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The effects of algebra blocks on student achievement in algebra

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THE EFFECTS OF ALGEBRA BLOCKS ON STUDENT ACHIEVEMENT
IN ALGEBRA

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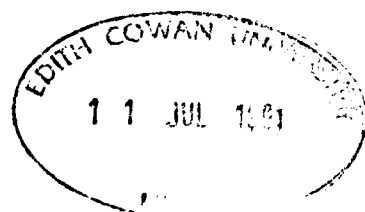
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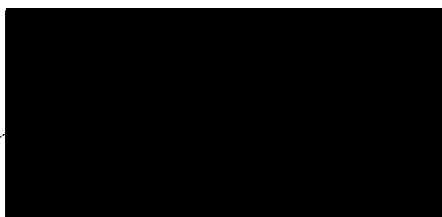
Abstract

This study examined the effects of Algebra Blocks on student understanding for the concepts of binomial expansion and trinomial factorisation. The purpose of the study was to illuminate the use and effectiveness of Algebra Blocks in the teaching-learning process. Two year nine classes with similar mathematics levels were taught binomial expansions and trinomial factorisations. The experimental class was taught using Algebra Blocks whilst the untreated class acted as the control group. After eight lessons of instruction, both classes were tested on their understanding of both concepts. To provide qualitative data, three randomly chosen students from each class were interviewed on their understanding of both concepts. The experimental results were statistically significant ($p<.01$) for trinomial factorisations, but there was no statistical significance ($p<.05$) for the binomial expansion section. The interviews supported the use of Algebra Blocks as their use appeared to provide the students with conceptual imagery. The study supported the implementation of Algebra Blocks into the mathematics classroom as Algebra Blocks were found to be an effective teaching aid.

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<u>Acknowledgements</u>

Introduction

The purpose of this study is to outline the difficulties associated with teaching and learning algebra and to illuminate the use and effectiveness of Algebra Blocks in the teaching-learning process.

Background to the Problem

Algebra is often a difficult subject for students as they can not grasp its abstractness and its seemingly unrelated web of complex algorithms.(Lovitt,Marriot & Swan,1984). At present, students entering high school are experiencing very limited success in learning algebra as the gap between the arithmetic, concrete orientation of mathematics experienced in primary school and the symbolic, abstract approach taken to teaching algebra in high school, is too large. Research by Swinson (in Swinson,1982) confirms this statement. As a result of the symbolic, abstract approach adopted in teaching secondary algebra, students can not visualise their answers and usually see no relationship between two similar examples or to their real world experiences. Consequently, this leads students into rote learning a number of algorithms to use for different examples, without the knowledge of why they are applying a particular algorithm. What eventuates is instrumental learning - "using rules without reason." (Skemp,1976,p.20). Students use an algorithm to obtain a solution, but upon query of their solution they

can not justify that their solution is correct or incorrect. This approach to learning algebra appears to commonly occur among mathematics students in all classrooms.

As a mathematics educator, the solution to this problem is to teach towards relational understanding, that is, "knowing both what to do and why." (Skemp,1976,p.20). To achieve this aim students must first be able to visualise their solutions. The solutions obtained must be given meaning in relation to the student's experiences to enable the students to develop relational understanding. The use of concrete aids in teaching is seen by the majority of educators as a great assistance in helping students understand and visualise concepts more clearly. Research (Fennema,1972) has shown that concrete aids assist in developing links between the concrete and symbolic representations of the concept. This leads to a better understanding in the student of how the concept was developed and gives meaning to the solution as it relates to a concrete experience.

Algebra Blocks are a concrete , mathematical aid which have existed in a variety of forms for some considerable time but they have not been widely utilised. Algebra Blocks consist of pieces of coloured wood which represent mathematical symbols through the concept of the area of a rectangle. One side of the Algebra Block represents the positive symbol and the other side is the negative value of the symbol. An example of their use can be seen in appendix A.

Algebra Blocks are not widely utilised , as this may

be due to the teachers' doubt of their effectiveness. This research project will provide the teacher with the necessary evidence to decide whether to use or not to use Algebra Blocks in the teaching of algebra. If Algebra Blocks are proved effective , then teachers may change their traditional, barren methods of teaching algebra. The students will gain by having a more effective and efficient means of solving algebraic problems.

The extent to which the results of this research project can be implemented into other topics of algebra is dependent upon how closely the algorithms are related to either binomial expansions or trinomial factorisations. This study is limited to measuring the effects Algebra Blocks have on the learning of binomial expansions and trinomial factorisations. In algebra topics such as expansions and factorisations of algebraic expressions and the solving of simple linear equations, Algebra Blocks could be easily implemented. However, in concepts that are not as closely related, such as solving simultaneous equations or indices, Algebra Blocks would have an extremely limited application.

The purpose of this research is to examine the effects the use of Algebra Blocks have on achievement in algebra, particularly in teaching binomial expansions and trinomial factorisations.

Definition of Terms

Algebra Blocks are an area based representation of the mathematical symbols: x^2 , x and 1 . Algebra Blocks are a manipulative concrete aid which can be used in a number of algebra topics such as binomial expansions, trinomial factorisations, linear equations and expansions and factorisations of algebraic expressions.

Binomial expansion refers to multiplying two binomial expressions of the type (x + a) or (x - a), where 'a' is an integer, so as to find the simplest form.

Trinomial factorisation refers to writing an expression of the type x^2 + bx + c , where 'b' and 'c' are integers, as a multiplication of two binomial expressions.

Understanding refers to knowing both what to do and why. The term 'knowing what to do ' refers to knowing which processes to use to obtain the correct solution. Knowing why to do it refers to knowing reasons for selecting the processes chosen to obtain the solution. For understanding to take place links need to be made between the new concept and related knowledge and imagery. Understanding will be measured in the research by an individual score on both parts of the post-test plus information collected from the interviews.

Review of Literature

The process of teaching is dependent upon the learning process. The knowledge of how a student learns will affect the choices of teaching strategies adopted. The learning theories described by Piaget(1950), Bruner(1966) and Dienes(1971) have direct implications to the teaching of mathematics. These three learning theories will each be described briefly and then the general implications of the theories for the teaching of mathematics will be discussed. The effects these implications have on the teaching strategies adopted for the teaching of algebra will follow this discussion.

Learning Theories Relating to Mathematics. The theory of learning described by Piaget(1950) links cognitive development with biological development. This implies that children can only learn a concept when they are cognitively developed to do so. This theory identifies four stages of cognitive development, which determine what can be learnt and hence what should be taught. Piaget relates each stage to an approximate age of the student. The stages are:

1. Sensorimotor (0 - 2 years) - child develops bodily control.

2. Preoperational (2 - 7 years) - words associated with concrete objects; view everything in relation to themselves.

3. Concrete Operational (7 - 12 years) - children learn concrete concepts but have difficulties with

abstract notations.

4. Formal Operations (above 12 years) - children no longer rely on concrete representations to represent ideas. Abstract operations with symbols can be carried out mentally.

(Copeland,1984; Piaget,1950; Sawrey & Telford,1968,p.42)

Piaget's theory of learning has many followers in the field of cognitive psychology (Brainerd,1978; Gage & Berliner,1979; Sawrey & Telford,1968) and it is equally well received by mathematical psychologists (Bell,Costello & Kuchemann,1983; Copeland,1984; Resnick & Ford,1981). Research by Low (1980) supports Piaget's theory of linking cognitive development to biological development, by pointing out that "one possible source of difficulty is that many concepts are introduced either at a time or in a way which is unsuited to the children's level of cognitive development." (p.8)

Though Piaget gives chronological ages for the stages, there is little supporting evidence that agrees with these age distinctions. Shayer's (in Swinson,1982) research shows that of junior high school pupils (ages 12-15 yrs.), probably in excess of fifty percent do not operate at Piaget's formal level and some never will. Biggs and Collis (1982) modified Piaget's theory into six stages, where the one stage of concrete operations was divided into the three stages of - Early concrete (7 - 9 yrs.), Middle concrete (10 - 12 yrs.) and Concrete generalisations (13 - 15 yrs.) - with formal operations not beginning until the age of sixteen. Though there is little agreement in when students reach a certain level of

Piaget's cognitive development theory, followers of Piaget do agree upon the fact that children go through all these stages, though some children, as Shayer points out, do not make it to formal operations. As this study is concerned with secondary students, only the concrete and formal operational stages are of relevance. Piaget's theory has many implications for the teaching of mathematics which will be discussed later.

There are however, theories of intellectual development which disagree with Piaget on the link between cognitive and biological development. Guilford (in Higgins,1973) views the structure of the intellect as relatively static which goes against Piaget's view of a dynamic theory where cognition develops with biological age. Bruner(1966) believes that there are three stages of growth in which children come to represent the world. These are :

1. Enactive stage - where holding, touching, moving and so forth is needed to provide experience of the concept with the object.

2. Iconic stage - information is carried by imagery, that is, by visual and diagrammatic representations.

3. Symbolic stage - language and written symbols are used.

Though there appear to be links between Bruner's theory and Piaget's theory on how a child develops a concept, there is no link in Bruner's theory between intellectual and biological development. Contrary to Piaget, Bruner (1966) has suggested that " any idea or problem or body of knowledge can be presented in a form

simple enough so that any particular learner can understand it in a recognisable form." (p.44). By following Bruner's theory of instruction mathematical concepts can be logically developed in the child.

Another theory of learning, developed by Dienes (1971), regards mathematics as a building up of structures. In order to classify structures and identify relationships, students need to analyse, abstract, generalise and construct knowledge. Dienes believes teachers should present mathematical concepts in the form of multiple embodiments, that is, in as many equivalent forms as possible. This will allow the student to form a clear imagery and hence understanding of the concept. Dienes believes mathematical concepts are learnt in progressive stages. In Dienes (1973), he describes his stages of learning as :

1. Free play - students manipulate and experiment with physical and abstract representations of the concept.

2. Games - after a period of free play , students discover regularities, relationships and constraints associated with the concept.

3. Searching for communalities - students discover common properties that distinguish between concepts.

4. Representation - students develop or receive a diagrammatic or verbal representation of the concept.

5. Symbolisation - students formulate appropriate verbal and mathematical symbols to represent the concept.

6. Formalisation - students use the concept to solve and apply mathematics problems.

The two theories presented by Bruner and Dienes are

structure orientated, whereby the student follows a series of stages to learn a concept. Gagne (in Bell,1978,p.110) supports this mechanistic view when he describes his 'Phases of a Learning Sequence '. These styles of learning theory adopt a logical approach where links need to be made with prior knowledge to advance to a higher level of the skill or concept.

The three theories examined, that is, Piaget's, Bruner's and Dienes, all attempt to explain the complex process of learning. However, no single theory is a complete model of the learning process. To provide the student with the best teaching strategies, the teacher needs to be aware of the implications of each theory.

