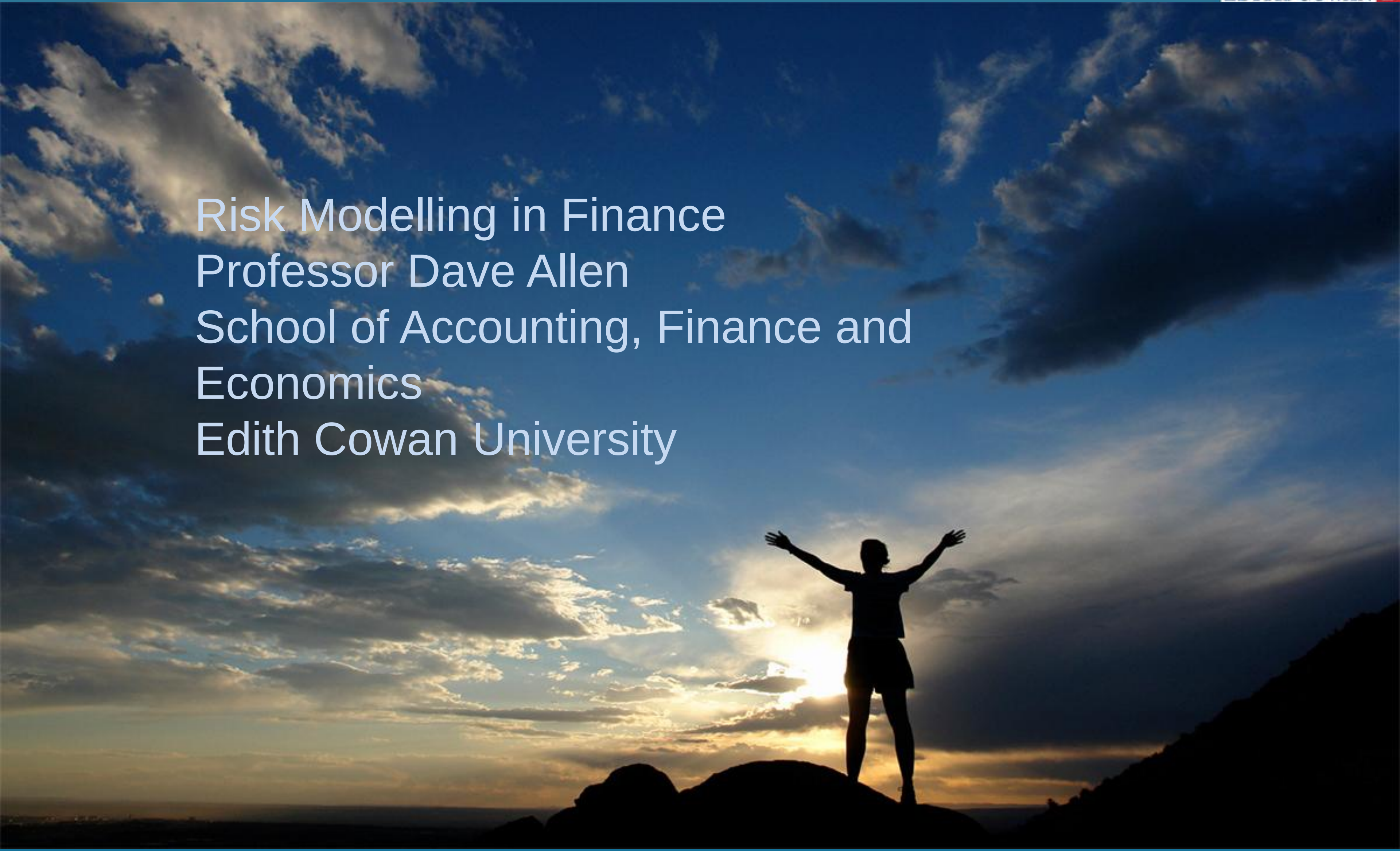
2010

Risk Modelling in Finance

Dave Allen
Edith Cowan University



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Professor Dave Allen
School of Accounting, Finance and Economics
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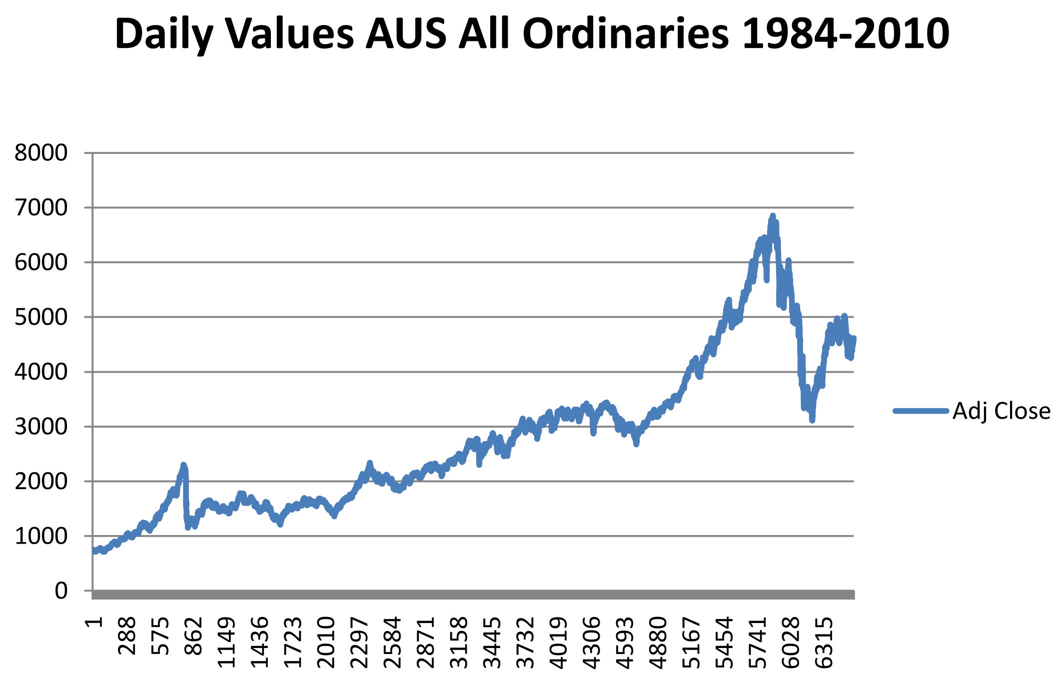
Introduction

Below we have a graph of the Australian All ordinaries Index

Clearly its value has changed a lot in the last year.

Over a longer time period



Introduction

Suppose we download daily closing values from August 3rd 1984 to August 10th 2010 from Yahoo finance http://au.finance.yahoo.com/

We have 6582 observations of the closing values of the index in an excel spreadsheet.

By taking the natural logarithm of the daily price changes we can calculate the daily gains or losses we can obtain from investing in this series.

$$r_t = \ln \frac{p_t}{p_{t-1}}$$

Introduction

If we averaged these daily returns we would have a value of 0.00284 per cent per day or about 7.1% per year.

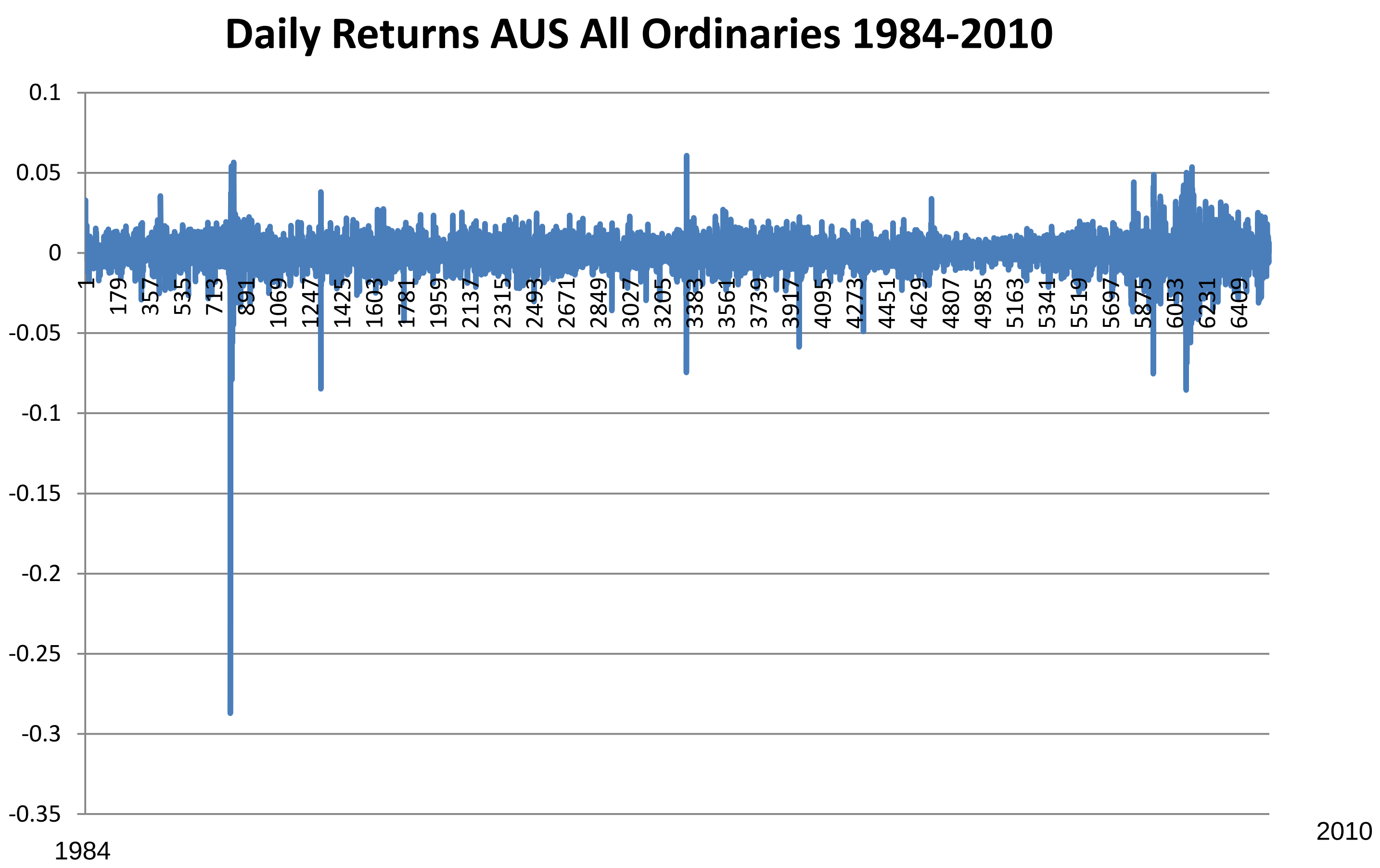
$$\bar{r}_t = \frac{1}{N}\sum_{1}^{N} r_t$$

The standard deviation of the daily return is about 0.01% in daily terms.

