1-1-1995

Some issues in the sliding mode control of rigid robotic manipulators

Sanjay Rao
Edith Cowan University

SOME ISSUES IN THE SLIDING MODE CONTROL OF RIGID ROBOTIC MANIPULATORS

by

SANJAY RAO

B.E.(Computer Engg.), UNIVERSITY OF POONA, INDIA

A Thesis Submitted in Partial Fulfilment of the Requirements for the

Award of

Master of Science (Computer Science)

Department of Computer Science

School of Information Technology and Mathematics

Edith Cowan University,

Perth, Western Australia

October, 1995

ABSTRACT

This thesis investigates the problem of robust adaptive sliding mode control for nonlinear rigid robotic manipulators. A number of robustness and convergence results are presented for sliding mode control of robotic manipulators with bounded unknown disturbances, nonlinearities, dynamical couplings and parameter uncertainties. The highlights of the research work are summarized below:

- A robust adaptive tracking control for rigid robotic manipulators is proposed. In this scheme, the parameters of the upper bound of system uncertainty are adaptively estimated. The controller estimates are then used as controller parameters to eliminate the effects of system uncertainty and guarantee asymptotic error convergence.

- A decentralised adaptive sliding mode control scheme for rigid robotic manipulators is proposed. The known dynamics of the partially known robotic manipulator are separated out to perform linearization. A local feedback controller is then designed to stabilize each subsystem and an adaptive sliding mode compensator is used to handle the effects of uncertain system dynamics. The developed scheme guarantees that the effects of system dynamics are eliminated and that asymptotic error convergence is obtained with respect to the overall robotic control system.

- A model reference adaptive control using the terminal sliding mode technique is proposed. A multivariable terminal sliding mode is defined for a model following control system for rigid robotic manipulators. A terminal sliding mode controller is then designed based on only a few uncertain system matrix bounds. The result is a simple and robust controller design that guarantees convergence of the output tracking error in a finite time on the terminal sliding mode.

Declaration

I certify that this thesis does not incorporate without acknowledgment any material previously submitted for a degree or diploma in any institution of higher education; and that to the best of my knowledge and belief it does not contain any material previously published or written by another person except where due reference is made in the text.

Sanjay Rao

Date: 14/2/96

Acknowledgment

I would like to thank my supervisor Dr. Zhihong Man for his guidance, support and encouragement throughout the course of this work.

My gratitude also extends to Dr. Jim Millar (Chairperson, Dept. of Computer Science) and all the staff members and fellow post-graduate students in the Department of Computer Science, for their kind reception and co-operation during my Masters candidature at Edith Cowan University.

Finally, I would like to thank my parents and friends for their love, understanding and support during this period.

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Chapter 1

INTRODUCTION

1.1 Background

In the past few decades, robots have played a significant role in the ever-escalating need for automation. Most applications are limited to simple, low-precision and low-speed tasks. Low-speed robots offer a more pragmatic solution today as they have minimal dynamic interaction which allows the control model to be effectively linearized and decoupled.

Most of the present-day industrial robots use PID-type controllers which are generally completely error-driven. The PID control scheme uses independent joint-controllers for each link of the robotic manipulator (Craig, 1986). The main drawback of this scheme comes from the inherent lack of an adequate decoupling mechanism for errors that are caused due to joint couplings and other link interactions. The errors caused by these dynamical interactions are suppressed by the control law.

A robot controller used in high-speed operations must be able to handle system nonlinearities and dynamical joint couplings. An adequate compensation mechanism must be provided for unmodelled errors, external disturbances and noise. The controller must also be capable of handling parameter variations like unplanned payload changes and cater for real-time control at high control bandwidths.

In order to overcome the control problem associated with a highly nonlinear plant like, a robot manipulator, several linearization schemes were developed in Luh (1983), Desa and Roth (1985), Whitehead et al. (1985), Kreutz (1989). However, these schemes were based on several restrictive assumptions. The symmetric positive-definite inertia matrix and the vector of coriolis, gravitational and centrifugal forces were assumed to be exactly known and any violation of these assumptions could result in the failure of the linearization control.

The computed torque input method uses an independent input for each degree of freedom and provides a global feedback linearization scheme for robotic manipulators. But this approach did not provide good results when the difference between the computed torque and the actual robot dynamic parameters were significant. (Silva de and MacFarlane, 1984)

Spong and Vidyasagar (1987) used a feedback compensator to deal with system uncertainties and external disturbances. This is ensured by placing the poles of the closed loop system sufficiently far in the left half-plane. The known dynamics are then used to design a nominal system model. However, the output tracking error still cannot converge to zero.

Adaptive control is useful when the dynamics of the manipulator are unknown, or change due to uncertainties. Adaptive control uses an on-line adjustment mechanism, based on several useful properties of the robotic manipulator, to change its parameters depending upon the changes detected in the system dynamics. Various adaptive control schemes have been proposed (Crag et al., 1986., Slotine and Li, 1987, 1988., Middleton and Goodwin, 1988).

In Dubowski and Des Forges (1979), a model-reference adaptive control scheme was proposed. The error between a reference model, and the actual robot response is used by the adaptive controller to update the servo parameters in real-time. This scheme has several disadvantages. Firstly, the feedback control must be realised through some independent means. Also, the adaptive law, which is independent of the robot model, assumes that some nonlinear terms of the robotic manipulator are constant (Silva de and MacFarlane, 1984).

The work in Crag et al. (1986) proposed the use of a dynamical equation of the robotic manipulator within a linear function of the unknown parameters. The estimated parameters are then used to design the controller. This scheme ensures convergence of the output tracking error to zero with all signals constrained within established bounds. But, it still needs a measurement of the acceleration for the adaptive mechanism. Furthermore, the estimate of the inertia matrix must remain uniformly positive-definite. This latter constraint is removed in Ortega and Spong (1989) where an estimate of the inertia matrix and other unknown parameters which have a fixed value are used for feedback. The output tracking error convergence is ensured by an adaptive additive signal that compensates for the

error in the estimates. However, it still requires all signals to remain bounded. Amestegui et al. (1987) use a different parameter estimation technique which does not require the bounded condition of the above schemes. Middleton and Goodwin (1988) proposed a scheme which does not require measurement of the joint acceleration but it still requires boundedness of the inverse of the estimates of the inertia matrix.

In all the above adaptive schemes no mechanism is provided to specify the transient error. In addition, it is well-known that the non-uniform nature of asymptotic stability can lead to a loss of stability and a large deviation from the desired response, due to small changes in dynamics or the presence of small unmodelled disturbances

Sliding mode control or variable structure control was pioneered by Emelyanov and several other researchers in the early 1960's in the Soviet Union (Emelyanov, 1962, 1966). The plant under consideration was a linear, second-order system modelled in phase variable form. Sliding mode control has been used for robot control since the late 1970's (Young, K-K.D., 1978) and has since evolved to be an effective method for the control of robotic manipulators with large system uncertainties and bounded input disturbances.

In the work of Young (1978, 1988), Abbass and Chen (1988) and Morgan and Ozguner (1985) it is shown that robustness and convergence can be obtained by using linear sliding mode techniques based on the upper and lower bounds of all unknown system parameters.

Morgan and Ozguner (1985) and Abbass and Ozguner (1985) presented decentralised sliding mode control schemes. These schemes were modifications to the Young controller and use a simplified controller design where local controllers are used for each subsystem. Unfortunately, the chattering that occurs in the control input due to the control action can cause excitation of undesired high-frequency dynamics. To counter the chattering problem, Slotine and Sastry (1983) proposed the boundary layer technique. Further details of the boundary layer controller can be found in chapter 2.

Further research in decentralised sliding mode control yielded several new and improved schemes. In the scheme of Fu and Liao (1990), five parameters of the uncertain bounds need to be adaptively estimated in each local controller. However, as the number of links increase, the controller design gets increasingly complicated. Leung et al. (1991) proposed a generalised scheme where only five parameters are estimated for any n-link robotic manipulator. However, it does not address the problem of eliminating the effects of bounded input disturbances. In the work of Man and Palaniswami (1994), a sliding mode controller is designed for any n-link rigid robotic manipulator using only four uncertain system matrix bounds. Robustness and asymptotic convergence properties are obtained using an upper bound of the input disturbances.

The terminal sliding mode technique has been developed based on the idea of terminal attractor in Zak (1988, 1989). Unlike the linear sliding mode control in Utkin (1977), Young (1978, 1988) and Man and Palaniswami (1993, 1994), the terminal sliding mode technique has a nonlinear term of the velocity error. By

suitably designing the controller, the terminal sliding variables can reach the terminal sliding mode in a finite time and the output tracking error can then converge to zero in a finite time on the terminal sliding mode.

1.2 Contributions of the thesis:

The thesis investigates the following three control algorithms for rigid robotic manipulator control using linear and terminal sliding mode control techniques.

- A robust adaptive tracking control for rigid robotic manipulators.
- A decentralised adaptive sliding mode control scheme for rigid robotic manipulators.
- Model following control using terminal sliding mode control technique for rigid robotic manipulator.

The contents of the thesis are organised as follows:

Chapter 2 provides a brief survey of variable structure theory and its application to robotic manipulators. The fundamentals of sliding mode control design, robustness analysis for linear and nonlinear systems are reviewed.

Chapter 3 considers a robust adaptive tracking control for rigid robotic manipulators. A linearised error system, based on a nominal system model is described, and a robust sliding mode control scheme using an uncertain bound is briefly reviewed. A new robust adaptive tracking control scheme for rigid robotic manipulators is proposed where an adaptive tracking mechanism is used for the estimation of the uncertain bound. The estimate is then used as a controller

parameter to eliminate the effects of large system uncertainties and to obtain asymptotic error convergence. Error convergence and robustness with respect to uncertain system dynamics are discussed in detail.

Chapter 4 considers a decentralised adaptive sliding mode control scheme for rigid robotic manipulators. An adaptive mechanism is proposed to estimate the upper bound of system uncertainties. This estimate, which is updated in the Lyapunov sense in each subsystem, is then used as a local controller parameter to guarantee asymptotic convergence and eliminate the effects of uncertain dynamics. This results in a simple and robust design for the sliding mode controller.

Chapter 5 considers a new model following control using terminal sliding mode technique for rigid robotic manipulators. A new model following control scheme using terminal sliding technique is investigated based on the idea of terminal attractors in Zak (Zak, M.,1988, 1989). A multivariable terminal sliding mode is defined for a model following control system of rigid robotic manipulators. A controller is then designed based on only a few uncertain system matrix bounds. This scheme results in a simplified robust controller design that guarantees convergence of the output tracking error in a finite time on the terminal sliding mode.

Chapter 6 provides a brief overview of the results of the three schemes proposed in chapters 3, 4 and 5 respectively.

Chapter 2

A Survey of Sliding Mode Control Theory and its Application to Robotic Manipulators

2.1 INTRODUCTION

In Chapter 1 we briefly discussed various schemes applied in the control of rigid robotic manipulators. This chapter provides a survey of sliding mode control theory and its application to the control of robotic manipulators. In broad terms, a sliding mode control system may be regarded as a combination of subsystems, where each subsystem has a fixed structure and its own region of operation in the system space. The control law defines a time-varying surface embedded within the state space of the dynamical system such that the system trajectories are forced to remain in the vicinity of this imaginary surface.

Sliding motion occurs when a system state is repeatedly forced across the switching surface that passes through the state of equilibrium, the origin. The system trajectory then appears to "slide" asymptotically to the origin. When sliding

motion occurs on all the sliding surfaces, together, the system is said to be on the sliding mode.

The main objective of the sliding mode controller is to force every trajectory to come in contact with and remain at the intersection of m sliding surfaces in the n-dimensional joint space, where $n>m$. The motion of the system on the sliding mode is effectively confined to a certain subspace of the full state space, thus making the system equivalent to a lower order system called the equivalent system. Once on the sliding mode, the system response is robust to parameter variations or unmodelled system characteristics. (Utkin and Young, 1978).

In this chapter, sections (2.2)-(2.4) discuss the conditions for existence of a sliding mode using a linear system model and provides a brief discussion on the implications of using sliding modes on non-linear systems. Section (2.5) gives an insight into the robustness of sliding mode control systems. Section (2.6) explains the boundary layer control. Section (2.7) reviews the terminal sliding mode control technique which ensures finite-time convergence on the sliding mode. Section (2.8) explains the application of sliding mode control to rigid robotic manipulator control.

2.2 SLIDING MODE CONTROL FOR LINEAR SYSTEMS

The primary function of the sliding mode controller is to ensure convergence of every trajectory towards and onto the intersection of the sliding surfaces. The design of the controller consists of 3 steps (Thukral and Innocenti, 1994)

1. Choice of a set of switching surfaces.

2. Determination of the behaviour of the control law.

3. Determination of the switching logic to be used with the sliding surfaces which passes through the origin.

The following sections discuss the application of the sliding mode technique to linear systems.

2.2.1 The Linear System Model

Consider the linear, time-invariant plant

$$\dot{X}(t) = AX(t) + Bu(t) \qquad (2.1)$$

where $X \in R^n$ and $u \in R^m$ represent state and control vectors respectively and n>m. A and B are constant system matrices and B is assumed to be of full rank m. The pair (A, B) are assumed to be completely controllable.

The function of the sliding mode controller is to perform the following tasks:

1. To drive the system trajectories from any arbitrary initial position towards the sliding mode. (reaching mode)

2. To guarantee that that all motion therein, remains on the sliding mode. (sliding mode)

The reaching mode is realised using a suitable high-gain control action. The high-gain switch action forces the control from any arbitrary initial condition onto the sliding surfaces.

This control law is of the form

$$u = -k \quad when \quad S(X) \geq 0$$

$$u = k \quad when \quad S(X) < 0$$

where k is a positive constant, such that, the reaching mode attains the reaching condition S(X)=0.

The constant k must ensure that the trajectory reaches the sliding mode in the shortest time possible. The value of k generally depends on the upper bound of the system input that would fully compensate the dynamics of the controlled system and achieve the desired state while providing robustness to parameter uncertainties.

The application of the sliding mode technique begins with the design of a set of switching plane variables such that system response is asymptotically stable and has the required transient characteristics. The switching plane variables generally used are linear functions of the system states. This scheme called the *linear sliding mode technique*, is based on the assumption that asymptotic convergence to the origin can be guaranteed by ensuring that the tangential component of the switching variable always points towards the switching surfaces. (Utkin, 1977)

The sliding mode design defines m sliding surfaces which form a set of intersecting hyperplanes that pass through the origin. These switching surfaces can be defined as:

$$S_i = C_i X \qquad i = 1..m$$

or S=CX

where, $C_i \in R^n$ is a constant vector, called the switching plane variable vector and $\dot{X} \in R^n$ is the state vector in phase variable form.

The system (2.1) is said to be on the sliding mode when the state reaches and remains on the intersection of the m hyperplanes.

$$S = \bigcap_{i=1}^{m} S_i = \{X : C_i X = 0\}, \quad i = 1..m$$

Once on the sliding mode, the switching control law is de-activated and the trajectories converge asymptotically to the origin, governed only by the sliding mode parameters. Sliding mode control is thus a means of ensuring asymptotic convergence of the system trajectory on the sliding mode.

The Lyapunov second or direct method is a generalised means of proving system stability. It provides a time-domain method based on the system model. In the time-invariant case, the stability problem becomes one of determining the stability of the equilibrium state which is assumed to be the origin of the system space.

For analysis of sliding mode control systems, the Lyapunov candidate function

$V = \frac{1}{2} S^T S$ is generally used.

According to the Lyapunov direct method

$V > 0$

$$\dot{V} = S^T \dot{S} < 0 \quad or \quad S_i^T \dot{S}_i < 0 \quad i = 1..m,$$

$$V(0) = 0$$

is a sufficient but not necessary condition for asymptotic convergence of the state trajectories of the system.

2.2.2 Equivalent Control

Equivalent control is used for describing the system dynamics on the sliding mode (Hung et al., 1993). It is based on the fact that $\dot{S}(x) = 0$ is a necessary condition for the state trajectories to stay on the sliding surface S(X)=0.

The origin, where the intersecting hyperplanes meet, i.e. where $S_i = 0 \quad and \quad \dot{S}_i = 0$ (i=1..m) can be expressed as:

$$\dot{S} = C\dot{X} = 0 \tag{2.2}$$

substituting (2.1) in (2.2) we get:

$$\dot{S} = C(AX + Bu) = 0 \tag{2.3}$$

$$u_{eq} = -(CB)^{-1} CAX(t) \qquad \text{where} \qquad |CB| \neq 0.$$

and u_{eq} is called equivalent control.

Equivalent control represents the state of the input required to ensure that the state trajectory stays on the sliding surface S=0.

The system response on the sliding mode can now be described as:

$$\begin{aligned}\dot{X}(t) &= AX(t) - B(CB)^{-1}CAX(t) \\ &= [I - B(CB)^{-1}C]AX(t)\end{aligned} \tag{2.4}$$

The system defined in (2.4) is called the equivalent system.

From the above discussion it is clear that the equivalent system is independent of the control input X. The sliding variable vector C_i uses the control input only as a parameter to drive the system from an arbitrary initial condition onto the sliding mode. The matrix C can therefore be designed with no prior knowledge of the control inputs.

The non-singularity property of CB ($|CB| \neq 0$) implies that N(C) (null space of C) and R(B) (range space of B) are complementary regions of the state space i.e. $N(C) \cap R(B) = \{0\}$. Therefore, the behaviour of the equivalent system is unaffected by the control input when sliding motion occurs within N(C). On the other hand, if $|CB| = 0$ then $N(C) \cap R(B) \neq \{0\}$ and the resulting motion depends on Bu in (2.1). Utkin (1977) has shown that if $|CB| = 0$ then equivalent control is either not unique or does not exist. Therefore sliding motion cannot be achieved if the non-singularity condition is not satisfied.

2.3 A SIMPLIFIED HYPERPLANE MODEL

The system model defined in (2.1) assumes that the input B is of full rank m. Dorling and Zinober (1986) proposed a scheme which simplifies the design of sliding mode control systems.

There exists an orthogonal nxn transformation matrix T such that:

$$TB = \begin{bmatrix} 0 \\ B_2 \end{bmatrix}$$ where B_2 is mxm and non-singular.

Define a transformed state variable y = Tx such that

$$\dot{Y}(t) = TAT^T Y(t) + Bu(t) \tag{2.5}$$

and the sliding condition is

$$CT^T y(t) \equiv 0 \tag{2.6}$$

If Y is partitioned such that

$$Y^T = \begin{bmatrix} Y_1^T & Y_2^T \end{bmatrix} \quad where\ Y_1 \in R^{n-m}, \quad Y_2 \in R^m$$

TAT^T *and* CT^T are partitioned as :

$$TAT^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{2.7}$$

$$CT^T = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \tag{2.8}$$

then (2.5) and (2.6) can be written as:

$$\dot{Y}_1(t) = A_{11} Y_1(t) + A_{12} Y_2(t) \tag{2.9}$$

$$\dot{Y}_2(t) = A_{21} Y_1(t) + A_{22} Y_2(t) + B_2 u(t) \tag{2.10}$$

On the sliding mode:

$$C_1 Y_1(t) + C_2 Y_2(t) \equiv 0 \tag{2.11}$$

$$Y_2(t) = -FY_1(t) \tag{2.12}$$

where, $F = \frac{C_1}{C_2}$

The equation may be represented as:

$$\dot{Y}_1(t) = (A_{11} - A_{12}F)Y_1 \tag{2.13}$$

The equivalent system is an (n-m)th order system thereby simplifying the system dynamics on the sliding mode.

From (2.12) and (2.13) we can say that the system dynamics are governed by C.

A suitable choice C can guarantee desirable performance. The following section discusses two methods of designing the sliding mode parameter matrix C.

2.3.1 Hyperplane design schemes

The equivalent system behaviour on the sliding mode depends on an appropriate choice of F, where $F=C_1/C_2$ and consequently of the sliding mode parameter matrix C. This section discusses two methods of sliding mode parameter design.

By Quadratic Minimisation:

Utkin and Young (1978) proposed a design in which a cost functional is minimised. This cost functional consists of an integrand which is a linear quadratic regulator (LQR) of the state X(·).

If t_s denotes the time at which sliding motion begins, then the cost functional is defined as:

$$J(u) = \frac{1}{2}\int_{t_s}^{\infty} X^T(t)QX(t)\ dt \tag{2.14}$$

where Q>0 is a constant, symmetric matrix.

The main objective is to minimise J, assuming a known initial condition $X(t_s)$ such that $X(t) \rightarrow 0$ as $t \rightarrow \infty$.

The performance index is then reduced to the transformed state space q by partitioning the product

$$T^T QT = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \tag{2.15}$$

compatibly with Y and defining

$$Q^* = Q_{11} - Q_{12}Q_{22}^{-1}Q_{21} \tag{2.16}$$

$$A^* = A_{11} - A_{12}Q_{22}^{-2}A_{21} \tag{2.17}$$

$$V(t) = Y_2(t) + Q_{22}^{-1}Q_{21}Y_1(t) \tag{2.18}$$

The LQR is now in the standard form:

$$J = \frac{1}{2}\int_{t_s}^{\infty}\left[Y_1^T Q^* Y_1 + v^T Q_{22} v\right]dt \tag{2.19}$$

$$\dot{Y}_1 = A^* Y_{11} + A_{12}v(t) \tag{2.20}$$

After solving the appropriate Riccati matrix P from (2.19) we get

$$F = Q_{22}^{-1}[Q_{21} + A_{12}^{T}P] \tag{2.21}$$

By Eigenstructure Assignment:

The equivalent system can be written as:

$$\dot{X}(t) = (A - BK)X(t) \tag{2.22}$$

where $K = (CB)^{-1}CA$.

Assuming that the sliding motion has commenced on N(C), the state variables must remain in N(C) during the sliding motion such that

$$C[A - BK] = 0 \tag{2.23}$$

$$R(A - BK) \subseteq N(C)$$

Let λ_i(i=1..n) be the eigenvalues of the equivalent system with corresponding eigenvectors V_i then we have

$$C[A - BK]V_i = \lambda_i CV_i = 0 \tag{2.24}$$

Therefore, either $\lambda_i = 0$ or $V_i \in N(C)$

Assuming that the motion on the equivalent system, A-BK=A_{eq}, has n-m distinct, non-zero eigenvalues then the corresponding eigenvectors $\{V_i : i = 1..n-m\}$ determine the null space of C, since dim[N(C)]=n-m.

i.e. CV=0 V=[v_1,...,v_{n-m}]

But, C is not uniquely determined because the equation CV=0, (V_i: i=1..n-m)

has m^2 degrees of freedom.

This is clear if we define,

$$W = \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \mathrm{TV} \tag{2.25}$$

where the partitioning of W is compatible with that of Y

$$\therefore \quad 0 = CT^T.TV = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = C_2 \begin{bmatrix} F & I_m \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \tag{2.26}$$

where $F = \dfrac{C_1}{C_2}$ and I_m is an m-dimensional unit matrix

thereby yielding the equation

$$FW_1 = -W_2 \tag{2.27}$$

Therefore, if W_1 is non-singular then a unique value of F is determined by the expression (2.26).

Dorling and Zinober (1986) have shown that the eigenvectors of matrix (A-BK) are not freely assignable. At most *m* elements may be arbitrarily assigned. The remaining *(n-m)* elements must be determined using the assigned elements allowing a degree of adjustment to be carried out by inspection. Other eigenvector assignment schemes can be found in Moore(1976), Klein and Moore(1977) and Sinswat and Fallside(1977).

2.4 SLIDING MODE CONTROL OF NONLINEAR SYSTEMS

In the previous sections we discussed the application of the sliding mode control technique to a linear, time-invariant system. In the nonlinear case, the fundamentals of sliding mode control are similar to those of linear systems. The control law is also easily derived. However, sliding mode analysis and the corresponding switching function derivation become a difficult problem. The following sub-sections discuss the implications of applying the technique to nonlinear systems.

2.4.1 The Nonlinear System Model

Consider the nonlinear, time-varying plant.

$$\dot{X}(t) = f(t, X) + B(t, X)u(t) \tag{2.28}$$

where, $X(t) \in R^n$ is the state vector and $u(t) \in R^m$, $f(t,X) \in R^n$ and $B(t,X) \in R^{nxm}$ are control input vectors. Further, each entry in f(t,X) and B(t,X) is assumed to be continuous, with continuous bounded derivatives with respect to X.

In sliding mode control of non-linear systems, the fundamental problem lies in the derivation of the switching law. One possibility is to subject the dynamical system to various state transformations, whereby, the differential equations of the system are expressed in simple canonical forms. The reaching law can then take advantage of the characteristics of the canonical forms. The next section explains two such canonical transformations for sliding mode stability analysis. Further details may be found in Hung et al. (1993).

