

2011

Enhancing the teaching and learning of computational estimation in year 6

Paula Mildenhall
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**ENHANCING THE TEACHING AND
LEARNING OF COMPUTATIONAL
ESTIMATION IN YEAR 6**

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B Ed (Hons)
M Ed**

**Submitted in fulfilment of the requirements of
the Doctor of Philosophy in the
FACULTY OF ARTS AND EDUCATION
at Edith Cowan University**

April 2011

USE OF THESIS

The Use of Thesis statement is not included in this version of the thesis.

ABSTRACT

There have been repeated calls for computational estimation to have a more prominent position in mathematics teaching and learning but there is still little evidence that quality time is being spent on this topic. Estimating numerical quantities is a useful skill for people to be able to use in their everyday lives in order to meet their personal needs. It is also accepted that number sense is an important component of mathematics learning (McIntosh, Reys, Reys, Bana, & Farrell, 1997; Paterson, 2004) and that computational estimation is an important part of number sense (Edwards, 1984; Markovits & Sowder, 1988; Schoen, 1994).

This research hoped to contribute towards establishing computational estimation as a more accepted and worthwhile part of the mathematics curriculum. The study focused on a professional learning intervention, which used an action research approach, and was designed to develop teachers' pedagogical content knowledge of computational estimation. The study utilised a multiple case study model set within a social constructivist and sociocultural paradigm to investigate the teachers' involvement in this intervention. Case studies selected were completed focussing on three of the teachers and their classes.

After the analysis of the individual cases, a cross-case analysis was conducted. From this cross-case analysis it was noted that, whilst each individual teacher's response was different, some general findings emerged. The findings showed that all of the teachers' pedagogical content knowledge for computational estimation increased and they were able to understand the estimation strategies to a certain extent. Most of the teachers thought that these strategies were worthwhile to teach. The teachers selected learning tasks that they thought were pedagogically appropriate and these approaches included; meaningful tasks where estimation was the main computational choice, judging reasonableness of answers in all mathematics computations and the explicit teaching of the estimation strategies. The students were engaged in these different tasks and their computational estimation performance improved and this improvement was statistically significant. As there was also a statistically significant correlation between students using reasoned estimation strategies and students selecting the best estimate, it may be hypothesised that the students' enhanced

awareness of estimation strategies increased the students' estimation performance. At the end of the study, the students also had a much broader and positive perception of computational estimation.

DECLARATION

I certify that this thesis does not, to the best of my knowledge and belief:

- i. incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education;
- ii. contain any material previously published or written by another person except where due reference is made to in the text; or
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I would also like to declare that in this thesis a professional proof reader was utilised to provide low level proof reading of spelling, punctuation and grammar.

I also grant permission for the Library at Edith Cowan University to make duplicate copies of my thesis as required.

ACKNOWLEDGEMENTS

I would like to thank my supervisors Prof Mark Hackling and Dr Paul Swan. I will be eternally grateful for their mentoring during this process. Their combination of wise words and continuous encouragement made it a truly authentic learning experience.

I would also like to thank the Graduate Research School, they also helped to make it a once in a lifetime experience. I would particularly like to mention Dr Danielle Brady, Dr Susan Hill, Heather Williams, Dr Silvia Torezani. I so appreciated them, giving their precious time at different stages of the journey.

I was very grateful to have the collegial support of my fellow PhD students in the post graduate lab, Dr Angela Fitzgerald, Michelle Ellis, and Erasmus Norviewu-Mortty and Pru Smith.

Sadly, my mother passed away during the final stages of writing up of my thesis. I would like to acknowledge her love and her continuous support of me following my dreams, however wild they have been.

Finally, I would like to thank my husband Pete and two fantastic sons who were teenagers at the time and loved me dearly during this sometimes-difficult endeavour. I truly could not have done it without them.

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CHAPTER 1: INTRODUCTION

This chapter sets the study in context, identifies the problem to be addressed, explains the rationale and significance of the study and culminates in the listing of the research questions.

Background

Supporting students' development of numeracy is an urgent priority for Australian education and it is also a priority in the United States of America (National Council of Teachers of Mathematics, 2006) and the United Kingdom (Wintour & Meikle, 2007). An important component of being numerate is being able to undertake computational estimation. There are two important aspects of computational estimation that contribute to numeracy. In order to be numerate it is vital that students possess a sound number sense. This term is a relatively recent one and has been described by McIntosh, Reys, Reys, Bana and Farrell, (1997) as:

A person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for managing numerical situations. (p. 3)

The ability to estimate numerical quantities is an integral component of number sense (McIntosh et al., 1997) and educators are becoming increasingly aware of how estimation can contribute to the development of number sense (Lemaire, Arnaud, & Lecacheur, 2004). Estimating numerical quantities is a useful skill for people to use in their everyday lives in order to meet their personal needs. In a study designed to find out the types of calculations carried out by adults it was revealed that "almost 60% of all calculations required only an estimate" (Northcote & McIntosh, 1999, p. 20). Tomorrow's citizens will also need to be able to check the reasonableness of the calculations they undertake using technology, so estimation is a vital checking device to ensure that these technologies are producing the correct answers (Swan, 2002).

The development of computational estimation is included in a general way in the Australian curriculum for primary mathematics (Australian Curriculum and

Assessment Reporting Authority, 2011). It is also a requirement in the United States of America in the NCTM principles (2006) concerning grades three to five (ages 7 to 10):

- Compute fluently and make reasonable estimates;
- Develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results;
- Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to the students' experience. (p. 2)

This study implemented a professional learning intervention that was designed to increase teachers' understanding of how computational estimation strategies could be developed in Year 6 mathematics and created a structured process, which enabled the participants to reflect on their new teaching approaches.

Problem

Despite the general consensus amongst educators that computational estimation is an important part of the mathematics curriculum, it has been noted that it has not been emphasised at school level (Reys & Bestgen, 1981). It is perplexing that computational estimation appears to be such an unpopular topic. Children and teachers appear to find estimation dissatisfying. Teachers find it ambiguous and students think that mathematics can only have one correct answer (Yoshikawa, 1994). There are a number of reasons why estimation may be unpopular. Swan (2002) hypothesised that teachers and students value exact calculation more highly than estimation. Dehaene (1997) believed that estimation is difficult for students. Research by Alajmi and Reys (2007) found that teachers had negative perceptions of estimation due to their belief that mathematics was concerned with procedures and exact answers. The lack of teaching about estimation in classrooms may also be due to the fact that research has failed to show teachers how to develop this topic area (Sowder, 1992).

Rationale

It is vitally important that students be given opportunities to develop the ability to use computational estimation strategies so that they may become numerate citizens of the future. Sound numeracy is vital in order to function effectively in the work

force and also to be empowered in their personal lives (Australian Curriculum and Assessment Reporting Authority, 2010). In today's society, most exact calculations can be made using technology, but checking devices are still needed and estimating these exact calculations is a way of checking that the technology is working correctly and that they have used the technology correctly. Recent research has asserted that estimation is an integral component of number sense (Baroody & Coslick, 1998; McIntosh et al., 1997). Students who possess number sense have the flexibility to interpret and solve number problems with understanding, which is an important skill to possess in attempting to gain expertise in the area of mathematics.

Significance

There is a significant gap in the literature regarding how to enable students to use computational estimation in the primary mathematics classroom (Reys & Reys, 2004). This research will contribute towards understanding what knowledge teachers need in order to teach computational estimation effectively and how they can be supported through a teacher professional learning intervention that addresses teachers' beliefs and pedagogical content knowledge.

It is extremely important to provide teachers with direction as to how to develop students' understanding of computational estimation situations because of the satisfaction that comes from really understanding something. It gives students confidence and involvement in their learning (Hiebert et al., 1997). Personal experience suggests that the outcomes can be very positive for young students when their number sense is developed and it is hoped that this research will contribute towards more students being empowered in this way.

Purpose and Research Questions

The purpose of this study was to investigate the impact of a professional learning intervention that was designed to enhance the teaching and learning of computational estimation. There were two main foci in this study: The teachers and the students. The following questions were therefore explored:

1. How did the teachers' development of beliefs and pedagogical content knowledge

about computational estimation inform their teaching approaches?

2. How did the teaching approaches impact on students' beliefs about estimation, their mathematical knowledge and their computational estimation abilities?

CHAPTER 2: LITERATURE REVIEW

Underpinning the entire study are the theories of social constructivism and sociocultural theory. Social constructivism and sociocultural theory underpin the study due to the Researcher's beliefs as to how knowledge is created and the belief that there is not one objective reality. An explanation of this theory and its relevance to the study is outlined as the first area of consideration.

The literature regarding computational estimation is the next section of the review as this is the mathematical focus of the study. Unfortunately the literature on computational estimation is somewhat fragmented. As a research topic, it gained some prominence in the 1980s but since then it has lost some impetus, with only a trickle of new research conducted in the 1990s (Trafton, 1994) and not much more from the year 2000. This section is followed by a review of teacher and student beliefs and knowledge and the impact these have on teaching and learning. The last section of the chapter is devoted to reviewing professional learning models and factors that influence their effectiveness in enhancing teachers' beliefs and pedagogical content knowledge. The chapter concludes with the presentation of the study's conceptual framework.

Social Constructivist and Sociocultural Theory

Social constructivism and sociocultural theory fundamentally addresses the question "What is knowledge?" and "How does it develop?" Social constructivist theory is complex (Hacking, 1999) but central to this learning theory is that students construct knowledge for themselves (Ernest 1991). This is in contrast to behaviourist theories which were predominate in the pre Second World War era. The behaviourist tradition relied primarily on the use of direct teaching of carefully sequenced material in order for learning to take place. This view does not take into account the true complexity that learning with understanding involves (Palincsar, 2005). Behaviourists reduced the study of learning to the input of new material and the "gradual solidification" of material (Sfard, 2008, p. 69). Due to this lack of acknowledgement of the complexity of human learning, behaviourist learning theory has gradually been rejected by many.

Constructivist learning theories

Constructivist theories, in contrast to behaviourist learning theories, assume that learning takes place when learners are fully engaged and active (Beck & Kosnick, 2006). Learners are involved in the learning experience in order to construct new knowledge for themselves. This may mean questioning, experiencing and reflecting on this new learning experience in light of their previous understanding (Piaget, 1967). Noddings (1990) argued that these knowledge networks are under continual development. Though engaging in activity there is the transformation of existing structures so that learning is enhanced. Beck and Cosnik (2006) state that “Constructing our own knowledge is necessary in part because that is how the mind works” (2006, p. 9) . Constructivists attempt to understand the learner’s mind and internal processes, and they focus on creating optimal student participation in the learning process (Vighnarajah, Wong, & Abu Bakar, 2008). Constructivists view learning as sense-making and focus on gradual individualisation rather than on the acquisition of rote knowledge that is prevalent in behaviourist theories (Sfard, 2008). The central tenet of constructivism is the notion of knowledge being individually constructed rather than discovering an ‘external’ reality (Ernest, 1989; Glasersfeld, 1989; Piaget, 1967).

While cognitive constructivism focused on the individual, social constructivism and sociocultural theory brought to the fore the importance of the social setting. Brown (2001) asserted that these more individually focussed constructivist theories are somewhat inadequate as they do not take account of role of social interactions in learning and how new ideas are co-constructed through conversation with others.

These social constructivist views are consistent with those of Vygotsky (1933) who was one of the first learning theorists to introduce sociocultural theory. There was little awareness of the Russian educators’ perspectives until recently and in the 1980s this theory was applied to understand cognition in educational contexts. There are some differences between social constructivism and sociocultural theory although there are many connections between the two and therefore both theories are complementary. Staples (2006) asserts that the major differences between social constructivism and sociocultural theory is the stronger emphasis in sociocultural theory on the roles of cultural tools and artefacts and the historical contexts of

present social activity.

Scaffolded learning

Central to social constructivist and sociocultural theory is the notion that the teacher scaffolds the learning for the student until individualisation occurs (Sfard, 2008). Vygotsky (1933) believed that our higher functions are socially mediated and that language and tools are central to learning. When learners interact with an adult or more able person, they often operate at a level slightly higher than their cognitive ability and this was deemed by Vygotsky as the zone of proximal development. Vygotsky (1933) argued that the level of a child's mental development could be raised by the assistance of teachers or more experienced peers in the tasks of problem solving. When engaging with a problem, which the learner would not be able to solve alone, the teacher provides whatever support is necessary so that the learner is able to complete the task. Palincsar (2005) makes the important point that in social constructivist theory learning precedes development. This assertion is in contrast to Piagetian theory where development occurs and then the learning follows this.

Social context

Social constructivism recognises that social interaction is an integral part in this active construction of knowledge (Anghileri, 2006; Carpenter, Fennema, Franke, Levi, & Empson, 1999; Ernest 1991; Piaget, 1967; Treffers, 1991). Sawyer (n.d) asserts that socioculturalists believe that it is impossible to consider the individual without considering the social context within which the individual is situated and in this way it allows the learner to understand the process they are trying to learn (Trent, Artiles, & Englert, 1998). Sfard (2008, p. 78) states that in social cultural learning theory "rather than being an acquirer of goods, the learner is now seen as a beginning practitioner trying to gain access to a well-defined, historically established form of human doing".

Overall, sociocultural theory makes the assumption that all mental actions "are inevitably situated in cultural, historical and social settings" (Mortimer & Scott, 2003, p. 120). This theory places the teacher at the forefront for creating the social context within which students are to learn mathematics. Whereas in transmissive philosophies i.e., behaviourism, the role of the teacher was to transmit information,

sociocultural theory outlines how they teacher creates a specific culture where students engage in social activity before individualising knowledge for themselves (Jaworski, 1996). In social constructivist theory, learners work together in a collective learning environment, talking and collaborating, in order to create something that did not exist before. This collaborative and supportive environment allows the learner to “take risks and develop ownership of their learning” knowing that they can think and discuss ideas without negative consequences (Beck & Kosnick, 2006, p. 12).

Sociocultural theory emphasises the social setting of the classroom and the creation of a community of learners (Rogoff, 1995). It is becoming apparent that that learning which draws on the collective group is more successful than working alone (Palincsar, 2005) .

Within the social setting the learner must consider how previous knowledge will inform their understanding of new knowledge (Beck & Kosnick, 2006). In order to learn using this process it means that the learner needs to spend time continually reflecting on his learning process and discourse. Where there are differences between the discourse in a social setting and that in a learning setting, this because each subject area has its own social language in which the learner has to be enculturated. This process is indispensable in order to learn with understanding (Sfard, 2008).

Tools and symbols

Sociocultural theory emphasise that the learning is mediated through tools, such as concepts and language, and artefacts, and as students master their use of tools their learning increases (Vygotsky, 1933). The use of signs allows humans to not follow natural biological growth but embark upon an entirely different process based on the culture in which the learner is situated (Vygotsky, 1933). This knowledge gaining process is attained through various semiotic mechanisms such as language and psychological tools (John-Steiner & Mahn, 1996) . Palincsar (2005, p. 353) explains that “These semiotic means are both the tools that facilitate the co-construction of knowledge and the means that are internalized to aid future independent problem-solving activity” .

The major semiotic tool in social constructivist learning theory is language

(Palincsar, 2005). Language is connected with thought and therefore lies at the heart of our understanding of the world (Sfard, 2008). Furthermore, teachers play an important role in mediating different types of classroom discourse. The teacher can engage in various types of discourse for instance modeling may be used in the early stages where the learner imitates the practice of the more experienced participant (Sfard, 2008) whereas less intrusive language may be used as the learner increases in their understanding (Trent et al., 1998).

In conclusion, this theory suggests that students' prior knowledge and active engagement in learning on authentic and purposeful tasks, supported with rich discourse within a community of learners, are important contributors to successful learning outcomes (Cobb, Boufi, McClain, & Whitenack, 1997). This study is underpinned by social constructivism and sociocultural theory due to the researchers' belief that the primary mathematics classrooms need to be engaging in sense making learning experiences. A component of this development of number sense involves being able use estimation in computations and as a higher order skill a sociocultural approach offers the best support to the learner of computational estimation.

Computational Estimation in the Primary School

Computational estimation was virtually neglected as a research topic during the 'new math' era of the 1960s and 1970s and it was only in the 1980s did research begin to focus on the topic again (Sowder, 1992). An example of this is the NCTM year book in 1986, which chose estimation as its topic after a gap of 50 years (Schoen, 1986). Any research on computational estimation that has taken place throughout this time has been hindered by different researchers holding contrasting epistemological beliefs and the lack of classroom practice to observe due to its absence as a mathematics topic (Trafton, 1994).

Computational estimation

Computational estimation is differentiated from numerosity estimation and measurement estimation (Sowder, 1992) and yet these three terms appear intertwined. Numerosity refers to the amount of discrete objects in a set (Sowder,

1992) and Sowder identified Fermi problems as a type of numerosity problem. Measurement estimation refers to the process of measuring that does not involve the use of any measurement implementation. This is done mentally although this process often calls on images and manipulation of these images (Bright, 1976).

Verschaffel, Greer and De Corte (2007) identified a connection between the three types of computational estimation, stating that a factor which impedes the development of all types of estimation generally is the difficulties that students have with mental number lines. They would like more “studies on the relationships between number line, computational and numerosity estimation” (Verschaffel et al., 2007, p. 582). This assertion that computational estimation has links with other types of defined estimation has important ramifications for the research study as it was suggested that computational estimation problems could be set in measurement and numerosity contexts i.e., how far is it from the school to the park and how many piano tuners are there in Chicago?. The professional learning intervention used measurement contexts and it has already been noted in the literature, and that measurement contexts can be useful for understanding decimals (D Clarke, personal communication 12/3/2008). As Clements noted “measurement is one of the principal real-world applications of mathematics. It bridges two critical realms of mathematics: geometry or spatial relations and real numbers” (1999, p. 1). It has also been noted that children in the 5-11 age group often need such representations as linear number lines in order to construct their understanding of the estimation of fractions for their use in computational estimation (D Clarke, Roche, & Mitchell, 2008). Sowder (1992) also acknowledged that measurement and computational estimation overlap.

At times, the term approximation is used rather than estimation and attempts have been made to distinguish between the two (Hall, 1984; Sowder, 1992). They are difficult to distinguish (Usiskin, 1986) and there is lack of agreed definitions (Dowker, 2003; Schoen, 1994). For this proposal, it is a priority to define clearly, what computational estimation is rather than distinguishing the difference between approximation and estimation.

Reys and Bestgen defined computational estimation as an “interaction of mental computation, number concepts and technical arithmetic skills such as rounding and

place value. It is a mental process which is performed quickly and which results in answers that are reasonably close to a correctly computed result” (Reys & Bestgen, 1981). A very similar definition to this was provided by Dowker (1992, p. 45) who defined computational estimation as “making reasonable guesses as to the approximate answers to arithmetic problems, without or before actually doing the calculations”. A recent and succinct definition is “finding an approximate answer to arithmetical problems without actually (or before) computing the exact answer” (Lemaire, Lecacheur, & Farioli, 2000, p. 1).

A definition created by the Researcher does not necessitate the estimation to be done quickly as stated by Reys and Bestgen (1981) nor to be undertaken at a certain stage of the arithmetic process as stipulated by Dowker (1992) but still acknowledges the process of approximation as was outlined in the definition by Lemaire, Lecacheur & Farioli (2000) and Dowker (1992). This definition also asserts that estimation is done to simplify computations. The definition is “a process in which some or all of the numbers in an arithmetic problem are approximated to simplify the computation of the estimate” (Mildenhall, 2009, p. 153). The Researcher’s definition was used in this study.

Estimation as a component of number sense

There have been various definitions of number sense, which are shown below in Table 2.1.

Table 2.1: Number sense definitions

Definition of number sense	Defined by
An intuition about numbers that is drawn from all the varied meanings of number. It has five components: (1) having well-understood number meanings, (2) developing multiple relationships among numbers, (3) understanding the relative magnitudes of numbers, (4) developing intuitions about the relative effect of operating on numbers, (5) developing referents for measures of common objects	(National Council of Teachers of Mathematics, 1989 pp. 39-40)

Definition of number sense	Defined by
The sensation that we describe as a sense of number ... rather than being the primary source of the discourse on numbers... is the outcome of the relevant discursive practice. With experience, the stories about numbers ... become so familiar and self-evident that we are able to endorse or reject new statements about them in a direct, non-reflective way. Such immediacy of decision, when no rationalisation is necessary to make us certain of our choices is the general defining characteristic of situations in which we say that we have sense of something.	(Sfard, 2008, p. 51)
<ul style="list-style-type: none"> a) The concept of number, especially with respect to relative sizes of numbers and the ways that they can be decomposed and combined b) The relationships among and between numbers, such as $\frac{1}{2} = 0.5 = 50\%$ c) The properties of the numbers under the various operations and the effect on numbers of each operation d) The role of numbers as measures of various quantities in real world settings and especially the homomorphism between the numbers under operations in the world of mathematics and the quantities under appropriate transformations in the real world setting 	(Schoen, 1989 p. 67)
A person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient ways of managing numerical situations	(McIntosh et al., 1997, p. 3)

All of these definitions identify the necessity for 'understanding' numbers. The term 'understanding' though has different meanings for different users (Skemp, 1976). Therefore, the other aspects of the definitions are important to elaborate what the authors specifically mean by understanding. Most of the authors identify that understanding that different representations of the numbers are equal to other representations i.e., $\frac{1}{2} = 0.5$ is an important component of number sense. As well as understanding numbers, it is important to be able to understand the connections between addition, subtraction, multiplication and division. If someone possesses

number sense, they are able to use this understanding in order to calculate effectively. McIntosh et al. (1997) and Schoen (1989) and the National Council of Teachers of Mathematics definition all stressed the importance of being able to use this knowledge in the real world. Sfard (2008) asserted that if this deep understanding has occurred the number knowledge may appear intuitive whereas Sfard asserts that the number sense holder has in fact individualised and connected many different aspects of number and how to calculate with these. Within the mathematics community there is some debate as to whether it is possible to easily define number sense (Dehaene, 1997; Silver, 1994; Verschaffel et al., 2007). Despite this uncertainty concerning the definition of number sense, this research study has used this term as it provides a clear framework for the development of computational estimation.

This study is asserting that estimation is a component of number sense as defined by McIntosh et al. (1997) and it is shown in Table 2.1. Part of the rationale for the focus on computational estimation in this study is the fact that it has been identified that developing estimation will further develop number sense as estimation is an integral component of number sense (Dolma, 2002; Trafton, 1994; Verschaffel et al., 2007). The research literature asserts that proficiency in estimation is part of possessing proficiency in number sense (Trafton, 1994; Verschaffel et al., 2007). Baroody and Coslick (1998 p. 7-4) maintained that “number sense permits flexibly switching among different representations of numbers and flexibly switching among estimation or mental – computation strategies”. McIntosh et al. stated that “those who view mathematics in this way [possess number sense] continually utilise a variety of internal “checks and balances” to judge the reasonableness of numerical outcomes” (1997, p. 3). The process of estimation is one where the learner analyses and reflects on numbers, strategies and solutions, and therefore it is an appropriate strategy for judging the reasonableness of computations.

There are obviously links between mental computation and computational estimation (Dowker, 2003; Hazekamp, 1986; McIntosh, 2004; Yoshikawa, 1994). Reys stated “There are two distinguishing characteristics of mental computation. It produces an exact answer, and the procedure is performed mentally, without using external devices such as pencil and paper. Mental computation is an important component of estimation in that it provides the corner-stone necessary for the diverse numeric

processes used in computational estimation” (1984, p. 548). From that statement it is clear that when you estimate you undertake mental computation whereas the reverse is not true. Leutzinger, Rathmel and Urbatsch (1986), explained that:

Mental computation is not a separate estimation process; rather, it is sometimes needed when children estimate by comparing and partitioning with known quantities. Children need instruction and practice [sic] with mental computation before they can use it efficiently for estimation. (p. 89)

For students, whilst there are many links between the two, you can be good at one and poor at the other (Dowker, 2003; Reys, 1984). Estimation may be undertaken in mental computation situations when an exact computation would be too complex or unnecessary (Usiskin, 1986). This has particular significance when students are just beginning to learn about a new concept and it means they can concentrate on the operation and gradually build up their knowledge networks about the place value of the numbers (Dowker, 2003; Reys, 1984).

Computational estimation strategies

The computational estimation strategies may be formulated under three broad categories - reformulation, translation and compensation (Reys, Rybolt, Bestgen, & Wyatt, 1982). Dowker asserted that these headings are insufficient to define the “essential components of estimation” (2003, p. 256) and are so broad they may be difficult for teachers and students to work with. Various researchers over the last 30 years (Dowker, 2003; Levine, 1982; McIntosh et al., 1997; Reys, 1984) have described different estimation strategies using different terms. Attempts to indicate when different estimation strategies should be introduced have been made by past researchers. It has been proposed that in the intermediate grades of 3 to 5, front end, rounding to compatible numbers and benchmarks strategies are appropriate (Reys & Reys, 2004).

After evaluating these different terms a synthesis of the terms was undertaken and then these strategies were evaluated for their appropriateness in the primary school curriculum.

When these estimation strategies were considered, it was decided that any strategies that required a second step, that is an adjustment in order to make the estimate more

precise, would not be introduced to the Year 6 students. This was because it may be too complex for the primary students to master as had been found by Lemaire et al. (2000).

The first strategy named for inclusion was the most well-known strategy rounding. The next strategy was nice or compatible numbers and this strategy involves the process of evaluating the computation as a whole. Front end loading is probably one of the most simple strategies so this was included in this taxonomy. Recently the value of the benchmarking strategy has been highlighted when developing a conceptual understanding of fractions (D Clarke & Roche, 2009) and therefore this was a very important strategy to include in a taxonomy of computational estimation strategies suitable for the primary school. When discussing with primary students the reasonableness of answers, De Nardi (2004) suggested that it may be suitable to ascertain whether the solution would be within a certain range. In upper primary, this would be particularly appropriate when working with decimals.

It was also decided that an intermediary estimation strategy would be included. Lovitt and Clarke (1992) had suggested that a suitable learning activity for novice estimators might be to intuitively state which number in a set was the average. Dowker (2003) also noted in her research that young students often were able to make an appropriate estimate but not explain their reasoning. Therefore, it was decided that intuition was worth naming as a strategy.

Many activities appropriate to this age group i.e., how many jellybeans in the jar, require students to take a sample of the quantity to solve this. There was no strategy that fitted this process exactly. When considering an efficient way that the students would solve this strategy, the term 'sample' was created. Figure 2.1 outlines the terms that were used on the professional learning intervention.

Strategy	Identified by	Explanation of how numbers are approximated	Possible Operation of calculation	Example
Rounding	Dowker (1992) Trafton (1986) Reys (1984) Reys et al.(1991)	Initially it is necessary to decide to what place you will be rounding a number, i.e., the nearest tenth, whole number, tens, hundreds, thousands, or ten-thousands. You may then round up, round down, round up and down the numbers set depending upon the context or the parameters set in the learning context. You may round one or all of the numbers in the problem. The new approximated numbers are then used to solve the arithmetic problem.	Addition Subtraction Multiplication Division	256 + 9 Rounded to nearest 10 260 + 10
Nice (Compatible) numbers	Dehaene (1997) Reys (1984; R. Reys, 1986) Allinger and Payne (1986) Dowker (1992) Leutzinger et al. (1986)	Numbers are converted to ‘more compatible numbers’ but not using rounding strategies. These may be : A. Compatible numbers B. Useful fractions for computing with C. Easy percentages for computing with. The newly approximated numbers are then used to solve the arithmetic problem.	Addition Subtraction Multiplication Division	A. 27 + 49 + 38 + 81, take the 27 and the 81 and say that is about 100 B. $76 \times 89 \approx 75 \times 88 = 0 \cdot 75 \times 88 \times 100 = \frac{3}{4} \times 88 \times 100$ C. 43% of 34.50 $\approx 50\%$ of 34.50
Front end loading	Dowker (1992) Trafton (1986) Allinger and Payne (1986)	Only the leading digits are focused on and computed with.	Addition Subtraction Multiplication Division	Add 3000 and 4000 in the calculation 3421 + 4112

Benchmarking	McIntosh (2006) Carter (1986) Reys et al. (1991) Booth & Sieglar (2006) Carlow (1986) (D Clarke & Roche, 2009)	A benchmark is the identification of a familiar quantity or amount which is used when estimating an unfamiliar quantity or amount. A version of this strategy encourages children to use concrete material to create an estimate with these anchors or groups of objects and build up visual perceptions of 10, 20, and 100. The newly approximated numbers are then used to solve the arithmetic problem.	Addition Subtraction Multiplication Division	$7/8 + 11/12$ Using the familiar benchmark of $1 \approx 1 + 1 = 2$
Range (Estimating by bracketing)	De Nardi (2004)	A framework approach is taken to calculate a range. Using another strategy approximate numbers are formed taking a lower number result and upper number result as a range of where the answer should be between.	Addition Subtraction Multiplication Division	$6 \times 3 \cdot 7$ is between $6 \times 3 = 18$ $6 \times 4 = 24$ So the answer should be in the range of 18-24.
Intuition	Mildenhall (2009)	Students undertake a quantitative judgement based on their past experience but they cannot say how they arrive at their estimate.	Addition Multiplication Division Subtraction	$594 + 602 \approx 2 \times 600$

Figure 2.1: Taxonomy of computational estimation strategies suitable for the primary school

Teaching Computational Estimation

Students' estimation capabilities are emerging in Year 6 (Dowker, 2003; Reys & Reys, 2004; Schoen, 1994; Siegler & Booth, 2005; Vance, 1986) and this study researched different teaching approaches, which developed these emerging concepts and skills. This was a difficult task as "robust useful theories of teaching do not yet exist" (Hiebert & Grouws, 2007, p. 373) and the true significance of estimation has not been appreciated by the educational community (Trafton, 1994) which means there is a lack of domain specific recent research available (Reys & Reys, 2004).

Most research that has taken place has recorded what students can do concerning computational estimation rather than how to teach computational estimation. Lemaire et al. (2000) observed that computational estimation was not taught in French schools. When testing how fifth grade students could estimate without instruction, they were able to use the estimation strategies of rounding and truncation. Few fifth grade students used the compensation strategy. McIntosh et al. (1997) noted that computational estimation was not taught in Taiwanese schools and instead there was a focus on exact pencil and paper computations. In the Taiwanese study, it was found that students performed extremely highly in paper and pencil computations. This emphasis on routine algorithms, in the Taiwanese schools, did not produce students with an ability to estimate whole numbers and decimals. When asked the question $\frac{12}{13} + \frac{7}{8}$ in a written test that required exact answers, 61% of students were able to obtain the correct answer. When asked the same question in a timed test that only allowed for an estimation, only 25 % of students were able to obtain the correct answer. McIntosh et al. deduced from this study that routine algorithms do not appear to develop estimation ability (1997). McIntosh et al. asserted, as a result of this study, that "certainly more curriculum development, case studies and action research are needed to develop effective practices" (McIntosh et al., 1997, p. 53).

It is important to note that estimation skills, where they have been taught in the primary school, have normally been taught in an isolated manner (Dowker, 2003). Bobis (1991) conducted a quazi-experimental study and found that fifth grade

students' estimation performance improved when they were instructed how to use different estimation strategies using worksheets with teacher direction as the mode of delivery.

Reys and Reys (2004) asserted that in the 1980s and 1990s in the US, estimation was normally taught as an isolated skill and this did not encourage its implementation as an integral component of the primary mathematics curriculum. These researchers observed that where resources were moving towards a problem-centred, contextual approach the guidance for teachers as to how to use the estimation strategies within this approach was missing. They suggested that this area needs more attention (2004, p. 104).

In this domain it may be more productive to develop the concepts and skills in meaningful contexts (Reys & Reys, 2004). It has also been asserted that "One reason is that it is easier for students to appreciate the value of estimation when it is used in the right situations; they realise that using estimation can save effort or energy" (Yoshikawa, 1994, p. 61). In support of teaching computational estimation in meaningful contexts is the research by Reys (1986). He found in testing a range of ages, grade 5 students to adults, they were much more competent at answering questions that were set in context than those presented symbolically.

There have been few research studies which have investigated which strategies should be taught to primary school students. Reys and Reys suggested that in grades 3-5, front end, rounding to compatible numbers and benchmarks strategies are appropriate (Reys & Reys, 2004). D Clarke recommended that the benchmarking strategy be introduced to develop the conceptual understanding of fractions (2009). Neill (2006) conducted a study where he worked with Year 8 students and after they received instruction for eight weeks, they were able to implement the estimation strategies when working on computational estimation problems and develop a tool kit of estimation strategies. Research conducted by Star and Rittle-Johnson (2009), introduced fifth and sixth grade students to examples of solutions to estimation problems. They found that those students who received instruction comparing alternative solution strategies were more effective problem solvers than those students who received instruction that focused on one estimation strategy at a time.

Principles of Effective Mathematics Teaching and Learning

Due to the scarcity of research on how to teach computational estimation in the primary school, it was necessary to consult the research literature concerning mathematics teaching generally as well as the literature on computational estimation. This synthesis was useful to guide the creation of the professional learning program that was implemented to support teachers, develop learning tasks and strategies for teaching about estimation.

Problematic mathematical tasks are provided as learning experiences

Franke, Kazemi and Battey (2007) asserted that “starting with a good task is necessary for providing opportunities to engage students in high level thinking” (p. 234). Ernest explained that mathematics instruction developed from a social constructivist perspective should “consist primarily of problem posing” (1991, p. 265). He also advocated that mathematics taught with a social constructivist philosophy should “be centrally concerned with human mathematical problem posing and solving ... inquiry and investigation should occupy a central place in the school mathematics curriculum” (Ernest 1991, p. 283).

Problem posing and problem solving allow students to develop metacognitive skills and independence (Lowrie, 2002). Instead of imposing “an understanding of mathematics on children”, mathematics teaching needs to be a thoughtfully implemented investigative approach (Baroody, 2003, p. 28). Baroody asserted that using this investigative approach the “teacher usually poses a worthwhile task (one that is challenging and complex) as a way of exploring, learning and practising basic concepts and skills; teachers may take advantage of teachable moments” (p. 20). An example of problem solving as the stimulus for the construction of new mathematical knowledge was seen in the professional development program Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Franke, Levi, & Empson, 1999). This program stressed that if presented with specific problems chosen by the teacher, children are able to “construct viable solutions to a variety of problems” (p. 4). Students constructed these solutions using a variety of manipulatives and models and out of this problem solving process emerged the understanding (Hiebert et al., 1997). Sfard describes a type of activity, which leads the student to mathematical

understanding as “an exploration” (2008, p. 226). Sfard cautioned however that understanding mathematics is a difficult process and students may not be immediately capable of engaging in this explorative process. Teachers need expertise in creating the learning conditions in which students may use explorations to build their understanding of mathematics.

In many school situations there is a hidden curriculum implied in such phrases as “work quietly, neatly or obtain correct answers”. If students are provided with a problem, unfortunately the focus is not on solving the problem but on following certain classroom procedures (Brown, Collins, & Duguid, 1989; McIntosh et al., 1997).

Learning experiences are set in context

Abstract learning devoid of a context provides children with learning which may not be transferred for use in the real world (Silver, 1994). If children are to be numerate they need to have robust knowledge of number (McIntosh et al., 1997) and embedded learning may facilitate this robust learning (Brown et al., 1989; D Clarke, 2003). Carraher, Carraher, and Schliemann’s research (1985) found that the children working in Brazil’s markets were able to solve problems they were unable to do in school contexts. Similarly Lave’s (1977) ground breaking research described the learning undertaken by apprentices in Liberia who gradually became experts through engaging primarily in authentic meaningful activities.

This awareness of the process of learning in meaningful contexts may be transferred into the school setting (Silver, 1994; Streefland, 1991). Freudenthal founded a movement in the Netherlands where more attention was placed on embedding problems in a context so that students could create images that would help them understand mathematics (Gravemeijer & Terwel, 2000). Sfard (2008), asserted that mathematical understanding is developed through creating a number of representations of abstract mathematics and learning experiences set in contexts can act as a realisation of abstract mathematics and contribute towards students’ understanding of the abstract mathematics (p. 157).

Bobis et al. (2005) stated that children come to school with a relatively sophisticated understanding of number sense. One reason for this may be that they start from their

informal understanding of number, which has been developed in relevant contexts such as the home (Anghileri, 2006; Carpenter et al., 1999; D Clarke & Cheeseman, 2000; Mulligan, 2004).

Whether the tasks need to be completely authentic and within the immediate experience of the students has recently been questioned (Nicol & Crespo, 2005). Instead of assessing whether the contextual task is relevant to the students in order to assess its suitability, it needs instead to be judged on the criteria of “how it engages students' desires to think about and do the mathematics featured in the task” (Nicol & Crespo, 2005).

For the purpose of this study, it had been decided to use a definition of a meaningful task as a “real or imagined situation in which a mathematical task is embedded. This embedding of the mathematics in some way is aimed at making tasks seem more realistic or understandable or at providing substantive information to support the posing of the task” (Sullivan, Zevenbergen, & Mousley, 2003, p. 109).

A cautionary note, concerning learning set in meaningful contexts, is made by Askew, Bibby and Brown (2004, p. 37) who stated, “the main message that emerged is that while practical work and real contexts can be useful, they need to be chosen carefully, and be accompanied by careful dialogue with students to establish the extent of their understanding”. Just placing computations in context may not be enough. Sfard (2008) also asserted that practical activities are only beneficial to students as a stepping stone to developing understanding of abstract mathematics. She explained that where the focus for the learning is on achieving results in the contextual activity and discussions are not producing a mathematical narrative this may not necessarily lead to mathematical understanding.

Metacognitive processes are developed

Teachers need to develop students' metacognitive processes in primary mathematics classrooms. Metacognition is a social constructivist learning strategy that has been defined in many different ways (Lesh & Zawojewski, 2007) but it is useful for this proposal to be aware of one definition provided by Flavell (1976) who invented the word metacognition and stated that:

Metacognition refers to one's knowledge concerning one's own cognitive processes or anything related to them, e.g., the learning-relevant properties of information or data. For example, I am engaging in metacognition if I notice that I am having more trouble learning A than B; if it strikes me that I should double check C before accepting it as fact. (p. 232)

There are three processes which constitute metacognitive activities. These are planning, monitoring and self regulation (McKeachie, Pintrich, Lin, & Smith, 1986). Vygotsky (1933) believed that talk in young children is used as a metacognitive tool. As students get older they still engage in self-talk but this becomes internalised. Young children, particularly, do not have the skills to engage in complex metacognitive decisions (Anghileri, 2006). Maturity is needed for students to engage independently in executing inquiry type activities. Another important aspect of metacognition is the ability to reflect on your learning throughout the learning process (Hiebert et al., 1997; Treffers, 1991). Ertmer and Newby (1996) explained that “as a powerful link between thought and action, reflection can supply information about outcomes and the effectiveness of selected strategies, thus making it possible for a learner to gain strategy knowledge from specific learning activities” (p. 14). In order to develop number sense reflective thinking is necessary and McIntosh et al. (1997) stressed that teachers need to provide a “climate where reflection and evaluation are important elements in the work” (p. 44). Anghileri (2006) asserted that as well as individual reflection, collective reflection greatly enhances learning. This collective reflection has been used to extend students' mental computation strategies (Trafton & Theisson, 2004) and it may be effective for increasing students' awareness of computational estimation strategies.

In the 1990s, research found that students' problem solving could be more successful if their metacognitive processes were developed (Schoenfeld, 1992). Schoenfeld's work (1985) was with high school students but this metacognitive awareness can be developed from an early age in mathematical learning (Anghileri, 2006) if teachers scaffold and support students' emerging metacognitive abilities (Ertmer & Newby, 1996). They may do this by using strategies such as “brainstorming, joint collaboration (between teacher and student and among students), feedback, guided questioning, and cognitive structuring (the organization [sic] and generalization [sic]

of information)” (Zakin, 2007, p. 4). Bell (1993) identified the connection between learning and doing in his research investigating fraction misconceptions. Through intense discussion, the students in his research were able to develop their conceptual understanding of number and fractions. Sfard (2008) asserted that learners, in order to fully understand mathematics, must be guided by teachers and that examples are very useful in this respect. Students must not be placed in authentic tasks and deserted. Instead, teachers need to play an active role throughout the learning process (Askew, 2004; Hiebert et al., 1997; Rowe, 2006).

Teachers build on what students already know

Active construction of knowledge, which has deep meaning for the learner, is at the heart of all types of constructivist learning theory. This active construction of knowledge is a process undertaken by the learner not the teacher and it involves the assimilation of new information into existing schema or the accommodation of existing schema to fit with the new information (Piaget, 1967). These processes are analogous to Vygotsky’s process of internalisation (Mortimer & Scott, 2003). Ausubel (1968) perceived that the choice of activities which students experience is crucial if they are able to build up their understanding. Ausubel (1968, p. 130) explained that “by employing optimally effective methods of ordering the sequence of the subject matter, constructing its internal logic and organization [sic], and arranging practice trails” effective learning can be promoted. Hiebert and Carpenter (1992, p. 69) stated that “networks of mental representation are built gradually as new information is connected to existing networks”. Askew (2004, p. 178) noted that effective teachers had an “interest in what students have previously learned, how to make sense of students’ interpretations of the lessons and how this might be taken into account in planning and teaching”.

These different aspects that constitute effective teaching approaches of mathematics are components that may be intertwined and used in order to create a pedagogy that will provide students with a classroom learning environment that is conducive to creating learners with deep conceptual understanding of primary mathematics.

Student Beliefs

It is hard to distinguish between beliefs and knowledge as in many ways they are intertwined (Thompson, 1992). Philipp (2007) distinguishes between them by arguing that knowledge has a connotation of consensuality. Thompson (1992) and Pehkonen and Torner (1999) also asserted that knowledge normally has some agreed criteria with which the information is validated. In contrast beliefs are much more individual (Philipp, 2007). It is acknowledged that there is difficulty in defining beliefs. Therefore for this study it was decided to use Rokeach's (1972) very simple definition which is "A belief is any simple proposition, conscious or unconscious, inferred from what a person says or does, capable of being preceded by the phrase I believe that... The content of a belief may describe the object of belief as true or false, correct or incorrect; evaluate it as good or bad; or advocate a certain course of action or a certain state of existence as desirable or undesirable" (p. 113).

There is a growing awareness of the true complexity of learning which acknowledges the importance of students' beliefs in the learning process (Schoenfeld, 1992). Researchers have asserted that if change in students' thinking in mathematics is to occur, students' beliefs need to be an important focus (Brinkmann, 2001). For this study there was the acknowledgment that some students hold negative beliefs concerning estimation (Schoen, 1994) and it is the Researcher's view that students' negative beliefs about computational estimation have impacted on their engagement with learning. Pehkonen and Törner (1999) believe that students' beliefs are an important component of their mathematics learning and that negative beliefs inhibit students from being active in their learning. Eynde, De Corte and Verschaffel (2002) defined beliefs in mathematics as "the implicitly or explicitly held subjective conceptions students hold to be true, that influence their mathematical learning and problem solving" (p. 16).

McLeod (1992) held similar views to this and identified four important categories of student beliefs about mathematics learning. These were beliefs about; mathematics, self, mathematics teaching and social context.

Eynde et al., (2002, p. 27) explained that "students' mathematics-related beliefs, are constituted by their beliefs about the class context, beliefs about the self, and of

course beliefs about mathematics education”. Spangler (1992) asserted that there is a cyclical process to beliefs, which are influenced with experiences, which contribute to beliefs which in turn further contribute to how students approach learning tasks.

Research has explored different beliefs that students have about mathematics (Frank, 1988; Kloosterman & Cougan, 1994; Spangler, 1992; Stodolsky, Salk, & Glaessner, 1991) and by focussing on past research studies, it is possible to identify some of the beliefs that students have articulated. Overall McDonough (2008) found that the eight and nine year old students’ mathematical beliefs were very complex. Spangler (1992) found that many students believe that there is only one correct answer in mathematical questions, whereas obviously in open tasks there are often more than one correct answer. She also found that students perceived that mathematics was useful in the outside world and believed that it was important for computational aspects of real world tasks. Stodolsky, Salk and Glaessner (1991) asked 60 fifth grade students about their beliefs about school mathematics and found that when students were asked what mathematics was, their answers mostly concerned the four operations with far less mention of such topics as probability or geometry. Frank (1988) conducted a study with 27 students who were mathematically gifted middle school students in the United States. The findings revealed five major beliefs. These were that, mathematics was about computational algorithms, mathematical problems should be able to be done quickly, mathematics was all about right answers, and mathematics was a set of rules to be passively received and that it was the teacher’s role to transmit this mathematical knowledge. Frank’s findings have been replicated in other research studies. Szydlik (2000) also found that many students believed that school mathematics is about following the instructions of the teacher. Schoenfeld (1989) found that in his study of 230 students in Years 10-12, they believed that mathematics problems should be able to be completed in a few minutes and should not take any length of time.

Recent research has now revealed the connection between students’ beliefs and achievement (House, 2006; Nezahat, Mahir, Mehmet, & Hakan, 2005). Research has revealed that students who believed that success was a product of studying achieved higher scores (House, 2006; Randel, Stevenson, & Witruk, 2003). Particularly relevant to this study is the finding by Op’t Eynde and De Corte(n.d) that students

who have positive perceptions of mathematics and themselves as competent were more likely to be high achievers.

Teachers' Beliefs

The research on teachers' beliefs points to the complexity of adult beliefs. What teachers believe directly impacts on their teaching, although at times there may be some mismatch of beliefs and practice (Philipp, 2007; Thompson, 1992).

Researchers have found that beliefs impact upon teaching approaches but that espoused beliefs are often different to enacted beliefs due to the social setting of the school (Anderson, Sullivan, & White, 2004; Ernest 1989). Karaac and Threlfall (2004) identified that teachers' school settings can block teachers from implementing their professed beliefs in their practice and that they may resolve a personal conflict in what they perceive as appropriate to teach in the school setting with different personal beliefs by separating the school setting from the ideal. Beswick (2005) highlighted the importance of context as a factor to explain why some actions do not fit with beliefs. A study by Berswick (2005) with 25 teachers found that nearly all of the teachers responded that they held beliefs that were consistent with constructivist teaching. However, the teachers who held constructivist views did not always create a student-centred classroom, which she attributed to the contextual demand of curriculum coverage. The recognition that belief systems operate within different contexts explains the complexities of this area and that there is not always a clear linear relationship between beliefs and practice.

These difficulties and complexities should not deter educators from attempting to encourage teachers to engage in a process of reflecting on their beliefs and practice. One study that was shown to be effective in changing teachers' beliefs is CGI (Carpenter et al., 1999). Although the change was difficult and did not happen immediately new beliefs emerged through "learning about children's thinking" (Carpenter et al., 1999, p. 109). The researchers believed that the change occurred because the professional learning model included the teachers reflecting on their students' learning of mathematics. They could observe that "students were capable of inventing strategies and doing more ... The children increasingly solved harder problems and reported their thinking; the teachers listened and understood children

thinking better” (Fennema et al., 1996, p. 14). Keady (2007) was able to identify five stages of changing beliefs which the teachers in his study passed through during a professional learning intervention which took place over one year. The teachers who were able to look critically at their beliefs and improve on their own practice were the teachers with the best science pedagogical content knowledge. Other studies which have managed to change teachers’ beliefs are the Victorian Early Years Numeracy Project and the Count Me In Too Project in New South Wales (Bobis et al., 2005). Beliefs are particularly important for teaching of estimation, as teachers need to have a positive perception of this component of mathematics if they are going to support its introduction into the primary curriculum (Alajmi, 2009).

Teacher Knowledge

Teachers are only able to teach using the knowledge they possess. The National Council of Teachers of Mathematics stated that teachers need “ a sound knowledge of mathematics and how children learn mathematics” (National Council of Teachers of Mathematics, 2006, p. 1). Whilst content knowledge is important, the knowledge needed by effective teachers includes how to teach that subject (Shulman, 1986). Shulman (1986, p. 9) changed the traditional concept of effective teaching knowledge by unpacking the types of knowledge needed into three aspects; subject matter knowledge, curricular knowledge and pedagogical knowledge. This created new lines of research and produced further work which explored pedagogical content knowledge (Grossman, 1990). Hill, Ball and Schilling (2008) conducted research into PCK for mathematics teaching and broadened the categories of knowledge required for effective teaching (Figure 2.2). They created this PCK model in anticipation that it would support research into effective instruction. One new category was KCS. This is “content knowledge intertwined with knowledge of how students think about it”(Hill et al., 2008, p. 375) . This aspect was mentioned in Shulman’s model and has just been formally categorised as shown in Figure 2.2. This is important for a subject such as computational estimation which has been perceived as difficult to learn. The researchers noted the importance of the knowledge of the students and they concluded that this is an area that requires more research.

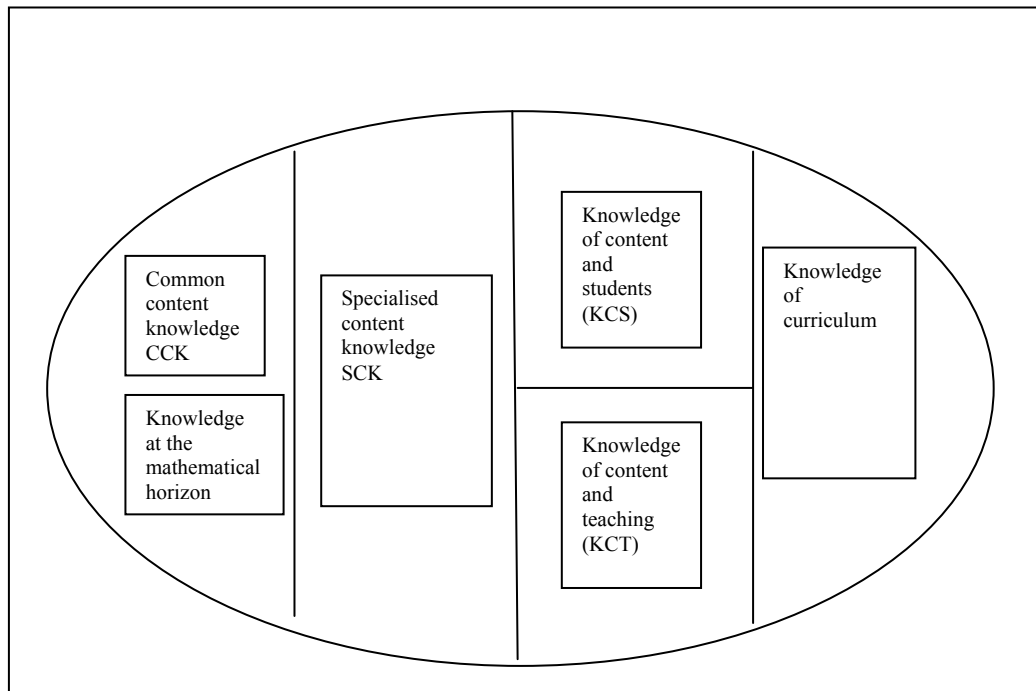


Figure 2.2: Domain map of knowledge for mathematics teaching (Hill et al., 2008)

Sowder (2007) also embraced the ideas of Shulman (1986) and Grossman (1990) and stated that the necessary types of knowledge needed for teaching mathematics were:

- an overarching knowledge and belief about the purposes for teaching (mathematics);
- knowledge of students' understandings, conceptions, and potential misunderstandings (in mathematics);
- knowledge of mathematics curriculum and curricular materials; and
- knowledge of the instructional strategies and representations for teaching particular topics. (p. 164)

She also stressed that pedagogical content knowledge is often limited in teachers because they lack the basic mathematical knowledge, which is the basis for developing pedagogical content knowledge. Ball and Bass (2000) asserted that within mathematics certain topics are taught using specific representations that support the teaching of difficult subjects. For effective teaching of domains of content in mathematics, teachers need to develop specific PCK for those domains of

knowledge. This area is very important for computational estimation as there is a lack of research literature which maps the PCK specifically for computational estimation.

Chick, Baker, Pham and Cheng (2006) created a framework for PCK which drew on the research literature as well as their own research findings. The researchers concurred that Shulman's initial outline of PCK encompassed most aspects of knowledge needed to teach effectively to students. Their framework clearly delineated between "Clearly PCK" which is the blend of content and pedagogy, "content knowledge in a pedagogical context" which is the content focus of the knowledge and "pedagogical knowledge in a content context" which is the pedagogical focus (Chick et al., 2006, p. 61).

The study documented in this thesis focused on investigating the PCK required for teaching computational estimation. The research into teaching mathematics to primary school students suggests that activities set in meaningful contexts such as real world scenarios or models such as number lines may be effective components of teaching computational estimation. The literature also suggests that the content knowledge in a pedagogical context may include certain estimation strategies and using estimation as a metacognitive framework in order to check the solutions of calculations.

Pedagogical content knowledge for primary mathematics teaching is not necessarily gained through undertaking a mathematics undergraduate degree. This was confirmed in Askew's study, which found no correlation between teachers with a mathematics undergraduate degree and those who were assessed as highly effective teachers. He found that highly effective teachers had a "good knowledge not only of how students learn mathematics in general and the understandings of the particular students being taught, but also knowledge of effective activities and ways to explain aspects" (Askew, 2004, p. 178). The type of knowledge that is needed was described by Ma (1999) after observing the contrasting teaching approaches of American and Chinese teachers. The effective Chinese teachers possessed a profound understanding of fundamental mathematics - knowledge which is "intellectually demanding, challenging and exciting" (Ma, 1999, p. 116). Ma (1999) illustrated how the Chinese teachers picked representations that would illustrate the mathematical

topic to be taught. One Chinese teacher suggested that the “equations $1\frac{3}{4} \div \frac{1}{2} =$ can be represented from different perspectives ... here is $1\frac{3}{4}$ kg of sugar and we want to wrap it into packets of $\frac{1}{2}$ kg each” (p. 80). In contrast she described how the American teachers picked unsuitable ways to teach fractions because their “deficiency in understanding the meaning of division by fractions determined their inability to generate an appropriate representation” (p. 70). Bobis (2004) noted that the teachers working on the Count Me In Too project realised how important mathematical pedagogical content knowledge was for their teaching to be effective:

They made comments such as “the importance of arrays to teach multiplication and division” and their “better understanding of place value”. Some also mentioned how their involvement in the program had impacted on the way they themselves perform mental computation and how they now “pass this on to their children”. (p. 168)

In the primary mathematics, classroom teachers need to possess fundamental, connected mathematics knowledge. In order to teach computational estimation effectively teachers need well-developed number sense, a deep understanding of estimation strategies, the conditions under which each can be applied and a repertoire of learning tasks through which computational estimation can be developed which the students may use in the process of undertaking computational estimation.

Professional Learning

Professional development was often based on a deficit model but gradually there has been a new perspective that acknowledges that the teachers must themselves have agency in their own learning and hence the term professional learning now replaces the older term of professional development. Teachers are now viewed as active learners “shaping their professional growth in professional programs and in practice (D Clarke & Hollingsworth, 2002, p. 948) . Guskey (1986) asserted that it important to recognise that change is difficult for teachers and will take time. Guskey (1986) also asserted that an important factor that contributes towards successful professional learning is the consideration of what motivates teachers to engage.

Sprinthall, Reiman, and Theis-Sprinthall (1996) held the view that professional learning needs to be based on promoting teacher growth, having an appreciation of the classroom complexity, having a solid knowledge base and reacting thoughtfully to teachers. Other factors of effective teacher professional learning are providing on-going support (Hackling, Goodrum, & Rennie, 1999), teachers working in collaboration (Bray, 2002; Keady, 2007) and time spent with students in order to reflect on how the learning in the professional learning situation can be incorporated into the classroom (Bobis et al., 2005; Fennema et al., 1996; Willis, Treacy, & Western Australian Department of Education and Training., 2004).

One study that was an effective professional learning program was Cognitively Guided Instruction program (Carpenter et al., 1999). It successfully improved student's learning outcomes. In order to create this change the professional learning program changed the teachers' beliefs about mathematics. CGI facilitated this change by presenting the participants with certain principles and encouraging the participants to trial these ideas back in the classroom. Guskey's views concur with this, maintaining that it may not be possible to change beliefs and then improve practice but that beliefs may change as a result of improved practice(Guskey, 1986) . He presented this model of development in a linear fashion. Clarke and Hollingsworth (2002) elaborated upon this model however in order to represent the true complexity of professional learning asserting that a linear model is too simplistic. Within this complexity Clarke and Hollingsworth outline the important of reflecting on new practices and new student outcomes as a vehicle for changing teacher beliefs and the interconnectedness of these. This is shown in Figure 2.3

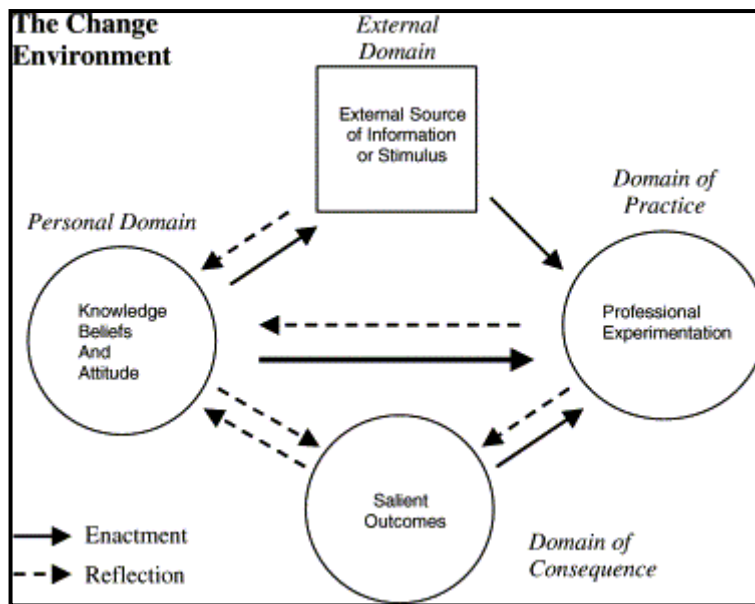


Figure 2.3: Model of interconnected growth (Clarke & Hollingsworth, 2002)

This model shows that providing teachers with models of new practice, time for individual reflection and time for reflection on the impact of suggested new practice is essential for effective professional learning.

The aim of this study was to provide a learning experience that was authentic and valuable for the teachers in enhancing their actual classroom practice. An action research approach was therefore created in order to create this type of authentic learning which would allow participants time to reflect on their teaching and share their ideas with others. The professional learning intervention created an intertwined process of professional development workshops, collaborative teacher reflection and trialling in the classroom. This has been described as action learning (McGill & Beaty, 2002) and was described as “a continuous process of learning and reflection, supported by colleagues, with an intention of getting things done” (McGill & Beaty, p. 11). This model of providing extra important material for learners within an action research process was envisaged as useful by McGill and Beaty (p. 233) who perceived that these various aspects would enrich the action learning process. This professional learning model of action research combined with professional development workshops is shown in Figure 2.4. The teachers followed three cycles of the action research process.



Figure 2.4: Model of intertwined action learning, adapted from McGill and Beaty (2002)

Develop PCK for computational estimation

In this professional learning intervention, the findings from the literature review were synthesised so that certain principles could be formulated to underpin the design. Four principles were created and this influenced the three days of professional learning through the Researcher’s choice of reflective academic reading, discussions and professional learning activities, as well as the design of the tasks for the students. As there was little recent research on how to teach

computational estimation, it was not possible to simply present the teachers with a pre-existing pedagogical framework for computational estimation. Instead, general principles of how to teach mathematics effectively were presented. These principles were that mathematics teaching and learning is effective when it is; active (Franke et al., 2007), metacognitive (McKeachie et al., 1986), and contextual (Gravemeijer & Terwel, 2000). The fourth principle was that numerical estimation is an integral part of number sense (McIntosh et al., 1997).

As it was established in the pre-study teacher interviews that none of the teachers knew about the variety of computational estimation strategies, these had to be introduced. Initially the terms were defined and used by the Researcher and, at this stage, the teachers were peripheral participants in the discourse. As the professional learning intervention progressed, the teachers (or adult learners in this case) gradually individualised their understanding of the strategies. As Sfard (2008) asserted, this interest in the new discourse should not be assumed and the agreement to act as a learner should never be taken for granted (p. 5).

The teacher workshops focussed on content knowledge about the computational estimation strategies and how to teach mathematics effectively. The teachers were introduced to six estimation strategies; front end loading, range, compatible numbers, rounding, intuition, and benchmarking (Mildenhall, 2009). They were presented with suggested teaching activities and it was also recommended that 'estimation as a checking device' become part of the normal expectations of teachers and students in the mathematics lessons, that is the sociomathematical norm (Yackel & Cobb, 1996). Ways to enable estimation to become part of the classroom culture were explored in the professional learning program. As well as having specific units focused on estimation, the teachers were encouraged to develop ways that enabled estimation to become an integral part of all the student's calculation activities. It was hoped that eventually the students would automatically create an estimate in their heads whenever they met an exact calculation.

Within this professional learning program the aspects of PCK that were focused on are summarised. Using Chick et al.,'s (2006) framework headings it possible to identify how different aspects of PCK were developed. These aspects of PCK are shown in Figure 2.5.

Aspect of PCK	Computational estimation
Clearly PCK	Contexts such as real world scenarios and number lines
	Problem based learning experience
Content knowledge in a pedagogical context	Computational estimation strategies
	Computational estimation as a metacognitive tool in order to check reasonableness of answers

Figure 2.5: PCK to be developed using Chick’s framework

Facilitating the action research process

As well as professional development sessions, the intervention also created an action research process. The teachers could reflect on their present practice and the new content knowledge that they had been given on the professional learning intervention and gradually undertake the process of individualisation of the new knowledge introduced on the professional learning intervention (Sfard, 2008). Patton (2002) asserted that this type of inquiry can change practice and encourage practitioners to engage in more systematic and reflective practice.

