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The relationship between cooperative small group composition and the learning of a mathematical concept in the primary school years: A pilot study

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USE OF THESIS

THE RELATIONSHIP BETWEEN COOPERATIVE SMALL GROUP COMPOSITION AND THE LEARNING OF A MATHEMATICAL CONCEPT IN THE PRIMARY SCHOOL YEARS: A PILOT STUDY.

by

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Abstract

In recent years small group cooperative learning has been given increasing attention by researchers. This interest has been a result of a growing awareness of the benefits that small group cooperative learning can bring to the learning process. These benefits include gains in areas of academic achievement, self-confidence as a learner, cross-cultural/cross-racial relationships, social acceptance of mainstreamed students, and improved attitudes towards school and learning.

A particular focus of North American researchers has been small group cooperative learning in mathematics. Little work had been done in this area in Western Australian schools and with the changed emphasis in the Western Australian primary school mathematics syllabus away from rote learning and pen and paper calculations toward discovery learning a local study seemed appropriate.

This pilot study proposed to investigate the relationship between the composition of cooperative small groups, heterogeneous or homogeneous, and the learning of a mathematical concept in the primary school years. The literature in this area was surveyed with emphasis on the rationale for small group cooperative learning, different kinds of small group cooperative learning focusing on the Groups of Four model, heterogeneous and homogeneous group composition, and group composition in mathematics related to expected achievement and social outcomes. The conceptual framework for this pilot study emerged from both the literature in this area and the direction being taken by a team of W.A.C.A.E. researchers who are investigating small group cooperative learning techniques.

Data were collected using a quasi-experimental analysis of variance design. Year 6 students in two classes participated in the study. Two mathematical concepts were introduced to each class of students with students learning one concept in a heterogeneous group and the other concept in a homogeneous group. The two classes learned the same concept at the same time but used contrasting group composition techniques. A post-test was applied at the completion of instruction for each mathematical concept. An analysis of variance was used to analyse the data from the post-tests.

The research hypothesis for the pilot study was that in the small group cooperative learning situation of the Groups of Four model heterogeneous group composition would result in greater higher-order learning achievement than homogeneous group composition. This hypothesis was not supported by the findings of the pilot study. The data indicated that while the students had developed higher-order thinking skills in the concept areas, the type of group composition had not affected the amount of higher-order learning which occurred. Heterogeneous group composition was not proven to be better than homogeneous group composition, and homogeneous group composition was not proven to be better than heterogeneous group composition. While the findings of the pilot study should be viewed with caution, if they are to be taken literally then there are significant implications for teachers, students, mathematics education, and researchers.

The thesis concluded with recommendations for further research which emerged from the data collected during the pilot study.

Declaration

I certify that this thesis does not incorporate, without acknowledgement, any material previously submitted for a degree or diploma in any institution of higher education and that, to the best of my knowledge and belief, it does not contain any material previously published or written by another person except where due reference is made in the text.

Signed [redacted]

Date 11 [redacted] 1990

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Chapter 1

Introduction

Overview of the Chapter

The chapter will commence with a description of the background of the research problem, followed by a justification for the pilot study in terms of the significance of the problem and its potential contribution to educational theory and practice. The research problem will be stated and the major terms used within the body of the thesis will be defined.

Background

This thesis focuses on small group cooperative learning, especially as applied to the relationship between group composition and the learning of academic concepts in mathematics. This question is an important one for, although small group cooperative learning has been a feature of classrooms in Western democracies for at least 40 years, researchers have had difficulty in arriving at a consensus view of the benefits of small group cooperative learning and appropriate ways to structure such learning. This caused a decline in research activity in this small group cooperative learning in the 1970s and early 1980s.

In recent years, with the propagation of Marilyn Burns' (1981) Groups of Four model of small group cooperative learning in North American schools, researchers have turned again to an examination of small group cooperative learning. This research has been spearheaded by Professor Tom Good at the University of Missouri, Columbia and his work was joined in 1989 by a group of researchers at the Western Australian College of Advanced Education.

Classroom research into small group cooperative learning techniques has investigated themes including achievement (Good et al., 1988; Webb, 1982a), group interaction (Webb, 1980, 1982a, 1982b), social skills (Good et al., 1988), and group composition (Good et al., 1988; Noddings, 1989; Webb, 1982a). The present study focused on one area of small group cooperative learning in mathematics, namely, group composition and its relationship to achievement. More specifically, group composition was viewed in terms of groups composed on a heterogeneous basis and a homogeneous basis.

One area of the curriculum in which small group cooperative learning techniques have been given increasing attention by classroom researchers is mathematics (Good et al., 1988). Mathematics, traditionally, is seen by students as a highly structured subject, one in which material is presented by the teacher or through a textbook, and the 'quick, right answer' has been given pre-eminence. Increasingly, researchers have been finding that small group cooperative learning techniques are an alternative means by which students can come to understand the concepts of mathematics. Higher levels of student outcomes can be achieved while the amount of interpersonal competition among students can be reduced (Noddings, 1989; Parker, 1984).

Significance of the Study

Very little research on small group cooperative learning in the primary classroom has been undertaken in Australia. One purpose of this pilot study was to contribute to the Australian research effort now starting in the general area of small group cooperative learning as a result of world researchers, such as Good, calling for crossvalidation studies. The present study was a contributing pilot study on one critical aspect of small group cooperative learning.

Currently, a team of researchers from the Western Australian College of Advanced Education (W.A.C.A.E.) is conducting classroom effectiveness studies focusing on small group cooperative learning techniques. Figure 1 illustrates the research direction the W.A.C.A.E. team is pursuing.

Small Group Cooperative Learning Techniques

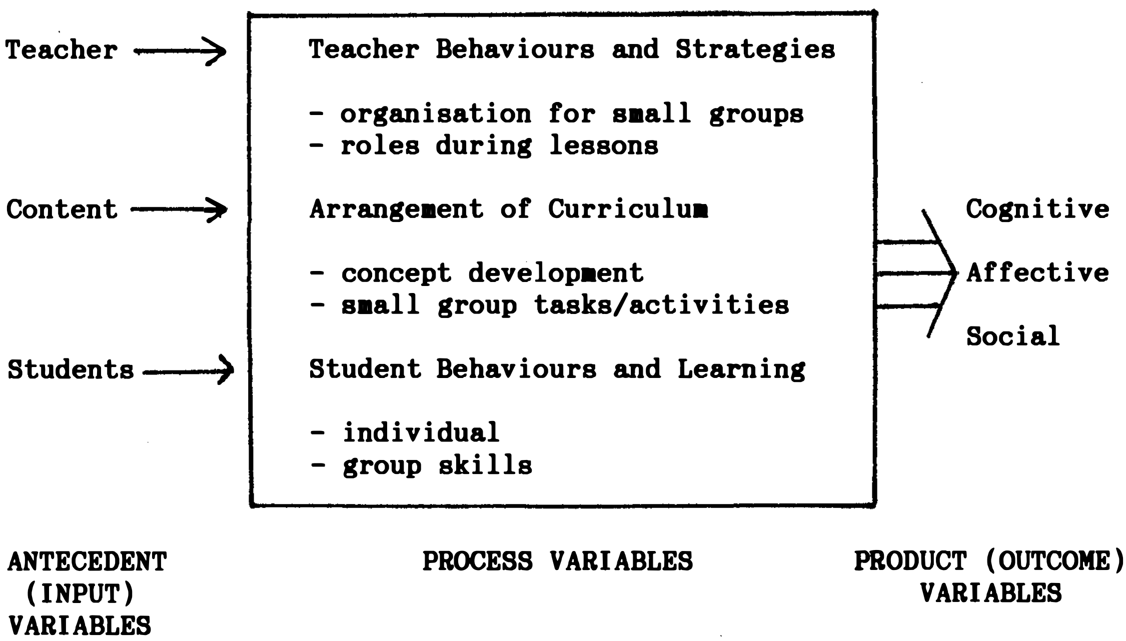

Figure 1. Conceptual framework for the classroom effectiveness studies: Small group cooperative learning project

The present pilot study fits the conceptual framework in Figure 1 in regard to teacher behaviours or strategies - organisation for small groups. However, in all other respects this pilot study was independent of the W.A.C.A.E. research.

Statement of the Problem

The purpose of this study was to investigate, on a pilot basis, the relationship between the composition of small groups in small group cooperative learning and the outcomes in terms of higher-order thinking skills in a Year 6 class.

Definition of Terms

The major terms which are used within the body of this thesis are defined below.

1. Group composition, at the most general level, may be considered to be either heterogeneous groups or homogeneous groups.
2. Heterogeneous groups are groups composed of students of mixed abilities.
3. Homogeneous groups are groups composed of students in which the range in ability has been restricted, ie. groups of high-achievers, groups of low-achievers, or groups of average-achievers.
4. Cooperative learning is defined as a form of interaction in which students work together to attain a shared goal, with shared effort and shared reward (Johnson & Johnson, 1986; Slavin, 1987).
5. The Groups of Four model is an approach to small group cooperative learning where students work in groups of four to solve problems and explore tasks/topics. The Groups of Four model of small group cooperative learning will be explained more fully in Chapter 2.
6. A mathematical concept, for the purpose of this study, will involve the development of higher-order thinking skills within a mathematical system (eg. arithmetic, algebraic, geometric).

7. Higher-order thinking skills, for the purpose of this study, involve analysis, synthesis, and evaluation skills according to Bloom's Taxonomy of cognitive learning (Barry & King, 1988, p. 14).

8. Problem solving involves a mathematical problem where the answer is not immediately apparent, and where the method of solution is not immediately obvious.

Summary of the Chapter

This chapter outlined the research problem, including the background of the problem and the significance of the pilot study to educational research. The major terms used in the body of this thesis were defined.

Chapter 2

Review of the Literature

Overview of the Chapter

This review of literature related to the topic of small group cooperative learning in mathematics and group composition will begin with a discussion of small group cooperative learning. The purpose of using small group cooperative learning techniques will then be discussed followed by the kinds of small group cooperative learning techniques commonly used, focusing on the Groups of Four model. The review of literature will then outline group composition techniques followed by a review of the research on the working of cooperative groups in mathematics.

Small Group Cooperative Learning

Small group work is a strategy for keeping students involved with their own work while still catering for a wide range of abilities (Cohen, 1986, p. 6). Currently there are two major forms of small group work in common use in schools. These are ability-based groups where students work independently of each other and groups which focus on cooperative learning where students work together to master a task or produce a product. In this study the focus of investigation involved only small group cooperative learning. Consequently, the review of literature is centred around the small group techniques of cooperative learning.

The use of small group cooperative learning appears to have emerged from the Dewey/Vygotsky view that intelligence is enhanced by social interaction. Broad purposes of small group cooperative learning which are derived from this view are the facilitation of cognitive development, social/democratic development, and moral development (Noddings, 1989, p. 608).

As a result of the work of Dewey and Vygotsky, cooperative learning groups are usually student-centred with an emphasis on group processes, problem solving, attitudes, and social development (Noddings, 1989, p. 610). Groups are generally heterogeneous in composition, although, as will be discussed later, they can be homogeneous in composition (Behounek, Rosenbaum, Brown & Burcalow, 1988).

Cohen (1986, p. 9) has suggested that, for small group cooperative learning to be an effective technique for developing conceptual learning, two basic conditions must be met. First, the task should require higher-order thinking and, secondly, the group must have the intellectual skills, vocabulary, and relevant information to succeed in the task. In addition to Cohen's suggestions, there should also be properly prepared task instructions (Davidson, 1990; Noddings, 1989).

Johnson, Johnson, Holubec and Roy (1984) identified four basic elements that they suggested must exist for small group cooperative learning to be truly cooperative. These elements are:

1. positive interdependence among group members;
2. face to face interaction among students;
3. individual accountability for mastering the assigned material; and

4. interpersonal and small group skills. (Johnson et al., 1984, p. 8)

Noddings (1989, p. 613) has suggested that student training and practice in small group processes and procedures is particularly important for success. Students need to understand the rules associated with small group cooperative learning and learn to interact constructively with other students in their group. In summary, the students need to be taught how to work, cooperate, and communicate effectively in groups. The students need to use interpersonal and small group skills as outlined by Johnson et al. (1984, p. 8).

According to Noddings (1989, p. 620), Parker (1984), and Good et al. (1988) teachers, like students, require training in small group cooperative learning techniques. In particular, teachers need to learn to respond to student needs rather than initiate contacts with students. Teachers also need to structure the goals of learning so that students are concerned with the performance of other group members as well as their own performance, leading to positive interdependence among group members. (Johnson et al., 1984, p. 8) The teacher becomes a facilitator of learning, providing an introduction to the lesson before the students explore the task. The teacher then helps the students clarify their findings in the lesson conclusion (Cohen, 1986; Noddings, 1989).

Rationale for Small Group Cooperative Learning

Small group cooperative learning is a process which provides students with an opportunity to become actively involved in their own learning and to perceive an element of control over their own learning (Davidson, 1990). Good et al. (1988) indicate that the increased interactions among students may enhance group members' communication skills, including the ability to identify and clarify their own thinking, and to defend and justify their own beliefs.

This view is supported by researchers such as Cohen (1986) who highlights the fact that students in a cooperative group communicate about the task with each other. The communication may involve students "asking questions, explaining, making suggestions, criticizing, listening, agreeing, disagreeing, or making joint decisions." (Cohen, 1986, p. 3) Students may also use non-verbal interactions such as hand gestures or facial expressions. These group skills may be transferred to many adult and student work situations.

According to Parker (1984, p. 1) and Davidson (1990, p. 7), having students working cooperatively toward common goals can lead to significant gains in areas of academic achievement, self-confidence as a learner, social relationships, cross-cultural/cross-racial relationships, social acceptance of mainstreamed students, and improved attitudes towards school and learning.

Cooperative learning in small groups requires students to focus on higher-order thinking skills such as discovery, concept development, and problem solving rather than the rote learning of facts (Cohen, 1986; Good et al., 1988; Parker, 1984). "Small group learning, with pupils cooperating in the study of subject matter, can lead to superior achievement in higher-order thinking. This gain. . . is not at the expense of information loss." (Parker, 1984, p. 5)

In summary, small group cooperative learning appears to have educational and social advantages which may help students in their future working lives as well as in their school years.

Kinds of Small Group Cooperative Learning

The degree of educational and social benefit the student receives may be affected by the kind of small group cooperative learning technique involved. There are a variety of small group cooperative learning techniques discussed in the literature. Some of the best known and most widely researched small group cooperative learning techniques are outlined by Good and Brophy (1987). These include 'Learning Together', 'Jigsaw', 'Jigsaw II', 'Student Team Learning' and 'Group Investigation'.

For this study, the small group cooperative learning technique which will be used is an application of the Group Investigation technique which was developed by Sharan and Sharan (1989). The Group Investigation method involved students working cooperatively in groups for problem solving, inquiry, discussion, cooperative planning, projects, and the development of higher-order thinking skills. Marilyn Burns (1981) adapted and refined the Group Investigation method into what has become known as the Groups of Four model of small group cooperative learning. This model has been developed in some detail and reported in the literature by Burns (1981) and Parker (1984). The Groups of Four model involves group problem solving with an emphasis on enhancing the development of higher-order thinking skills.

Burns (1981, p. 50) has stated that four students working together in a group allows for a diversity of ideas without hampering individual participation . Burns has also listed benefits to learning as a result of the Groups of Four model. These include maximising interactions that occur among students, providing a learning environment that serves students' intellectual development, group exploration of ideas and exchange of thoughts, discussion, becoming aware of other students' points of view, and providing a way to implement the curriculum in such a way that there is room for errors which may lead to a potential for new understandings (Burns, 1981, p. 51).

The Groups of Four model of small group cooperative learning is based on three rules for students to follow. These rules are:

1. Each member of the group is responsible for his or her own work and behaviour.
2. Each member of the group must be willing to help any other group member who asks for help.
3. You may only ask the teacher for help if all four group members have the same question.

