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Teachers of mathematics teach mathematics differently : a case study of two teachers

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TEACHERS OF MATHEMATICS TEACH MATHEMATICS DIFFERENTLY: A CASE STUDY OF TWO TEACHERS

VOLUME 1

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USE OF THESIS

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I certify that this thesis does not, to the best of my knowledge and belief:

(I) incorporate without acknowledgement any material previously submitted for a degree or diploma in any institution of higher education;

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VOLUME 1

ABSTRACT

This thesis investigates the different approaches adopted by two teachers for teaching mathematical content at the upper primary level of education. Questions have been raised by researchers about the impact teachers' philosophical background may have on their perception of how mathematics should be taught. Similarly questions have been asked about the role of content in mathematics education in relation to the process of education. The two teachers held different beliefs about what they were doing when they were teaching mathematics and why they were teaching that way. Their methodological emphases were different; one could be described as being more learner-centred and the other as more content-centred.

This case study research analysed classroom observations, and interviews with the teachers and the students, collected over a twelve month period. The results indicated a difference in perception being expressed by the students in each class, about the mathematics they were being taught and its function in their own lives. The outcomes of this study were concerned with the impact of each teaching methodology. The qualitative nature of the research provides readers with data which may help them to make informed choices about approaches to teaching mathematics. Most importantly this study highlights the factors which may have an impact on how a teacher elects and/or feels constrained to deliver set mathematics curriculum.

A special feature of this thesis is the analytical tool and framework that was used as a lens to view and discuss the large amount of data generated. The usefulness of the seven *Principles of Teaching and Learning* (Bickmore-Brand, 1989) has been demonstrated through their application as indicators of effective classroom practice.

INTRODUCTION

In education there has been an increasing interest in the application to secondary schooling of methodologies used in the primary and early childhood sectors. Changes in curriculum and the effect on students have long interested the author. The impetus for this study came from the author's observation of children, including her own, who seemed not only to enjoy mathematics but also to find it untraumatic in the early years of primary school. However, as they moved through the primary school, negative attitudes seemed to be established until eventually these became associated with the subject throughout secondary schooling.

A study to investigate differences in methodologies was also stimulated by an experience to which the reader may relate. The author was teaching a Year 7 class at the time and was the subject of a research study into the application of counselling techniques in the classroom and the degree of empathy a teacher showed the children during three different lessons over a ten-week period. The author was both audio- and video-taped while she taught Art, Mathematics and Creative Writing. What became quite apparent was her shift in the degree of autonomy she allowed the children in each subject. Especially noticeable was the open and student-centred approach she adopted for both writing and art lessons in contrast with the subject-centred mathematics lessons. The change in her teaching style was due, in part, to the degree of confidence she had with the content as well as to her perception of what her students needed to know before starting high school. Implied values about the process and content of each subject also varied for the different content areas.

The author includes these personal anecdotes because they had an influence on the methodology adopted for this research. Her aim was to attempt to describe, by a range of classroom data, any differences in teaching methodology which may be apparent when teachers with different perspectives teach similar content.

As a University lecturer in Primary Education for the past seventeen years, the author has been very conscious of the different messages being given to the student teachers throughout their training. They are exposed to many different approaches as they move from lecturer to lecturer in the various disciplines including, for example, health education, science education, physical education, mathematics education, language arts education, music education, art education, including the different inputs from special education, English as a Second Language education, Catholic education and any other electives they choose to take. Student teachers face the task of deciding which approach best suits their philosophy and with which they feel most comfortable. They often encounter conflicting principles of teaching and learning.

It was partly this situation which gave the impetus to the author's Masters thesis, which presented a substantial literature review attempting to pull together the common threads from these disciplines. The initial eighteen common themes were condensed to seven principles about teaching and learning—context, interest, modelling, scaffolding, metacognition, responsibility and community

(Bickmore-Brand 1989). These Principles about teaching and learning have provided a framework which has been applied in this thesis for reflecting on the teaching and learning occurring in two classrooms. The thesis addresses the topic "Teachers of mathematics teach mathematics differently."

Teachers of mathematics teach mathematics differently: A case study of two teachers

Chapter One	**Chapter Two**	**Chapter Three**	**Chapter Four**	**Chapter Five**	**Chapter Six**
Literature Review	*Conceptual Framework*	*Methodology*	*Data Analysis*	*Major Findings*	*Emerging Themes*
The interface of language learning theory with mathematics education • Current trends in mathematics education • Teaching and learning principles created from language learning Principles associated with mathematics education research			Context Interest Metacognition Scaffolding Modelling Responsibility Community	Context Interest Metacognition Scaffolding Modelling Responsibility Community	Relationship between major research questions and emerging themes

Figure 1. Structured Overview of Thesis.

Overview of Thesis Layout

Figure 1 provides a structured overview for the layout of the thesis showing the interrelationship between the chapters.

In Chapter One the research context for this study is provided, reflecting upon the work of writers who have raised questions about the relationship between ideologies and practice. The Literature Review is in two parts. The first section of the first part reviews work related to the philosophy of mathematics education and its impact on teaching methodology. The second section discusses constructivist ideas and raises questions about the role and function teachers might have when working with students. The third section includes selected discussion about language and mathematics. The second part of the Literature Review is a synthesis of the research which underpins the seven Principles for teaching and learning, which are used to provide a framework for analysing the data presented in this thesis. In this review, the work of key informing the areas covered by each of these Principles, is described.

In Chapter Two the Conceptual Framework of this study is presented, showing the relationship between the literature and the research questions and methodology used.

The Methodology for the data collection is described in Chapter Three and substantiates the details from each stage and type of data collected. This chapter concludes with a rationale for the use of the teaching and learning

Principles as a lens through which the data were analysed and the information organised.

The main Analysis of the data is found in Chapter Four, using the teaching and learning Principles as a framework. Each Principle forms a subsection and each concludes with a discussion interpreting the results.

The major findings pertaining to each of the Principles are located in Chapter Five and are positioned within the related literature. Chapter Six provides a macro picture and attempts to draw out the major themes which have emerged from the research as a whole. It also addresses the relationship between the major research questions and the emerging themes. It concludes with a statement which describes the contribution of this study to current mathematics educational research.

Due to the interpretive nature of the study it has been important to signal to the reader what orientation the author might have regarding her philosophical stance before reading of her judgements on those of the teachers in this study. The philosophical position taken by the author in this thesis is consistent with Nesher's (1988) *pedagogical realism*. It is based on pedagogy which is consistent in the first instance with the constructivist view that there is a need for the learner to construct knowledge through interaction with the environment, but it also encompasses the need to develop a repertoire of mathematical understandings. The author believes this is consistent with the model developed by Clements and Lean (1988), which draws a relationship between the familiar real world concepts, the formal mathematical language and symbolic manipulation. Details of this model are discussed further in Chapter One.

Chapter One

LITERATURE REVIEW

Introduction

One of the major challenges in mathematics education is that of understanding the culture of mathematics classrooms and finding ways to support students in their construction of mathematical concepts. As Ellerton and Clements (1991) remarked in their review of classroom mathematics research:

> Mathematics is now being seen not so much as a God-given body of objective knowledge, but rather as something which is socially determined; that is to say, mathematics is seen as being based on human rationality and not as a transcendental body of knowledge waiting to be discovered ... This realisation has lead mathematics educators ... to speak of teachers of mathematics as "mathematical enculturators." (pp. 50-51)

The tensions between the differing beliefs mathematics educators hold about mathematics and how it should be taught is a theme which will continue to recur throughout both the literature review and the study.

The Interface of Language Learning Theory with Mathematics Education

School mathematics is about constructing shared mathematical meanings and accessing a system of potential mathematical meanings. Although most mathematical representations are non-verbal (diagrammatic, symbolic, etc), classroom discourse requires students to share their meanings through the medium of verbal language. It is therefore useful to look at language acquisition to gain some insight into the nature of the classroom discourse which facilitates the acquisition of mathematical meaning. This thesis aims to inform the debate on the interface between language learning and mathematics education.

In 1991 Ellerton and Clements published a comprehensive review of the research surrounding language and school mathematics. They described what appeared to be disconnected interest groups, and the conduct of parallel research by the groups, oblivious of the other's work. These groups had in common an interest in the role of language in the mathematics classroom and reflect a growing body of research concerning the interface between language and mathematics.

> During the 1970s and 1980s there was an amazing growth in the number of publications specifically concerned with identifying language factors that influence mathematics learning ... However ... 'language and mathematics learning' researchers tended to work in separate camps, being largely unaware of the efforts of those often closely related fields of endeavour. There was an obvious need for investigation and development of links between the various research thrusts. (pp. 18-19)

The model that Ellerton and Clements used to plot the relationship between the various approaches to language and mathematics education embed social, cultural, linguistic and cognitive factors. They emphasise the important role played by language in the teaching and learning of mathematics. They argue that, provided learners have prior knowledge of the language and concepts embedded within a message, then mathematical meanings can be communicated unambiguously through words. This is in contrast with radical constructivist notions (for example, von Glasersfeld, 1990a), and in particular with Wheatley's (1991) claim that "ideas and thoughts cannot be communicated in the sense that meaning is packaged into words and 'sent' to another who unpacks the meaning from the sentences" (p. 10). The issue of communication in von Glasersfeld's interpretation is that concepts can only be "abstracted" from experience. Both positions would agree that interaction with the teacher can help students develop their interpretation of the concepts. Research surrounding language and mathematics is of particular relevance to this thesis.

The emphasis in mathematics education research is towards an integration of the social, cultural and cognitive aspects of learning the language of mathematics as Laborde (1988) noted in her report on the major issues discussed at the Sixth International Congress on Mathematical Education in Budapest. She referred to the following different subgroups which focused on language and mathematics:

1. language in the mathematics classroom;
2. the relationship between the development of language structures and the development of mathematical structures;

3. the spoken and written texts of the classroom;
4. the ways in which teachers use language to negotiate meaning in the classroom;
5. the question and answer routines of classroom discourse; and
6. the interplay of cultural factors in mathematics teaching.

The first four of these six groupings, in fact, provide a useful framework for the discussion on the interface between language learning theory and mathematics education which follows.

Language in the mathematics classroom: Natural learning. One of the confusions which exist when discussing language and learning mathematics is the use of the term natural learning. Although Cambourne (1988) made a major impact on education across Australia in his application of natural language learning conditions, the emphasis was on the conditions for learning language *naturally* not on learning *natural* language. The linguists and genrists' who have written subsequently (Christie, 1990; Cope & Kalantzis, 1990; Derewianka, 1990; Martin, 1990) labour the disadvantage students would have if they operated in life with only the (natural) language they had acquired in their first language setting. What can usefully be taken from this, the author believes, is the concept that certain conditions are desirable for the acquisition of language (Cambourne, 1988) and if mathematical language discourses are to be acquired then mathematics classrooms too might benefit from the application of similar conditions. Ellerton and Clements (1991) reported on the movement across Australia in the late 1980s of the restating of Cambourne's conditions for learning by various educators "so that they fitted the contexts of school mathematics" (p. 124). They also signalled that Cambourne's "natural" language learning approach to mathematics had not been tested experimentally. This discussion on language acquisition and the application of certain conditions should not be confused with the adoption of language strategies such as process writing and journal writing which were evident in primary mathematics classrooms in the 1980s. Genrists such as Derewianka (1990) would concur with Cambourne that it is not the language which is natural but the learning.

Language in the mathematics classroom: Language strategies. Recent application of language strategies in mathematics classrooms (for example, journal writing and process mathematics) has left some mathematics teachers wondering if they are teaching mathematics or English. As Mousley (1990a) expressed it "Mathematics teachers generally spend little class time on teaching students how to set out mathematical reports, yet the English teachers would not see this as their responsibility" (p. 57). Stephens (1993) draws attention to the assistance students may need in order to present their mathematical writing for investigative projects as part of their VCE (Victorian Certificate of Education) assessment. A significant part of upper school mathematics requires students to conduct extensive independent investigations, involving comprehensive research. The literacy demands may be quite high, as students undertake extensive reading, and prepare their written reports. Stephens describes the teacher's role as not only supporting students in their mathematical investigations but extending that role "well beyond providing information and resources ... it must be seen as assisting students to come to terms with the specific demands of report writing" (p. 166).

The application of language-arts strategies continues to be debated by mathematics educators (see Ellerton & Clements, 1991; Pengelly, 1988; Waywood, 1993). The author shares Marks and Mousley's (1990) concerns about the application of process writing in the mathematics classroom. It must be remembered that the process writing approach was to some extent superseded by the genre movement which in turn was heavily influenced by the work of linguists such as Halliday (1985) and Christie (1990). The contribution of linguists to analysis of classroom mathematics discourse follows.

The spoken and written texts of the classroom. Green (1988) suggests that language must be adapted not only for different social contexts but for different subjects and different texts within each subject. As Green (1988) states

> For the present purposes, literacy will be defined flexibly and operationally as a situation-specific competency with regard to written language. The main concept to be introduced is the notion of subject-specific literacy—that is, the particular literacy, or set of literacy competencies, that is inextricably part of the operation of specific subject areas as contexts for learning and meaning. (p. 157)

Mathematics classroom discourse studies have drawn from the writings and terminology of linguists. For example teachers now consider the notion of a specific genre or register for mathematics which must be mastered in order to access the mathematical potential of what is being learned.

A significant contribution has been made by Lemke (1989) who studied secondary mathematics classrooms and concluded that, to learn mathematics, students needed to adopt the syntax and see the semantic differences between the language they bring to the classroom (their everyday language) and the more formal language of the classroom. He suggests that "thematic patterns" can be found in a variety of situations or resources such as textbooks, dialogue between a teacher and a student, and so on. He described these thematic patterns as having a fluency in speaking and writing the language of the subject which is recognisable as mastery of the use of its fundamental concepts and principles.

> Language patterns are emphasised here because we know more about the semantics of language than about any other human resource for making meaning. Many thematic patterns can also be expressed in the language of mathematics or through various kinds of diagrams, but the semantics of language seems to form a common denominator for all systems. (p. 137)

Lemke goes on to suggest that the emphasis on language patterns has developed because researchers have a greater understanding about the semantics of language than they have about other ways in which people make meaning.

These thematic patterns, Lemke (1988) suggests, have a predictable activity structure or genres. They are typically written forms, such as algorithms, proofs and diagrams, with journal writing a recent feature of classrooms. The speech forms tend to be mental (oral arithmetic), marking homework, demonstrating a proforma or strategy to students, and teacher-student turn taking, which usually follows a fairly closed questioning format, with interaction occurring only with a few target students. Tobin and Gallagher (1987) have commented on similar observations in many teacher-centred classrooms.

Pimm (1987) appropriates the linguistic notion of communicative competence, and argues for "communicative mathematics teaching."

> The general notion of communicative competence involves knowing how to use language to communicate in various social situations—how to use language appropriate to context. In other words, it requires an awareness of the particular, conversational or written, context-dependent conventions operating, how they influence what is being communicated and how to employ them appropriately according to context. (p. 4)

This in itself raises issues concerning what is valued in the mathematics classroom as appropriate discourse. Is it getting the semantics right? Or is it adopting the syntax in all of its non verbal and verbal forms? As Chapman (1992) acknowledges.

> In any school subject, the weighting attached to what is said is important. Mathematics, in particular, is typically regarded as a factual subject and thus it is likely to have a high modality structure. Its content matter is certainly as important as its processes. Mathematics involves "realities", such as functions, rules and angles. It is a discourse that asserts claims to; "truths"; for example, in the form of theorems and proofs. These are the "stuff" of school mathematics. They are part of the knowledge that is produced in the classroom. (p. 57)

This thesis attempts to inform this concept of how mathematics is framed in the classroom and the language which a teacher might use to help students construct mathematical understandings.

The relationship between the development of language structures and the development of mathematics structures. The idea that mathematics is a language, composed of texts, is becoming increasingly accepted (Mousley & Marks, 1991; Reeves, 1990). The systemic linguist, Kress (1985), examined the interplay of the language and the form of the message, and concluded that "discourse carries meanings about the nature of the institution from which it derives; genre carries meanings about the conventional social occasions on which texts arise" (p. 20). There has been increased use of children's language in mathematics classrooms where children, for example, keep logs about what they are studying, or write letters about the mathematics they have been studying (see Ellerton 1988; Waywood, 1993).

Many research studies have focused on the ways in which meaning can be hampered when students do not have access to the formal language forms within which certain mathematical concepts are encoded (Carraher & Schlieman, 1985; Dawe, 1983; Enemburu, 1989; MacGregor, 1989). Marks and Mousley (1990) describe mathematics as having several mathematical genres with which the student must become competent e.g., graphs, algorithms, textbook exercises and recount. "Just as in language learning, separate attention may be devoted to descriptive, expressive and persuasive forms of writing (and others), so in mathematics there is a whole complex of procedures appropriate to different types of algebraic situation" (Bell, 1988, p. 148).

Similarly teachers need to be concerned that their own language communicates accurately. Austin and Howson (1979) discuss the problems in teaching algebra which is necessarily full of symbolism. While a greater use of symbolism could lead to an improvement in the student's competence in manipulating symbols, it could at the same time lead to a decrease in their ability to understand the underlying meanings. The discussion in this thesis concerned with "scaffolding" reflects on how each teacher in this study used the language of their mathematical registers during classroom interactions.

An investigation of the social production of language and thinking and how mathematical meanings are generated in both home and school practices have been developed by Walkerdine (1988). Walkerdine draws on principles found in the literature of psycholinguistics, semiotics and discourse theory (de Saussure, 1974; Olson, 1977; Vygotsky, 1978; Wells, 1981) to support her research, and views the classroom as a site for particular social practices. She concludes that;

> The "truth" about children's "mathematical development" is produced in classrooms, and that all learning can be understood as taking place within social practices in which the relation between signifier and signified is constantly problematic. (p. 9)

The ways in which teachers use language. Because classroom mathematics teaching is to a large extent, "talk," be it teacher talk or student talk, it is the intention of this thesis to use language learning principles as a lens through which to discuss the classroom interactions. We use language to make sense of our environment, and therefore language becomes an important tool for coming to terms with the content of each of the subject areas.

Chapman (1993) concludes "that teachers and learners use language to talk about mathematics, to talk mathematically, to 'do' mathematics; in sum, to enact the social processes that constitute school mathematics" (p. 2).

The study by reading researcher Fry (1988) uses techniques for monitoring eye movements and applies them to student's reading of mathematical material. It would seem from his study that successful problem-solvers are readily able to identify relevant information, but that processing is highly individualised and personal. This issue of the idiosyncratic nature of how information is processed will be taken up in more depth in the "interest" section of the data analysis chapter. The whole issue of the amount of reading the subject of mathematics requires raises questions about the literacy capacity of the students and to what extent comprehension of the problem is an integral part of a student's ability to succeed in mathematics (Newman, 1977). Clements (1982) reported an interview protocol which categorised errors that students made on a set of mathematical questions as reading, comprehension, transformation, process skills, encoding or careless/ motivation errors. These studies found that almost half of all errors made could be attributed to comprehension errors.

Conclusion. The heart of the interface of language acquisition research with mathematics education lies in the composition of the mathematical register or mathematical language within which mathematical meanings are exchanged. Debate will no doubt continue over whether learning and literacy learning can be equated. This thesis adopts the position of Green (1988) and Lemke (1988) that language, and in particular the semantics of language, is currently our best researched resource for making meaning of the world. The role of the teacher as a scaffolder of that language development is a key question that this study seeks to explore.

Current Trends In Mathematics Education

In his introduction to the proceedings from the twenty-seventh annual conference of the Mathematical Association of Victoria, Ken Clements (1990) highlighted some of the prominent issues influencing mathematics education as he saw them. The key issues he identified were: raising the female profile in senior and tertiary institutions; assessing co-operative group methods; challenging the use of computers in the lower grades; the degree to which constructivism can be applied in the mathematics classrooms; the role of skills in problem-solving; and finally the accountability of our education system and the relationship of national curriculum and testing programs. Three of these issues—constructivism, group methods and problem—solving will emerge as important for this thesis. In this literature review, these issues will be considered under the broad headings of philosophy and constructivism.

Philosophy

Many mathematics teachers never make explicit their philosophical rationale for teaching their subject and yet their belief systems and values strongly impact on their methodological choices (Pateman, 1989). Hersh (1986) claims that it is the teacher's view of the nature of mathematics which has the greatest impact on practice "not on what he or she believes is the best way to teach" (Hersh quoted in Grouws, 1992, p. 42). In other words one's philosophy of teaching mathematics is intimately linked to one's philosophy of mathematics.

The question "What is the nature of mathematics, and what is its relationship with the real world?" has become one of the major themes in contemporary discussion about the teaching and learning of mathematics. Frye (1970) captures this in his description of knowledge as "not something one has: it is something one is" (p. 3), and his comment that "education is concerned with two worlds: the world that man lives in and the world he wants to live in" (p. 17). Education authorities in Victoria have been concerned to "develop a philosophical and educational position for mathematics teaching and learning" (Victorian Curriculum and Examinations Board, 1989, p. 31).

The answers to these issues have shifted from the Platonist and formalist perspectives to social semiotic perspectives found in the radical constructivist literature. The subsequent direction for the foundation of mathematics drew from the radical constructivist perspective and in particular the metaphysical principle described by von Glasersfeld (1988, 1991). In von Glasersfeld's (1990a) words:

> The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viable; cognition serves the subject's organisation of the experiential world, not the discovery of an objective ontological reality. (p. 23)

Knowledge in these terms is the result of our own construction of the world as we experience it and our consequent projection of this knowledge on the world. It is not seen as a matching process between the personal knowledge of an individual and some objective reality.

Five schools of philosophical thought have grown out of the debate surrounding "what is mathematics?" (Ernest, 1985)—namely Logicism, Platonism, Formalism, Falliblism and Constructivism. Emphasis in this literature review will be given to constructivism but the others will be discussed briefly in order to position them in this study.

Logicism. Logicism purports that all mathematics can be reduced to logical concepts. One example is from Whitehead and Russell's (1910) *Principia Mathematica* which explored the perspective that all mathematics could be derived from a set of axioms in a logicist program (Pateman, 1989). One of the most well-known applications of this school of thought to education came in the form of the New Math with its set theory and its heavy emphasis on detailed mathematical terminology. Also associated with logicism is the tradition of Euclidean geometry, and theorem and proof approaches to perceived mathematical truths. Classrooms today are questioning the premature introduction of formal symbolic mathematics to primary school children. Similarly Euclidean geometry has been excluded from secondary curricula with claims that the logical structure is rarely understood. Lastly the practice of theorem and proof patterning while still being used in schools has been criticised for its poor transferability and suppression of metacognitive critical thinking (Usiskin, 1981).

Platonism. Platonism views mathematical knowledge as having an objective existence, a static view of knowledge as a pre-existing entity. Davis (1986) provides a summary of the beliefs which underpin the Platonic and Kantian traditions.

> 1. The belief in the existence of certain ideal mathematical entities such as the real number system;
> 2. The belief in certain modes of deduction;
> 3. The belief that if a mathematical statement makes sense, then it can be proven to be true or false;
> 4. The belief that fundamentally, mathematics exists apart from the human beings that do mathematics. Pi is in the sky. (p. 165)

Support for this thinking was often attributed to the widespread acceptance of Euclidean geometry and Newtonian views. Although both Euclidean and Newtonian approaches have been central to theory and practice of mathematics education for centuries, they have been increasingly challenged over the past two decades, particularly by constructivist scholars.

The nineteenth century saw the invention of non- Euclidean geometries such as "space-filling curves" and "continuous nowhere differentiable curves" (see Hersh, 1986, p. 15). Among some philosophers (Kitcher, 1984) there was an acknowledgment that much of what underpinned geometrical theorems was culturally specific and expressed what was the reality for a particular human's experience. According to Brandau (1988), educators, when confronted with this uncertainty in mathematics, felt angry, confused and highly vulnerable as teachers. They experienced a loss of control and feelings of uncertainty about how to teach the subject.

Formalism. Formalists came to the rescue in the early twentieth century when the foundation of mathematics seemed to be floundering in the backlash of the demythologising of Platonism (see commentaries by philosophers—Davis & Hersh, 1981; Ernest, 1991; Hersh, 1986; Lakoff, 1987). Hilbert's formalist program aimed to translate mathematics into uninterrupted formal systems (Ernest, 1991). However formalism views mathematics as a meaningless game which makes no attempt to try to connect reality with mathematics during the teaching process (Pateman, 1989). There is much evidence which is strongly critical of the presentation of mathematics, especially in textbooks, as the rote learning of rules and formulas, without any concern for understanding. A key critic of the formalist perspective is Lakoff (1987) who argues that it incorrectly equates formal mathematical logic with human reasoning.

The advent of *Godel's Incompleteness Theorem* (Hersh, 1986) accompanied a shift away from formalist perspectives recognising that the mental activities of mathematicians were largely intuitive and not reflective of the symbol system or refined proof descriptions which found their way into textbooks. One outcome of the formalist influence is that the classroom learning culture tends to be more concerned with reproduction than conceptualisation. It downplays the interpretations and meanings students are ascribing to their mathematical activities and perpetuates the myth that the locus of the "right answer" lies external to the students themselves, that is in the form of the "answers in the back of the book" or "the teacher."

Falliblism. Falliblism attempts to describe rather than prescribe mathematics (Lakatos, 1976). It elaborates on what mathematicians do when they are practising mathematics, and the ways in which they apply what they know. To interpret an educational application of this philosophy would be to explore this utilitarian view of mathematics and trace the historical contribution mathematics has made to society.

Constructivism. Constructivism views mathematical knowledge as constructed in the mind of the learner, rather than existing as some entity waiting to be discovered. Although the teaching and learning of mathematics aims at developing shared meanings and understandings, learning itself is still a personal and unique experience.

Constructivism, during the 1980s (Cobb 1986; Labinowicz, 1985; von Glasersfeld, 1987) began what was to continue into the early 1990s. As Noddings (1990) recalls "The word has become the battle cry for a reconsideration of our problems and our best road toward the solution (p. 2)".

The mathematics education literature concerning constructivism can be found, for example, in the writings of Cobb (1986), Confrey (1987), Dorfler (1987, 1989), Kamii (1987), Labinowicz (1985) Steffe (1987, 1990) and von Glasersfeld (1987, 1990a). It reveals a struggle to identify the implications of constructivist theory for how teachers should teach and students learn in mathematics classrooms.

Romberg and Carpenter (1986) observed that the constructivist movement came at a time when research was questioning traditional approaches to the teaching and learning of mathematics. They went on to comment that traditional teaching styles were characterised by "extensive teacher-directed explanation and questioning followed by student seatwork on paper-and-pencil assignments" (p. 851). Taylor (1992) summarises the limitations of traditional mathematics in the following way:

> First, in traditional classrooms, mathematics is promoted as a record of knowledge which is divorced from processes of inquiry by fragmentation into subjects, topics, lessons, facts and skills. Second, daily lessons are geared towards students passively absorbing the record of knowledge in relation to their established cognitive frameworks. Third, teachers' roles are determined largely by managerial concerns for covering the syllabus, maintaining control and order, and instructing the whole class, rather than by considerations of a conception of mathematical knowledge or an understanding of how individual learning occurs. (p. 5)

The extent to which these limitations still apply in mathematics classrooms needs to be investigated.

The report of the National Research Council (1989) in the United States of America describes traditional mathematics teaching as something which *teachers prescribe* and *students transcribe.* Similarly in Australia recommendations in the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990) encourage educators to move away from a transmission view of learning towards more constructivist approaches. The view of mathematics and the approaches to teaching and learning suggested in this Statement have implications for how mathematics learning should be supported. For example, the Statement suggests a classroom learning environment which encourages practical activity, the appropriate use of technology, and discussion. Mathematics can no longer be regarded as a chalk-and-talk subject from the perspective of the teacher, or as a textbook, pencil-and-paper subject from the perspective of the students.

Although constructivist ideologies have been embraced by many mathematics educators, there are different interpretations of the term "constructivism" (Ellerton & Clements, 1991). As a radical constructivist, von Glasersfeld (1987) takes the extreme perspective that knowledge does not exist outside the actions and experience of the learner, it is

> the result of an individual subject's constructive activity, not a commodity that somehow resides outside the knower and can be conveyed or instilled by diligent perception or linguistic communication ... language is not a means of transporting conceptual structures from teacher to student, but rather a means of interacting that allows the teacher here and there to constrain and thus to guide the cognitive construction of the student. (von Glasersfeld, 1990b, p. 37)

Thus, according to von Glasersfeld, concepts cannot be transmitted from teacher to learner.

Constructivism in education is not a single entity, but rather is something which can be, and in fact has been, interpreted in many ways. Phillips (1995) acknowledges that there "is an enormous number of authors, spanning a broad philosophical or theoretical spectrum, who can be considered as being in some sense constructivist" (p. 6). Taylor (1992), for example, describes a number of forms of constructivism.

- *cognitive constructivism* as a social learning theory (Zimmerman, 1981);
- *critical constructivism* in mathematics teacher education (Taylor, 1991);
- *contextual constructivism* in science education (Cobern, 1991);
- *cultural constructivism* (Scott, Cole, & Engel, 1992);
- *C_1 and C_2 constructivism* in mathematics and mathematics education (Lerman, 1989);
- *ecological constructionism* in educational research (Steir, 1992);
- *generative learning model* for science education (Osborne & Wittrock, 1983);
- *personal construction of knowledge* in science education (Driver, 1988);
- *personal constructionism* in science education (Northfield & Symington, 1991);
- *radical constructivism* in mathematics education (von Glasersfeld, 1991);
- *social constructivism* as a philosophy of mathematics education (Ernest, 1991);
- *social constructivism* in science and mathematics teacher education (Tobin, 1990);
- *social constructionism* and language in education (Gergen, 1992);
- *social constructivist perspective* on mathematics education (Bauersfeld, 1992).

The diversity of this list makes it clear that there is no single definition of constructivism. Strike's (1987) response to the burgeoning branches of constructivism was to find a common thread between different constructivists. One such thread he has identified was children's misconceptions. Phillips (1995) regards talk of student misconceptions as problematic because it implies a given "set of 'correct' conceptions that all learners should have" (p. 10).

At the heart of constructivism is the debate about what constitutes mathematical knowledge. Wheatley (1991), for example, in describing radical constructivism, comments that "we do not find truth but construct viable explanations of our experiences" (p. 10). Cobb (1990a) discusses practical issues and claims that constructivism "concerns the social and physical characteristics of settings in which students can productively construct mathematical knowledge" (p. 8). Each of these notions of constructivism has implications for the degree of "ownership" the learner associates with the mathematical knowledge gained. In contrast, traditional school mathematics was owned by teachers, examiners and textbook writers.