This type of process, where teachers are engaged in planning phases of action research within professional development workshops has been used successfully by other researchers (Perrett, 2003). Wilson and Cooney (2002, p. 132) asserted that that the “development of a reform-orientated teacher so characterised, is rooted in the ability of the individual to doubt, to reflect and to reconstruct” and professional learning can be an effective tool for stimulating this type of change. The three-staged process of doubting, reflecting and reconstruction was part of the professional

development program for cognitively guided instruction (Fennema et al., 1996).

Freire (1993), a proponent of empowering learners to engage in change, rejected the notion of the students passively receiving information and instead asserted that it is necessary for students to think for themselves. His vision for humanist education was not the imposition of educational plans but work alongside people in their particular context.

The action research process in this research incorporated an intense process of reflection. This reflection included the shared discussions at the professional learning workshops, individual reflections whilst trialling the estimation ideas back in the classroom and being interviewed three times by the Researcher. It was anticipated that the process of reflection would allow the teachers to engage in an authentic learning process. It was also envisaged that at times perspectives presented on the professional learning would be in conflict with the teachers' present beliefs. The action research process, creating time away from the school setting, would allow teachers to work through this disequilibrium or cognitive conflict (Keady, 2007; Sfard, 2008).

Conceptual Framework for the Proposed Study

An elaborated conceptual framework for this proposed study is presented in Figure 2.6. Underpinning the study was social constructivist theory and sociocultural theory due to the Researcher's personal beliefs that reality is a personal construction (Patton, 2002, p. 96). The study was predominately qualitative, focussing on representing the different world views of the participants involved in the research. Through reviewing the literature, it was recognised that teacher professional learning would be an appropriate vehicle for developing teachers' PCK in the specific mathematics domain of computational estimation. This professional learning was quite complex as the literature review revealed that there is a gap in recent literature as to how to teach and learn computational estimation. Due to this complexity, it was decided to use both a cyclical action research approach combined with professional development workshops. Within the action research process the teachers would reflect on students' estimation performance and beliefs and this would impact on their planning of subsequent tasks. From synthesising the literature it was envisaged

that this action research process and professional development workshops would enhance teacher beliefs and pedagogical content knowledge of computational estimation.

The conceptual diagram had at its central point the practice of teaching and learning. This classroom interaction is where students' computational estimation performance and beliefs about computational estimation may be impacted on by the teaching approaches.

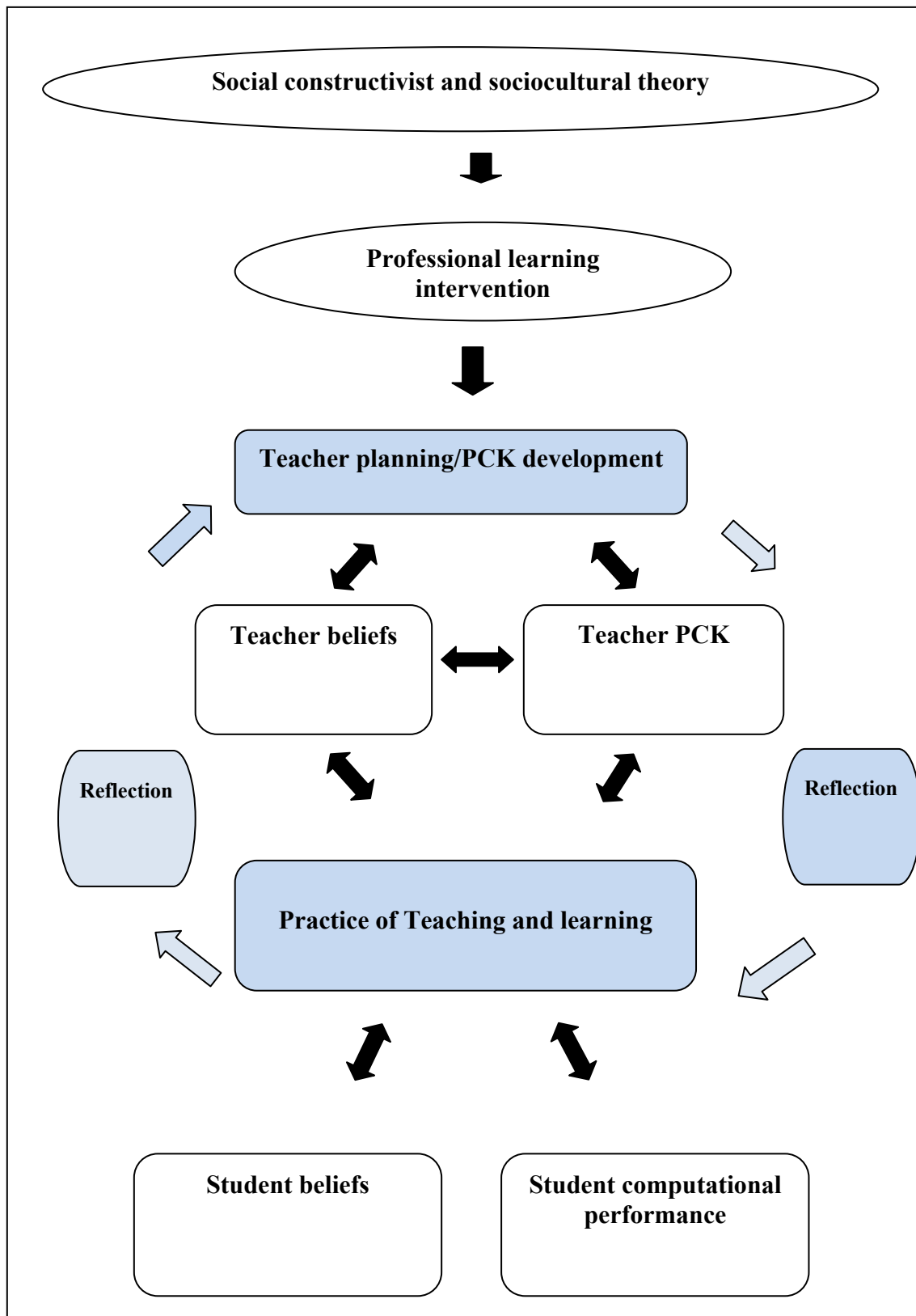


Figure 2.6: Elaborated conceptual framework for the study

CHAPTER 3: METHODOLOGY CHAPTER 3: METHODOLOGY

Constructivist theory underpinned the methodology for the professional learning intervention. Constructivists assert that “All tenable statements depend on one worldview, and no worldview is uniquely determined by empirical or sense data about the world” (Patton, 2002, p. 97). Within a constructivist paradigm, truth is not something out there but is instead “the most informed and sophisticated construction on which there is consensus amongst individuals most competent (not necessarily most powerful) to form a construction” (Guba & Lincoln, 1989, p. 86).

Due to the complexity of classroom research, involving teachers and students, it was decided that an experimental approach, involving the controlling of extraneous variables, was not suitable (Stringer, 2008). This research was aiming to research an authentic process in depth and present the teachers’ and students’ perspectives. This research focussed on the teachers’ beliefs, PCK and teaching approaches and students’ beliefs and learning of computational estimation. It has been noted that beliefs are quite difficult to research but there is the suggestion that qualitative research is most suitable to collect data on this area (Martino & Zan, 2001), as it is possible to understand the emotion behind the statements of beliefs when using a more qualitative approach.

Multiple Case Study Design

The unit of analysis for this research was individual case studies that were organised within a multiple case study design. This design was selected as the research was focussed on investigating teaching and learning processes within specific contexts (Merriam, 1998). A case study is a bounded system and, in this research project, the bounded system was the teacher and pupils involved in the professional learning intervention (Miles & Huberman, 1994; Patton, 2002). Merriam (1998) asserted that case studies are ideally suited to studying processes such as school interventions and this can include both description and development of causal implications. The case study research method was useful to “gather comprehensive, systematic, and in-depth information about each case of interest” (Patton, 2002, p. 447). Initially there were three individual case studies and then a cross-case analysis was conducted. By

comparing and contrasting the different cases, the interpretation and understanding of how to develop computational estimation would be far more comprehensive than if only one case was focused on (Merriam, 1998). Soy also outlined how cross-case analysis can focus on patterns across the cases. “When a pattern from one data type is corroborated by the evidence from another, the finding is stronger ... In all cases, the Researcher treats the evidence fairly to produce analytic conclusions answering the original "how" and "why" research questions” (Soy, 1997 p. 1). The cross-case analysis increased the validity of the study and allowed the findings to be made with more certainty (Miles & Huberman, 1994).

In order to address the research questions there was a mixture of data collection approaches in this multiple case study, including quantitative and qualitative data (Miles & Huberman, 1994). Patton (2002) asserted that “human reasoning is sufficiently complex and flexible that it is possible to research predetermined questions and test hypothesis about certain aspects of a program while being quite open and naturalistic in pursuing other aspects of a program” (p. 253).

The methodology also utilised an action research approach due to its commitment to involving its participants in a transformative process, empowering them to look critically at their practice and recording their perspectives regarding how to teach computational estimation. This was especially important as there is little recent research as to how to teach computational estimation in the primary school.

Lewin (1946) was the first to use the phrase action research. He envisioned a type of research that would lead to social action. He asserted that “research that produces nothing but books will not suffice”(Lewin, 1946, p. 144). Lewin (1946) explained that the process would involve planning, executing and evaluating in a number of steps. In Figure 3.1 there is a diagrammatic depiction of this spiral process, focussing on the steps of plan, act and reflect. Proponents of action research recognise that this cycle is at the heart of this approach (McNiff & Whitehead, 2005; Riggall, 2009; Stringer, 2008). Riggall described the cycle as a process where “the researcher plans a change, then implements it and then reflects on it. This completes an action research cycle but the process does not end there. More change is planned based on the learning from the earlier cycle, which is then implemented, observed and then reflected on” (Riggall, 2009, p. ix).

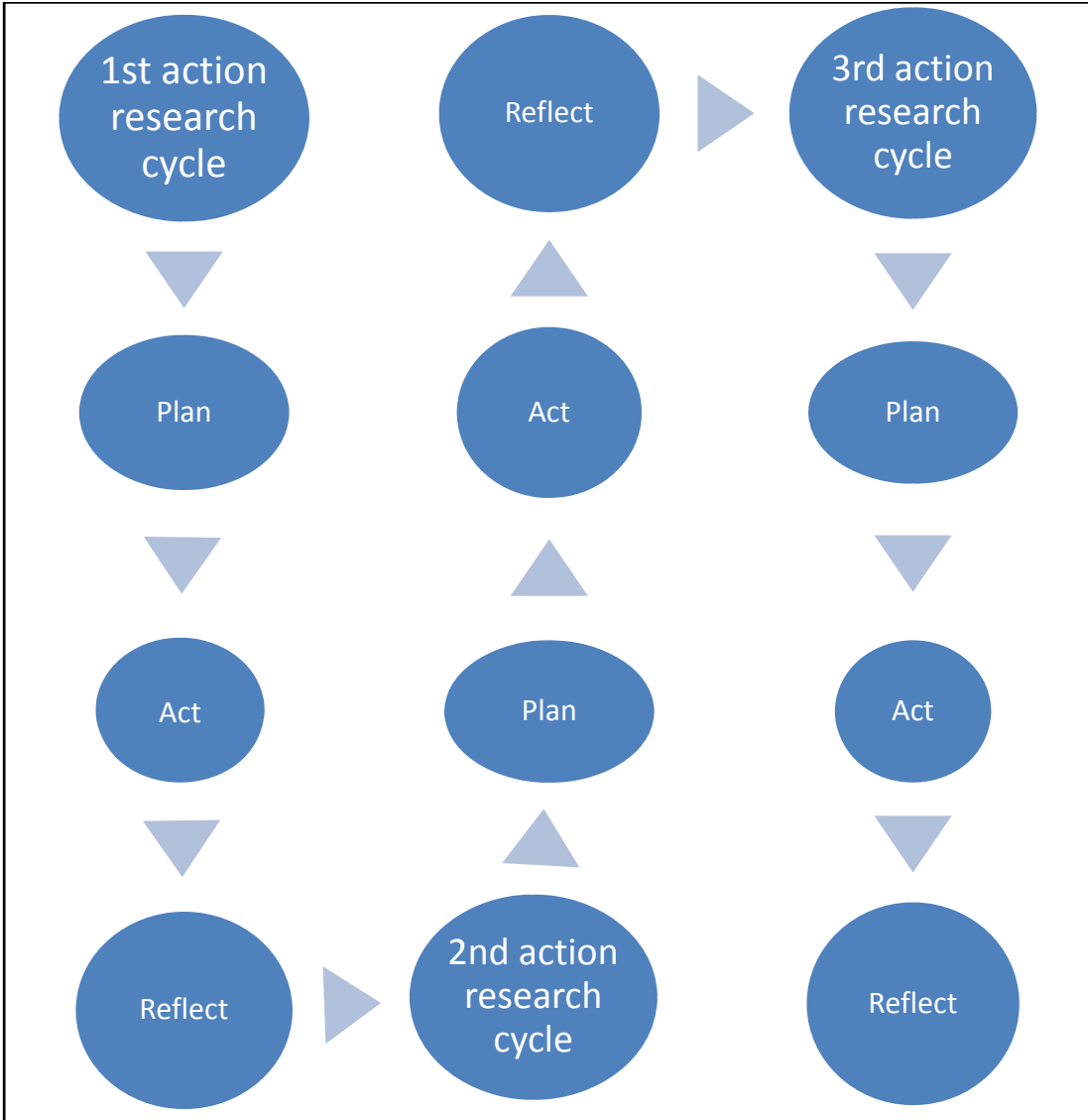


Figure 3.1: Action research cycle

The action research process was focussed on the professional learning days, with the researcher fully involved in all of these professional learning days. This type of action research approach with the professional learning program at the heart of the focus has been adopted successfully by other effective programs McGill and Beaty (p. 233). This was a collaborative approach recognising the different aims of practising teachers and the Researcher.

In this study, Year 6 teachers responded to the invitation to consider how to teach computational estimation as a component of number sense with the aim of

improving their own practice. At the first professional learning workshop, the Researcher encouraged the teachers to reflect on how they taught students to estimate when calculating. This facilitating role was important in empowering the teachers to think critically. It has been noted in schools that often practices are “widely followed because of their being valued, but they are often valued because of their being widely followed” (Sfard, 2008, p. 205). A similar view was held by Freire (1993) who focused on educating illiterate Brazilians. He perceived that certain disadvantaged members of society may not realise that their society is in need of change. He specifically worked with Brazilian peasants and he transformed their literacy levels by empowering them to change their situation in a way that was relevant to their context.

There were three action research cycles in this research study, which involved the sharing of individual perspectives in a group setting at each cycle of the action research process. In the final twilight session there was a final sharing of what each person, including the Researcher, had learnt during the process.

Selection of the Participants

It was logical to choose the teachers from which the study “could learn the most from” (Patton, 2002, p. 233). For this study, this entailed selecting teachers who had some expertise in the area of primary mathematics. This was particularly important as there is little recent research literature on how to effectively develop computational estimation (Reys & Reys, 2004) so the process was dependent on the expertise of the teachers. Six teachers were invited through the purposeful sampling technique of nomination (Stough & Palmer, 2003). Principals from the low fee independent schools were asked to nominate teachers who:

- had taught for at least three years,
- had perceived competence and confidence in teaching primary mathematics,
- were interested in investigating how to develop estimation and number sense
- planned to teach Year 6 and were able to participate in the study.

From the six teachers who were nominated and joined the professional learning intervention, further purposeful sampling took place to focus on three of the teachers for the case studies who represented different levels of engagement in the program

(Patton, 2002) .The student participants were the Year 6 pupils of the teachers who participated in the professional learning intervention. The teachers were asked to suggest between four and six students in each class for the focus groups depending on the interest and agreement of the students and students' parents. The teachers selected students who were keen to express their ideas and “shed light on the phenomenon being studied” (Hatch, 1995, p. 66) and were representative of the class in terms of the perceived spread of mathematical ability.

Research Foci

Within each case study, there were two main foci to the research. One concerned the teachers' involvement in the professional learning intervention and the other focus was the students' involvement.

Procedures

Before the study began, during December 2009, the teachers in the professional learning intervention and their schools were visited. During this visit, the teachers were interviewed and their beliefs and knowledge about mathematics were recorded using a digital recorder. The Researcher then transcribed these recordings and these transcripts were stored within the NVIVO 8(QSR, 2008) software program.

The study of the teacher professional learning intervention took place over one year and involved three one-day workshops. At the beginning of each day, the teachers shared their individual perspectives and this session was recorded digitally. During the rest of the day, a critical friend took observational notes of the professional learning workshops. Each afternoon the teachers then worked through student tasks, which were selected as potentially worthwhile in order to develop students' performance in computational estimation. The tasks were evaluated and adapted by the teachers, so that they were suitable for each of their personal school contexts and that they were designed to be utilised in a way that the teachers thought were pedagogically appropriate.

Before the students began their work on estimation, the Researcher conducted the student focus group interviews. These were conducted at the school and then the

recordings were transcribed by the Researcher and also entered into NVIVO 8(QSR, 2008). The teachers administered the pre-test using the written protocol provided by the Researcher. The Researcher collected and scored these tests and the results were entered into PASW(SPSS inc, 2010).

Back in their classrooms, the teachers were encouraged to develop a culture where computational estimation was a part of their students' mathematical repertoire as well as implementing the focussed extended estimation tasks each term. During the implementation of the tasks, the Researcher visited the classrooms for one session where the extended tasks were being taught and collected qualitative data through participant observation and student work samples. The observations of the classroom, which included transcription of sections of the discourse in the lessons recorded on a digital recorder were entered into NVIVO 8 (QSR, 2008).

The teachers reflected on this initial implementation of the extended learning tasks at the end of Term 1 and in the second professional learning day collaboratively planned the next two tasks. This redesign of the tasks used the knowledge gained about how students learned to estimate, to further improve the teaching approaches used for the next estimation tasks. At the end of the end of Term 2, interim interviews were conducted with the teachers, which were transcribed by the Researcher and entered into NVIVO 8 (QSR, 2008). In this session, the teachers and Researcher shared with each other what they had learnt about how to teach computational estimation and number sense, whilst being involved in the process.

At the end of Term 4, the final data collection took place. The final interviews with the teachers and the focus group interviews with the students also took place in the schools and these interviews were transcribed by the Researcher and entered into NVIVO 8. Teachers administered the post-tests to their students (Appendix I). The scoring of the test entailed marking each item either correct or incorrect. To clarify that the Researcher accepts that estimation is not about one correct answer, it is pertinent to mention there that as the students were instructed to provide the "best estimate", therefore the estimate that was closest to the exact answer was the one that was marked as correct.

As well as scoring answers as correct or not, the explanations of how the students answered the questions were also categorised. The Researcher used the categories of

strategy type and whether it was a reasoned estimation strategy or not. The criteria for how these explanations were categorised is listed in Figure 3.2.

Strategy	Criteria
Reasoned estimation strategies	
Rounding	Students described replacing the exact numbers in the questions with ones that were rounded to multiples of 10 etc. This rounding had not been done with the other numbers particularly in mind i.e. $34 + 55$ rounded to $40 + 60$, which would make it a nice number.
Nice numbers	Students described taking the different numbers in the question and replacing them with numbers that were compatible with each other.
Benchmarking	Students replaced the numbers in the question with numbers are used a reference point i.e., 1 for the $11/12$ s .
Font end loading	Students explained that they focused on the front digits.
Range	Students explained “It was more than or between one number and another.”
Unreasoned estimation strategies	
Exact	Students had written down the exact numbers in the question and often shown some type of algorithm.
Guess	Students did not offer any other strategy stating they had guessed.
Intuition	Students described their reasoning as something that they couldn’t reason or explain but not a straight guess.

Figure 3.2: Criteria for assessing student responses

Judgements were made from reading the students’ explanations. Where it was not possible to make a judgement, not enough information was recorded. In Figure 3.3 an example is shown of each judgement of an estimation strategy.

An example of the rounding strategy

	<p>4. Without calculating the exact answer, circle the best estimate for:</p> <p style="text-align: center;">45×105</p>	<p>A 4000</p> <p><input checked="" type="radio"/> B 4600</p> <p>C 5200</p>	<p>45×100</p> <p>(Rounding)</p> <p>Estimation Strategy</p> <p>Rounding</p>
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An example of the nice numbers strategy

Question	Correct Answer	How I worked it out
<p>1. About how many days have you lived?</p>	<p>A 300</p> <p><input checked="" type="radio"/> B 3000</p> <p>C 30'000</p> <p>D 300'000</p>	<p>$400 \times 11 =$ $4'400$ c</p> <p>I used nice numbers strategy</p> <p>Estimation Strategy</p> <p>NICE NUMBERS</p>

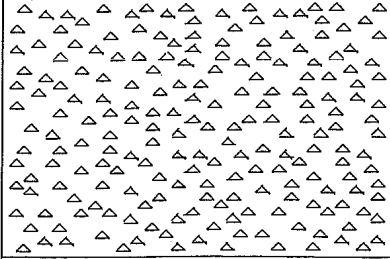
An example of the benchmarking strategy

<p>2. Without calculating the exact answer, circle the best estimate for :</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\frac{12}{13} + \frac{7}{8} =$ </div>	<p>A 1 <input checked="" type="radio"/> B 2 C 19 D 21</p>	<p>$\frac{12}{13}$ is nearly 1 and $\frac{7}{8}$ is nearly 1 $1+1 = 2$ Estimation Strategy BENCHMARKING</p>
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An example of the front end loading strategy

<p>5. Without calculating the exact answer, circle the best estimate</p> <p style="text-align: center;"> Estimation Strategy FRONT END LOADING 65 + 81 </p>	<p><input checked="" type="radio"/> A 165 B 300 C 200 D 360</p>	<p>1. Looked at the 20 30 60 80 and added them</p>
--	--	---

An example of the sample strategy

<p>3. About how many triangles are there here? (Circle the nearest answer.)</p> 	<p>A 20 B 50 C 100 D 200 E 500</p>	<p>1 estimated how much was half of the triangles then 1 times by 2.</p> <p>Estimation Strategy SAMPLE</p>
---	--	--

An example of the range strategy

<p>6. $6 \times 3.7 =$ Is the answer between</p> <p>22 24</p>	<p>A 23-26 B 18-24 C 23-28 D 16-18</p>	<p>$6 \times 3 = 18$ rounded down $6 \times 4 = 24$ ↑ 3.7 round up</p> <p>Estimation Strategy RANGE</p>
--	--	---

An example of the exact strategy

Question	Correct Answer	How I worked it out
1. About how many days have you lived?	<p>A 300</p> <p>B 3000</p> <p><input checked="" type="radio"/> C 30 000</p> <p>D 300 000</p>	<p>I used 365 days in a year then timesed it by 12.</p> <p>Estimation Strategy EXACT</p>

An example of the guess strategy

<p>2. Without calculating the exact answer, circle the best estimate for :</p> <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> $\frac{12}{13} + \frac{7}{8} =$ </div> <p>Estimation Strategy Guess</p>	<p>A 1</p> <p><input checked="" type="radio"/> B 2</p> <p>C 19</p> <p>D 21</p>	<p>guess 5</p> <p>$\frac{13}{8}$</p>
---	--	---

Figure 3.3: Examples to show student response of different strategies

This process of data collection described previously has been summarised and is shown in Figure 3.4.

Duration	Activity	Data Collection	Teachers' role	Researcher's role
Action research cycle 1				
4 hrs	Interview teachers	Semi-structured interview	Respond to questions	Collect questions
5 hrs	Interview students	Focus groups interviews	Organise class time	Conduct interviews
8 hrs	Test students	Estimation test	Administer test	Collect tests
1 day	Planning and reflecting : Collaborative planning of project overview and in depth planning of first term of work	Planning documents	Conveys the teacher's perspective and context of school for field work Participant in planning day	Deliver the research findings Collaborative planning of unit
TERM1 10 weeks	Acting and reflecting: Unit 1	Observation in class Collection of artefacts	Implement and reflect on the teaching	Observe teaching and learning
Action research cycle 2				
1 day with teachers	Planning and reflecting: Collaborative professional learning day	Planning documents Observations of meetings	Conveys the teacher's perspective Participant in planning day	Collaborative planning of units. Sharing reflections of Term 1
TERM 2 10 weeks	Acting and Reflecting: Unit 2	Observations in class Collecting of artefacts	Teacher implementing and reflecting on units	Observe this teaching and learning
4 hrs	Interview teachers	Unstructured interview	Interview	Undertaking the interview

Duration	Activity	Data Collection	Teachers' role	Researcher's role
Action research cycle 3				
1 day with teachers	Planning and reflection: Collaborative professional learning day	Planning documents Observations of meetings	Teacher to plan next topic in light of first two topics	Collaborative planning of units Sharing reflections of Term 1
TERM 3 10 weeks	Acting and reflecting: Unit 3	Observation in class Collecting artefacts	Teacher implementing and reflecting on topics	Observe this teaching and learning
Final reflections				
5 hours	Interview students	Focus group interviews	Organise time allocation	Undertake interviews
8 hours	Test students	Estimation test	Administer tests	Collect tests
4 hours	Interview teachers	Semi-structured interviews	Be interviewed	Undertake interviews

Figure 3.4: Procedure of data collection

Instruments

As the classroom is inherently complex it was necessary to select data collection instruments which were able to capture and describe some of this complexity. As this was a multiple case study some instrumentation was designed before the study began so that comparison between the cases was possible (Miles & Huberman, 1994). By using multiple methods it was more likely to produce a more complete view of the case study (Patton, 2002). Therefore this research study has selected a variety of data collection methods, which have been described below.

Researcher's audit trail journal

The Researcher's thoughts and perspectives were considered to be an important source of data. As this study is underpinned by social constructivist theory it is important to be aware of the perspectives of the Researcher and to document interpretations of events as they occur (Merriam, 1998). This audit trail journal began from the beginning of the study and as well as reflections on the data collected, documented the methodological decisions and the reasons (Richards, 2005). An excerpt of this is shown in Appendix A.

Semi-structured teacher interviews

Initially the teachers were interviewed in order to find out about their present teaching approaches, their knowledge of teaching computational estimation and their beliefs about computational estimation. The interviews were semi-structured in that the Researcher had "freedom in the sequencing of questions, in their exact wording, and in the amount of time and attention given to different topics" (Robson, 2002, p. 278). The interview questions are presented in Figure 3.5.

Questions:

1. Supposing I walked into your mathematics classroom. What does your normal mathematics lesson look like?
2. Assessment is an important issue. Could you tell me how you find out if your children in your class have learnt something in mathematics?
3. When you hear the word 'estimation' what do you think of?
4. The undergraduate students often return from 'prac' saying that it is difficult to fit topics like estimation into the curriculum when it is so overcrowded. How have you tackled this issue generally and in relation to estimation?
5. As well as an overcrowded curriculum can you perceive any other difficulties with teaching estimation in mathematics?
6. The most common estimation strategy taught is rounding. Do you teach rounding or any other strategies? Are there other strategies that you are aware of?

[If the answer is 'no' or 'only rounding' go to question 10]

If yes -

7. A child in Mr Clarke's Year 4 class wanted to find an estimated answer for the question $21 + 28 + 19 =$. Describe what estimation strategies the child could use to solve this problem.
8. Ms Fot's class were posed with the question: *Can you give me a quick estimated answer to the question $4/10 + 8/9$?* Two children gave the answer fairly quickly without having to reach for pen and paper or a calculator. What ideas do you have as to how they did this?
9. How have you planned your teaching of estimation strategies?

All respondents to answer the next questions

10. Is there anything else you would like to say?

Questions to be asked in 2nd and 3rd cycle

11. What if anything do you think you have learned from this program?
12. How are your beliefs about computational estimation changing?

Figure 3.5: Teacher interview questions

At the end of Term 2, a less structured interview took place. There were two questions which directed them to reflect on their changing beliefs and increasing pedagogical content knowledge but overall it was more of an "informal conversational interview" (Patton, 2002, p. 343). This was undertaken mid-way through the professional learning program to allow the teachers to share what was of

importance to them. This allowed for “flexibility, spontaneity and responsiveness to individual differences and situational changes” (Patton, 2002, p. 343). At the end of the action research cycle, the semi-structured interviews were repeated with the teachers with additional questions added where appropriate. A transcript of one of the semi-structured teachers’ interviews is shown in Appendix B.

Focus group interviews

The students were also interviewed through the use of focus group discussions to investigate their beliefs and knowledge about computational estimation (Johnson & Christensen, 2004) in action research cycle 1.

Focus groups were chosen as opposed to one-to-one interviews which may be intimidating for this age group (Patton, 2002). These focus group interviews were modelled on McDonough’s research which investigated young children’s beliefs in mathematics with a variety of questions selected so that the children’s beliefs about computational estimation could be recorded. They were given a prompt i.e., a photograph, or mathematics question and asked to respond to it in a certain way. The focus group questions for students are presented in Figure 3.6.

Questions:

1. Your friend has a different answer to you on a maths problem. What do you do and why?
2. An alien lands on Earth and wanted to know what mathematics is (show a cartoon of an alien). What would you tell him?
3. Estimation - write down all the things you can tell me about this.
4. Bill had to work out an estimated answer to $43 + 28$ in his head. He said that the answer was about 70. How did he calculate this ?
5. Bill then worked out an estimated answer to $11/12 + 7/8$ in his head. He gave one of these three answers about $1/2$, about 2 or about $18/20$. What was his answer - how did he work it out?

6. The shopping bill showed the amount below and I paid with the money in my hand (\$20)

\$ 5.45 + \$4.80 + \$6.15 + \$5.16 (Show this receipt on a poster for 30 seconds)

Look at the receipt for 30 seconds and then tell me if I had enough money by estimating?

How did you work out your estimate?

7. Andrea got $5/11$ in a mathematics test. In your head calculate what would her mark be as an approximate percentage? How did you get this percentage?
8. **About** how many children are in your school? How can you work this out mentally?
9. Bill sat working on one mathematics question for 15 minutes. Do you think he is clever?

Yes or No

Why ?

10. Do you think that McDonalds could benefit from employing a mathematician?

(adapted from McDonough, 2008 & Spangler 1992)

Figure 3.6: Student interview questions

These focus group interviews were repeated at the end of the study. Due to the length of time between interviews, it is unlikely that students remembered the questions and it allowed for a direct comparison of the answers. An example from a focus group interview is in Appendix C.

Students' written work

The students produced various forms of written work in the year including symbolic and written recording of calculation processes and written descriptions of their solution procedures. These were produced throughout the year in the classroom and were dated and collated so that it provided further insights as to how the professional learning intervention and the tasks influenced the student learning outcomes.

Professional learning, planning, observation and documentation

Collaborative discussions of the professional learning workshops were recorded and transcribed and reflective notes were created by the Researcher. An excerpt of these notes is shown in Appendix D.

The suggested learning activities, which included the expected learning outcomes, activities, resources and assessment opportunities of the computational estimation tasks, were created and were part of a handbook which was used by the participants. This was collected at the end of the professional learning as a record of the initial input from the facilitators and guest contributors. An excerpt from this handbook is shown in Appendix E and two of the suggested extended learning tasks are shown in Appendix F.

A critical friend, who was a postgraduate doctoral education student, acted as an additional Researcher to record observations of the professional learning workshops and an excerpt of this is shown in Appendix G.

Participant observation of classroom

It was necessary to observe the teachers and students, in their classrooms, as they implemented the teaching approaches. When collecting the data in this stage, insights from Leach and Scott's research (2002) methodology were utilised. They focussed, not only on the activities, but also "how these activities were staged" (p. 138). The Researcher observed the implementation of one of the extended computational estimation tasks in each classroom as a participant observer and an example of how this observation was recorded is shown in Appendix H. The amount of involvement of the Researcher needs to be considered to ensure the demands on the Researcher are not too high (Robson, 2002). The Researcher was predominately observing but

where students and teachers needed support this was given. This was important if the positive relationships with the teachers and students were to be maintained. An excerpt from some classroom observation transcripts is shown in Appendix H.

Computational estimation test

In order to further strengthen the internal validity of the research, the students' change in estimation performance, as a component of number sense, was measured quantitatively. Students in all three cases were given a pre - and post computational estimation test (CET) (Appendix I). The students in all three case study classes were presented with estimation-related questions. Three of these questions were taken from a number sense test, which has been used in internationally recognised research (McIntosh et al., 1997) and two were drawn from other sources and adapted so that they followed the same format as the number sense test. The questions were selected to test the variety of computational estimation strategies that may be learnt by the students during the study. These questions were:

Question 1: How many days have you lived?

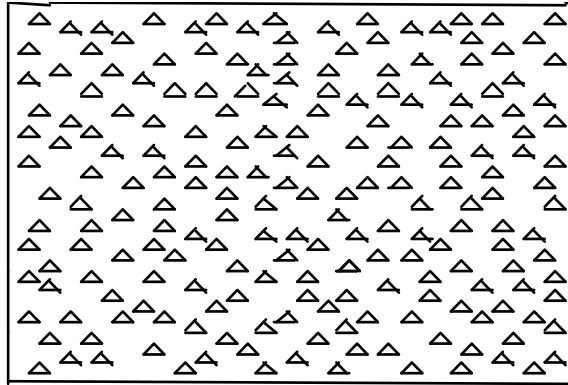
A Number Sense Item Bank for the Number Sense Test (McIntosh et al., 1997) was collated after pilot testing using interviews to ensure correct wording and that they were not able to be answered using a rote learned procedure. This question above, was taken from this bank of questions. It was used in the NST in the USA, Australian and Swedish components of the study although the Swedish study used slightly different numbers. This question was selected for this research, as it would test whether students could use such estimation strategies as rounding or frontend loading after interpreting the context and assessing that a multiplication calculation was required.

Question 2: Without calculating the exact answer, circle the best estimate for $\frac{12}{13} + \frac{7}{8}$

This question, was taken from the Taiwanese component of the NST (McIntosh et al., 1997) and was used in a non-comparative component of the overall study. They used this question in the Taiwanese component of the study so that a comparison could be made between how students answered this with time restrictions where they would have to estimate the answer and how they answered this where they had no

time restriction so that they could implement routine procedures to obtain the correct answer. This question would test whether the students were able to use a benchmarking strategy.

Question 3: About how many triangles are there?



The question was also taken from the NST (McIntosh et al., 1997). It was used in the USA, Australian and Swedish components of the studies. This question was included in this research study as it was envisaged that students would use the sample strategy in order to solve it. Once the students had obtained a sample of triangles in the picture, the students would use that number to estimate how many triangles there were in total, using multiplication.

Question 4: $45 \times 105 \approx$

Question 4 was also taken from the NST (McIntosh et al., 1997) although it was only used in the Australian component of the study and only used with 10 year old students. When answering this question, students could use a rounding, frontend loading or nice numbers strategy to answer it. As all of the answers in the multiple choice selection were in the ballpark, some compensation was required in order to ascertain which answer was actually the 'best estimate'.

Question 5: Without calculating the exact answer, circle the best estimate for :

- | |
|-----|
| 27 |
| 38 |
| 65 |
| +81 |

Question 5 was created so that it was possible to see if students chose to use the nice numbers or frontend loading strategy. As there was not an item in the NST item bank

that specifically tested the use of the nice numbers or front end loading strategy, a question from an article by Reys (1986) was adapted by the Researcher using the same multiple choice format. The wording of the question was pilot tested before adding this to the set of questions.

Question 6: $6 \times 3.7 \approx$ is the answer between

Question 6 was designed so that students would need to be able to use the range strategy within a computation that involved approximating a decimal number to two whole numbers. It followed the same format as the other questions taken from the NST and some students were interviewed in pilot testing to ensure that the wording was easily understood.

All of the questions used the format of the NST and the test was administered at the beginning and end of the study. The format of the test was multiple choice and there was a 30 second time limit so that students had to estimate as they did not have time to perform an exact calculation. In order to reduce any anxieties related to taking these tests the students were assured that the results would not be used to judge them at school in any way. The administration of the test followed the 1997 protocols of the NST. Specifically, the instructions were read by the class teacher and the students were asked to spend no more than 30 seconds on each item. Students responded directly by writing in one colour on the test pages. They were not permitted to write any other information. This requirement was designed to prevent any mechanical paper and pencil procedures being used to arrive at the solution. When the test was completed the students were given five minutes to go back and describe how they answered each question, using a pen of a different colour. The Computational Estimation Test is shown in Appendix I.

Data Analysis

The data, both qualitative and quantitative, were analysed from a constructivist perspective (Patton, 2002). Hennig (2010) asserted that it is informative for research which is underpinned by constructivism to use inferential statistics. Taken from this constructivist paradigm the statistical significance of quantitative data collection is expected to be considered as a supporting piece of evidence but not as a stand alone judgement that the findings are actually significant (Marinez-Pons, 1999). This is

particularly important as this focus for the study was a professional learning intervention where there was no attempt to control variables.

It was important to acknowledge the subjectivity of the Researcher in the research process who possessed a particular world view, particularly that the aim of school mathematics is for students to understand and individualise the mathematics that they are learning. Despite this subjectivity it is imperative that the claims made in this study are credible. To facilitate this an audit trail or log trail was implemented throughout the research so that the outcomes could be justified and made transparent (Richards, 2005). To further increase the internal validity, certain strategies suggested by Merriam (1998) were incorporated:

- Triangulation - a variety of data collection methods was utilised and a critical friend was asked to attend the professional learning workshops and give her perspective;
- Long term observation - the research study ran for 12 months;
- Researcher biases – the study has attempted to make transparent the subjective perspectives that the Researcher held.

The critical friend recorded her observations and thoughts in a separate document and these were used to triangulate the data. The critical friend also was used to discuss potential findings and these thoughts were recorded in the researcher's log. The research project was designed so that the data could be interpreted whilst the Researcher was still in the field. This has been termed interim analysis (Johnson & Christensen, 2004). Throughout the implementation stages, analysis of the case studies was undertaken. The literature was also revisited at these points in order to support the Researcher's interpretations as they emerged. The research questions provided a focus for the analysis in Table 3.1:

Table 3.1: Data analysis

Research question	Data analysis
1. How do the teachers' development of beliefs and pedagogical content knowledge, about computational estimation inform their teaching approaches?	1. Themes created which identify and explore the relationship between beliefs, PCK and teaching
2. How do the teaching approaches impact on students' beliefs about estimation and mathematical knowledge and their computational estimation abilities?	2. Themes created which identify and explore the relationship between teaching approaches and the beliefs and abilities of the students about computational estimation. 2. Pre- post-test results analysed using PASW (SPSS INC, 2010)

A content analysis of the qualitative data was conducted using a coding system (Johnson & Christensen, 2004). Themes which became apparent and best answered the research questions within each case study were identified using the multiple sources of data collection i.e., interviews, evidence of children's work, classroom observation, and planning documentation. The use of multiple sources of data contributed to the internal validity of the study and provided triangulation (Patton, 2002). Each case was analysed initially as its own case using the computer software NVIVO 8(QSR, 2008).

The process of coding began with "a priori coding" as certain areas of focus had been decided in creating the research questions (Bazeley, 2007). Using NVIVO 8(QSR, 2008), which used the terminology of nodes in place of coding, parent nodes of teacher beliefs, teacher PCK, the teaching approaches and the students' beliefs and student performance of computational estimation were created. Analytical approach to data analysis

Nvivo (QSR, 2008) is a software product designed to enhance the quality of qualitative data analysis . It is specifically designed so that:

Qualitative research software like NVivo, helps people to manage, shape and make sense of unstructured

information. It doesn't do the thinking for you; it provides a sophisticated workspace that enables you to work through your information.

With purpose built tools for classifying, sorting and arranging information, qualitative research software gives you more time to analyze your materials, identify themes, glean insight and develop meaningful conclusions (p.1).

This software allows all the data, including audio and scanned documents to be organised in a single location. It does not restrict how the data is analysed. It was the researcher's intention to use the data management system to support the analysis. Initially all of the qualitative data was stored in NVIVO so that accessibility could be enhanced. This virtual folder storage system is shown in Figure 3.7

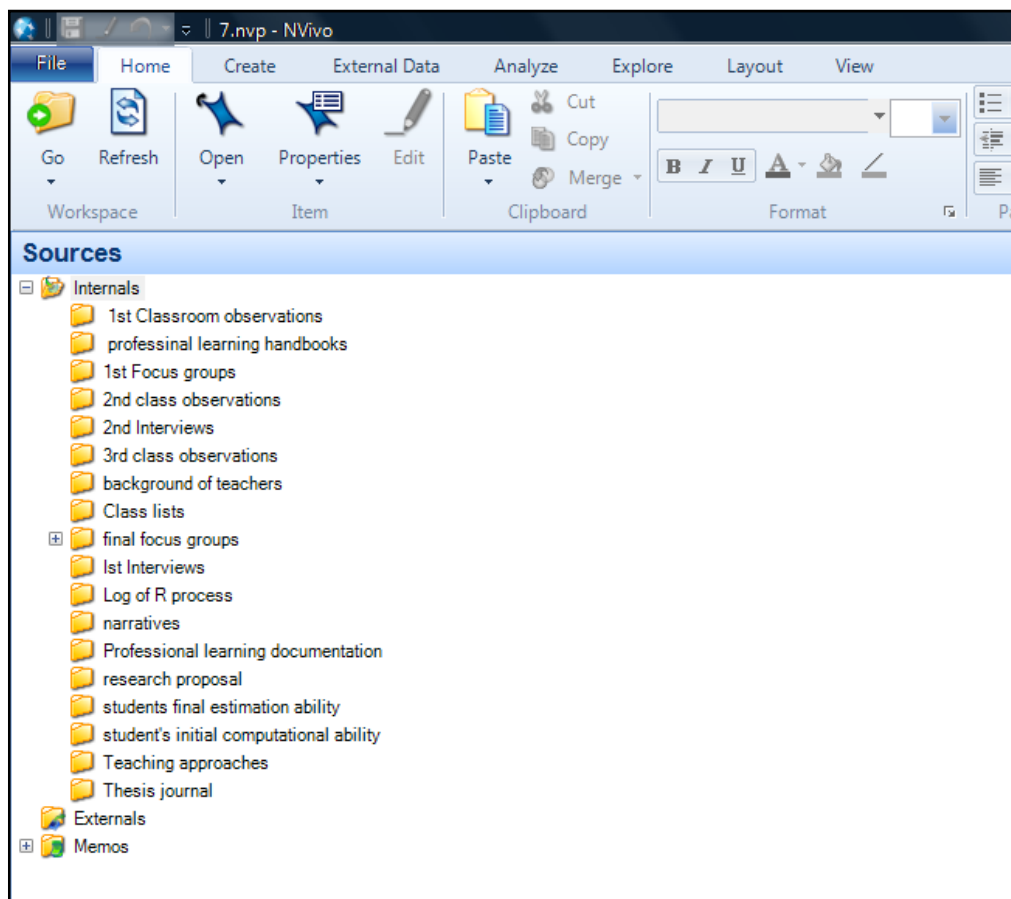


Figure 3.7: Initial organisation of qualitative data

The advantage of using NVIVO software is that the data can be analysed as a case

and in themes simultaneously. In this way it was possible to have all the data from one case presented together as shown in Figure 3.8.

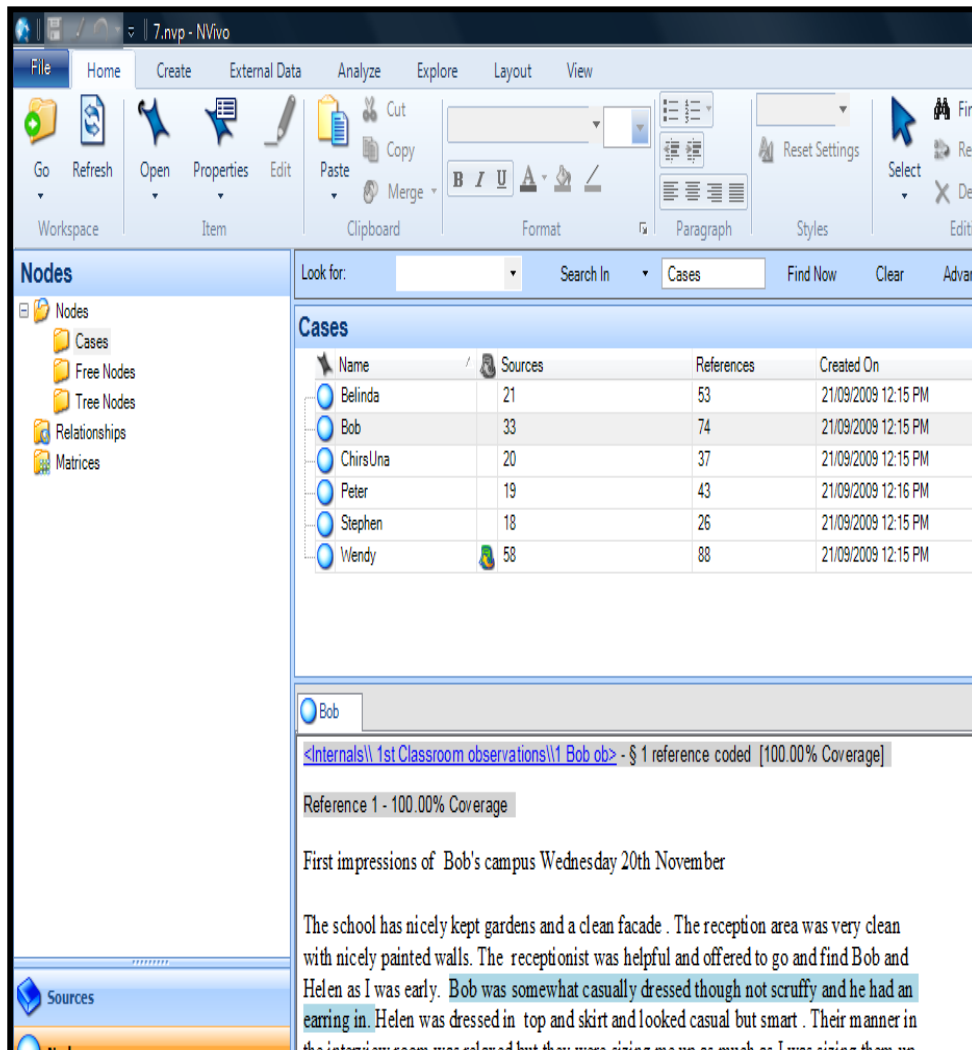


Figure 3.8: Organisation of data in cases

Once the data was organised, general inductive coding then took place in order to consider the individual teachers' and students' responses (Thomas, 2003). The multiple sources of data were analysed using NVIVO 8(QSR, 2008) so that themes or nodes could be identified from more than one source of data (Figure 3.9).

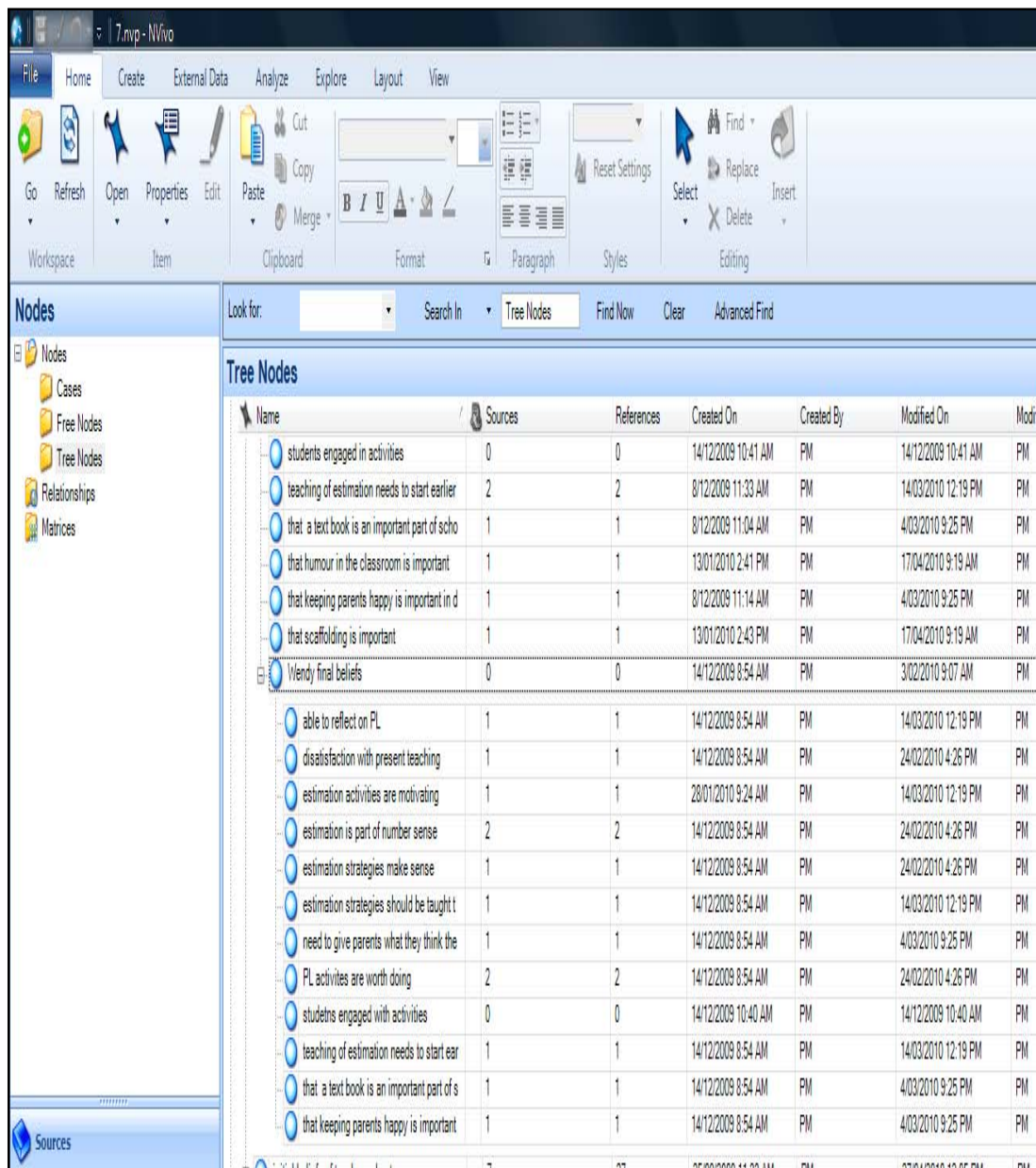


Figure 3.9: Data organised in themes

All sources of data related to a particular case were coded together so that it was possible to create an individual case matrix to identify responses to the different *a priori* codes (Bazeley, 2007). Using the NVIVO 8(QSR, 2008) modelling capability these could be represented visually which facilitated the analysis process and this is shown in Figure 3.10.

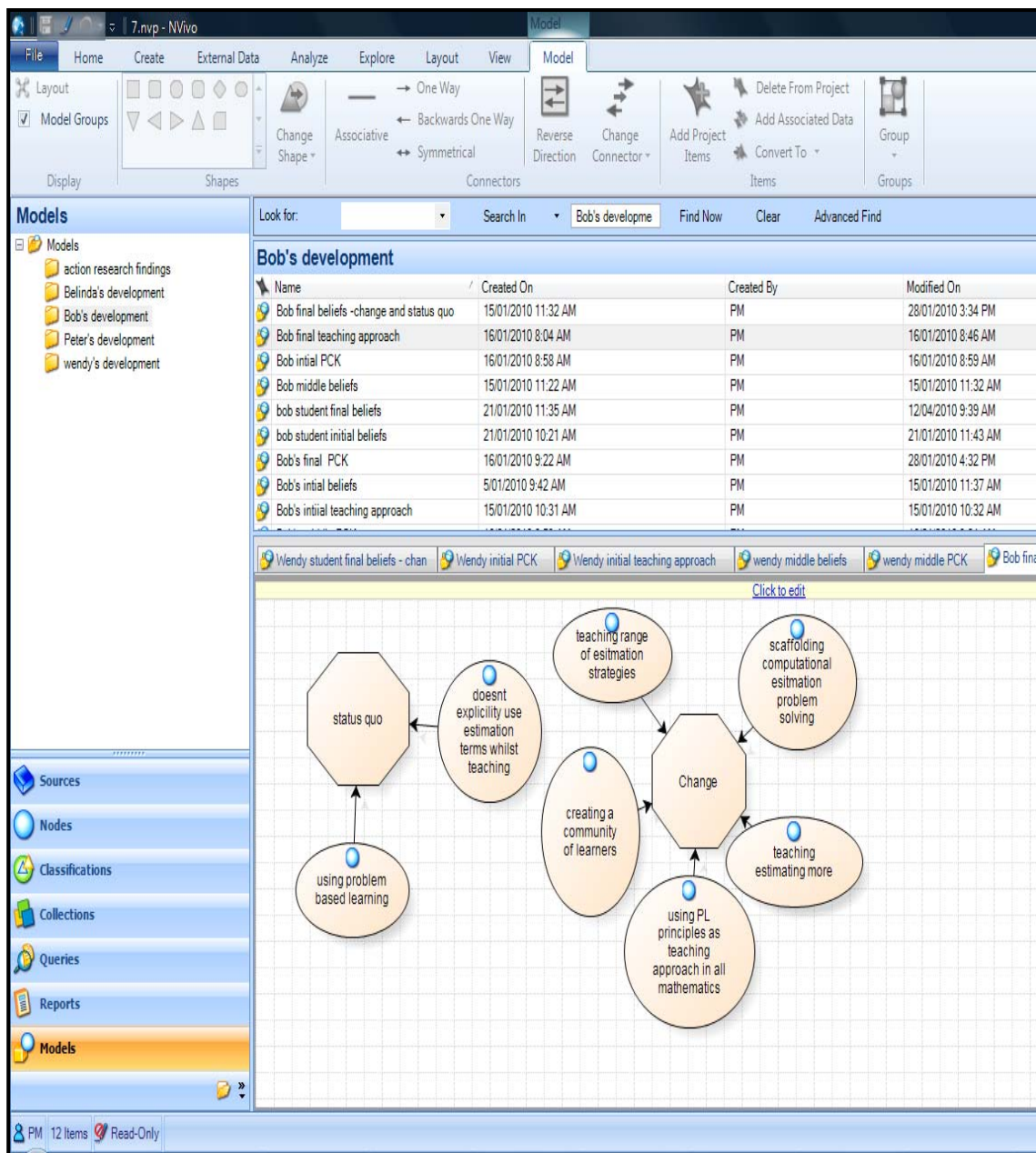


Figure 3.10: Visual models to support data analysis

These themes were captured using a process described by Erickson (1986). He suggested finding initial evidence and then grouping these findings into assertions and then grouping these assertions into general assertions (see Appendix K). This process was followed to generate a chain of evidence from the data to support the overall findings.

The second stage of the analysis explored how the professional learning affected the teachers and students. This process was not intended to prove causality but instead to

provide insights into potential changes and the variables impacting on teachers' professional learning. This process is captured succinctly by Patton (2002):

When careful study of the data gives rise to ideas about causal linkages, there is no reason to deny those interested in the study's results the benefit of those insights. What is important is that such statements be clearly qualified as what they are: interpretation and hypothesizing. (p. 475)

Within this constructivist framework, some inferential statistics were included as a mechanism for increasing the evidence available to support the findings. These statistics were able to show where the results were not due to chance (Marinez-Pons, 1999). However, interpretations need to be made with the understanding that the testing did not take place under experimental conditions. The pre- and post-test quantitative data were collated and analysed using PASW (SPSS Inc, 2010) in order to calculate changes in estimation performance on the CET. The analysis focussed on the pre- and post-test mean scores for each question and the estimation strategies used by the students. At this stage, a paired two-tailed t-test was conducted on the mean pre and post-test scores to determine if the improvement was statistically significant.

Once a detailed analysis of the individual cases had been undertaken then cross-case comparisons were made. Miles and Huberman (1994) explain that the aim of this is:

Reassuring yourself that the events and processes in one well-described setting are not wholly idiosyncratic. At a deeper level, the aim is to see processes and outcomes across many cases, to understand how they are qualified by local conditions, and thus to develop more sophisticated descriptions and more powerful explanations (1994, p. 172).

The benefit of undertaking a cross-case analysis was so that it was possible to establish some common findings and highlight the condition under which this occurred (Bazeley, 2007). In this way, it was possible to identify findings that occurred in all three cases. Where aspects differed, it was possible to consider why these were different.

By creating a "meta matrix" it was possible to undertake a systematic comparison of the different cases (Miles & Huberman, 1994, p. 177). From this meta matrix it was possible to identify possible commonalities and differences and offer some general

insights into how the process of the professional learning intervention unfolded.

Statistical data was also used to support the multiple case studies, in providing evidence about impacts on the students' performance whilst being involved in the professional learning intervention. Student performance data from the three classes were combined in order to calculate overall mean improvement in computational estimation from the pre- and post-tests. The performance of all students from the three classes was also considered question by question in order to evaluate students' performance improvement on the different questions.

Whilst conducting the cross-case analysis it was noticed that there may be an association between the students' increased estimation strategy awareness and computational estimation performance. Therefore, a Pearson Chi-square test was undertaken on each question using PASW (SPSS INC, 2010) and a Pearson product correlation test was undertaken on the students' use of estimation strategies and their performance in selecting the best estimate in the post-test.

Ethics

This study required the involvement of practicing teachers and students. Formal permission to approach schools was therefore sought and gained from the University Human Research Ethics Committee and was granted. Ethically it was important that the teachers and their schools consented freely to this research. Letters were sent to invite principals to take part in this program and four principals agreed that this program would be beneficial to their school. Six teachers from these four schools then expressed an interest in being involved in the research.

At the beginning of the school year the children in the teachers' classrooms were provided with an explanatory information letter, and consent form which they returned to the school teacher, explaining if they were happy or not to be involved in the research project. Children who did not want to take part still were involved in the professional learning activities as the teaching suggestions were all included in the WA curriculum framework. As the research progressed it was important to respect all of the teachers and their students, therefore all points of view were taken as equally valid.

All data that was collected was anonymous and was allocated a number and a pseudonym. A high level of confidentiality was maintained through this project and the de-identified data was kept in a locked filing cabinet at the university with the researcher the only person who had access to it.

Limitations

Although the focus of the action research was the professional learning process, a limitation of the research was that there was not more time spent on the classroom observation. When inviting the teachers to participate in the study the Researcher did not want the imposition on the teachers to be too great. It was important not to discourage the teachers' involvement in this innovative research project and it was the Researchers' perception from having been a primary school teacher herself, that been observed is inherently stressful. If there had been more time spent observing the different lessons there could have been more analysis of the discourse between the teachers and the students. When considering how this discourse could have been collected, video recording and analysis would have greatly added to understanding of how students learn computational estimation and allowed for more retrospective investigation of what occurred in the classroom (Sfard, 2008).

Part of the quantitative data collection used a multiple choice assessment test. Multiple choice testing certainly has limitations (Roberts, 2006). Students may not perform in the test as they would in a more relaxed mode of assessment method. In an interview method, it is also possible to probe incorrect answers to find out why the student has chosen a certain answer and find out more about the estimation strategies used. There were insufficient resources to conduct individual interviews with all of the students. The multiple choice test therefore was a pragmatic choice of data collection that allowed for all the students' computational estimation performances to be evaluated.

In order for the data collection to be meaningful and manageable, the scope of the study was only one type of school and one year group. Consequently, the findings

are limited to these contexts and in future studies it would be valuable to consider other types of schools and other year groups.

Summary

As outlined in Chapter 1, the purpose of this study was to investigate Year 6 teachers in an action research based professional learning program and determine its impact on the teaching and learning of computational estimation as a component of number sense. It was decided that an in-depth study would be appropriate using predominantly qualitative methods using a multiple case study design.

This applied research (Johnson & Christensen, 2004) endeavours to build on the experimental and clinical research which has been undertaken in the past (Dolma, 2002) and provide insights for mathematics educators as to how a teacher in a professional learning intervention may support the development of computational estimation in the primary school curriculum.

The next three chapters present the three case study findings in narrative form and this is followed by the cross-case analysis.

CHAPTER 4: CASE STUDY WENDY

Background

Wendy (pseudonym) is in her fifties and has been teaching for about 25 years. She started out teaching in government schools for three years, worked part-time when her children were small, and then has been at her present school for 22 years where she is now the Director of Curriculum (Teacher interview 1, 3/12/2008). When asked what she particularly wanted to gain from attending the professional learning days she stated that she wanted to discover more about place value as it is something “that children struggle with a lot and I don’t think they see the point of estimation” (Teacher interview 1, 3/12/2009).

The school, Green Meadow School (pseudonym), is a relatively large, K-12 low fee independent school with three classes in each year group and with few behavioural difficulties. It is co-educational with no Indigenous students (Australian Curriculum Assessment and Reporting Authority, 2010). The school grounds are spacious with sporting fields and well-maintained lawns. In Australia, as part of providing information to parents about the performance of schools, the government has created an Index of Socio-Educational Advantage (ISEA). Using nationally available statistics, 15 variables were used to create the metric with a mean of 1000 and this statistic provides an indication of the socio-economic status of the parents of students that attend the school. Green Meadow School’s ISCEA score was 1118, which is considered relatively high.