The classroom is arranged so that students can easily work and communicate with each other in their groups of four students.

During Groups of Four sessions the teacher's role is mainly as a facilitator. The teacher circulates around the groups observing the various interactions and helping when an entire group has a question. The teacher does not intrude on a group unless the group is having difficulty with the problem itself or there is difficulty within the group. The teacher's intervention should only be to determine the nature of the problem, offer assistance, and move on when the group commences working again. The teacher is also responsible for summarizing the results for the whole class when the groups have finished exploring a problem. This is the time where faulty generalisations are corrected and the students' ideas are taken further. (Burns, 1981; Parker, 1984)

Approaches to the Composition of Groups in Small Group Cooperative Learning

There are two major approaches to the composition of groups in small group cooperative learning: homogeneous group composition and heterogeneous group composition. There has been considerable debate in the literature about the merits of each approach, and because of the importance of group composition to this study the literature will be examined in some detail.

Homogeneous groups. Homogeneous groups are usually teacher-selected on the basis of performance. In terms of cooperative learning in small groups, Good and Brophy (1987) have found that homogeneous groupings produced mixed results. While groups containing average-achievers worked and interacted well together "groups in which the students either were all high-achievers or all low-achievers did not work well together or interact much about academic achievement" (Good & Brophy, 1987, p. 439).

Webb (1982b) has suggested that homogeneous group composition may be more suitable for average-achieving students. She conducted a study to examine peer interaction and learning in cooperative small groups. The results indicated that average-achievers in homogeneous groups achieved higher results and received more explanations related to the group task from other group members than average-achievers in heterogeneous groups. Webb concluded that this may suggest that in heterogeneous groups the high-achievers may perceive a responsibility toward the low-achievers, and therefore try to help them, but not toward average-achievers, and thus tend to ignore them (Webb, 1982b, p. 653).

Heterogeneous groups. The matter of how heterogeneous groups are formed is of much interest to educators. Davidson (1990) has highlighted three different approaches in relation to the composition of heterogeneous groups.

The first approach is to randomly assign students to groups (Davidson, 1990, p. 9). For example, Burns (1981) has advocated random assignment to groups as in the Groups of Four model of small group cooperative learning. Random group composition is also supported in the literature by Parker (1984) and Good et al. (1988).

A second approach to heterogeneous group composition is to allow students to form their own groups (Davidson, 1990, p. 9). A concern about using this approach is raised by Cohen (1986, p. 61) who fears that isolated or low-achieving students will not be chosen until last, or will be rejected by their group.

Johnson et al. (1984, p. 28) have stated that "having students select their own groups is often not very successful". In their discussion of this statement the authors cite many incidents of high-achieving students working with other high-achieving students, students of similar races working together, students of similar gender working together, and minority students working with minority students. In this respect, allowing students to form their own groups may undermine the stated gains of cooperative learning in the areas of social relationships, cross-cultural/cross-racial relationships, and social acceptance of mainstreamed students.

The third approach to forming heterogeneous groups has been highlighted by Davidson (1990, p. 9). The essence of this approach is to use teacher-selection based on performance, race/ethnicity, or gender. Johnson et al. (1984) recommend teacher selection of groups which include high-, low-, and average-achieving students in the same group. Noddings (1989) advocates this approach, supporting a group composed of one high-achieving student, one low-achieving student, and two average-achieving students.

Good and Brophy (1987) and Webb (1982b) have produced some evidence that a group composed by the teacher, consisting of one high-, one low-, and two average-achieving students, as advocated by Noddings, is not the most suitable group formation technique. Research by Good and Brophy (1987), among others, has indicated that the composition of heterogeneous groups is more beneficial for high- and low-achievers (Good & Brophy, 1987; Parker, 1984; Webb, 1982b). The students who tend to ask questions (ie. high-achievers) and give, or are given explanations (ie. low-achievers) by the group members, seem to learn more in a small group cooperative learning situation.

Good and Brophy (1987) have found that in a heterogeneous group consisting of one high-, one low-, and two average-achieving students much of the interaction within the groups involved "tutoring of the low-achievers by the high-achievers with the average-achievers remaining relatively passive" (1987, p. 438). However, when they compared these findings against their findings for homogeneous groups (presented in the previous section) Good and Brophy supported the use of heterogeneous groups, emphasizing the need to train students how to act during group activities (1987, p. 439).

Robertson, Graves and Tuck (cited in Davidson, 1990, p. 369) state that "random grouping allows for heterogeneity and sends the message that all students are equally valued as group members". They suggest that heterogeneous groups reflect the composition of the whole class. Noddings (1989) expands upon this idea by expressing the view that heterogeneous groups "working in a fully cooperative way may be best for a wide variety of tasks that call forth multiple abilities" (p. 616).

Webb (1982a) conducted a study on group composition, group interaction, and achievement in cooperative small groups which found that, on average, students in heterogeneous groups scored higher on achievement tests than students who worked in homogeneous groups. The study also found that "asking a question and receiving no answer was detrimental to achievement, and uniform-ability groups produced more of this behaviour than mixed-ability groups" (p. 481).

Cooperative Small Group Learning In Mathematics

Both heterogeneous groups and homogeneous groups are used in mathematics. In some cases heterogeneous groups may be the most suitable type of group composition for mathematics activities and, in other cases, homogeneous group composition may be more suitable (Johnson & Johnson cited in Davidson, 1990; Noddings, 1989). Researchers support the use of heterogeneous and homogeneous groups for various aspects of mathematics, depending upon the material or the topic to be covered (Johnson & Johnson cited in Davidson, 1990; Noddings, 1989). The research also supports the use of cooperative learning techniques focusing on higher-order thinking as being beneficial to the understanding of mathematical concepts (Good et al., 1988; Johnson & Johnson cited in Davidson, 1990; Noddings, 1989).

Mathematics, although traditionally a highly structured area of the curriculum, is one subject in which the use of small group cooperative learning is becoming increasingly important. The current Western Australian mathematics syllabus "recognises the need for student learning through problem solving" (Ministry of Education, 1989, p. i). The motto of this syllabus document is: "Let the learner learn before the teacher teaches". The use of small group cooperative learning in mathematics allows for active learning, opportunities for peer interaction, and opportunities for conceptual thinking. The development of skills and knowledge, especially in the areas of problem solving, statistics, measurement and estimation, can be enhanced using this technique (Good et al., 1988; Kerslake, 1989; Rosenbaum, Behounek, Brown & Burcalow, 1989). Good et al. (1988) concluded that students involved in small group cooperative learning situations "were more active learners and more motivated and enthusiastic about mathematics" (p. 45).

Davidson (1990) has defined four characteristics of small group cooperative learning in mathematics which draw upon the four basic elements of cooperative learning outlined by Johnson et al. (1984):

1. A mathematical task for group discussion and resolution.
2. Face-to-face interaction in small groups.
3. An atmosphere of cooperation and mutual helpfulness within each group. (Davidson, 1990, p. 8)
4. Individual accountability.

Davidson (1990, pp. 4-5) also listed the following advantages of using cooperative groups during mathematics :

* Small groups provide a social support mechanism for the learning of mathematics.
* Small group learning offers opportunities for success for all students in mathematics.
* Mathematics problems are ideally suited for group discussion.
* Mathematics problems can often be solved by several different approaches.
* Students in groups can help one another master basic facts and necessary computational procedures.
* The field of mathematics is filled with exciting and challenging ideas that merit discussion.
* Small groups provide a forum for asking questions, discussing ideas, making mistakes, learning to listen to others' ideas, offering constructive criticism, and summarising discoveries in writing.
* Mathematics offers many opportunities for creative thinking, exploring open-ended situations, making conjectures and testing them with data, posing intriguing problems, and solving non-routine problems.

* Students in groups can often handle challenging situations that are well beyond the capabilities of individuals at that developmental stage.

Noddings (1989) regards group composition (heterogeneous/ homogeneous) as crucial to the academic learning expected to take place on any mathematical task. She has advocated matching group composition to the task. The research, in her opinion, states that measurement activities, simulations, games, and problem solving activities are all suitable activities for heterogeneous groups (Noddings, 1989, p. 613). However, Noddings has suggested that when "teachers want to use small groups for ordinary academic exercises . . . homogeneous ability (or achievement) groupings may be appropriate. The solution of textbook word problems is an excellent example." (Noddings, 1989, p. 615)

Noddings (1989) has also supported the idea of homogeneous grouping as being more beneficial for average-achieving students in mathematics. Noddings has indicated that groups organised for higher-order tasks in mathematics should be homogeneous in order that all members can feel free to interact with one another. Moreover, she has stated that "it may well be that both high- and low-ability students would perform well in homogeneous groups if the tasks were appropriately differentiated" (Noddings, 1989, p. 615).

Parker (1984) and Good et al. (1988), after investigating research studies concerning mathematics education, appear to have reached the conclusion that a breakdown has occurred in the area of learning problem-solving skills. Students are able to perform academic exercises, such as word problems that appear in mathematics textbooks involving addition, subtraction, multiplication, and division, but are less capable of solving problems. Parker has stated that, "in order to be a successful and contributing member in our modern technological society, students will need skills in problem solving" (1984, p. 3). Small group cooperative learning attempts to deal with the issue of developing problem-solving skills by encouraging students "to explore ideas, justify their viewpoints, share discoveries, talk through problems, and synthesize knowledge" (Parker, 1984, p. 4).

Problem-solving tasks typically make "higher cognitive demands on students and often require several steps and a variety of methods for solution" (Good et al., 1988, p. 15). Good et al. (1988) found that most of the lessons observed in their study of small group cooperative learning situations focused on higher-order thinking skills such as problem solving. They also found that most of the groups were heterogeneous in composition.

Johnson and Johnson (cited in Davidson, 1990, p. 113) have advocated that when students "are working on problem solving tasks and learning how to communicate mathematically, heterogeneous groups are the most appropriate [form of group composition]".

In summary, this review of the literature on small group cooperative learning in mathematics has demonstrated that small group cooperative learning has considerable benefits. However, the literature is not so clear as to whether homogeneous or heterogeneous grouping is preferable. Hence, the question will be given more attention as the central focus of the study.

Summary of the Chapter

The review of literature was related to the topic of small group cooperative learning. The chapter commenced with a discussion of small group cooperative learning followed by a rationale for the use of small group cooperative learning. Different kinds of small group cooperative learning were listed with major emphasis placed on the Groups of Four model of small group cooperative learning. Two approaches to the composition of groups in small group cooperative learning, namely, heterogeneous and homogeneous group composition were discussed in terms of various ways the groups can be formed, achievement outcomes expected, and social outcomes expected. A discussion about small group cooperative learning in mathematics concluded the chapter.

Chapter 3

Research Design

Overview of the Chapter

In this chapter the procedural aspects of the experimental section of the pilot study will be discussed under the headings of conceptual framework, statement of the hypothesis, research design, subjects, mathematical concepts, teaching programmes and instruments, procedure, assumptions of the study, limitations of the study, ethical considerations, and a summary of the chapter.

Conceptual Framework

As the review of literature indicated, the composition of groups was considered to be a contentious aspect of small group cooperative learning. The teacher's behaviours and strategies in the composition of groups constitutes a key component of the process variables prevailing where small groups are used during lessons. Accordingly, the study focused on one feature of the conceptual framework developed by the W.A.C.A.E. Classroom Effectiveness Studies team (1990) which was outlined in Figure 1, namely, organisation for small groups.

The conceptual framework for this study was derived from the wider W.A.C.A.E. research team's conceptualisation. As Figure 2 illustrates, the conceptual framework comprises a set of antecedent variables, one aspect of the set of process variables, and one major product variable. The antecedent variables which were considered important for this study included the teacher, the component of curriculum used in the study, which was mathematics, and the students.

The key process variable within the organisation for small groups was group composition, either homogeneously formed groups or heterogeneously formed groups. The major product variable under consideration for this study was higher level cognitive achievement.

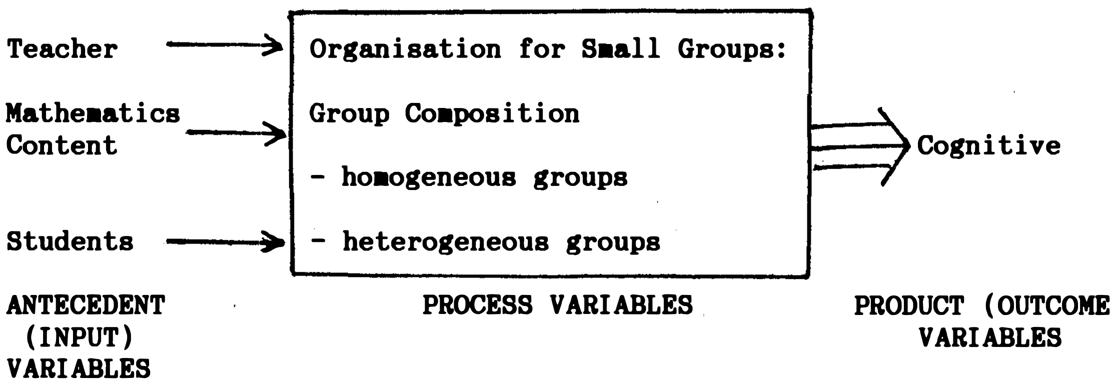

Figure 2. Conceptual framework

The study was designed on the premise that a teacher could control the composition of small groups when teaching new content through small group cooperative learning techniques. Depending on how the groups were formed, students would experience different learning conditions resulting in different achievement rates of higher-order cognitive learning. Teachers were to use matching behaviours and strategies in their teaching, and the treatment of the curriculum was to be approximately equivalent. Presumably the different ways of grouping the students would have had an impact on student process variables. However, this study was concerned with investigating whether or not a difference in achievement resulted from two different ways in which groups were formed rather than describing the nature and extent of differences in how students behaved.

The study involved students of a similar socio-economic status, in the same grade level at school. Within both classes, each student received the same instructions and the same problems to solve. This was done in an attempt to minimise the 'student' variable.

The concept areas chosen for this study were new to the students; therefore, pre-testing was unnecessary as it was assumed that students had no previous experience in these concept areas. The literature stated that problem solving activities promoted higher-order thinking and learning achievement. Programmes were designed so that the mathematical concepts were presented as a series of problem solving activities. All students undertook the same concepts during the same time period and completed the same tasks and activities in an effort to minimise the 'content' variable effects.

The product variable was cognitive, namely, the development of higher-order thinking skills. The researcher attempted to minimise all variables leading to the development of higher-order thinking skills except the process variable of group composition, and the student behaviour within small groups variable. The literature pointed to heterogeneous group composition as being more effective for problem solving tasks, higher-order thinking tasks, and tasks that call for multiple abilities. The literature also suggested that students working in a heterogeneous group scored higher on average than students working in homogeneous groups.

Statement of the Hypothesis

The hypothesis adopted for this pilot study was that in the small group cooperative learning situation of the Groups of Four model, heterogeneous group composition would result in greater higher-order learning achievement than homogeneous group composition.

This hypothesis was derived from the purpose of the pilot study which was to investigate the nature of the relationship between the composition of small groups in cooperative learning and the outcomes in terms of higher-order thinking skills in a Year 6 class.

If small group cooperative learning situations in mathematics generally involve skills such as problem solving, concept development, and discovery, then the approach to small group cooperative learning which seemed most appropriate to use was the Groups of Four model.

When students are learning a new concept in mathematics, practical hands-on problem solving is often involved rather than the solving of word problems from textbooks. The research presented in the review of literature suggested that for the learning and understanding of higher-order thinking skills, heterogeneous group composition may be more effective than homogeneous group composition.

The aim of this pilot study was to provide empirical evidence to support the contention that there is a positive relationship between a group composition approach and higher-order learning achievement of mathematical concepts.

