	5		
10	5	2	13	5	4	1	5	5	6	1	5	5	1	5	5	14	5	
2	1	5	1	1	14	5	1	13	5	2	33	1	2	1	16	23	5	30

5																		
14	23																	
14	14	5																
14	4	2	10															
2	2	3	3	4														
1	5	2	14	2	23													
5	5	20	5	5	1	33												
20	2	16	6	5	16	5	5											
5	5	5	14	2	2	16	5	5										
2	14	5	3	10	5	5	14	16	6									
5	5	1	5	14	5	3	5	23	5	3								
5	2	5	33	3	16	5	5	5	1	13	6							
20	5	2	16	2	5	14	14	16	5	13	16	5						
5	2	5	2	20	20	5	1	14	4	3	1	13	5					
5	5	4	5	5	5	1	2	23	1	1	6	23	5	2				
4	5	5	10	5	5	10	5	4	1	20	5	3	33	1	14			
10	5	5	5	20	5	5	10	13	2	24	1	5	33	1	23	1		
5	4	14	1	5	5	3	5	5	33	20	14	30	1	5	5	3	2	
1	10	5	5	6	13	1	10	1	5	13	5	3	5	2	1	5	1	5

Panel (b)

20																		
5	5																	
5	5	2																
4	5	2	5															
2	1	16	3	6														
5	5	5	5	5	5													
5	5	2	2	2	13	5												
3	4	5	1	5	5	20	36											
1	26	5	5	1	3	1	5	2										
1	1	2	4	14	14	5	5	5	5									
3	14	5	1	5	13	5	1	2	4	1								
5	5	2	5	2	14	5	5	5	3	20	23							
20	33	16	4	4	1	1	2	5	3	13	3	3						
3	14	5	23	1	33	3	23	26	6	5	4	1	5					
3	13	36	5	26	1	26	34	13	1	13	5	1	16	33				
13	14	5	5	30	1	1	5	14	4	34	16	1	1	1	13			
23	5	2	2	5	3	10	20	23	5	5	14	33	5	5	5			
1	4	1	33	5	5	23	26	2	2	5	34	1	13	36	23	26	3	
2	1	2	5	14	1	5	2	1	5	1	13	16	6	3	16	3	1	5

2																		
14	1																	
2	1	14																
20	1	13	16															
20	5	14	14	13														
2	1	6	26	13	5													
2	20	14	2	1	20	33												
5	5	3	5	5	1	5	13											
5	2	5	3	5	24	5	5	6										
5	4	3	1	5	33	3	1	5	3									
14	5	26	33	23	5	13	20	1	1	13								
14	36	5	16	5	5	1	1	23	1	23	23							
5	5	3	2	5	5	13	5	1	13	1	5	1						
5	20	14	3	16	13	2	4	14	4	13	4	5	16					
2	5	5	5	33	2	5	14	3	3	1	1	3	6	4				
5	5	4	4	14	36	5	4	5	1	23	33	23	1	5	23			
2	1	5	5	10	4	36	5	34	5	1	1	5	5	5	5	23		
5	5	2	24	2	10	5	33	2	5	36	33	4	16	36	3	1	1	
5	2	2	5	5	5	14	4	36	1	2	5	3	13	5	5	5	5	5

Panel (c)

Figure A1: Panels (a), (b) and (c) display the pre-GFC, GFC and post-GFC c-vine (on the left) and d-vine (on the right) dependence structure matrices of the gold portfolio, respectively.

4																		
3	2																	
16	16	5																
3	5	3	5															
20	5	5	5	5														
4	13	20	24	14	5													
4	33	16	4	1	2	20												
10	23	30	5	5	10	23	5											
34	16	5	16	14	20	1	16	1										
5	14	5	14	5	10	10	4	3	10									
10	3	5	1	5	20	5	5	5	5	5								
1	2	20	4	10	5	5	5	5	6	6	13							
6	10	4	10	1	5	20	5	20	1	2	1	1						
10	3	13	13	4	10	16	20	2	5	10	14	5	4					
20	1	5	5	10	5	23	5	10	3	2	13	16	1	5				
10	5	2	5	20	10	5	5	14	2	1	2	5	14	4	2			
13	5	2	5	2	5	5	2	1	3	14	20	3	4	10	5	2		
5	14	14	10	5	5	2	3	20	5	2	2	1	20	3	5	10	14	
5	14	2	5	14	20	14	2	1	2	20	20	2	10	5	20	14	5	20

4																		
4	24																	
14	5	5																
5	4	1	5															
2	2	3	5	24														
4	1	13	5	5	4													
5	13	4	6	4	5	33												
20	3	3	13	1	20	5	13											
3	3	3	1	5	34	1	5	5										
5	3	5	13	5	13	5	3	33	33									
13	5	1	5	5	20	26	5	23	13	1								
16	13	5	34	1	33	5	2	16	2	4	14							
6	5	5	5	1	1	16	5	23	6	5	3	14						
5	16	2	5	34	23	26	5	2	5	33	1	3	1					
14	3	2	2	5	1	23	5	20	2	13	3	16	5	13				
14	5	5	20	1	23	5	1	5	20	1	5	16	14	14	5			
5	1	13	3	5	5	26	3	14	16	5	13	16	3	3	2	2		
14	20	16	1	13	10	1	5	2	14	20	14	2	10	1	16	4	1	
14	2	40	4	5	5	5	2	3	14	1	2	14	5	5	3	14	3	2

Panel (a)

24																		
16	1																	
14	5	5																
33	4	20	5															
13	5	5	3	26														
4	1	3	33	5	5													
2	33	5	5	4	5	5												
5	14	1	13	6	6	20	5											
1	5	5	16	5	3	1	36	5										
6	5	5	30	13	20	20	1	13	5									
1	5	5	6	2	5	5	5	20	14	33								
5	5	5	1	5	2	5	6	13	5	2	23							
3	1	20	23	23	13	5	3	5	5	5	2	5						
2	5	2	2	5	3	5	33	5	5	5	1	5	5					
5	5	3	5	5	1	14	33	5	5	14	3	5	3	10				
5	14	2	16	5	5	16	3	5	5	14	2	1	5	5	2			
16	20	33	3	5	5	13	10	14	2	14	10	33	3	5	2	2		
33	33	2	2	13	5	14	1	5	16	5	2	1	2	10	2	5	20	
5	5	1	20	5	5	2	19	5	20	1	1	5	14	20	14	2	1	5

1																		
6	5																	
4	1	23																
14	1	36	33															
4	3	34	14	24														
5	6	26	5	1	4													
26	1	6	23	5	33	16												
5	4	5	3	1	33	33	3											
2	5	13	20	5	3	1	26	36										
5	5	1	16	3	5	6	1	26	14									
3	16	2	6	5	6	33	26	5	14	5								
3	3	5	3	2	5	13	10	2	5	20	1							
5	16	5	5	1	4	13	5	5	4	5	5	3						
3	5	13	1	2	5	1	3	3	5	5	13	16	1					
33	2	5	14	1	2	13	5	1	23	10	5	3	33	5				
5	2	33	2	16	3	16	33	5	20	3	14	16	5	5	10			
1	2	3	2	2	13	14	4	13	1	5	13	4	14	5	2	14		
2	23	5	2	2	20	2	3	26	5	5	1	4	1	5	1	5	5	
5	5	1	5	5	5	1	14	5	5	1	14	2	2	20	5	5	20	10

Panel (b)

19																			
20	18																		
18	20	17																	
17	17	20	16																
16	16	16	20	15															
15	15	15	15	20	14														
14	14	14	14	14	20	13													
13	13	13	13	13	13	20	12												
12	12	12	12	12	12	12	20	11											
11	11	11	11	11	11	11	11	20	10										
10	10	10	10	10	10	10	10	10	20	9									
9	9	9	9	9	9	9	9	9	9	20	8								
8	8	8	8	8	8	8	8	8	8	8	20	7							
7	7	7	7	7	7	7	7	7	7	7	7	20	6						
6	6	6	6	6	6	6	6	6	6	6	6	6	20	5					
5	5	5	5	5	5	5	5	5	5	5	5	5	5	20	4				
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	20	3			
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	20	2		
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	20	20	
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Panel (c)

Figure A2: Panel (a) displays the full sample period (on the left) and pre-GFC (on the right) r-vine dependence structure matrices of the gold portfolio. Panel (b) displays the GFC and post-GFC r-vine dependence structure matrices of the gold portfolio. Panel (c) displays the order of the gold stock return series according to the r-vine tree structure.

3																		
14	6																	
19	14	13																
1	16	5	16															
16	2	5	36	14														
13	1	6	3	3	1													
14	4	16	3	2	5	13												
9	3	2	3	3	13	4	5											
14	16	20	3	14	13	3	5	6										
20	5	13	16	16	14	13	5	3	14									
16	5	5	16	16	5	13	16	1	33	2								
14	3	5	5	2	24	5	5	1	3	5	3							
3	16	14	5	16	16	2	14	16	16	4	1	13						
2	13	2	14	13	1	13	14	4	2	4	36	33	13					
7	14	5	5	26	24	1	5	14	16	5	5	13	3	33				
3	14	5	16	5	4	14	13	4	4	13	2	1	16	3	33			
1	3	5	16	14	5	5	1	14	14	2	20	33	33	33	36	5		
16	26	13	5	26	2	3	5	36	14	20	3	23	1	23	26	13	2	
20	5	3	3	16	3	4	13	1	14	3	1	14	1	5	13	5	13	34

5																		
5	1																	
20	1	10																
16	14	1	16															
20	16	2	14	5														
16	16	14	2	16	14													
14	20	13	1	5	1	2												
16	5	14	3	3	5	24	1											
3	14	16	13	14	5	3	6	13										
19	1	5	4	5	4	10	4	5	34									
14	14	9	5	5	5	5	13	1	23	26								
9	2	4	3	4	16	3	6	3	13	6	5							
14	2	5	23	5	33	3	5	16	33	33	24	1						
1	5	1	13	6	5	23	13	5	16	2	3	2	5					
20	14	16	13	6	14	5	6	16	5	16	16	3	3	4				
1	5	3	4	13	6	14	16	13	2	33	5	26	3	5	23			
4	3	3	1	1	3	16	5	3	1	23	3	13	16	33	33	2		
3	2	16	3	5	23	3	5	5	14	33	4	5	5	5	20	5	2	
16	26	16	1	5	13	6	16	5	13	5	23	13	16	14	3	2	14	1

Panel (a)

14																		
19	3																	
17	14	5																
17	20	1	14															
5	3	5	16	16														
20	14	3	1	14	10													
5	3	14	3	3	16	14												
17	1	5	1	5	1	16	16											
1	3	3	1	3	14	4	2	16										
5	14	14	16	16	3	14	20	14	13									
3	3	13	3	16	13	20	5	2	10	5								
14	5	5	2	13	2	1	5	4	5	4	3							
5	16	5	14	13	14	3	5	5	14	36	13	2						
7	14	5	1	5	13	5	1	2	5	23	3	5	3					
5	16	14	5	3	5	3	3	1	36	13	26	23	36	3				
7	3	7	5	23	23	23	14	36	6	13	14	13	33	13	5			
3	3	5	3	5	14	6	13	5	10	3	3	23	20	13	5	3		
20	1	4	6	16	14	1	3	23	3	5	14	5	1	3	33	33	4	
14	14	1	2	2	14	4	3	6	5	14	1	34	13	6	13	16	6	1

16																		
1	3																	
17	5	13																
7	2	5	13															
14	17	4	33	16														
14	2	1	5	16	16													
14	5	1	20	5	3	20												
14	3	3	1	5	20	16	2											
14	3	3	6	1	1	2	1	4										
14	5	20	4	5	5	3	1	3	13									
1	5	3	14	1	10	3	2	23	34	5								
14	3	14	14	5	3	5	1	6	5	2	20							
14	4	20	3	1	20	14	2	5	20	1	14	5						
14	20	4	3	13	5	1	20	2	1	14	4	5	5					
19	14	1	3	1	14	5	1	3	10	10	1	36	1	14				
14	14	2	14	3	4	1	3	2	3	1	4	20	6	13	6			
16	1	3	5	14	1	14	5	20	16	14	4	13	13	16	13	3		
3	5	16	5	3	14	5	13	13	14	16	5	5	13	33	5	5	14	
14	5	5	6	23	4	3	3	13	2	5	1	34	5	1	5	5	33	6

Panel (b)

5																		
7	14																	
17	14	5																
17	14	5	14															
1	5	3	2	13														
20	14	14	6	14	5													
5	3	5	5	3	20	16												
20	14	5	10	19	13	5	3											
1	5	14	16	13	2	5	5	6										
14	1	3	14	4	16	5	16	2	4									
3	16	26	5	5	16	16	3	23	6	14								
17	2	2	5	1	5	13	2	5	16	1	33							
1	23	14	33	23	2	3	2	33	14	5	2	5						
2	5	20	10	1	14	36	1	5	14	5	2	5	26					
2	14	3	14	4	16	16	13	16	1	13	33	20	13	1				
2	3	20	2	1	1	16	5	14	6	13	26	3	24	5	4			
14	2	5	26	3	16	3	2	5	2	1	16	4	16	13	33	2		
16	10	14	3	5	14	1	3	5	26	3	5	3	13	14	13	5	34	
20	16	5	13	16	5	16	13	2	16	4	5	5	5	2	33	3	3	3

5																		
2	14																	
17	14	1																
19	7	20	2															
1	5	16	3	1														
20	7	5	14	2	5													
3	20	5	5	5	13	5												
14	20	13	10	5	5	5	20											
1	14	1	2	13	23	14	1	5										
14	14	14	5	20	1	14	5	16	14									
16	13	3	3	3	20	23	1	26	5	1								
16	3	5	5	5	1	5	20	14	2	2	2							
5	2	5	2	1	2	1	10	5	5	1	23	6						
13	17	3	14	4	5	1	5	5	1	5	5	5	5					
1	5	20	33	13	1	14	6	3	16	5	3	3	14	1				
17	5	36	2	14	5	5	1	5	20	23	1	10	20	1	4			
2	5	1	2	1	16	5	2	5	5	13	23	33	23	4	36	36		
16	14	13	3	14	13	5	3	16	5	1	14	14	23	5	23	13	33	
16	5	20	20	2	1	5	3	4	3	2	13	4	4	5	5	14	23	34

(c)

Figure A3: Panels (a), (b) and (c) display the pre-GFC, GFC and post-GFC c-vine (on the left) and d-vine (on the right) dependence structure matrices of the iron ore-nickel mining portfolio, respectively.

14																		
5	33																	
16	5	1																
16	3	1	16															
14	5	5	16	23														
16	5	5	3	33	5													
1	3	2	4	14	1	5												
20	5	1	5	13	5	20	20											
16	5	5	16	16	16	16	5	5										
1	1	1	3	20	2	3	1	1	6									
2	13	4	4	14	2	26	3	1	16	16								
14	1	5	16	16	16	14	13	5	5	16	13							
20	5	1	5	1	13	33	1	1	13	6	13	3						
16	6	5	5	16	16	16	1	3	2	14	5	14	1					
16	5	5	5	16	3	24	5	5	2	14	2	16	5	5				
5	5	24	1	23	5	5	5	5	5	5	1	1	13	5	5			
5	33	2	5	1	23	20	14	34	10	3	3	3	1	1	1	36		
3	5	5	14	16	16	16	5	40	2	14	3	5	13	13	19	20	1	
26	1	5	14	14	2	2	5	5	2	13	3	33	1	20	2	5	1	34

16																		
14	5																	
5	5	5																
2	5	1	5															
14	16	1	23	1														
5	5	5	5	6	5													
1	20	1	5	1	5	23												
14	5	20	13	1	3	1	5											
14	3	16	16	1	1	33	13	5										
2	13	13	20	4	5	5	3	1	3									
5	16	13	13	5	13	2	2	30	1	5								
5	16	14	14	13	13	34	5	36	13	5	3							
16	13	5	14	5	23	5	3	13	23	5	3	33						
16	16	13	13	1	1	3	36	5	16	23	13	3	1					
5	5	5	1	1	5	33	23	34	23	1	1	2	36	14				
33	16	23	14	3	13	14	3	14	13	16	5	13	14	23	5			
20	1	16	3	4	1	14	13	26	3	1	16	5	2	1	3	1		
14	16	1	14	2	2	20	2	5	3	1	13	5	4	34	5	5	5	
3	3	23	5	5	5	3	5	26	13	5	5	33	1	5	2	1	33	23

Panel (a)

5																		
1	5																	
1	2	5																
14	5	4	5															
7	10	5	1	3														
3	2	5	2	20	2													
20	5	4	5	5	5	13												
14	14	33	4	5	5	13	13											
16	16	3	3	5	13	13	13	5										
2	5	20	5	5	4	10	23	36	1									
14	20	1	5	1	1	1	13	1	4	1								
14	5	1	5	5	2	3	5	5	5	1	13							
7	5	24	1	5	13	5	33	3	3	5	2	1						
1	10	20	3	5	1	1	4	3	1	2	23	5	1					
3	2	10	5	2	1	4	5	34	5	10	33	1	13	5				
1	2	5	1	3	5	5	1	23	4	4	4	23	5	5	20			
14	5	5	20	14	5	5	20	16	1	5	24	1	5	5	3	5		
1	14	23	5	3	5	23	5	1	5	3	14	3	4	1	34	36	26	
1	5	14	5	1	5	5	16	5	33	14	1	13	3	2	5	5	14	33

5																		
2	4																	
10	20	10																
2	5	1	10															
5	10	1	5	2														
2	3	2	13	3	3													
7	5	5	5	5	13	3												
14	14	3	14	34	5	10	3											
1	2	5	2	16	1	5	3	3										
14	5	14	3	13	33	23	13	1	1									
20	16	3	13	3	2	2	5	40	26	10								
20	14	5	1	5	5	16	16	3	5	23	5							
3	14	5	1	16	1	3	5	13	2	3	5	2						
1	16	13	5	20	14	2	4	5	16	3	1	5	16					
5	14	2	2	3	5	13	1	23	13	2	5	5	5	5				
5	5	1	5	5	5	5	13	3	23	5	1	16	4	16	3			
20	2	2	14	20	5	14	26	24	3	5	5	5	5	5	1	13		
5	16	5	1	16	1	5	3	1	1	1	1	16	5	14	20	14	20	
5	5	5	1	26	34	23	4	5	1	33	33	4	23	3	34	1	1	1

Panel (b)

1																		
20	5																	
3	2	5																
20	1	23	26															
5	5	16	14	5														
3	2	5	5	3	5													
20	1	1	23	2	1	1												
5	16	3	5	3	5	1	3											
20	14	5	5	16	6	5	16	5										
20	5	20	2	2	3	6	3	2	3									
1	2	5	5	3	5	5	5	3	3	10								
14	16	3	13	16	16	5	16	3	2	3	1							
2	5	5	5	2	5	13	20	1	3	26	5	13						
10	5	5	5	20	4	5	23	5	36	5	5	1	23					
5	5	36	6	23	14	3	23	5	5	1	1	23	5	2				
3	2	5	5	1	1	5	1	3	5	5	5	1	33	4	1			
3	14	5	20	23	13	23	5	1	16	5	5	33	16	10	1	13		
14	20	10	2	16	1	14	4	4	1	20	1	14	1	1	5	24	33	
3	5	14	36	1	36	23	3	3	5	16	5	1	5	4	2	6	4	5

5																		
5	2																	
14	10	2																
2	20	10	2															
5	5	5	4	5														
5	5	5	13	40	40													
5	1	40	34	3	5	5												
20	3	5	1	26	5	1	5											
14	13	3	5	5	16	5	40	1										
14	20	5	3	14	1	5	1	5	6									
2	5	20	3	3	5	1	3	5	33	5								
20	5	16	16	26	1	5	16	1	3	3	13							
17	20	20	16	3	5	23	1	5	5	1	33	5						
3	2	14	1	16	4	5	3	5	3	3	1	5	2					
3	14	14	5	20	2	3	5	5	13	4	5	5	5	13				
20	5	3	5	36	13	20	2	5	5	1	5	5	6	5	33			
16	5	5	1	4	30	5	1	5	24	36	36	5	33	13	36	3		
5	20	14	5	5	23	3	16	33	3	1	1	13	14	6	23	10	3	
1	2	1	5	5	23	1	3	5	1	5	5	1	14	1	5	3	2	1

Panel (c)

Figure A4: Panels (a), (b) and (c) display the pre-GFC, GFC and post-GFC c-vine (on the left) and d-vine (on the right) dependence structure matrices of the coal-uranium energy portfolio, respectively.