2.4.2 Canonical forms for sliding modes

Reduced form:

The state vector X is partitioned into X_1 and X_2, where $X_1 \in R^{n-m}$ and $X_2 \in R^m$.

The input matrix B takes the form:

$B = \begin{bmatrix} 0 & B^* \end{bmatrix}^T$ where B^* is an invertible mxm matrix.

The reduced form of the system model is then given as:

$$\dot{X}_1 = A_1(X) \tag{2.29}$$

$$\dot{X}_2 = A_2(X) + B^*(X)u \quad where \quad u \in R^m \tag{2.30}$$

Consider a general m-dimensional sliding equation:

$$S(X)=S(X_1,X_2)=0 \tag{2.31}$$

Theoretically, it is possible to solve for X_2 in terms of X_1 using $X_2=W(X_1)$.

where, W is a constant.

Therefore, $S(X)=X_2-W(X_1)$ (2.32)

The problem of defining a switching function S(X) is to find $W(X_1)$ such that the sliding motion is asymptotically stable.

Controllability form:

The state vector is in the form:

$$X = \begin{bmatrix} X_1^T \\ \cdot \\ \cdot \\ X_m^T \end{bmatrix}, \quad X_i \in R^{n_i}, \text{ where i=1..m, and } \sum_{i=1}^{m} n_i = n \tag{2.33}$$

The m system inputs are partitioned into m subsystems, each represented in controllable canonical form.

The controlled canonical form of each subsystem is:

$$\dot{X}_i = A_i X_i + \alpha_i(X) + \beta_i(X)u \quad where \quad i = 1..m \tag{2.34}$$

$$A_i = \begin{bmatrix} 0 & I_{n_i=1} \\ 0 & 0 \end{bmatrix} \quad where \quad A_i \in R^{n_i x n_i} \tag{2.35}$$

$$\alpha_i(X) = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ 0 \\ \alpha_{i0}(X) \end{bmatrix} \quad where \quad \alpha_i \in R^{n_i x n_i} \tag{2.36}$$

$$\beta_i(X) = \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \beta_{i0}(X) \end{bmatrix} \quad where \quad \beta_i \in R^{n_i x n_i} \tag{2.37}$$

The overall system dynamics are given by:

$$\dot{X} = AX + \alpha(X) + \beta(X)u \tag{2.38}$$

where,

$$A = \begin{bmatrix} A_1 & 0 & 0 & \cdot \\ 0 & A_2 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & A_m \end{bmatrix} \tag{2.39}$$

$$\alpha(X) = \begin{bmatrix} \alpha_1(X) \\ \cdot \\ \\ \alpha_m(X) \end{bmatrix} \tag{2.40}$$

$$\beta(X) = \begin{bmatrix} \beta_1(X) \\ \beta_2(X) \\ \cdot \\ \beta_m(X) \end{bmatrix} \tag{2.41}$$

This canonical form where the system is decomposed into m subsystems facilitates the use of a decentralised sliding mode scheme with decoupled sliding functions of the form:

$$S_i = C_i X_i, \quad i = 1..m \tag{2.42}$$

where, X_i is a substate vector in phase variable form.

The equations of the subsystems are:

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= x_{i3} \\ &\cdot \\ &\cdot \\ \dot{x}_{i(n_i+1)} &= x_{in_i} \end{aligned} \tag{2.43}$$

where,

$$\dot{x}_{in_i} = \alpha_i(X) + \beta_i u_i , \quad i = 1..m \tag{2.44}$$

$$\begin{aligned} &\therefore c_i x_i = 0 \quad \textit{becomes} \\ &x_{i1}^{(n_i)} + c_{i1} x_{i1}^{(n_i-1)} + ..+ c_{i(n_i-1)} x_1 + c_{in_i} x_i = 0 \end{aligned} \tag{2.45}$$

The stability of the sliding mode in each of the m subsystems is guaranteed by choosing the elements of C_i to match the desired characteristic equation.

2.5 ROBUSTNESS ANALYSIS OF SLIDING MODE CONTROL

The main objective of sliding mode control system design is to ensure convergence on the sliding mode. Even the basic task of transferring objects of variable masses and inertial properties, along a prescribed trajectory can introduce a large perturbation of the dynamic model parameters of the manipulator. The function of reducing sensitivity to external disturbances and parameter uncertainties lies with the controller.

Assuming that the nominal system matrix A has an uncertainty ΔA due to external disturbances, then the state equation may be expressed as:

$$\dot{X}(t) = (A + \Delta A)X(t) + Bu + Df \tag{2.46}$$

where $f \in R^n$ is a bounded external disturbance vector and D is compatibly dimensioned.

Spurgeon (1991) has shown that the robustness of (2.28) can be ensured only if the following rank condition is satisfied.

rank[B:D]=rank[B:ΔAT]=rank[B] (2.47)

where T is the matrix of the basis vectors of the equivalent system's subspace.

On the sliding mode,

$$C\dot{X} = 0$$

$$C(A + \Delta A)X + CBu_{eq} + CDf = 0$$

$$u_{eq} = -(CB)^{-1} C(AX + \Delta AX + Df) \quad (2.48)$$

the equivalent system can be expressed as:

$$\dot{X}(t) = [I - B(CB)^{-1} C](AX + \Delta AX + Df) \quad (2.49)$$

$$C\dot{X} = 0$$

(2.49) is also called the invariance condition.

If there exists $\Delta\tilde{A}$ *and* $\Delta\tilde{f}$, where $\Delta\tilde{A}$ *and* $\Delta\tilde{f}$ are estimates of ΔA and Δf respectively, such that the matching conditions $\Delta A = B\Delta\tilde{A}$ and $f(t) = B\Delta\tilde{f}$ are satisfied then the sliding mode is invariant.

For nonlinear systems:

$$\dot{X} = A(X,t) + \Delta A(X,P,t) + B(X)u + \Delta B(X,P,t)u + f(X,P,t) \quad (2.50)$$

where ΔB is the uncertainty in B and P is an uncertain parameter vector, then it has been shown in Gao and Hung (1993) that invariance holds true if the following matching conditions are satisfied:

$$\Delta A(X,P,t) = B(X,t)\Delta\tilde{A}(X,t) \tag{2.51}$$

$$\Delta B(X,P,t) = B(X,t)\Delta\tilde{B}(X,P,t) \tag{2.52}$$

$$f(X,P,t) = B(x,t)\Delta\tilde{f}(X,P,t) \tag{2.53}$$

For certain $\Delta\tilde{A}$, $\Delta\tilde{B}$ and $\Delta\tilde{f}$

2.6 BOUNDARY LAYER CONTROL

Sliding mode theory is based on the assumption that the sliding mode control law is operated at continuous switching times which are theoretically infinite, thereby ensuring that the trajectory stays on the switching surface. In practice, however, finite sampling rates are possible which causes the state to move away from the switching surface until the end of the sampling interval. The control action then forces the trajectory back onto the switching surface. The net result is a chattering input, which has a detrimental effect on the overall system performance. In applications where, for example, the robot has to perform an autonomous sampling operation, this chattering could cause extensive tool wear, sample degradation and actuator saturation. (Venkataraman and Gulati, 1993).

The "boundary layer" technique (Slotine and Sastry, 1983) defines a region around the switching surface such that any trajectory starting outside the region has the full amplitude of control applied to it, but within the boundary layer, it receives a proportionally reduced control amplitude. The discontinuous control signals therefore have the effect of being smoothed out within the boundary layer and

thereby reduces the chattering of the state trajectory close to the sliding surface. Robust tracking can therefore be achieved to within a predefined accuracy.

The main drawback of boundary layer control is that the trajectories within the boundary layer are only an approximation of the desired dynamics on the sliding surface. The conditions for guaranteed accuracy are provided in Slotine (1984) for continuous control. However, in the discrete-time case, the deviation of the trajectory from the switching line is a function of the width of the boundary layer and the sampling frequency (Richards and Reay, 1991). Another drawback of this scheme is that the manifolds must be designed off-line, using bounds on the uncertainties and the expected system response in the vicinity of the sliding surface. In the following section we discuss the terminal sliding mode control technique which eliminates the input chattering problem.

2.7 TERMINAL SLIDING MODE TECHNIQUE

Linear sliding mode uses a high-gain control switch to force convergence towards the sliding surface to satisfy the condition S=0. However, the system does not actually stay on the sliding surface since $\dot{S} <> 0$, thus resulting in the chattering effect explained in the previous section. Linear sliding mode ensures exponential stability with full model information and asymptotic stability on the presence of uncertainties.

The terminal sliding mode technique (Venkataraman and Gulati (1993)) takes into consideration the rate of change of system nonlinearities rather than magnitude. The chief advantage of using terminal sliding mode instead of linear sliding mode

can be attributed to its convergence time which is controllable and finite, while providing improved precision. Convergence to equilibrium is achieved without applying the high-gain switching laws used in the linear sliding mode technique.

2.7.1 Terminal Attractors

The idea of using terminal attractors was proposed by Zak, M. (1988) to enhance the convergence properties of dynamical systems. Venkataraman and Gulati (1993) proposed a scheme which applies the idea of terminal attractors to the design of sliding mode control of robotic manipulators.

The idea of a terminal attractor can be demonstrated using a cubic parabola:

$$\dot{x} = -x^{1/3} \tag{2.54}$$

with its equilibrium point defined at $x_{equil}=0$

Integrating between limits t_{init} and t_{equil}

$$t_{equil} - t_{init} = \frac{3}{2} x_i^{2/3}$$

This implies that the system settles to equilibrium in finite time.

For the system (2.54), x_{equil} is the terminal attractor.

Consider a first-order terminal attractor:

$$\dot{x} + X(x) = 0 \tag{2.55}$$

where x is bounded for bounded X and Sgn(X)=Sgn(x).

Also, $\frac{\partial X}{\partial x} \to \infty \; as \; x \to 0$

For Lyapunov analysis, the Lyapunov function candidate V is assumed to be bounded for bounded x.

i.e. $\|V(x \neq 0)\| > 0 \; and \; \|V(x = 0)\| = 0$, if

$\dot{v} + V(v) = 0$, such that V(·) has the terminal attractor property, then the dynamical system is considered to be terminally stable.

2.7.2 Terminal Sliding Control

Consider the system

$$\ddot{x} = f(x) + u \tag{2.56}$$

The terminal attractors are of the form:

$$\dot{x} = \alpha x^{\frac{\beta_n}{\beta_d}} \quad where \quad \alpha > 0 \quad and \quad \beta_n, \beta_d = (2i+1) \; where \; i \in I \; and \; \beta_d > \beta_n \tag{2.57}$$

The control law is of the form:

$$u = \ddot{x}_d - \alpha \frac{\beta_n}{\beta_d} e^{\frac{\beta_n}{\beta_d} - 1} \dot{e} - f \tag{2.58}$$

Substituting (2.56) in (2.58) we get the closed-loop system

$$\ddot{e} + \alpha \frac{\beta_n}{\beta_d} e^{\frac{\beta_n}{\beta_d} - 1} \dot{e} = 0 \tag{2.59}$$

where $e=(x-x_d)$ and where x and x_d are the actual and desired trajectories respectively.

The sliding surface for the above system is defined as:

$$S_i = \dot{e}_i + \alpha e_i^{\frac{\beta_n}{\beta_d}} = 0 \tag{2.60}$$

where i denotes initial conditions.

(2.59) and (2.60) represent the terminal stability of the system defined in (2.56).

The surface S_i is called the *terminal slider* and the control law u is called the *Terminal Slider Control.*

Substituting $\dot{e}$ in (2.58), in terms of e, we have

$$u = x_d + \alpha^2 \frac{\beta_n}{\beta_n} e^{\frac{2\beta_n}{\beta_d} - 1} - f \tag{2.61}$$

For the control u to be bounded for a bounded e,

$$\frac{2\beta_n}{\beta_d} - 1 > 0 \quad i.e. \frac{\beta_n}{\beta_d} > \frac{1}{2} \tag{2.62}$$

The initial condition S_i must always be zero. This is ensured by a continuous redesign of each trajectory. In linear sliding mode control, the reaching mode is implemented using a high-gain switch which forces the trajectories from an

arbitrary initial state onto the sliding surface. The terminal sliding mode models the behaviour between the initial condition and the sliding surface as a dynamical system.

Consider a control law of the form:

$$u = \ddot{x}_d - \alpha \frac{\beta_n}{\beta_d} \dot{e} - \gamma s^{\frac{\partial n}{\partial d}} - f \tag{2.63}$$

Sustituting (2.63) in (2.56) we get:

$$p = \dot{s} + \gamma s^{\frac{\partial n}{\partial d}} = 0 \tag{2.64}$$

(2.64) specifies the finite time steady state convergence of s from any arbitrary initial condition s_i, after which the system reaches e=0 on the terminal sliding mode similar to the linear sliding mode.

2.8 SLIDING MODE CONTROL OF MANIPULATORS

The previous sections provided a brief overview of sliding mode control for linear and nonlinear systems. A robotic manipulator plant is a typical example of a nonlinear system. In recent years, many researchers have investigated the application of sliding mode technique to robotic manipulators. Robustness and convergence results have been provided by Young (1978), Morgan and Ozguner (1985), Slotine and Sastry (1983).

2.8.1 Robot Link Dynamics

The dynamic model of an n-link rigid robotic manipulator can be expressed in Euler-Lagrange formulation as:

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i.j} f_{ijk}(q)\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k \qquad k = 1..n \tag{2.65}$$

where d_{kj} are the coefficients of the inertia matrix D(q), $\phi_k(q)$ are the gravitational forces and τ_k are the input torques. The coefficients f_{ijk} of the coriolis and centrifugal terms are defined as:

$$f_{ijk} = \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{ij}}{\partial q_k}\right\} \tag{2.66}$$

and f_{ijk} are known as Christoffel symbols.

In matrix form the expression (2.65) is written as:

$$D(q)\ddot{q} + F(q,\dot{q})\dot{q} + G(q) = \tau \tag{2.67}$$

where $D(q) \in R^n$ is the symmetric, positive-definite manipulator inertia matrix, $q \in R^n$ is the joint angle/displacement vector, $F(q,\dot{q})\dot{q} \in R^n$ represents the coriolis and centripetal torques, $G(q) \in R^n$ is the vector of gravitational torques.

The k,jth element of the matrix F is written as

$$f_{kj} = \sum_{i=1}^{n} \frac{1}{2}\left(\frac{\partial d_{ki}}{\partial q_i} + \frac{\partial d_{ki}}{\partial dq_i} - \frac{\partial d_{ij}}{\partial q_k}\right)\dot{q}_i \tag{2.68}$$

and the component of G (q) is ϕ_k.

The nonlinear model defined above have some fundamental properties which can be exploited to facilitate control system design.

Property 1: The inertia matrix D(q) is symmetric, positive-definite and both D(q) and $D(q)^{-1}$ are uniformly bounded as a function of q.

Property 2: There is an independent control input for each degree of freedom.

Property 3: The Euler-Lagrange equation for the robotic manipulator is linear in the unknown parameters. All the unknown parameters are constant (for example, link masses, link lengths, moments of inertia, etc.) and appear as coefficients of known functions of the generalised co-ordinates. By defining each coefficient or a linear combination of them as a separate parameter, a linear relationship results so that we may write the matrix equation as:

$$D(q)\ddot{q} + F(q,\dot{q})\dot{q} + G(q) = Y(q,\dot{q},\ddot{q})\theta = \tau \qquad (2.69)$$

where Y is an nxr matrix of known functions called as regressor functions, and q is an n-dimensional vector of unknown parameters as shown in Spong and Vidyasagar (1989). The robotic manipulator system in (2.67) can be expressed in the generalised form $\dot{X} = AX + Bu$. Therefore, the basic sliding mode theory can be used to simplify controller designs which exhibit robustness, by using the structural properties mentioned above.

2.9 CONCLUSION

In this chapter we have briefly surveyed sliding mode control theory and its application to linear and nonlinear systems with an emphasis on its application to robotic manipulators. Though sliding mode control has been researched extensively in the area of robotic manipulator control, there are many issues still left unresolved before it can be practically feasible.

The following chapters of the thesis propose several new and improved schemes for sliding mode control of rigid robotic manipulators. These schemes show improved robustness on the sliding mode.

Chapter 3

A Robust Adaptive Sliding Mode Tracking Control for Rigid Robotic Manipulators

3.1 INTRODUCTION

Sliding mode control is one of the most important approaches for dealing with rigid robotic manipulators with nonlinearities, uncertain dynamics, and bounded input disturbances. The most distinguished feature of the sliding mode control technique is its ability to provide strong robustness for control systems that renders closed loop systems completely insensitive to nonlinearities, uncertain dynamics and bounded input disturbances in the sliding mode. In Young (1978, 1988), Abbass and Chen (1988) and Morgan and Ozguner (1985) it is shown that robustness and convergence can be obtained for robotic manipulators by using sliding mode technique based on the upper and the lower bounds of all unknown system parameters. The main drawback of using the above control schemes is that all the upper and the lower bounds of unknown parameters need to be obtained prior to controller design. This could result in the sliding mode controller design becoming complicated if the controlled systems have many unknown parameters.

The design of the sliding mode controller is greatly simplified in Man and Palaniswami (1994), where, for any n-link rigid robotic manipulator which may have many uncertain parameters, only four uncertain system matrix bounds and an upper bound of the input disturbance are required in the sliding mode controller design to obtain robustness and convergence. Also, only one uncertain bound is used in the design of the sliding mode compensator for rigid robotic manipulators with uncertain dynamics.

Various schemes for the design of sliding mode controller without requirement of the prior knowledge of the uncertain bounds have been investigated by many researchers. For example, in Fu and Liao (1990) and Leung, Zhou and Su (1991), an adaptive mechanism is developed to estimate the uncertain bound parameters and the estimates are then used as controller parameters to guarantee that the effects of the system uncertainties can be eliminated and asymptotic error convergence can be obtained for robot control systems. However, the drawback of the scheme in Fu and Liao (1990) is that five parameters of the uncertain bounds need to be adaptively estimated in each local controller design. The use of this scheme in a controlled robot having many links complicates the controller design of the overall system and increases processing time. In the adaptive sliding mode control scheme of Leung, Zhou and Su (1991), although only five uncertain system matrix bounds are estimated for any n-link robotic manipulator, it does not address the problem of eliminating the effects of bounded input disturbances. Therefore, for practical applications, the above adaptive sliding mode tracking control schemes still needs further improvements.

In this chapter, a new robust adaptive sliding mode tracking control scheme is proposed for rigid robotic manipulators with both uncertain dynamics and bounded input disturbances for achieving robustness and asymptotic error convergence based on Fu and Liao (1990), Man and Palanaiswami (1994) and Leung, Zhou and Su (1991). The robotic manipulator is treated as a partially known system, the known dynamics are separated out to perform linearization, and the dynamical uncertainties are assumed to be upper bounded by an unknown positive function. Then, a nominal feedback controller is designed to stabilise the nominal system model and an adaptive sliding mode compensator is introduced to eliminate the effects of the unknown parameters of the plant. A key feature of this scheme is that only three uncertain parameters of the upper bound of the system uncertainties are estimated for any n-link rigid robotic manipulators with both uncertain dynamics and bounded input disturbances. The estimates are then used as compensator parameters to guarantee that the effects of the system uncertainties are eliminated and asymptotic error convergence are obtained for robot control systems. It can also be seen that the proposed scheme in this chapter can be easily used for practical implementation as opposed to the schemes in Man and Palaniswami (1994) and Leung, Zhou and Su (1991). In addition, simulation results show that the estimated uncertain bound using the proposed adaptive mechanism is non-conservative and that the amplitude of the control signal is greatly reduced.

This chapter is organised as follows: In section 3.2, an n-link rigid robotic manipulator as a partially known system is formulated and a robust sliding mode control using only one uncertain bound in Man and Palaniswami (1994) is briefly

discussed. In section 3.3, a new robust adaptive tracking control scheme for rigid robotic manipulators is proposed where an adaptive mechanism for the estimation of only three parameters of the uncertain bound is introduced. Error convergence and robustness with respect to uncertain dynamics are discussed in detail. In section 3.4, a two-link rigid robotic manipulators is simulated in order to examine the proposed control scheme.

3.2 PROBLEM FORMULATION

Consider the dynamics of an n-joint rigid robotic manipulator system described by the following second order nonlinear vector differential equation

$$M(q)\ddot{q} + h(q, \dot{q}) = u(t) + d(t) \tag{3.1}$$

where q(t) is the nx1 vector of joint angular positions, M(q) is the nxn symmetric positive definite inertia matrix, $h(q, \dot{q})$ is the nx1 vector containing coriolis, centrifugal forces and gravity torques, u(t) is the nx1 vector of applied joint torques (control inputs) and d(t) is the nx1 vector of the bounded input disturbances.

Let a robotic manipulator system described by equation (3.1) have some known parts and some unknown parts, which can be expressed as:

$$M(q) = M_0(q) + \Delta M(q) \tag{3.2}$$

$$h(q, \dot{q}) = h_0(q, \dot{q}) + \Delta h(q, \dot{q}) \tag{3.3}$$

where $M_0(q)$ and $h_0(q,\dot{q})$ are the known parts, $\Delta M(q)$ and $\Delta h(q,\dot{q})$ are the unknown parts. Using expressions (3.2) and (3.3), dynamic equation (3.1) can be written in the following form:

$$M_0(q)\ddot{q} + h_0(q, \dot{q}) = u(t) + \rho(t) \tag{3.4}$$

where $$\rho(t) = - \Delta M(q)\ddot{q} - \Delta h(q, \dot{q}) + d(t) \tag{3.5}$$

The following system with no uncertainties is defined as the "nominal system"

$$M_0(q)\ddot{q} + h_0(q, \dot{q}) = u_1 \tag{3.6}$$

In this chapter, the following assumptions are made:

Assumption 3.1: $M_0(q)$ is invertible for all q.

Assumption 3.2: The nominal system in expression (3.6) is stabilizable.

Assumption 3.3: The system uncertainty r(t) is bounded by a positive function:

$$\| \rho(t) \| < b_0 + b_1 \| q(t) \| + b_2 \| \dot{q}(t) \|^2 \tag{3.7}$$

where b_0, b_1 and b_2 are positive numbers.

Two steps are considered in the development of the robust tracking control for robotic manipulators in expressions (3.1)-(3.3) in this chapter. First, a nominal feedback controller is designed to stabilise the nominal system and a sliding mode compensator is then designed to eliminate the effects of both uncertain dynamics,

and the bounded input disturbances, so that the output tracking error of the closed loop system with large system uncertainties asymptotically converges to zero.

Let q_r represent the desired trajectory that the robotic manipulator system must follow and the output tracking error be defined as $e(t) = q - q_r$.

Using nominal system equation (3.6), we get the following linearized error system:

$$\dot{e} = Ae + Bv \tag{3.8}$$

where $$e = \left[\varepsilon^T, \dot{\varepsilon}^T \right]^T \tag{3.9}$$

$$A = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \tag{3.10}$$

$$B = \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{3.11}$$

$$v = M_0(q)^{-1}(u_1 - h_0(q, \dot{q})) - \ddot{q}_r \tag{3.12}$$

Lemma 3.1: The error e(t) in error dynamics equation (3.8) for nominal system (3.6) asymptotically converges to zero if the following nominal feedback control law is used

$$u_1 = h_0(q, \dot{q}) + M_0(q)(Ke + \ddot{q}_r) \tag{3.13}$$

where $K = [- K_1, - K_2]$, $K_1 \in R^{nxn}$, $K_2 \in R^{nxn}$, and matrix K is designed such that

$$A_1 = A + BK \tag{3.14}$$

is an asymptotically stable matrix.

Proof: See Singh (1986) and Man (1993).

Next, let the control input in dynamic equation (3.1) have the following form:

$$u(t) = u_1 + u_0 \tag{3.15}$$

where u_1 is designed for the nominal system of expression (3.6). u_0 is a compensator to be used to eliminate the effects of the system uncertainties in expression (3.5).

Using expressions (3.4), (3.6), (3.8) and (3.13), we get the error dynamic equation for the closed loop robotic manipulator system in the following form:

$$\dot{e} = A_1 e + BM_0(q)^{-1}u_0 + BM_0(q)^{-1}\rho(t) \tag{3.16}$$

In order to use the sliding mode technique to design the compensator u_0, we define a set of sliding variables in the error space passing through the origin.

$$S = Ce \tag{3.17}$$

where $C = [C_1, C_2]$, matrices $C_1 \in R^{nxn}$ and $C_2 \in R^{nxn}$ are nonsingular and

$$Re\lambda(-C_2^{-1}C_1) < 0 \tag{3.18}$$

Remark 3.1: It has been seen that the key problem for the design of the controller in expression (3.15) is how to design the compensator u_0 for achieving robustness and convergence. If the upper bound of the system uncertainties in expression

(3.7) is known, the design of robust sliding mode compensator can be summarised in the following theorem Man and Palaniswami (1994).