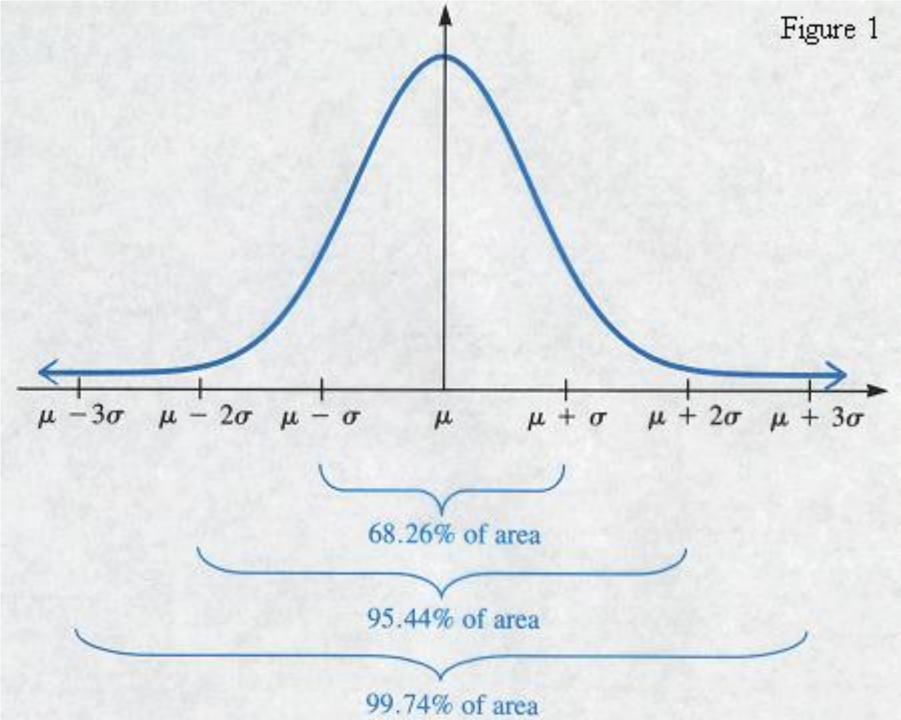
$$\sigma_r = \sqrt{\frac{1}{N}\sum_{1}^{N}(r_t - \bar{r}_t)^2}$$

This is one way of looking at risk. If the distribution was normal we would expect an outcome of the average or mean plus or minus two standard deviations in 95 cases out of 100.

The graph on the next slide shows there is a great deal of volatility. We can see big downward spikes such as the 1987 Crash.


Daily Returns AUS All Ordinaries 1984-2010
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3
-0.35
1
179
357
535
713
891
1069
1247
1425
1603
1781
1959
2137
2315
2493
2671
2849
3027
3205
3383
3561
3739
3917
4095
4273
4451
4629
4807
4985
5163
5341
5519
5697
5875
6053
6231
6409
1984
2010

We can use these concepts to think about risk



Introduction

If the distribution was normal we have some useful properties.

Unfortunately distributions of financial returns are often too fat in the tails and too peaked in the middle to be normal.

We are also assuming that the mean and standard deviation do not change over time.

In this talk I shall consider how ways of thinking about these issues and about measuring risk which may have changed over time.

I shall start with some very old ideas and then move through to more recent work.

Introduction

- In this presentation I shall start with a review of some of the early work with implications for current approaches to modelling risk:
  - First some historical and classical asides
  - then work in the 18th Century by David Hume and Nicholas Bernouilli.
  - I shall then proceed to consider early 20th Century thought on the topic by Keynes, Knight and Ramsey.

- Introduction (contd)
- Next we consider Markowitz's development of portfolio theory in 1952 and the attached treatments of risk.
- The next component will consider risk modelling per se in financial econometrics
  - The ARCH/GARCH literature
  - Realised Volatility
  - Implied Volatility and the VIX index

- Value at Risk (VaR) and Conditional Value at Risk (CVaR)
- Then I shall look at some recent developments using quantile regression analysis and Engle and Manganelli's (2004) CAViaR model.
- Finally, I shall look at forecasts of volatility and make a few comments about the onset of the financial crisis.

Risk What is it?

- For example, according to Wikipedia: The term *risk* only emerged relatively ‘recently’. "In the Middle Ages the term *riscium* was used in highly specific contexts, above all in the sea trade and in its ensuing legal problems of loss and damage." In the 16th Century the vernacular use of *rischio* was derived from the Arabic word "رزق", "rizk", meaning 'to seek prosperity'. It was introduced to European usage by North African trading links.
- The term risk appeared in the English language in the 17th Century imported from Europe and eventually usage moved away from the concept of good and bad fortune to the more modern usage

The original concern was with extreme outcomes

Some 18th Century views

- David Hume in ‘A Treatise on Human Nature’, first published in parts in 1739 and 1740, writes in Section 11; on “Of Probability, and of the idea of Cause and Effect”.
- In this section he talks of three important relations, identity, the situations in time and place, and causation. He argues that we may “consider the relation of contiguity as essential to that of causation; one object is associated with another, priority of time in the cause before the effect”.
- (This was later taken up in a time series econometrics sense by Clive Granger in his concept of Granger Causality)

Hume

- However, Hume then cautions that belief in causality comes from observation; “since it is not from knowledge or any scientific reasoning, that we derive the opinion of the necessity of a cause to every new production, that opinion must necessarily arise from observation and experience”.

David Hume

- “One would appear ridiculous, who would say, that it is only probable the sun will rise to-morrow, or that all men must dye; though it is plain we have no further assurance of these facts, than what experience affords us”.
- He concludes that the supposition that the future resembles the past is derived from habit, based on past experience.
- The problem is when change accelerates as is the case in the modern world. Past experience may be a very poor guide, as we have seen in the GFC.

St Petersburgh paradox

- An 18th Century conundrum.
- Consider the following game of chance: you pay a fixed fee to enter and then a fair coin is tossed repeatedly until a tail appears, ending the game. The pot starts at 1 dollar and is doubled every time a head appears. You win whatever is in the pot after the game ends. Thus you win 1 dollar if a tail appears on the first toss, 2 dollars if a head appears on the first toss and a tail on the second, 4 dollars if a head appears on the first two tosses and a tail on the third, 8 dollars if a head appears on the first three tosses and a tail on the fourth, etc. In short, you win 2^{k-1} dollars if the coin is tossed k times until the first tail appears.

St Petersburgh paradox

- What would be a fair price to pay for entering the game? To answer this we need to consider what would be the average payout: With probability 1/2, you win 1 dollar; with probability 1/4 you win 2 dollars; with probability 1/8 you win 4 dollars etc. The expected value is thus:

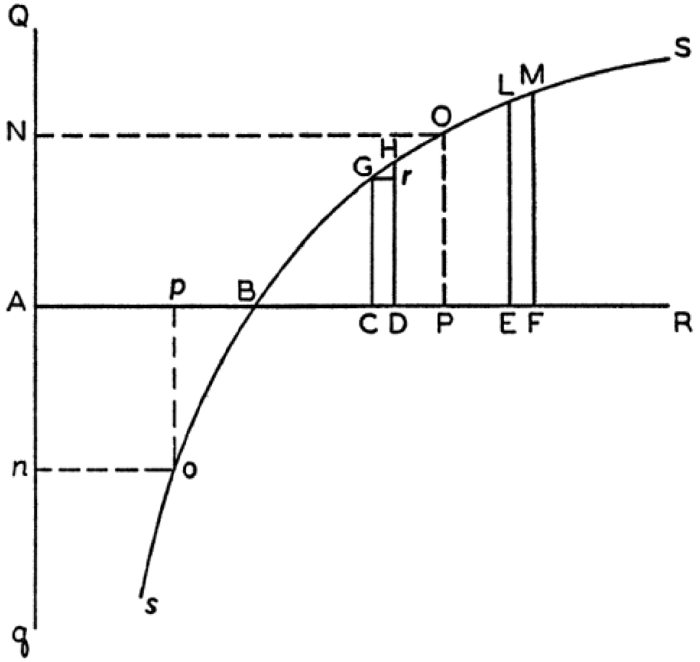
$$
\begin{aligned}
E &= \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{8}\cdot 4 + \frac{1}{16}\cdot 8 + \cdots \\
&= \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \cdots \\
&= \sum_{k=1}^{\infty} \frac{1}{2} = \infty .
\end{aligned}
$$

Nicholas Bernoulii

- Bernouilli suggests "the determination of the value of an item must not be based on its price, but rather on the utility it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate"

Bernoulli developed expected utility theory as a decision criteria under conditions of risk and resolved the St Petersburg paradox.

Figure 1. Bernouilli's expected utility of outcomes



- Bernoulli solved the problem by suggesting that the decision would be made on the basis of the expected utility of the outcome.
- The expected utility of a small probability of a large gain would not be given the same weight as the near certainty of paying a large amount to play the game. The utility would also be a function of a person's initial wealth.
- Variants of these ideas were subsequently taken up in the development of portfolio theory.
- This idea of the importance of an individual's attitudes to risk was another element in subsequent work.