Implications of the Learning Theories for Mathematics Teaching. In the theories of learning developed by Piaget, Bruner and Dienes, the need to relate knowledge to the environment through the manipulation of concrete objects is inherent. Piaget, Bruner and Dienes believe that the manipulation of concrete objects forms the basis of mathematics. (Dickson,Brown & Gibson,1984,p.12). This is seen in Piaget's third stage, that is, concrete operations, in Bruner's first stage - enactive stage, and in Dienes' first three stages - free play, games and searching for communalities. The use of concrete objects aids in the learning process as they seed the ground for later developments of the concepts. Research by Behr et al (in Swinson,1982), Parham (in Suydam,1986) and Raphael and Wahlstrom (1989) all support the view that the use of manipulative aids assist in the learning of skills, concepts and principles resulting in better mathematics

achievement. Both educational and mathematical psychologists support the use of concrete aids in teaching.(Beattie,1986a; Herbert,1985; Higgins,1973; Keats,Collis & Halford,1978; Sigel & Cocking,1977). It is suggested that aids that are used in teaching mathematical concepts need to be concrete rather than abstract and they should be objects that can be manipulated. (Brainerd,1978,p.278; Henry,1982; Swinson,1982).

Traditionally, mathematics teaching methods have relied heavily on an expository approach, whereby students are trained mainly in facts and computational skills.(Keats et al,1978). Little emphasis is given to students solving problems, forming judgements or understanding the interrelationship of concepts. Instead, mathematical skills and concepts are taught using the explain-practice-memorize (Greenwood & Anderson,1983) or "tell 'em - drill 'em - test 'em" (Ransley,1980,p.11) teaching approach. This method of teaching does not allow for the way in which students learn. It assumes that all students are at Piaget's formal operations and so are capable of dealing with abstractions and learning like adults. However," the success of Piaget's studies depends upon showing that children are not miniature adults." (Higgins,1973,p.66). As the theories of learning have shown, students learn in a hierarchic structure where concrete aids are used to develop the concepts until the concept is formalised in the students mind.

The teaching method that needs to be adopted, based on the theories of Piaget, Bruner and Dienes, is discovery learning. This approach to teaching allows the student to

manipulate the objects at will to discover any relationship and communalities between concepts. Effective thinking in mathematics is dependent upon understanding, not on rote learning words and phrases from a text. (Herbert, 1985). Students need to be taught for relational understanding - "knowing both what to do and why "(Skemp, 1976, p.20) rather than instrumental understanding - "using rules without reason." (Skemp, 1976, p.20). Discovery learning teaches toward relational understanding of concepts whereas the expository approach adopts the method of instrumental understanding where algorithms are rote learnt. The differences in methods of teaching are summed up perfectly by James Newman (in Schminke & Arnold, 1971, p.1):

> " There are two ways to teach mathematics. One is to take real pains towards creating understanding - visual aids, that sort of thing. The other is the old British system of teaching until you're blue in the face."

This statement is a true indication of the effects the discovery learning approach and the expository approach have on student understanding. Though discovery learning is harder to teach, that is, more preparation time and effort is required, it does create better long term understanding than the expository approach.(Worthen, 1968). This is consistent with the view on understanding presented by Skemp (1976). Additionally , the manipulative aids used in discovery learning help motivate students, stimulate them to think mathematically and

informally introduce difficult concepts and ideas in
mathematics. (Herbert,1985).

Manipulatives need to be used in mathematics as they
are able to illustrate or develop a concept or skill and
they assist in students discovering patterns and making
generalisations.(Berman & Friederwitzer,1989; Thorton,
1986; Worth,1986). Manipulatives organise students
thinking allowing students to see relationships.
(Thorton,1986). Additionally manipulatives assist in the
learning of algorithms as they provide a structure for
students to follow the flow of the developing procedure or
algorithm. (Beattie,1986a; Thorton,1986). However,
research by Suydam (1984) indicates that though most
teachers believe manipulatives should be used for
mathematics instruction and that the use of manipulatives
does develop understanding, this belief is not being
translated into practice as the use of manipulatives
diminishes through the grades.

Consequently, mathematics needs to be taught using
concrete aids or manipulatives. The concrete aid can be
linked to its symbolic representations using language.
This leads to the formalisation of the concept and as such
it can be used in its abstract form. This view is
supported by Swinson(1982).

Manipulatives assist in bridging the gap between the
level of mathematics which is being taught and the
cognitive level of the student. That is, the
manipulatives assist the student in visualising the
abstract ideas, represented by symbols, making the concept
more meaningful. Research by Fennema (1972) supports the

use of concrete aids in teaching mathematical concepts.
Manipulatives assist in bridging the gap between the
student's concrete environment and the abstract level of
mathematics.(Beattie,1986b; Fennema,1973; Heddens,1986;
Hynes,1986). However, the manipulative concrete objects
can not be used in isolation, there needs to be a clear
link with the final symbolic form of the concept.
(Bright,1986; Swinson,1982). Consequently, the inherent
disadvantage of using manipulative aids is that they are
solely an intermediate measure until students can attach
meaning to the concrete generalisations.(Low,1982).
Therefore, an instructional model using manipulatives
needs to move from concrete (such as manipulation of
blocks), to representational (drawing diagrams), to
symbolic. (Beattie,1986b; Lewis,1985).

However, the advantages of mathematics do not lie in
the manipulation of concrete objects, but in the
manipulation of abstract symbols, which is the final goal
all teachers seek.(Higgins,1973). Once the student can
remember the structure of the concept and can manipulate
the abstract symbols, the physical objects should only be
used if the student strikes difficulties. The concrete
aids act solely as a foundation on which the concept is
built and so can be returned to at any time. (Harrison &
Harrison,1986; Higgins,1973; Lovitt et al,1984;
Swinson,1982).

The Effects of these Implications on Teaching
Algebra. Algebra constitutes a large portion of secondary
mathematics. Due to its broadness there is no absolute
definition of 'Algebra' though some define 'Algebra' as

"the study of generalisations,"(Schools Council,1973,p.1)
or simply as generalised arithmetic. Algebra is, by its
very nature, abstract. It involves the manipulation of
symbols and coefficients to obtain a required solution.
As Algebra is an abstract concept it occurs at the highest
levels of the learning theories of Piaget, Bruner and
Dienes. In Piaget's theory abstract thought occurs in
formal operations; in Bruner's theory it occurs at the
symbolic stage and in Dienes' theory it occurs in the
symbolisation and formalisation stages. Due to Algebra's
abstractness, the teaching of Algebra to lower secondary
students must begin at a concrete level. As Shayer has
previously pointed out, as many as fifty percent of junior
high school students do not reach formal operations. This
in itself is evidence enough to suggest that many students
would not understand the concept of algebra if it was
merely presented to them in its abstract form. At present
the symbolic abstract approach taken to teaching algebra
is too large a jump for most students beginning secondary
school from their arithmetic orientation of mathematics
experienced in primary school.(Swinson,1981 in Swinson,
1982).

At present students entering high school are
experiencing very limited success in learning algebra.
(Booth,1986; Swinson,1981 in Swinson,1982). A smooth and
effective transition for students from primary school
arithmetic to secondary algebra is required. (Briggs,
Demana & Osborne,1986). Students are finding algebra
concepts and the manipulation of the algebraic symbols
difficult. (Berman & Friederwitzer,1989; MacGregor,1986;

Sawyer,1988). In research by Kuchemann (1978) and Low (1982) students were found to be having difficulty with understanding that a variable is represented by a symbol. Similar supporting evidence is found in Keats et al (1978). It appears mathematics teachers assume students can grasp the idea of a variable representing any number, though this is simply not the case. It is at this point that teachers of algebra need to recall and apply the learning theories of Piaget, Bruner and Dienes. As has been discussed previously, these three theorists believe in developing a concept, such as 'variable', from a concrete foundation to its abstract symbolism. This transition from the student's concrete environment to the abstract level of mathematics can be achieved by the use of manipulatives. (Beattie, 1986b; Fennema,1973; Heddens,1986; Hynes,1986).

As algebra is an abstract mathematical subject and many students are not at the cognitive level to deal solely with abstractions when algebra is initially introduced in secondary school (Shayer in Swinson,1982); it seems logical to use manipulatives as a stepping stone for better understanding. Many mathematics educators (Fremont, 1969; Henry,1982; Lumbard,1963; Mason & Broom, 1979a,b; Miller,1973; Sobel & Maletsky,1975) have developed concrete aids and manipulatives to assist in the understanding and learning of many algebra topics and their related algorithms. However, the ideas developed by these mathematics educators have not been researched to ascertain whether these manipulatives do actually assist in student understanding. Instead the presumption is that

all the relevant research on manipulatives in general -
which points to the fact that manipulatives increase
understanding - can be applied directly to these ideas.
As a consequence the research on the effect that algebra
manipulatives have on understanding is limited.

At present, in the algebra topics of binomial
expansion and trinomial factorisation, enquires suggest
that a traditional abstract teaching approach is being
implemented. Most of the research identified so far has
contradicted the use of this approach as being appropriate
to introducing and teaching these topics. Instead, the
research points to an approach that uses physical objects
that can be manipulated, as the ideal method of
introducing abstract concepts. In the topics of binomial
expansion and trinomial factorisation, many mathematics
educators (Hollingsworth & Dean,1975; MacDonald,1986;
Williams,1986) have developed puzzles and aids to assist
in students understanding the processes and algorithms of
the two topics. However, Algebra Blocks are the only true
manipulative aid that has been developed to assist
students in their understanding of binomial expansions and
trinomial factorisations. As the topics of binomial
expansions and trinomial factorisations form the
cornerstone for the development of many other mathematical
concepts, it is perceived essential that any aid that can
assist in clarifying understanding would be of benefit to
the student.

Algebra Blocks have been in existence for many years
in a variety of forms but they have not been widely
utilised. Knight (1957) first introduced Algebloc as an

invention by E.Van Lierde, but this initial attempt could only deal with positive symbols. Mathematic educators in Bidwell(1972), Fremont(1969), Henry(1982), Lumbard(1963), Miller(1974) and Sobel and Maletsky (1975) all attempted to introduce similar models that could deal with both positive and negative symbols. These models however were not practical as they did not allow for an easy, logical manipulation of the blocks, particularly when dealing with negative variables. Hence, the models did not provide the logical patterning that would allow the concepts to be easily recognised and understood. However, Lovitt et al (1984) successfully designed a model that easily used Algebra Blocks to represent both positive and negative symbols. To the researcher's knowledge there has been no research on this particular model of using Algebra Blocks. Consequently the effects of the use of Algebra Blocks on teaching both binomial expansion and trinomial factorisation is as of yet unknown. Fremont (1969) and Lovitt et al (1984) suggest that as examples become more difficult the use of physical objects or diagrams becomes burdensome. Hence the student is encouraged to discover patterns from completed examples, so as a more formal method of symbolisation can be adopted. This will create a more effective and efficient means of solving algebra problems.

Summary. The three theories of learning described by Piaget, Bruner and Dienes seem to have the most direct implication on teaching mathematics. All three theorists point to the use of concrete aids as assisting in the learning of concepts. These concrete aids need to be both

physical objects and easy to manipulate. Research by both Fennema (1972) and Swinson (1982) points to concrete manipulatives assisting in bridging the gap between concrete representations and abstract symbolisations.

As algebraic concepts, by their very nature, are abstract, concrete aids are required to assist in the process of conceptualisation. The topics of binomial expansion and trinomial factorisation are a corner stone in the secondary algebra course and so students' progress in later years is assisted by the mastery of these topics. Algebra Blocks have been designed as a concrete aid to assist in teaching these two topics. The model described by Lovitt et al (1984) for implementing Algebra Blocks into these two topics is the most appropriate model, as it uses a simple mechanical procedure which leads directly to the required answers.