Specifically, this experiment set out to:

1. construct and provide equivalent programmes of work for the learning of two mathematical concepts;
2. establish heterogeneous and homogeneous groups for the purpose of learning two mathematical concepts;
3. randomly assign students, in class groups, to a particular sequence of treatment as shown in the experimental design;
4. test the students' performance in higher-order learning achievement of mathematical concepts with an immediate post-test;

5. verify the general aim of the pilot study by finding a relationship between group composition and the learning of a mathematical concept.

Given the research hypothesis, the null hypothesis was that in the small group cooperative learning situation of the Groups of Four model there would be no difference in higher-order learning achievement between heterogeneous group composition and homogeneous group composition.

Research Design

A quasi-experimental analysis of variance design was used in this pilot study. Two classes of students received instruction on two mathematical concepts, namely, exponents and percent. The independent variable was group composition, namely, heterogeneous or homogeneous. The dependent variable was the understanding of mathematical concepts.

No control group was used in this pilot study as the study focused on group composition in a small group cooperative learning situation and involved comparing two experimental groups rather than comparing an experimental group to a group receiving no treatment. Instead, the subjects themselves were their own controls. Gay (1987, p. 279) describes using subjects as their own controls as "exposing the same group to the different treatments, one treatment at a time". This helped to control for subject differences as the same subjects received both treatments.

The class groups were assigned randomly to the order of group composition treatments (heterogeneous/homogeneous) and to the order of concepts using standard randomisation procedures. The researcher assumed that the students were already assigned randomly to the Year 6 class groups/year level. The individual students worked together in small groups of four students within each class group.

The analysis of variance design used in the study could be termed a hierarchical design containing one within-group variable. One class group was exposed to one order or sequence of group composition treatment while the other class group was exposed to a second order or sequence of group composition treatment. At the completion of each treatment all the subjects were tested to determine the amount of learning in terms of higher-order thinking skills, and an alpha level of 0.05 was used to determine significant differences between groups.

The variability among students had five potential sources, namely, practice effects, class group effects, small group effects, treatment effects, and residual individual differences.

Subjects

As the literature revealed a necessity for teachers and students to be trained in small group cooperative learning techniques and group processes, two volunteer teachers who had completed a Bachelor of Education unit which covered these areas were selected to participate in this pilot study. The two teachers taught at different primary schools in the Perth metropolitan area, although the two schools were located in areas of similar socio-economic status.

The subjects for the present study consisted of the students in the classes taught by the two teachers. Both teachers taught a Year 6 class. The total number of students involved in the study was fifty (N=50). There were 16 students in one class (hereafter referred to as Class A) and 34 students in the other class (hereafter referred to as Class B).

Mathematics Concepts

The researcher met with the two classroom teachers prior to the commencement of the experiment in order to determine two mathematical concept areas which could be used for the experimentation phase of the pilot study. The determining criteria for selection were that the concepts were:

1. as yet not introduced to the students;
2. of comparable difficulty;
3. unrelated;
4. of a short term nature, that is, the development of the concepts could be achieved in two lessons.

Only three possibilities met the above requirements, namely, exponents, percent, and volume. Of these possibilities exponents and percent were chosen. Although neither class had commenced the volume activities prescribed in the Year 6 mathematics syllabus, volume as a concept had been introduced in previous years in the mathematics syllabus used by both schools. Neither exponents or percent had been included in the mathematics syllabus until Year 6 and so these two concepts more adequately met the stated criteria.

Teaching Programmes and Instruments

The two teachers used the same programmes developed by the researcher in order to maintain consistency in both content and method between schools and classes. These programmes detailed the mathematical concepts to be introduced, the tasks and/or worksheets to be used, and the style of group composition to be used. The programmes outlined the structure of each lesson and detailed the steps for the teacher to follow during each lesson. In addition, each teaching programme provided the teacher with a cover sheet detailing classroom organisation, group organisation in terms of being either heterogeneous or homogeneous, and the materials and resources necessary for each lesson such as sufficient copies of each worksheet/activity for each group. Accompanying each programme of work was a post-test.

In developing the programmes the researcher examined the syllabus content for that concept area. Programmes of work were developed using the content related to the concept and the researcher's knowledge of the Groups of Four model. The programmes of work consisted of two one-hour lessons for each concept. The programmes were developmental in nature so that even low-ability students would have a chance of gaining some mastery.

Research cited in the review of literature revealed that problem solving tasks usually involved higher-order thinking skills which are the 'product' variable of the present study. For this reason, the researcher based the programmes of work on problem solving tasks.

Each lesson plan (two per programme; four in total) detailed the main mathematical ideas being explored, the development of the lesson, and the expected responses to be elicited from the students. The format of each lesson is illustrated in Figure 3. The development of each lesson was organised into two sections, namely, teacher and small group, as shown in Figure 3. The teacher section detailed the tasks and instructions for teachers to follow and the small group section detailed the small group problem solving activities.

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED RESPONSES
	Teacher * ~~Review~~ small group activity. * Introduce the next small group activity.	
Extend new learning	Small Group * In your groups find a way to illustrate 35% Is this less than or greater than 50%? * [If groups are stuck - suggest drawing two 10 *x* 10 grids.]	* Will vary.

Figure 3. Structure of teaching programmes.

The structure of each lesson varied from lesson to lesson. However, two basic lesson structures were used and these are outlined in Figures 4 and 5.

Time		Activity
10 min.	1.	Teacher introduction of topic. Review of prior knowledge. Teacher introduces work to be explored.
40 min.	2.	Students work in cooperative groups to explore the tasks and complete the activity.
10 min.	3.	Conclusion: Review of lesson.

Figure 4. Lesson structure: Example 1

Time		Activity
10 min.	1.	Teacher introduction of topic. Review of prior knowledge. Teacher introduces work to be explored.
15 min.	2.	Students work in cooperative groups to explore the task.
5 min.	3.	Whole group: Teacher and students review the task.
5 min.	4.	Teacher introduces the next task.
15 min.	5.	Students work in cooperative groups to explore the task.
5 min.	6.	Whole group: Teacher and students review the tasks.
5 min.	7.	Conclusion: Review of lesson.

Figure 5. Lesson structure: Example 2

In all, three teaching programmes were developed for the teachers to follow. The first programme developed was a familiarisation programme for both teachers and students. The other two programmes were exponents and percent. Each programme will be discussed in the following section of the chapter.

Familiarisation programme. In order to meet the requirement outlined in the research of familiarising students and teachers with small group cooperative learning, familiarisation lessons were developed by the researcher which were unrelated to the concepts of either exponents or percent. The familiarisations lessons were devised to acquaint and expose both students and teachers with the small group cooperative learning model of the Groups of Four and with the overall format of the experimental lessons. The familiarisation lessons used both examples of lesson structures as outlined in Figures 4 and 5.

The familiarisation programme consisted of two lessons and the programme is included as Appendix 3. In the first lesson the students were introduced to the Groups of Four model of cooperative learning. The rules of working in Groups of Four were also discussed with the students.

The activity for the first familiarisation lesson focused on consecutive numbers and number patterns using a problem solving approach. The second familiarisation lesson consisted of a series of problem solving activities based on the number strand of the Western Australian Mathematics Syllabus.

Prior to the implementation of the familiarisation lessons the researcher provided a copy of the programme to a tertiary lecturer who was conversant with the Groups of Four model of small group cooperative learning. This was done in order to establish that the planned lessons were consistent with the Groups of Four model of small group cooperative learning and were appropriate to use at the Year 6 level.

Before the actual familiarisation phase of the study commenced the researcher met with the two classroom teachers on a one-to-one basis. During these meetings the researcher inducted the teachers on the Groups of Four model as proposed by Burns (1981) and Parker (1984). The programme was discussed with the teachers and any questions raised by the teachers were answered by the researcher.

Similarly, at the completion of the familiarisation phase the researcher reviewed the lessons with the teachers. At this time any problems that had arisen were dealt with.

During the familiarisation phase the students worked in both heterogeneous and homogeneous groups. These group placements were retained for the experimental phase of the pilot study.

Exponents programme. The researcher devised a programme of work which focused on exponents. This programme is included as Appendix 4. The lesson structure utilised was the second example of lesson structure as shown in Figure 5. The programme was developed after examination of the Western Australian Mathematics Syllabus and verified by a mathematics education tertiary lecturer for purposes of content validity. The researcher then met with the classroom teachers and briefed them on the programme of work to be implemented.

The first lesson on exponents aimed at developing the notion of exponent as being used to indicate "to the power of" as in 10^2 = 10 to the power of 2. In the second lesson on exponents the focus was on the relationship between exponents, place value, expanded numerals and exponential notation.

Percent programme. The researcher undertook the same steps in developing the programme for percent, verifying validity of the programme, and briefing of the teachers as carried out for the exponents programme. The percent programme is included as Appendix 6.

The first lesson on percent was aimed at developing the notion of percent as meaning "in every hundred" and the relationship between percent, decimals, and fractions. Emphasis in the second lesson was on the everyday applications of percent, particularly in the areas of deposits and interest, and discounts.

Post-tests. Each post-test was related directly to the accompanying programme of work. The post-tests for exponents and percent are included as Appendices 5 and 7, respectively.

When developing the post-tests, the researcher developed a Table of Specifications as recommended by Gay (1987) to construct a multiple-choice test for each mathematical concept. The format used for the Table of Specifications is outlined in Figure 6.

UNIT OBJECTIVES	TOPICS	MARKS	NUMBER OF QUESTIONS	LEVEL OF OBJECTIVES

Figure 6. Outline of Table of Specifications

This format allowed a proportionate emphasis in the test to be placed upon higher-order thinking skills such as analysis, synthesis, and evaluation according to Bloom's Taxonomy of Cognitive Objectives. Other areas of Bloom's Taxonomy of Cognitive Objectives were also included in the tests to provide balance, and to ensure that the test was not too difficult.

The researcher chose to use a multiple-choice test as "they are the best form of objective type questions . . . . because they can measure knowledge, understanding, and thinking skills in most subjects". (Barry & King, 1988, p. 187) When generating the test items, the researcher referred to the Table of Specifications and also took into account guidelines for construction of multiple-choice tests as discussed in Barry and King (1988, pp. 187-188). These guidelines included:

1. Use a stem that is simple and meaningful.
2. Ensure that the stem and the alternatives are grammatically correct.
3. Avoid the use of negatives.
4. Use plausible alternatives and order them in sequence - alphabetically, numerically, or chronologically.
5. Use four or five alternatives only.
6. Ensure there is one clear, correct answer.
7. Ensure that the answers are in random order across the test.

The easiest items were placed at the beginning of the tests so that each student could experience success in the test.

The researcher chose to create a 26-item multiple choice test so that the reliability of the test could be established using split-half reliability procedures. The split-half reliability procedures involved a test being able to be split into two equal halves, hence there are 26 items in the test.

In order to establish content validity of the tests the researcher provided the tests along with the teaching programmes to be examined for content validity.

Procedure

In schematic form, the procedure for the study was as shown in Figure 7. The study was conducted in three phases, namely, the familiarisation phase, the experimental phase, and the post-test phase.

The study required the assistance of two volunteer teachers. Two teachers who met the requirements were approached informally and appraised of the research without being appraised of the direction of the research hypothesis. With the support of their respective principals the two teachers agreed to participate. A copy of the letter seeking approval from the principals is included as Appendix 1.

		CLASS A	CLASS B
		*** Familiarisation Phase ***	
	Lesson 1	Homo/mc1	Hetero/mc1
Experi-	Lesson 2	Homo/mc1	Hetero/mc1
mentation		*** First post-test ***	
Phase	Lesson 3	Hetero/mc2	Homo/mc2
	Lesson 4	Hetero/mc2	Homo/mc2
		*** Second post-test ***	

Homo = homogeneous group
Hetero = heterogeneous group
mc1 = mathematical concept number 1 (exponents)
mc2 = mathematical concept number 2 (percent)

Figure 7. A schematic representation of the procedural design

The familiarisation phase was the period of time during which the familiarisation programmes were implemented.

The experimentation phase was the period of time during which the actual programmes of treatment were implemented. This occupied two weeks, with each group learning one concept in a homogeneous group and one concept in a heterogeneous group. One mathematical concept and the associated post-test were administered each week.

Group composition. When the students were required to work in heterogeneous groups, the groups were assigned using a system of proportional stratified sampling. The students in each class were ranked on the basis of their mathematical abilities (from highest to lowest) by their respective classroom teachers. The researcher used this information to form heterogeneous groups consisting of one high-, one low-, and two average-achieving students. Although this method of forming heterogeneous groups had claimed advantages and anticipated difficulties, as discussed in the review of literature, this method was chosen as it also allowed the groups to be gender balanced. Each group consisted of at least two males and two females as far as possible. This was deemed necessary to lessen possible differences in results being attributed to gender differences as discussed in Chapter 2.

Researchers such as Good and Brophy (1987) and Webb (1982b) found that a group comprised of one high-, one low-, and two average achieving students may be more beneficial for high- and low-achievers than for average-achievers. However, Robertson, Graves, and Tuck (cited in Davidson, 1990) viewed heterogeneous groups as reflecting the composition of the whole class. Consequently, the researcher decided that a group consisting of one high-, one low-, and two average-achievers would be used as this method appeared to reflect a 'normal' class situation.

Homogeneous groups were also formed on the basis of the rank a student was given by the class teacher for mathematical ability. The researcher used this information to form homogeneous groups which also were gender balanced with two males and two females wherever possible. The research suggests that average-achievers perform better in homogeneous groups than high- and low-achievers.

Appendix 2 shows the rank order of students and placements of students into both heterogeneous and homogeneous groups. Within each class group the students worked in groups containing four students. Where there were not enough students to complete a group of four, a group of five students was used.

Testing. The post-test phase involved all students completing the same test. Two post-tests were administered. The first post-test was administered after the students had completed the activities for mathematics concept number 1 (exponents) and the second post-test was administered after the students had completed the activities for mathematics concept number 2 (percent). The post-tests were collected from the classroom teachers on completion of each concept. The researcher marked the tests and recorded the results. The teachers were given copies of their students' results, including a test analysis for each concept area.

At the completion of these three phases, the data gathered were analysed.

Assumptions of the Study

The following assumptions were made in relation to this research project:

1. Socio-economic status has been accepted in research domains as a proxy variable for I.Q. and/or achievement. As the sample was being chosen from schools with a similar socio-economic status the researcher assumed that the students in both classes would have had a similar educational and social background.
2. An assumption made on the basis of the educational background of the students in the two classes was that the students would have similar academic abilities.

3. The researcher assumed that students in Year 6 at both schools had a similar background in relation to mathematics education as both schools had used the same mathematics syllabus.
4. The concepts chosen were new to the students.
5. The teachers involved followed the provided teaching programmes closely to ensure consistency between classes.
6. Students were assigned randomly to their whole class groups.

Limitations of the Study

The following limitations applied to the research project:

1. The researcher took into account the prescription of Gay (1987) that a researcher should remain separate from the experimental phase of the study. As a result, the researcher did not observe the lessons and hence could not verify:
   a) the passivity effect of average-achievers;
   b) the teacher's presentation;
   c) the social/affective outcomes.
2. For effective small group cooperative learning to take place, the literature and previous research had shown that teachers required training in enhancing group processes. To fulfil this requirement, two volunteer teachers were chosen to participate in this study. Both teachers had completed a Bachelor of Education unit which provided a background in small group cooperative learning techniques. Although the use of volunteer teachers and their classes placed a limitation on the study in that the sample was not chosen randomly, the researcher was more certain of reducing possible teacher effects by using teachers familiar with small group cooperative learning.

3. Slight differences may have occurred between the sample and the population from which the sample was chosen. A small sample has been shown to be more likely to produce results different from the population than a large sample (Gay, 1987, p. 114). In this research the sample size was 50 students, taken from two primary classes in the Perth metropolitan area. Given that this sample size was quite small, some generalizability problems may arise when applying the findings of the study to a wider population. This problem is escalated by the fact that the sample was not totally random and was instead chosen on the basis of using the classes of the two volunteer teachers.