Theorem 3.1: Consider the error dynamics in expression (3.16) for the robotic manipulator system in expression (3.1) with assumptions 3.1-3.3. If the compensator u_0 is designed such that

$$u_0 = \begin{cases} \dfrac{(S^T C_2 M_0(q)^{-1})^T}{\|S^T C_2 M_0(q)^{-1}\|^2} w & \|S\| \neq 0 \\ 0 & \|S\| = 0 \end{cases} \qquad (3.19)$$

where

$$w = -S^T C A_1 e - \|S\| \|C_2 M_0(q)^{-1}\| (b_0 + b_1 \|q\| + b_2 \|\dot{q}\|^2) \qquad (3.20)$$

and S is the sliding variable vector defined in expression (3.17) and (3.18),

then the output tracking error e(t) asymptotically converges to zero.

Proof: See Man and Palaniswami (1994) or Man (1993).

Remark 3.2: It can be seen from theorem 1 that prior knowledge of the uncertain bound in expression (3.7) is required in the compensator design. Although a method for off-line estimation of the bound parameters in expression (3.7) was developed in Grimm (1990) and Man (1993), the estimates of the uncertain bound

parameters were very conservative, and the control input signals, using the conservative estimate as the controller parameters, are then very large. Therefore, application of the above robust control scheme in practical situations are still difficult.

3.3 A ROBUST ADAPTIVE TRACKING CONTROL SCHEME

In this section, we propose a novel approach to avoid the requirement of the prior knowledge of the upper bound of the system uncertainties r(t) in expression (3.7).

Now we let $\hat{b}_0$, $\hat{b}_1$ and $\hat{b}_2$, be the estimates of b_0, b_1 and b_2 in expression (3.7) which are updated by the following adaptive laws:

$$\dot{\hat{b}}_0 = k_0 \| C_2 M_0(q)^{-1} \| \| S \| \tag{3.21}$$

$$\dot{\hat{b}}_1 = k_1 \| C_2 M_0(q)^{-1} \| \| S \| \| q \| \tag{3.22}$$

$$\dot{\hat{b}}_2 = k_2 \| C_2 M_0(q)^{-1} \| \| S \| \| \dot{q} \|^2 \tag{3.23}$$

where k_i ($i = 0, 1, 2$) are arbitrary positive numbers and $\hat{b}_i$ ($i = 0, 1, 2$) have arbitrary positive initial values. Then we have the following convergence and robustness results.

Theorem 3.2: Consider the error dynamics in expression (3.16) for the robotic manipulator system in expression (3.1) with assumptions 3.1-3.3. If the compensator u_0 is designed such that

$$u_0 = \begin{cases} \dfrac{(S^T C_2 M_0(q)^{-1})^T}{\| S^T C_2 M_0(q)^{-1} \|^2} w & \| S \| \neq 0 \\ 0 & \| S \| = 0 \end{cases} \tag{3.24}$$

where $w = -S^T C A_1 e - \| S \| \| C_2 M_0(q)^{-1} \| (\hat{b}_0 + \hat{b}_1 \| q \| + \hat{b}_2 \| q \|^2$

κ_i *and* $\hat{b}_i$ $(i = 0, 1, 2)$ are updated by the adaptive laws in expressions (3.12) - (3.23),

then, the output tracking error e(t) can asymptotically converge to zero.

Proof: Defining a Lyapunov function

$$V = \frac{1}{2} S^T S + \frac{1}{2} \sum_{i=0}^{2} k_i^{-1} \tilde{b}_i^2 \tag{3.25}$$

where $\tilde{b}_i = b_i - \hat{b}_i$ (3.26)

$$\dot{\tilde{b}}_i = - \dot{\hat{b}}_i \tag{3.27}$$

and differentiating V with respect to time, we have

$$\dot{V} = S^T\dot{S} - \sum_{i=0}^{2} k_i^{-1}\tilde{b}_i\dot{\hat{b}}_i$$

$$= S^T\left[CA_1 e + CBM_0(q)^{-1}u_0 + CBM_0(q)^{-1}\rho(t)\right] - \sum_{i=0}^{2}\kappa_i^{-1}\tilde{b}_i\dot{\hat{b}}_i$$

$$= S^T CA_i e + S^T C_2 M_0(q)^{-1}u_0 + S^T C_2 M_0(q)^{-1}\rho(t) - \sum_{i=0}^{2}\kappa_i^{-1}\tilde{b}_i\dot{\hat{b}}_i$$

$$= S^T CA_i e + w + S^T C_2 M_0(q)^{-1}\rho(t) - \sum_{i=0}^{2}\kappa_i^{-1}\tilde{b}_i\dot{\hat{b}}_i$$

$$= -\|S\|\left\|C_2 M_0(q)^{-1}\right\|(\hat{b}_0 + \hat{b}_0\|q\| + \hat{b}_3\|\dot{q}\|^2) + S^T C_2 M_0(q)^{-1}\rho(t) - \sum_{i=0}^{2}\kappa_i^{-1}\tilde{b}_i\dot{\hat{b}}_i$$

$$= -\|S\|\left\|C_2 M_0(q)^{-1}\right\|(\hat{b}_0 + \hat{b}_0\|q\| + \hat{b}_3\|\dot{q}\|^2) + S^T C_2 M_0(q)^{-1}\rho(t)$$

$$-\|S\|\left\|C_2 M_0(q)^{-1}\right\|((b_0 - \hat{b}_0) + (b_1 - \hat{b}_1) + (b_2 - \hat{b}_2)\|\dot{q}\|^2)^2$$

$$-\left\|C_2 M_0(q)^{-1}\right\|\|S\|(b_0 + b_1\|q\| + b_2\|\dot{q}\|^2) + \left\|C_2 M_0(q)^{-1}\right\|\|S\|\|\rho(t)\|$$

$$= -\,h\,\|S\|_1 < 0 \qquad \text{for } \|S\| > 0 \tag{3.28}$$

where,

$$h = \|C_2 M_0(q)^{-1}\|\,((b_0 + b_1\|q\| + b_2\|\dot{q}\|^2) - \|r(t)\|) > 0 \tag{3.29}$$

Expression (3.28) is the reaching condition for the sliding variable vector S to reach the sliding mode in a finite time

$$S = Ce = 0 \tag{3.30}$$

On the sliding mode, error dynamics of the closed loop system has the following form

$$\dot{\varepsilon} = -C_2^{-1}C_1\varepsilon \tag{3.31}$$

Therefore, the tracking error e(t) converges to zero asymptotically.

Remark 3.4: Unlike sliding mode control schemes in Young (1978, 1988), Abbas and Chen (1988), Morgan and Ozguner (1985), Fu and Liao (1990), Man et al. (1992, 1993, 1994), Slotine and Sastry (1983), Corless and Leitmann (1981) the prior knowledge of the upper bound of the system uncertainty is not required in the sliding compensator design in this scheme. An adaptive mechanism is introduced to estimate the upper bound of the system uncertainty and the estimates are then used as controller parameters to guarantee that the effects of large system uncertainties can be eliminated and asymptotic error convergence can be obtained for rigid robotic control systems.

Remark 3.5: It can be seen that, compared with Fu and Liao (1990) and Leung, Zhou and Su (1991), the proposed scheme in this chapter can be easily implemented for practical application because, for any n-link rigid robotic manipulator, only three parameters of the upper bound of system uncertainties are

adaptively estimated in the Lyapunov sense to guarantee the good tracking performance and strong robustness.

Remark 3.6: The adaptive and robust properties of the sliding mode compensator in expression (3.23) can be explained as follows: When the output tracking error is large due to the effects of system uncertainties r(t) in expression (3.5), the estimates of $\hat{b}_i$ (i = 0, 1, 2) can be automatically increased according to the update law in expressions (3.21) - (3.23). The control gain can be increased in expression (3.23). Therefore, the effects of uncertain dynamics can be eliminated, the sliding variable vector S driven to zero, and the output tracking error can then asymptotically converge to zero in the sliding mode.

Remark 3.7: It can be seen from expressions (3.21) to (3.23) that three parameters of the upper bound of the system uncertainties are estimated in the Lyapunov sense. It is not necessary for the estimates to converge to their true values because the values of the estimates are increased until the sliding variable vector S converges to zero. Therefore, the true value of the upper bound of system uncertainties is not required.

Remark 3.8: To eliminate chattering in the control input, the following boundary layer compensator can be used in place of the sliding mode compensator in expression (3.24).

$$u_0 = \begin{cases} \dfrac{(S^T C_2 M_0(q)^{-1})^T}{\|S^T C_2 M_0(q)^{-1}\|^2} w & \|S^T C_2 M_0(q)^{-1}\| \geq \delta \\ \dfrac{(S^T C_2 M_0(q)^{-1})^T}{\delta^2} w & \|S^T C_2 M_0(q)^{-1}\| < \delta \end{cases} \tag{3.32}$$

where d is a positive number.

The above boundary layer compensator offers a continuous approximation to the discontinuous variable structure compensator in expression (3.24) inside the boundary layer and guarantees attractiveness to the boundary layer and ultimate boundedness of the output tracking error to within any neighbourhood of the origin. This will achieve optimal trade-off between the control bandwidth and tracking precision. Therefore, the chattering and sensitivity of the controller to parameter uncertainties and input disturbances can be eliminated. But the drawback is that nonzero error exists. The detailed discussion on the boundary layer technique can be found in Slotine and Sastry (1983) and Corless and Leitmann (1981).

3.4 A SIMULATION EXAMPLE

A simulation example with a two-link robotic manipulator is performed for the purpose of evaluating the performance of the proposed control scheme. The dynamic equation the two-link robotic manipulator model is given by

$$\begin{bmatrix} \alpha_{11}(q_2) & \alpha_{12}(q_2) \\ \alpha_{12}(q_2) & \alpha_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} \beta_{12}(q_2)\,\dot{q}_1^{\,2} + 2\,\beta_{12}(q_2)\,\dot{q}_1\,\dot{q}_2 \\ -\,\beta_{12}(q_2)\,\dot{q}_2^{\,2} \end{bmatrix}$$

$$+ \begin{bmatrix} \gamma_1(q_1,\ q_2)\,g \\ \gamma_2(q_1,\ q_2)\,g \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (3.33)$$

where $\alpha_{11}(q_2) = (m_1 + m_2)\,r_1^2 + m_2\,r_2^2 + 2\,m_2\,r_1\,r_2\cos(q_2) + J_1$

$$\alpha_{12}(q_2) = m_2\,r_2^2 + m_2\,r_1\,r_2\cos(q_2)$$

$$\alpha_{22} = m_2\,r_2^2 + J_2$$

$$\beta_{12}(q_2) = m_2\,r_1\,r_2\sin(q_2)$$

$$\gamma_1(q_1,\ q_2) = -\left((m_1 + m_2)\,r_1\cos(q_2) + m_2\,r_2\cos(q_1 + q_2)\right)$$

$$\gamma_2(q_1,\ q_2) = -\,m_2\,r_2\cos(q_1 + q_2) \qquad (3.34)$$

The parameter values are

$$r_1 = 1\ \text{m},\quad r_2 = 0.8\ \text{m}$$

$$J_1 = 5\ \text{kg.m},\ J_2 = 5\ \text{kg.m}$$

$$m_1 = 0.5\ \text{kg},\ m_2 = 1.5\ \text{kg} \qquad (3.35)$$

Desired reference signals are given by

$$q_{r1} = 1.25 - \frac{7}{5}e^{-t} + \frac{7}{20}e^{-4t}$$

$$q_{r2} = 1.25 + e^{-t} - \frac{1}{4} e^{-4t} \tag{3.36}$$

In order to constrain the error dynamics in the sliding mode from the start to the end, we consider a situation characterised by the same initial values on the system and its reference signal Young (1978). In this example, the initial angular positions and velocities are selected as

$$[\, q_1(0), \quad q_2(0)\,]^T = [\, q_{r1}(0), \quad q_{r2}(0)\,]^T = [\, 0.2, \quad 2\,]^T$$

$$[\, \dot{q}_1(0), \quad \dot{q}_2(0)\,]^T = [\, \dot{q}_{r1}(0), \quad \dot{q}_{r2}(0)\,]^T = [\, 0, \quad 0\,]^T \tag{3.37}$$

The nominal values of m_1 and m_2 are assumed to be

$$\hat{m}_1 = 0.4\text{kg}, \quad \hat{m}_2 = 1.2\text{kg} \tag{3.38}$$

and the other system parameters are assumed to be known. The nominal system is then built from the known system dynamics.

In this example, we let the desired error dynamics of the closed loop nominal system have the following form:

$$\dddot{\varepsilon}_i + 5\ddot{\varepsilon}_i + 4\varepsilon_i = 0 \qquad i = 1, 2 \tag{3.39}$$

Then, using pole placement method for expression (3.14) in lemma 3.1, the feedback matrix K can be designed as follows

$$K = \begin{bmatrix} -4 & 0 & -5 & 0 \\ 0 & -4 & 0 & -5 \end{bmatrix} \tag{3.40}$$

Sliding mode is prescribed as

$$S = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} e = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.41}$$

Runge-Kutta method with a sampling interval DT = 0.01s is used to solve the nonlinear differential equation numerically. Figures 3.2(a) and 3.2(b) show the output trackings and Figures 3.3(a) and 3.3(b) show the control inputs of joint 1 and joint 2, respectively, using the proposed robust adaptive tracking controller. The effects of the system uncertainties are eliminated and good tracking performance is obtained. Figures 3.4(a) and 3.4(b) show good performance of the closed loop system using the boundary layer compensator in expression (3.33) with the adaptive update law in expression (3.14) (d = 0.05). Figures 3.5(a) and 3.5(b) amply demonstrate the good output tracking capability of the boundary layer controller.

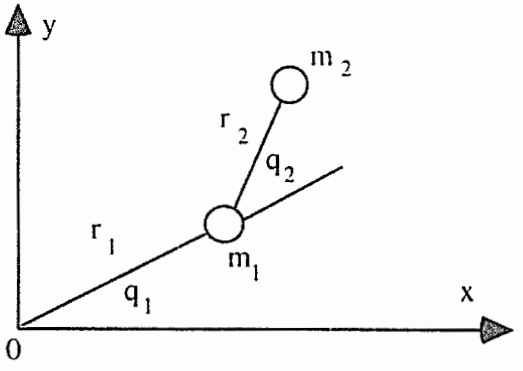

Fig. 3.1 Two-link robotic manipulator model

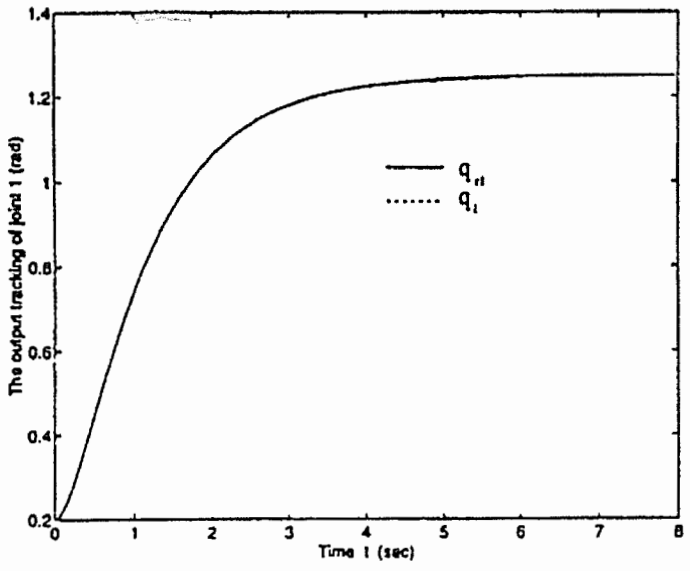

Fig. 3.2 (a) The output tracking error of joint 1 with input disturbances

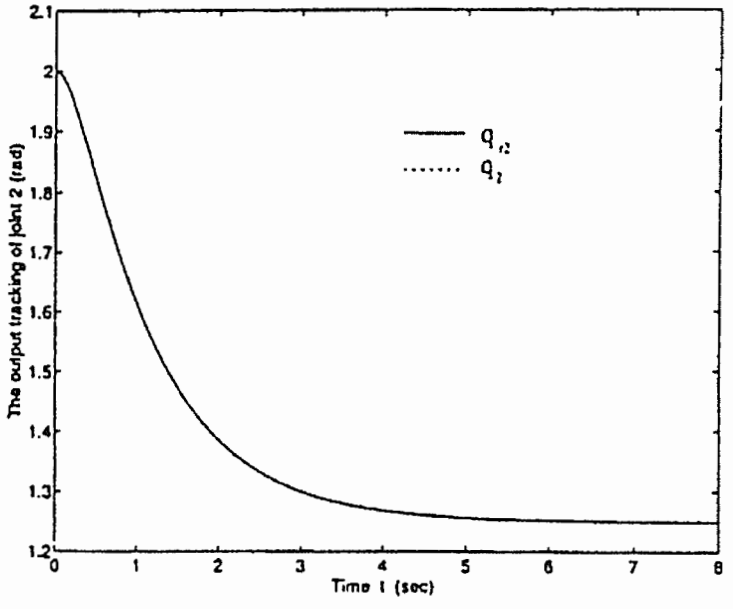

Fig.3.2 (b) The output tracking error of joint 2 with input disturbances

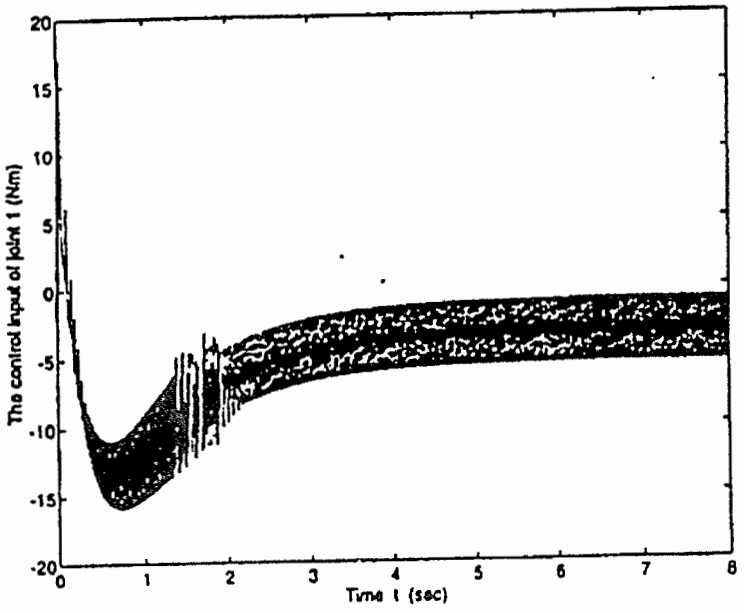

Fig. 3.3 (a) The control input of joint 1

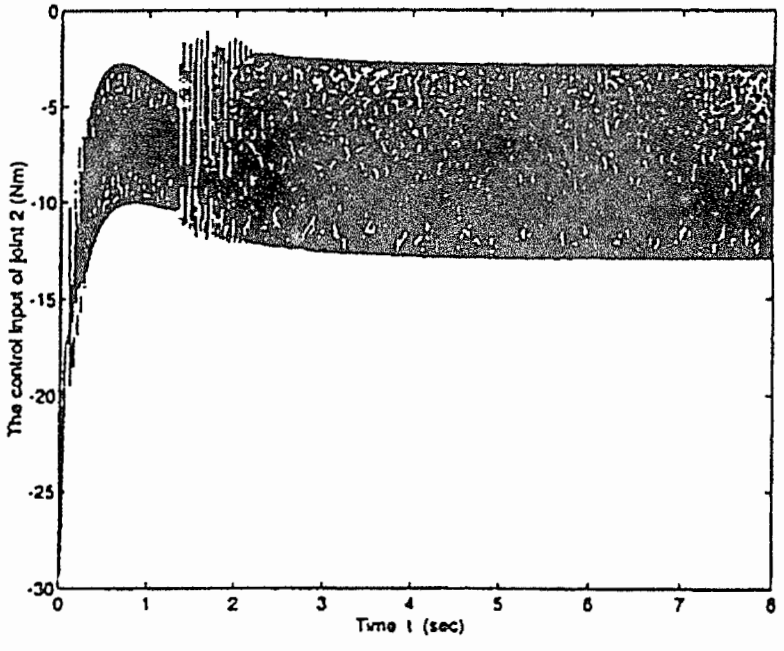

Fig. 3.3 (b) The control input of joint 2

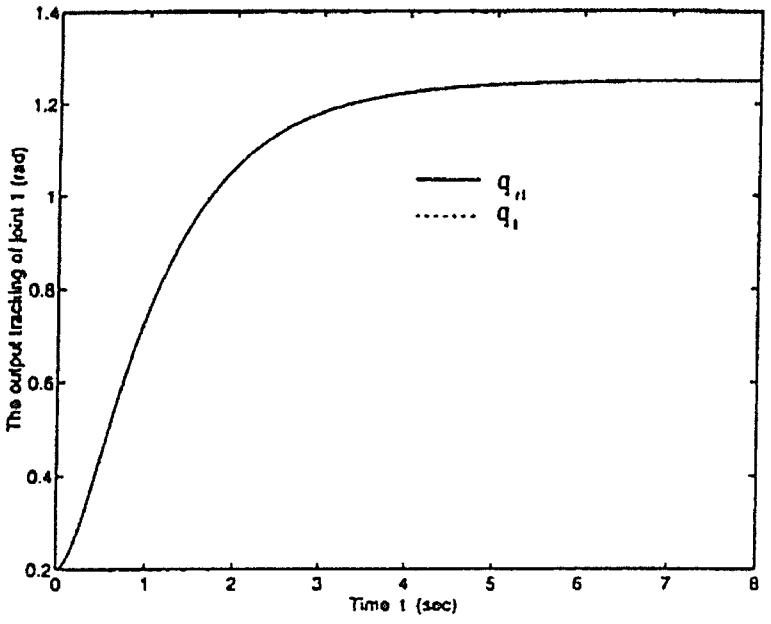

Fig.3. 4 (a) The output tracking of joint 1 using a boundary layer controller

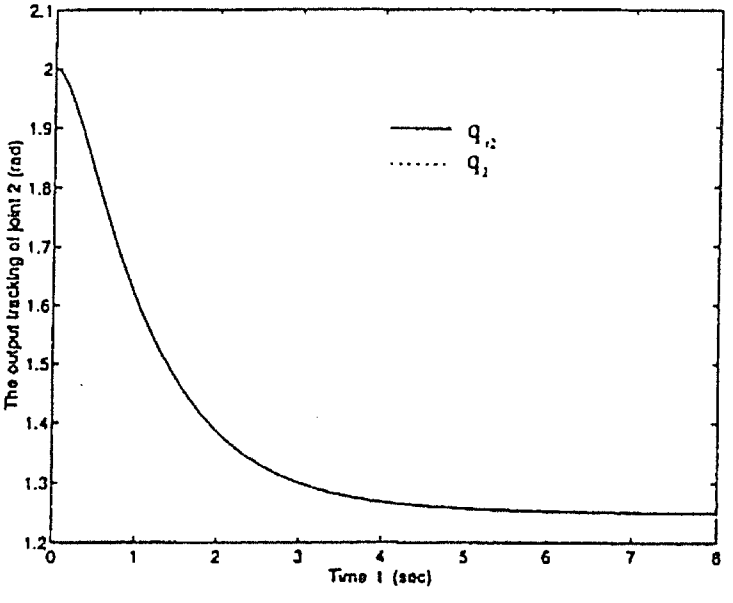

Fig. 3.4 (b) The output tracking of joint 2 using a boundary layer controller

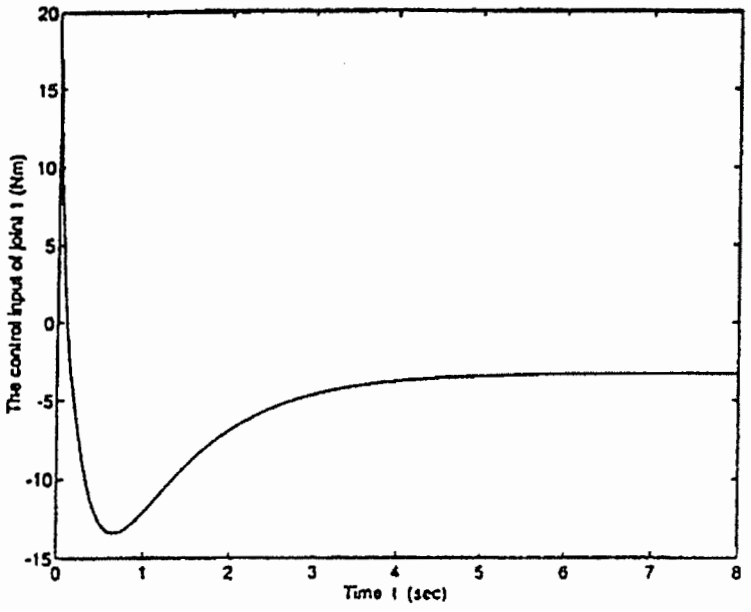

Fig. 3.5 (a) The control input of joint 1 using a boundary layer controller

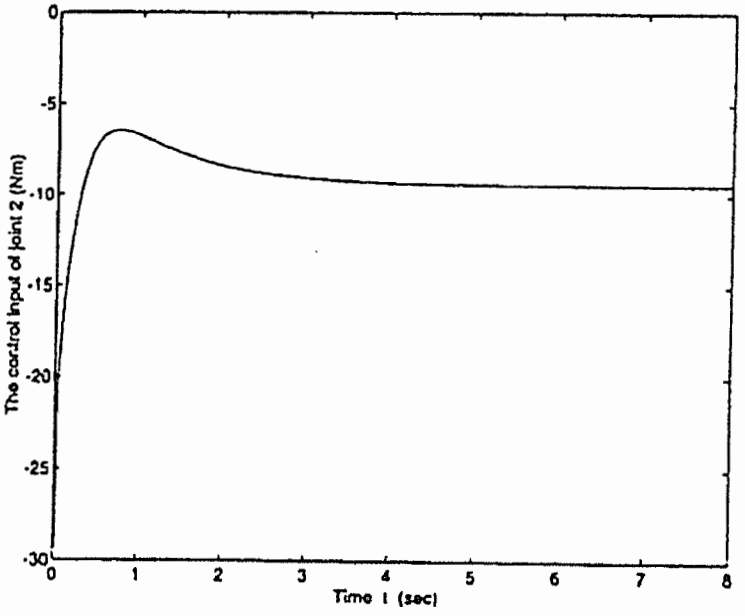

Fig. 3.5 (b) The control input of joint 2 using a boundary layer controller

3.5 CONCLUSION

A new robust adaptive tracking control scheme using sliding mode technique for rigid robotic manipulators was developed. Unlike most sliding mode control schemes, this scheme requires no prior knowledge of the uncertain bound. By adaptively estimating only three parameters of the uncertain bound in the control gain, the effects of system uncertainties can be eliminated, asymptotic error convergence can be obtained, and the amplitude of the control signals can be significantly reduced. The scheme can easily be implemented for practical applications. The results of a simulation performed on a two-link rigid robotic manipulator were presented to demonstrate the effectiveness of the proposed control scheme.