The 2008, Australian National Assessment Program Literacy and Numeracy (NAPLAN) testing revealed that their Year 5 students received an average score of 509 in mathematics compared with the national average of 476 and this cohort were the 2009 Year 6 students involved in the study. Wendy’s class of 32 students comprised 14 girls and 18 boys. She taught the students for all subjects and the class was of mixed ability.

Wendy's Views about Mathematics Teaching

When asked in the initial interview, in the year prior to commencement of the study, to illustrate one of her typical mathematics lessons, Wendy chose to describe a lesson with the Year 2 students that she was currently teaching. In this lesson, she used the Interactive White Board (IWB) to teach a division algorithm using virtual images on the IWB to explain the algorithm to the students: The students had to go to the IWB and put the images into groups. Wendy used a number of pictures that could not be divided evenly:

We started with easier ones, easier pictures and we got the kids to come up and put them into groups and we actually wanted the kids to realise that sometimes there are remainders. Even though they are only Year 2s. We got them to put them into groups and said what do we do now there is one left over? It would depend what it was as to what we did with it. We got them to do some written algorithms. We got the brighter ones to go back and do the written algorithms so we could repeat the whole lesson again with the struggling ones and they were more confident to do it on the board (Teacher interview 1, 3/12/2008).

Wendy had become excited about the new technology and found it helpful to use virtual manipulatives on the IWB in her mathematics lessons to illustrate the concepts she was explaining (Teacher interview 1, 3/12/2008).

Wendy's openness to describe this lesson, which followed the pedagogical approach of modelling a mathematical procedure and then instructing the students to implement and practise that procedure, suggests that this type of mathematical teaching may be the sociomathematical norm of her classroom (Yackel & Cobb, 1996). This pedagogical approach appears to involve combining an explanation of how to use a procedural algorithm, focussing on the digits rather than the magnitude of the number, with some concrete manipulatives used as a demonstration tool to show the magnitude of the numbers.

In the initial interview Wendy indicated that she would like to know more about how to teach primary mathematics. She was concerned that some of her students were not reaching their mathematical potential (Professional Learning day 1 observation, 18/2/2009). Each of the professional learning days began with a reflection session. In this first session, the teachers shared their views about how they thought mathematics should be taught and Wendy explained that she was worrying about

some aspects of teaching, as explained below:

If they don't do well with their end of year tests you worry, but you are still dealing with them in the class and there is no benchmarks that you are looking for and I really think that Maths is neglected. I find Maths easy but I would like to help those kids that are struggling and I would like to know where it starts, where to go back to instead of bandaid by the end of Year 6. You have got the end of year tests and NAPLAN [National Assessment Program Literacy and Numeracy] you are working towards the test rather than their problems (PL1 observation, 18/2/2009).

She was particularly concerned with how she was teaching place value and perceived that even recent teaching approaches that had been suggested to her were not helping her students to understand the numbers that they were working with. A dice activity, which involves rolling the dice to generate a number and then placing that number wherever it may create the largest magnitude, has been designed to encourage the students to realise that the same digit can hold different magnitudes depending upon its place. Wendy explained how she believed that that this activity was not contributing towards students' developing number sense:

Place value is something that children struggle with a lot ... I think place value is very hard to teach and a bit hard for the kids to get the concept. They love the dice rolling and make the biggest number but they still don't understand the place value even though in an activity like that they can do it and they can read the number after a while but they still don't understand what that number really is (Teacher interview 1, 3/12/2008).

Wendy's Views about Teaching Computational Estimation

In her initial interview, Wendy explained that teaching algorithms procedurally was her predominate computational teaching approach in mathematics and within this approach she hoped that students would use estimation to anticipate the ballpark of the answers to their algorithms. She scaffolded this for her students by asking: "What do you think it [the answer] is going to be?" (Teacher interview 1, 3/12/2008). Wendy described how she used computational estimation in her teaching:

With word problems, you need to use a lot of estimation. [Pause]... and especially with kids, struggling with word problems so you say 'Is your answer going to be bigger or smaller?' That is your first estimation (Teacher interview 1, 3/12/2008).

Despite saying this to the students, Wendy did not explain how she taught students to check their work, suggesting that there were not many of these types of activities. She mentioned that her students struggled to see why they should estimate (Teacher interview 1, 3/12/2008). It is logical to surmise that if students were solving algorithms by only considering the digits, that is, focussing on the computational ritual, then it would be much easier to check this answer by redoing the algorithm in the same way.

Key Finding 4.1: Wendy's teaching approaches of computational estimation involved informing students of the importance of estimating before completing routine algorithms.

A commercial textbook was used in Wendy's school. Students completed the pages of the textbook at the same time as each other, using the same procedure (Teacher interview 1, 3/12/2008). Wendy taught rounding as an algorithm in the way that it was prescribed in the text. It was Wendy's belief that this approach is not very authentic:

Wendy: Sometimes it becomes very fake and I think that's where the problems come. Estimate this - why? I think that there has to be purpose for estimation or there is no point in doing it.

Researcher: So where it is just a textbook exercise then kids are turned off from it?

Wendy: And the textbook will give you a right answer for it and you are thinking that's not right. It's an estimate! (Teacher interview 1, 3/12/2009).

Key Finding 4.2: Wendy perceived that the rounding exercises in her school's textbooks were not authentic and did not engage her students.

Wendy's Pedagogical Content Knowledge

Educators are becoming aware that there is specific PCK required for teaching different components of mathematics. Effective teaching of specific mathematics topics requires that teachers possess pedagogical frameworks of these mathematical components. At present, Wendy did not have a pedagogical framework for teaching computational estimation, although she was aware of some estimation activities such

as “how many MABS [multi base arithmetic blocks] do you think will fit into this room?” (Teacher interview 1, 3/12/2008). She was also not aware of any computational estimation strategies other than rounding:

Researcher: The most traditional strategy is rounding.

Wendy: Rounding doesn't always work. Do you round up or round down?

Researcher: Are there any other strategies that you have been aware of?

Wendy: Like guess and check?

Researcher: A bit like that.

Wendy: Not particularly, no (Teacher interview 1, 3/12/2008).

Key Finding 4.3: The only computational estimation strategy Wendy mentioned was the rounding strategy but she was not aware of the other strategy names.

Key Finding 4.4: Wendy did not have a pedagogical framework for teaching computational estimation using a variety of computational estimation strategies.

When conducting a search for curriculum resources about teaching the range of computational estimation strategies to primary school students in Australia on the World Wide Web (June, 2010) the Researcher found few references to suitable documents. The Researcher anticipated, therefore, that many teachers would have limited PCK about computational estimation and this was the main reason that a professional learning project had been created. This problem of schools not focussing on such areas as computational estimation as an integral component of number sense teaching was noted in the Researcher's audit trail journal:

Through discussions with my supervisors, it was quickly realised that so little computational estimation is being undertaken here in WA that the results [of just testing students and finding out what strategies they knew] may be very inaccurate. If we had proceeded down this line we would have only found out what students could do intuitively rather than what they were capable of if they had been taught (Audit trail journal, 13/6/2009).

Students' Beliefs about Computational Estimation

When they were interviewed at the beginning of the year, Wendy's students suggested that they viewed mathematics as something with one correct answer, that was done quickly (Student focus group 1, 10/2/2009). When they were asked what they would do if someone had a different answer to them, Bill's (pseudonym) answer implied that he perceived school mathematics as being about one correct answer:

Researcher: Your friend has a different answer to you on a maths problem. What do you do and why?

Bill: You might just give them a hint or something, that they may have got it wrong but you don't tell them the exact answer because that means that they won't learn anything (Student focus group 1, 10/2/2009).

When asked if they thought someone who spent 15 minutes on a question was clever, most of the students thought that amount of time was too long to spend on a problem. Alison explained her thoughts in the following response:

Researcher: Bill sat working on one mathematics question for 15 minutes. Do you think he is clever?

Alison: I don't think he was very clever. He should have gone on to the next one and then if he had spare time he should have gone back to that one. Otherwise he could spend the whole time that he was meant to be doing the test, just doing the question and he would get a worse mark than if he had done all the other ones and gone back to it (Student focus group 1, 10/2/2009).

The students also talked about mathematics as being about equations and the four operations.

Key Finding 4.5: The students in Wendy's focus group believed that mathematics is something about the four operations, something with one right answer and is done quickly.

One of the students mentioned that mathematics is used in the real world. When the students were asked to write down what they thought estimation meant, the students who replied used the word 'guessing' to most clearly describe their understanding of the term although they appreciated that it was more than a guess.

"Estimation is something that you do before knowing the answer, estimation is guessing in a mathematical way" (Emily, Student focus group 1, 10/2/2009). Bill

wrote “it is if you get a hard maths question, trying to guess the answer before you work it out, e.g., 20×21 estimate = 400”. These students did not think that an estimate was a correct answer (Student focus group 1, 10/2/2009).

Key Finding 4.6: The students in Wendy’s focus group believed that estimation is a mathematical guess.

Students’ Computational Estimation Competence before the Professional Learning Intervention

All the students in Wendy’s class were asked to complete the Computational Estimation Test (CET) (see Appendix I), and their responses to the six estimation multiple choice questions revealed some interesting insights into the students’ estimation competence at the beginning of the project. Figure 4.1 revealed that in all six questions student performance in selecting the best estimate was higher than their performance in selecting a reasoned estimation strategy.

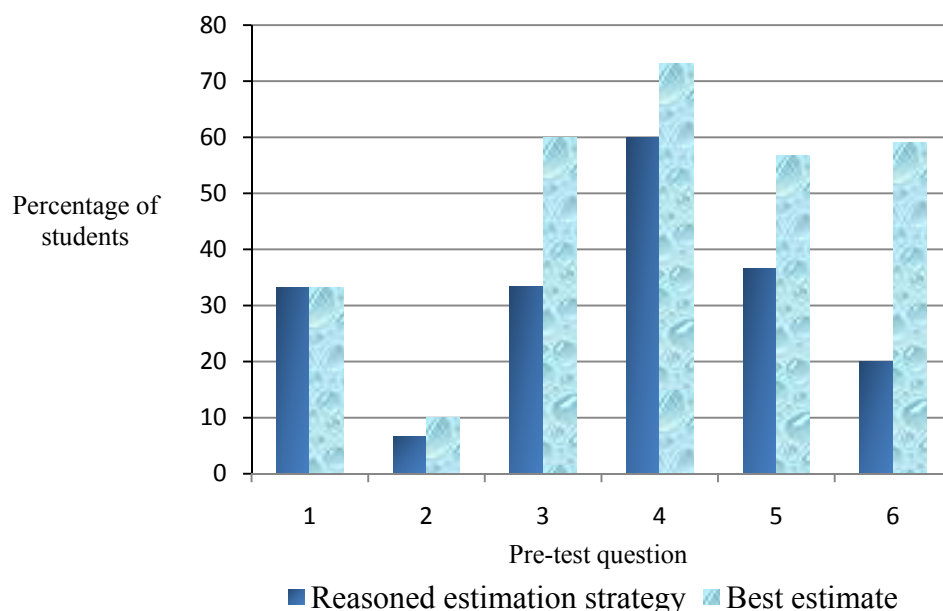


Figure 4.1: Percentage of students using a reasoned estimation strategy and identifying the best estimate before the professional learning intervention.

Estimation Question 1

To solve this problem it was expected that a student would round their age down to 10 years old, round the number of days in the year down from 365 to 300 and then undertake the multiplication, 300 multiplied by 10. As Table 4.1 shows, only 33% of the students in Wendy's class stated that they had lived for 30 000 days, which was the best estimate (B). The other responses were (A) 300, (C) 3000 and (D) 300 000.

This question was also included in the Number Sense Test (NST) (McIntosh et al., 1997) which was administered in 1997 to a representative sample of Australian students which consisted of 167 students in the 10 year old test and 168 students in the 12 year old test. In this test, 35% of 10 year olds and 38% of 12 year olds were able to arrive at the best estimate. This would suggest that the students' ability to answer this question was similar to the Australian sample in the NST (McIntosh et al., 1997).

As this study was interested in students' use of computational estimation strategies, it is worthwhile considering what strategies were used to answer the question. Table 4.1 shows that only 33% of the students used rounding to mentally solve the problem and as this strategy is central to the successful completion of the task this may explain why so many of the students did not arrive at a suitable estimate. This assertion is supported by the fact that 85% of students who used the rounding strategy were able to arrive at the best estimate. This would suggest that students may be more successful at finding the best estimate on this type of question if they were taught to use the rounding strategy in authentic problems. Table 4.1 also shows how 33% of the students in Wendy's class attempted to answer this without approximating the numbers. Due to time constraints, trying to use exact calculations would be very inefficient and therefore it is not surprising that only 10% of students attempting to answer it 'exactly' managed to obtain the best estimate.

Table 4.1: Per cent of students selecting various answers to Question 1, and computational estimation strategies used (n=30)

Answer	Per cent					
	Rounding	Guess	Not enough information	Exact	Intuition	Total
1A	0	0	0	0	0	0
1B*	20	0	7	3	3	33
1C	10	7	3	23	7	50
1D	3	7	0	7	0	17
Total	33	14	10	33	10	100

Note. * denotes the best estimate

Estimation Question 2

To solve this problem the students had to simplify these relatively complex fractions into easily computed whole numbers. It was expected that the students would approximate the two fractions to the whole numbers one and one, which would produce an appropriate estimate of two. This question was designed to explore whether students were able to undertake computational estimation with fractions using the benchmarking strategy and relating the fractions to easily visualised whole numbers. This question has been used since the early 1980s when in the 1981 North American national testing (Post, 1981) it was found that only 24% of 13 year olds could calculate the best estimate of 2 . As Table 4.2 shows, only 10% of Wendy's students were able to select the best estimate for this question suggesting that they were not able to use this strategy and that most students in the class lacked a conceptual understanding of fractions.

Table 4.2 shows that 10% more students went for answer C (19) rather than D (21) suggesting that most students added the 12 + 7 which were both numbers in the fraction. In order to gather more information about how students answer these types of questions, they were asked similar ones in the focus group interviews (Student focus group, 12/2/2009). One of the children explained that to add $\frac{11}{12}$ and $\frac{7}{8}$ you would work on them as whole numbers: 12 plus 8 = 20 and 11 plus 7=18 so wouldn't it be closer than $\frac{18}{20}$ (Peter, Student focus group, 12/2/2009).

Table 4.2: Per cent of students selecting various answers to Question 2, and computational estimation strategies used (n=30)

Answer	Per cent				
	Bench- marking	Guess	Whole number thinking	Intuition	Total
2A	0	0	0	0	0
2B*	7	0	3	0	10
2C	0	13	37	0	50
2D	0	7	30	3	40
Total	7	20	70	3	100

Note. * denotes the best estimate

It appears therefore that students did not know how to approximate complex fractions to more easily visualised benchmarked whole numbers and this assertion is supported by the fact that only 7% of the students used this strategy and that 70% used whole number thinking to answer this question (Table 4.2).

Estimation Question 3

To solve this problem, students had to count how many triangles there were but due to the time constraints, they were not able to count them individually. As Table 4.3 shows, 59% of students were able to obtain the best estimate. This was a similar finding to the results of the Number Sense Test (McIntosh et al., 1997) where 54% of 10 year olds and 62% of 12 year olds obtained the best estimate for this question. Table 4.3 shows that 30% selected the incorrect answer 100 (C) which is an underestimate, whereas only 6% selected the overestimate 500, (E), and no students selected the extreme underestimate 50 (A). Table 4.3 also shows that 56% of those students who got the answer correct used an intuitive strategy suggesting that this is an efficient strategy albeit one that has little reasoning attached to it.

Table 4.3: Per cent of students selecting various answers to Question 3, and computational estimation strategies used (n=30)

Answer	Per cent					Total
	Range	Sample	Guess	Not enough information	Intuition	
3A	0	0	0	0	0	0
3B	0	3	0	0	0	3
3C	3	10	7	0	10	30
3D*	3	13	7	3	33	59
3E	0	0	3	0	3	6
Total	6	26	17	3	46	98

Note. * denotes the best estimate

Estimation Question 4

To solve this problem, students were expected to multiply 45 by 100 and then compensate as the 105 was rounded down. Table 4.4 shows that 73% of students selected the best estimate (B), which is 4600. This result is 13% higher than in the Number Sense Test (McIntosh et al., 1997) for 10 year olds (they did not give this question to the 12 year old cohort). This high success rate suggests that they had been instructed in the past how to multiply by 10 and on the rounding strategy in symbolic questions.

Only 6% of students selected the underestimate of 4000 (A) and only 20% selected the estimate that was the greatest overestimate, which was 5200 (C). This question requires a similar approach to Question 1 i.e., using the rounding strategy and then multiplying, but they did not have to interpret the context. The students were far more successful at this question where the multiplication was set in symbolic terms. Possibly, they could simply implement a learnt procedure rather than have to interpret a context, which would require a more in-depth understanding of the mathematical concept of multiplication.

As Table 4.4 shows, the most popular strategy used to answer this question was rounding. Of the 56% of those students who selected the best estimate, 54% used the rounding strategy. This suggests that, at the beginning of the study, most of the students were able to use rounding to solve a two digit by a three-digit multiplication.

Table 4.4: Per cent of students selecting various answers to Question 4, and computational estimation strategies used (n=30)

Answer	Per cent					Total
	Rounding	Front End loading	Guess	Not enough information	Intuition	
4A	3	0	3	0	0	6
4B*	40	3	13	10	7	73
4C	13	0	3	3	0	19
Total	57	3	20	13	7	100

Note. * denotes the best estimate

Estimation Question 5

To solve this problem students could either focus on the front-end digits to produce an underestimate, they could use the nice number strategy and add those together, i.e., $20 + 80$ and $40 + 60$, or they could round the numbers to produce a quick estimate. As adding two digit numbers is an early computational skill it is expected that there would be competency in this area. Table 4.4 shows that 56% of students were able to select the best estimate.

Considering how straightforward this sum is, it is surprising that the percentage of students selecting the best estimate was not higher. This suggests that students may rely on their procedural algorithms to answer this type of question and not consider the value of the digits. The second most popular answer was 165 (A) which was quite a large underestimate. There was not one predominate strategy used, suggesting that students had not been taught to focus on a particular type of estimation strategy for this type of computation, instead possibly relying on algorithms to answer this type of question (Table 4.5). When asked a similar question in the focus group interviews some of the students were able to identify a front end loading strategy and a rounding strategy, suggesting that some students use these strategies intuitively without being taught them (Student focus group 1, 12/2/2009).

Table 4.5: Per cent of students selecting various answers to Question 5, and computational estimation strategies used (n=30)

Answer	Per cent							Total
	Rounding	Benchmark	Range	Front end	Guess	Not enough information	Intuition	
5A	0	0	0	3	3	7	3	16
5B	0	0	0	0	3	3	7	13
5C*	13	3	3	14	10	10	3	56
5D	0	0	0	0	3	3	7	13
Total	13	3	3	17	19	23	20	98

Note. * denotes the best estimate

Estimation Question 6

This task was designed to determine if students could identify an estimated answer within a range. This could be undertaken by reasoning that the answer should be between 6×3 and 6×4 . Therefore the answer would be somewhere between 18 and 24. As shown in Table 4.6, over half of the students, 59%, were able to select the best estimate.

Only 7% of students thought that it could be 16 -18 (D). The fact that few students selected D means that probably most students knew that $6 \times 3 = 18$ and that the answer could not be less than that. Table 4.6 shows that rounding was the most popular strategy but 36% of students were not able to articulate how they obtained the best estimate.

Table 4.6: Per cent of students selecting various answers to Question 6, and computational estimation strategies used (n=30)

Answer	Per cent					Total
	Rounding	Guess	Not enough information	Exact	Intuition	
6A	7	0	3	3	3	16
6B*	23	10	23	0	3	59
6C	10	3	3	0	0	16
6D	0	0	7	0	0	7
Total	40	13	36	3	6	98

Note. * denotes the best estimate

The overall mean test score on the computational estimation pre-test was 2.9/6. The students were much more proficient at estimating when a similar question was presented symbolically, i.e., in Question 4, 73% of students selected the best estimate, rather than when a similar multiplication question was presented in a context i.e., in Question 3, 33% of students selected the best estimate. Figure 4.1 shows that the students had varying degrees of ability in using reasoned estimation strategies before the professional learning intervention began. Relatively few students were able to use an estimation strategy to solve a multiplication calculation set in a context and they were more successful at using a reasoned estimation strategy when it was presented symbolically and in that situation, they undertook the rounding strategy, which they had spent time on in previous mathematical lessons.

Key Finding 4.7: Students had a much higher competency when estimating the answer to symbolic mathematical problems than contextual problems.

Most students were unable to estimate the answer when adding two fractions with unlike denominators. Virtually none of the students were able to use an estimation strategy to convert complex fractions into easily visualised whole numbers.

Key Finding 4.8: Almost all of the students were not able to use an estimation strategy to convert complex fractions into easily visualised whole numbers.

The students did not appear to be proficient at estimating two digit addition calculations. It is surprising that over half the students were not able to use a reasoned estimation strategy in order to add four two digit numbers as these are numbers that are worked on regularly from Year 2/3 in the primary school.

Key Finding 4.9: Around half of the students found it difficult estimating when adding two-digit numbers quickly and less than half the students were able to use a reasoned estimation strategy.

Response to the Action Research Professional Learning Process

This research engaged six teachers in three cycles of professional learning using an

action research approach which is considered a suitable approach for teachers to reflect on their practice (Somekh, 2006). The teachers worked in this collaborative research group and focussed on the question: How to teach computational estimation as a component of number sense? This was a genuine question due to the absence of any recent research concerning how to teach computational estimation in the primary school. The following findings demonstrate how the teachers reflected on this question and how changes in their beliefs and PCK about computational estimation impacted upon their teaching approaches. In order to provide as rich a description of this process as possible, the students' responses to these teaching approaches were also considered as part of the case study. This narrative includes excerpts from a variety of sources including the professional learning handbooks, which outlines the three action research cycles.

The first research cycle: Reflecting and planning

The research was to be collaborative using the expertise of all the teachers and this aim was spelt out at the beginning of the research project:

I will come up with ideas and things and you may sit there and think well that's not what I see in the classroom and that is really to be valued and hopefully by the end we can really improve on what research has said in the past (Facilitator, PL1 observations, 18/2/2009).

One difficulty of this professional learning program was establishing the credibility of the program when so little is known about successful avenues for teaching computational estimation. The researcher reflected on this at the beginning of the professional learning program:

This is a genuine activity [asking the teachers how to teach computational estimation] in that there are hardly any curriculum resources available for the teaching of computational estimation. This lack of resources made my position difficult at the beginning of the research. I had flagged a topic which there is little written about and not many people in western Australian primary schools are involved in so it was not possible to know for sure that the Year 6 students would gain from this process. This made everyone, myself included, a little nervous. Yet as the time went on and students in the different classes could understand the concepts my confidence grew. Of course it would have been nice to have had this confidence at the beginning of the PL and conveyed this to the teachers but until one is prepared to step out and try things, genuine conviction cannot be attained (Audit trail journal, 7/09/2009).

The first day focussed on the first professional learning principle, that is, mathematics teaching is effective when the focus is on active learning. The professional learning began using established research literature (Carpenter et al., 1999) which had documented how very young students were able to solve addition and subtraction problems without being explicitly told what procedure to use. Wendy reacted very positively to this statement below, despite the approach not being part of her usual teaching style:

PL Facilitator: The idea is that research has shown that 74% of kindergarten students can work on their intuitive knowledge. “Carla has \$7. How much more will she need to buy a puppy for \$11”. So you might or might not find it interesting that kindies can resolve that ...

Wendy: I do think we underestimate what good kids can do and I think that we turn them off. We try and teach them ‘this is number’ (PL1 observations, 18/2/2009).

The initial professional learning day was the first time that Wendy had heard about using computational estimation strategies other than rounding (Teacher interview, 3/12/2008, PL1 observations, 18/2/2009) but she was open to hearing about this new information. The professional learning facilitator introduced the estimation strategies to the teachers using an extended version of the computational estimation test (CET) (PL1 observations, 18/2/2009). This test had been created for students but as the strategies were new to the teachers, it was also suitable as a learning activity for them. By working through the questions, it allowed the teachers to begin to identify how the computational estimation strategies could be used. On the question “How many triangles?”, initially Wendy said, “Is it intuition?” The professional learning facilitator was able to highlight to Wendy the benefit of the sample strategy rather than using the less sophisticated intuition strategy (PL1 observations, 18/2/2009). Due to time constraints, it was not possible to provide Wendy and the other teachers with a detailed exploration of all the strategies on the first day but they were able to get a general overview and so they could begin considering how they might incorporate into the curriculum (PL1 observations, 18/2/2009).

In the afternoon, the teachers were introduced to the part of the action research process that involved trialling the ideas presented in the professional learning workshops. It was explained that the research literature indicated that estimation

might be useful in the primary classroom in two main areas. The first was as a checking tool when doing exact calculations and the teachers were shown the flow chart created by Bobis (1991) which suggested that students work through a process of estimate, calculate and evaluate. The second suggestion from the literature was that at times it is worth presenting problems where estimation is the main computational choice (Yoshikawa, 1994). The teachers were asked to consider some suggested learning tasks where estimation was the main computational choice and adapt these to suit their personal context.

The first research cycle: Acting and reflecting

The teaching approaches presented on the first professional learning day were quite different to Wendy's normal teaching approach and so she needed time to reflect on how they could be incorporated into the classroom. On the first visit by the Researcher to Wendy's school, Wendy did not seem to have had time to reflect on the professional learning day (Audit trail journal, 6/3/2009). This necessity to reflect on professional learning in a series of cycles rather than it just be a 'one shot' professional learning activity made it a fairly intense process but in her interview Wendy reflected how, in the end, she felt this was beneficial:

Researcher: But did you think about the ideas discussed on the professional learning or did you feel when you got back you were just too busy to implement them?

Wendy: I knew I had to implement them 'cos [because] you were coming which was good because it meant that I found a time without too many interruptions to do those well (Teacher interview 3, 18/11/2009).

Wendy did manage to find the time, despite being extremely busy, to reflect on what she thought would be appropriate to teach the class about computational estimation. She was happy for one of these lessons to be observed and this observed lesson was one of the first she taught about computational estimation. The class was set up with groups of tables, and Wendy was at the front where her IWB was located. She started with a number sense game using the IWB, working with the whole class (Classroom observations 1, 23/3/2009). The discourse in the classroom for this part of the lesson was predominately teacher controlled with closed questioning.

- Teacher: Maud, you have done so well you can spin the next one. Largest [number] again, the first number is 2 [Child spins the next number].
- Teacher: Right it is on 10, so that is 0 [The spinner didn't have 0].
- Teacher: 5[spins again], 5 [spins again], 3 [spins again], 6 [spins again]. How many numbers do we need?
- All students: 1
- Teacher: Put your hand up if you want a big number; put your hand up if you want a small number. Just wait a minute you are not listening. Yes, right shhh.
- Students: Hurray. 7.
- Teacher: Simon, did you make the biggest number? What number did you make?
- Simon: Seven million six hundred and fifty five thousand three hundred and twenty.
- Teacher: Well done. Who made that one? (Children put their hands up).
- Teacher: Fantastic (Classroom observations 1, 23/3/2009).

After the game, Wendy introduced the newspaper learning activity that was suggested in the professional learning workshop. The aim of this activity was to convey to the students that computational estimation made sense in the real world. Continuing with a whole class teaching approach for the introduction to the main part of the lesson, Wendy initiated a relatively closed questioning type of discussion. She found an article in the day's newspaper, which was about the issue of the State's roads and it had a large amount of estimated data within the article.

The students were all listening intently and some of the students answered questions revealing that they were able to identify the differences between numbers, which looked as though they were exact, and those that appeared to be an estimate. The following is an excerpt from some of the recorded discourse:

- Teacher: What about the 2990 km of regional roads?
- All students: No.
- Teacher: Probably it is fairly accurate 'cos otherwise they would probably say 3000 km of regional roads wouldn't they so we will call that one. Right we will choose someone else to hold the pen [New child comes to the front].

Teacher: Okay, see the paragraph down on the third column, Professor Cameron estimated that the point-to-point cameras would simply issue 850 000 speeding fines. Do you think that is accurate or estimate?

Students: Estimate (Classroom observations 1, 23/3/2009).

As it was a new topic to both teacher and students, it is understandable that they were just beginning to explore the concept of estimation. An example of this is that the issue of measurement always being an estimate was not addressed in this discussion. However, they did begin to see that numbers that were multiples of tens were often used and that there are different reasons why estimation is useful. Wendy explained that the students were particularly interested in the idea that estimation could be used to deceive people about the true amount (Informal teacher interview, 27/3/2009).

After the whole class introduction, the class worked in pairs to explore the differences between exact numbers and estimates. In this section of the lesson, the discourse was student to student. Wendy scaffolded this by providing a pre-prepared table with the heading to place the numbers in the first column. In this first column, the students had written in one colour if it was an estimated number and another colour if it was an exact number. In the next column, the students had to copy the words from the article which had signalled to the students that it was estimation. Figure 4.2 shows how the activity encouraged students to consider the estimation language that is used in the real world i.e., 'more than or about'. The work sample showed that the student Nigel recorded that when decimal points are included by the journalist then it probably is an exact number.

Estimate

Exact

Estimation in Everyday Life.

Number	Estimation words used in the article.
250	A colony of 250 animals
48	Lelit Janchoom
2	About 2 Baht
8¢	8¢ each
6500	Bought him for 6500 baht
£115m	Some £115m has meanwhile bought
£24.4m	exact season cost Chelsea (500,000) way back in August 2005 (decimal point)
Six months	out for six months
24 hours	struggle against Liverpool 24 hours
£45m	£45m-plus spent on a team
£33m	£33m Brazilian for whom City beat
\$14.7	\$14.7 billion dollar (decimal point)
2000	more than 2000 students
800	about
400	more than
50	just
\$100 million	more than
\$108.8 million	\$108.8 million (decimal point)
\$128.3 million	\$128.3 million (decimal point)
\$73.6m	\$73.6 (decimal point)
4	4 gold medals
\$12.6 million	\$12.6 million (decimal point)
80,000	more than
\$5 million	more than
45,000	up to

Nigel

Figure 4.2: Example of estimation and exact numbers found in the newspaper

The students worked in pairs to complete this task. After a while, some of the students started to go off task a little, so it was beneficial that Wendy drew them back into a whole group discussion, as noted in the observation record:

I think it was very good that the teacher actually drew the students back to make them think about what they had found in the articles and certainly lots of the students were able to talk about estimation words and purposes for estimation (Classroom observations 1, 23/3/2009).

At this stage Wendy used the IWB to summarise and record what words the students had found in the newspapers and as she recorded the words she asked the students whether the words suggested whether the numbers were exact or were estimates (see Figure 4.3).

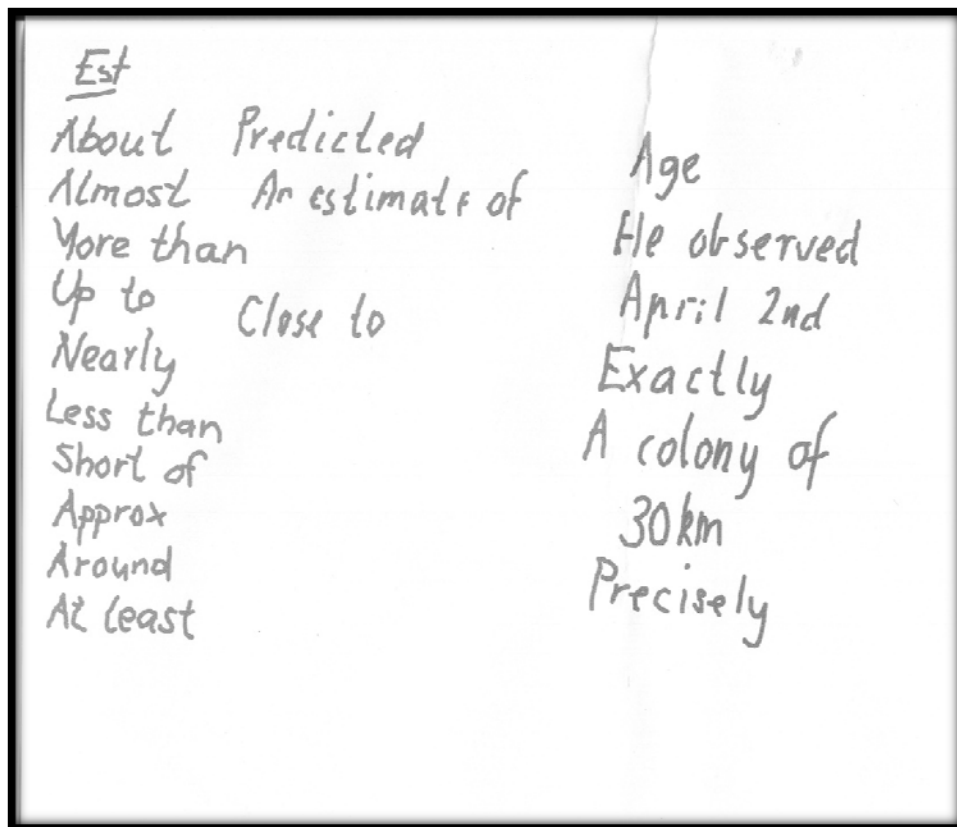


Figure 4.3: Collated words, estimates and exact, on the IWB

The lesson went for most of the afternoon session and Wendy was appearing far more confident and seemed to be enjoying teaching this aspect of mathematics. When the Researcher visited Wendy shortly afterwards a change was noted and this was recorded in the audit trail journal:

I went out to pick up the work from Monday and Wendy had changed in her attitude towards the study. She was smiling and friendly (Audit trail journal 27/3/2009).

Wendy sent an email to inform the Researcher about the students completing the shopping role play, which was designed to encourage computational estimation using mental calculations (Email from Wendy, 8/4/2009), and her comments were much more positive in the second professional learning day: “You were there for the newspaper activity. That was really good. They really saw the value of the estimation activity” (PL 2 observations, 13/5/2009).

Acting on her set of beliefs as to what she thought were suitable teaching approaches for school mathematics, Wendy still taught from the textbook. On an informal visit, she shared some concerns she had about her teaching from the textbook, as noted in the audit trail journal:

She was very concerned about a lesson that she had just taught. It was taken from the textbook that they have to follow and was a procedural exercise multiplying three-digit numbers by a one-digit number. They were all doing the same activities and she was trying to explain to them how to do it in a procedural fashion. It was a great contrast to the type of activity that they have been working on in the professional learning (Audit trail journal, 27/3/2009).

The second research cycle: Reflecting and planning

The second professional learning day was mostly concerned with explaining the estimation strategies to the teachers and addressing the third principle of the professional learning, that effective teaching uses appropriate contexts as a representation for abstract mathematic concepts.

As Wendy arrived for the professional learning workshop, it was clear from her demeanour that she was much more enthusiastic about the project (PL2 observations, 13/5/2009). In her initial reflection at the beginning of the day, Wendy shared how she believed that mathematics should be relevant to children:

- Wendy: We had this page in the maths book so we are trying to get that finished too and it was a practical example in the text book otherwise you need to do practical activities that was practical for them even though the text book tried to do practical activities it wasn't real life to them.
- Researcher: So that is very interesting about whether it is interesting to them. Is it real world to them?
- Wendy: The surf shop and stuff like that [activities from the PL] were more relevant to them than the maths book [text book] saying the carpenter was cutting things (PL2 observations, 13/5/2009).

Wendy had asked the students towards the end of Term 1, to estimate how they would spend a million dollars. They chose the items from a variety of sources and when they found the exact price, they did not have to write it down exactly - they simply estimated. After completing these activities, the teacher provided work samples for the Researcher to use. As Wendy did not focus on the different estimation strategies at this early stage, it was logical that the students only used rounding as a strategy for estimating, even if they were quite flexible as to how they rounded. It is of interest that the student in the example below was inconsistent in their use of rounding. Figure 4.4 shows one student's response to this activity.

Purchase	Research on prices	Created estimate	Running total of estimates
King Charles Spaniel Puppy	www.pupst4sale.com.au	\$1 000 00	\$1 000 00
House	www.homenet.com.au	\$740 000	\$741 000
Car	www.toyota.com.au/HybridSynergyDrive	\$90 000	\$831 000
Phone	www.1.au.shopping.com	\$360	\$831 360
Digital Piano	www.piano.world.com.au	\$1 800	\$833 160
Laptop	www.laptop.com.au	\$1 476	\$834 660
P.M.H donation	/	\$75 300	\$909 960
Home less Charity	/	\$75 300	\$985 260

Figure 4.4: Work sample showing how estimates can be used when calculating purchases worth one million dollars

One student, Alex (pseudonym), wrote that when estimating with money it is often worthwhile rounding up (Figure 4.5). This was discussed in the professional learning workshop as a strategy that avoids the shopper going over budget.

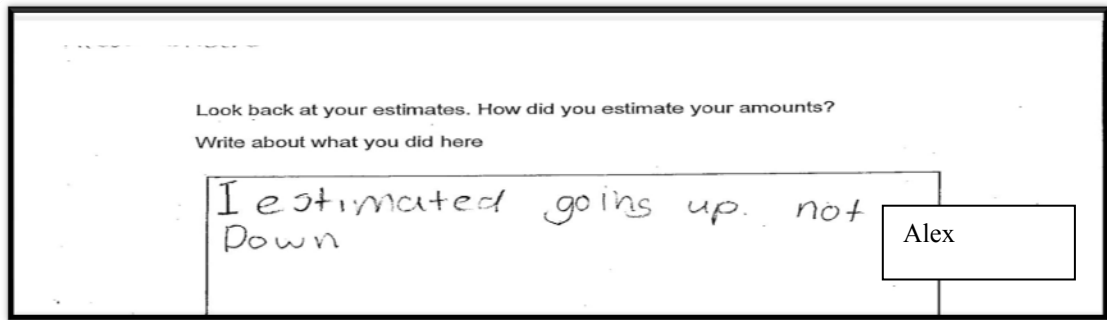


Figure 4.5: Work sample showing student articulating why how he rounded up when estimating

Wendy was open to and interested in learning about the estimation strategies and believed that, as well using the strategy names provided, it may help the students to include the term 'balanced rounding' where one number is rounded up and the other number is rounded down (PL2 observations, 13/5/2009).

During a further session in the professional learning workshop on the value of contexts, which included the use of games, the issue of using large numbers in the primary mathematics curriculum arose. Wendy felt comfortable using problems set in context with large numbers, although during a discussion about this she had made the point that, in her opinion, it was a good idea to start with working on problems with smaller numbers when a new concept was being introduced. Wendy thought that once students were comfortable with the concept it was then effective pedagogy to move to larger numbers rather than working with large numbers from the outset. In this excerpt below she explains that rather than ask the question, "Could you fill the room with cubes?" to the students, which is asking the students to conceptualise large numbers, it would be better initially to ask the students to think about a smaller amount such as one cubic metre first:

Contributor : Don't be afraid of getting into the large numbers, kids will really get into it, to help kids come to that kind of thing 'cos whether you like it or not large numbers are an important part of our understanding.

Contributor: Would a million of those fit inside this room? [Shows a MAB cube with a thousand cubes].

Belinda: Yes.

Contributor: Why?

Belinda: Cos I know the size of a hundred block and I know the size of a thousand block.

Wendy: I tried to make a cubic metre so the kids can visualise it. When you move from the simple to the large number the process is the same, but the comprehension is much harder (PL2 observations, 13/5/2009).

In the afternoon Wendy worked through the suggested learning tasks and considered how she would choose to implement them. Wendy and her colleague seemed to think that the Fermi problems would be suitable for Year 6 and decided that their students would benefit from working with large numbers (PL2 observations, 13/5/2009). An excerpt of the unit plan is shown in Appendix F.

The second research cycle: Acting and reflecting

The second observation of Wendy's teaching took place near the end of term (Classroom observation 2, 1/7/2009). Wendy conducted a reflective lesson in order to discuss the students' understanding of the activities that she had been teaching. Wendy had learnt some estimation strategies in the professional learning workshop and now started using the more formal strategy vocabulary with her students. Wendy gave the students explicit explanations of the meanings of the computational estimation strategies using the IWB. She used the paper posters that had been provided at the workshop (Appendix J) and scanned them so that they could be viewed easily by the whole class on the IWB. Having the written descriptions meant that the students' introduction to this mathematical language was correct and clear. What emerged in the second observation reveals the readiness of students to incorporate new mathematical language into their discourse. The children in Wendy's class started using the terms immediately. Wendy asked the students to explain when they had used various strategies in the different activities:

Wendy: Front end loading, who can tell me when you use front end loading?

Scott: You need to focus on the front two digits, say you have got 435 and 328 first number which is 4 that's 400 and 3 which is 300 and then when you plus them together you get a rough estimation of what total could be.

Wendy: Thanks Scott. When might we want that one?

Scott: Like higher numbers.

Ellie: When we did our millionaire project (Classroom observations 2, 1/7/2009).

Key Finding 4.10: Wendy explained the estimation strategies explicitly using formal terminology.

Wendy then used the IWB to find an estimation game that was available on the internet. This game was not one suggested at the professional learning workshop but the workshops had emphasised that it was important that teachers pick what they thought was pedagogically appropriate. The game involved the students using their estimation strategies in a variety of computational scenarios. The game required the students to initially round the numbers in the computation, as it was too difficult for them to be calculated mentally, undertake the operation and then place the number within a predetermined range. In one example, the student had rounded the numbers to 30×60 and then the student placed the answer into the can on the right at the bottom of the screen, which was in the correct range (Classroom observations 2, 1/7/2009). Wendy had said in the first professional learning workshop that she believed that games were very worthwhile and here she used the game to engage the students to estimate using the rounding and range strategy. One of the last activities of the lesson involved fractions. Working as a whole class, the students had to decide whether the fractions that were shown on the IWB were more or less than a half using the benchmark strategy. Wendy was quickly able to access some virtual manipulatives available on the IWB to provide the students with a representation of the symbol to support the students' learning process.

When talking to Wendy on the following day (Teacher interview 2, 2/7/2009) she was able to reflect on how she now believed that computational estimation could be a strategic number sense process:

Wendy: Yeh, I see a lot more point in estimation, I used to think it was for kids to make sure their answer was near the correct answer and I used to teach it that's why you did it like that but now I see that there is another point to it ...its been really good because the kids that are actually seeing that it not all about getting the right answer some of the time and using the strategies.

Researcher: They were able to identify those strategies.

Wendy: And I think they are seeing the value of estimating gradually whereas before when it was a textbook thing they don't see the point.

Researcher: So are you finding that it is spilling over into the everyday work.

Wendy: Yes but not with all of them (Teacher interview 2, 2/7/2009).

Wendy believed that she had understood the estimation strategy terminology and found it to be straightforward. She was able to explain in the interview how students could undertake computational estimation using different strategies.

Researcher: A child in Mr Clarke's Year 4 class wanted to find an estimated answer for the question $21 + 28 + 19 =$. Describe what estimation strategies the child could use to solve this problem?

Wendy: Rounding, nice numbers (Teacher interview 2, 2/7/2009).

Key Finding 4.11: Wendy was able to answer questions concerning the use of computational estimation strategies, suggesting that she understood these estimation strategies.

Wendy asserted that the students had found the computational estimation terminology beneficial when problem solving as noted on the second professional learning day:

Wendy said that her students enjoyed using the grown up terms of the estimation strategies and they had been able to understand and use them. She said that the students had enjoyed the Fermi problems (PL2 Observations, 29/7/2009).

Key Finding 4.12: Wendy believed that students are motivated by computational estimation activities.

The critical friend, who made observational notes and attended the workshops, in order to increase the internal validity of the study, also noted that it did seem at this stage that Wendy had two sets of beliefs running. One set of beliefs were her beliefs concerning 'school mathematics' (Richards, 1991). The other set were 'transitional beliefs' that appeared to be growing from being part of the professional learning intervention and set in the context of an 'ideal world' i.e. "normal maths vs. estimation work" (Critical friend's observations, 29/ 7/2009).

The third research cycle: Reflecting and planning

To further develop the PCK of the teachers, the third professional learning day

focussed on the second principle of the professional learning, that is, with primary school students, the teacher needs to take a central role in scaffolding the metacognitive learning of the students.

The teachers were asked to consider how estimation has a role to play in this process. It was explained that estimation as a checking device is part of metacognition and that if students were used to estimating it might become part of their metacognitive toolbox. The Professional Learning facilitator produced a Power Point to highlight how teacher prompts such as: “Ask questions such as do I need an exact or an estimated answer? may be used when problem solving” (PL3 PowerPoint, 29/7/2009). Wendy explained that some of her students did not want to estimate as they were rushing to finish and that students do not like looking at their own errors (PL3 Observations, 29/7/2009).

The suggested learning tasks provided in the workshop indicated that the teachers could take the students to the nearest park and Wendy became anxious at this stage about taking the students off campus and all the implications of this. The plans only suggested that it would be a possibility to go off the school grounds to make it more realistic and she was reassured that these were only suggestions and that she should implement it however she felt was the most appropriate.

The third research cycle: Acting and reflecting

In the third observation of Wendy’s teaching, she was working on an activity using computational estimation to plan a trip to the park (Classroom observations 3, 22/10/2009). She took the activity straight from the professional learning workshop materials. She was worried that her teaching approach would not give the students enough scaffolding but the planned guided inquiry booklet provided clear guidance to the student as to how to organise the activity. Wendy introduced the main task of planning a barbeque telling them that they had to “choose which one (park) is best, budget for the food, work out how much it will cost, make a Power Point” (Classroom observations 3, 22/10/2009). She reminded the students of the estimation strategies by putting the descriptions of the strategies up on the IWB and indicating how the students could use these when working on the problems:

Teacher: We have to bring in the estimation strategies we have been talking about because the prices will vary from day to day we can only have an estimate 'cos we can't really have an accurate amount. You need to use friendly numbers if we are looking at prices and something cost \$3.99 and you need 10 or 12 or 15 packets of them what would you probably do with the \$3.99?

Sarah: Make it in to a 4.

Teacher: Make it into a?

Sarah: 4.

Teacher: 4 what?

Sarah: 4 dollars.

Teacher: You need to remember all of those estimation strategies and work out which estimation strategy works out best for things (Classroom observations, 22/10/2009).

Key Finding 4.13: Wendy modelled the thinking involved using estimation strategies in number sense explorations.

When Wendy was talking to the students, she focussed on making the numbers friendly or easier to compute. She never discussed how precise the estimate would be.

Key Finding 4.14: Wendy focussed her computational estimation teaching on the problems set in meaningful contexts where the main computational choice was estimation and students focussed on making the numbers easier in the problems.

Wendy put the students into groups of four and they began work immediately. There was quite a high noise level but the students were all engaged, talking about which park they would visit. Having an individual handbook allowed each student to be able to personally keep track of the task requirements and it made the organisational issues of the lesson very simple. Working collaboratively, all the groups managed to progress through the task using the estimation strategies to budget for a trip to the park (Classroom observations 3, 22/10/2009).

Key Finding 4.15: Wendy used the teaching approach of organising the class to work in small groups in order to facilitate discussion between the students about the computational estimation problems.

As the students completed this task, they estimated in the calculations but they did not overtly discuss that they were using the estimation strategies. The following discussion by Bill with his group shows how he used the estimating strategies to work out the approximate cost of the trip:

Bill: So there will be 62 kids and round about \$3.00 for the juice
 boxes. $\$3 \times 11 = \33 for juice boxes [there were 6 in each pack]
 (Classroom observation 3, 22/10/2009).

Key Finding 4.16: Wendy's students used computational estimation language when discussing how to solve the problem.

The lesson ended without a plenary. This meant there was no opportunity to discuss what strategies had been used, nor to consider other representations of the calculations that had taken place i.e., the number line of these computational estimations. The lesson had been very successful in that it engaged the students to complete a task, which involved many estimated calculations. They were engaged in the work up to the end of the lesson but the students had to leave for recess so there was no opportunity for a whole class reflection. This lack of time meant that unfortunately there was not an opportunity for the teacher to summarise the calculations in the task and focus the students on the abstract mathematics. As some of the students had not finished putting the plan on to a Power Point the teacher suggested that this be completed later. Wendy then emailed some of the Power Points through to the Researcher (Figure 4.6).

The Food



For the picnic lunch we had to organise food costs. The foods and drinks we are taking are hot dogs (sausages in bread rolls), an apple and a juice box.

Sausages = \$36.00 = 9 packs
 Juice Boxes = \$36 = 12 packs
 Apples = \$28 = 7 packs
 Bread Rolls = \$48 = 12 packs

To pay for the picnic everyone would need to bring \$2.20

Simon, Jack, Maud,
and Frank

Figure 4.6: Group’s presentation of using estimates to plan a trip to the park

Key Finding 4.17: Wendy’s students were able to use estimations as a main computational choice in extended problem task.

Final Pedagogical Content Knowledge, Beliefs and Teaching Approaches

In the professional learning twilight session, Wendy continued to express her dissatisfaction that some of her students were not appearing to be developing number sense (PL4 observations, 14/10/2009). She and the other participants in the professional learning intervention discussed that there needed to be some fundamental changes in how their primary schools approached the teaching of mathematics. In this excerpt below the other participants’ speech has been included so that it is possible to understand the context more fully:

Stephen: It shows they [the students] are not thinking about the process.

Wendy: They [the students] are not thinking about numbers.

Belinda: Well they are thinking about the process and only thinking about half of it - they then think that will I do, they are not thinking, going back and thinking is that sensible?

Stephen: Which means we must be doing something along the way that is wrong.

Wendy: I agree ...

Wendy: Part of it is that I think they don't care enough, a lot of ours [students] (PL4 observations, 14/10/2009).

This dissatisfaction with the 'school mathematics' approach was also raised when the final test results were discussed in the final interview. Wendy was very disappointed that the students, even after a large amount of time spent on fractions in the classroom, using the prescribed text book, still did not seem to understand them (Teacher interview 3, 18/11/2009).

Due to the lack of recent research in this mathematical area, this study was asking the genuine question, 'Should computational estimation be taught to Year 6 students?' It is therefore very important to determine if the teachers could understand the strategies themselves, as it would not be possible to include them into the primary curriculum if the teachers could not understand them. In the final interview, Wendy explained that she was very comfortable with the computational estimation strategies: "Oh I only knew rounding really [at the beginning] so now I know lots, when I can remember them!". She was also able to solve estimation problems using a variety of estimation strategies (Teacher interview 3, 18/11/2009).

Key Finding 4.11: Wendy was able to solve estimation problems, indicating that she had a good understanding of computational strategies.

She also believed that all of the computational estimation strategies should be taught to Year 6 students.

Researcher: What strategies do you think are worth teaching to students in Year 6?

Wendy: I think they are all important (Teacher interview 3, 18/11/2009).

Key Finding 4.18: Wendy perceived that the computational estimation strategies were worth teaching to Year 6.

She had developed pedagogies that she thought were appropriate for teaching computational estimation. She believed that number sense was very important and that students should start learning about computational estimation with small numbers when they are younger.

What I think is so important is the number sense aspect of it like yeah ... and the sense of how much things are, if you have got 56×28 and you put the answer in the 100s and so many kids do ... Using lots of practical activities; Starting with numbers the children understand and can relate to (in the lower grades). Building to higher numbers; games; journaling of their understanding: Children need to understand why they are estimating (PL4 observation, 14/10/2009).

Key Finding 4.19: Wendy's pedagogical approaches for teaching computational estimation as a component of number sense were; games, practical activities set in problem-solving contexts, modelling and scaffolding tasks, and journaling.

Key Finding 4.20: Wendy believed that computational estimation was an important component of number sense.

Wendy had observed the students enjoying the estimation activities and she believed that the students had found computational estimation engaging (Teacher interview, 18/11/2009).

There was a growing set of beliefs which appeared to be emerging from being involved in the professional learning but she had an equally powerful set of beliefs about 'school mathematics' and she expressed both of these beliefs in the final teacher interview. These beliefs reflected that she viewed 'school mathematics' as providing parents with a consistent program in order to provide equity and harmony:

It gives us evidence that we have taught maths and not only that though because the kids are in different classes the parents get all upset - that they are all working in the same book they feel that at least their kids aren't missing out (Teacher interview 3, 18/11/2009).

The school mathematics in this setting was a text book (Pascal Press, 2009) which had activities for all the students to complete at the same pace, often emphasising one procedural way of solving different aspects of mathematics. Estimation activities involved written directions to estimate first and then complete the algorithm but with no instructions as to **how** to estimate. Unfortunately, estimation as a checking device requires number sense and as algorithm proficiency does not teach this, students would probably find it easier to redo the algorithm as a checking device so they may be reluctant to estimate.

Key Finding 4.21: Wendy believed that in the real world of her school context, a text book approach and its focus on procedural teaching of estimation i.e., rounding exercises and estimate before algorithms, maintained harmony with parents and therefore was the best pragmatic pedagogy even though she described it as a ‘band aid’ approach.

Students’ Final Beliefs about Computational Estimation

Most of the students in the focus group at the end of the school year still perceived mathematics as something with one correct answer and able to be done in a short time. When asked to describe mathematics the only student that answered stated “It’s like patterns ‘cos in Maths you always find patterns and they help you to work out stuff” (Alison, Student focus group 2, 18/11/2009). When asked what they thought about a student working on a question for 15 minutes that answers had not differed from when the project began. Below is Francis’ response:

Researcher: Bill sat working on for 15 minutes do you think he is clever?

Francis: No, because with a maths test you could go through, if you don’t know a question, you go through all the ones you do know and when you have time you could go back to it after (Student focus group 2, 18/11/2009).

Key Finding 4.22: The students believed that mathematics is something about patterns, that is done quickly and about one correct answer.

When analysing the focus group answers to the question, “What would you do if someone has a different answer to you on a mathematics question?” they indicated that the students still perceived mathematics as something that had one correct answer (Student focus group 2, 18/11/2009).

The focus group students were asked to draw a concept map to show what they thought estimation meant. Their answers indicated that their beliefs about estimation had grown. Bill, before the professional learning intervention, had thought that estimation was mostly a guess. Now he could see that estimation was more than a guess and something that could be useful in mathematics. In Figure 4.7 it is possible to identify how Bill’s awareness of computational estimation was limited to something as a checking device.

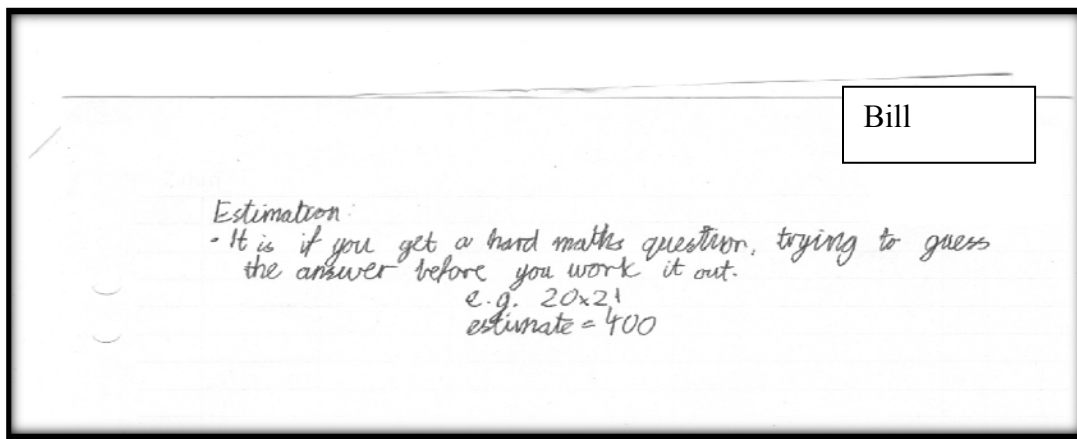


Figure 4.7 Bill's concept map of computational estimation before the professional learning intervention

Bill's concept map after the study is shown in Figure 4.8. His understanding of what estimation was had broadened to include something that could be a computational choice in its own right that used a variety of estimation strategies and not just as a checking device when performing routine algorithms.

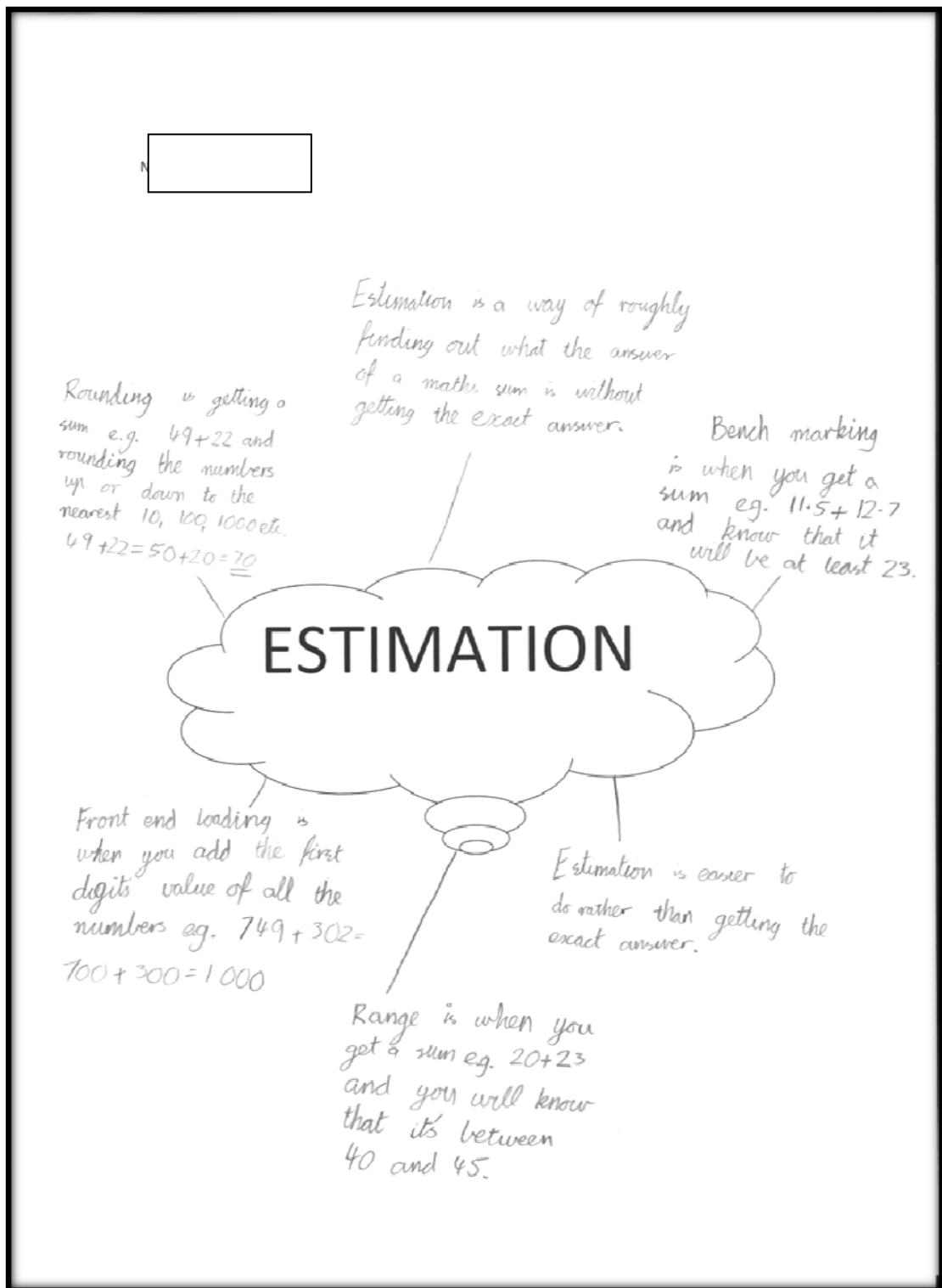


Figure 4.8: Bill's concept map of estimation after the professional learning intervention

The students had enjoyed the estimation activities. Most of the focus group now possessed positive beliefs about computational estimation, and they felt it made them

more confident in mathematics:

Researcher: What do you think about learning estimation this year? How have you found it?

Peter: I thought it was fun especially with the surf shop cos we were selling things to our friends.

Francis: I'm not too keen on maths but I reckon when you made it more fun like with the M and Ms and the surf shops it made it more fun and you could interact with other people and it made it easier for you to like kind of work out stuff.

Most of the students in the focus groups had used the estimation strategies to make the mathematics easier, although some were disappointed that in making it easier it removed some of the challenge for them.

Researcher: Do you enjoy estimating or do you prefer exact answers?

Bill: I enjoy both but to be honest I probably prefer exact answers.

Researcher: And why do you prefer exact answers?

Bill: It is more challenging because I think estimating is a bit easy (Student focus group 2, 18/11/2009).

Key Finding 4.23: The students believed that estimation is; more than a guess, fun, makes mathematics easier, involved a variety of estimation strategies, and helps make sense of mathematics although it can remove some of the challenge.

Students' Computational Estimation Performance after the Professional Learning Intervention

The students undertook the Computational Estimation Test (CET) for a second time at the end of the professional learning intervention. This was conducted during the latter half of Term 4 and the teachers conducted the tests using the same procedure as the pre-test. This provided data about whether the teaching approaches of computational estimation had affected students' computational estimation performance. Figure 4.9 shows that there was an increase in the percentage of students selecting the best estimate on all test questions, except for Question 4, which was the same. The mean pre-test score was 2.93/6 and the post-test mean score was 3.9/6. Therefore the mean student test score improved by 0.97 and this difference was highly significant (paired samples t test (29) = 3.778, $p < 0.001$).