4. *As no pre-test was used the researcher had no measure of students' prior knowledge in either of the concept areas.* However, the researcher decided that as the concepts were new to the students a pre-test was unnecessary. If a pre-test had been used the students may have been alerted to the direction of the research, resulting in a bias in the results.

5. The "Hawthorn Effect" may have had some bearing on the results obtained for this study. The Hawthorn Effect is a term used to describe any situation "in which subjects' behaviour is affected not by the treatment per se, but by their knowledge of participation in a study" (Gay, 1987, p. 275). The students in this study were not told the research direction, but were informed that the researcher was investigating a new way of presenting mathematics. This may have biased results but both the teachers and researcher deemed that an explanation was necessary to explain a new practice and a new set of rules in the class.

6. The novelty of mathematics lessons being presented in a different way may have increased the students' intrinsic motivation, and so raised the students' achievement levels.

Ethical Considerations

Confidentiality of all participants in the study was guaranteed by the researcher. In order to adhere to this guarantee the following steps were taken:

1. The schools involved in the study were not identified by school name or location.
2. The teachers involved in the study were not identified in the study except where referred to as the teacher of Class A or Class B.
3. The students who participated in the study were not identified in the study except where referred to by a number from 01 to 50, or as a member of Class A or Class B.

Summary of the Chapter

This chapter examined the procedural aspects of the pilot study. The conceptual framework which emerged from the literature and direction of a W.A.C.A.E. research team on classroom effectiveness studies was presented. This was followed by restating the research problem and a statement of both the experimental and null hypotheses. The quasi-experimental research design was presented followed by a description of the subjects involved in the pilot study. The method and criteria for selection of the mathematical concepts to be explored were then discussed and a description of the teaching programmes and instruments was provided. The actual procedure followed was then described and the chapter concluded with a discussion of both the assumptions and the limitations of the pilot study.

Chapter 4

Data Analysis

Overview of the Chapter

This chapter on data analysis will discuss the methods used by the researcher to determine the validity and reliability of the post-tests, the procedure followed to collate the data, and the method used to analyse the data.

Validity of the Post-Tests

Validity, in educational measurement terms, is the degree to which a test measures what it is supposed to measure. There are several different types of validity: content, construct, concurrent, and predictive. For this kind of study content validity was the most crucial. Gay (1987, p. 130) has stated that "content validity is of prime importance for achievement tests as a test score cannot accurately reflect a student's achievement if it does not measure what the student was supposed to learn".

Gay (1987, p. 542) has defined content validity as "the degree to which a test measures an intended content area; it is determined by expert judgement and requires both item validity and sampling validity". Item validity refers to whether the items on the test represent measurement in the intended content area. Sampling validity refers to how well the test samples the total content area (Gay, 1987, p. 129).

Content validity cannot be given a quantitative value and cannot be computed. Rather, content validity is determined by an expert who reviews the test and the process used in developing the test, taking into account both item and sampling validity.

For this pilot study, the programmes of work, the Tables of Specifications, and the proposed post-tests were submitted to a mathematics education tertiary lecturer. This lecturer reviewed all the relevant information and recommended some minor alterations to the post-tests. Attention was given to the recommended alterations.

Reliability of the Post-Tests

Reliability, in terms of educational measurement, is the degree to which a test consistently measures whatever it measures, and is expressed numerically, usually as a coefficient. The reliability coefficient can range between a positive, perfect reliability of +1.00 and a negative, perfect reliability of -1.00. A coefficient of 0.00 normally means there is no reliability.

There are several ways of determining reliability. The different procedures for determining reliability in common usage were examined and split-half reliability procedures were adopted to determine the reliability of the post-tests. Split-half reliability procedures, while requiring only one application of the test, also provided the researcher with a measure of internal consistency.

The researcher applied the following procedure adapted from Gay (1987, p. 139) to the data in order to calculate split-half reliability of the post-tests:

1. The total test was administered to both class groups.
2. The researcher divided the total test into two comparable halves by including all odd items in one half and all even items in the other half.
3. The researcher computed each subject's score on the two halves of the test - each subject consequently had two scores for each test; a score for the odd items and a score for the even items.

4. The researcher correlated the two sets of scores using the Pearson r formula:

$$r = \frac{\Sigma XY - \frac{(\Sigma X)(\Sigma Y)}{N}}{\sqrt{\left[\Sigma X^2 - \frac{(\Sigma X)^2}{N}\right]\left[\Sigma Y^2 - \frac{(\Sigma Y)^2}{N}\right]}}$$

5. The researcher evaluated the results.

As a split-half reliability coefficient only represents the reliability of a test half as long as the actual test, a correction formula needed to be applied in order to determine the reliability of the total test. The correction formula used by the researcher was the Spearman-Brown prophecy formula:

$$r \text{ (total test)} = \frac{2r \text{ (split half)}}{1 + r \text{ (split half)}}$$

Standard error of measurement (SE_M) was also calculated for each post-test. Standard error of measurement is an "estimate of how often you can expect errors of a given size" (Gay, 1987., p. 142). A small standard error of measurement indicates high reliability, while a high standard error of measurement indicates low reliability. Standard error of measurement was calculated using the following formula:

$$SE_M = SD\sqrt{1 - r}$$

The researcher calculated standard error of measurement as a secondary source of reliability data, and as an aid to the interpretation of post-test data in relation to the scores obtained by the students.

Collation of the Post-Test Data

The post-tests were marked by the researcher. Accuracy was ensured by double checking the marking of each test and the recording of each test score. As the post-tests used were both of an objective nature with only one correct answer per question, no scorer interpretation was required. The scoring system consisted of one mark per correct question. For this reason, the establishment of scorer reliability was not necessary.

After the two post-tests were scored the results were transferred to summary data sheets which are included as Appendix 8. The recording of the scores in a systematic manner facilitated examination of the data and data analysis procedures.

In order to facilitate computer analysis of variance the data were coded in a numerical way. The steps employed by the researcher in this process were:

a) Each subgroup was assigned a number as follows:

    (1) Class B: Exponents - heterogeneous groups.

    (2) Class A: Exponents - homogeneous groups.

    (3) Class A: Percent - heterogeneous groups.

    (4) Class B: Percent - homogeneous groups.

b) Each student was assigned an ID number from 01-50.

c) Scores for each student on each post-test were assigned numerically from 01 to 26, depending on each student's score.

The researcher used the information presented on the data summary sheets to construct a frequency distribution and frequency polygon for each concept area in order to examine the descriptive statistics related to the post-test scores such as the mean, median, range, and standard deviation.

Analysis of the Post-Test Data

Analysis of variance was adopted as the overall statistical procedure in the analysis of the performance scores gained by the students on the two post-tests. This section of the chapter will describe analysis of variance briefly and will then outline the procedure followed by the researcher when computing the analysis of variance.

Description of analysis of variance. The concept which underlies analysis of variance is that the variance of scores can be attributed to two sources, namely, variance between groups and variance within groups. For variance of scores to occur the dependent variable would be affected by the independent variable. The dependent variable for the pilot study was the understanding of mathematical concepts. Gay (1987, p. 392) stated that "randomly formed groups are assumed to be essentially the same at the beginning of a study on a measure of the dependent variable". The independent variable was group composition, namely, heterogeneous or homogeneous. At the completion of the study, after administration of the independent variable, also referred to as the treatment, analysis of variance was used to determine whether the between groups variance (treatment) differed from the within group variable (error) by more than could be expected by chance (Gay, 1987, p. 392).

Analysis of variance is calculated as an *F* ratio. If the *F* ratio is deemed to be significant then the null hypothesis is rejected. However, if the *F* ratio is not deemed to be significant the null hypothesis is not rejected.

__Outline of procedure.__ For the pilot study analysis of variance was used to determine whether there was a significant difference between the means of the post-tests at an alpha level of 0.05 (α=0.05).

The commercially available computer software package Statistical Analysis System (SAS) was used to compute the analysis of variance. The coded data, as described in the section on Collation of Data in this chapter, was entered into the computer. The computer was then used to perform the analysis of variance using the General Linear Models procedure of analysis of variance. During this procedure the post-test mean score for each concept was compared on a class basis.

The null hypothesis for this study could be interpreted as: Class A should perform equally as well as Class B on both concepts. If the null hypothesis is rejected then according to the direction provided by the research hypothesis, the researcher would expect the results of the analysis of variance to indicate that:

a) mc1 (exponents): Class B should perform better than Class A.

b) mc2 (percent): Class A should perform better than Class B.

mc = mathematics concept

__Summary of the Chapter__

This chapter examined the procedures the researcher used in order to determine validity of the post-tests, followed by a discussion of reliability procedures applied to the post-tests. The method used to collate the data from the post-tests was then outlined and the chapter concluded with a discussion of the analysis of variance procedure followed by the researcher.

Chapter 5

Results and Discussion

Overview of the Chapter

The following chapter will present the results for this pilot study in terms of validity, reliability, descriptive statistics, and analysis of variance. Interpretations of the results will also be included. General observations based on the post-test data will then be presented followed by a discussion of the results.

Results of Content Validity

The post-tests were reviewed by a mathematics education tertiary lecturer who declared them to have content validity for the purposes of this pilot study.

Interpretation of the Validity Results

As the post-tests were deemed to have content validity, they were considered to have measured what the students were supposed to learn. In this respect, the results gained by each student on each post-test tended to reflect each student's achievement in the learning of the respective mathematics concepts.

As the dependent variable for this study was the understanding of two mathematical concepts, post-tests which contained both item validity and sampling validity, that is, content validity, were essential to the outcome of the study. In order to attribute any significant differences in achievement to the independent variable the dependent variable needed to be measured accurately and in an appropriate way. By having established content validity of the post-tests any significant differences in achievement could be attributed to the independent variable of group composition.

Results of Reliability Procedures

The results of the split-half reliability procedures, application of the Spearman-Brown correction formula, and Standard Error of Measurement (SE_M) procedures for each post-test are presented in Table 1.

Table 1

Results of Reliability Procedures

Concept Area	Split-half Reliability	Corrected Reliability	SE_M
Exponents	r = 0.32	r = 0.48	3.20
Percent	r = 0.42	r = 0.59	1.98

Interpretation of Reliability Results

The results presented for the reliability of the post-tests indicated that there was a positive relationship between the two halves of each post-test.

The post-test for exponents was reliable to an acceptable level. If the test was administered again the students would obtain a similar score to that achieved on the first administration of the test. The standard error of measurement score of 3.20 indicated that a student's true score on the exponents post-test could range three marks in either direction from the same student's obtained score. This is considered to be acceptably small on a 26-item test (Gay, 1987).

The post-test for percent was also deemed reliable. If the percent post-test was administered a second time, the two scores obtained by each student would be similar. A standard error of measurement score of 1.98 was considered by the researcher to be acceptably small for a 26-item test (Gay, 1987).

The reliability coefficients of 0.48 and 0.59 were considered satisfactory compared to normal educational measurement standards of reliability. The purpose of assessing split-half reliability was to check that the tests did contain internal consistency and that the two halves of each post-test were positively related. In terms of this pilot study, the recorded levels of split-half reliability and the standard error of measurement results were judged acceptable. This meant that when viewing each student's score and using the scores gained for the purposes of analysis of variance, the score of each student was sufficiently close to the student's true score. In this sense, the results gained were deemed reliable.

Descriptive Statistics

Most statistical analyses such as the analysis of variance procedures planned for this study are based on the assumption that the scores represent a normal distribution. In order for analysis of variance procedures to be applied to the data gathered in this study descriptive statistics were calculated to determine whether or not the scores gained on the two post-tests represented a normal distribution.

The data for each student on both post-tests were collated and frequency distributions and polygons based on the results of the two post-tests were devised. Following a report of the descriptive statistics for each post-test, an interpretation of the results will be presented.

__Frequency distributions and polygons.__ The frequency distribution for the score gained by each student on the exponents post-test is presented in Table 2. Only 49 scores were available as one student from Class B was not present when the exponents post-test was administered.

__Table 2__

__Frequency Distribution Based on 49 Exponent Test Scores__

Score	Frequency of Score
6	1
7	4
8	3
9	6
10	6
11	3
12	2
13	3
14	4
15	3
16	4
17	1
18	1
19	3
20	1
21	2
22	2

The data from Table 2 were transferred to a frequency polygon which is presented in Figure 8. As Figure 8 shows, the scores achieved on the exponent post-test ranged from 6 through to 22, with a mean of 12.88. The scores are positively skewed but with a standard deviation of 4.45 which is moderate and, with the measures of central tendency approximately the same, the distribution is acceptable.

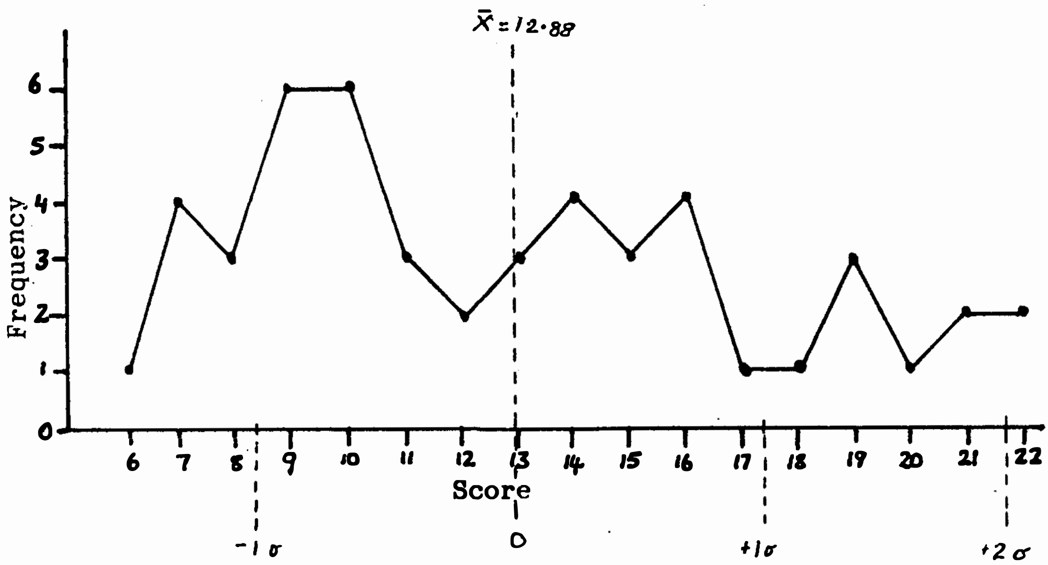

Figure 8. Frequency polygon based on 49 exponent test scores

The frequency distribution for the score gained by each student on the percent post-test is presented in Table 3. Only 48 scores were available as two students from Class B were not present when the percent post-test was administered.

The data from Table 3 were transferred to a frequency polygon which is presented in Figure 9. As Figure 9 shows the scores achieved on the percent post-test ranged from 11 through to 24, with a mean of 18.10. The scores represent a normal distribution with a standard deviation of 3.09 which is moderate, and with the measures of central tendency approximately the same.