Though concrete aids are seen by most educators as assisting in the learning process there has been no research found to confirm the effects Algebra Blocks have on achievement in algebra. As the traditional teaching method for teaching binomial expansions and trinomial factorisations is based on the manipulation of abstract symbols, an approach that concentrates on introducing the concepts through concrete representations may assist in creating greater achievement and understanding.

Research Hypotheses

1. <u>Null Hypothesis:</u>

The use of Algebra Blocks in teaching Year Nine students binomial expansions produces no difference in student understanding than their non-use.

<u>Alternate Hypothesis:</u>

The use of Algebra Blocks in teaching Year 9 students binomial expansions produces greater student understanding than their non-use.

2. <u>Null Hypothesis:</u>

The use of Algebra Blocks in teaching Year 9 students trinomial factorisations produces no difference in student understanding than their non-use.

<u>Alternate Hypothesis:</u>

The use of Algebra Blocks in teaching Year 9 students trinomial factorisations produces greater student understanding than their non-use.

Methodology

Methods of Implementation

The research project adopted a quasi-experimental design as the students were not randomly allocated into their classes. In addition, there were six clinical interviews conducted to add qualitative information to the quantitative data. The design of the research consisted of two Year Nine classes, one class was the experimental group and the other class was the control. The experimental group, Class A, was taught the topics of binomial expansion and trinomial factorisation with the aid of Algebra Blocks. The control group, Class B, was taught the same two topics in the traditional, expository teaching method. Thus the treatment being tested was the use of Algebra Blocks on achievement and understanding.

The topics of binomial expansion and trinomial factorisation are introduced in the Mathematics Development unit, Unit 3.4. All the students in the two classes had previously completed the Mathematics Development unit, Unit 3.3. Based on the progress of the students through the Mathematics Unit Curriculum and their results for Unit 3.3, the two classes should have had similar mathematical abilities. However, this research could not base its results on hearsay or perceived performance, so a pre-test was implemented before the teaching of the two topics began. Upon conclusion of teaching the two topics of binomial expansion and trinomial factorisation, the respective teachers

implemented a thirty-five minute post-test.

The two research classes were taught by two qualified teachers. Both teachers were trained by the researcher to adopt similar teaching methods. Class A was taught by Teacher A and Class B was taught by Teacher B. Teacher A was trained in the use of Algebra Blocks and followed guidelines for using Algebra Blocks developed by Lovitt et al (1984).Teacher B was asked to adopt the typical expository approach to teaching the two topics. (See Appendices B and D for tests).

After the post-test had been conducted a random selection of three students from each class were interviewed on their understanding of the topics. These interviews took approximately fifteen minutes and in which time students were asked to answer questions pertaining to their understanding of binomial expansions and trinomial factorisations and to explain their answers. These interviews provided descriptive information on the understanding of the topics that the students possessed. The data obtained from these interviews came in the form of either the student giving written answers or the researcher taking notes from observations and discussions with the students. The interviews were semi-structured to provide some flexibility to enable the interviewer to probe the student's thoughts at points of interest.(See appendix F for interview protocol).

To ensure the external validity of the research the Hawthorne effect needed to be counteracted. As with any research measuring differences in performance if the participants are aware that they are being observed or

tested they are most likely to perform at their best. To achieve this goal both teachers informed their classes that they (the students) were involved in researching the effectiveness of a new teaching strategy and that the students' achievement in the topics were going to determine this effectiveness. The aim of informing both classes was to ensure neither class was advantaged by being affected by the Hawthorne effect. Hence, it was assumed both classes had been affected by the Hawthorne effect and as a result the Hawthorne effect should have been counteracted.

The design for this research was chosen to be a quasi-experimental design plus randomly selected interviews for a number of reasons. Firstly, there was a need to obtain results about the students' achievement in the two topics so that a comparison between classes could be made. Hence , a procedure of testing the students' ability after the learning phase and analysing the data statistically was required. Secondly, though the analysis of data would give a result on the achievement patterns the data could not accurately reflect the students understanding and visualisation of the topics. Consequently , some interviews were required to broaden the scope of the data and enable analysis of the students' perceptions and visualisation of the topics. The interviews could also obtain data on the students' attitudes to the teaching strategies adopted and give some insight into the students' perceptions of the use and effectiveness of Algebra Blocks. Thirdly, no other research methodology could obtain the required information

about both the students' achievement and their
understanding of the two topics and still give reliable
and valid results.

Population

The population for the research project consisted of
two Year Nine classes which were in the process of
completing the Mathematical Development unit, Unit 3.4.
Classes A and B consisted of 32 and 30 students
respectively , all of whom had successfully completed Unit
3.3.   The students in each Unit 3.4 class had been
randomly chosen for their class from the pool of students
who had previously passed Unit 3.3.   Based on the progress
of the students through the Mathematics Unit Curriculum
and their results for unit 3.3 , the two classes appeared
to have had similar mathematical levels.   The classes
contained students of mixed ability and each sex, with
students lying between the ages of thirteen and fourteen.
Though the students formed mixed ability classes, the
average ability of either class would be higher than that
of the whole Year Nine school population as this group of
students were studying the stronger mathematics courses.

The school chosen for the research supported a
Catholic ethos and a multicultural population with
concentrations in English, Italian and Asian.   The
socio-economic status of the school was estimated to be
middle class.   The decision to choose this school for the
implementation of the research was based on the
researcher's sound knowledge of the school, the manner in
which it operated and its willingness to cooperate.   Also

the research required two Unit 3.4 classes, as it was in

Unit 3.4 that binomial expansions and trinomial

factorisations were introduced. The school met this

requirement as it was teaching three Unit 3.4 classes when

the research was implemented.

Three different teachers , all male , taught the

three Unit 3.4 classes. Only two of the three classes

were required to implement the research and these were

chosen for different reasons. Class A was chosen as the

class to be taught with Algebra Blocks as the teacher was

willing to use the Algebra Blocks in his lessons. Class B

was chosen between the remaining two classes as this

teacher was willing to cooperate.

The population was limited to the two classes so that

the number of students being treated was approximately

equal to the number of students untreated. The third

class was not considered as another variable, that is , a

different teacher , would have lessened the predictive

validity and reliability of the results. For simplicity

and reliability only two classes were used to implement

the research.

Instrumentation

The research required two tests , a pre-test and a

post-test. (See Appendices D and B). The pre-test was a

school designed assessment test. This test measured the

student's ability in the content covered in Unit 3.4

before the teaching of binomial expansions began. The

test covered some general aspects of algebra such as

operations with negatives and solving linear equations ,

along with some arithmetic. The pre-test was a gauge of
the starting abilities of the students in the subject of
algebra. A pre-test was necessary to compare the starting
abilities of the students in each class. The pre-test was
marked by the classes' respective teachers.

The post-test intended to measure the effect Algebra
Blocks had on student achievement in the topics of
binomial expansion and trinomial factorisation. The test
consisted of sixteen questions - eight requiring binomial
expansion and eight requiring trinomial factorisation.
The sixteen questions were chosen in consultation with the
two teachers involved in the research. From the
discussions with the teachers each question was assured of
its validity in measuring the student's achievement in
expanding binomials and factorising trinomials. Only
sixteen questions were chosen as any fewer would not
obtain reliable results and any more questions would take
the students too long to answer.

The post-test had two sections - binomial expansion
and trinomial factorisation - and within each section
examples were graded in difficulty. The grading of the
test questions was as suggested by the teachers involved.
The binomial expansion section only contained examples
with a positive coefficient of 1 in front of the variable
'x'. That is , (x+3) could have been a binomial
expression used as the variable 'x' was only being
multiplied by the number one. However,(2x +3) or (-x +3)
were not binomial expressions used in the test as the
variable 'x' was being multiplied by the numbers 2 and -1
respectively. The possible combinations of multiplying

these binomial expressions were tested, that is , binomial expressions that were multiplying a : positive by positive (eg.(x + 2)(x + 3)) ; positive by negative (eg.(x+2)(x-3)); negative by positive (eg.(x-2)(x + 3)); and negative by negative (eg.(x - 2)(x - 3)). The test contained two examples of each of the four combinations for multiplying binomials , thus making eight questions. This gave the test validity in measuring the students' achievement in each type of question.

The trinomial factorisation section tested only monic trinomials (in a monic trinomial the coefficient of the x^2 term is 1). Only monic trinomials were tested as this was all the Unit 3.4 objectives warranted. The four combinations of trinomial expressions , that is, positive:positive (eg.x^2 + 2x +1), negative:positive (eg.x^2 - 2x +1), positive:negative (eg.x^2 + 2x -3) or negative:negative (eg.x^2 -2x-3), which can be factorised into simple binomial expressions, were tested. Again , two examples of each of the four combinations were tested to obtain valid results.

The binomial expansion and trinomial factorisation sections of the post-test were designed in this way for two reasons. Firstly , it allowed for testing the students' abilities in the full array of binomial and trinomial examples which were allowed by the unit objectives , thus providing the test with face validity. Secondly , it allowed for a split halves reliability test to be conducted to test the reliability of each section of the test. Using odd versus even numbered test items , the students scores for both their odd test and even test were

correlated for both sections of the post-test. After taking the shortened test into account the reliability of the binomial expansion sub-test was r = 0.97 and for the trinomial factorisation sub-test r = 0.91.

The marking of the post-test was done by two colleagues, both of whom were qualified mathematics teachers. Both markers each marked all the test papers and compared the results for any discrepancies in students' marks. If any discrepancies existed, the two markers discussed the situation between themselves and came to a common agreement on the students' marks. The markers awarded marks based on the criteria in the marking key. (See Appendix C). The markers did not know which class the individual papers belonged to and they were neutral to the situation. This marking procedure had a small subjective element involved , however this subjectivity lay in the markers and not the researcher.

A different marking key , whereby either zero or one mark was awarded for an incorrect or correct answer respectively , was considered. Though this approach was considered extremely objective, it did not allow for silly errors or misinterpretations that often occur in mathematics and in particular algebra. With a "hit or miss" approach to marking the results obtained would not have been a reliable measure of the student's ability, as the student may have had a full understanding of the processes involved but may have made an error in transcribing the information. Therefore the option to reward students with some marks for obviously understanding the processes involved but for some reason

obtaining the wrong answer, was adopted. Additionally, having neutral markers working in liaison was considered to produce the most valid means of accurately measuring the students' achievement in the topics.

Data Collection

The pre-test results were collected from the respective teachers with a number representing a student and a corresponding number representing the student's mark. The post-test was implemented in the classroom after a total of eight fifty minute teaching lessons in the topics of binomial expansion and trinomial factorisation. Each class was given thirty-five minutes to complete the test , after which the test was collected by the teachers. Each test paper had the student's identification number on it corresponding with the number given to them after the pre-test. The test papers were handed on to the markers who followed the guidelines set down in the marking key.

The training of the teachers and markers was necessary to obtain predictive validity and reliability in the test results. The teachers were trained in one-to-one discussions on the following points. Firstly, both teachers were asked to adopt the same approach to setting out both binomial expansions and factorising trinomials, so that they would teach the concepts in a similar way. (The setting out of the solutions to expanding binomials and factorising trinomials can be seen in the letters to the teachers in Appendix G). Secondly, a common approach to factorising trinomials was needed. Each teacher was asked to adopt the split method approach (see letters to

teachers in Appendix G) to achieve a desirable level of comparability in teaching methods. Thirdly, each teacher was asked to use examples from Fundamental Mathematics Book 2 , chapters sixteen and seventeen. Fourthly, emphasis was placed on the teacher acting naturally in front of the class , but also to ensure to as great a degree as possible to achieve a positive , happy mood in each class. These points were necessary to achieve some comparability between the two class teachers' teaching methods and their classroom environments.