Panel (a) – c-vine (left)

17																		
17	2																	
5	5	5																
14	4	5	2															
2	2	5	5	13														
14	3	14	3	2	23													
14	20	5	20	1	13	14												
14	1	5	1	20	20	14	2											
14	20	20	1	20	5	6	4	1										
2	20	14	5	1	20	26	16	14	1									
14	10	14	14	4	2	20	3	3	2	5								
1	5	3	2	5	5	20	5	14	5	14	23							
20	2	2	14	20	5	16	5	3	5	2	5	5						
14	1	14	20	20	1	20	5	2	5	13	2	5	4					
20	5	5	16	5	13	26	5	5	3	1	1	1	13	4				
2	2	14	20	20	3	5	20	5	2	5	4	2	1	10	1			
2	20	5	5	5	5	2	20	3	10	20	26	5	13	5	1	5		
1	23	3	10	20	3	20	5	5	33	14	13	5	6	4	4	3	36	
9	20	3	3	4	2	3	5	5	13	33	5	14	3	14	5	5	2	16

Panel (a) – d-vine (right)

20																		
20	5																	
2	20	5																
14	20	5	14															
7	5	2	5	1														
2	2	2	5	20	2													
20	20	20	5	5	14	1												
7	7	20	2	20	5	4	14											
14	5	20	1	20	20	3	6	6										
3	3	20	4	5	2	1	3	4	5									
14	5	2	20	3	2	13	2	36	4	4								
16	16	20	5	5	5	13	5	6	20	2	20							
14	20	20	1	2	3	13	20	14	1	3	1	14						
14	14	20	1	1	20	3	14	1	13	3	3	20	10					
3	14	14	20	3	20	14	3	20	23	36	6	2	14	5				
20	4	5	1	5	3	23	3	5	13	5	5	33	36	20	2			
5	14	20	1	5	2	1	23	5	23	23	23	5	1	6	5	2		
2	5	3	14	14	14	23	14	6	3	5	13	23	2	14	1	5	2	
5	16	4	3	5	5	5	1	5	5	5	13	14	5	1	5	1	5	1

Panel (a)

Panel (b) – left

9																		
14	5																	
5	5	33																
16	2	6	20															
17	1	2	5	13														
3	5	16	5	13	5													
13	5	16	2	2	13	14												
14	14	13	4	16	3	23	14											
14	14	4	5	16	5	23	5	5										
20	2	14	3	5	5	5	33	2	1									
3	5	2	14	6	14	3	3	3	2	1								
5	4	3	20	5	5	16	5	5	1	14	5							
1	2	2	3	14	4	23	16	1	2	2	14	5						
1	1	1	2	14	14	16	16	2	33	6	2	2	33					
1	1	1	40	3	23	13	1	3	1	6	1	13	4	5				
14	4	3	2	3	6	5	16	5	13	13	36	5	3	16	13			
20	3	13	5	16	14	5	16	1	4	5	2	13	23	13	3	26		
5	33	5	5	5	5	1	5	1	23	1	16	5	1	10	1	36	1	
2	3	16	5	4	1	3	2	5	13	33	5	34	6	1	3	1	33	3

Panel (b) – right

16																		
20	36																	
20	16	2																
4	1	3	1															
16	3	13	5	16														
3	16	14	10	2	2													
3	16	20	5	13	33	13												
3	2	6	3	5	5	5	5											
20	3	20	23	5	16	5	6	5										
5	24	14	16	14	5	4	3	6	33									
3	14	2	3	6	4	5	3	1	1	5								
14	16	23	3	14	16	3	2	4	1	13	16							
14	10	4	3	1	36	5	3	14	3	1	13	16						
1	20	9	2	13	5	3	1	3	5	16	3	5	3					
17	4	4	4	6	5	23	5	1	13	14	13	6	3	1				
20	1	5	1	10	23	5	6	5	24	23	33	5	6	2	33			
14	20	2	2	16	5	6	16	1	3	1	5	2	14	33	5	23		
14	5	2	5	23	1	3	23	5	13	13	13	26	2	14	33	2	16	
1	23	1	33	5	33	5	3	14	5	5	1	10	36	16	5	3	33	5

Panel (b)

Panel (c) – left

2																		
5	14																	
5	5	5																
2	13	5	5															
2	20	5	1	1														
14	3	16	1	5	5													
5	20	5	3	1	14	2												
7	5	1	20	5	20	2	5											
17	5	5	5	16	5	26	36	1										
2	20	5	33	16	2	26	16	13	1									
3	4	5	14	3	14	4	16	5	13	2								
14	5	3	2	1	33	24	3	13	6	5	5							
20	2	2	13	26	20	16	5	16	3	16	5	5						
14	3	4	20	5	5	5	1	2	16	3	26	23	3					
13	26	16	2	5	5	5	26	14	23	26	2	5	16	6				
20	14	1	20	5	20	5	2	16	14	1	10	1	5	6	16			
5	1	5	5	5	5	16	5	34	10	2	16	1	5	3	5	5		
5	1	1	2	16	2	13	13	14	1	13	23	24	3	13	5	5	5	
1	1	1	3	4	5	1	5	5	3	13	5	4	20	26	34	13	20	4

Panel (c) – right

14																		
1	5																	
14	14	5																
14	20	16	5															
7	1	1	13	1														
5	14	3	6	5	13													
2	5	20	13	5	10	14												
2	1	5	2	2	13	24	1											
5	20	5	2	5	3	20	6	33										
5	20	2	2	14	5	6	4	1	5									
2	5	5	10	3	3	5	13	3	26	23								
20	3	3	14	5	2	14	1	6	13	1	14							
20	36	13	5	23	5	1	5	5	16	3	3	2						
20	2	3	20	1	16	13	5	5	5	3	5	5	2					
1	20	14	5	3	20	20	3	14	6	5	1	3	2	13				
14	14	5	16	14	5	2	5	5	4	1	16	36	5	2	16			
14	3	5	2	5	20	6	5	16	13	13	4	36	23	4	5	16		
5	2	13	16	9	5	33	1	23	26	23	2	23	5	36	13	5	14	
5	3	1	13	3	3	5	6	5	3	5	33	14	3	5	5	4	3	34

Panel (c)

Figure A5: Panel (a) displays the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the oil-gas energy portfolio, respectively. Panel (b) displays the d-vine Kendall tau correlation matrix of the same portfolio and sample period.

5																		
3	10																	
5	5	20																
16	5	3	3															
5	5	16	5	33														
5	10	5	5	3	13													
5	5	30	40	24	5	3												
5	13	3	2	5	26	16	5											
1	5	20	2	1	14	10	1	3										
16	4	16	1	16	5	5	3	5	6									
5	5	20	5	3	5	1	5	3	20	14								
3	40	30	5	3	5	4	5	1	5	5	5							
13	36	14	3	33	33	1	16	4	24	3	14	3						
1	5	5	5	5	5	13	33	5	5	13	1	14	1					
5	10	20	5	3	5	5	30	2	5	2	5	5	1	13				
14	13	4	1	3	4	14	16	5	5	5	1	6	5	1	6			
2	10	20	5	1	16	1	33	4	4	5	5	3	3	5	13	20		
13	5	20	2	5	5	5	5	1	5	3	4	5	16	16	5	3	3	
1	13	5	16	5	1	6	4	3	5	14	40	5	5	4	1	3	5	10

5																		
20	3																	
20	5	1																
2	5	5	14															
20	5	5	3	4														
20	2	5	5	10	26													
5	10	10	5	5	5	33												
10	5	5	5	13	4	6	24											
10	5	14	20	2	20	5	1	1										
14	14	5	33	16	1	23	2	3	16									
3	2	10	20	5	5	5	1	33	1	3								
5	3	20	20	5	14	5	5	5	5	14	3							
5	13	16	16	5	5	13	16	33	13	3	3	4						
2	20	4	33	4	1	13	3	5	3	33	13	13	3					
20	14	13	1	14	13	14	1	5	4	6	16	33	3	1				
14	5	16	20	2	16	13	5	13	3	33	1	5	4	14	2			
5	5	10	5	2	16	5	16	14	6	1	3	6	26	1	1	5		
5	1	1	13	1	1	3	5	5	13	1	5	5	23	1	30	4	13	
40	24	14	3	6	5	2	5	23	2	5	3	6	14	20	14	5	36	16

Panel (a)

16																		
5	16																	
20	14	3																
20	16	2	1															
5	14	2	2	14														
14	6	10	5	5	10													
1	13	6	16	1	5	1												
20	4	5	20	5	2	33	33											
14	10	2	2	5	16	5	3	14										
14	5	3	3	3	6	13	5	14	33									
5	16	2	20	14	2	10	1	23	5	13								
2	2	14	3	5	5	23	1	3	3	3	5							
14	14	20	20	5	16	1	5	5	20	5	10	5						
5	14	2	13	5	20	10	5	5	13	6	5	33	14					
1	5	10	23	1	33	10	6	1	20	4	5	5	5	1				
20	5	10	5	20	5	5	5	5	13	1	5	33	2	5	5			
5	10	16	14	14	36	13	26	20	5	14	5	1	5	5	5	13		
13	26	10	1	20	10	5	36	5	13	26	5	23	2	1	5	2	16	
14	14	14	3	20	16	23	5	5	26	16	3	16	5	5	3	5	3	34

5																		
5	5																	
5	5	5																
2	10	5	10															
5	5	5	5	2														
5	3	2	5	10	13													
14	20	3	5	23	5	23												
14	5	1	26	3	5	1	33											
14	20	4	10	2	5	10	5	5										
14	10	3	4	5	14	33	24	5	23									
3	2	16	3	16	1	1	23	1	13	3								
14	14	5	10	4	5	20	23	36	23	1	5							
20	3	14	13	14	20	2	5	13	1	5	33	1						
20	5	14	4	4	5	5	13	5	23	1	33	1	1					
5	14	5	3	16	2	14	5	2	5	14	13	5	1	5				
3	20	1	10	3	2	13	5	5	3	1	3	26	36	5	1			
1	5	14	14	5	5	13	1	36	5	33	3	5	23	16	6	26		
5	16	20	5	3	14	16	13	3	3	5	5	5	4	2	23	1	1	
20	5	5	2	5	16	5	16	14	2	16	16	2	5	5	14	13	5	5

Panel (b)

2																		
5	36																	
14	1	5																
5	20	6	1															
20	1	5	5	5														
3	3	5	16	20	20													
1	33	2	6	6	20	5												
5	13	16	33	3	4	20	16											
20	3	5	16	5	3	16	5	3										
1	13	1	2	16	1	2	1	1	33									
5	5	10	20	5	20	2	16	1	33	10								
1	1	5	16	5	14	2	5	5	16	13	5							
20	3	5	5	5	5	16	1	1	3	4	16	13						
7	3	5	10	5	3	3	23	13	3	20	5	36	2					
5	13	5	14	20	14	2	5	10	3	6	5	5	16	2				
3	13	5	5	33	3	2	1	1	24	5	5	14	5	5	3			
14	6	13	13	5	1	14	5	5	1	33	5	33	2	13	23	13		
20	5	1	20	5	5	20	16	10	5	16	5	20	33	5	5	5	5	
5	5	5	3	3	5	20	5	5	1	5	5	23	5	5	5	5	3	5

5																		
5	10																	
10	14	5																
14	23	16	2															
14	10	4	20	5														
20	14	3	1	5	16													
20	5	5	5	5	5	5												
5	17	1	14	3	13	13	1											
2	4	14	14	4	3	1	5	10										
5	5	3	13	20	3	20	20	20	5									
20	3	1	4	20	5	5	5	5	5	20								
5	20	16	16	13	1	20	16	2	5	5	5							
5	1	5	5	23	34	5	10	5	1	16	5	1						
5	14	5	2	14	6	5	13	1	13	5	5	5	33					
1	5	1	20	5	9	3	5	3	3	14	2	13	33	3				
14	5	5	5	23	1	5	2	13	5	5	5	3	5	23	1			
20	3	1	5	14	3	5	3	13	33	24	3	5	3	3	2	5		
5	5	5	20	5	20	5	1	33	13	3	23	5	4	5	1	5	10	
16	3	13	33	5	2	5	5	5	13	33	6	5	5	6	1	26	13	1

Panel (c)

Figure A6: Panels (a), (b) and (c) display the pre-GFC, GFC and post-GFC c-vine (on the left) and d-vine (on the right) dependence structure matrices of the mix-metals leptokurtic portfolio, respectively.

20																		
3	33																	
20	4	4																
13	13	5	33															
2	10	5	3	33														
2	20	3	5	3	1													
14	3	16	23	13	13	1												
3	3	14	6	6	14	16	20											
16	14	13	13	20	13	3	5	5										
5	13	3	6	13	1	3	2	14	20									
2	14	20	16	36	2	14	13	5	14	2								
3	2	2	4	3	6	16	3	5	10	23	14							
1	2	14	14	3	14	4	2	3	20	20	4	14						
16	2	10	4	20	20	3	2	10	13	14	1	6	3					
3	20	2	4	14	10	14	14	20	4	3	2	1	13	20				
13	2	1	5	20	2	20	3	20	16	10	5	20	20	5	2			
2	20	2	2	20	5	5	1	4	5	3	20	2	1	2	10	20		
3	20	2	20	5	2	20	10	2	16	14	14	3	5	20	2	2	5	
16	14	7	7	2	2	17	14	20	20	1	20	2	20	17	14	14	2	1

2																		
5	5																	
13	5	5																
36	2	2	5															
13	20	3	24	5														
5	14	26	5	5	14													
16	13	3	36	33	16	14												
13	13	6	16	6	6	1	23											
16	2	1	23	6	13	5	1	5										
13	3	1	5	4	16	33	5	24	4									
36	5	2	33	14	16	6	3	20	16	5								
16	1	13	3	13	16	5	3	13	5	1	16							
5	16	1	1	33	2	33	14	3	36	23	5	5						
5	4	13	5	5	16	3	6	16	5	3	16	2	13					
5	16	23	5	2	2	6	5	13	14	14	20	16	6	2				
5	13	5	3	6	16	6	3	16	6	20	1	2	5	14	23			
3	5	5	33	16	4	13	3	16	3	20	3	16	14	5	4	14		
3	5	3	14	4	1	14	13	14	16	3	5	1	14	20	14	1	16	
20	20	14	7	9	3	2	1	19	3	14	14	1	19	1	14	5	1	5

Panel (a)

13																		
3	6																	
3	4	1																
1	16	2	36															
3	2	3	14	14														
13	14	13	16	5	14													
36	5	33	33	20	14	5												
5	16	6	1	13	3	5	13											
3	6	4	26	13	6	36	5	3										
24	5	14	3	33	5	3	3	3	14									
3	33	5	13	3	33	5	5	4	14	3								
3	10	14	6	1	23	1	14	1	2	14	1							
23	5	2	1	3	23	16	14	3	4	20	13	5						
5	1	23	1	16	3	4	3	6	14	13	5	5	14					
3	13	6	10	5	16	6	16	3	1	1	16	1	13	1				
2	2	2	14	1	1	14	2	1	5	16	16	36	16	14	16			
14	5	20	14	3	14	3	20	2	14	16	4	2	5	5	5	3		
1	1	16	3	1	5	5	5	1	3	3	20	1	3	20	5	5	3	
5	17	14	19	1	7	14	14	17	14	5	5	3	20	14	17	7	14	10

16																		
36	2																	
2	26	14																
2	5	6	4															
3	5	33	3	16														
2	1	5	14	1	16													
34	2	6	5	23	14	2												
2	23	23	16	20	5	16	13											
13	5	5	13	14	1	5	13	16										
36	33	2	16	5	1	1	16	13	23									
6	2	24	2	13	13	16	4	2	14	13								
5	24	23	2	1	2	4	3	1	13	5	5							
5	5	16	5	3	16	33	16	14	5	16	5	16						
3	36	36	16	3	2	5	3	1	5	14	14	16	3					
24	23	36	3	3	5	5	5	4	2	1	4	6	13	6				
1	23	26	5	14	5	13	1	2	2	2	14	10	1	33	3			
36	13	1	5	3	14	5	1	5	2	20	10	3	5	14	5	5		
14	2	16	5	20	5	1	14	14	17	5	14	5	20	14	3	14	14	
2	5	16	20	5	20	1	2	5	2	2	7	17	17	17	16	20	20	20

Panel (b)

0.046																		
0.036	-0.012																	
0.049	0.025	0.013																
0.032	0.020	0.034	-0.01															
0.050	0.037	0.011	0.011	-0.010														
0.024	0.048	0.033	0.050	0.016	0.024													
0.062	0.073	0.017	-0.018	0.006	0.024	0.003												
0.041	0.025	0.019	0.028	0.012	0.024	0.008	0.071											
0.016	0.045	0.031	0.020	0.057	0.022	0.009	-0.002	0.089										
0.020	0.016	0.024	0.010	0.014	0.026	0.023	0.008	0.040	0.081									
-0.004	0.039	0.040	0.013	-0.009	0.030	0.028	0.018	0.019	0.026	0.108								
0.017	0.006	0.054	0.014	0.032	0.018	0.048	0.010	0.025	0.056	-0.010	0.092							
0.007	0.027	0.042	0.037	0.036	0.032	0.023	0.040	0.018	0.051	0.050	0.017	0.080						
0.019	0.012	0.082	0.035	0.070	0.053	0.050	0.077	0.104	0.021	0.044	0.054	0.014	0.030					
0.064	0.053	0.023	0.047	0.030	0.057	0.075	0.056	0.118	0.025	0.022	0.028	0.016	0.024	0.074				
0.025	0.073	0.064	0.054	0.066	0.060	0.086	0.026	0.069	0.033	0.073	0.052	0.086	0.065	0.097	0.038			
0.045	0.041	0.102	0.084	0.057	0.084	0.111	0.031	0.024	0.065	0.04	0.080	0.058	0.070	0.099	0.107	0.099		
0.031	0.098	0.178	0.164	0.110	0.112	0.148	0.082	0.075	0.182	0.121	0.074	0.044	0.105	0.214	0.164	0.137	0.060	
0.108	0.181	0.303	0.305	0.265	0.344	0.339	0.138	0.188	0.136	0.137	0.164	0.104	0.172	0.286	0.336	0.272	0.360	0.171

Panel (c)

Figure A7: Panels (a) and (b) display the r-vine dependence structure matrices of the iron ore-nickel mining portfolio for each of the four period scenarios. Panel (c) displays the r-vine Kendall tau correlation matrix corresponding to the full sample period scenario.

5																		
1	5																	
5	14	14																
5	5	5	4															
5	20	10	1	16														
5	5	5	16	5	16													
13	5	1	5	5	20	1												
3	1	5	1	10	5	10	5											
5	33	36	13	2	5	10	20	4										
5	5	1	5	14	2	20	10	1	3									
3	5	5	1	14	14	20	2	5	5	5								
20	13	10	1	20	5	2	5	5	33	33	5							
13	10	5	1	16	20	3	20	24	34	1	20	2						
5	5	5	5	3	3	14	5	1	3	14	10	2	4					
5	20	5	20	20	2	16	20	5	14	5	3	2	2	20				
3	20	3	16	1	10	16	1	1	20	1	5	20	14	5	20			
5	20	6	5	2	1	2	5	5	20	5	20	20	5	14	20	2		
5	20	10	16	2	10	13	14	5	5	20	2	3	20	2	2	14	3	
1	20	2	10	3	3	20	20	20	14	20	5	1	2	14	14	14	14	20

5																		
23	1																	
5	23	3																
16	34	16	1															
5	23	33	5	13														
5	3	3	13	1	33													
1	26	16	2	5	2	5												
5	2	5	6	23	5	26	16											
5	1	13	20	4	1	3	23	14										
23	16	34	5	5	26	6	5	5	1									
33	24	3	5	20	10	16	3	23	5	5								
23	5	33	5	5	10	1	2	1	20	5	3							
13	23	23	5	5	5	3	5	36	5	2	1	13						
13	5	14	13	5	16	3	13	26	5	34	20	5	5					
26	33	2	5	2	13	3	5	5	33	13	5	33	5	5				
14	5	5	14	2	16	13	16	6	33	5	16	13	4	16	13			
5	5	14	14	3	13	5	3	5	3	10	16	14	5	1	14	1		
5	6	3	16	16	16	13	2	5	5	5	20	3	5	34	1	3	1	
3	5	16	5	5	5	2	14	2	5	13	3	14	20	23	5	14	14	5

Panel (a)

13																		
6	5																	
1	3	33																
13	5	5	23															
14	2	5	5	2														
5	34	1	1	3	5													
5	23	1	4	3	10	1												
3	3	3	6	3	5	3	3											
23	23	3	16	24	5	5	10	2										
5	23	1	20	5	14	16	5	26	14									
5	4	26	5	2	5	5	13	5	5	3								
5	1	1	2	5	19	13	26	5	3	16	13							
2	5	2	16	14	10	5	1	26	5	5	16	5						
16	16	33	1	5	1	20	6	23	5	34	36	5	14					
3	3	3	5	13	16	13	3	5	5	5	1	10	5	13				
3	16	3	16	5	3	14	13	5	5	3	1	2	24	1	5			
13	1	1	1	5	16	5	20	5	5	14	1	1	20	13	5	5		
2	20	5	14	14	1	1	20	5	2	10	3	5	3	5	6	14	4	
1	1	7	20	3	5	14	14	14	20	5	14	5	5	20	20	20	20	5

26																		
16	4																	
1	1	5																
2	6	5	4															
23	10	5	14	5														
5	23	5	36	6	4													
5	13	5	14	5	5	20												
1	1	1	5	16	5	5	5											
36	33	3	5	4	5	5	5	5										
14	13	2	1	5	5	1	3	33	5									
5	6	6	16	10	2	4	5	5	33	5								
16	14	5	5	13	4	10	1	33	1	36	13							
5	5	1	1	1	3	20	3	2	3	5	33	5						
10	1	5	14	33	4	20	13	5	3	5	5	5	5					
5	14	14	5	3	5	13	16	3	33	1	23	36	2	5				
40	5	16	16	3	4	3	1	16	16	5	5	5	3	3	10			
5	3	16	5	5	16	2	3	20	16	3	16	5	5	5	5	16		
3	6	14	5	5	5	5	14	20	5	20	14	16	13	2	20	5	6	
5	14	20	5	5	1	3	20	17	20	2	14	2	5	3	1	14	20	5

Panel (b)

0.028																		
-0.008	0.061																	
0.127	0.001	0.004																
0.179	0.129	0.041	0.004															
0.154	0.262	0.043	-0.008	0.001														
0.118	0.140	0.041	0.001	0.068	0.004													
0.005	0.130	0.017	0.052	0.099	0.107	0.006												
0.014	-0.009	-0.014	-0.003	0.134	0.202	0.129	0.019											
0.021	-0.013	-0.001	0.002	0.013	0.288	0.166	0.084	0.005										
0.202	-0.031	-0.017	-0.022	0.032	0.040	0.220	0.090	-0.002	0.007									
0.028	0.233	0.005	0.004	0.073	0.034	0.058	0.110	0.003	-0.012	0.046								
0.225	0.030	0.028	0.014	0.062	0.040	0.020	0.069	-0.006	-0.018	-0.007	0.179							
0.004	0.267	0.052	0.012	0.020	0.059	0.03	0.064	0.000	-0.023	-0.001	0.047	0.222						
0.048	0.023	0.030	0.011	0.035	0.021	0.036	0.132	0.005	0.026	0.001	0.165	0.018	0.015					
0.036	0.049	0.047	0.037	0.155	0.006	0.008	0.070	0.017	0.035	0.050	0.031	0.000	0.036	0.082				
0.034	0.062	0.003	0.000	0.006	0.225	0.030	0.056	0.027	0.058	0.009	0.066	0.062	0.028	0.073	0.094			
0.251	0.045	0.004	0.025	0.219	-0.007	0.092	0.103	0.031	0.062	0.023	0.071	0.083	0.066	0.053	0.074	0.066		
0.589	0.347	0.098	0.002	0.179	0.199	0.022	0.002	0.088	0.119	0.028	0.092	0.033	0.080	0.084	0.083	0.104	0.045	
0.101	0.117	0.065	0.071	0.129	0.068	0.126	0.178	0.168	0.196	0.105	0.242	0.051	0.294	0.160	0.188	0.130	0.145	0.175

Panel (c)

Figure A8: Panels (a) and (b) display the r-vine dependence structure matrices of the coal-uranium energy portfolio for each of the four period scenarios. Panel (c) displays the full sample period scenario r-vine Kendall tau correlation matrix of the same portfolio.