Table 3

Frequency Distribution Based

on 48 Percent Test Scores

Score	Frequency of Score
11	1
12	1
13	2
14	1
15	4
16	5
17	7
18	7
19	5
20	3
21	6
22	1
23	2
24	3

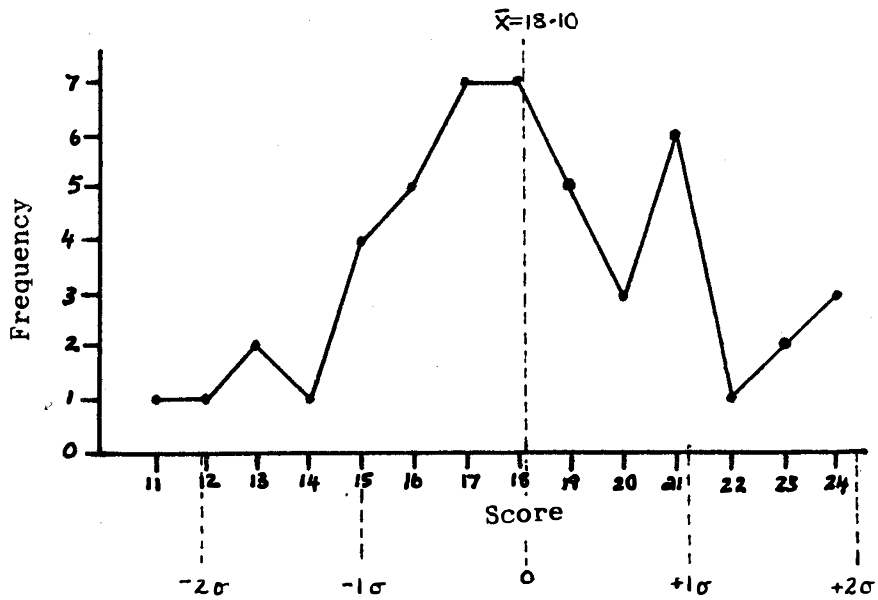

Figure 9. Frequency polygon based on 48 percent test scores

<u>Combination of the post-test results.</u> Measures of central tendency and measures of variability were determined for the post-test scores. The median, mean, range, and standard deviation of the scores for each post-test are presented in Table 4. The results in Table 4 represent descriptive statistics for Class A, Class B, and Class A and Class B combined.

Table 4

Measures of Central Tendency and Variability

Class	Group Composition	Exponents	Group Composition	Percent
A	Homo	M : 10.00 $\bar{X}$: 11.87 R : 7-20 SD: 3.74	Hetero	M : 18.00 $\bar{X}$: 16.93 R : 12-20 SD: 2.60
B	Hetero	M : 13.00 $\bar{X}$: 13.36 R : 6-22 SD: 4.68	Homo	M : 18.00 $\bar{X}$: 18.59 R : 11-24 SD: 3.14
A + B		M : 12.00 $\bar{X}$: 12.88 R : 6-22 SD: 4.45		M : 18.00 $\bar{X}$: 18.10 R : 11-24 SD: 3.09

hetero = heterogeneous groups
homo = homogeneous groups
M = median
$\bar{X}$ = mean
R = range
SD = standard deviation

<u>Interpretation of the Descriptive Statistics</u>

From the data displayed in Tables 2 and 3 and Figures 8 and 9 the distribution of scores appeared to be relatively normal or symmetrical. This was confirmed by establishing measures of central tendency for the post-test scores. Gay (1987, p. 354) states that for a symmetrical distribution the median and mean are the same or very close. Table 4 demonstrated that within each class the median and mean were similar, and overall the median and mean for both classes were very close. This indicated that the distribution of scores for the post-tests was essentially normal or symmetrical.

When scores represent a normal distribution, it is true to say that:

a) $\bar{X} \pm 1SD$ = approximately 68% of the scores;

b) $\bar{X} \pm 2SD$ = approximately 95% of the scores; and

c) $\bar{X} \pm 3SD$ = approximately 99^{+}% of the scores.

The results for both the post-tests indicated that the scores were consistent with the standard pattern of scores. This method of analysing the measures of central tendency and variability was used as a double check to ascertain that the scores represented essentially a normal distribution.

Most statistical analyses are based on the assumption that scores are normally distributed. Because the scores obtained from both post-tests were essentially normal in terms of distribution the analysis of variance techniques were considered to be appropriate and any results gained would be accurate.

Analysis of Variance Results

After establishing that the scores of the two post-tests essentially represented a normal distribution, an analysis of variance (ANOVA) was undertaken. Using the General Linear Model of Analysis of Variance an *F* ratio of 15.91 was computed. The analysis of variance procedure then determined whether the *F* ratio was significant or not at an alpha level of 0.05. Table 5 displays the results of the analysis of variance. Means with the same Duncan Grouping in Table 5 were not significant.

Table 5

Results of ANOVA

Group	*N*	Mean	Duncan Grouping
2	16	11.875	A
1	33	13.364	A
4	34	18.636	B
3	14	16.929	B

Group 1: Class B - exponents - heterogeneous
Group 2: Class A - exponents - homogeneous
Group 3: Class A - percent - heterogeneous
Group 4: Class B - percent - homogeneous
N = number of subjects

For this study the researcher was concerned with comparing the means of Groups 1 and 2, and Groups 3 and 4. The mean of Group 1 was found to be not significantly different from the mean of Group 2. The mean of Group 3 was found to be not significantly different from the mean of Group 4. A summary of these results is presented in Figure 10.

	Class A		Class B
Exponents	Group 2 $\bar{X}$ = 11.875	← not significant →	Group 1 $\bar{X}$ = 13.364
Percent	Group 3 $\bar{X}$ = 16.929	← not significant →	Group 4 $\bar{X}$ = 18.636

Figure 10. Summary of the ANOVA results

Interpretation of Analysis of Variance Results

The research hypothesis for this pilot study was that in the small group cooperative learning situation of the Groups of Four Model heterogeneous group composition would result in greater higher-order learning achievement than homogeneous group composition.

The null hypothesis was that in the small group cooperative learning situation of the Groups of Four Model there would be no difference in higher-order learning achievement between heterogeneous group composition and homogeneous group composition.

The interpretation of the results of the analysis of variance will be discussed using the headings of exponents and percent. For each of these mathematical concept areas the results will be restated, and related to both the research and null hypotheses.

Exponents. For the pilot study to be consistent with the research hypothesis, Class B was expected to perform better than Class A on the mathematical concept of exponents, that is, Group 1 would perform better than Group 2. However, the results demonstrated that there was no significant difference between the means of Group 1 and Group 2. For the area of exponents the null hypothesis was not rejected and, therefore, the research hypothesis was not supported.

<u>Percent.</u> In order for the results to be consistent with the research hypothesis, Class A was expected to perform better than Class B on the mathematical concept of percent, that is, Group 3 would perform better than Group 4. The results demonstrated that there was no significant difference between the means of Group 3 and Group 4. Therefore, in the mathematical concept area of percent the null hypothesis was not rejected and, consequently, the research hypothesis was not supported.

Overall, the null hypothesis was not rejected and therefore the research hypotheses were not supported. The results of the pilot study indicate that in the small group cooperative learning situation of the Groups of Four Model there was no difference in higher-order learning achievement between heterogeneous group composition and homogeneous group composition.

General Observations Based on the Post-Test Data

The following section of the chapter will describe some trends and patterns noted by the researcher when examining the data. These trends and patterns are additional to the original intentions of the study and therefore do not have any significant bearing on the research hypotheses. However, the researcher decided that the trends and patterns were important enough to be included in the thesis.

The post-test scores for both classes were examined on a small-group by small-group basis. The students' scores were organised into Groups of Four according to the group arrangements used during the experimentation phase of the study. This was done so that account could be taken of the scores within each small group in order to identify any patterns or trends which emerged.

A trend was noticed in Class B. In four of the eight heterogeneous groups the high-achiever in the group scored 75% or better. In two instances where the high-achiever scored over 75% the low-achiever in the group scored 40% or better on the post-test. On a third instance the high-achiever was not present for the administration of the post-test but the low-achiever in the group scored 50% on the test which may suggest a continuation of the trend. The fourth instance where the high-achiever scored over 75% the low achiever's score (under 40%) was inconsistent with the observed trend. Conversely, when the high-achievers in Class B scored less than 75% on the post-test the low-achievers in the same groups scored less than 40%. This occurred in all four heterogeneous groups where the high-achiever scored less than 75%. However, when the post-test scores for Class A were examined for heterogeneous groups the same trends were not apparent, although judgement was restricted as two of the four high-achievers in Class A were not present for the administration of the post-test.

The second trend noticed by the researcher was based upon the information in Table 6. In this study the gender of the teacher appeared to be related to the post-test mean scores when compared on a gender basis. The researcher calculated the means for the females and males in both concept areas on a class basis. The results of this comparison are displayed in Table 6. In Class A, which was taught by a female teacher, the females achieved a higher mean score than males regardless of the mathematical concept or group composition. The males in Class B, which was taught by a male teacher, achieved a higher mean score than females regardless of the mathematical concept or group composition.

Table 6

Gender Based Means For Class A and Class B

Class	Exponents		Percent	
	M	F	M	F
A	11.30	14.00	15.75	18.16
B	14.29	10.50	19.76	13.88

The above trends and patterns based on the post-test data were not directly related to the area of research undertaken by the pilot study but they were deemed worthy of reporting as they may suggest areas for further research.

Discussion of the Results

A brief restatement of the results will be provided at the beginning of this section. This will be followed by a discussion of five possible contributing factors to the results gained.

Restatement of the results. Content validity was established for the post-tests prior to their implementation. Split-half reliability was computed and, while not producing reliability levels generally considered as high in research domains, was deemed satisfactory for the purposes of the pilot study.

Descriptive statistics including the median, mean, range, and standard deviation of the post-test scores were calculated and reported. The descriptive statistics indicated that the post-test scores essentially represented a normal distribution.

Analysis of variance procedures were applied to the post-test data. The results of the analysis of variance indicated that in both the mathematical concept areas there were no significant differences in higher-order learning achievement between the groups and the null hypothesis was not able to be rejected. As a consequence, the research hypothesis that in the small group cooperative learning situation of the Groups of Four model heterogeneous group composition would result in greater higher-order learning achievement than homogeneous group composition was not supported.

Variables which may have affected the results. As the null hypothesis was not rejected it was speculated that some factors of an inter-class nature may have intervened in the results. One contributing factor may have been the gender issue discussed in the previous section of the chapter. Another major factor may be differential levels of abilities between the two classes.

Although an assumption underlying the study was that the two classes would be relatively equal on the basis of socio-economic status, there did appear to be a difference between the two classes. Class B students scored higher on both mathematical concept post-tests than Class A students. This may be an indication that the students in Class B were more academically advanced than the students in Class B at the commencement of the experiment but as no pre-tests were applied there is no information available to support this statement other than the post-test scores. On the exponents post-test the mean score for Class B was 13.364 and for Class A was 11.875. On the percent post-test the mean score for Class B was 18.636 and for Class A was 16.929. While there were no reported significant differences in these mean scores, Class B students performed better than Class A students in both post-tests.

A factor contributing to the non-rejection of the null hypothesis may have been the degree to which the teacher was able to follow the programmes provided by the researcher. The researcher had emphasized the importance of following the programmes in order for there to be consistency in instruction for both the classes. Although neither teacher reported any difficulties following the programmes, the possibility arises that one class teacher may have provided extra work for the students in the relevant mathematical concept areas.

Another factor which was discovered only after the experiment was completed was that in Class B a relief teacher replaced the normal classroom teacher for a major portion of the percent phase of the experiment. The regular classroom teacher only returned in time to administer the percent post-test, which meant that the relief teacher took the two lessons focused on the concept of percent. Although the regular classroom teacher left instructions on the correct procedure to follow during these lessons there was no guarantee that these instructions were followed. It is possible that the relief teacher may have been unaware of the importance of following the prescribed lessons or prescribed group composition and may have deviated from or expanded upon the content of the percent programme. If this did occurr the percent post-test results may have been distorted in favour of Class B as these students were in homogeneous groups for the concept area of percent. The research hypothesis for the mathematical concept of percent was interpreted as Class A should perform better than Class B. If direction or instruction other than that which was provided, in the programme of work for percent was provided Class B students may have scored higher than normally would have been expected on the percent post-test. If results for Class B were unintentionally raised this could have affected the results of the study for the mathematical concept of percent. However, on the basis of subjective assessment it is unlikely that this occurred.

The type of content in the familiarisation lessons was different to the type of content in the experimental lessons. The familiarisation lessons used a problem solving approach without an emphasis on a particular concept area. The experimental lessons used a problem solving approach to expand knowledge and higher-order learning achievement in a particular mathematical concept area. The reason for this difference was that the major aim of the familiarisation phase was to train the teachers and students in the Groups of Four model of small group cooperative learning. However, it is possible that the variation between the content of the familiarisation and experimental lessons may have influenced the results as a change in the nature of the lessons may have prevented students from associating skills learned during the familiarisation phase with the skills required for the experimental lessons.

The mathematical concept of exponents was treated first and the mathematical concept of percent was treated second. The mean scores of the classes for the exponents post-test and the percent post-test were significantly different. The percent post-test mean scores were significantly higher than the mean scores on the exponents post-test. Practice effects may have been a contributing factor in this phenomenon. The students had received more experience in the Groups of Four model of small group cooperative learning when they commenced the lessons for percent than they had received prior to commencing the lessons for exponents. The students would have been more experienced in the Groups of Four model, working in small groups, and more familiar with the lesson structure when completing the percent lessons. This may have had a bearing on the results.

Five possible factors contributing to the non-rejection of the null hypothesis have been discussed. These were differential levels of abilities between the two classes, the degree to which the teacher was able to follow the programmes of work, the effect a relief teacher may have had on the results, the effect the type of content in the familiarisation and experimental lessons may have had, and practice effects. The first three factors discussed were all inter-class factors. The difference in type of content in the familiarisation lessons and the experimental lessons was due to lack of clear direction from the literature, while the practice effects factor was a result of the order of treatment and may have applied in reverse if the exponents treatment had occurred second. The gender of the teacher may have also been a contributing factor.

Summary of the Chapter

The results of the study were presented in this chapter. Content validity and split-half reliability were calculated and both found to be at satisfactory levels. Descriptive statistics were used to determine that the scores represented a normal distribution. This was important as analysis of variance procedures are based on normal distributions of scores. The results of the analysis of variance were presented. The null hypothesis was not rejected and therefore the research hypothesis was not supported. General observations of trends apparent in the data were discussed, followed by a discussion of the possible contributing factors in the non-rejection of the null hypothesis.

Chapter 6

Conclusions, Implications, and Recommendations

Overview of the Chapter

The chapter will commence with a summary of the pilot study and report the major conclusions. The results of the study will be related to the relevant literature. An evaluation of the research design will be undertaken prior to a discussion of the implications of the study. The chapter will conclude with several recommendations for further research.

Summary of the Pilot Study

In recent years small group cooperative learning has been given increasing attention by researchers. This interest has been a result of a growing awareness of the benefits that small group cooperative learning can bring to the learning process. These benefits include gains in areas of academic achievement, self-confidence as a learner, cross-cultural/cross-racial relationships, social acceptance of mainstreamed students, and improved attitudes towards school and learning.

A particular focus of North American researchers has been small group cooperative learning in mathematics. Little work had been done in this area in Western Australian schools and with the changed emphasis in the Western Australian primary school mathematics syllabus away from rote learning and pen and paper calculations toward discovery learning a local study seemed appropriate.

This pilot study proposed to investigate the relationship between the composition of cooperative small groups, heterogeneous or homogeneous, and the learning of a mathematical concept in the primary school years. The conceptual framework for this pilot study emerged from both the literature in this area and the direction being taken by a team of W.A.C.A.E. researchers who are investigating small group cooperative learning techniques.

Data were collected using a quasi-experimental analysis of variance design. Year 6 students in two classes participated in the study. Two mathematical concepts were introduced to each class of students, with students learning one concept in a heterogeneous group and the other concept in a homogeneous group. The two classes learned the same concept at the same time but used contrasting group composition techniques. A post-test was applied at the completion of instruction for each mathematical concept and an analysis of variance was used to analyse the data from the post-tests.

The data indicated that while the students had developed higher-order thinking skills in the concept areas, the type of group composition had not affected the amount of higher-order learning which occurred.

Conclusions of the Study

The conclusion of the study was that in the small group cooperative learning situation of the Groups of Four model there was no difference in higher-order learning achievement between heterogeneous group composition and homogeneous group composition.

The research hypothesis that in the small group cooperative learning situation of the Groups of Four model heterogeneous group composition would result in greater higher-order thinking than homogeneous group composition was not supported. As a result, the debate about whether to use heterogeneous or homogeneous groups in small group cooperative learning situations in mathematics was not resolved by this pilot study, nor was it the intention of this pilot study to do so. Rather, as stated previously, one of the purposes of the pilot study was to contribute to the growing educational research data in the area of small group cooperative learning.