Chapter 4

A Decentralised Adaptive Sliding Mode Control for Rigid Robotic Manipulators

4.1 INTRODUCTION

Decentralised sliding mode control is a powerful method for the control of rigid robotic manipulators. The basic principle of the decentralised sliding mode control developed in Abbass and Ozguner (1985), Ozguner, Yurkovich and Abbass (1987), and Xu, et al. (1990), Morgan and Ozgunner (1985) is that the upper bound of dynamical interactions and the upper and the lower bounds of all unknown parameters in each subsystem are assumed to be known. A set of local sliding mode controllers are then designed to drive subsystems to move in their local sliding modes. In the local sliding modes, the desired system dynamics for the overall system, are completely insensitive to system uncertainties, dynamical interactions and bounded external disturbances.

However, in many practical situations, the following problems using the above decentralised sliding mode control schemes have been noted. First, the designs of real time local sliding mode controllers based on the upper and the lower bounds of

unknown parameters are very complicated and time-consuming if the controlled system has many unknown parameters. Second, the upper bound of the dynamical interactions in each subsystem is unknown because the maximum value of the norm of the dynamical interactions is variable in different cases. Although a conservative bound of the norm of the dynamical interactions can be used, the result is a set of high gain local sliding mode controllers which are not suitable for practical applications. Therefore, the issue of removing the requirement of the prior knowledge of uncertain bounds of both dynamical interactions and unknown parameters in the local sliding mode controller design has been a challenging topic in the area of decentralised sliding mode control for rigid robotic manipulators.

In this chapter, a decentralised adaptive sliding mode control for rigid robotic manipulators is proposed. A rigid robotic manipulator is treated as a partially known system and the known dynamics of each subsystem are separated out to perform linearisation. The nominal model of each subsystem is then stabilised by a local feedback controller, and the effects of uncertain dynamics are then compensated by a local adaptive sliding mode compensator. A key feature of this scheme is that prior knowledge of the uncertain dynamics in each subsystem is not required. An adaptive mechanism is introduced to estimate the uncertain bound for each subsystem. The estimate is then used as a parameter of the local sliding mode compensator to guarantee that effects of the uncertain system dynamics are eliminated and asymptotic error convergence is obtained for the overall robotic control system. In addition, the estimate of the uncertain bound in each subsystem is updated in Lyapunov sense. After the local sliding variable in each subsystem reaches its sliding mode, the estimate is held constant to keep the local error dynamics in its local sliding mode.

Unlike the schemes in Abbass and Ozgunner (1985), Ozgunner, Yurkovich and Abbass (1987), and Xu, et al. (1990), Morgan and Ozgunner (1985), the local controller design is greatly simplified in this scheme due to the fact that only one on-line estimated uncertain bound is used in the local sliding mode compensator design, rather than the upper and the lower bounds of all unknown parameters.

This chapter is organised as follows: In section 4.2, the system model and control objectives are formulated. In section 4.3, a decentralised adaptive sliding mode control scheme is developed. The error convergence and robustness are discussed in detail. In section 4.4, a simulation example on a two-link robotic manipulator is given in support of the theoretical results.

4.2 PROBLEM FORMULATION

The dynamics of a rigid robotic manipulator are generally described by the following second-order nonlinear differential equation

$$M(q)\ddot{q} + h(q, \dot{q}) = u(t) \qquad (4.1)$$

where q(t) is the nx1 vector of joint angular positions, M(q) is the nxn symmetric positive-definite inertia matrix, $h(q,\dot{q})$ is the nx1 vector containing coriolis, centrifugal forces and gravity torques, and u(t) is the nx1 vector of applied joint torques (control inputs).

The robotic manipulator system described by expression (4.1) can be decoupled into the following n interconnected subsystems.

$$m_{ii}\ddot{q}_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} m_{ij}\ddot{q}_j + h_i = u_i \qquad i = 1, ..., n \qquad (4.2)$$

where m_{ii} is the ith diagonal element of matrix M(q) and is always positive due to the positive-definiteness of M(q).

h_i and u_i are the ith elements of $h(q,\dot{q})$ and u(t), respectively.

$\sum_{j=1}^{n} m_{ij}\ddot{q}_j$ (where $j \neq i$) represents the dynamical interconnections.

Considering the imprecise knowledge of system parameters, m_{ii} and h_i can be written as:

$$m_{ii} = m_{ii0} + \Delta m_{ii} \qquad (4.3)$$

$$h_i = h_{i0} + \Delta h_i \qquad (4.4)$$

where m_{ii0} (> 0) and h_{i0} are the known parts, Dm_{ii} and Dh_i are the unknown parts.

Using expressions (4.3) and (4.4) in expression (4.2), we have

$$m_{ii0}\ddot{q}_i + h_{i0} + d_i = u_i \qquad i = 1, ..., n \qquad (4.5)$$

where

$$d_i = \sum_{\substack{j=1 \\ j\neq i}}^{n} m_{ij}\ddot{q}_j + \Delta m_{ii}\ddot{q}_i + Dh_i \qquad (4.6)$$

represents uncertain dynamics of the ith subsystem.

Remark 4.1: It has been noted from expression (4.6) that the uncertain term d_i is not only related to the ith local subsystem, but also related to other interconnected subsystems. In this scheme, the uncertain term d_i is assumed to be upper bounded.

A.4.1 $$|d_i| < \bar{d}_i \tag{4.7}$$

where $\bar{d}_i$ is an unknown positive number.

It will be seen in the next section that the prior knowledge of the above uncertain bound is not required in the local controller design, and a new adaptive mechanism is introduced to estimate the above uncertain bound in the Lyapunov sense in order to guarantee good tracking performance.

4.3 CONTROLLER DESIGN

Based on the known parts of subsystem in expression (4.5), the ith nominal subsystem model is defined as

$$m_{ii0}\ddot{q}_i + h_{i0} = u_{i1} \tag{4.8}$$

and the output tracking error is defined as

$$\varepsilon_i = q_i - q_{ir} \tag{4.9}$$

where q_{ir} is the ith desired reference signal for q_i to follow.

The local controller design for each subsystem is divided into two parts in this chapter. First, a local nominal feedback controller is designed to make the output tracking error of the nominal subsystem asymptotically converge to zero. Then, a local sliding mode compensator is introduced to deal with the effects of system

uncertainties in each subsystem so that the output tracking error of each subsystem asymptotically converges to zero.

Now, defining an error vector $e_i = [\varepsilon_i, \quad \dot{\varepsilon}_i]^T$ and using expression (4.8), we get the linearized error dynamic equation for the ith nominal subsystem as follows.

$$\dot{e}_i = A_i e_i + B_i v_i \tag{4.10}$$

where

$$A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{4.11}$$

$$B_i = [\, 0, \quad 1 \,]^T \tag{4.12}$$

$$v_i = - m_{ii0}^{-1} h_{i0} + m_{ii0}^{-1} u_{i1} - \ddot{q}_{ri} \tag{4.13}$$

Lemma 4.1: Error vector $e_i(t)$ in error dynamics equation (4.10) for the ith nominal subsystem asymptotically converges to zero if the following local nominal feedback control law is used

$$u_{i1} = m_{ii0} k_i e_i + h_{i0} + m_{ii0} \ddot{q}_{ri} \tag{4.14}$$

where k_i is designed such that

$$A_{i1} = A_i + B_i k_i \tag{4.15}$$

is a asymptotically stable matrix.

proof: See Leitman (1981), Bremer (1985) and Singh (1986).

Next, we consider the local sliding mode compensator design.

Let $$u_i = u_{i1} + u_{i0} \tag{4.16}$$

where u_{i1} is the local nominal feedback controller given in expression (4.14), and u_{i0} is the local sliding mode compensator.

Using expressions (4.14) and (4.16) in expression (4.5), the error dynamic equation for the ith local subsystem can be written in the following form

$$\dot{e}_i = A_{i1}e_i + B_i m_{ii0}^{-1} u_{i0} - B_i m_{ii0}^{-1} d_i \tag{4.17}$$

For the design of the local sliding mode compensator, a set of local switching plane variables are defined as

$$S_i = C_i e_i \qquad i = 1 \ldots n \tag{4.18}$$

where $C_i = [\, c_{i1},\ c_{i2}\,]$, $c_{i1} > 0$ and $c_{i2} > 0$.

In addition, as mentioned in section 4.2, the upper bound of the uncertain term d_i in each subsystem is unknown. To avoid the requirement of the prior knowledge of the upper bound of the uncertain dynamics in the local controller design, the following adaptive mechanism is used to estimate the uncertain bound.

$$\hat{d}_i = a_i c_{i2} m_{ii0}^{-1} | S_i | \tag{4.19}$$

where $\hat{d}_i$ is the estimate of $\bar{d}_i$, which has an arbitrary positive initial condition and a_i is a positive constant number. Then we have the following robustness and convergence results.

Theorem : Consider the error dynamics in expression (4.17) for the ith subsystem in expression (4.5). If the ith local sliding mode compensator u_{i0} is designed such that

$$u_{i0} = - c_{i2}^{-1} m_{ii0} C_i A_{i1} e_i - \hat{d}_i \text{sign}(S_i) \tag{4.20}$$

Then the output tracking error asymptotically converge to zero in view of the overall system.

Proof: Defining a Lyapunov function

$$V_i = \frac{1}{2} S_i^2 + \frac{1}{2} a_i^{-1} \tilde{d}_i \tilde{d}_i \tag{4.21}$$

where $\tilde{d}_i = \bar{d}_i - \hat{d}_i$ (4.22)

$$\dot{\tilde{d}}_i = - \dot{\hat{d}}_i \tag{4.23}$$

and differentiating V with respect to time, we have

$$\dot{V} = S_i \dot{S}_i - a_i^{-1} \tilde{d}_i \dot{\hat{d}}_i$$

$$= S_i C_i (A_{i1} e_i + B_i m_{ii0}^{-1} - B_i m_{ii0}^{-1} d_i) - \alpha_i^{-1} \tilde{d}_i \dot{\hat{d}}_i$$

$$= S_i C_i A_{i1} e_i + S_i c_{i2} m_{ii0}^{-1} u_{i0} - S_i c_{i2} m_{ii0}^{-1} d_i - a_i^{-1} \tilde{d}_i \dot{\hat{d}}_i$$

$$= - | S_i | c_{i2} m_{ii0}^{-1} \hat{d}_i - S_i c_{i2} m_{ii0}^{-1} d_i - a_i^{-1} \tilde{d}_i \dot{\hat{d}}_i + a_i^{-1} \hat{d}_i \dot{\hat{d}}_i$$

$$= - | S_i | c_{i2} m_{ii0}^{-1} \hat{d}_i - S_i c_{i2} m_{ii0}^{-1} \tilde{d}_i d_i - \tilde{d}_i c_{i2} m_{ii0}^{-1} | S_i | + \hat{d}_i c_{i2} m_{ii0}^{-1} | S_i |$$

$$= - S_i \, c_{i2} \, m_{ii0}^{-1} \, d_i - \bar{d}_i \, c_{i2} \, m_{ii0}^{-1} \, | S_i |$$

$$\leq - h_i \, | S_i | \qquad (4.24)$$

$$\text{where } h_i = c_{i2} \, m_{ii0}^{-1} \, (\bar{d}_i - | d_i |) > 0 \qquad (4.25)$$

Expression (4.24) is the sufficient condition for the local switching plane variable S_i to reach the local sliding mode

$$S_i = C_i e_i = 0 \qquad (4.26)$$

On the local sliding mode, we have

$$\dot{\varepsilon}_i = - c_{i2}^{-1} c_{i1} \varepsilon_i \qquad (4.27)$$

Expression (4.27) means that the output tracking errors e_i ($i = 1 \ldots n$) asymptotically converge zero.

It is well known that sliding variable vector S in view of the overall system is given

$$\text{by} \qquad S = [S_1 \quad \ldots \quad S_n]^T \qquad (4.28)$$

The sufficient condition for the switching plane variable vector S to be globally stable is given by

$$S_i \dot{S}_i < 0 \qquad i = 1 \ldots n \qquad (4.29)$$

Therefore, asymptotic error convergence can be guaranteed in view of the overall system.

Remark 4.2: Unlike the decentralised sliding mode control schemes in Abbass and Ozguner (1985), Ozguner, Yurkovich and Abbass (1987), and Xu, et al. (1990), Morgan and Ozguner (1985), prior knowledge of the uncertain dynamics of each subsystem is not required in the sliding compensator design used in this scheme. An adaptive mechanism is introduced to estimate the upper bound of the local uncertain dynamics. The estimate is then used as a control parameter of the local sliding mode compensator to guarantee that effects of the large system uncertainties are eliminated and asymptotic error convergence is obtained for the overall rigid robotic control systems

Remark 4.3: The adaptive property of the local sliding mode compensator in expression (4.20) can be explained as follows:

When the output tracking error e_i is large due to the effects of arbitrary bounded uncertainties, the estimate of $\bar{d}_i$ can be automatically increased according to the update law in expression (4.19) or the following expression.

$$\hat{d}_i = \hat{d}_i(0) + a_i^{-1} \int_0^t m_{ii0}^{-1} c_{i2} \mid S_i \mid dt \qquad (4.30)$$

The control gain can then be increased. Therefore, the effects of uncertain dynamics can be eliminated, the local sliding variable vector S_i can be driven to zero, and the output tracking error e_i can then asymptotically converge to zero on the local sliding mode.

Remark 4.4: It can be seen from expression (4.30) that the estimate of the uncertain boundand in each subsystem is not directly related to the uncertain dynamics and it is updated in the Lyapunov sense. After the local sliding variable reaches its sliding mode, the estimate of the uncertain boundand in each subsystem will be a constant to keep the local error dynamics in the local sliding mode.

Remark 4.5: The robustness property of the proposed control scheme is obvious. First, although each subsystem in expression (4.5) has uncertain dynamics, the proposed decentralised controller can make the sliding variable vector in expression (4.28) converge to zero in a finite time (see expression (4.24)). Secondly, in the sliding modes, the overall system is completely insensitive to nonlinearities, dynamical couplings and parameter uncertainties. The behaviour of the error dynamics is determined only by the sliding mode parameters in expression (4.18).

Remark 4.6: Compared with the decentralised sliding mode control schemes in Abbass and Ozguner (1985), Ozguner, Yurkovich and Abbass (1987), and Xu, et al. (1990), Morgan and Ozguner (1985), the design of the local controller in this scheme is greatly simplified in the sense that only an on-line estimated, uncertain bound is used as a control parameter in the local sliding mode compensator rather than the upper and the lower bounds of all known system parameters.

Remark 4.7: If the robot system in expression (4.1) has bounded input disturbance vector, the ith element of the input disturbance vector can be combined into the uncertain term in expression (4.6). Then the sliding mode compensator has the same form as in expression (4.20). The difference is that the control parameter $\hat{d}_i$ in

expression (4.30) will be larger in order to eliminate the effects of the input disturbances.

Remark 4.8: The local sliding mode compensator in expression (4.20) gives a discontinuous chattering signal across the local sliding mode $S_i = 0$, which may excite undesirable high-frequency dynamics. To eliminate the chattering, the following local boundary layer compensator can be used in place of the sliding mode compensator in expression (4.20).

$$u_{0i} = \begin{cases} - c_{i2}^{-1} m_{ii0} C_i A_{i1} e_i - \hat{d}_i \, \text{sign}(S_i) & |S_i| \geq \delta_i \\ \\ - c_{i2}^{-1} m_{ii0} C_i A_{i1} e_i - \hat{d}_i \, (S_i / \delta_i) & |S_i| < \delta_i \end{cases} \tag{4.31}$$

$$\text{where, } \dot{\hat{d}}_i = \begin{cases} a_i \, c_{i2} \, m_{ii0}^{-1} |S_i| & |S_i| \geq d_i \\ \\ 0 & |S_i| < d_i \end{cases} \tag{4.32}$$

and d_i is a positive number.

The above boundary layer control law offers a continuous approximation to the discontinuous control law inside the boundary layer and guarantees attractiveness to the boundary layer and ultimate boundedness of the output tracking error to within a neighbourhood of the origin. But the drawback is that nonzero error exists (Corless and Leitmann (1981) and Slotine and Sastry (1983)).

4.4 A SIMULATION EXAMPLE

A simulation example with a two-link robotic manipulator is performed for the purpose of evaluating the performance of the proposed control scheme. The full dynamic equation of the simulated manipulator model is given by

$$\begin{bmatrix} \alpha_{11}(q_2) & \alpha_{12}(q_2) \\ \alpha_{12}(q_2) & \alpha_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} \beta_{12}(q_2)\,\dot{q}_1^2 + 2\,\beta_{12}(q_2)\,\dot{q}_1\,\dot{q}_2 \\ -\,\beta_{12}(q_2)\,\dot{q}_2^2 \end{bmatrix}$$

$$+ \begin{bmatrix} \gamma_1(q_1, q_2)g \\ \gamma_2(q_1, q_2)g \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where,

$$\alpha_{11}(q_2) = (m_1 + m_2)\,r_1^2 + m_2\,r_2^2 + 2\,m_2\,r_1\,r_2\cos(q_2) + J_1$$

$$\alpha_{12}(q_2) = m_2\,r_2^2 + m_2\,r_1\,r_2\cos(q_2)$$

$$\alpha_{22} = m_2\,r_2^2 + J_2$$

$$\beta_{12}(q_2) = m_2\,r_1\,r_2\sin(q_2)$$

$$\gamma_1(q_1,\ q_2) = -\left((m_1 + m_2)\,r_1\cos(q_2) + m_2\,r_2\cos(q_1 + q_2)\right)$$

$$\gamma_2(q_1,\ q_2) = -\,m_2\,r_2\cos(q_1 + q_2)$$

The parameter values are

$$r_1 = 1\ \text{m},\quad r_2 = 0.8\ \text{m}$$

$$J_1 = 5\ \text{kg.m},\ J_2 = 5\ \text{kg.m}$$

$$m_1 = 0.5\,\text{kg},\ m_2 = 1.5\,\text{kg}$$

For the use of the decentralised control scheme proposed in section 4.3, each link is considered as a subsystem. Desired reference signals for two subsystems to follow are given by

$$q_{r1} = 1.25 - \frac{7}{5}e^{-t} + \frac{7}{20}e^{-4t}$$

$$q_{r2} = 1.25 + e^{-t} - \frac{1}{4}e^{-4t}$$

In order to constrain the error dynamics of each subsystem in its sliding mode from the start to the end, we consider a situation characterised by the same initial values on each subsystem and its local reference signal, Man (1993) and Young (1988). In this example, the initial angular positions and velocities are selected as

$$[\, q_1(0),\ \dot{q}_1(0)\,]^T = [\, q_{r1}(0),\ \dot{q}_{r1}(0)\,]^T = [\, 0.2,\ 0\,]^T$$

$$[\, q_2(0),\ \dot{q}_2(0)\,]^T = [\, q_{r2}(0),\ \dot{q}_{r2}(0)\,]^T = [\, 2,\ 0\,]^T$$

The estimates of m_1 and m_2 are assumed to be

$$\hat{m}_1 = 0.4\text{kg},\ \hat{m}_2 = 1.2\text{kg}$$

and the other system parameters are assumed to be known. The nominal model of each subsystem is then built from the known subsystem dynamics.

The nominal feedback matrices k_1 and k_2 in expression (4.14) or (4.15) are designed as

$$k_1 = [\ -4,\ \ -5\ ]$$

$$k_2 = [\ -4,\ \ -5\ ]$$

The local sliding variables S_1 and S_2 are defined as

$$S_1 = 20\varepsilon_1 + 2\dot{\varepsilon}_1$$

$$S_2 = 20\varepsilon_2 + \dot{\varepsilon}_2$$

The initial values of the estimates of the uncertain bounds for two subsystems are selected as $\hat{k}_1(0) = 15$ and $\hat{k}_2(0) = 20$

The computer simulation with a sampling interval $\Delta T = 0.001s$ is performed. Fig.4.1(a) - Fig.4.2 (b)show the output trackings and the control inputs by the use of the local sliding mode compensator in expression (4.20). It can be seen that, although each subsystem has uncertain dynamics, good tracking performance is achieved, where the dashed lines indicate the reference trajectories and the actual trajectory are indicated by the solid lines.

To eliminate the chatterings observed in Fig.4.2 (a and b), the local boundary layer compensator in expression (4.31) is implemented. The simulation results in Fig.4.3(a) - Fig.4.4(b) show that not only the problem of chattering is eliminated, but also the amplitude of the control inputs is greatly reduced by using the boundary layer controller.