The markers were trained together in a discussion of the requirements necessary to provide reliability and validity in the marking. The markers were asked to be objective and most importantly consistent in their awarding of marks. Also emphasis was placed on the need to follow the guidelines set out in the marking key before marks were awarded or deducted.

Ethical issues needed to be considered in relation to the access of the data. To comply by the school's ruling on conducting research in the school, the students were identified only by a number. As the students' achievement in schooling is a private matter no names were sought or required for the research. With anonymity the problem of data access was overcome and so the markers had free access to the students' papers.

Data Analysis

The data collected from the pre-test was analysed using a two-tailed, two-sampled t-test with sixty degrees of freedom and α =0.05. The null hypothesis being

tested was that there was no difference between the means for the two classes in the pre-test. The alternate hypothesis was that the means were not equal.

The data collected from the post-test was first analysed in a tabulated form. Each class had their results tabulated showing each student's score for each individual question in the binomial expansion and trinomial factorisation sections along with their respective totals. Additionally, a one-tailed, two-sampled t-test with sixty degrees of freedom and α = 0.05 was used to analyse the significance of the results obtained for each test. The alternate hypothesis for each objective was that the use of Algebra Blocks produces greater understanding than their non-use.

<u>Results</u>

<u>Statistical Significance</u>

The pre-test results (Table 1) indicate that class B scored slightly higher than class A on the pre-test , however this difference in the means was not statistically significant at the 0.25 level. As the sample size used (n= 62) was relatively large an estimate from the normal distribution curve showed that there would still be no statistical significance in these pre-test results at p<0.39.

After the treatment period, the results from the post-test (table 2) show that in the binomial expansion section of the test , the treated class , class A , scored a higher average total. However, this difference in the means was not statistically significant at the 0.05 level. Hence , as $t_{observed}$ < $t_{critical}$ at the 0.05 level , the null hypothesis was accepted . That is , based on the results from the post-test there was no difference between the two classes' understanding of expanding binomials.

Similarly , the treated class scored a higher average total on the trinomial factorisation section of the post-test.(Table 2). On this occasion the difference in the means was significant at the 0.01 level as $t_{observed}$ > $t_{critical}$. Therefore the alternate hypothesis was accepted , that is , the use of Algebra Blocks produces greater student understanding in factorising trinomials than their non-use.(The full complement of marks given for each student on each question can be seen in Appendix H).

Table 1: Pre-test results.

Class A	Class B
(n = 32)	(n = 30)
mean = 52.25	mean = 53.6
standard	standard
deviation = 16.63	deviation = 22.84

$$t_{observed} = -0.2673$$
$$t_{critical} = -2.0000$$

(Means and standard deviations are in terms of percentages)

Table 2 : Post-test results.

Group Test	Class A (n = 32)	Class B (n = 30)
Binomial	mean = 88.4	mean = 82.2
	standard	standard
	deviation = 22.6	deviation= 26.5

$$t_{observed} = 0.9934$$
$$t_{critical} = 1.671$$

Trinomial	mean = 70.2	mean = 49.5
	standard	standard
	deviation = 29.3	deviation= 33.8

$$t_{observed} = 2.5892$$
$$t_{critical} = 2.390$$

(Means and standard deviations are in terms of percentages)

Results from Interviews

The general perception from the interviews was that the students interviewed from class A could visualise and in particular, represent their answers to the given questions via diagrams. Students interviewed from class B could solely represent their answers in the symbolic mode and could not see how their answers could be represented either diagrammatically or by an embodiment.

When the interviewed students were asked what they understood by the algebraic expression $(x + 2)(x + 3)$ and to draw a diagram of this expression ; the three students interviewed from class A could all draw a picture of a rectangle with sides $(x + 2)$ and $(x + 3)$ and relate the answer of this expansion back to the area of the rectangle, that is , $x^2 + 5x + 6$ as the area of the rectangle with side lengths $(x + 2)$ and $(x + 3)$. However, none of the three students interviewed from class B could draw a diagram of this expression. Even after prompting each of the students interviewed from class B by drawing diagrams of rectangles with integer sides and finding their areas , only one of the three students could relate the expanded form of this expression back to the area of a rectangle with side lengths $(x + 2)$ and $(x + 3)$.

Similarly, when dealing with trinomials, all three students interviewed from class A could represent $x^2+9x+20$ as the area of a rectangle , with each binomial expression from its solution : $(x+4)(x+5)$, representing the lengths of the rectangle's sides. However , the three students interviewed from class B could not represent the trinomial

x^2 + 9x + 20 as the area of a rectangle with the lengths of the rectangle's sides being represented by the trinomial's factorisation.

The students interviewed from class A all used the appropriate algorithm to solve the questions given in the interviews. However, two of the three students went back to drawing diagrams of the Algebra Blocks when faced with a difficult question or when they knew they had the wrong solution. In one example , one of the students interviewed had trouble with adding -4x and 3x , but once he drew a diagram of the question this minor problem was easily resolved as the student could see the answer in the diagram.

All the students interviewed from class B attempted to solve the questions given using the learnt algorithms. When one student was asked why he simplified (x + 3)(x +2) to x(x + 2) + 3(x + 2) he could not explain or provide reasons for choosing to do this step. It appeared that the student had rote learnt the algorithm but had no understanding of what each step in the algorithm represented.

When a student interviewed from class A was asked how he knew that (x+3)(x+2) was equal to x^2 + 3x +2x + 6 ; he simply drew the following diagram (see Figure 1) to show how x^2 + 3x + 2x + 6 adds up to a rectangle with sides (x+3) and (x+2).

It appeared from the interviews that the students interviewed from class A had some embodiment or visual representation of the concepts to fall back onto when a trouble spot in the solution was reached. This was in

<u>Figure</u> <u>1</u> : <u>Diagram showing student's understanding of:</u>

<u>(x+3)(x+2)</u> = <u>x²</u> + <u>3x</u> + <u>2x</u> + <u>6.</u>
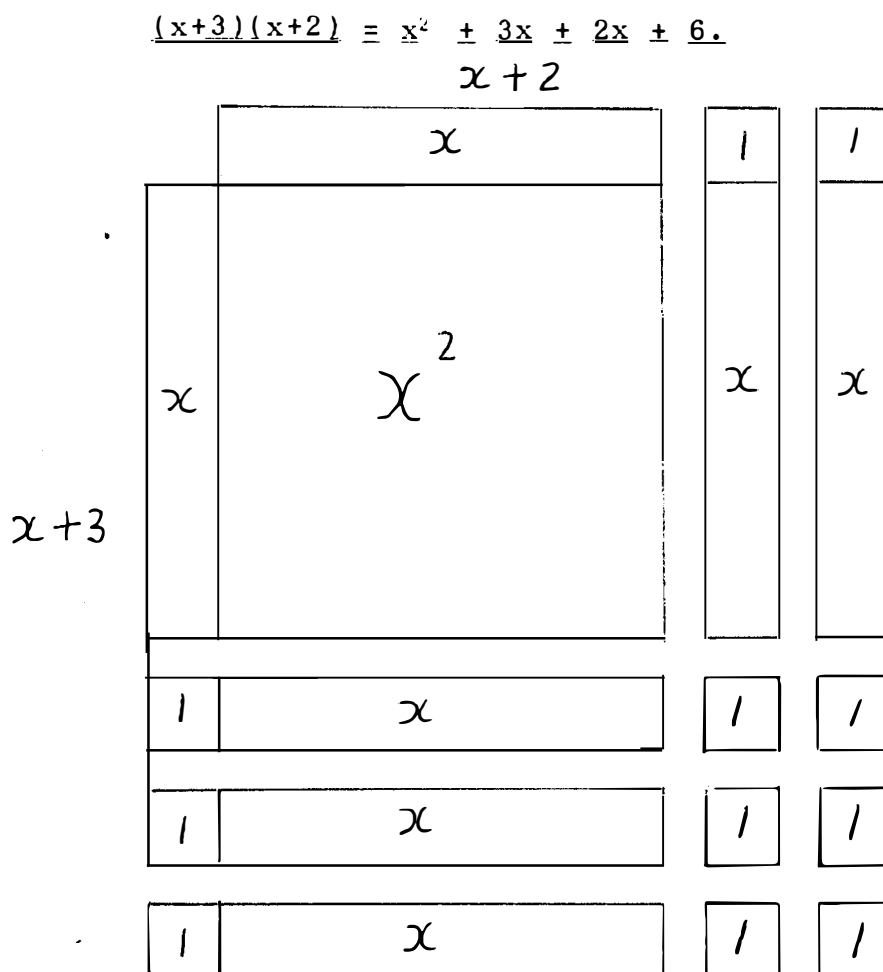
stark contrast to students interviewed from class B who
had no other means by which to represent their solutions.
Consequently, these students had to work back through
their solutions when a trouble spot was reached and look
for incorrect symbols , signs or processes. Though this
process of going back through the symbolic representation
of the solution was often quicker than drawing diagrams of
the solution to find the error, it appeared that when a
student interviewed from class B could not understand or
find where they had gone wrong they had no other means of
checking their solutions. Subsequently , these students
would often provide the wrong answer. Students
interviewed from class A always had the diagrammatic

representation of their solution to look back onto so they could see where they had made the mistake and subsequently translate the correction back into the symbolic solution.

In the examples from the post-test , many students from class B made fundamental mistakes because they could only represent the solution in the symbolic mode. For example , in the binomial expansion section of the post-test, five of the thirty students from class B added or subtracted the numerals in each binomial expression instead of multiplying them ; for example they would write, $(x+\underline{4})(x+\underline{7}) = x^2+11x+11$. (The underlined numerals should have been multiplied to give 28 , but the students have added them to give 11). This type of error occurred 16 times in the answers given by students from class B. This type of error also occurred among students in class A but the error occurred only eight times in the answers given by students from class A.

A common misconception of factorising trinomials by students from class B was :

$$x^2 +10x +21$$
$$= (x+5)(x+5)$$

OR

$$x^2 -14x +48$$
$$= (x+7)(x+7)$$

Here the students were simply obtaining the numerals in the two binomial expressions by dividing the coefficient of the "x" term by two and ignoring the positive and negative signs. This did not appear to cause a problem in the students from class A.

The students interviewed who used Algebra Blocks said that they all enjoyed their use and thought they were of some value to their understanding of the concepts. The brightest student of the three students interviewed from class A preferred to rely on the symbols and learnt algorithms but said that he still enjoyed the use of Algebra Blocks and could use them to visualise his answers.

The topics of binomial expansion and trinomial factorisation are often easy topics for teachers to teach but students find the concepts difficult to learn and understand. The teacher who used the Algebra Blocks said he enjoyed the experience and found that the students seemed to enjoy using them. The teacher reported that many students continued to draw diagrams after the use of Algebra Blocks had ceased.

Summary

The results from the experiment indicated that Algebra Blocks produced greater student achievement when used to factorise trinomials but there was no difference in student achievement when they were used to expand binomials. The results from the interviews appeared to support the use of Algebra Blocks in teaching both concepts as they create better student understanding and visualisation. The students who used Algebra Blocks appeared to have multiple embodiments of the same concept providing a better understanding.