- Holton (2004) argues that the work of Hume precedes two main strands in 20th Century thinking about risk: subjective probability and ‘operationalism’. The former suggests that beliefs about probability summarise individual calculations but are not scientific, or independently verifiable. They are specifications of individual degrees of belief.
- Keynes (1921) took the view that probabilities are ‘rationally determinate’.
- “Part of our knowledge we obtain direct; and part by argument. The theory of probability is concerned with the part which we obtain by argument, and it treats of the different degrees in which the results so obtained are conclusive or inconclusive”.


TIME

Frank Knight 1921

- suggested that there are two fundamentally different ways of arriving at the probability judgment of the form that a given numerical proportion of X's are also Y's. The first method is by a priori calculation, and is applicable to and used in games of chance. This is also the type of case usually assumed in logical and mathematical treatments of probability.
- It must be strongly contrasted with the very different type of problem in which calculation is impossible and the result is reached by the empirical method of applying statistics to actual instances This meant that for all practical purposes, this type of scientific probability was not met in business decisions: the difference between risk and uncertainty.

Knight suggested a simple scheme for separating three different types of probability situation:

1. A priori probability.
2. Statistical probability. Empirical evaluation of the frequency of association between predicates, not analyzable into varying combinations of equally probable alternatives
3. Estimates. The distinction here is that there is no valid basis of any kind for classifying instances.

- The latter is prevalent in business situations.

Ramsey 1926

- Suggested that objective probabilities in decision making are impossible: Ramsey appeals to Hume as the foundation for his argument; “Among the habits of the human mind a position of peculiar importance is occupied by induction. Since, the time of Hume a great deal has been written about the justification for inductive inference. Hume showed that it could not be reduced to deductive inference or justified by formal logic. So far as it goes his demonstration seems to me final; and the suggestion of Mr Keynes that it can be got round by regarding induction as a form of probable inference cannot in my view be maintained.

Thus, we have a number of important antecedents to the treatment of risk in modern finance.

- Hume stating that everything is down to Human experience and the belief that history will repeat itself.
- Bernoulli developing a calculus of expected utility.
- Ramsey and Knight saying that we cannot exactly quantify uncertainty.

Portfolio theory and modern finance

These concepts were drawn upon by Markowitz (1952)

- Harry Markowitz (1999) in a paper on the early history of portfolio theory mentions that in the *Merchant of Venice*, one of Shakespeare's characters in the play; the merchant Antonio says:

  "My ventures are not in one bottom trusted,

  Nor to one place; nor is my whole estate

  Upon the fortune of this present year;

  Therefore, my merchandise makes me not sad"

  Act 1, Scene 1

- "Returns on securities are uncertain events rather than random variables subject to known probabilities. This implies only that the expected returns, variances of returns, and covariances of return referred to in this chapter should be interpreted as based on probability beliefs rather than on objective probabilities".
- Variance had been suggested as a measure of economic risk by Irving Fisher (1906) in *The Nature of Income and Capital*. Furthermore, Marschak (1938) had suggested using covariance matrices as an approximation for utility of consumption of commodities and Marschak was one of Markowitz's supervisors at the University of Chicago.

The important thing to remember is that Markowitz developed a subjective version of portfolio theory that was based on subjective beliefs.

It involved optimisation of the return on a portfolio given risk levels.

It was later the subject of extension to the development of the capital asset pricing model by the assumption of common beliefs and a number of other assumptions.

Markowitz Continued.

Markowitz notes that Roy (1952) suggested a portfolio selection model at the same time that he developed his approach. He developed an efficient set in a similar fashion but advised choosing a single portfolio on this set that maximises $(u_p - d)/\sigma^2_p$ where *d* is a disaster level return the investors wishes to avoid falling below. He suggests that Roy does not receive full credit for this.

The portfolio expected return is calculated as:-

$$\mathrm{E}(R_p) = \sum_i w_i \mathrm{E}(R_i) \qquad (1)$$

Where R_i is return and w_i is the weighting of asset *i*.

And portfolio variance can be written as:-

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij} \qquad (2)$$

where i≠j. Alternatively the expression can be written as:

$$\sigma_p^2 = \sum_i \sum_j w_i w_j \sigma_i \sigma_j \rho_{ij}$$

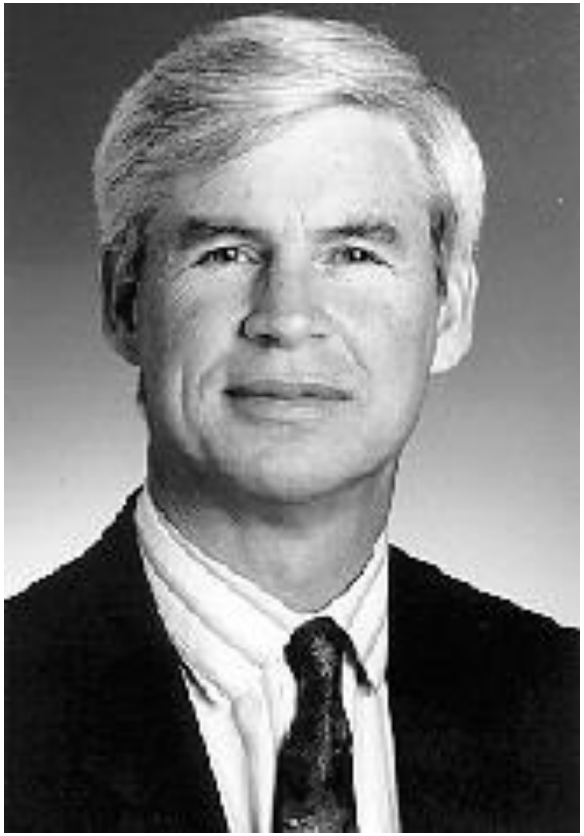
- In contrast to Sharpe (1964) Tobin assumed that one can lend at the risk free rate but not borrow. Sharpe's more sweeping assumption that all can borrow or lend at the risk free rate together with the other assumptions about homogeneity of expectations etc leads to the ubiquity of the capital market line when describing the efficient set. Markowitz (1999) points out that Tobin's assumptions were more cautious whereas, Sharpe's together with Lintner (1965) and Mossin (1966) lead directly on to the Capital Asset Pricing Model which was to revolutionize financial economics.

Risk Modelling in financial econometrics

Robert Engle was one of the first to explore the modelling of volatility and was awarded the joint Nobel Economics prize in 2003 for his work on ARCH Modelling.

He noted in his Nobel address that "Optimal behavior takes risks that are worthwhile. This is the central paradigm of finance; we must take risks to achieve rewards but not all risks are equally rewarded. Both the risks and the rewards are in the future, so it is the expectation of loss that is balanced against the expectation of reward." In this speech he drew a direct connection between, his work, the previously discussed work on portfolio theory and the CAPM, and the development of option pricing models.

Professor Robert Engle



In 1982 when engaged in modelling inflation, Engle developed the autoregressive conditional heteroskedasticity or ARCH model. Engle provided the following explanation in his Nobel address: "The ARCH model described the forecast variance in terms of current observables. Instead of using short or long sample standard deviations, the ARCH model proposed taking weighted averages of past squared forecast errors, a type of weighted variance. These weights could give more influence to recent information and less to the distant past.

The big advance was that the weights could be estimated from historical data even though the true volatility was never observed.

In 1986 Bollerslev formulated the generalized autoregressive conditional heteroskedasticity (GARCH). This generalizes the autoregressive ARCH model to an autoregressive moving average model. The weights loaded on past squared residuals are assumed to reduce in a geometric fashion at a rate estimate from the data set.

The GARCH forecast variance is made up of three components in the case of the standard GARCH(1,1) model, which uses one lag of past forecasts and past error sizes. One component is the intercept which is an average of the long run variance. The second is the forecast for the previous period and third is the size of the previous error.

The conditional mean return can be specified as:

$m_t = E_t\text{-}_1[r_t]$ (3)

the conditional variance as:

$h_t = E_{t-1}[r_t - m_t]^2$ (4)

If we let $E_{t-1}[u]$ be the expectation of some variable u, given the information set available at time t-1. This might be referred to as Z_{t-1}]. This suggests that Rt is generated by the following process:

$$R_t = m_t + \sqrt{h_t e}_t \qquad (5)$$

The specification of conditional variance in a GARCH (p,q) model is:

$$\varepsilon_t = h_t^{1/2} u_t \qquad (9)$$

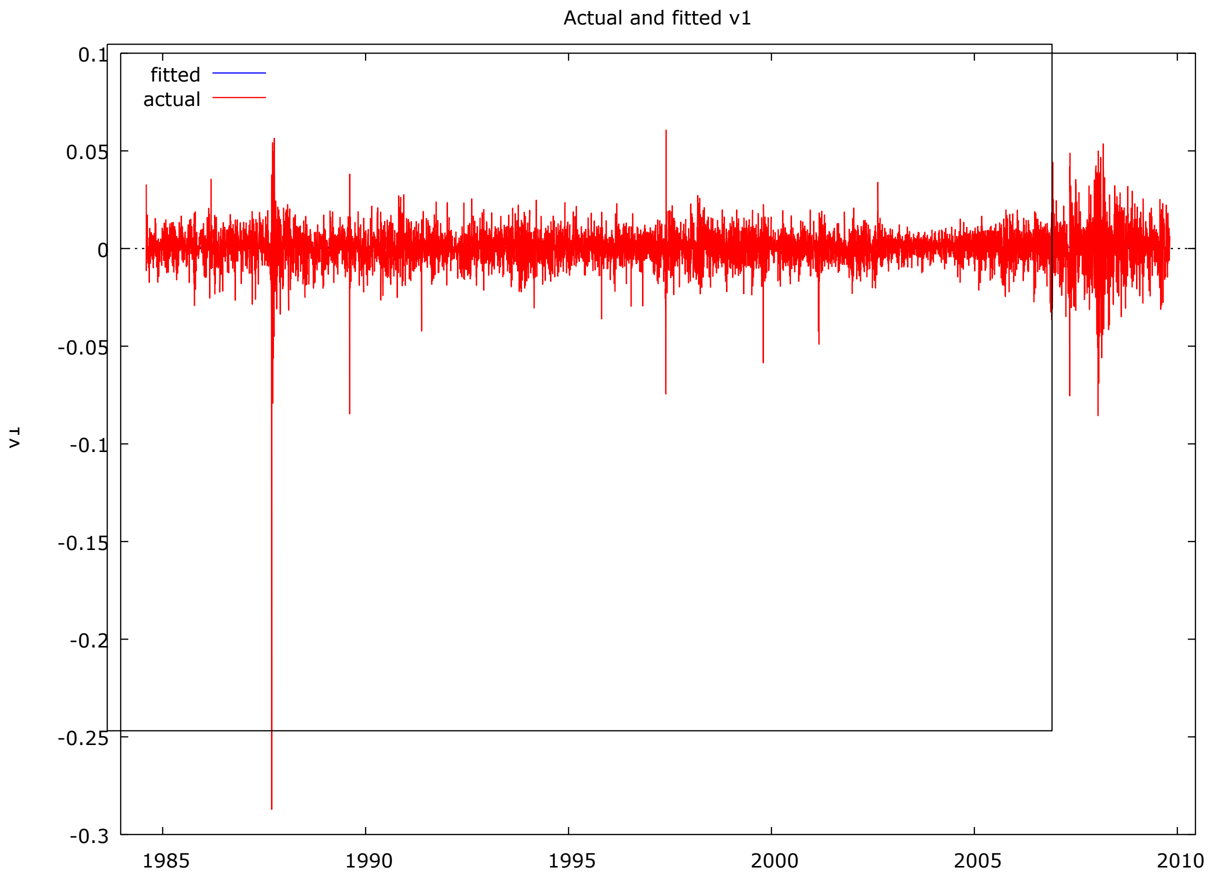
$$h_t = \omega + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-1}^2 + \sum_{i=1}^{q} \beta_i h_{t-1} \qquad (10)$$