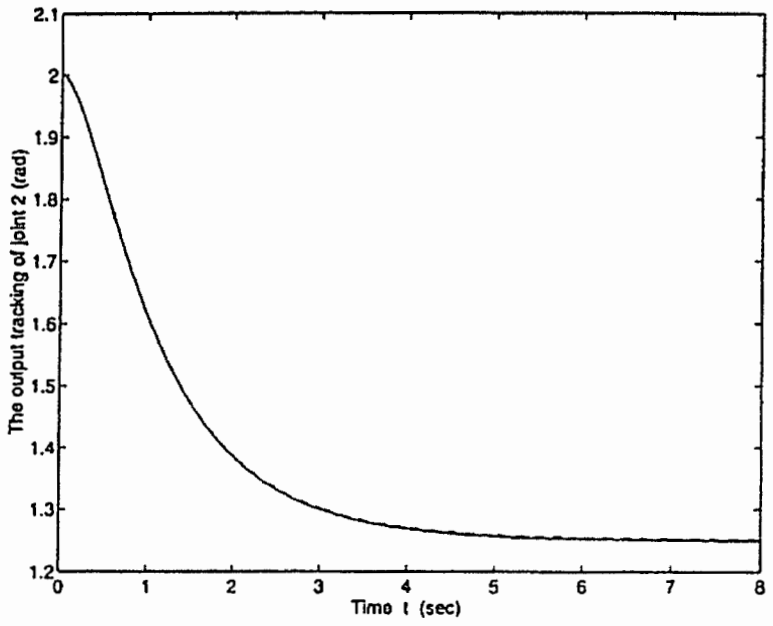

Fig.4.1 (a) The output tracking of joint 1 using a sliding mode compensator

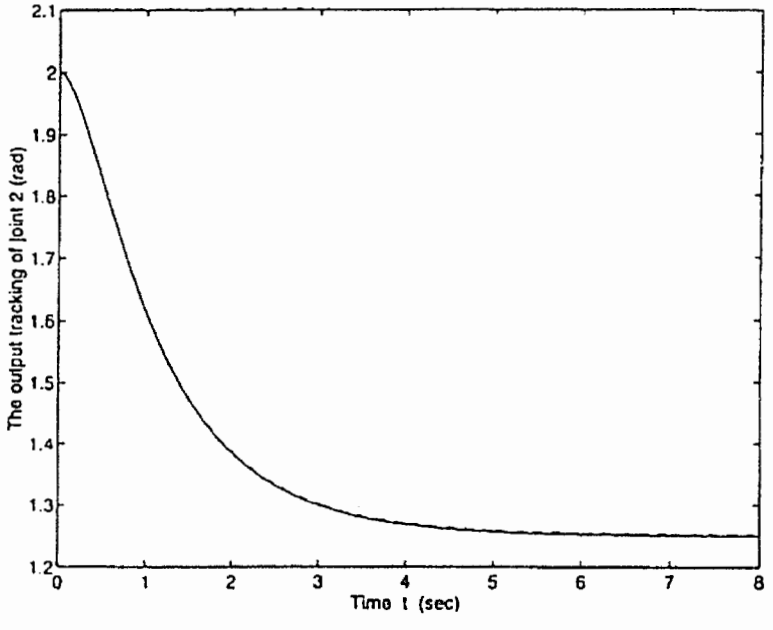

Fig. 4.1. (b) The output tracking of joint 2 using a sliding mode compensator

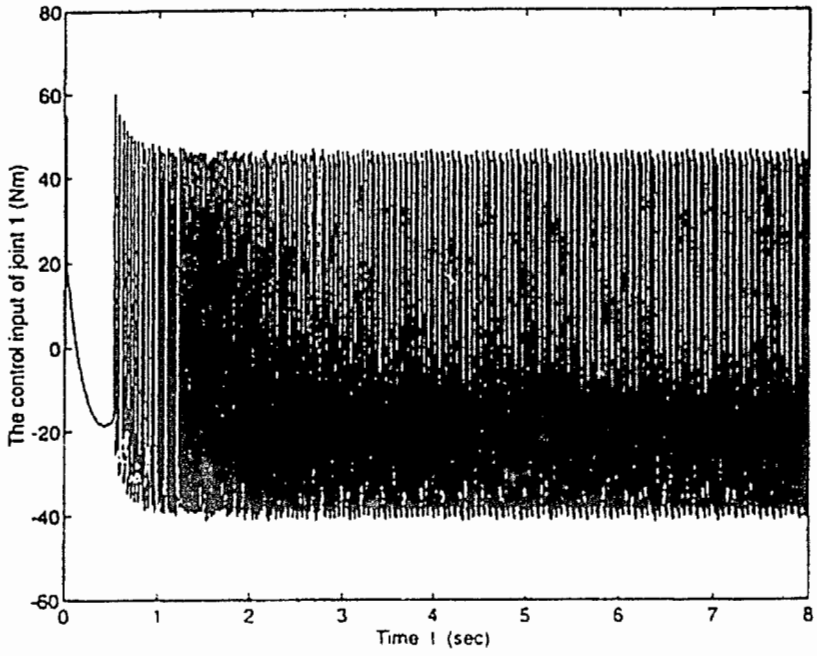

Fig.4.2(a) The control inputs of joint 1 using a local sliding mode compensator

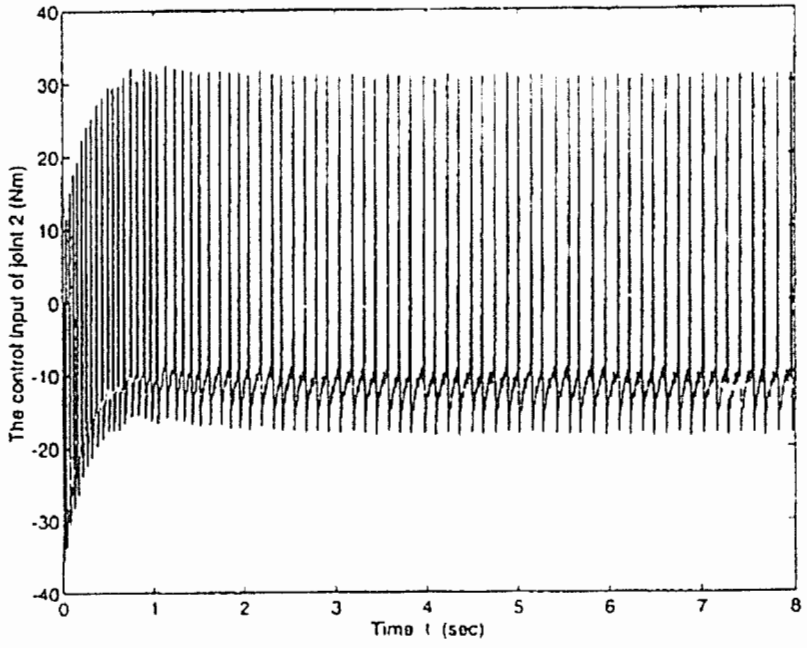

Fig. 4.2 (b) The control inputs of joint 2 using a local sliding mode compensator

- - - desired output
---- actual output

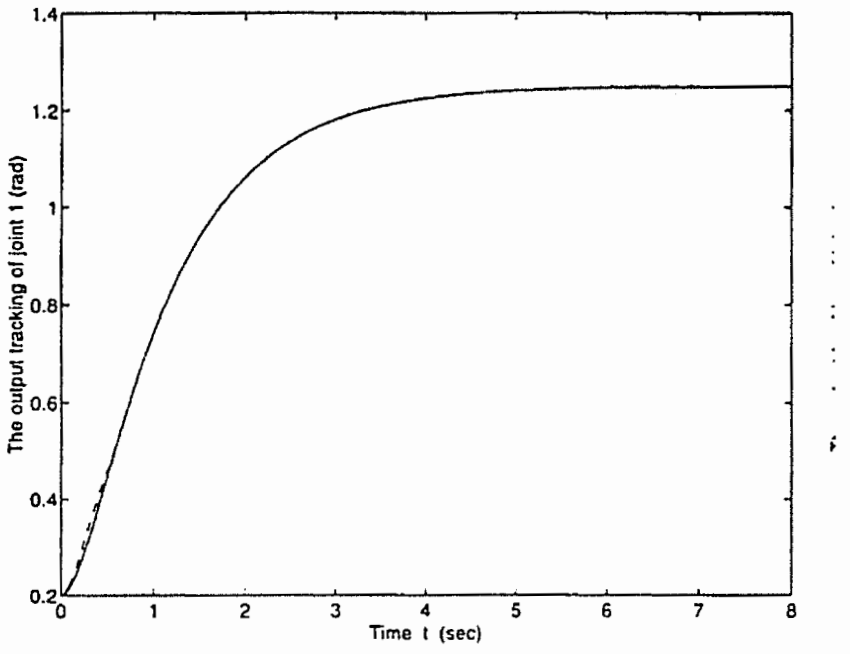

Fig. 4.3 (a) The output tracking of joint 1 using a local boundary layer compensator

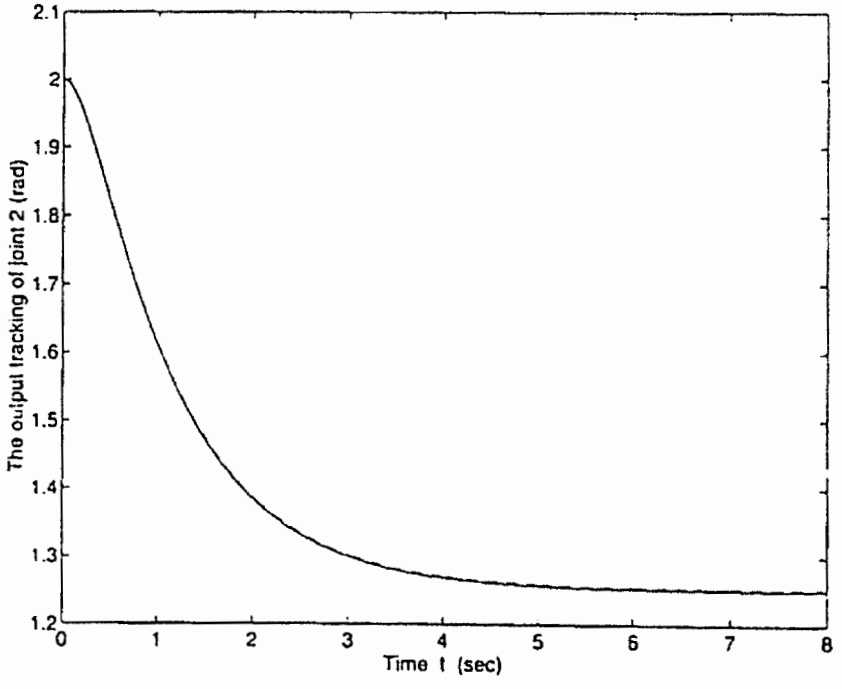

Fig. 4.3(b) The output tracking of joint 2 using a local boundary layer compensator

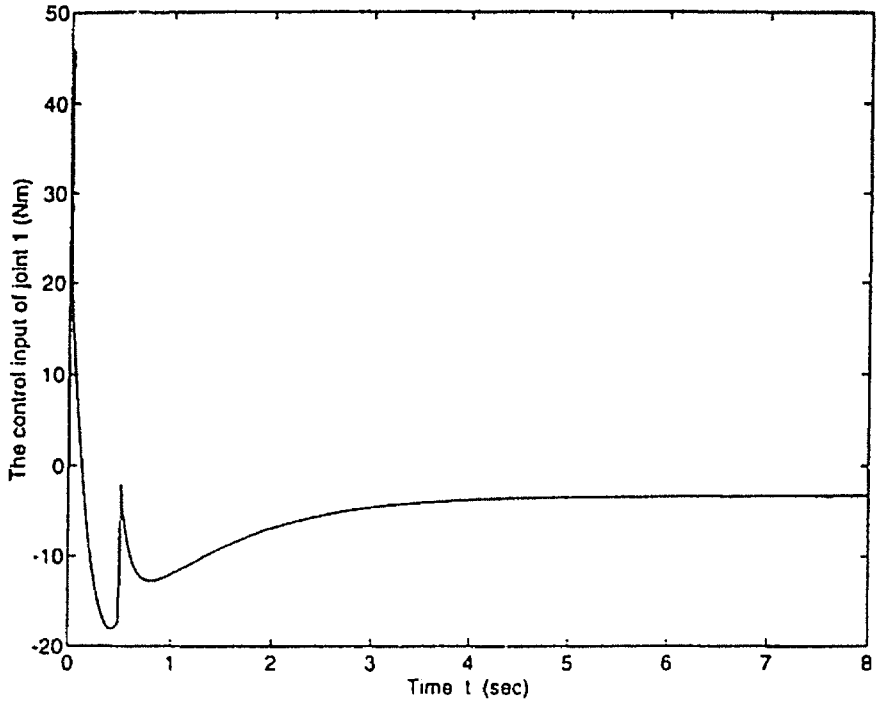

Fig. 4.4 (a) The control inputs of joint 1 using a local boundary layer controller

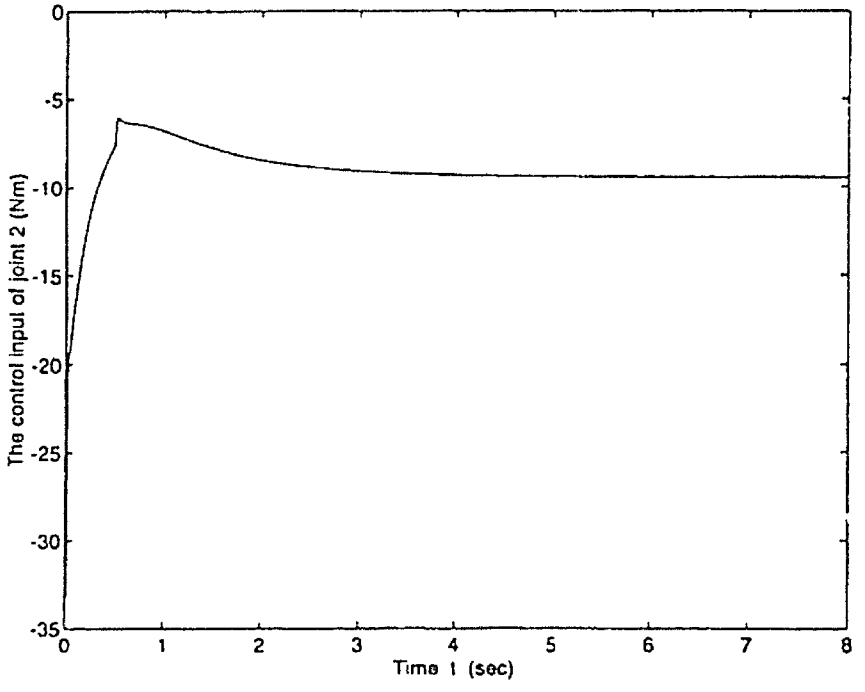

Fig. 4.4 (b) The control inputs of joint 2 using a local boundary layer controller

4.5 CONCLUSION

A decentralised adaptive sliding mode control scheme for rigid robotic manipulator is investigated in this chapter. The main contribution of this scheme comes from the fact that prior knowledge of the uncertain dynamics in each subsystem is not required. An adaptive mechanism is introduced to estimate the uncertain bound for each subsystem. The estimate is then used as a local controller parameter to guarantee that effects of the uncertain dynamics can be eliminated and asymptotic error convergence can be obtained in view of the overall robotic control system. A simulation example is used in support of the proposed control scheme.

Chapter 5

A Model Following Control Using Terminal Sliding Mode Technique for Rigid Robotic Manipulators

5.1 INTRODUCTION:

In robot control engineering, problems such as unmodelled dynamics, parameter uncertainties and external disturbances affect the control system design and the quality of control. In order to improve the performance of robot control systems, many robust adaptive control schemes have been developed for rigid robotic manipulators in Crag et al. (1986), Spong and Ortega (1990), Amestegui et al. (1987), and Middleton and Goodwin (1988). These adaptive control schemes are designed under the condition that all signals remain bounded (Ortega and Spong (1989)). Considering the fact that asymptotic stability has not been proven to be uniform, small changes in dynamics may result in loss of stability. These adaptive control schemes are therefore inadequate for satisfactory performance of robot control systems with uncertainties and disturbances.

In recent years, the sliding mode technique has provided an efficient method for the control of robotic manipulators with large uncertainties and bounded input disturbances. The work in Morgan and Ozguner (1985), Young (1978, 1988), and Yeung and Chen (1988) has shown that the robustness and convergence can be established for robotic manipulators with large system uncertainties by using the

sliding control theory, based on the upper and the lower bounds of unknown parameters. Leung et al. (1991) and Man and Palaniswami (1993) show that asymptotic error convergence of the sliding mode control system for robotic manipulators can still be designed based on only a few uncertain system matrix bounds rather than on the upper and the lower bounds of all unknown parameters.

In this chapter, we present a model following control scheme using the terminal sliding mode technique for rigid robotic manipulators based on the idea of terminal attractor in Zak (1988, 1989). A multivariable terminal sliding mode is first defined for the model following control system of rigid robotic manipulators, and the relationship between the terminal sliding variable vector and the error dynamics of the closed loop system is established for the stability analysis of the error dynamics. Then a robust terminal sliding controller is designed based on a few structural properties of rigid robotic manipulators. Unlike the linear sliding mode control schemes in Utkin (1977), Young (1977, 1988), and Man and Palaniswami (1993, 1994), the terminal sliding variable vector has a nonlinear term of the velocity error. By suitably designing the controller, the terminal sliding variable vector can converge to zero in a finite time, and the output tracking error can then converge to zero in the terminal sliding mode in a finite time.

Similar to the conventional linear sliding mode control schemes, the proposed terminal sliding mode control scheme is robust to large uncertain dynamics and bounded disturbances. Further, the controller design is greatly simplified such that only a few uncertain bounds of the controlled robot system are required as the controller parameters.

In section 5.2, the dynamics of rigid robotic manipulators and definition of the terminal sliding mode are formulated. Section 5.3 describes the proposed model following control scheme using terminal sliding mode technique for rigid robotic manipulators with uncertain dynamics, and the controller design and convergence and robustness analysis are discussed in detail. Section 5.4 presents a modified scheme to handle robotic manipulators with both uncertain dynamics and bounded unknown disturbances in the control input. Section 5.5 presents a simulation example based on a two-link robotic manipulator in support of the proposed control schemes, and section 5.6 gives conclusions.

5.2 PROBLEM FORMULATION:

A rigid robotic manipulator is defined as an open kinematic chain of rigid links, where each degree-of-freedom of the manipulator is powered by an independent torque. Using the Lagrangian formulation, the dynamic equation of an n-degree of freedom rigid robotic manipulator can be described as follows:

$$J(q)\ddot{q} + F(q, \dot{q}) + G(q) = u(t) \qquad (5.1)$$

where q ($\in R^n$) is the vector of n joint angular positions as the system output, $J(q)$ ($\in R^{nxn}$) is a symmetric positive-definite inertia matrix, $F(q,\dot{q})$ ($\in R^n$) is the vector of coriolis and centrifugal forces, $G(q)$ ($\in R^n$) is the vector of gravitational torques and $u(t)$ ($\in R^n$) is the vector of input torques (control inputs).

Define $X = [q^T \quad \dot{q}^T]^T$, expression (5.1) can be written in terms of state variables as:

$$\dot{X} = A X + Bu \tag{5.2-a}$$

$$A = \begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ B_1 \end{bmatrix} \tag{5.2-b}$$

where A ($\in R^{2nx2n}$) is a system matrix, A_1 and A_2 are nxn matrices, B ($\in R^{2nxn}$) is an input matrix and $B_1 = J(q)^{-1}$. Considering the classical dynamics of nonlinear robotic systems, it is easily seen that the matrix A is a function of q and its derivative, and the symmetric positive-definite matrix B_1 is a function of q whose norm is uniformly bounded independent of q. The parameters in matrices A_1, A_2 and B_1 are assumed to be unknown.

For model following control design, the following linear reference model is used

$$\dot{X}_m = A_m X_m + B_m r(t) \tag{5.3-a}$$

$$A_m = \begin{bmatrix} 0 & I \\ A_{m1} & A_{m2} \end{bmatrix} \quad B_m = \begin{bmatrix} 0 \\ B_{m1} \end{bmatrix} \tag{5.3-b}$$

where $A_{m1} = -\text{diag}(a_{i1})$, $A_{m2} = -\text{diag}(a_{i2})$ and $B_{m1} = -\text{diag}(b_{i1})$ $(1 \le i \le n)$ are known constant matrices determined from an engineering point of view, and r(t) is a nx1 reference input vector assumed to be nonzero.

To ensure the equality $X = X_m$ for an arbitrary reference input vector r(t), the following matching conditions are assumed to be satisfied (Miyasato and Oshima (1989))

A.5.1 $$(I - B B^{+}) B_m = 0 \tag{5.4-a}$$

A.5.2 $$(I - B B^{+})(A_m - A) = 0 \tag{5.4-b}$$

where,

$$B^{+} = (B^{T} B)^{-1} B^{T} \tag{5.4-c}$$

and the control input for the perfect model following is then given by

$$u = - K_x X + K_r r \tag{5.5}$$

where

$$K_x = - B^{+}(A_m - A) \tag{5.6-a}$$

$$K_r = B^{+} B_m \tag{5.6-b}$$

$$B^{+} = \begin{bmatrix} 0 & B_1^{-1} \end{bmatrix} \tag{5.6-c}$$

In order to design the model following control system using terminal sliding mode technique for the rigid robotic manipulator in expression (5.1), we set the control input in the following form:

$$u = \Theta_1 X + \Theta_2 r + \Theta_4 e + \Theta_5 \tag{5.7}$$

where $\Theta_1(\in R^{nx2n})$, $\Theta_2(\in R^{nxn})$, $\Theta_4(\in R^{nx2n})$ and $\Theta_5(\in R^n)$ are discontinuous controller gain matrices and e(t) is the output tracking error defined by

$$e(t) = X_m - X \tag{5.8}$$

Now, differentiating equation (5.8) with respect to time, we have

$$\dot{e} = \dot{X}_m - \dot{X} \tag{5.9}$$

The dynamics of the output tracking error can then be obtained by using expressions (5.2) - (5.9) as follows:

$$\dot{e} = A_m X_m + B_m r - A X - Bu$$

$$= A_m e - \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 \Theta_4 e + \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 \left[B^+ (A_m - A) - \Theta_1 \right] X$$

$$+ \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 (B^+ B_m - \Theta_2) r - \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 \Theta_5 \tag{5.10}$$

A set of terminal sliding variables in the error space passing through the origin can be defined as:

$$S = C \tilde{e} \tag{5.11}$$

where $C = [C_1 \quad C_2]$

$$= \begin{bmatrix} c_{11} & I & & \\ \cdot & & \cdot & \\ & \cdot & & \cdot \\ & & c_{nn} & I \end{bmatrix} \tag{5.12}$$

$$\tilde{e} = \left[\varepsilon_1^p \ \ldots \ \varepsilon_n^p \ \ \dot{\varepsilon}_1 \ \ldots \ \dot{\varepsilon}_n \right]^T \tag{5.13}$$

Remark 5.1: In expression (5.13), $p = p_1/p_2$, where positive integers p_1 and p_2 are selected such that:

$$p_1 = (2l + 1), \ l = 0, \ 1, 2, \ldots \tag{5.14-a}$$

$$p_2 = (2m + 1), \ m = 1, 2, \ldots \tag{5.14-b}$$

$$p_2 > p_1 \tag{5.15}$$

It is shown later that the selections of p_1 and p_2 in expressions (5.14) and (5.15) can guarantee $0 < p < 1$ and the tracking error ε_i can then converge to zero in the terminal sliding mode in a finite time, for all bounded initial conditions.

Remark 5.2: Vector $\tilde{e}$ in expression (5.11) can also be written into the following form:

$$\tilde{e} = e + \Delta \tilde{e} \tag{5.16}$$

where

$$\Delta \tilde{e} = \left[\varepsilon_1^p - \varepsilon_1, \ \ldots, \ \varepsilon_n^p - \varepsilon_n, \ \ 0, \ \ldots, \ 0 \right]^T \tag{5.17}$$

Using expressions (5.13), (5.16) and (5.17), the multivariable terminal sliding variable vector S in expression (5.11) can be written into the following form:

$$S = C\,\tilde{e}$$

$$= C(e + \Delta\,\tilde{e}\,)$$

$$= Ce + C_1(\,\tilde{\varepsilon}\, - \varepsilon) \qquad (5.18)$$

where $\tilde{\varepsilon} = \left[\, \varepsilon_1^p \;\ldots\; \varepsilon_n^p \,\right]^T \qquad (5.19)$

It will be seen later that it is convenient to use expression (5.18) of the terminal sliding variable vector S in controller design and convergence analysis.

Remark 5.3: The ith element of S in expression (5.11) can be written into the following form:

$$s_i = c_{ii}\;\varepsilon_i^p + \dot{\varepsilon}_i \qquad c_{ii} > 0 \qquad (5.20)$$

Similar to the conventional sliding mode control technique, if the controller is designed such that s_i ($i = 1, \ldots n$) converge to zero, then we say that the terminal sliding variables s_i ($i = 1, \ldots n$) reach the terminal sliding mode

$$c_{ii}\,\varepsilon_i^p + \dot{\varepsilon}_i = 0 \quad (i = 1, \ldots n) \qquad (5.21)$$

It is shown in Zak (1988, 1989) that $\varepsilon_i = 0$ is the terminal attractor of the system (5.21). Let the initial value of ε_i at time $t_i(0) = 0$ be $\varepsilon_i(0)$ ($\neq 0$) and parameter p be

chosen as shown in remark 1, then the relaxation time t_i for a solution of the system (5.21) is given as follows:

$$t_i = c_{ii}^{-1} \int_{\varepsilon_i(0)}^{\varepsilon_i \to 0} \frac{d\varepsilon_i}{\varepsilon_i^{p}} = \frac{\varepsilon_i(0)^{1-p}}{c_{ii}(1-p)} \tag{5.22}$$

Expression (5.22) also means that, on the terminal sliding mode in expression (5.21), the output tracking error converges to zero in a finite time. The details on the terminal attractor and its applications can be found in Zak (1988, 1989).