Current discussion is focused upon the issue of how the word *knowledge* is used (Cobb, 1995; Driver, Asoko, Leach, Mortimer, & Scott, 1994; Driver & Scott, 1995; Smith, 1995). Smith (1993, 1994) uses *knowing* to indicate the subjective meanings of the individual described by constructivists, and the word *knowledge* to indicate socially negotiated and accepted forms of language, described by socio-cultural constructivists.

Phillips (1995) in developing a framework for comparing constructivisms describe a certain polarisation between the individual psychology dimension of constructivism and what can be described as social constructivism:

> Some constructivists—Piaget and Vygotsky would be quintessential figures here—have been concerned with how the individual learner goes about the construction of knowledge in his or her own cognitive apparatus; for other constructivists, however, the individual learner is of little interest, and what is the focus of concern is the construction of human knowledge in general. (p. 7)

Further to this range of views of what constructivism can be found in the feminist epistemologies (Harding, 1993; Longino, 1993) who emphasise the socio-political nature of the construction of knowledge. Harding (1993) draws attention to the way in which society is stratified, be it by race, gender, class and so on. The activities of those in power "both organize and set limits of what persons who perform such activities can understand about themselves and the world around them" (p. 54). This view of constructivism would suggest a direct need to impact on pedagogic policy.

Constructivism for the classroom. The constructivist agenda has been successful in highlighting the importance of attending to how children know and think about various mathematical ideas and procedures, and the language that teachers and children use in the construction of knowledge. The key issue for mathematics educators is that they are still struggling to describe what a constructivist classroom might look like. As Putnam, Lampert and Peterson (1990) write

> In general, constructivist researchers have sought to understand the development or construction of mathematical knowledge that takes place through an individual's more or less natural interactions with the environment. It is not readily apparent, however, how this perspective should be applied to teaching and instruction. (p. 90)

More research is needed concerning the application of the constructivist knowledge about how children learn, for mathematics instruction.

Cobb (1990a) has probably offered one of the most concrete interpretations of constructivism for the classroom to be put forward by mathematics education researchers. He highlights five points:

> First, the claim that students can discover mathematics on their own is an absurdity ... Second, students do not learn mathematics by internalizing it from objects, pictures, or whatever ... Third, the instructional representation approach and its internalization account of learning does not address the very real danger that students will make a separation between mathematics in school and in everyday settings ... Fourth ... we can view mathematics as a communal practice composed of taken-to-be-shared mathematical actions ... mathematics is a normative conceptual activity ... and learning mathematics can be seen as a process of acculturation into that practice. Fifth ... mathematical thought is a process by which we act on conceptual objects that are themselves the product of our prior conceptual actions. (pp. 10-12)

Cobb's ideas may be constructive in as much as they challenge mathematics educators to reflect on particular aspects of classroom practice.

Social constructivism, according to Wood, Cobb and Yackel (1995), seems to be a more palatable version of constructivism for classrooms since "It is useful to see mathematics as both cognitive activity constrained by social and cultural processes, and as a social and cultural phenomena that is constituted by a community of actively cognising individuals" (p. 401). Confrey (1990) suggests that teachers pay attention to how the student has constructed an idea and if necessary "assist the student in restructuring those views to be more adequate from the student's and from the teacher's perspective" (p. 109). The implications for classroom practice is to focus on how children reorganise and reconstruct their experiences both physical and social. Tharp and Gallimore (1988) refer to the term "guided reinvention", where the child is transforming what is internalised, and captures ideas from both the cognitive constructivists and the social constructivists.

Solomon (1987) criticised constructivism for being rationalist and not addressing the social context in which the learning is taking place. Wheatley (1991) responded to such criticisms by commenting that in the classroom it was important for teachers to provide opportunities for meaning to be negotiated and consensus reached about mathematical understandings.

The idea of learning being jointly constructed during classroom interactions becomes pivotal in any discussion about pedagogy and is developed later in this literature review with reference to the writings of Chapman (1992), Lemke (1987, 1988, 1989) and Walkerdine (1988). From a constructivist perspective, as Smith (1995) explores, the "shared understandings" that are built up within classroom discussions are similar to Confrey's (1995) notion of "agreement to agree." It represents the location of the individual's construction within the context of the social activity taking place in the classroom.

This version of constructivism has recently been called the "joint approach" (Cobb & Bauersfeld, 1995) and the "emergent perspective" (Cobb, Jaworski, & Presmeg, 1995). It acknowledges the psychological constructivist's idea of the individual's reorganisation of concepts, but that this occurs during the participation of practices within the learning community, for example the classroom.

Simon (1995) was keen to see how constructivism translated into the reality of classroom practice. He experimented with teaching his university class in ways consistent with constructivist principles. He saw a unit of work planned for 1-2 periods extend into 8 periods and subsequently noted that learning involves a complex network of connections not the parcelling up of one skill or an idea at a time. Learning did not proceed linearly, but had several sites in a person's web of understanding on which that learning could be built. He also learned that even though he did not have a thorough knowledge of a particular domain, as his students engaged in problem situations, his own understanding of the particular concept developed.

The implication for the classroom from Phillips' (1995) point of view is to focus on more than the individual's construction of knowledge. He stresses the need to ensure that any constructivist epistemology should include the opportunity for the learners to construct knowledge in such a way that they enhance the inclusivity of the "knowledge-constructing communities" and "empower long-silenced voices" (p. 12).

Although it is generally believed that constructivist style leads to better understanding than a transmissionist style (Sierpinska, 1994), there is still ongoing discussion as to whether everything in mathematics can be reconstructed. She advocated that "some things in mathematics just have to be 'told' ... there is no way of making the students reconstruct some more advanced concepts in mathematics" (p. 68).

Taylor (1992) observed one teacher's attempt at applying a constructivist pedagogy and comments on the communications dilemmas which occur:

> A radical constructivist view of linguistic communications helps to explain why teacher pronouncements of the wrongness of students' mathematical ideas, and the reiteration of clarifying explanations, are likely to be ineffective for many students. Students are able to interpret teacher communications only from within their extant conceptual frameworks. Radical constructivism requires teachers to attempt to infer the nature of students' conceptual structures and operations ... and provide learning activities that facilitate their purposeful reconstruction, rather than assuming that clearer explanations of the teacher's conceptual structures will enable students more readily to see the teacher's intended meaning. (p. 169)

The ability of the teacher to build on what the student brings to the classroom rather than the focus being on the student doing the connecting with what the teacher constructs as mathematical knowledge is under investigation in this study.

Teachers who have recently experimented with a new branch of geometry called "fractals" would be aware of the accompanying message that mathematics is not a fixed body of knowledge but is a constantly developing discipline (Mousley, 1990b, p. 42). Mathematics frequently has its origins in order to satisfy some practical requirements, but inevitably, because of the nature of the human mind it seems to escalate and transcend its original function. Debat around this issue of whether mathematics should be real or abstract can be resolved by agreeing that it is both.

It is the intention of this thesis to investigate the location of the learner's mathematical understandings in the classroom discourse.

Language in the constructivist classroom. The language in the mathematics classroom, be it spoken or written, is unpacked by the learner. Different learners will do the unpacking in different ways depending on the prior knowledge of the terminology and concepts being mentioned (Bickmore-Brand, 1993; Ellerton & Clements, 1992). As von Glasersfeld (1983) explains, the reason we do in fact succeed in communication is not because we share an identical match with the speaker's representation, but our "understanding" of each other occurs since "it is quite sufficient that the communicators' representations be compatible in the sense that they do not manifestly clash with the situational context or the speaker's expectations" (p. 53).

As Mousley and Marks (1991) report, the ability to calculate correct answers to mathematical textbook questions is often partly related to the learner's ability to read and interpret the questions. The work of Orr (1987) depicts students for whom the language associated with the mathematics they are doing in school conflicts with the language they use outside of school, that is it differs in its structures, constructions and conventions. This conflict means that students appear to lack understanding of formal structures.

Pimm's (1987) work on the complexity of the mathematical writing system discusses the lack of redundancy in the writing system. "Elegance is measured in part by brevity and in part by simplicity. Accessibility plays no part ... pupils need considerable training on *how* to read mathematics" (p. 184).

The value therefore is in the language exchanges or social interaction in learning which occurs in the classroom (Yackel, Cobb, Wood, Wheatley & Merkel, 1990). Wheatley (1991) sees the social construction of literacy as an aspect usually undervalued by many radical constructivists. Kamii (1984) acknowledges that while children construct their mathematical concepts from within (Piaget & Inhelder, 1963), she sees the importance of constructing their ideas in a dialogue with other children and their teacher.

Concluding Discussion

In relation to the paradigm shifts that have influenced mathematics education, there is an accompanying scepticism from writers, for example, Higginson (1989) and Watson (1989) when they quote Kuhn (1970) and Wittgenstein (1958, 1976, 1978). They serve to unsettle what could appear to be a well defined field. Not only were they challenging "What is mathematics?" they were also questioning "What counts as mathematics?". Wittgenstein considered the foundations of mathematics, and was intent on stripping away the myths that mathematics was objective and that it was in some way beyond scrutiny.

As Nesher (1988) suggests, mathematics learned in school has a completely different agenda from the utilitarian one. It is aimed at a set of concepts which, although perpetuating the culture, remains general and abstract and largely devoid of specific context. Feynman (1985) captures what he sees as the results of the present system:

> After a lot of investigation, I finally figured out that the students had memorized everything, but they didn't know what anything meant ... So, you see, they could pass the examinations, and "learn" all this stuff, and not know anything at all, except what they had memorized ... Finally, I said that I couldn't see how anyone could be educated by this self-propagating system in which people pass exams, and teach others to pass exams, but nobody knows anything. (pp. 212, 213, 218)

The pedagogical implications are for mathematics educators to reconsider the content of what is being taught in classrooms and whether the teacher and learner share the same logic reasoning system, and is this the logic and reasoning system we find reflected in mathematics?

Researchers` into cross-cultural learning are frequently coming across different ways of organising the world, for example in studies of different aboriginal groups (Christie & Harris, 1985). They remind us that there is an inculturation of mathematical knowledge through the institutions of that culture's society, and can take a very "man-made" form (Alcoff & Potter, 1993). There can be philosophical "blind spots" which may still block communication in the mathematics classroom, for example, when a second language learner is working out the relationship between what they are being presented with and their existing knowledge.

To take on board the message of the constructivist philosopher is to encompass awesome consequences for the content and the delivery of mathematics education. In 1990 Pateman and Johnson tried to capture the challenge of the discussions at that time. Teachers will need to address,

> content (which can hardly be rigidly prescribed in advance by the constructivist teacher), methodology (which probably needs to be idiosyncratic to children and context), and assessment (particularly difficult for those so used to competitive ratings). The constructivist teacher will need to be somewhat of an opportunist, and also an able elementary mathematician willing to continue to learn both about mathematics and children in the attempt to develop them as autonomous creators of their own mathematics. (p. 351)

In Phillips' (1995) view pedagogy will need to go further than that. It will need to acknowledge that there are considerable constraints impinging on the knowledge-constructing activities presented in classrooms. For Phillips, he would see the current direction of constructivism including the potential to "improve the natûre and operation of our knowledge-constructing communities, to make them more inclusionary and to empower long-silenced voices" (p. 12) (see also Goldman, 1992).

At the crux of the research reported in this thesis is the extent to which a teacher's beliefs about how mathematics should be taught impacts on how that teacher operates in the mathematics classroom, and on the messages students make explicit regarding the subject of mathematics and the way it is being presented in their classroom.

Teaching and Learning Principles Generated from Language Learning Principles and Associated Mathematics Education Research

The following part of the Literature Review provides a synthesis of the research which has informed the development of the seven teaching and learning Principles put forward by Bickmore-Brand (1989). The seven Principles —labelled[1] context, interest, modelling, metacognition, scaffolding, responsibility and community—will be used as a framework for analysing the data collected in this thesis. Relevant mathematics education research has also been described.

Principle 1: Context—Creating a Meaningful and Relevant Context for the Transmission of Knowledge, Skills and Values

Introduction

The premise behind this teaching/learning Principle is that most learning occurs naturally within a context which makes the value of the learning obvious to the learner and motivates the learner to acquire the skill. Advocates of this Principle recommend the need to create holistic contexts for skills development (see, for example, Bickmore-Brand, 1989; Cambourne, 1988; Goodman, 1983; Holdaway, 1986; Smith, 1988).

Making Sense of the World

Cambourne (1988) developed a comprehensive set of conditions for learning, based on natural language acquisition. He notes that we accomplish

[1] Note that the labels have been chosen to describe a cluster of associated constructs which form the basis of the framework. The order in which the principles are presented is arbitrary and does not infer a hierarchy.

incredibly complex rules of language from the cultural context into which we are born and raised. These rules, although embedded within the culture, come to us through an immersion which is "always whole, usually meaningful and in a context which makes sense or from which sense can be construed" (p. 34).

The immersion component of children's language learning in relation to children's understanding of print has been highlighted by Goodman (1983). She indicates the wholeness of this meaning-making process for children and that their approximations enable them to refine their ability to cope with less contexualised situations, or contexts which are different. Smith (1988) observes that, when children are immersed in language, they attend to what makes sense and ignore what is useless.

One of the by-products of using language is that it can develop our understanding about how language can be used. Although opportunity for the constructive use of language does occur in classrooms, children frequently receive negative messages about language. For example, some learn that reading and writing can be meaningless and laborious, and that large sections of text do not necessarily make sense.

Holdaway (1986) and Smith (1988) both advocate the need to create a community of learners in the classroom for this process of language acquisition to be maximised (this will be taken up in the *"community"* language/learning Principle). Holdaway (1986) has this to say about the community context needed for learning, and the acceptance and adoption of only what is meaningful:

> Language is not only the most social of human skills, it is the necessary condition for specifically human society of any kind. It is learned in strong communal settings and fueled by social satisfactions. Conversely, when a particular linguistic item does not embody communal meaning, it is not likely to be learned in any useful way. (p. 69)

Thus context is viewed as central to the development of meaningful language.

Holistic nature of learning.

Although behaviourists and psychologists disagree with one another in many areas, it could be argued that they agree on the ultimate importance of involving learners wholly in the process of acquiring knowledge (see, for example, Bruner, 1986; Kelly, 1955; Piaget & Inhelder, 1969; Sadler & Whimbey, 1985; Skinner, 1956). Kemp (1985) likens the holistic nature of language acquisition to learning to drive a car:

> Automatic bike-riding and car-driving and, more especially speaking, are skills acquired by practising the totality of the art rather than their specific ingredients. One doesn't learn to pedal a bike without being able to balance on it, and balance is maintained by pedalling. One doesn't learn to decode language symbols if meaning isn't their basis, and it is meaning that should make the response to reading a set of symbols self-confirmatory or self evidently wrong. (p. 50)

Thus textbooks and curriculum which presents information in chunks or in a disjointed way, can be criticised for not establishing meaningful contexts. Sadler and Whimbey (1985) feel that this lack of integration makes it difficult for learners to relate or generalise to other contexts. They suggest that the mastery of a skill needs to be regarded as developmental, with progression to the next step not necessarily implying perfection of the step before.

Learning was regarded by Bruner (1986) as a spiral process, in which the learner begins to cope with information in increasingly less contexualised or in different contexts. Applebee and Langer (1983) are critical of the artifical boundaries of subject areas

> Rather than separating students' learning of subject-area content from their developing thinking and language skills, such activities integrate new learning with ways in which students express their knowledge. (p. 175)

Thus how information is presented in schools can assist students to make meaning. Mousley (1990b) stresses that in order to understand the integration of mathematical concepts it will assist students if the problems used in classroom mathematics are "drawn out of children's daily lives" (p. 40) so they can make appropriate links. Mousley continues: "It seems a pity that studies of ratio, proportion and similar shapes are frequently abstracted away from the reality of our students' lives" (p. 39).

Making the purpose for learning explicit.

Athey (1983) describes learning contexts as being socially negotiated and that it is desirable for learners to be wholly involved rather than simply being asked to learn by rote. Boomer (1988), an advocate for the use of real contexts for learning, notes the importance of concepts being taught within a context which can be transferred to other contexts. For this reason it is important that students recognise the intention behind the activity—students need to see how an activity fits into the broader context, and to understand the purpose of doing it at all (Applebee & Langer, 1983).

The presentation of knowledge in this approach would be more consistent with "action knowledge" (Barnes, 1976), rather than "relational" (Skemp, 1976), where the students so understand the underlying principles of what they are learning that they can reapply them to solve problems in different contexts.

Social contexts of learning.

The everyday mathematics of the student's world differs from culture to culture and D'Ambrosio (1984, 1985) used the expression "ethnomathematics" to describe the mathematics that people use in their everyday lives. Hence the kinds of mathematical knowledge children bring to school may be quite different from the mathematical concepts which they encounter in classrooms. The cultural context of any school setting can in fact, be a powerful source of mathematics for classroom applications.

When school mathematics is taught without reference to real-world contexts, students often experience difficulty in linking what they do in class with their personal worlds. As Chapman (1992) states:

> The shared meanings of mathematics include mathematical techniques that is, procedural knowledge, as well as conceptual knowledge. Knowing how to perform actions in certain ways is a strong feature of school mathematics. Standard written algorithms often provide a focus for mathematics lessons. Knowing how to "do" long division, for example, really means knowing how to do it the "correct" way. Factorising binomial expressions involves performing a pattern of actions that is likely to be practiced and repeated many times in the mathematics classroom. The method is as important as the result. Actions such as these are ways of making meaning in mathematics. They are semiotic practices that make sense in mathematics. They are also an important part of what presently constitutes school mathematics. (p. 37)

Thus from an anthropological perspective (Bishop, 1992; Shweder, 1983; Zaslavsky, 1992), learning mathematics can be seen as a process of acculturation into that practice. Bishop (1992) would argue that it is a myth that mathematics is a culturally neutral phenomenom, and notes that what can be referred to as "western mathematics" is "one of the most powerful weapons in the imposition of western culture" (p. 33). He discusses the inappropriateness of some of the problem-solving exercises that were presented in Tanzanian colonial textbooks, and acknowledges that "appropriateness" was entirely judged in terms of cultural transmission" (p. 36).

In 1987 de Lange developed what he called *The Realistic Mathematics Education Model* which was based on students coming to mathematical understandings through their efforts to solve problems from real life. The primary focus is the intersection between how the student perceives mathematical ideas in real world contexts and the mathematical world.

School contexts for learning.

The term "context" is used in the literature to encompass a range of meanings, including linguistic, cognitive and social factors. In the development of Bickmore-Brand's (1989) seven Principles, the "context" Principle adopted Green's (1988) notion that subject areas are "contexts for learning and meaning" (p. 2) and the knowledge, skill or values being imparted within these subject areas must fit within the wider understanding that children can access through their own culture. As Lemke (1990) discusses, people make meaning about, or make sense of, an action or event by "connecting" it to a context.

Anderson (1990) remarks "Do we intentionally mislead or confuse our students by failing to include meaningful context or by presenting mathematical models as reality?" (p. 327). Resnick (1987, p. 16) has identified four areas of discontinuity between learning done in school and mathematical activities done out of school. First, school tasks are predominantly individual, whereas in mathematical tasks associated with the real world, calculations are often socially shared. Second, schools discourage the use of mathematical aids, whereas in society people generally access cognitive tools for efficiency. Third, schools tend to value abstract and symbolic processing, whereas in society mathematics is usually situational and directly connected with objects. Fourth, schools aim at giving students a broad knowledge base, whereas society tends to require a competence which is location specific. Nesher (1988) adds another area which acknowledges that schools are environments which are designed intentionally to promote certain knowledge and with it norms and certain social knowledge (p. 56). Nesher (1988) also expresses her concern about the lack of transferability of school mathematics into the real world of the student. She decries the proportion of school time which a student spends working on "exercises that do not teach him anything" (p. 72), (see also Robitaille & Garden, 1989).

In Germany in the early 1970s education underwent change by introducing streaming into what was a traditional education system. At the time, Freudenthal (1973) was quite outspoken about the unnaturalness of separation in a society where people mostly worked in heterogeneous groups. He was even more concerned about the learning processes students were experiencing. He advocated "mathematics for everybody", where there needed to be a closer connection between everyday reality and mathematics in the classroom.

This demand for relevance in school mathematics is not new. Kilpatrick (1992) noted the comment of Caldwell and Courtis (1925) that "the subject matter of arithmetic is today being continuously modified to eliminate all elements of little direct use in the life of the child" (p. 68). In the 1990s, senior secondary mathematics courses include an abundance of contextualised topics (see, for example, articles written about the Victorian Certificate of Education in Clements (1990)). Willis chaired the project "Algebra for All Australians" which attempted to relate the content to real situations through the ideas of algebraic modelling (see *"modelling"* Principle in this chapter). Fitzsimons (1990) describes the growing trend towards real data applications for senior secondary mathematics. See also Clarke, (1995); Evans, (1992); Gnanadesikan, Schaeffer, and Swift (1987), Harris, (1992); Landwehr (1990); and Landwehr, Swift and Watkins (1987), for other examples of mathematics being taught in context.

Sommers (1992) asked students to develop mathematical projects about activities which were of interest and relevance to their own communities. She hoped that by preparing statistical reports on topics such as "daily traffic" or "salaries and averages in professional sports," students would see some of the ways in which mathematics is used in everyday life.

Van den Brink (1988) recommends that "people" contexts rather than "object" contexts need to be employed in early childhood teaching. In this way, the concept development is linked to a familiar context such as toys, animals or people. Clements and Del Campo (1990) are concerned that teachers should attempt to assist students to create links between the language and symbols of the mathematics studied in school, and the real-world context. Extensive studies of children's fraction concepts (see, for example, Clements & Lean, 1988; 1994; Ellerton & Clemènts, 1994) suggest that there is no guarantee that having "fraction-related knowledge" in one real-world context (e.g. being able to pour, from one full glass, three equal quantities in three other glasses) will carry over to other real world contexts or that it will be related to the concept of one-third.

Wright (1994) is critical of the current trend in mathematics to over-emphasise the use of real world applications in mathematics classrooms. He claims that it is not necessary for young children to solve real world problems to understand arithmetic, and similarly he does not agree that engaging children in problem-solving strategies will necessarily involve them in real world problems. His recommendations are echoed in the "interest" and "metacognition" sections of this chapter. Wright believes that teaching emphases should be on engaging children in solving problems which they find challenging and which require reflective thinking on their own mathematical processing. Such problems may involve real life situations but should not necessarily be required to do so.

Textbook Contexts.

Almagor and Berman (1990) discuss the need for students to see mathematics as applicable to the real world. They point out that many examples in text books bear little relation to the students' interests or worlds even when they are set in practical contexts. They emphasise, therefore, the importance of believable applications. Costello (1993) agrees that students do not necessarily relate to the applications provided by their textbooks, but suggests that a broad investigation such as one centred around the zoo, is more likely to cater for a diverse range of interests.

Pengelly (1987), wishing to remain faithful to mathematical structures and concepts, encourages the development of links between mathematical understandings and real-life situations. Ellerton's (1988) work on the perceptions that learners hold about mathematics noted how bound they were to a textbook/ teacher/ examiner-oriented view of mathematics. Clements's (1984) statement has special relevance to the language found in textbooks. He felt that it could be argued that many children are "likely to experience more language difficulties in the mathematics classroom than in any other place which they are required to attend on a regular basis" (p.146).

In his list of the five main points in the application of constructivism to the classroom, Cobb (1990a) questions the practice of teaching the rules before giving students any opportunity to learn by applying them to a real life situation. The accompanying idea in Bickmore-Brand's *"context"* Principle is that a student be given a sense of the overall "big" picture before starting to work on detailed examples. Presmeg (1986) recommends providing a visual representation of a problem until practice of a procedure or application of a formula leads to the student being able to internalise the concept.

Conclusion.

A range of meanings can be associated with the term context. One interpretation, adopted by Chapman (1992), and used by Bickmore-Brand (1993) seems to imply that only one context—the real life context from which the mathematics was drawn—is valid for mathematics classrooms. However it would appear important to incorporate Lemke's (1987) notion that meaning making is interdependent on the social practices of the community within which it is embedded. Thus many children experience difficulty with mathematics when these connections outside the mathematics classroom are not made explicit. Praeger (1993) alludes to the decontextualised nature of mathematics education which has often resulted in a poor attitude towards mathematics among our society:

> If, as seems to be true, mathematics empowers the individual and society, why do so many people seem to regard mathematics with either fear or disinterest? Mathematics is often 'seen as a body of knowledge to be memorized rather than a way of knowing' (Mary Barnes in Haysom, 1988). It is therefore regarded as stagnant and isolated from reality. At the same time, those talented in mathematics are seen as strange or perhaps threatening. (p. 1)

Thus, if students are to see mathematics as a resource for making sense of our world, then learning mathematics needs to be interwoven with the real world from which it is drawn.

Principle 2: Interest—Realising that the Starting Point for Learning must be from the Knowledge, Skills and or Values Base of the Learner

Introduction

The premise behind this teaching/learning Principle is that in order for learning to take place, the learner has to connect the information in some way to what s/he already knows (Cobb & Steffe, 1983; Steffe & Cobb, 1988). The consequence of such a procedure is that knowledge will be idiosyncratically processed and stored by each individual (Bickmore-Brand, 1989). This Principle has been well established by educational research (see for example, Bickmore-Brand, 1989; Bruner, 1983; Goodman, 1983; Kelly, 1955; Piaget & Inhelder, 1969; Vygotsky, 1962).

Connecting to What is Known

According to Kelly (1955), we construct our concept of the world and test it against the real world. This learning is a tensely active process, and new knowledge may often challenge our existing knowledge. Our ability to accommodate new information or experiences into existing conceptual structure will depend upon how dearly we hold onto our constructs (Bawden, 1985; Papert, 1980). Individual differences in how we view the world will also be influenced by how varied our life's experiences are and whether they provide an appropriate source for new constructs. Children do not necessarily "see" or "remember" or "copy" what they are exposed to but reconstruct it for themselves (Copeland, 1984). Von Glasersfeld (1983) suggests that "truth" is considered to be derived from a variety of paths of action, and so any one construction may be equally as valid as another. Eric Smith (1995) expresses it in this way:

> The issue of importance here is that children *reorganize* and *reconstruct* experiences of their physical and social environment. The mental plane is not isomorphic with the external plane of action and speech; as the external plane is internalized, transformations in structure and function occur. In this respect Vygotskian theory is similar to the Piagetian perspective in recognizing that the child cannot be a passive recipient of knowledge. (p. 28)

Athey (1983) and Frank Smith (1988) both point out the, at times frustrating, reality that the communication of ideas between people goes through the filter/ distortion system of individuals.

One of the major influences on mathematics education is psychology (Grouws, 1992). Educational psychologists concern themselves with how (mathematics) content is taught and learned; in particular, psychologists address the processing of concepts and the conditions which enhance or detract from learning, and consider how and when concepts are developed (Bruner, 1978; Piaget, 1973). To a large extent language learning research was also influenced by cognitive psychologists, and subsequently those concerned with learning to read, for example, the schema theorists (Pearson & Johnson, 1978; Rumelhart, 1981; Tierney & Pearson, 1981). The underlying assumption about learning is that new information needs to be connected with prior knowledge, however scant this may be.

Resnick (1980) coupled constructivist ideas with information-processing strategies and recommended that the teacher needs to "make contact with already stored knowledge" (p. 136) of their students. Teachers, aware of the need to make connections with what students already know provide student-centred activities which are either concrete, or analogies via which the learner can frame the new information. Ausubel (1968), in the epigraph of his book *Education psychology: A cognitive view*, said "If I had to reduce all of educational psychology to just one principle, I would say this: The most important single factor influencing learning is what the learner already knows" (p. vi).

Constructivists believe that the learner will feel a sense of "ownership" of the mathematical knowledge when it is actively linked to his or her own world—"When someone actively links aspects of his or her physical and social environments with certain numerical, spatial, and logical concepts a feeling of 'ownership' is often generated" (Ellerton & Clements, 1991, p. 56).

Cobb (1987a) expressed concern about basing pedagogy on the information-processing model, particularly when it continues to perpetuate the metaphysical realist assumptions that signify the objectified existence of mathematical knowledge (see Cobb's (1987a) critique of the schema theorist). The teacher's role becomes one of translating the "public information" of that culture for the students who are then in the role of decoding the information and then encoding it into their own schemas or "private knowledge." Rather than the teacher presenting the information as reified knowledge, the pedagogical shift is towards the meanings and processes of the students' construction of new mathematical knowledge.

The underlying philosophy of Cognitively Guided Instruction (CGI) is the view that teachers need to make instructional decisions about linking new information to the student's existing knowledge (Fennema, Carpenter & Peterson, 1989a). The model developed by these researchers shows a clear relationship between what the teacher believes about how students learn and about teachers' beliefs concerning their own content knowledge and the instructional practices which evolve. This approach shares some elements in common with the model developed by Weaver (1980); see the "community" section in this chapter.

In his summary of where mathematics education now stands and where it may be going, Davis (1992) begins his recommendations with the statement "Build on the ways that students actually think" (p. 724), which recognises the importance of how students learn. It is this theme which is central to the *"interest"* Principle.

Mousley (1990a) describes a significant study of the introduction of a new unit of work to the Years 11 and 12 Victorian Certificate of Education (VCE) mathematics assessment. The nature of the project demanded that both teachers and students go beyond traditional approaches to the teaching and learning of mathematics, and was responsive to individual student interests and needs, which "could no longer be assumed to be the same across a class" (p. 54). The Victorian Ministry of Education's (1988) curriculum document *The Mathematics Framework: P-10*, in referring to the kinds of mathematical experiences which would be appropriate for Years P- 10, state that:

> Mathematics can make sense when it is linked to what students know, or want to know. We must build students' mathematical knowledge, not just out of books, but out of their interests and experiences ... the history of mathematics reflects that mathematical advances were made by people solving problems which interested them, problems which were practical in nature. (p. 12)

It is not uncommon for students to be able to do quite complex mathematical calculations in their day-to-day lives but experience great difficulty when placed in a classroom to perform a "sum." An extensive study conducted in New Zealand (Young-Loveridge, 1989) looked at the early years of schooling and the lack of relatedness between what children already knew and what was being taught. Young-Loveridge observed that:

> Large numbers of children were taught certain concepts (e.g., rote counting, enumeration, pattern recognition, ordinal numbers, numeral recognition) even though they already knew them, but were not taught addition and subtraction which they could also do. (p. 60)

This is a finding which is consistent with the notion that what is taught in schools is not well matched with the understandings that the learner brings to the classroom.