Usual requirements are that $\omega > 0$, α and $\beta > 0$. These are sufficient conditions for the conditional variance to be positive. The conditional variance depends on the average volatility, constant value ω, the error/reaction coefficient α and the lag/persistence coefficient β.

The conditional variance depends on the average volatility, constant value ω, the error/reaction coefficient α and the lag/persistence coefficient β. The ARCH term is is the which represents news about volatility from previous periods and the GARCH term, which is the last period's forecast variance .

Both parameters (α and β) are sensitive to the historic data used to estimate the model. The size of the parameters α and β determine the short run dynamics of the volatility. The closer the GARCH lag coefficient β is to unity the greater the persistence of shocks to the conditional variance. A large ARCH error coefficient α causes volatility to react to market movements. The sum of the two components must be less than unity if the process is to be stationary.

A GARCH(1,1) model's output of the volatility of the All Ords 1984-2010



- **Recently attention has switched to realised volatility models which take advantage of the availability of intraday real time trading data.**

Realised volatility

The advantage of applying realized volatility metrics constructed from high-frequency intraday returns, are that they permit the use of traditional time-series methods for modeling and forecasting. McAleer and Medeiros (2003) provide a review of realised volatility models.

Realised volatility

If we set a day as a unit of measurement, as usual, and sample the continuously compounded intraday returns of day *t* at frequency *M*,

$$r_{jt} = p_{t-1+j/m} - p_{t-1+(j-1)/M}$$

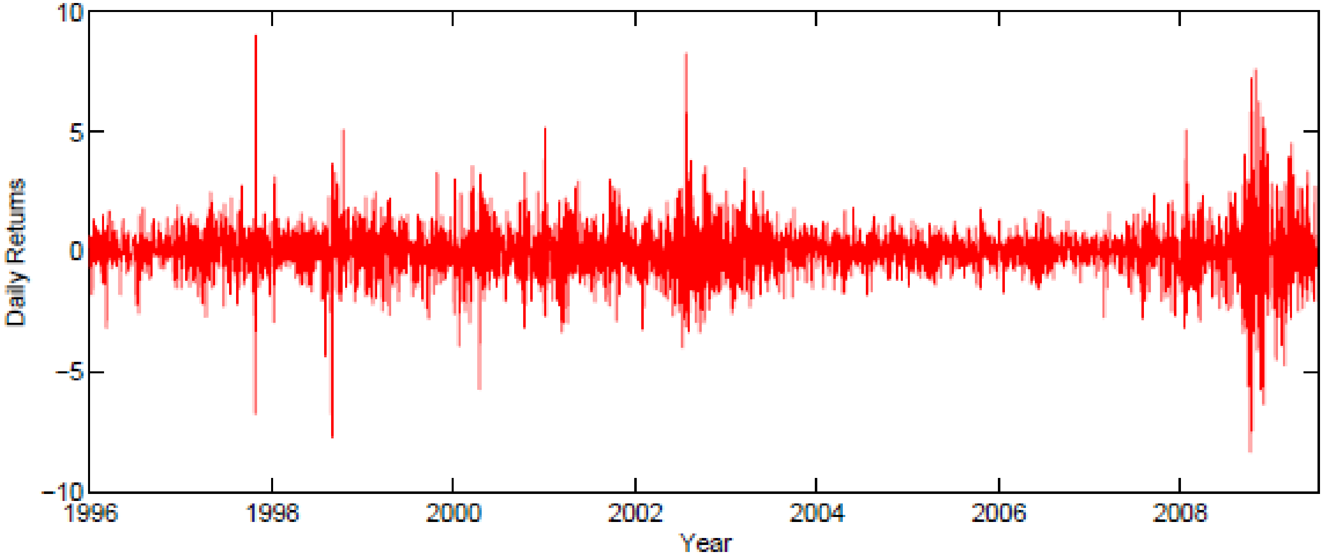
The realised quarticity over day t can be defined as:

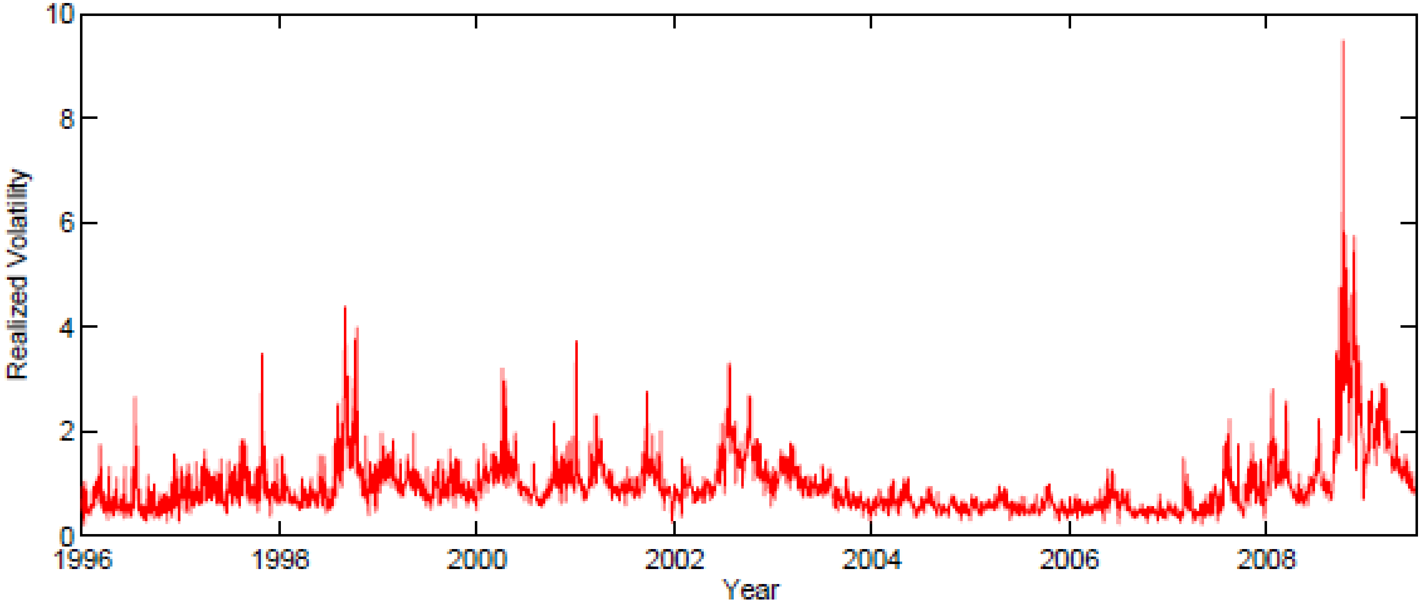
$$RQ_t = \frac{M}{3}\sum_{j=1}^{M} r_{t,k}^4 \;\; \rightarrow \int_0^t \sigma\,(s)^4 ds$$

A metric developed from this is used to sample realised quarticity for S&P 500 Index data, obtained from SIRCA using high frequency intraday data obtained from SIRCA's Taqtic Reuters database for January 2nd 1996 to March 26th 2007.

- D.E.Allen, M. McAleer and M. Scarth, “Realised Volatility Risk”, Working paper, (Dec 2009).
- In the following few slides I will show some estimates of the realised volatility of the S&P 500 index.
- We propose a dually asymmetric realized volatility (DARV) model, which incorporates the important fact that realized volatility series are systematically more volatile in high volatility periods. Returns in this framework display time varying volatility, skewness and kurtosis.
- I shall not go into the mechanics of the model. I shall just show some graphs of the output.