Remark 5.4: For the sake of simplicity, the terminal sliding mode parameter matrix C_2 in expression (5.12) is chosen as a unity matrix. However, matrix C_2 can be chosen to be a different diagonal matrix for different convergence requirements of the error dynamics in the terminal sliding mode.

For further analysis, the following assumptions on system matrix bounds are used (Leung, et al. (1991), Miyasato and Oshima (1989), Man and Palaniswami (1993)).

A.5.3 $$\lambda_{min}(C_2 B_1 C_2^{T}) > k_1 \tag{5.23}$$

A.5.4 $$\| B_1 \| \leq k_2 \tag{5.24}$$

A.5.5 $$\| B^{+}(A_m - A) \| \leq k_3 \tag{5.25}$$

A.5.6 $$\| B^{+} B_m \| \leq k_4 \tag{5.26}$$

where k_1, k_2, k_3 and k_4 are positive numbers.

Remark 5.5: According to the mechanical characteristics of rigid robotic manipulators and the boundedness of the reference model, the above assumptions are valid.

5.3 TERMINAL SLIDING MODE CONTROLLER DESIGN:

For the design of the terminal sliding mode control system for robotic manipulator (5.1), we have the following theorem.

***Theorem 5.1*:** Consider the error dynamics in expression (5.10) where the matching conditions in expressions (5.4-a)-(5.4-b) and the assumptions of uncertain system matrix bounds in expressions (5.23) - (5.26) are satisfied. If the controller gain matrices in expression (5.7) are designed such that

$$\Theta_1 = \begin{cases} \dfrac{k_2 k_3 \|C_2\|}{k_1 \|S\| \|X\|} C_2^T S X^T & \|S\| \|X\| \neq 0 \\ 0_{2nxn} & \|S\| \|X\| = 0 \end{cases} \tag{5.27}$$

$$\Theta_2 = \begin{cases} \dfrac{k_2 k_4 \|C_2\|}{k_1 \|S\| \|r\|} C_2^T S r^T & \|S\| \|r\| \neq 0 \\ 0_{nxn} & \|S\| \|r\| = 0 \end{cases} \tag{5.28}$$

$$\Theta_4 = \begin{cases} \dfrac{\|C\| \|A_m\|}{k_1 \|S\| \|e\|} C_2^T S e^T & \|S\| \|e\| \neq 0 \\ 0_{nx2n} & \|S\| \|e\| = 0 \end{cases} \tag{5.29}$$

$$\Theta_5 = \begin{cases} \dfrac{\| C_1 \| \, \| \varepsilon_r - \dot{\varepsilon} \|}{k_1 \, \| S \|} S & \| S \| \neq 0 \\ 0_{nx1} & \| S \| = 0 \end{cases} \tag{5.30}$$

where parameter p in expression (5.13) satisfies expression (5.14) and (5.15) as well as expression (5.31)

$$p > 0.5 \tag{5.31}$$

and $$\varepsilon_r = \text{diag}(\, p \varepsilon_1^{p-1} , \quad \ldots, \quad p \varepsilon_n^{p-1} \,) \, \dot{\varepsilon} \tag{5.32}$$

then the output tracking error $\varepsilon(t)$ will converge to zero in a finite time.

Proof: Consider the following Lyapunov function

$$v = \frac{1}{2} S^T S \tag{5.33}$$

and differentiating v with respect to time, we have

$$\dot{v} = S^T C \, \dot{\tilde{e}}$$

$$= S^T [\, C \dot{e} + C \Delta \, \dot{\tilde{e}} \,]$$

$$= S^T (C A_m e - C_2 B_1 \Theta_4 e)$$

$$+ S^T [\, C_2 B_1 B^+ (A_m - A) X - C_2 B_1 \Theta_1 X \,]$$

$$+ S^T (C_2 B_1 B^+ B_m r - C_2 B_1 \Theta_2 r)$$

$$+ S^T [C_1 \varepsilon_r - C_1 \dot{\varepsilon} - B_1 \Theta_5] \qquad (5.34)$$

Using expressions (5.27) - (5.30), four terms in expression (5.34) satisfy the following inequalities

$$S^T(CA_m e - C_2 B_1 \Theta_4 e)$$

$$= S^T C A_m e - S^T C_2 B_1 \frac{\|C\| \|A_m\|}{k_1 \|S\| \|e\|} C_2^T S e^T e$$

$$< S^T C A_m e - \|S\| \|C\| \|A_m\| \|e\|$$

$$\leq 0 \qquad (5.35\text{-a})$$

$$S^T [C_2 B_1 B^+ (A_m - A) X - C_2 B_1 \Theta_1 X]$$

$$= S^T C_2 B_1 B^+ (A_m - A) X - S^T C_2 B_1 \frac{k_2 k_3 \|C_2\|}{k_1 \|S\| \|X\|} C_2^T S X^T X$$

$$< S^T C_2 B_1 B^+ (A_m - A) X - k_2 k_3 \|S\| \|C_2\| \|X\|$$

$$\leq 0 \qquad (5.35\text{-b})$$

$$S^T(C_2B_1B^+B_m r - C_2B_1\Theta_2 r)$$

$$= S^T C_2 B_1 B^+ B_m r - S^T C_2 B_1 \frac{k_2 k_4 \|C_2\|}{k_1 \|S\| \|r\|} C_2^T S r^T r$$

$$< S^T C_2 B_1 B^+ B_m r - k_2 k_4 \|S\| \|C_2\| \|r\|$$

$$< 0 \qquad (5.35\text{-c})$$

$$S^T \left[C_1 \varepsilon_r - C_1 \dot{\varepsilon} - B_1 \Theta_5 \right]$$

$$= S^T C_1 \left[\varepsilon_r - \dot{\varepsilon} \right] - \frac{S^T B_1 S}{k_1 \|S\|} \|C_1\| \|\varepsilon_r - \dot{\varepsilon}\|$$

$$\leq S^T C_1 \left[\varepsilon_r - \dot{\varepsilon} \right] - \|S\| \|C_1\| \|\varepsilon_r - \dot{\varepsilon}\|$$

$$\leq 0 \qquad (5.35\text{–d})$$

Then using expressions (5.35-a) - (5.35-d) in expression (5.34).we have

$$\dot{v} < S^T C A_m e - \|S\| \|C\| \|A_m\| \|e\|$$

$$+ S^T B_1 B^+ (A_m - A) X - k_2 k_3 \|S\| \|X\|$$

$$+ S^T C_2 B_1 B^+ B_m r - k_2 k_4 \|S\| \|C_2\| \|r\|$$

$$< - \|S\| \left(k_2 k_4 \|C_2\| \|r\| - \frac{S^T}{\|S\|} C_2 B_1 B^+ B_m r \right)$$

$$= - \kappa \|S\| \qquad (5.36)$$

where

$$\kappa = \left(k_2 k_4 \|C_2\| \|r\| - \frac{S^T}{\|S\|} C_2 B_1 B^+ B_m r \right) > 0 \quad \text{for } \|S\| \neq 0 \qquad (5.37)$$

Expression (5.36) is the sufficient condition for the terminal sliding variables S to reach the terminal sliding mode S = 0 in a finite time. On the terminal sliding mode, the output tracking error can then converge to zero in a finite time according to expressions (5.21) and (5.22).

Remark 5.6: Theorem 5.1 shows that, although the parameter uncertainties, nonlinearities and dynamical couplings exist in the robotic manipulator system in expression (5.1), the controller can still be designed by using a few uncertain system matrix bounds in A.5.3 - A.5.6 to guarantee that the terminal sliding variable vector S converges to zero in a finite time and then the output tracking error converges to zero in a finite time on the terminal sliding mode..

Remark 5.7: The proposed terminal sliding mode control system exhibits good robustness to large system uncertainties, nonlinearities and dynamical interactions due to the fact that only a few uncertain system matrix bounds are used in the controller design instead of the upper and the lower bounds of all unknown system parameters.

Remark 5.8: After the error dynamics reach the terminal sliding mode in expression (5.21), the signal vector ε_r in expression (5.32) can be written as follows:

$$\varepsilon_r = \mathrm{diag}\left(p\varepsilon_1^{p-1}, \ \ldots, \ p\varepsilon_n^{p-1} \right) \dot{\varepsilon}$$

$$= \begin{bmatrix} p\varepsilon_1^{p-1} & & \\ & \ddots & \\ & & p\varepsilon_n^{p-1} \end{bmatrix} \begin{bmatrix} -c_{11}\varepsilon_1^{p} \\ \vdots \\ -c_{nn}\varepsilon_n^{p} \end{bmatrix}$$

$$= \left[-c_{11} p\varepsilon_1^{2p-1} \ \ldots \ -c_{nn} p\varepsilon_n^{2p-1} \right]^T \qquad (5.38)$$

Expression (5.38) shows that, mathematically the positive number p in expression (5.13) satisfies expressions (5.14) and (5.15). But, in order to guarantee the terminal convergence of variable ε_i, the number p must satisfy expression (5.31) such that the signal vector ε_r in expression (5.38) must be bounded as the output tracking error ε_i converges to zero on the terminal sliding mode.

5.4 A MODIFIED CONTROL SCHEME:

In this section, we modify the controller in section 5.3 to handle the case of a robotic manipulator with both uncertain dynamics and bounded unknown disturbances in the control input. In this case, the system model in expression (5.1) can be expressed in the following state variable form

$$\dot{X} = A X + Bu + h \tag{5.39}$$

where A and B are defined as before, $h = \begin{bmatrix} 0 & h_1^T \end{bmatrix}^T$, h_1 ($\in R^n$) is a vector of disturbances and matrix h is assumed to satisfy the following inequality

A.5.7 $$\| B^+ h \| \leq k_5 \tag{5.40}$$

where k_5 is a positive number.

To ensure the equality $X = X_m$ for an arbitrary reference input r(t) in system (5.39), the following matching condition is assumed to be satisfied together with ***A.5.1*** and ***A.5.2***.

A.5.8 $$(I - BB^+) h = 0 \tag{5.41}$$

Control input for the perfect model following is then given by

$$u = - K_x X + K_r r + K_h h \tag{5.42}$$

where K_x and K_r are defined as before and $k_h = -B^+$.

In a similar way with the control law (5.7), the following control law is proposed to handle the case with bounded unknown input disturbances

$$u = \Theta_1 X + \Theta_2 r + \Theta_3 + \Theta_4 e(t) + \Theta_5 \tag{5.43}$$

where controller gain matrices Θ_1, Θ_2, Θ_4 and Θ_5 are obtained as in equations (5.27) - (5.30), and Θ_3 ($\in R^n$) will be designed to eliminate the effects of disturbances.

Similar to expression (5.10), the error dynamics in this case is written as

$$\dot{e} = \dot{X}_m - \dot{X}$$

$$= A_m X_m + B_m r - A X - Bu - h$$

$$= A_m e - \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 \Theta_4 e + \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 [B^+ (A_m - A) - \Theta_1] X$$

$$+ \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 (B^+ B_m - \Theta_2) r + \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 (-B^+ h - \Theta_3)$$

$$- \begin{bmatrix} 0 & I \end{bmatrix}^T B_1 \Theta_5 \qquad (5.44)$$

For the stability analysis of error dynamics (5.44) and the design of the controller gain matrix in control law (5.43), we have the following theorem.

Theorem 5.2: Consider the error dynamics (5.44) for nonlinear robotic system (5.1) with bounded unknown input disturbances. If the matching conditions in (5.4-a), (5.4-b) and (5.41), and the assumptions on the uncertain system matrix bounds in (5.23) - (5.26) and (5.40) are satisfied, and if the control law (5.43) with controller gain matrices (5.27) - (5.30) and (5.45) is used

$$\Theta_3 = \begin{cases} \dfrac{k_2 k_5 \|C_2\|}{k_1 \|S\|} C_2^T S & \|S\| \neq 0 \\ 0_{nx1} & \|S\| = 0 \end{cases} \tag{5.45}$$

then the output tracking error $\varepsilon(t)$ converges to zero in a finite time.

Proof: Using $2v = S^T S$, we have

$$\dot{v} = S^T \dot{S} = S^T C \dot{e}$$

$$= S^T (C A_m e - C_2 B_1 \Theta_4 e)$$

$$+ S^T [C_2 B_1 B^+ (A_m - A) X - C_2 B_1 \Theta_1 X]$$

$$+ S^T (C_2 B_1 B^+ B_m r - C_2 B_1 \Theta_2 r)$$

$$- S^T (C_2 B_1 B^+ h + C_2 B_1 \Theta_3)$$

$$+ S^T [C_1 \varepsilon_r - C_1 \dot{\varepsilon} - B_1 \Theta_5] \tag{5.46}$$

Noting that

$$= - S^T (C_2 B_1 B^+ h + C_2 B_1 \Theta_3)$$

$$= - S^T C_2 B_1 B^+ h - S^T C_2 B_1 \frac{k_2 k_5 \| C_2 \|}{k_1 \| S \|} C_2^T S$$

$$\leq - S^T C_2 B_1 B^+ h - k_2 k_5 \| S \| \| C_2 \|$$

$$< 0 \qquad (5.47)$$

then, using expression (5.27) - (5.30) and (5.47), we have

$$\dot{v} < - \kappa \| S \| \qquad (5.48)$$

Expression (5.48) is the sufficient condition for the terminal sliding variable vector S to reach the sliding mode in a finite time. The output tracking error $\varepsilon(t)$ can converge to zero in a finite time on the terminal sliding mode.

5.5 A SIMULATION EXAMPLE:

Consider a two-link robotic manipulator model as shown in Figure 5.1. The links are of length r_1 and r_2, the mass m_1 and m_2, respectively. The mass is assumed to be concentrated at a point at the end of each link. The position state variables are the angles q_1 and q_2. Additional moments of inertia J_1 and J_2, about the centres of gravity of each link are also included in the model. The dynamic equation is given by (Young (1988))

$$\begin{bmatrix} \alpha_{11}(q_2) & \alpha_{12}(q_2) \\ \alpha_{12}(q_2) & \alpha_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} \beta_{12}(q_2)\, \dot{q}_1^2 + 2\, \beta_{12}(q_2)\, \dot{q}_1\, \dot{q}_2 \\ - \beta_{12}(q_2)\, \dot{q}_2^2 \end{bmatrix}$$

$$+ \begin{bmatrix} \gamma_1(q_1,\ q_2)\, g \\ \gamma_2(q_1,\ q_2)\, g \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

where $\alpha_{11}(q_2) = (m_1 + m_2)\, r_1^2 + m_2\, r_2^2 + 2\, m_2\, r_1\, r_2 \cos(q_2) + J_1$

$$\alpha_{12}(q_2) = m_2\, r_2^2 + m_2\, r_1\, r_2 \cos(q_2)$$

$$\alpha_{22} = m_2\, r_2^2 + J_2$$

$$\beta_{12}(q_2) = m_2\, r_1\, r_2 \sin(q_2)$$

$$\gamma_1(q_1,\ q_2) = -((m_1 + m_2)\, r_1 \cos(q_2) + m_2\, r_2 \cos(q_1 + q_2))$$

$$\gamma_2(q_1,\ q_2) = -m_2\, r_2 \cos(q_1 + q_2)$$

The parameter values are:

$$r_1 = 1\ \text{m},\quad r_2 = 0.8\ \text{m}$$

$$J_1 = 5\ \text{kg.m},\ J_2 = 5\ \text{kg.m}$$

$$m_1 = 0.5\ \text{kg},\ m_2 = 1.5\ \text{kg}$$

The reference model for the manipulator to follow is given as:

$$\dot{X}_m = A_m X_m + B_m r$$

where,

$$X_m = \begin{bmatrix} q_{r1} & q_{r2} & \dot{q}_{r1} & \dot{q}_{r2} \end{bmatrix}^T$$

$$A_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & 0 & -5 & 0 \\ 0 & -4 & 0 & -5 \end{bmatrix} \quad B_m = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $\quad r(t) = \begin{bmatrix} 5 & 5 \end{bmatrix}^T$

Since we are interested in trajectory tracking and hope that the transient response is determined entirely by the sliding motion, we consider a situation characterised by the same initial values of both, the reference model state and the plant state. In the simulation, we assume the initial values of $X(t)$ and $X_m(t)$ to be

$$X(0) = X_m(0) = \begin{bmatrix} 0.2 & 2 & 0 & 0 \end{bmatrix}^T$$

The terminal sliding mode is defined as

$$\varepsilon_1^{3/5} + \dot{\varepsilon}_1 = 0$$

$$\varepsilon_2^{3/5} + \dot{\varepsilon}_2 = 0$$

The matrix bounds in assumptions A.5.3 - A.5.7 are chosen as

$$k_1 = 0.2,\ k_2 = 1,\ k_3 = 1,\ k_4 = 1 \text{ and } k_5 = 1.5.$$

Runge - Kutta method with a sampling interval $\Delta T = 0.01s$ is used to solve the above nonlinear differential equations numerically. Fig.5.2 - Fig.5.4 show the output trackings, tracking errors, and input torques by the use of control law (5.7) with controller gain matrices (5.27) - (5.30). Fig.5.5 - Fig.5.7 show the trajectories of the same signals in the case of input disturbances ($h_1(t) = [\sin(10t) \quad \sin(10t)]^T$) with control law (5.43) with controller gain matrices (5.27) - (5.30) and (5.44). It is easy to see that good tracking performance is achieved. The effect of chattering, and thereby, amplitude of the control inputs is reduced by the use of boundary layer controller (Slotine and Sastray (1983), and Slotine (1984)) ($\delta_1 = 0.1$, $\delta_2 = 0.15$, $\delta_4 = 0.025$) as can be seen in Fig.5.8 - Fig.5.10 for the case of system with disturbances in Fig.5.11 - Fig.5.13 ($\delta_3 = 0.05$, $\delta_5 = 0.05$). The boundary layer controller offers a continuous approximation to the discontinuous control law inside the boundary layer, and guarantees attractiveness to the boundary layer and ultimate boundedness of the output tracking error to within a neighbourhood of the origin depending on $\delta_i (i = 1,...5)$.

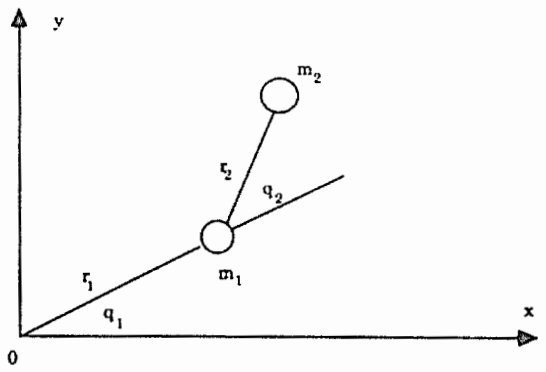

Fig.5.1 Two-link robotic manipulator model

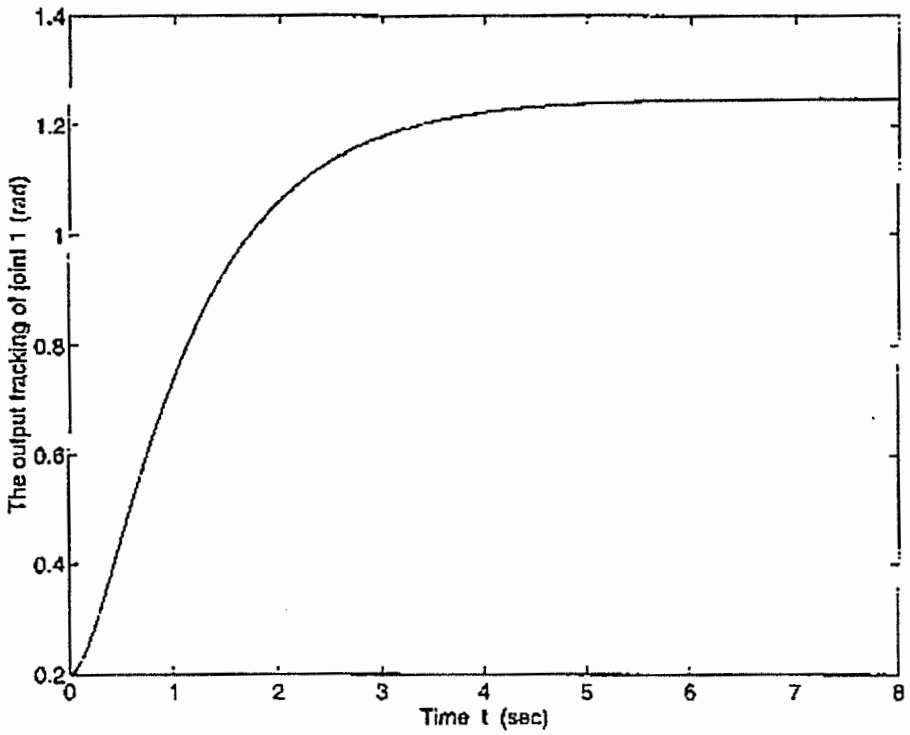

Fig.5.2-(a) The output tracking of joint 1

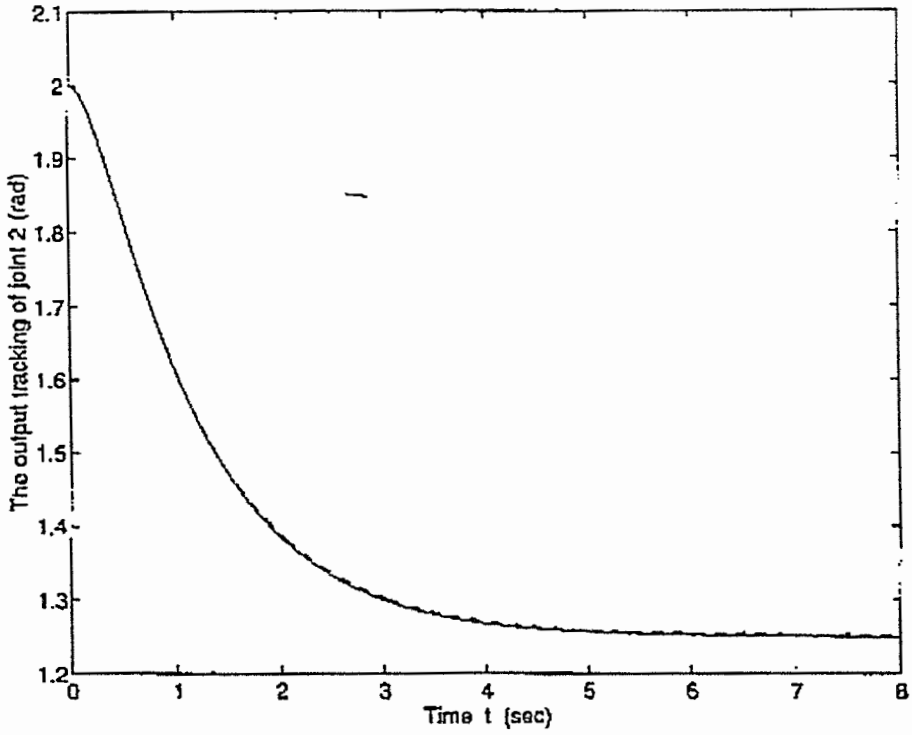

Fig. 5.2-(b) The output tracking of joint 2

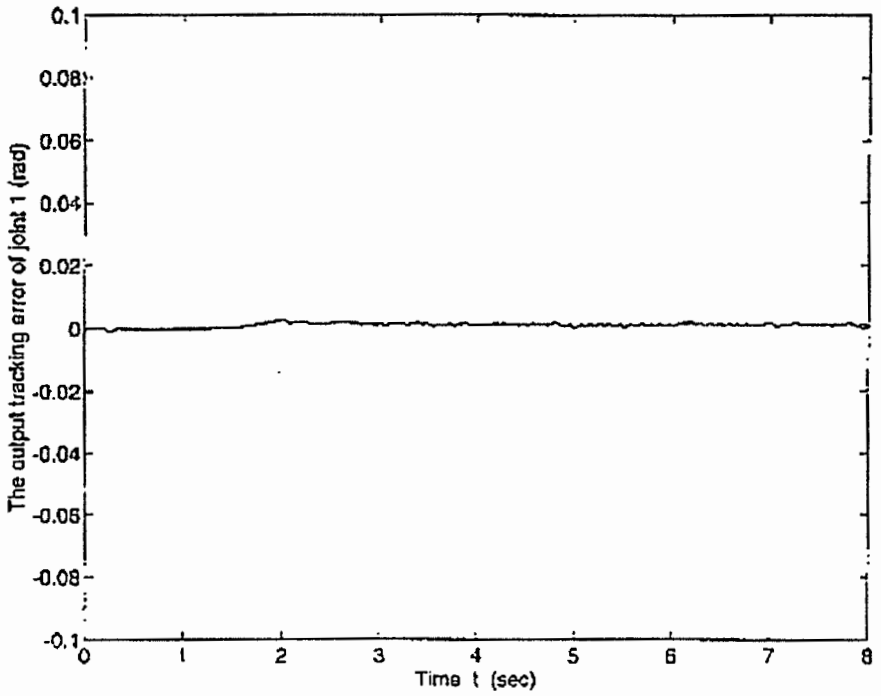

Fig. 5.3-(a) The output tracking error of joint 1.