40																		
1	1																	
1	20	1																
5	1	4	10															
5	33	23	5	5														
1	20	2	5	5	14													
5	5	3	20	2	20	20												
16	5	13	5	1	3	20	14											
14	10	5	14	5	14	13	5	14										
5	13	14	5	1	5	5	2	14	2									
6	4	5	2	14	3	1	6	3	14	5								
1	20	2	2	14	5	20	33	3	2	3	3							
5	3	6	20	3	23	14	36	2	14	5	4	20						
5	3	5	13	1	3	5	16	1	3	5	20	4	5					
5	23	5	4	2	3	3	23	3	1	2	5	3	5	5				
5	3	10	4	14	13	5	20	1	5	20	20	13	14	20	2			
13	13	14	4	3	4	13	3	14	1	5	2	5	3	5	1	5		
5	5	5	20	20	5	13	14	5	3	2	2	20	2	2	2	2	2	
5	5	14	14	9	2	20	14	3	14	20	17	3	14	17	20	2	7	20

13																		
2	5																	
2	13	36																
23	33	16	1															
33	3	3	13	5														
5	33	36	5	1	23													
30	5	13	23	36	1	16												
2	13	13	3	5	13	36	23											
2	16	26	4	1	13	14	3	5										
16	5	5	13	2	20	5	16	14	5									
10	13	24	3	16	16	1	13	3	3	5								
2	1	1	23	33	13	6	1	23	23	23	3							
1	13	5	2	33	14	23	16	6	33	5	5	1						
5	5	3	1	23	4	5	4	5	5	1	1	5	30					
16	3	6	2	1	3	14	14	3	6	6	33	2	20	3				
14	33	3	4	1	5	5	2	5	2	6	1	5	1	4	3			
1	16	16	14	1	2	2	14	5	5	1	3	3	3	3	16	1		
5	3	6	1	14	16	16	1	1	2	4	33	34	5	16	9	2	4	
3	16	3	20	16	3	14	14	20	14	17	30	20	5	2	1	2	3	20

Panel (a)

26																		
5	1																	
20	23	10																
20	3	1	13															
5	1	5	5	16														
20	2	14	13	5	5													
5	1	2	5	1	2	13												
16	14	16	5	13	14	5	5											
1	3	14	33	23	16	1	20	5										
2	5	13	33	5	3	5	10	2	5									
1	14	10	1	1	3	5	5	4	13	16								
3	6	3	13	5	3	1	16	13	2	16	33							
6	5	5	5	1	23	5	5	5	14	14	4	1						
14	33	6	5	1	1	5	14	16	13	13	3	16	5					
5	3	5	3	1	13	4	5	4	3	5	3	5	3	3				
1	6	5	14	23	5	4	2	14	5	1	14	5	4	36	13			
1	5	5	1	23	2	2	3	5	20	14	1	13	5	14	5	13		
14	14	14	20	33	14	5	20	1	5	3	5	1	5	16	5	2	5	
14	14	9	19	5	20	14	20	5	7	7	1	20	1	9	2	5	14	14

3																		
14	26																	
3	5	13																
3	1	34	23															
5	13	26	1	16														
5	5	16	6	2	5													
3	5	6	5	5	14	33												
1	3	3	4	1	17	1	6											
16	20	3	5	26	16	4	1	13										
2	5	1	33	5	13	1	14	5	1									
1	34	5	13	3	14	3	2	33	6	1								
14	16	3	13	13	16	5	1	3	23	23	33							
16	3	10	3	2	33	5	16	3	26	3	13	24						
36	2	2	5	3	16	2	1	13	16	14	5	2	13					
14	4	13	1	20	1	13	14	14	1	3	5	33	4	13				
34	13	14	5	2	5	2	3	5	3	20	20	5	5	16	5			
1	5	5	20	16	16	16	3	3	5	5	4	14	5	16	13	2		
5	20	3	4	2	23	5	5	1	20	20	14	14	14	14	14	2	2	
2	5	1	3	20	5	20	20	2	5	2	2	5	7	1	5	20	14	14

Panel (b)

-0.029																		
0.004	0.025																	
0.004	0.037	0.043																
0.007	0.005	0.034	0.053															
0.013	-0.02	-0.007	0.061	-0.024														
-0.01	0.033	-0.008	0.023	0.039	0.03													
0.049	0.016	0.039	0.042	0.003	0.036	0.064												
0.001	0.01	0.01	0.068	0.007	0.004	0.086	0.024											
0.001	0.155	-0.013	0.034	0.031	0.016	0.027	0.019	0.088										
0.004	0.017	0.074	0.051	0.008	0.016	0.004	0.014	0.057	0.068									
0.002	0.02	0.039	0.129	0.02	0.021	0.094	0.011	0.011	0.039	0.054								
0.02	0.048	0.019	0.006	0.049	0.013	0.05	-0.003	0.017	0.007	0.042	0.007							
0.016	0.016	0.028	0.095	0.019	-0.017	0.016	-0.003	0.093	0.027	0.027	0.013	0.042						
0.029	0.005	0.055	0.022	0.037	0.029	-0.023	0.025	0.037	0.054	0.049	0.044	0.023	0.023					
-0.035	-0.015	0.061	0.037	0.019	0.027	0.019	-0.015	0.035	0.041	0.034	0.089	0.011	0.05	0.056				
0.027	0.014	0.063	0.037	0.048	0.032	0.043	0.096	0.062	0.051	0.071	0.115	0.029	0.066	0.068	0.092			
0.009	0.018	0.047	0.032	0.052	0.021	0.024	0.047	0.045	0.053	0.123	0.086	0.043	0.05	0.099	0.092	0.073		
0.024	0.028	0.066	0.083	0.094	0.092	0.028	0.075	0.079	0.06	0.157	0.17	0.097	0.07	0.122	0.136	0.152	0.069	
0.061	0.045	0.145	0.131	0.159	0.478	0.26	0.137	0.147	0.145	0.3	0.299	0.105	0.146	0.293	0.271	0.411	0.289	0.198

Panel (c)

Figure A9: Panels (a) and (b) display the r-vine dependence structure matrices of the oil-gas energy portfolio for each of the four period scenarios. Panel (c) displays the full sample period scenario r-vine Kendall tau correlation matrix of the same portfolio.

1																		
14	5																	
5	16	5																
3	13	4	13															
3	1	5	5	13														
3	14	13	10	5	4													
5	33	3	5	13	5	20												
5	5	5	4	20	10	5	20											
5	3	5	10	20	2	5	23	5										
33	23	10	13	5	5	13	5	5	5									
5	5	6	5	5	3	5	20	5	20	13								
5	5	6	10	20	5	5	3	5	20	5	5							
20	1	4	2	5	20	3	1	10	14	5	20	5						
5	3	14	20	13	5	20	5	20	5	1	3	5	20					
5	5	4	10	23	5	2	5	5	10	6	5	10	5	5				
20	5	10	5	2	16	10	20	20	20	3	5	20	5	5	5			
5	5	10	10	14	20	14	5	5	20	3	2	5	5	2	5	2		
10	10	5	5	1	5	2	2	2	1	20	20	2	2	2	5	2	4	
1	5	10	10	20	2	20	20	20	5	14	3	1	20	1	5	5	20	5

3																		
13	3																	
16	1	5																
5	3	3	5															
1	3	3	1	5														
5	16	16	36	23	5													
1	23	23	10	1	5	5												
3	5	3	13	14	5	5	1											
5	13	3	1	5	13	5	6	33										
5	1	33	4	3	3	1	1	5	5									
3	16	3	33	5	16	23	3	2	5	14								
1	33	16	14	5	5	5	2	5	5	5	1							
26	5	23	5	6	5	5	3	1	5	5	5	5						
2	5	16	5	1	5	5	13	5	23	5	10	5	5					
13	2	5	5	5	36	2	33	5	4	13	5	2	5	26				
16	5	3	13	5	20	33	1	5	13	10	14	20	14	5	5			
1	4	5	13	5	2	5	16	26	5	5	5	13	5	16	2	20		
13	3	16	5	3	20	10	16	6	20	2	5	20	2	14	5	5	4	
5	13	14	1	20	5	20	13	10	3	5	1	5	4	2	14	1	5	20

Panel (a)

1																		
6	33																	
16	5	5																
5	5	1	5															
1	2	2	1	23														
4	2	5	20	1	2													
5	5	13	10	1	33	23												
1	14	13	2	16	33	33	5											
5	5	2	23	24	4	5	1	1										
36	16	10	5	1	2	24	10	10	33									
1	16	20	2	3	5	3	5	5	33	1								
5	13	1	3	14	16	10	1	23	5	4	5							
5	14	33	3	5	14	2	5	26	16	1	26	16						
1	5	5	5	3	20	5	5	10	36	6	14	5	16					
14	2	5	3	16	16	1	10	5	5	5	34	2	36	16				
30	5	5	16	16	1	5	13	5	16	5	4	14	13	4	14			
1	2	2	5	3	14	10	5	3	5	20	10	3	5	4	3	4		
5	1	5	5	5	20	5	10	3	2	2	5	14	5	5	10	3	20	
5	20	20	3	14	20	10	1	10	14	20	14	5	14	14	14	16	14	14

13																		
23	14																	
13	4	33																
23	5	1	5															
5	5	4	16	5														
5	2	5	20	16	13													
3	16	16	1	20	2	1												
13	5	3	3	5	1	5	5											
2	3	2	5	1	5	23	34	13										
5	23	5	5	33	5	5	4	20	1									
5	5	3	4	16	5	5	5	14	13	5								
5	1	3	3	5	1	5	5	20	16	6	6							
5	3	3	5	5	10	5	1	3	13	5	23	5						
16	13	2	13	6	1	5	3	5	34	5	33	33	5					
4	20	5	36	3	34	1	36	3	1	23	5	1	26	5				
1	5	20	16	1	5	3	10	3	3	5	13	13	1	13	5			
2	20	20	5	1	23	5	3	3	5	5	1	3	5	2	14	3		
14	2	10	10	5	5	5	20	20	3	4	3	1	3	13	2	3	5	
3	3	5	5	20	5	5	5	1	20	5	2	16	2	14	7	20	5	20

Panel (b)

0.040																		
0.005	-0.025																	
0.024	0.004	-0.019																
0.014	0.005	0.004	0.009															
0.006	0.009	-0.018	0.023	0.029														
0.014	0.003	0.005	0.064	0.020	0.011													
0.049	-0.007	0.002	0.035	0.044	0.053	0.046												
0.044	0.011	0.050	0.003	0.057	0.078	0.024	0.039											
0.009	0.008	-0.013	0.046	0.031	0.040	0.019	-0.006	0.027										
-0.027	-0.007	0.031	0.010	0.032	0.009	0.014	-0.023	0.048	0.086									
0.054	-0.012	0.003	0.050	0.030	0.023	0.032	0.028	0.050	0.081	0.012								
0.154	-0.019	0.002	0.026	0.047	0.051	0.042	0.011	0.050	0.079	0.009	0.078							
0.053	-0.007	0.002	-0.002	0.045	0.035	0.029	-0.012	0.056	0.007	0.061	0.077	0.058						
0.102	0.009	0.002	0.128	0.027	0.037	0.048	0.062	0.096	0.073	0.039	0.032	0.116	0.072					
0.081	0.044	0.002	0.057	-0.005	-0.002	0.051	0.061	0.091	0.122	0.012	0.074	0.056	0.108	0.074				
0.329	0.058	0.082	0.047	0.049	0.049	0.099	0.055	0.104	0.073	0.020	0.099	0.072	0.069	0.103	0.220			
0.332	0.101	0.052	0.073	0.083	0.099	0.217	0.084	0.098	0.120	0.057	0.137	0.101	0.121	0.228	0.113	0.439		
0.084	0.098	0.041	0.029	0.112	0.118	0.110	0.096	0.092	0.067	0.110	0.173	0.295	0.297	0.132	0.448	0.246	0.029	
0.088	0.063	0.104	0.094	0.160	0.136	0.126	0.217	0.237	0.178	0.302	0.122	0.132	0.174	0.122	0.114	0.125	0.153	0.381

Panel (c)

Figure A10: Panels (a) and (b) display the r-vine dependence structure matrices of the leptokurtic portfolio for each of the four period scenarios. Panel (c) displays the full sample period scenario r-vine Kendall tau correlation matrix of the leptokurtic portfolio.

10																		
2	10																	
14	1	3																
20	4	1	1															
5	2	1	26	1														
2	5	2	3	33	33													
14	20	1	5	5	13	5												
20	2	5	26	1	5	2	3											
3	1	1	5	23	14	2	36	5										
2	5	14	5	5	4	5	2	2	10									
5	3	1	5	3	23	1	5	5	4	14								
10	2	20	5	6	14	3	5	10	2	20	10							
2	5	2	5	26	5	1	13	33	13	5	5	10						
2	20	5	5	4	2	3	13	36	26	2	5	5	5					
2	2	5	5	5	6	2	2	5	24	14	13	5	5	5				
5	5	10	5	5	13	5	13	3	13	5	23	23	5	10	16			
5	5	10	10	5	34	4	5	5	23	5	5	23	1	5	10	14		
5	5	2	20	5	5	33	5	36	2	26	13	33	26	5	5	5	5	
5	2	5	5	5	5	5	26	26	6	20	20	5	4	1	5	1	36	24

3																		
3	5																	
2	24	10																
5	16	16	5															
5	5	5	1	2														
1	2	1	5	6	14													
1	5	16	24	5	13	30												
3	33	30	2	2	3	16	5											
2	13	5	5	23	24	40	1	1										
2	5	1	1	1	2	33	23	14	1									
2	2	2	2	23	5	23	40	5	33	5								
2	5	13	4	16	14	24	23	33	1	2	36							
2	2	3	33	5	14	1	23	3	16	5	2	1						
5	16	14	14	5	5	4	6	13	5	14	13	26	23					
7	5	5	1	16	2	26	16	5	1	1	13	3	1	23				
14	14	5	4	13	5	1	36	33	3	5	13	5	13	13	13			
14	3	1	1	3	34	5	5	5	2	14	33	14	3	1	3	13		
14	5	5	14	14	5	1	5	4	23	3	16	1	4	36	26	5	23	
5	5	13	3	23	2	2	2	5	3	23	1	6	5	16	24	26	2	13

Panel (a)

2																		
5	1																	
2	5	5																
2	5	5	1															
5	16	3	14	4														
5	2	5	1	5	3													
5	5	36	13	5	3	3												
1	6	3	23	5	5	1	23											
2	3	5	1	33	2	5	5	5										
7	4	5	1	2	3	5	3	5	4									
2	2	2	16	13	13	5	5	1	3	5								
20	20	14	3	14	5	36	16	5	16	23	1							
1	1	5	5	1	14	14	5	5	4	33	1	5						
20	1	3	5	5	1	5	5	14	5	10	1	36	4					
5	14	2	1	5	3	5	14	2	13	1	2	33	3	33				
5	14	14	1	1	20	13	23	4	5	5	13	4	5	4	26			
10	2	5	1	6	5	14	4	5	24	5	13	5	23	5	5	33		
2	33	14	5	5	1	16	4	5	16	4	5	36	13	13	23	5	5	
30	5	5	5	5	1	3	5	14	13	6	23	39	23	39	5	16	5	5

5																		
2	5																	
7	5	1																
2	2	13	5															
3	14	3	33	1														
1	26	4	5	3	5													
5	5	33	5	33	14	13												
17	2	4	5	2	1	3	5											
20	2	3	13	5	1	1	13	1										
20	5	2	5	3	13	6	5	6	1									
1	14	3	5	5	5	23	5	4	33	5								
14	14	3	14	3	2	1	2	1	26	3	5							
5	1	5	23	3	3	1	16	2	1	2	4	1						
5	5	5	13	33	33	1	3	23	5	5	5	1	4					
34	26	5	13	3	3	33	3	13	3	14	14	1	5	2				
5	10	1	13	23	5	26	36	26	5	5	2	1	2	5	5			
2	1	5	13	13	6	5	33	26	5	33	5	14	26	5	33	14		
10	10	33	6	1	3	14	5	13	16	13	13	13	13	3	6	2	5	
5	10	5	2	16	5	1	5	5	1	13	1	14	10	30	5	1	5	34

Panel (b)

0.234																		
0.294	0.129																	
0.101	0.039	0.006																
0.177	0.071	0.051	0.026															
0.928	0.016	-0.009	-0.011	0.016														
0.308	0.034	0.055	0.023	-0.016	-0.005													
0.164	0.088	-0.010	0.071	-0.028	0.019	0.021												
0.159	0.099	0.110	-0.013	0.066	0.052	0.026	0.008											
0.096	0.065	0.047	0.037	-0.016	0.056	0.024	-0.017	0.007										
0.120	0.044	0.066	0.100	0.082	0.018	0.095	0.093	0.071	0.075									
0.101	0.006	0.014	0.032	0.008	-0.002	-0.011	0.023	-0.026	0.004	0.004								
0.108	0.179	0.044	0.024	0.006	0.013	0.036	0.045	0.036	0.164	0.140	0.034							
0.460	0.069	0.225	0.048	-0.008	0.017	-0.009	0.010	-0.020	0.033	0.226	0.225	0.112						
0.644	0.151	0.036	0.069	0.017	-0.002	0.016	0.036	-0.005	-0.016	0.014	0.116	0.100	0.029					
0.614	0.331	0.127	0.040	0.093	0.002	0.015	0.014	0.028	-0.015	0.012	0.020	0.116	0.044	0.028				
0.503	0.247	0.161	0.080	0.053	0.021	0.029	0.024	0.013	0.022	0.022	-0.003	-0.009	0.039	0.060	0.005			
0.347	0.402	0.214	0.164	0.040	-0.002	0.000	-0.012	-0.026	-0.001	-0.033	-0.016	-0.003	0.004	0.046	0.052	0.002		
0.593	0.370	0.358	0.220	0.182	0.065	-0.008	0.060	-0.010	0.008	-0.009	0.022	-0.019	-0.007	-0.011	0.069	0.065	0.036	
-0.612	-0.019	-0.252	-0.255	-0.151	-0.175	-0.060	-0.001	0.000	0.000	0.039	0.028	0.033	0.000	0.004	0.051	0.004	0.000	-0.002

Panel (c)

Figure A11: Panels (a) and (b) display the d-vine dependence structure matrices of the manufacturing benchmark portfolio for each of the four financial period scenarios. Panel (c) displays the full period scenario d-vine Kendall tau correlation matrix of the manufacturing benchmark portfolio.

5																		
5	5																	
33	1	3																
16	3	30	1															
36	4	2	14	1														
26	33	5	14	1	1													
5	10	3	3	1	23	13												
13	26	10	13	13	1	1	5											
13	5	2	5	20	24	14	2	13										
3	20	4	1	5	3	5	36	14	5									
5	26	5	5	5	34	1	5	33	13	30								
5	2	5	5	1	10	5	5	2	36	10	30							
1	20	5	1	1	3	2	16	5	2	4	2	4						
23	33	5	4	34	5	33	5	1	14	10	5	1	30					
1	1	5	1	24	1	13	5	13	4	10	20	2	5	5				
14	5	5	10	10	5	33	5	13	36	5	5	3	5	5	5			
1	13	2	1	3	5	1	2	4	5	5	20	2	2	2	9	2		
2	1	5	6	4	14	1	5	5	3	5	5	33	5	5	5	5	5	
13	7	2	13	20	5	2	1	20	19	14	5	20	1	5	1	1	1	3

5																		
13	13																	
13	16	3																
13	2	23	3															
3	3	2	3	5														
36	23	2	3	23	40													
1	5	2	3	1	5	5												
5	33	5	2	1	1	5	3											
5	3	14	36	5	36	5	1	16										
5	14	1	5	14	26	16	14	5	24									
33	4	16	6	1	3	40	4	16	3	5								
5	10	16	5	5	3	1	2	3	26	23	5							
13	14	24	2	14	2	20	2	5	4	2	16	14						
1	3	4	3	5	6	1	40	5	6	10	5	6	13					
2	2	3	3	5	5	13	5	20	14	14	3	1	3	16				
13	5	4	26	1	5	1	2	1	13	1	5	5	5	33	2			
33	1	13	34	1	6	10	16	6	16	2	2	33	5	5	2	3		
5	5	5	1	1	5	2	5	1	1	10	5	16	5	33	5	5	5	
19	16	3	16	6	13	3	5	7	14	14	13	16	5	3	3	1	13	16

Panel (a)

33																		
33	13																	
1	1	34																
6	23	6	23															
23	16	13	5	5														
3	14	1	36	33	3													
1	6	5	1	4	14	23												
3	3	1	1	3	33	1	3											
3	20	3	6	5	4	5	2	14										
5	1	3	13	23	5	24	4	4	5									
5	20	5	2	13	13	5	14	14	6	34								
4	5	3	16	5	13	3	1	23	5	16	5							
4	3	4	1	3	2	5	14	5	5	5	2	33						
1	23	4	5	13	5	5	5	20	5	13	13	5	5					
1	33	5	5	1	5	5	3	1	23	5	3	1	5	5				
5	1	1	5	5	5	5	16	5	20	5	5	1	5	2	5			
6	5	5	34	40	10	5	5	2	1	6	5	5	2	2	2	5		
3	4	1	5	2	5	16	23	5	5	3	3	33	5	5	5	5	5	
2	13	5	3	33	5	5	14	1	14	3	19	7	5	1	5	5	1	1

5																		
1	2																	
1	34	1																
5	14	36	16															
1	3	34	33	5														
5	16	23	34	3	5													
14	14	33	5	5	13	13												
26	5	14	1	1	4	30	5											
5	33	23	23	3	5	3	13	6										
5	5	13	4	5	33	3	1	6	5									
3	3	5	2	5	5	5	1	23	13	33								
1	20	5	5	6	3	5	23	2	5	5	5							
14	2	36	5	5	4	6	10	2	5	33	13	4						
1	1	1	13	23	4	14	13	3	13	5	5	5	10					
5	5	4	5	13	1	4	5	5	3	14	1	13	20	5				
2	33	1	33	1	5	5	6	5	5	33	13	14	5	5	20			
26	5	5	5	1	2	5	5	2	2	5	33	2	34	23	5	2		
34	4	1	5	5	5	5	36	5	5	1	1	5	13	13	6	5	5	
33	14	20	20	2	5	14	1	1	16	17	14	1	2	5	1	1	6	14

Panel (b)

0.027																		
0.014	0.010																	
-0.002	0.006	0.011																
0.001	0.003	-0.039	0.004															
0.000	0.011	-0.011	0.004	0.036														
-0.001	-0.007	0.028	0.004	-0.003	-0.014													
-0.012	0.083	0.012	0.010	0.026	-0.008	0.006												
0.003	-0.005	0.076	0.007	0.024	-0.014	0.011	-0.053											
0.002	0.083	0.031	0.022	0.055	-0.005	0.032	0.006	0.026										
0.001	0.064	0.026	0.011	0.083	0.010	0.049	-0.016	0.060	0.026									
0.011	-0.004	0.079	0.048	0.024	-0.002	0.013	0.100	-0.010	0.017	-0.078								
-0.006	0.049	0.051	0.05	0.056	0.047	0.039	0.045	0.058	-0.010	0.061	-0.038							
0.005	0.073	-0.025	0.021	0.074	0.010	0.108	0.012	0.082	0.121	0.013	0.036	0.014						
-0.002	-0.006	0.137	0.003	-0.021	0.080	-0.020	0.113	0.011	0.027	0.110	0.076	0.018	-0.072					
0.009	0.032	0.162	0.019	-0.003	-0.005	0.023	0.176	0.024	0.014	0.150	0.117	0.023	0.116	-0.099				
0.001	0.030	0.176	0.027	0.038	0.023	-0.036	0.276	0.026	-0.006	0.164	0.152	0.014	0.195	0.249	-0.100			
-0.003	0.006	0.347	0.003	0.016	0.021	0.028	0.431	0.024	0.027	0.351	0.332	-0.007	0.341	0.435	0.442	-0.279		
0.007	0.005	0.413	0.001	0.000	0.003	0.005	0.529	0.057	0.003	0.370	0.332	-0.002	0.411	0.552	0.571	0.598	-0.368	
0.002	0.254	0.097	0.006	0.089	0.061	0.308	0.085	0.176	0.160	0.049	0.140	0.151	0.091	0.067	0.077	0.075	0.004	0.063

Panel (c)

Figure A12: Panels (a) and (b) display the c-vine dependence structure matrices of the manufacturing benchmark portfolio for each of the four financial period scenarios. Panel (c) displays the Kendall tau correlation matrix of the same portfolio corresponding to the full sample period scenario.