The following chapter will discuss the results , placing them in perspective with the learning theories and

relevant literature. Additionally, a discussion on the limitations, implications and further research will immediately follow the discussion of results.

Conclusion

Discussion of Results

The results indicate that Algebra Blocks had a positive effect on student understanding in both expanding binomials and factorising trinomials. Although the statistical data indicated Algebra Blocks had no effect on improving the students' understanding of expanding binomials, the interviews appear to contradict this suggestion. From the interviews the students who used Algebra Blocks appeared to have formed some imagery of the concept of expanding binomials and linked this concept back to prior mathematical knowledge, that is, the area of a rectangle. Hence, the students who used Algebra Blocks had a more informed understanding of what is meant by expanding binomials.

In the trinomial factorisation section both the statistical data and the student interviews overwhelmingly supported the use of Algebra Blocks in producing a greater understanding of the concept. From the interviews there was a clear, distinguishing gap between the imagery that the students who used Algebra Blocks had of the concept compared to those students who did not use Algebra Blocks. The students who used Algebra Blocks could represent the concept diagrammatically and so they could work with a visual image. Additionally, this imagery was related to prior mathematical knowledge, that is, the area of a rectangle. Therefore, the students could learn the concept of factorising trinomials in a way suggested by Skemp (1971) - by interrelating the new mathematical

concept (trinomial factorisation) with other concepts
(area of a rectangle) to form knowledge and understanding.

The results obtained for using Algebra Blocks
supports the belief held by Dienes (1971) and Bruner
(1966) for teachers to teach new concepts in the form of
multiple embodiments, that is, to teach the concept in a
variety of forms. Students who did not use Algebra Blocks
could solely manipulate symbols, whereas those students
taught with Algebra Blocks had the opportunity to
manipulate Algebra Blocks, diagrams and symbols. This
choice in concept representation assisted in the students'
understanding of the concepts as they could learn in their
preferred mode of representation and subsequently transfer
their answer into the symbolic mode.

The interviews support the views held by Beattie
(1986a) and Thorton (1986) that manipulatives assist in
the learning and understanding of algorithms. This was
noted in the results when a student who did not use
Algebra Blocks could not give an explanation for doing the
step - (x+2)(x+3) = x(x+2) + x(x+3). Clearly, the student
could perform the algorithm but it appeared the student
did not actually understand the reasons behind the steps.
This student was being taught -and consequently learning -
instrumentally, that is, "using rules without reason."
(Skemp,1976,p.20). In contrast the students using Algebra
Blocks appeared to have a relational understanding of the
concept, that is, "knowing both what to do and why."
(Skemp,1976,p.20). These students could link the new
concept to prior knowledge and follow the reasons behind
the algorithm's steps by the manipulation of the Algebra

Blocks. This relational understanding was seen in the interviews when a student was able to explain, using an Algebra Block diagram, that $(x+2)(x+3) = x^2+2x+3x+6$.

From the interviews and observations made from the students' answers in the post-test, it appeared the algorithm for factorising trinomials was a source of difficulty for many students. Many students from class B had misconceptions about the use of the algorithm for factorising trinomials as they did not understand the relationships involved in the algorithm. However, students from class A appeared to have less trouble understanding and implementing the processes involved in factorising a trinomial. It is believed that the manipulation of Algebra Blocks played an important role in establishing the procedure and structure for students to follow the flow of the algorithm for factorising trinomials. When students who used Algebra Blocks had problems finding the factors of the trinomial, the students chose to revert back to drawing diagrams to organise their thinking and logically follow through the procedure. The role Algebra Blocks played in creating conceptual understanding was a major difference in each class's understanding of the algorithm for factorising trinomoials. These results support the findings of Beattie (1986a), Berman and Friederwitzer (1989), Thorton (1986) and Worth (1986), all of whom believe that manipulatives assist in organising the student's thinking; developing conceptual understanding; discovering relationships; and learning algorithms.

The results of this study reflect the theoretical

framework on which this research was built, that is, the need to teach from concrete to abstract as suggested by Piaget, Bruner and Dienes. In the study, students from class B were solely taught the two concepts in their abstract, symbolic mode, whereas the students using Algebra Blocks had the opportunity to see the concept develop from the concrete Algebra Blocks, through the intermediate diagrammatic mode and finally to manipulating the abstract symbols. The results indicate the mode of instruction that goes from concrete to abstract produces greater student understanding. Therefore, the results confirm the views held by Beattie (1986a), Fennema (1973), Heddens (1986) and Hynes (1986), that is, to introduce mathematical concepts with manipulatives to bridge the gap between the student's concrete environment and the abstract level of mathematics being taught.

The experimental results obtained from the binomial expansion section suggest there was no difference between the two classes' understanding of the concept. This result may have been affected by the students' lack of experience with using Algebra Blocks. Observations made from the students' test answers and a discussion with teacher B suggested that the algorithm for expanding binomials was not a difficult algorithm to learn by a purely symbolic, expository method. However, the results from the interviews appear to conclude that this symbolic, expository approach does not provide the much needed understanding of each step of the algorithm. It is therefore likely that the students who used Algebra Blocks when expanding binomials understood the steps of the

algorithm better than the untreated class. Subsequently, these students may have transferred this knowledge to the trinomial factorisation section - as binomial expansions and trinomial factorisations are inverse procedures. This transfer of knowledge, along with the added experience the students had in using Algebra Blocks may have provided the particularly favourable experimental results in the trinomial factorisation section.

The student interviews suggested that the Algebra Blocks provided a source of enjoyment and motivation. Also both the interviews and experimental results (particularly for trinomial factorisations) suggested that Algebra Blocks assisted in producing greater student understanding of the two concepts. Hence the results of this study support the view held by Herbert (1985) that manipulatives assist in providing : motivation; stimulation to think mathematically; and a means of introducing difficult concepts.

Limitations

The results of this study need to be viewed in the light of certain limitations in the design of the research methodology. Firstly, two teachers were involved in the study, whereas ideally one unbiased teacher would have been more suitable. However, the two teachers used were "trained" to adopt similar teaching approaches, but their different classroom behaviours and attitudes towards their students would have affected the classroom learning environment.

Secondly, the number of students involved in the

research was small in comparison with the state-wide population. This would not have had adverse affects on the results obtained, but it would affect the study's external validity or generability, that is, the ability to accurately generalise from the research results obtained to the larger state-wide population. However, sufficient details have been given about the sample population used for the research so it is up to the individual teacher to decide if the sample population reflects the attributes of their class.

Thirdly, there is evidence that manipulatives assist in concept retention (Skemp 1971). Therefore, a limitation of this study was the absence of the administration of a concept retention test approximately six to eight weeks after the post-test was implemented. Based on results from past studies a concept retention test may have produced statistically significant experimental results in favour of using Algebra Blocks for both binomial expansion and trinomial factorisation.

Nevertheless, the instruments used in this study to test student understanding were very reliable and valid. Using a split-halves reliability test for both sections of the post-test, the sub-test reliabilities were calculated as $r = 0.97$ for the binomial expansion section and $r = 0.91$ for the trinomial factorisation section. From discussions with the teachers involved in the research the test questions were considered to be valid, that is, the questions were measuring what they purported to measure.

No research study is perfect as there are many intervening variables that the researcher either can not

control or was not aware of their influence, which impinge on the reliability and validity of the results. The results of this study need to be viewed with this statement in mind.

Implications

Viewing the results in the light of the limitations mentioned, certain implications can be drawn from the results of this study. Firstly, the use of Algebra Blocks in algebra instruction - particularly binomial expansions and trinomial factorisations - should become more frequent. Research by Scott (1987) concluded that there was a perceived increase in the use of mathematics materials once the material kits are purchased for teachers and a variety of activities are provided. Hence, Algebra Block kits need to be purchased by the schools or the students need to be encouraged to make their own set from card and the teachers made aware of the use and effectiveness of Algebra Blocks in algebra instruction. There is no point in mathematics educators designing these manipulatives and researching their effectiveness on student understanding if the results are not communicated to the teachers so that they can implement the ideas into the classroom.

Any new development which succeeds in producing greater student understanding of mathematics concepts must be worth implementing into the classroom. Algebra is the backbone of the secondary school mathematics curriculum, therefore, to provide students with a better imagery of some algebra concepts - as Algebra Blocks appear to

achieve in binomial expansions and trinomial factorisations - can only lead to better understanding and better results in other aspects of the mathematics curriculum.

Mathematics educators must continue to move with the changes and developments in their field, otherwise students might still be using the abacus instead of a calculator. Algebra Blocks appear to provide the students with the necessary imagery of the concepts that so many educators believe leads to better conceptual understanding. Additionally, Algebra Blocks provide a source of enjoyment which seems to be lacking in many mathematics classes. Therefore, this research implies that as Algebra Blocks have proven to be an effective and enjoyable learning aid, they should be implemented into the mathematics classrooms.

Further Research

This research concentrated on the use of Algebra Blocks in teaching binomial expansions and trinomial factorisations. Further research could study the effect Algebra Blocks have on student understanding on a number of other concepts, such as : collecting like terms, expanding and factorising simple algebraic expressions, understanding the meaning of 'variable ', solving linear equations and solving quadratic equations. These studies may provide further evidence for using Algebra Blocks.

Further research could take place looking at the effects Algebra Blocks have on concept retention. Within a study on concept retention a larger sample of interviews

may provide more insight into the student's thought processes, development of imagery and the effect the student's imagery has on concept retention.

This study has evaluated the effectiveness of Algebra Blocks on algebra in the classroom setting. Similar studies could attempt to evaluate the effectiveness of other manipulatives in the area of mathematics. Additionally, a research study may design a new manipulative aid and evaluate its effectiveness.

Though this study concluded that Algebra Blocks are an effective manipulative aid in assisting in gaining greater student understanding, it needs to be made clear that the Algebra Blocks were only an intermediate step to bridge the gap between the student's concrete environment and the abstract level of mathematics being taught. For clearly, the advantages of mathematics do not lie in the manipulation of blocks but rather in the manipulation of symbols. Therefore, this study supports the view held by Heddens (1986), that is, " the need for a careful sequencing of activities to lessen students' dependence on the concrete level and increase their facility with the abstract level is crucial." (p.17).

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QUADRATIC REPRESENTATIONS

Lesson Q. 1

The blocks are an AREA based representation.