General Discussion of the Study in Relation to the Literature

The following section of the chapter will relate the design structure, results and findings of the pilot study to the relevant literature. The discussion will focus around the issues and literature presented in Chapter 2.

Design structure. Cohen (1986, p. 9) suggested two preconditions necessary for small group cooperative learning to be an effective technique for the development of conceptual learning. The first precondition was that the task should require higher-order thinking. The dependent variable of the research design used for this pilot study was the understanding of mathematical concepts. A mathematical concept was defined in Chapter 1 as involving the development of higher-order thinking skills such as analysis, synthesis, and evaluation. In this respect, Cohen's first precondition was fulfilled.

The second precondition was that the group must have the intellectual skills, vocabulary, and relevant information to succeed in the task. The two programmes of work planned for the students were submitted to a mathematics education tertiary lecturer in order to establish content validity. The vocabulary and information to be used formed an integral part of establishing content validity. The two programmes of work were based on the Year 6 mathematics syllabus, with sequential progress through each mathematical concept. This involved the assumption that the students would have the necessary intellectual skills. Therefore, the second precondition imposed by Cohen was fulfilled.

Noddings (1989) and Davidson (1990) suggested properly prepared task instructions in addition to Cohen's suggested preconditions. The two programmes of work provided to the teachers included task instructions which were assumed to be appropriate on the basis of content validity.

Johnson et al. (1984, p. 4) listed four elements identified in their research that should exist in order for small group cooperative learning to be truly cooperative. These elements were positive interdependence among group members, face-to-face interaction among students, individual accountability for mastering assigned material, and interpersonal small group skills. Using the Groups of Four model of small group cooperative learning for mathematics instruction and including post-tests to cater for individual accountability fulfilled the requirements identified by Johnson et al. (1984).

The literature expressed a need for the training of teachers (Good et al. 1988; Noddings, 1989; Parker, 1984) and students (Good & Brophy, 1987; Noddings, 1989) in small group cooperative learning. In line with this expression of need, a familiarisation phase was included in the study.

Webb (1982a) stated that, on average, students in heterogeneous groups scored higher on achievement tests than students who worked in homogeneous groups. This idea was adopted by the researcher as the basis of the research hypothesis of the study.

Good and Brophy (1987) and Webb (1982b) argued against teacher formed groups composed of one high-, one low-, and two average-achieving students on the grounds that this form of group composition was more academically beneficial for high- and low-achievers, as the high-achievers would perceive a need to tutor the low-achievers, but not perceive the same need in regard to the average-achievers. Conversely, Johnson et al. (1984) and Noddings (1989) supported a group composed of one high-, one low-, and two average-achievers. For the purpose of this study, the mode of one high-, one low-, and two average-achievers was adopted. The linkage reported in Chapter 5 between high-achievers doing well and low-achievers doing well in this form of group composition supported the literature on this topic.

Results and findings. The literature revealed a debate among researchers in regard to the composition of small cooperative learning groups. Webb (1982b), Good and Brophy (1987), and Noddings (1989) suggested that average-achievers would perform better in homogeneous groups and remain relatively passive in heterogeneous groups. The pilot study did not support the literature in this respect. The results of the two post-tests indicated that the average-achievers scored similar results regardless of the group composition technique used. This finding is not meant to imply that this would always be the case.

As no process data were gathered in the pilot study no information was available from the study to support or reject the view of Good and Brophy (1987) that average-achievers remain relatively passive in heterogeneous groups as compared to homogeneous groups.

Good and Brophy (1987) suggested that high-achievers and low-achievers would not perform as well in homogeneous groups as they would perform in heterogeneous groups. The findings of the present study did not support the literature, as the results of each post-test indicated that the high- and low-achieving students scored similar results on both post-tests. This would lead to the conclusion that performance was relatively unaffected by group composition. However, this is not meant to imply that the finding of this pilot study would always be supported. The reservations regarding teacher formed groups of one high-, one low-, and two average-achieving students expressed by Good and Brophy (1987) and Webb (1982a) discussed in the previous section of the chapter were supported in several of the groups from Class B. However, the relationship between high-achievers and low-achievers doing well as noted in Class B was not apparent in the results from Class A. Overall, a group composed of one high-, one low-, and two average-achieving students was supported by the study as an effective method of composing heterogeneous groups for instruction.

The literature reviewed discussed group composition in relation to the subject area of mathematics. Many of the researchers supported the use of cooperative learning techniques focusing on higher-order thinking as being beneficial to the understanding of mathematical concepts (Good et al., 1988; Johnson and Johnson cited in Davidson, 1990; Noddings, 1989). This view was supported by the findings of the pilot study. The students in both Class A and Class B were exposed to each concept area for a relatively short period of time yet the students demonstrated a satisfactory understanding of the two concept areas. The focus on higher-order thinking seemed to aid the students' concept attainment and development, which supported the findings reported in the literature.

In regard to group composition, Noddings (1989) proposed that homogeneous groupings were more beneficial for average-achieving students working on mathematical tasks requiring higher-order thinking. Conversely, Good et al. (1988), Parker (1984), and Johnson and Johnson (cited in Davidson, 1990) proposed that heterogeneous groupings were more beneficial for all students when working on mathematical tasks requiring higher-order thinking. No significant difference was found between heterogeneous group composition and homogeneous group composition in higher-order learning achievement. Therefore, the results of the study did not support the literature.

Noddings (1989, p. 613) regarded group composition as crucial to the academic learning expected to take place on any mathematical task and advocated matching group composition to the task. She stated that heterogeneous groups were more suited to measurement activities, simulations, games, and problem solving activities and that homogeneous groups were more appropriate for ordinary academic purposes in mathematics such as solving textbook word problems. The ideas expressed by Noddings (1989) were not supported by the findings of this pilot study in relation to matching group composition to the task. However, the findings of this study suggest it may be more advantageous to use heterogeneous groups for the learning of standard concepts and skills in mathematics no significant differences in achievement scores were indicated between heterogeneous and homogeneous group composition, but there are known affective gains associated with heterogeneous grouping.

The issue of gender was referred to in Chapter 5 when discussing a linkage between the gender of the teacher and the gender-based mean scores for each class. In surveying the literature no evidence was found of research that involved the gender of the teacher and gender achievement in small group cooperative learning. The possibility arises that this pilot study has opened up a further area for research which could in turn extend the literature on small group cooperative learning.

Noddings (1989), Johnson et al. (1984), Parker (1984), and Good et al. (1988) suggested that student and teacher training in small group cooperative learning techniques was important. This contention was supported by the pilot study. The familiarisation phase was designed to provide students and teachers with practice in small group processes and procedures. Because there was little guidance in the literature as to how much training should be undertaken it was assumed that in the pilot study two lessons would be sufficient; one lesson involving heterogeneous group composition and one lesson involving homogeneous group composition. In retrospect, the familiarisation phase may have been of more benefit to the teachers and the students if it had extended over a longer time period. A longer familiarisation period should continue until some stage of all-round readiness is observed among teachers and students working in small groups. In this respect the pilot study may have extended the literature concerning familiarisation of teachers and students in small group cooperative learning.

The literature may have also been extended in the area of the type of content used in familiarisation and experimental programmes.

As discussed in Chapter 5, the familiarisation and experimental lessons should have been similar and both focused on mathematical concepts. This is recommended for any further studies resulting from this research.

The preceding section of Chapter 6 reviewed the pilot study in relation to the literature. Aspects of the research design were related to the literature, as were the results and findings of the study.

Evaluation of the Research Design

The quasi-experimental design devised for the pilot study involved two class groups of students receiving instruction on two mathematical concepts. The classes undertook the same series of lessons in each concept area, using the same problem solving activities and the teachers followed the same programmes of work. The only difference between the two classes was that they used contrasting group composition techniques. At the completion of instruction for each concept a post-test was administered. The post-test scores were analysed to determine if any significant differences in higher-order learning achievement (the dependent variable) resulted from the different forms of group composition (the independent variable).

The research design was operational and enabled statistical results to be determined. The design fulfilled the intentions of the research. The lack of significant differences in the mean scores of each class did not seem to be related to a fault in the research design.

In addition to the problem of academic level differentials discussed in Chapter 5, a further problem in the design was the time limit imposed on the study. The results gained may have varied if additional time had been allowed for the familiarisation phase and the experimental phase of the procedural design. Extra time in the familiarisation phase would have allowed the teachers and the students to be more comfortable not only with the Groups of Four model of cooperative learning but also with its application to specific mathematical concepts. Extra time during the experimental phase may also have allowed for a deeper conceptual understanding to develop.

If this research design was to be replicated a recommendation would be to introduce a pre-test phase into the procedural design in order to determine differential abilities between classes. The mathematics concepts introduced were new to the students but some students may have had some earlier experience in the concept areas and a pre-test would be useful to establish any prior knowledge the students may have. As discussed in Chapter 5, one class seemed to be more academically able than the other class. Academic differences would be identified through the use of a pre-test. A pre-test would be needed to examine the students' conceptual knowledge before commencing to write the programme of work.

If a pre-test was introduced to the research design then analysis of variance would no longer be the most appropriate statistical analysis device. Analysis of Covariance (ANCOVA) could be used instead. Analysis of covariance compares the mean score of the post-test to the mean score of the pre-test to establish any significant differences in the scores.

The overall structure of the research design appeared sound in terms of gathering product data rather than process data. The research design functioned as planned. With modifications allowing for a pre-test and allowing more time at each stage of the procedural design, the research design would be recommended for further use.

Implications of the Study

If the results of the pilot study are to be taken literally, then one conclusion which could be drawn from this study is that there is no difference between heterogeneous groups and homogeneous groups in the academic domain. Heterogeneous groups were not proven to be more effective than homogeneous groups, and homogeneous groups were not proven to be more effective than heterogeneous groups. Another conclusion of the study was that small group cooperative learning was effective in the teaching of two mathematical concepts.

As the preceding discussion has indicated these conclusions should be treated with caution. However, it is intriguing to speculate upon some possible implications for teachers, students, mathematics education, and researchers.

Teachers. If, as the study indicated, there is no significant difference in higher-order learning achievement as a result of group composition then the implications for classroom teachers are significant.

Mathematics teachers often group students according to the ability level of each student, placing students with similar abilities into a group. This occurs not only within a class but also between classes where 'streaming' is used and in multi-level classrooms. A belief widely held by teachers is that the teacher can best cater for each student's needs academically using homogeneous groups in mathematics. The finding of this pilot study impacts upon this belief as the study has shown no academic advantage for homogeneous groups or heterogeneous groups in mathematics. Furthermore, the literature highlighted in Chapter 2 pointed to various affective gains which have resulted from heterogeneous grouping, including gains in areas such as cross-cultural/cross-racial relationships, social acceptance of mainstreamed students, and improved attitudes toward school and learning (Davidson, 1990; Parker, 1984). If heterogeneous group composition and homogeneous group composition produce no significant differences when compared on academic domains then heterogeneous group composition would be a superior form of grouping because of the known gains in the affective domain. If the findings of the pilot study are supported by further research then it would seem that teachers should plan for greater use of heterogeneous groups in mathematics.

Various teaching strategies are used by teachers in mathematics. If the results of the study are valid then small group cooperative learning is important in developing higher-order thinking regardless of the group composition approach. Therefore, teachers should make more use of small group cooperative learning as a teaching strategy.

A clear implication of the study is that training should be provided for teachers in small group cooperative learning techniques both at pre-service and in-service levels. The role of the teacher has emerged as an important variable in the Groups of Four model of small group cooperative learning and there was a need expressed by many researchers for the training of teachers in small group cooperative learning techniques. This expression of need was supported by the findings of the pilot study and demonstrated by the need for extra familiarisation lessons. The teacher was considered as a possible contributing factor in the non-rejection of the null hypothesis.

Students. The implication of the findings of the pilot study on students may be quite important. Low-achieving students and high-achieving students in homogeneous groups often suffer the stigma of belonging to the bottom group or the top group. If achievement is unaffected by group composition, students should be placed in heterogeneous groups so that other students would no longer be able to easily identify and label other students as belonging to a certain group. Students' expectations of each other would be likely to alter in keeping with new grouping practices. By placing students in heterogeneous groups, pressure may be eased on a student who continually fails or continually is expected to succeed at a high level.

Mathematics. The pilot study has supported the use of small group cooperative learning techniques with the new Western Australian mathematics syllabus. The implication of this finding on the classroom teacher is that a problem solving approach to mathematics in schools is effective and practical. As problem solving develops conceptual thinking, active learning and opportunities for peer interaction, small group cooperative learning should be adopted by teachers and used in their mathematics instruction. Moreover, the study indicated that more use could be made of heterogeneous grouping in learning routine concepts and skills in mathematics.

Researchers. The existing literature on group composition and mathematics recommended one form of group composition over another on academic grounds. The form of group composition recommended varied among educational researchers. As the results of the present study demonstrated no significant academic advantage for one group composition approach over another, the question arises as to whether group composition really affects achievement or not. While in view of the limitations of this study the answer to this question must remain an open one, it is believed that there are grounds and directions for further research on this important topic. In the meantime, the results of this pilot study provide some grounds for reflection upon widely held beliefs and grouping practices in Western Australian schools.

Recommendations for Further Research

Several areas have been highlighted as areas for further research in the area of small group cooperative learning rising from this pilot study.

1. A replication of this pilot study would be appropriate, bearing in mind the need to control for the various factors which have emerged during this study. These include the introduction of a relief teacher during the experimental phase, differential ability levels between the two classes, the need to allow sufficient time for the study to be conducted, and extending the familiarisation phase until the teachers and the students reach some stage of all-round readiness. A modified research design which includes a pre-test is recommended.
2. A study could be conducted in a similar style to this pilot study but in a different area of the curriculum, controlling for the same factors as mentioned above, and including a pre-test phase.

3. This study could be replicated but expanded upon to include observers to collect data about how each group functions during the experimentation phase. This should provide valuable insight into the affective outcomes of small group cooperative learning as well as the academic involvement of average-achieving students in heterogeneous groups, and academic involvement of high- and low-achieving students in homogeneous groups.
4. A study which investigated the importance of the teacher in the small group cooperative learning process would be valuable. The teachers would need to be fully in-serviced in small group cooperative learning techniques and about their role as a facilitator during small group cooperative learning.
5. The issue of gender referred to in Chapter 5 appeared to be worthy of further investigation. The role of the teacher is important and if achievement outcomes of the students are related to the gender of the teacher then any data collected in this area would provide valuable information to researchers in the area of small group cooperative learning.
6. Inter-small-group differences as discussed in Chapter 5 may be a better basis for experimental comparison than inter-class differences. A study investigating inter-small-group differences could also take into account the academic performances of high-, low-, and average-achievers in both heterogeneous groups and homogeneous groups.
7. A study could be carried out to investigate mathematical tasks or concepts which are suited/not suited to small group cooperative learning.

Summary of the Chapter

A summary of the pilot study was provided. The major conclusion resulting from the study was presented. A synopsis of the relevant literature was followed by a general discussion relating the results gained to the literature. This was followed by an analysis of the research design. The implications of the study were discussed and recommendations for areas of further research were presented.

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APPENDICES

Appendix 1

Cover Letter

WESTERN AUSTRALIAN COLLEGE OF ADVANCED EDUCATION

MOUNT LAWLEY CAMPUS

[redacted]
Mount Lawley
Telephone (09) [redacted]
Facsimile (G2/G3) [redacted]

[redacted]
Mount Lawley
Western Australia 6050

27 August 1990

Principal
School
Avenue
WA

Dear

I am writing to ask your permission for an Honours student at the W.A. College to do a small piece of research on group composition in class.

By way of background, was in my B.Ed. class on Effective Teaching in the Classroom in Semester One, and as part of our work we looked at small group co-operative learning. As a knowledge of small group learning is essential for this research study, and as has a year six, he would fit in with this study rather nicely. I have sounded him out to this effect and as he was positive, I have decided to ask your permission for the exercise to go ahead.