5																		
6	40																	
26	13	24																
5	33	1	40															
1	5	1	5	1														
14	5	16	13	24	24													
4	20	13	5	2	5	36												
5	5	1	5	5	33	3	5											
4	23	5	5	5	2	13	20	36										
5	23	1	5	5	5	5	24	5	1									
1	23	3	4	20	20	5	5	40	5	5								
5	10	23	13	13	5	5	5	5	3	26	13							
13	4	5	13	23	13	2	5	13	2	16	2	1						
13	5	5	10	5	1	5	2	23	1	5	3	3	13					
5	5	4	20	20	33	3	3	23	3	1	2	5	14	5				
10	5	4	10	1	5	1	5	14	5	5	5	1	36	2	10			
2	1	5	5	5	1	2	20	1	14	2	2	20	3	36	20	5		
1	2	13	2	2	1	14	5	5	5	20	5	2	20	3	2	2	5	
2	10	10	14	1	5	2	5	3	3	19	20	7	2	14	1	1	5	5

2																		
6	23																	
2	5	24																
23	33	23	33															
34	2	14	14	20														
36	5	5	1	3	5													
20	33	1	1	33	5	13												
26	1	1	33	1	5	2	5											
2	2	16	1	5	40	5	5	23										
16	16	2	23	5	5	1	16	23	20									
16	13	5	6	4	2	2	1	1	5	2								
1	3	3	16	2	5	5	2	2	13	16	5							
10	16	13	5	23	5	1	1	14	5	23	4	20						
26	5	3	3	16	16	2	2	13	36	4	1	16	2					
23	5	6	5	1	13	14	3	36	10	3	2	5	10	2				
2	2	2	1	5	13	1	5	33	16	5	5	5	26	2	16			
3	2	3	26	2	5	5	5	14	1	16	5	1	2	13	5	34		
14	3	23	3	13	5	5	5	1	5	13	1	40	33	5	5	1	1	
5	5	3	5	5	3	5	7	19	2	3	2	23	5	5	14	14	5	2

Panel (a)

5																		
5	5																	
33	33	1																
4	5	23	3															
33	1	3	5	5														
6	1	4	3	33	36													
3	20	5	3	4	5	1												
26	5	2	5	4	5	1	1											
16	13	13	1	3	6	1	5	13										
1	3	3	33	1	1	5	5	5	5									
5	33	23	24	26	5	33	5	5	13	5								
2	5	16	24	3	20	20	14	5	5	1	5							
4	5	36	1	36	17	16	5	5	5	16	5	3						
16	5	1	14	5	5	23	5	5	5	5	2	13	10					
6	1	14	13	36	5	5	5	3	10	10	20	10	5	5				
5	5	3	16	33	2	1	4	5	1	20	2	33	5	5	5			
16	3	5	2	5	13	5	16	4	10	5	5	13	17	13	5	5		
4	1	1	4	4	26	5	5	5	16	5	10	13	13	2	5	5	2	
1	1	14	2	7	5	2	5	2	5	2	5	3	5	10	5	5	14	5

2																		
5	14																	
5	1	14																
5	6	5	13															
6	5	2	5	14														
5	1	5	24	1	23													
5	1	13	5	5	2	6												
5	1	5	3	2	1	3	13											
2	5	13	1	13	1	1	13	5										
23	23	3	5	2	26	4	1	5	3									
1	1	5	5	6	1	34	14	5	26	13								
2	5	5	3	5	23	13	1	10	1	33	5							
2	5	1	5	23	5	16	26	13	5	33	1	13						
10	1	33	1	2	5	1	3	5	1	16	14	1	5					
5	1	3	33	16	6	2	5	13	5	4	33	5	5	14				
13	1	13	13	3	2	14	13	5	13	5	5	13	1	2	5			
14	16	5	5	5	24	2	3	1	5	5	3	2	5	30	5	1		
5	34	26	5	1	2	13	14	16	20	3	2	5	30	16	2	1	34	
7	5	34	5	5	13	14	14	5	2	1	5	5	1	5	20	17	1	2

Panel (b)

-0.03																		
0.02	-0.063																	
-0.014	0.016	-0.006																
0.057	-0.005	0.006	-0.041															
-0.005	0.059	0.009	0.025	0.019														
0.01	0.063	0.001	0.019	-0.02	-0.002													
0.042	0.103	0.006	0.084	0.022	-0.011	-0.003												
0.063	0.157	-0.011	0.082	0.08	-0.001	0.007	0.027											
0.017	-0.013	0.047	0.105	0.079	0.015	0.026	0.053	-0.001										
0.056	-0.016	0.014	0.192	0.145	0.11	0.082	-0.013	-0.057	0.005									
0.021	-0.006	0.01	0.015	0.2	0.213	0.063	0.1	-0.115	0.037	-0.01								
0.024	0.069	-0.002	0.026	0.023	0.302	0.146	0.047	-0.251	0.019	-0.015	0.002							
0.028	0.016	0.067	0.015	-0.013	0.002	0.282	0.164	0.011	0.038	0.019	0.018	0.003						
0.009	0.133	0.031	0.063	-0.007	0.002	-0.02	0.336	-0.006	0.043	0.021	0.023	0.014	0.002					
0.183	0.232	0.005	0.024	0.093	-0.001	0.011	0.032	-0.006	0.028	0.103	0.072	-0.018	0.017	-0.011				
0.209	0.286	0.001	0.163	0.025	0.128	0.021	0.04	0.002	0.02	0.053	0.063	0.065	-0.011	0.015	0.213			
0.359	0.059	-0.016	0.155	0.259	0.011	0.096	0.069	-0.003	0.06	0.059	0.043	0.062	0.02	-0.013	0.252	0.361		
-0.003	0.21	0.005	0.149	0.296	0.002	0.078	0.114	0.056	0.064	0.081	0.104	0.098	0.088	0.042	0.649	0.665	-0.021	
0.644	0.108	0.114	0.049	0.091	0.429	0.097	0.14	0.014	0.096	0.16	0.176	0.254	0.308	0.164	0.086	0.125	0.099	0.539

Panel (c)

Figure A13: Panels (a) and (b) display the c-vine dependence structure matrices of the manufacturing portfolio for each of the four financial period scenarios. Panel (c) displays the full sample period scenario c-vine Kendall tau correlation matrix of the same portfolio.

14																		
7	2																	
17	20	5																
17	20	2	20															
3	20	5	16	20														
20	5	14	14	20	1													
20	20	20	3	14	20	5												
2	5	5	10	20	13	14	14											
1	3	3	3	10	2	20	20	2										
20	14	14	16	2	14	2	20	2	2									
3	14	13	16	14	16	14	20	2	2	2								
14	2	10	2	2	2	1	3	5	1	4	36							
20	16	14	16	13	16	3	20	4	20	4	13	4						
2	2	2	2	5	1	13	5	2	33	6	5	10	6					
2	20	20	20	3	3	3	5	20	20	1	33	5	6	14				
7	2	2	14	5	5	16	5	4	6	13	16	10	33	14	14			
3	20	5	3	20	2	5	2	3	2	2	16	23	2	5	23	4		
3	2	10	4	16	2	20	5	6	3	1	3	5	13	5	24	6	2	
20	20	20	2	16	5	14	13	2	20	3	1	2	10	2	13	16	13	26

14																		
1	7																	
7	1	2																
14	20	4	5															
14	5	5	3	2														
20	20	5	5	20	3													
20	14	5	5	20	3	2												
16	20	3	2	1	2	2	4											
14	1	5	16	6	5	4	20	5										
2	16	20	20	20	13	1	16	20	2									
3	3	3	17	5	10	2	2	2	5	20								
20	16	1	2	20	20	2	2	1	13	2	13							
14	17	5	10	13	10	20	2	2	2	2	3	10						
14	14	1	16	2	13	5	5	13	4	6	1	20	13					
16	14	5	1	2	3	16	5	10	20	1	5	2	2	23				
20	5	20	1	5	3	2	16	20	1	3	5	2	16	20	5			
20	16	14	20	3	20	5	3	1	2	13	2	2	2	3	5	2		
20	14	14	20	2	5	5	20	10	5	20	4	2	5	2	2	5	2	
14	20	20	5	3	20	5	5	5	20	14	3	5	3	14	13	16	13	1

Panel (a)

0.272																		
0.303	0.107																	
0.339	0.148	0.127																
0.286	0.214	0.092	0.084															
0.111	0.123	0.074	0.041	0.083														
0.172	0.105	0.05	0.057	0.063	0.036													
0.153	0.081	0.054	0.033	0.076	0.1	0.039												
0.344	0.107	0.094	0.065	0.065	0.033	0.033	0.03											
0.137	0.057	0.047	0.051	0.068	0.101	0.046	0.108	0.018										
0.164	0.074	0.051	0.062	0.043	0.09	0.023	0.054	0.028	0.054									
0.086	0.053	0.036	0.055	0.024	0.058	0.048	0.065	-0.003	0.048	0.028								
0.336	0.164	0.111	0.076	0.075	0.026	0.048	0.014	0.065	0.01	0.018	-0.002							
0.136	0.063	0.063	0.049	0.034	0.098	0.051	0.117	0.023	0.133	0.029	0.031	0.01						
0.36	0.137	0.05	0.091	0.088	0.075	0.021	0.051	0.045	-0.011	0.007	-0.011	0.072	0.009					
0.265	0.083	0.065	0.074	0.041	0.026	0.032	0.015	0.077	0.047	0.008	-0.024	0.021	0.009	0.017				
0.305	0.113	0.162	0.097	0.04	0.009	0.011	0.046	0.027	0.029	0.017	0.026	0.036	-0.023	0.024	0.009			
0.074	0.064	0.042	0.025	0.092	0.078	0.033	0.043	0.022	0.051	0.025	0.039	-0.006	0.024	-0.01	-0.012	0.007		
0.09	0.072	0.056	0.03	0.03	0.061	0.076	0.018	0.015	0.027	0.041	0.048	0.031	0.037	0.012	-0.014	0.005	0.003	
0.169	0.092	0.111	0.073	0.039	0.059	0.049	0.024	0.002	0.053	0.067	0.055	0.019	0.03	-0.002	0.014	0.013	0.011	-0.007

Pane (b)

Figure A14: Panel (a) and (b) display respectively the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the iron ore-nickel mining portfolio. Panel (b) displays the c-vine Kendall tau correlation matrix of the same portfolio.

Panel (a), c-vine (left)

1																		
20	5																	
3	2	5																
14	5	23	3															
5	5	20	5	1														
3	2	20	20	3	5													
20	5	33	1	5	13	13												
20	5	5	20	20	5	5	14											
14	20	14	5	5	5	14	5	5										
20	5	20	5	3	5	5	23	5	5									
14	2	5	5	14	4	1	1	3	2	5								
14	16	1	5	20	1	3	1	5	2	3	13							
2	5	20	5	5	3	3	5	3	5	34	14	5						
5	10	5	10	33	5	5	6	3	36	5	1	33	1					
3	5	5	5	14	5	5	3	5	36	5	34	1	13	10				
20	10	5	5	20	13	5	4	20	1	4	5	1	23	14	3			
3	5	5	20	6	10	5	20	5	13	5	26	33	33	14	5	13		
14	20	5	20	3	3	5	1	5	10	4	1	3	2	1	5	23	3	
3	5	14	5	5	1	1	24	16	33	14	13	33	3	14	16	4	5	1

Panel (a), d-vine (right)

5																		
2	20																	
20	2	5																
14	2	3	4															
20	16	5	14	3														
5	2	20	5	5	5													
5	20	14	20	36	33	23												
2	3	14	14	2	13	3	13											
20	20	3	20	16	3	36	3	2										
20	5	20	5	4	5	1	1	33	3									
20	13	16	14	10	10	3	20	20	20	5								
5	1	5	3	3	5	5	3	13	10	5	5							
5	20	20	20	5	5	5	20	3	5	2	5	5						
20	5	1	5	14	14	5	1	5	2	1	5	24	26					
14	20	3	3	1	20	1	14	3	13	5	3	5	2	3				
16	14	2	5	20	1	16	36	13	2	5	16	1	2	4	6			
10	1	1	5	5	14	6	1	10	10	5	14	5	1	5	1	13		
14	1	1	5	26	5	13	16	5	1	1	5	4	13	1	5	1	5	
16	10	20	20	20	5	5	5	23	1	5	3	5	5	26	13	10	2	6

Panel (a)

0.101																		
0.116	0.590																	
0.076	0.502	0.166																
0.196	0.105	-0.004	0.007															
0.242	0.462	0.178	0.100	0.013														
0.103	0.501	0.158	0.075	0.061	0.046													
0.120	0.033	-0.006	0.01	0.135	0.002	0.003												
0.164	0.483	0.131	0.057	0.095	0.064	0.060	0.004											
0.160	0.218	0.022	0.066	0.060	0.042	0.039	0.022	0.088										
0.055	0.553	0.251	0.141	0.031	0.057	0.064	-0.005	0.064	0.011									
0.130	0.132	0.031	0.054	0.039	0.038	0.028	0.008	0.022	0.026	-0.015								
0.188	0.143	0.025	0.056	0.069	0.034	0.020	0.028	0.027	0.055	0.010	0.037							
0.294	0.106	0.038	0.031	0.115	0.023	0.010	0.030	0.023	0.049	-0.006	0.021	0.055						
0.035	0.297	0.117	0.038	-0.002	0.042	0.038	0.004	0.009	-0.001	0.017	-0.010	-0.008	0.001					
0.046	0.547	0.201	0.144	0.013	0.069	0.067	0.005	0.075	-0.003	0.125	-0.008	0.016	0.010	0.055				
0.052	0.209	0.017	0.072	0.039	0.007	0.017	0.005	0.039	0.022	0.002	0.072	0.005	-0.004	0.004	0.003			
0.065	0.521	0.154	0.115	0.016	0.080	0.063	0.055	0.083	0.011	0.107	-0.008	-0.007	-0.015	0.005	0.103	0.006		
0.145	0.256	0.063	0.085	0.043	0.016	0.015	-0.004	0.061	0.053	0.016	0.069	0.035	0.026	0.017	0.019	-0.007	0.014	
0.006	0.240	0.002	0.056	0.011	-0.005	-0.003	-0.005	0.00	-0.003	0.003	0.004	-0.002	0.004	0.005	0.001	0.005	0.061	-0.004

Panel (b)

Figure A15: Panel (a) displays the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the coal-uranium energy portfolio. Panel (b) displays the c-vine Kendall tau correlation matrix of the same portfolio.

19																		
7	1																	
1	5	5																
14	5	5	2															
2	19	5	5	1														
14	3	4	3	20	13													
3	5	5	20	5	13	5												
1	5	5	1	5	5	5	5											
14	1	3	5	16	5	4	1	5										
7	14	14	14	20	20	5	36	16	1									
14	1	16	20	6	2	5	14	5	3	14								
1	5	3	2	20	23	1	5	3	3	16	4							
1	20	5	14	20	5	5	10	3	1	2	5	33						
14	1	14	20	20	13	3	14	5	3	13	5	3	1					
23	13	1	16	5	5	3	36	13	33	1	13	1	23	1				
2	5	1	20	20	1	5	20	3	5	5	5	3	1	10	5			
7	20	14	13	2	6	6	5	3	1	20	5	26	5	23	1	5		
13	13	1	2	20	5	16	5	5	6	1	1	5	26	5	5	20	23	
9	14	5	20	1	5	3	5	3	5	6	10	14	5	14	10	16	2	5

14																		
3	3																	
14	16	5																
14	1	20	5															
5	20	5	5	1														
1	3	5	5	1	5													
20	5	14	10	5	13	1												
5	20	13	3	5	6	23	5											
1	1	5	16	5	1	4	1	33										
1	4	20	13	33	5	5	6	1	5									
2	5	4	10	4	1	13	3	5	3	1								
19	19	5	20	2	5	5	13	3	3	1	14							
14	1	5	3	14	5	13	5	16	1	6	2	14						
14	14	5	3	1	20	2	1	14	13	23	5	13	2					
5	14	13	5	5	16	4	14	1	2	13	5	14	6	14				
20	14	10	2	2	5	5	5	3	16	20	5	5	24	13	16			
20	2	16	1	5	1	23	3	1	23	3	5	13	5	1	3	16		
2	20	5	33	1	26	24	5	6	5	26	16	20	5	5	5	5	5	
5	5	3	1	23	1	5	33	5	33	5	23	1	5	1	1	33	5	13

Panel (a)

0.132																		
0.182	0.068																	
0.175	0.137	0.068																
0.204	0.081	0.091	0.094															
0.215	0.174	0.033	0.053	0.081														
0.184	0.121	0.134	0.064	0.099	0.066													
0.209	0.045	0.123	0.118	0.127	0.047	0.082												
0.195	0.085	0.053	0.032	0.063	0.033	-0.011	0.071											
0.189	0.179	0.085	0.011	0.062	0.074	0.052	0.052	-0.009										
0.264	0.056	0.229	0.068	-0.006	0.097	0.036	0.014	0.092	0.093									
0.412	0.132	0.087	0.153	0.030	0.009	0.049	0.031	0.072	0.020	0.064								
0.310	0.105	0.105	0.100	0.116	0.015	-0.006	0.043	0.008	0.052	0.070	0.049							
0.250	0.210	0.125	0.041	0.038	0.113	0.018	0.047	0.017	0.052	0.013	0.010	0.035						
0.198	0.178	0.115	0.108	0.063	0.121	0.052	0.035	0.023	0.028	-0.015	0.022	0.014	0.004					
0.249	0.081	0.054	0.086	-0.021	0.055	0.040	0.081	0.098	0.190	0.040	0.154	0.040	0.013	0.021				
0.242	0.114	0.091	-0.007	0.006	-0.019	0.034	0.023	0.022	0.017	0.091	0.065	0.015	-0.027	0.037	0.033			
0.255	0.241	0.034	0.013	-0.027	0.076	-0.038	0.030	-0.012	-0.034	0.045	0.40	0.028	0.082	0.032	0.019	0.054		
0.469	0.145	0.142	-0.011	0.010	-0.009	-0.046	0.020	0.034	-0.054	-0.036	0.027	0.221	0.078	0.079	-0.031	-0.050	-0.041	
0.315	0.186	0.004	-0.010	-0.002	0.006	-0.012	-0.003	0.045	-0.006	0.009	-0.003	-0.008	0.124	0.008	-0.008	-0.002	0.014	0.002

Panel (b)

Figure A16: Panel (a) displays the GFC period scenario c-vine (on the left) and d-vine (on the right) dependence structure matrices of the oil-gas energy portfolio. Panel (b) displays the Kendall tau correlation matrix of the same portfolio.

5																		
20	10																	
20	10	20																
3	10	16	20															
5	5	2	20	14														
5	5	10	4	10	10													
5	20	5	16	10	5	16												
20	5	20	20	5	26	1	5											
1	10	2	20	20	20	5	1	20										
14	20	20	1	20	5	13	1	2	13									
5	10	20	20	2	20	14	10	13	2	5								
1	10	20	5	20	20	13	5	20	5	1	5							
2	5	16	20	5	16	5	33	3	20	20	13	5						
20	10	20	5	20	20	1	10	5	5	10	20	14	10					
5	10	2	20	2	20	10	10	3	5	4	10	5	5	5				
20	10	2	20	20	20	10	10	24	10	5	10	10	5	20	5			
20	10	16	5	20	16	10	33	20	4	20	5	1	1	13	13	1		
1	10	20	10	2	20	5	10	5	5	3	10	5	33	10	5	5	23	
20	20	14	20	20	2	6	10	1	5	5	5	5	5	5	5	5	3	5

10																		
2	5																	
20	20	10																
20	5	5	1															
20	3	5	3	1														
14	5	1	5	1	5													
20	1	5	20	20	10	16												
20	2	20	3	16	5	3	3											
5	20	1	5	5	20	5	10	10										
5	14	20	2	20	10	16	3	5	33									
20	20	5	20	20	5	24	20	5	10	5								
5	20	20	1	20	34	5	13	2	5	5	5							
2	5	33	5	16	20	13	23	13	14	20	5	5						
20	5	5	5	14	5	5	3	1	20	3	5	5	1					
2	14	10	5	4	5	14	10	1	1	20	5	3	3	1				
10	10	10	20	5	10	10	4	5	23	5	4	1	10	5	20			
10	2	20	14	20	5	20	5	3	5	3	13	20	5	3	20	5		
2	10	2	20	5	20	1	13	10	5	14	1	5	1	10	5	5	5	
10	5	5	5	13	5	14	14	10	14	5	4	3	13	5	10	26	3	16

Panel (a)

0.052																		
0.126	0.073																	
0.237	0.061	0.076																
0.122	0.042	0.112	0.112															
0.125	0.317	0.139	0.107	0.112														
0.049	0.013	0.054	0.005	0.047	0.041													
0.063	0.031	0.022	0.003	0.032	0.056	0.002												
0.217	0.038	0.073	0.092	0.075	-0.021	-0.006	-0.022											
0.122	0.083	0.103	0.090	0.126	0.177	0.062	-0.008	0.051										
0.302	0.057	0.054	0.087	0.068	0.012	0.007	0.012	0.078	0.019									
0.105	0.283	0.205	0.114	0.134	0.430	0.002	0.043	0.017	0.074	0.031								
0.132	0.062	0.115	0.101	0.107	0.187	0.005	0.069	0.040	0.106	0.040	0.121							
0.136	0.055	0.059	0.087	0.024	0.039	-0.009	-0.003	0.024	0.072	0.055	0.026	0.046						
0.145	0.125	0.065	0.108	0.074	0.109	0.011	0.038	0.059	0.113	0.083	0.084	0.007	0.073					
0.119	0.117	0.174	0.098	0.127	0.336	0.045	0.069	0.013	0.130	0.010	0.221	0.089	-0.017	0.06				
0.153	0.092	0.106	0.096	0.078	0.151	0.042	0.045	-0.008	0.107	0.059	0.108	0.056	0.078	0.069	0.101			
0.160	0.079	0.077	0.060	0.057	0.024	0.056	-0.009	0.048	0.036	0.060	0.052	0.022	0.063	0.039	0.018	0.058		
0.088	0.117	0.162	0.069	0.063	0.252	0.038	0.064	0.057	0.115	0.021	0.218	0.109	-0.029	0.052	0.199	0.066	-0.009	
0.174	0.120	0.134	0.111	0.135	0.185	0.001	0.062	0.051	0.089	0.075	0.118	0.049	0.022	0.058	0.094	0.037	0.024	0.095

Panel (b)

Figure A17: Panel (a) and (b) display the full sample period c-vine (on the left) and d-vine (on the right) dependence structure matrices of the mix-metals leptokurtic portfolio, respectively. Panel (b) displays the c-vine Kendall tau correlation matrix of the same portfolio.