Rarely in teaching practice does the teacher or the textbook encourage students to develop concept schema by attempting to link new concepts with other mathematical or scientific concepts. At times, there may be a cursory association of a given concept with aspects of the students' everyday experiences (Ellerton & Clements, 1991; Pateman & Johnson, 1990; Pengelly, 1990). According to Brown, Collins and Duguid (1989) students are participants in an academic culture which may not necessarily reflect what "just plain folks" do with mathematics:

> The general strategies for intuitive reasoning, resolving issues, and negotiating meaning that people develop through everyday activity are superceded by the precise, well-defined problems, formal definitions, and symbol manipulation of much school activity. (p. 35)

The literature on the teacher or learner's inability to connect what the learner already knows to the classroom is well documented (Balacheff, 1991; Bero, 1994; Cobb, 1985; Ellerton & Clements, 1987; Pengelly, 1990; Sierpinska, 1996; Steffe & Cobb, 1988; van Dormolen, 1993). This is particularly the case in the learning environments of second language, languages other than English, Aboriginal and disadvantaged learners (Bishop, 1992; Carrahar, 1991; D'Ambrosio, 1991; Enemburu, 1989; Saxe, 1988).

This is in fact consistent with the ideas embedded within the *"interest"* Principle. In many mathematics classrooms, students have not been able to make sense of, or attach meaning to school experiences. In an attempt to survive, many resort to rote learning, either to satisfy the teacher, or to pass tests and examinations, or both.

Learners Generating Their Own Rules

The importance of encouraging learners to generate their own rules about a concept was a theme taken up by Kamii (1989). She pointed out that it has been tempting for those designing mathematics courses to develop conceptual understanding along "logical" paths. But, as Kamii recommends, children can invent their own procedures, and generate their own rules for conducting a certain procedure (see Kamii, 1989 for discussion on children's invention of procedures for two-digit addition and place value).

Although some of these self-made rules can interfere with the development of other rules which may have more currency, through hypothesising and testing, the learner discovers or self-generates a concept (Green, 1988). In the process the learner identifies which attributes can be generalised to newly encountered examples, and is able to discriminate between examples and non-examples (Tennyson & Park, 1980). It is inevitable that the learner will make mistakes in this process but Goodman (1983) prefers to conceive of these errors as "miscues" or as "misperceiving" and consider them as opportunities, as a window might be, to view into the learner's mind.

This was echoed in the constructivist literature pertaining to science teaching which was concerned about reconstructing students' misconceptions, or distorted preconceptions (Posner, Strike, Hewson, & Gertzog, 1982). Research on students' conceptions (Ausubel, 1968; Bawden, 1985; Kelly, 1955; Solomon, 1987) indicates that students' beliefs about the nature of the world is based largely on their experience and that even when presented with scientifically-based and rational explanations, many have difficulty "letting go" of their old beliefs and adopting a new perception. This notion of "cognitive conflict" (Ellerton & Clements, 1991) has its origins in Piagetian understandings of cognitive development. In order for a student to change their ideas about a concept, either he or she has to either feel dissatisfied with his or her own knowledge, or become attracted to the benefits of entertaining a shift in their thinking. Skemp (1986) develops this in the following way:

> A schema is of such value to an individual that the resistance to changing it can be great, and circumstances or individuals imposing pressure to change may be experienced as threats—and responded to accordingly. Even if it is less than a threat, reconstruction can be difficult, whereas assimilation of a new experience to an existing schema gives a feeling of mastery and is usually enjoyed. (p. 42)

Skemp is therefore suggesting that assimilation of a new concept is more likely to occur when the learner modifies an existing schema rather than reconstructs a concept completely.

Observations of children performing real-life problem-solving tasks showed that children preferred to use their own routes or have their own strategies even when school routines have been taught and rehearsed (Ginsburg, 1982; Saxe & Posner 1983; Scribner, 1984). Carraher and Schlieman (1985) found that most students displayed a wide range of self-generated strategies and competencies in performing mathematical computations in real world settings (for example, as candy sellers on the streets of Brazil). These strategies and competencies are rarely evident in the classroom when the same students perform algorithms involving the same properties of the numeration system.

Davies (1993) investigated factors which inhibited the idiosyncratic interpretations of experience and found, that although children began school confident in their own ability to perceive and reason independently, this independence increasingly deferred to the teacher's knowledge the further students moved through the school system. Wheeler (1982) pointed out that, as human beings, we are already equipped to think mathematically, just as we are to speak, and that mathematics is everyone's "birthright" (p. 23).

Most radical constructivist pedagogy is concerned with how students organise their own concept base when exposed to new information: "This is the primary reason why radical constructivists focus on students' activities rather than on abstract relationships that they can *see* in students' environments as sources of knowledge" (Cobb, 1987a, p. 29). One of the focal points for this thesis is concerned with how teachers handle the divergencies produced when students generate their own rules to solve a problem or share their own misconceptions with the class.

Transfer of Knowledge

Perception of experience is not enough according to Kolb (1984), who maintains that learning and therefore knowing, requires both a grasp or figurative representation of the experience and some transformation of that representation. This is in line with the work of other writers—Boomer (1988) and Barrett (1985)—who see the transfer of knowledge to other situations as not only efficient learning, but desirable. Athey (1983) refers to the most basic form of knowing—associative learning—and quotes Gagné (1970) and Piaget (1969) who both suggest that association plays an important role in our ability to store a signal in our mind.

One aspect of the *"interest"* Principle is starting where the learner is at. Therefore when a student has a particular interest in a particular subject, for example, art, sport or music, the teacher can capitalise on helping the student to make the links between the two areas (see Almagor & Berman, 1990; Clements & Del Campo, 1987; Costello, 1993; de Mestre, 1987; Gentile, 1980).

Even though one aim in mathematics education is for the learner to be able to use the register, or appropriate language for that situation, it is likely that considerable trial and error on the part of the student will be needed. As Brown (1958) suggests, the teacher can be aware that

> the word is "an invitation to a concept" inviting attention to itself and to the related verbal and non-verbal context when it is heard again; but it must be heard again and again, and in varying contexts as well. (p. 16)

This is consistent with Boomer's (1988) recommendation that the more of one's own language that one can explore a new idea with, and the more one can represent the new idea to one's own experiences (through talking, writing, drawing, modelling, and so on) then the more likely it will be that one will come to understand. This elevates the importance of the learner's own personal language and learning style in the classroom, because it is through students' personal language that they can relate their own lives to the subject matter being taught (Pinnell, 1985). Athey (1983) sees language as being limited to the extent to which it can be tied in with the child's concrete experiences, and suggests that other representations such as painting, fantasy, play and so on can be called upon. Newman (1983) talks about not being able to have control of the medium unless you can make meaning in that medium. One of the themes taken up in this thesis is the degree of idiosyncracity with which learners solve non-routine problems.

Smith (1995) discusses the contrast between students' understandings concerning the historically developed cultural mathematical tools and "constructivist analyses that focus on the emergence of ways of languaging and symbolizing within the local community" (p. 26). He refers to the work of Driver, Asoko, Leach, Mortimer and Scott (1994) who argue that mathematics and science are symbolic activities.

Register

The linguistic term "register" has been described by Halliday (1978) as a "set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p. 195).

The "language of the learner" is a key factor in the "interest" section as is the "language of mathematics" and the teacher's ability to help students develop skills in using the mathematical register. The "language of the teacher" will be dealt with in more detail in the "scaffolding" section.

Communicating in mathematics. It is now widely accepted that the specialised language of mathematics can interfere with the way of communication of ideas to those who are less mathematically literate. Clements (1982) concluded that mathematics is probably the hardest of the school registers for students to access. As noted in the previous section on "context," Clements (1984) states that children are likely to experience more difficulties with the language in the mathematics classroom than almost any other place which they are likely to frequent. Discussions in this past decade (see Hunting, 1988; Marks & Mousley, 1990; Reeves, 1986; and Watson, 1993) about the relationship of language to mathematics, frequently deal with the highly specialised vocabulary of mathematics which involve a reinterpretation of everyday language (Bickmore-Brand, 1993).

A distinction can be drawn between oral mathematics, in which the focus is more on the semantic structure, and less on syntactic expression, and written mathematics, in which an unfamiliar word order or syntax can induce a loss of meaning for the reader (Grando, 1988). Students may need assitance to express their meaning in syntactically appropriate written forms. There is increasing support for the practice of using the learner's own language to clarify his/her thinking, in particular about the mathematics he/she reads, and use his/her own language to make linkages and to understand new concepts (Del Campo & Clements, 1987a; Hersh, 1990; Hughes, 1986; McIntosh, 1988; Reeves, 1986; Robinson, 1986; Waters & Montgomery, 1992, 1993).

Pimm (1987) discussed the notion of describing mathematics as a "language" as a metaphor for understanding mathematics in linguistic terms—for structuring "the concept of *mathematics* in terms of *language*" (p. xiv). His later work (1991) draws attention to the difference between written and spoken mathematics, the former relying on complex symbol systems and the latter which uses "natural" language which assists the student to reflect on and to "conjure and control" the mental images in mathematics (p. 23).

Clements and Lean (1988) have emphasised the importance of learners making connections between familiar concepts, with the formal mathematical language, and the manipulation of symbols (p. 222).

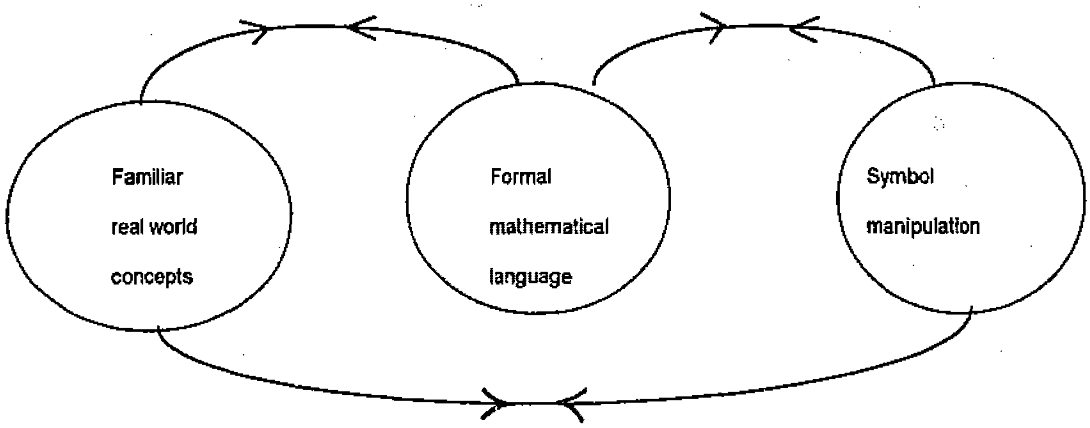

Figure 2. Establishing Links in Cognitive Structure (Clements & Lean, 1988, p.222).

It is possible that unless these links are made students learn only fragmented pieces of information which are associated with what can be termed "school mathematics." Thus a major concern is that "if links are not initially drawn by children between their informal knowledge and the written symbols, children may develop separate systems of arithmetic, one that operates in school and one that operates in the real world, and they will not readily see the connections between them" (Grouws, 1992, p. 83; see also Carraher et al., 1987; Cobb, 1988; Ginsburg, 1982; Lave, 1988). Research continues to explore how these relationships can best be developed within the school setting (e.g., Bero, 1994;

Bickmore-Brand, 1990; Chapman, 1993; Clements & Del Campo, 1989, 1990; Gawned, 1990; Harris, 1991; Reeves, 1990; Watson, 1990).

Specialist vocabulary. Mathematics has its own specialised vocabulary as well as making use of standard words in non-standard ways. Students are often unable to adapt to the use of different semantic structures (Carpenter, Hiebert & Moser, 1981) or are distracted by an unfamiliar syntax (Goldin, 1992), even when similar information has already been provided.

To help children become aware of the language of mathematics, a number of teachers asked students to write definitions of specialised terminology (Abel & Abel, 1988; Cosgrove, 1982; Evans, 1984; Ganguli, 1989; Havens, 1989; Keith & Keith, 1985; MacGregor, 1990; Myers, 1984). Peters (1993) refers to a class teacher whose students produced a manual to which they could refer if they forgot the meaning of specific terms. Students' comments revealed that the development of the manual had had a useful metacognitive function, "When I know it I can go back to it. Even my old maths books from last year and stuff, I've got them in my desk and we've done, like some of it's revision for me and I look in there and I've got some stuff and now I know how to do it" (p. 158).

Language is not neutral, and words have different meanings for each person depending on his/her experiences. Language is so closely linked to concepts that it is difficult to separate a concept from its name, as Vygotsky's (1962) famous example of an experiment where children were told a dog could be called "cow," showed.

Part of the daily routine task of the mathematics teacher is to help students learn the vocabulary and syntax of the mathematics register. Chapman (1992) demonstrates this with several examples:

> The reinterpretation of existing words is a trait of school mathematics and of mathematics generally. The word 'show', for example, in the context of the mathematics lesson becomes an instruction to prove or justify, rather than its everyday or non-mathematical meaning, to display or point out. The words 'set', which has a number of non-mathematical meanings, takes on specific properties in mathematics. (p. 41)
> ... While certain features of the mathematics register, such as those described above, can be isolated and identified, the language used in the mathematics classroom cannot be regarded as a fixed or distinct set of words. Words alone do not carry the meanings of school mathematics. To operate and control the register of school mathematics, learners need to master its complex systems of meaning relations: its 'ways of thinking'. (p. 43)

As Chapman suggests, far more complex structures than vocabulary are being communicated in mathematics classrooms.

Family of registers. Green (1988) uses register to include a number of different registers operating within one subject register. For example, Green talks of a family of registers, where not only is formal language used, but the register of teaching, where different kinds of language are used for different activities within the lesson, is also present.

Pimm (1987) introduces the term "register" in relation to how language is modified during attempts to communicate mathematical ideas (p. 196). Pimm makes a useful distinction between "talking for myself" and "talking for others." He raises the following useful questions about classroom practice: What kind of talk is worthwhile in mathematics? Is it the register of mathematics, that is the appropriate ways of speaking and writing about mathematics, or is it the body of knowledge that is mathematics? The classroom transcripts collected during this thesis may help to provide insights into these questions.

Success in school mathematics. This above discussion is not to suggest that the language used in the classroom should be reduced to looking at vocabulary alone. Being successful in school is attributed by Lemke (1988) to learning how to think and talk appropriately and fluently, in the mathematics register.

> It is not "superior intelligence" that makes for academic success in science or other fields. It is superior fluency in using the language of the subject: superior mastery of its genres, of its thematics, and of the techniques of combining these flexibly in practical use. (Lemke, 1988, p. 98)

Teaching in line with Lemke's ideas will provide experiences in mathematics classrooms which will assist students to recognise how their personal experiences link with formal mathematical language and concepts being presented in schools. In turn, students need to be able to recognise particular mathematical concepts when these are expressed in symbolic form (Clements, 1994). This thesis will analyse how two teachers developed the language of mathematics with each of their classes.

A Register Continuum. Before leaving this discussion of register it is important to include reference to writers who see movement between registers as a part of coming to understand the mathematical concepts associated with the language. Although scholars such as Pengelly (1987) have discussed the importance of incorporating natural language forms into the mathematics classroom, she emphasises that teachers should never lose sight of the major goal of students gaining an appreciation and understanding of appropriate mathematics concepts (Pengelly, 1987).

As learners grapple with the terminology and associated concepts, they will continue to connect the new information with what they already know. Hodge and Kress (1988) note that learners shift between what they describe as "less mathematical language" and "more mathematical" language. Chapman (1992) talks about language being "less" or "more" mathematical as expressed as part of continuum. Walkerdine (1982, 1988) refers to these shifts as being along the continuums of metaphoric and metonymic axes of mathematical discourse.

Given the importance of language in the mathematics classroom, there can be little doubt that any research which investigates classroom discourse needs to examine closely the ways in which teachers use mathematical language with their students.

Interest

Research is continuing to show that mathematical concepts could be introduced to children at an earlier age if they were to be connected to contexts in which students are interested. Van den Brink (1988) gives the previously cited example of successful concept development in the early years of schooling, when mathematical processes are linked with people, animals and toys.

Much has come out of research into interactional learning which supports the value of using the child's interest as a foundation for future concept development. Lehr's (1985) research is most comprehensive. He cites the works of Bruner (1978), Cazden (1983), Clark (1976), Halliday (1978), McKenzie (1985), Vygotsky (1962) and Wells (1981), and provides evidence of studies where children's interest successfully drove the curriculum content (see Bruner, 1978; McKenzie, 1985). Lehr was quick to point out, however, that this framework should not be taken to suggest that teachers do not have a clear sense of the cognitive, affective and physiological goals for their classrooms. Rather, he suggests, teachers are able to dove-tail their goals and agendas with the children's interests.

Steffe (1990) recommends that teachers "decentre" and take greater account of their students' backgrounds and interests. More particularly, "mathematics teaching should consist of interactive communication within a consensual domain of teacher and student experience" (Ellerton & Clements, 1991, p. 87).

Summary

This *"interest"* Principle attempts to capture some of the concerns which Kieren (1994) discussed when he reflected upon mathematics education research carried out over the past 25 years. He felt that we now have better insights into individual differences and divergencies between students. He believes, however, that mathematics education research needs stronger theoretical underpinnings. On the other hand, theory generating research, such as that carried out by Chapman (1992), helps to build our understanding on the interplay between *construction* and *sharing*, rather than treating them as distinct aspects. She recommends that attention be drawn to "meaning making," as the student's ideas are developed, shared, negotiated and constructed.

It would, however, be simplistic to imply that the key message to emerge from this part is that a child-centred curriculum is a pre-requisite for effective learning. In reality, students generally have little or no say in determining what counts as knowledge and how knowledge should be shaped in schools. Wells (1981) has been quite outspoken about the difference between home contexts and those provided by schools. He noted, for example, that decisions are rarely made in schools as a direct response to children's initiatives.

Aside from the focus on the personal construction of meaning and the development of an understanding of conventional mathematical register is the issue that is central to this thesis—namely, how driven should the curriculum be by the learner's interest?

Principle 3: Metacognition—Making the Learning Processes Explicit

Introduction

Metacognition has been variously defined as thinking out-loud (Ericsson & Simon, 1980), being aware of one's knowledge concerning one's processing (Flavell, 1976), and self-regulatory procedures including "on line" decision-making (Palincsar & Brown, 1984). Schoenfeld (1992), when discussing problem-solving and metacognition, suggested that "problem-solving and metacognition were perhaps "the two most overworked and least understood buzzwords of the 1980s (p. 336)." He refered to the move in the late 1980s toward "cognitive and social perspectives on human behaviour, in the theme of enculturation" (p. 347), where understanding strategies and aspects of metacognitive behaviour was included in the specifics of domain knowledge. In other words it is not simply what you know, it is how you use it, when you use it and ultimately whether you choose to use it. Thus although there is no single definition of metacognition, the term is generally used when attempting to express what is going on "inside a person's head".

This teaching/learning Principle highlights the importance of encouraging students to articulate their thinking processes. It is through such articulation that students should be able to judge the soundness of their thinking. Vygotsky (1962) used the term "verbal thought" to describe how learners conduct an inner dialogue with themselves. Students, hearing the speech patterns of the teacher or peers can internalise these and make them part of their own inner speech (Vygotsky, 1978). Mead (1962) also emphasised the notion that thinking originates and develops through experience with verbalisation, and that we tend to conduct an inner dialogue with ourselves.

Asking children to think aloud can also assist the adult/expert to gain insights into possible strategies that may have prompted the "miscues" (Goodman, 1983) children make. Research suggests that students already know the extent of their misunderstanding before they seek the help of their teacher (Newman & Schwager, 1992; Rohrkemper & Bershon, 1984). There is some evidence to suggest that learners who have been identified as helpless by their teachers, actually have the needed ability and skills, but consistently display negative self-involved inner speech (D'Amico, 1986). Newman and his colleagues (Newman, 1990; Newman & Goldin, 1990) conducted research on the self regulatory behaviour of students and noted that their beliefs about self influenced their willingness to seek help (see also Anthony, 1996; Herrington, 1992). Fennema and Sherman (1977) found that students who were able to articulate the aspects of classroom mathematics that was personally useful to them generally gained higher test scores.

The issue of self correction is an important one. It is based on the premise that learners will recognise their own errors. However, in order for this level of awareness to be developed the learner needs to have been exposed to feedback on two levels—on the one hand, internal monitoring and on the other, outside evaluation (Newman, 1988). The first form of feedback has to lead into the development of an internal mechanism, by which students can validate their meanings as well as the development of their processes of learning. In other words, they not only comprehend but have become critical thinkers. McCormack and Pancini (1990) describe "being metacognitive" as meaning that

> we become aware of and responsible for the way we approach new ideas and new skills. Being metacognitive means that we do not have to be just passive victims of our past habits or experiences—we can take control of the way we go about learning. (p. 19)

This line of thinking has emerged from the research of Bawden (1985), who quoted the learning principles of Carl Rogers (1961) and included the notion of self-evaluation. Bawden argued that evaluation of the process and the product resides to a large extent within the learner. Sadler and Whimbey (1985) believe that students need to be assisted to become critical thinkers, and through this approach to become more responsible for their own learning.

Flavell (1976) considers self-regulation processes as the "active monitoring and consequent regulation and organisation of processes (p. 232). Garofalo and Lester (1985) describe these as managerial behaviours—"organising information of data, planning solution attempts, executing plans and checking results" (p. 168). Encouraging students to articulate their thinking throughout problem-solving activities is considered helpful in developing self regulation.

However research findings (Anthony, 1994; Baird & Northfield, 1992; Peterson, 1988; Schoenfeld, 1987) continue to indicate that many students exhibit passive learning behaviours, leaving them, as Anthony (1996) believes, poorly placed to reach their learning goals.

Students Verbalise

In being critical of their own and others' dialogue, the learner is in a position to exercise "reflective observation abilities" (Kolb, 1984), reflecting on and observing these experiences from many perspectives. Such reflective processes will also involve learners in evaluating, in a Rogerian (1961) sense, the logic of what they are constructing as they attempt to create concepts which integrate their observations into logically sound models of "abstract conceptualisation" (Bawden, 1985, p. 6). Journal writing and other written genres are being used in mathematics classrooms to encourage metacognitive thinking, where students are encouraged to not only write down their reflections about mathematics they are learning, but to write about their ability to approach certain tasks and concepts (Wasam-Ellam, 1987a, 1987b: Waywood, 1993).

The capacity of children to stand back enough from their language and analyse it is obviously a question which needs to be addressed. Southwell (1993) noted that reflection was a way of re-evaluating an experience in an attempt to make sense of and position it with what is already known. She argued that it is a highly personal and complex act because it relies on both cognition and feelings. "Having a student describe a process followed provides a good opportunity for the teacher to assess the student's achievement and to diagnose any difficulties the student might be having (p. 229)." There will be limitations on a learner's capacity to reflect on his/her reasoning processes at different stages of concept development. Clearly, too, such reflection is likely to depend on the language ability of individual learners.

> At all levels, including adolescence and in a systematic manner at the more elementary levels, the pupil will be far more capable of "doing" and "understanding in actions" than of expressing himself verbally ... "Awareness" occurs long after the action. (Gruber & Vonech, 1977, pp. 726-732)

Learning is enhanced when there is interaction with others who are using hypothesising and self-correcting language and strategies (Hall, 1986). Kamii (1982) examined Piaget's research into learning development and noted that children develop autonomy from adults who when exchanging points of view with children "have the effects of motivating the child to construct the rules of conduct for himself through the co-ordination of viewpoints" (p. 77).

Grouws (1992) reports a study by Carpenter et al (1989) which examined how accessible students' strategies became when they had the opportunity to talk about how they worked through their informal knowledge to solve problems:

> Once children's informal strategies were readily accessible and were objects of discussion, symbols were introduced as ways of representing knowledge the children already had. In this way, symbols were linked to the children's intuitive knowledge about addition and subtraction. (Grouws, 1992, p. 82)

Research continues to demonstrate that students appear to learn more effectively if they are given opportunities to verbalise their thought processes either in writing or orally.

Students Sharing Their Verbalising With Their Peers

The previous discussion recommended that teachers encourage their students to verbalise during their problem-solving. This need not only be achieved on a one-to-one basis with the teacher, but peers and even the whole class can also participate in the articulation of ideas. Students can share and be exposed to feedback when opportunities are created in the classroom for open sharing and dialogue during problem-solving tasks. Forman and Cazden (1983) quote a study carried out with problem-solving sessions involving pairs and individuals, where pairs were more successful than individuals at solving problems (the issue of co-operative learning is taken up in the section on "community" in this chapter).

Frid and Malone (1995) highlight negotiation which occurs when peer discussion is valued and encouraged. They argue that the classroom social context plays a key role in developing the rules and norms of classroom practice. They note that "if students are not provided with opportunties to develop skills at verbalising their mathematical understandings, then it is inappropriate to expect them to have capacities to do so" (p. 145).

Conjecture regarding the beliefs that students and teachers hold both about mathematics learning and about the nature of mathematics is receiving increased attention (Clarke, 1986; Cobb, 1994; Day, 1996; Frid & Malone, 1995; Tobin & Tippins, 1993). The mathematical meanings that students construct during classroom discourse may be problematic if they are discordant with the teacher's intended outcomes. For example, the failure of many students to check the validity of their solutions may suggest that for mathematics problems students perceive that it is the performance of the solution which is the main aim of mathematics tasks rather than the outworking of their construction of the concept (Anthony, 1996; Ball, 1988; Lee & Wheeler, 1987; Schoenfeld, 1985a; Stodolsky, 1985).

Transfer of School Mathematics To Real Life

A distinction is drawn by Douady (1985) between cognition and metacognition. Mathematical knowledge does not remain as an "object" when the learner is able to use his/her mathematical understandings as a "tool" in an effort to understand the mathematical concepts involved in a given situation. Clarke (1995) also refers to the appropriate use of mathematical "tools," and argues that:

> One measure of sophistocated mathematics is the student's familiarity with a mathematical object or procedure in many diverse contexts, since this familiarity will facilitate the solution of non-routine problems and optimise the transfer and application of the mathematics. (p. 27)

Some students know their mathematical content and problem-solving strategies but make metacognitive mistakes of inappropriate "managerial" decisions which render their knowledge as ineffective (Anthony, 1996; Lester, Garfalo & Kroll, 1989; Schoenfeld, 1983, 1992).

Increasing numbers of research studies in the past decade have demonstrated students' capacities to perform mathematical tasks in out-of-school settings. However, because these experiences are not necessarily valued by many teachers, there is little opportunity to display this knowledge in the classroom setting (see, for example, Bishop, 1992; Carraher, 1991; Carraher, Carraher & Schliemann, 1985; Resnick, 1984; Schliemann & Carraher, 1990). Most of these studies illustrate how students were able to do quite complex tasks in out-of-school settings, and yet when the same processes were encountered by the students in a formal mathematics classroom context, they were unable to apply the appropriate algorithm.

Lean, Clements and Del Campo (1990) highlight the need to establish links in the classroom between everyday concepts and formal mathematical language. Unless the teacher makes explicit the language structures required in the discipline of school mathematics, meaning can be constrained (Mousley & Marks 1991). Students need access to the most appropriate, powerful and "sophisticated" genres not just in mathematics classrooms, but in all subject classrooms.

Teacher Talk

Schoenfeld (1992) refers to the works of Palincsar and Brown (1984) in the area of reading environments, Scardamalia and Bereiter (1983) for writing environments, and Schoenfeld (1985b) for mathematical environments, in an attempt to identify elements which might be common to productive learning environments. They concluded that, not only was domain-specific knowledge required, but also an understanding of strategies and aspects of metacognitive behaviour. Significant too was their finding that students needed to experience the "gestalt" of the discipline in ways that practitioners do. This, therefore has implications for the teachers' use of their own language in the classroom.

The suggestion, also consistent with the recommendations to teachers in the Californian State Department's *Mathematics Framework* (1991) is that adults model what their thinking aloud might sound like while they compose and draft or problem solve in front of the children (see also Cambourne 1988; Hunkins 1987; Palincsar & Brown, 1989; Sadler & Whimbey 1983). Lester et al. (1989) refers to the importance of teachers modelling good "executive behaviours." Such explicit demonstration exemplifies the meta-textual knowledge teachers draw upon in order to achieve certain goals. At times, the thinking aloud may take the form of a recommended procedure or advance organisers (Ausubel, 1968; Hunkins, 1987; Polya, 1945), especially "when they [students] need more assistance formulating abstract ideas to anchor content" (Joyce, Showers & Rolheiser-Bennett, 1987, p. 18). This theme is echoed in the writings of Schoenfeld (1985b; 1987).

The role of the adult in this metacognitive teaching/learning Principle can be divided into three areas of adult assistance—scaffolding (see specific section in this chapter), modelling (see specific section in this chapter), and direct instruction in metacognitive strategies (Cazden, 1983). Direct instruction is when "the adult not only models a particular utterance but directs the child to say or tell or ask " (Cazden, 1983, p. 14). Cazden argues that, when adults use this form of dialogue it may be construed that the content is probably especially valued. Greeno (1980) suggests that strategic principles need to be made explicit during instruction in problem-solving of geometry, but should not however be interpreted as the teacher's imposition of prescribed steps for students.

A teacher's explicit and often ritualistic rephrasing and refining of the learner's expressions at times forms part of what Furniss and Green (1991) term a "joint construction" (Furniss and Green also refer to the dialogue as "aided instruction" or "collaborative construction" (p. 48)). Pateman and Johnson, (1990) also note that this dialogue is based on responses from learners, who in turn are being given feedback about their processing and expression. Chapman (1992) refers to a teacher signalling ways of speaking that are appropriate to a particular subject-area, and that these are developed as part of the social practices of the classroom (p. 41). Leinhardt and Putnam (1987) use the description—"lesson parser" which signals to the students recognisable parts of the lesson which they can expect to experience.

A considerable amount of research has looked at classroom discourse, and in particular teacher's questions (Barnes, Britton & Rosen, 1971; Christie, 1985; Clarke & Peterson, 1986; Comber, 1990; Fennema, Carpenter & Peterson; 1989b; Lubinski, Thornton, Heyl, & Klass, 1994; Painter, 1986; Voigt, 1985). One of the features which characterises many mathematics classrooms is that of teachers asking questions to which they already know the answers. This ritual is, however, readily understood by the students who know that the teacher already knows, and that the teacher is really only testing them. Charles (1995), on a video developed for use in professional development sessions with teachers, discusses this issue and recommends that teacher-student dialogue should be consistent with the qualities of a "good conversation."

Ainley (1988) categorises teachers' questions into four types—pseudo-questions, genuine questions, testing questions and directing questions. Directing questions have the greatest metacognitive potential. Teachers can structure questions in order to try to activate the student's schema, or to open up new areas for exploration, or to check whether the students have followed a particular line of logic. Such questions give students a very different message about mathematics from testing questions which reinforce the belief that there is clearly one right answer.

The kinds of messages that students receive from their teacher about appropriate discourse when solving problems is one of the issues under investigation in this thesis.