These inferential statistics are expected to be interpreted from a socially constructed perspective due to the nature of the design of the study and its focus on rich data in a naturalistic setting without any attempt to control variables (Hennig, 2010).

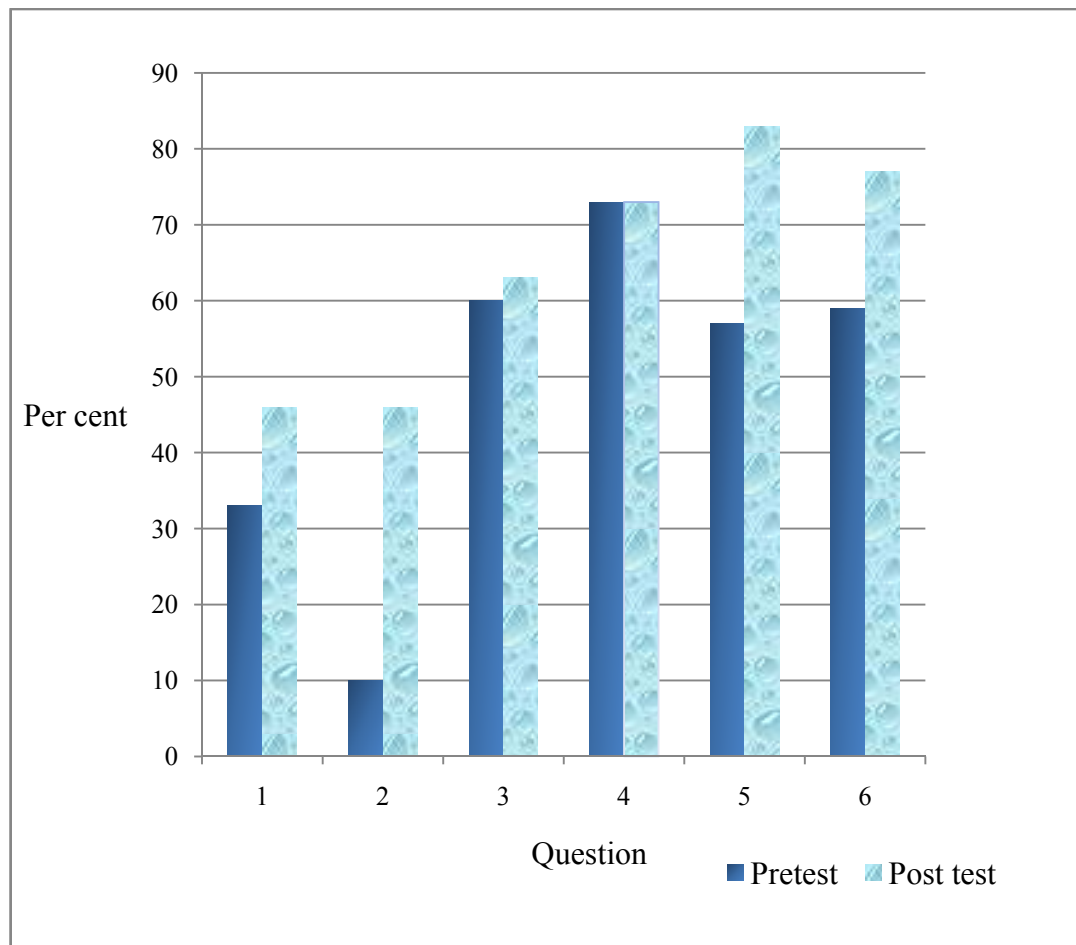


Figure 4.9: Per cent of students selecting the best estimate on the computational pre- and post-test

As this research focussed on how the particular teaching approaches affected the students' performance in using computational estimation strategies, it was of interest to compare differences in students' use of reasoned estimation strategies. This analysis focussed on whether the students were able to articulate what estimation strategy they used, rather than trying to calculate the answer exactly, estimating intuitively or guessing.

Table 4.7 shows students' use of reasoned estimation strategies in the pre and post-tests.

Table 4.7: Percentage of students using estimation strategies and selecting the best estimate in pre and post-tests

Question	Reasoned strategy used			Best estimate		
	Pre-test	Post-test	Increase	Pre-test	Post-test	Increase
1.	33	76	43	33	46	13
2.	7	43	36	10	46	36
3.	33	90	57	60	63	3
4.	60	83	23	73	73	0
5.	37	93	57	57	83	26
6.	40	83	43	59	77	18

As each of the test questions was developed to identify the students' performance using different estimation strategies it is worthwhile to highlight some of the changes on the individual questions.

Estimation Question 1

Table 4.7 shows that in Question 1, 76% of the students used a reasoned estimation strategy in the post-test compared to 33% of students in the pre-test. This suggests that having taken part in activities which involved rounding in a context, Wendy's students were now more able to apply reasoned estimation strategies to solve problems within a context and this enabled more students to select the best estimate. Table 4.7 also shows that a higher proportion of Wendy's 10 year old students (46%) selected the best estimates than the students that completed the Number Sense Test (McIntosh et al., 1997) (35% -10 Year olds, 38% - 12 Year olds). This suggests that by providing tasks using meaningful contexts, the students may have become more proficient at these than in normal classrooms where there was not the same focus on context. This assertion is supported by the fact that there was a 13% improvement in question 1 in the post-test compared with the pre-test, suggesting that more students in the class were able to estimate in problems within a context after the study had

taken place.

Estimation Question 2

Question 2 provided students with the opportunity to use benchmarking to the nearest whole number on a task involving the addition of fractions. Table 4.7 shows that only 7% of students were able to articulate that they had used a reasoned estimation strategy on Question 2 in the pre-test, suggesting that they had little experience with focussing on fraction computations in any other way than algorithmically. In the post-test, 43% of students were now able to use a reasoned estimation strategy suggesting that Wendy's teaching had broadened the students' approaches to the calculations of fractions. It appeared that nearly half of the students were able to consider the approximate value of the fraction in relation to the nearest whole number.

In the post-test 46% of students were able to obtain the best estimate of 2 (B) and this was a 36% improvement on the pre-test results. This revealed that nearly half of Wendy's students, after the professional learning intervention, showed that they considered fractions in relation to a benchmark of the nearest whole number and then easily computed the numbers. This assertion is supported by the fact that only 25% of 12 year olds were able to select an appropriate answer in the Taiwanese component of the number sense research (McIntosh et al., 1997) compared to 46% of students in Wendy's class in the post-test.

Estimation Question 3

In Question 3, 33% of students used a reasoned estimation strategy in the pre-test whereas after being involved in the professional learning 90% of students were now able to use these types of strategies. Wendy had spent time teaching the students how to use estimation strategies in quantitative situations that were similar to this question and it appears that the students were able to use the knowledge gained from this instruction in order to answer this question.

Interestingly this increase in using articulated estimation strategies does not appear to be more effective than the intuitive unreasoned strategies that the students used before the study began. There was only a 3% increase in the number of students able to select the best estimate.

Estimation Question 4

When answering Question 4 in the post-test, over half of the class were able to use a reasoned estimation strategy before the study began but this had increased to 83% by the end of the study (Table 4.7). Table 4.7 also shows that in the analysis of students' responses to Question 4 in the pre- and post-test there was not any improvement in how many students selected the best estimate. This question was presented symbolically and as the socio-mathematical norm of the class was to practise solving many symbolic equations, this may explain why there was a high level of proficiency at the start of the study. The students in Wendy's class had been taught to make the computations 'friendlier' without concern for the precision of the estimate. All the suggested estimates, 4000 (A), 4600 (B) and 5200 (C) were in the 'ball park' for estimating 45×105 and could have been attained by estimating in ways encouraged in the professional learning project. Therefore, although many students used an appropriate reasoned estimation strategy of rounding, they may have not undertaken any compensation. They possibly could have rounded down i.e., $40 \times 100 = 4000$ or rounded the first number up i.e., $50 \times 100 = 5000$.

Estimation Question 5

On question 5 in the post-test, there was also a 57% increase in students using a reasoned estimation strategy, and nearly all of the students were able to identify the best estimate (Table 4.7). This would suggest that there is value in teaching upper primary school students to use reasoned estimation strategies when estimating the addition of two digit numbers. These estimation strategies could be either front end loading to produce an underestimate or the nice number strategy and adding those together i.e., $20 + 80$ and $40 + 60$ or round the numbers.

Estimation Question 6

In Question 6 there was a 43% increase in students using a reasoned estimation strategy. Wendy had spent little time teaching students about estimating within a range but it appears that raising students' awareness of estimation generally has resulted in more students using an estimation strategy on this question. This general increased awareness of estimation has resulted in an 18% increase in students able to select the best estimate. The students' computational estimation performance improved. The mean student test score improved significantly ($p < 0.001$).

Key Finding 4.24: Students' computational estimation performance improved overall and this was statistically highly significant.

Overall, it appears that the students' performance in using reasoned estimation strategies had greatly improved during the study. This raised awareness of estimation strategies and time spent practising using estimation strategies appears to have increased the students' ability to select the best estimate. The students were still more competent at estimating in the symbolic question in comparison to the question presented contextually.

Key Finding 4.25: Students are much more proficient at estimating multiplication problems, which are purely symbolic, and not set in context. Nearly three quarters of the students were able to apply estimation strategies when problems were set in a context.

The students' performance on selecting the best estimate and their use of reasoned estimation strategy when adding fraction with unlike denominators improved.

Key Finding 4.26: About half the class were able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators after being instructed in how the benchmarking strategy may be used in this process. Far more students were then able to use this benchmarking strategy.

The only question where there was no improvement in the students' computational estimation performance was Question 4, where all of the answers were in the 'ball park'. This may reflect that Wendy did not choose to spend time teaching how some estimations were more precise than others.

Key Finding 4.27: Students showed no improvement in estimating to a high degree of precision when assessing an estimated answer of a multiplication of a two digit by three-digit number.

Chapter Summary

As shown in Figure 4.10, in response to the professional learning intervention, Wendy developed new PCK and this may have impacted upon her beliefs to a certain extent. The action research approach enabled Wendy to investigate some new

teachings approaches, which she implemented in the classroom setting. This process did appear to impact on students' beliefs and computational estimation performance. Following the cyclical process of the professional learning, trialling the new teaching approaches and the subsequent impact of these on student beliefs and learning outcomes, this process appeared to further develop Wendy's PCK and beliefs. A table summary of the key findings related to the beginning and end of the case study are shown in Appendix H.

Teacher PCK

At the beginning of the professional learning Wendy had little PCK for computational estimation as a component of number sense. The only estimation strategy that Wendy was aware of was rounding and how this strategy could be used for checking written computation algorithms. The professional learning workshops addressed content knowledge about the range of estimation strategies and provided Wendy with an opportunity to develop her own understanding of these. Wendy worked through the estimation tasks provided in the workshops and evaluated the benefits of the various estimation strategies. Back in her own classroom, her understanding of the new strategies was reinforced through teaching them to her students. Wendy developed a sound pedagogical content knowledge of the full range of strategies and had a framework for how she could teach them. This framework included games, practical activities set in meaningful contexts and journaling.

Assertion 4.1: Wendy's engagement in the action research process broadened Wendy's PCK of this subject area in that she understood the strategies and developed a pedagogical framework for how these could be taught in Year 6.

Teacher's beliefs

It can be suggested that Wendy's beliefs were impacted upon through engaging in the professional learning intervention. As the action research process was an integral component of the study, Wendy was able to reflect on this process and articulate her changing ideas. Wendy believed that the school system of following a textbook was appropriate for school mathematics, although she did point out the disadvantages of

this system i.e., that it did not engage the students. Wendy did believe that the ideas suggested at the professional learning workshops held some value and the effect of this was to create two distinct sets of beliefs. Wendy did gradually believe that computational estimation as a computational choice had value in an ‘ideal world’. She also felt that the problem solving approach was much more motivating for the students.

Assertion 4.2: Wendy’s developing PCK of computational estimation as a computational choice and checking device impacted her beliefs and she now believed that computational estimation could develop number sense in an ideal world and that computational estimation strategies were worthwhile.

Teaching approaches

Wendy’s school had an structured system of following a text book (Pascal Press, 2009) in the primary school section of the school. As head of curriculum in the school, Wendy had been involved in making this decision and therefore was supportive of this approach. Her normal mathematics time allocation was spent teaching from the textbook and that mostly involved completing computational algorithms. In the second cycle of the action research Wendy did perceive, however, that in an ‘ideal world’ computational estimation could be a computational choice in its own right and that students should be introduced to the variety of estimation strategies. With these beliefs in place, Wendy created extra time in order to evaluate the suggested learning tasks from the professional learning workshops. The cyclical processes of the professional learning intervention allowed Wendy to trial and refine new teaching approaches of computational estimation. Wendy created learning tasks that involved explorations so that the students would be able to learn how to use computational estimation in problem situations. Wendy explicitly introduced the definitions of computational estimation strategies to the students, including the formal names for them, using the IWB as a presentation tool.

When Wendy introduced the explorations, which incorporated the strategies, she scaffolded the students’ learning by modelling one approach to solving the problem. She also explained to the students that the estimations should focus on making the numbers friendlier. She then allowed the students time to spend time working in groups exploring how to solve the problems.

Finally, as Wendy engaged in the third action research cycle, these impacts seemed to be strengthened. The fact that Wendy was able to understand and use the estimation strategies enhanced her teaching as she was confident about these strategies.

Assertion 4.3: Wendy's teaching approaches appeared to be impacted upon her developing beliefs that computational estimation was important in developing number sense and her developing PCK of computational estimation strategies. Wendy developed two teaching approaches that she thought were appropriate in an ideal world; estimating in problem situations and directly teaching the estimation strategies. Wendy maintained two contexts – the ideal world and the real world. In the real world of the classroom she often continued to use procedural approaches to teaching estimation.

Students' beliefs

The two different teaching approaches outlined appeared to have produced contradictory beliefs in the students. As the students were following the textbook in their normal program and completed routine algorithmic tasks at the end of the professional learning intervention, they still perceived mathematics as something about one right answer and something that is done quickly. They were also engaged in activities introduced by Wendy, which focussed on problems that required computational estimation as the main computational choice, and these activities do appear to have affected their beliefs. The students now had a much broader and more positive perception of estimation. These positive beliefs grew as the study progressed and as they perceived the value in the different teaching approaches.

Assertion 4.4: Wendy's teaching approach of creating extra problem based computational estimation learning tasks appeared to impact on the students' beliefs and broaden their perception of mathematics and estimation. Their perceptions of computational estimation appeared to be very positive.

Analysis of the pre and post-tests indicate that the students' competence in estimation improved significantly during the duration of the study. Many more students were able to use an estimation strategy when selecting the best estimate in two digit addition questions, suggesting that being explicitly taught appropriate strategies such as front end loading were beneficial for the students when estimating

with this type of operation.

Assertion 4.5: Wendy's teaching approach of creating extra problem based computational estimation learning tasks appeared to improve students' estimation performance and this improvement was statistically significant. Wendy's focus on making the numbers easier meant that the students did not focus on the precision of the estimate. Wendy's teaching approach of directly teaching the estimation strategies appeared to increase the students' awareness of the estimation strategies.

Conclusion

It is clear that through developing Wendy's PCK she started to investigate how she could teach computational estimation in an ideal world and that this appeared to influence the students' beliefs and computational estimation performance. It appeared, however, that due to the school having a textbook approach embedded within the mathematics curriculum which was perceived as very effective by the parents, these changes would not probably impact upon Wendy's teaching overall nor spill over to impact the school curriculum.

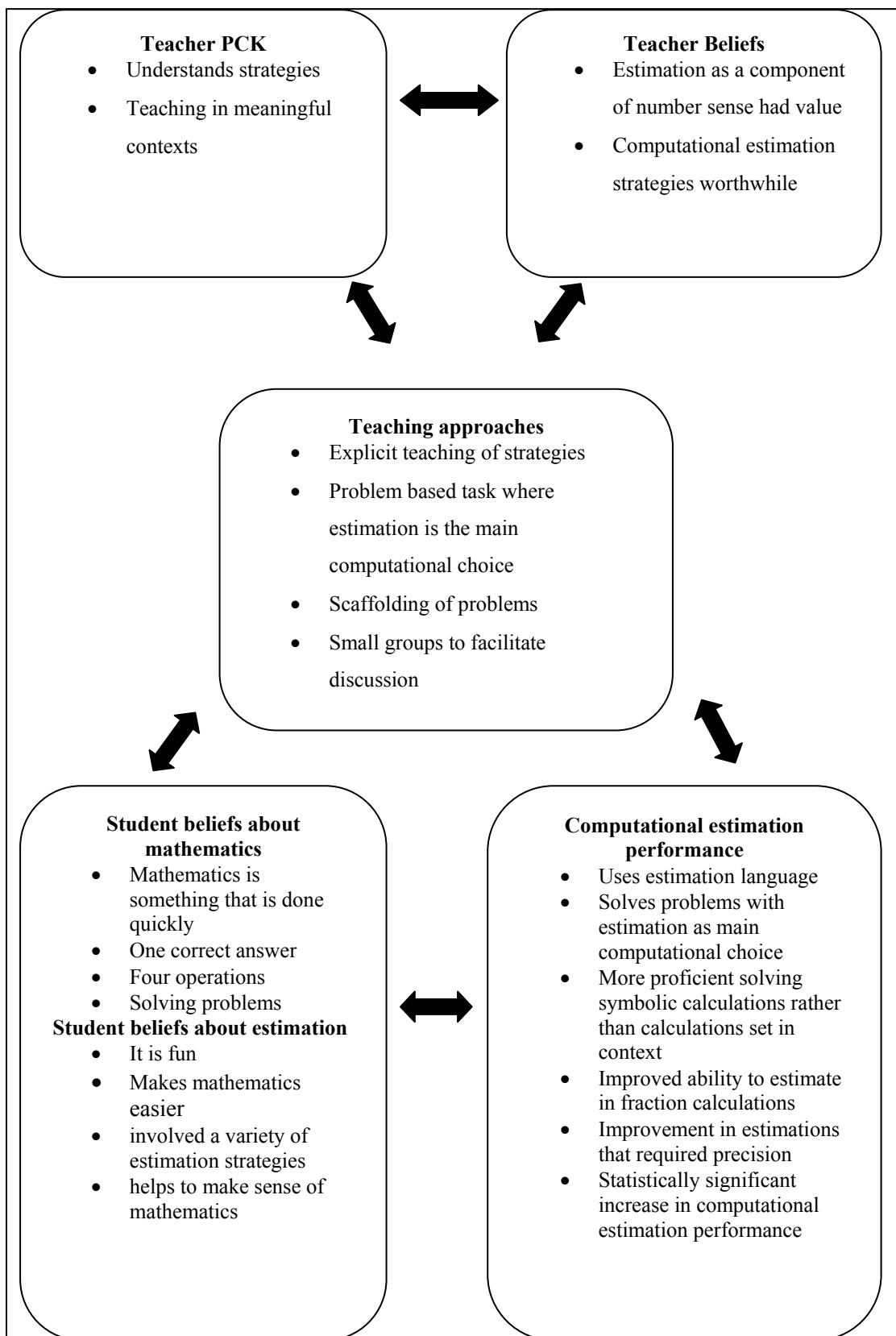


Figure 4.10: Model to show the impact of the professional learning intervention on Wendy and her class

CHAPTER 5: CASE STUDY - PETER

Background

Peter (a pseudonym) was a teacher who had come into the profession as a mature aged student who studied for his BEd degree at a university in Perth. He had been teaching for nine and a half years and has taught at Sandilands School for eight years. At his present school, he taught Year 6 but whilst the students were having other specialist lessons, he also worked in the upper school where he taught high school media (Teacher interview 1, 4/12/2008).

The school, Sandilands School (pseudonym), is a K-12 low fee independent school, situated on one campus. It is a relatively new school and it has a strong commitment to support a specific ethnic community in the metropolitan area. As an example of the close links with the ethnic community, the government from the country of origin from this ethnic community had provided the school with various resources including teachers to work at the school. The school buildings were designed to reflect the country of origin and were well maintained, although there were plans to develop the school further. These plans included doubling the size of the school buildings and building a new library. It was a relatively small school considering the age range; there were only 42 staff and 493 students. It was co-educational and there were no students identified as indigenous (Australian Curriculum Assessment and Reporting Authority, 2010). The socioeconomic backgrounds of the parents was slightly above average, with an ICSEA value of 1038 (Australian Curriculum Assessment and Reporting Authority, 2010) where the average is 1000. It drew students from a large geographical area within the metropolitan region and all the students were polite and courteous and in school uniform. Peter's class was one of two Year 6 classes and it was of mixed ability. Peter taught the class for all subjects. There were 32 students in the class, and this included 18 boys and 15 girls.

On the 2008 NAPLAN testing, Year 5 students had a mean score of 474 in mathematics compared with the national average of 476 and this cohort were the 2009 Year 6 students involved in the study.

Peter's Views about Mathematics Teaching

In the preceding school year, before the professional learning program began, Peter was interviewed. He was welcoming and comfortable with showing me his present classroom and the students working in a mathematics lesson before the interview began. He had organised his classroom so that there were rows of tables facing the front. The students were working in silence and were marking their work.

When I asked Peter what he would like to gain from the professional learning workshops, he said, "Anything, information, ideas, resources" (Peter, Teacher interview 1, 4/12/2009). Peter stated that at present he really enjoyed teaching mathematics. His early experiences of mathematics had been negative but after a very positive time in his mathematics education units at university, he completely changed his opinion of the subject (PL1 observation, 18/2/2009). Despite perceiving these units at university as positive, he appeared cynical about the difference between what was taught in the classroom and how experts say mathematics should be taught (Teacher interview 1, 4/12/2008). Peter explained how the parents' wishes were paramount in all decision making on what was taught in the school:

Researcher: Do you feel that parental expectations or other people's expectations affect your practice?

Peter: A lot more here, because it is a community cultural based school, the parents have a lot of say, remember that the [specific ethnic group] community runs the school.

When explaining how he believed mathematics should be taught in the first interview, just after Peter showed his class working in rows without talking, he stated, "Hopefully I would incorporate lots of group work (he laughs) 'cos that's what you want me to say" (Teacher interview 1, 4/12/2008). Peter was reassured in the initial interview that for this research study it was important that he stated what he genuinely believed, as these views would be fully respected. The Researcher stated, "No, I want you to say honestly [how you teach]. It is not about clichés, blah, blah, blah" (Teacher interview 1, 4/12/2008).

When Peter started explaining how he taught mathematics, he stated that "Do you know when you walk into my maths class to tell you the truth I prefer quiet and I

don't know if I am a bit old fashioned like that" (Teacher interview 1, 4/12/2008). This appeared to be in keeping with the other classes in the school. The class next door was noted to be also working quietly on routine algorithms (Classroom observation 2, 24/6/2009).

Key Finding 5.1: Peter believed that students learn more effectively when working without talking.

His focus was teaching students mental maths with a number focus: "The biggest thing with my Year 6 that comes through is they still don't know their times tables, still can't add up a die quickly, that's my biggest focus" (Peter, Teacher interview 1, 4/12/2008). Despite explaining that generally he preferred the classroom to be quiet, he did make exceptions to this as one of his teaching approaches was facilitating the use of games. They had 'buddy' classes once a week where his Year 6 class paired up with younger students and he found it very useful in these sessions, to encourage his students to teach the younger students using these games. Peter was conscious that as he focussed on teaching number concepts that sometimes he did not manage to teach the other mathematical areas (Teacher interview 1, 4/12/2008). Peter elaborated more about his present mathematics teaching approaches in the second teacher interviews and he explained how the students worked from a textbook (Teacher interview 2, 24/6/2009). Peter explained that the students worked through this textbook at their own pace (Teacher interview 2, 4/12/2008). This textbook, selected by the school and used throughout the primary school, included some problem solving but many of the exercises focussed on students completing standard written algorithms in the four operations (Teacher interview 2, 24/6/2009). The students worked through the exercises at their own pace, so there was no opportunity to engage in the extra activities suggested in the teacher's guide.

Peter assessed the students in his class formatively and summatively. He organised the formative assessment through a variety of mechanisms:

Over the shoulder, depends on what I am assessing at the time... triples, today's number, mental maths, straight away I am getting an idea and comparing their last results instantly. I collect their work" (Teacher interview, 4/12/2008).

The class also undertook summative assessments and these assessments were part of

a whole school policy. The school held a testing week in week seven of each term and Peter used these results to make some summative judgements (Teacher interview 1, 4/12/2008).

Peter's Views on Teaching Computational Estimation

In Australia it appeared that there were few recent curriculum resources that provide guidance on how to teach computational estimation strategies in the primary school, so the Researcher had anticipated that the teachers would not have a pedagogical framework for teaching computational estimation. Peter's answers about computational estimation in the initial interview implied to the Researcher that his level of understanding was similar to what would be expected of most primary school teachers (Teacher interview 1, 4/12/2008).

Key Finding 5.2: Peter did not have a pedagogical framework for teaching computational estimation using the variety of computational estimation strategies.

Peter was not aware of the formal strategy names for computational estimation other than rounding: "I try and teach as many strategies like when we do mental maths on the board. Round up, round down" (Teacher interview 1, 4/12/2008).

Key Finding 5.3: The only computational estimation strategy Peter mentioned was the rounding strategy and he was not aware of the other strategy names.

Peter believed that estimation had a place in his mental maths lessons. He would tell the students to estimate first, although he did not spend time discussing any different strategies:

Estimation - I always incorporate it into my mental maths all the time. Estimation always, let's estimate the difference between that building and that building, what is 12 X 13? Estimate first.

Key Finding 5.4: Peter believed that computation estimation is useful as a checking device before doing exact mental computations.

In the second teacher interviews, Peter explained that his normal practice entailed using the text book 'Maths for WA' (R.I.C Publications, 2008), which included students working through exercises that developed the students' expertise using the rounding strategy. The students worked through these exercises at their own pace (Teacher interview 2, 24/6/2009).

Key Finding 5.5: Peter's students completed routine textbook exercises that involved rounding.

Peter's Students' Beliefs about Computational Estimation

The Researcher interviewed a representative sample of the class in order to find out what they believed about estimation and mathematics. The students in Peter's focus group thought that mathematics was something that was done quickly. When they were asked if they thought someone one who spent 15 minutes solving a mathematical problem was clever, one student answered, "If he took 15 minutes [for] maths mmm. It should depend on the year but normal questions should take about 2 to 3 minutes." Other students in the group gave similar answers (Focus group interview 1, 10/2/2009).

Most of the students in the focus group seemed to think that mathematics was about one exact answer. When they were asked what they would do if someone had a different answer to them, most of the students thought that they should check their answer. Not one child said something that conveyed that they thought that two different answers could be acceptable. The three students that answered stated:

John: I could ask him. Why do you have a different answer, how did you do it?

Jack: I would go over it with the teacher and I would like double-check it they may have incorrectly marked that or maybe they were cheating.

Jason: You would do it a different way to see if it still was the same (Focus group interview 1, 10/2/2009).

The students gave a variety of answers as to what mathematics was. One of the students in the focus group perceived mathematics as something that was about problem solving “it’s solving problems with numbers” (Focus group interview 1, 10/2/2009). Most of the students in the interview thought mathematics was about the four operations:

Researcher: An alien lands on earth and wanted to know what mathematics was (students shown a cartoon of an alien), what would you tell him?

John: I would tell him it's problem solving with numbers.

Josh: We would teach him how to add, subtract, divide and multiply.

Jason: If you met him, you could tell him all about it as an example on a piece of paper.

Researcher: And what would you write on the piece of paper?

Jason: I would write a sum like $2 + 2$.

Key Finding 5.6: Students in the focus group believed that mathematics is about problem solving, the four operations, something with one correct answer and is done quickly.

When the students in Peter’s class were asked if they thought that McDonalds could benefit from employing a mathematician they had a discussion between themselves as they acknowledged that the mathematician would be employed for more than simple computations:

Jack: Maybe in the farms they would need to count how many chickens they would need to have and if they need say chicken nuggets. How much chicken nuggets they can make with that certain amount of chickens.

Jason: The drive through people they have to work out how much dollars they have and how much change they need.

Jack: Well they have computers to do that.

Jason: Well what if they don't?

John: Well, like Jason said, they could approximate how many chickens they would need or how many nuggets they will need.

It was of interest in the study to try to identify what the students' different beliefs about estimation were, as all the students in the focus group used the word guess in their answer. When students talked about estimation, they focused on the strategy term "round off" (Focus group interview 1, 10/2/2009).

Key Finding 5.7: The students in the focus group perceived estimation as a type of mathematical guess.

Students' Overall Estimation Competence before the Professional Learning Intervention

All the students in Peter's mixed ability Year 6 class were asked to complete the Computational Estimation Test before the study began, using the same procedures as the students in the other classes involved in this case study. This was important as it revealed the specific competencies of the students before their teacher became involved in the professional learning intervention. Below is an in-depth analysis of the students' responses to the six computational estimation questions.

Estimation Question 1

In the pre-test 30% were able to calculate the best estimate (B). This is slightly lower than the results in the Australian sample where 35% of 10 year olds and 38% of 12 year olds were able to select the best estimate in the Number Sense Test (NST) (McIntosh et al., 1997). This problem was set in a context and this relatively low result may have been because the students were less experienced at solving problems set in context. This suggestion is supported by the information Peter revealed in the second teacher interview that the school had a policy of students working independently on a textbook where many of the problems were presented symbolically.

There was a variety of estimation strategies used by Peter's class in order to answer Question 1. Table 5.1 shows that there were two most popular strategies used and neither of these was a reasoned estimation strategy. In response to Question 1, 30% of students attempted to try to answer it exactly without approximating the numbers first. This would have been difficult to do because of the time limit. This approach by the students suggests that the students may be used to answering mathematical

questions exactly. There were also 30% of students who could not explain how they estimated, suggesting that these students did not possess any estimation strategies that they could use to answer this question.

Table 5.1: Per cent of students selecting various answers to question 1, and computational estimation strategies used (n=20)

Answer	Per cent						Total
	Rounding	Range	Guess	Not enough information	Exact	Intuition	
1A	0	0	0	10	0	0	10
1B*	10	0	5	0	15	0	30
1C	5	5	5	5	15	10	45
1D	0	0	0	10	0	0	10
NA	0	0	0	5	0	0	5
Total	15	5	10	30	30	10	100

Note.

* denotes the best estimate

NA denotes no answer indicated

Estimation Question 2

Students were not given enough time to compute this calculation exactly using such strategies as finding common denominators. This question required the students to approximate the relatively complex fractions into easily visualised whole numbers resulting in an extremely simple calculation. Table 5.2 shows that only 10% of students in Peter's class were able to calculate an appropriate estimate, suggesting that few students possessed conceptual understanding of fractions. This is compared to 25% of students who were able to calculate an appropriate answer in the Taiwanese component of the NST (McIntosh et al., 1997) and 24% of 13 year olds in North American national testing in 1981 (Post, 1981).

Most of Peter's students were not able to access an estimation strategy to answer this question in the short time frame. Although they heard the question read out to them so that they knew they were dealing with fractions, they reverted to whole number thinking. Similar numbers of students selected the answers 19 and 21 suggesting that some added the two top numbers and the same number added the two bottom numbers.

Table 5.2: Per cent of students selecting various answers to question 2, and computational estimation strategies used (n=20)

Answer	Per cent					Total
	Rounding	Guess	Not enough information	Exact	Whole number thinking	
2A	0	0	0	0	0	0
2B *	5	0	0	5	0	10
2C	0	10	0	0	35	45
2D	0	5	0	0	35	40
NC	0	0	5	0	0	5
Total	5	15	5	5	70	100

Note. * denotes the best estimate

Estimation Question 3

As shown in Table 5.3, 65% of students chose the best estimate of 200 (D).

Table 5.3: Per cent of students selecting various answers to Question 3, and computational estimation strategies used (n=20)

Answer	Per cent							Total
	Rounding	Range	Sample	Guess	Not enough information	Exact	Intuition	
3A	0	0	0	0	0	0	0	0
3B	0	0	0	0	0	5	0	5
3C	0	0	10	5	0	0	5	20
3D*	5	5	20	10	5	5	15	65
3E	0	0	0	0	10	0	0	10
Total	5	5	30	15	15	10	20	100

Note. * denotes the best estimate

On question 3, very few of the students selected the underestimates of 20 (A) and 50 (B). In the NST (McIntosh et al., 1997) 54% of 10 year olds and 62% of 12 year olds obtained the best estimate so Peter's class were more competent than the students in that study. This appeared to suggest that the students had the intuitive ability to process this type of visual information without it being a focus in the school curriculum. Peter's class did not predominantly select one strategy; slightly more students selected the sample strategy (15%) than intuition (10%) but it did not appear

that the class had been taught one particular way of answering this question. Some students attempted to count the exact number of triangles.

Estimation Question 4

Table 5.4 shows that 45% of students were able to select the best estimate of 4600 (B). On Question 4, Students appeared to find this question easier than Question 1, which required a similar mathematical understanding but Question 1 was set in a context and therefore also required the students to interpret the problem first. This suggests that the students found it easier to estimate with a symbolic question rather than when it was set in a context. Peter's students' results were considerably lower than the NST (McIntosh et al., 1997), where 60% of 10 year old students selected the best estimate. Nearly a third of the class selected the overestimate 5200 (C).

Thirty per cent of the students were unable to describe how they estimated and 20% of the students guessed their estimate. Only 25% of students used a reasoned estimation strategy in the pre-test, suggesting that whilst the students exhibited competency in selecting the best estimate, many could not articulate the estimation strategies that they used.

Table 5.4: Per cent of students selecting various answers to Question 4, and computational estimation strategies used (n=20)

Answer	Per cent						Total
	Rounding	Front end loading	Guess	Not enough information	Exact	Intuition	
4A	5	0	5	5	0	0	15
4B*	10	0	10	10	5	10	45
4C	5	5	5	5	10	0	30
NA	0	0	0	10	0	0	10
Total	20	5	20	30	15	10	100

Note.

* denotes the best estimate

NA denotes no answer selected

Estimation Question 5

There is an expectation that many Year 6 students would be able to estimate with two-digit additions, as this is one of the more simple computations. Table 5.5 shows

that 50% of Peter's class were able to select the best estimate. A quarter of the class thought that the answer was the underestimate, 165 (A), which was quite a large underestimate, suggesting that these students were not experienced at estimating with two-digit numbers before the study began.

Table 5.5: Per cent of students selecting various answers to Question 5, and computational estimation strategies used (n=20)

Answer	Per cent					Total
	Front end loading	Guess	Not enough information	Exact	Intuition	
5A	5	5	5	10	0	25
5B	5	0	0	10	0	15
5C*	10	0	5	25	10	50
5D	0	0	5	0	0	5
NA	0	0	5	0	0	5
Total	20	5	20	45	10	100

Note.

* denotes the best estimate

NA denotes no answer indicated

Estimation Question 6

Table 5.6 reveals that 50% of students were able to select an appropriate answer to this question, which was designed to ascertain whether students could estimate within a range. Nearly half of the class selected the overestimate C (23-28). This suggests that many students found it difficult to estimate when calculating numbers that included decimals. When considering how the students solved this problem it suggests that most students did not have a reasoned estimation strategy in their repertoire to answer this question. Table 5.6 shows that 30% of students guessed the answer to this question and 35% of students could not articulate how they answered the question.

Table 5.6: Per cent of students selecting various answers to Question 6, and computational estimation strategies used (n=20)

Answer	Per cent						Total
	Rounding	Range	Guess	Not enough information	Exact	Intuition	
6A	0	0	5	0	0	0	5
6B*	10	10	10	15	5	0	50
6C	0	0	15	20	5	5	45
6D	0	0	0	0	0	0	0
Total	10	10	30	35	10	5	100

Note. * denotes the best estimate

Figure 5.1 shows that the students had varying degrees of ability in using reasoned estimation strategies before the professional learning intervention began.

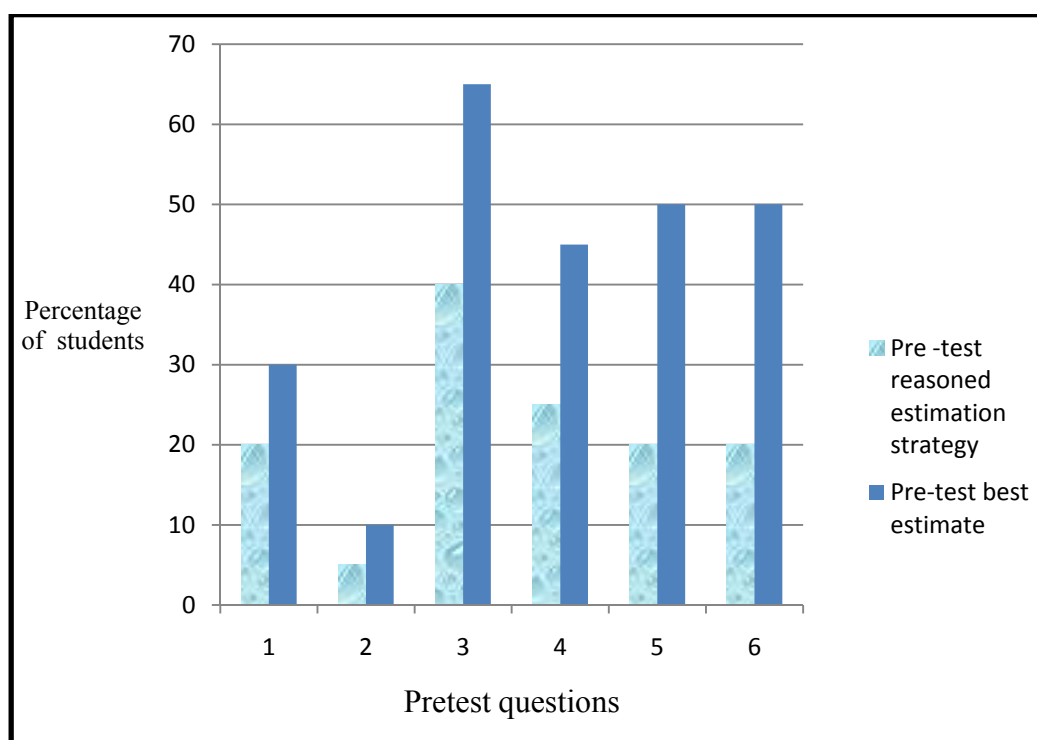


Figure 5.1: Percentage of students using a reasoned estimation strategy and identifying the best estimate before the professional learning intervention

Relatively few students were able to use an estimation strategy to solve a multiplication calculation set in a context. The students were most successful at using a reasoned estimation strategy when it was presented symbolically and, in that situation, most used the rounding strategy.

Key Finding 5.8: Students were much more proficient at estimating multiplication problems that are purely symbolic and not set in context.

Most of Peter's students were not able to select a best estimate when presented with a calculation involving fractions.

Key Finding 5.9: Generally, students were not able to use an estimation strategy to convert complex fractions into easily visualised whole numbers.

It is surprising that over half the students were not able to use a reasoned estimation strategy in order to add four two-digit numbers as these are numbers worked on regularly from Year 2 in the primary school.

Key Finding 5.10: Half of Peter's students found it difficult estimating when adding two digit numbers quickly and a minority of the students was able to use a reasoned estimation strategy.

Finally, few students were able to estimate using a reasoned estimation strategy in order to calculate a range. The overall mean test score on the computational estimation pre-test was 2.6/6.

Peter's Response to the Action Research Professional Learning Process

It was important that this study captured in detail the teacher's responses to the professional learning intervention as it unfolded and the use of a narrative allowed for the longitudinal nature of the study to be conveyed. Peter attended the three professional learning days held at the University during school time but he was unable to attend the twilight reflective session at the end of the study. He was able to convey his final views, however, in the final interview, which took place at the end of the study. It is important to reiterate that each teacher's perception of how to teach computational estimation was fully respected, appreciating that each individual's experiences shaped their particular world view (Patton, 2002).

The first research cycle: Reflecting and planning

At the beginning of the professional learning day, the members of the group were invited to share their experiences and Peter was the final person to share his

perspectives. He appeared to lack confidence and spoke more about his personal mathematics ability rather than how he thought mathematics should be taught. As he was the last contributor, he had heard everyone else explain their teaching approaches so it was interesting that he chose not to share his approaches:

I am from Sandilands and I teach high school math as well. I am a mature aged student through uni. I carried a great wall about mathematics to the extent that my wife knew I wouldn't even try. It wasn't until the lecturer broke that down for me and I guess I carry that through. I love teaching maths now. It is one of my favourite subjects and yet if you knew me before, I didn't, so I guess that's where I teach from (PL1 Observation, 18/2/2009).

The first principle that was presented in the professional learning was that effective mathematics teaching promotes active learning. In the group discussion it appeared that Peter understood this approach (PL1 Observation, 18/2/2009). Despite understanding the approach, Peter was not sure these activities would work with younger students (the example used in the workshop was with Kindergarten students undertaking problem-solving activities). The critical friend who acted as an independent observer of the professional learning days noted "Peter discounting activities based on beliefs about abilities in individual students i.e., "kindy students can't count to 100" (Critical friend observations, 18/2/2010).

Peter, despite saying that he was quite comfortable with the computational estimation strategies, did not believe that it would be appropriate to introduce these computational estimation strategies to Year 6 students. This was noted in the observations:

Peter felt that young students may find it confusing to name these strategies. I said that I was open to these perspectives. I did question that didn't he use that approach with mental computation strategies? Both Bob and Peter didn't answer although they did say that they shared "how did you do that?" at the end (PL 1 observation, 18/2/2009).

The critical friend involved as an additional Researcher also noted:

They [Peter and Bob] just didn't see the value in giving the students the language and prescribing a way of solving it instead telling them to say, "I just estimated". Problem-based learning allows students freedom to explore, just do it, not caught up in naming (PL critical friend's observations, 18/2/2009).

Key Finding 5.11: Peter did not believe in teaching formal estimation strategies.

In the afternoon, the groups were presented with the suggested learning tasks that were to be adapted to suit the individual teacher's contexts. The teachers had been encouraged in the morning to work with other teachers in the group whom they did not know (PL1 observations, 18/2/2010) but in the afternoon they were encouraged to work in their school teams so that they could discuss how their individual school contexts would affect their planning. The number of teachers attending from each school had not been stipulated (PL handbook, 2009) but all the other participants had another colleague from their school to discuss things. This meant that in the afternoon, Peter joined another pair of teachers from one school but obviously he was not able to discuss his particular school context (PL1 observations, 18/2/2009). The critical friend noted that she believed that this was disadvantageous for Peter's development and she voiced this concern at the end of the first professional learning workshop:

Sarah [the critical friend] had considered how the professional learning workshop develops teacher PCK. She wrote in her notes that working in school pairs was effective, the shared relevance and context made it more meaningful. Sarah elaborated on this in discussion with me at the end of the day to explain that she felt that working with others in the school teams was very productive in order to develop understanding. We also discussed how Peter is actually at a disadvantage with this as he is working without anyone from his school (PL1 observation, 18/2/2009).

The teachers were asked to consider how they would teach computational estimation in their classroom, taking into consideration that research findings suggested that estimation has a place as part of checking exact calculations and in problem solving, where estimation could be the main computational choice (Yoshikawa, 1994).

The first research cycle: Acting and reflecting

At the end of the professional learning workshop it was explained to the teachers that they needed to trial the professional learning activities in a way that they believed was pedagogically appropriate (PL1 observations, 18/2/2009). They were asked to contact the Researcher when they were teaching the tasks, so that these teaching approaches would be observed.

The school term had nearly ended and Peter had not contacted the Researcher. When there were only two weeks left before the end of term, contact was made via email to inquire what progress Peter was making (Email from Researcher, 26/3/2009). If Peter did not consider that the computational estimation tasks were useful to implement in the classroom this opinion needed to be respected. Peter informed the Researcher that he had been very busy with other activities and had not managed to fit the activities into term 1 (Email from Peter, 7/4/2009). He appeared not to have engaged in the reflective cycle envisaged by the professional learning intervention (Audit trail journal, 29/3/2009). Peter was asked in a later interview if he believed that he had difficulty engaging in the action research professional learning process. He explained that he had many other priorities at the school and that his thoughts had not always been focussed on reflecting on the professional learning workshops.

Researcher: So you think that at times you have had other issues to think about other than the curriculum you teach?

Peter: Without a doubt, too much.

Researcher: Now the professional learning has been about asking the question about the teaching and learning of computational estimation. Do you think there has been competing priorities?

Peter: Yes, there's always other things. There is always something stopping you running your program or, I don't know, there are always other priorities in front of you (Teacher interview 3, 29/10/2009).

Due to this initial lack of engagement, it was of interest to find out if Peter perceived that this was due to the way the professional learning intervention had been organised. In the final interview, Peter was asked if the professional learning program could have been improved at all. His answers appeared to imply that he perceived that the professional learning workshops were satisfactory:

Researcher: How could the professional learning workshops have supported you further?

Peter: They [the different aspects of the professional learning workshops] were all positive ... Probably didn't share too many of the ideas outside the contact of the group of ECU, we didn't except for the sharing of the jelly beans ... Let's see, I don't know, you gave us lesson plans, you gave us ideas, you supplied us with resources. I don't think I could have asked for any more to tell you the truth (Teacher interview 3, 29/10/2009).

Peter's focus appeared to have been on managing the day-to-day concerns of the school context and he was not able consider how the ideas presented in the research literature could be relevant for his teaching:

You probably could [have more sharing of ideas from the professional learning workshops] but it's another thing, there are so many things to do and so many people to contact that, I don't know, sometimes you want to do it but you prioritise in your classroom. Unfortunately, there is big difference between the ideal world and the reality (Teacher interview 3, 29/10/2009).

The second research cycle: Reflecting and planning

When the teachers gathered back on the second professional learning workshop, the day began with each teacher sharing their thoughts. It is important to reiterate that within this forum each person's views were equally valued. Peter reported on his trialling in the classroom and he explained that he had been using estimation only as a checking device in his mental mathematics but he explained that his students were not engaged using this teaching approach:

Estimation is a way of pre- and post-checking, estimation is always pushed in the morning in mental maths cos that is what it is ... and a lot of exposure to activities that try and prompt that. Normally they will do the sum and then just round the number and say that is my estimation so I guess that is not going to change until they work out for themselves that, em, estimation is going to help them when they fail (PL 2 Observations, 13/5/2009).

In this reflective session, Peter also shared his belief that he perceived primary school teachers did not play an important role in developing students' estimation skills. He believed that students would develop these skills anyway:

Bear in mind that all adults estimate when we are older, we all do it now so I think it is just this year [that they don't want to estimate]... They all develop through failure (PL2 observations, 13/5/2009).

Key Finding 5.12: Peter believed that students would develop estimation skills regardless of the teaching approach.

After the teachers shared their individual experiences of engaging with the professional learning process, there was a general discussion. At this point, Peter did begin to start considering some of the ideas about estimation and engaging in some of the discussion. Peter made the point that he believed that it was important to begin the estimation work further down the school:

We did talk about what happens in early years [in the last professional learning workshop]. You know kids are throwing two dice and they are told 9, 10, 11, 12 rather than about 10 and 10 (PL2 observation 13/5/2009).

During the second professional learning day, the teachers were provided with a comprehensive picture of the estimation strategies. It was explained to the group that in the research literature there were certain estimation strategies that may be suitable for the primary school. After the introduction, the group were asked for their initial thoughts. In the first professional learning workshop, Peter had not thought there was value in naming the computational estimation strategies and in this workshop Peter still believed that teaching and naming these computational estimation strategies in the primary school was inappropriate:

Facilitator: What you think of the names [of the computational estimation strategies]? If they were going to become commonplace, so like bridging ten. What do you think those terms would mean? I am keen to hear whether (at this point I see that the group is looking uncomfortable, so I clarify my assumption that people name mental strategies generally). Do you get kids to talk about bridging ten?

(No immediate reply)

Peter: I think we had this discussion last time. I think the thing is that kids have to find out for themselves. If you are talking about it i.e., instead of $9 + 9$ you say $20 - 2$. You wouldn't name that, compensation? I just say, "How did you do it?" I don't think you need it. Especially weaker students, you give any extra information, you just over load them.

Facilitator: I guess I am thinking it would actually even be a trigger for them (PL2 observation, 13/5/2010).

During the day, Peter appeared to be engaged in the session about how games could be an effective teaching approach in the primary classroom and was an active participant in this session. This is logical as he mentioned frequently how he perceived that there was great value in using games as a vehicle for learning mathematics (PL2 observation, 13/5/2010).

In the afternoon, Peter worked with two teachers from another school who had trialled many more of the suggested learning activities and these other teachers took the lead in the planning. Together they produced a plan that consisted of students solving Fermi problems working with larger numbers and solving more abstract problems i.e., how much water you have drunk in your life time (PL2 observation, 13/5/2010) .

The second research cycle: Acting and reflecting

During Term 2, Peter's pre-service teacher was taking all of the mathematics classes and Peter explained to the Researcher that she would be teaching the students during the visit to the school by the Researcher (Classroom observation 1, 6/5/2009). Peter had given the pre-service teacher the freedom to design her own program. As the professional learning facilitator, I believed that this would be beneficial for the pre-service teacher to teach some of the estimation activities if she thought that they were suitable. This would give Peter the opportunity to observe some of the type of activities that he was unsure about pedagogically and be able to discuss these with another teacher who had worked in his context. When visiting the school during Term 2 the classroom was still organised in the same way that Peter had set it up in the previous year i.e., he had the classroom organised in rows with the teacher's table at the front (Classroom observation 1, 6/5/2009). The computational estimation strategy word wall, which had been provided on the professional learning days, had not been put up on the wall.

The pre-service teacher gave the students experiences of undertaking computational estimation using the concrete referent of M and M's (Classroom observation 1, 6/5/2009). In this lesson, one student used the sample strategy of realising that it was possible to use a known quantity of 45 M and Ms in a 50 ml jar to calculate how many there may be in 100 ml, although this strategy was not formally named (John,

Classroom observation 1, 6/5/2009). Another student explained his strategy of calculating how many “M and Ms were in the jar.” “95 ’cos that one was 5 less than 50 [in the 50 ml jar]” (Brandon, Classroom observations 1, 6/5/2009). In a second lesson undertaken by the pre-service teacher, she used the jellybeans as a concrete referent for the students to estimate. Peter joined in this lesson towards the end (Classroom observation 2, 24/6/2010). Peter explained to the class at the end of the lesson that this jar of jellybeans would be going to another school so that they could try to estimate how many jellybeans there were in the jar (Classroom observation 2, 24/6/2010).

The second teacher interview took place at the school immediately after the lesson taken by the pre-service teacher where the students had been estimating how many jellybeans were in the jar. Peter had allowed the pre-service teacher to teach mathematics in a way that she had thought was the most effective. He explained that she had influenced his beliefs about teaching in that she focused on students explaining their thinking and not just focusing on the correct answer:

The pre-service teacher has been teaching and one thing I will put more emphasis on, making sure I go through “how did you do that?” You know even more. I think I was quite good at it before anyway cos I came through uni later and that is one thing that they were pushing but I definitely more conscious of doing that (Teacher interview 2, 24/6/2010).

Peter maintained that if he had been teaching the program he would not have used the selected computational estimation activities and taught problem-based activities, where estimation was the computational choice. Instead, he explained that he would have taught computational estimation as a checking device in his mental mathematics and maths textbook work. He explained that, “If I was teaching this term, I wouldn’t have been teaching estimation [a computational choice in its own right]. I would have been using it as a [checking strategy] strategy for all maths [mental maths and textbook] you have to, it has to be” (Teacher interview 2, 24/6/2010).

Peter explained in this second interview that when he taught computational estimation as a checking device in his mathematics lessons, the students were not motivated to use estimation and in fact would simply write in an estimate after they had worked out the exact answer:

The hardest part of it is estimation as a checking tool and whether they do it and or whether they find the answer and then rounding it afterwards. I think that is the hardest part. Teaching them the value of it – for a Year 6 kid it's extra work (Teacher interview 2, 24/6/2010).

Key Finding 5.13: Peter believed that exercises in his school's textbooks that required students to use an estimate, as a checking device did not engage his students.

Peter maintained his belief that students would eventually start to use estimation even if they were not keen to use it in the primary school: "I think that it [estimation] comes anyway. I am sure that high school kids they would be checking and estimating" (Teacher interview 2, 24/6/2010).

In this second interview, Peter explained how his normal focus in the second term with Year 6 was teaching multiplication with decimals. His description of the problems the students have with this teaching approach revealed how Peter's normal pedagogical approach was on following procedures such as counting places and putting in the decimal point to obtain the correct answers:

Peter: Multiplication with decimals for some reason, don't know some get it, some don't get it, we do it and we have been doing it for the last week and a half and we have and for some reason some just don't get it in year 6.

Researcher: I guess it is quite complex isn't it?

Peter: Not really, just count the places and put it in the spot (Teacher interview 2, 24/6/2010).

He explained however that the students found this approach surprisingly difficult to master. Although the procedure was quite simple, it was difficult for the students to remember (Teacher interview 2, 24/6/2010).

Peter did not appear to be developing his PCK of computational estimation. When he was asked what computational estimation strategies he was aware of, he appeared not to have engaged in the workshop session and developed his content knowledge of the computational estimation strategies:

Researcher: What estimation strategies other than rounding are you aware of?

Peter: I am the one who believed there shouldn't be terms for these things.

Researcher: But you said // (interruption)

Peter: Building on, I can't remember it, see it's not me doubling (Teacher interview 2, 24/6/2010).

In the second teacher interview, Peter was also asked how he could estimate when adding fractions with unlike denominators. At the professional learning workshops, the teachers had been shown the strategy of how to estimate with more complex fractions by approximating them to the nearest benchmarked whole numbers but Peter did not appear to remember this:

Researcher : Ms Fot's class were posed with the question: Can you give me a quick estimated answer to the question $4/10 + 8/9$?

Two children gave the answer fairly quickly without having to reach for pen and paper or a calculator. What ideas do you have as to how they did this?

Peter: They probably round the fraction 9th into a fraction that was similar to the 10th and then just added them (Classroom observation, 24/6/2009).

In the second teacher interview Peter explained that he did perceive that his beliefs about how to teach mathematics had been altered whilst being involved in the professional learning intervention. Peter did appear to genuinely begin to reconsider some of his teaching approaches as he now perceived that students needed to have a deep understanding of number that they would be able to use flexibly. He stated that he now believed that "number sense needs to be brought out and taught more and it is valuable and much earlier grade one, doubling, groups and pattern" (Teacher interview 2, 24/6/2009).

Key Finding 5.14: Peter was beginning to consider that number sense activities had value in the Year 6 classroom.

When Peter arrived at the third professional learning workshop, he seemed to be more relaxed (PL3 observations, 29/7/2009). Peter explained to the rest of the group that the pre-service teacher had been teaching the class for most of the mathematics activities. He explained that at this stage, his class were estimating before their computations. He did discuss how his teaching now included more discussion with

the students so that they could explain their thinking. He believed this to be beneficial, as the students were learning from each other and explaining how they were estimating (Critical friend's observational notes, 29/7/2009).

Over lunch, Peter discussed with the teacher from the school who was involved in the guessing the jelly beans competition. They had each organised their classes so that each child had estimated how many jellybeans there were in the other class's jar. They shared the estimates and assessed which student had arrived at the best estimate (PL3 observations, 29/7/2009).

In the afternoon, Peter worked enthusiastically on the planned trip with the teachers from Hill View School and at the final discussion of the day at the professional workshop; he explained that he was going to run it as a fundraising barbeque (PL3 observations, 29/7/2009). At the end of the day, Peter asked when it would be convenient to arrange a time to watch his computational estimation activities. This was the first time that he had done this.

The third research cycle: Acting and reflecting

The classroom looked completely different when visiting it for the third time (Classroom observation 3, 26/8/2009). The tables were not organised in rows anymore but were now in groups. They had butchers paper on them. Peter did not involve the class in an introductory activity. Instead he explained to the whole class that they were going to plan an excursion and that initially they were to undertake a brainstorm in their groups. (Classroom observation 3, 26/8/2009).

Key Finding 5.15: Peter began to use the teaching approach of organising the class to work in small groups in order to facilitate discussion about the computational estimation problems.

Peter did scaffold their learning at this point and guide the students as to what areas they may need to consider in their groups using question and answer:

- Teacher: What is some of the things we have to think about when we plan an excursion?
- Fred: Price.
- Teacher: Good, what else?
- Scott: Where the place is.

Teacher: Location, pick a new cloud, pick a corner. Okay that shouldn't take that long.

All right, so we have got price, location, what else should we think about?

John: When.

Teacher: When (Classroom observation 3, 26/8/2009).

At that point, Peter gave the class time to complete their group brainstorm. Whilst they were discussing these aspects in their groups, Peter walked around the classroom observing the groups' progress. He read some of the students' work and made comments on them to the group. He was particularly concerned that they not get involved in too much detail at this point. The class were extremely excited about this activity (Classroom observation 3, 26/8/2009).

After about five minutes, Peter then explained what he wanted the finished task to look like. He wanted the students to complete a proposal for the trip. He did specify that the students had to include the route, the activity and the costs:

There is a lot of work here. You have to complete this excursion package and I am going to go through quickly what you have to do. All right, you have to make a written proposal. You will give me half a page, the aim of the excursion, the route as explained on a map and I have got maps here and I will give them to each group. What time you are going to start, what time the buses are going to come, what time we are going to finish, all those things we need to think about and a budget time table of all the costs (Classroom observation, 26/8/2009).

When explaining the task Peter did set some parameters such as departure and arrival times. There were a few questions from the class regarding other aspects, which potentially could have been part of a class negotiation for this type of task. The class were extremely excited and engaged in this activity and they were all very keen to plan the task. Peter provided the students with lots of resources to help their planning but inherent in this type of task in the primary classroom is the fact that not all the costs are available which makes it ideal for an estimation activity (Classroom observations 3, 26/8/2009). When explaining to the students that they would not know all the costs, Peter revealed that he perceived estimation to be a type of guess: "now you won't have the cost of some of the things, so you are going to have to

guess.” The task was scaffolded to a certain extent, as Peter provided his own booklet for the students to fill in and this included pre-prepared tables to complete different aspects such as itineraries (Classroom observations 3, 26/8/2009).

As the students completed the task, they were all engaged and there was lots of discussion about where they would visit. They were very interested in the non-mathematical elements such as where they should go. Peter did not interrupt them as they progressed and worked through the tasks, although as he walked round he did ask them what progress they were making. Jack’s group was observed having problems deciding how long the bus trip would take, although they did not raise this with the teacher. The students required an understanding of rates and they appeared not to have a clear procedure as to how they would solve it. Figure 5.2 shows how they finally decided that the coach would travel at one km/minute.

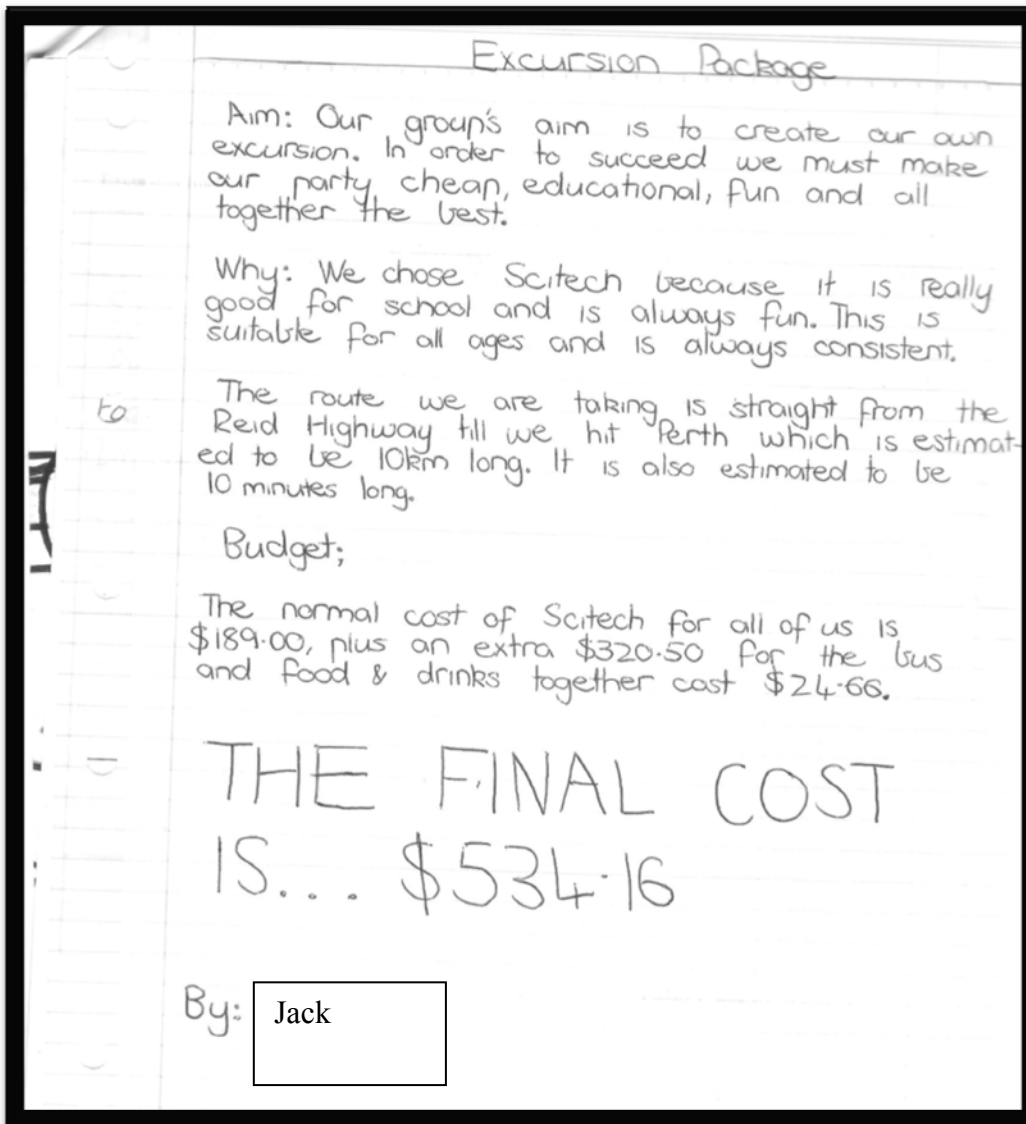


Figure 5.2: Jack and his group estimate cost and times of the planned trip

Students around the class were naturally estimating using language such as, “We will get there about 1.50” (Edward, Classroom observations 3, 26/8/2009). Many of the students were also putting times into half hours (Classroom observations 3, 26/8/2009).