2																		
14	5																	
14	5	5																
10	5	5	1															
3	5	5	5	20														
33	6	2	5	4	36													
5	5	2	2	5	1	26												
33	23	13	20	2	33	6	4											
2	1	1	6	3	1	13	5	23										
2	24	5	5	33	14	5	23	13	13									
33	5	13	3	2	2	13	13	13	13	20								
2	5	2	3	5	5	5	2	5	23	20	5							
5	13	1	5	13	4	4	2	5	33	5	5	16						
3	4	5	5	14	5	2	5	2	4	14	5	13	1					
5	5	1	5	2	14	13	2	2	23	23	2	2	2	16				
16	2	1	5	4	13	2	2	2	1	33	10	14	1	14	13			
5	3	2	6	10	5	5	5	13	5	34	4	5	5	2	3	1		
20	2	1	1	2	2	1	5	1	14	1	2	20	2	5	5	2	5	
14	2	2	14	14	19	2	2	1	5	5	2	7	2	2	2	5	20	20

26																		
2	16																	
2	23	2																
5	13	1	10															
3	23	3	16	10														
2	5	2	16	5	2													
14	14	2	5	2	4	20												
20	13	2	1	33	5	1	13											
13	2	4	13	14	5	13	3	24										
5	14	1	6	5	5	6	5	4	20									
5	14	10	3	16	5	5	2	4	13	1								
2	4	2	20	5	20	3	1	2	13	2	13							
2	13	1	5	2	13	5	20	13	5	6	13	5						
14	14	5	14	20	2	2	5	10	40	4	14	13	3					
6	5	4	5	2	33	10	10	36	10	13	3	20	13	5				
2	1	1	5	2	4	2	2	1	5	5	1	2	3	2	2			
5	5	5	23	1	5	14	5	2	14	14	2	2	2	2	2	2		
3	2	2	23	16	2	4	5	5	4	2	5	1	5	13	1	5	13	
5	2	2	3	3	2	14	2	1	5	2	7	1	2	14	1	20	2	1

Panel (a)

0.03																		
0.00	0.20																	
0.00	0.19	0.25																
0.03	0.04	0.14	0.03															
0.02	0.30	0.09	0.03	0.06														
-0.02	0.02	0.28	0.03	0.00	-0.01													
-0.01	-0.02	0.04	0.09	0.04	0.01	-0.01												
0.00	-0.02	0.02	0.10	0.05	0.00	0.01	0.01											
0.05	0.01	0.01	0.02	0.02	0.04	0.00	0.02	-0.01										
-0.02	-0.02	-0.01	0.02	-0.01	0.01	-0.01	0.00	0.03	0.01									
-0.02	0.03	0.04	0.03	0.03	0.02	0.01	0.02	0.01	0.01	0.02								
0.04	0.03	0.05	0.04	-0.02	0.02	0.05	0.02	0.18	-0.01	0.04	-0.03							
0.03	0.02	0.07	0.07	0.02	0.02	0.04	0.01	0.07	-0.01	0.01	0.03	0.02						
0.04	0.02	-0.02	0.02	0.03	0.00	0.03	0.04	0.32	0.01	0.00	0.04	0.02	0.06					
0.05	0.02	0.04	0.04	0.05	0.05	0.04	0.02	0.09	-0.01	0.00	0.05	0.06	0.03	0.03				
0.02	0.03	0.07	0.06	0.03	0.04	0.10	0.08	0.08	-0.01	0.00	0.07	0.06	0.08	0.02	0.03			
0.04	0.03	0.09	0.03	0.03	0.08	0.11	0.08	0.03	0.25	0.00	0.09	0.09	0.09	0.11	0.03	0.06		
0.07	0.36	0.07	0.06	0.06	0.13	0.13	0.11	0.04	0.01	-0.01	0.12	0.17	0.14	0.13	0.14	0.12	0.10	
0.09	0.09	0.15	0.10	0.10	0.20	0.22	0.21	0.12	0.08	0.19	0.26	0.26	0.28	0.20	0.21	0.22	0.15	0.21

Panel (b)

Figure A18: Panel (a) displays the full sample period r-vine (on the left) and c-vine (on the right) dependence structure matrices of the retail benchmark portfolio. Panel (b) displays the Kendall tau correlation matrix of the same portfolio.

5																		
5	5																	
33	1	3																
16	3	30	1															
36	4	2	14	1														
26	33	5	14	1	1													
5	10	3	3	1	23	13												
13	26	10	13	13	1	1	5											
13	5	2	5	20	24	14	2	13										
3	20	4	1	5	3	5	36	14	5									
5	26	5	5	5	34	1	5	33	13	30								
5	2	5	5	1	10	5	5	2	36	10	30							
1	20	5	1	1	3	2	16	5	2	4	2	4						
23	33	5	4	34	5	33	5	1	14	10	5	1	30					
1	1	5	1	24	1	13	5	13	4	10	20	2	5	5				
14	5	5	10	10	5	33	5	13	36	5	5	3	5	5	5			
1	13	2	1	3	5	1	2	4	5	5	20	2	2	2	9	2		
2	1	5	6	4	14	1	5	5	3	5	5	33	5	5	5	5	5	
13	7	2	13	20	5	2	1	20	19	14	5	20	1	5	1	1	1	3

5																		
6	40																	
26	13	24																
5	33	1	40															
1	5	1	5	1														
14	5	16	13	24	24													
4	20	13	5	2	5	36												
5	5	1	5	5	33	3	5											
4	23	5	5	5	2	13	20	36										
5	23	1	5	5	5	5	24	5	1									
1	23	3	4	20	20	5	5	40	5	5								
5	10	23	13	13	5	5	5	5	3	26	13							
13	4	5	13	23	13	2	5	13	2	16	2	1						
13	5	5	10	5	1	5	2	23	1	5	3	3	13					
5	5	4	20	20	33	3	3	23	3	1	2	5	14	5				
10	5	4	10	1	5	1	5	14	5	5	5	1	36	2	10			
2	1	5	5	5	1	2	20	1	14	2	2	20	3	36	20	5		
1	2	13	2	2	1	14	5	5	5	20	5	2	20	3	2	2	5	
2	10	10	14	1	5	2	5	3	3	19	20	7	2	14	1	1	5	5

Panel (a)

-0.03																		
0.02	-0.063																	
-0.014	0.016	-0.006																
0.057	-0.005	0.006	-0.041															
-0.005	0.059	0.009	0.025	0.019														
0.01	0.063	0.001	0.019	-0.02	-0.002													
0.042	0.103	0.006	0.084	0.022	-0.011	-0.003												
0.063	0.157	-0.011	0.082	0.08	-0.001	0.007	0.027											
0.017	-0.013	0.047	0.105	0.079	0.015	0.026	0.053	-0.001										
0.056	-0.016	0.014	0.192	0.145	0.11	0.082	-0.013	-0.057	0.005									
0.021	-0.006	0.01	0.015	0.2	0.213	0.063	0.1	-0.115	0.037	-0.01								
0.024	0.069	-0.002	0.026	0.023	0.302	0.146	0.047	-0.251	0.019	-0.015	0.002							
0.028	0.016	0.067	0.015	-0.013	0.002	0.282	0.164	0.011	0.038	0.019	0.018	0.003						
0.009	0.133	0.031	0.063	-0.007	0.002	-0.02	0.336	-0.006	0.043	0.021	0.023	0.014	0.002					
0.183	0.232	0.005	0.024	0.093	-0.001	0.011	0.032	-0.006	0.028	0.103	0.072	-0.018	0.017	-0.011				
0.209	0.286	0.001	0.163	0.025	0.128	0.021	0.04	0.002	0.02	0.053	0.063	0.065	-0.011	0.015	0.213			
0.359	0.059	-0.016	0.155	0.259	0.011	0.096	0.069	-0.003	0.06	0.059	0.043	0.062	0.02	-0.013	0.252	0.361		
-0.003	0.21	0.005	0.149	0.296	0.002	0.078	0.114	0.056	0.064	0.081	0.104	0.098	0.088	0.042	0.649	0.665	-0.021	
0.644	0.108	0.114	0.049	0.091	0.429	0.097	0.14	0.014	0.096	0.16	0.176	0.254	0.308	0.164	0.086	0.125	0.099	0.539

Panel (b)

Figure A19: Panel (a) displays the full sample period r-vine (on the left) and c-vine (on the right) dependence structure matrices of the retail benchmark portfolio. Panel (b) displays the Kendall tau correlation matrix of the same portfolio.

5																		
20	10																	
20	10	20																
3	10	16	20															
5	5	2	20	14														
5	5	10	4	10	10													
5	20	5	16	10	5	16												
20	5	20	20	5	26	1	5											
1	10	2	20	20	20	5	1	20										
14	20	20	1	20	5	13	1	2	13									
5	10	20	20	2	20	14	10	13	2	5								
1	10	20	5	20	20	13	5	20	5	1	5							
2	5	16	20	5	16	5	33	3	20	20	13	5						
20	10	20	5	20	20	1	10	5	5	10	20	14	10					
5	10	2	20	2	20	10	10	3	5	4	10	5	5	5				
20	10	2	20	20	20	10	10	24	10	5	10	10	5	20	5			
20	10	16	5	20	16	10	33	20	4	20	5	1	1	13	13	1		
1	10	20	10	2	20	5	10	5	5	3	10	5	33	10	5	5	23	
20	20	14	20	20	2	6	10	1	5	5	5	5	5	5	5	5	3	5

10																		
2	5																	
20	20	10																
20	5	5	1															
20	3	5	3	1														
14	5	1	5	1	5													
20	1	5	20	20	10	16												
20	2	20	3	16	5	3	3											
5	20	1	5	5	20	5	10	10										
5	14	20	2	20	10	16	3	5	33									
20	20	5	20	20	5	24	20	5	10	5								
5	20	20	1	20	34	5	13	2	5	5	5							
2	5	33	5	16	20	13	23	13	14	20	5	5						
20	5	5	5	14	5	5	3	1	20	3	5	5	1					
2	14	10	5	4	5	14	10	1	1	20	5	3	3	1				
10	10	10	20	5	10	10	4	5	23	5	4	1	10	5	20			
10	2	20	14	20	5	20	5	3	5	3	13	20	5	3	20	5		
2	10	2	20	5	20	1	13	10	5	14	1	5	1	10	5	5	5	
10	5	5	5	13	5	14	14	10	14	5	4	3	13	5	10	26	3	16

Panel (a)

0.052																		
0.126	0.073																	
0.237	0.061	0.076																
0.122	0.042	0.112	0.112															
0.125	0.317	0.139	0.107	0.112														
0.049	0.013	0.054	0.005	0.047	0.041													
0.063	0.031	0.022	0.003	0.032	0.056	0.002												
0.217	0.038	0.073	0.092	0.075	-0.021	-0.006	-0.022											
0.122	0.083	0.103	0.090	0.126	0.177	0.062	-0.008	0.051										
0.302	0.057	0.054	0.087	0.068	0.012	0.007	0.012	0.078	0.019									
0.105	0.283	0.205	0.114	0.134	0.430	0.002	0.043	0.017	0.074	0.031								
0.132	0.062	0.115	0.101	0.107	0.187	0.005	0.069	0.040	0.106	0.040	0.121							
0.136	0.055	0.059	0.087	0.024	0.039	-0.009	-0.003	0.024	0.072	0.055	0.026	0.046						
0.145	0.125	0.065	0.108	0.074	0.109	0.011	0.038	0.059	0.113	0.083	0.084	0.007	0.073					
0.119	0.117	0.174	0.098	0.127	0.336	0.045	0.069	0.013	0.130	0.010	0.221	0.089	-0.017	0.06				
0.153	0.092	0.106	0.096	0.078	0.151	0.042	0.045	-0.008	0.107	0.059	0.108	0.056	0.078	0.069	0.101			
0.160	0.079	0.077	0.060	0.057	0.024	0.056	-0.009	0.048	0.036	0.060	0.052	0.022	0.063	0.039	0.018	0.058		
0.088	0.117	0.162	0.069	0.063	0.252	0.038	0.064	0.057	0.115	0.021	0.218	0.109	-0.029	0.052	0.199	0.066	-0.009	
0.174	0.120	0.134	0.111	0.135	0.185	0.001	0.062	0.051	0.089	0.075	0.118	0.049	0.022	0.058	0.094	0.037	0.024	0.095

Panel (b)

Figure A20: Panel (a) displays the entire series c-vine (on the left) and d-vine (on the right) dependence structure matrix of the mix-metals leptokurtic portfolio. Panel (b) displays the c-vine estimated Kendall tau correlation matrix of the same portfolio.

5																		
3	10																	
5	5	20																
16	5	3	3															
5	5	16	5	33														
5	10	5	5	3	13													
5	5	30	40	24	5	3												
5	13	3	2	5	26	16	5											
1	5	20	2	1	14	10	1	3										
16	4	16	1	16	5	5	3	5	6									
5	5	20	5	3	5	1	5	3	20	14								
3	40	30	5	3	5	4	5	1	5	5	5							
13	36	14	3	33	33	1	16	4	24	3	14	3						
1	5	5	5	5	5	13	33	5	5	13	1	14	1					
5	10	20	5	3	5	5	30	2	5	2	5	5	1	13				
14	13	4	1	3	4	14	16	5	5	5	1	6	5	1	6			
2	10	20	5	1	16	1	33	4	4	5	5	3	3	5	13	20		
13	5	20	2	5	5	5	5	1	5	3	4	5	16	16	5	3	3	
1	13	5	16	5	1	6	4	3	5	14	40	5	5	4	1	3	5	10

5																		
20	3																	
20	5	1																
2	5	5	14															
20	5	5	3	4														
20	2	5	5	10	26													
5	10	10	5	5	5	33												
10	5	5	5	13	4	6	24											
10	5	14	20	2	20	5	1	1										
14	14	5	33	16	1	23	2	3	16									
3	2	10	20	5	5	5	1	33	1	3								
5	3	20	20	5	14	5	5	5	5	14	3							
5	13	16	16	5	5	13	16	33	13	3	3	4						
2	20	4	33	4	1	13	3	5	3	33	13	13	3					
20	14	13	1	14	13	14	1	5	4	6	16	33	3	1				
14	5	16	20	2	16	13	5	13	3	33	1	5	4	14	2			
5	5	10	5	2	16	5	16	14	6	1	3	6	26	1	1	5		
5	1	1	13	1	1	3	5	5	13	1	5	5	23	1	30	4	13	
40	24	14	3	6	5	2	5	23	2	5	3	6	14	20	14	5	36	16

Panel (a)

16																		
5	16																	
20	14	3																
20	16	2	1															
5	14	2	2	14														
14	6	10	5	5	10													
1	13	6	16	1	5	1												
20	4	5	20	5	2	33	33											
14	10	2	2	5	16	5	3	14										
14	5	3	3	3	6	13	5	14	33									
5	16	2	20	14	2	10	1	23	5	13								
2	2	14	3	5	5	23	1	3	3	3	5							
14	14	20	20	5	16	1	5	5	20	5	10	5						
5	14	2	13	5	20	10	5	5	13	6	5	33	14					
1	5	10	23	1	33	10	6	1	20	4	5	5	5	1				
20	5	10	5	20	5	5	5	5	13	1	5	33	2	5	5			
5	10	16	14	14	36	13	26	20	5	14	5	1	5	5	5	13		
13	26	10	1	20	10	5	36	5	13	26	5	23	2	1	5	2	16	
14	14	14	3	20	16	23	5	5	26	16	3	16	5	5	3	5	3	34

5																		
5	5																	
5	5	5																
2	10	5	10															
5	5	5	5	2														
5	3	2	5	10	13													
14	20	3	5	23	5	23												
14	5	1	26	3	5	1	33											
14	20	4	10	2	5	10	5	5										
14	10	3	4	5	14	33	24	5	23									
3	2	16	3	16	1	1	23	1	13	3								
14	14	5	10	4	5	20	23	36	23	1	5							
20	3	14	13	14	20	2	5	13	1	5	33	1						
20	5	14	4	4	5	5	13	5	23	1	33	1	1					
5	14	5	3	16	2	14	5	2	5	14	13	5	1	5				
3	20	1	10	3	2	13	5	5	3	1	3	26	36	5	1			
1	5	14	14	5	5	13	1	36	5	33	3	5	23	16	6	26		
5	16	20	5	3	14	16	13	3	3	5	5	5	4	2	23	1	1	
20	5	5	2	5	16	5	16	14	2	16	16	2	5	5	14	13	5	5

Panel (b)

2																		
5	36																	
14	1	5																
5	20	6	1															
20	1	5	5	5														
3	3	5	16	20	20													
1	33	2	6	6	20	5												
5	13	16	33	3	4	20	16											
20	3	5	16	5	3	16	5	3										
1	13	1	2	16	1	2	1	1	33									
5	5	10	20	5	20	2	16	1	33	10								
1	1	5	16	5	14	2	5	5	16	13	5							
20	3	5	5	5	5	16	1	1	3	4	16	13						
7	3	5	10	5	3	3	23	13	3	20	5	36	2					
5	13	5	14	20	14	2	5	10	3	6	5	5	16	2				
3	13	5	5	33	3	2	1	1	24	5	5	14	5	5	3			
14	6	13	13	5	1	14	5	5	1	33	5	33	2	13	23	13		
20	5	1	20	5	5	20	16	10	5	16	5	20	33	5	5	5	5	
5	5	5	3	3	5	20	5	5	1	5	5	23	5	5	5	5	3	5

5																		
5	10																	
10	14	5																
14	23	16	2															
14	10	4	20	5														
20	14	3	1	5	16													
20	5	5	5	5	5	5												
5	17	1	14	3	13	13	1											
2	4	14	14	4	3	1	5	10										
5	5	3	13	20	3	20	20	20	5									
20	3	1	4	20	5	5	5	5	5	20								
5	20	16	16	13	1	20	16	2	5	5	5							
5	1	5	5	23	34	5	10	5	1	16	5	1						
5	14	5	2	14	6	5	13	1	13	5	5	5	33					
1	5	1	20	5	9	3	5	3	3	14	2	13	33	3				
14	5	5	5	23	1	5	2	13	5	5	5	3	5	23	1			
20	3	1	5	14	3	5	3	13	33	24	3	5	3	3	2	5		
5	5	5	20	5	20	5	1	33	13	3	23	5	4	5	1	5	10	
16	3	13	33	5	2	5	5	5	13	33	6	5	5	6	1	26	13	1

Panel (c)

Figure A21: Panels (a), (b) and (c) display the pre-GFC, GFC and post-GFC c-vine (on the left) and d-vine (on the right) dependence structure matrices of the mix-metals leptokurtic portfolio, respectively.

5																		
20	10																	
20	10	20																
3	10	16	20															
5	5	2	20	14														
5	5	10	4	10	10													
5	20	5	16	10	5	16												
20	5	20	20	5	26	1	5											
1	10	2	20	20	20	5	1	20										
14	20	20	1	20	5	13	1	2	13									
5	10	20	20	2	20	14	10	13	2	5								
1	10	20	5	20	20	13	5	20	5	1	5							
2	5	16	20	5	16	5	33	3	20	20	13	5						
20	10	20	5	20	20	1	10	5	5	10	20	14	10					
5	10	2	20	2	20	10	10	3	5	4	10	5	5	5				
20	10	2	20	20	20	10	10	24	10	5	10	10	5	20	5			
20	10	16	5	20	16	10	33	20	4	20	5	1	1	13	13	1		
1	10	20	10	2	20	5	10	5	5	3	10	5	33	10	5	5	23	
20	20	14	20	20	2	6	10	1	5	5	5	5	5	5	5	5	3	5

10																		
2	5																	
20	20	10																
20	5	5	1															
20	3	5	3	1														
14	5	1	5	1	5													
20	1	5	20	20	10	16												
20	2	20	3	16	5	3	3											
5	20	1	5	5	20	5	10	10										
5	14	20	2	20	10	16	3	5	33									
20	20	5	20	20	5	24	20	5	10	5								
5	20	20	1	20	34	5	13	2	5	5	5							
2	5	33	5	16	20	13	23	13	14	20	5	5						
20	5	5	5	14	5	5	3	1	20	3	5	5	1					
2	14	10	5	4	5	14	10	1	1	20	5	3	3	1				
10	10	10	20	5	10	10	4	5	23	5	4	1	10	5	20			
10	2	20	14	20	5	20	5	3	5	3	13	20	5	3	20	5		
2	10	2	20	5	20	1	13	10	5	14	1	5	1	10	5	5	5	
10	5	5	5	13	5	14	14	10	14	5	4	3	13	5	10	26	3	16

Panel (a)

0.052																		
0.126	0.073																	
0.237	0.061	0.076																
0.122	0.042	0.112	0.112															
0.125	0.317	0.139	0.107	0.112														
0.049	0.013	0.054	0.005	0.047	0.041													
0.063	0.031	0.022	0.003	0.032	0.056	0.002												
0.217	0.038	0.073	0.092	0.075	-0.021	-0.006	-0.022											
0.122	0.083	0.103	0.090	0.126	0.177	0.062	-0.008	0.051										
0.302	0.057	0.054	0.087	0.068	0.012	0.007	0.012	0.078	0.019									
0.105	0.283	0.205	0.114	0.134	0.430	0.002	0.043	0.017	0.074	0.031								
0.132	0.062	0.115	0.101	0.107	0.187	0.005	0.069	0.040	0.106	0.040	0.121							
0.136	0.055	0.059	0.087	0.024	0.039	-0.009	-0.003	0.024	0.072	0.055	0.026	0.046						
0.145	0.125	0.065	0.108	0.074	0.109	0.011	0.038	0.059	0.113	0.083	0.084	0.007	0.073					
0.119	0.117	0.174	0.098	0.127	0.336	0.045	0.069	0.013	0.130	0.010	0.221	0.089	-0.017	0.06				
0.153	0.092	0.106	0.096	0.078	0.151	0.042	0.045	-0.008	0.107	0.059	0.108	0.056	0.078	0.069	0.101			
0.160	0.079	0.077	0.060	0.057	0.024	0.056	-0.009	0.048	0.036	0.060	0.052	0.022	0.063	0.039	0.018	0.058		
0.088	0.117	0.162	0.069	0.063	0.252	0.038	0.064	0.057	0.115	0.021	0.218	0.109	-0.029	0.052	0.199	0.066	-0.009	
0.174	0.120	0.134	0.111	0.135	0.185	0.001	0.062	0.051	0.089	0.075	0.118	0.049	0.022	0.058	0.094	0.037	0.024	0.095

Panel (b)

Figure A22: Panel (a) and (b) display the entire series c-vine (on the left) and d-vine (on the right) dependence structure matrices of the mix-metals leptokurtic portfolio, respectively. Panel (b) displays the c-vine Kendall tau correlation matrix.