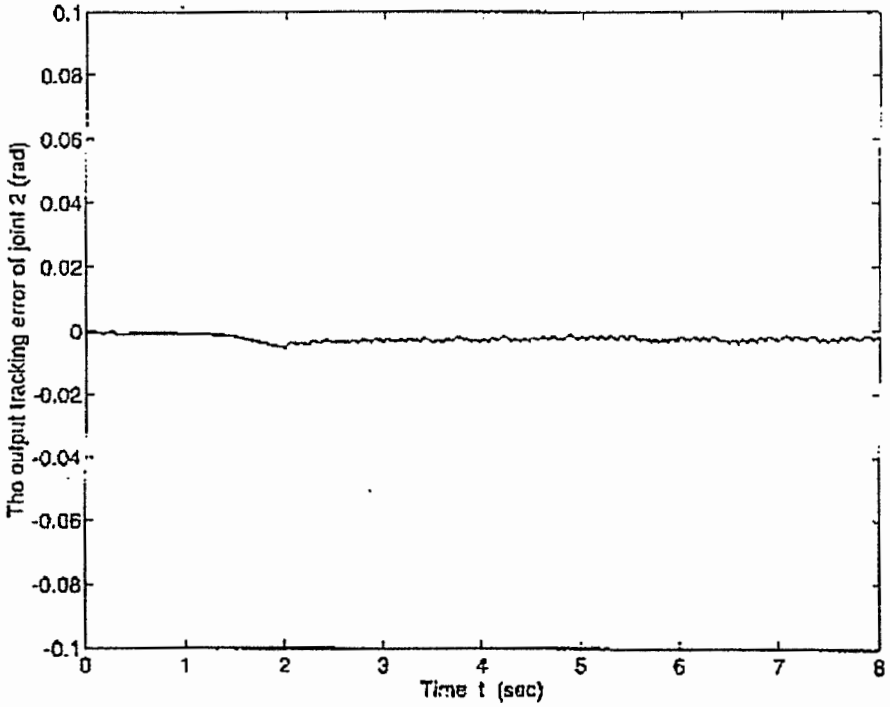

Fig. 5.3-(b) The output tracking error of joint 2

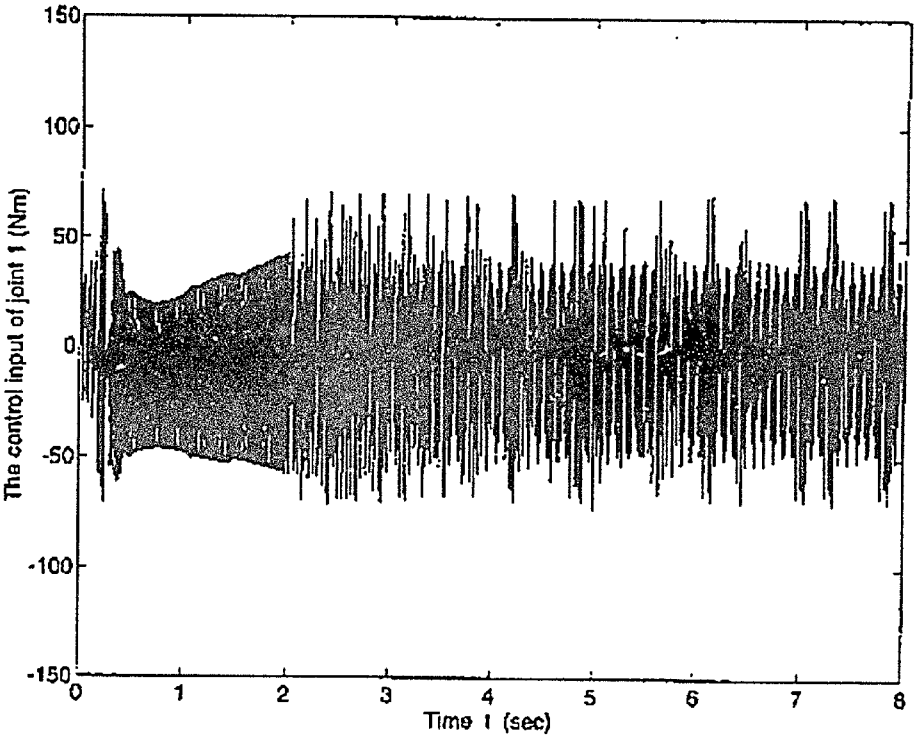

Fig.5.4-(a) The control input of joint 1.

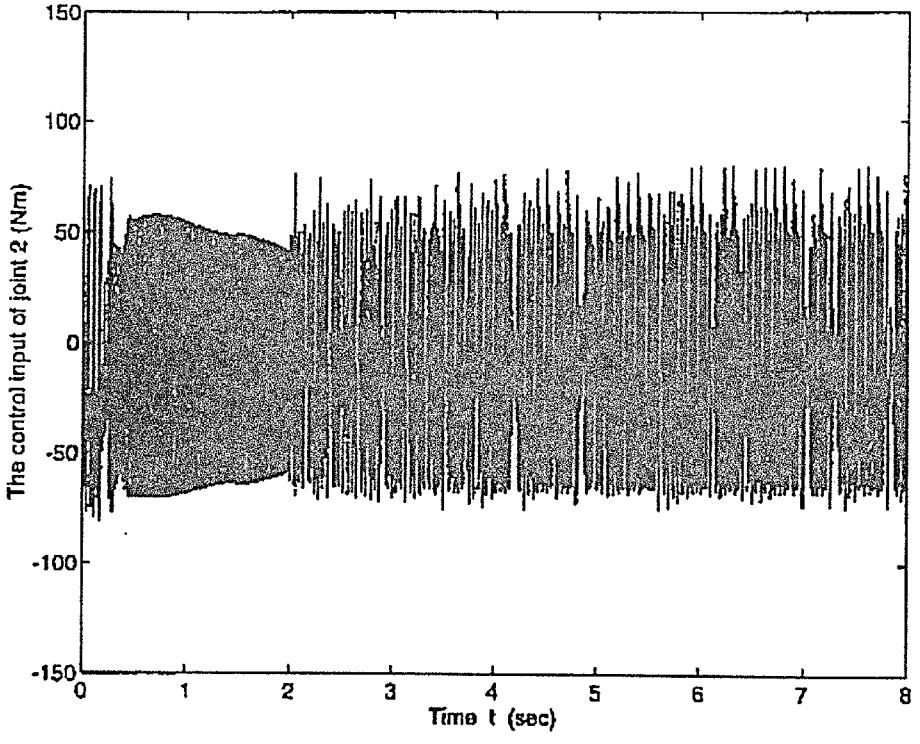

Fig.5.4-(b) The control input of joint 2

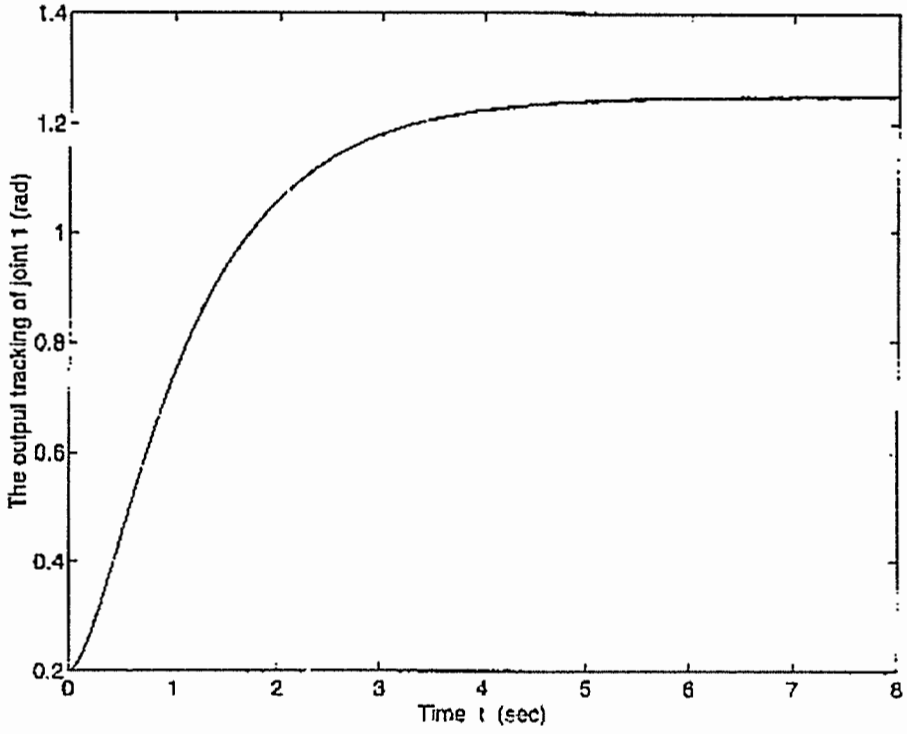

Fig.5.5-(a) The output tracking of joint 1 with input disturbance

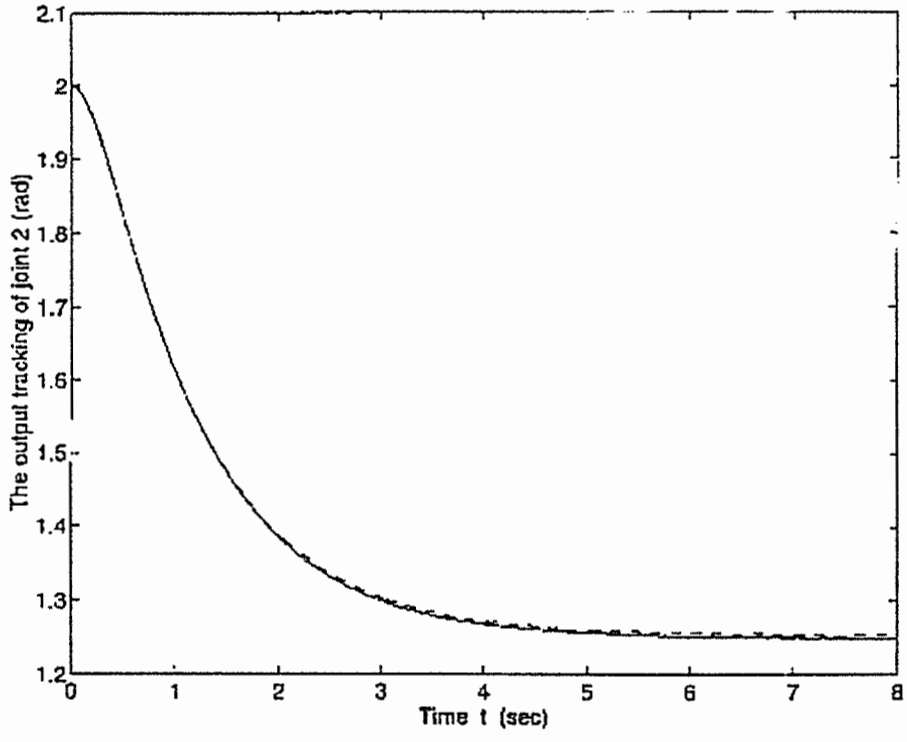

Fig.5.5-(b) The output tracking of joint 2 with input disturbance

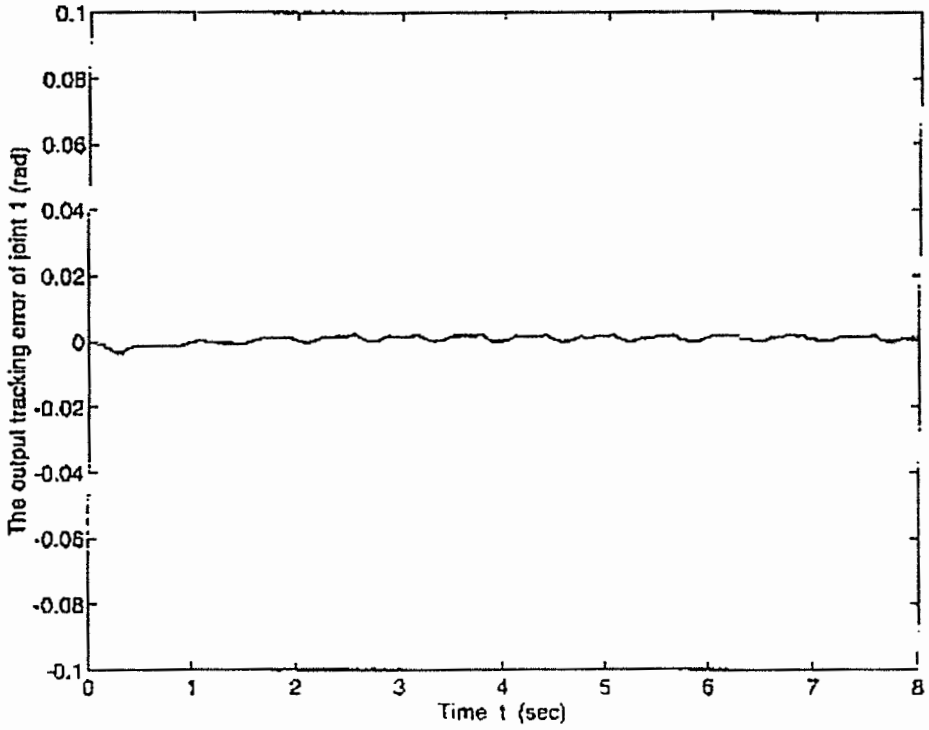

Fig.5.6-(a) The output tracking error of joint 1 with input disturbance

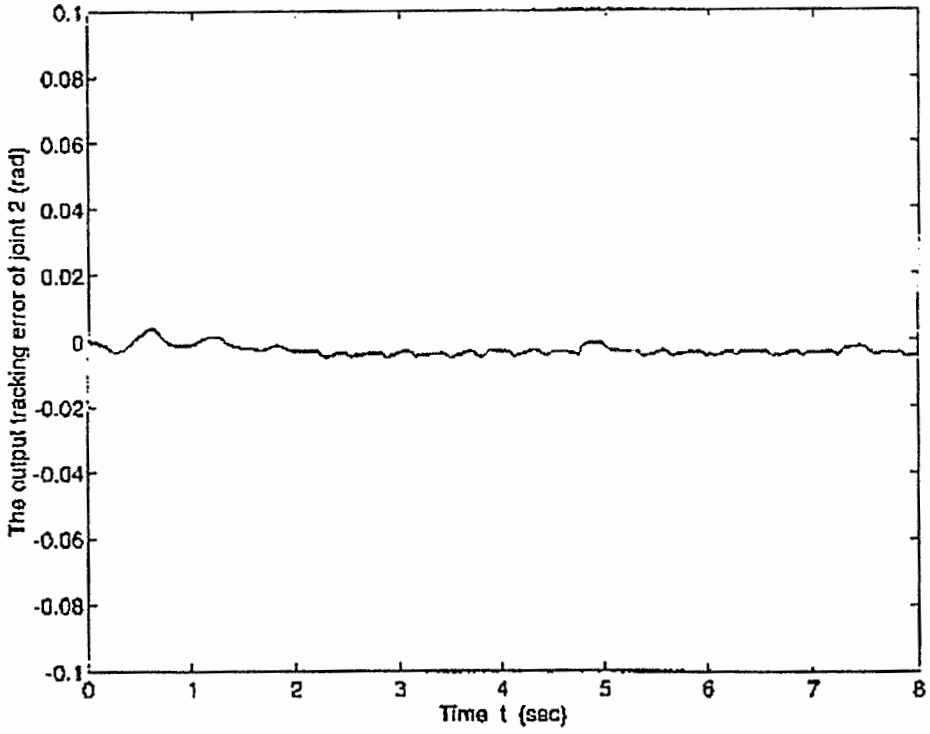

Fig.5.6-(b) The output tracking error of joint 2 with input disturbance

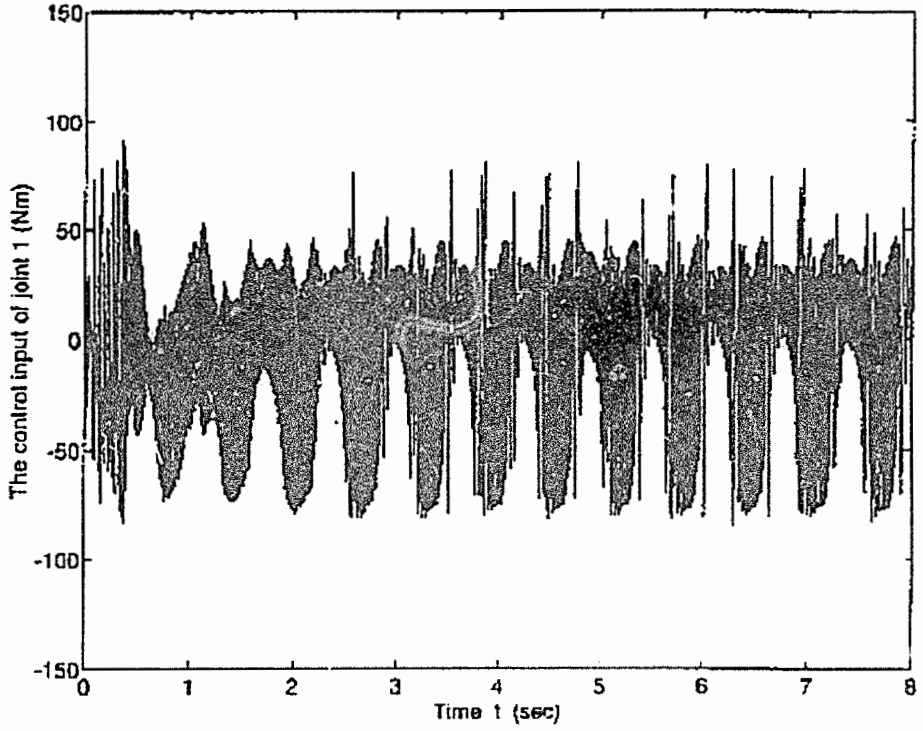

Fig.5.7-(a) The control input of joint 1 with input disturbance

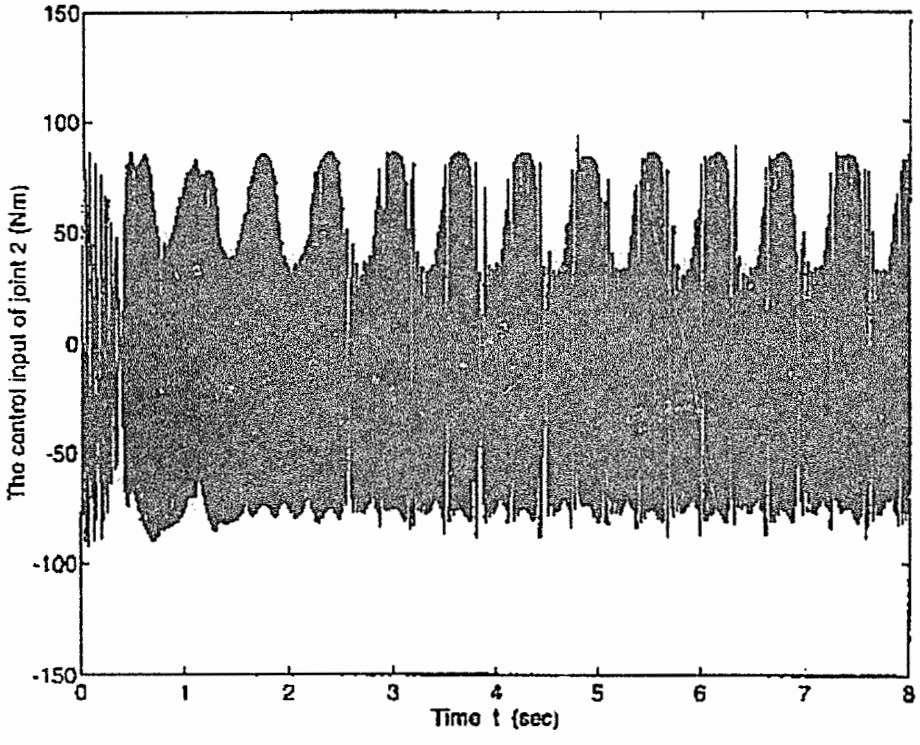

Fig.5.7-(b) The control input of joint 2 with input disturbance

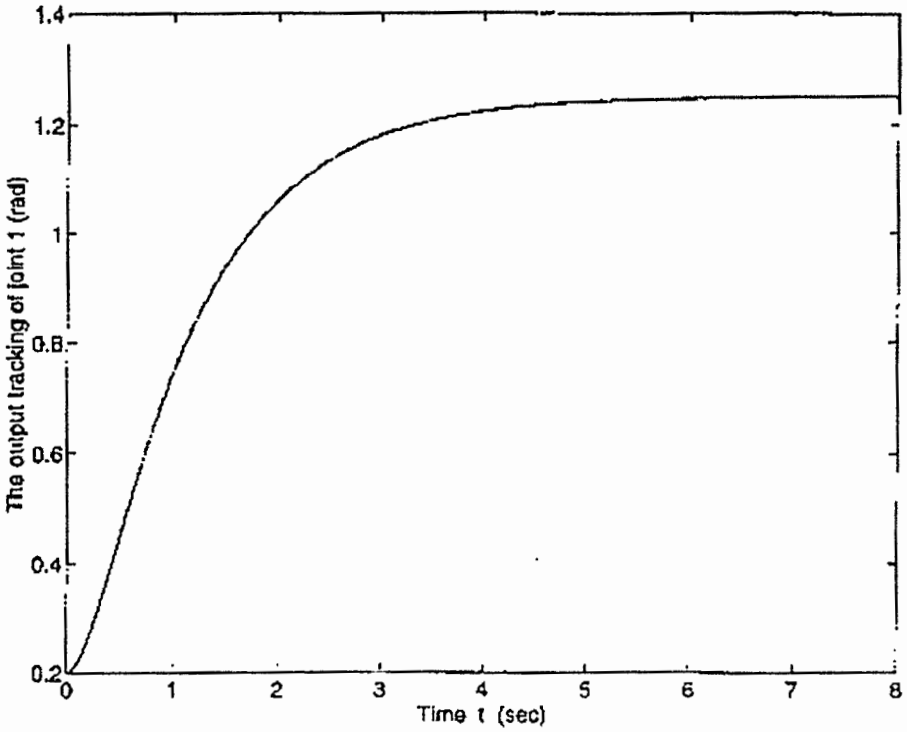

Fig.5.7-(a) The output tracking of joint 1 (using a boundary layer controller)

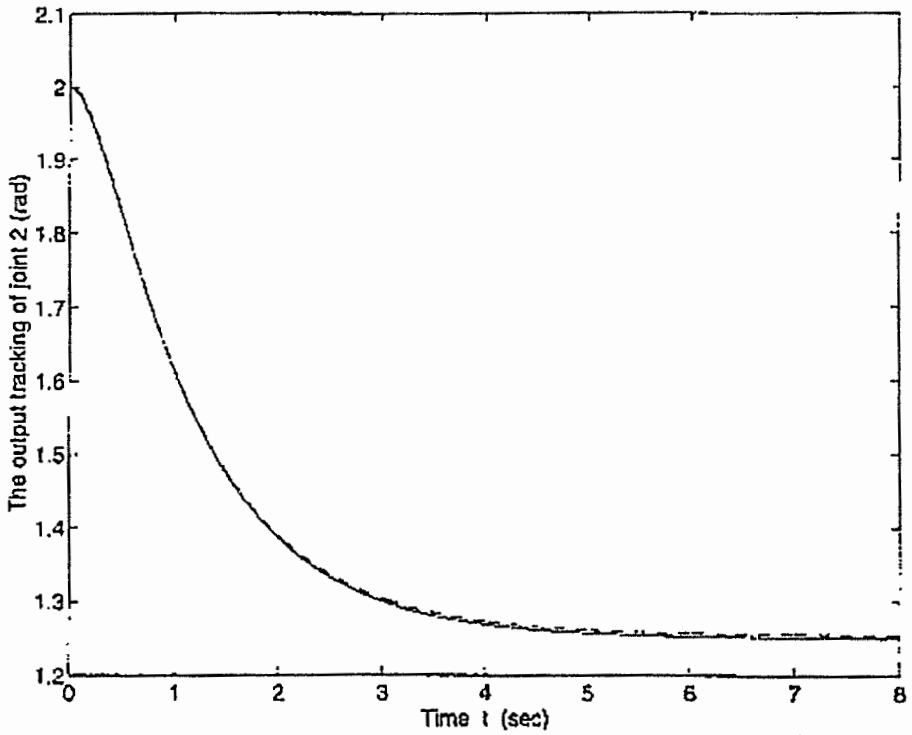

Fig.5.7-(b) The output tracking of joint 2 (using a boundary layer controller)