Key Finding 5.16: Peter’s students used computational estimation language when discussing how to solve the mathematical problem.

The groups worked on this task for one hour and they nearly all engaged for this time. They were asked to complete their proposal by the end of the session.

As a plenary to the lesson, Peter asked each group to explain their proposal and describe their estimates. The first group nominated two people to come out to the

front. Lily was able to explain the trip carefully and focussed on the estimated computations in the planning:

The aim of the excursion is for people to have a chance and learn the art of roller-skating. It is also for people to learn about marine life and marine biology. For the planetarium, it is to observe what happens in space, the earth and the sun and the stars constellation.

We will start at Sandilands . This will be approximately 20 km. Hillarys boat harbour is \$ 280, AQUA is \$10 each.

People in the bus, \$2.50 each and to get to Morley is \$10 which is 5kms which is \$50. Morley to Hillarys is \$10 per km and 13 km and its \$130. Hillarys to school is 10 dollars/km, 20 km, 200 dollars. All up it will cost \$1062. (Lily, Classroom observations 3, 26/8/2009).

They presented their proposal, which consisted of a front cover page. This page shows the great interest the students had in the non-mathematical aspects of the trip and that this engagement contributed towards the students creating mathematical calculations. Inside the proposal it included a timetable of the suggested itinerary. The students used estimated times that could be calculated easily. The group of students then included a budget of the costs of the day trip (Figure 5.3).

Key Finding 5.17: Peter's students were able to use estimations as a main computational choice on extended problem solving tasks.

Peter made sure that all the class listened carefully and praised them for their efforts but he did not undertake any extended conversation about the mathematics nor the estimates used.

Peter: Is that your whole excursion ? Okay, what did you do about your lunch?

Lily: Well we have to bring our own food because some people have to bring special place settings.

Peter: Well done, give them a clap.

The group went and sat down (Classroom observations 3, 26/8/2009).

Final Pedagogical Content Knowledge, Beliefs and Teaching Approaches

In the final interview, Peter reflected on his teaching approaches at the end of the professional learning intervention (Teacher interview 3, 29/10/2009). Peter explained that the school was planning to reconsider its whole school teaching approach of mathematics:

That is going to be the focus next year. We are going to try and get a maths coordinator, the school is going to change in its format next year ... That has only been announced in the last, em, two weeks and one of the focusses will be the maths curriculum (Teacher interview 3, 29/10/2009).

Peter stated that he was now more conscious of developing computational estimation into every mathematics lesson. "I am more conscious of it [computational estimation] now, every maths lesson I am thinking about it and how can I put it [computational estimation] in, even if I just talk about it a little bit" (Teacher interview 3, 29/10/2009).

Peter stated that he was also spending more time discussing with the students how they had obtained the answers in their thinking, not just finding out whether they had achieved the correct answer (Teacher interview 3, 29/10/2009).

When the students were working on mathematical problems in different contexts, he now believed that computational estimation was an important computational choice in its own right:

Yeh if you really want to analyse it, we only estimate, we never give exact. So even how old you are it is still an estimation ... so it is more important than what you think ... (Teacher interview 3, 29/10/2009).

Peter perceived that when teaching he was now asking students just to calculate an estimate at times and not just use estimation as a checking tool:

Researcher: How has your teaching impacted their use [the students] of estimation?

Peter: They [the students] have been told to think about it and not just as a checking tool, as an actual tool.

Peter was still not teaching the students the computational estimation strategies formally and instead was asking the students to estimate without any discussion of how to estimate (Teacher interview 3, 29/10/2009). He explained this in the final teacher interview:

So it seems to me that when we start to define them [computational estimation strategies] we confuse the weaker ones that we are trying to pull through, 'cos they are trying to learn new definitions rather than learn the procedure or different ways that they can do it (Teacher interview 3, 29/10/2009).

Peter explained that another reason he did not think it was worthwhile to introduce formal strategy terms was that when the students were explaining their process it was difficult for him to make sense of this and then to provide a formal strategy term for this process immediately. He appeared to believe that the benefit of sharing this information was more for the other students in the class who did appear to follow the computation processes more easily than he did:

Sometimes I say to a kid, "How did you do that?" and I just can't even keep up but another kid says "Yeh that is how I do it ". How are you supposed to give a definition to that and it does happen, that is one of the reasons why I don't like naming them (Teacher interview 3, 29/10/2009).

Key Finding 5.18: Peter's pedagogical approaches for teaching computational estimation included; solving problems where estimation was the main computational choice, and students reflecting on reasonableness in computations in mental mathematics by using estimation as a checking tool, but not formally teaching estimation strategies.

Peter's PCK about computational estimation was beginning to develop. His content knowledge was increasing, as he now knew some of the computational estimation strategies. He was able to name the front end loading strategy. He was also able to use the benchmarking strategy when estimating with fractions and saw the value in the strategy even though he called it a rounding strategy.

Researcher: Mr Fot's class were posed with the question: Can you give me a quick estimated answer to the question $9/10 + 8/9 = ?$

Peter: Well he rounded, that it is almost a whole number and almost a whole number so I actually did that one with my class it was really good. I even did it with my year 12s which was really good but none of them got it no one everyone tried to go common denominator cos that is how we have been taught (Teacher interview 3, 29/10/2009).

Key Finding 5.19: At the end of the project Peter had a growing understanding of the estimation strategies although he did not use all the formal estimation strategy terms.

Students' Final Beliefs about Computational Estimation

At the end of the study, the students in the focus group were gathered together in order to find out if their beliefs about computational estimation and mathematics had changed whilst their teacher had been involved in the professional learning intervention. When the students in the focus group were asked what they would do if someone had a different answer to them, the students' answers implied that they

normally expected there to be one correct answer. One student did allude to the fact that he now thought he might compare strategies: “You could identify their strategies and why they chose that strategy” (John, Student focus group 2, 29/10/2009).

When asked to describe what mathematics was, all the responses referred to the four operations and numbers; “Where numbers say in addition you would add them to make a different number and multiplication and division” (John, Student focus group 2, 29/10/2009).

The students in the focus group still seemed to think that mathematics was something that should not take 15 minutes. Jack’s response reflected how he perceived that if you had to take that long on the question it would mean you were finding it unusually difficult:

No because he could just skip that question and go to another question, so there is more percentage of him getting a higher mark than just sitting on it and if he has enough time to finish it all, he has time to come back and finish it if he hasn’t done.

Key Finding 5.20: Students in the focus group believed that mathematics is about working out problems, is about one correct answer, and something that is done quickly.

When the students in the focus group were asked how they perceived the estimation activities the students who responded commented that they had been fun, suggesting that they did perceive that these extra activities were an extra addition to normal mathematics:

Jack: I think they are quite fun. It is always very challenging and with estimating. There is not much chance of you always getting it right every time, but by following simple and effective strategies you can come quite close to the answer and I think that’s what challenges you and estimation can be done all different ways. It can be fun. It can be on paper, it can be jellybeans and that’s what makes it fun.

Jason: I think he covered it.

Josh: Interesting.

Jason: Quite fun (Student focus group 2, 29/10/2009).

As part of the data collection, the students were also asked to draw concept maps about their perceptions of computational estimation. At the beginning of the study, most students in the focus group perceived estimation to be a type of guess and their definition of estimation was quite limited. One student's pre-intervention concept map is shown in Figure 5.4. It revealed that the student had quite a limited conception of estimation. In the post-intervention concept map, the students' responses indicated that they did not associate estimation with the contextual problems. Not one student in the focus group mentioned these learning tasks nor did they refer to affective perceptions that they may have developed from these different types of learning experiences. Instead, they focused mostly on estimation as a checking device. Their experiences of estimation after the professional learning intervention did appear to have broadened as shown in Figure 4.5 and the word 'guess' was only mentioned by one of the students on their concept maps. They did not mention any of the computational estimation strategies that they now used, suggesting that the class did not have a vocabulary to describe the strategies that they may have created.

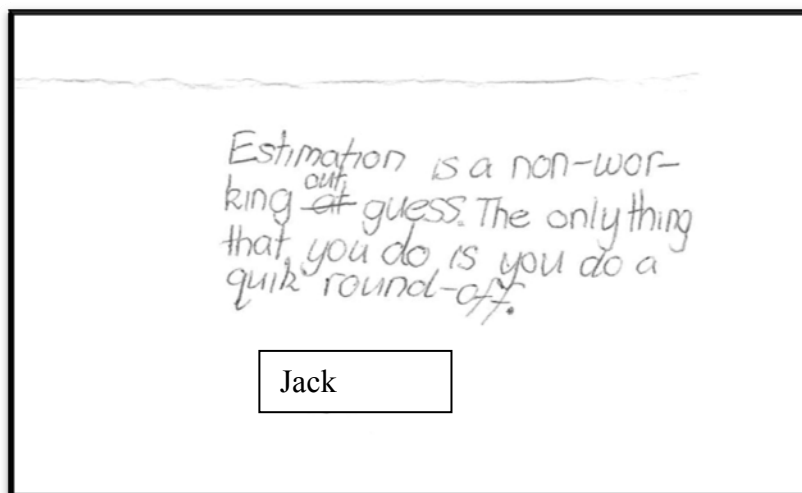


Figure 5.4: Jack's pre study concept map of estimation

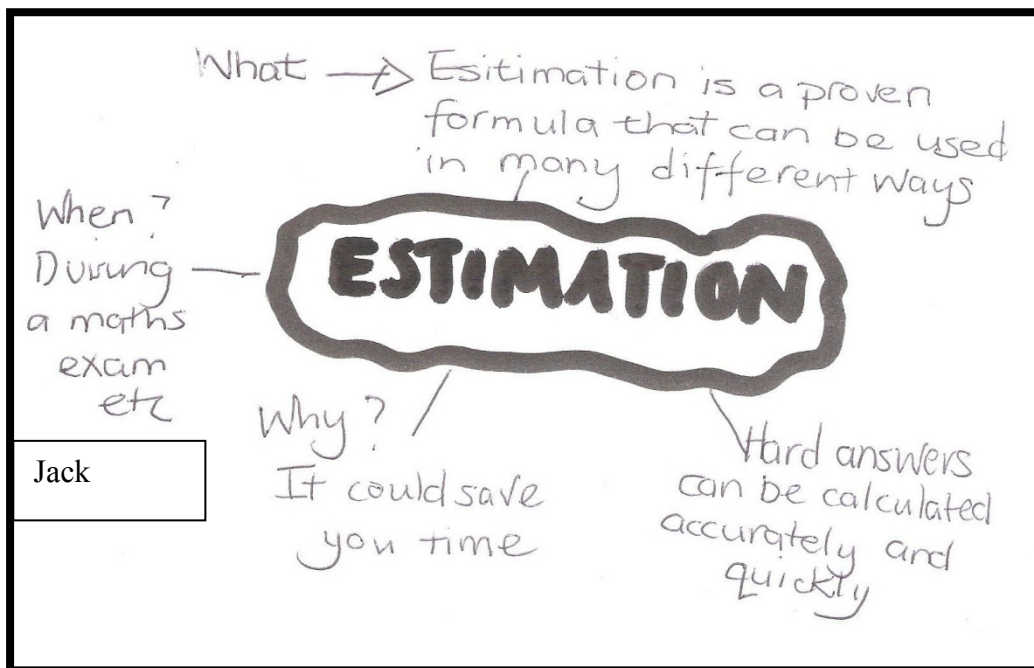


Figure 5.5: Jack's post-test computational estimation concept map

Key Finding 5.21: Students in the focus group believed that learning about computational estimation is “fun”, is about the rounding strategy and useful as a checking tool.

Students’ Computational Estimation Performance after the Professional Learning Intervention

At the end of the study, which was at the end of the school year, the students completed the CET for the second time. The mean in the pre-test score was 2.6 and the mean in the post-test score was 3.4. Therefore, the mean student test score improved by 0.8 and this result was not statistically significant.

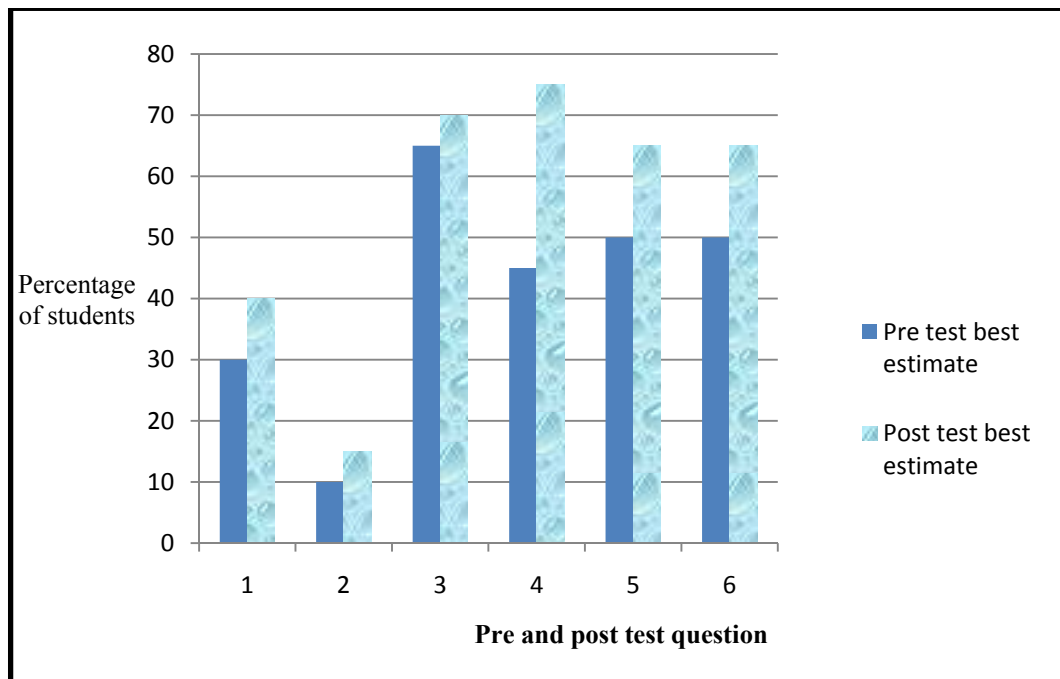


Figure 5.6: Difference in pre and post-test results for the computational estimation test

Estimation Question 1

On Question 1, Figure 5.6 shows that 40% of students were now able to select the best estimate (B) 3000 in the post-test. This result reflects an improvement of 10%. This suggests that Peter’s class may have benefited from undertaking some activities set in meaningful contexts (Figure 5.6). Although the improvements in selecting a best estimate in a context were less than the improvements in the question presented symbolically, when considering how Peter’s students answered the estimation it became apparent that many students evaluated the range considering the question. Many students stated that it had to be more than 300 because that was less than one year and they had lived for eleven years. This ability to try to produce an answer which made sense may have arisen from spending time in the classroom considering how different students solved problems rather than simply being presented with one procedural way of solving the problem. It appeared that the pre-service teacher’s pedagogy and the professional learning interventions focus on sense making activities might have been the catalyst for this change in the students’ thinking especially as the students appeared to have great potential for learning. Table 5.7 shows that in the post-test, 60% of students used a reasoned estimation strategy. Within the reasoned estimation strategies, 25% of those used the range strategy specifically.

Estimation Question 2

In Question 2, the students needed to possess a conceptual understanding of fractions, so that they would be able to identify that both the fractions in the computation were close to one. The computation then becomes very straightforward. In the post-test, only 15% of the students were able to answer this correctly.

This suggested that most students in Peter's class had not spent time developing the benchmark strategy. At the end of the research study, 70% of the students selected C (19). The students, unable to draw on a reasoned estimation strategy, possibly reverted to whole number thinking and simply added the two digits, which represented the numerators. Other evidence supports this suggestion. In the final focus group, the students were asked a similar question. The two students that answered both suggested adding the face value of the digits in the fraction:

Researcher: Bill then worked out $11/12$ and $7/8$ and he said the answer was about $1/2$, 2, or $18/20$ s. What was his answer and how did he work it out.

John: $18/20$ because he added 12 and 8 which equals 20 and he added 11 and 7 which is equal to 18, $18/20$.

Jason: Like John he added 8 with 12 equals 20 $18/20$ simplify it so half it so $9/10$, yeh and that's how he got it (Student focus group 2, 29/10/2009).

Estimation Question 3

In Question 3, Table 5.7 shows that 70% were able to select the best estimate (D). This was greater than the result in the Macintosh's example (54% for 10 year olds and 62% for 12 year olds). Overall 45% of the students used a reasoned estimation strategy, which was only a 5% improvement on the pre-test. The pre-service teacher spent time scaffolding a process where the students knew how many there were in a sample of a numerical quantity and then used that knowledge to select the total amount. There were 30% of the students that used the range strategy focussing on making sense of how many triangles there could be in total. None of the students was identified as having used a sample strategy even though the pre-service teacher had modelled this strategy although she did not name it.

Estimation Question 4

In the post-test Table 5.7 shows that 75% of the students were able to select the best estimate. This is an improvement of 35%. This question required the students to work on an estimation presented symbolically and required a precise estimate: The three multiple-choice answers (A) 4000 (B) 4600 and (C) 5200 are all estimates that may be considered as suitable. Peter spent a lot of time with his class in introductory mental arithmetic sessions that focussed on mentally computing numbers not presented contextually and worked on computational estimations presented in the same format in the textbook. Peter believed that computational estimation was important as a checking device for symbolic computations and spent most of the school year focussing on this. This may have led the students to spend more time considering how precise the estimate was in comparison to the exact answer. Peter only began to focus on computational estimation as the main computational choice towards the end of the professional learning intervention and even when he did discuss this with his students, he did not overtly mention to the students that they should make the numbers easier. This approach appears to have produced students who produced a high proficiency in selecting a best estimate in a symbolic format and requiring a level of precision.

Estimation Question 5

In this question students needed to estimate two digit addition calculations. In the post-test, 65% of students were able to select the best estimate. This means that 35% were still not able to select the best estimate in this calculation and as this is one of the first operations that students would work with it is surprising that more students did not select the best estimate. The students were more competent at estimating in a multiplication calculation than an addition calculation. It may be that it is more common in school to teach students to multiply a number by 10 than it is to estimate in addition calculation. Peter did not explicitly teach the students how to use front-end loading strategy, which would have been an appropriate strategy. There were 45% of the students who were able to use a reasoned estimation strategy at the end of the study, which was a 20% improvement. About a third of the students were therefore still not able to use a reasoned estimation strategy when estimating a two-digit addition calculation.

Estimation Question 6

In this question, 65% of students were able to select the best estimate when estimating within a range. This was a 15% improvement in comparison to the pre-test. This question requires conceptual understanding of decimals and due to the time restrictions, the students in Peter's class would not be able to solve it using a procedural approach that Peter explained using in the class. There were 50% of students able to use a reasoned estimation strategy at the end of the task. Of these students, 25% used the range strategy. This result means that half the class were not able to select and use a reasoned estimation strategy when estimating a multiplication calculation which included decimals and required the answer to be presented within a range.

Overall, it appears that the students' performance in using reasoned estimation strategies and selecting a best estimate might have improved however, the level of improvement was not statistically significant ($p = 0.076$).

Key Finding 5.25: Peter's students' computational estimation performance improved overall but statistically this was not significant.

The time spent judging the reasonableness of exact calculations, and learning to use the rounding strategy in the text book appears to have increased the students' ability to select the best estimate in symbolic calculations where the rounding strategy could be used.

Key Finding 5.24: Peter's students are much more proficient at estimating multiplication problems, which are purely symbolic and not set in context.

Key Finding 5.26: Peter's students' ability to select the best estimate on a multiplication calculation that required an answer with some precision improved.

The students appeared to have difficulties selecting the best estimate on questions where other estimation strategies were required such as the benchmarking strategy.

Key Finding 5.22: Generally, students in Peter’s class were not able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and few students were able to use the benchmarking strategy.

The students did appear to have developed awareness of selecting an estimate that is more precise than others are. This may reflect that Peter spent time considering how close an exact answer was to an estimate and generally creating a classroom culture where calculations were expected to make sense.

Table 5.7: Percentage of students using a reasoned estimation strategy in pre and post- tests

Question	Reasoned strategy used			Best estimate		
	Pre-test	Post-test	Increase	Pre-test	Post-test	Increase
1.	20	60	40	30	40	10
2.	5	10	5	10	15	5
3.	40	45	5	65	70	5
4.	25	45	20	45	75	35
5.	20	45	25	50	65	15
6.	20	50	30	50	65	15

Chapter Summary

In response to the professional learning intervention, Peter’s engagement in developing his PCK was impeded by his beliefs that the ideas presented were not relevant to his classroom context. Peter believed that estimation was useful as a checking device and he encouraged his students to use this strategy. As Peter began to perceive that the professional learning process had value he began to investigate some new teaching approaches, which he implemented in the classroom setting (see Appendix L). This process did appear to impact upon students’ beliefs and to a limited extent student performance of computational estimation (see Figure 5.7).

Pedagogical content knowledge

The Researcher's personal experience suggested that most primary school teachers do not spend time teaching the variety of computational estimation strategies. It was not surprising, therefore, that Peter did not know about the computational estimation strategies at the beginning of the professional learning workshops. In the second interviews, Peter still had difficulty naming any of the computational estimation strategies and he was unable to describe how students could solve different estimation problems using the strategies. This may have been because he did not believe that the computational estimation strategies were not appropriate to teach to students and therefore it would not be time well spent learning these strategies. It was also the case that Peter had many day-to-day demands which limited the time available to reflect on the professional learning workshops. It did appear that Peter became more positive about the estimation strategies and in the final interviews Peter was able to name some of the strategies and he was now able to confidently answer how students could solve estimation problems using the strategies.

Despite beginning to understand these strategies, due to his belief that they were not appropriate to teach to primary school students, he did not develop any pedagogical approaches for teaching these strategies. His pedagogical approaches were mainly designed to integrate computational estimation as a checking device for the mathematics lessons that required exact answers. The students were taught the rounding strategy in the textbook, so they were able to use this strategy when required to estimate. Peter was beginning to develop the pedagogical approaches of setting estimation tasks in meaningful contexts although he did not integrate the teaching of estimation strategies into this teaching approach.

Assertion 5.1: Peter did not engage in the professional learning process due to his beliefs that his context would not benefit from it, so therefore the impact on his PCK was limited at the beginning. As the reflective process continued, Peter's PCK of the program began to develop, in that he began to consider how computational estimation could be taught in Year 6 within a number sense framework. His beliefs that the computational estimation strategies did not have value still impeded Peter's development of the content knowledge about the strategies.

Teacher's beliefs

At the beginning of the professional learning intervention Peter appeared to believe

that students would become proficient in mathematics regardless of the approach the teacher took. He was very strongly aware that the parent wishes were paramount so he may have organised his classroom in a way that maintained harmony in the school environment i.e., quiet, working on mental mathematics and students knowing their tables. He appeared to believe that ideas presented by the research were therefore not relevant to the school context. Peter perceived that he had competing priorities and focussed on the day-to-day demands of the real classroom instead of reflecting on the theories in the research literature and professional development activities presented in the professional learning workshops. As the professional learning progressed, Peter did appear to begin to evaluate his present beliefs. This change may have been supported as the parents, who were extremely influential, began to sense a need for change. The pre-service teacher's pedagogy also appeared to support this change. Peter began to identify aspects from the professional learning workshops that had value in his classroom practice and therefore he began to form a broader conception of how to teach computational estimation. He began to believe that problems set in meaningful contexts could be beneficial to students. The professional learning workshops has defined computational estimation as a component of number sense and by the end of the intervention he believed that this needed to be emphasised. At the beginning of the professional learning workshops, he did not believe that the computational estimation strategies should be taught to the students and this belief did not change.

Assertion 5.2: Peter's beliefs that his school context would not benefit from developing his PCK, hindered Peter's engagement in this reflective process. Towards the end of the program, Peter's beliefs did change as he began to perceive that the process had value and that students would benefit from developing number sense although he did not believe that the estimation strategies had benefit for Year 6 pupils.

Teaching approaches

At the beginning of the professional learning intervention, Peter's main teaching approach of computational estimation was to tell students to estimate before conducting exact calculations in mental mathematics sessions at the beginning of lessons. As the professional learning intervention progressed, Peter gradually encouraged students to use estimation as a checking device in all his mathematics

lessons. This included when students were working through the textbook (R.I.C Publications, 2008). Towards the end of the professional learning, Peter tried a new teaching approach of creating a task where students had to work collaboratively on an estimation task set in a meaningful context.

Assertion 5.3: Initially it appeared that Peter was resistant to changing teaching approaches due to believing that parental expectations were paramount. Peter's beliefs did change and he came to believe that computational estimation was important in developing number sense. This change impacted upon his teaching approaches. He began to develop two teaching approaches; estimating in problem situations and estimation as a checking device.

Students' beliefs

As the students were following the textbook in their normal program, it was therefore to be expected that, at the end of the professional learning intervention, they would still perceive mathematics as something about one right answer and something that is done quickly. Peter's teaching approach to use on estimation as a checking device for exact answers this appears to have impacted on the students' beliefs as they saw estimation as something which could be used for this in mathematics. Towards the end of the research study, the students were engaged in activities that focussed on problems where estimation was the main computational choice, and these activities did appear to have impacted upon the students as they now had a more positive perception of estimation. Peter did not teach the students about the variety of computational estimation strategies and therefore the students still believed that the only estimation strategy available was rounding.

Assertion 5.4: Peter's beliefs that estimation was important as a checking device appeared to impact on the students, as they believed that estimation was important for improving their mathematics. Peter's teaching approach of creating extra problem-based computational estimation learning tasks appeared to impact on the students' beliefs as they perceived these new experiences of estimating to be fun.

Students' computational estimation competencies

Peter's students' computational estimation performance did improve, although the improvement in computational estimation was not statistically significant. Statistical analysis has to be interpreted within the fact that the testing was undertaken within a

naturalistic setting without attempting to control variables. It is therefore most useful to consider the individual question competencies. As Peter focussed on teaching students how to solve mathematical computations symbolically, it is logical they were more competent at estimating in this type of question than the question set in a context. At the end of the study, three quarters of the class were able to select the best estimate when presented with a symbolic multiplication computation.

At the beginning of the study, few students were able to select an appropriate estimate when adding fractions with unlike denominators. There was only a slight increase in students' performance at the end of the study. Peter did not teach the students how to use the benchmark strategy and it appears that few students had a reasoned estimation strategy available to answer this question.

There was a 15% improvement in students able to select a best estimate when estimating the addition of two digit numbers. However, at the end of the study, a third of the class were still unable to select the best estimate. As this calculation is very straightforward, it would be anticipated that students would have a higher level of competency in this area. This lower performance may be due to half the class not using a reasoned estimation strategy in order to answer this question.

Assertion 5.5: Peter's teaching approach of students evaluating the reasonableness when calculating and of creating extra problem-based computational estimation learning tasks appeared to improve students' estimation performance although the improvement was not statistically significant. The students appeared very receptive to the new teaching approaches which included sense-making activities. Peter's decision not to teach the formal estimation strategies appeared to limit the students' use of reasoned estimation strategies when calculating.

Conclusion

Peter's day-to-day demands in the school context appeared to impede his ability to engage fully in the reflective process in the first half of the professional learning program. Peter did appear to begin to reflect on the professional learning process about half way through the year and he started to investigate how he could teach computational estimation differently to his previous approach. As this reflection

process began mid way through the school year and due to the fact that Peter did not believe that all the ideas presented in the research literature were relevant to his school context, these changes in his practice only had a limited influence on the students' beliefs and computational estimation performance. At the end of the year, the parents also wanted the school to engage in a process to evaluate how mathematics was taught in the school. Peter, having engaged in the professional learning intervention, and arrived at a stage of critically evaluating his own teaching, meant that he was well placed to be part of the evaluation process in the coming year.

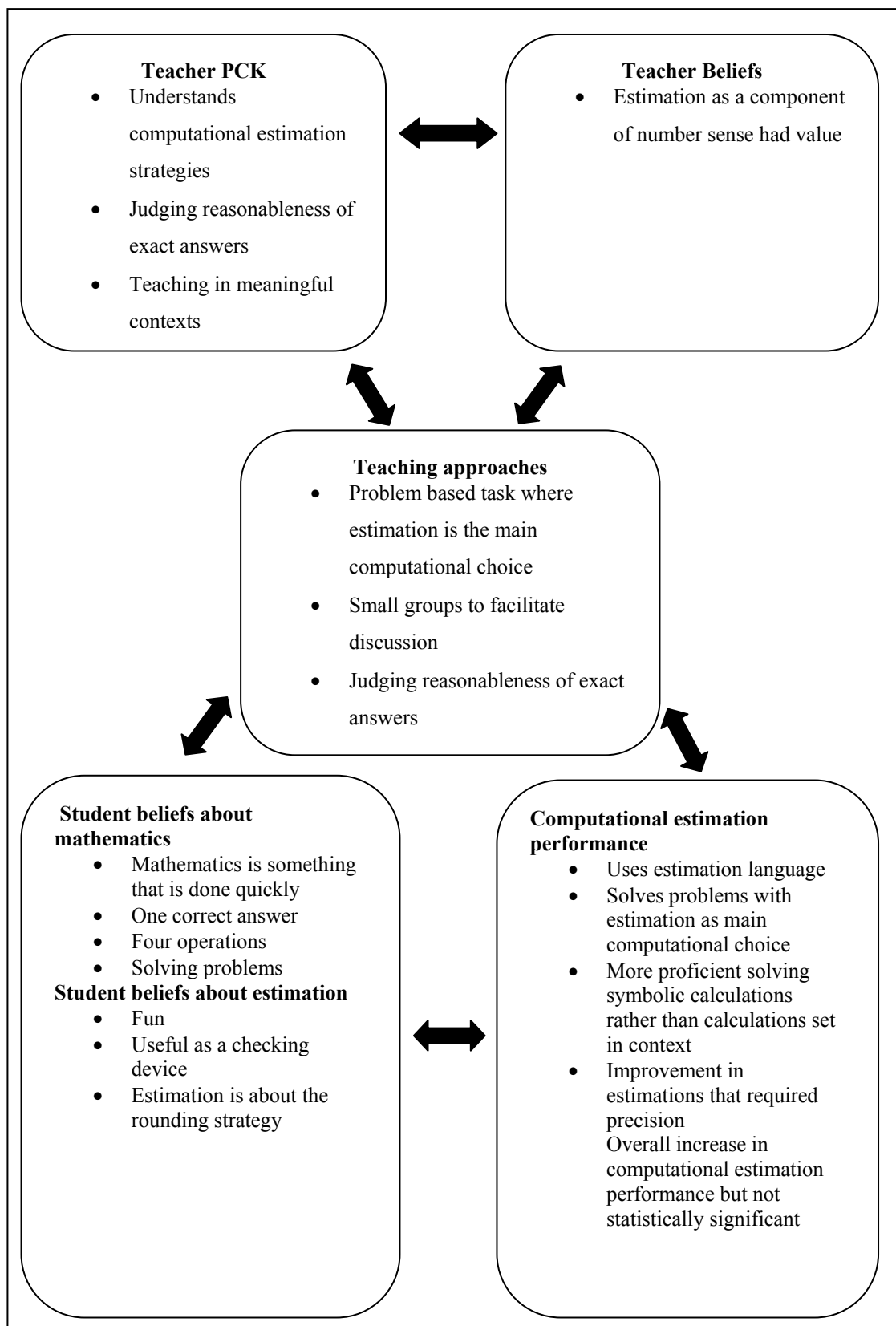


Figure 5.7: Model to show the impact of the professional learning intervention on Peter and his class

CHAPTER 6: CASE STUDY - BOB

Background

Bob had been teaching for 16 years and he is the upper primary coordinator in the school. He had worked in other jobs, including sheep shearing and the army (Teacher interview 1, 20/11/2008). He was enthusiastic about his teaching of mathematics and this enthusiasm was conveyed when he was asked what his thoughts were about the upcoming project:

I am looking forward to working in the maths area. It's something that for a couple of years I don't think there has been a push in it in schools (Teacher interview 1, 20/11/2008).

The school, Hillview School (pseudonym), is a K-12 low fee independent school with two campuses. There are 145 full time teaching staff and 1876 students on both campuses. The school buildings on the campus were in good condition, single storey with a well-stocked library and the grounds are well maintained. It was co-educational. The students generally had English as their first language and only one per cent of the students were identified as Indigenous (Australian Curriculum Assessment and Reporting Authority, 2010). The socioeconomic backgrounds of the parents, was about average, with an ICSEA value of 1003 (Australian Curriculum Assessment and Reporting Authority, 2010) where the average is 1000.

The 2008 NAPLAN testing revealed that their Year 5 students received an average score of 478 in mathematics compared with the national average of 476 and this cohort was the 2009 Year 6 students involved in the study. The school had made the decision to stream the classes for mathematics in Year 6. Bob's class was one of two Year 6 classes and his class was the higher ability of the two. There were about 30 children in each classroom but Bob had 25 students in his streamed mathematics class.

Bob's Views about Mathematics Teaching

At the beginning of the professional learning project Bob believed that mathematics should be engaging for students and he believed that primary school students should

be encouraged to “think outside the square” (Teacher interview 1, 20/11/2008). He perceived that there were two different aspects to his teaching. The first was a problem-solving approach and the second was teaching routine algorithms. He believed that it was very important that students understood the mathematics before they attempted the algorithms: “There is a place for repetition of algorithms, etc but only once the kids understand what the algorithm’s actually doing” (Teacher interview 1, 20/11/2008).

Bob was concerned that some students in his class received tuition out of school, which encouraged rote learning, and this was of particular concern with students who had Asian ethnic backgrounds. Bob was concerned that these students were not developing a deep understanding of mathematics and were just learning to follow a procedure:

A prime example of this is the kids that go to Kumon – they are drilled on algorithm by algorithm, by rote process. You give them an abstract problem or a word-based problem and they sit there staring at you and they say where’s the maths. They cannot make the transfer between the abstract to this set of numbers that are put on the board. (Teacher interview 1, 20/11/2008).

Bob was not obliged by the school to use a prescribed textbook and he had the freedom to choose the learning tasks for his students that he believed were appropriate for their needs. When Bob was asked to describe a typical mathematics lesson in the first interview, he explained that he had two different types of approaches in his teaching. The first would be to provide learning tasks that would facilitate active learning and the second was to provide learning tasks that focussed on students being taught procedural algorithms:

If we are doing an open task, say if I was introducing them to the 4 4s kind of problem, we were working with before, kids working individually, pairs or groups of three. Try to look at what was going on and how they make the different numbers using only 4,4s, and the mathematical equations that they know. ...

If we were doing a consolidation session where they had to be doing set algorithms, unfortunately they still need to know how to do algorithms with multiplication and division and that sort of thing, so you would have

examples maybe on the board or on a sheet and you would get them to work through those examples (Teacher interview 1, 20/11/2008).

Key Finding 6.1: Bob believed that mathematical tasks for students should be problem based and students should develop a deep understanding rather than simply master routine algorithms.

Bob assessed the students in his class formatively with no summative assessments. When he was asked in the initial interview how he knew if the students in his class had learnt something in mathematics he explained his assessment process as follows:

I suppose there are a number of ways to see if they have grasped the concept. I like to discuss it with them, interact with them with their working. (Teacher interview 1, 20/11/2008).

Bob's Views on Teaching Computational Estimation

Bob did not have a defined pedagogical framework for teaching computational estimation using a variety of computational estimation strategies. He was not aware of the formal strategy names for computational estimation. He summarised his understanding of computational estimation before the study began: "You were using the strategies before [personally] but you hadn't given the strategy a name" (Teacher interview 3, 12/11/2009). He did have an intuitive sense of estimating and some ideas about how estimating activities could be taught using a cross-curricular approach and within a measurement context:

Estimating with SOSE, how much rainfall you think you have received based on various areas that they know. If we were looking at packaging something, estimating, thinking of how many packages would fit into the container based on manipulating the objects and having a look (Teacher interview 1, 12/11/2008).

Key Finding 6.2: Bob had an intuitive sense of the computational estimation strategies but did not know the formal terms for these strategies.

Despite knowing about some of the activities that would develop estimation, he explained that computational estimation was not a component of mathematics that he covered specifically and therefore he was unable to outline clearly how he would teach these skills (Teacher interview 1, 12/11/2008).

Key Finding 6.3: Bob did not have a pedagogical framework for teaching computational estimation using a variety of computational estimation strategies.

He only mentioned the benefits of estimating in passing to his students. When they were completing their algorithms, he would say to the students, “Make sure you estimate before you do it and that is all you do” (Teacher interview 3, 12/11/2009). He appeared to lack confidence when explaining his beliefs about computational estimation. In the excerpt below Bob stated that he believed that computational estimation is not taught due to time constraints in the busy primary curriculum:

Estimation is one of those things. It's like health or RE and something has to go. It's the thing that gets dropped. We will pick it up somewhere, we will pick it up next year, we will pick it up the year after and that it's unfortunate that everybody says that. It keeps getting dropped and at the end it hasn't been done (Teacher interview 1, 20/11/2008).

Key Finding 6.4: Bob believed that there was no time in the primary curriculum to teach computational estimation.

Bob's Students' Beliefs about Computational Estimation

At the beginning of the study, most of students in the focus group thought that mathematics was something that was done quickly. When they were asked if they thought someone one who spent 15 minutes solving a mathematical problem was clever, they all said, “No, No, No” (Student focus group 1, 10/2/2009). Most of the students in the focus group seemed to think that mathematics was about one correct answer. When they were asked what they would do if someone had a different answer to them not one child said something that conveyed that they thought that two different answers could be acceptable. One of the students, Adam, stated:

I would go back and check my answer and if it was the same as their answer then I would do it like that and then if it turned out to be the same as my answer so then I would write it down (Focus group interview 1, 10/2/2009).

Key Finding 6.5: Most of the students in Bob's focus group perceived mathematics as something about one correct answer, something involving working out problems and is done quickly.

Most of the students in the focus group also perceived mathematics as something that was about problem solving, although they did not elaborate as to what they perceived the word 'problem' to actually mean. As an example Jane stated "It's solving problems but with numbers" (Focus group interview 1, 10/2/2009).

Some of the responses implied that mathematics could be used in the real world and their responses included this statement:

Researcher: Do you think that McDonalds could benefit from employing a mathematician?

Adam: I think they could then they could use the mathematician to work out how much it would cost to hire a dietician to find out how much rubbish they put in the food.

Hannah: I think they would be 'cos they might be counting how much money and figuring how much they would have in a year and then the next year (Focus group interview 1, 10/2/2009).

When the students were asked what they thought the word estimation meant, all of the students in the focus group used the word "guess" in their answers, although the students in the focus group thought there was some reasoned mathematics attached to this. One student's example was as follows: "Estimation is where you guess instead of finding out the real answer. Say if you had to estimate you would have to guess how similar something is or maybe different" (Hannah, Student focus group 1, 10/2/2009).

A few of the children in the focus group did mention that they did not perceive an estimation to be correct:

Jane: It is a guess on whatever. You don't have to be right

Adam: Guessing, not a good understanding. Shooting before aiming. A mathematical guess (Student focus group 1, 10/2/2009).

Key Finding 6.6: The students in Bob’s focus group perceived estimation as a type of guess with some mathematical reasoning attached to it .

Students’ Estimation Competence before the Professional Learning Intervention

All the students in Bob’s streamed Year 6 class were also asked to complete the Computational Estimation Test (CET) and their responses to the six estimation multiple-choice questions revealed the specific competencies of Bob’s class at the beginning of the project before the teachers participated in the professional learning intervention.

Estimation Question 1

As Table 6.1 shows 42 % were able to calculate the best estimate (B) in the pre-test. This is compared to 35% of 10 year olds and 38% of 12 year olds in the Number Sense test (NST), suggesting that students in Bob’s class had greater competency in solving contextual estimation problems than the students in the Australian sample of the Number Sense Test (McIntosh et al., 1997). The result that that nearly half the class were able to select the best estimate suggested that Bob’s students had some previous experience of how to complete these types of contextual problems before the professional learning intervention began.

The most popular computational estimation strategy used by Bob’s students at the beginning of the project to answer Question 1 was rounding (59 %) as shown in Table 6.1 and this was the only reasoned estimation strategy the students explained that they used. The second most popular strategy use was students answering it exactly and 18 % of the students attempted to answer the question in this way.

Table 6.1: Per cent of students selecting various answers to Question 1, and computational estimation strategies used (n=17)

Answer	Per cent				
	Rounding	Guess	Not enough	Exact	Total
1A	0	0	0	0	0
1B*	24	6	6	6	41
1C	35	0	12	12	59
1D	0	0	0	0	0
Total	59	6	12	18	101

Note. * denotes the best estimate

Estimation Question 2

This question required the students to approximate the relatively complex fractions to easily visualised whole numbers, demanding that students possess a conceptual understanding of fractions. In this question, Table 6.2 shows that 35 % of the students were able to calculate an appropriate estimate. This is compared to 25% of students who were able to calculate an appropriate answer in the Taiwanese component of the NST (McIntosh et al., 1997) and 24 % of 13 year olds in American national testing in 1981 (Post, 1981). This suggested that Bob's students were more competent at answering this type of question than many students in the other studies, although many students in the class still selected an answer that was not close to the acceptable estimate, suggesting that these students did not have a conceptual understanding of fractions.

When considering the students' written explanations of what strategies they used to the question, the fact that 67 % of the students who selected the best estimate (B), guessed the answer may mean that their conceptual understanding of fraction is lower than Table 6.2 suggested. There were 29% of the students still used whole number thinking when considering the answer to this question.

Table 6.2: Per cent of students selecting various answers to Question 2, and computational estimation strategies used (n=17)

Answer	Per cent						Total
	Rounding	Bench- marking	Guess	Not enough information	Exact	Whole number thinking	
2A	0	0	6	0	0	0	6
2B *	6	6	24	0	0	0	36
2C	0	0	12	6	0	29	47
2D	0	0	6	0	6	0	12
Total	6	6	48	6	6	29	101

Note. * denotes the best estimate

Estimation Question 3

Due to time constraints the students were unable to count how many triangles there were exactly, therefore they had to use some other estimation strategy in order to select the best estimate. Bob's students were generally competent at selecting a best estimate in the pre-test with 77% of students able to obtain the best estimate of 200 (D). None of the students selected the underestimates of 20(A) and 50 (B) (Table 6.3).

Table 6.3: Per cent of students selecting various answers to Question 3, and computational estimation strategies used (n=17)

Answer	Per cent			Total
	Sample	Guess	Intuition	
3A	0	0	0	0
3B	0	0	0	0
3C	6	6	0	12
3D*	41	24	12	77
3E	6	0	6	12
Total	53	30	18	101

Note. * denotes the best estimate

Estimation Question 4

In this question, 53 % of students were able to select the best estimate of 4600 (B) and this is 12 % higher than the contextual multiplication problem (Table 6.4) and

therefore it appeared that the students had a higher level of competency in answering multiplication questions that were set in symbolic form than contextually.

In the NST 60% of 10 year old students selected the best estimate which suggested that Bob's above average ability class were less competent at this type of abstract question than the students in the Australian sample of the NST (McIntosh et al., 1997). When considering the estimates that were not as close, similar numbers of students selected the underestimate (A) 4000 and the overestimate (C) 5200.

Table 6.4 shows that the majority of students (65%) used the appropriate of strategy of rounding and this strategy was the only reasoned estimation strategy selected; 18% of students were only able to guess the answer.

Table 6.4: Per cent of students selecting various answers to question 4, and computational estimation strategies used (n=17)

Answer	Per cent					Total
	Rounding	Guess	Not enough information	Exact	Intuition	
4A	12	6	6	0	0	24
4B*	47	6	0	0	0	53
4C	6	6	0	6	6	24
Total	65	18	6	6	6	101

Note. * denotes the best estimate

Estimation Question 5

In Question 5, students had to estimate the sum of four two-digit numbers. The CET asked students to answer the questions within a specific time limit, so that they did not have time to calculate the answer exactly. In this way, in order to obtain a best estimate, they would have to use an estimation strategy. In the pre-test 71% were able to select the best estimate, which suggested that nearly three quarters of the class were competent at estimating addition calculations of two digit numbers before the research began.

When considering what strategies the students utilised in order to answer the question, there was a variety of strategies identified. Table 6.5 shows that 35 % of students used the reasoned estimation strategies of rounding and front end loading.

There were 30% of students admitted guessing the answer to the question.

Table 6.5: Per cent of students selecting various answers to Question 5, and computational estimation strategies used (n=17)

Answer	Per cent					Total
	Rounding	Front end loading	Guess	Not enough information	Intuition	
5A	0	6	6	6	12	30
5B	0	0	0	0	0	0
5C*	6	24	24	0	18	72
5D	0	0	0	0	0	0
Total	6	30	30	6	30	102

Note. * denotes the best estimate

Estimation Question 6

Finally, in the last question, Table 6.6 reveals that 73% of students were able to select an appropriate answer to this question which was designed to ascertain whether students could estimate within a range. This suggested a high competency in calculating a range before the study began. Twenty seven per cent of students selected the overestimate C (23-28). When considering how the students solved this problem, 73 % considered the range within which the answer would fall, with 73 % of students who selected the best estimate using this strategy (Table 6.6). There were 20% of students who used the range strategy were unable to select the best estimate.

Table 6.6: Per cent of students selecting various answers to Question 6, and computational estimation strategies used (n=17)

Answer	Per cent			Total
	Range	Guess	Not enough information	
6A	0	0	0	0
6B*	53	7	13	73
6C	20	7	0	27
6D	0	0	0	0
Total	73	14	13	100

Note. * denotes the best estimate

The pre-test was administered in order to assess the students' performance at estimating before the professional learning intervention began. The students showed competency at using estimation strategies when tackling problems set in context i.e., Questions 1 and 3, suggesting that they had experience in solving these types of problems.

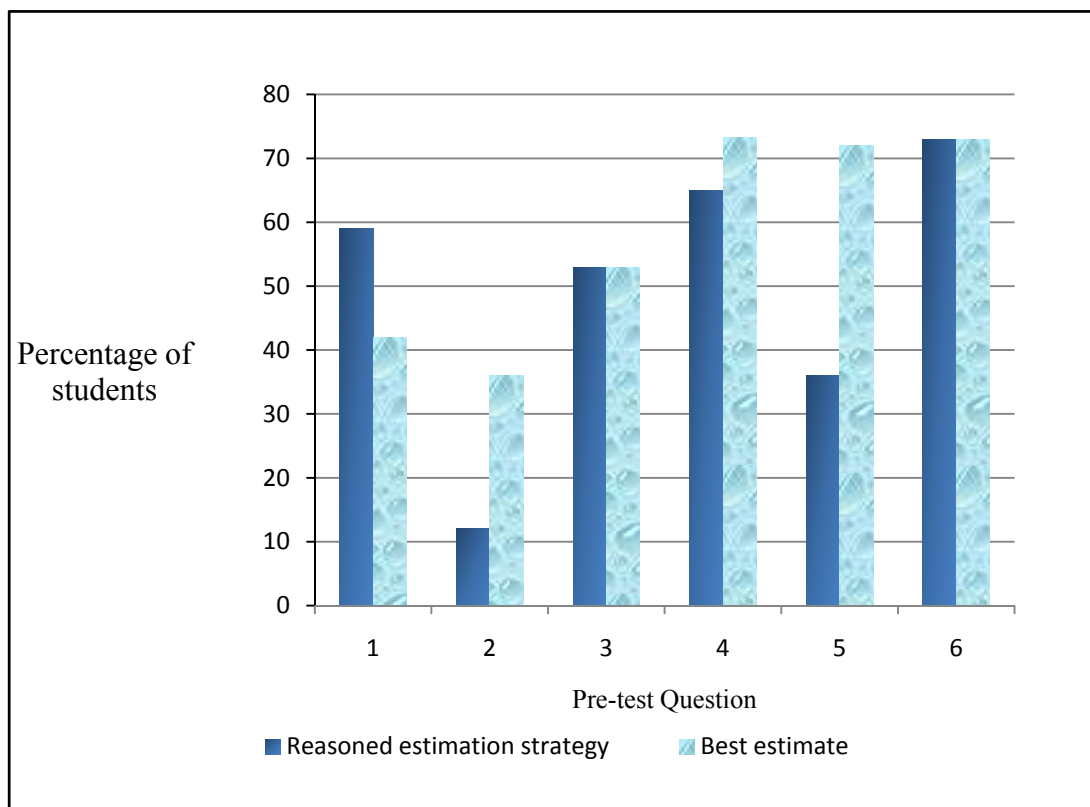


Figure 6.1: Percentage of students using a reasoned estimation strategy and identifying the best estimate before the professional learning intervention

Despite more students in Bob's class being able to select an appropriate answer when estimating with fractions than other similar studies, nearly two thirds of students still did not select the best estimate, suggesting that many did not possess a deep conceptual understanding of fractions.

Key Finding 6.8: Over half of the students were not able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators under time pressure and few students used a reasoned estimation strategy.

Bob's students showed high levels of competency at estimating when adding two

digit numbers, although 64% of students could not articulate how they estimated when estimating this sum.

Key Finding 6.9: The majority of the students were able to select the best estimate when adding four, two digit numbers although less than half of the students used a reasoned estimation strategy to answer this question.

The students had similar estimation abilities when working on a multiplication question that was presented symbolically as in a context. Forty one percent of students were able to select the best estimate on Question 1 which was a multiplication question that was set in a context and 53 % of students were able to select the best estimate on Question 4 that was presented symbolically).

Key Finding 6.7: Students had a higher competency when estimating the answer to abstract mathematical problems than contextual problems where both questions required students to multiply in the calculation.

The overall student performance before the study began was higher than the other two case studies with the mean CET score of 3.41/6.

Response to the Action Research Professional Learning Process

The narrative below details Bob's involvement in this professional learning intervention. Bob was involved in all three cycles of the professional learning although he was unable to attend the last twilight reflective session due to being held up in traffic. He did send through a written reflection at the end of the project.

The first research cycle: Reflecting and planning

On the first professional learning day Bob was open about his previous experiences and was keen to share these with the group (PL1 observations, 18/2/2009). He stated in the initial interview that he wanted his students to "think outside the square" (PL1 observations, 18/2/2009) so therefore the first principle that was presented in the professional learning workshop, that effective mathematics teaching promotes active learning was aligned to his present teaching beliefs (Audit trail journal, 27/11/2009).

When the computational estimation strategy terms were introduced, he seemed wary of them. He was sitting next to Peter who did not agree with formalising the names of the teaching strategies and using these in their teaching (PL1 observations 18/2/2009). The critical friend involved as an additional Researcher noted:

They [Peter and Bob] just didn't see the value in giving the students the language ... Problem based learning allows students freedom to explore, just do it, not caught up in naming (Critical friend observations, 18/2/2009).

Bob was asked to consider how he would teach computational estimation in the classroom. They were presented with suggested learning tasks that were to be adapted to suit the schools' contexts. Bob was fortunate that he had another teacher to work with on the tasks and appeared to enjoy the practical nature of the suggested tasks (PL1 observations, 18/2/2010).

The first research cycle: Acting and reflecting

It appeared that Bob was able to reflect on the professional learning day and began to adopt some of these new ideas into his teaching very quickly. On an informal visit to the school a week later, he was very enthusiastic when discussing the project with the Researcher. Despite being wary of the computational estimation strategies at the professional learning workshop, he introduced the strategies to the students using the laminated word wall that had been given out on the first day (Figure 6.2). The word wall simply stated the names of the strategies rather than any explanations of the strategies. The Researcher reflected on this in her audit trail journal after the visit:

The PL was on the Wednesday and he had expressed some reservations about the strategies but he relayed to me that on the Thursday he had already given them the tests and introduced and talked about the strategies to his students. Bob informed me that as soon as he got back he had told them about the different strategies and that he was very excited about it. He was using the word wall. He had fixed blu tack to the back and was displaying them in the classroom. His body language showed his enthusiasm and his confidence in using the strategies (Audit trail journal, 23/2/2009).

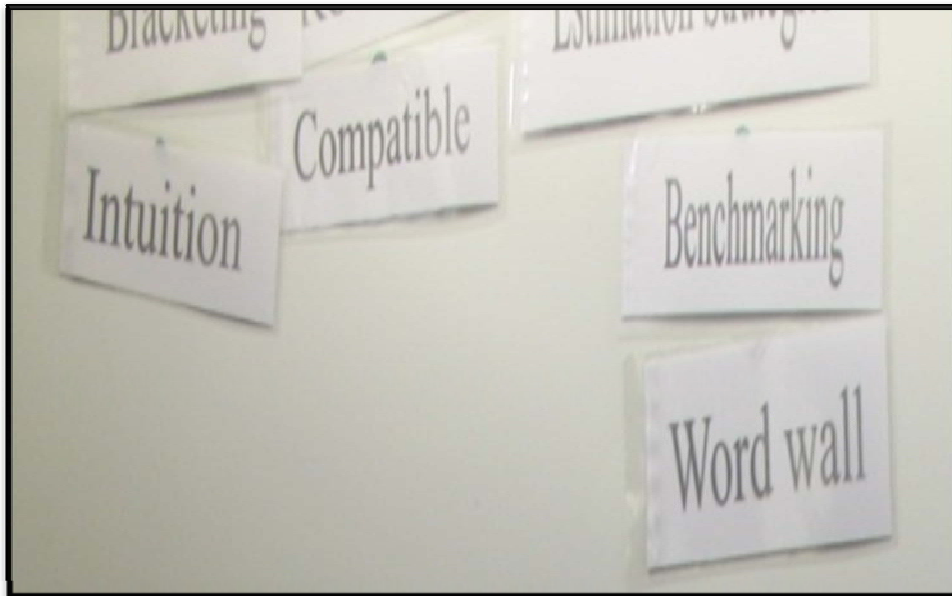


Figure 6.2: Bob's estimation strategy word wall

Key Finding 6.10: Bob explicitly described the computational estimation strategies to the students at the introduction of the work on computational estimation.

On the Researcher's first visit to Bob's classroom, the Researcher noted that it was organised with the tables in three large groups. His desk was at the side but when he spoke to the whole class, he stood at the front where the IWB and normal white board were situated. For the first lesson that was observed, Bob conducted a lesson that focused on the meaning of estimation using newspapers as a resource. He introduced the lesson by reminding the students what they had been doing in the last estimation lesson, rather than doing a separate warm up activity. Bob discussed with the students about the previous lesson of estimating how many M and Ms there were:

Teacher: We estimated and we came up with some weird and wonderful numbers. Did you know roughly how many were in the container?

Josh: 200.

Teacher: Did you know that to start with? No, I didn't tell you that to start with. Guess and then you are trying to guess two things or estimate, what were they, Emily, any ideas?

Emily: White and green.

Teacher: Yes, white and green but you are also trying to work out, not just white but in total? We all decided straight off that there were more white ones.

- Emily: More green.
- Teacher: More green, how did you know that?
- Emily: They just looked like more green (Classroom observation 1, 5/3/2009).

The students were set the newspaper learning task that aimed to show students that estimation had a real world purpose. Bob started the lesson with some whole class discussion. The excerpt below highlights some of this discussion:

We then started talking about the language we use when we estimate. It might be a guess or an educated guess. ...What I want you to do is to find an old newspaper article - find a new one each and tear it out, get a lead pencil, a red or a green one that will stand out from the newsprint and highlight or underline any words that you think there that would relate to estimation (Classroom observation1, 5/3/2009).

Bob then gave the students an opportunity to work in pairs so that they could discuss this task with each other and begin to construct their own understanding of the word estimation. The researcher observed that:

They were mostly engaged. A few students got overly interested in the articles so they did not look for estimation words. The activity definitely provided students with the opportunity to see that estimates are used in real world activities and are worthwhile (Classroom observation 1, 5/3/2009).

Bob then drew the class back together into a whole class discussion, where he summarised what had been the focus of the lesson. He produced an Excel spreadsheet from the class's responses to show how common the different estimation words were (Figure 6.3).

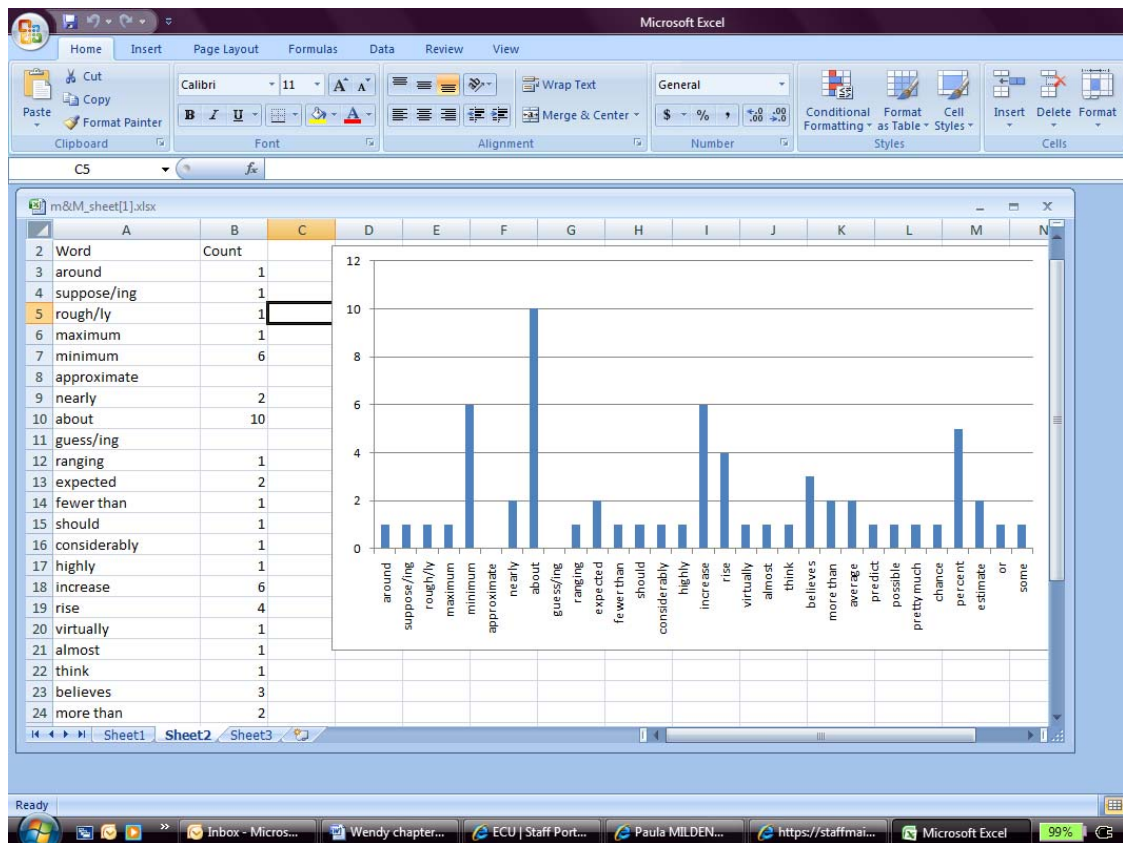


Figure 6.3: Excel spreadsheet showing estimation words

The second research cycle: Reflecting and planning

When Bob reported on his trialling in the classroom in the reflective session at the beginning of the second professional learning day, he explained that he had encountered difficulties with some parents who thought that the estimation lessons were not mathematics (PL2 observations, 13/5/2009; Critical friend’s observations). It was a setback for Bob and affected his confidence:

She [a parent] was most upset this sort of thing was not maths ... maths is about numbers on the page. I am sorry that it’s a stereotype but a lot of these Asian parents, all they do is learn to apply formula, crunch numbers (PL2 Observations, 13/5/2009).

Bob’s beliefs appeared to be less certain at this point. This difficulty with some parent’s perception with what they thought mathematics was appeared to affect his enthusiasm.

When the group discussed how estimated numbers could be represented in comparison with the exact number, pedagogically Bob did not perceive that there was value using the graphical number line to show students a different representation of estimated numbers. He much preferred using concrete representations of numbers. In response to this, the professional learning facilitator asked the group if they thought there was a problem with only creating concrete representations as they do not show the linear representation (Opfer & Siegler, 2007) but Bob did not seem to think that this mattered. Below is the excerpt where he explained his thoughts:

Bob said that he would use physical collections of sweets or objects and talk about which was the nearest. I asked what he thought about that not showing the linear representation — he didn't seem to feel that mattered (PL 2 observations, 13/5/2009).

In the afternoon session at the professional learning workshop, the teachers were presented with some suggested learning tasks for their students. The teachers' evaluated these, working in two groups. Bob's group worked on the Fermi problems and when they gave feedback as to what sort of Fermi problems they thought would be suitable for Year 6, they thought that it would be pedagogically appropriate to use objects that the students could physically see and were concrete as a stimulus for the lesson (PL 2 observations, 13/5/2009).