5																		
5	5																	
33	1	3																
16	3	30	1															
36	4	2	14	1														
26	33	5	14	1	1													
5	10	3	3	1	23	13												
13	26	10	13	13	1	1	5											
13	5	2	5	20	24	14	2	13										
3	20	4	1	5	3	5	36	14	5									
5	26	5	5	5	34	1	5	33	13	30								
5	2	5	5	1	10	5	5	2	36	10	30							
1	20	5	1	1	3	2	16	5	2	4	2	4						
23	33	5	4	34	5	33	5	1	14	10	5	1	30					
1	1	5	1	24	1	13	5	13	4	10	20	2	5	5				
14	5	5	10	10	5	33	5	13	36	5	5	3	5	5	5			
1	13	2	1	3	5	1	2	4	5	5	20	2	2	2	9	2		
2	1	5	6	4	14	1	5	5	3	5	5	33	5	5	5	5	5	
13	7	2	13	20	5	2	1	20	19	14	5	20	1	5	1	1	1	3

5																		
6	40																	
26	13	24																
5	33	1	40															
1	5	1	5	1														
14	5	16	13	24	24													
4	20	13	5	2	5	36												
5	5	1	5	5	33	3	5											
4	23	5	5	5	2	13	20	36										
5	23	1	5	5	5	5	24	5	1									
1	23	3	4	20	20	5	5	40	5	5								
5	10	23	13	13	5	5	5	5	3	26	13							
13	4	5	13	23	13	2	5	13	2	16	2	1						
13	5	5	10	5	1	5	2	23	1	5	3	3	13					
5	5	4	20	20	33	3	3	23	3	1	2	5	14	5				
10	5	4	10	1	5	1	5	14	5	5	5	1	36	2	10			
2	1	5	5	5	1	2	20	1	14	2	2	20	3	36	20	5		
1	2	13	2	2	1	14	5	5	5	20	5	2	20	3	2	2	5	
2	10	10	14	1	5	2	5	3	3	19	20	7	2	14	1	1	5	5

Panel (a)

-0.03																		
0.02	-0.063																	
-0.014	0.016	-0.006																
0.057	-0.005	0.006	-0.041															
-0.005	0.059	0.009	0.025	0.019														
0.01	0.063	0.001	0.019	-0.02	-0.002													
0.042	0.103	0.006	0.084	0.022	-0.011	-0.003												
0.063	0.157	-0.011	0.082	0.08	-0.001	0.007	0.027											
0.017	-0.013	0.047	0.105	0.079	0.015	0.026	0.053	-0.001										
0.056	-0.016	0.014	0.192	0.145	0.11	0.082	-0.013	-0.057	0.005									
0.021	-0.006	0.01	0.015	0.2	0.213	0.063	0.1	-0.115	0.037	-0.01								
0.024	0.069	-0.002	0.026	0.023	0.302	0.146	0.047	-0.251	0.019	-0.015	0.002							
0.028	0.016	0.067	0.015	-0.013	0.002	0.282	0.164	0.011	0.038	0.019	0.018	0.003						
0.009	0.133	0.031	0.063	-0.007	0.002	-0.02	0.336	-0.006	0.043	0.021	0.023	0.014	0.002					
0.183	0.232	0.005	0.024	0.093	-0.001	0.011	0.032	-0.006	0.028	0.103	0.072	-0.018	0.017	-0.011				
0.209	0.286	0.001	0.163	0.025	0.128	0.021	0.04	0.002	0.02	0.053	0.063	0.065	-0.011	0.015	0.213			
0.359	0.059	-0.016	0.155	0.259	0.011	0.096	0.069	-0.003	0.06	0.059	0.043	0.062	0.02	-0.013	0.252	0.361		
-0.003	0.21	0.005	0.149	0.296	0.002	0.078	0.114	0.056	0.064	0.081	0.104	0.098	0.088	0.042	0.649	0.665	-0.021	
0.644	0.108	0.114	0.049	0.091	0.429	0.097	0.14	0.014	0.096	0.16	0.176	0.254	0.308	0.164	0.086	0.125	0.099	0.539

Panel (b)

Figure A23: Panel (a) displays the full sample period r-vine (on the left) and c-vine (on the right) dependence structure matrices of the retail benchmark portfolio. Panel (b) displays the full sample period r-vine Kendall tau correlation matrix.

APPENDIX B: Plots of the fitted vine copula models

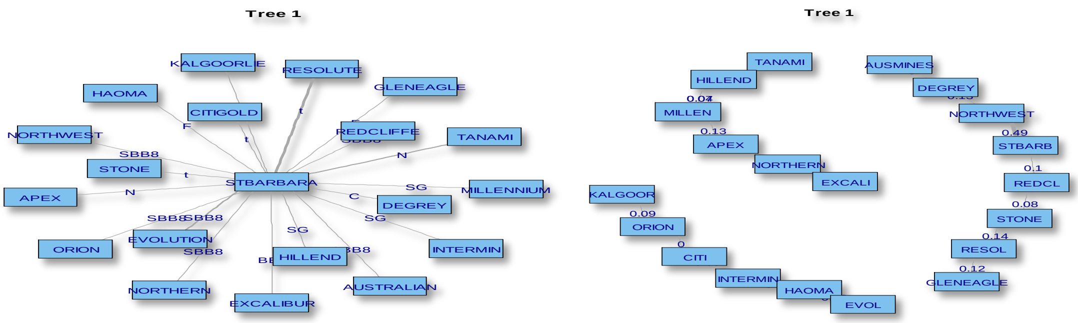

Figure B1: The first tree of the c-vine (on the left) and d-vine (on the right) copula models fitted to the gold portfolio based on the full sample period. The letters in between the rootstock and the rest of the stocks from the c-vine refer to the bivariate copulas used to model the dependence. The numbers in between the names of the stocks from the d-vine are the Kendall tau correlation values.

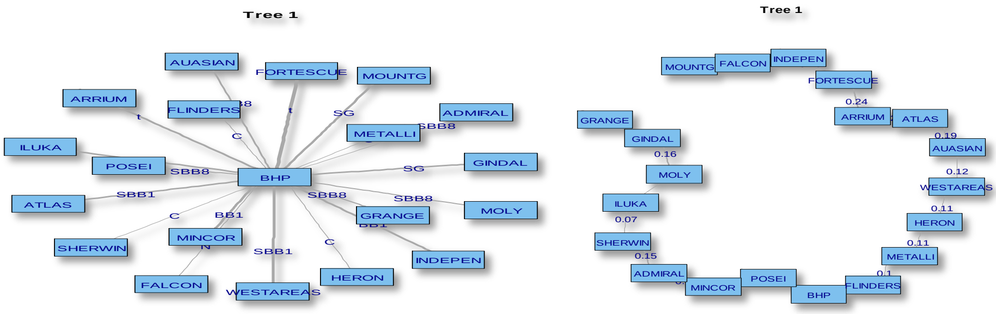

Figure B2: The first tree of the c-vine (on the left) and d-vine (on the right) copula models fitted to the iron ore-nickel mining portfolio based on the full sample period. The letters in between the rootstock and the rest of the stocks from the c-vine refer to the bivariate copulas used to model the dependence. The numbers in between the names of the stocks from the d-vine are the Kendall tau correlation values.

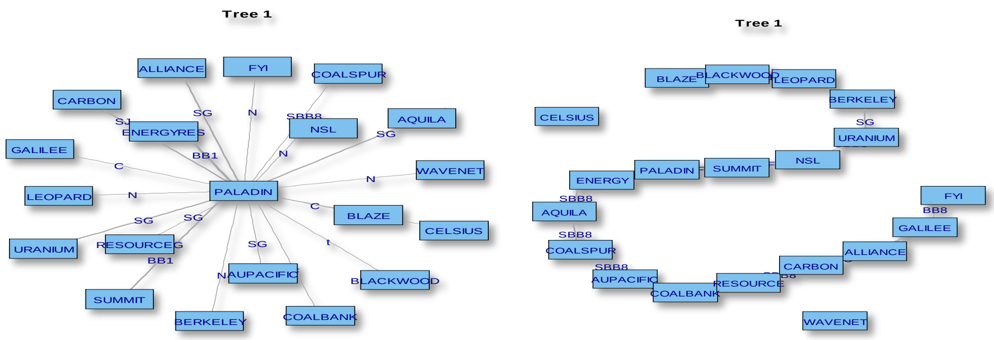

Figure B3: The first tree of the c-vine (on the left) and d-vine (on the right) copula models fitted to the coal-uranium energy portfolio based on the full sample period. The letters in between the rootstock and the rest of the stocks from the c-vine refer to the bivariate copulas used to model the dependence. The numbers in between the names of the stocks from the d-vine are the Kendall tau correlation values.

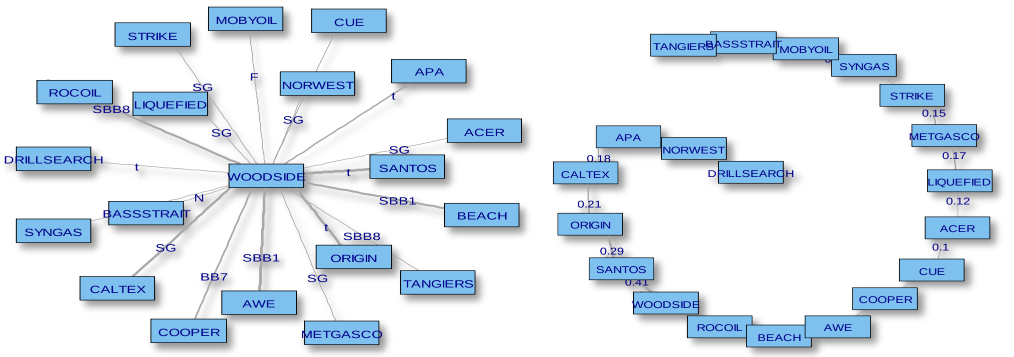

Figure B4: The first tree of the c-vine (on the left) and d-vine (on the right) copula models fitted to the oil-gas energy portfolio based on the full sample period. The letters in between the rootstock and the rest of the stocks from the c-vine refer to the bivariate copulas used to model the dependence. The numbers in between the names of the stocks from the d-vine are the Kendall tau correlation values.

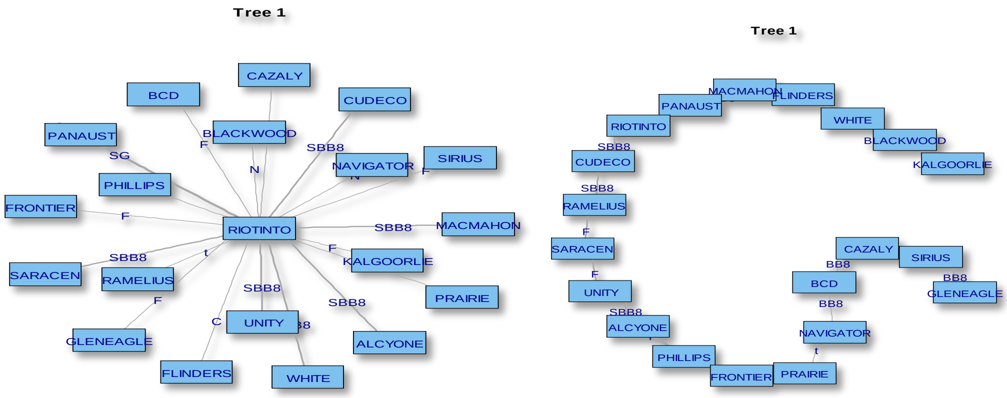

Figure B5: On the left, the first tree of a c-vine application to the mix-metals leptokurtic portfolio using the full sample period scenario. On the right, the first tree of a d-vine applied to the same data and period scenario.

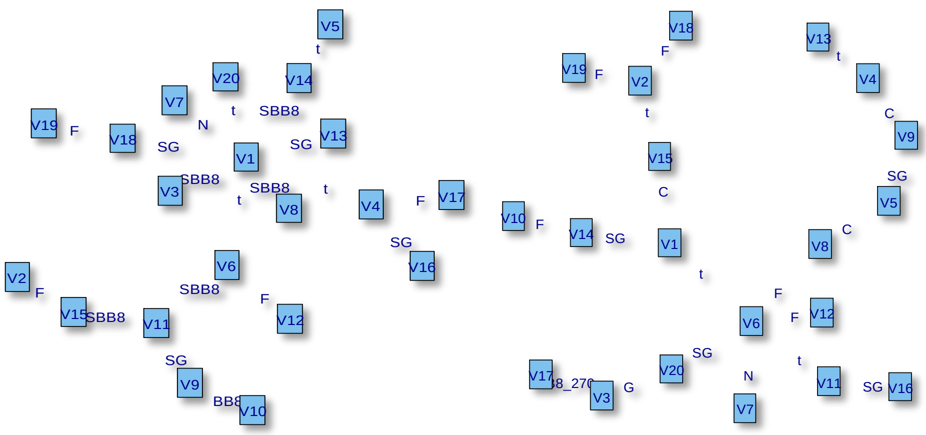

Figure B6: On the left, the first tree of an r-vine application to the gold portfolio under the full sample period. On the right, the first tree of an r-vine application to the gold portfolio under the pre-GFC sample period.

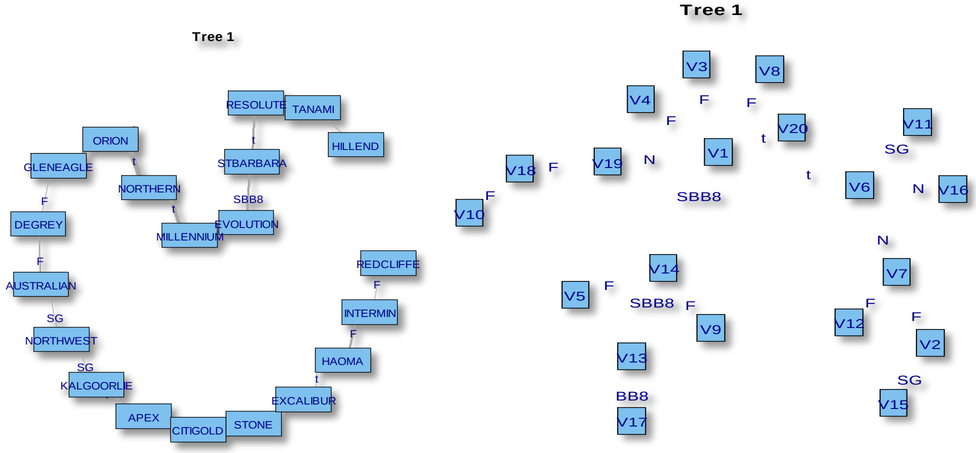

Figure B7: On the left, the first tree of a d-vine fitted to the gold portfolio under the post-GFC sample period. On the right, the first tree of an r-vine fitted to the gold portfolio under the post-GFC period scenario.

APPENDIX C: Portfolios' efficient frontiers

Gold | Iron ore-nickel | Coal-uranium | Oil-gas | Mix-metals | Retail | Manufacturing

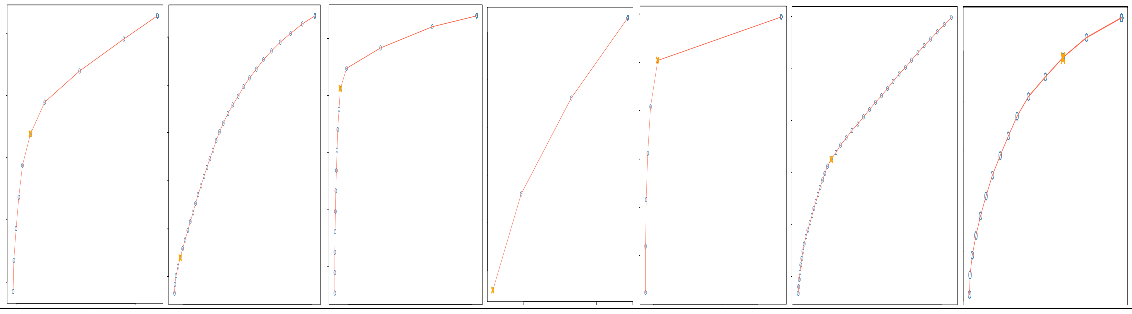

Figure C1: This table depicts the efficient frontiers of the portfolios modelled under the *CDaR* risk measure and based on the full sample period.

Gold | Iron ore-nickel | Coal-uranium | Oil-gas | Mix-metals | Retail | Manufacturing

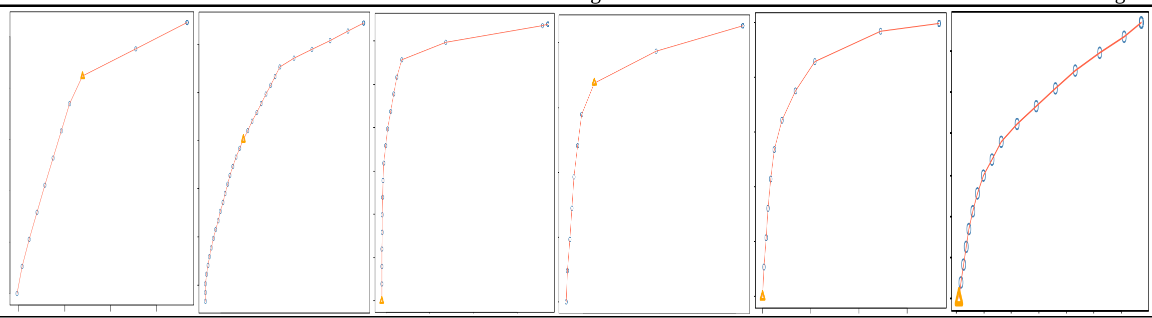

Figure C2: This table depicts the efficient frontiers of the portfolios modelled under the *Minimax* risk measure and based on the full sample period.

Gold | Iron ore-nickel | Coal-uranium | Oil-gas | Mix-metals | Retail | Manufacturing

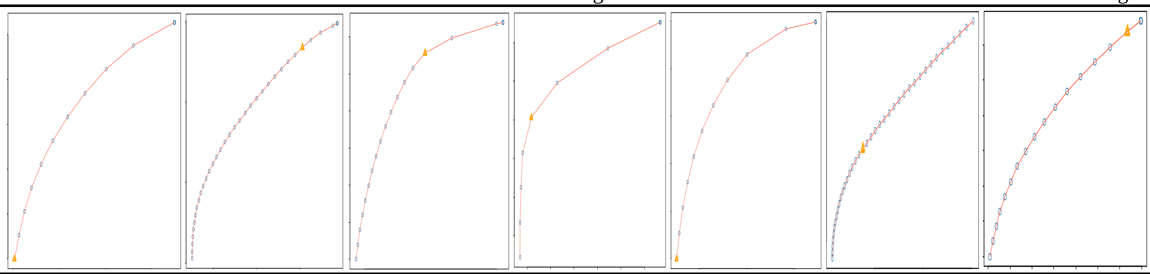

Figure C3: This table depicts the efficient frontiers of the portfolios modelled under the *MAD* risk measure and based on the full sample period.

Gold | Iron ore-nickel | Coal-uranium | Oil-gas | Mix-metals | Retail | Manufacturing

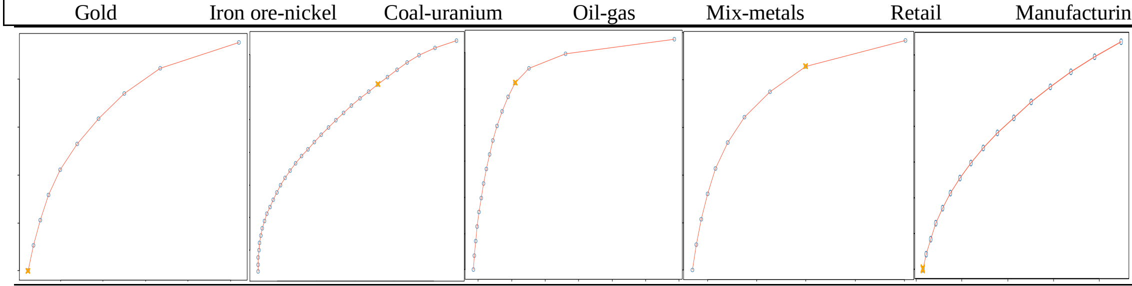

Figure C4: This table depicts the efficient frontiers of the portfolios modelled under the *Variance* risk measure and based on the full sample period.

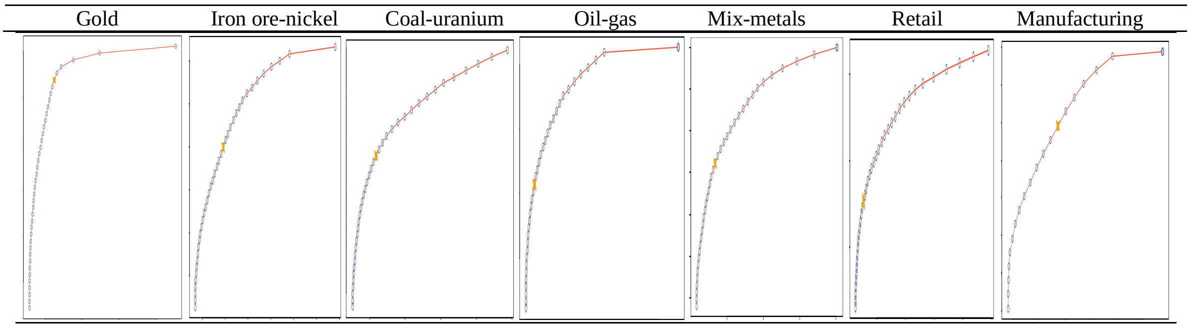

Figure C5: This table depicts the efficient frontiers of the portfolios modelled under the *CVaR* risk measure and based on the pre-GFC sample period.

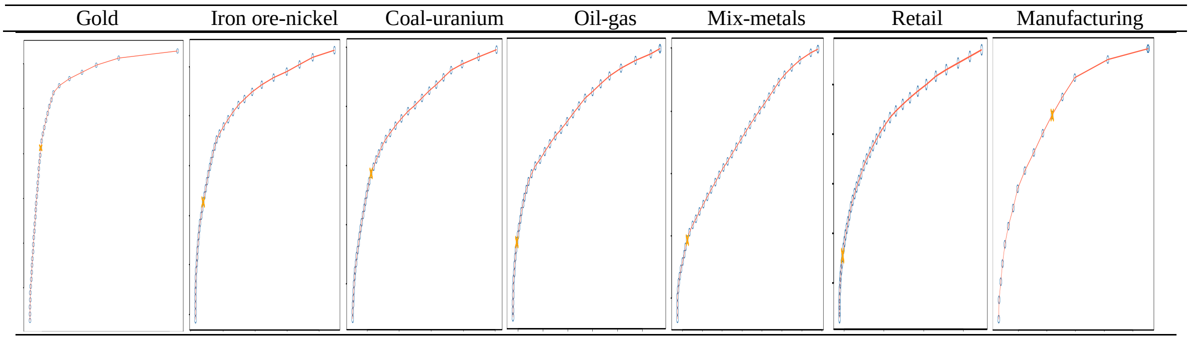

Figure C6: This table depicts the efficient frontiers of the portfolios modelled under the *CDaR* risk measure and based on the pre-GFC sample period.

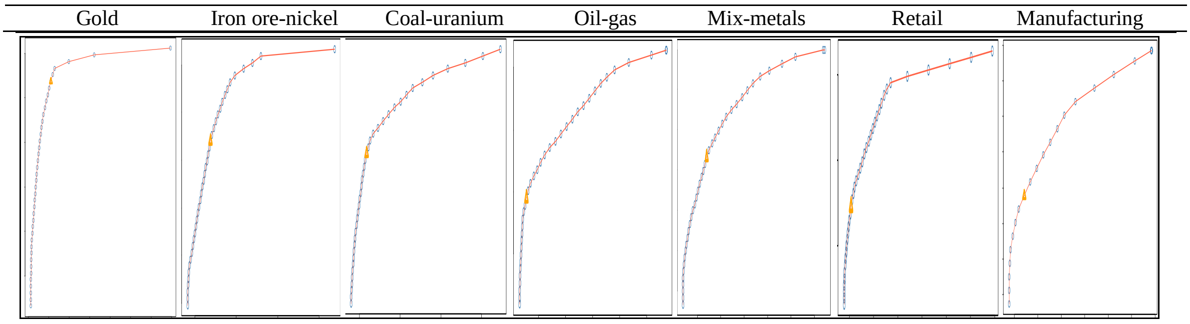

Figure C7: This table depicts the efficient frontiers of the portfolios modelled under the *Minimax* risk measure and based on the pre-GFC sample period.