Classroom Talk

The emphasis of classroom talk shifts from teacher-centred to learner-centred when students are encouraged to verbalise and develop their exploratory and expository language (Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Cobb, Wood, Yackel & McNeal, 1992). A comparison between classrooms in which the focus has moved away from teacher-dominated talk and classrooms in which teacher-dominated talk is still the preferred emphasis, suggested that there were large differences in the quality of students' written work (Boero, 1988). This quality was reflected not only in the expository text produced but also in its complexity. This finding is supported by research comparing high and low achieving students—high-achieving students are more reflective than low-achieving students (El- Faramawy, 1988). Dialogue which has a reflective metacognitive function is supported by constructivists like Steffe and D'Ambrosio (1995), who have suggested that "regarding the thoughts of another for further consideration, especially as something to understand, critique, or compare and contrast with one's own thoughts, provides a perspective on the role of social interaction in the mathematics classroom that is quite valuable" (p. 156).

Barrett (1985) draws attention to two aspects of knowledge, and terms them "public" and "private" knowledge. Once learners are in a position where they have to represent their cognitive experiences or personal (private) knowledge, they go through a process of reflection and re-interpretation which "disembeds it from the 'concrete' context, cognitively speaking, and both consolidates and changes it as it becomes exposed to the public" (p. 74). The potential benefits have been indicated by Schoenfeld (1987) and Goos (1994) in their separate studies on metacognition in problem-solving. Their results reflect the success of peer collaboration as an effective practice for developing the metacognitive behaviour of self-regulation among learners.

The benefit to the learner of being able to transfer what has been learned to a new context is widely recognised, and many writers recommend that this be given greater attention in classroom talk (Barrett, 1985; Case & Sandieson, 1988; Hiebert & Lindquist, 1990; Kieren, 1988; Lampert, 1986, 1989). Explicitly discussing the transfer of what has been learned to a new context enables learners not only to interpret what is before them, but to share the significant components from that context and then to transfer these to different contexts. In essence, transfer skills are demonstrated by an ability to generalise, and are consistent with Boomer's (1988) transition and transformation principles. Boomer (1988) refers to students showing how they were able to apply what they knew from one medium to another. Similarly Baker (1991) maintains that metacognitive components in skills training increases the potential for success in transfer. The ability to apply these metacognitive strategies often creates the distinction between experts and novices (Anthony, 1996; Weinstein & Mayer, 1986). Of interest to educators are the findings of Leinhardt (1989) who observed that "expert" teachers seemed to possess knowledge which was well developed and interconnected and accessible enough to enable them to be responsive flexible teachers compared with their "novice" counterparts (see also Borko & Livingston, 1989; Livingston & Borko, 1990).

Students Discover for Themselves

Most children come to school with well-established thinking skills which have enabled them to survive in a real and meaningful world. In order to be successful in our education system they need to learn to turn language and their thinking upon themselves, so that they are not only able to direct their own thought processes, but they are also able to distinguish between possible interpretations of what they hear (Wilson & Wing Jan, 1993). In order to survive both inside and outside the school system they may also find it helpful if they are able to represent their ideas in other forms which are less dependent on the context from which the information was originally drawn (Dalton, 1985; Dalton & Boyd, 1992).

Students can be assisted by their teachers to become aware of what they already know (Novak, 1986) and to evaluate their thinking. In this way teachers can assist students to become empowered by their capacity to learn how to learn, and to recognise what works best for themselves. Noddings (1993) has concerns that "turning students loose 'to construct' will not in itself ensure progress toward genuinely mathematical results" (p. 38), and hence the role of the teacher can be quite crucial.

Probably the most explicit work concerning metacognitive classroom practices has come from Cobb (1990b). He listed four points to describe metacognitive approaches:

> 1. Explaining how an instructional activity that a small group has completed was interpreted and solved;
> 2. Listening and trying to make sense of explanations given by others;
> 3. Indicating agreement, disagreement, or failure to understand the interpretations and solutions of others;
> 4. Attempting to justify a solution and questioning alternatives in situations where a conflict between interpretations or solutions has become apparent. (p. 208)

The socio-constructivist model (Yackel, Cobb, Wood, Wheatley, & Merkel, 1990) encourages effective communication between teacher and student. Teachers who adopt this model assume that what the student contributes will be personally meaningful and that it is up to teachers to assist the students to verbalise their ideas in mathematically meaningful ways. The intent is to work with the students as they try to express the concepts with which they are grappling. The teacher's interactions become an overt model for the students with respect to possible ways of engaging in dialogue about mathematical issues.

Critical Numeracy

Kameenui and Griffin (1989) demonstrate that school mathematics rarely calls upon students to use their metacognitive skills in genuine ways, or suggests to students that they are causal agents of their own cognitive processing. Pupils are asked for responses but rarely are put in a position where they need to contemplate the social consequences of their responses (Bruss & Macedo, 1985; Freire, 1994). Castles (1992), in his manual for interpreting statistics, exemplifies the idea that the location and representation of numerical information may be closely linked with politics and power relationships. "There is no denying that statistics can be used, deliberately or unconsciously, to mislead. The first concern is bias within the statistical collection itself" (p. 39). He raises many cautionary points about interpreting data and suggests that readers "keep your wits about you. In using data, be sceptical and critical" (p. 41).

In schools students rarely exhibit "critical numeracy" behaviours, which may be as a result of not being encouraged to do so. Schoenfeld's (1989) extensive study of problem-solving behaviours among high school students reveals a great deal about the situational or school mathematical contexts in which students find themselves. Schoenfeld observed that, when given an unfamiliar problem out of context, students invariably failed to monitor their strategies, and to assess whether their initial strategies should be abandoned, sustained or modified. Rather, they tended to pursue a dead-end path, without realising the implications, largely because they do not see themselves as being in control of the learning. Steffe and D'Ambrosio (1995) recommend that rather than pose problems or tasks, teachers should construct situations which the students regard as genuine problems. In this way students are placed in a position where their critical thinking skills will need to be called upon.

Critical numeracy advocates (Goddard, Marr & Martin, 1991; Johnston, 1994) argue that students also need to be aware of the range of motives teachers may have in presenting mathematical information in the ways they have and in the context in which they have chosen to place it. This is consistent with a branch of reading pedagogy which defines reading as "critical social practice" (Baker & Luke, 1991; Freebody, Ludwig, & Gunn, 1995),

> Reading is a social practice using written text as a means for the construction and reconstruction of statements, messages and meanings. Reading is actually "done" in the public and private cultural spaces of everyday community, occupational and academic institutions. Reading is tied up in the politics and power relations of everyday life in literate culture. (Luke, 1995, p. 167)

In the same way, mathematics education may need to be investigated for the ways in which it defines mathematics, although writers like Bishop (1992), Joseph (1992) and Willis (1989) have had some impact on raising awareness of critical social practice in mathematics education.

Integration of Mathematical Themes Across the Curriculum

In the primary school, because few subjects are taught by specialist teachers, there are numerous opportunities for teachers to develop themes across content areas, and to assist students to make the connections. Making these links explicit for students has been included in this *"metacognition"* Principle.

Mousley (1990b) notes that measurement concepts are often taught as though they are single entities. She sees great potential for integration within the mathematics curriculum. "Length, for instance should be studied in conjunction with other lengths (such as height, perimeter), time, angle, money, volume, area, temperature and other measurement concepts" (p. 40). She is critical of the way in which fractions have been excised into a separate area of study. Behr, Harel, Post and Lesh (1993) suggest an integration of the subconstructs of rational number, (for example, invariance of arithmetic operations, composition, decomposition, and conversion of units), and connecting the multiplicative field to other conceptual fields. These writers note that in current curricula equivalence is often treated as an isolated topic. It is possible for students to study a unit of work on for example, fractions, for this to be followed by a unit on decimals, without any explicit linking to what they have learned in ways which may enhance their understanding of the new concept.

Even when problem-solving has involved real-life situations it may be taken from an artificial story problem and presented in such a way that the students develop separate systems of arithmetic—one to assist them in their informal problem-solving and the other to solve the school arithmetic (Carraher et al., 1987; Cobb, 1988; Ginsburg, 1982). Tobin and Phillips (1990) discuss integration of statistical concepts in the form of "projects," and comment that "project work in all areas of the school curriculum would be beneficial" (p. 81). If the pertinent mathematical skills are extracted from the proposed activities for the unit(s) of work in both for example, science and mathematics, a network of concepts and skills can be planned for (Mousley, 1990a).

Teachers have experimented with incorporating mathematics in art and in music (Clements, 1990). By using mathematics as a resource for their art or music teachers hoped that students might see how their mathematics relates to different contexts.

The notion of integration of mathematical themes across the curriculum has been included within the *"metacognition"* Principle because of the need to make explicit to students the links between mathematics and other areas. Such linking would seem to address the importance of reducing boundaries between domains, and would encourage "the search for similarities between tasks within larger domains of knowledge"(Hiebert, 1992, p. 77).

Summary

The discussion on metacognition has included a broad association of research which is concerned with the development of a learner's capacity to be aware of his/her own cognitive processing and regulate its use. This teaching/learning Principle recognises the value of having students articulate their thinking processes during problem-solving tasks. It also suggests that the teacher encourage the awareness of cognitive processing being used by both the teacher and the students.

Comment was also made concerning the delivery of the content in classrooms. It was suggested that this was being done in ways which may work against the development of student's metacognitive self-regulation strategies. The research suggested that certain problem-solving tasks being used in schools may not provide students with opportunity to develop their critical numeracy skills. Similarly there is a lack of integration in how concepts are presented across domains. A suggestion was made that concepts may be being presented to students in a form that may make it difficult for them to make links between the information both within the subject and across the curriculum. This thesis will present data related to the development of metacognitive strategies in two classrooms and the impact each teacher's instruction has on the solution of a non-routine problems by the students.

Principle 4: Scaffolding—Challenging Children To Go Beyond Their Current Thinking, Continually Increasing Their Capacities

Introduction

The *"scaffolding"* Principle is based on the assumption that, with adult/expert support, learners can be stretched beyond what they might normally achieve without such support. There is no expectation that the learners will become independent learners independently. Scaffolding is essentially a hand-holding strategy, tailored to meet the needs of the individual, and can be provided at any stage when assistance might be beneficial for the learner's development. Since the aim of scaffolding is to build on what the child appears to know in order to stretch the child, the nature and form of scaffolding will vary, as will the time needed for scaffolding to be in place. The roles and responsibilities of the teacher and the student will vary, as will the joint construction of meaning, the power of the teacher's modelling, the self-destructive nature of scaffolding and its timing and predictability.

Stretching the Learner

Vygotsky (1962) is recognised as the founder of the label "scaffolding" when it is used to refer to the specific interactions which occur between adult/expert and child/learner described above. His reported research demonstrates how children's intellectual and linguistic development arise through their interaction with the significant adults in their lives. He saw these adults as having an important function assisting the children to reach their potential development.

He believed that the children would not have been able to develop in the same way just by virtue of age or maturation. Vygotsky termed this the "zone of proximal development," where instruction leads the child to focus on particular aspects of learning, in a joint problem-solving context which eventually will be independently handled by the child. This same idea can also be identified in the self concept work of the psychologist, Luft (1969), where he describes a dual process—one of self-disclosure by an individual on the one hand, and on the other, one of receiving feedback which helps the individual to reach his/her potential.

In this thesis, discussions about classroom discourse where the teacher continues to refine the student's expression towards more mathematically appropriate language (Chapman, 1992) has been included in the section on "scaffolding." Cambourne (1988) used the term "raising the ante" to describe the situation when a teacher's response attempts to take the student forward in his/her thinking and expression. Some writers have made comment concerning the power that school genres can have in directing students' thinking (Mc Cormack, Waller, & Flower, 1987; Ongstad, 1994; Ricoeur, 1981; Sørbye, 1994).

The Social Construction of the Teaching Situation

Some students are able to complete particular tasks when they follow along with the teacher, but they have difficulty tackling the same tasks for homework. According to Palincsar and Brown (1989), this difference could be attributed to the fact that many teachers provide prompts and clues as they complete tasks in the classroom. Although such prompts and clues may not be specifically directed to a particular student, but rather to the class in general, they lie within what Palincsar and Brown describe as band width. The students' learning is being socially supported by the culture established in the classroom by the teacher.

Chapman (1992) argues that "the relatively less successful student relies on the transformational language shifts between less mathematical language and more mathematical language" (p. 5).

She provides examples of how the teacher might develop this with a student:

> The essence of Arthur's answer is correct, but the teacher puts it into a more 'proper' sentence structure. Stuart's term "the same" is restated by the teacher as "the same amount". The speakers are apparently making sense with each other as they develop a more mathematical way of talking. (p. 46)

In this example a dialectic model is operating where the student's learning is socially constructed. Austin and Howson (1979) make special note of the "language of the teacher," and refer to the importance of the teacher's role in helping to develop fluency in the mathematical language register.

Scaffolding Can Act as a Framework or Platform for the Next Step in the Learning

Independent studies by Cairney (1987) and Zubrich (1987) examine how the adults/experts tailor their dialogue in an effort to create a shared construction of meaning. The above writers were influenced by Bruner's (1978) concept of scaffolding as a temporary framework providing a platform for the next step toward more "adult" communication. Bruner observed mothers who tried to prevent their children from "slipping back", by at the same time demanding more complex performances in their language (Lehr, 1985).

Ninio and Bruner (1978) have been particularly influential in their advocacy of scaffolding. They describe details of a dyad involving a mother with a young infant, in which the two are seen jointly constructing meaning, in spite of the major differences in language abilities of the two. In other words, the mother's scaffolding is at a level the child can manage and in the context of a presumably mutually satisfying interaction. As Holzman (1972) described interactions of this type: "The child finds out by the response of the adults what he is assumed to mean by what he is saying" (p. 321). Wells (1981) terms this a "negotiation of conversational meaning" (Wells, 1981, in Lehr, 1985, p. 667), observing the same interactions patterns in classrooms.

Evidence that scaffolding is taking place is not always reflected in dialogue, but can take the form of a protocol. In fact, this is probably the most familiar form of scaffolding used in mathematics classrooms; Polya's (1981) four-phase framework for problem-solving. Such a framework is used by some teachers to scaffold to help students develop appropriate problem-solving skills.

As in metacognition teacher's questioning of students tends to be one of the most readily available forms of scaffolding in the classroom. Apart from questions which are designed to test students' understanding, questions can, as suggested by Ainley (1988), perform three functions (see also the previous discussion in the *"metacognition"* Principle): (a) "structuring", which can activate students' existing knowledge in order to connect with new information; (b) "opening-up", which suggest further exploration such as "What would happen if...?"; and (c) "checking," which encourage students to reflect on their own logic and processing such as "Do you agree with...?" (p. 93). Ultimately the learner may be in a position to ask these questions of themselves or other learners, which leads into the next section—"the switching of roles of learners and teachers."

Interactionists (see, for example, Bauersfeld, 1988; 1992; 1995; Voigt, 1985; 1994; 1995) discuss the various assumptions, patterns and routines occuring during classroom discourse. Certain kinds of classroom interactions, such as "funelling" focusing, reciting, and "concrete-to-abstract" practices can have a profound effect on the quality and extent of the learning occurring in the classroom (Bauersfeld, 1995; Brousseau, Davis & Werner, 1986; Bruner, 1996; Krummheuer, 1995; Voigt, 1985; 1995). If these classroom interactions are sensitive to the learner's conceptual understandings and build on these, they have the potential to act as a scaffold for the learner.

The Switching of Roles of Learners and Teachers

The notion of "turn taking" between teachers and learners has been explored by Stern (1975) and Snow (1976). Wells (1981) also noted this feature of turn taking and the instructional role the adult takes on during a dialogue. It is hoped that the questioning and responses of the "expert" will enable the learner to eventually internalise this form of dialogue and ultimately ask these questions of themselves (Scollon, 1976; Staton, 1984). In this way learners have been able to draw upon the teacher's language as a resource (Kreeft, 1983-84).

Tizzard and Hughes (1984) point out that the role of the adult/expert/teacher should be viewed as a flexible one in which the learners may be encouraged to adopt the teacher's language while the adult takes on a more passive and non-directive role.

Taylor's (1992) study of a Year 12 mathematics teacher notes the difficulty a teacher had in adopting a "teacher as learner" role, and attempts to refine his "teacher as informer" role. According to Taylor, this teacher was unable to adjust to the role of "teacher as learner" because of personally constraining beliefs about his "technical curriculum rationality" which appeared to keep him in the role of "teacher as controller." There was, however, some shift from "teacher as transformer" to "teacher as interactive transformer." Alro and Skovsmose (1996) emphasised that the negotiation of meaning in a classroom where the teacher has an "absolutist" view of mathematics may be more concerned with students' clarifying the meaning they suppose is in the mind of the teacher or textbook, rather than constructing mathematical meaning for themselves.

In classrooms where co-operative teaching/learning procedures have been adopted, students attempt to bridge the gap between their peers' knowledge and that of the content being presented (Dansereau, 1987; Singer, 1978).

The Teacher Regulates the Level of Difficulty for the Learner

Research carried out by Snow (1983) and Thomas (1985) analysing adult-child discourse suggests that an adult will often continue and extend the topic that the child introduces (Snow, 1983; Thomas, 1985). In order to regulate the level of difficulty of the interaction for the child, the adult will use a predictable structure to the dialogue to allow for more complex concepts to be discussed (Clark, 1976; Snow, 1983; Thomas, 1985). The adult will be aware of what the child can do and will insist on certain levels of performance, they "raise the ante" and gently increase the challenge. Others writers have contributed along the same lines in regard to this idea of adjusting the level of difficulty between teacher and students during classroom interactions (see for example, Bruner, 1983; Cambourne, 1988; Parrett & Chafe, 1974).

The Self Destructive Nature Of Scaffolding

As Bruner (1986) would describe the teacher/student interactions of scaffolding as "the loan of consciousness that gets the children through the zone of proximal development" (p. 132). Inevitably the adult/expert gradually withdraws, and as the child/learner develops genuine understanding of the concept, the scaffolding is taken away or no longer used—it self-destructs (Cazden, 1983).

There are differing views on who is in control of removing the scaffolds. Cambourne and Turbill (1986) perceive that the control for the learning is exercised by the learner and not the adult (this view has support from Graves, (1983) and from Harste, Woodward and Burke, (1984)). "As learning occurs the scaffolds are removed by the children and others serving a different function may be erected" (Cambourne & Turbill, 1986). Alro and Skovsmose (1996) describe classroom discourse as being very much in the control of the teacher, who determines what children may discuss in the mathematics classroom and frames the knowledge and how it will be unfurled.

Regardless of whether it is the child who is in control of the self-destruct aspect of scaffolding or the adult, there is no doubt that some modelling of the language will have to be present in the child's environment. As Herber and Nelson-Herber (1987) comment: "Students should not be expected to become independent learners independently. Rather they should be shown how to become independent, and this *showing how* should be a natural part of instruction" (p. 584). Noddings (1990) suggests that although students inevitably perform constructions the mathematics that they produce may not necessarily be adequate, accurate or powerful.

Predictable Routines as Part of the Classroom Culture

In an attempt to maximise the effectiveness of scaffolding, Applebee and Langer (1983) propose that the structure of an activity be made more explicit. This can be achieved if the context is predictable (Bruner & Ratner, 1978; Cambourne, 1988; Ninio & Bruner, 1978; Wells, 1981) and the questioning and modelling is structured so that the children have an opportunity to internalise it, and can eventually function without the external support. The professional development package for secondary teachers called *Stepping Out* (Education Department of Western Australian, 1996), provides detailed guidelines for teachers to develop scaffolds in this way. For example, these guidelines provide writing frameworks to help teachers make explicit what they expect of students in relation to a given piece of work. The guidelines also suggest ways in which teachers can provide support for students as they attempt to interpret text.

Mousley and Marks (1991) discuss the use of writing frameworks such as these in the mathematics classroom. For example the "procedure" genre (Martin, 1985) is recognisable in mathematics texts—"First write ...", "Now take away ...", "Have a look to see which pronumeral is easiest to ..." (p. 5). Martin (1985) commends the practice of having students write in these genres because "Such a task enables students to clarify both the nature of mathematical processes and the logical orders in which these might be carried out. ... Students need to develop greater control of the more sophisticated expository genres that are valued in mathematical culture (p. 6)."

Halliday (1981) has included "structure" as one of his five criteria for the application of scaffolding in the classroom, where he believes that the framework of an activity should be predictable enough to provide support for the students. In a similar way, Halliday advocates that the modelling and questioning components of an activity should take on a recognisable "routine."

The notion of "routines" was discussed by Trevarthen (1980) when he pointed out that: "the routines of action and the rules behind them are accepted because of a co-operative motive, but they do not create the motive" (Trevarthen, 1980, in Searle, 1984, p. 482). Routines should not be used to justify making children restructure their experience to fit their teacher's structure. Learners should have transformational freedom in deciding how to arrive at and express the concept (Chapman, 1992).

Summary

Over the last 20 years, teachers have been confronted with various movements including mastery learning, discovery learning, problem-solving, and constructivism. Teachers, therefore, are faced with the dilemma of deciding what is most appropriate for them as teachers, and for their classroom. The author believes that many teachers today find this dilemma very real. They do not feel comfortable about leaving children to deduce the strategies and mathematical concepts for themselves, neither do they wish to resort to teaching concepts and strategies by direct instruction followed by sustained practice by the children. Scaffolding would appear to utilise components from each.

The power of the scaffold is first, in the perception of the teacher's role and relationship with the learner. Second, but no less important it is in the teacher's own conceptual understanding of what is being taught and his/her ability to assess the students' response in relation to their concept development. This thesis aims to explore how this might look in two different classrooms.

Principle 5: Modelling—Providing Opportunities to See the Knowledge, Skills and/ or Values Being Used by a 'Significant' Person

Introduction

Most of the self-concept research carried out by psychologists such as Combs (1962), Jourard (1964), and Purkey (1970) acknowledges that feedback provided by parents, siblings and "significant others" assists in shaping our concept of ourselves—namely what is worth valuing and not valuing about ourselves. It goes without saying that the most effective modelling is likely to occur within a supportive context, especially when the mentor is a person with whom the learner feels closely bonded. This has ramifications for the relationships which teachers form with their students. A teacher is in a powerful position to influence a child's growth, and this seems to be the case regardless of age (Purkey, 1970).

A large body of research (Barsalou, 1992; Bereiter & Scardamalia, 1993; Cazden, 1993; Gardner, 1991; Gee, 1992; Heath, 1983; Lave & Wenger, 1991; Perkins, 1992; Rogoff, 1994; Street, 1984) suggests that mastery of a knowledge/ skill or concept requires an immersion in a community of learners engaged in authentic versions of such practice. The term "situated practice" has been attached to this notion (see, for example, Barsalou, 1992; Eiser, 1994; Gee, 1992; Harre & Gillett, 1994; Margolis, 1993; Nolan, 1994).

According to these writers children should be engaged in real contexts in which the model of teacher's and children's participation is authentic, and the skills which students develop when working within these contexts are being used for a meaningful purpose.

Holdaway (1986) commented about authenticity and the way it affects the roles of the teacher and learner. He noted that, "in terms of the spectator/performer relationship, the learner at this stage acts as a fascinated "spectator" while the skilled person is active, not as a "performer," but as a genuine user of the skill" (p. 59). The notion of the teacher as a performer needs to be viewed in the light of the many spectators in the classroom. Further, the notion of spectator carries with it the potential danger that children may perceive that the teacher is merely demonstrating particular skills rather than modelling genuine applications of the skill which might also be useful in their own lives. This thesis will explore the reasons for differences in the way each teacher modelled genuine use of the skill she was teaching.

Modelling in Mathematics

The term "modelling" is used in several ways in the mathematics and the mathematics education literature. For example, the VCE Mathematics Study Design (Victorian Curriculum & Examinations Board, 1989) uses the term "modelling" to describe part of the problem-solving process. Those solving problems—and this could be teachers or learners—may wish to set up a model or a simulation in order to formulate a mathematical description of the problem. Problem-solving and modelling activities are described as providing a vehicle for the development of mathematical concepts and skills.

Schoenfeld (1992) when discussing the theme of enculturation suggests that "environments be designed to take advantage of social interactions to have students experience the gestalt of the discipline in ways that practitioners do" (p. 347). Thus Schoenfeld is suggesting that modelling by practitioners is an important part of enculturation.

Although "modelling" in the two examples given is associated with different meanings, both share the notion that, through a process of modelling, users can potentially gain increasing control over their environment.

Chapman (1992) discusses the way teachers "introduce and model 'mathematical' words and language structures which are privileged over other language forms. Learning mathematics involves learning its register" (p. 39). This message is enhanced when the teacher, as Brandau (1988) did, talked with her university students of her struggle to solve a mathematical problem and overtly showed and encouraged them to share in her struggle. Therefore modelling may imply overt instruction but does not imply direct transmission, and is preferably negotiated as part of the social rules and norms of the classroom (Frid & Malone, 1995). Cope & Kalantzis, (1995) describe the process in this way:

> Rather it includes all those active interventions on the part of the teacher and other "experts" that scaffold learning activities, that focus the learner on the important features of their experiences and activities within the community of learners. (p. 37)

The modelling, whether it be by the teacher or other "experts" denotes endorsement of the mathematical understandings which are being verbalised.

The Model and the Learner

Children sense when adults adopt demonstration as a teaching strategy because it signals the value of the learning (Cazden, 1983). This is not to suggest that such demonstrations should be carried out in the same way and repeated. It is important to note that the same information might be presented in a mutiplicity of ways and with a wide range of materials (Cambourne, 1988). However, not all modelling is conscious nor intended as Holdaway (1979) observed:

> There is a non-instructional display of use and value by more mature members of the learner's close community. The most powerful forms of social motivation arise from the admiring observation of functional skills, demonstrated by the "significant others", and occurring in a context of cultural belonging. (p. 58)

The most researched by-product of this unconscious message is the "hidden curriculum" and the role it plays in communicating a particular set of social values.

This is not to say that learners will only imitate what has been modelled consciously or unconsciously to them. Smith (1988) raises the issue that although children are immersed in an environment rich with different adult models they are unlikely to reproduce exactly the same language used by any one model. For example, children do not end up talking in the same way as their school teachers. Independent studies of Cope and Kalantzis (1995) and Macmillan (1995) on situated learning, explore findings that raise concerns that situated learning can lead to mastery in practice, in which some learners spend a "good deal of time pursuing the 'wrong' leads, so to speak" (Cope & Kalantzis, 1995, p. 36). Neither does "situated practice" necessarily create learners or communities who can "critique what they are learning in terms of historical, cultural, political, ideological, or value-centred relations" (p. 36).

The relationship with a particular model, and possibly also with peers, continues to be an important ingredient in helping children to develop mathematical knowledge, and gain fundamental skills and values. Smith (1988) discusses the richness of this modelling:

> Learning is vicarious, but the only way I can account for the enormous amounts of unwitting learning that children accomplish, much of which is apparently error-free on the first trial, is that children actually learn from what other people do—provided they are the kind of people the children see themselves as being. (p. 8)

It would appear from this that the power of modelling in children's learning environments should not be under-estimated.

Summary

The discussion on "modelling" has centred around two main themes. First, the shaping of the learning should be dependent upon the kind of roles and relationships which exist between the "expert" and the learner. Second, it is dependent on the situational context in which the learning is taking place.

It would seem that learning occurs not only at a functional level which achieves meaning and the desired communication but at another level in which children also learn to recognise the situations in which it is appropriate to use that communication (Cambourne, 1988; Chapman, 1992; Isler, 1980; Macmillan, 1995; Smith, 1988). During the situated practice the child is likely to generalise from the modelling they have been exposed to and trial it in other situations (Cambourne, 1988). The adult is there to model and provide feedback regarding the child's attempts and its appropriateness (Cope & Kalantzis, 1995).

The *"modelling"* Principle draws upon the teacher's capacity to provide scaffolds for the students in their learning. Gallimore and Tharp (1990) and Rogoff (1989) describe the process as one where the child learns through experience with the cultural tools that they see modelled in that social context. Through joint problem-solving with more skilled partners, the child learns how to mediate the everyday learning with the schooled knowledge.

Principle 6: Responsibility—Helping Children to Develop the Capacity to Accept Increasingly More Responsibility for Their Learning

Introduction

This teaching/learning Principle deals with the issue of who is responsible for the learning which is occurring in the classroom. Constructivists, educators and commentators on constructivism are agreed that inevitably learners must construct their own meanings, and that this also happens in traditional education settings in which teachers attempt to transmit knowledge (see, for example, Kelly, 1955; Piaget, 1973; von Glasersfeld, 1990b).

As Rogers (1961) predicts, personal growth occurs as students commit themselves to risk-taking experiences in their pursuit of unknown solutions to their problems. However, many students will often only tend to assert their rights as individuals when they are encouraged to do so (Barrett, 1985; Freire, 1994).

One concern often noted is that learners will not learn adequately if left to their own devices, (see for example, Hersh, 1986; Hiebert, 1992). According to Hersh, the question of leaving the responsibility of learning entirely to the learner tends to be associated with an abandonment of the teacher's responsibility. However, in parallel to the discussion in the "scaffolding" section, where adult support is gradually withdrawn, here, the notion of "responsibility" implies that learners take increasingly more responsibility for doing a particular task as the "expert" gradually lets go. At times, roles may be reversed, and tutor becomes tutee (Papert, 1980).

Who is in Charge of the Learning?

Cobb (1990a) is critical of controlled mathematical teaching which purports to make "discovery" of a concept easier for the child. He argues that "what is taught is rarely what is learned" (p. 7). Although Davis (1989) accepts that students need to be more committed to their learning, he reinforces the premise that mathematical concepts are socially constructed, (an idea which is further developed in the "community" section), and voices a concern about leaving children to do the learning entirely on their own. This tension has also been explored by Ellerton and Clements (1991) who ask the question about students, "Do they have the background to choose wisely, and how and when should the teacher guide?" (p. 57).

Gradual Release of Responsibility to the Learner

Herber and Nelson-Herber (1987) offer the following explanation about students being responsible for their own learning: "Independence does not mean isolation. It has to do with who's in charge rather than who one's with" (p. 586). They make the point that we always need support from others if we are in a process of continually expanding our knowledge of the world (see also Kelly, 1955). Gray (1987) and Hall (1986) as does Smith (1988), see the opportunity for much independent learning to take place when there is a supportive environment. The adult can assist in children's personal construction of knowledge, by extending the student's meanings and taking account of the student's point of view (Cambourne 1988; Wells 1981). The writers are not suggesting that the "hand holding" model is the only path to follow. Weeks (1987) structures her learning activities so that students have the freedom to accept responsibility at different rates: "When children are given choices, they may need time to learn how to use that freedom without interfering with the freedom of others" (p. 149).