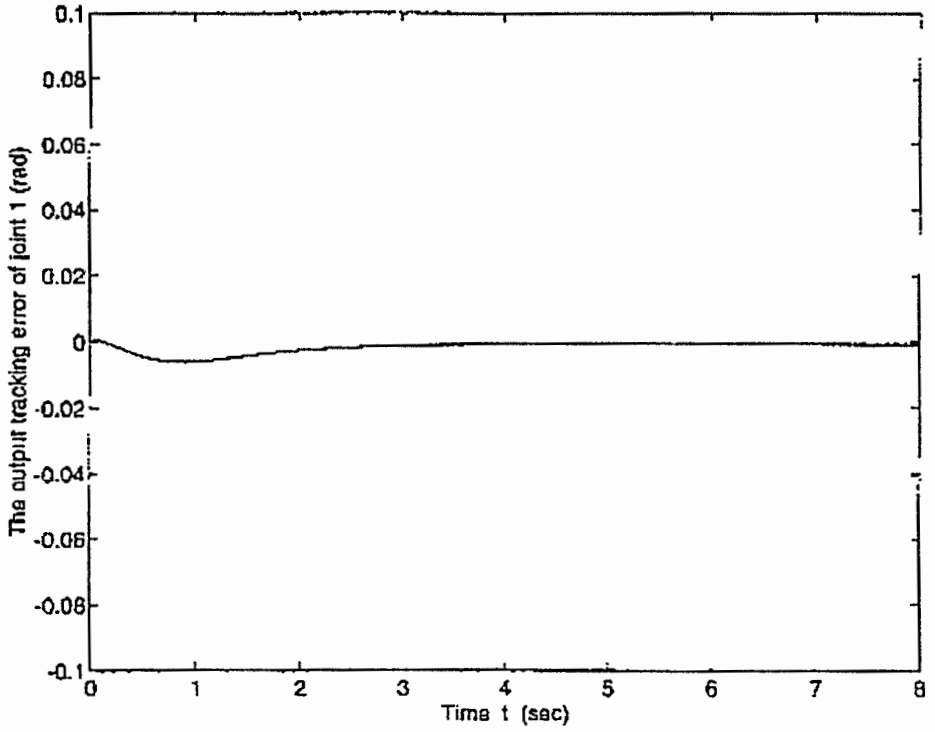

Fig.5.9-(a) The output tracking error of joint 1 (using a boundary layer controller)

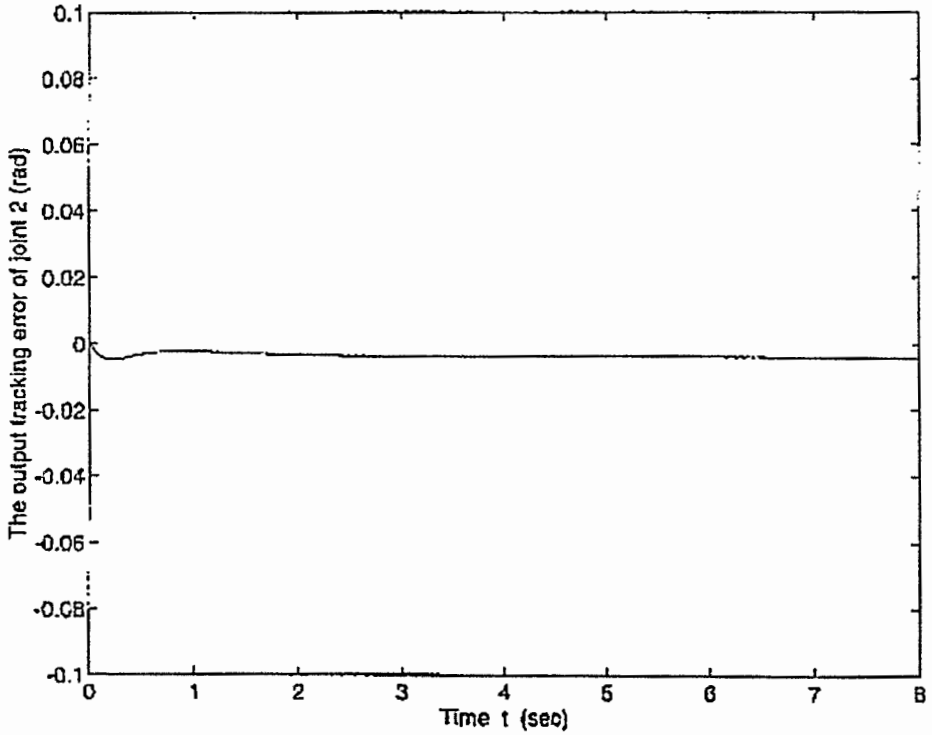

Fig.5.9-(b) The output tracking error of joint 2 (using a boundary layer controller)

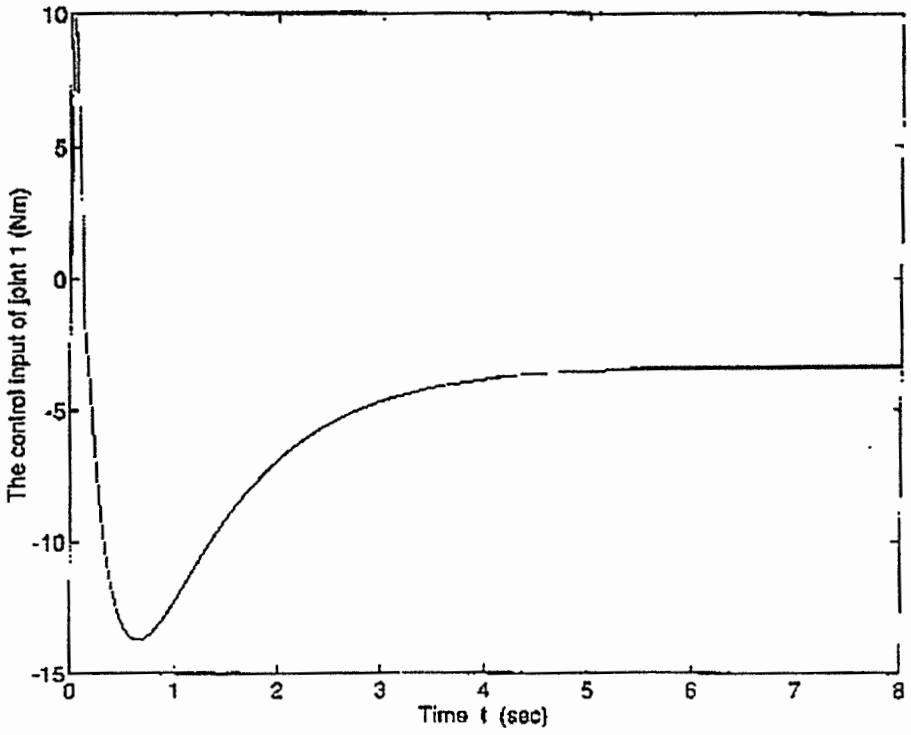

Fig.5.10-(a) The control input of joint 1 (using a boundary layer controller)

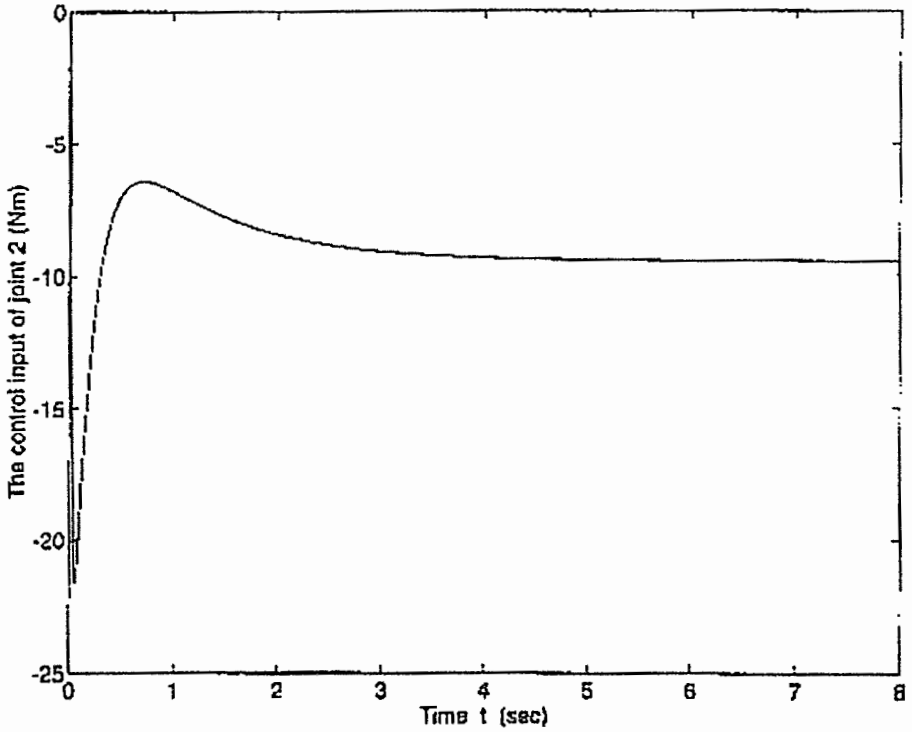

Fig.5.10-(b) The control input of joint 2 (using a boundary layer controller)

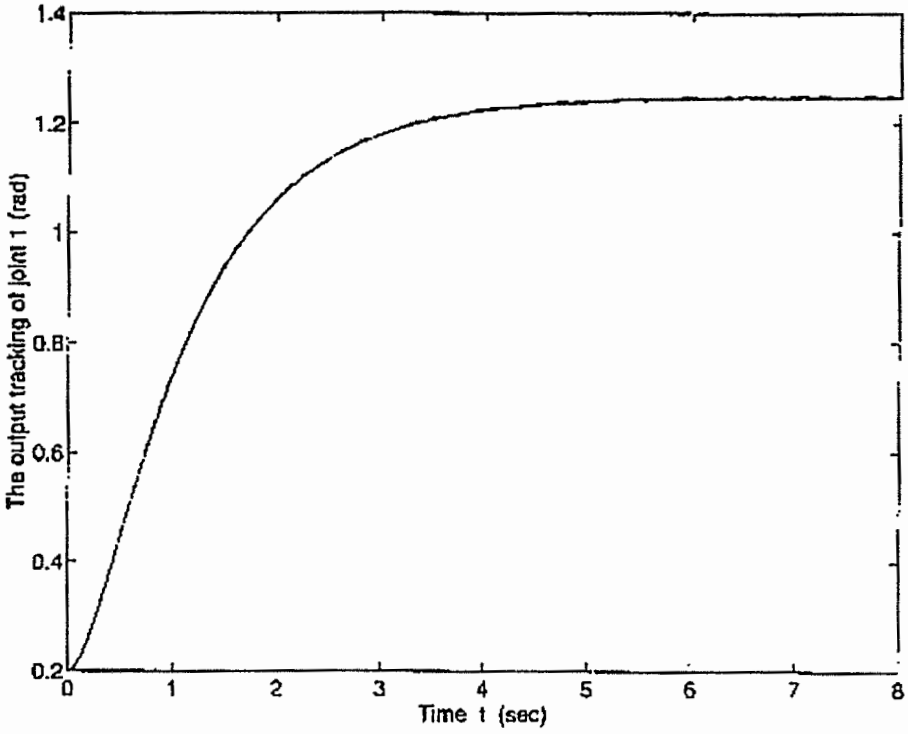

Fig.5.11-(a) The output tracking of joint 1 with input disturbance (using a boundary layer controller)

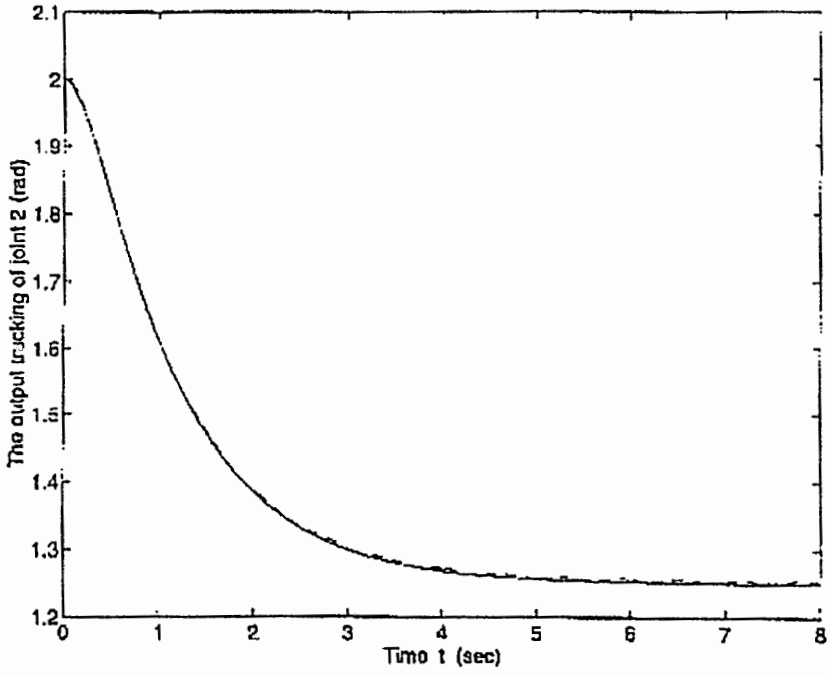

Fig.5.11(b) The output tracking of joint 2 with input disturbance (using a boundary layer controller)

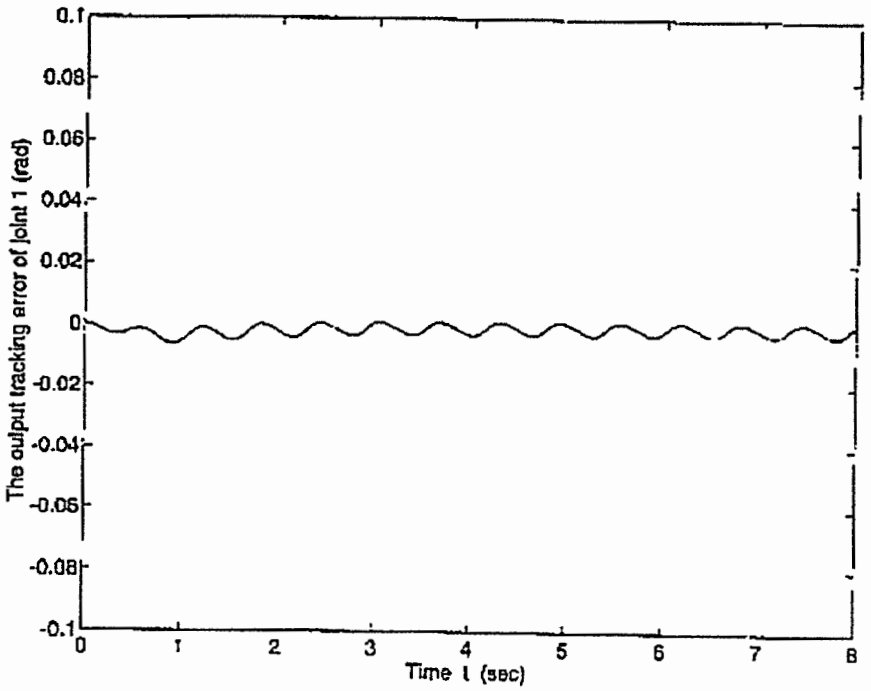

Fig.5.12-(a) The output tracking error of joint 1 with input disturbances (using a boundary layer controller)

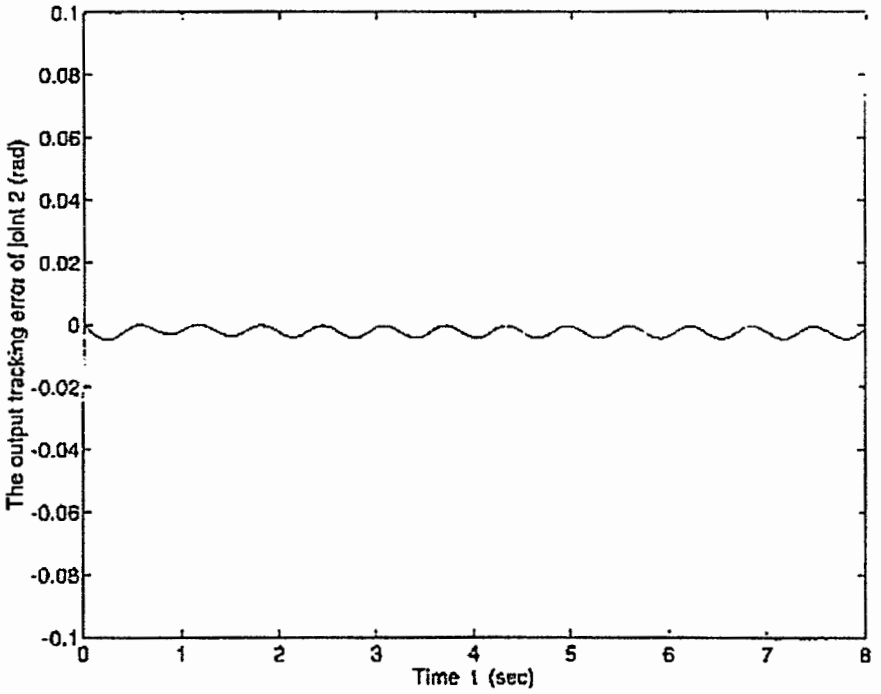

Fig.5.12-(b) The output tracking error of joint 2 with input disturbances (using a boundary layer controller)

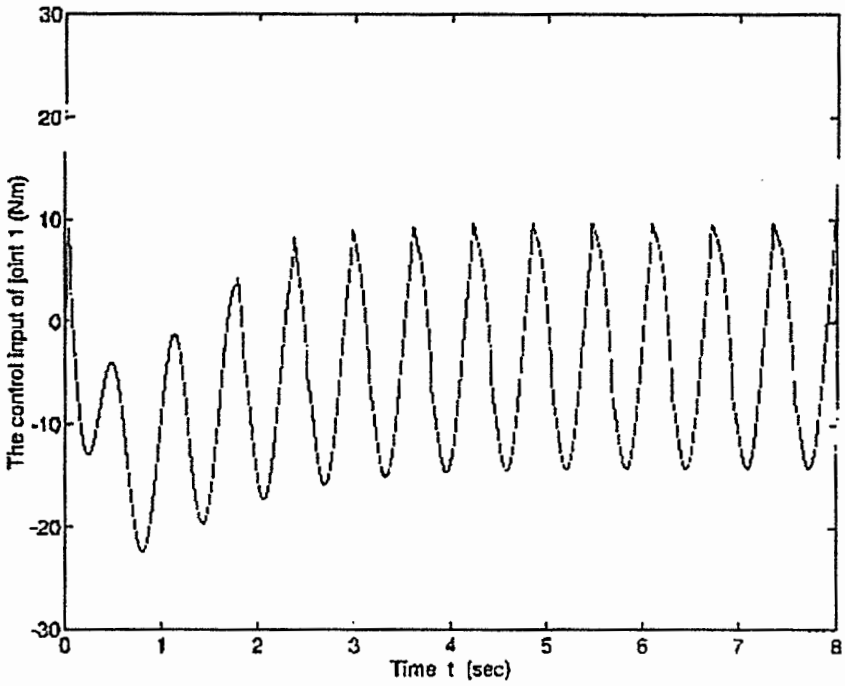

Fig.5.13-(a) The control input of joint 1 with input disturbances (using a boundary layer controller)

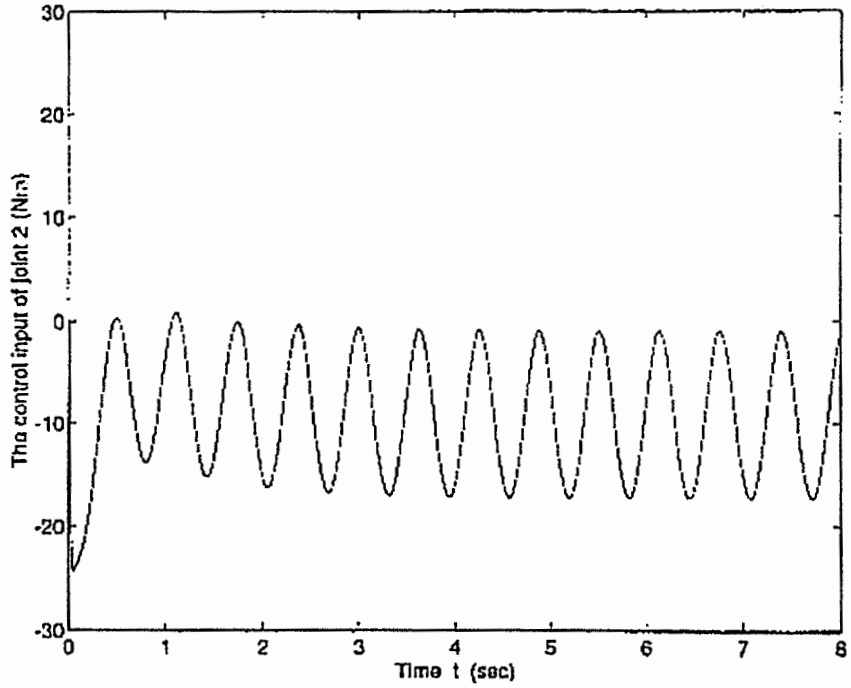

Fig.5.13-(b) The control input of joint 2 with input disturbances (using a boundary layer controller)

5.6 CONCLUSION

In this chapter, a model following control scheme using terminal sliding mode technique for rigid robotic manipulators is exploited. The main feature of this chapter is the design and definition of a terminal sliding mode controller using only a few uncertain system matrix bounds. It guarantees robustness to large uncertainties and bounded input disturbances, and the error convergence in a finite time is obtained on the terminal sliding mode. Simulation results are provided to demonstrate the effectiveness, simplicity and practicality of the proposed control schemes.

Chapter 6

Conclusion

6.1 Summary

The sliding mode control technique has proved to be a powerful technique in the control of highly nonlinear systems like robotic manipulators. Chapter 1 of this thesis shows the evolution of control schemes in the field of robotics where simple feedback and adptive controllers were insufficient to solve the control problems in robotic manipulators. The main factors affecting this were the nonlinearities, parameter uncertainties, nonlinear couplings and disturbances. Sliding mode control proved to be a powerful technique in the solution to the robot control problem and has instigated considerable research efforts in the field. The terminal sliding mode technique based on the idea of terminal attractors in Zak (1988) has shown great promise due to its finite-time convergence and robustness properties.

Chapter 2 provides a brief survey of sliding mode control theory and its application to linear and nonlinear systems. It also provides a discussion of the terminal sliding mode control and its application to the control of robotic manipulators.

Chapter 3 proposes a new robust adaptive tracking controller for rigid robotic manipulators. The chief advantage of using this scheme is that it does not require prior

knowledge of the uncetain bound. The scheme uses adaptively estimated values of only 3 parameters of the uncertain bound in the control gain. These parameters are then used to eliminate system uncertainties, obtain asymptotic convergence and reduce the amplitude of the control signal.

In Chapter 4, a decentralised adaptive sliding mode control scheme is proposed. This scheme requires no prior k: owledge of the uncertain dynamics of each subsystem. This scheme guarantees elimination of the effects of uncertain system dynamics and asymptotic error convergence using local feedback controllers to stabilize each subsystem and an adaptive compensator to handle the effects of system uncertainties.

Chapter 5 proposes a new terminal sliding mode technique. A terminal sliding mode controller is designed based. This scheme uses only a few uncertain system matrix bounds for the design of the controller. The result is a simple and robust controller that guarantees finite-time output tracking error convergence on the terminal sliding mode. The result is a simple, effective and practical control scheme.

In summary, this thesis has provided several new and improved linear and terminal sliding mode control schemes aimed at achieving robustness and convergence against system nonlinearities, parameter uncertainties, nonlinear couplings and external disturbances in the control of robotic manipulators. Robustness and Lyapunov stability analyses were provided for each of these schemes. Simulation results were used to demonstrate the tracking and convergence capabilities of the schemes.

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