The second research cycle: Acting and reflecting

After the first professional learning workshop, the Researcher had given the teachers a word wall with the names of estimation strategies on them. On reflection, it was decided to provide a more comprehensive resource, so the Researcher provided the teachers with posters that also included a definition as well as the name of the computational estimation strategy. When entering the classroom it was observed that Bob still had the original word wall signs up and had not replaced them with the posters and adapted names that had been provided (Classroom observation 2, 27/7/2009). For the second observation, the students were still in their configuration of three groups. The observed lesson began with a whole class discussion and as part of this Bob used humour in the classroom as an overt teaching approach: "Enrico Fermi loved to challenge his students by setting them seemingly impossible questions, okay much as I do to you sometimes [everyone laughs]" (Classroom observation 2, 27/7/2009).

Key Finding 6.11: When teaching computational estimation Bob used humour to engage students in computational estimation lessons.

He based the lesson around the Fermi problems as outlined in the suggested learning tasks. Initially Bob's approach was to model the problem process to the students:

Teacher: These are estimates. These are problems based on one piece of information and you extrapolate the rest. How many families own a piano? There are 600 000 families. How many do you think may have a piano? [The students are all looking at the board and are looking very interested in what is happening].

Francis: Possibly a half or a third maybe.

Teacher: A third, okay so one in three owns a piano - how many pianos?

Francis: 200 000 pianos (Classroom observation 2, 27/7/2009).

Bob followed this process all the way through to the end of the problem, working out approximately how many piano tuners there were in Chicago. It was noted in the audit trail journal how effective this strategy appeared to be in scaffolding the problem solving process for the students:

Lessons were effective where the teacher provided a worked example of the types of exploring that was going to be completed in the task (Audit trail journal, 2/9/2009).

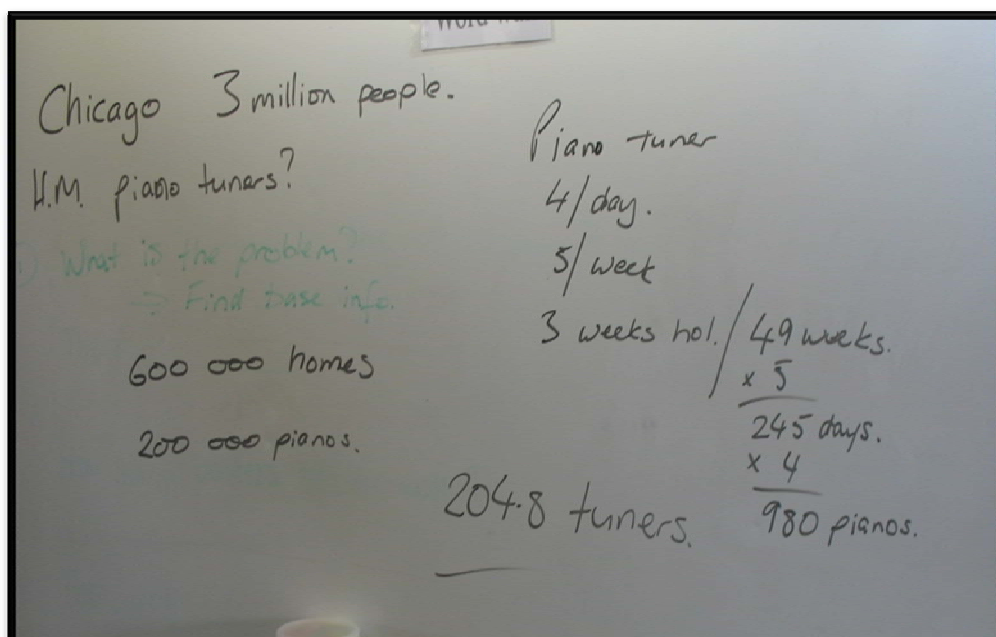


Figure 6.4: Bob modelling Fermi estimation problem

Bob recorded his calculations as he went so that the students were able to see how he organised his thinking as shown in Figure 6.4. The students appeared to be listening carefully. In this explanation, Bob used estimates but he did not talk about what strategies he used to create the estimates in the calculations. Bob did explain to the students that when estimating there is not one exact number that needs to be used in the calculations. Bob pointed out that when Enrico Fermi solved the problem, he estimated that there were four in a family rather than five as Bob's class had done but he explained that this does not matter and "doesn't make your solution wrong, doesn't make his solution wrong, doesn't make our solution right, doesn't make his right" (Classroom observation 2, 27/7/2009). In the suggested lesson plans provided at the professional learning workshops, there was a scaffolded problem process that the students could follow to guide them whilst they undertook the estimation problems. Bob instructed the students to use this series of prompts and Figure 6.5 shows how Josh, a student in Bob's class, described his group's response to this.

FERMI PROBLEM SOLVING STEPS

1. **WHAT IS THE PROBLEM ?**
 Recognise what the problem is asking and identify if the computations need to be exact, estimates or both. Our problem is
 How many tonnes of apples will jinnay have?
2. **ESTIMATE/ EXACT** We are going to solve our Fermi problems using estimation . We are going to find out these estimates .
 by taking 3 4x4 cm squares and counting the number of trees in them we then divided that by 3 to have an average 4x4 cm square then we found out how many 4x4 squares and then that by the number of trees. The units of measure are i.e. hours spent on an activity.
3. **CALULATE** Calculations need to be undertaken. We are going to calculate

Josh

Figure 6.5: Students' scaffolded problem-solving sheet

Bob asked the students to work in groups and try to work out how many apple trees there were in a picture of an orchard:

I want each group to come up with a method of solving the problem. The only thing I am going to tell you is that you get a hundred kilos of apples from each tree. That is the piece of information that is a given. In this one here the given was 3 million people in Chicago in this it is 100kg of fruit per tree (Classroom observation 2, 27/7/2009).

The students all appeared to be engaged in the problem. There was some noise as the students discussed how they were going to solve it and at one point Bob called the

class back together to discuss their progress and clarify their thoughts. Towards the end of the lesson he called the group back, reminding them that in his class “How you went about it is often more important than with what you ended up with” (Classroom observation 2, 27/7/2009).

In the following transcript taken from the classroom observations, the first group explained how they tackled the problem:

Nick: Our method was we found we counted two sides of the orchard and timesed them together. We all counted the two sides and then we added them together and then averaged out our answers so we are fairly confident what the answer was for trees in the orchard.

Teacher: Why did you count them all separately and then find an average from that?

Nick: ‘Cos we got one person to count them.

Teacher: So you’re allowing for what?

Nick: Human error (Classroom observations 2, 27/7/2009).

Bob went around all of the groups, listening to the different ways the groups estimated how many apple trees there were in the orchard. There was a variety of solutions, although most of them were a type of sampling. At that point in the professional learning intervention the estimation strategy ‘sample’ had been suggested to the group but Bob used the word bracketing to describe this process. Bob finished the lesson, challenging the students to select a Fermi question of their own and solve it at home for homework (Classroom observation 2, 27/7/2009). Bob also showed the class the jellybeans he had received from the other class at Sandilands School. He explained that they were going to estimate how many jellybeans were in the jar they had received from Sandilands and that they were going to fill a jar to pass on to Sandilands school (Classroom observation 2, 27/7/2009).

In the second teacher interviews, Bob explained that he was finding the ideas from the professional learning workshops blending into the rest of his mathematics teaching (Teacher interview 2, 28/7/2009). He was focussing on open tasks as they “allow you to build skills where students need them” (Teacher interview 2, 28/7/2009). He had discussed the term number sense with his students and it appeared that he was able to change the classroom culture to incorporate an estimation culture:

I think it an interesting term to get through to the kids. When we first started talking about it to the kids, number sense, and the reasonableness of the solution, there were lots of them who did not have much understanding of what was going on but after a little while they very soon click in to what you are talking about and the two terms became almost interchangeable ... I have these guys giving an estimate approximation for just about everything they do before they start so they can see if their answers are reasonable which is something they weren't doing before (Teacher interview 2, 28/7/2009).

Key Finding 6.12: Bob believed that computational estimation as a component of number sense should be an integral component of all mathematic lessons.

Bob perceived that the estimation strategies had value and explained how he remembered them: “Oh I can just look at the board and look at my little cards. I have left them up on the board ‘cos we are always referring to them, singling it [the computational estimation strategy] out, which one we are using, why we are using” (Bob, Teacher interview 2, 28/7/2009). Bob’s PCK of computational estimation appeared to be growing, as he was also able to use formal strategy names and answer questions as to how students would estimate:

Researcher: A child in Mr Clarke’s Year 4 class wanted to find an estimated answer for the question $21 + 28 + 19 =$. Describe what estimation strategies the child could use to solve this problem.

Bob: If they are competent at rounding they could very quickly round the numbers and do rounding. Once you have done a few of these, you can look at it very quickly and say okay chunk and answer together just by looking at what you have. You know you could do front end loading but look at where the second digit is to go up or down. You could also get them to bracket [my term for compatibles/nice numbers] a couple of numbers together $21 + 28$, you know that they are going to be about 50 and the other is about 20 so that is about 70 (Teacher interview 2, 28/7/2009).

Bob’s beliefs about teaching computational estimation appeared to be much clearer through being involved in the professional learning and the ideas that had been presented:

Nothing that cropped up that I've thought oh I wouldn't have thought that. It has confirmed a lot of things that I thought for a while and helped me solidify things I have been doing for ages but are common sense as far as I'm concerned but then I have been teaching for ages but talking to some of the younger staff here and at other schools it's not things that they have been aware of 'cos that's not what they were doing when they were doing maths. This whole idea of a common sense approach to things has gone out the window (Teacher interview 2, 28/7/2009).

The third research cycle: Reflecting and planning

The third professional learning workshop focussed on metacognitive aspects of mathematics learning to further develop the PCK of the teachers. The professional learning emphasised how estimation plays an important role in the effective mathematics learner who used estimation as a checking tool throughout the learning process. Bob looked very relaxed when he arrived, reflecting a new apparent confidence. In his initial shared reflection session of the trialling back in the classroom he described how the students' parents were becoming supportive of the estimation work:

They went home and thought how to solve this. This really had the parents involved. One Dad had come in and said I can't understand how to solve this... The parent was then hooked and really interested (PL3 observations, 29/7/2009).

The critical friend also noted that Bob was creating a classroom culture where estimation was an integral component:

Bob said that students [were] starting all probs [problems] with an estimate first. That it was okay to be wrong; some risk taking in having a go/guess – creating an environment /culture in classroom where you can take risks (Critical friend observations, 29/7/2009).

The pedagogical content knowledge that was being developed in this third professional learning day concerned the teachers' understanding of fractions, including developing teachers' understanding of how useful the benchmarking strategy could be. The guest professional learning contributor provided tasks which showed the different representations that could be used, including the number line and paper strips. Bob was very engaged in this session (PL3 observations,

29/7/2009). Bob immediately used the paper strips with his class after the professional learning workshop. He sent an email to the Researcher shortly after the professional learning day, explaining how successful this activity was:

I've got my top group folding the fractions strips at the moment. We made the 'simple' fractions and shown how to use for simple addition and subtraction examples. They are now trying to work out how to fold to get sevenths and elevenths, some very creative ideas coming out! (Email from Bob, 3/08/2009).

The professional learning workshop concluded with the teachers planning the estimation units around a trip to the park. Bob worked in the collaborative group with the other teacher from his school and Peter from Sandilands School, Bob and his team evaluated the suggested learning tasks and decided that he would use the basic format suggested but would extend his students to make it more of a challenge (PL3 observations, 29/7/2009).

The third research cycle: Acting and reflecting

When the Researcher went to the school for the third lesson observation Bob's confidence had grown tremendously and he was extremely relaxed (Classroom observation 3, 13/8/2009). His class was still organised in three large groups with his desk situated at the side. Bob used the same approach for introducing the lessons as he had done previously by discussing the previous estimation work. Bob then presented the class with the suggested trip to the park learning activity although he did not use the scaffolded booklet that had been provided. Instead, he described the problem to the students and negotiated, with the students, how to solve the problem. He did use some of the photos that had been provided to give the students visual clues as to what food items could be purchased. He had stated that he was going to make the challenge more difficult for the students in the discussion at the professional learning workshop (PL3 observations, 29/7/09) but he made it quite straightforward for the students. Below is a short extract of this discussion, which took place so that Bob could establish the parameters of the problem and explain to the students how they were to explore this problem in small groups:

Teacher: In your little groups you have to work out the following things, how much food would you need, how long will it take us to get to the park you decide to go to and we are walking - it's like do you remember the Fermi problem, you make some assumptions here.

Now how long would it take you to walk to the park? How far can I walk in an hour? Does anybody know? What is the average distance that you would walk in an hour?

Nick: Two.

Hannah: Three and a half .

Teacher: Hannah says three and a half; well do we have an advance on three and a half? (Classroom observation 3, 13/8/2009).

Key Finding 6.13: Bob used the teaching approach of organising the class to work in small groups in order to facilitate discussion between the students about the computational estimation problems.

Bob continued to use humour as a successful teaching strategy to engage the students. When explaining the Fermi problem, he jokingly added that one factor might be: “If you are taking the footy to kick yeh and if you kick it up into a tree how much time you will take to get it out the tree [Students’ laugh]”. Bob modelled for the students how to estimate but he did not overtly use the named strategies. The students followed, using a similar discourse to Bob, and estimated using the strategies but did not overtly mention the names of the strategies either (Classroom observation 3, 13/8/2009).

The students were motivated by the problem and they were very engaged in creating the trip out. They were interested in the non-mathematical details of the trip such as which park would be the most suitable, as well as calculating the mathematics involved in the problem.

In the observational notes made, it was recorded that students Frank, Edward, Isabelle and Hannah were overheard reminding each other that it was only an estimate they needed and they were using estimation language such as “let’s round it up” (Classroom observation 3, 13/8/2009). In this same group of students, they were trying to work out how far it was to the park. In the classroom observations it was noted that:

They [the group the Researcher is observing] then wanted to work out how far it is to Whiteman Park. She [the student] knows that it takes her mum 13 minutes to get there but she does not know how far that is. The teacher listens to this discussion and prompts the student’s thinking saying:

Teacher: Let's assume your mum is going at the speed limit 80 km / hr.

Teacher: Let's say that it is about $\frac{1}{4}$ hour [rather than 13 minutes]
(Classroom observation 3, 13/8/2009).

The group began calculating what a quarter of 80 was, deciding that it would take about 20 minutes to get there. Bob used the estimation strategy of nice numbers here as $\frac{1}{4}$ is much easier to work with than $\frac{13}{60}$ but he did not explicitly draw the student's attention to this (Classroom observation 3, 13/8/2009).

Key Finding 6.14: Bob modelled different computational estimation strategies in his teaching but he did not use the formal estimation strategy terms.

The learning task actually needed more much time than was available in one session. Although the students could probably have benefited from more time, Bob drew the class together as a plenary and asked the students to report back. They had written their ideas on butcher's paper and this allowed them to organise their ideas and present them to the class. In Figure 6.6 it is possible to identify how one of the groups planned their route to the park and then estimated the itinerary using rounded numbers i.e., 25 or 30 minutes.

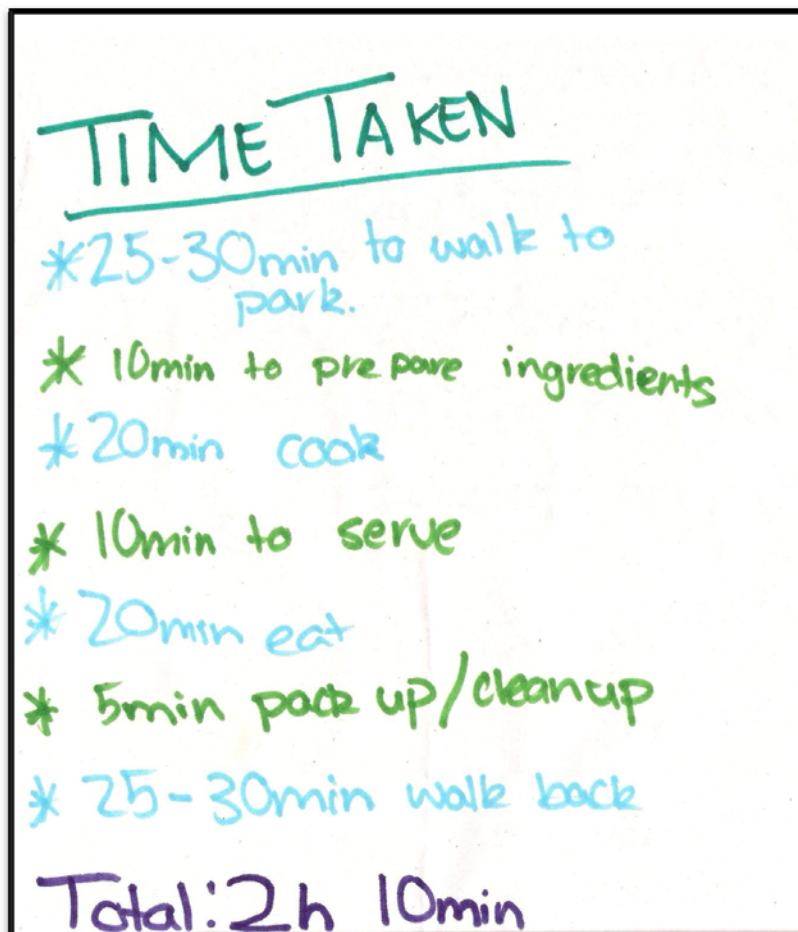


Figure 6.6: Jill's itinerary

The students stood in their groups at the front in turns to report back while the rest of the class listened. Below is an excerpt of one group's response and in this response there is an example of benchmarking using $\frac{1}{2}$:

So we estimated that it is 500 – 700 m to the park, 'cos my house is about just under 1 km to the school and the park is splat bang in the middle of my way to school, so that is about $\frac{1}{2}$ way. It takes an hour to my house, so we estimated a $\frac{1}{2}$. If we took about 20 minutes to walk to the park so we did 20 min there, 20 min back, so we took 40 minutes. So we then got the bbq ready, we got the bbq and it took 5 min to heat up - it is one of those free ones, and em then the cooking time was 10 min until the sausage were done and yeh we took 20 min to eat 'cos Jessica takes 20 minutes to eat a large hot dog (Classroom observation 3, 13/8/2009).

Key Finding 6.26: Students were able to use estimations as a main computational choice in extended problem task.

When the students gave feedback on their proposed trip to the rest of the group many of the students used estimated numbers in the calculations. Another example of estimation being used by the student was in the explanation that his group would take their own BBQ fuel:

Nick: Bbq fuel \$1 petrol, so that is a bit under a litre of fuel to get there.

Teacher: Nick, how did you know that it is a bit under a litre if it cost you a dollar.

Nick: Well I know the fuel prices roughly are \$1.20 ish.

Teacher: So if you have \$1 it is not quite a full litre.

Key Finding 6.24: Bob's students used computational estimation language when discussing how to solve the problem.

Bob did not discuss with the students how they had estimated or the estimation strategies that had been used by the different groups. All the groups were able to report to the class and share their progress and all but one had completed a proposal on the butcher's paper provided. As well as calculating how long it would take to get to the park the class were also required to calculate their food costs.

Final Pedagogical Content Knowledge, Beliefs and Teaching Approaches

In the final interview, Bob explained how estimation as a component of number sense had become part of his classroom culture:

They have a lot more idea, if you say to them now, okay look before we start, let's just come up with an approximation of where we should end up and they are much happier to do it, they can see now the benefit. If you had seen it before 'oh I don't need to do that, I'll just work it out', they would make an error and have no idea they were actually wrong. They'd be calling out answers. I'd be looking at what, 'Well, that's what I got' [Bob putting on a student's voice] Well didn't you estimate first? "No" [Bob putting on a student's voice] and so now they are going off and doing it (Teacher interview 3, 12/11/2009).

Bob had been able to be fully engaged in the cyclical research process and he had been able to discuss with colleagues the issues that had been raised (Teacher interview 3, 12/11/2009). He was able to offer his developing expertise to other teachers in the group and this appeared to increase his self-efficacy. In the excerpt below Bob explained how the intensive nature of the professional learning intervention meant that he was able to support other teachers:

Bob: Sometimes you are able to offer them a bit of advice and say I have done that I found this worked and give it a try.

Researcher : Yeah.

Bob: And then the next time you see them they say I tried that and it worked, thanks for that, so that was good (Teacher interview 3, 12/11/2009).

Bob appeared to convey that he perceived that some of the research literature that he was presented with on the professional learning was not very accessible to teachers. He perceived that what was happening in academia was not always helpful in the real world of the classroom (Teacher interview 3, 12/11/2009).

When asked what could have made the professional learning intervention more supportive he felt that it would have been beneficial to have had more interaction as he found the sessions a long way apart. He also wondered if the group could have taken advantage of new technologies such as blogs, so that there could have been more communication between the members:

At times, I thought it would be great to go back for another session. They seemed a little bit far apart, you know you did something and you thought we have got 10 weeks to get together, you sort of forget what we are discussing. Yeh, maybe more sessions, maybe setting up a professional learning network somewhere, a blog for people to access or a link that we can tap into em would have been handy (Teacher interview 3, 12/11/2009).

A significant group of stakeholders at Bob's school was the parents and their views appeared to be very important to Bob (PL2 observation, 13/5/2009). The parent's gradual increasing support for estimation appeared to increase Bob's willingness to engage with the professional learning process. He had encountered some difficulties with parents at the beginning with this new approach to teaching mathematics but as the school year progressed the parents became very supportive once they could see how engaged the students were (Teacher interview 3, 12/11/2009).

The cyclical research process initiated at the professional learning workshops of trialling and evaluating worked particularly well in Bob's context as he had the freedom to teach whatever he thought was most suitable for the students. He was therefore able to evaluate the ideas that were suggested in the professional learning workshops and trial certain teaching approaches that he thought were pedagogically appropriate (Teacher interview, 12/11/2009).

At the beginning of the professional learning, Bob had believed that using open tasks and problem solving was an effective teaching approach when working with exact numbers (PL1 observations, 18/2/2009). When it was suggested that the estimation strategies could be taught using this type of approach, Bob found it straightforward to incorporate this new knowledge into his present beliefs. Bob described how his classroom was now one where the students were interested in sharing their learning: "They [the students] are enjoying it more, developing the ability to become critically honest with each other as to how their work looks but in a positive sense (Teacher interview 3, 12/11/2009).

Key Finding 6.24: Bob's students were working as a community of learners justifying their computational solutions.

Bob believed that the estimation strategies were worth teaching although he did seem to think that the formal terminology of categorising the estimation strategies was not as important as understanding the general concept of estimating numbers.

Key Finding 6.15: Bob believed that computational estimation strategies are worthwhile to teach Year 6 students.

An important teaching approach that Bob used was scaffolding students' learning in problem solving, so that each student in his class had some direction on how to complete the task. He explained that when teaching computational estimation he believed that it was necessary to:

Provide a scaffold to the learner to follow it in the first instance. If you just say to the learner, "Get on with it", they can spend ages just thinking what the problem is asking, whereas if you break it down for them into some sections and you say to them tackle this bit first, then you can come up with something for this, then you can try to

work out how it relates to this part, then onto this part. You have given them a pathway whereas if you don't have that, especially with more complex problems, they have got no idea (Teacher interview 3, 12/11/2009).

Key Finding 6.16: At the end of the professional learning intervention, Bob had developed new pedagogies for teaching computational estimation: Practical activities set in meaningful contexts, scaffolding problem-solving tasks, students reflecting on exact computations in mathematics by using estimation as a checking tool .

In the final interview Bob was able to show his growing content knowledge of computational estimation. Bob was able to estimate answers to problems that the students may be asked to solve and he was very confident in this:

Researcher: A child in Mr Clarke's Year 7 class wanted to find an estimated answer for the question $21\ 014 + 2811 + 19112 =$

What estimation strategy would you have used to add it up?

Bob: I would have used front end loading it for that one. The simple thing is to front end load it and if you had a quick look you'd realise 2800 make that 3, $21 + 3 + 19$, bang 43 thousand and you are done (Teacher interview 3, 12/11/2009).

Bob was able to discuss most of the strategies but he was not able to remember all their names, suggesting that he believed the formal terms for these estimation strategies were less important. This was observed in his teaching, where he used the strategies but did not accompany this with naming the formal names. The school appeared to consider Bob's approach to teaching mathematics very effective and they wanted to use his expertise, asking him to be the mathematics coordinator for the primary school in the following year.

Key Finding 6.17: Bob was able to answer questions concerning the use of estimation strategies, suggesting that he understood these computational estimation strategies.

Students' Final Beliefs about Computational Estimation

At the end of the professional learning intervention, the student focus group was

gathered together to find out how their perceptions of computational estimation had changed. When the students in the focus group were asked what they would do if someone had a different answer to them, all three of the students who answered the question stated they would check their answer, implying they were expecting one correct answer to the question. One of the students did mention that it would be worthwhile to identify if they were both reasonable, suggesting that some of the computational estimation work in the class, had influenced the student's beliefs. Adam explained:

I'd do the same as Hannah— if I knew they had a different answer I'd check both answers see how reasonable they are (Focus group interview 2, 12/11/2009).

The students who described how they perceived mathematics explained that mathematics was something that was done quickly. One student described mathematics as “simple ... drills” (Adam, Focus group interview 2, 12/11/2009).

When the students were asked what they would tell an alien that mathematics was, the students gave similar answers to their first focus group interview before the professional learning intervention began. The answers conveyed that they still perceived mathematics as something that was about working out problems:

Adam: I'd probably tell him that it's a way of working problems out using equations.

Hannah : I would probably tell them that it is a way of joining two numbers together or measuring the base or area.

Nick: I'd probably tell them it's different ways of using different numbers to create other numbers in certain formulations or something.

Key Finding 6.18: Bob's focus group believed that mathematics is about working out number problems, one correct answer and was done quickly.

From the students' responses it was clear that some of the students were uncomfortable with the way mathematics had been different during the study compared with other years at school. One student in the focus group, Adam, articulated the differences between mathematics before which he described as drills (Student focus group 2, 12/11/2009) and the mathematics that involved estimating in a problem-solving situation. Adam found this change quite uncomfortable for various reasons. He explained in the final focus group interviews, “I am not one for

estimating, I normally have a rough idea of what I am after, I don't normally go through these estimation processes ... I had no need to ask whether I estimate or not 'cos the answers were quite simple" (Adam, Student focus group 2, 12/11/2009).

Adam believed that mathematics was "a way of working problems out using equations" (Student focus group 2, 12/11/2009) and was about correct answers which were "black and white" and that computational estimation work did not match this perception.

Key Finding 6.19: Some students held negative beliefs about estimating.

The estimation work introduced a broader view of mathematics to the students and Adam found this difficult. He explained:

Yeah I think that is one of the reasons that I don't like Fermi problems 'cos there is no one particular answer which defines the purpose of mathematics. Mathematics is either black or white (Student focus group 2, 12/11/2009).

When they were asked to write down all they could about estimation on a concept map, the focus group's maps all had more appropriate ideas written down after the professional learning intervention. The concept maps reflected a growth in the articulation of the different experiences of estimation. Most of the maps at the end of the study included the strategies' names and included references to the fact that estimation could be used in real life. The concept maps also referred to the fact that estimation made the mathematics easier. At the beginning of the study, Hannah essentially perceived estimation as a guess (Figure 6.7). After the study, she appeared to have a broader understanding of estimation. She appeared to perceive estimation as something which made mathematics easier, that had many uses and something that had different answers (Figure 6.8). She was also aware of the different computational estimation strategies that she could use. Hannah appreciated that computational estimation could be used in problem solving situations in and out of school. Bob spent time at the beginning of the study discussing with the students what computational estimation was and he used resources such as newspapers so that the students could see that estimation was used in the real world. Time spent on this appears to have supported the students' conceptual awareness of computational estimation.

Estimation is where
you guess instead of
finding out the real
Answer. Say if you
had to estimate you
would have to guess
how long something is or
maybe different.

Figure 6.7: Hannah's pre-study concept map of estimation

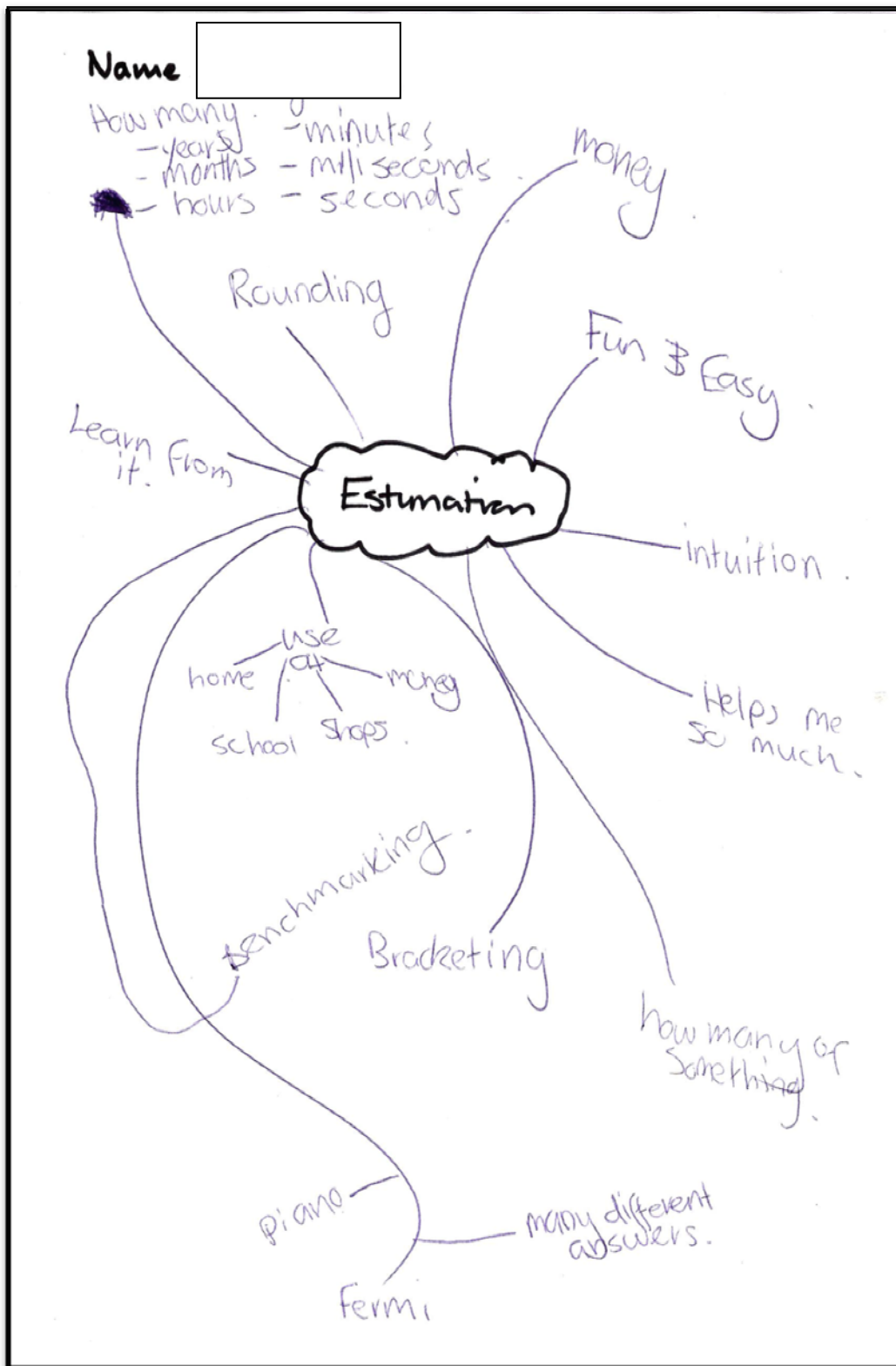


Figure 6.8: Hannah's post study estimation concept map

In the focus group, Hannah also discussed how she liked the fact that, in the types of mathematics they had been involved in, there could be a number of answers that were reasonable:

I really liked the Fermi problems cos you don't have to be exactly right 'cos there wasn't one answer, there was lots of right answers.

Other students in the focus group discussed how the estimation work in Bob's class had made the mathematics easier:

Jane: If you are able to estimate with the process then it is really easy to do and it helps me out.

Hannah: I agree with Jane, it gives you a rough guide of where you are going and what numbers to work with.

Nick: The actual estimation is quite easy 'cos you are usually rounding, well if you are using rounding you are usually rounding numbers into easy numbers to work with and then once you have got your end product it is easier to do the easy detailed problem and get the right answer.

Key Finding 6.20: The students believed that estimation is more than a guess, makes mathematics easier, helps to make sense of mathematics and can remove the exactness and enjoyment of mathematics.

Students' Computational Estimation Performance after the Professional Learning Intervention

All the students in Bob's class were asked to complete the computational estimation test again at the end of study and their responses to the six estimation multiple choice questions revealed some interesting insights into the possible growth of the students' estimation performance. In Bob's class, there was an improvement on all questions at the end of the study (Figure 6.9).

The mean pre-test score was 3.41/6 and the mean post-test score was 4.53/6. Therefore the mean student test score improved by 1.12 and this result was statistically highly significant (paired samples t test (16) = 3.271, $p \leq 0.005$). As mentioned in the earlier case study, these inferential statistics are expected to be interpreted from a socially constructed perspective due to the nature of the design of the study and its focus on rich data in a naturalistic setting (Hennig, 2010).

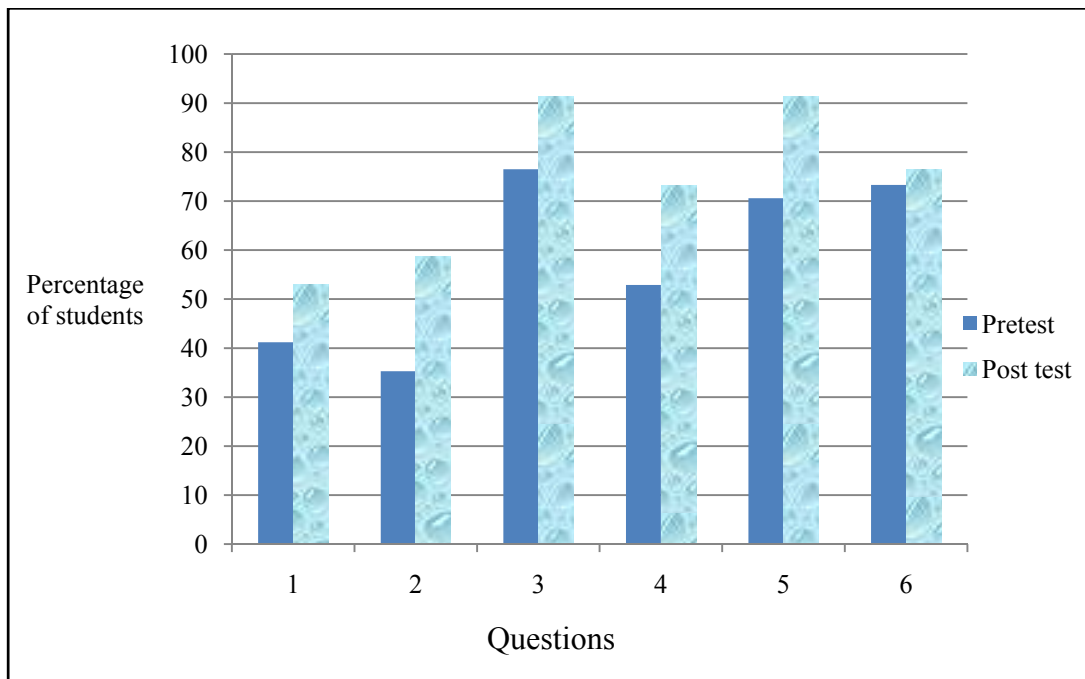


Figure 6.9: Difference in pre and post-test results on the CET

As the research focussed on how the particular teaching approaches affected the students’ performance in using computational estimation strategies, it was of interest to compare differences in students’ use of reasoned estimation strategies. Table 6.7 showed that the students’ performance as using a reasoned estimate on all questions improved.

Table 6.7: Percentage of students using estimation strategies and selecting the best estimate in pre and post- tests

Question	Reasoned estimation strategy			Best Estimate		
	Pre-test	Post-test	Increase	Pre-test	Post-test	Increase
1.	59	77	18	41	53	12
2.	12	41	29	35	59	24
3.	53	71	18	77	91	14
4.	65	82	18	53	73	20
5.	35	82	47	71	91	20
6.	73	82	9	73	77	4

Estimation Question 1

On Question 1, Table 6.7 shows that 53% of students were now able to select the best estimate (B) 3000 in the post-test. This result reflects an improvement of 12%, suggesting that Bob's students had benefited from estimating in contextual situations. Bob had started teaching most of his new mathematical content in a problem-solving situation rather than explaining to students how to follow a mathematical procedure (Teacher interview 3, 22/10/2009). This post-test level of competency is much higher than the Australian sample in the NST (McIntosh et al., 1997), where 38% of 12 year olds in the NST were able to select the best estimate.

Question	Reasoned estimation strategy			Best Estimate		
	Pre-test	Post-test	Increase	Pre-test	Post-test	Increase
1.	59	77	18	41	53	12
2.	12	41	29	35	59	24
3.	53	71	18	77	91	14
4.	65	82	18	53	73	20
5.	35	82	47	71	91	20
6.	73	82	9	73	77	4

Table 6.7 shows that the number of students who were able to use a reasoned computational estimation strategy to answer this question in the post-test was 77% and this was an increase of 18 % in comparison with the pre-test result of 59 %. It is important to note that many students in Bob's class were able to use a reasoned estimation strategy before the study began, suggesting that the school may have developed students' ability to estimate when presented with a multiplication problem set in a context.

Estimation Question 2

On Question 2, 59% of students were able to select the best estimate (B) of 2 (Table 6.7). This is compared to 35% of students who were able to complete this in the pre-test. This improvement of 24% suggested that Bob's teaching during the study, that included using some of the activities suggested by the guest contributor's session on fractions, had increased the students' ability to estimate with fractions. Whilst acknowledging that this group were the most capable students in a streamed class, this result of 59% reflects a high competency level when compared to the 25% of students who were able to answer this question with an appropriate answer in the

NST (McIntosh et al., 1997).

When considering what strategies were used by Bob's class there was a 29% improvement on students using a reasoned estimation strategy. This suggested that there was benefit by Bob explicitly teaching the estimation strategies such as benchmarking earlier in the study, in combination with providing activities such as the 'fraction strips' which developed a conceptual understanding of fractions. 41 % of students used a reasoned estimation strategy in the post-test compared with only 12 % of the students using a reasoned estimation strategy in the pre-test. The difference between the pre- and post-test appears to have occurred due to the reduction in students guessing the answer.

Estimation Question 3

The students' performance when answering Question 3 in the post-test was extremely high (Table 6.7) with 94% of students able to select the best estimate of 200 (D). This was greater than the result in the NST research study (54% for 10 year olds and 62% for 12 year olds). This result reflects a growth of 18% when comparing pre and post-test results. When considering what strategy the students used to answer the question, Table 6.7 shows that 71% of students used a reasoned estimation strategy. Bob had spent time teaching such strategies as sampling. In the observed lesson in Term 2, Bob's students investigated different strategies that could be used to answer this type of question and it appears likely that this learning approach has impacted upon the students positively. Bob's students were skilled at selecting the best estimate of a number of discrete objects before the study began. By being involved in the study, it appears to have led to the situation where nearly all of the students in Bob's class could select the best estimate when solving a problem set in a numerosity context.

Estimation Question 4

When analysing the answers to Question 4, Table 6.7 shows 73% of students were able to calculate the best estimate (B) in the post-test, which is an improvement of 20%. This would suggest that the students were more competent at estimating two-digit numbers multiplied by three-digit numbers at the end of the professional learning intervention. As all of the answers (A) 4000, (B) 4600 and (C) 5200 are

estimates that may be considered as an estimate with satisfactory precision in the primary classroom, it is important to note that Bob's students did appear to focus on the precision of the estimate. This may have been because Bob integrated estimation in to his all of his number lessons and used computational estimation as a checking device when working on exact calculations. This may have led the students to spend more time considering how precise the estimate was in comparison to the exact answers.

Estimation Question 5

At the end of the study, 94% of students were able to select the best estimate on Question 5. This is an improvement of 21%. This high level of competency suggested that most students in the class were now able to estimate when adding four, two-digit numbers. This may be because Bob did teach the computational estimation strategies explicitly at the beginning of the study, even if he did mention them less as the study progressed. There were a very high percentage of students who now used a reasoned estimation strategy, with an increase of 47%. The two most popular strategies used to answer the question were rounding and front end loading (35% and 29 % of the students respectively). Both of the strategies are efficient strategies and allow this estimation to be completed in the short time frame that was available. One student used the sample strategy to answer this question. The student thought that all the numbers were around 50 and then multiplied 50×4 . Only 35 % of students were unable to articulate how they estimated the answer.

Estimation Question 6

Finally, as shown in Table 6.7, 77 % of the students managed to estimate when multiplying a whole number by a decimal number and arrive at an appropriate answer (B). There was only a 3% improvement between the pre-test and post-test answers. When considering this small improvement it is important to appreciate that there was already a high level of competency in this type of question before the study began. At the end of the professional learning intervention, Bob's class became more competent in all six questions requiring the students to select a best estimate, including the question that required some precision.

Key Finding 6.24: Bob's students' computational estimation performance improved overall and statistically this was highly significant.

He spent time teaching students how to estimate in problem solving contexts although the students' performance when estimating in the symbolic question was still higher than in the contextual question.

Key Finding 6.23: Bob's students are much more proficient at estimating multiplication problems, which are purely symbolic and not set in context.

The students' performance on selecting the best estimate and their use of reasoned estimation strategy when adding fractions with unlike denominators improved. This may have been because Bob introduced the benchmarking strategy at the beginning of the project and therefore had a greater repertoire of strategies to draw on to answer this question.

Key Finding 6.21: Over half the class were able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators after being instructed in how the benchmarking strategy may be used in this process. Far more students were then able to use this benchmarking strategy.

Finally, nearly all the students in the class were able to select the best estimate when adding four two-digit numbers.

Key Finding 6.22: Nearly all the students were able to select the best estimate when adding four two-digit numbers and around two thirds of the class were able to use a reasoned estimation strategy.

Chapter Summary

After being involved in the professional learning intervention, Bob developed new PCK about computational estimation and this impacted upon Bob's beliefs about the specific area profoundly (see Figure 6.10). This profound change was partly enabled

due to Bob's beliefs about mathematics teaching in general being aligned with the professional learning principles (see Appendix M). The action research approach enabled Bob to investigate some new teachings approaches which he eventually implemented in to his entire mathematics program. This process did appear to impact on students' beliefs, although there was some resistance amongst students who did not want to change their approaches to learning mathematics. Overall, the student performance of computational estimation increased and the students' ability to use reasoned estimation strategies also increased (Table 6.7.) The cyclical reflective process created by the professional learning meant that these developments occurred gradually over the school year.

Teacher's PCK

At the beginning of the professional learning, Bob did not have a pedagogical framework for teaching computational estimation using computational estimation strategies. Through being informed about the computational estimation strategies in the professional learning workshops, Bob was able to develop his personal content knowledge in this area. Bob already had developed PCK of teaching mathematics using a problem solving approach. Therefore it was straightforward for him to develop PCK specifically for computational estimation.

Through trialling the teaching activities, Bob developed effective teaching approaches. He developed two main teaching approaches; the first entailed a problem solving approach set in meaningful contexts. The second was the use of estimation in all mathematics including problems where exact answers were required. In this approach estimation acted as part of the metacognitive aspects of learning where Bob encouraged the students to reflect on their calculations using computational estimation. Bob did not mention the formal strategy terms with the students regularly nor did he develop a completely sound understanding of the formal strategy names, so these names did not become a central component of his PCK of computational estimation.

Assertion 6.1: The provision of research literature and workshop activities about computational estimation strategies and how computational estimation could be a computational choice in its own right broadened Bob's PCK of this subject area in that he understood the strategies and developed a pedagogical framework for how these could be taught in Year 6.

Teacher's beliefs

The professional learning intervention did appear to impact on Bob's beliefs. At the beginning of the study, Bob believed that there was little time to teach estimation. At the professional learning workshop, it was proposed that computational estimation was not an extra in a busy curriculum that was an optional area of instruction but instead was a central component of number sense teaching. Through engaging in the reflective process, Bob was able to trial the new teaching approaches in the classroom and this led him to believe that it was worthwhile integrating computational estimation into all of his mathematics. He now believed that with some problem-solving tasks students could be asked to produce estimates rather than exact answers and when students were working on exact answers, estimation was a useful tool in order to check the reasonableness of their results.

Assertion 6.2: Bob's developing PCK of computational estimation as a computational choice and checking device impacted upon his beliefs and he now believed that computational estimation was an integral component of developing number sense.

Teaching approaches

At the beginning of the professional learning intervention, Bob had two main teaching approaches. He used to teach new concepts using a problem solving approach. He also used to teach standard written algorithms once they had a good grasp of the concept. Through engaging in the reflective process implemented in the intervention, Bob began to focus much more on teaching using the problem solving approach. Using his new PCK about computational estimation, he created learning tasks that involved students only required to produce estimates. He also developed a belief that estimation should be an integral component of all mathematics teaching and therefore he created an estimation culture within his classroom using estimation to check exact answers. Bob did develop content knowledge of the estimation strategies. At the beginning of the study, he did introduce the estimation strategies to the students but towards the end he appeared to place a lower priority on using the formal computational estimation terms and this appeared to impact upon his teaching approaches. When teaching the students how to estimate calculation he focussed on modelling how he estimated and then expected the students to follow in a similar manner.

Assertion 6.3: Bob's developing beliefs that computational estimation was important in developing number sense and developing an understanding of estimation strategies impacted his teaching approaches. Bob developed two teaching approaches: estimating in problem situations and estimation as a checking device. Bob was encouraged to continue these teaching approaches due to support from parents who were important stakeholders in his school.

Students' beliefs

At the beginning of the professional learning intervention, the students in Bob's class believed that mathematics was about one correct answer and solving problems. They perceived estimation as a type of guess but also as something with a type of reasoning attached to it. Bob taught the students mathematical concepts primarily through solving problems. Therefore it followed that the students perceived mathematics at the end of the study as something that is about problems. It appeared that some of the students might also have begun to appreciate that, through engaging in open tasks, some mathematics problems can have more than one acceptable answer. Bob had integrated estimation into all of his mathematics and it appeared that some students might have resisted the change in approach and maintained their entrenched beliefs that mathematics is clear-cut with one exact answer. Most of the students, however, as a result of engaging in the variety of estimation activities in the class, now believed that estimation was more than a reasoned guess and instead something that made mathematics easier and helped to make sense of mathematics.

Assertion 6.4: Bob's teaching approach of integrating estimation into all of his mathematics appeared to impact the students' beliefs and broaden their perception of mathematics and estimation. Some students appeared resistant to the change in approach to mathematics teaching and learning.

Students' computational estimation performance

There was an improvement in computational estimation performance on the CET and this result was statistically significant. At the end of the professional learning intervention, Bob believed that computational estimation had an important part to play in many aspects of mathematics. This broad implementation appears to have provided his students with a repertoire of skills in order to estimate in a variety of situations. Of particular interest is that students appeared to pay attention to how

precise the estimate was and this may have been possibly due to Bob evaluating the differences between exact answers and estimated answers. Students had been taught quite explicitly how to use the benchmarking strategy at the beginning of the study and this may have resulted in more students being able to solve this in problems which required the addition of fractions. The explicit teaching of the front end loading strategy at the beginning of the study and then being able to practise this in problem solving situations may also have resulted in the fact that nearly all students could estimate when selecting the best estimate in a two-digit addition problem. Bob did not explicitly refer to the strategies often during the study but the students appeared to make use of the strategies even if the socio- mathematics norm was not to name them.

Conclusion

Bob broadened his present beliefs that mathematics teaching should be problem based to incorporate computational estimation. His changed beliefs and PCK appeared to impact on all of his teaching. This was able to occur because Bob did not have any restriction on what he taught and was able to trial the new ideas presented in the research literature.

These teaching approaches appeared to impact on students' beliefs and increased their computational estimation performance (Figure 6.10). Bob's school community also appeared to perceive this teaching and learning to be of great benefit and they asked Bob to share his new PCK on estimation and mathematics, becoming a mathematics curriculum leader of the school.

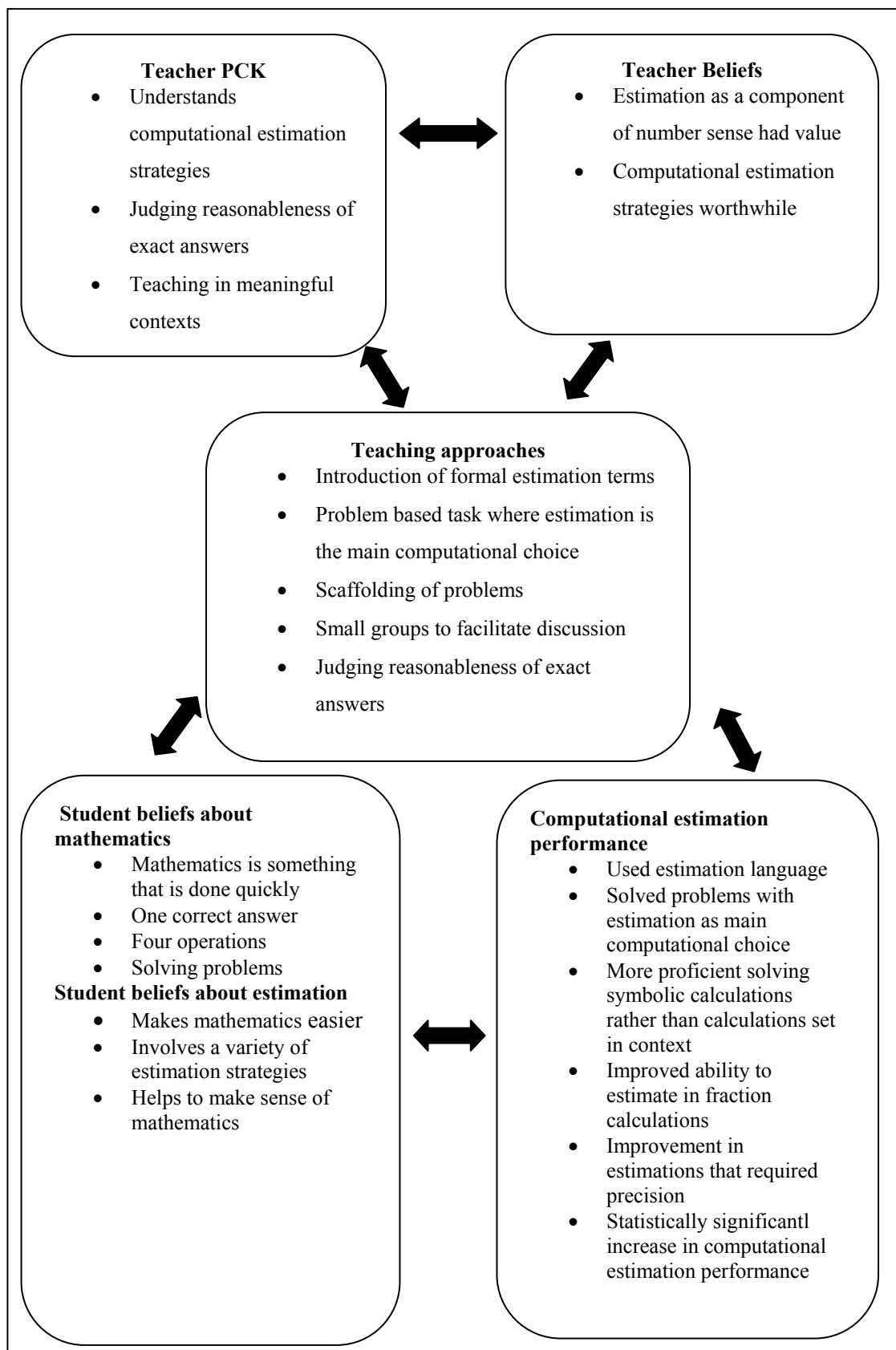


Figure 6.10: A model to show the impact of the professional learning intervention on Bob and his class

CHAPTER 7: CROSS-CASE ANALYSIS AND DISCUSSION

This chapter considers the commonalities within and differences between the three case studies. There were changes in all three case study teachers in response to the professional learning intervention. These changes all resulted in different teaching approaches and therefore the impact on the students was different in each case study. The students all experienced some computational estimation activities and these affected their computational estimation performances. The study investigated how the development of the teachers' beliefs and pedagogical content knowledge informed their teaching approaches and how their teaching approaches influenced the students' beliefs and student performance of computational estimation. The cross-case analysis focused on these areas and discusses how the teachers' responses were similar and how in some cases the professional learning intervention produced different responses from the teachers and students. As a result of this the professional learning intervention produced learning outcomes which were different for both teachers and students. This is consistent with the socially constructive perspective taken in this study, which states that each person's prior beliefs determine what learning outcomes are generated. It is also important to acknowledge the worldview of the Researcher that subjectively interpreted events as 'successful' where they led to students engaging in an active learning process that would lead to an understanding of mathematics (Sfard, 2008). From this worldview, interpretations of the findings were made. Drawing on these interpretations, general assertions were made as to the suggested impacts of the professional learning intervention and the conditions under which these occurred.

When designing the study the research literature was synthesised and a conceptual framework was created. This main body of the original framework is shown in Figure 7.1.

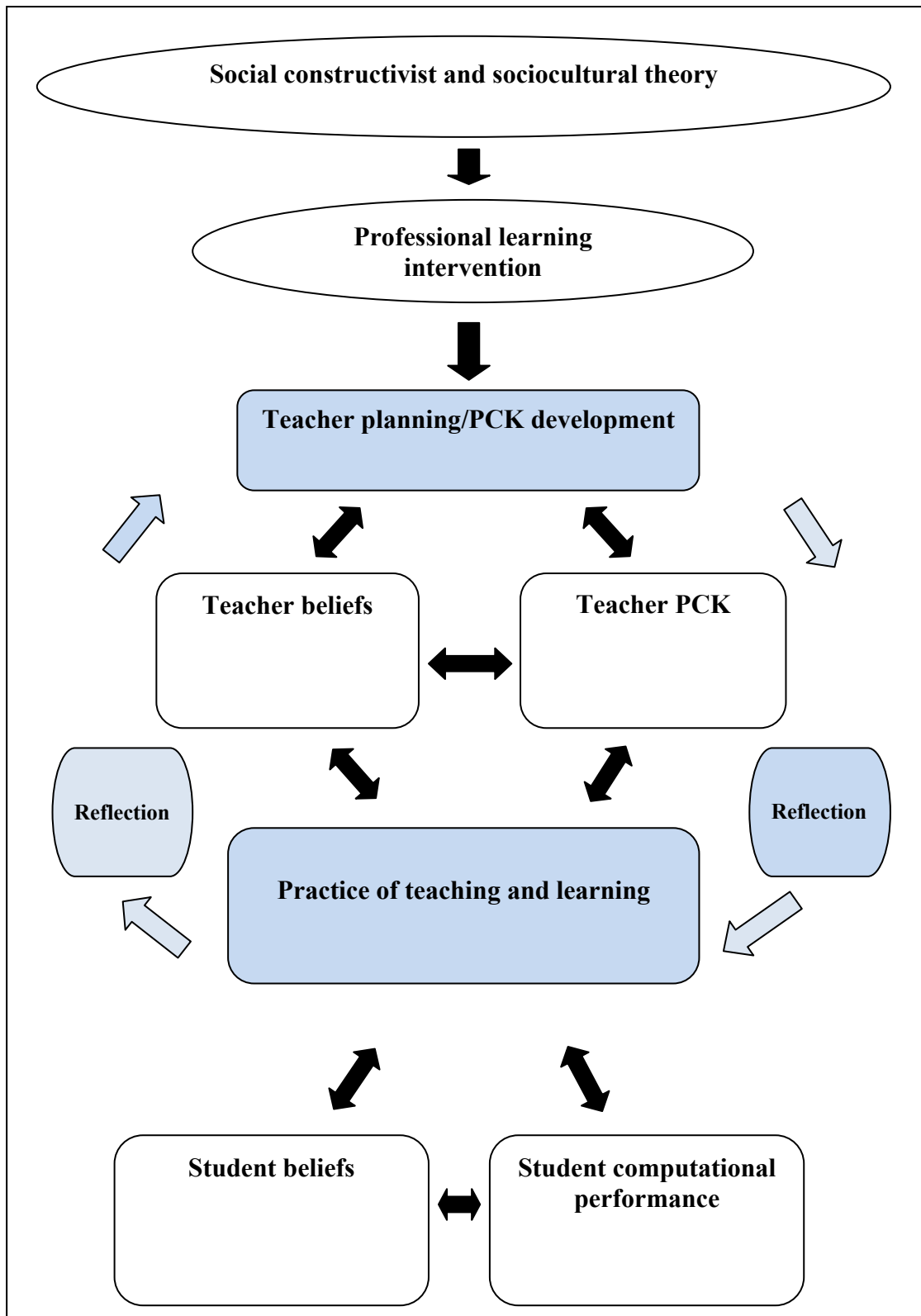


Figure 7.1: Conceptual framework for the study

This chapter will interpret the findings of this study in terms of the existing research literature and the conceptual framework. In doing this it will be possible to outline how this study has generated new knowledge and has contributed to the understanding of the teaching and learning of computational estimation.

Context of the Multiple Case Study

The context of this study was very important and the setting of the three case studies influenced the findings. Using purposeful sampling the Researcher invited metropolitan primary schools with few perceived student behavioural difficulties to take part in the research. As there had been little recent research into how to teach computational estimation, it was considered appropriate to research a setting where the students did not have major behavioural difficulties and were generally able to spend time in the classroom learning mathematics. There were few curriculum resources to guide the teachers so it was a demanding mathematics topic to teach and it would obviously have been more demanding if the teachers had extra behaviour management issues to deal with. The teachers were competent and confident teachers of mathematics, had at least three years teaching experience, and were willing to share their previously acquired knowledge to contribute to the question of how to teach computational estimation. The participating teachers all taught Year 6 students in low fee, non-government schools. These schools were established by religious groups and despite being privately run, still received government funding based on the socioeconomic status of the parents of the school (Independent Schools of Australia, 2007). The main difference between the three classes was that Bob's class was a streamed class whereas Peter's and Wendy's were of mixed ability, but they had many similarities in terms of their school cultures. These commonalities amongst the research participants undoubtedly affected the collaborative discussions. Any consensus reached about how to teach computational estimation was made with reference to their personal contexts, so generally the findings meant they were pertinent to these types of school cultures. The important contextual factors are summarised in Table 7.1.

Table 7.1: A summary of the teachers' contexts

Factors	Wendy	Bob	Peter
School	Low fee independent	Low fee independent	Low fee independent
Organisation of class	Mixed ability	Top stream of 2 classes	Mixed ability
Curriculum	School text book	Broad outline provided by school	School text book
ISEA value (mean 1000)	1118	1003	1038
Years of teaching	25	16	9
Number of students in the class	32	25	32
Mean Year 5 NAPLAN score 2008	476	478	474

How the Development of the Teachers' Beliefs and Pedagogical Content Knowledge Informed their Teaching Approaches

The development of the teacher beliefs and PCK was influenced by the extent to which the teachers engaged in the action research process. This development of teacher beliefs was a very personal process for the teachers and despite some commonalities across the cases, each teacher's beliefs remained different from one another (Assertion 4.2; Assertion 5.2; Assertion 6.2). Observing the teachers' PCK development revealed how all the teachers were able to understand the strategies to a certain extent and establish new pedagogies for teaching computational estimation (Assertion 4.1; Assertion 5.1; Assertion. 6.1).

Engagement in the professional learning process

Gaining the respect of the teachers so that they would be willing to engage in this learning process was made more difficult by the lack of recent research available about how to teach computational estimation. Only as the teachers became aware that the computational estimation strategies could be taught to Year 6 did the confidence in the process emerge in the group (Audit trail journal, 11/ 8/2009). Bob and Wendy appeared open to the ideas presented at the professional learning

workshops (PL1 observations, 18/2/2009). Peter did not appear to engage fully in the professional learning process until he had seen the pre-service teacher develop number sense activities with his students. In the third day of professional learning, Peter was more engaged in the professional learning process and then trialled some of the activities back in the classroom (PL3 observations, 29/7/2009).

There was also a difference in the extent to which the teachers perceived the need to change their teaching. Bob and Peter appeared to be more satisfied with their present teaching approaches and did not mention the need to investigate how they could change their teaching in the initial interview (Teacher interview 1, 20/11/2008; 4/12/2008). Wendy appeared to be less satisfied with her present teaching approach and was keen to use the action research process to enable more students in her class to understand mathematics (PL 1, 18/2/2009).

Wendy and Bob did not alter their beliefs about their fundamental approach to teaching mathematics whilst being involved in the professional learning intervention. As Bob taught using a problem-based approach normally, it was a logical process for him to incorporate estimation into problem-based learning and it fitted neatly into his present pedagogical approaches. Wendy had a pragmatic set of beliefs about the goals of school mathematics, that teaching routine algorithms using a textbook was appropriate in her school context as it created harmony amongst parents who were important stakeholders in the school even though she described this teaching approach as ‘band aiding’ the problems that students had. She therefore considered what was the most effective way to teach in an ‘ideal world’ in the action research process. In this way, there was also no conflict in beliefs for her. This phenomenon of compartmentalising beliefs in different contexts has been observed in previous research (Beswick, 2005; Karaağ & Threlfall, 2004).