Accepting Responsibility

Smith (1981) considers the difference in society's expectations for the home and the school. In the natural language learning situation of the home, there is no anxiety that the child's immature attempts will become a permanent fixture of the child's repertoire. He suggests that learning in schools should parallel what it is in the home context—essentially a social act, where in the main the learner is not even aware of what he/she is learning. The idiosyncracy of learners will mean that they will take from each classroom experience something different (see "interest" section in this chapter). One of the major problems confronting educators is that children become socialised into the culture of the classroom and of the school, and many become "dependent learners" who wait for someone else to direct their learning. Alro and Skovsmose (1996) describe how school discourse influences the roles of teacher and students in regard to responsibility when they write: "the students do not need to take full responsibility for their answers—the teacher will always provide the right algorithm or the right result" (p. 7).

Most children in schools have little say in determining what is valued as knowledge, and decisions in schools are rarely made as a direct response to children's initiatives (Searle, 1984). Cambourne (1988) is also concerned about the classroom consequences of not placing responsibility in the hands of the learners and purports that:

> School learning is probably their first experience of someone else deciding for them what shall be learned and when it shall be learned. In this sense it is probably their first experience of learning which does not conform to what they have been doing prior to attendance at school. It is probably their first taste of what I call 'aberrant learning'—learning which is not what the brain was designed to do. (p. 63)

The experiences of schools set up what Cambourne believes to be dependent learning where students become dependent on someone else directing their learning.

The idea that school is a preparation of students for citizenship is embedded within the Principle of *responsibility*. Accompanying this is the implication that it is the responsibility of both the student and the teacher. As Hersh (1986) writes "The student must be given the responsibility to decide what role she wishes mathematics to play in her life, whereas the teacher must be sure that the decision is an informed one. Ultimately they (the students) must decide" (p. 186-187).

Ownership

Ellerton and Clements (1991) explore ownership in classrooms where teachers "establish teaching-learning environments in which students would feel that they 'owned' the mathematics they learned. because they themselves had created it" (p. 56). The classroom interactions do not assume there is a body of knowledge that is owned by the teacher, textbook and/or worksheet writers, external examiners, or by great, mysterious figures of the past (like Pythagoras and Euclid). This concept of ownership of the mathematics they are learning is the ultimate phase students pass through as they broaden their conceptions of knowledge.

As Steffe (1990) points out "A particular modification of a mathematical concept cannot be caused by a teacher any more than nutriments can cause plants to grow" (p. 392). The following example demonstrates how one teacher balances the dilemma of who is in control in the classroom. Peters (1993) observed that the mathematics teacher in her study valued independent work habits in which students took responsibility for their own behaviour and use of time, solved their own problems, and chose how they went about tasks and with whom they worked. She discussed with the teacher in her study the issue of control and the need for the children to have some measure of control over, and responsibility for, their learning but ultimately, she suggested, it was she, as the teacher, who had the responsibility for providing guidance and direction.

> Although I might make some decisions about it, that I say these are my decisions and that's non negotiable, but then actually give them some freedom ... to do things. But hold up models or examples knowing that they will pick up that idea without actually saying "You have to do this." ... It makes them feel a little more in control. It's part of their self-esteem too. (p. 72)

Despite initiatives in the upper secondary schools to include more individualised projects (Australian Education Council, 1990; Victorian Curriculum and Examinations Board, 1989), many teachers prefer to use teacher-selected, curriculum-set, or textbook-prepared topics (Rice & Mousley, 1990). Mousley (1990a) notes that student-centred pedagogy seems to elude schools, even when the potential lies in the curriculum for students to accept more responsibility. Peters (1993) believes that striving towards student ownership of mathematical problems by encouraging the generation of problem-posing situations rather than presenting students with only problem-solving situations can lead to a greater capacity to solve problems both in and out of school.

Role Issues

The most desirable learning situations are more likely to arise when students take responsibility for their own learning, and motivation is intrinsic. The traditional role of the teacher is necessarily challenged if not changed when students are accepted as being responsible for their learning (Holdaway, 1992). The difficulty comes when the self-esteem of the teacher is bound up with their perception of their role as the "authority" figure (Sullivan & Mousley, 1994). The teachers perceptions of their role is at the heart of this thesis, together with how this perception influences the kinds of choices each teacher made for the approaches she used with her students.

The dilemma for teachers trying to shift into a more student-oriented pedagogy is the students' lack of skills and poor conditioning for this different mode of operating (Sato, 1988). Sato developed a model which allowed gifted students to move into greater independence. Some of the components of Sato's model are as follows:

reproduction → to → production
externally directed learning → to → internally directed learning
consumers → to → producers
problem solvers → to → problem finders
question answerers → to → question askers.

The emphasis in this model is towards a much more *proactive* role on the part of the students, rather than a *reactive* one, responding to agendas that the teacher is likely to have set.

Summary

If educators embrace the ideas which have emerged in this teaching/learning Principle of Responsibility—from linguists, psychologists, child-developers and natural language advocates—a multi-faceted role for teachers emerges. Bickmore-Brand (1989) recommended that teachers be responsive and able to change their roles in appropriate ways dictated by the learner's needs.

Principle 7: Community—Creating a Supportive Learning Environment Where Learners Feel Free to Take Risks and be Part of a Shared Context

Introduction

The ideas contained in this teaching/learning Principle refer not only to the roles of the teacher or the learners, but also have ramifications for the whole school environment. When a shared context is created, Smith (1988) described it in what he termed a sense of belonging to a club or a community of learners. Other writers have discussed the benefits of being immersed in a community of learners (Barsalou, 1992, Bereiter & Scardamalia, 1993, Gee, 1992, Heath, 1983). Part of the assumption behind this Principle is that we will be understood by those with whom we communicate. As Holdaway, (1979) commented:

> Sharing even clumsily in the skillful activity of "significant others" establishes membership in the group and fixes valuing of the skill. The resultant sense of cultural identity supports a growing assurance of personal identity as one who belongs with, and needs to master, the skill. (p. 59)

By creating a community atmosphere in a classroom the learner may be in a position to share problems and successes with other members of the group (Boomer, 1988; Cairney, 1987).

The importance of creating a shared context of meaning should not be underestimated (Wells, 1981). A shared context is important, for example, in making "public" what Barrett (1985) talks about as "private" or personal knowledge. Pimm (1987) refers to this as "talking for oneself" and "talking for others." Burnes and Page (1985) believe that a teacher who creates and encourages a shared context in the classroom is going to be in a better position to select appropriate materials and instructional procedures. According to Burnes and Page, a shared context helps to reduce the distance between what learners bring with them to the classroom and the content of what is being taught. The classroom climate that the teacher sets up for children's learning will not only reflect what the teacher knows (e.g., about mathematics), but what he/she believes about how children learn (Lubinski, Thornton, Heyl & Klass, 1994).

Vygotsky (1962, 1978) introduced the idea that the mind is a product of the social life in which one finds oneself. Initially the dialogue may be between two individuals—for example the teacher and the child—and is interpersonal. However, Vygotsky argued that eventually those speech patterns become internalised as a product of these experiences and become inner speech or intrapersonal dialogue and may be used to self-regulate one's actions. It is therefore useful to extrapolate this concept to the classroom. For example, in small group settings, where each group member's speech is being modelled aloud and becomes available for internalisation by other members, individuals may build up intrapersonal dialogue from these group experiences.

The notion that children learn through interaction with others was assessed in a longtitudinal project by Jones, Thornton and van Zoest (1992). They reported that, through discussions with their peers, students developed better understandings of the problem, and shared and refined their own solution attempts as well as their mathematical understandings and strategies.

Classroom as Culture

The classroom provides varied and complex opportunities for the social construction of meaning. As Bishop (1985) notes,

> each individual person in the classroom group creates her own unique construction of the rest of the participants, of their goals, of their interactions between herself and the others and of all the events, tasks, mathematical contents which occur in the classroom. (p. 26)

Consequently the classroom becomes a culture in its own right. Habermas (1970, 1972, 1978, 1984) moves away from an emphasis on each individual constructing meaning in an isolated environment to a recognition of the individual as part of the social milieu. The "knowers" are therefore bound with their culture, suggesting that all knowledge is mediated by social interactions.

Lave (1988) and Pimm (1991) are also interested in this connection between culture and cognition and the everyday life activities in which the learner may be engaged or may wish to be engaged. Lave's focus on "context" has already been discussed in a previous section of this chapter. According to Pimm (1987), "mathematics is, among other things, a social activity, deeply concerned with communication" (1987, p. xvii).

Social influences are a major determinant in what counts as knowledge in classrooms (see discussions from German interactionist literature, for example, Bauersfeld, 1988; 1992; 1995; Voigt, 1985; 1994; 1995). Knowledge is constructed "intersubjectively" and is socially negotiated between all participants in the classroom. Those who are considered to be "significant others" in the socio-emotional dynamics of the classroom have the potential to play an important role in this process. Solomon (1987) describes this as "lifeworld knowing" (p. 67).

Personal construct pedagogies fall short when they fail to take into account the dynamics of classroom interaction which shape the construction of personal meaning. It is interesting to note Bruner's (1986) shift from the notion of an egocentric learner as discoverer to a position which embraces the impact of the community in the learning process.

> My model of the child in those days was very much in the tradition of the solo child mastering the world by representing it to himself in his own terms. I have come increasingly to recognise that most learning in most settings is a communal activity, a sharing of the culture. It is not just that the child must make knowledge his own, but that he must make it his own in a community of his own who share his sense of belonging to a culture. It is this that leads me to emphasise not only discovery and invention but the importance of negotiating and sharing. (p.127)

Thus learning does not occur through an internalisation of what is being learned, but through a negotiation and sharing of this knowledge.

Bauersfeld (1988, 1995) emphasises the importance of developing a mathematics classroom culture in which students are especially aware of their mathematical concepts and operations. He envisages a mathematics curriculum in which the focus has moved away from "subject-matter" knowledge to one on nurturing a classroom community in which students and teacher jointly construct and negotiate meanings. Pedagogy needs to take account of individual learners, but must also embrace the socio-cultural context of each classroom learning environment. (see Cobb, 1989; Cobb, Wood & Yackel, 1990; Cobb, Wood, Yackel & McNeal, 1992; Cobb, Yackel & Wood, 1992).

Social Construction of Meaning

The *community* Principle represents an attempt to value the contextual features of the classroom, while at the same time recognising the cultural background and prior learning that the individuals bring with them to any classroom discourse. Even teacher's beliefs and views seem to be shaped by the experiences in the classroom. as they respond to the demands and problems of teaching (Ernest 1988).

Classroom studies conducted by Yackel. Cobb, Wood, Wheatley and Merkel (1990), have made it possible to document the shift in students' thinking when the context was one which facilitates cooperation between students grappling with mathematical ideas. The researchers conclude that this interaction shaped "what is learned and how it is learned" (p. 20). Teachers attempting to adopt constructivist ideas have aimed at establishing learning environments which nurture interest and understanding through cooperation and through social interaction which tolerates dissonance (Pateman & Johnson, 1990).

Teachers' beliefs about mathematics will also influence the development of the social context in the classroom as Nickson (1992) concludes:

> If mathematics is characterised in terms of openness and of questioning and testing ideas, what is necessarily involved is a sharing of ideas and problems among the pupils and between pupils and teacher. Although it is clear that specific concepts and skills have to be learned, the learning and application can be more purposeful because of this shared nature. The context becomes more purposeful rather than artificially contrived. (p. 104)

Nickson clearly values the notion of developing a community of learners. The research of Alro and Skovsmose (1996) investigated school discourse and observed that teacher's expectations about their role in turn influences the kinds of knowledge that is permitted to be talked about in classrooms.

Cooperative Learning

Slavin (1987) found a higher achievement score for students working in cooperative learning situations when compared with that for students working in traditional classrooms. Students' efforts were attributed to a combination of cooperative and competitive incentives which motivated them to encourage one another in the learning environment. Wheatley (1991) argued, as have Gooding and Stacey (1991), that the strength of problem-centred learning is that students can operate at their own cognitive levels and use their preferred learning styles.

Traditionally, mathematics classrooms are very competitive places (Mousley & Clements, 1990), often characteristically different from other learning environments (Clarke & Clarke, 1990; Stigler & Baranes, 1988).

As Davidson (1990) reports, debate continues to surround the relationship between cooperative learning and performance. Davidson conducted a major review of some 70 studies comparing cooperative learning with traditional whole class teaching. The findings came out strongly in favour of cooperative and small group approaches (see also Davidson & Kroll, 1991). On the other hand, a number of research studies have produced results which reject the idea that cooperative learning leads to greater individual gains (see Gooding, 1990; Healy, Hoyles & Sutherland, 1990; Pirie & Schwarzenberger, 1988).

A major stumbling block for the implementation of cooperative learning in upper schools remains in the assessment area. Even when "who" and "what," are being assessed, have been made explicit at the outset, workload inequities continue to exist (Clarke, 1995).

State curricula have embraced the notion of cooperative group learning (see for example the *Mathematics Framework* document issued by the Victorian Ministry of Education (1988) and the Education Department of Western Australia's *Student Outcome Statements* (1995), and *A National Statement on Mathematics for Australian Schools* (AEC, 1990)).

Clearly, more research is needed on the role of cooperative group learning in the mathematics classroom. "How To ... " documents such as Dalton's (1985) "Adventures in Thinking," although very practical for problem-solving across the curriculum, lack a strong research rationale. Ellerton and Clements (1991) raise the question that may need to be asked when applying cooperative learning in mathematics classrooms—under what circumstances is peer discussion likely to be valuable? (p. 103). The research reported in this thesis may be able to shed some light on the question of peer discussion when classroom interaction patterns are analysed.

Risk-Taking

Associated with the concept of creating a "community" of learners is the concept that learning is a negotiation of meaning which should empower the students to take risks concerning their own conceptual development. Risk-taking in order to construct more mature mathematical ideas, with a by-product of ownership by the learner of the mathematics, can be found in the research of Del Campo and Clements (1990) and Pateman (1989).

Brandau (1988) captured a classroom atmosphere which is most consistent with the ideas behind the teaching/learning principle of *"community"*. She reported:

> The way I teach is risky. I'm trying to create a certain atmosphere ... one that allows for honesty, risk taking, and freedom ... a safe one that allows students to express their fear about mathematics ... And yet such expression gives them the freedom to move beyond their fears and to learn the mathematics they feel they never learned ... growth occurs when we risk and show vulnerability. (p. 198-199)

Mistakes become viewed in a different way, where they are not regarded as "faults," but rather part of the normal process of learning and doing mathematics.

Empowerment

In computing lessons where the teacher is often involved in a one-to-one situation, many students are quick to pass on what they have learned to others. Discoveries are shared and valued by one another (Green, 1988; Newman, 1977). Cobb (1990b) is quite specific about classroom practices which value the learner's voice, encourage differing opinions, and support risk taking which may take students into alternative interpretations. Although he celebrates difference, he does not undervalue the need to work toward a consensus in which various mathematical ideas are consolidated. The Cockroft Report (1982) was quite explicit about the need to provide opportunities for students to share their ideas with their teacher and peers and be given the opportunity to develop and refine their understandings. Stacey (1990) recorded the following comments made by a teacher after experimenting with independent project work for a VCE unit:

> Students need to be brought gradually to the point where they can define and plan a complete investigation. Collecting their own data, seeking their own references, planning how to present results, for example, are all decisions which contribute to autonomy: (p. 64)

Stacey acknowledges that autonomy cannot be expected of students right from the outset. Students need to be in a learning environment where they can gradually take control of their learning.

The Place of Errors

Another aspect of the community Principle which can be considered to be learner-centred in its pedagogy is the way in which errors are viewed. Goodman (1973) reconstructed errors in oral reading as "miscues"—windows into the reader's mind. In a similar approach Wheatley (1991) views student errors not so much as mistakes waiting to be corrected or erased from the student's behaviour, but as "rich sources of information about children's thinking. They indicate the meanings children have given to the associated ideas" (p.14).

Both mathematics and science education researchers (Hewson & Hewson, 1988; Posner, Strike, Hewson & Gertzog, 1982; Resnick, Nesher, Leonard, Magone, Omanson & Peled, 1989) argue that errors are a natural consequence when a learner is attempting to integrate new procedures with prior knowledge. Unless the learning environment is such that students will realise the inconsistencies between their attempts at a solution and approaches which lead to acceptable solutions, it is possible that they may continue to sustain both concepts as different and isolated pieces of knowledge.

Resource

The ideas being developed in this section about "community" suggest that all members of the class become resources for one another. This would be typical of a classroom in which students can try out their ideas on one another and extend their own understandings as they listen to, compare and challenge one another or seek to justify their own solutions. Greeno (1988) notes that collaborative practice depends on communication with others and "knowing how to use the resources that are available in the situation" (p. 24). Clements (1990) provides many examples of teachers exploring new ways of working with students over their investigations, with students being a resource for one another.

Learning Styles

Where a shared community exists, there is opportunity for free exchanges of opinion and the expertise and contributions of individual members can be maximised. In her article "There's more than one way to learn", Burns (1989) explores the idea that performance at school may be an issue of how well our learning style matched with the teaching style. For some students learning is achieved through sensory-based experience and for others their perceptions are based on thinking things through in a more abstract way. A teacher may find students learn more when there is a change of roles rather than the traditional role where the learner is dependent on the teacher. As described in the "responsibility" section, there should be multiple roles, rather than the traditional stance between teacher and student. Rather than an abandonment of instruction, however, there is a suggestion of instructional diversity (Holdaway, 1979) and the application of a variety of professional techniques, which cater for a range of learning styles and backgrounds.

Teacher-Student Relationships

Wells (1981) comments that teachers in classrooms that facilitate learning focus upon collaborative learning and shared meanings. In a facilitated classroom learning community, the students experience open-ended, exploratory conversations about issues and topics which arise from shared activities and interests, that have been negotiated along with the intentions and meanings. When individuals are given the opportunity to challenge each other then they are more likely to re-organise their viewpoint (Piaget in Copeland, 1984).

Boomer (1988), well known for his writings about the negotiation of learning within classroom contexts, encourages this less didactic instruction, greater risk-taking and open questioning, to create a class of problem solvers where all participants look to one another as a resource (see also Tobin & Fraser, 1988). Sadler and Whimbey (1985) describe this environment as one where students feel free to take risks, where their efforts are valued and where failure along the way is forgivable. This approach is based on the assumption that "collaboration" maximises resources, and that children learn from one another.

It is interesting to note that as children experience this sense of belonging to a community of learners, the community itself brings about its own form of control far more powerfully than authoritarian approaches (McDermott, 1977). The learning relationships between teacher and students are dependent on trust. This trust can get easily threatened and even sabotaged in schools because there is so much concern for getting it right or getting a good mark. Reports have been made of students even being competitively unsupporting of one another (Brophy, 1987).

The learning that occurs in the classroom is affected by the teacher-student relationship. Cobb (1987b, 1988) observed that students adjusted their goals and activities to fit their teachers' expectations, and adopted a view of mathematics consistent with how mathematics was handled in class. Weaver's (1980) model of how teacher's beliefs and definitions of reading, made an impact upon the classroom dynamics has been adapted for mathematics in Figure 3.

Children's success at mathematics
⇓ ⇑
Children's mathematics strategies
⇓ ⇑
Children's definitions of mathematics
⇓ ⇑
Instructional approach
⇓ ⇑
Textbooks' and/or teacher's definitions of mathematics

Figure 3. Relationship of teacher's beliefs to the classroom context (after Weaver, 1980, p. 4).

The definitions which teachers hold about their subject area stem from their beliefs about the content and how it should be taught (Steiner, 1987; Weaver, 1980). It influences the learning environment they create and inevitably the students' perceptions about the subject and the strategies they use. It is therefore significant to focus on the ways in which mathematics teachers create experiences of mathematical learning for their students.

Concluding Discussion

In conclusion, the ideas put forward in these Principles are that the shared meanings of mathematics include not only conceptual knowledge, but also mathematical techniques, that is, procedural knowledge or knowing how to perform actions in certain ways. Arising from this is a key issue—that many learners have difficulty in making meaning from these semiotic practices which make sense in mathematics. Students have trouble seeing the connections between much of what we do in schools. The issue of whether mathematics classrooms can be places to experience mathematical learning in a context of the learning community will be a focus in this study of two classrooms.

Chapter Two

THE STUDY AND ITS CONCEPTUAL FRAMEWORK

The review of mathematics education literature has demonstrated an increasing research interest in the relationship between language and the teaching of mathematics. In both the mathematics education and the language education literatures can be found issues, still unresolved, concerning the selection of content, the role of the teacher and the culture most conducive for learning.

Figure 4 attempts to capture some of the issues that are common to both the mathematics education and language education fields. Both fields have an interest in the issue of context and the degree to which curriculum can have application to the real world. Associated with this is the issue of modelling and its role in teaching and learning mathematics. The issue of the individual's construction of concepts and how teachers accommodate this within the classroom is also a shared concern, and of particular concern to mathematics education. Related to this issue is how directive a teacher should be in facilitating the development of metacognitive learning strategies in their students. Both literatures raise issues concerning the degree of control learners can take for their learning and the kind of learning culture which can best facilitate the development of autonomous learners.

Language Education	Mathematics Education
Context- to what extent can the curriculum be shaped to embed real world contexts into the content?	1. What is the role of context in learning? Related to this is the issue of the extent to which "real world" situations need to be incorporated into the curriculum.
Modelling- to what degree do students see their teachers engaging in genuine use of the skills they are imparting?	2. What is the role of modelling in the teaching and learning of mathematics?
Interest- to what extent can teachers cater for the idiosyncracity of the learner?	3. How driven should the content of the curriculum be by the learner's interests? Related to this is the issue of learner-centred curriculum versus content-centred curriculum.
Scaffolding- what kinds of language facilitate the learner's growth in conceptual understanding?	4. What is the role of direct instruction in the teaching of problem-solving strategies? Related to this are two issues:- First, the teacher's knowledge, skills, and perceptions about the subject area, and how teachers communicate with the students. Second, the metacognitive skills of learners.
Metacognition- what degree can students transfer the strategies explicitly taught in classrooms to novel situations?	
Responsibility- how can teachers develop in children the capacity to accept increasingly more responsibility for their learning?	5. To what extent and by what means can the learner take responsibility for his/her own learning? Related to this are the issues of ownership and/or empowerment within the classroom environment.
Community- what role does a teacher have in a classroom environment where children negotiate, feel free to take risks and act as a resource for one another?	

Figure 4. Some common issues which emerged from the review of the fields of language and mathematics education.

This Study

This research involved a detailed study of two teachers—one with a strong language-arts background, and the other with a strong mathematics education background. Through observations of these two teachers as they worked with the students in their respective upper primary mathematics classrooms, the research aimed to inform the debate around the issues identified in Figure 4. The overarching focus of the thesis is: "Teachers of mathematics teach mathematics differently." Even though both teachers were teaching the same or closely similar content, they had differing pedagogical and philosophical emphases. One teacher believed that her main task was to prepare her students for higher mathematical studies and the other, to prepare students to function in society.

The Principles for Teaching and Learning (Bickmore-Brand, 1990) will operate as a lens through which classroom interactions and practices in each mathematics classroom are analysed and discussed. Each Principle will be coded and given subcategories for the purpose of using the data analytical tool, Non-nUmerical Data Indexing, Searching and Theorising (NUDIST) (Richards & Richards, 1990).

Issues in Teaching and Learning- Major Research Questions

This study will therefore investigate how mathematical concepts are dealt with in the classroom, what learning environments are provided, what is selected as content and what attitudes about the value of mathematics the students develop. The aim of the research is to contribute to the body of understanding in mathematics education about these issues. The following research questions become the major focus for the study.

1. What is the role of context in learning?
2. What is the role of modelling in the teaching and learning of mathematics?
3. What is the role of direct instruction in the teaching of problem-solving strategies?
4. How driven should the content of the curriculum be by the learner's interests?
5. To what extent and by what means can the learner take responsibility for his/her own learning?

Through the investigation of the two teachers in this study the research analysis will reflect upon the factors influencing the teaching of mathematics.

Data Sources : An Overview

Because of the wide range and quantity of data collected, it has not been possible to utilise all of the information. The main data drawn upon for the data analysis have been:

- Three lesson samples from each teacher—"typical" lesson series (of a sequence of events on the same topic), March lesson and August lesson;
- Initial-Teacher and Post-Teacher Interviews;
- Initial-Student and Post-Student Interviews;
- Diary;
- Field Notes;
- Lesson Transcripts;
- Samples of teachers' programs;
- Textbooks, worksheets and materials used during lessons; and
- Progressive Achievement Test (PAT) and Placement Test J standardised assessments.

Before reading the Data Analysis Chapter 4 it is recommended that the reader become familiar with the three lessons for each teacher— "typical lesson series" (Appendix 1), March lesson (Appendix 2) and August lesson (Appendix 3). These formed the basis for the NUDIST analysis and the interrater comparisons. All graphs and tables describing and contrasting the approaches used by the teachers have been drawn from the NUDIST analyses of these lessons.

Each Principle is dealt with to a large extent independently from the next. For this reason, the same data examples may be used more than once to illustrate a component or subcategory of each Principle. This repetition is intentional.

Chapter Three

METHODOLOGY

The Sample: The Teachers and the Students

This study involved the use of a range of methodologies to investigate two upper primary school classrooms. Two teachers who teach at the upper primary level, and who are respected by their profession, were selected—one in the area of mathematics with an interest in language and the other with strength in language and an interest in mathematics. Both teachers worked in the private school system (both Anglican schools) and as such were not bound by the content prescribed in the Western Australian Education Department's Mathematics Syllabus. The Syllabus did, however, form the basis for the teachers' choice of content. In other words, although the teachers had a commitment to ensuring that the children were adequately prepared for the senior school they would soon enter (part of the same school), they did not feel obligated to achieve all of the criteria embedded in the content written for and by the State School System.

Upper primary level classes were chosen because teachers at this level are quite concerned with preparing children for high school.

The First Classroom

Lyn is a Year 6 teacher who is recognised in the language arts field as exemplary in her classroom practice. She has presented guest lectures at the tertiary level and videos have been made of her teaching in lower grades. Her reputation appears to have arisen from (a) her competence in applying recent theory and research to her teaching; and (b) her capacity to articulate her theory and practice; and (c) her integration of mathematics, sciences and humanities curriculum content in her programming. She has also presented papers at national conferences and run ELIC (Early Literacy In-Service Course) professional development for teachers. She is a "key" language arts teacher in her school. Her move to the upper primary school level was experimental and was initiated by the Principal, who had concerns that her teaching methodology might not suit the others in the team in his private co-educational (mostly boys) school. This was the second year of this trial.

There were 26 mixed ability children in Lyn's Year 6 class of whom 6 were girls.

The Second Classroom

The second Year 6 teacher is Michelle, who is recognised in the mathematics teaching field as being exemplary in her classroom practice, and who also integrated recent language theory into her classroom programming. This teacher has also given guest lectures at the tertiary level to teacher education students and has written and presented papers for the Western Australian branch of the Australian Association of Mathematics Teachers. She has participated in the organisation of "Maths Activity Day" (MAD) run by Edith Cowan University. She was chosen for this study because of her reputation for her ability to articulate theory and practice. She is a "key" mathematics teacher in her school and takes the top mathematics group where four Year 6 and 7 classes have been ability grouped. She has also conducted "House Maths" for which students from several different year levels meet every fortnight to participate in a range of non-curriculum mathematics activities.

The classroom observation of Michelle's teaching was not with her "home" class but the top ability mathematics class (Year 6/7) which she taught for one or two 45-minute periods daily, depending on the day of the six-day cycle. There were 26 above average ability children in this class of Michelle's, of whom all were girls.

Data Collection

Data collection centred on (a) regular field observations over a nine-month period as a participant observer in the classroom; and (b) regular interviews with the key participants in the study. The consistent presence of the researcher in the classroom as a participant observer helped to ensure that the researcher became an accepted part of the classroom routine and culture. Data collected in this ethnographic framework included keeping field notes, and collecting samples of children's work as well as copies of classroom mathematics materials. Video- and audio-tapes were recorded, and a personal diary was kept. The data were collected within the following twelve-phase framework.

First phase. Selection of teachers. Two teachers were selected after consultation with university academics, professional associations and the teachers themselves.

Second phase. Parent information. Permission was gained from the participating schools, and an explanation of the research given to the Principal, the teachers directly involved in the study, and any interested staff. In each school, Parent-teacher Interview Nights provided opportunities for the study to be discussed with the parents of the students in each class.

Third phase. Initial-Teacher Interviews. The teachers were interviewed at their respective schools at the end of the year prior to their teaching the class in the study. This was to give them a chance to feel more comfortable with the author at a time when they would have felt more in control of their teaching context than they would at the start of the following year.

The teachers were interviewed to ascertain their beliefs in relation to language and mathematics, and to language and mathematics teaching and learning. Questions which probed each teacher's understanding of, and their view of, mathematics education and language-learning Principles, were asked. The teachers were asked to respond to the questions on a rating scale. The interviews were audio-taped for later transcription.

The interview questions were designed to give insights into the match and/or mismatch between a teacher's beliefs and practice. The interview schedule included questions which asked the teachers about their choice of content, their perceptions about mathematical abilities, and their own confidence, as teachers, in the area.

Interview questions. The questions used in the interview schedule have been reproduced below, together with a brief rationale for their inclusion.

1. Which Year level or mathematics level do you teach? (This was to ascertain the Year level change, if any, between that year and the subsequent one in which the author was to observe them teaching.)

2. How do you currently programme for mathematics on a weekly basis? (This was to ascertain whether the teacher used an integrated approach, or whether she preferred to work on a specific topic or concept over a period of time.)

3. Where do you draw your content from? (This was to determine whether the teacher followed the set W. A. Education Department syllabus or one set by the school, or whether she followed a theme, or integrated her mathematics lessons with another subject(s) area(s) or followed a text book(s).)

4. What key resources do you use? (This was to ascertain whether the teacher used mostly Education Department produced material or commercially produced worksheets and/or textbooks, or whether teacher-made or child-made material was used.)

5. Can you describe the children with above average mathematical ability in your class? What skills, attitudes or behaviours do you see them exhibit? (This was to gain insights into what each teacher perceived as important attributes in mathematics learning and presumably ones which she would be trying to develop in her classroom.)

6. Can you describe the children who have lower mathematical ability in your class? What skills, attitudes or behaviours do they exhibit? (This was asked in

order to gain insights into what each teacher perceived as possible factors working against effective mathematics learning in her classroom.

7. What are the characteristics of a good mathematics teacher? (This question was aimed at revealing the beliefs and values placed by each teacher on the craft of mathematics teaching. The teacher's responses would then be linked to the year's observations of actual classroom practice.)

8. What would be the ideal way you would like to teach mathematics? (This was to ascertain if the teacher was experiencing any constraints which could be associated with her teaching context, such as ability grouping or class size or composition.)

9. What changes would you like to see in your current mathematics programme? (This was to ascertain how reflective each teacher was about her practice and how empowered she felt to operate in ways which were consistent with her beliefs.)