$1 \begin{array}{c} \square \\ 1 \end{array}$
$\therefore$
Area = 1

$1 \begin{array}{c} \rule{3cm}{0.4cm} \\ x \end{array}$
Area = x

x
Area = x^2

$-1 \blacksquare \atop 1$
Area = -1

$1 \blacksquare \atop -x$
Area = -x

x
$-x$
Area = $-x^2$
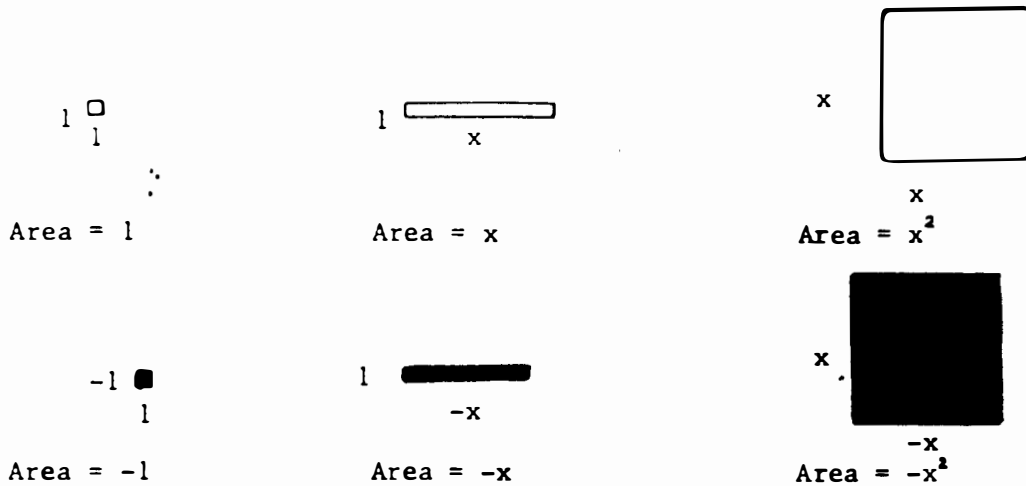
Pre-requisite skills

    (i)   knowledge that area of rectangle = length x width

    (ii) ability with integer arithmetic

Prior experiences using blocks in linear algebra is **not essential,** though would clearly be an advantage.

Algebra	Area representation
$x^2 + 4x - 2$	
$-2x^2 + 3x$	
$2(x^2 + 1)$	2 lots of $x^2 + 1$
$4 - x^2$	NOTE: $4-x^2$ is easily shown to be the same as $-x^2 + 4$
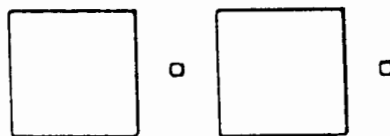
* Source: Lovitt,C.,Marriot,C.,& Swan,K.(1984). Lessons in algebra using algebra blocks. Laverton,Victoria: EDU-DOMES.

Lesson Q. 2

A fundamental observation needs to be stressed

i.e.　☐　+　■　= 0　i.e. nothing (1 + -1 = 0)

▭ + ▬ = 0 　　(x + -x = 0)

☐ + ■ = 0 　　$(x^2 + -x^2 = 0)$

Example:　Simplify $3x^2 + 5x - 2 - 2x^2 - 6x + 5$

Achieved by putting in each succesive term then removing nothings.

becomes
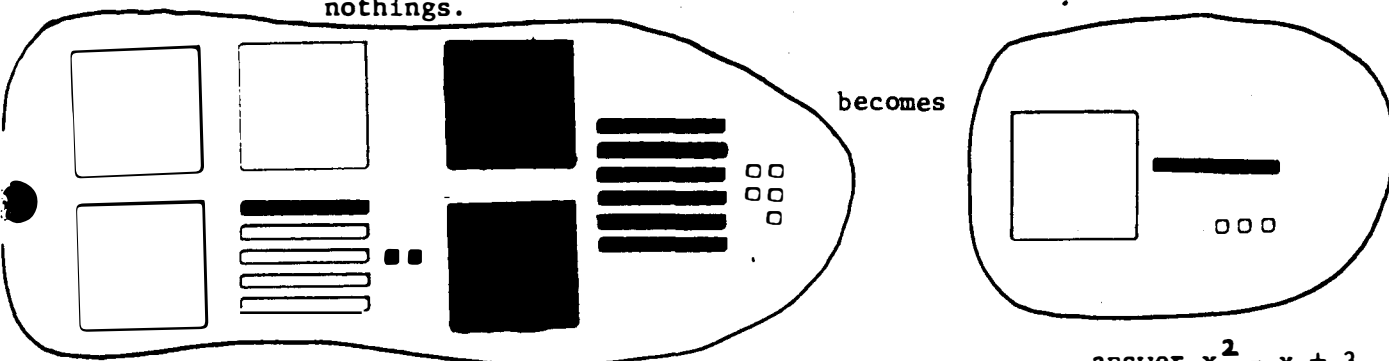
answer $x^2 - x + 3$

BACKGROUND

To simplify 2x - 3x we could employ either of two methods

Method 1　2x + -3x
　　　i.e. put in 2x and -3x
　　　and simplify

Method 2　2x - 3x
　　　put in 2x and take out 3x

take out nothings

leaves -x

Can't take out 3x so must add a nothing

now remove 3x

answer -x
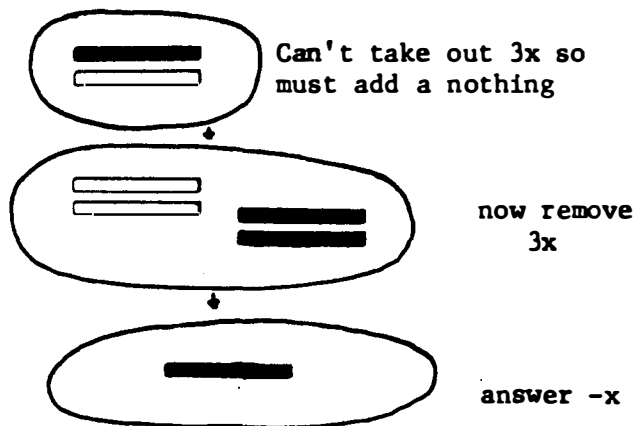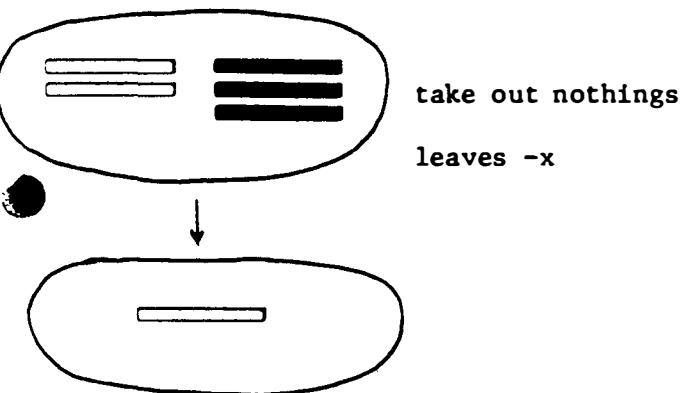
Both methods work but we will use method 1 since it is more closely linked to formal pencil and paper methods.

Lesson Q. 3

Background: (x + 2)(x - 3)

The area representation parallels the formal expansion
method using the distributive property.

Step 1 Set up the multipliers (factors) on
the frame.
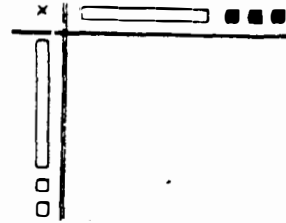
Step 2 (x + 2)(x - 3) means <u>all</u> of the (x + 2) is to be multiplied
by <u>all</u> of the (x - 3). Systematically finding the <u>12</u>
<u>separate products</u> produces the following <u>rectangle.</u>
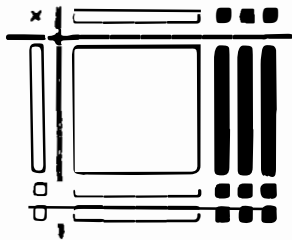
x lots of x - 3 i.e. x(x - 3)

2 lots of x - 3 i.e. 2(x - 3)

Analysis of each step

(x + 2)(x - 3)

a rectangle is created of <u>width</u> (x + 2)
and <u>length</u> (x - 3)

= x(x - 3) + 2(x - 3)

This can be seen as two separate
rectangles namely x lots of (x - 3)
and 2 lots of (x - 3) (see diagram)

= x^2 - 3x + 2x - 6

Each of the two rectangles is composed
of smaller parts

= x^2 - x - 6

Simplify by removing 'nothings'
Remember ⊏⎯⎯⎯⊐ + ⊏⎯⎯⎯⊐ = 0

Focus on Distributive Property:

The diagram above has 12 products. This in fact illustrates the

distributive property taken to extremes.

i.e. (x + 2)(x - 3) = (x + 1 + 1)(x - 1 - 1 - 1) which produces 12 products

e.g. the larger square ▢ (x^2) is the product of x . x

each unit ▢ is the

product of 1 . -1.

QUADRATIC EXPANSIONS

Worksheet for Lesson Q. 3

Expand the following using blocks. Show all 4 lines of working as in examples. MAKE SURE YOU CAN 'SEE' EACH LINE OF WORKING IN THE DIAGRAM.

<u>Type 1</u>   e.g. $(x + 3)(x + 1)$
$$= x(x + 1) + 3(x + 1)$$
$$= x^2 + x + 3x + 3$$
$$= x^2 + 4x + 3$$

1. $(x + 2)(x + 4)$
2. $(x + 1)(x + 5)$
3. $(x + 3)(x + 3)$
4. $(x + 0)(x + 4)$ i.e. $x(x + 4)$
5. $(x + 6)(x + 2)$

<u>Type 2</u> e.g. $(x + 3)(x - 1)$
$$= x(x - 1) + 3(x - 1)$$
$$= x^2 - x + 3x - 3$$
$$= x^2 + 2x - 3$$

1. $(x + 4)(x - 2)$
2. $(x + 5)(x - 1)$
3. $(x + 3)(x - 3)$
4. $(x + 7)(x - 1)$
5. $(x + 5)(x - 4)$

<u>Type 3</u> e.g. $(x - 3)(x + 1)$
$$= x(x + 1) - 3(x + 1)$$
$$= x^2 + x - 3x - 3$$
$$= x^2 - 2x - 3$$

1. $(x - 4)(x + 2)$
2. $(x - 6)(x + 1)$
3. $(x - 2)(x + 2)$
4. $(x - 5)(x + 3)$
5. $(x - 6)(x + 4)$

<u>Type 4</u> e.g. $(x - 3)(x - 1)$
$$= x(x - 1) - 3(x - 1)$$
$$= x^2 - x - 3x + 3$$
$$= x^2 - 4x + 3$$

1. $(x - 4)(x - 2)$
2. $(x - 3)(x - 1)$
3. $(x - 2)(x - 2)$
4. $(x - 4)(x - 3)$
5. $(x - 6)(x - 5)$

Lesson Q. 5

The purpose of this lesson is to develop factorization as the exact reverse of expansion. Both are based on forming a rectangle.

EXPANSION

$(x + 2)(x + 4)$

$= x(x + 4) + 2(x + 4)$

$= x^2 - 4x + 2x + 8$

$= x^2 + 6x + 8$

$x^2 + 6x + 8$

$= x^2 + 4x + 2x + 8$

$= x(x + 4) + 2(x + 4)$

$= (x + 2)(x + 4)$

The formal method of factorization requires the 6x to be split into two parts. In the blocks this is represented by the splitting of the 6x so that a rectangle can be formed. Finding the split can be systematically arrived at with the following method which we shall call the SPLIT METHOD.

e.g. 1 $x^2 + 6x + 8$

?   ?   The problem becomes;

How to split the 6x so that 8 units will complete the rectangle.