The project is a pilot study on the relationship between small group composition and the learning of a mathematical concept. The general idea is that two classes from two schools will participate in the study. Each class will learn two mathematical concepts, with students learning one concept in similar ability groups and another concept in mixed ability groups. The results of the two groupings will then be compared.

All told there will be six lessons taken from programme, so there should be no disruption to classroom learning. The lessons will be planned by the Honours student, Sue Eaton, but taken by This should ensure that the lessons are presented properly and the data will be reliable.

I think the study could be a useful one because the student is very good and the topic is practical and relevant. What is more, no Australian research seems to have been done in the area. Of course, if you are agreeable to the project a copy of the final report will be given to the school.

I would appreciate it if you could give some consideration to this request and I will phone you later this week for your response, or to answer any queries you might have.

With best wishes

KEVIN BARRY
Senior Lecturer
Department of Education Studies

Appendix 2

Student Ranks and Groups

Class	Class Rank	Student Number	Sex	Homogeneous Group	Heterogeneous Group
A	1	1	F	1	2
	2	2	M	1	1
	3	3	M	1	4
	4	4	F	1	3
	5	5	F	2	1
	6	6	M	2	4
	7	7	M	2	4
	8	8	F	3	3
	9	9	M	2	2
	10	10	M	3	2
	11	11	F	3	1
	12	12	M	3	3
	13	13	F	4	2
	14	14	M	4	3
	15	15	M	4	4
	16	16	M	4	1
B	1	17	M	5	12
	2	18	M	5	9
	3	19	F	5	11
	4	20	F	5	8
	5	21	M	6	10
	6	22	M	6	5
	7	23	F	6	6
	8	24	F	6	7

Class	Class Rank	Student Number	Sex	Homogeneous Group	Heterogeneous Group
B	9	25	F	7	5
	10	26	M	7	6
	11	27	M	7	9
	12	28	M	7	8
	13	29	F	7	8
	14	30	F	8	12
	15	31	M	8	11
	16	32	M	8	11
	17	33	F	8	9
	18	34	F	8	12
	19	35	F	9	6
	20	36	M	9	7
	21	37	M	9	10
	22	38	M	10	9
	23	39	F	9	6
	24	40	F	10	5
	25	41	F	10	10
	26	42	M	10	7
	27	43	M	11	12
	28	44	F	11	9
	29	45	M	11	6
	30	46	M	12	5
	31	47	M	12	8
	32	48	F	11	10
	33	49	F	12	7
	34	50	F	12	11

Appendix 3

Familiarisation Programme

FAMILIARISATION LESSON 1: CONSECUTIVE NUMBERS

CLASSROOM ORGANISATION:

* Arrange desks into clusters of four.

GROUP ORGANISATION:

* Mixed-ability groups

MATERIALS/RESOURCES:

* Chart of rules for working in groups-of-four (provided).

* Large sheets of paper for students to record results on.

FAMILIARISATION LESSON 1: CONSECUTIVE NUMBERS

Main Ideas	Development of the Lesson	Replies
Introduction to Groups-of-Four	<u>Teacher</u> * Assign students to pre-determined mixed-ability groups * Tell the students that they are going to be working in these groups-of-four in maths today. They have to work as a 'team' and cooperate with each other * When the students are working in these groups they have 3 rules to follow - as outlined on the chart. 1. *Each member of the group is responsible for his or her own work and behaviour.* This means that each student has responsibilities and it is their responsibility to meet them. These include: - if you do not understand something the first thing you do is ask your group for help. - if you do understand something, do not take over and give answers. - listen to other peoples' ideas. - contribute to the group effort. 2. *Each member of the group must be willing to help any other group member who asks for help.* - each group member has three willing helpers close by at all times. - only give help when asked - help, not just by giving answers, but by trying to find questions that will help someone. 3. *You may only ask the teacher for help if all four group members have the same question.* - seek help from one another first. - only if no-one in your group has any ideas can you ask the teacher for help. * Ask the students if they have any questions about any of the rules. (Answer all questions on the basis of the Groups-of-Four article by Marilyn Burns.)	

Main Ideas	Development of the Lesson	Replies
Introduction to lesson on Consecutive Numbers	<u>Teacher</u> * Each group is going to find all the ways to write the numbers from 1 to 25 as the sum of consecutive numbers. * "Consecutive numbers" means numbers in a row. For example: 1, 2, 3, 4, 5 are consecutive numbers. So are 6 & 7. The numbers 1, 4, & 7 are not consecutive numbers. * [Demonstrate on the blackboard what the students are to do, using 9 as an example] For example, I'll use the number 9. 9 = 4 + 5 (4 & 5 are consecutive numbers) 9 = 2 + 3 + 4 * Some numbers will not work. Try the number 4. * When you have found all the ways to write the numbers from 1 to 25 as the sum of consecutive numbers see if you can find a pattern for the numbers that will not work. * Also see if you can find any other patterns and write a sentence which describes each pattern you find. * [Provide each group with a large sheet of paper.] Tell each group to write group members' names on the sheet of paper. This sheet of paper is for them to record what they do on. NOTE: do not tell students how to approach sharing the work and do not tell them how to record.	
Students work on the task	<u>Small Groups</u> * [Each group is to complete the task working together. They can allocate the work however they want to, and can record in their own way.] * Groups who finish the task early can: - extend the current activity up to 35; - find three consecutive numbers whose sum is 114 (37, 38, 39); 468 (155, 156, 157); 1113 (370, 371, 372).	* Ways of writing 1 to 25 as sums of consecutive numbers. * Summary statements/ sentences for patterns.
Discussion	<u>Teacher</u> * [When all groups have completed the task have them put their results up somewhere where the whole class can see them. Allow time for each group to examine the results of other groups and compare results and recording styles.]	

Main Ideas	Development of the Lesson	Replies
	* DISCUSS: What the results show and some of the patterns the students have discovered in their groups. How did each group work to find out which consecutive numbers could be summed to produce a number between 1 & 25? Who did the recording? Comparisons between groups and differences between groups.	
Lesson Conclusion	<u>Teacher</u> * Review the summary statements /sentences about the patterns. (If there is an incorrect statement - correct it.)	

FAMILIARISATION LESSON 2: PROBLEM SOLVING ACTIVITIES

CLASSROOM ORGANISATION:

* Arrange desks into clusters of four.

GROUP ORGANISATION:

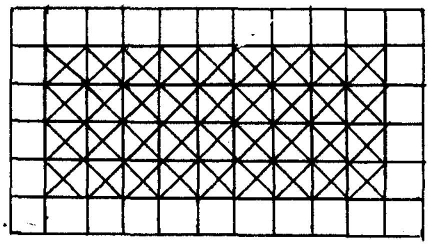
* Uniform-ability groups

MATERIALS/RESOURCES:

* Chart of rules for working in groups-of-four (provided).
* Handouts of activities.
* Squared paper
* Paper to use to work out the problems encountered.

FAMILIARISATION LESSON 2: PROBLEM SOLVING ACTIVITIES

Main Ideas	Development of the Lesson	Replies
Review of Groups-of-Four	<u>Teacher</u> * Assign students to pre-determined uniform-ability groups. * Review rules for working in groups-of-four (using the chart): 1. Each member of the group is responsible for his or her own work and behaviour. 2. Each member of the group must be willing to help any other group member who asks for help. 3. You may only ask the teacher for help if all four group members have the same question.	
Problem Solving	<u>Teacher</u> * [Introduce the first problem solving activity.] * "Let's all shake hands". (On handout - one copy for each group.) Twenty friends meet for the first time in many years. They all shake hands with each other. How many handshakes all together? * Suggest to the students that they act it out in their groups with 1 person, 2 people, 3 people, 4 people and keep a record. Look for a pattern and extend this up to 20 people. * [Allow 10-15 mins.]	
	<u>Small Groups</u> * [Students work together in their groups to solve the problem.] * Record the information, and look for a pattern	* 20 people = 190 handshakes.
Conclusion of Task 1	<u>Teacher</u> * Accept responses from groups as to how many handshakes.	
	* Discuss the various ways the groups used to form an answer (what strategy they used).	* Possibly: -system -diagrams -pattern -formula $\frac{n(n-1)}{2}$ = handshakes

Main Ideas	Development of the Lesson	Replies
Introduction to Task 2	<u>Teacher</u> * This activity is very different from the last one. You have to find a rectangle using square paper where the number of squares on the outside of the rectangle equals the number of squares on the inside of the rectangle.  The number of □ should equal the number of ⊠. In this example that has not occured. * [Can give the hint that there are only two rectangles where this occurs and they are both smaller than 15 *x* 15.]	
	<u>Small Groups</u> * [Use squared paper to find solutions] * Each group is provided with a handout of the task. Record results on a piece of paper. * [Allow 10 to 15 mins.]	* Solutions: 8 *x* 6 12 *x* 5
Conclusion of Task 2	<u>Teacher</u> * DISCUSS: The answers/solutions from each group and how the solution was reached. How well did the group work together? How the group work was shared.	
Conclusion of Lesson	<u>Teacher</u> * Review the activities completed and the ways in which the groups are working together.	

Appendix 4

Programme for Exponents

EXPONENTS: LESSONS 1 AND 2

CLASSROOM ORGANISATION:

* Arrange desks into clusters of four.

GROUP ORGANISATION:

*

MATERIALS/RESOURCES:

* Chart of rules for working in groups-of-four (provided).
* Large sheets of paper for students to record results on.
* Handouts of activities.

EXPONENTS: LESSON 1

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
	<u>Teacher</u> * Revise the multiplication principle that *factor x factor = product.*	
Introduction to new learning	* Study the statement (1 *x* 10). What can you tell me about this statement? [In particular where 10 is used as a factor.]	* 1 set of 10 1 lot of 10 2 factors:1,10 Product is 10
	* Try (2 *x* 10). * [Introduce the first small group activity]	
	<u>Small Group</u> * Tasks:	
Practice new learning	1. What other numbers are there where 10 is used as a factor once?	1*x*10, 2*x*10, 3*x*10 4*x*10, ... 9*x*10, 23*x*10
	2. What can you say about these numbers?	* 10 once. 2 factors in this numeral.
	3. What do they all have in common?	* 10 as a factor Products <100 Products have a final digit of 0.
	<u>Teacher</u> ~~* Review~~ of small group activity. * Introduce the next activity.	
	<u>Small Group</u>	
Extend new learning	* Can you think of a number where 10 is used as a factor twice. Write down as many as you can think of.	10*x*10=100, 2*x*(10*x*10)=200,.. 9*x*(10*x*10)=900 23*x*(10*x*10)=2300
	* What have all these numbers got in common?	* 10 as a factor twice. 2 zeros in product. >100
	<u>Teacher</u> * Review small group activity. * [Introduce the next small group activity with the tasks written on the outside of an envelope and an extra task written inside the envelope - to only be completed when the other tasks are completed.]	

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
	<u>Small Group</u> * Think of numbers where 10 is used as a factor three times and write them down.	1x(10x10x10),... 9x(10x10x10)... 23x(10x10x10)
	* What do these numbers have in common?	* 10 as a factor three times. 3 zeros in the product. >1000
	* Special Activity: Find different ways of writing these numbers with factors of 10. [eg. (1x10), (9x10), 1x(10x10), 5x(10x10), 8x(10x10), 3x(10x10x10), 7x(10x10x10), 4x(10x10x10x10).]	
	<u>Teacher</u> * Review the small group activities from above. Accept all answers, and list all possible suggested short forms on the blackboard/overhead. * Allow the class to decide which shortened form they like best. * Demonstrate the actual short form: 2x(10x10x10)=2000 $2x10^3=2000$ OR $2x2x2=2^3$; $6x6=6^2$; $12x12x12x12=12^4$ * Introduce the term "exponent" which tells us the number of times 10 is used as a factor. * Introduce small group activity.	
	<u>Small Group</u> * Complete the following table: 10^4 = 1x(10x10x10x10) = 10000 10^3 = = 10^2 = = 10^1 = = 10^0 = =	
Conclusion	<u>Teacher</u> ~~* Review~~ the small group activity. * Conclusion - Main Points: 1 The term exponent tells us how many times 10 is used as a factor. 2 Can be written the long way or the short way. 3 Exponents are used not only when 10 is used as a factor but also when other numbers are used as a factor too. eg. $2^3=2x2x2=8$; $4^2=4x4=16$; $5^4=5x5x5x5=625$	

SPECIAL ACTIVITY

Find different ways of writing these numbers with factors of 10.

a. (1×10)

b. (9×10)

c. $1 \times (10 \times 10)$

d. $5 \times (10 \times 10)$

e. $8 \times (10 \times 10)$

f. $3 \times (10 \times 10 \times 10)$

g. $7 \times (10 \times 10 \times 10)$

h. $4 \times (10 \times 10 \times 10 \times 10)$

10^4	=	$1 \times (10 \times 10 \times 10 \times 10)$	=	10 000
10^3	=		=	
10^2	=		=	
10^1	=		=	
10^0	=		=	

Write statements about the following:

a. 10^2 b. 10^4 c. 10^3 d. (3×10^3) e. (7×10^4)

eg: 10^2 = ten to the power of two

EXPONENTS: LESSON 2

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
Introduction	<u>Teacher</u> * As a review of the previous lesson, introduce small group activity.	
Practice learning	<u>Small Group</u> * Write these numbers in exponent form: 100, 1000, 30, 4000, 10000, 1	* 1×10^2, 1×10^3, 3×10^1, 4×10^3, 1×10^4, 1×10^0
	<u>Teacher</u> ~~* Review~~ the first small group activity. * Introduce the second small group activity	
Extend new learning	<u>Small Group</u> * Write down all the ways you can think of to say/read these exponent numerals. Try 10^2	* ten squared, Ten to the power of 2. Ten exponent 2 Ten used as a factor twice.
	<u>Teacher</u> * Accept all answers and put on blackboard. Discuss possibilities. * Tell the students that an exponent, as in 10^2, names how many factors of 10 in this number which we call the POWER of a factor - ten to the power of 2. * Introduce the next small group activity.	
	<u>Small Group</u> * Write statements about the following: 10^2, 10^4, 10^3, 3×10^3, 7×10^4 eg. 10^2 = ten to the power of two.	
	<u>Teacher</u> * Review small group activity. * Introduce the next small group activity.	

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES

<u>Small Group</u>

* Complete the following table:

	1×10
100	
	1×10×10×10
10 000	
	1×10×10×10×10×10

* What patterns seem to show up?

EXPECTED REPLIES:
* Relationship between the zeros in the compact and expanded numerals. Place value.

* Now study this chart. Write down any patterns you can see.

TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES.
10 000	1 000	100	10	1
(1×10×10×10×10)	1×10×10×10	1×10×10	1×10	1
10^4	10^3	10^2	10^1	10^0.

EXPECTED REPLIES:
* Names number of zeros in that power of 10. The exponent seems to name the number of places to the left of the one's place.

<u>Teacher</u>

~~* Review~~ small group activity.

* Introduce the following activity.

<u>Small Group</u>

* In writing expanded numerals for any compact numeral, what use might we make of exponential numeration?

eg. 368 = 300 + 60 + 8
= (3×100) + (6×10) + 8
= (3×10×10) + (6×10) + 8
= $(3\times 10^2) + (6\times 10^1) + 8$

* Try 23465, 7328, 435.

Conclusion

<u>Teacher</u>

~~* Review~~ the small group activity.

* Conclusion - Main Points:
  * 10^2 reads as ten to the power of two.
  * There is a relationship between the number of zeros in the compact and expanded numerals.
  * Exponents are related to place value.
  * Expanded numerals can be related to exponential numeration.

Complete this chart.

Make sure you all discuss each one.

	1×10
100	
	1×10×10×10
10 000	
	1×10×10×10×10×10

What patterns can you see?