10. Can you identify areas in which improvements could lift your current programme? (This was asked in order to gain insights into any constraints that the teacher felt and any aspirations she might have for self-improvement.)

11. Are you aware of any underlying philosophy with which you strive to be consistent when teaching mathematics, or other subjects? (This was asked in order to gain some understanding about which educational fields might be influencing each teacher's approach to teaching, and to reveal details of her current professional reading.)

12. Who and what influences your professional practice? (This was asked in order to reveal which teaching practices were most valued by each teacher, and to ascertain what major factors influenced her practice.)

13. What forms of professional development have you taken part in during the last 5 years? (This was to ascertain which educational fields have influenced the teacher and how recent her professional development has been, as well as how broadly she seeks out professional assistance.)

14. Is there any area of your personal teaching on which you are currently focusing? (The aim of this question was to reveal how reflective she was about her teaching, and to assist my interpretation of her methodology.)

15. What do you believe about how children learn language? (This was to reveal which underlying principles inform her teaching and to compare with previous answers about educational influences on her methodology. Each teacher's responses will be related to observations of her classroom practice.)

16. What do you believe about how children learn? (This was asked in order to begin to understand the type of learning environment and conditions each teacher believed was important to maximise learning and to see whether this is consistent with any language and/or mathematics learning theories.)

17. What do you believe about how people learn mathematics? (The reason for asking this question was similar to that given for Question 15, but in a mathematical context .)

18. What impact do parents have on your teaching in general, and on your teaching of mathematics in particular? (This was asked in order to ascertain how

much autonomy each teacher felt and whether the classroom environment was inclusive or exclusive of parents and to what extent. Responses would be compared with classroom observations.)

19. What impact does school policy have on your teaching in general, and on your teaching of mathematics in particular? (This was to ascertain whether the teacher felt she had autonomy in her selection of content and style. For example, a policy decision may be made in the school to use ability groups, or the school may require certain content to be covered before children can progress to the next year level.)

20. What impact do other staff at your school have on your teaching in general, and on your teaching of mathematics in particular? (This was to reveal to what extent each teacher felt professionally isolated or supported.

21. Are there any aspects of mathematics you feel form a necessary base to help support children who are about to enter Year 8? (This was asked in order to ascertain the extent to which each teacher felt obligated to ensure that each child has received certain concepts.)

22. In what areas of the mathematics course that you currently teach do you feel most confident about? (This was to gauge how confident each teacher felt about her own mathematical skills.)

23. In what areas of the mathematics course that you currently teach do you feel less confident about? (This was to ascertain each teacher's confidence in mathematics, and the degree to which this may have been apparent when

teaching particular topics. Each teacher's responses to this question would be compared later, with her practice.)

24. Are there any areas of primary school mathematics that you would feel hesitant to teach at this time? (This was to ascertain each teacher's depth and range of mathematical content knowledge, and whether the teacher might feel threatened by a child with very strong mathematical skills.)

25. Which age level do you feel most comfortable teaching? (This was asked in order ascertain whether each teacher had a preferred level, and whether this matched the year level at which she then taught.)

The remaining questions related to the seven Principles developed from the model proposed by Bickmore-Brand (1990). These Principles are summarised under the Principles for Teaching and Learning in Language Education in Appendix 4 (Bickmore-Brand's Pedagogical Principles Explained). Each teacher was asked to show her responses on a scale, in order to indicate the extent to which she was committed to different beliefs and practices. The 0-100% scale gave the teacher the opportunity to indicate her priorities with respect to that practice enabling a nil response to be made, whereas the -10...0...+10 scale was used to indicate her *attitude* toward that practice which could demonstrate both positive, negative and neutral positions.

26. To what extent do you attempt to develop a sense of there being a mathematics community in your class? Show your answer on a scale from 0-100%. (This question was asked in order to gain insights into the learning environment which each teacher created. Responses might also indicate

whether a teacher thought it may be possible to create such an environment for mathematics.)

27. To what extent do you believe teachers need to create a feeling of community in the class in order to teach mathematics? Show your answer on a scale from 0-100%. (This question was included in order to consider the extent to which teachers' beliefs match their practice.)

28. To what extent do you use direct instruction in your mathematics lessons? Show your answer on this scale from -10...0...+10. (The use of both a positive and negative scale was to provide for a wide range of responses.)

29. To what extent do you believe direct instruction to be important in mathematics teaching? Show your answer on a scale from 0-100%. (Each teacher was asked to indicate her response on a scale of 0-100 to capture the value she placed on direct instruction.)

30. To what extent do you attempt to make mathematics relevant to the child's immediate world? Show your answer on a scale from 0-100%. (Comparison of the scaled responses with later classroom observations made it possible to consider any differences between each teacher's practice and her perceptions about her practice.)

31. To what extent do you believe that mathematics instruction needs to be relevant to each child's immediate world? Show your answer on the scale from -10...0...+10 (This was to be plotted on a scale for later comparison with classroom observations.)

32. If scaffolding is the teacher's capacity to lead the student to the next level in their development through questioning and prompting ... to what extent do you practice it? Show your answer on the scale from 0-100%. (This question included providing a brief definition of scaffolding. Teacher's responses would be compared later with the practice observed.)

33. To what extent do you believe teachers should operate in this way (Refers to Q. 32) in mathematics lessons? Show your answer on the scale from -10...0...+10. (This was included to help ascertain whether each teacher's beliefs matched the practice the author was to observe or whether each felt that this methodology was more likely to be adopted by other teachers or for other subjects.)

34. To what extent do you use metacognitive strategies with the children? Show your answer using a 0-100% scale. (This was to be plotted on a 0-100 scale to indicate the value each teacher placed on a relatively "new" strategy. If the term "metacognition" appeared to be unfamiliar to the teacher, then it was explained during the interview.)

35. To what extent do you think that metacognitive strategies should be incorporated into mathematics classroom practice? Show your answer on a scale from -10...0...+10. (This was included to ascertain if each teacher applied a strategy across subjects or only when teaching mathematics.)

36. To what extent do you cater for the individuality of the learner in your classroom? Show your answer using this scale from -10...0...+10. (This has been included to ascertain the degree to which each teacher perceives herself to be

responding to children's individual needs. Comparisons will be made later with observations of each teacher's practice.)

37. To what extent do you believe individuality in teaching to be important? Show your answer using this scale from -10...0...+10. (This was included to ascertain the degree to which each teacher perceived herself to be responding to children's individual needs, and has enabled comparisons to be made with each teacher's practice.)

38. To what extent do you provide opportunities for children to accept responsibility for their learning i.e. owning and developing the curriculum, and accepting responsibility for their decisions? Show your answer using this scale from -10...0...+10. (This was included to ascertain each teacher's beliefs about children accepting responsibility for their learning. Comparisons have been made with later classroom observations.)

39. To what extent do you believe children should be given opportunities to accept responsibility in the mathematics classroom? Show your answer using this scale from -10...0...+10. (This plotting of each teacher's beliefs on a scale has allowed later comparison with classroom observations.)

Fourth phase. End of year classroom observations. Each teacher was observed at the end of the year (the year preceding the main study) and particular note taken of her relationships with her students and the implementation of her beliefs about language/mathematics learning as expressed in her earlier interview.

Both teachers' approaches to evaluation, their record keeping and their program was also observed. This set of classroom observations was carried out with the assumption that, by the end of the year, most classrooms would be operating in a routine with which the teachers were comfortable. Such an approach would give each teacher an opportunity to show the author their teaching style under optimal conditions compared with the start of the year. Teacher-student relationships could be observed and how teachers' beliefs and practices, as expressed in their interview (Third phase) might be reported on. These observation sessions gave the teachers the opportunity to get to know the author and to question any aspects of the research.

Fifth Phase. Classroom observations at start of next year. Each school was visited at the start of the year when both teachers had new classes. Any pre-testing that was occurring was noted. The classroom tone and establishment of routines was observed.

The presence of the author in the classroom from the start of the year helped to establish the routine of the participant-observer role for the children. It would also reveal aspects of the teacher's priorities (such as pre-testing), which behaviours the teacher reinforced (both positive and negative) and the children's adjustment to and manipulation of the classroom climate.

Sixth Phase. Initial-Student Interviews. The children were individually interviewed (and videoed) to ascertain their attitudes towards mathematics, their perceptions of their mathematical ability and the function of mathematics in their lives. The interview also provided data on the academic background of their parents and the student's perception of the function of mathematics in their families' lives (work and home duties, all family members), the students' perception of their own competence on a range of mathematical problems and their performance on a problem-solving task (planning a class party) (see Appendix 5 for Initial-Student Interview Schedule).

The children's answers were used to build a profile about each child in relation to; (a) his/her self image as a mathematical learner and user; (b) his/her beliefs about mathematics; (c) his/her skills and understandings about the function of mathematics; and (d) the home and classroom contexts.

Before the interview with the children took place, each teacher was given the opportunity to examine the questions, and to comment on whether she was comfortable with what was being asked of the children. Each teacher was then free to access the information and to view the tapes of the children's responses.

Seventh phase. Small group problem-solving. Each class was given the task of planning a class party (they could spend $150 for the party). The children were video-taped in their planning a class party without teacher input. The children were selected randomly and placed in 5 small groups. How they operated in these groups was observed for their capacity to co-operate, think laterally, work methodically and achieve a solution to a problem which, it was anticipated, they would be motivated to solve. Each group was observed separately and was asked to keep their ideas confidential until the video-taping was completed. The idea behind this approach was to observe the children operating without teacher intervention or influence. The task was not given to them by the teacher nor was she present.

The children's behaviours were then compared with how they operated in the normal mathematics classroom, both when they were asked to work independently and when the students worked in pairs or small groups. The observations also provided insights into the students' capacities to accept responsibility for the mathematical solutions they developed, and the degree to which they collaborated with peers with whom they, at that stage, had not been used to working. Each teacher viewed the video and shared it with the class.

Eighth phase. Video-taping the whole class problem-solving. Each class teacher was then video-taped with her whole class, as they continued to plan the class party. Each class had been given $150 cash to spend on the party. This activity was designed to provide the author with an opportunity to observe how individual children operate in a whole-class setting—who emerged as leaders, who encouraged creative thinking, who went along with the crowd and were less dominant than in the small group setting.

The teacher was observed, and her methodology for problem-solving, her capacity to scaffold the children's decision-making, the way in which she created a classroom community atmosphere and the degree to which children were given responsibility for their ideas, noted.

Ninth phase. The class party. The implementation and celebration of the class party was observed by the author who was also a participant. This approach was adopted in order to observe the extent to which the teacher recognised that the activity provided opportunities to apply mathematical skills in what may be described as a real-world context. This procedure also gave the author the opportunity to observe the children in an informal setting, as well as to talk to the teacher and observe the class dynamics.

It was also important to follow the planning aspect through to completion and hold the party so that the children could not write off their planning efforts as merely a school "exercise" or simulation.

Tenth phase. Observation of mathematics lessons. Each teacher was observed teaching a mathematics lesson on a regular weekly basis, over a 9-month period. The intention was that presence of the author in the classroom would become fairly natural and unobtrusive because of its regularity. The author became a participant observer taking notes about the classroom dynamics—its routines, teacher-child interchanges, teacher-directed aspects and child-initiated occasions, problem-solving strategies, the content taught and the evaluation approaches adopted.

Eleventh phase. Post-Student Interviews. After the main body of the observation data collection had been completed, all children in both classes were individually interviewed (video-taped) on their perceptions of the function of mathematics in their lives, their perceptions of their own mathematical ability on a range of problems, and on their capacity to do a novel problem-solving task (The students were asked *"On what day and in what year will your 21st birthday fall?"*). Similar questions to those asked at the beginning of the year regarding the children's attitudes and perceptions about; (a) their self image as a mathematical learner and user; (b) their beliefs about mathematics; and (c) their skills and understandings about the function of mathematics, were asked (see Appendix 6 for interview questions). The novel problem-solving task was to observe any obvious approaches they may have learnt from instruction which they were now applying to this novel, but assumed-to-be relevant situation.

Twelfth phase. Post-Teacher Interview. Each teacher then viewed the video taken of her classroom and was interviewed about her perception of, and reaction to, each child's responses (see Tables 3a and 3b). The intention of this aspect of the data collection was to verify whether the teacher agreed with each child's perception of his/her mathematical ability and concept knowledge. It also provided an opportunity for each teacher to reflect on the effect they had had over the year on the child's attitudes and perceptions.

Relationship between the Major Research Questions and the Data Collection

It should be reiterated that only the issues which emerged from the language and mathematics education literatures that shared some common concerns have been identified in this study (see Figure 4). Figure 5 attempts to identify which data are likely to inform the research issues generated from both fields. In order to streamline the study the six research issues appearing in the centre column will be the major research questions for this study.

Language Education	Mathematics Education	Data Collection
Context- to what extent can the curriculum be shaped to embed real world contexts into the content?	1. What is the role of context in learning? Related to this is the issue of the extent to which "real world" situations need to be incorporated into the curriculum.	Context • field notes for examples of teaching skills in context; • diary entries, textbook and worksheet samples pertaining to this observation; • post-student interviews with all children; concerning the relevance of what they had been exposed to and their everyday life; • initial- and post-teacher interviews about their beliefs and practices concerning context; • Placement Test J and PAT assessment.
Modelling- to what degree do students see their teachers engaging in genuine use of the skills they are imparting?	2. What is the role of modelling in the teaching and learning of mathematics?	Modelling • field notes for examples of explicit modelling by teacher or a student; • field notes for implicit demonstration of using mathematics to solve real-life problems by the teacher; • post-student interviews for ways students believed their teacher used mathematics in her everyday life; • initial- and post-teacher interviews about their beliefs and practices concerning modelling.

Interest- to what extent can teachers cater for the idiosyncracity of the learner?	3. How driven should the content of the curriculum be by the learner's interests? Related to this is the issue of learner-centred curriculum versus content-centred curriculum.	Interest • field notes for any content which has been selected and/or delivered with links to the knowledge, skills or values-base of the learner; • field notes for examples of when the idiosyncracity of the learner has been catered for; • post-student interviews for the individual way each child solves the 21st birthday problem, compared with the first problem of sharing the $150 from the initial interview with each child; • initial and post-teacher interviews about their beliefs and practises concerning catering for individuals.

Scaffolding- what kinds of language facilitate the learner's growth in conceptual understanding?	4. What is the role of direct instruction in the teaching of problem-solving strategies? Related to this are two issues:- First, the teacher's knowledge, skills, and perceptions about the subject area, and how they communicate with the students. Second, the metacognitive skills of learners.	Scaffolding • classroom transcripts of teacher-talk which builds on children's current thinking and expression; • initial- and post-teacher interviews about the nature of scaffolding in their classrooms.
Metacognition- what degree can students transfer the strategies explicitly taught in classrooms to novel situations?		Metacognition • classroom transcripts of specific learning strategies being taught or made explicit; • post-student interviews with all children for evidence of awareness of what they have learnt; • post-student interviews with all children comparing each child's perception of what s/he has learned with the teacher's evaluation of their learning; • post-student interviews with all children to see how they rank themselves in the class and comparison with initial interview responses; • problem-solving strategies exhibited by the students during the 21st birthday novel question; • initial- and post-teacher interviews about their beliefs and practices concerning direct teaching of problem-solving strategies.

Responsibility- how can teachers develop in children the capacity to accept increasingly more responsibility for their learning?	5. To what extent and by what means can the learner take responsibility for his/her own learning? Related to this are the issues of ownership and/or empowerment within the classroom environment.	Responsibility • field notes to see changes in degree of responsibility children are being given; • initial- and post-teacher interviews about their beliefs and practices concerning responsibility.
Community- what role does a teacher have in a classroom environment where children negotiate, feel free to take risks and be a resource for one another?		Community • field notes to see where teacher encourages a community of learners; • post-student interview to see who the children go to for help; • post-student interview to see how the class as a whole goes about problem-solving; • initial- and post-teacher interviews about their beliefs and practices concerning community.

Figure 5. The relationship between the major research issues which emerged from the language and mathematics education literatures and the data collected, analysed and reported in this thesis.

Factors Influencing the Teaching of Mathematics: A Case Study of Two Teachers and Their Classrooms

Treatment of Data

The data analysis will attempt to build descriptors of the classroom practices which reflect each of the seven language teaching and learning Principles that have been adopted as a framework for this study. The extent to which the teacher with a language education background and the teacher with a mathematics education background applied each Principle in their mathematics classroom will then be tabulated.

For the purposes of using the seven Principles as an analytical tool the following summary has been provided. Coding details and hierarchical representation of the ideas in each Principle can be found in Appendix 7.

The coding and key headings for these components act as a coding device for the Non-nUmerical Data Indexing, Searching, and Theorising (NUDIST) data analysis (see Richards & Richards, 1990) conducted on observation data for this study.

Codings and Key Headings Used in NUDIST Data Analysis

1. Context—teacher creates meaningful and relevant contexts for the acquisition of knowledge, skills and values.

    1.1 Real world—teacher provides opportunities for students to acquire knowledge, skills and values in real-world situations.

1.1.2 Textbook—teacher makes use of textbook examples which simulate or exemplify real-world situations in order to develop certain knowledge, skills and values.

1.2 Big Picture—teacher makes explicit the ways in which the learning of some specific knowledge, skill or value fits into the larger context from which it was drawn.

1.3 Purpose—teacher makes explicit the reason for a particular focus on the knowledge, skill or value being experienced.

1.4 Social Contexts—teacher makes explicit ways in which classroom activities relate to the way that knowledge, skill or value is used in society.

2. Interest—teacher realises the starting point for learning must be from the knowledge, skills and or values base of the learner.

2.1 Connect—teacher explicitly relates the content being presented in the classroom with the student's own knowledge base.

2.1.2 Cultural/Gender—teacher takes account of the background of the learner.

2.3 Register—teacher develops with students the appropriate mathematical register.

2.3.1 Link New Terminology—teacher builds the mathematical terminology from the student's own language base.

2.4 Different Styles—teacher caters for a range of different learning styles and adjusts her teaching methodology

2.4.1 Generate Own Rules—teacher actively encourages the solutions to problems developed by individual children.

2.4.2 Various Mediums—teacher uses an approach that offers a variety of mediums, and allow students to express their understandings through a variety of mediums.

3. Metacognition—teacher makes explicit the learning processes which are occurring in the classroom.

3.1 Explicit Modelling—teacher verbalises his/her own thinking strategies and ways of problem-solving.

3.1.2 Integrate—teacher indicates how the content being presented relates to other subject areas.

3.1.2.1 Links—teacher makes links between what has previously been taught and the knowledge under discussion.

3.1.2.2 Shifts From One Representation To Another—teacher demonstrates the concept in various forms and contexts.

3.1.3 Revisit—teacher provides opportunities for students to go over a concept.

3.1.4 Students Verbalise—teacher provides opportunities for students to demonstrate to their peers how they solved a problem.

3.1.4.3 Discover For Themselves—students are given opportunities to work out their own ways of solving problems

3.1.4.6 Students Share—students are given opportunities to share in small groups their thinking strategies.

3.2 Maths Transfer to Real Life—teacher makes explicit how the concepts they are learning at the time relate to real life.

3.3 Materials—teacher uses a range of materials to accompany explanations about a concept or procedure.

3.3.1 Concrete-Abstract—teacher starts with more concrete ideas and materials before progressing to the concept in its abstract and symbolic form.

3.3.4 Open-ended—teacher questions and provides problems which have an open-ended procedure for the solution.

3.4 Critical Numeracy—students are given opportunity to assess the motives behind the function of mathematics as it is used in society.

3.5 Feedback—teacher provides feedback not only about the solution but the way the student went about solving the problem.

3.5.4 Joint Construction—teacher and the class are engaged in the joint construction of meanings and problem-solving processes.

3.5.5 Frameworks—teacher and the students construct frameworks or protocols which might assist students in their thinking strategies.

4. Scaffolding—challenging children to go beyond their current thinking, continually increasing their capacity.

4.1 Stretch—teacher builds upon the student's understanding and through his/her response, challenge the learner to go beyond his/her current thinking.

4.1.1 Time Varies—teacher acknowledges that some students will need longer scaffolding assistance than others

4.1.2 Regulate Difficulty—teacher provides a range of scaffolds which vary according to the ability of the individual.

4.2 Framework—teacher provides support in the form of protocols which may assist the learner to become independent.

4.2.1 Routine—teacher provides support in the form of routines which in their predictability provide security for the learner to experiment within.

4.2.2 Peers—teacher provides opportunities for peer tutoring to enable the students to scaffold one another.

4.2.3 Situations Constructed—teacher provides situations within which the skills can be developed and the skills practiced.

4.3 Self Destruct—teacher allows the support to self destruct as new scaffolds are put into place.

4.4 Generalise—teacher allows students to generalise what supports they find helpful across to other contexts.

4.5 Transformational Freedom—teacher encourages the students to express themselves in increasingly more mathematical language.

4.5.1 Switching Roles—teacher encourages students to take on the role of expert.

5. Modelling—teacher provides opportunities for students to see the knowledge, skills and or values in operation by a "significant person".

5.1 Real Life—teacher shares with the students the genuine use of the skill for his or her daily life.

5.1.2 Overt—teacher explicitly models the genuine use of the skill for his or her daily life.

5.1.2.1 Language—teacher makes explicit by sharing out aloud his/her thinking strategies about the skill he or she is involved in with the students.

5.1.2.2 Informal—teacher talks aloud and shares his/her thinking strategies in spontaneous informal situations with students.

5.1.2.3 Grappling—teacher talks aloud and shares his/her own struggles when thinking through a problem or making a decision.

5.2 Charts—teacher provides examples for the students to base their own work on.

5.3 Students—teacher provides opportunities for the students to model genuine use of the skill being under focus in the class.

5.3.1 Compare—teacher provides opportunities for the students to compare their own examples with other peers.

5.4 Many Uses—students will have opportunities to see the skill in action in a variety of circumstances.

6. Responsibility—teachers develop in children the capacity to accept increasingly more responsibility for their learning.

6.1 Gradual Release—teacher provides opportunities for students to increase the degree of independent mastery or control of the learning.

6.1.1 Hand-holding—teacher provides structures which support students and does not expect them to become independent on their own.

6.2 Accept—learners demonstrate a willingness to take increasingly more control of their learning.

7. Community—teacher creates a supportive classroom environment where children feel free to take risks and to be part of a shared context.

7.1 Support—teacher creates a community in which all participants feel supported.

7.1.5 Risks—teacher creates a community where students feel free to take risks and make approximations.

7.1.5.1 Teacher Faults—the classroom climate is such that the teacher feels free to admit any faults she may have made during instruction.

7.1.5.2 Start With Errors—the class members feels sufficiently comfortable with one another that they are able to work as a group on the errors that any one of them may have made.

7.2 Learning Styles—teacher values the contribution of students with different learning styles and caters for them.

7.2.1 Levels—teacher acknowledges that there will be different levels of achievement within the class but that they can all make a contribution.

7.3 Resource—class operates in such a way that the students act as resources for one another and are not solely reliant upon the teacher.

7.3.1 Collaborative—students work in collaboration with one another. Where competition is used it will be negotiated with and by the students.

7.4 Empower—students feel empowered to make decisions about the curriculum and management of their learning.

7.4.1 Negotiation—students develop skills in being able to negotiate their own learning with the teacher and with one another.

7.5 Expectation Of Success—members of the class will share a positive attitude about the potential of the individuals and group.

7.5.1 Celebrate Success—teacher provides opportunities for the class to celebrate successes in learning achievements of both individuals and groups.

7.5.2 Quality—teacher and class make explicit their opinions concerning the calibre of work being produced by each member.

7.5.3 Work Expectations—teacher is explicit and open about his/her work expectations and classroom behaviour.

NOTE: During the trialing of the NUDIST codings some categories were eliminated, and hence some categories discussed in Chapter 4 appear not to be in numerical sequence.

Chapter Four

DATA ANALYSIS PART ONE

Introduction

The task of classroom observation is a formidable one. When empirical research methods give way to qualitative data collection then not only can the amount of data generated become extremely large, but the analysis of such data can be unwieldy. As described in the previous chapter, the types of data collected were field notes, lesson transcripts, teacher programs, student work samples, researcher diary entries, textbooks, and interviews, collected via print, handwritten notes and as video- and audio- tapes. The aim of such broad spectrum data methodology was to attempt to illuminate and interpret the process of education in two diverse contexts. In order to interpret such a socio-cultural form of data collection (Evertson & Smylie, 1987; Florio-Ruane, 1987), it was necessary to overlay a framework or lens through which the data could be viewed.

Atweh (1993) discusses the use of Halliday's (Halliday & Hassan, 1989) Functional Theory of Language framework as a practical tool to deal with the large amount of data gathered. He admits that even in such circumstances, not only were not all of the data used, but not all the constructs of that theory were used. During the initial stages of the data collection the author was predisposed to interpreting the data from a range of foci, for example, gender issues, the use of technology in the classroom, self-concept and school achievement ...

However, over the observation period, as the reflection through the Diary entries built up, it seemed that the grounded theory which seemed to be emerging reflected elements of the learning Principles developed during the author's Master's research. For this reason it was decided to use the framework as a lens through which the data could be viewed.

Mousley, Sullivan and Waywood (1993), in referring to the work of Snowden and Keeves (1988), raise the issue that although qualitative research produces a richness in detail, capturing interactions which are often iterative, it is difficult for the researcher "to categorise, to compare verbally or statistically, to chart or graph, and to decide what to report or omit" (p. 416). For this reason the author, as others (see Mousley, Sullivan & Waywood, 1993) have done, chose to use NUDIST software for the analysis of data. Initial categories for the NUDIST analysis were framed by the seven Principles. NUDIST software permits successively finer classifications within major classifications. This enabled a hierarchy of ideas which were associated with each of the seven Principles to be developed.

Despite the detailed analysis made possible through NUDIST analysis, it is not necessarily suggested that the results be read as one would empirical data analysis, Clarke (1992) argued that "reductionist approaches to educational research frequently deny the fundamental interrelatedness and socially-situated nature of educational constructs" (see Clarke, Frid & Barnett, 1993, p.153).

Lerman (1994a) argues that research on teachers' beliefs and practices cannot separate out the cognitive and emotional responses of the researcher from the research methodology. The data are then re-examined from various perspectives in an attempt to identify commonalities to what were initially value-laden, personal conceptualisations (Delamont, 1992). In other words this study took raw data in the form of classroom observations which were inevitably collected through the field-note lens of the author's own construct system, it then overlayed the students' perceptions and teachers' perceptions of those experiences. An inherent danger in any research situation which includes a participant observer is the threat to the credibility of the findings if they are regarded as one person's interpretation of events.

This introduction therefore acknowledges the inescapable fact that the data gathered has been filtered through the author's own construct system (Lerman, 1994b). It should be noted, however, that data has been drawn from multiple sources within both classrooms, and that this triangulation, as well as the inter-rater reliability exercise which took place after the main round of analysis had been completed, adds to the validity of the study.

Computer Coding of Raw Data

The NUDIST analysis was carried out on the complete set of field notes according to a coding system of categories and subcategories of each Principle (see Appendix 7 for an overview of each node and subnodes/subcategories.) The field notes were word processed in a form which was suitable for coding with NUDIST. The text is unformatted and split into collections of sentences or phrases which capture what can be described as a unit of meaning. These units are numbered in sequence and called text units (TU).

In order to analyse the text units, a hierarchical coding system was developed. Each Principle was described as a separate node and given a coding number, for example *Context (1), Interest (2), Metacognition (3)* ... Subsidiary ideas relating to each of these nodes were coded as subnodes. For example *Materials* used in association with a text unit identified as metacognition would be coded *(3.3)*. Each sentence or unit of meaning in the field notes was subsequently coded according to the nodes and subnodes it most strongly exemplified. For example, text unit 142 read *"teacher encouraged cross group discussion."* This was coded in the following way: *metacognition- students share (3.1.4.6), modelling- compare (5.3.1)* and *community- resource (7.3).* Each text unit from the field notes could be identified by sequential number and node number:

142 *teacher encouraged cross group discussion*[2]
(3 1 4 6) (5 3 1) (7 3)

Note that it is possible for one text unit to receive several coding categories.

The collections of associated ideas under each node were examined to determine whether they should be moved to a different subnode, or whether the subnode should be deleted as redundant. Note that a node may still exist even if

[2] In order to help identify which classroom is being referred to, the two classrooms have been assigned a different font e.g., Michelle and *Lyn*.

there are no text units which are coded under its category. For a complete overview of the nodes and sub nodes see Appendix 7.

NUDIST enables the researcher to request details about the number of incidences of each node and subnode, on a lesson-by-lesson basis. It was therefore possible to identify the nodes and subnodes which were predominant for each teacher and in which lessons. It was also possible to follow any changes which occurred over the time frame of the field-note observations.

Inter-Rater Reliability Check for Data Analysis

It was recognised that the bias of the researcher could exert a significant influence on the categorisations and descriptors chosen (and hence the nodes/subnodes). The following procedures were therefore put in place.

First, the initial Principles were drawn from a large body of research and similar themes have emerged in the works of other writers (see, for example, Boomer, 1988; Cambourne, 1988; Sullivan & Mousley, 1994; and the Mathematics Curriculum Teaching Program (MCTP) materials (Lovitt & Clarke, 1988, 1988-89)). These Principles formed the basis for the categories of the nodes and subnodes.

Second, each classroom teacher in the study was given a selected lesson series (a series of events on the same topic, see Appendix 1), taken from the field notes, which was ratified by them as "typical" of their practice at that time.

Third, three independent moderators were asked to code a set of sample text units, using the same coding system as had been used in the NUDIST analysis. They were given Appendices 4 and 8 to assist them in classifying the text units. In order to make this inter-rater reliability exercise for the analytical tool less cumbersome and time consuming, three samples were chosen from each set of field notes for each teacher. Two samples were chosen by selecting, at random, a lesson three weeks into the term (March lesson sample, ML) and three weeks before the observations were concluded (August lesson sample, AL). The last sample to be coded was the deliberately selected "typical" lesson series which each of the teachers had ratified (Typical lesson series sample, TL). In other words, Lyn's class sample lesson used the text units 357-417, which represented a typical series of events in that class. A comparably-sized set of text units was also taken from Michelle's field notes (text units 302-360) to form her typical series of events. The text units for each lesson sample were returned to a more prose format for the purpose of readability (see Appendices 1, 2 & 3.). As described above, each rater gave each text unit a node and subnode coding, according to the Principle it exemplified.

Each moderator who was asked to undertake the inter-rater reliability check had a different relationship to the task. The first was a tertiary lecturer and academic skills adviser and had been in each of Michelle's and Lyn's classrooms assisting with the videotaping in the early stages of the study; he was therefore familiar with the context of the study. Les was selected because, as a lecturer in education and an academic skills adviser, he had an interest in trialing the analytical tool.