Figure 1. S&P 500 Jan 1996 - Mar 2007




10
8
6
4
2
0
Realized Volatility
1996
1998
2000
2002
2004
2006
2008
Year

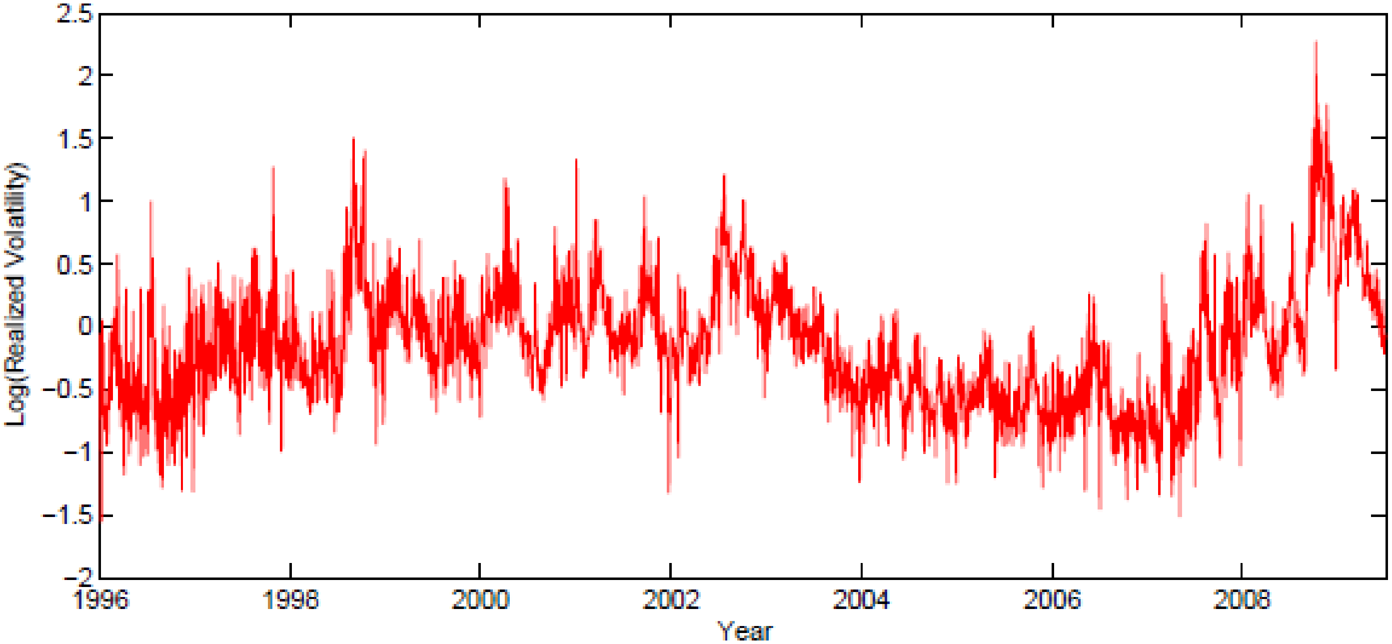

2.5
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
Log(Realized Volatility)
1996
1998
2000
2002
2004
2006
2008
Year

Figure 2. Why shocks to volatility matter. The difference between conditional forecasts from a GARCH (1,1) model and direct estimates from high frequency data, S&P 500 Index data, Jan1996 - Mar 2007.

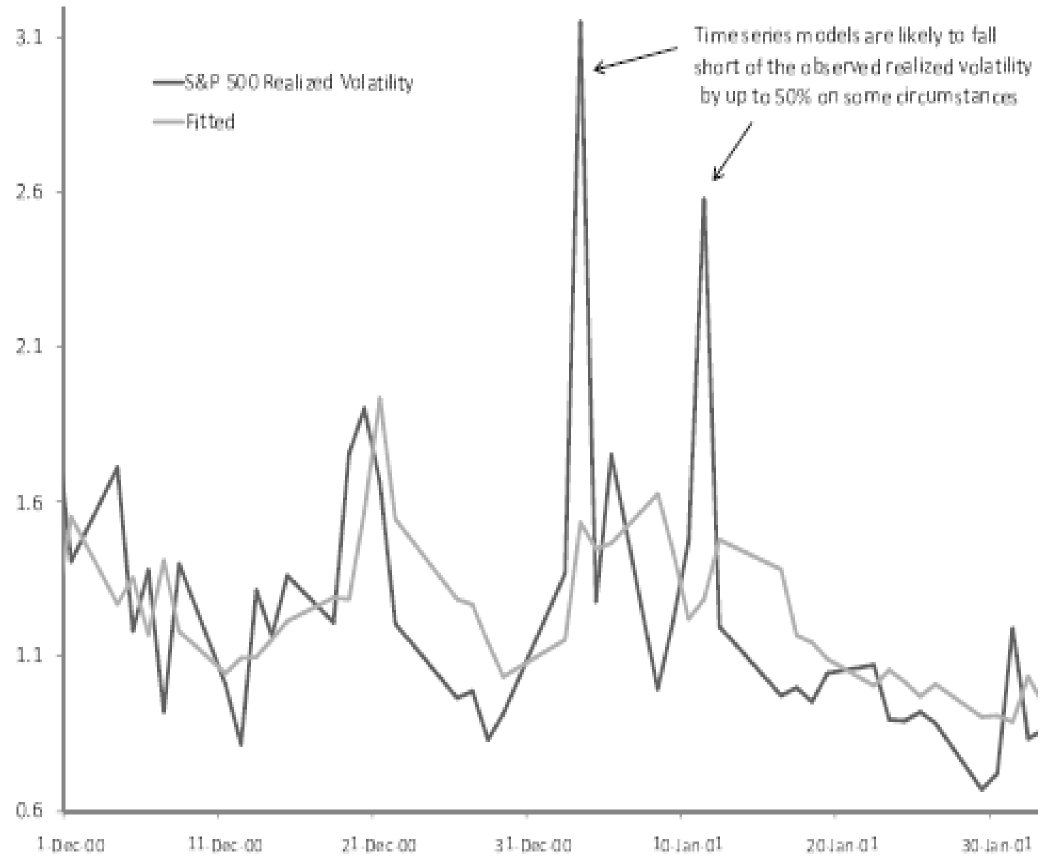


Figure 3 Persistence in the Volatility of volatility S&P500 : autocorrelation.

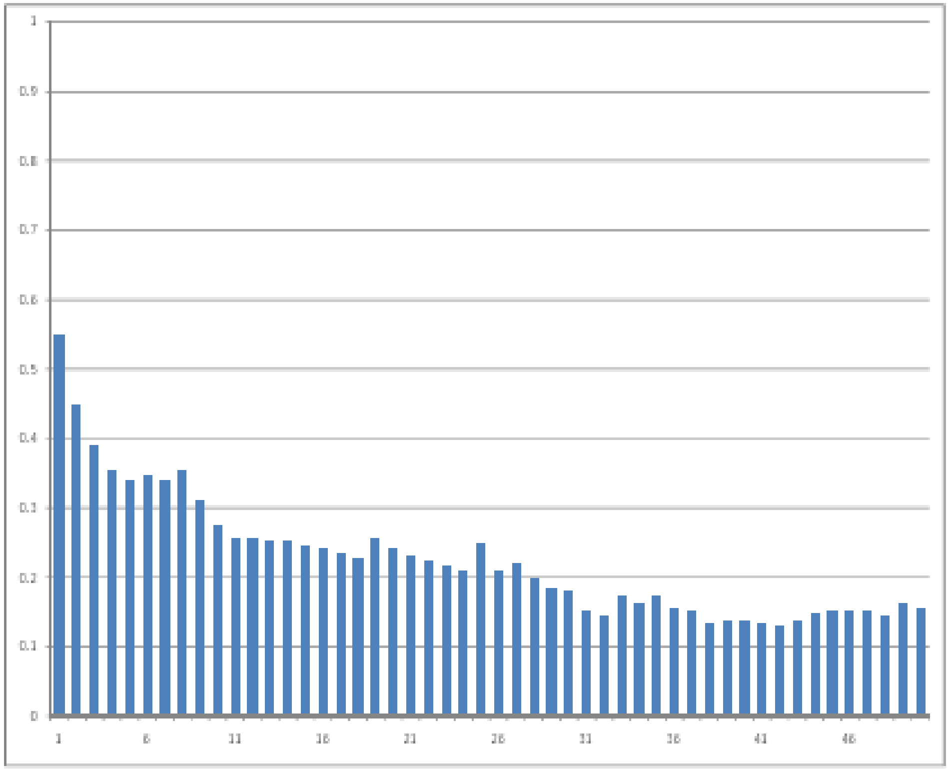
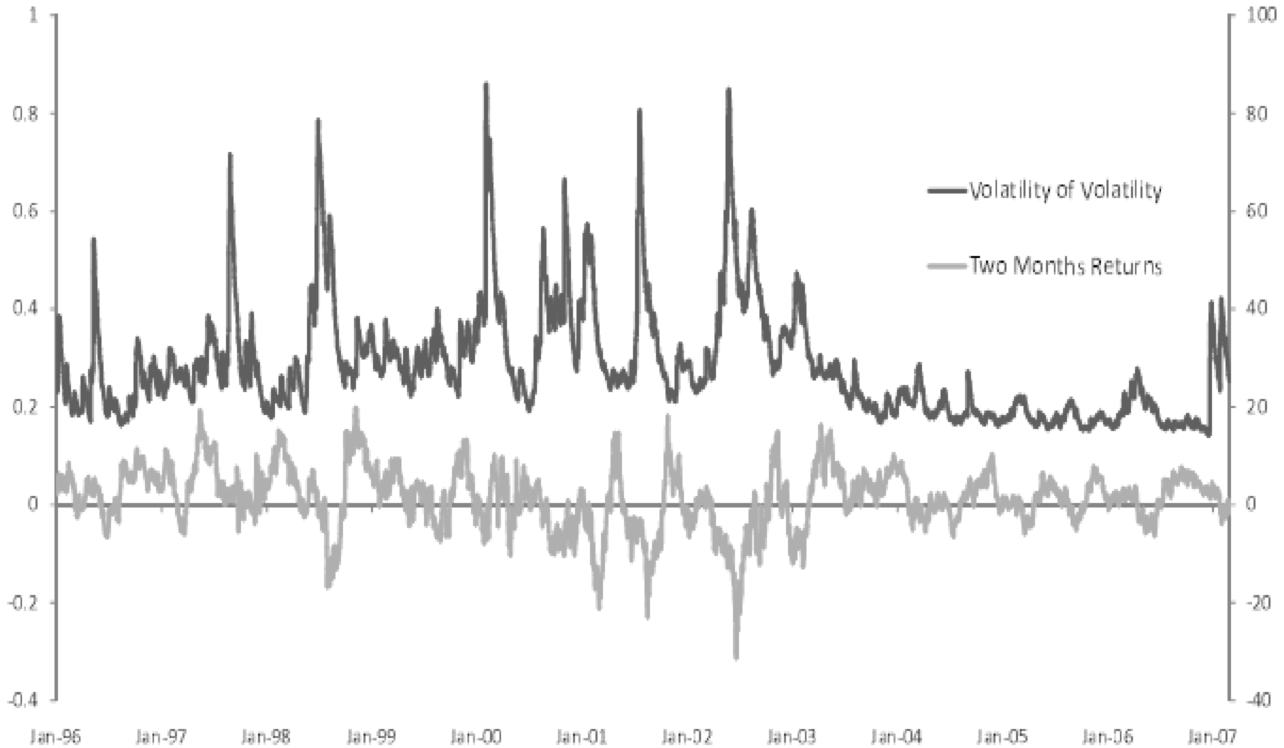


Figure 4. S&P500 Estimated volatility of volatility.