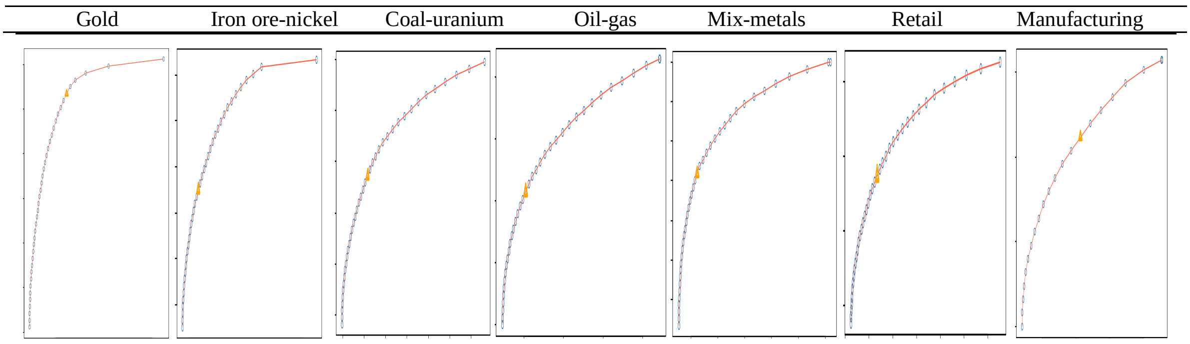

Figure C8: This table depicts the efficient frontiers of the portfolios modelled under the *MAD* risk measure and based on the pre-GFC sample period.

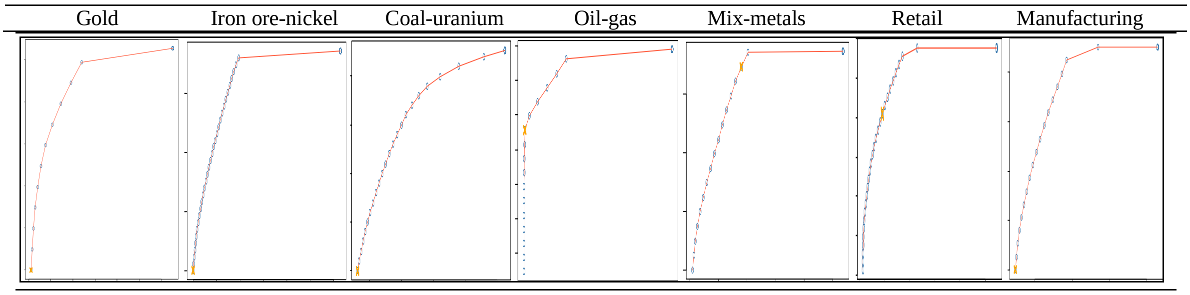

Figure C9: This table depicts the efficient frontiers of the portfolios modelled under the *CVaR* risk measure and based on the GFC sample period.

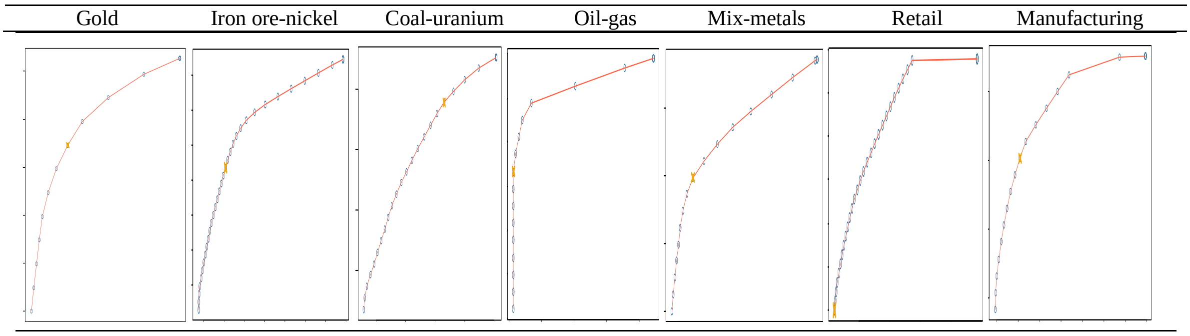

Figure C10: This table depicts the efficient frontiers of the portfolios modelled under the *CDaR* risk measure and GFC sample period.

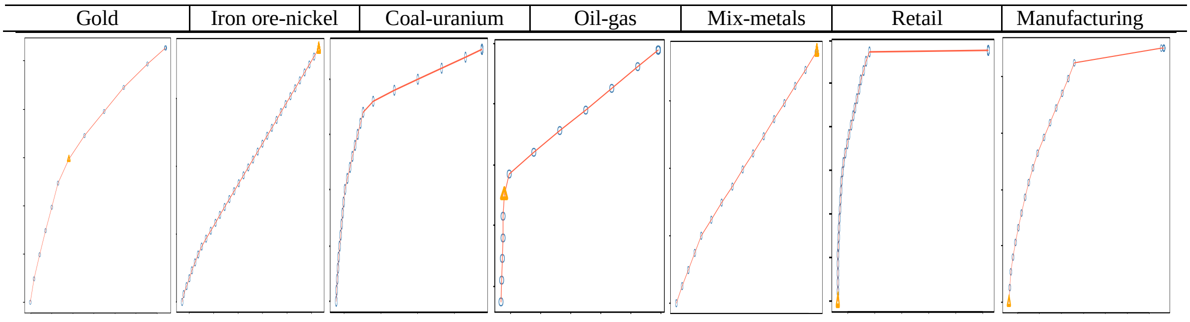

Figure C11: This table depicts the efficient frontiers of the portfolios modelled under the *Minimax* risk measure and based on the GFC sample period.

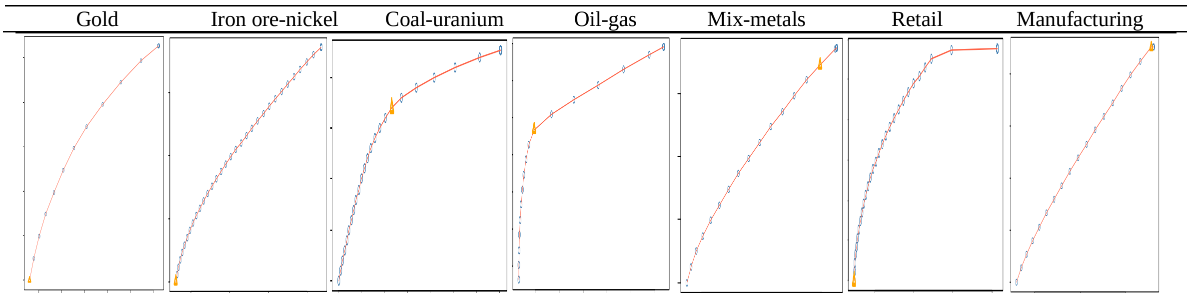

Figure C12: This table depicts the efficient frontiers of the portfolios modelled under the *MAD* risk measure and based on the GFC sample period.

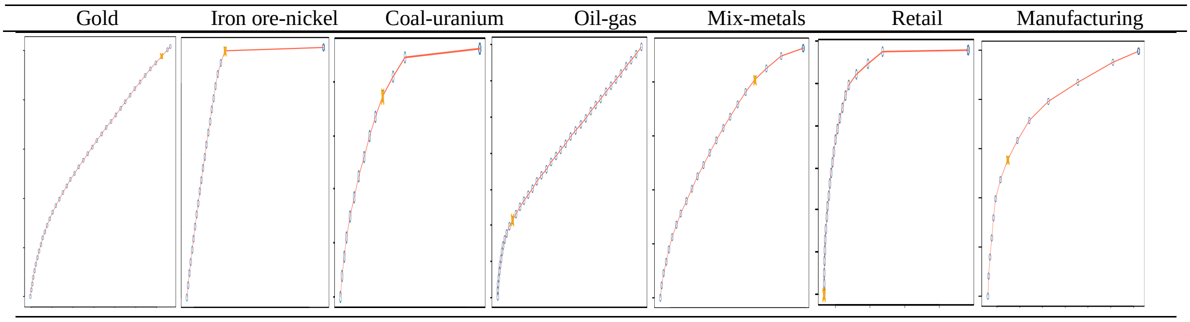

Figure C13: This table depicts the efficient frontiers of the portfolios modelled under the *CVaR* risk measure and based on the post-GFC sample period.

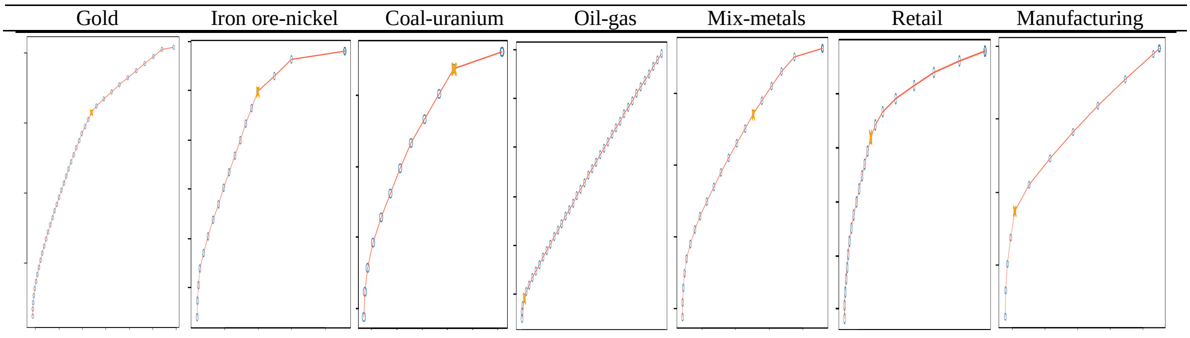

Figure C14: This table depicts the efficient frontiers of the portfolios modelled under the *CDaR* risk measure and post-GFC sample period.

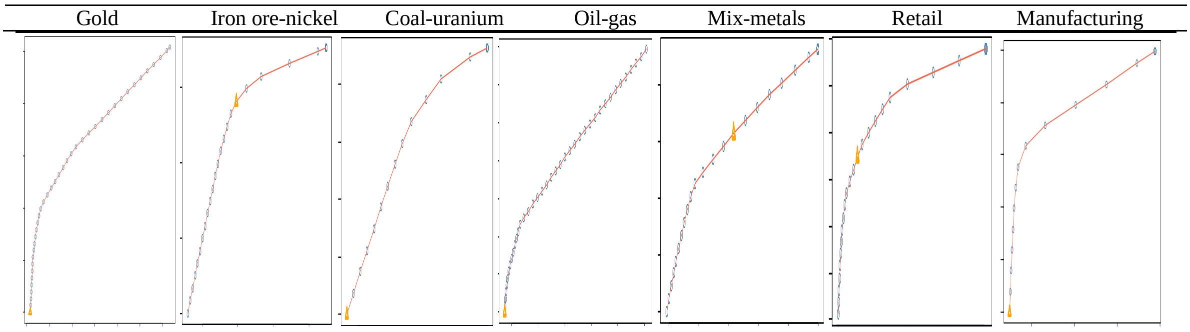

Figure C15: This table depicts the efficient frontiers of the portfolios modelled under the *Minimax* risk measure and based on the post-GFC sample period.

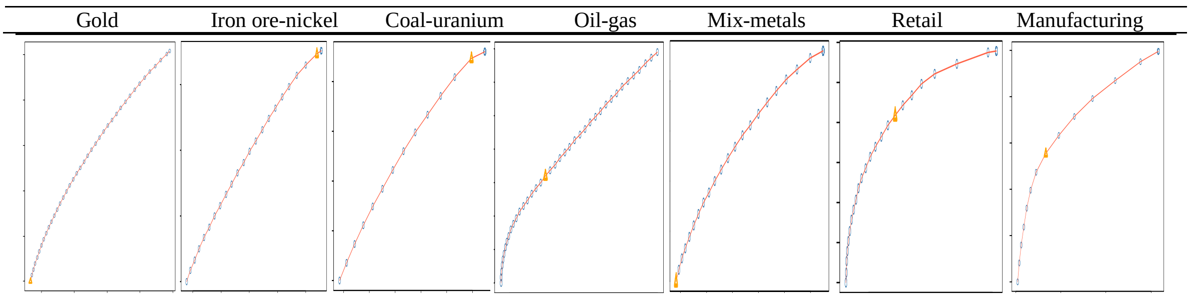

Figure C16: This table depicts the efficient frontiers of the portfolios modelled under the *MAD* risk measure and based on the post-GFC sample period.

APPENDIX D: Optimal weights for the pre-GFC, GFC and post-GFC

Table D1: Optimal weights of the gold portfolio (pre-GFC)

Gold stocks	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
SBMX	0	0.00	0	4.22	2.78	1.40	1.75	2.33
NWRX	3.08	20.31	5.1	6.83	7.56	7.71	5.64	5.82
NSTX	5.56	6.10	13.26	4.73	4.25	6.72	6.95	4.85
SHKX	8.78	0.00	7.42	8.41	6.82	6.79	7.86	8.00
DEGX	0	0.00	4.76	2.36	3.25	2.07	2.59	1.87
RSGX	3.92	0.00	0	5.11	4.37	2.68	3.35	4.47
AXMX	8.63	0.00	2.55	8.49	6.47	5.23	6.54	7.86
ORNX	4.62	0.00	1.83	3.17	4.94	2.91	3.64	4.24
RCFX	8.6	1.12	0	5.56	5.99	4.74	5.04	6.72
EXMX	13.55	16.91	20.42	8.03	9.35	13.88	12.84	10.31
TAMX	3.28	0.00	0	4.55	6.17	2.80	3.50	4.67
GLNX	2.33	0.00	0	4.09	2.88	1.86	2.33	3.10
MOYX	4.21	22.37	1.61	4.33	5.28	7.97	3.86	4.61
EVNX	3.97	0.00	9.66	2.18	2.47	3.66	4.57	2.87
AUZX	0	0.00	0	2.45	2.24	0.94	1.17	1.56
HEGX	0.15	3.82	7.41	3.17	1.74	2.67	3.12	1.69
KMCX	5.43	4.93	0	3.97	4.12	3.87	3.38	4.51
IRCX	19.47	24.45	19.28	10.31	12.33	16.05	15.35	14.04
HAOX	4.4	0.00	6.71	4.83	4.21	4.03	5.04	4.48
CTOX	0.01	0.00	0	3.2	2.78	2.02	1.50	2.00
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	3.181	19.839	4.256	1.117	0.023	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the gold sector portfolio for the pre-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D2: Optimal weights of the gold portfolio (GFC)

Gold stocks	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
SBMX	1.12	0.00	0	1.64	0.00	0.55	0.69	0.92
NWRX	1.81	0.00	0	0.93	0.00	0.55	0.69	0.91
NSTX	0.11	0.00	0	3.53	1.55	1.04	1.30	1.73
SHKX	11.57	0.00	9.83	10.28	14.40	9.22	11.52	12.08
DEGX	2.62	0.00	22.48	2.97	3.28	6.27	7.84	2.96
RSGX	8.45	0.00	0	12.69	15.29	7.29	9.11	12.14
AXMX	0	0.00	0	0.00	0.00	0.00	0.00	0.00
ORNX	0	0.00	0	0.00	0.00	0.00	0.00	0.00
RCFX	0	0.00	0	1.08	0.00	0.22	0.27	0.36
EXMX	0	0.00	0	2.72	0.00	0.54	0.68	0.91
TAMX	2.6	0.00	0	7.28	6.55	3.29	4.11	5.48
GLNX	0	0.00	0	0.00	0.00	0.00	0.00	0.00
MOYX	1.02	0.00	0	0.00	0.00	0.20	0.26	0.34
EVNX	10.95	47.86	13.2	8.54	10.84	18.28	10.88	10.11
AUZX	0	0.00	0	0.00	0.00	0.00	0.00	0.00
HEGX	22.35	0.00	30.05	13.90	16.56	16.57	20.72	17.60
KMCX	8.48	23.80	15.87	9.48	10.78	13.68	11.15	9.58
IRCX	11.16	26.97	0	7.64	8.10	10.77	6.73	8.97
HAOX	12.9	1.38	8.56	15.74	9.95	9.71	11.79	12.86
CTOX	4.86	0.00	0	1.58	2.70	1.83	2.29	3.05
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	5.962	116.086	11.71	1.969	0.08	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the gold sector portfolio for the GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D3: Optimal weights of the gold mining portfolio (post-GFC)

Gold stocks	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
SBMX	8.23	12.75	24.17	0.00	4.06	9.84	9.12	4.10
NWRX	3.70	7.57	9.67	5.95	6.81	6.74	6.53	5.49
NSTX	16.27	15.31	16.48	17.03	15.00	16.02	16.20	16.10
SHKX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DEGX	6.10	0.15	0.00	4.72	4.46	3.09	3.82	5.09
RSGX	10.62	0.00	0.00	12.94	11.32	6.98	8.72	11.63
AXMX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ORNX	0.00	0.00	4.02	1.07	0.00	1.02	1.27	0.36
RCFX	0.06	0.00	0.60	0.12	0.09	0.17	0.22	0.09
EXMX	0.00	0.02	1.48	0.64	0.15	0.46	0.57	0.26
TAMX	7.75	13.19	11.72	10.09	7.39	10.03	9.24	8.41
GLNX	0.30	0.00	0.00	1.24	2.68	0.84	1.06	1.41
MOYX	1.76	9.27	0.00	3.21	3.83	3.61	2.20	2.93
EVNX	27.60	22.90	18.03	21.36	24.60	22.90	22.90	24.52
AUZX	1.97	0.00	1.83	1.11	1.20	1.22	1.53	1.43
HEGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KMCX	3.72	0.16	0.00	0.73	0.00	0.92	1.11	1.48
IRCX	3.75	18.69	0.00	7.36	8.95	7.75	5.02	6.69
HAOX	8.17	0.00	8.79	2.62	1.53	4.22	5.28	4.11
CTOX	0.00	0.00	3.19	9.81	7.92	4.18	5.23	5.91
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	4.12	20.167	5.983	1.41	0.037	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the gold sector portfolio for the post-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D4: Optimal weights of the iron ore-nickel mining portfolio (pre-GFC)

Ore-nickel stocks	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
BHPX	6.59	10.02	0.00	5.91	12.48	7.00	6.25	8.33
GBGX	0.00	0.00	0.00	0.00	0.68	0.14	0.17	0.23
MCRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WSAX	0.00	0.00	0.00	1.82	5.56	1.48	1.85	2.46
AGOX	0.00	0.00	0.00	0.00	0.54	0.11	0.14	0.18
FMSX	5.00	1.53	0.00	5.08	2.37	2.80	3.11	4.15
GRRX	7.07	0.00	0.00	5.17	5.74	3.60	4.50	5.99
ARHX	5.48	7.49	0.00	3.12	1.17	3.45	2.44	3.26
ARI	25.84	16.29	42.23	17.07	29.02	26.09	28.54	23.98
FCNX	2.81	2.72	6.40	5.49	1.26	3.74	3.99	3.19
POSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HRRX	0.00	4.97	0.00	1.94	3.06	1.99	1.25	1.67
MGXX	0.00	0.00	0.00	1.91	4.00	1.18	1.48	1.97
ADYX	0.00	0.00	0.00	1.75	1.15	0.58	0.73	0.97
FMGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ILUX	42.91	54.71	44.11	43.22	23.14	41.62	38.35	36.42
IGOX	0.00	0.00	0.00	0.00	2.87	0.57	0.72	0.96
SHDX	2.88	0.00	7.26	3.78	1.10	3.00	3.76	2.59
MLMX	1.43	2.28	0.00	1.68	2.98	1.67	1.52	2.03
MOLX	0.00	0.00	0.00	2.07	2.88	0.99	1.24	1.65
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	2.971	15.674	5.147	0.965	0.014	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the iron ore-nickel sector portfolio for the pre-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D5: Optimal weights of the iron ore-nickel mining portfolio (GFC)

Ore-nickel stocks	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
BHPX	35.80	0.00	65.80	38.85	45.07	37.10	46.38	39.91
GBGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MCRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WSAX	1.46	0.00	0.00	0.20	0.27	0.39	0.48	0.64
AGOX	0.00	0.00	0.85	0.00	1.20	0.41	0.51	0.40
FMSX	12.36	11.61	11.19	10.98	8.43	10.91	10.74	10.59
GRRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ARHX	0.00	0.00	0.00	2.03	0.00	0.41	0.51	0.68
ARI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FCNX	1.76	0.00	0.00	3.21	0.00	0.99	1.24	1.66
POSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HRRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MGXX	0.00	0.00	0.00	0.00	1.34	0.27	0.34	0.45
ADYX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FMGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ILUX	28.68	45.60	19.29	19.88	19.81	26.65	21.92	22.79
IGOX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SHDX	18.74	42.78	2.88	23.81	23.89	22.42	17.33	22.15
MLMX	1.19	0.00	0.00	1.04	0.00	0.45	0.56	0.74
MOLX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	5.428	39.559	10.387	1.756	0.063	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the iron ore-nickel sector portfolio for the GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D6: Optimal weights of the iron ore-nickel mining portfolio (post-GFC)

Ore-nickel stocks	CVaR (LP)	CDaR (LP)	Minimax (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
BHPX	39.88	30.65	29.05	39.80	41.21	36.12	37.49	40.30
GBGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MCRX	0.00	0.00	3.12	0.00	0.00	0.62	0.78	0.00
WSAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AGOX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FMSX	0.00	5.52	0.00	3.16	1.84	2.10	1.25	1.67
GRRX	10.04	18.25	5.33	3.93	6.66	8.84	6.49	6.88
ARHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ARI	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FCNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
POSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
HRRX	9.75	5.59	0.00	5.13	6.68	5.43	5.39	7.19
MGXX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ADYX	3.26	8.86	21.41	1.30	1.98	7.36	6.99	2.18
FMGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ILUX	37.07	30.06	41.09	38.33	37.62	36.83	38.53	37.67
IGOX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SHDX	0.00	0.00	0.00	2.14	1.10	0.65	0.81	1.08
MLMX	0.00	1.06	0.00	6.21	2.91	2.04	2.28	3.04
MOLX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	4.003	31.116	5.898	1.32	0.031	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the iron ore-nickel sector portfolio for the post-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D7: Optimal weights of the coal-uranium energy portfolio (pre-GFC)