The second was also a tertiary lecturer in the field of mathematics education and very familiar with the Principles. Anne was selected because of her experience in the language and mathematics area, and because she had an interest in trialing the analytical tool. Anne had also worked with the Principles used in the coding system in depth when applying them in the course of her own research, to a range of secondary mathematics lessons. Anne was not familiar with either of the schools, nor did she know either of the teachers.

The third was a classroom primary teacher, who was actively involved in several professional organisations. Linda had also taught in both schools and knew both of the teachers in the study very well. She was conversant with current pedagogy, having attended professional development courses, and she has been a committee member of the state literacy association.

The raw data are available for closer scrutiny, and examples from the text units are used throughout this thesis. Readers therefore have the opportunity to form their own impressions of the data interpretation and application of the analytical tool. The dilemmas of computer-based qualitative data analysis are discussed in detail by Mousley, Sullivan & Waywood (1993).

Differences Apparent in the Coding of the Four Raters

Even though the three moderators were given my descriptions of the categories and extended sub—category coding for each category (see Appendices 4 & 8), differences in interpretation occurred. For example, Les understood *metacognition* to be of a higher order level of processing and therefore failed to code this category when all other markers did. He also interpreted *modelling* to be any teacher demonstration rather than my given definition which implied an authentic modelling by the teacher or peer who was genuinely using the skill.

Anne tended to code *context* somewhat more frequently than all other markers. She interpreted any incident in which the students used real world examples as *context* even if that specific text unit did not explicitly refer to the context. This was because she knew that they were still using real world contexts, from having read the preceding text units. Other markers tended to rank what was more explicitly referred to in relation to context.

In discussions with Linda over these differences she explained that she felt a pedagogical leaning towards Lyn's philosophy, particularly that associated with features which create a positive climate in the classroom. Linda believes a teacher-dominated classroom destroys the children "... it disempowers, or rather neutralises many of the resources in the classroom."

Linda has also used the textbook used in Michelle's classroom, but, in her opinion, she feels it is culturally inappropriate for our classrooms. She commented that the classroom activities Lyn provided were similar to her own practice. Linda felt that in coding the field notes she tended to have less categories for each text unit for Michelle, but many more for Lyn. Linda noted that the quality of the statements the author had made in her field notes about Michelle were less detailed than those made about Lyn.

As mentioned above. Les explained that he saw *metacognition (3)* in a different light compared with the author's definition. For example, when he saw *explicit teacher modelling (3.1)*, (the author's coding for the subnode under the node— *metacognition*), he would score it under *modelling overt (5.1.2)*. For example he gave *5.1.2* to the following text unit for which the author coded *3.1*.

Michelle demonstrated long division on the board.

This form of coding resulted in an increase in his overall scoring for *modelling*. Les saw *metacognition* as related to higher levels of thought processes and strategies. He also had trouble with deciding the "*hand- holding*" *(6.1.1)* aspect of *responsibility (6)*. He felt he could have placed the incidences within *community (7)* under *support (7.1)*, and argued that activities coded under these categories may not necessarily have lead students to be independent in the way *responsibility* was defined. However, he appears to have inflated this aspect as this example shows:

> *Michelle completed checking the answers for the workbook problems in a whole class forum, reinforcing the concept with magnetic fractions when difficulties arose.*

Les coded this as *responsibility- hand-holding (6.1.1)* when other coders coded it under *community- support (7.1)*.

Les felt that *metacognition* was not being developed as much as it could be in Lyn's lessons because of what he described as her "haphazard nature."

Anne's scores for *context*—especially *big picture (1.2)* and *purpose (1.3)*— tended to be higher than for other raters. The notion behind these two subnodes was to reflect how often the teacher *explicitly* showed the students how a particular mathematical concept might fit into the big picture of where their class (or where the concept) was heading, and explicitly saying *why* they were working on this section anyway. Examples of this "mis-coding" in terms of their relationship to the author's definitions, are as follows, taken from Lyn's lesson 5.8.92

> *Lyn took the problem of one student and built on that and encouraged the whole class to share in solving the problem.*

Anne scored it as *context- purpose (1.3)* among other classifications (*2.1, 2.4, 3.1, 3.5, 3.1.2, 3.1.4.6, 5.3.1, 7.1.5.2, 7.2, 7.3.1, 7.4.1* see Appendix 4 and Codings and Key Headings Used in NUDIST Data Analysis, Chapter 3, for a ready reference to coding categories and definitions). None of the other raters scored it as *context* (Jennie—*3.1.4, 5.3.1, 5.2.2.2, 7.1.5.2, 7.2.1, 7.2,* Linda—*3.1, 4.1, 7.1.5.2* and Les—*3.1, 4.2, 5.3,*).

> *She got a student to explain her dressage arena to the class.*

Anne scored this text unit from Lyn's lesson as *context—purpose (1.3)* and *context—big picture (1.2)* among other classifications. When discussing this aspect of her coding with Anne at a later time she realised that she was coding both overt and covert examples of *context- big picture (1.2)* and *context- purpose (1.3)*. In Michelle's lesson for example:

> *The teacher's idea behind this activity was to assist students to understand the denominator by colouring them first before doing the fraction sums.*

Anne coded *context—purpose (1.3) and context—big picture (1.2).*

Anne also coded *context—real world (1.1)* and *social context (1.4)* into text units when this was more implied than actually stated. For example the following extract, from Lyn's August lesson (AL)

> ***The teacher encouraged the students to join together and share with each other, acknowledging to the students that she was not the only expert in the class especially as they know their own sports in more detail***

Anne coded it in this way—1.1, 1.4, 2.1, 2.4, 5.3.1, 6.2, 7.1, 7.2, 7.3, 7.4.1. None of the other raters coded it as context (Jennie—2.1, 3.1.4.6, 4.4, 4.5.1, 5.4, 5.3.1, 7.2, 7.3, 7.3.1, Les—6.1, 7.1.5, Linda—2.4.1, 3.1.4.6, 4.5.1, 6.2, 7.2, 7.2.1, 7.3.1) (see Appendix 4 and Codings and Key Headings Used in NUDIST Data Analysis, Chapter 3, for a ready reference to coding categories and definitions.)

Anne tended to include frequent strings of *context* codings for the one text unit in her coding. For example, she coded *1.1,1.2,1.3,1.4*, for anything that resembled *real world context* regardless of whether context was explicitly included in that text unit. Anne would also put "using the *textbook*" under the category of *context* regardless of whether it was related to a *real life context,* because her view was that this is still a *context*, albeit a classroom *context*. In discussion with her about her coding she acknowledged that she did not see a great deal of difference between *context—real world (1.1), context--big picture (1.2), context—purpose (1.3) and social context (1.4),* and scored them in clusters. She raised the issue of whether her background studies in sociology might not have influenced her tendency to perceive these categories differently from the definitions the author had provided.

With regard to *metacognition—critical numeracy (3.4)* Anne tended to score this aspect higher than other scorers because she felt that much of teachers' work with children is drawing their attention to the role and function of the mathematics in that context. For example Anne coded the following two text units as *metacognition-critical numeracy (3.4)*,

> *Lyn checked out whether students knew how to work out 'average' speeds. She said they would stay with whole numbers for now and explained that they would be taught how to calculate averages with decimals later in the year...*
>
> *Once students had entered their information from their investigation, they rotated to another group to enter their information onto the sheet*

The other markers tended to code *metacognition- critical numeracy (3.4)* when the teacher was more overtly signalling to the students this feature of mathematics as was the case with this activity from Michelle's lesson:

> *Carla wanted to use the calculator to check her working out. The teachers encouraged the whole class to do this too.*

Anne would also attribute *scaffolding—stretch (4.1)* to any leading or developing actions taken by the teacher. This was also the author's interpretation, but both Les and Linda tended to score this aspect less frequently.

These examples bring out the subjectivity inherent in NUDIST analysis and ultimately led the author to include all ratings and not only her own in the data analysis in order to value the diversity of each person's interpretation. (Tables 1-

7 in Appendix 9 present the individual score of each rater including the author for each Principle, node/category and subnote/subcategory.)

Teacher Verification of the "Typical" Lesson Series

As previously mentioned the "typical" lesson series was chosen from the field notes from each class, using a comparable-sized sequence of text units, which represented what the author believed to be typical of each teacher. The text units were returned to a more prose format for the purpose of readability. When ascertaining the validity of the selection of this "typical" lesson series sample, both teachers readily confirmed that the series of events on the same topic could be regarded as "typical." The following transcript was from a discussion with Michelle after she had read the "typical" lesson series sample.

Michelle: No I think the lessons were a fair example of what you saw.

Jennie: That's what I really need quite a definite confirmation from you.

Michelle : It was a typical number lesson, I guess, I think there were other lessons that were more materials based, more active where the children were more active, but it was a typical number lesson-where we sort of started off with discussion, chalk talk then work on to the textbook. So yes it was a typical number lesson. I use different strategies in other concepts and different things.

I think number lessons tend to be very formal and very, um your typical chalk-talk, sit at the seat and do the exercises. I think that's the only way to teach certain number concepts. Those concepts then can be transferred to your more fun activities and you're getting into space and measurement, you need the number initially because that's the only way to teach very formal concepts I think. Then when I am teaching more activity- space, measurement, the children are more actively involved and they are the *ones recording data, listing numbers, looking for patterns, and coming up with some sort of solution and I tend to take a bit of a back seat and let them work it out. So, yeah I think the lesson plans would be very different if you were looking at something less formal.*

Michelle was keen to describe areas of mathematics which she felt she would teach differently. This included her approach to the use of groups in teaching mathematics. These comments are included at this point in the discussion in order to establish Michelle's position before the interpretation of the data is presented to the reader.

Michelle: Yeah I try to use group work. I probably didn't use it as much as I should have. Looking back I think I could probably have made more use of it. Its a good way to get through to children that are struggling, if they are not understanding me, then there's a good chance they'll understand their peers- for the fact that they can talk on their own level and their own terms. I think I could have used it a lot more than I did, looking back.

Jennie: That interested me because you had them all ability grouped anyway and I wonder whether that was a factor.

Michelle: Well it was a bit, although, within ability groups you did have, you know the brighter children and those that aren't quite coping as well. Yeah that was a large factor too, they were streamed already and you didn't get the two extremes of a range of abilities. I think its an important tool that we should use more often.

Jennie: When the PAT test was done on your children they were clustered around the upper stanines.

Michelle: There weren't any extremes, or maths problems within that group. There were some very bright kids, and I guess the average to upper range there, which I guess which is why I didn't use it as much, but I should have. And I think if I were doing things again that's something I would change.

In her Post Interview, Lyn described her approach to teaching in the following way:

Lyn: Yes that was very much what I did do yeah, that's very much that sort of thing, that I would take the topic, do the group work, do the discussion look through the words that the children will use through'it, the vocabulary, set them some tasks go back and do it, come back if they get stuck, and work on that, yeah very much so, and in fact very much, still very much, like that but taking more care in, not more care, taking, being more aware in myself that I am meeting the needs of all the children and the biggest awareness I would think is making maths pleasurable and taking that pressure off, taking that stress off maths, that its not "I don't like maths" that maths can be fun and that maths is useful and that. At the time of doing all this, it was a very new type concept within the whole school in the area that I was in, so yes that was treading ground that everybody was not familiar with perhaps. In fact the lovely side of this that now this far down the track, the guy next door to me has now just done a paper in Darwin "With love, like maths" the whole thing and its this sort of saying sort of thing. So yes this is very typical of a lesson and I still work in that type of way. Probably not as hard and there's probably not as much now for me to be concerned that they are not doing maths and I haven't had the feedback from any of them, when they've left. In lots of ways when they got to Year 7 that was the concern I guess that with anxious all that might come out of that when they get to Year 7 is that they know and it panned out that those boys who were probably weak when they came were still those and those in the middle, that same sort of flow through, but that they were game to have a go at maths and were not frightened.

Lyn, like Michelle, commented on improvements she could make on her own teaching as this section of the Post Interview reveals:

Lyn: But I can see lots of things now that in hind sight I would do very differently.
Jennie: Yeah, well tell me some of them.
Lyn: I would have a clearer picture in my whole mind, of what I was going to do.
Jennie: Right.

Lyn: All the way through, rather than what am I going to do next, where is this going to lead me, and it could be because I now have younger children. With the older children, I made those expect-, those judgments, that they would just know that. With younger children I would be either be more direct (unclear)
Jennie: Mm. What year have you got now?
Lyn: Year 4/5 composite. When they get a bit older they appear that they should have got these concepts, because they're now in Year 6 but in actual fact they don't. I would have made those judgments for Year 6 (unclear)
Jennie: So when you're looking at that you're main criticism of yourself is-
Lyn: That I really didn't plan it through more logically, in my own mind, that I didn't go from this to this to this, but I guess when probably you have completed, you can look back ... and I had never done that before, that lesson, I'm just trying to think, that was all new.

These comments by each teacher about her teaching style have been included at this point to provide a backdrop for the presentation of the data from the direct classroom observations of each of the lessons (March, August and "typical" lesson series) for Michelle and Lyn (see Appendices 2, 3 & 1).

Procedural Steps in the Establishment of a Composite Representation of the Data

The data analyses and results have been based on coding the data by NUDIST software. Rather than relying solely on the author's own coding of the data, the following steps have been followed to take account of the coding from the other three raters.

Step 1. The text units were returned to a more prose format for each lesson and titled March Lesson (ML), August Lesson (AL) and "Typical" Lesson Series (TL). These were coded by each rater, and tallies made of all code occurrences for each lesson and each teacher.

Step 2. Tables were constructed to indicate relative percentages of codings recorded for each node/category and subnode/subcategory for each teacher. This tabulation was chosen rather than a literal tallying of codings recorded within each incidence (text unit), because it had the potential to indicate trends by each teacher towards different Principles. Rather than use all the lessons it was thought that the other lessons represented random samples and did not always stand alone in the way that the "typical" lesson series did. This lesson had also been the specific lesson ratified by each teacher as having been "typical" of their classroom (see Tables 8-13 in Appendix 9).

Step 3. Tables 1-7 (see Appendix 9) indicate each rater's coding for each Principle and a comparison between Lyn and Michelle's totals for each Principle for the "typical" lesson series.

Step 4. All codings—all raters' and the author's—for each teacher, were collated and averaged to indicate a percentage for both Lyn and Michelle. Discussion of the data can then be based on a composite view—that is to say, on a composite collection of the various factors identified from a range of viewpoints (see Tables 1-7 in Appendix 9).

Step 5. The data have been presented as pie charts at the beginning of the discussion on the data analysis of each Principle in Part Two of this chapter (see Figures 7a, 7b, 8a, 8b, 9a, 9b, 10a, 10b, 11a, 11b, 12a, 12b, 13a and 13b).

Although these figures present the information as a percentage, it should be noted that this was arrived at by calculating the combined totals from each rater who frequently ascribed more than one node and subcategory classification to an event/text unit. The totals of these incidences were combined and then averaged for each teacher. The data analysis discussion will therefore consider the percentages represented in Figures 7a, 7b ... 13b in the light of the number of incidences recorded for each teacher on each node and subcategory.

Triangulation in the Presentation of the Data Analysis

Field notes, however, represent only one type of data collected for this thesis. Chapter 4 (Part Two) presents the analysis of data from a range of sources within the framework provided by the seven Bickmore-Brand Principles. Wherever appropriate other data collected has been inserted into the discussion.

Triangulation of data has been possible through pre- and post- interviews with each teacher, through students' comments and excerpts from lesson transcripts, and through students mathematics test results. The juxtaposing of different sources of related data is an attempt to capture the interrelatedness and socially-situated nature of the classrooms under investigation. The intention is that the data analysis in Chapter 4 (Part Two), provides data from a range of perspective's which will enable informed discussion to be initiated.

CHAPTER FOUR: PART TWO

Overall Balance of Each Principle For Each Teacher

Figure 6 shows the overall balance of each Principle for each teacher in their "typical" lesson series. The percentages are the average of the combined scores of the raters taken from the "typical" lesson series for each Principle. The data in Figure 6 indicate, for example, that the two teachers differ markedly in their emphasis of *Context* (Lyn—14.5%, Michelle—7.2%). Lyn's percentage was drawn from 103 incidences, whereas Michelle's totalled 28. (Incidences have been derived by totalling the combined scores of the raters. See Tables 1-7 in Appendix 9). Figure 6 will be referred to throughout Chapter 4 as each Principle is discussed and analysed.

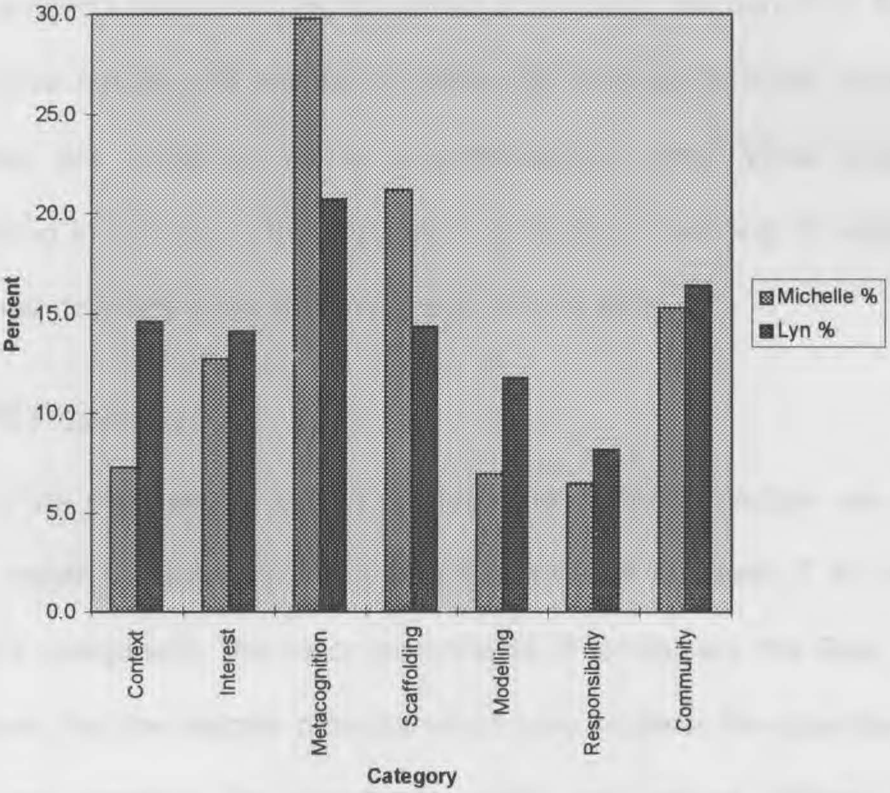

Figure 6. Summary of NUDIST Categories for each Principle as Classified by Four Raters for the "Typical" Lesson Series.

Context: Teacher Creates Meaningful and Relevant Contexts for the Acquisition of Knowledge, Skills and Values

Introduction

The premise behind this Principle is that most learning occurs naturally within a context which makes the value of the learning obvious/explicit to the learner. Learning takes place within a social context. The presentation of curriculum into separate subject areas, and the further compartmentalisation within subjects seems to have the effect of removing learners from the contexts which give meaning to the teaching/learning process. In many cases learning activities are presented in a decontextualised form. What seems to be happening in schools is the separation of students' learning of subject content from their developing thinking, language and life skills.

NUDIST categories

For the purpose of NUDIST analysis the *Context* Principle was broken up into 4 major components and 1 subcategory (see Appendix 7 for overview of NUDIST categories). The major components of context are: the *Real World (1 1)* situations that the teacher provides which give students the opportunity both to learn and practice the knowledge, skills and values related to specific mathematical concepts within a meaningful context; the *Big Picture (1 2)* that the teacher makes explicit the relationship between the learning currently under exploration and its place within the larger context from which it has been derived or to which it might be applied; the teacher can make the *Purpose (1 3)* of the learning explicit to the learner, therefore reinforcing what is being done in school as meaningful; and the teacher can draw students' attention to the ways in which what they are learning changes within the various *Social Contexts (1 4)*.

The diagram in Appendix 7 shows the hierarchical relationship between the node of *Context* and the subcategories (subnodes). The next level under *Real World (1 1)* is the way in which a teacher may use a *Textbook (1 1 2)* to simulate or provide an example of a real world context, in order to develop the students' learning of particular knowledge, skills and /or values related to that context.

Table 1 (see Appendix 9) indicates the combined totals for each NUDIST component and subcategory for *Context* in the "typical" lesson series of Lyn and Michelle as located by the four raters. Figure 6 represents the average of the combined totals of the four raters for subcategories of *Context* in the "typical" lesson series for each teacher.

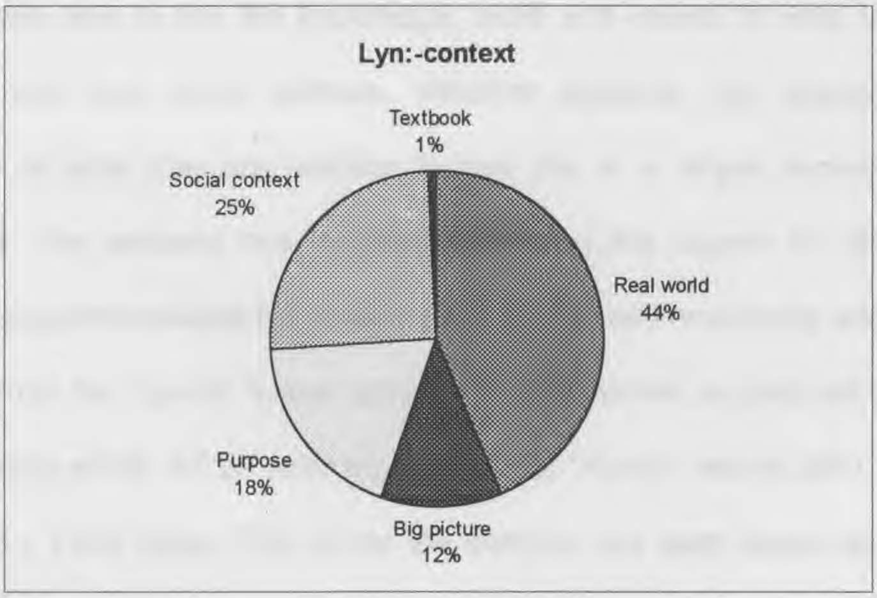

Figure 7a. Summary of NUDIST categories for Context classified by four raters for Lyn.

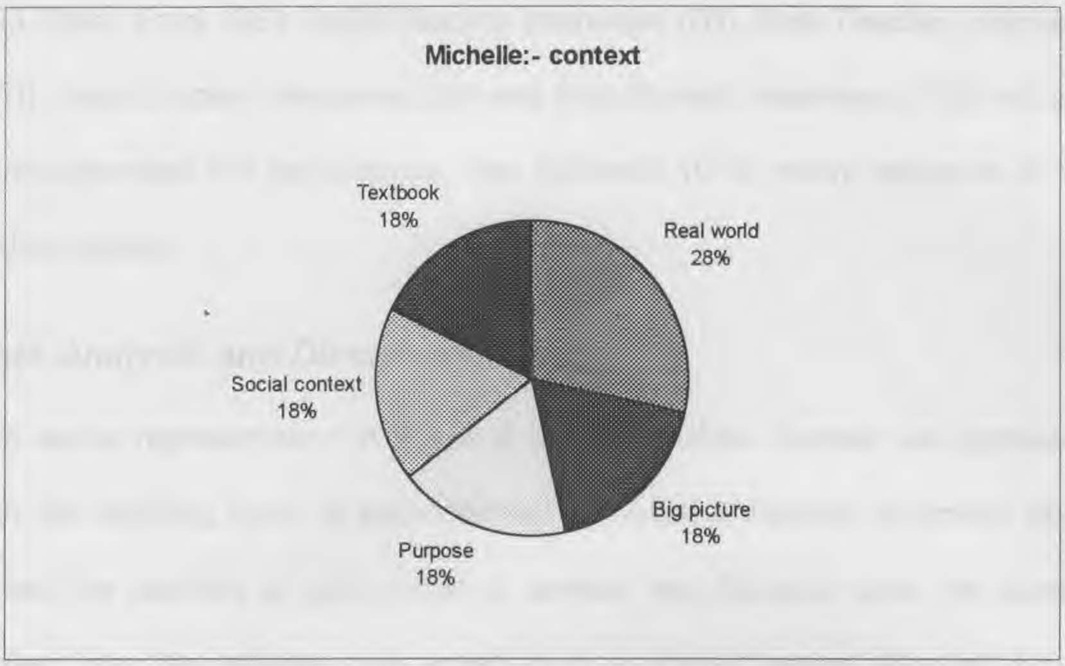

Figure 7b. Summary of NUDIST categories for Context classified by four raters for Michelle.

Terms of Reference for Classroom Observations

Each classroom was examined to report on the degree to which each teacher was able to link the knowledge, skills and values of what they were teaching into real world contexts. Whether students can appreciate the relevance of what they are learning to real life, or a larger context, will be discussed. The analysis has included examining the degree to which each teacher compartmentalised her teaching and the content area being addressed.

Data from the "typical" lesson series will be presented, as also will data from other lessons which will be referred to either as "March" lesson (ML), "August" lesson (AL), Field Notes (TU). Since the analysis has been based on the field notes, reference will be made to text units- TU, which derived from the treatment of these field notes as part of the NUDIST analysis. Data from lesson transcripts

(LT), Diary Entry (DE), Initial-Teacher Interviews (ITI), Post-Teacher Interviews (PTI), Initial-Student Interviews (ISI) and Post-Student Interviews (PSI) will also be incorporated into the analysis. See Appendix 10 for ready reference to this coding system.

Data Analysis and Discussion

The above representation in Figure 6 of the Principle *Context* was consistent with the teaching focus of each teacher. Lyn used a thematic approach which based the teaching of skills within a context, and Michelle used the content drawn from the syllabus and linked it to a context where she felt it was appropriate. The discussion which follows refers to the pie chart Figures 7a and 7b, Summary of NUDIST categories for *Context* classified by the four raters for Lyn and Michelle.

Real world (1 1)

This component of context refers to the teacher's use of situations which allowed the students to develop their knowledge, skills and values in a real world context. A code of *(1 1)* was also recorded if the teacher showed how the learning fits into a real world context.

The overall impression of Lyn's teaching style was that context was a major emphasis (44%, see Figure 7a). Lyn went to great lengths to provide real world situations within which she taught the mathematical subskills. This number could even be considered underscored, considering the period covered during the data collection of this research—she would use a real world topic, for example, a

Mini Olympic Games to be held at the school, and that would constitute a block of a month or more of mathematics lessons, rather than a series of different real world applications for each concept taught.

These examples taken from Lyn's "typical" lesson series and field note transcripts give some indication of these activities:

TL *Lyn decided that she will use the forthcoming camp at Collie as the context for the new term's work in mathematics. Students will be travelling from Guildford where the school is, to Collie, and Lyn will be focusing on the mathematics involved in travelling to the campsite (approximately 200 kms away). Students are to plan the most cost-efficient route and to make decisions on the mode of transport so that plans can be made for the camp itself.*

TU 32 *Lyn describes that what they're doing is like setting up a business* (Students are involved in the production and sale of snack foods, and Lyn is getting them to explore what overheads they may have in order to set a fair price.)

TU 107 *Use of realia in the classroom e.g. newspaper advertisements very common*

TU 181 *students to be managers of the party lunch- been given menu and $150*

TU 373 *last night's homework on blackboard "Find out where you see these speed limits? When you might use them?- 10 kph, 20 kph ... 110 kph. Any other speed limits?*

When asked in the Post-Teacher Interview about how she used the camp trip in the mathematics lessons (see Appendix 1 for "typical" lesson series on planning the most cost-efficient route and form of transport to their class campsite at Collie some 200 kms away), Lyn's answer revealed the potential for a wide range of knowledge, skills and values to be developed from this topic which had its roots in a real world context:

PTI *the camp trip, my first one was to plan the possible routes you could take, if we were going independently. I began by saying that we're going to Collie for our camp and they had to use a map or their Atlas and find the possible routes. It was really just getting them to understand where Collie was, and to look at what direction, how they could go, and whether it would be feasible for them to just go by car or by bus or by train or by plane, or by boat, all those sorts of options, and what was possible and what would not be suitable for children. Using maps, they had to work out how far each route would be and many of the children, when I reflect back, many of the children, had all these interesting ways that they'd go further, out to York and ... Some even wanted to go by boat, and how would they go about hiring a yacht so it really brought in a whole, other dimension. In doing this, when we actually left, for the camp, the children were aware of the camp, of where it was, of the quickest way of going and the most economical- money-wise for children. They had previous knowledge of the time, the distance, and an understanding of the program on that first day, it's much clearer than- "Yeah we're going to camp, that's great but what are we going to do?" The direction, the understanding of the exact direction of whether we were going north, the exact location of Collie, not just that its "down there", but exactly where it is down there ...*

In Lyn's classroom the knowledge, skills and values were integrated into the large topics or themes on which she bases her program. This had the effect of connecting topics which appeared separate in the syllabus documents. For example, the Collie trip had students working out scales, reading maps, finding averages, reading timetables, investigating available and preferred forms of transport, and involved them in a range of basic mathematical and literacy processes. Thus in order to work out the most cost efficient route and form of transport to their camp, the students had to seek the knowledge, skills and values which were a necessary part of solving the problem.

Although the proportion of observed occurrences of Michelle's use of knowledge, skills and values taught in a real world context was considerably lower than Lyn's, it was however, a higher proportion than observed for Michelle's use of the other subcategories of *Context* (see Figures 7a and 7b). Michelle valued the use of context in her teaching from the point of view that it would make mathematics more relevant for her students. She commented upon this in her Initial-Teacher Interview:

ITI *I think maths must be relevant to the child's world in everything that we do. I think its like any learning we must have a reason for learning and if you can't see the relevance or importance of it then why learn it. Children need to see that they are going to use this maths that we do in the classroom outside.*

The field notes on Michelle's lessons, however, suggest that any examples in these lessons were limited in scope and frequency. Examples tended to receive brief mention rather than be interwoven with the mathematical concepts involved. The following examples illustrate this point.