Implied volatility

Black-Scholes OPM

Another way of deriving measures of volatility is via an option pricing model and its implied standard deviation (ISD). This started with the first closed-form option pricing model derived by Black and Scholes (1973) and has been expanded to various generalizations. If *f* denotes the option pricing model and c is the price of the option; then

$$c = f(S, X, \sigma, R, T)$$

Where

S = price of the underlying security

X = the exercise price

σ = volatility (standard deviation of the return on the underlying security)

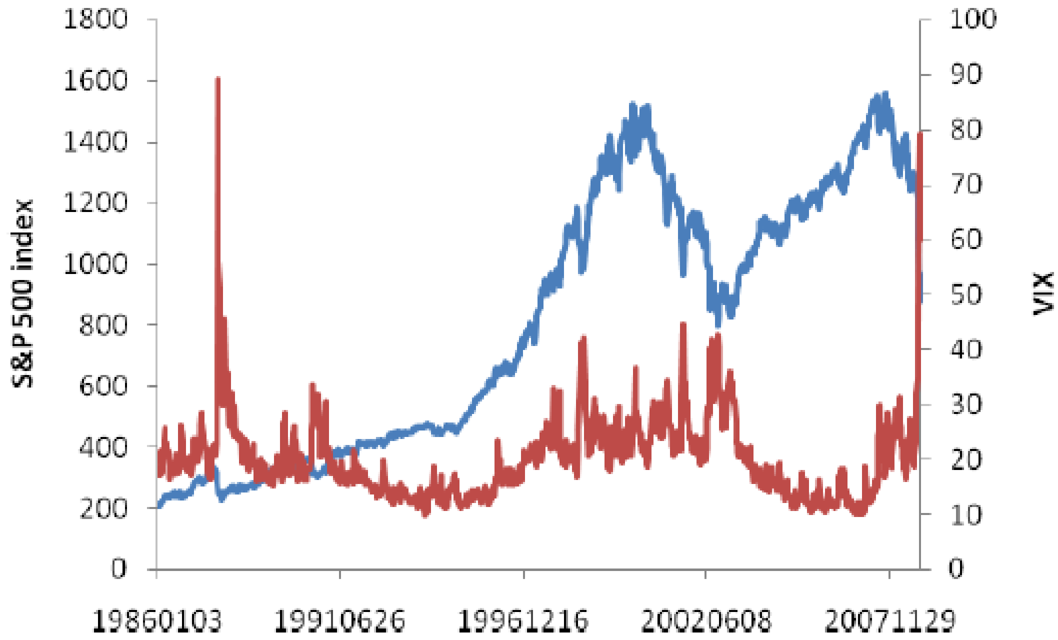
R= risk-free interest rate

T = Time until option expiry.

The VIX index

A common measure of volatility is the VIX index. The VIX is the Chicago Board Options Exchange Volatility Index which measures the implied volatility of S&P 500 index options. A high value represents a more volatile market with more expensive options, given that option prices increase with greater volatility, and the VIX is a made up of a weighted blend of prices for a range of options on the S&P 500 Index.

The VIX Index and S&P 500 Index. Friday closing levels during the period January 3rd 1986 to October 31st 2008. (Source: R.E. Whaley, (2009) "Understanding the VIX", *The Journal of Portfolio Management* Spring 2009, Vol. 35, No. 3,



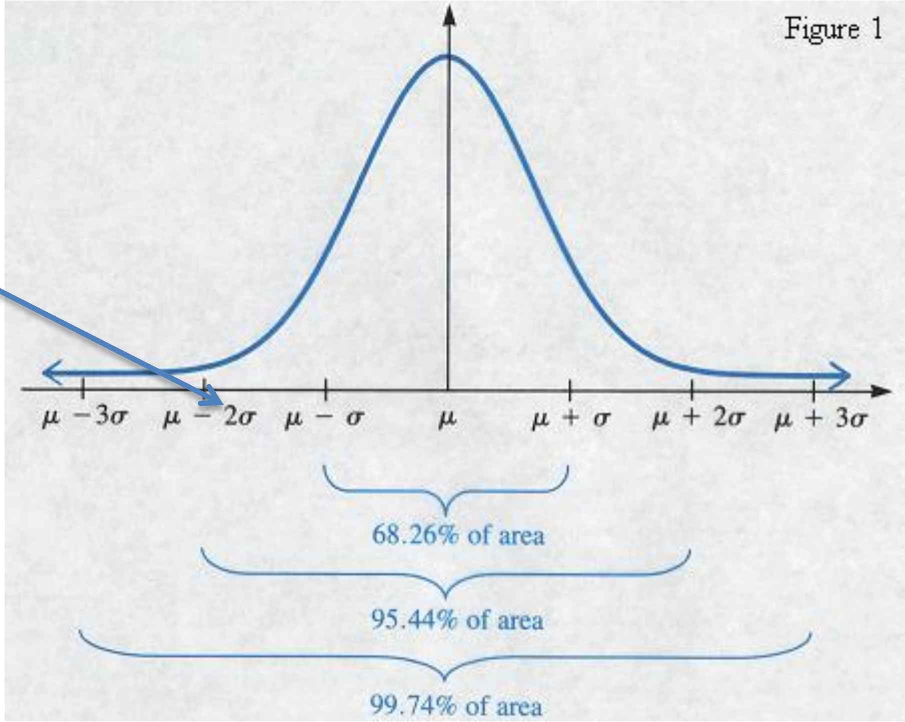
Value at Risk

Value at Risk (VaR) is a procedure designed to forecast the maximum expected loss over a given period at an **Value at Risk** expected confidence level.

The use of VaR has become all-pervasive in a relatively short period of time despite its conceptual and practical shortcomings. VaR received its first broad recommendation in the 1993 Group of Thirty Report. Subsequently its use and recognition have increased dramatically, particularly when the Basel Committee on Banking Supervision adopted the use of VaR models, contingent upon certain qualitative and quantitative standards.

- We can use our previous diagram to show 2.5% VaR and CVaR

Outcome breaching this value VaR 2.5%



The entire area to the left under the curve at this point is CVaR 2.5%

- A description of the various methodologies for the modelling of VaR can be seen at http://www.gloriamundi.org/ . The predominant approaches to calculating VaR rely on a linear approximation of the portfolio risks and assume a joint normal (or log-normal) distribution of the underlying market processes.
- There is a comprehensive survey of the concept by Duffie and Pan (1997), and discussions in Jorion (1996), and RiskMetricsTM (1996).

- Nevertheless, despite its popularity, VaR has certain undesirable mathematical properties; such as lack of sub-additivity and convexity; see the discussion in Arztner et al (1997, 1999). In the case of the standard normal distribution VaR is proportional to the standard deviation and is coherent when based on this distribution but not in other circumstances. The VaR resulting from the combination of two portfolios can be greater than the sum of the risks of the individual portfolios.

- An attractive alternative to VaR is CVaR – Conditional-Value-at-Risk. Pflug (2000) proved that CVaR is a coherent risk measure with a number of desirable properties such as convexity and monotonicity w.r.t stochastic dominance of order 1, amongst other desirable characteristics. Furthermore, VaR gives no indication on the extent of the losses that might be encountered beyond the threshold amount suggested by the measure.
- By contrast CVaR does quantify the losses that might be encountered in the tail of the distribution. This is because a portfolio’s CVaR is the loss one expects to suffer, given that the loss is equal to or larger than its VaR.
- Robert Powell and I have done a good deal of joint work using CVaR as a metric.
- Allen and Powell (2009) have compared changes in Bank default risk in the United States and United Kingdom over time, including the current crisis period. A common approach used by Banks to measure the probability of default among customers is the KMV / Merton structural model which measures distance to default. We use this same approach to measure the distance to default of the Banks themselves.

- **Step 1. We obtain balance sheet and market capitalisation figures**
- **Step 2. We calculate Standard Deviation and Conditional Standard Deviation (Cstdev) of Asset returns**
- **Step 3. Using Merton's formula, we calculate DD and PD.**

$$DD = \frac{\ln(V/F) + (\mu - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}} = 1.38$$

- Where DD is measured as the number of standard deviations

$$PD = N(-DD) = 0.084$$

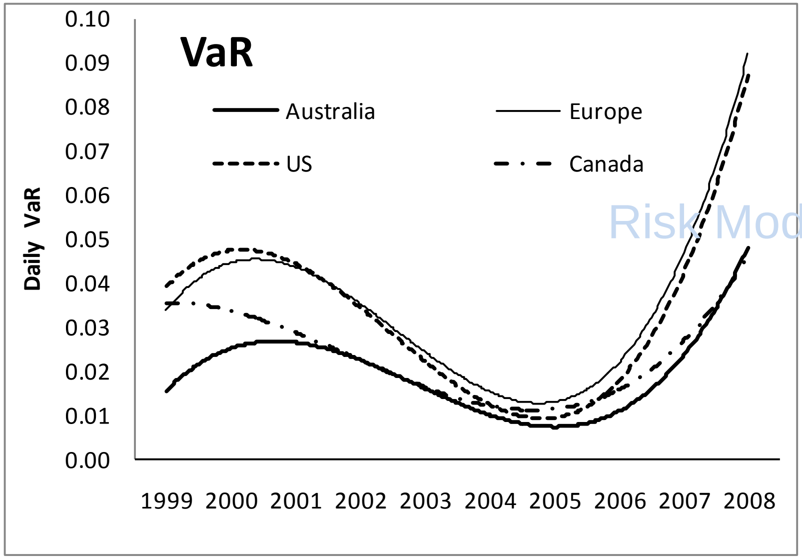
Step 4. We substitute the Cstdev figures calculated in Step 2 into Merton's formulae to calculate Conditional Distance to default (CDD) and Conditional Probability of Default (CPD) .