TEN THOUSANDS	THOUSANDS	HUNDREDS	TENS	ONES
10 000	1 000	100	10	1
(1×10×10×10×10)	1×10×10×10	1×10×10	1×10	1
10^4	10^3	10^2	10^1	10^0

WRITE DOWN ANY PATTERNS YOU CAN SEE.

$$368 = 300 + 60 + 8$$
$$= (3\times100) + (6\times10) + 8$$
$$= (3\times10\times10) + (6\times10) + 8$$
$$= (3\times10^2) + (6\times10^1) + 8$$

IN YOUR GROUP TRY:

a. 23 465

b. 7 328

c. 435

d. 1 020

Appendix 5

Test for Exponents

Name: ____________________

EXPONENTS

** Please circle the correct answer.

** If you circle the wrong answer by mistake then cross out the wrong answer and circle the correct answer.

1. The numeral 10^3 equals
   A. 3*x*10
   B. 10*x*3
   C. 10*x*10*x*10
   D. 10*x*10*x*10*x*3

2. The exponent digit in the numeral ($2x10^3$) is the digit
   A. 2
   B. 1
   C. 0
   D. 3

3. The numeral (1*x*10*x*10) equals
   A. 20
   B. 100
   C. 21
   D. 101

4. Ten to the power of four is written as
   A. 10^4
   B. 10*x*4
   C. 104
   D. 4^{10}

5. The 2 digit in the numeral 10^2 tells us
   A. the number of times 10 is used as a factor.
   B. the number of digits in that numeral.
   C. that ten is to be multiplied by two.
   D. that ten is to be divided by two.

6. The numeral 200 equals

A. $2+10^3$
B. $2+10^2$
C. $2x10^3$
D. $2x10^2$

7. The numeral 500 equals

A. 5x10
B. 5x10x10
C. 5x10x10x10
D. 5x10x10x10x10

8. If the numeral 10^1 is 10 then the numeral 10^0 is

A. 0
B. 1
C. 10
D. 100

9. The numeral (5x10x10x10) equals

A. 500
B. 5 000
C. 5 111
D. 50 000

10. The numeral $(3x10^2)$ is read as

A. three lots of ten to the first power.
B. three lots of ten.
C. three lots of a hundred and two.
D. three lots of ten to the second power.

11. The expanded numeral (3x100)+(4x10)+6 can be shown as

A. 10^3+10^4+6
B. $(3x10^3)+(4x10^1)+6$
C. $(3x10^2)+(4x10^2)+6$
D. $(3x10^2)+(4x10^1)+6$

12. The numeral 4216 equals

A. $(4x10^2)+(2x10^2)+(1x10^1)+6$
B. $(4x10^3)+(2x10^2)+(1x10^1)+6$
C. $(4x10^3)+(2x10^2)+(1x10^1)+(6x10^1)$
D. $(4x10^3)+(2x10^2)+6$

13. The numeral $(7x10^6)$ equals

A. 70 000 000
B. 7 000 000
C. 700 000
D. 706

14. Numbers in which ten is used as a factor twice

A. are less than 100
B. are less than 200
C. come between 100 and 200
D. are over 99

15. The expanded numeral $(4x10^3)+(2x10^1)+3$ equals

A. 421
B. 4 201
C. 4 023
D. 43 213

16. The expanded numeral (3000+400+60) equals

A. $(3x10^2)+(4x10^1)+(6x10)$
B. $(3x10^3)+(4x10^3)+(6x10^1)$
C. $(3x10^3)+(4x10^2)+(6x10^1)$
D. $(3x10^3)+(4x10^2)+6$

17. Ten to the power of one is ten times smaller than

A. 10
B. 20
C. 100
D. 1 000

18. Where there are no factors of 10 in a number, like 2, we can write that number as

A. $2x10^1$
B. $2x10^0$
C. $2x10^{-1}$
D. $2x10$

19. In the numeral 10^3 the one digit is how many places to the left of the ones place?

A. 1
B. 2
C. 3
D. 4

20. Numbers that have a factor of 10 and come between 1 000 and 10 000 use ten as a factor how many times?

A. once
B. twice
C. three times
D. four times

21. In place value, the numeral 10^3 is one place to the left of the numeral 10^2 because

A. 10^3 is ten times smaller
B. 10^3 is ten times greater
C. 10^3 is one more than 10^2
D. 10^3 is bigger than 10^2

22. The numeral 10^4 is one hundred times greater than

A. 10^3
B. 10^2
C. 10^1
D. 10^0

23. The numeral $10^2=100$ because the one digit in 100 is how many places to the left of the ones place?

A. 1
B. 2
C. 10
D. 100

24. The numeral $(9x10^3)$ equals

A. $(9x10^1)x10^2$
B. $(9x10^1)x10^1$
C. $(9x10^1)x10^3$
D. 10^1x10^2

25. The expanded numeral $(7x10x10^3)$ equals

A. $70x10^4$
B. $7x10x30$
C. $7x3x10^2$
D. $7x10^4$

26. The number 2^6 equals

A. 8
B. 12
C. 26
D. 64

Appendix 6

Programme for Percent

PERCENTAGES: LESSONS 1 AND 2

CLASSROOM ORGANISATION:

* Arrange desks into clusters of four.

GROUP ORGANISATION:

*

MATERIALS/RESOURCES:

* Chart of rules for working in groups-of-four (provided).
* Large sheets of paper for students to record results on.
* Handouts of activities.

PERCENTAGES: LESSON 1

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
Review of Groups-of-Four	<u>Teacher</u> * Assign students to pre-determined groups. * Review the rules for Groups-of-Four: 1 Each member of the group is responsible for his or her own work and behaviour. 2 Each member of the group must be willing to help any other group member who asks for help. 3 You can only ask the teacher for help if all four members of your group have the same question.	
Introduce new learning	<u>Teacher</u> * Study this number: 50% What can you tell me about this number?	* ½ of something 50 percent 50 % $\frac{50}{100}$
	* Does anyone know what the word 'percent' means? "per" means "in every" "cent" means "hundred" SO 'percent' means "in every hundred". * Now what can you tell me about 50%?	* means 50 in every 100.
	* How could I write 50% as a fraction?	$\frac{50}{100}$ (or $\frac{5}{10}$, $\frac{1}{2}$)
Practice new learning	<u>Teacher</u> * In your groups complete the following chart.	

50%	$\frac{50}{100}$	50 in every 100
10%		in every 100
100%		in every 100
60%		in every 100
25%		in every 100

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
	<u>Small Group</u> * Complete the task described above.	
Introduce new learning	<u>Teacher</u> * Review the small group activity. * How could we write 50% as a decimal? Work this out in small groups.	
	<u>Small Group</u> * [Short discussion time.]	* 0.50 (or 0.5)

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
	<u>Teacher</u> * [Accept all responses from the groups - these may include 50.0] * Discuss correct response in terms of 50% as 50 in every 100, therefore, 50% written as a decimal is 0.50 (50% is half of 100%, therefore the decimal is less than a whole number.) * Introduce small group activity 2. [You may need to clarify that 0.5=0.50]	Students will have been taught that. $\frac{50}{100} = 0{\cdot}50$ $\frac{25}{100} = 0{\cdot}25$
Practice new learning	<u>Small Group</u> * In groups complete the following chart-talk about each one.	

50%	$\frac{50}{100}$	0.50
30%		
25%		
75%		
100%		
120%		

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
	<u>Teacher</u> * ~~Review~~ small group activity. * Introduce the next small group activity.	
Extend new learning	<u>Small Group</u> * In your groups find a way to illustrate 35%. Is this less than or greater than 50%? * [If groups are stuck - suggest drawing two 10 *x* 10 grids.]	* Will vary.
Conclusion of lesson	<u>Teacher</u> * ~~Review~~ the small group activity. Ask orally whether 60% would be greater than or less than 50%? * Conclusion - Main Points: * Percent means 'in every hundred' . eg. 50% means 50 in every 100 75% means 75 in every 100 28% means 28 in every 100 * Percentage can also be expressed as either a fraction or a decimal. eg: <u>Homework Task:</u> ~~Look through~~ a newspaper or shop brochure and find examples where percentages are used. Bring 2 or 3 of these examples along to the next class.	* greater than

Can you complete this chart ?

Percent	Fraction	Words.
50%		in every hundred.
10%		
100%		
60 %		
25 %		

Percent	Fraction	Decimal.
50%	$\frac{50}{100}$	0·50
30%		
25%		
75 %		
100 %		
120%		

Complete this table in your groups.

Make sure you discuss each one.

PERCENTAGES: LESSON 2

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
Review and practice of learning	<u>Teacher</u> * Assign students to groups. * Review previous lesson by setting a group activity involving completing the following chart: <u>Small Group</u> (chart below)	
	<u>Teacher</u> ~~* Review~~ the small group activity. * Introduce the next small group activity.	
Extend learning	<u>Small Group</u> * Each group is to place their examples of percentages into groups (eg. banks, sales, etc.) and arrange them onto a chart. * Display the chart to the other groups when all groups are finished.	
	<u>Teacher</u> * Discuss different places where the percentages are used, using the charts as a starting point. See if the students know of any other places.	* banks-interest rates, sales, ads, test mark population data elections, pie graphs, etc.
	* [Develop the idea of a bank deposit at a rate of 10% per year. $10 for every $100 per year.] If you have $100 in a bank account at an interest rate of 10%, what would happen to the amount of money you have?	* It would increase $10 every $100.
	* Introduce the next small group activity.	

Chart (Small Group):

50%		
1%		
		0.45
	$\frac{78}{100}$	
18%		

MAIN IDEAS	DEVELOPMENT OF THE LESSON	EXPECTED REPLIES
	<u>Small Group</u> * Complete this chart: Interest rate of 10% $10 for every $ 100 for every $ 200 for every $ 300 for every $1000 for every $1700 for every $2200	
	<u>Teacher</u> * Ask - If a shop offers a discount of 5% off all goods, what could that do to prices? * Introduce small group activity.	* Prices go down by 5%.
	<u>Small Group</u> * [Handout 1 copy per group of the Trendy Kids advertisement offering 'up to 50% off selected stock' and the associated tasks.]	Answers 1-$22.00 2-$10.00 3-$7.50 4-$18.00 5-$40.00.
	<u>Teacher</u> ~~* Review~~ the small group activity, especially numbers 4 and 5.	4. → 10% discount on $20.00 is $2.00. $20.00 - $2.00 $18.00.
Conclusion	<u>Teacher</u> * Conclusion - Main Points: * percent is used in a variety of every day situations. * 10% deposit gives $10 for every $100 increase * 5% discount - prices go down. * various discounts on clothing items effects prices in various ways. Size of discount is related to the original price.	5 → 20% discount on $50.00 is $10.00 so $50.00 - $10.00 $40.00. OR 10% of $50 is $5 so 20% is $10.00 etc.

TRENDY KIDS

Trendy Kids have the following items on sale.

1. A girls dress, which is usually priced at $44·00. In the sale this dress is discounted by 50%. Sale price: __________

2. Boys shoes, normally $20·00, but with a 50% discount will cost __________

3. Baby's shoes, normally $15·00, but are discounted by 50%. Sale price: __________

4. Shirts which normally cost $20·00 are discounted by 10%. Sale price __________

5. Jeans which normally cost $50·00 are discounted by 20%. Sale price __________

Percent.	Decimal.	Fraction
50%		
1%		
	0·45	
		$\frac{78}{100}$
18%		
	0·05	

Please complete this chart.

INTEREST RATE OF 10%

$10 for every $100

$ for every $200

$30 for every $

$ for every $1000

$ for every $1700

$220 for every $

Appendix 7

Test for Percent

Name: ______________________

PERCENT

** Please circle the answer you think is correct.

** If you change your mind about an answer, cross out the wrong answer and circle the answer you think is correct.

1. The word *percent* means
   - A. a hundred.
   - B. in every hundred.
   - C. in every one.
   - D. one in a hundred.

2. The symbol used for percent is
   - A. %
   - B. ^o\o
   - C. $\frac{o}{o}$
   - D. o|°

3. 60 percent written as a fraction is
   - A. $\frac{60}{1}$
   - B. $\frac{60}{10}$
   - C. $\frac{60}{100}$
   - D. $\frac{6}{100}$

4. 32 percent written as a fraction is
   - A. $\frac{32}{1}$
   - B. $\frac{32}{10}$
   - C. $\frac{32}{100}$
   - D. $\frac{320}{100}$

5. 75 percent written as a fraction is
   - A. $\frac{750}{100}$
   - B. $\frac{75}{100}$
   - C. $\frac{75}{10}$
   - D. $\frac{75}{1}$

6. $\frac{54}{100}$ written as percentage is
   - A. 0.54%
   - B. 0.45%
   - C. 45%
   - D. 54%

7. $\frac{9}{10}$ written as a percentage is

A. 90%
B. 9%
C. 0.9%
D. 0.09%

8. 43% written as a decimal is

A. 430.0
B. 43.0
C. 4.3
D. 0.43

9. 70% written as a decimal is

A. 0.007
B. 0.07
C. 0.70
D. 7.0

10. 1% written as a decimal is

A. 0.001
B. 0.01
C. 0.1
D. 1.0

11. 0.58 written as a percentage is

A. 5.8%
B. 8.5%
C. 58%
D. 85%

12. 1.00 written as a percentage is

A. 0.01%
B. 1%
C. 10%
D. 100%

13. Common places where you can see percentages used are

A. shops and paintings
B. paintings and banks
C. libraries and paintings
D. banks and shops

14. Where are you most likely to find a sign saying 25% off?

A. doctor
B. library
C. shop
D. paintings

15. If you deposited $100 at a rate of 10%, how much money would you have after 1 year?

A. $110
B. $100
C. $90
D. $10

16. A dress costs $60 normally but in a sale it is discounted by 20%. How much will it cost now?

A. $80
B. $48
C. $40
D. $12

17. 48% is greater than

A. 32%
B. 49%
C. 53%
D. 71%

18. 50% is less than

A. 12%
B. 43%
C. 49%
D. 63%

19. $\frac{38}{100}$ is greater than

A. 83%
B. 61%
C. 46%
D. 23%

20. 0.79 is less than

A. 80%
B. 79%
C. 78%
D. 62%

21. If a desk costing $1200 is on sale with a 10% discount, what will it cost to buy?
    A. $120
    B. $1000
    C. $1080
    D. $1320

22. If a painting costs $90 to buy after a 10% discount is taken off, what would the painting have cost originally?
    A. $190
    B. $100
    C. $99
    D. $95

23. A video costing $150 is reduced by 10% in a sale. How much discount will you get?
    A. $1.50
    B. $5.00
    C. $15.00
    D. $50.00

24. If $300 is deposited at a rate of 10%, how much interest will the money earn in a year?
    A. $30
    B. $15
    C. $10
    D. $3

25. A person earns $50 on an account earning 10% interest per year. How much money did the person have in the bank to start with?
    A. $5000
    B. $500
    C. $50
    D. $5

26. A person earns $170 on an account earning 10% interest in one year. How much money does the person now have altogether?
    A. $1530
    B. $1700
    C. $1780
    D. $1870

Appendix 8

Data Summary Sheets

Class	Student Number	Exponent Test Result	Percent Test Result
A	1	12	17
	2	17	-
	3	13	-
	4	16	16
	5	20	19
	6	9	18
	7	9	19
	8	10	20
	9	9	18
	10	7	15
	11	11	18
	12	16	20
	13	15	19
	14	8	13
	15	10	13
	16	8	12
B	17	21	24
	18	22	21
	19	14	16
	20	-	17
	21	19	21
	22	22	24
	23	12	19
	24	15	20

Class	Student Number	Exponent Test Result	Percent Test Result
B	25	21	24
	26	13	21
	27	16	16
	28	16	22
	29	14	15
	30	18	21
	31	19	21
	32	10	15
	33	11	18
	34	9	21
	35	10	17
	36	19	16
	37	14	23
	38	14	23
	39	7	15
	40	7	18
	41	10	17
	42	10	17
	43	15	18
	44	8	17
	45	9	18
	46	11	19
	47	13	17
	48	9	16
	49	7	11
	50	6	14