Peter believed that some expert opinions on how to teach mathematics were not very relevant for his classroom practice (Teacher interview 1, 4/12/2008), so initially it appeared that he focussed on the day-to-day demands of his classroom rather than trialling the new ideas that were suggested concerning computational estimation. It was only when a pre-service teacher modelled how some of the ideas suggested on the professional learning workshops were relevant, that Peter faced some disequilibrium or cognitive conflict about the students’ understanding of the mathematics they were learning (Keady, 2007; Sfard, 2008). Peter gradually began

to articulate that he was reevaluating his beliefs. In this way, he began to recognise the importance of developing number sense (Key Finding 5.14). The key findings and assertion have been interpreted in terms of the literature concerning the engagement of the teachers in professional learning to construct General Assertion 1.

General Assertion 1: The participating teachers engaged at different levels in the professional learning intervention and this appeared to depend on how valuable they believed estimation would be to enhance their practice and how similar the teachers believed the professional learning goals were in to their beliefs about the goals of school mathematics. Optimum engagement in the professional learning program occurred where the teachers respected the process, believed that developing knowledge about teaching estimation would be worthwhile and were free to trail the process with their students.

Development of PCK and beliefs about computational estimation

All three teachers' beliefs and PCK about teaching computational estimation to Year 6 students were broadened whilst being involved in the professional learning intervention (Assertion 4.1, 4.2; Assertion 5.1, 5.2; Assertion 6.1, 6.2). Previous research also reports the development of teacher beliefs (Carpenter et al., 1999; Keady, 2007) and PCK (Bobis et al., 2005) in a professional learning environment.

A summary of the beliefs and PCK developed by the teachers is shown in Table 7.2. The table shows the five themes that emerged from the key findings. The blank sections indicate that this theme was not a key finding in the particular case whereas the shaded sections indicate that this theme was a key finding. This visual representation means that common themes to all three cases are easily identifiable.

Table 7.2: Summary of developed PCK and beliefs in the multiple case studies

Theme	Wendy	Peter	Bob
Content knowledge - Understanding Strategies	(KF 4.11) Understands computational estimation strategies	(KF 5.19) Some understanding of the estimation strategies	(KF 6.17) Understands computational estimation strategies
Beliefs – Number sense is important for Year 6 students	(KF4.20) Computational estimation was an important component of number sense in the ideal world	(KF 5.14) Beginning to consider that number sense activities had value in the Year 6 classroom	(KF 6.12) Computational estimation as a component of number sense should be an integral component of all computation lessons
Pedagogical approaches - Computational strategies	(KF 4.18) Computational estimation strategies worth teaching to Year 6 in ideal world	(KF 5.11) Doesn't believe in teaching formal estimation strategies to Year 6	(KF 6.15) Computational estimation strategies worth teaching to Year 6
Pedagogical approaches - Judging reasonableness with exact calculations	(KF 4.21) In the real world of her school context, a text book approach and its focus on procedural teaching of estimation and exact calculations, maintained harmony amongst parents	(KF 5.18a) Reflecting on exact computations by estimating answers	(KF 6.16) Reflecting on exact computations by estimating answers
Pedagogical approaches Teaching in meaningful contexts	(KF 4.19) Tasks set in meaningful contexts where estimation was the main computational choice could be a valuable in an ideal world	(KF 5.18b) Practical activities set in meaningful contexts	(KF6.16) Tasks set in meaningful contexts where estimation was the computational choice were valuable as a central teaching approach

At the beginning of the professional learning intervention, the teachers knew about the rounding strategy and Bob appeared to have some intuitive understanding of other strategies. However, none of the teachers knew any strategy names other than rounding. In the initial teacher interviews, none of the teachers mentioned how they would teach the variety of computational estimation strategies (Key Finding 4.4; Key Finding 5.2; Key Finding 6.3). The results of this research study are consistent with those of Alajmi (2009) who found that most of the Kuwaiti teachers interviewed in his research only the used rounding strategy.

At the beginning of the professional learning intervention, computational estimation did not play a large role in the three teaching repertoires. Wendy and Peter taught computations predominantly through routine algorithms. Wendy believed that it was part of the normal instruction to mention to her students that they should estimate the result so that they could check if their answers were reasonable (Key Finding 4.1). She indicated that her students did not find this engaging. Peter explained at the beginning of the professional learning intervention that he believed that estimation had a place as a checking device when he was asking the students to compute mentally at the beginning of mathematics lessons. He also believed that Year 6 students did not find estimation as a checking device to be engaging (Key Finding 5.4). Bob believed that problem-based learning was an important teaching approach and that routine algorithms should play a lesser part in mathematics instruction to Year 6. Within this approach, however, Bob believed that there was no time to teach computational estimation (Key Finding 6.4).

Many of the activities on the first day of professional learning focussed on the teachers engaging in activities to facilitate their understanding of estimation as a component of number sense (PL Handbook, 2009). By the end of the professional learning intervention all of the teachers believed in the value of teaching estimation as a component of number sense (Assertion 4.2; Assertion 5.2; Assertion 6.2).

The professional learning workshops also provided opportunities for the teachers to develop content knowledge about the computational estimation strategies. After the strategies were described, the teachers worked on mathematical problems to develop their understanding of the strategies. Content knowledge of the estimation strategies is an important component of the PCK of computational estimation (Hill et al., 2008; Shulman, 1986). It appeared that all the teachers were able to develop their knowledge of the estimation strategies to a certain extent. Wendy and Bob were able to name most of the estimation strategies by the second teacher interviews, which took place at the end of Term 2 and this is shown in Table 7.2. They were also able to explain how students could solve estimation problems and why they may have used various computational estimation strategies (Key finding 4.11; Key Finding 6.17). Peter was not able to name any of the strategies at the end of Term 2 and was not able to explain how he would use the estimation strategies to solve estimation problems. At the end of the professional learning intervention, Peter was able to remember the name of some computational estimation strategy names and was able

to explain how you would use the estimation strategies, even though he did not use the formal strategy terms when he did this (Key Finding 5.19). Peter still was using a textbook that covered instruction on how to use the rounding strategy; otherwise, he encouraged students to solve the estimation problems without directing them to certain strategies (Assertion 5.1). To the Researcher's knowledge, there have been no other research studies which have recorded the impacts of introducing these estimation strategies as a component of number sense through a professional learning process, so it is difficult to evaluate this process in comparison to other research studies.

Bob and Wendy stated that teaching the formal computational estimation strategy terms was pedagogically appropriate (Key Finding 4.18, Key Finding 6.15). These strategies included benchmarking, nice numbers, front-end loading, rounding, sample, and intuition (Mildenhall, 2009). This range of strategies is similar to the list suggested by Reys (2004). He asserted that in Grades 5-8 (a similar age range to this study), front-end loading, compatible (nice) numbers, rounding and benchmarking strategies were suitable. Reys and Reys suggested that in K-2, estimating quantities is a suitable emphasis, which is similar to the intuition and sampling strategies suggested in this research study. All the strategies listed above were observed being used by the students. No students involved in this study had heard of the strategies before the study began. If the students had been introduced to some of the strategies at an earlier age, this might influence what strategies would be suitable for this age range. At the end of the professional learning intervention, Bob and Wendy both believed that computational estimation should be taught to Year 6 as a component of number sense (Assertion 4.2; Assertion 6.2). As Table 7.2 shows one of the three teachers in this study did not believe that different estimation strategies had value as a teaching approach and this is similar to the research by Alajmi's (2009) research who found that 46% of his sample of teachers believed that computational estimation and the estimation strategies should not be included in the Kuwaiti mathematics curriculum.

The first and second professional learning workshops focussed on the value of setting problem tasks in meaningful contexts so that students could evaluate the purpose of estimation (PL 1 Handbook). This approach was taken based on the research literature which demonstrated that using meaningful contexts provides valuable learning environments that lead to the understanding of mathematics (Bobis

et al., 2005; Gravemeijer & Terwel, 2000; Silver, 1994). Before the research began, the teachers had not taught estimation as a computational choice in meaningful contexts. After being involved in the professional learning intervention the teachers argued that computational estimation should be taught in meaningful contexts as a computational choice in its own right (Table 7.1) (Assertion 4.1; Assertion 5.1; Assertion 6.1). These contexts used for solving Fermi problems and planning a trip to a local park appeared to be motivating for the students. The students were able to understand the reason why they were estimating and use this purpose to decide how they were going to estimate.

Researchers are now asserting that setting computational estimation tasks in contexts is valuable (Reys & Reys, 2004; Trafton, 1994) but so far there has been little guidance from research as to how this should be structured (McIntosh et al., 1997; Reys & Reys, 2004). Past research often focused on how to teach computational estimation through teaching discrete skills (Bobis et al., 2005; Reys & Bestgen, 1981). Case and Sowder (1990) investigated how primary school students could estimate using addition algorithms. Only a few recent research studies have considered teaching computational estimation in a problem-solving context (Neill, 2006; Nohda & Yabe, 1994).

Table 7.2 shows that Bob and Peter asserted that it was also important to focus on computational estimation as a checking tool when calculating exact numbers (Key Finding 5.18; Key Finding 6.16). Table 7.2 also shows that Wendy did not use this strategy in her teaching. When she was not doing the extra estimation tasks she reverted to following the textbook exactly and focussing on procedural teaching of algorithms (Key Finding 4.21). This practice of developing overall number sense in order to develop estimation ability was noted as an effective approach in research conducted by Trafton (1986). He interviewed seventh and eighth grade students and he found that students who simply conducted some computational estimation without considering the magnitude of the numbers were unable to judge whether an answer was reasonable.

The key findings and assertions have been interpreted in terms from the literature concerning the development of the teachers' beliefs and PCK to generate the following general assertions.

General Assertion 2: The teachers' beliefs about computational estimation developed whilst being involved in the professional learning intervention. At the beginning of the professional learning intervention the teachers did not believe that estimation had an important place in the primary mathematics curriculum due teaching algorithms and time pressures. At the end of the professional learning intervention, the teachers believed that teaching computational estimation as a component of number sense had value and most of the teachers believed that teaching the computational estimation strategies to Year 6 was worthwhile.

General Assertion 3: The PCK of the teachers developed whilst being involved in the professional learning intervention. At the beginning of the intervention the teachers had little PCK of computational estimation. As the teachers began to believe that estimation was worthwhile, this knowledge grew. At the end of the intervention, the teachers were able to name and understand computational estimation strategies. Most of the teachers also developed the pedagogical approaches of using the formal estimation strategy terms and using the strategies, setting estimation problems in meaningful contexts and providing students with opportunities to check the reasonableness of exact calculations.

How the developed PCK and beliefs informed the teaching approaches

Using the key findings from the individual case studies, it was possible to synthesise the different teaching approaches of computational estimation developed within this multiple case study as a set of six themes. These are summarised in Table 7.3. As previously stated, the blank sections indicates that this theme was not a key finding in the case study whereas the shaded sections indicate that this theme was a key finding in the case study.

Table 7.3: Summary of teaching approaches developed in the multiple case study

Themes	Wendy	Peter	Bob
Explicit teaching of strategies	(KF 4.10) Introduced the language used to describe the computational estimation strategies in extra mathematics time		(KF 6.10) Introduced the language of the computational estimation strategies at the introduction of the study
Problem based task where estimation is the main computational choice	(KF 4.14) Problems in meaningful contexts, teaching focussed on making numbers easier in extra mathematics time	(KF 5.16) Beginning to trial practical activities set in meaningful contexts where the computational choice is only an estimation	(KF 6.16) Scaffolded all computational estimation problems where estimation is the main computational choice
Scaffolding of estimation problem solving	(KF 4.13) Teacher modelling use of estimation strategies in problems in extra mathematics time		(KF 6.14) Models use of estimation strategies without explicitly using formal terms to describe strategies when problem solving (KF 6.16) Scaffolded all computational estimation problems where estimation is the main computational choice
Small groups to facilitate discussion	(KF 4.15) Using small group to facilitate discussion when exploring computational estimation problems in extra mathematics time	(KF 5.15) Beginning to trial working in small groups in order to facilitate discussion	(KF 6.13) Small group used to facilitate discussion exploring computational estimation problems

Themes	Wendy	Peter	Bob
Judging reasonableness	(KF 4.21) Routine exercise teaching rounding and algorithms using the text book in normal program	(KF 5.18) Students judging reasonableness of exact calculations.	(KF 6.12) Computational estimation as a component of number sense integral component of all computation lessons to judge reasonableness
Routine teaching of rounding procedurally from text book	(KF 4.21) Routine exercise teaching rounding and algorithms using the text book in normal program	(KF 5.5) Routine exercise teaching rounding in text book	

Past research studies have found that beliefs (Fennema et al., 1996; Yackel & Cobb, 1996) and teachers' PCK inform their teaching (Bobis et al., 2005). It is possible to hypothesise as to how the developed beliefs and PCK summarised in Table 7.2 may have informed the teaching approaches summarised in Table 7.3. It is important to qualify that these perspectives are the Researchers' interpretation of the data (Patton, 2002) and that the relationships between these different factors are quite complex.

The teachers' beliefs about school mathematics appeared crucial in shaping their teaching approaches in this study. Although the teachers' perspectives were all valued, the workshops presented a perspective from the research literature that valued students developing number sense. Number sense is an emerging term to describe students who can work flexibly with number in quantitative situations. It requires students to perceive that, when calculating, these numbers should make sense (Silver, 1994). The teachers' valuing of number sense in their context shaped their computational estimation.

As Table 7.2 shows, by the end of the study all of the teachers believed in the value of estimation and number sense (Key Finding 4.20; Key Finding 5.14; Key Finding 6.12). It was discussed at the professional workshops that setting computational estimation tasks in contexts would promote students making sense of their estimated calculations as suggested by leading mathematics educators (Reys & Reys, 2004; Yoshikawa, 1994). It was observed that Wendy and Bob, and to a lesser extent Peter,

used their developed pedagogical content knowledge of teaching computational estimation using tasks set in meaningful contexts in their classrooms (Table 7.3). The meaningful learning contexts that were explored at the workshops and then used by the teachers in their classrooms included identifying estimations in newspapers, spending approximately one million dollars, a shopping game set in a surf shop, solving Fermi problems and planning a trip to a local park. These contexts appeared to be motivating for the students, even when they were beyond the students' day-to-day experiences.

Generally, the teachers scaffolded these problem-solving tasks and modelled for the students how they could solve the estimation problems (Key Finding 4.13; Key Finding 6.14). The professional learning workshops were structured so that the teachers were encouraged to consider the needs of the students in designing their teaching tasks (PL handbook, 2009). The workshops focussed on how the teachers should adapt the tasks to suit the needs of their students (PL handbook, 2009). Through observing the teachers' classroom practice, it revealed that the teachers appeared to consider that the students had not encountered the strategies before in meaningful contexts and adapted the tasks so that initially they modelled for the students how they could solve the estimation problems (Key finding 4.13; Key finding 6.14). This teaching approach which is similar to the cognitive apprenticeship model (Brown et al., 1989) and is perceived by Sfard as an essential step in learning with understanding (Sfard, 2008). She asserts that when learning a new concept the students are only able to engage in the task with a large amount of scaffolding from the teacher. From this perspective, students engage in tasks they do not understand as a "peripheral participant" and this is a necessary step towards personal construction of knowledge. Recent research by Star and Rittle-Johnson concurs with this perspective that modelling solutions is an effective teaching approach. Their research has shown the effectiveness of teaching students new estimation strategies through providing them with worked examples on how to solve problems and the different strategies that could be used. The fifth and sixth-grade students involved in their study who compared different solutions strategies were more flexible problem solvers than students who studied estimation strategies one at a time (Star & Rittle-Johnson, 2009).

As Bob and Wendy gained a solid understanding of the computational estimation strategies, it appeared that they were keen to share this new knowledge with their

students. As Table 7.3 shows, Wendy and Bob introduced these explicitly to the students shortly after learning about the strategies themselves (Key Finding 4.10; Key Finding 6.10). Wendy and Bob both believed that the computational estimation strategies were worthwhile. In the first cycle of the action research process, Wendy and Bob explicitly introduced the estimation strategy terms. This gave the teacher and the students the language to discuss the different ways that they had estimated and the benefits of using different estimation strategies.

Bob tended to use the new terminology less as the action research process progressed and appeared to focus less on the formal terms for the estimation strategies. The finding that most of the teachers in this research study were able to understand the computational estimation strategies is an important finding as this is obviously a necessary prerequisite if these strategies are to become commonplace in primary mathematics teaching (Mildenhall, Hackling, & Swan, 2010). Peter was not as proficient at understanding the computational estimation strategies and this appeared to impede his teaching. He did not formally teach the students the estimation strategies (Table 7.3) and he explained in the final teacher interviews how he found it difficult to understand students' reasoning. This is an important perspective and if this were a common concern amongst other primary teachers, it would be worth considering how teachers could be supported in this area.

Working directly with Year 8 students over a period of eight weeks, Neill (2006) , established his pedagogical framework for teaching computational estimation. This is shown in Figure 7.2. Neill recommended using a combination of explicit teaching of the strategies (or methods) and problem-based learning. Neill asserted that it is appropriate to solve a problem first, acknowledge all students' strategies and then highlight a target method. This is a different order from the teaching approach used by Bob and Wendy who introduced a whole variety of strategies at the beginning of the school year and modelled a few estimation strategies when introducing the tasks to the students.

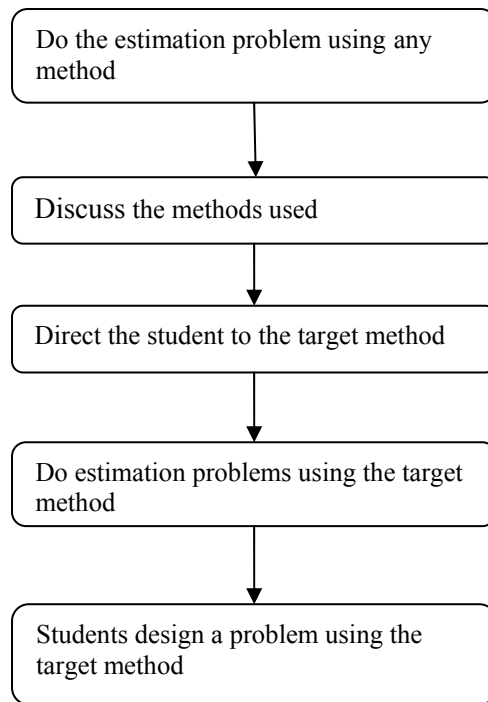


Figure 7.2: Neill's (2006) model for introducing computational estimation strategies

A final factor in how the teachers developed PCK and beliefs informed their teaching was the teachers' beliefs about their personal school context. Wendy and Peter had a school textbook to follow that was supported by the parents who were important stakeholders at the school. During the school year, they continued to teach estimation in the way the textbook suggested as well as trial their new teaching approaches that they believed were pedagogically appropriate (Key Finding 4.21; Key Finding 5.5). Teachers professing beliefs as to how mathematics should be taught but at the same time being guided by beliefs about the goals of school mathematics has been found in other research studies (Beswick, 2005; Karaa & Threlfall, 2004). This relationship between the beliefs about the school context, beliefs about teaching computational estimation in an ideal world, PCK and teaching approaches is shown in Figure 7.3.

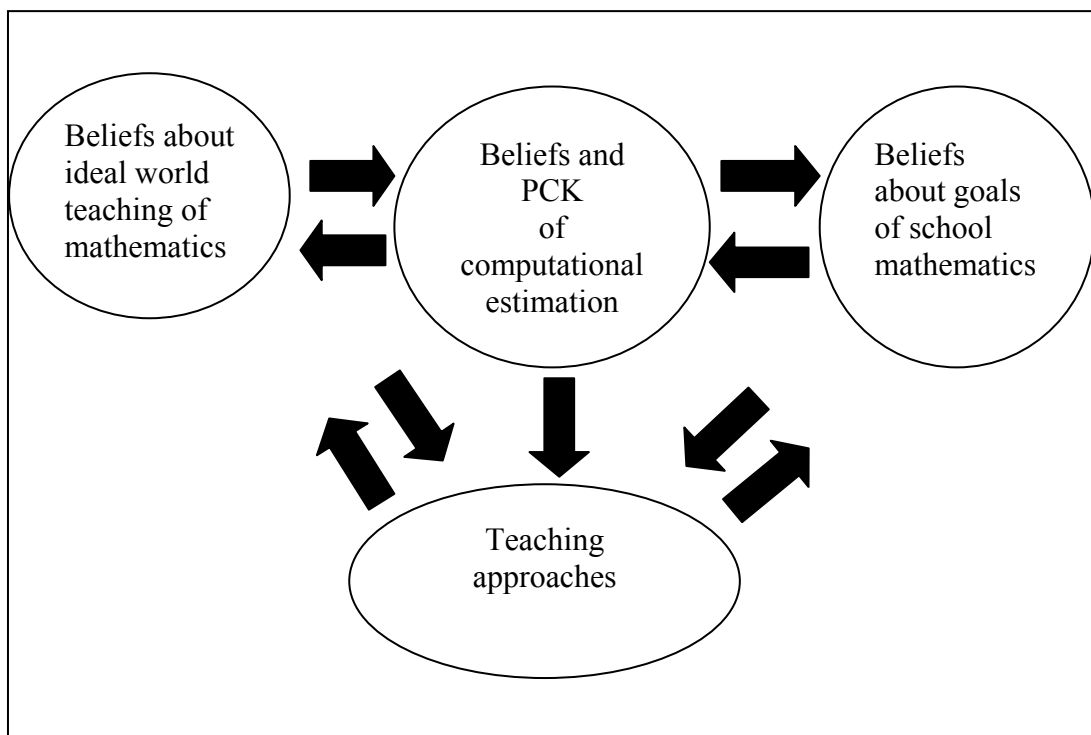


Figure 7.3: The relationship between idealistic beliefs, pragmatic beliefs about the goals of school mathematics (Karaaç & Threlfall, 2004) and teaching approaches

There appeared to be an important link between the teachers' beliefs and pedagogical knowledge of computational estimation and what pedagogical approaches they implemented in the classroom. If curriculum writers were hoping to include computational estimation for the first time, it would be necessary to provide professional learning programs where teachers are provided with opportunities to critically reflect on their present beliefs and PCK of computational estimation. In Wendy's case, her goals of school mathematics were different to her perception of the goals of teaching mathematics in the ideal world. This factor also means that for the optimum implementation of computational estimation in a school, the whole school community, principals, parents, teachers and students need to believe that computational estimation as a component of number sense has value. The key findings and assertions have been interpreted in terms of the literature to generate the following General Assertion.

General Assertion 4: The beliefs and PCK developed in the professional learning process appeared to shape the teachers' approach to teaching of computational estimation. Their beliefs in the value of estimation as a component of number sense appeared to mean that they provided learning tasks in meaningful contexts where estimation was the main computational choice. This belief also may also have caused the teachers to encourage students to check the reasonableness in their exact calculations using estimation. Two of the teachers believed that the estimation strategies had value for Year 6 and they introduced these to the students using the formal terms. Wendy and Bob's new estimation content knowledge informed their teaching, as they were now able to teach the students about these strategies. Wendy's new PCK did not impact on the majority of her mathematics teaching as her beliefs about the pragmatic goals of teaching in the real school context was not the same as her beliefs as to what was pedagogically appropriate in an ideal world.

The Impact the New Teaching Approaches had on Students' Beliefs and Computational Estimation Performance

The teachers all trialled new teaching approaches whilst being involved in the professional learning intervention. It has been possible to offer insights as to how those teaching approaches may have affected the students' beliefs and their computational estimation performance.

The development of students' beliefs

It was of interest in this study to gather evidence of students' beliefs about estimation before and after the professional learning intervention. Focus groups from each class were interviewed and from these responses, key findings were identified. A summary of key findings related to the students' beliefs is shown in Table 7. 4. The unshaded section indicates that this theme was not a key finding in the case study whereas the shading indicated that this theme was a key finding in the case study.

Table 7.4: A summary of the developed students' beliefs in the multiple case studies

Themes	Wendy	Peter	Bob
Mathematics is about one correct answer and done quickly	(KF 4.22) Mathematics is about something about patterns, that is done quickly and is about something that is about one correct answer	(KF 5.20) Mathematics is about working out problems, is about one correct answer and something that is done quickly	(KF 6.18) Mathematics is about working out problems, one correct answer and something that is done quickly
Estimation			
Awareness of estimation	(KF 4.23) More than a guess		(KF 6.20) More than a guess
Fun	(KF 4.23) Fun	(KF 5.21) Fun	
Making mathematics easier	(KF 4.23) Make mathematics easier	(KF 5.21) Useful as a checking device	(KF 6.20) Make mathematics easier
Negative			(KF 6.20) Can remove the exactness of mathematics and therefore less enjoyable
Estimation strategies more than rounding	(KF 4.23) Involved a variety of strategies	(KF 5.21) Estimation is about the rounding strategy	(KF 6.20) Involved strategies

When the study began, the students believed that mathematics was something that was done quickly, was about the four operations and solving problems. The fact that they perceived that mathematics was something that was done quickly, suggests that the students normally worked on routine algorithmic tasks rather than extended problems. The students also expected there to be one right answer to mathematical questions. Generally, this perception of mathematics did not change by the end of the professional learning intervention. At the end of the study, the students still perceived that mathematics was something that was done quickly and was about one exact answer (Key Finding 4.20; Key Finding 5.20; Key Finding 6.18). These findings about how students perceived mathematics are consistent those findings of Frank (1988).

Students' understanding of estimation in all three classes was quite limited at the

beginning of the professional learning intervention. One of the common words that the students used to describe estimation was a type of mathematical guess. The students did not mention that they perceived estimation as a computational choice in its own right but did mention it as useful as a checking device.

Most of the students at the end of the professional learning intervention believed that the estimation work had been an extra set of activities, on top of the normal mathematics curriculum. However, their beliefs about estimation had broadened. They now believed estimation could include the variety of estimation strategies and could be a computational choice in its own right when problem solving. The focus group students from Peter's class, however, still believed that the only estimation strategy that they could use was rounding. Generally, the students had very positive perceptions of estimation and the estimation activities they had been involved in (Assertion 4.4; Assertion 5.3; Assertion 6.4). Most students found the work easier than when working with exact numbers and they described this positively. This students' positive perception of estimation was not found in research by Yoshikawa (1994). Interviewing Japanese students, he found that most of the 159 Grade 4 and 5 students avoided using computational estimation. The key findings and assertions have been interpreted in terms of the literature to support the generation of the General Assertion 7.

The development of students' computational performance

Students appeared to be more competent at estimating by the end of the professional learning intervention. As shown in Table 7.5 a common theme to all three cases was that the students were using estimation language when solving problems where estimation was the main computational choice. There was also an impressive improvement in students' computational estimation performance by the end of the professional learning intervention (Assertion 4.5; Assertion 5.5; Assertion 6.5) and this is summarised in Table 7.5.

Table 7.5: A summary of the students' computational estimation performance

Theme	Wendy's class	Peter's class	Bob's class
Using computational estimation language	(KF 4.16) Students used computational estimation language when discussing how to solve the problem	(KF 5.16) Students used computational estimation language when discussing how to solve the mathematical problem	(KF 6.27) Students used computational estimation language when discussing how to solve the problem
Solving problems using only estimations	(KF 4.17) Students were able to use estimations as a main computational choice in extended problem task	(KF 5.17) Students were able to use estimations as a main computational choice in extended problem task	(KF 6.26) Students were able to use estimations as a main computational choice in extended problem task
More proficient at estimating symbolic calculations and problems set in context	(KF 4.22) Students are much more proficient at estimating multiplication problems which are purely symbolic and not set in context	(KF 5.24) Students are much more proficient at estimating multiplication problems which are purely symbolic and not set in context	(KF 6.23) Students became more proficient when estimating a multiplication mathematical calculation which are purely symbolic than when it was set in context
Improvement when estimating with fractions	(KF 4.23) Nearly half the class were able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and far more students could use a reasoned estimation strategy to estimate when adding fractions	(KF 5.22) Few students in Peter's class were able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and few students were able to use the benchmarking strategy	(KF 6.21) Over half the class were able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators. Far more students were then able to use this benchmarking strategy

Theme	Wendy's class	Peter's class	Bob's class
Computational estimation performance	(KF 4.26) Students' computational performance improved overall and statistically this was highly significant	(KF 5.25) Students' computational estimation performance improved overall but statistically this was not significant	(KF 6.24) Students' computational performance improved overall and statistically this was highly significant
Estimating with precision	(KF 4.25) Students showed no improvement in estimating to a fairly high degree of precision when assessing an estimated answer of a multiplication of a two digit by three digit number	(KF 5.26) Students' ability to select the best estimate on a multiplication calculation that required an answer with some precision improved	(KF 5.25) Students' ability to select the best estimate on a multiplication calculation that required an answer with some precision improved

Over the three classes, the students' mean pre-intervention test score of 2.94/6 increased to 3.89/6 following the intervention and this improvement was statistically highly significant. The students' estimation performance increased on all six questions.

The number of students using reasoned estimation strategies also increased significantly following the intervention (Key Finding 4.22; Key Finding 5.22; Key Findings 6.21). When conducting this cross-case analysis it also became apparent that by the end of the study there may be a correlation between students who used these reasoned estimation strategies in order to answer the question and those whose were more proficient at selecting the best estimate. A Pearson product correlation coefficient was computed to assess the relationship between students using reasoned estimation strategies and selecting the best estimates. There was a strong and significant correlation between the two variables, $r = 0.499$, $n = 67$, $p = 0.001$. Increases in students' reasoned estimation strategy use were correlated with increases in estimation performance.

Generally, the students had a higher competency at solving the symbolic estimation problem (Question 4) than the problem set in context (Question 1). In the pre-test, there were more students who were able to select the best estimate in the symbolic

problem, compared to the contextual question. At the end of the study, the students improved in both types of questions but the student performance answering the question using symbolic representation remained higher. These are comparable results to the NST study (McIntosh et al., 1997). The finding is contradictory to the finding by Reys (1986) who found that in a research study with participants ranging from Grade 5 to adults, they were able to answer a question set in a context that they could not answer a decontextualised question. In the question used in the research by Reys (1986), the context possibly gave the students a mental image of the fractions involved, which supported the students and could act as a realisation of the mathematical concept (Sfard, 2008). In this CET, the symbolic question ($105 \times 45 \approx$,) could be answered if students had practised multiplying symbolic representations of numbers by multiples of 10 and were able to consider the general magnitude of the number. They possibly did not need the support of a context to help them answer the question. Instead, the context in Question 1 may have actually made the question more complicated in that they had to complete additional steps to solve it. Other mathematical Researchers have recommended using symbolic and other representations such as real-life scenarios, and pictures in order for students to develop a deeper understanding of the mathematical concept (Herrington, 1990; Sfard, 2008).

Where the students were expected to estimate in a symbolic calculation in the CET on Question 4, there was some precision required in order to select the best estimate. At the beginning of the study, 59% of the students could select the best estimate. Overall, there was a 15% improvement, although Peter's and Bob's students' improvement was far greater than that of Wendy's class.

Few students were also able to select the best estimate when adding two fractions with unlike denominators at the beginning of the study (16%). At the end of the study, it would appear that the students had improved when estimating with fractions. In the post-test 40% of the students were able to select the best estimate, suggesting that the students' ability to estimate with fractions had greatly improved whilst their teachers were involved in the professional learning intervention. This is a particularly noteworthy improvement considering that only 25% of 12 year old students were able to answer this question on the Taiwanese component of the NST (McIntosh et al., 1997). Students in Bob's and Wendy's classes improved more than in Peter's class. It also appears that the students in Wendy and Bob's classes were

more able to use reasoned estimation strategies when estimating with fractions. There were far more students who used a reasoned estimation strategy in Wendy's and Bob's classes compared to Peter's class. A Pearson chi square test was conducted on the results of Question 2 that involved the estimation of fractions. It was performed to examine the relationship between students selecting a reasoned estimation strategy and selecting the best estimate. The relationship between these was strong and significant, $X^2(1, N = 67) = 34.87, p = 0.000$, because awareness of the strategy enhanced the students ability to select a successful strategy. Students using a reasoned estimation strategy in Question 2 were more likely to select a best estimate.

At the beginning of the study, 60% of the all the students were able to select the best estimate when adding four two-digit numbers. This is possibly lower than expected as two-digit addition calculations are one of the first calculations that students undertake in the primary school. At the end of the study, 80% of the students were able to select the best estimate, which represented a 20% improvement. The improvement in this question was greatest in Wendy's class. This may have been due to the fact that she taught the front end loading strategy explicitly and spent time discussing how this could be used. It may be that this strategy could be taught early on in the primary school as students are being taught to add two digit numbers. As Clarke (2009) suggests that the benchmarking strategy helps students understand fractions, then the front end loading strategy may help students understand two and three digit whole numbers. The key findings and findings from the literature concerning how the students' computational estimation performance developed have been summarised in the following general assertions.

General Assertion 5: The students' estimation performance increased and this was statistically significant. In response to the intervention, students improved in their ability to use estimation language and solve estimation problems. The use of estimation strategies increased and this appeared to be directly related to students' increased success in solving estimation problems and selecting the best estimate in the post-test.

General Assertion 6: At the end of the case study, the student performance was higher on the symbolic question on estimation than the problem set in context. More students could estimate with fractions with unlike denominators and most students could select the best estimate when adding two-digit numbers.

The impact of the teaching approaches on student beliefs

Research is revealing that there is a relationship between students' beliefs and their achievement (House, 2006; Nezahat, Mahir, Mehmet, & Hakan, 2005). Therefore, students' beliefs are an important factor in increasing students' computational estimation performance. The students' beliefs about mathematics did appear to be very entrenched despite the teachers trialling approaches that involved extended tasks which had more than one answer. At the end of the study, the students still perceived mathematics to be something that involved one correct answer and was done quickly (Key Finding 4.20; Key Finding 5.20; Key Finding 6.18).

The new teaching approaches of the participating teachers did appear to impact the students' beliefs in various ways. They appeared to perceive the new estimation activities as a supplement to normal activities. After experiencing these estimation tasks, they had a much more positive perception of estimation. The students had all worked on estimation tasks where estimation was the main computational choice and when working with exact numbers, they were able to simplify these in the estimations (Key Finding 4.21; Key Finding 6.20). This appeared to impact on their beliefs about estimation in that they believed estimation could be a computational choice in its own right and that this process made the problem solving easier.

Wendy and Bob taught the computational estimation strategies explicitly and this teaching approach appears to have impacted on the students' beliefs. The students from these classes believed that estimation strategies available were more than rounding and they did include using other strategies such as front end loading and benchmarking in their concept maps of estimation. The key findings have been interpreted in terms of the literature concerning how the new teaching approaches impacted on students beliefs to generate the following assertion.

General Assertion 7: The students' overall beliefs about mathematics did not change in response to the intervention. Their beliefs about estimation as a separate component did appear to have changed. At the beginning, the students believed that estimation was a mathematical guess that involved rounding, whereas at the end they perceived that estimation involved a variety of strategies and could be a computational choice in its own right. They also believed that estimation was fun and made mathematics easier. The changed beliefs about estimation appeared to be in response to explicit teaching of a range of estimation strategies and using tasks set in meaningful contexts.

The impact of the teaching approaches on students' computational estimation performance

The teachers' involvement in the professional learning intervention appeared to improve the students' estimation performance. The teaching approaches that appeared to impact student performances were, the use of meaningful contexts, introducing the students to the variety of computational estimation strategies, and using estimation to judge the reasonableness of estimation in all calculations.

All of the teachers used a variety of tasks set in meaningful contexts to teach computational estimation (Key Finding 4.14; Key Finding 5.16; Key Finding 6.16). This teaching approach appeared to provide students with learning tasks that developed their understanding of how they could use estimation as a computation choice (Key Finding 4.17; Key Finding 5.17; Key Finding 6.26). The students in the focus groups explained how they perceived these teaching approaches had influenced their learning (Key Finding 4.23; Key Finding 5.21; Key Finding 6.20). Two of the teachers scaffolded when to use the different strategies to solve estimation problems (Key Finding 4.13; Key Finding 6.16) and structured the lessons so that the students worked collaboratively (Key Finding 4.13; Key Finding 6.16). It was observed that the students were able to use estimation language when working in meaningful contexts (K.F.4.16; Key Finding 5.16; Key Finding 6.27). A link between using contexts as a teaching approach and the increase in students' computational estimation performance was observed in Bob's classroom. Bob used contexts for most of his estimation and exact calculation work and his students made the greatest improvement in mean test score. In addition, in support of this assertion,

Peter only began using contexts towards the end of the intervention and his students' mean improvement on the CET was the lowest. The benefits of contexts used to teach computational estimation was found in research by Nohda and Yabe (1994). They found that when Year 5 Japanese students worked on a task in a problem context they were able to use simpler numbers to make reasonable estimates.

Introducing the estimation strategies appears to have impacted on the students' estimation performance. At the beginning of the professional learning intervention, two of the teachers explicitly introduced the estimation strategies to the students (Key finding 4.10; Key Finding 6.10). This highlighted to the students that there were different ways of estimating other than rounding. When the students were asked to explain how they estimated in the CET at the end of the professional learning intervention, more students provided explanations of reasoned estimation strategies. This increase in using estimation strategies in the classroom and in the CET may have been because they had been provided with a number of different strategies that could be used when estimating and therefore were able to use these strategies rather than just guess or use the previously learnt rounding strategy. The teachers providing the students with descriptions of different reasoned estimation strategies, which they appeared to use, may then also have increased their estimation performance. This assertion is supported by the statistical evidence that there was a significant correlation between students using a reasoned estimation strategy and their ability to select the best estimate. The relationship between students using a reasoned estimation strategy and selecting the best estimate was particularly strong for students' performance in Question 2 where students were estimating with fractions. In this question, students using a reasoned estimation strategy were much higher in Bob and Wendy's class where the benchmarking strategy had been introduced explicitly. Other research has found that explicitly teaching these strategies has improved computational estimation performance. Bobis (1991) using quazi-experimental research also found that, after instruction from worksheets and teacher direction, Year 5 students' computational estimation performance improved. The teachers were instructed to use the worksheet activities to teach the students rounding, truncating and benchmarking and she found in follow up interviews that students used these strategies. Students particularly improved on their ability to benchmark fractions. Clarke (2009) found when interviewing Year 5 students, those students who had a sound conceptual understanding of fractions used the

benchmarking strategy. Whilst his research did not focus on the results of teaching students the benchmarking strategy, he asserted that it would be beneficial for all students to know about the successful strategy of benchmarking. He therefore recommended that in the teaching of fractions, the explicit sharing of the benchmarking strategy is undertaken in the context of whole-class discussion.

Peter's and Bob's classes spent time instructing their pupils to use estimation to check the reasonableness of exact calculations (Key Finding 4.21; Key Finding 6.12). By the end of the professional intervention, encouraging students to make sense of the mathematics was an integral part of their teaching approaches in all of their mathematics. These sense-making teaching approaches appear to have impacted on the students' computation estimation performance and encouraged the students to consider if the calculations were sensible. In the CET many of Peter's students explained how they considered the contextual question (Question 1) and considered which estimates would make sense and this led to a 10% improvement in the students' performance on Question 1. Both Peter's and Bob's students improved in their performance on Question 4, where it was important to consider the precision of the estimate. Wendy spent less time checking the reasonableness of exact calculations as she used the set textbook that taught algorithms procedurally (Key Finding 4.21) and, where the textbook occasionally required an estimate, it did not specify that the student use the estimate to consider the reasonableness of the exact answer in the calculation. As Wendy's students did not improve on Question 4, which required an estimate with some precision, it may have been because Wendy spent less time than Peter and Bob evaluating the reasonableness of calculations in her everyday mathematics work.

Bob appeared to create a community of learners (Yackel & Cobb, 1996) in his classroom for all his mathematics. Bob spoke of how his students would now discuss the mathematics with each other and justify their solutions (Assertion 6.5). The incorporation of his teaching approaches of computational estimation, within this sociomathematical norm, appeared to be particularly effective in developing estimation and number sense. In the CET his students' performance increased by at least 12% on every question and this included questions that required a precise estimate and one that required the use of the benchmarking strategy.

Neill (2006) recommended using a combination of teaching approaches. He

combined a variety of strategies, working on estimation problems and discussion of the solutions found that the Year 8 students were able to demonstrate a greater number of estimation strategies. They had a greater understanding of how estimation could be used (Neill, 2006). The key findings have been interpreted in terms of the literature to generate the following general assertions.

General Assertion 8: As the teachers had provided tasks where the students used estimation as the main computational choice, at the end of the study the students appeared to have developed a deeper understanding of computational estimation, learnt that they could use estimation as a computational choice in its own right and could solve estimation problems. Most of the teachers had also introduced a number of different estimation strategies in their teaching, and this appeared to be linked to the fact that the students realised that estimation was not just about rounding but involved a number of different strategies. As there was a statistically significant correlation between students using reasoned estimation strategies and students selecting the best estimate, it may be hypothesised that the increased students' awareness of estimation strategies increased the students' estimation performance.

General Assertion 9: Bob's teaching approaches, which developed estimation strategies and number sense within a community of learners (Yackel & Cobb, 1996) and incorporated estimation as a computational choice in its own right and as a tool to check the reasonableness of exact answers appears to have been particularly effective in enhancing students' estimation performance.

Due the complexity of the real classroom, it was only possible to speculate about the possible impacts of the professional learning on the teachers and students but these insights have been provided where they were deemed relevant. The relationships between these general findings are represented in Figure 7.4.

Figure 7.4 is a conceptual model based on the findings from the study. It recognises

that learning is a social activity and that the personal construction of knowledge occurs through engaging in this social activity (Ernest 1991). The teachers engaged at different levels in the reflective process (General Assertion 1) and this process developed the teachers' beliefs and PCK. Through engaging in these professional development workshops it appeared that the teachers came to believe that estimation as a component of number sense had value and that the estimation strategies were worthwhile. These new beliefs may have influenced the pedagogical approaches that they developed. These pedagogical approaches included tasks set in meaningful contexts, using estimation to judge the reasonableness of calculations and the explicit teaching of the strategies (General Assertion 2; General Assertion 3).

The teachers trialled these teaching approaches in their classrooms and these different teaching approaches appeared to influence the students' beliefs about the nature of mathematics and estimation to a certain extent. Whilst the students' beliefs about mathematics generally did not change, their beliefs about estimation did appear to broaden as they had much more positive perceptions of computational estimation at the end of the study. These new teaching approaches also appeared to improve the students' estimation performance. Their estimation language was enhanced, their use of reasoned estimation strategies increased and they were more likely to select the best estimate (General Assertion 5; General Assertion 6).

In the final chapter of the thesis, the general assertions that have been created in this chapter will form the basis for the conclusions drawn from this research and importantly answer the research questions that were established at the beginning of the study.

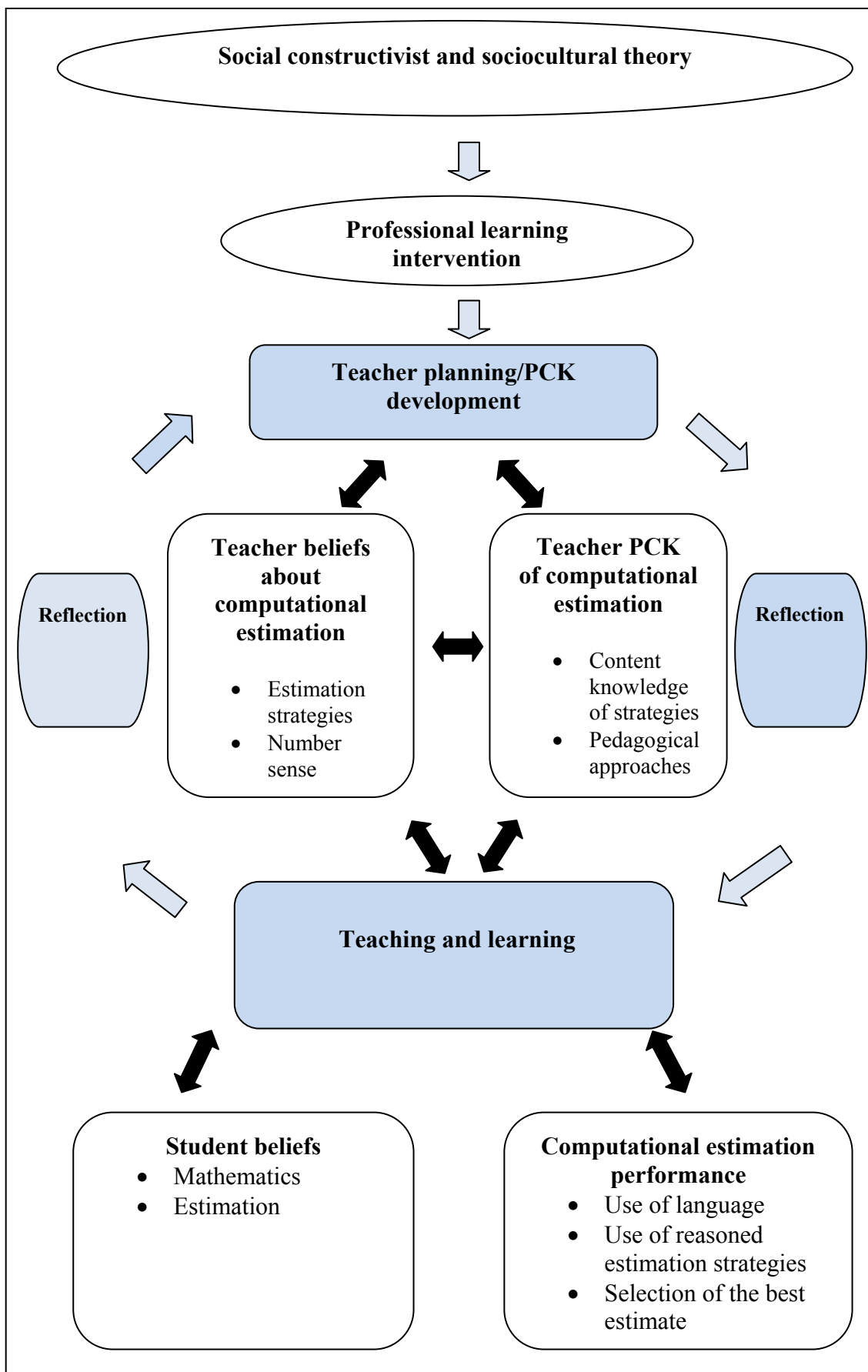


Figure 7.4: Conceptual framework for the study

CHAPTER 8: CONCLUSION AND IMPLICATIONS

Introduction

This study was set in the context of a professional learning intervention, which involved teachers from low fee independent metropolitan schools. The study investigated experienced teachers' engagement in a professional learning program focussed on teaching computational estimation as a component of number sense to Year 6 students.

Conclusion

The general assertions, created in Chapter 7, have been used to answer the research questions in this chapter and form the basis of the conclusions of the thesis.

Research question 1. How did the teachers' development of beliefs and pedagogical content knowledge about computational estimation inform their teaching approaches?

The professional learning intervention invited Year 6 teachers to consider how to teach computational estimation as a component of number sense. The intervention used professional development workshops combined with an action research approach. The participating teachers engaged at different levels in the professional learning intervention and this appeared to depend on how valuable they believed the professional development would be to enhance their practice and how similar the teachers believed the professional learning goals were to their beliefs about the goals of school mathematics (General Assertion 1). When considering how the professional learning intervention informed the teachers' practice it was decided to investigate specifically how their beliefs developed due to previous research stating that teachers often had negative perceptions of computational estimation. Previous research has also identified a link between sound PCK and effective teaching. Therefore it was also decided to focus on how the developed PCK impacted on the teaching of computational estimation.

At the beginning of the professional learning intervention, most of the teachers did not believe that estimation had an important place in the primary mathematics

curriculum due to their focus on teaching procedural algorithms and pressures. Through engaging in the year-long professional learning process, all of the teachers came to believe that teaching computational estimation as a component of number sense had value. A large amount of time was spent at the professional learning workshops discussing the different estimation strategies. The strategies discussed were: rounding, front-end loading, nice numbers, benchmarking, intuition, sample and range. By the end of the study most of the teachers believed that teaching the computational estimation strategies to Year 6 was worthwhile (General Assertion 2).

The PCK of the teachers also developed whilst being involved in the professional learning interventions. At the beginning of the intervention the teachers had little PCK of computational estimation which was to be expected with few available curriculum resources for teaching computational estimation as a component of number sense available. At the end of the intervention, the teachers were able to name and understand most of the computational estimation strategies. Most of the teachers also developed the pedagogical approaches of using the formal estimation strategy terms and using the strategies, setting estimation problems in meaningful contexts and providing students with opportunities to check the reasonableness of exact calculations (General Assertion 3).

The beliefs and PCK developed in the professional learning process appeared to shape the teachers' approach to teaching of computational estimation. Their beliefs in the value of estimation as a component of number sense are likely to have influenced their decision to provide learning tasks in meaningful contexts where estimation was the main computational choice. The meaningful context that the teachers selected included games, planning scenarios and some real world contexts that were outside the day-to-day experiences of the students. This belief in estimation as a component of number sense may have also caused the teachers to encourage students to make sense of their calculations and check the reasonableness of them using estimation. Generally, the teachers also believed that the estimation strategies had value for Year 6 and they introduced these to the students using the formal terms. The implementation of new PCK was impeded where the pragmatic goals of teaching in the real school context were not the same as the beliefs as to what was pedagogically appropriate in an ideal world (General Assertion 4).

Research question 2. How did the teaching approaches impact on students' beliefs about estimation, mathematical knowledge and their computational estimation abilities?

One component of the professional learning intervention was an action research process where the teachers' trialled teaching approaches that they thought were pedagogically appropriate back in their own classrooms. The study investigated how these new teaching approaches impacted upon the students' beliefs and knowledge of computational estimation. As this research was designed to record an authentic professional learning process involving a real classroom, the analysis was quite complex and it was only possible to provide insights into how the teaching approaches impacted on the students.

Overall, as a result of the professional learning intervention, there was an impressive increase in the students' estimation performance that was statistically significant. The intervention did not appear to impact upon students beliefs about mathematics although the approaches did appear to broaden the students' perceptions of estimation as an individual component of mathematics (General Assertion 7).

The students' overall beliefs about mathematics did not change in response to the intervention. At the end of the study, the students still perceived that mathematics was something that was done quickly and was about one exact answer (General Assertion 7). Their beliefs about estimation did appear to have changed. At the beginning, the students believed that estimation was a mathematical guess that involved rounding, whereas at the end they perceived that estimation involved a variety of strategies and could be a computational choice in its own right. The changed beliefs about estimation appeared to be in response to explicit teaching of a range of estimation strategies and using tasks set in meaningful contexts (General Assertion 7).

The teachers selected tasks set in meaningful contexts such as Fermi problems and planning real life scenarios. As students had little prior knowledge of computational estimation as a component of number sense, most of the teachers used a heavily scaffolded teaching approach to model for the students how estimation could be a computational choice in its own right. Most of the teachers also introduced a number of different estimation strategies in their teaching, initially by introducing the formal estimation terms through the use of word walls or posters and then by using these

estimation strategies in the tasks set in the meaningful contexts (General Assertion 3).

Through engaging in these tasks set in meaningful contexts, it appeared that, by the end of the study, the students had a deeper understanding of computational estimation, including that it could be a computational choice in its own right. This deeper understanding appeared to increase their ability to solve estimation problems (General Assertion 8). Particularly noteworthy improvements in their computational estimation performance were the increasing number of students who could select the best estimate when estimating with fractions with unlike denominators and the number of students who could select the best estimate when estimating the solution involving two-digit addition problems (General Assertion 6). Interestingly, despite situating many of the estimation tasks in meaningful contexts, the students' performance was still higher on the symbolic question than the problem set in context on the CET (General Assertion 6).

These teaching approaches also seem to have impacted on the students realising that estimation was not just about rounding but involved a number of different strategies. As there was a statistically significant correlation between students using reasoned estimation strategies and students selecting the best estimate, it may be hypothesised that the teachers increasing students' awareness of estimation strategies increased the students' estimation performance (General Assertion 8).

Due to the impressive increases in computational estimation performance and the use of estimation by the students in Bob's class, this research suggests that his combination of teaching approaches may be linked to the effective development of estimation and number sense. He created a community of learners that encouraged the students to discuss the mathematics with each other and justify their solutions (Yackel & Cobb, 1996). He then incorporated estimation into this classroom culture. Within this culture, he introduced the estimation strategies at the beginning of the study using a word wall so that the students and himself could refer to these. He set tasks in meaningful contexts for the students to solve where estimation was the main computational choice. As part of the classroom culture created by Bob, he also asked students to use estimation as a tool to check the reasonableness of exact answers (General Assertion 9).

Implications

There are certain implications that may be drawn from the outlined conclusions. These include implications for research, practice and teacher professional learning and these have been outlined below.

Implications for research

This study was a small-scale piece of case study research which built on previous assertions about the value of estimation and number sense (McIntosh et al., 1997; Silver, 1994) in the pursuit of investigating how estimation could be taught as a component of number sense. It had been asserted that more studies are needed to investigate how estimation as a component should be taught (McIntosh et al., 1997; Silver, 1994). As there have been so few research studies recently as to how computational estimation should be taught, this piece of research has only been able to add limited insights into how to teach computational estimation. There needs to be many more research efforts. Future research needs to probe more deeply into the discourse between the students and the teacher. Through this analysis it may be possible to identify the types of discourse that support students developing a deep understanding of how to use the computational estimation strategies in mathematical problems. Video analysis would be a very appropriate form of data collection that would allow for retrospective analysis of the discourse (Sfard, 2008). These types of studies could also reconsider the estimation strategies suggested in this research study and the age group at which they should be introduced. It would also be of interest to investigate how different types of contexts affect the types of teaching approaches that are pedagogically appropriate for students and if the strategies that were found to be appropriate in this research were applicable to other contexts.

Future directions could be to build on this small scale research where one age group was considered, to investigate the benefits of working in situations where there was a whole school approach to teaching computational estimation and also monitor students' progress over a number of years.

Implications for practice

This research has demonstrated the importance of student and teacher beliefs about

computational estimation and number sense (Assertion 4.1; Assertion 5.1; Assertion 6.1). Traditionally teachers have had a negative view of estimation (Dehaene, 1997) and this may have contributed to the neglect of this subject area. In this research, where the goals of school mathematics were different to the goals of teaching mathematics in the ideal world, this impeded the inclusion of computational estimation into the everyday curriculum (General Assertion 1). If there were to be any efforts to make computational estimation an integral part of the primary curriculum there would need to be a process where the vision created in curriculum documents was shared and understood by principals, mathematics educators, teachers and parents.

This research recommends focussing closely on Bob's teaching approach of computational estimation and it suggests that it may be a possible model for practice. He used a number of different teaching approaches and created a community of learners that valued estimation in all areas of mathematics. Bob often used the teaching approach of modelling how one estimation strategy could be used to solve a problem and then allowing the students to explore similar problems using strategies that they thought were worthwhile. This pedagogical approach may have resulted in improved computational estimation performance as Bob's students' computation estimation performance was very significant.

Implications for teacher professional learning

This research demonstrated that teachers were reluctant to change their present practice. Many of the teachers in this professional learning intervention predominantly taught procedural algorithms at the beginning of the professional learning intervention. Research had suggested that algorithms are ineffective in developing students' reasoning about computational estimation (McIntosh et al., 1997) so it was not one of the teaching approaches recommended at the professional learning workshops. For the teachers to change their approach of teaching procedurally, it required the teachers to consciously reflect on their present practice in comparison to their beliefs about teaching mathematics. If this research study had been organised as a one-off workshop, the teacher would have been given the support necessary to make fundamental changes in the classroom. An implication of this research therefore for professional learning is to recommend that where major paradigm shifts are part of the professional learning, then one-shot professional

learning programs are less likely to be successful.

This professional learning intervention used a combination of the action research process and providing workshops that developed the teacher content knowledge. At times the teachers engaged in mathematical tasks, acting as though they were students. This allowed teachers time to practise using this content knowledge before using it with their students. The workshops also provided the initial lesson plans for the teachers to adapt. This amount of support was greatly valued by all the teachers in the professional learning intervention. They were all busy teachers with many curriculum areas to cover and they found this type of detailed support advantageous for the teachers and they did not believe that it curtailed their teaching, as they were free to adapt the documents.

If the professional learning process were to be a learning process for the teachers it was important that the professional learning facilitator gained the respect of the teachers. This was difficult in the early stages of the professional learning intervention as there was little guidance available as to how to teach computational estimation. Using videos of teachers teaching students how to solve computational problems set in meaningful tasks would be very beneficial in the professional learning workshops. It would show the teachers how some teachers have approached the teaching of computational estimation and instill more confidence in the teachers in this approach.

The collegial aspect of the professional learning intervention was perceived by the teachers as the most beneficial aspect of the professional learning intervention. They found being able to share frustrations and successes extremely useful. This professional learning intervention brought together teachers from similar types of school and this appeared to facilitate discussion as teachers understood concerns and ideas that were related to their schools.

Final Conclusion

For many years, mathematics educators have been asserting that computational estimation has value in the primary curriculum (Neill, 2006; Reys & Bestgen, 1981; Silver, 1994) and individual researchers have produced findings in order to support its implementation into the curriculum (Bobis, 1991; Mack, 1988). It is perplexing

therefore, that computational estimation does not appear to be an integral component of the primary curriculum. This research showed that in one setting teachers were able to undertake a professional learning process and in this environment, most of the teachers were able to understand the strategies and develop pedagogical approaches to teach these strategies to their students. The Year 6 students involved in the study engaged in tasks to develop their understanding of computational estimation. Most of these students believed that estimation was something that made mathematics easier and they had positive perceptions of the estimation tasks.

If researchers, curriculum writers, parents, principals, mathematics educators and students can collaborate in a cohesive manner, then computational estimation may take its rightful place in the primary mathematics curriculum and students may gain the benefits from being able to use computational estimation and have an awareness of the variety of estimation strategies.

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APPENDICES

Appendix A

Excerpt from Audit trail journal

The research consultant at ECU made the good point that curriculum innovations are often top down and it is quite unusual for this to be happening at a grass roots level and therefore it is quite different from many other curriculum reforms.(July 2009)

Must read through this article

[Models of continuing professional development: a framework for analysis](#)
[Journal of In-service Education](#)

Volume 31 Number 2, 2005; Pages 235–250

It has occurred to me that the students have been very engaged in some of these activities that I have been observing. I must go through the final interview for students and add a question about their engagement as one thing I am noticing is their concentrated focus. I want to find out about their perceptions on this factor etc 11/8/2009

The professional learning aspect - development

The relationship between myself and the other research participants in this research is fluid. I have been considering lately, about the idea of professional learning being where nobody should be offering leadership and instead it should be some sort of cooperative. Instead I see this research where there is strong leadership but who takes the leadership role is dynamic i.e., sometimes in discussion the teachers will take this role reflecting on their practice on offering this insight to the whole group, including the Researcher, or the facilitator guiding the group, taking the times to guide the group to explore certain principles and the group not getting distracted. The presentation of research findings to the group is presented as a catalyst for the teachers' own practice. This is a genuine activity in that there are hardly any curriculum resources available for the teaching of computational estimation. This lack of resources made my position difficult at the beginning of the research. I had flagged a topic which there is nothing written about and not many people in Western Australian primary schools are involved in. This made everyone, myself included a little nervous. Yet as the time went on and students in the different classes could understand the concepts my confidence grew. Of course it would have been nice to have had this confidence at the beginning of the PL and conveyed this to the teachers but until one is prepared to step out and try things genuine conviction cannot be attained. (7/9/2009)

Appendix B

Final teacher interview with Bob

Date 12/11/2009

Time :10.20 am

At the teachers' school

Researcher: I found the focus group interesting and they were all talking about the benefits when xxxx said I prefer exact answers I want to be precise.

Bob: He wants to be precise.

Researcher: 'It's black and white' well that to me was really valuable, he said all that I have done in the past is exact answers

Bob: He is such an earnest little character, a few like him and the others in that they want to be precise and all of a sudden they are saying before you go to that, come up with an approximation so that before you come up with that you can see whether it is in the realms of possibility or not .

Bob: It was funny, doing some of the bits and pieces and talking to them about things reminded me, you would have seen the seven up series from the UK with the British Researchers started in the late 40,50 s where they went back

Researcher: Yeah where they went back//

Bob: When they were 14,21,28,35,42,49 and they will be heading towards 56 it was done a while ago - a lot of them have dropped out but it was really interesting to see their progression and how some of them, you know, one of them went on to be an astrophysicist of Berkeley or something, you know all sort of carry on

Researcher: You can see it as a catalyst even now

Bob: Having done some of the child psych stuff, I wonder

Researcher: You'll see where they end up

Researcher: (Looking at the tests) .With the tests CET, they seemed to have more of an idea than they did in the beginning

Bob: They have a lot more idea. If you say to them now "okay look before we start let's just come up with an approximation of where we should end up "and they are much happier to do it, they can see now the benefit. If you had seen it before 'Oh I don't need to do that, ,I'll just work it out , 'who would make an error and have no idea they were actually wrong they'd be calling out answers – I'd be looking at "what " "Well that's what I got " "well didn't you estimate first " "no" and so now they are going off and doing it .

Researcher: Yeah

Bob: And now they feel almost out of place if they are the one not.