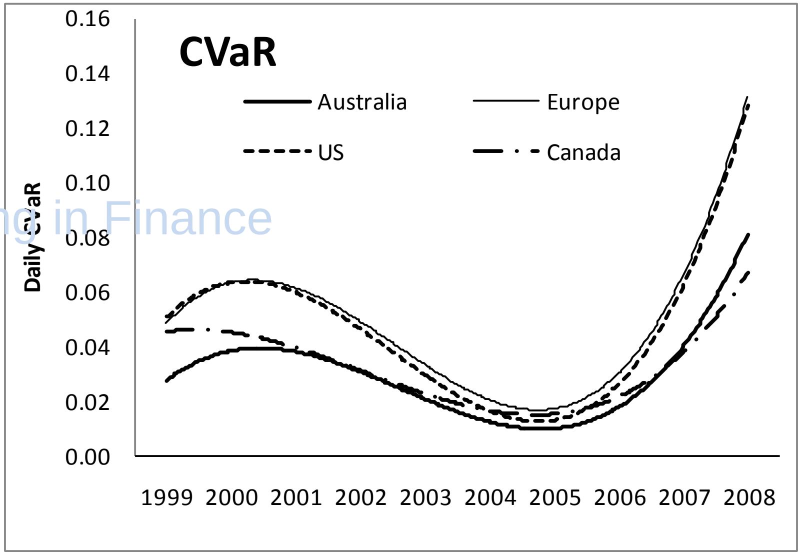
$$CDD = \frac{\ln(V/F) + (\mu - 0.5\sigma_V^2)T}{CStdev_V\sqrt{T}} = 0.31$$

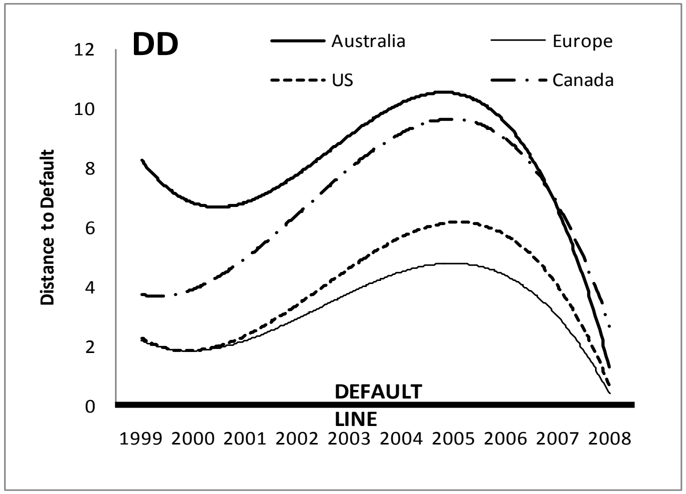
$$CPD = N(-CDD) = 0.38$$

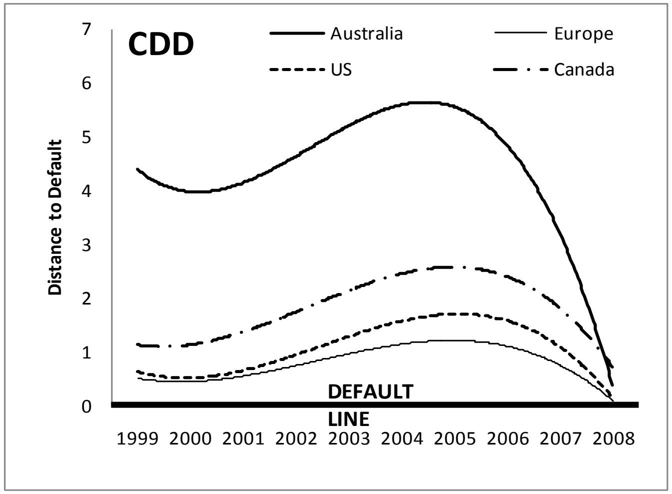
We repeat the process for prior years

VaR and CVaR Comparison between Bank Portfolios.









- The figure compares the yearly DD and CPD Australian figures in Table 5 to the other 3 regions using 3 point polynomial trend lines. Daily data for the European and US banks is obtained from Datastream, and DD and CDD for each year is calculated in exactly the same manner as for Australia in table 5, using the same 12 month windows

How well do the models forecast?

Poon and Granger (2005) reviewed over 90 studies that make comparisons between two or more models.

Historical volatility works as well as any other model.

Implied standard deviations are good forecasts in the short term (a month or so)

None work well over longer horizons

The purpose of the forecast is important: horses for courses.

- A new approach: applying quantile regressions.

The ideas behind quantile regressions.

- Boscovitch in the 18th Century considered fitting regression lines using deviations around the median, subsequently variations on the idea were investigated by Laplace and Edgeworth.
- Koenker observes: “reading the marks left by Laplace, Edgeworth, Fisher, Fr´echet,Kolmogorov, Tukey and Huber on the merits of the median all reveal a noble quality of mind”.

- The idea has been slowly catching on in finance

In finance some recent examples include:

- Bassett, G., and H. Chen. 2001. Portfolio style: Return-based attribution using quantile regression. In *Empirical Economics.* Springer, 1405–41.
- L. Ma and L. Pohlman, "Return forecasts and optimal portfolio construction: a quantile regression approach", *The European Journal of Finance,* Vol. 14, No. 5, July 2008, 409–425
- Engle, R., and S. Manganelli. 2004. CAViaR: Conditional autoregressive value at risk by regression quantiles. *Journal of Business and Economic Statistics 22: 367–81.*

- Quantile regression promises to be a more effective tool than OLS, when it comes to analysing the extremes of a distribution. The behaviour of the tails of a distribution is more efficiently described by quantile regression.
- Quantile regression as introduced in Koenker and Bassett (1978) is an extension of classical least squares estimation of conditional mean models to the estimation of an ensemble of models for conditional quantile functions.
- The central special case is the median regression estimator that minimizes a sum of absolute errors. The remaining conditional quantile functions are estimated by minimizing an asymmetrically weighted sum of absolute errors.

- The symmetry of the piecewise linear absolute value function implies that the minimization of the sum of absolute residuals must equate the number of positive and negative residuals, thus assuring that there are the same number of observations above and below the median.
- The other quantile values can be obtained by minimizing a sum of asymmetrically weighted absolute residuals, (giving different weights to positive and negative residuals). Solving

$$\min_{\xi \epsilon \mathcal{R}} \sum \rho_\tau (y_i - \xi)$$

Where $\rho_\tau(\cdot)$ is the tilted absolute value function as shown in Figure 1

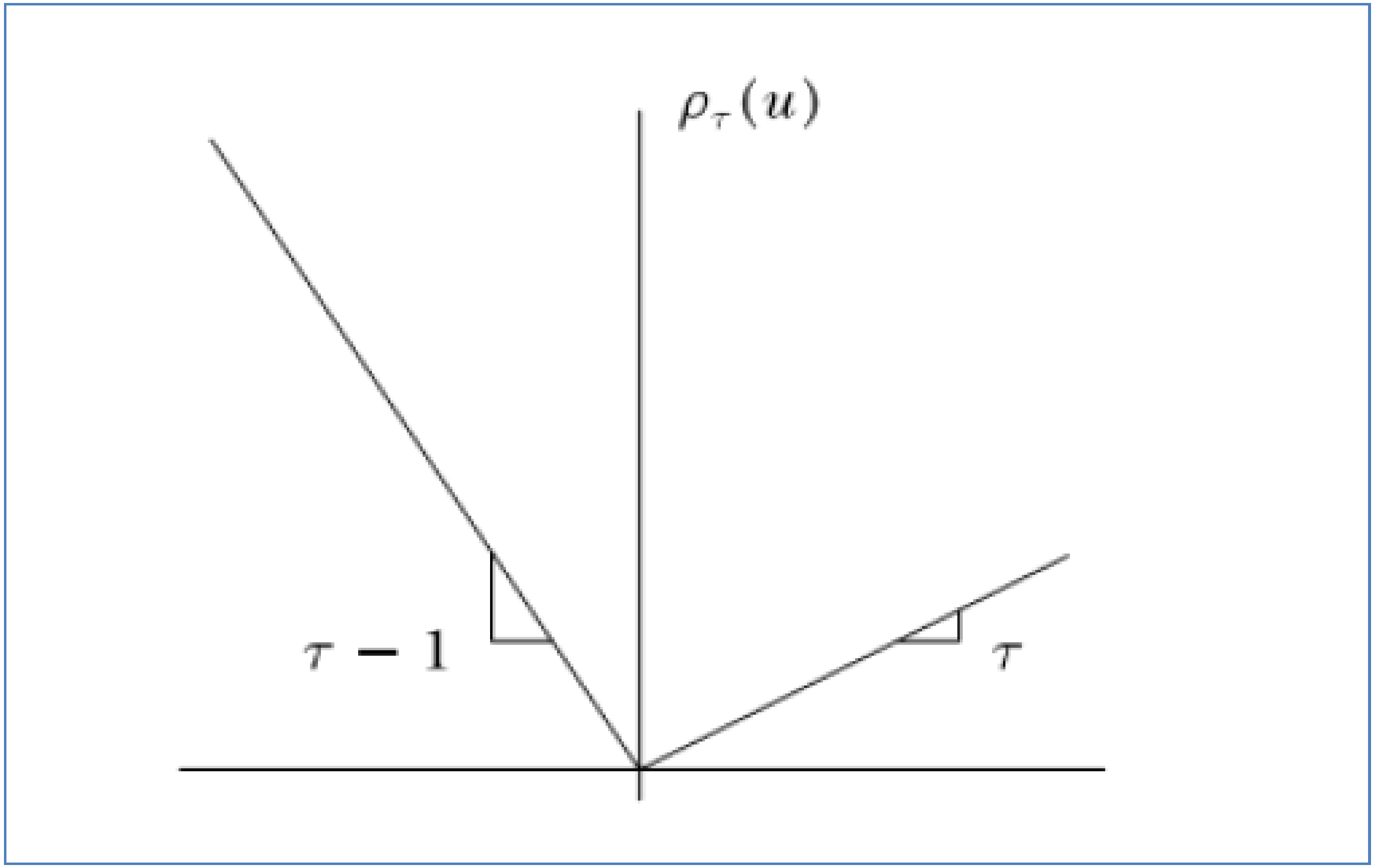


Figure 1: Quantile Regression ρ Function

- To obtain estimates of the other conditional quantile functions, we replace

absolute values by $\rho_\tau(\cdot)$ and solve

$$\min_{\xi \in \mathcal{R}^p} \sum \rho_\tau(y_i - \xi(x_i, \beta))$$

- The resulting minimization problem, when is formulated as a linear function of parameters, can be solved very efficiently by linear programming methods