(i)   line all 6x along the length   (ii)   move one x to the width

i.e. a 6, 0 split   i.e. a 5, 1 split
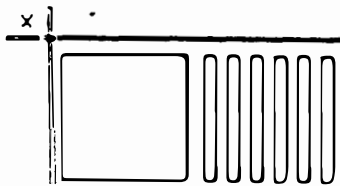
needs 0 units to complete rectangle (it is already a rectangle)
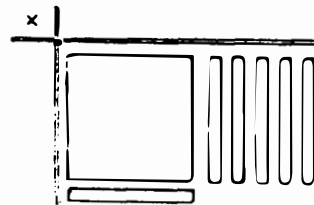
needs 5 units to complete rectangle

(iii)   move another x to the width

i.e. a 4, 2 split
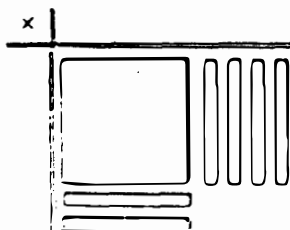
needs 8 units. Hence 4, 2 is the split we are seeking

The rectangle can often be found quickly by inspection or trial and error. However, in more difficult cases a systematic method of searching for factors is necessary. With the SPLIT method IF FACTORS EXIST YOU WILL QUICKLY FIND THEM and students will never be uncertain about what to try next.

Lesson Q.5 (cont.)

e.g. 3 $x^2 - 9x + 20$

? ?   How to split the ⁻9x so that ⁺20 will complete
          the rectangle.

(i)   line −9x along the length
      i.e. a⁻9, 0 split

(ii) move a − x to the width
      i.e. an⁻8,⁻1 split
                        etc.

      The search is summarised
      in this table

SPLIT	UNITS NEEDED FOR RECTANGLE
-9, 0	0
-8, -1	+8
-7, -2	+14
-6, -3	+18
-5, -4	+20

Hence −5, −4 is the
split we are
seeking
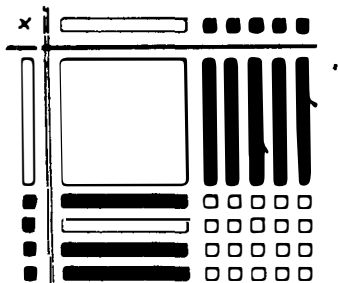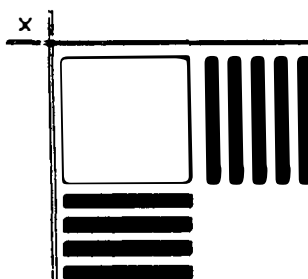
$x^2 - 5x$

$-4x + 20$

$x^2 - 9x + 20$

$= x^2 - 5x - 4x + 20$

$= x(x - 5) - 4(x - 5)$

$= (x - 5)(x - 4)$

<u>Notes</u> a) $x^2 - 5x$ and $-4x + 20$ can be seen as two separate rectangles

b) $x^2 - 5x = x(x - 5)$

and $-4x + 20 = -4(x - 5)$

i.e. Each of these two rectangles
has factors

c) The <u>common factor</u> of (x − 5) is seen as putting together 2
rectangles of the same length i.e. (x − 5). This produces
a larger rectangle of the same length (x − 5) but which

Appendix B
BINOMIAL EXPANSION AND TRINOMIAL FACTORISATION TEST

This test involves answering the sixteen (16) questions below. The first eight (8) questions require binomial expansion whilst the second eight (8) questions require factorising trinomials. You have thirty five (35) minutes in which to answer the questions.

Good luck !

1. $(x + 2)(x + 3)$

2. $(x + 4)(x + 7)$

3. $(x - 1)(x + 5)$

4. $(x - 4)(x + 6)$

5. $(x + 6)(x - 5)$

6. $(x + 4)(x - 5)$

7. $(x - 2)(x - 4)$

8. $(x - 4)(x - 9)$

9. $x^2 + 10x + 21$

10. $x^2 + 8x + 15$

11. $x^2 - 8x + 12$

12. $x^2 - 14x + 48$

13. $x^2 + x - 30$

14. $x^2 + 3x - 10$

15. $x^2 - 5x - 24$

16. $x^2 - 13x - 30$

Appendix C

MARKING KEY

Marks will be allocated on the basis of the following:

a) One mark will be allocated for each correct step. This implies a 3 mark maximum for binomial expansions and a 4 mark maximum for trinomial factorisations with a 0 mark minimum for both. No half marks shall be awarded.

b) If the answer is correct and no working has been shown, then full marks shall be awarded. Conversely, if the answer is incorrect and no working has been shown , then no marks shall be awarded.

c) One mark will be deducted if the student transcribes information from one line to the next incorrectly. This implies every time a wrong sign , number or variable is written a mark will be deducted, unless the mistake is a continuation of a previously made mistake. So if the student continues to use the correct processes after making an error the error will only count once.

Steps for solving Binomial Expansion :

eg. $(x + 3)(x - 4)$

$= x(x - 4) + 3(x - 4)$ ... STEP 1

$= x^2 - 4x + 3x - 12$ ... STEP 2

$= x^2 - x - 12$ ... STEP 3

Steps for Factorising Trinomials :

e g. $x^2 - x - 12$		SPLIT	UNITS NEEDED
$= x^2 - 4x + 3x - 12$	STEP 2	1,0	0
$= x(x - 4) + 3(x - 4)$	STEP 3	2,1	-2
$= (x - 3)(x - 4)$	STEP 4	-3,2	-6
		-4,3	-12
			...STEP 1

Mathematical Development 3.4

TEST 1

1. Calculate:
   (a) -6 + 7 - 19
   (b) (4 ÷ 8) x (5 + 2)
   (c) -2 x 12 - 25

   (d) -5² - 3
   (e) (-4)² + 1
   (f) (-36 - 4) - 8

2. I have overdrawn my bank account and it shows -$42.
   What would be the new balance if I were to deposit:
   (i) $40 ?
   (ii) $21 ?
   (iii) $68 ?

3. Write each of the following numbers as a product of prime factors:
   (a) 12
   (b) 44

   (c) 150
   (d) 142

4. Complete the following pyramids:
   (a)   (b)   (c)
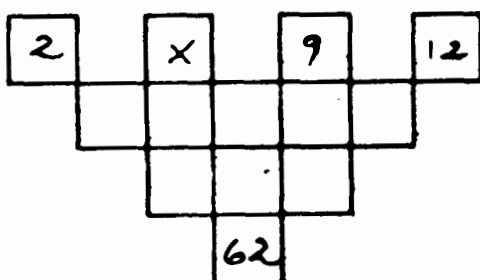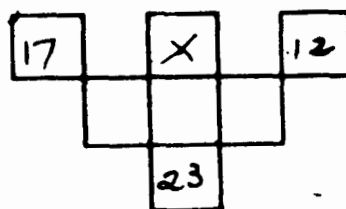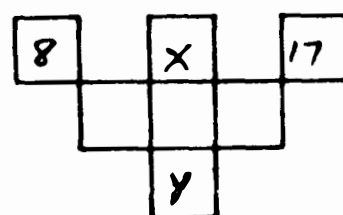
5. Solve these equations:
   (a) x + 27 - 5

   (b) 12 + 4x = 60 - 4x

   (c) -3 - 5x - -13

   (d) $-\dfrac{2}{3}x - 12$

   (e) 1.2x - -3.6

   (f) $\dfrac{1}{2}x - \dfrac{3}{4} - \dfrac{1}{4}$

6. Graph the following pairs of linear equations on the same set of axes and clearly state the point of intersection of the two lines:

   (a) y - x - 2   and   y - -x + 1
   (b) y - $\frac{1}{5}$x + 2   and   y - -2x + 2
   (c) y - 2   and   x - -3

ce.ecement 3 4   TEST 1 - SOLUTIONS   55   tables w

a) $-.8$ d) -28

c) -28 e) 17 ⑥   6.

-49 f) $-.7$

? (i) $-\$2$

(ii) $-\$21$ ③
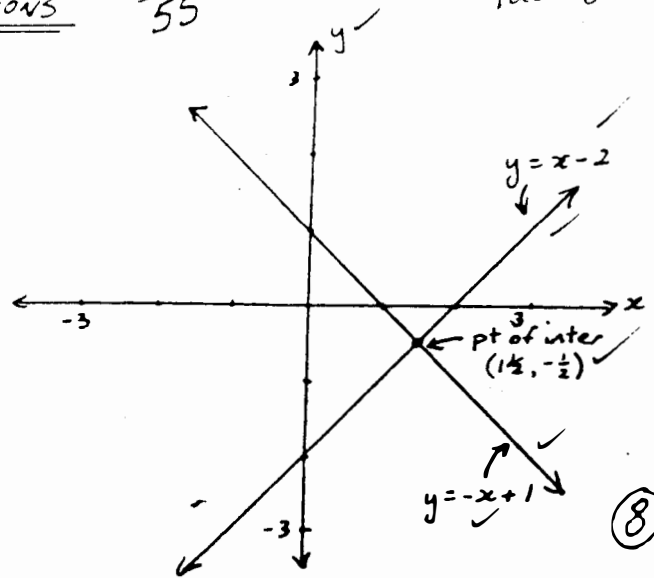
(iii) $\$26$

3. (a) $2 \times 2 \times 3$ or $2^2 \cdot 3$

(b) $2 \times 2 \times 11$ or $2^2 \cdot 11$

(c) $2 \times 3 \times 5 \times 5$ or $2 \times 3 \times 5^2$

(d) 2×71
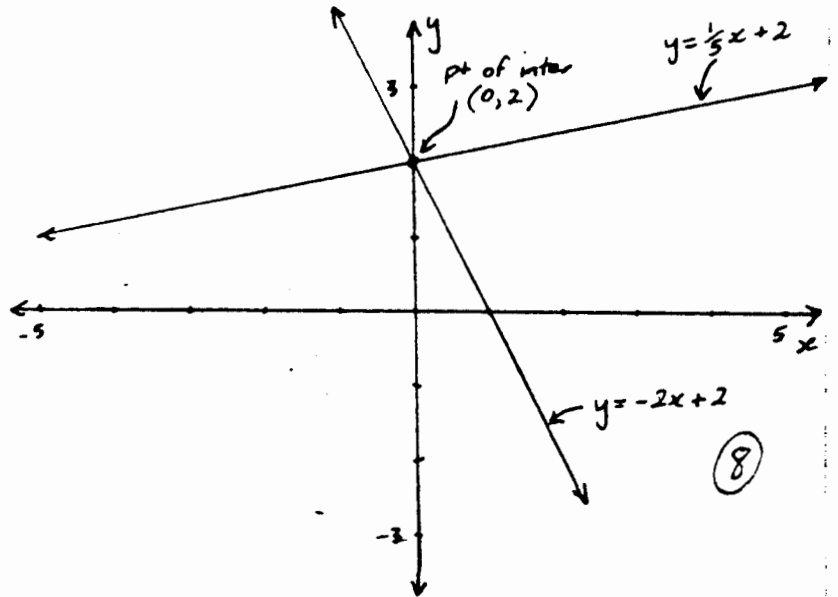
4. (a) $3x + 41 = 62$

$\therefore x = 7$

(b) $2x + 29 = 23$

$\therefore x = -3$

(c) $2x + 25 = y$ ⑥

5. (a) $x = -22$

(b) $x = 6$

(c) $x = 2$

(d) $x = -18$ ⑥

(e) $x = -3$

(f) $x = 2$
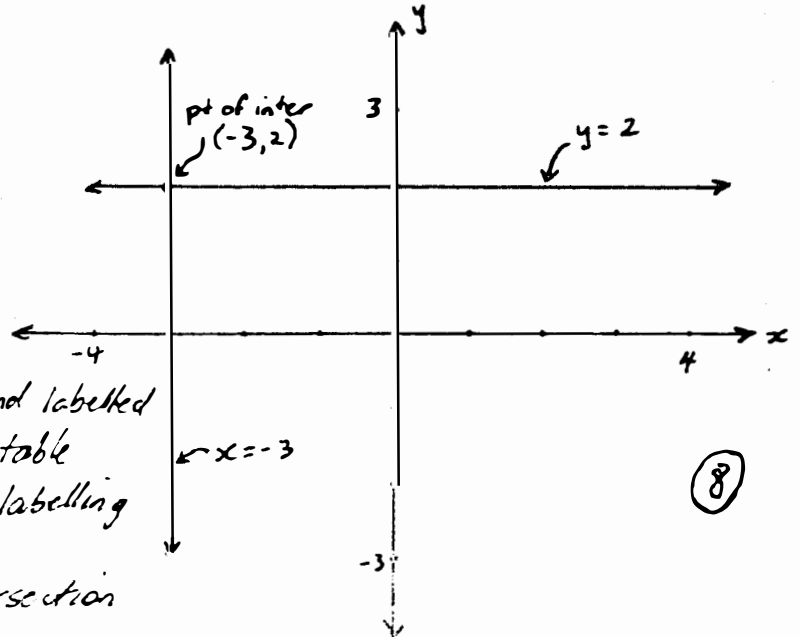
For each graph - 2 marks each line and labelled
                    - 1 mark for each table
                    - 1 mark for correct labelling
                      etc of axes
                    - 1 mark correct intersection