Researcher: It seems that you have been quite flexible in what you have been able to do in the maths this year - Is there an official policy?

Bob: We come up with our program as long as we teach certain concepts throughout the year, meet certain end points, how you approach it and how you come around it .It is fairly flexible and having the top group I like to set them challenges so instead of saying "this is how this works" well if I do this and I get this answer how did I get there? Where are we going to end up and let them explore with things. The 44 problems is still at the back, we can revisit them every now and again and there is some that are really clued up with all sorts of number strategies , that don't come in until the year 10 curriculum but these guys are looking it having a play , some of them manipulate it, some say 'no too hard for me' or not my thing but those that are, we can stretch them and have some fun

Researcher: And do you have a specific maths curriculum leader in school or not?

Bob: Emm that's evolving at the moment and at the moment and at this point in time it looks like it might be becoming me .

Researcher : Okay yeah

Bob: But it's evolving. We have had different things in place with different teachers, as they go to persons for different areas but with the change in the school structure it's been put in place at the moment

things are going to develop a little bit more .

Researcher : Great I just wondered about that you know. Do you feel that you have a lot of parental expectations at this school?

Bob: I think parents at any school have expectations and I don't think we are any different from any other school .Em we may be not as bad as if you were at xxxx College or xxxx or something like that however we do have a number of parents who will push for the best for the kids . NAPLAN results were out the other day and we had several parents saying "little Jonny got this and he is sitting here what do I do to get him into the top of the group" and it's really hard to explain to them that it is one test on one day. It gives us a snap shot, it shows us where the cohort is - as a whole sits but as for showing us where little Johnny it doesn't tell us huge amounts. He could have had a bad day, some of the ESL kids don't understand a word the wording of the question so their marks drop, others manage to just pick it . They are multiple choice and they just guess it correctly and who knows.

Researcher: So you would say concerned parents but not someone who is paying 10 grand a term ?

Researcher: Do you feel your priorities in your teaching role this year - have you been able to be focussed on maths or do you think there are lots of other things to worry about ?

Bob: I think, now I think my focus is about right as I try and balance everything out, try and be flexible with what you do. If something is going really well I try and let it run for longer, if it's not curtail it and go somewhere else em reevaluate what you are doing .

Researcher: So you haven't found you have had lots of other issues other than the teaching - that's what I was getting at

Bob: No

Researcher: The professional learning has had the sharing of ideas with colleagues, the reading of published research, games and activities to work in the classroom and suggested lesson plans and you were able to go back and reflect on those. How did you find each of those , which was the most useful and why ?

Bob: The sharing of the ideas up at Joondalup were brilliant, some of the conversation that came out of those was really, really valuable. Seeing the stuff that we do, problems that you have, everybody is having the same dramas and also to talk to some of the others. I have got the top maths group here but just how good are they ? You talk to the others and say I am doing this with mine and you see the looks on their faces they say “ you what , I couldn’t do that with mine” . It shows I have got some really strong mathematicians in my group who are really highly capable students and that is a bit of a buzz. Like some of the Fermi numbers and the Fibonnacci sequence.

Researcher : Yeh that is right

Bob: And Goldbach with these guys and they get a real kick out of it and they have shown their parents and I have had their parents come in and say huh I never understood it and now “Wilson” has come home and explained it to me and now I get it – it’s really good (When I went out to school I observed how he was teaching the difference between prime and composite numbers through a problem solving approach).

Researcher: Yeah

Bob: It’s really good

Researcher: And you wouldn’t have known that but for talking to other people and them sharing the problems they are having.

Bob: Sometimes you are able to offer them a bit of advice and say I have done that, I found this worked and give it a try.

Researcher: Yeah

Bob: And then the next time you see them they say I tried that and it worked, thanks for that so that was good

Bob: Some of the publications I found I am never a technical reader em some of them I found them a little heavy going, a little out there and you are looking at some of them going you’re have taken 10 pages to say what you could have said in a page.

Researcher: Yes (I laugh)

Bob: Come on I don't have time for all of this and I think this is with a lot of academics they just like the sound of their own voice someone says you have to write a 10 thousand word journal article and they have 1 thousand words that explains it all, so they just pad it out.

Researcher: Yeh and it is what you find useful - so that would be less useful you would think. You think that sharing ideas was the most important thing .

Bob: The suggested lesson plans came in handy cos they gave you a jumping off point. Saying we'll start here and we will go and explore this they are always handy to have

Researcher: Actually someone else said that having a starting off point //

Bob: It might be different for a secondary maths teacher but for a primary maths teacher with there is time factor and resources, its huge that's the biggest thing. If someone has the sample lesson, they can run with it – adapt it and play with it that's right run with it. Whereas just give the concept you think where the hell do I go with this?

Researcher: How could the professional learning have supported you further ?

Bob: At times, I thought it would be great to go back for another session. They seemed a little bit far apart you know you did something and you thought we have got 10 weeks to get together. You sort of forget what we are discussing yeh maybe more sessions, maybe setting up a professional learning network somewhere, a blog for people to access or a link that we can tap into em would have been handy.

Researcher : You are quite technically literate, there you have people that are at different levels you can see it you have to appreciate to use it .

Bob: Some of those web 2 objects are great to play with but unless you actually look around and you have got someone to drive it. I have tried here get class blogs going and other bits and pieces, even the network administrators put that many hurdles in, you give up.

Researcher: Has it been very different how you taught estimation last year to how you taught it this year ?

Bob: I think I have used it a lot more this year. There is a week or two and then you leave it and you say "make sure you estimate before you do

it “ and that is all you do it . Whereas there has been much more language in it Are you chunking ? Are you front end loading ? Are you rounding ? What are you doing? And building it in to every maths lesson rather than estimation strategies as part of your number work.

Researcher: Yeah – which strategies do you think are most worth teaching to the students ?

Bob: Front end loading, I think was really a valuable one. The idea of chunking (nice numbers) was good em, just rounding in general. The ability to look at numbers and say its roughly 1000 or its roughly 500 or if I round it to 10 it is this or if I round it to 100 it's this - just they see where number parts come in that was interesting .

Researcher: Even just to make it easier

Bob: We can come up with an approximation and we can come up with four different approximations of adding up four numbers in the thousands depending on where you round them to.

Researcher: So you didn't have to get too technical with the words?

Bob: As long as they got some part of it, okay we are on the road and then you keep progressing and working at what it going on

Researcher: The most common strategy is rounding what strategies are you now aware of ?

Bob: Chunking, Front-end loading, emm long pause .Sampling which is really quite interesting - we are actually doing some stuff at the moment in chance and data where they are writing a survey saying is this a census or is this a sample? great what is the difference and with graphing, looking at discrete and indiscrete data graphing in that and getting them to see that using different data because the data fits not because that you will use that graph not because it looks pretty but because it fits. I am trying to convince them that plotting the height of vegetables as they grow is not a bar graph “Ah yes but when I measured it on this date it was this but when I measured it yesterday it was this “. Well in between, something happened. It didn't just go up three cm over night. It is not like counting people who like chocolate m and ms versus strawberry m and ms.

Researcher : And it is that real world idea that it has that real world purpose //

Bob: Just getting them to see that has been useful, and through the estimation, through the graphing. It has all opened their eyes to okay this is real and it does have a benefit and it stops some of the “ why are we learning this?” questions, why do we have to learn how to do a graph, why do I have to learn how to estimate?

Researcher: A child in Mr Clarke year 7 class wanted to find an estimated answer for the question

$$21\ 014 + 2811 + 19112 =$$

What estimation strategy would you have used to add it up?

Bob: I would have front end loading it that one. The simple thing is to front end load it and if you had a quick look, you'd realise 2800 make that 3 21 +3+19 bang 43 thousand and you are done.

Researcher: Ms Fot's class were posed with the question: Can you give me a quick estimated answer to the question $9/10 + 8/9 =$

Bob: You could almost use a visual strategy for that and go 9/10 is almost a whole and 8/9 is almost a whole (he has forgotten the word benchmarking) almost 2 wholes which makes almost 2 wholes.

Researcher: How would you rate your awareness of estimation strategies at the beginning compared with at the end on a scale of one to five?

Bob: I tended to use a lot of estimating myself any way, just the stuff that I do but I think that I am now aware of what the strategies are – you were using the strategies before but you hadn't given the strategy a name

Researcher: Yeah

Bob: Whereas now you can put a label to put on it , okay now have done some front end loading or rounding or chunking (nice numbers) or I have done some whatever .

Researcher: So in a way it is a bit difficult to put it in a scale . It's not been none to a lot it, is sort of a change of how you have looked at it .

Bob: It's been a lot to a lot but now there are labels

Researcher: Good .You have said before [in this interview] that the groups' [class's] perceptions of maths has changed its wider than they thought before

Bob: Yeah they are more open to try things, they just love a challenge and they love to be stretched. We have been doing some number pattern work and a few of them spent hours to come up with these fiendish number patterns to test their mates. I came up with 20 from $+1+1$ to $+1-2$ sort of things we went into a whole range of sequences, so they have now changed and come up with a whole range of square roots and cube roots and also sorts of carry on and arguing with each other cos it doesn't work and yes it does you mean that was a 3 and not an 8 no wonder your pattern didn't work

Researcher: So it is their view of maths that it becoming broader ?

Bob: Broader – they are enjoying it more, developing the ability to become critically honest with each other as to how their work looks but in a positive sense. Before there was an almost one upmanship whereas now they doing it almost to test their mates but have fun with it, but if they can't get it there is no "silly billy", its "I did this, can you see what I have done ?"

"Now ah now I get it, let me try me see if I can try". They are more willing to try to help each other and push each other along rather than get ahead of this pack.

Researcher: Yeah when you have the problem solving, it leads you not just to be ahead of the pack, not just 18/20 rather than 12/20.

Bob: They just want to support each other more, which is a nice positive to come out of it

Researcher: And you use humour quite a lot?

Bob: You have got to have fun with it, you have got to use humour. You make the stories for your problems humorous. You use their names and their situations as much as possible to make it real or applicable to them.

Researcher : And I notice at a few times at the beginning with the computational estimation work you gave them quite a clear idea of how you would

solve this problem – now I am beginning to think as I get to the end of it [this research intervention] that that for people at the beginning of something or for people who haven't got an idea [of how to solve a mathematical problem] that would be a really beneficial teaching tool - what do you think about that ?

Bob: You need to give, as with any learning situation you need to provide a scaffold to the learner to follow it in the first instance. If you just say to the learner “get on with it” they can spend ages just thinking what the problem is asking, whereas if you break it down for them into some sections and you say to them tackle this bit first, then you can come up with something for this, then you can try to work out how it relates to this part, then onto this part. You have given them a pathway, whereas if you don't have that, especially with more complex problems, they have got no idea.

Researcher: Yeah its fine if they have got some idea and it's almost like a practice situation but maybe if they have got no idea the teacher has to do that ?

Bob: If we are teaching them double digit multiplication and you just went from single digit to double digit then remember single digit and its 5×5 and you get 25. Well that one its 25×25 – you still do your 5×5 now we go from here, now we are going to multiply by tens and you break it down and put it into the multiplication square, you can write it down as four separate algorithms and then combine it and you give them a number of strategies and we say all of this - that is what we are doing in here.

Researcher: Yeah

Bob: It's like long division, short division I still show it to mine. I say to them when we do short division that is what we do in our head written down. It makes it so much easier. Then if you struggle with it keep going with long division either in your head, to write it down

Researcher: Yeah it is just an aid to you.

Bob: Nobody is going to look at you and say to you what are you doing ? It is just an aid to you, a tool .

Researcher: And even with open problem solving, children need to be given

guidance on how to set work out cos that can really confuse you .

Bob: Yeh, if you haven't followed a logical pattern. You have still got a pile of numbers on a page and you are going "I can't see your solution". Set it out in some sort of logical order so that if you do make an error you can find it so can we can find your solution. Some children have a jumble of numbers and there is this thing circled in the middle of the page and that is my answer.

Researcher: And then it is hard to follow.

Bob: Yes it is hard for me to follow and hard for you to follow .

Researcher: Is there anything else you would like to say?

Bob: The whole unit, the sessions at ECU – everything it has been great.

Appendix C

Focus group : Sandilands School

Time:12 pm

The students were friendly and polite in fact they were extremely polite and my first impression is that as a group, they were all very keen to please and a little unworldly

Interviewer: Paula Mildenhall

Names: Child 04,Child 23, Child 12, Child 22, Child 19,

Researcher: Question 1. Your friend has a different answer to you on a maths problem, what do you do and why?

Child 23: I could ask him. Why do you have a different answer, how did you do it?

Child 22: I would go over it with the teacher and I would like double-check it, they may have incorrectly marked that or maybe they were cheating.

Child19: You would do it a different way to see if it still was the same

Researcher: Like you would do it a different way to check it

Child 19: Yes

Researcher: Question 2. An alien lands on earth and wanted to know what mathematics was (show a cartoon of an alien). What would you tell him?

Child 23: I would tell him it's problem solving with numbers

Child 12: We would teach him how to add, subtract, divide and multiply

Child 19: If you met him, you could tell him all about it as an example on a piece of paper.

Researcher: And what would you write on the piece of paper?

Child19: I would write a sum like $2 + 2$

Researcher: Question 3. What is estimation? Can you write what you think.

Child 12: To have a guess at, something like a question

Child 23: For e.g $3 \times 4 = ? 12$, you would have to guess or round off to where or what the answer might be

Child 04: [wrote nothing]

Child19: Estimation is similar to guessing, you estimate how long a table is if you don't have a ruler .

Child22: Estimation is a no working out guess. The only thing you do is you do a quick round off.

Researcher: Question 4: Bill had to work out an estimated answer to $43 + 28$ in his head . He said that the answer was about 70. How did he calculate this ?

Child 22: He quickly did 20 add 40 off by heart he knew that and then the 3 add 8 he did 70 he rounded off which is 71 so he rounded off to 70

Child19: He round the 28 and 43 to the nearest 10

Researcher: And what would that be?

Child19: 40 add 20, which is 60 and 3 add 8 which equals 11 which equals 71 but he just thought 3 add 8 equals 10 and thought of it as 70.
Child 23: He would round 28 to 30 and then 43 to 40 and then add 40 to 30 to make 70.

Researcher: Question 5. Bill then worked out an estimated answer to $11/12 + 7/8$ in his head. He gave one of these answers about $\frac{1}{2}$ or $18/20$. What was his answer how did he work it out?

Child12: About 18/20s

Researcher: Why ?

Child 12: Not quite sure

Child19: Add the denominator as 20 add the numerator as 18

Researcher: That's an idea because we denominator is important. Any other ideas

Child 22: Not sure, I am working it out now about 18/20

Researcher: Okay we will leave it there.

Researcher: Question 6. The shopping bill showed the amount below and I paid with the money in my hand (20) $\$5.45 + \$4.80 + \$6.15 + \5.16 sho this receipt on a poster for 30 seconds .Look at the receipt for 30 seconds and then tell me if I had enough money by estimating? How did you work out your estimate?

Child 19: I added the entire dollar and they were 20 so there were cents so it was over .

Child 22: I added up all the dollars first and then that equalled 19, 20 and then was also cents left over so that got me convinced that it would go over 20 dollars.

Child 23: I added all the dollars, 5 plus 5 equals 10 and then four plus 6 equal 10 so 10 plus 10 equals 20. So then were was cents left over so it would obviously be over 20 dollars .

Researcher: That's great.

Researcher: Question 7: Andrea got $5/11$ in a mathematics test. In your head calculate what would her mark be as an approximate percentage? How did you get this percentage?

Child 23: 40-50 %

Child 12: 50-60 %

Child 23: 50 % to 110 % and I minused both of them to 40 %

Child 22: $5/11$ is about 50 of 110% so I minused both of them by 10 and I got to 40 %

Child 23: I thought if 11 was 10 so 5 that's half so that would be about 50 % and then I thought I would have to take one of cos it was less than 50.

Child 22: I would say 45 % or even a tiny bit higher 'cos in approximate calculation it is more in the middle and when $5/11$ s which is like 110 take away 50 makes 40, I would say 40 or 45

Researcher: Good

Researcher: Question 8. About how many children are in your school ? How can you work this out mentally?

Child 12: going to the office, counting all the names 400 to 20 hundred and 90

Child 23: approximately estimate how many are in the class and then I would find out how many classes there are and then I would add all the classes up.

Researcher: Question 9. Bill sat working on one mathematics question for 15 minutes . Do you think he is clever?

Group: No said in unison

- Child 12: Have to work it out
- Child 22: If he was stuck on it for 5 minutes, he maybe should have made an approximate guess, of if it is an estimation, guess, round it ,he should have went on and then recalculated to make his estimation a bit more accurate .
- Child 19: If he took 15 minutes. It should depend on the year but normal questions should take about 2/3 minutes
- Child 23: He should have moved on to another question or done an estimation like Child 22 said but if that was a normal question, he would be quite bad at maths cos that would be quite easy to work out for some people .

Researcher: Question 10. Do you think that McDonalds could benefit from employing a mathematician?

- Child 22: Maybe in the farms they would need to count how many chickens they would need to have and if they need say chicken nuggets, how much chicken nuggets they can make with that certain amount of chickens.
- Child 19: The drive through people they have to work out how much dollars they have and how much change they need
- Child 22: Well they have computers to do that
- Child 19: Well what if they don't ?
- Child 23: Well, like Child 22 said, they could approximate how many chickens they would need or how many nuggets they will need. At the counter he will do a quick estimation in his brain how much a burger costs. He could estimate \$10 and if the person paid \$2- he could estimate about \$10 change.

Appendix D

Reflections from Professional learning workshop by Researcher

Date : 18th February 2009

Location:University

This was the first professional learning day. It was held in the library in the large meeting room. This room was chosen because it is quite dynamic in the decor and I wanted to motivate the teachers. It is a very trendy room with bright colours and lots of glass.

All the teachers were either on time or early and were friendly and enthusiastic. The room was very modern and pleasant to be in. They had coffee and lunch provided . We had various configurations and they sat at times with whole class discussions and at other times in two groups. xx acted as an observer/ critical friend .

Logically the teachers are going to teach estimation how they teach mathematics. So it was interesting at the beginning to hear how the felt about Mathematics ...

At this point at the beginning of the professional learning workshop, I wanted to set up how I wanted the group to work. Talking to all the teachers as we sat around the table (I have Power Points to support the discussion):

Researcher: In your show bag there is booklet that we are going to follow (I showed the booklet to everyone). Because it is collaborative I thought of setting up a google group or a web page but because it is so few of us, only 6 of you, it is just as easy to email so like that 44 suggestion if everybody if happy just to send off ideas with their email. What I wanted to start with today is to say that what I have are just ideas. I have been on the teaching journey, I have been hearing about all of yours. It is amazing how you try things - so the things I am presenting today at not a fait a compli. I really wanted it to be collaborative and to share ideas - I heard the grumpy old woman on just before Christmas and I just finished this Power Point (laugh) and the grumpy old woman was saying "if I hear one more thing about synergy I am going to bash that person. I thought oh no I nearly added on here sorry if you watch grumpy old women. On the other hand I think it s really lovely when people come from different perspectives and then work with a bit of synergy and you may have seen it a hundred times (show picture of the women with 2 faces) before but if you haven't could you a look it and write what you see

Appendix E

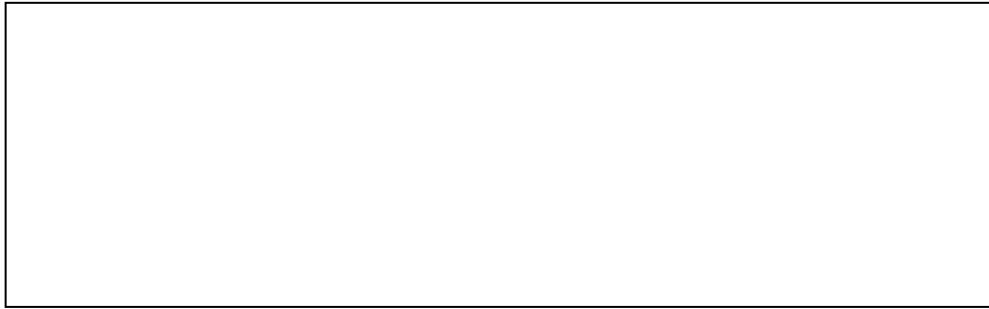
Excerpt from professional learning handbook

This course is about working with the expertise of teachers and academic Researchers in a collaborative manner. This course is designed to provide you with the latest insights from research, which will allow you to reflect on your teaching so that you may be able to teach more effectively. Your insights are valuable. Even if you have a different perspective to the research you come with great expertise at managing the complex activity of teaching. This professional learning is about a collaborative effort to explore different teaching approaches. At times we will have different perspectives but every perspective is worthy of being listened to and respected. Often, if different perspectives are fully embraced, it can create an amazing synergy - this is the aim of this project. Stephen Covey is a great proponent of this concept of synergy. In order to illustrate this point in his courses he often encourages his participant to look at the picture of this woman, which was first drawn in the 1800's



Picture source Young Girl-Old Woman Illusion." From [MathWorld](http://mathworld.wolfram.com/YoungGirl-OldWomanIllusion.html)--A Wolfram Web Resource. <http://mathworld.wolfram.com/YoungGirl-OldWomanIllusion.html>

What do you see?



This picture shows how important it is to open our minds to different viewpoints. I hope we all manage to open our minds and work with synergy in this program.

Throughout this professional learning program as you reflect you will possibly feel that you would like to spend time developing certain areas of your mathematics teaching of estimation and number sense in these sessions. It may be something about content or related to pedagogy. Please make these thoughts known to myself. This may include areas such as the benefits of enhancing your pedagogy....

Appendix F

Term 3 Unit: Fermi Estimation Problems		Curriculum links: Number 8.4 'estimate sums and products without prompting'	
Learning objectives and Children's learning outcomes Most children will learn to: Conduct an extended investigation Create a sample using estimation strategies Use multiplication with numbers greater than 2 digits .	Vocabulary	Warm up activities Bench mark development activities Estimating with fractions games More than less than game	Building on prior learning Check that children can already: Understand the concept of $\frac{1}{2}$ Understand the concept of multiplication (you may notice as they go that they need practice in some areas)
Lesson	Lesson focus, teaching notes and resources		Assessment for learning
Lesson 1	Warm up teaching games Intro: Sometimes when estimating we have to use an estimate of some familiar knowledge to work out some unfamiliar knowledge. Explain that there are some problems called Fermi problems Provide background to Fermi problems. Firstly lets solve this one together Ask how many students could stand in our classroom. ? Pool different ways of solving it. Suggest or allow		Students are able to use a sample rounding strategy to calculate and estimation of a quantity

	<p>students to share the idea of a sample .Create the sample. I.e. One square metre has six people in it. How many square metres in the room, in groups solve this? Ask one group to share their solution. (weaker students could use the calculator for the multiplication – it will take nothing away from estimating because we have to) Provide picture of an Apple orchard. Using a grid method calculate how many tree there are Then ask how much electronic technology time does the class use in a year What sort of electronic time will be counted (ie computer wii Nintendo emailing i.e. at school and at home Identify what our unit of measure is going to be : time spent in $\frac{1}{2}$ hour or one-hour blocks estimates i.e. 20 min = $\frac{1}{2}$ hour? This will use benchmarking of fractions –use benchmarking fractions games to support this Identify how we are going to cope with the amount varying: The amount of time spent on tech. will vary from day to day. Fill in the attached chart for a week Summerise the task ahead and what we have learnt today. Remind students of what a bench mark strategy is . Add this name to the word wall .</p>	
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<p>Lesson 2</p>	<p>One week later Students use their information in their groups to work out their calculations. They will work in groups of four. Encourage them to share results of electronic technology usage to enable them to work out how much the class will use in the year. To consider a week and then multiply by 50 for the year .</p> <p>How will we know if the answers we calculate are reasonable? Conduct calculations Check this data with census data provided (see MAWA link an worksheets below)</p>	
<p>Lesson 3</p> <ol style="list-style-type: none"> 1. WHAT IS THE PROBLEM? 2. ESTIMATE/ EXACT 3. Calculate. 4. CHECK 	<p>Read counting on Frank. Now that the students have the idea of Fermi problems they can pick one of their own and try and solve it. As Fermi problems are not about being accurate use estimates which should make it relatively easy to work out .Working in groups of four pick one of the following In groups of four: How many cups of water are there in a bath tub? How much paper does the school use in a week How many blades of grass on the school oval? How many bricks are there in the school?</p>	

<p>5. SHARE</p>	<p>How many kg of toothpaste have you squeezed out in your lifetime? (White , 2007) How long would it take to drive to the moon if you could? What distance will a ballpoint pen write? How many heartbeats will your heart make in a lifetime? Follow the guide in the worksheet to solve the problem</p>		
<p>Lesson 4</p>	<p>Finish calculations and reports and share with the class in turn. Vote for the best group</p>		
<p>Term 3 Unit: Planning and costing a school class outing –It’s very easy :It’s a walk in the park</p>		<p>W.A. Curriculum links: Number 8.4 Estimates sums and products without prompting or support to multiples of ten and can give upper and lower bounds on their estimates</p>	
<p>Learning objectives and Children's learning outcomes</p> <p>Most children will learn to: undertake computational estimation using addition and multiplication using numbers</p>	<p>Vocabulary</p> <p>Estimation strategies</p>	<p>Starting with what we know</p> <ol style="list-style-type: none"> 1. Fast facts p.39 Card capers (Paul Swan) 2. Discard card game 	<p>Building on prior learning</p>

with decimal fractions -			
Lesson	Lesson focus, teaching notes	Resources	Assessment
Lesson 1	<p>Planning a barbeque in a park within walking distance to your school</p> <p><i>You estimate because either you want to or you have to. Here you estimate because you have to. Estimation is often needed when you have to i.e. you are planning and you don't have the exact information.</i></p> <p>Explain the proposed visit and the parameters of the problem. Students will present their proposal in teams using a paper poster or Power Point. There is a booklet to support them. Brainstorm with students what the problem is, what they will need to work out, and how they will share the information. Break the extended task down into sections.</p> <p>Section 1: Which park to go to Section 2: How long the trip will take Section 3: What food to take</p> <p>Section 1</p> <p>Students need work out which park they wish to visit. Model for children how to create criteria to judge the park i.e.</p> <ul style="list-style-type: none"> • Play equipment • Distance • Catering /Toilets 	<p>Google map</p> <p>Power Point or poster</p> <p>Booklet</p>	

	<p>Complete criteria work sheet. They need to pick 3 parks using their knowledge in the group. If this was done over more than one day, they could always actually visit the park.</p> <p>Section 2</p> <p>The students will need to look at a map and estimate how long it will take to get there; once they have decided which park to go to. Use the Google map provided or look on Google maps. Students may work out how long it takes them to walk 1 km or work on an average of 4km/ hour. Check their approximations.</p> <p>Students complete itinerary – estimating times for eating etc .</p>		
<p>Lesson 3</p>	<p>Section 3</p> <p>Students will need to work out how much food they will need. Show photo pack (available as a Power Point or digital photos) with receipts. Ask “how many will be needed for the class?”</p> <p>Bring in strategies i.e. front end loading , compatible rounding, range strategy</p> <p>May use worksheet on buying items from the supermarket as practice.</p> <p>Show item for bbqs:</p>	<p>Power Point and photos</p>	<p>Students show that they can work with simple rates in estimation context</p>

	<p>Each student will eat one sausage + one hot dog roll + one juice box + one apple</p> <p>Show items on Power Point or work sheet</p> <p>Students need to present total cost of food as an estimate and price per class member</p> <p>Put all of this information:</p> <p>Section 1: Where to go</p> <p>Section 2: Itinerary</p> <p>Section 3: Food costs</p> <p>on to a poster or Power Point ready for the next lesson</p>		
Lesson 4	<p>Go around and listen to each of the presentation.</p> <p>They will have put in their calculations for checking.</p> <p>.</p> <p>The group will vote on which trip to follow</p>		
Lesson 5	<p>(Optional) Prepare and actually go on the trip. Record how long it took, how many sausages were eaten etc. (Maybe</p>		

	<p>take photos of the trip). On return have a class discussion about how valuable and useful the estimates were.</p> <p>Compare with estimates. Complete the table to explain the estimates you used. Were the estimates valuable ?</p>		
Lesson 6	<p>Extension math's project: travel agent project using Australian dollars for all calculation and only estimates required).</p>		

	<p>You are a travel agent. A customer requests that you plan a holiday for 2. He gives you the following requirements.</p> <p>You will have a total budget of \$50 000</p> <p>The holiday is for 2 adults</p> <p>The couple must be away for exactly 4 weeks</p> <p>They must visit at least 5 countries</p> <p>You will need to work in a group of 3</p> <p>As this is only plan and the exact prices will change you need only present estimates especially as they may not pick this holiday and you do not want to waste time on exact calculations .</p>	
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Appendix G

3rd Professional learning workshop

Date: 29/7/2009

Time 8.30- 3.00

Excerpt from critical friends notes

Helen: Personally enjoyed teaching it[estimation] – not just a maths lesson everything all at once.

[The lessons have] Lots of energy talk and fun - kids love it

The kids are;

Asking more questions, wanting to know how to do certain processes, all having a go

Area + volume, using volume + 100s and +1000s (The activity that was suggested in Professional learning workshop 2 where MAB's used to multiply by 100 and 1000).

Very concrete, knowledge /examples making it relevant in their experiences. Kids don't have the language though .

General comments

Difficulty to get kids to estimate:

Some overlap with checking as there is a rush to finish and to move onto something fun (spare time etc) as (a) value placed on fast finishing (b) difficulty in seeing own errors (e.g. editing)

Una - thought that students like knowing the names of the different estimation strategies - but do get a bit bogged with the terminology, change to symbols

Time, tension with trying to fit everything in. Balancing the teaching of maths - processes vs. problem solving, normal maths vs. estimation work

Guest facilitator [flagged] long- term planning important

Teachers [in professional learning] realising the estimation can be reinforced and revisited in the course of other maths work - doesn't need to be taught separately [integration]

Students enjoy the activities, though some teachers found that it was important to make the context relevant (tricky differences between what teachers think are relevant and what is relevant) weaker students need very concrete examples.

Risk taking, [the teachers trying to] create a classroom culture of having a go, that

it's okay to be wrong, showing that teachers even get it wrong sometimes. Fixed answers vs no fixed answer, tricky for some kids as they want a correct answer but for others it frees them up to have a go, not fixated on getting a correct answer .

Assessing learning in area of estimation difficult .

Appendix H

Wendy class observation excerpt

Date

Time: 1.30

Observation and Transcription: Paula Mildenhall

Overall impression

The class configuration has changed and it is now in a u shape - the class seems to be much more of a community of learners now as they are facing the front and each other and are able to talk to each other easily. One thing that seems to have facilitated this is the interactive white board. Another improvement is the installations of curtains, which allow the image on the IWB to be much darker and clearer. The only the down side is the hanging pictures which make it difficult for a lot of the children to see.

Reflection

The configuration appears to support whole class teaching. In this way, the IWB seems to have really influenced her teaching in that she is interacting with them as a whole class but maybe not as much in groups with them talking to each other. The children seem to have good sense of the strategies. They are able to use them very well.

Observation of lesson

The teacher begins the lesson talking to everyone. She looks much more confident than last time.

Teacher: Does anyone remember any of the [estimation] strategies that we were talking about?

Child 4: Eh, rounding

Child 8: Benchmarking

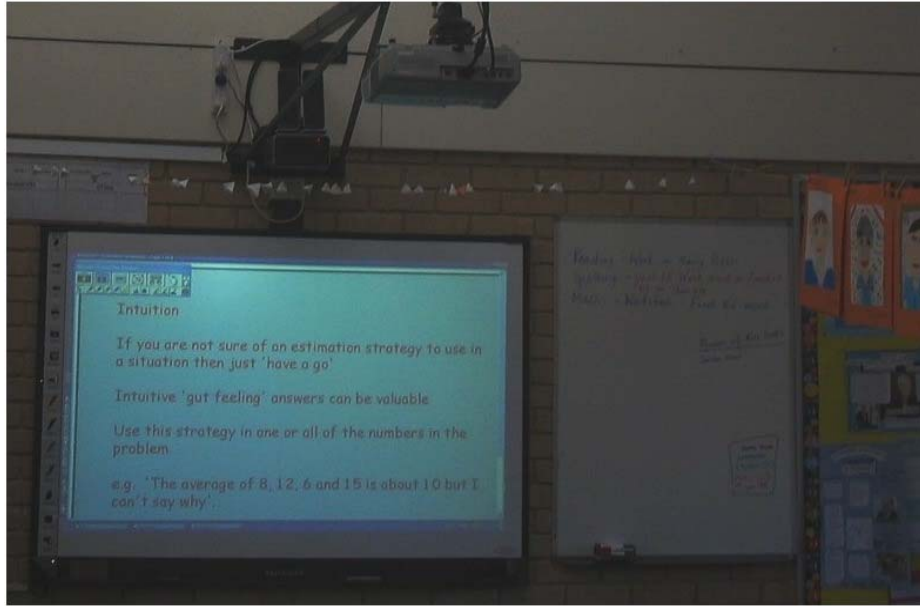
Child 2: Ranging and nice numbers

Child 11: Intuition

Child 30: Front-end loading

Teacher: Now we are just going to have a look and go through them again and then we are going to do a couple of activities to see which strategies you are using .

The teacher puts the explanations of the strategies up on the IW board. These are the posters that were provided by the Researcher and presented electronically.



Teacher: Right rounding, this is one that we have used most often - you use this usually to the nearest 10, sometimes you round up, sometimes you round down. A few of you are having trouble when we round to one decimal place and we need to go through it. Who feels confident with their rounding now, who needs help when it says to one decimal place?

A few hands go up but the teacher doesn't dwell on this question (feel that Wendy is a bit nervous with me watching her and rushing a little bit). Teacher then puts sampling up

Teacher: Who can give me an example of sampling?

Child 22: When we were doing the "how many people in the classroom, 20 could fit that way and 25 could fit that way, so you could times them to get the answer - you change it to 20 and 20 to make it easier.

Teacher: So you are using two strategies in one there okay (noticing that when you do, you also do rounding).

Teacher: Nice numbers (puts nice numbers strategy up on the IWB). Who can give me nice numbers and when you might use that one?

Child 17: If you have like 32 plus 74 em you could make it easier, so you could round like 32 would go to 30 and 74 would go to 70 and that is a hundred.

Teacher: That's not too bad but that is an example of rounding.

Child 9: When you look for things that are going to 10.

Teacher : Well done who has used that type of strategy ? Hands go up, Okay, hand down.

Wendy clicks on Front end loading which goes up on the board

Teacher: Front end loading , who can tell me when you use front end loading ?

Child 24 : You need to focus on the front two digits, say you have got 435 and 328 first number which is 4 that's 400 and 3 which is 300 and then when you plus them together you get a rough estimation of what total could be.

Teacher: Thanks Child 24, when might we want that one?

Child 24: Like higher numbers.

Child 6: When we did our millionaire project.

Teacher: When else did we use that kind of strategy? I know a few of you used it when you were doing the surf shop

Teacher: When might we use the range? (no one answers of a while)

Teacher: Think about when you are doing things, when we buy things your Mum says you can buy an outfit, something between this and that or if you are buying a couple of things, you might use it then. Try and see if sometime you use it.

Teacher: What about this one?

The teacher puts Intuition up on the IWB

Child 11: When you think maybe the answers a bit too difficult, or you just have a stab it .

Teacher: Yes, use it for the jelly beans.

Child 28: I used it for (couldn't catch wording, something to do with a maths problems)

Teacher: So then did you work it out accurately afterwards ?

Child 28: Eh yeh

Teacher: And then you used that just to check your answer, mm okay.

The teacher puts nice numbers up again on the IWB

Teacher: And this one, has anyone used this one? Can anyone think of a time they might use it

Child 9: This is one for nice numbers. If say you take 10×99 , you could just do build up to 100 and 10×100 and that's a thousand .

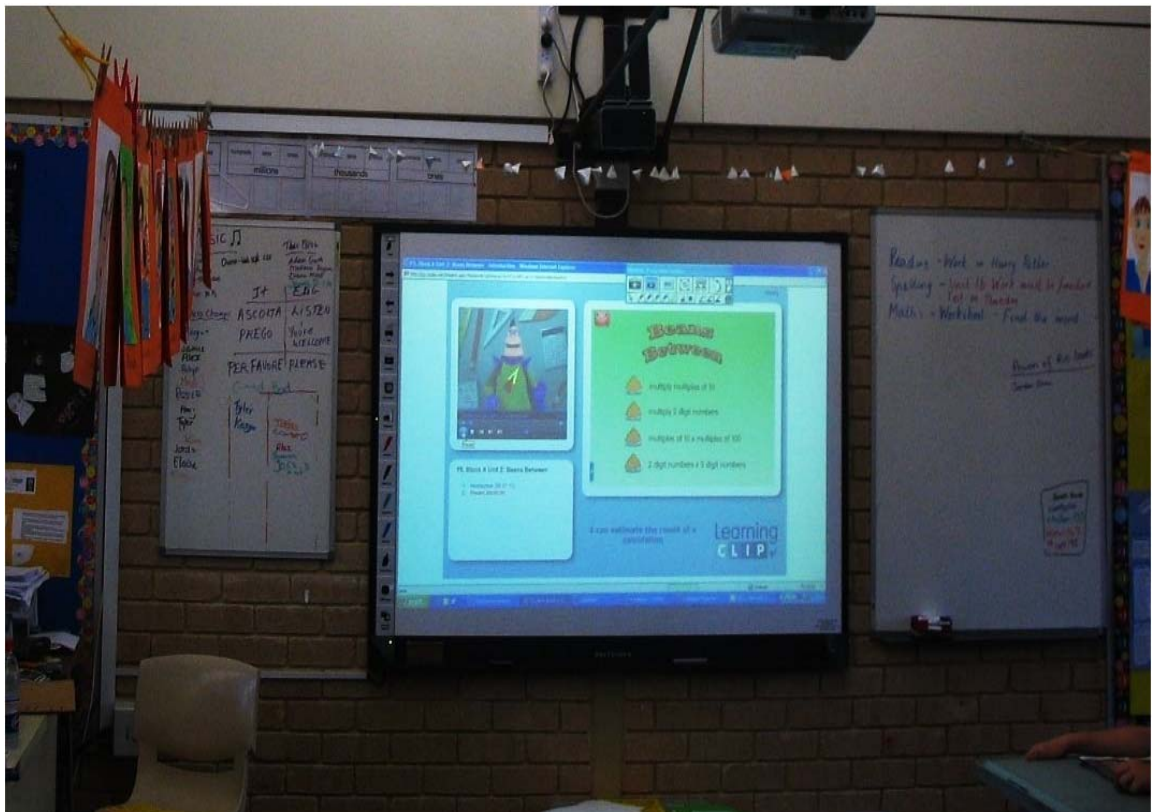
Teacher : Good boy Child 9.

Child 28: Isn't it already a nice number, 'cos 10 is a nice number ?

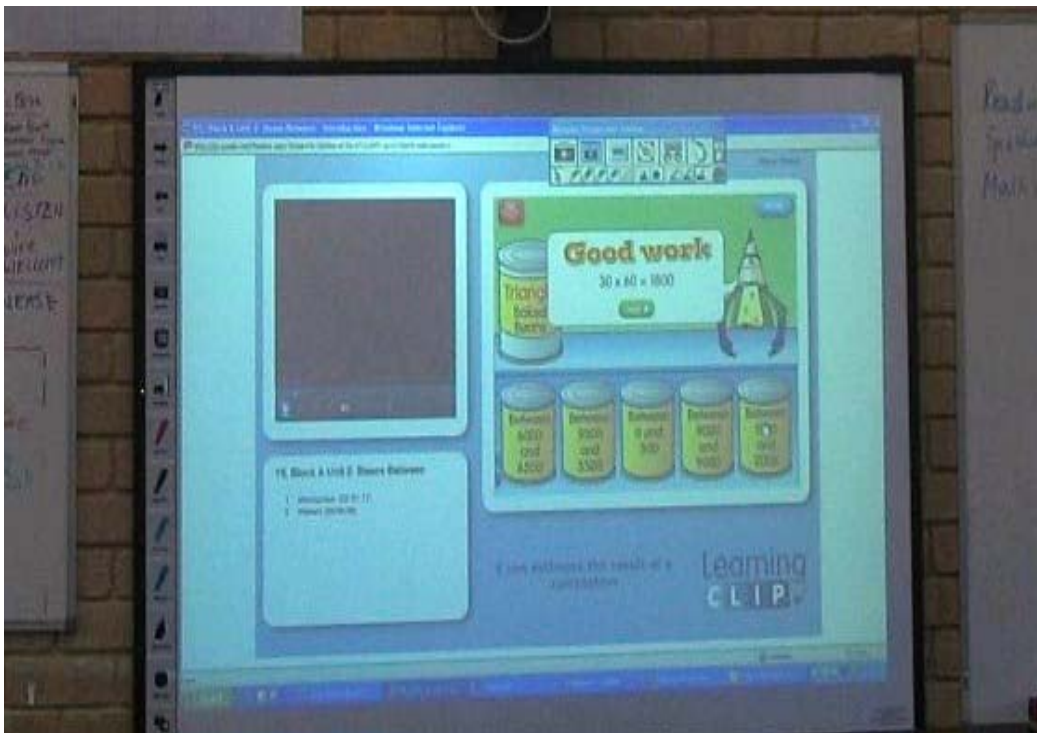
Teacher: Yes but it's even nicer 'cos 100 is easier to work with than 99 sometimes. Thanks Child 9

The teacher then played a game where they had to use a variety of estimation strategies in a multiplication problem including rounding and the range strategy. It was from an online source and used the IWB and was an interactive game about estimation. It was not in the suggested learning activities but the teachers had been encouraged to select what they thought were good activities.

The students all really loved the funny voice and they were able to follow the instructions.



The teacher selected Child 19 who read out the problem.



Child 19: In a contest, 30 contestants eat 60 cans each. About how many cans is that all together?

Teacher: Do you want to see if you can come and click on the right answer, come and do it on Webster and tell us what you are thinking as you are doing it .

Child 19: $6 \times 3 = 18$ and add the zeros make 1800.

Child then clicked on the can that stated the range 1000- 2000

Teacher: Well done, who else would like a turn ? Child 11 you have been working well come out and tell us what you are thinking.

A new problem is presented on the IWB.

Child 11: It is basically the same as Child 19, $7 \times 2 = 14$, so I think it is going to be 1400.

Teacher: So click on and see.

The cans had a label with the range the amount of cans should go in. Child 11 then pressed the can that was 1000 – 1500 and that was correct.

Teacher: Right it is gone a bit trickier, Child 22 you wanted a turn

The teacher brings up a new problem.

Child 22: Just looking at the numbers (99×59), 99 round that to 100 and then 59 round that to 50, so it is 500, but then it is going to be more I know so it has to be at least 500.

Teacher: Think Child 22, if you have 60 [she automatically corrects 50 to 60] \times 100 how many zeros did you have to add ?

Child 22: 60 – eh you have to add 3

Teacher: You add one for the 60, what is 60×10 ?

Child 22: 600.

Teacher: What is 60×100 ?

Child 22: So its 6000.

He pushes the button showing 6000 and he then gets it right.

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Appendix I

Instructions for the estimation test

1. Ask the students to write their names and other details on the paper.
2. Tell students: Today I want you to do the maths problems mentally. I will read each question while you follow me. Then I'll give you half a minute - 30 seconds - to do it, before asking you to go on to the next question.
3. Say "Now we are ready to start". Turn to the next page. Question 1 says ... and so on until the test is complete. Then give the students 5 minutes to go back and write in a coloured pencil how they solved it.

I will pick this up on a date convenient to you.

Estimation Test

Name

School.....

Class.....

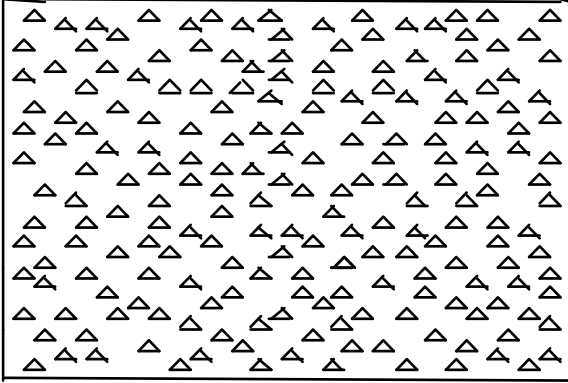
Here are some questions designed to help Paula Mildenhall in her research find out how you can estimate in term4.

Simply circle the best answer using a coloured pencil or pen.

There are 6 questions. You will have 30 seconds for each question. Your teacher will tell you when it is time to go to the next question.

Please make an estimate and do not calculate an exact answer.

When you have finished the questions you will then have 5 minutes to go back and work in your normal pencil and write a couple of sentences explaining how you worked each question out .

Question	Correct Answer	How I worked it out
<p>1. About how many days have you lived?</p>	<p>A 300 B 3000 C 30 000 D 300 000</p>	
<p>2. Without calculating the exact answer, circle the best estimate for :</p> <div style="border: 1px solid black; padding: 10px; width: fit-content; margin: 10px auto;"> $\frac{12}{13} + \frac{7}{8} =$ </div>	<p>A 1 B 2 C 19 D 21</p>	
<p>3. About how many triangles are there here? (Circle the nearest answer.)</p> <div style="border: 1px solid black; padding: 10px; width: 300px; height: 150px; margin: 10px auto;">  </div>	<p>A 20 B 50 C 100 D 200 E 500</p>	

<p>4. Without calculating the exact answer, circle the best estimate for:</p> <p style="text-align: center;">$45 \times 105 \approx$</p>	<p>A 4000</p> <p>B 4600</p> <p>C 5200</p>	
<p>5. Without calculating the exact answer, circle the best estimate for:</p> <p style="text-align: center;"> $\begin{array}{r} 27 \\ 38 \\ 65 \\ + 81 \end{array}$ </p>	<p>A 165</p> <p>B 300</p> <p>C 200</p> <p>D 360</p>	
<p>6. $6 \times 3.7 =$ Is the answer between</p>	<p>A 23- 26</p> <p>B 18-24</p> <p>C 23- 28</p> <p>D 16-18</p>	

Estimation with Nice Numbers

- Look at the numbers in the calculation to create an estimated answer
- Decide if you can replace the exact numbers with nice numbers
- Use these 'nice numbers' to calculate the estimated answer
- e.g, $34 + 72 \approx 30 + 70$

$$0.23 \times 40 \approx \frac{1}{4} \text{ of } 40$$

$$36 \div 5 \approx 36 \div 6$$

Appendix K

Changes to Wendy's beliefs, PCK and practice and to her students' beliefs and computational estimation competencies

Research Focus	Beginning of the case study	End of the case study
<p>Developments of teacher PCK about computational estimation</p>	<p>(K.F.4.3) Understands rounding strategy (K.F.4.4) No pedagogical framework for teaching computational estimation using the variety of strategies</p>	<p>(K.F.4.18) Computational estimation strategies worth teaching to Year 6 (K.F.4.11) Understands computational estimation strategies (K.F.4.19) Pedagogical approaches for estimation as a component of number sense – games, practical activities set in meaningful contexts and journaling (KF 4.21) In the real world, a text book approach and its focus on procedural teaching of estimation was appropriate. (K.F.4.19) Start introducing computational estimation from when students begin school. (A 4.1) Due to Wendy's engagement in the action research process broadened Wendy's PCK of this subject area in that she understood the strategies and developed a pedagogical framework for how these could be taught in Year 6.</p>

Research Focus	Beginning of the case study	End of the case study
Development of teacher beliefs about computational estimation	<p>(K.F. 4.) Estimation is used as a checking device when teaching algorithms</p> <p>(K.F. 4.2) Estimation exercises are not authentic and students are not engaged</p>	<p>(K.F.4.18) Estimation strategies are worthwhile to teach</p> <p>(K.F.4.12) Students are motivated by computational estimation tasks.</p> <p>(K.F.4.20) Wendy believed that computational estimation was an important component of number sense</p> <p>(K.F. 4.19) Tasks set in meaningful contexts where estimation was the computational choice could be a valuable in an ideal world</p> <p>(K.F. 4.21) Wendy believed that in the real world of her school context, a text book approach and its focus on procedural teaching of estimation maintained harmony with parents and therefore was the best pragmatic pedagogy</p> <p>(A. 4.2): Wendy’s developing PCK of computational estimation as a computational choice and checking device impacted her beliefs and she now believed that computational estimation could develop number sense in an ideal world and that computational estimation strategies were worthwhile. In the real context of her classroom Wendy believed that procedural teaching of estimation was the best pragmatic choice even though it was a band aiding approach.</p>

Research Focus	Beginning of the case study	End of the case study
<p>Development of teaching approaches for computational estimation</p>	<p>(K.F 4.2) Routine exercise teaching rounding in text book (K.F 4.1) Informs students to estimate before calculations</p>	<p>(K.F 4.21)Routine exercise teaching rounding and algorithms using the text book in normal program (K.F 4.13) Teacher modelling use of estimation strategies in problems in extra mathematics time (K.F 4.15) Using small group to engage students when exploring computational estimation problems in extra mathematics time (K.F 4.10) Explicitly describing the computational estimation strategies in extra mathematics time (K.F 4.14) Problems in meaningful contexts teaching focussed on making numbers easier in extra mathematics time. (A 4.3) Wendy’s teaching approaches appeared to be impacted by her developing beliefs that computational estimation was important in developing number sense and her developing PCK of computational estimation strategies. Wendy developed two teaching approaches that she thought were appropriate in an ideal world; estimating in problem situations and directly teaching the estimation strategies. Wendy maintained two contexts – the ideal world and the real world. In the real world of the classroom she often continued to use procedural approaches to teaching estimation</p>

Research Focus	Beginning of the case study	End of the case study
Impact on student beliefs about computational estimation and mathematical knowledge	<p>(K.F 4.5) Mathematics is about something about the four operations, with one right answer and is done quickly</p> <p>(K.F 4.6) Estimation is mathematical guess</p>	<p>(K.F 4.22) Mathematics is about something about patterns, that is done quickly and is about something that is about one correct answer</p> <p>(K.F 4.23) Estimation is:</p> <ul style="list-style-type: none"> • More than a guess • fun • makes mathematics easier • involved a variety of estimation strategies • helps to make sense of mathematics • can remove the challenge of mathematics <p>(A 4.4): Wendy's teaching approach of creating extra problem based computational estimation learning tasks appeared to impact the students' beliefs and broaden their perception of mathematics and estimation. Their perceptions of computational estimation appeared to be very positive.</p>

<p>Impact on computational estimation competencies</p>	<p>(K.F 4.7) Students had a higher competency when estimating the answer to symbolic mathematical problems than contextual problems where both questions required students to multiply in the calculation</p> <p>(K.F 4.8) Most of the students were not able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and few students used a reasoned estimation strategy</p> <p>(K.F 4.9) Around half of the students found it difficult to select the best estimate when adding two digit numbers and less than half of the students were able to use a reasoned estimation strategy</p>	<p>(K.F.4.16) Wendy's students used computational estimation language when discussing how to solve the problem</p> <p>(K.F.4.17) Wendy's students were able to use estimations as a main computational choice in extended problem task</p> <p>(K.F 4.25) Students are much more proficient at estimating multiplication problems which are purely symbolic and not set in context. Nearly three-quarters of the class were able to apply estimation strategies when problems were set in a context</p> <p>(K.F 4.26) Nearly half the class were able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and far more students could use a reasoned estimation strategy to estimate when adding fractions.</p> <p>(K.F.427.) Student showed no improvement in estimating to a fairly high degree of precision when assessing an estimated answer of a multiplication of a two digit by three digit number</p> <p>(K.F. 4.24)Wendy's students' computational performance improved overall and statistically this was highly significant</p> <p>(A. 4.5): Wendy's teaching approach teaching approach of creating extra problem based computational estimation learning tasks appeared to improve students' estimation performance although her focus on making the numbers easier meant that the students did not focus on the precision of the estimate. Wendy's teaching approach of directly teaching the estimation strategies appeared to increase the students' awareness of the estimation strategies.</p>
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Appendix L

Changes to Peter's beliefs, PCK and practice and to his students' beliefs and computational estimation competencies

Research focus	Beginning of the case study	End of the case study
Developments of teacher PCK about computational estimation	(K.F. 5.3)) Understood the rounding strategy (K.F. 5.2) No pedagogical framework for teaching computational estimation using the variety of strategies	(K.F. 5.18) Pedagogical approaches- practical activities set in meaningful contexts, not formally teaching estimation strategies, students reflecting on exact computations in mental mathematics (K.F. 5.19) At the end of the project Peter developed some understanding of the estimation strategies (A. 5.1) Peter did not engage in the professional learning process due to his beliefs that his context would not benefit from it, so therefore the impact of the provision of research literature and workshop activities, was limited. Towards the conclusion of the process Peter's PCK of this subject area did begin to develop, in that he began to consider how computational estimation could be taught in Year 6
Developments of teacher beliefs about computational estimation	(K.F. 5.4) Computation estimation used as a checking device before doing routine algorithms (K.F. 5.1) Students work more effectively independently without talking (K.F. 5.12) Students will develop	(K.F. 5.11) Doesn't believe in teaching formal estimation strategies (K.F. 5.4) Computation estimation useful before doing exact mental computations (K.F. 5.14) Number sense activities had value (K.F. 5.18) Estimation should be part of all

Research focus	Beginning of the case study	End of the case study
	<p>estimation skills regardless of the teaching approach (K.F. 5.11) Doesn't believe in teaching formal estimation strategies (K.F. 5.13) Didn't believe that exercises in his school's text book that required students to use an estimate as a checking device did not engage his students</p>	<p>mathematic lessons as a checking device (mental mathematic, textbook, problem solving). (A.2) Peter did not engage in the professional learning process due to his beliefs that his context would not benefit from it, so therefore the impact on his beliefs was limited at the beginning. As the reflective process continued and he began to engage in the process, Peter's PCK of this subject area did begin to develop and this process impacted his beliefs</p>
<p>Developments of teaching approaches of computational estimation</p>	<p>(K.F. 5.1) Students work independently without talking (K.F. 5.5) Routine exercise teaching rounding in text book (K.F. 5.4) Computation estimation useful before doing exact mental computations</p>	<p>(K.F. 5.15) Beginning to trial working in small groups in order to facilitate discussion. (K.F. 5.18) Beginning to trial practical activities set in meaningful contexts where the computational choice is only an estimation (K.F. 5.18) Students judging reasonableness of exact calculations (K.F. 5.5) Routine exercise teaching rounding in text book</p>
<p>Impact on student beliefs about computational estimation and mathematical knowledge</p>	<p>(K.F. 5.6) Mathematics is about problem solving, the four operations, something with one correct answer and is done quickly</p>	<p>(K.F. 5.20) Mathematics is about working out problems, is about one correct answer and something that is done quickly (K.F. 5.21) Estimation is useful as a checking</p>

Research focus	Beginning of the case study	End of the case study
	(K.F. 5.7) Estimation is mathematical guess	device, is about the rounding strategy and is fun (A.5.4) Peter's beliefs that estimation was important as a checking device appeared to impacts the students, as they believed that estimation was important at improving their mathematics. Peter's teaching approach of creating extra problem based computational estimation learning tasks appeared to impact the students' beliefs and make their perception of estimation more positive
Impact on computational estimation competencies	<p>(K.F. 5.8) Students are more proficient at estimating multiplication problems which are purely symbolic and not set in context.</p> <p>(K.F. 5.9) Most of the students were not able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and few students used a reasoned estimation strategy.</p> <p>(K.F. 5.10) Half of the students were not able to select the best estimate when adding two digit numbers and less than half the students were able to use a reasoned estimation strategy.</p>	<p>(K.F. 5.16) Peter's students used computational estimation language when discussing how to solve the mathematical problem</p> <p>(K.F. 5.17) Peter's students were able to use estimations as a main computational choice in extended problem task</p> <p>(K.F. 5.22) Few students able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and few students were able to use the benchmarking strategy.</p> <p>(K.F. 5.24) Students are much more proficient at estimating multiplication problems which are purely symbolic and not set in context</p> <p>(K.F. 5.23) About two thirds of the class were able to select the best estimate when adding two digit numbers and more students were able to use a reasoned estimation strategy</p> <p>(K.F. 5.25) Peter's students' computational estimation performance improved overall but</p>

Research focus	Beginning of the case study	End of the case study
		<p>statistically this was not significant (K.F. 5.26) Peter's students ability to select the best estimate on a multiplication calculation that required an answer with some precision improved.</p> <p>(A 5.5) Peter's teaching approach of estimating before and after calculating exact numbers and of creating extra problem based computational estimation learning tasks appeared to improve student's estimation performance. Peter's decision not to teach the formal estimation strategies appeared to limit the students' use of reasoned estimation strategies</p>

Appendix M

Changes to Bob's beliefs, PCK and practice and to his students' beliefs and computational estimation competencies

Research Focus	Beginning of the case study	End of the case study
Developments of teacher PCK about computational estimation	(K.F.6. 2) Intuitive understanding of other strategies (K.F.6.3) No pedagogical framework for teaching computational estimation using the variety of computational estimation strategies	(K.F.6.15) Computational estimation strategies worth teaching to Year 6 (K.F. 6.16) Pedagogical approaches practical activities set in meaningful contexts, scaffold problem solving, students reflecting on exact computations by estimating answers, (K.F. 6.17) Understands computational estimation strategies (A 6.1) :The provision of research literature about computational estimation strategies and how computational estimation could be a computational choice broadened Bob's PCK of this subject area in that he understood the strategies and developed a framework for how these could be taught in Year 6
Developments of teacher beliefs about computational estimation	(K.F.6.4) No time in the primary curriculum to teach computational estimation (K.F.6.1) Mathematical tasks for students should be problem based and students should develop a deep understanding rather than simply master	(K.F.6.12) computational estimation as a component of number sense should be an integral component of all computation lessons (K.F.6.15) Estimation strategies are worthwhile to teach

Research Focus	Beginning of the case study	End of the case study
	routine algorithms	<p>(K.F.6.16) Tasks set in meaningful contexts where estimation was the computational choice were valuable as a central teaching approach</p> <p>(A.6.2) Bob's developing PCK of computational estimation as a computational choice and checking device impacted his beliefs and he now believed that computational estimation was an integral component of developing number sense.</p>
Developments of teaching approaches of computational estimation	(K.F.6.4) No time to teach computational estimation	<p>(K.F. 6.10) Explicit description of the computational estimation strategies at the introduction of the study</p> <p>(K.F. 6.14) Models using estimation strategies without explicitly using formal terms to describe strategies when problem solving.</p> <p>(K.F. 6.16) Scaffolded all computational estimation problems where estimation is main computational choice</p> <p>(K.F.6.12) Computational estimation as a component of number sense integral component of all computation lessons to judge reasonableness</p> <p>(K.F. 6 .11) Humour used to engage students</p>

Research Focus	Beginning of the case study	End of the case study
		<p>(K.F. 6.13) Small group exploring computational estimation problems (A 6.3) Bob’s developing beliefs that computational estimation was important in developing number sense and developing understanding of estimation strategies impacted his teaching approaches. Bob developed two teaching approaches; estimating in problem situations and estimation as a checking device</p>
<p>Impact on student beliefs about computational estimation and mathematical knowledge</p>	<p>(K.F.6.5) Mathematics is about one correct answer, is done quickly, and is about working out problems (K.F.6.6) Estimation is a guess with some type of mathematical reasoning attached to it</p>	<p>(K.F.6.18)Mathematics is about working out problems, one correct answer and something that is done quickly. (K.F.6.19)Some negative beliefs about the new type of mathematics -estimating in a problem solving situation. (K.F. 6.20)Estimation is: More than a guess Makes mathematics easier Helps to make sense of mathematics Can remove the exactness and enjoyment of mathematics (A 6. 4): Bob’s teaching approach of integrating estimation into all of his mathematics appeared to impact the students’ beliefs and broaden their perception of mathematics and estimation. Some students appeared resistant to this change</p>

Research Focus	Beginning of the case study	End of the case study
Impact on computational estimation competencies	<p>(K.F.6.7) Students had a higher competency when estimating the answers to abstract mathematical problems than contextual problems where both questions required students to multiply in the calculation</p> <p>(K.F.6.8) Many of the students were not able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators and few students used a reasoned estimation strategy</p> <p>(K.F.6.9) Nearly three quarters of the students were able to select the best estimate when adding two digit numbers although less than half the students used a reasoned estimation strategy</p>	<p>(K.F.6.21) Over half the class were able to select an acceptable estimate when calculating the addition of two fractions with unlike denominators. Far more students were then able to use this benchmarking strategy</p> <p>(K.F.6.23) Students became more proficient when estimating an abstract multiplication mathematical calculation than when it was set in context.</p> <p>(K.F.6.22) Nearly all the students were able to select the best estimate when adding four two digit numbers and around two thirds of the class were able to use a reasoned estimation strategy.</p> <p>(K.F.6.24) Computational performance improved overall and statistically this was highly significant</p> <p>(K F 6.25) Bob's students are working as a community of learners justifying their computational solutions</p> <p>(K.F.6. 26) Students were able to use estimations as a main computational choice in extended problem task</p> <p>K.F.6.27) Students used computational estimation language when discussing how to solve the problem</p>

Research Focus	Beginning of the case study	End of the case study
		(A.6.5) Bob's teaching approach of integrating estimation into all of his mathematics appeared to improve students' estimation performance in a wide variety of areas and increase their awareness of the estimation strategies