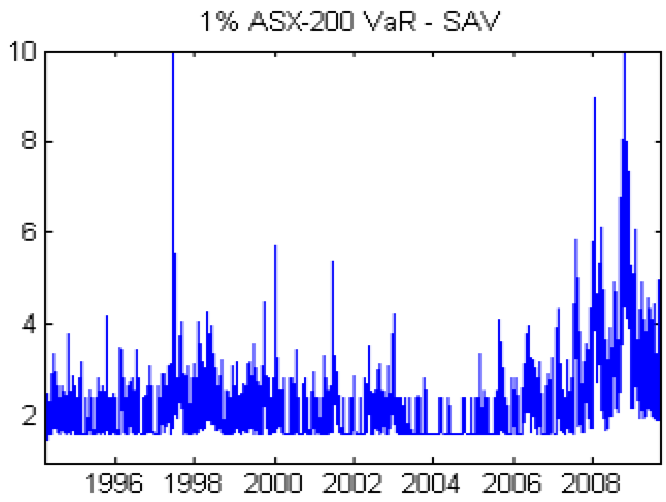
- Engle and Manganelli (2004), used the robust technique of quantile regression and proposed another method for calculation of Value at Risk which they termed as, Conditional Autoregressive Value at Risk by Regression Quantiles, or CAViaR.
- CAViaR, uses quantile regressions and instead of modelling the whole return distribution for calculation of VaR, it models the required quantiles of the return distribution directly. To predict the value at risk by modelling the lower quantiles, the model uses a conditional autoregressive specification, inspired by the fact that the distribution of volatilities over time is auto-correlated,

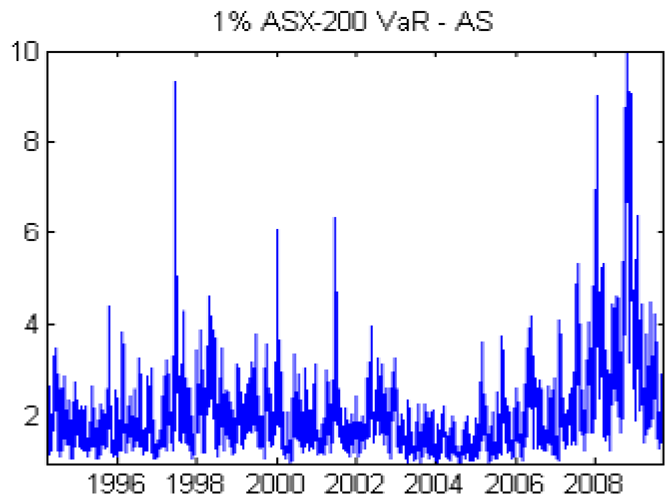
- We use various specifications of the CAViaR model:
- Adaptive $f_t(\beta) = f_{t-1}(\beta_1) + \beta_1\{[1 + \exp(G[y_{t-1} - f_{t-1}(\beta_1)])]^{-1} - \theta\}$
- Asymmetric absolute values $f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3|y_{t-1}|$
- Asymmetric slope $f_t(\beta) = \beta_1 + \beta_2 f_{t-1}(\beta) + \beta_3(y_{t-1})^+ + \beta_4(y_{t-1})^-$
- Indirect GARCH (1,1) $f_t(\beta) = (\beta_1 + \beta_2 f^2_{t-1}(\beta) + \beta_3 y^2_{t-1})^{1/2}$
- We compare these models with
- A GARCH (1,1) $\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1}$

- Morgans Risk Metrics $\sigma^2{}_t = \omega + (1-\lambda)\varepsilon^2{}_{t-1} + \lambda\sigma^2{}_{t-1}$
    - where $\omega = 0$ and λ is set to 0.94 for daily data
- Asymmetric power ARCH, or APARCH $\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^\delta + \sum_{j=1}^{p} \beta_j\sigma_{t-j}^\delta$

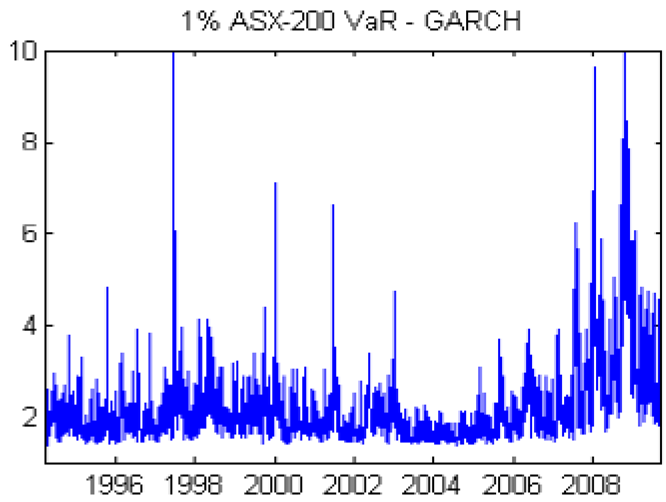
- I shall not dwell on the details of the models but show some graphs of the results.
- None of the models faired well during the period of the GFC.

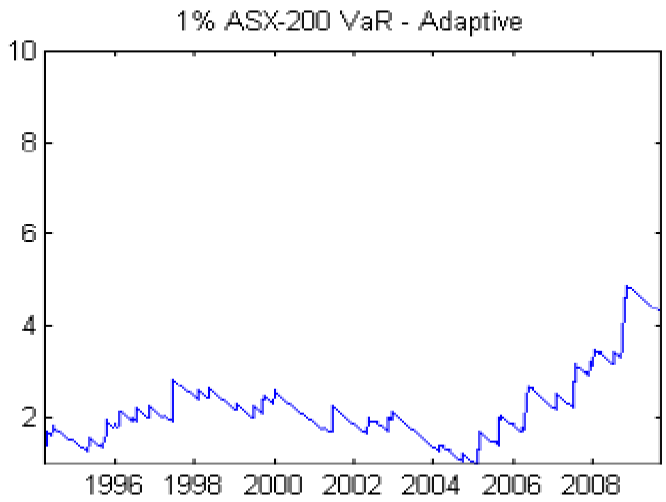
Variants of CAViaR

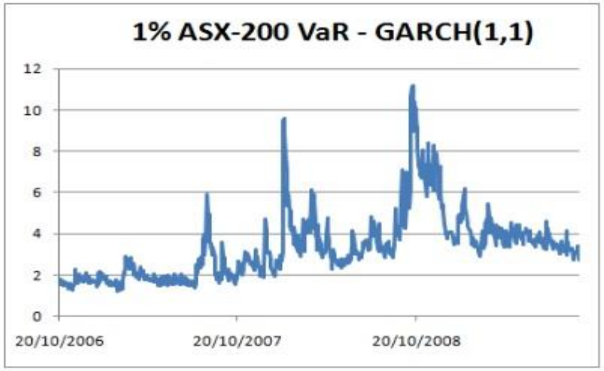




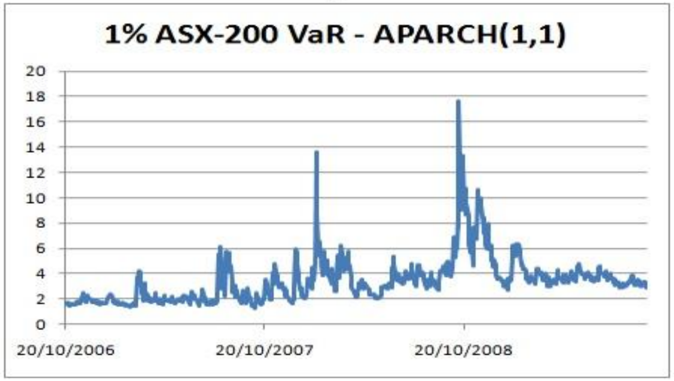
- Variants of CAViaR






1% ASX-200 VaR - GARCH(1,1)
12
10
8
6
4
2
0
20/10/2006
20/10/2007
20/10/2008


1% ASX-200 VaR - RiskMetrics
10
9
8
7
6
5
4
3
2
1
0
20/10/2006
20/10/2007
20/10/2008


1% ASX-200 VaR - APARCH(1,1)
20
18
16
14
12
10
8
6
4
2
0
20/10/2006
20/10/2007
20/10/2008

- the specification which works the best for the Australian market is the CAViaR Asymmetric Slope Model.
- None of the models work well during the GFC.

- Conclusion.
  - We have looked at a variety of different ways of trying to capture risk.
  - We are led back to Hume’s observation about experience.
  - If conditions are ‘normal’ then they seem to work reasonably well and we can rely on experience.
  - However, in ‘abnormal’ conditions such as the GFC when relationships change they do not work well at all.

- We do not, as yet appear to have a very satisfactory method for modelling uncertainty, especially in extreme circumstances.
- This is despite the sophistication of some of the models.
- Knight was correct when he talked about ‘uncertainty’.