Appendix F

Interview Protocol

1. What do you understand by $(x+2)(x+3)$?.

2. Draw a picture to represent $(x+2)(x+3)$.

3. Multiply $(x+3)(x-4)$; explaining each step, that is explain what you are doing and why.

4. What do you understand by factorising $x^2 + 9x + 20$?.

5. Draw a diagram to represent $x^2 + 9x + 20$.

6. Factorise $x^2 - 11x + 18$; explaining each step.

Appendix G

Dear Phil,

This note is written in anticipation of your assistance in my research project. It will assist as a future reference to what we have previously discussed.

Firstly, for the research project to be successful there needs to be an extremely limited amount of discussion between classes. This can be partially achieved by emphasising to the students that they are involved in some research involving the effectiveness of Algebra Blocks and if they discuss how these blocks are used with members of the other class, they will be jeopardising the results of the project.

Secondly, an introduction to the use of Algebra Blocks is needed. This should involve simply identifying each piece and then using them for addition and subtraction of like terms and multiplication of negatives.

Thirdly, a common approach to setting out both binomial expansions and trinomial factorisations is required. The setting out of the solution should be as follows:

Binomial Expansion	Trinomial Factorisation
$(x + 2)(x - 4)$	$x^2 - x - 6$
$= x(x - 4) + 2(x - 4)$	$= \quad x^2 - 3x + 2x - 6$
$= x^2 - 4x + 2x - 8$	$= \quad x(x - 3) + 2(x - 3)$
$= x^2 - 2x - 8$	$= \quad (x - 3)(x + 2)$

The purpose of setting out the work in this way is that the reason behind each step will be developed using the Algebra Blocks. Students should set out all four lines while learning as to consolidate their understanding.

Fourthly, the split method , as explained in EDU-DOMES page 31, is needed to be used by both classes so that comparibility exists in the teaching methods.
That is, for solving: $x^2 - 3x - 10$

SPLIT	UNITS NEEDED
-3,0	0
-4,1	-4
-5,2	-10

Fifthly, similar examples should be used in teaching each class. These can be found in Fundamental Mathematics Book 1 chapters 16 and 17.

All of these points are necessary to provide comparability between teaching methods so that the only variable is the use of Algebra Blocks in one class. This makes the research data both valid and reliable.

Attached is a copy of the test.
Thank you for your anticipated co-operation

Yours Faithfull

Bernard Roberts

Dear Paul,

This note is written in anticipation of your assistance in my research project. It will assist as a future reference to what we have previously discussed.

Firstly, for the research project to be successful there needs to be an extremely limited amount of discussion between classes. This will be achieved through Phil explaining to his class that they are involved in research. Additionally , to combat the Hawthorne effect your class will need to be informed that they too are involved in the research.

Secondly, as the students in the other class will be becoming familiar with Algebra Blocks , it is presumed desirable that your class revise the addition and subtraction of like terms and multiplication of negatives for half a lesson.

Thirdly , a common approach to setting both binomial expansions and trinomial factorisations is required. The setting out of the solution should be as follows :

BINOMIAL EXPANSIONS	TRINOMIAL FACTORISATIONS
$(x + 2)(x - 4)$	$x^2 - x - 6$
$= x(x - 4) + 2(x - 4)$	$= x^2 - 3x + 2x - 6$
$= x^2 - 4x + 2x - 8$	$= x(x - 3) + 2(x - 3)$
$= x^2 - 2x - 8$	$= (x - 3)(x + 2)$

The purpose of this setting out is to assist in student understanding when using the split method as described in the following point.

Fourthly, the split method as explained in EDU-DOMES, page 31 is needed to be used by both classes. That is , for solving
$x^2 - 3x - 10$

SPLIT	UNITS NEEDED
-3,0	0
-4,1	-4
-5,2	-10

Fifthly, similar examples should be used in teaching each class. These can be found in Fundamental Mathematics Book 1 chapters 16 and 17.

All these points are necessary to provide comparability between the two classes so that the only variable is the use of Algebra Blocks in one class. This makes the research data both valid and reliable.

Attached is a copy of the test.

Thank you for your anticipated co-operation.

Yours Faithfully,

Bernard Roberts

Appendix H

Class A : Results from Post-test

Cls	B1	B2	B3	B4	B4	B6	B7	B8	TOTAL	T1	T2	T3	T4	T5	T6	T7	T8	TOTAL
PA1	3	3	3	3	3	3	3	3	24	4	4	4	4	0				16
PA2	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PA3	3	3	3	3	3	2	2	2	21	4	4	4	4	2	4	0		22
PA4	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PA5	3	3	3	3	3	3	2	2	22	4	4	4	2	0	4	4		22
PA6	3	3	3	3	2	3	3	3	23	4	4	2	0	4	4	0		18
PA7	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PA8	2	2	2	3	2	2	2	2	17	4	4		4	0	0		0	12
PA9	3	3	3	3	3	3	2	3	23	4	4	4	4	4	4	4	4	32
PA10	3	3	3	3	3	3	3	3	24	4	4	3	0	0	4	4	4	23
PA11	1	1	0	0	0	0	0	0	2	0	0	0	0	0	0	0		0
PA12	3	3	3	3	3	3	3	3	24	4	4	4	4	0	4	4	4	28
PA13	3	3	3	3	3	3	3	3	24	4	4	4	4	2	0	4	4	26
PA14	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	0	28
PA15	3	3	3	3	2	3	3	2	22	4	4	4	4		4			20
PA16	3	3	3	3	2	3	3	3	23	4	4	4	4	4		4		24
PA17	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PA18	2	2	1	2	2	2	2	0	13	4	4	2		0	4		0	14
PA19	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PA20	3	3	3	3	3	3	3	3	24	2	4	4	4	4	4	4	4	30
PA21	3	3	3	3	3	3	3	2	23	4	4	4	4	4	4	4	4	32
PA22	2	0	1	1	1	1	0	0	6	0	4	0	0	0	2	0	0	6
PA23	3	3	3	3	3	3	3	3	24	4	4	4	4	0	4	4	4	28
PA24	3	3	3	3	1	2	3	3	21	4	4	0	4	1	1	0	4	18
PA25	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PA26	2	2	3	3	3	3	3	3	22	4		0			0			4
PA27	3	3	3	3	3	3	2	3	23	4	4	4	4	0	4	0	4	24
PA28	3	3	3	3	3	2	3	3	23	4	4	0						8
PA29	3	3	3	2	3	3	3	3	23	4	4	4	4	4	4	4	4	32
PA30	3	3	3	3	3	3	3	3	24	4	4	4		4	4	4	4	28
PA31	3	3	3	3	3	3	3	3	24	4	4	4	0	4	4	0	0	20
PA32	3	2	1	1	1	0	2	2	12	4	4	0		0	0	4	0	12
C.Sum	90	87	86	87	82	83	83	81	679	118	120	95	86	65	91	76	68	719
Mean									21.219									22.469
S.D.									5.422									9.377

Key
B = Binomial
T = Trinomial
C.Sum = Sum of Column

Appendix H

Class B : Results from Post-test.

Class	B1	B2	B3	B4	B5	B6	B7	B8	Total	T1	T2	T3	T4	T5	T6	T7	T8	Total
PB1	3	2	3	3	2	2	2	2	19	4	4	3		3	3	2	1	20
PB2	3	3	2	2	2	3	2	2	19	2	2	2	2	0	1	0		9
PB3	3	3	3	3	3	3	2	3	23	4	4	4	4	4	4	4	0	28
PB4	3	3	3	3	3	3	2	2	22	4	4	2	2	4	4	1	0	21
PB5	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PB6	3	3	3	2	3	3	3	3	23	4	4	4	4		4		4	24
PB7	3	3	3	3	3	3	2	2	22	0	0							0
PB8	3	3	2	2	2	3	2	2	19	2	2	0	0	0	1	0	0	5
PB9	3	3	3	3	2	1	3	3	21	4	4	4	4	4	4	2	0	26
PB10	0	0							0									0
PB11	3	3	2	1	1	1	3	2	16	0	0	0	0	0	0			0
PB12	3	3	3	3	3	3	2	3	23	4	4	0	0					8
PB13	3	3	3	3	2	2	3	3	22	4	4	0	0					8
PB14	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	2	0	26
PB15	2	2	2	1	2	3	2	2	16	4	4	4	4					16
PB16	3	3	2	2	3	3	3	3	22	4	4	0	0	0	4	0	0	12
PB17	1	1	1	1	1	1	1	1	8	0	0	0	0	0	0	0	0	0
PB18	3	3	3	3	3	3	3	2	23	4	4	4	4	2	2	2	0	22
PB19	3	3	3	3	3	3	3	3	24	4	4	4	4	4	2	4	4	30
PB20	3	3	3	3	2	3	3	3	23	4	4	2	2	0	2	0	0	14
PB21	3	3	3	3	2	2	2	1	19	0	0	0	0	0				0
PB22	3	3	3	3	1	1	0	3	17	2	3	0	2	0	4	4	0	15
PB23	3	3	3	3	3	3	3	3	24	4	4	4	4	4	4	4	4	32
PB24	3	3	3	3	3	3	2	3	23	4	4	4	4	4	4	0	4	28
PB25	3	3	3	3	2	3	2	3	22	4	4	4	4	0	4	4	4	28
PB26	3	3	3	3	3	3	3	2	23	4	4	0	0	0	2	4	0	14
PB27	3	3	3	3	3	3	3	3	24	4	4	2	4	0	4	4	0	22
PB28	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
PB29	3	3	2	3	3	3	3	3	23	4	4	3	0	0	4	1	3	19
PB30	3	3	3	3	3	3	3	3	24	3	3	1	0	0	3	3	3	16
C.Sum	81	80	76	74	69	73	68	71	592	89	90	59	56	37	68	45	31	475
Mean									19.73									15.83
S.D.									6.368									10.8