Coal-uranium Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
PDNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CBQX	0.00	0.00	2.18	0.28	1.71	0.83	1.04	0.66
CLAX	6.69	0.00	3.13	3.97	4.22	3.60	4.50	4.96
LRRX	7.60	0.00	16.72	6.24	5.93	7.30	9.12	6.59
AQAX	5.65	0.00	8.06	2.76	3.52	4.00	5.00	3.98
SMMX	18.11	20.97	20.17	16.50	20.02	19.15	18.70	18.21
GLLX	6.02	14.36	0.27	8.30	6.84	7.16	5.36	7.05
CPLX	5.67	0.00	4.70	8.03	5.75	4.83	6.04	6.48
RESX	0.80	2.94	1.48	3.36	1.10	1.94	1.69	1.75
CNXX	3.40	4.13	7.73	3.45	2.48	4.24	4.27	3.11
BWDX	15.69	9.81	0.00	9.88	12.57	9.59	9.54	12.71
UEQX	0.57	0.00	0.00	2.31	1.83	0.94	1.18	1.57
AGSX	0.00	1.25	0.00	1.61	1.56	0.88	0.79	1.06
EMAX	7.86	5.26	0.00	9.84	11.75	6.94	7.36	9.82
FYIX	11.04	8.44	15.05	5.16	5.42	9.02	9.17	7.21
BLZX	3.51	0.00	0.00	4.38	4.16	2.41	3.01	4.02
NSLX	0.99	0.00	2.74	5.92	2.16	2.36	2.95	3.02
AQCX	1.59	14.62	3.97	1.71	1.75	4.73	2.26	1.68
BKYX	0.00	0.00	0.00	1.68	1.19	0.57	0.72	0.96
WALX	4.81	18.23	13.80	4.60	6.06	9.50	7.32	5.16
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	3.059	17.108	4.028	0.994	0.018	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the coal-uranium sector portfolio for the pre-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D8: Optimal weights of the coal-uranium energy portfolio (GFC)

Coal-uranium Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
PDNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CBQX	4.46	0.00	7.59	8.22	7.59	5.57	6.97	6.76
CLAX	13.63	19.20	0.00	16.83	14.96	12.92	11.36	15.14
LRRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AQAX	3.41	0.00	3.64	11.58	10.66	5.86	7.32	8.55
SMMX	7.05	0.00	20.67	4.82	7.99	8.11	10.13	6.62
GLLX	0.00	0.00	0.24	3.02	3.37	1.33	1.66	2.13
CPLX	16.63	6.54	33.39	14.80	15.56	17.38	20.10	15.66
RESX	0.53	0.00	0.00	1.79	0.34	0.53	0.67	0.89
CNXX	12.98	23.39	0.00	7.47	7.57	10.28	7.01	9.34
BWDX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UEQX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AGSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
EMAX	26.23	48.10	7.77	19.48	21.06	24.53	18.64	22.26
FYIX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BLZX	3.38	2.77	0.00	1.65	1.03	1.77	1.52	2.02
NSLX	4.42	0.00	0.00	3.36	2.19	1.99	2.49	3.32
AQCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BKYX	0.00	0.00	8.85	3.30	2.54	2.94	3.67	1.95
WALX	7.27	0.00	17.86	3.67	5.13	6.79	8.48	5.36
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	5.751	55.146	8.561	1.698	0.057	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the coal-uranium sector portfolio for the GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D9: Optimal weights of the coal-uranium energy portfolio (post-GFC)

Coal-uranium Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
PDNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CBQX	0.00	26.41	0.00	1.08	0.00	5.50	0.27	0.36
CLAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LRRX	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00
AQAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SMMX	33.04	0.00	26.90	44.11	38.20	28.45	35.56	38.45
GLLX	8.82	4.81	0.00	1.75	0.00	3.08	2.64	3.52
CPLX	25.85	4.76	0.00	10.67	36.21	15.50	18.18	24.24
RESX	0.03	0.00	40.13	4.21	6.01	10.08	12.60	3.42
CNXX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BWDX	22.36	50.16	32.98	31.05	13.51	30.01	24.98	22.31
UEQX	0.00	0.00	0.00	0.41	0.00	0.08	0.10	0.14
AGSX	0.00	0.00	0.00	0.57	0.00	0.11	0.14	0.19
EMAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FYIX	1.71	0.00	0.00	1.25	0.00	0.59	0.74	0.99
BLZX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NSLX	0.00	13.85	0.00	0.30	0.00	2.83	0.08	0.10
AQCX	4.59	0.00	0.00	0.77	2.05	1.48	1.85	2.47
BKYX	0.00	0.00	0.00	0.81	0.00	0.16	0.20	0.27
WALX	3.61	0.00	0.00	3.00	4.02	2.13	2.66	3.54
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	4.172	38.512	8.305	1.303	0.049	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the coal-uranium sector portfolio for the post-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D10: Optimal weights of the oil-gas energy portfolio (pre-GFC)

Oil-gas Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
WPLX	6.21	0.00	0.00	0.06	2.65	1.78	2.23	2.97
AWEX	1.36	0.00	2.60	0.64	1.84	1.29	1.61	1.28
BPTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MOGX	3.47	0.83	6.02	2.45	2.65	3.08	3.65	2.86
NWEX	0.00	4.16	5.12	0.77	0.80	2.17	1.67	0.52
STOX	3.01	19.65	0.00	6.70	5.58	6.99	3.82	5.10
STXX	0.00	0.00	0.00	2.16	2.10	0.85	1.07	1.42
ACN	8.01	3.41	4.14	6.50	5.05	5.42	5.93	6.52
LNGX	0.04	0.00	0.00	3.64	3.01	1.34	1.67	2.23
CTXX	9.57	8.30	15.78	7.38	9.80	10.17	10.63	8.92
ORGX	22.23	18.38	25.88	19.75	21.93	21.63	22.45	21.30
CUEX	2.19	1.56	8.10	4.72	3.77	4.07	4.70	3.56
BASX	0.49	2.83	0.00	0.83	0.97	1.02	0.57	0.76
ROCX	3.27	0.89	0.77	0.00	2.09	1.40	1.53	1.79
MELX	2.62	0.00	1.20	1.33	2.76	1.58	1.98	2.24
TPTX	2.51	1.75	3.75	0.57	1.69	2.05	2.13	1.59
DLSX	1.48	0.00	1.44	1.84	1.30	1.21	1.52	1.54
APAX	33.55	38.23	22.04	34.02	26.92	30.95	29.13	31.50
SYSX	0.00	0.00	0.00	1.77	1.15	0.58	0.73	0.97
COEX	0.00	0.00	3.17	4.88	3.94	2.40	3.00	2.94
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	2.128	8.287	2.615	0.731	0.009	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the oil-gas sector portfolio for the pre-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D11: Optimal weights of the oil-gas energy portfolio (GFC)

Oil-gas Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
WPLX	5.78	0.00	0.00	6.33	8.16	4.05	5.07	6.76
AWEX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BPTX	0.00	18.03	0.00	0.00	0.00	3.61	0.00	0.00
MOGX	2.54	0.00	9.07	1.80	1.99	3.08	3.85	2.11
NWEX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
STOX	0.00	0.00	0.00	0.09	4.25	0.87	1.09	1.45
STXX	0.00	0.00	0.00	3.00	2.01	1.00	1.25	1.67
ACN	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
LNGX	0.00	0.00	0.00	0.76	0.53	0.26	0.32	0.43
CTXX	0.03	0.40	3.99	0.00	0.00	0.88	1.01	0.01
ORGX	58.41	64.38	68.40	58.85	50.97	60.20	59.16	56.08
CUEX	6.18	0.00	1.90	3.54	5.68	3.46	4.33	5.13
BASX	0.00	0.00	0.00	1.09	1.04	0.43	0.53	0.71
ROCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MELX	1.05	0.00	0.00	0.00	0.00	0.21	0.26	0.35
TPTX	3.01	0.00	0.00	2.43	0.11	1.11	1.39	1.85
DLSX	0.00	0.00	9.20	0.00	0.00	1.84	2.30	0.00
APAX	22.99	17.20	0.00	19.32	24.05	16.71	16.59	22.12
SYSX	0.00	0.00	7.44	0.24	0.41	1.62	2.02	0.22
COEX	0.01	0.00	0.00	2.55	0.79	0.67	0.84	1.12
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	3.556	26.033	6.997	1.182	0.029	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the oil-gas sector portfolio for the GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D12: Optimal weights of the oil-gas energy portfolio (post-GFC)

Oil-gas Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Minimax & CDaR
WPLX	11.59	0.00	0.00	4.93	2.66	3.84	4.80	6.39
AWEX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BPTX	0.07	0.00	0.00	0.00	0.00	0.01	0.02	0.02
MOGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
NWEX	0.29	0.00	1.17	1.19	1.72	0.87	1.09	1.07
STOX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
STXX	1.87	2.94	0.00	1.05	1.24	1.42	1.04	1.39
ACN	2.97	0.00	0.00	2.18	2.71	1.57	1.97	2.62
LNGX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CTXX	10.24	0.00	41.96	3.13	8.33	12.73	15.92	7.23
ORGX	10.96	17.53	16.35	7.87	12.42	13.03	11.90	10.42
CUEX	0.00	0.00	0.00	0.24	0.83	0.21	0.27	0.36
BASX	0.00	0.00	0.00	0.42	0.06	0.10	0.12	0.16
ROCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MELX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TPTX	5.63	4.90	0.48	3.90	4.62	3.91	3.66	4.72
DLSX	0.00	0.00	0.00	1.72	0.43	0.43	0.54	0.72
APAX	56.39	72.93	39.49	66.81	58.11	58.75	55.20	60.44
SYSX	0.00	1.23	0.00	0.00	0.00	0.25	0.00	0.00
COEX	0.00	0.47	0.55	6.55	6.89	2.89	3.50	4.48
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	2.656	10.704	4.166	0.889	0.014	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the oil-gas sector portfolio for the post-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D13: Optimal weights of the mix-metals portfolio (pre-GFC)

Mix-metals Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
RIOX	22.81	49.19	17.26	20.32	25.95	27.11	21.59	23.03
BCDX	17.87	28.62	18.35	15.67	16.07	19.32	16.99	16.54
CAZX	0.00	1.48	0.00	1.16	0.50	0.63	0.42	0.55
CDUX	1.54	0.29	1.90	0.19	0.01	0.79	0.91	0.58
FMSX	2.91	0.00	4.87	0.70	1.27	1.95	2.44	1.63
FNTX	1.79	0.00	3.22	0.52	0.85	1.28	1.60	1.05
GLNX	0.00	0.00	0.00	2.11	1.27	0.68	0.85	1.13
KMCX	1.06	8.94	3.64	1.30	2.47	3.48	2.12	1.61
MAHX	14.31	5.59	10.25	14.06	15.18	11.88	13.45	14.52
NAVX	0.89	0.00	0.00	1.74	1.76	0.88	1.10	1.46
PNAX	0.00	0.00	3.70	3.60	3.38	2.14	2.67	2.33
PHRX	1.45	0.00	3.55	1.78	2.77	1.91	2.39	2.00
PDZX	0.38	1.63	0.00	0.14	0.27	0.48	0.20	0.26
RMSX	4.16	2.75	5.45	1.09	1.45	2.98	3.04	2.23
SARX	2.08	0.00	3.24	1.33	0.09	1.35	1.69	1.17
SIRX	21.14	0.00	13.47	25.15	18.92	15.74	19.67	21.74
AYNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UMLX	0.00	1.51	0.00	0.00	0.00	0.30	0.00	0.00
BWDX	6.45	0.00	9.01	2.92	5.06	4.69	5.86	4.81
WECX	1.17	0.00	2.08	6.21	2.72	2.44	3.05	3.37
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	2.318	15.983	2.771	0.782	0.011	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the mix-metals sector portfolio for the pre-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D14: Optimal weights of the mix-metals portfolio (GFC)

Mix-metals Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
RIOX	6.65	0.00	0.00	15.25	17.75	7.93	9.91	13.22
BCDX	3.74	0.30	0.00	5.23	0.00	1.85	2.24	2.99
CAZX	0.00	0.00	0.00	0.26	0.00	0.05	0.07	0.09
CDUX	3.42	23.10	0.00	15.75	16.87	11.83	9.01	12.01
FMSX	15.35	8.22	12.97	13.55	10.03	12.02	12.98	12.98
FNTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GLNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
KMCX	8.02	17.09	0.00	5.75	9.89	8.15	5.92	7.89
MAHX	0.94	0.00	0.00	0.00	0.00	0.19	0.24	0.31
NAVX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PNAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PHRX	7.58	0.00	0.00	3.28	1.25	2.42	3.03	4.04
PDZX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RMSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SARX	19.37	0.00	61.89	10.81	14.66	21.35	26.68	14.95
SIRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AYNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
UMLX	13.81	9.18	0.00	9.57	11.44	8.80	8.71	11.61
BWDX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
WECX	21.11	42.11	25.15	20.54	18.11	25.40	21.23	19.92
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	5.864	93.664	12.762	1.829	0.072	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the mix-metals sector portfolio for the GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D15: Optimal weights of the mix-metals portfolio (post-GFC)

Mix-metals Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
RIOX	30.14	8.36	14.58	42.45	18.28	22.76	26.36	30.29
BCDX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CAZX	3.11	3.79	5.84	0.00	2.60	3.07	2.89	1.90
CDUX	5.84	7.44	10.05	1.99	0.00	5.06	4.47	2.61
FMSX	0.00	0.00	0.00	2.01	2.73	0.95	1.19	1.58
FNTX	4.47	0.00	0.00	3.52	10.22	3.64	4.55	6.07
GLNX	0.00	0.00	0.00	0.91	0.00	0.18	0.23	0.30
KMCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MAHX	5.52	31.55	0.00	5.32	16.64	11.81	6.87	9.16
NAVX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
PNAX	0.00	0.49	0.00	0.00	0.00	0.10	0.00	0.00
PHRX	0.00	0.00	0.00	0.61	0.00	0.12	0.15	0.20
PDZX	3.34	0.00	0.00	3.05	2.88	1.85	2.32	3.09
RMSX	14.26	0.00	10.99	5.32	11.33	8.38	10.48	10.30
SARX	7.63	20.43	36.93	4.85	22.56	18.48	17.99	11.68
SIRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AYNX	0.00	1.63	0.00	2.76	0.46	0.97	0.81	1.07
UMLX	2.30	4.82	2.31	1.04	0.00	2.09	1.41	1.11
BWDX	23.41	21.49	19.30	26.17	12.29	20.53	20.29	20.62
WECX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	3.914	24.233	6.441	1.261	0.04	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the mix-metals sector portfolio for the post-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D16: Optimal weights of the retail benchmark portfolio (pre-GFC)

Retail Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
CCLX	9.62	5.29	4.15	6.68	6.39	6.43	6.71	7.56
HILX	8.17	14.21	13.19	3.73	4.51	8.76	7.40	5.47
GWAX	3.14	0.00	10.35	2.96	2.97	3.88	4.86	3.02
MTUX	1.49	1.53	1.01	1.21	0.92	1.23	1.16	1.21
MTSX	7.30	25.49	0.00	13.24	9.17	11.04	7.43	9.90
WOWX	17.38	0.00	8.40	11.47	14.90	10.43	13.04	14.58
ARPX	10.71	0.00	10.23	15.66	12.99	9.92	12.40	13.12
CCVX	2.91	10.24	0.00	4.64	4.91	4.54	3.12	4.15
DJSX	0.00	0.00	0.00	1.30	1.58	0.58	0.72	0.96
DLCX	4.06	1.85	3.58	2.31	2.29	2.82	3.06	2.89
HVNX	0.00	0.00	0.00	0.33	0.41	0.15	0.19	0.25
JBHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RCG	3.07	1.38	3.65	1.06	0.97	2.03	2.19	1.70
SFHX	1.00	0.00	0.00	1.09	1.91	0.80	1.00	1.33
SULX	0.00	0.00	9.38	4.47	4.65	3.70	4.63	3.04
WESX	16.31	29.76	20.56	11.85	13.47	18.39	15.55	13.88
FANX	5.68	0.00	6.36	6.89	5.79	4.94	6.18	6.12
GZLX	3.00	7.65	0.00	8.26	6.72	5.13	4.50	5.99
FLTX	3.06	0.00	2.86	0.14	2.17	1.65	2.06	1.79
JETX	3.10	2.60	6.27	2.70	3.28	3.59	3.84	3.03
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	1.309	7.089	1.659	0.448	0.004	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the retail sector portfolio for the pre-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D17: Optimal weights of the retail benchmark portfolio (GFC)

Retail Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
CCLX	22.95	0.00	20.96	20.85	22.16	17.38	21.73	21.99
HILX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GWAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MTUX	18.83	20.65	17.09	12.14	12.92	16.33	15.25	14.63
MTSX	4.30	58.28	0.00	12.07	11.26	17.18	6.91	9.21
WOWX	7.70	0.00	0.00	1.18	0.78	1.93	2.42	3.22
ARPX	21.51	0.00	40.53	19.50	22.13	20.73	25.92	21.05
CCVX	1.28	0.00	0.00	2.16	1.08	0.90	1.13	1.51
DJSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DLCX	0.06	0.00	0.00	0.00	0.00	0.01	0.02	0.02
HVNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JBHX	3.09	0.00	0.00	8.89	9.07	4.21	5.26	7.02
RCG	7.11	21.07	7.32	7.62	6.15	9.85	7.05	6.96
SFHX	0.00	0.00	8.17	1.19	1.06	2.08	2.61	0.75
SULX	0.83	0.00	2.06	4.74	3.62	2.25	2.81	3.06
WESX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FANX	8.57	0.00	3.86	8.94	8.42	5.96	7.45	8.64
GZLX	3.76	0.00	0.00	0.72	1.34	1.16	1.46	1.94
FLTX	0.00	0.00	20.96	0.00	0.00	4.19	5.24	0.00
JETX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	2.58	26.864	3.537	0.879	0.014	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the retail sector portfolio for the GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D18: Optimal weights of the retail benchmark portfolio (post-GFC)

Retail Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
CCLX	33.21	30.20	33.92	19.47	20.55	27.47	26.79	24.41
HILX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
GWAX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MTUX	11.36	0.39	0.00	9.87	10.75	6.47	8.00	10.66
MTSX	11.25	2.25	8.36	2.20	2.73	5.36	6.14	5.39
WOWX	6.92	20.90	0.00	19.41	22.18	13.88	12.13	16.17
ARPX	24.57	39.91	40.81	17.48	19.61	28.48	25.62	20.55
CCVX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DJSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
DLCX	0.00	0.00	0.54	0.00	0.00	0.11	0.14	0.00
HVNX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JBHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
RCG	0.00	0.00	2.58	0.00	0.00	0.52	0.65	0.00
SFHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SULX	3.18	0.00	0.00	9.34	8.41	4.19	5.23	6.98
WESX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FANX	0.00	0.00	0.00	0.06	0.00	0.01	0.02	0.02
GZLX	9.50	6.35	13.79	22.16	15.77	13.51	15.31	15.81
FLTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JETX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	1.561	8.178	2.344	0.518	0.005	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the retail sector portfolio for the post-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D19: Optimal weights of the manufacturing benchmark portfolio (pre-GFC)

Manufacturing Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
SFCX	2.12	0.00	0.00	2.40	2.40	1.38	1.73	2.31
BLDX	2.76	0.00	6.76	2.06	2.98	2.91	3.64	2.60
BKWX	17.24	0.00	18.90	19.38	17.11	14.53	18.16	17.91
CSRX	3.54	11.28	6.42	5.10	5.59	6.39	5.16	4.74
JHXX	8.44	3.80	4.04	4.45	4.54	5.05	5.37	5.81
OLHX	3.86	7.47	5.11	3.12	3.79	4.67	3.97	3.59
CKLX	2.12	11.44	0.00	2.31	3.04	3.78	1.87	2.49
ANNX	4.68	12.53	4.23	14.21	12.39	9.61	8.88	10.43
SDIX	0.71	0.00	0.00	0.89	0.28	0.38	0.47	0.63
SOMX	0.00	0.00	0.00	0.54	0.19	0.15	0.18	0.24
UCMX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FWDX	8.15	0.00	10.50	9.71	8.78	7.43	9.29	8.88
FANX	6.71	0.00	2.84	5.47	6.50	4.30	5.38	6.23
KRSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ASBX	11.42	12.81	13.30	10.75	11.06	11.87	11.63	11.08
MHIX	2.67	11.74	2.94	2.29	2.07	4.34	2.49	2.34
CSLX	12.78	12.69	7.86	10.68	10.51	10.90	10.46	11.32
IDTX	7.32	0.00	12.39	3.92	4.66	5.66	7.07	5.30
CDAX	3.38	5.74	0.90	1.85	1.90	2.75	2.01	2.38
LGDX	2.12	10.48	3.81	0.84	2.22	3.89	2.25	1.73
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	1.429	10.776	1.821	0.519	0.005	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the manufacturing sector portfolio for the pre-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D20: Optimal weights of the manufacturing benchmark portfolio (GFC)

Manufacturing Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
SFCX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
BLDX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
BKWX	0.00	0.00	0	9.14	0.29	1.89	2.36	3.14
CSRX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
JHXX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
OLHX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
CKLX	1.76	0.00	0	4.48	0.03	1.25	1.57	2.09
ANNX	0.00	5.69	4.05	2.16	0	2.38	1.55	0.72
SDIX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
SOMX	18.23	11.59	31.74	6.38	16.72	16.93	18.27	13.78
UCMX	24.01	28.08	8.96	39.06	24.64	24.95	24.17	29.24
FWDX	0.00	0.00	0	1.27	0	0.25	0.32	0.42
FANX	16.96	0.00	0	9.97	15.55	8.50	10.62	14.16
KRSX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
ASBX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
MHIX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
CSLX	31.04	54.64	55.25	21.09	35.32	39.47	35.68	29.15
IDTX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
CDAX	8.01	0.00	0	6.44	7.44	4.38	5.47	7.30
LGDX	0.00	0.00	0	0.00	0	0.00	0.00	0.00
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	4.675	23.576	7.427	1.38	0.045	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the manufacturing sector portfolio for the GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.

Table D21: Optimal weights of the manufacturing benchmark portfolio (post-GFC)

Manufacturing Codes	CVaR (LP)	CDaR (LP)	Mini max (LP)	MAD (LP)	Var (QP)	MW	MW ex. CDaR	MW ex. Mini max & CDaR
SFCX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BLDX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
BKWX	0.00	0.00	1.61	0.00	0.00	0.32	0.40	0.00
CSRX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
JHXX	0.00	2.18	0.00	0.00	0.00	0.44	0.00	0.00
OLHX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CKLX	4.44	0.00	0.00	3.47	3.34	2.25	2.81	3.75
ANNX	27.91	55.79	15.82	24.73	24.89	29.83	23.34	25.84
SDIX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
SOMX	4.96	7.25	0.00	5.48	3.94	4.33	3.60	4.79
UCMX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
FWDX	24.25	7.22	47.50	16.57	20.46	23.20	27.20	20.43
FANX	0.00	0.00	4.56	0.00	0.00	0.91	1.14	0.00
KRSX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
ASBX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
MHIX	3.48	5.33	2.27	3.67	4.50	3.85	3.48	3.88
CSLX	21.39	0.00	17.35	24.57	24.19	17.50	21.88	23.38
IDTX	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CDAX	4.01	5.51	6.08	9.70	8.71	6.80	7.13	7.47
LGDX	9.56	16.73	4.82	11.80	9.97	10.58	9.04	10.44
P-Ret	0.042	0.042	0.042	0.042	0.042	NA	NA	NA
P-Risk	1.992	9.323	3.028	0.68	0.009	NA	NA	NA

Notes: This table reports the minimum risk optimal weights (%) of the manufacturing sector portfolio for the post-GFC period scenario. The abbreviations LP, QP, VaR and MW stand for the linear programing, the mean-variance quadratic programming, variance and mean of weights. The R-ret and P-Risk are the portfolio's return and risk, respectively. MW ex. *CDaR* and MW ex. *Minimax* & *CDaR* stand for mean of weights excluding the *CDaR* and, the *Minimax* and *CDaR* measures, respectively.