TL *She established that the students know what % means. She provided the example, the number of students who are here that went on camp is 82% She asks them how many that would mean were not here ...* (Lesson continued to develop with work on fractions generated by % with no further reference to any real world context) 30.3.92

TL *Continuation of work on %. (Other mathematics lessons on different content were conducted in the period between observations) Michelle stimulated the students schemas by discussing the proportion of students who went to camp who didn't like chicken.* (Lesson continued to develop with exercises on percentage with no real world context until the textbook is used about half way through the lesson period) 6.4.92

AL *She used the analogy of a break in an arm in order to explain the word "fraction". She discussed breaking a bone into many pieces or quantities.* (Lesson continued without reference to any real world context. Squares were coloured in to represent fractions and equivalence)

In the Post-Teacher Interview when Michelle was asked to respond to the following statement about her classroom practice, "The classroom mathematics is continually being related to the student's real world use of the skill or concept," she responded *always*. However, as can be seen by the lesson samples (ML, AL and TL) and in her classroom observation field notes, the author noted a number of missed opportunities where a connection with the real world could have been made. For example:

TU 152 *no real world application made explicit except in a token way in textbook showing oranges, even then the reality of real world problem which required this sharing of fractions of oranges was not even referred to by the teacher in passing.*

TU 367 *even contextualised examples are not really related to a real life problem that needs to be solved eg. you could have had interest rates for the 8.5% question.*

Even when not using a textbook Michelle tended to use constructed contexts which reflected textbook scenarios. The following example illustrates the type of content to which Michelle made reference

TU 46 *Mental story of Mr Fivemore- everything related to story eg. 5 dogs, 5 cats, 5 sons, 5 bank accounts etc. Mental questions eg. if Mr Fivemore has $50 in each account how much altogether*

Michelle valued the use of context, because she believed it assisted her students to connect the mathematical concepts they were learning to the real world. Instead of using a context within which the knowledge, skills and or values could be taught/learned, she would use it to introduce a topic as a way of motivating the students for the learning. However, the author observed that the real world example would then become disconnected from the rest of the lesson, which in most incidences became decontextualised (see Appendices 1-3 for Michelle's TL ("typical" lesson series), ML (March lesson) & AL (August lesson)). In many cases where the textbook was being used, the examples had a short real-world scenario from which the exercise questions may have been derived. The function of the context appeared not to provide a situation in which the knowledge skills and or values were a necessary part of solving problems integral to that scenario. The context seemed to act as a motivational part of the lesson and appeared to justify the activities which took place during the remainder of the lesson.

Big Picture (1 2)

This component of context refers to the ways in which the teacher makes explicit the relationship between the learning currently under exploration and the larger context from which it could have been derived or to which it can be applied (Lyn—12%, Michelle—18%, see Figures 7a and 7b).

Lyn's students had many opportunities to become aware of how the subskills of the mathematics curriculum content was related to real world contexts that were constructed to suit the classroom. Her students almost always encountered mathematical concepts in the context of a broad theme.

ML *"Find ways of discovering which foods are the most popular so we can have a menu suitable to all of us."* (Students had various ways of solving this problem, from ad hoc questioning of friends, to fully-blown surveys, the results of which were then graphed)

AL *Lyn began the lesson by reminding them about their latest maths project which was for them to choose a sport related to the forthcoming Olympic Games and demonstrate the mathematics in connection with that sport.*

TU 463 *"in this sport you will have to add, multiply and divide" says one student, becoming aware of the mathematics involved*

This aspect of the teacher making explicit how the mathematics was located within the "big picture" of what they were studying became clear from this lesson transcript of the Olympic Games series of lessons, in which students had invited another class into their classroom to share in a question-and-answer game:

LT 19.8.92 *We will sit them on the floor here and you're not to tell them the sport you have chosen. They can only ask you. They'll ask you questions and you can only say Yes or No. Remember you have all the mathematical knowledge about your sport they don't have, they have a lot of knowledge of Olympics because they have done a whole topic on Olympics. I don't know whether they have taken it from the mathematical angle or what they've taken or just an overview where you have done some in maths and you've done a comparison.*

From these examples it can be seen that the larger topics that Lyn used for developing her program around reinforced for students the big picture into which their mathematics lessons fitted.

Michelle's program tended to move from topic to topic, often weekly, based on different mathematical skills or concepts. The changes in topics were often not explicitly signalled to the students. The students were very aware that their teacher was preparing them for high school and that they had to get through a certain amount of content before the end of the year, as this comment suggests:

TL *Michelle introduced lesson by saying they needed to get this work done before end of term (2 weeks) and that it would be needed for next semester's work*

Michelle also made it explicit that specific mathematical expressions were needed in order to solve problems expressed in prose, for example:

ML *The teacher drew students' attention back to the textbook p. 42 (Addison Wesley purple) and said "Let's look at a word problem and get the maths out of it."*

The author discussed the idea of context with Michelle during the course of her classroom observations and noted that at the end of a series of lessons on fractions Michelle conducted an extended discussion with the students about how they could use what they had been learning in real life.

LT 11.8.92 *Michelle:- So you're converting your fractions to decimals again. We're actually doing that in a couple of weeks time so you're well ahead. Angharad?*
Angharad:- Like when you're measuring something when you're cooking. And you might have a cup of something and it doesn't have to be the things that you need so you can change it.
Michelle:- Mm. I wonder if I said to you, say for instance if you got the recipe off the back of a sugar packet and it said for you to use one quarter of this packet and there were twenty-five grams in that packet, how many grams are you going to use? If it said to you to use a quarter of a packet of sugar and there were... I'll make it easier there were 24 grams in that packet, which is highly unlikely. How many grams, er so, what fraction of that packet are you going to use?
Child:- Six grams
Michelle:- Six twenty-fourths or six grams good girl. Homework tonight is p. 75 questions 1-10.

Discussions between Michelle and the author after the lesson, made it clear that she had responded to my prompt to make more explicit the context in which the mathematics the students were learning could be applied. However, it still did not seem to reflect the essence of what was meant by the Principle of *Context*. Relating the mathematics to the real world was an "add on" rather than an integral part of the lesson. The author wondered if the rather abrupt ending of the discussion switching over to the homework may have signalled to the students rather token attention to the relationship between school mathematics and the "big picture" of real life.

Purpose (1 3)

This aspect of context refers to the extent to which each teacher made explicit the purpose of their teaching to their students. In othe, words, how explicit were the teacher's intentions in relation to what she was asking the students to do in class.

The pie charts (Figures 7a and 7b) indicate that the combined totals from the raters' coding of each teacher's "typical" lesson series, resulted in a similar pattern for Lyn—18% and Michelle—18%. However, there were qualitative differences in the ways in which each teacher made the purposes known to the students. The totals represented as a percent for *Purpose (1 3)* for Lyn was drawn from 19 incidences in Lyn's "typical" lesson series and Michelle's were drawn from 5 incidences in her "typical" lesson series (see Table 7 in Appendix 9).

Lyn's work was usually embedded within a major focus for the term and the way in which the mathematical content of a new topic would be related to the broad context was often discussed with the students. For example:

AL *Lyn began the lesson by reminding them about their latest maths projects which was for them to choose a sport related to the forthcoming Olympic Games and demonstrate the mathematics in connection with that sport ... They could also treat the project like* *a secret where the class has to guess which sport it was- different parts of the information would gradually be revealed during the presentation.*

TL *Lyn will be focusing on the mathematics involved in travelling to the campsite (approximately 200 kms away). Students are to plan the most cost- efficient route and to make decisions on the mode of transport so that plans can be made for the camp itself ... Lyn outlined the above task and then distributed a map of the area to each of the students (i.e. south western region of Western Australia) ... Lyn then gave the students the task of calculating the distance over time to travel along any route they chose from the school to Guildford, to the campsite at Collie. She explained to them they would be given school time to do this and that this would be what their maths was about this term.*

Consequently when students began work on looking at "scale," or finding the "averages of speed limits," (see Lyn's "typical" lesson series Appendix 1) they were more likely to see the purpose behind studying the particular mathematical concept in class. She discussed this in her Initial-Teacher Interview:

ITI *I feel if it is relevant to the child's immediate world and that they can see that it's part of what they are doing, then for them understanding will come far easier and they will see the purpose behind it, they will be able to link it to something they know and that they use all the time.*

Similarly during the planning of the class party she continued to remind the students of the purpose behind the mathematical activities they were doing (Listing favourite activities and foods for a party, surveying the class and costing materials). The following excerpts are taken from a whole class discussion about the planning of a class party:

LT (4.3.92) *Lyn: Right you've done the first stage where all of you were in a group ... I've looked through all of your groups and from your groups it will tell me the types of party you are going to have. I guess the consensus would be some sort of swimming party ... Right so we're interested in a party that has to cater for how many* *people? ... So as I send you off to look at your food you need to consider 1. Organising a balanced menu ... Right so we could conduct a survey to see how many people liked sandwiches ... So I want you [sandwich group] to prepare a survey sheet for us the next time ... We've got to reach a consensus because we haven't got $150 here and $150 here (points to each group) and $150 here and $150 here and $150 here. We've got $150 in toto ... Don't forget we have to come to a consensus, and Mrs Bickmore-Brand and I would like to be consulted, we have to think about what we really want and reach a consensus.*

During the whole year over which the lesson observations were conducted, the author did not see any lesson given by Lyn which was not explicitly connected to the real reason for doing the mathematics. Even when a worksheet was used, for example, it was based on a real-world context. During the trip to the campsite topic, for example, some students had been having trouble reading the bus, train and plane timetables they had brought into the class. The worksheet acted as a less complex timetable.

Michelle conveyed that the purpose of the mathematics lessons was to do with being able to develop further mathematical understandings, as these two sections from her lesson transcript on fractions shows:

LT (11.8.92) *Michelle: Alright, are you ready? Right girls eyes this way please. What we will be doing today is a continuation of what we did Monday and yesterday. Yesterday we were looking at equivalent fractions and we were working out how to find equivalent fractions of a simple fraction such as a third without actually having to draw those large graphs everytime you need to work out an equivalent fraction. What is that way that we came up with yesterday? Do you remember we saw a pattern between the different numbers? For instance we could tell instantly how many sixths is a third. Gabrielle?*

Michelle: OK So that's what we looked at yesterday. Now I know some of you had a little trouble with that so the plan today is to put you to work at your desk and then I'm going to mark the homework that you did last night ...

As mentioned above *(Big Picture 1 2)*, it was not until the last and third lesson in this series on fractions that Michelle explicitly discussed the purpose behind what they had been studying, as this section from the lesson transcript shows:

LT (11.8.92) *Michelle: Why on earth are we bothering to have a look at equivalent fractions? Why do we do it? Do you remember when we were talking about decimals we said where in the real world would we ever use decimal numbers, and we talked about accountants and we talked about calculator work and all of that. Where do we use equivalent fractions? Do we ever use it outside the classroom? Where Rebecca?*

The author made an entry in her diary (see DE below) which discussed the effect of going from one mathematical skill to another without an overall context to make the purpose explicit. The children had been doing "averages" in the previous week and were attempting to apply this knowledge to the new problem the teacher set in the following week. They were unaware that the purpose behind the lesson was to learn the new strategy of "using a table" to assist in calculating. It would be possible for them to discover this as they worked through the following set question:

It is noon at the sandwich shop, and Kathy and Jill are making sandwiches for the lunchtime rush. Today's specials are egg salad or ham and cheese. During the first 45 minutes, Kathy makes 5 egg salad sandwiches and Jill makes 7 ham and cheese sandwiches. During the second 15 minutes Kathy makes 9 egg salad and Jill makes 10 ham and cheese. During the third 15-minute period, Kathy makes 14 egg salad and Jill makes 14 ham

and cheese. In the fourth 15-minute period Kathy makes 20 egg salad, and Jill makes 19. If Kathy and Jill make sandwiches for an hour and a half at this rate, how many sandwiches will each one make during the last 15 minute period?

DE *Michelle's class 27.7.92*

A most interesting event occurred when she attempted to pursue a lesson on tables as a way to present information. When quizzed about how they might approach a question on calculating the number of sandwiches made in a school canteen, Angela had come up with a statement about averages being used in the Olympic Games (currently on the news) and even though Michelle ignored this, it must also have been in the minds of several others, in particular Melissa who wanted to interpret the question by finding the average and Carrie B. who also thought the question was asking for averages. The question did in fact ask "... at this rate (my emphasis, and often a cue to calculating an average), how many sandwiches will they have made in one and a half hours."

It showed two things—the ambiguity of questions and the urge for children to draw upon previous knowledge, which in this case, work that had been done last week i.e. work on averages.

The author shared with Michelle these observations about the students having difficulty working out the purpose behind the activity in terms of which mathematical skills they were required to bring to bear on the problem. She had this to say about the difficulties students may have encountered when changing topics.

PTI *Yeah that's common when you're changing topics, and the kids are drawing back on what you said yesterday, and thinking how does all this relate and how do we switch off from there to here. Um I think if there are many experiencing the same problem then you just address the whole class, but if it's just a few then you just need to speak to them on their own- "Yesterday we were doing ... this is something totally* *different, you need to do that in this way, this is something that we do differently." It is a problem, but I think most kids get around it.*

When asked about whether students could generalise about the purpose of using a skill when the teacher goes from one mathematical skill to another, Michelle responded in the Post-Teacher Interview with:

PTI *I think the only way you can do it, is to give them just a wide range of examples, and just give them different uses I guess, for the concept you're using. I think that's the only way you can do it is to give them a vast experience of it, and relate it as much as you can to their little worlds.*

Michelle worked from topic to topic based around a mathematical concept from which the students might assume the purpose of studying a topic was for their future lives as mathematics students. In order to tap into the purpose behind what Michelle was teaching the author asked her about what she wanted to make sure her students had before they left Year 7:

ITI *I think they need a basic understanding of our number system, numerations. I think they need to know how to add, subtract, multiply and divide and have a real understanding as I said, before they can't go onto high school maths if they haven't got their basic understanding.*

It would appear from this that the main purpose behind the mathematics was to help students develop basic mathematical skills (including numeration and the four operations) and to prepare them for the mathematics they were likely to encounter in high school. The responses from the students at the end of this section indicate that this purpose was successfully transmitted to the students.

An interesting comment came from the students in both classes during the early days of my classroom observation. The author had said she would give them $150 in order to plan a class party. She recorded the situation in her diary entry:

DE 19.2.92 *She [Lyn] phoned me later that night to tell me about the poor audio production and the next day. She is concerned that many of the children did not actually believe that there really was going to be a party! This says heaps for schools and the 'ropes' we put children through and their distrust of the carrot or lure in education.*

DE 19.2.92 *This reaction came through some of Michelle's class too although some had yet to be individually interviewed and discuss the party in the first instance.*

DE 20.2.92 *Found out from some of the girls via Carrie B. that they really wish that we were really going to have a party! Can't believe after all the build up that they would react this way.*

How each teacher chose to use the class party stimulation revealed much about their perceptions concerning the purposes for doing mathematics in their classroom. Lyn wanted to have the party later in the term after she had done a whole lot of work with the children, really maximising the opportunity for using real newspaper cuttings and supermarket visits for prices, and in the design facets of the party preparation. Michelle was keen to get the party over and done with and held it in a double math period at the end of February. She commented that she was reluctant to disadvantage the girls in any way by using up their lesson time, but at the same time wanted to accommodate the author.

Social Context (1 4)

This component of context refers to the opportunity the teacher makes to relate the content of the classroom to where it can be applied in the real world.

The average percentage given by the raters for this component of context for the "typical" lesson series reflects a large difference between their teaching styles (Lyn—25%, Michelle—18%, see Figures 7a and 7b). As the "typical" lesson series exemplifies, Lyn attempted to show students how what they were doing related to particular real life situations on more occasions during the "typical" lesson series than did Michelle. For example, in planning the trip to Collie in Lyn's class, the students had to calculate the distance over time to travel to the campsite:

DE (5.5.92) *Today's lesson was extending where she [Lyn] wanted them to calculate the time it would take to get there and they realised they would need how many kilometres per hour their mode of transport went and this led to a discussion on averaging and economy speeds and children were to go home and find out from car manuals the efficient speed and fuel consumption of their cars.*

TU 394 *Rotated groups and class will have entered onto sheet all the locations where different kph signs are used in real life*

AL *Set task of guessing the sport they've begun to come up with; teacher gives "Insight" section of West Australian with all the sports listed.*

TU 42 *Lyn pushes the idea of all the maths that is used in sport*

The very nature of Lyn's programming around a broad theme set up a situation in which the students needed mathematical processing in order to complete the tasks. The fact that they were doing activities which were part of their own society or culture as the content of their mathematics lessons had the potential to reinforce for the students, the links between the mathematics learned in school with the real world.

Michelle relied predominantly on the textbook for examples which, although they would often provide a social context, they were one step removed from actually using the mathematics for that social context. For example, the following text unit and field notes from Michelle's classroom illustrate this approach.

TU 172 "Subtracting Decimals–Using an electronic timer, a tennis serve was clocked at 62.483 meters per second (m/s). A baseball pitch had a speed of 41.556 m/s. How much greater was the speed of the tennis ball."

ML *The teacher got Linley to read* "Adding Decimals. Jason visited some European countries. He has 1 British pound note, 1 German, mark, and 1 Greek drachma. What is their total value in US dollars?"

There were times when Michelle would show her students the relationship between a mathematical concept being used in a social context and how it needed to be used in a mathematical context:

AL *She used the analogy of a break in an arm in order to explain the word "fraction." She discussed breaking a bone into many pieces or quantities. Michelle said that in maths the pieces have to be equal i.e. equal pieces.*

In reality there were very few instances where the connection between the social context and what the students were learning were made. The raters coded this aspect of Michelle's "typical" lesson series as a total of 5 incidences whereas Lyn's total was drawn from 26 incidences (see Table 1 in Appendix 9).

Textbooks/Worksheets (1 1 2)

This subcategory of context refers to the teacher's substitution of a real world context with a textbook or worksheet. The function of the textbook in this subcategory is to provide a simulation of a real world context from which to draw the mathematical skill development.

Even though Lyn did not use a textbook (Lyn's "typical" lesson series total of 1% came from the use of a worksheet) she said in her Initial-Teacher Interview, that her preference was for the Rigby Activity Book. This book presented topics related to one context and had associated activities. The author observed Lyn using worksheets to teach or reinforce a concept that the students were having difficulty with. For example, during the planning of the class party, the students needed to know the relationship of grams to kilograms in order to cater for 26 people, and a worksheet was used.

TU 376 *students to work from a prepared worksheet- students to work back in seats on kg's (weights)*

As discussed previously, in the planning of the most cost-efficient route to Collie the students had trouble reading authentic maps and timetables, so Lyn used worksheets which had limited variables on the map and timetable:

TL *During the week, the students had been given tasks to work on scales with various maps and worksheets to reinforce this new concept.*

TU 427-432 *Worksheet- timetable activity to consolidate. Students in stations, 2 groups working on their own one group with her on mat. Teacher asks students questions of what timetable tells you? Recap on yesterday's lesson asking which is central or important information eg. Hughes St or Lawsonville, relate problems to their Midland to Perth travel.*

The worksheets were used in order to give the students more experience in working with specific mathematical data and processes. This was before they started the actual calculating of the more complex amounts needed, in the first instance to plan their trip to Collie and in the second, to cater for the class party. Lyn discussed her use of textbooks and worksheets in her Post-Teacher Interview:

> PTI *No I don't use a textbook. Why? because I just have not found one which suited me that covered, that more, integrated approach to maths and so at the time of doing that there were very much still the texts of number crunching just not interesting texts that encourage thinking about maths ... I use a variety of textbooks for myself and take activities.*
>
> *When do you use worksheets?* [re Q 8 *Why did you use worksheets in this particular lesson series?*] *I use worksheets to supplement the activities that we have been doing, they do worksheets very much for homework because to satisfy parents that we do do homework and that the children are doing maths.*

It would appear that Lyn's use of worksheets is a somewhat reluctant one. What has emerged from this discussion is that the complexity of the real world tasks that she has the students working on need to be scaled down at times and time taken out for skill practice or less demanding versions of those real world tasks.

Michelle's percentage, as indicated on the pie chart (see Figure 7b), representing the raters' coding of the "typical" lesson series was 18%. Michelle usually used the set text (Addison Wesley) to practice a concept, and to set homework from. She stated in her Initial-Teacher Interview that she preferred the

Addison Wesley textbook series, from which to draw the content of her mathematics lessons:

ITI *It's a text that covers a lot of content, and you find you don't need to deviate only getting away from the text and reinforcing the concepts through some sort of concrete activities ... we wanted a text that we could continue through the years and we wanted something that the children became used to from the lower levels up into the upper levels and using the syllabus we found it was a problem because there are so many texts that you can use with the syllabus we wanted continuity through the grades ... and the language that the text used. It runs through the year levels beautifully and you find ... you don't have to spend that time becoming familiar with the text and the style of the book and the language that it uses because that has already happened.*

After the author had spent the year with Michelle it became apparent that the particular textbook was not always a desirable choice. The author had discussed this with Michelle on more than one occasion and she had this to say about the textbook in her Post-Teacher Interview:

PTI *I used the textbook mainly because it was a school-based decision. Every class had the same text through 1-7, so that was the main reason I guess I used the textbook, although I do like to use the textbook. That particular text did become extremely complex, and tended to put the children off because they start off very confident, the first few examples are easy and then suddenly they've got these enormous numbers to tackle. So I do like textbooks, I have my doubts about the one we used ... And worksheets I use as well just to consolidate the class work or to provide a format for the kids to record data and to collect their information. But I think textbooks are important, I think if I had a choice I would have a textbook, yep, absolutely because I think that keeps you on track, but you would need to choose them carefully.*

Once again the answer that Michelle gave about the choice of a resource which has limited real world contextual application revealed where her focus is in mathematics instruction. It would appear that Michelle's priority is to use a resource which will provide cohesion throughout the delivery of the syllabus.

Summary

The idea behind this Principle of *Context* is that knowledge, skills and or values are best learned within a context which indicates the value of the learning to the learner. Although each teacher valued the use of context and of relating what was being taught in school to the real world, their classroom practices reflected different priorities for this Principle.

Lyn was committed to context and most of what took place in her classroom had an inextricable connection with a real world context. By working within the social contexts she constructed, the students were involved in the learning of mathematical knowledge skills and values.

Michelle valued the need to connect the school world with the real world and used a textbook to provide contexts which could reinforce this to the students.

In order to ascertain the effect of each approach on the children's perceptions, the author interviewed each student at the end of the study, and asked them *"What do you think you have learned that will be useful for everyday life?"*

The overall response for both Lyn and Michelle's classes resulted in similar details, for example, decimals, percentages, multiplication, fractions. This was a surprising result given the difference in each teacher's approach—the skills and mathematical processes were implicit in the context in Lyn's class and were explicitly taught in Michelle's class. Exceptions to this response however did come from Lyn's class, and revealed a different depth about understanding the application of mathematics to everyday life. For example:

PSI *We have learnt to take care of problems and that. (Ben B)*
PSI *... don't take things for face value ... just don't rush ... find out how to do it before you try to do it. Well, a lot of the things that she did were, they were sort of, they had two meanings, they had a top, a surface meaning and then a lower meaning. (Paul S)*
PSI *For my horse riding to measure ... (Dean J.)*

The main applications that many of the students in Michelle's class could see were related to shopping. The exceptions were also revealing.

CP I'd be able to work things out at the shops and if I got a job where I had to add up money I'd be able to do that. And if I had to share it out as well, pay and cooking, I'd be a lot better with cooking. (Carrie P.)
GM I don't think that there's that much we'll learn that we'll use in everyday life, I just think you really have to know the basics, but it might still be good, you know just in case, you know, if you wanted to go on to university or something like that. (Georgia M.)

In the Post-Student Interview the students were asked: *"Do you think there is anything you have been taught that you doubt will be useful in your everyday life?"*. Most of the children in Lyn's class responded with comments like "no, not really". Some children in Michelle's class identified concepts such as volume, fractions and long division, as aspects of mathematics which they did not think would be useful.

CM A couple of things I think, I think some of the, I guess a lot of things I have, but like volume and *things that I haven't learnt yet, I don't know if, I mean I'll probably be able to use them somewhere I'm not quite sure for what. (Carrie B.)*

EB *Well I do think that some things are irrelevant but I can't really, I can remember sitting in class and thinking when are we going to use this, but I can't remember what. (Elizabeth B.)*

GM *Um ... I don't think like division of fractions and multiplication of fractions or I don't think long division, or anything like that. (Georgia M.)*

CR *I can't understand why you have to divide fractions by fractions. (Catherine R.)*

CV *Some things, not. Some things you can't picture yourself ... but some things, yeah. Well like doing, depends what job, but doing long division and stuff like that doesn't, I don't think you'll need it, but times table and maybe fractions and stuff, I suppose you'd use. (Chantelle V.)*

AW *Oh yeah, I don't think fractions, adding and taking fractions will be useful. (Alana W.)*

GH *Well fractions, coz I just like usually convert them to decimals. (Gabrielle H.)*

In summary then the data presented in this section suggest that Lyn placed a high priority on relating the mathematical concepts to the *Real World (1 1)* as her main teaching methodology, and her students reflect a similar emphasis with regard to the place of mathematics skills in daily life. The total number of incidences for this category derived by totalling the combined scores of the raters was 28 incidences for Michelle and 103 for Lyn (see Table 1 in Appendix 9). For Michelle, however, the incidences were relatively evenly spread across the subcategories within *Context*, with only a few more rated under the category *Real World (1 1)*. Michelle's students made less reference in their interviews to the function of the mathematics skills they are being taught for everyday life than did Lyn's students.

The difference in how the students in each class perceived what they were learning raises questions about the relevance of what is taught in the classroom, for the social context in which the students live. The students' attitudes towards mathematics were strongly linked to what they had experienced in the classroom.

Interest: Teacher Realises that the Starting Point for Learning Must be From the Knowledge, Skills and or Values Base of the Learner

Introduction

The premise behind this Principle is that in order for learning to take place, the learner has to connect the information in some way to what s/he already knows. The linguistic and cultural backgrounds of the learners will influence what students bring with them to the classroom. Teachers take account of these different starting points for learners in different ways, depending on their own philosophy and teaching style, and the particular mathematical content. The *Interest* Principle for the purpose of this analysis has been defined in terms of 3 major components and 4 subcategories, as summarised in Appendix 7. It may be helpful for the teacher to be aware of how the student's own experience, language and prior knowledge relates to the socially defined ideas of the classroom content. The very idiosyncratic nature of the way in which learners processes information, and finding out how students think can be difficult for teachers. In order for the learners to assimilate knowledge, they have to reshuffle and reconstruct their understandings. The degree to which any new information will displace what is there, will depend upon how "dearly" the learner holds on to those constructs. In other words the learner is not merely copying or remembering what is shown but rather reconstructing what was shown.

NUDIST categories

For the purpose of NUDIST analysis the *Interest* Principle was broken up into 3 major components and 4 subcategories (see Appendix 7). The major components of interest are: to *Connect (2 1)* the knowledge, skills and/or values being dealt with in the classroom to those of their learners; this involves connecting the *Register (2 3)* of the classroom discourse to the learner's own language; it also involves the acceptance of *Different Styles (2 4)* of learning, or going about a task.

The diagram in the Appendix 7 shows the hierarchical relationship between the major components and the subcategories of *Interest*. The next level under *Connect (2 1)* refers to the way in which teachers take into account the *Cultural and/ or Gender (2 1 2)* backgrounds of their students when attempting to relate their information with what the student knows. The next level under *Register (2 3)* refers to the specific dialogue the teacher uses. A teacher needs to *Link the New Terminology (2 3 1)* of the classroom discourse, which may be unfamiliar to the students, with the understandings the students bring to the classroom. When a teacher caters for the different styles of processing which may be exhibited in a class, she can also encourage the students to *Generate their Own Rules (2 4 1)* for solving a problem. The teacher can also provide *Various Mediums (2 4 2)* when presenting information to the students, which can assist them in their constructing the information.

Table 2 (see Appendix 9) indicates the combined totals for each NUDIST component and subcategory for *Interest* in the "typical" lesson series for Lyn and Michelle as located by four independent raters.

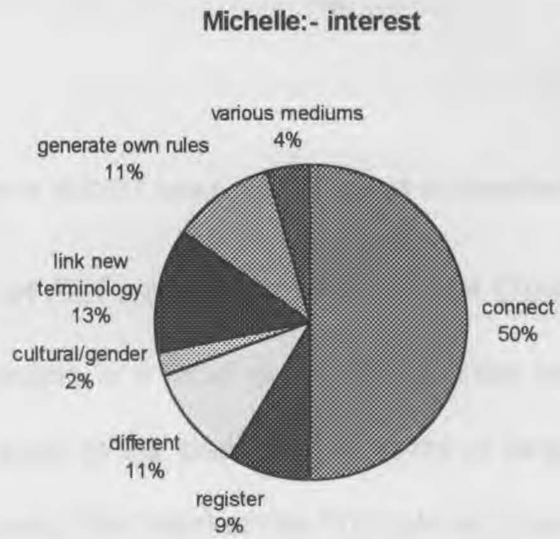

Figure 8a. Summary of NUDIST categories for Interest as classified by four raters for Michelle.

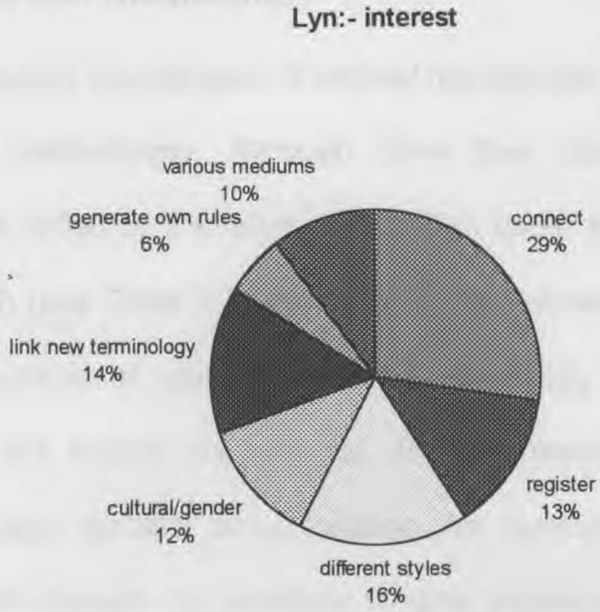

Figure 8b. Summary of NUDIST categories for Interest as classified by four raters for Lyn.

Interest—Terms of Reference for Classroom Observations

The *Interest* Principle is a label which tries to take into account what the learners bring with them to the classroom in terms of language, learning style and cultural background. The intent of this Principle is to recognise the proactive role the teacher can take in programming around the learner and relating the content to the learner's experience. It is also about valuing what the learner brings to the classroom and yet taking them beyond, if necessary, in order to construct what is being required by formal education.

Data Analysis and Discussion[3]

For each teacher this category of *Interest* represented a fairly high priority in their teaching methodology. Although more than double the number of incidences were coded by the raters for Lyn (98) under the *Interest* node, than for Michelle (46) (see Table 2 in Appendix 9), the percentage average of their total incidences across all categories differed only slightly (13% for Michelle and 14% for Lyn, see Figures 8a and 8b). In other words, even though both demonstrated their concern about relating the curriculum to the students' backgrounds and interests, Lyn exhibited a higher incidence of this.

One of Lyn's major teaching priorities was the way she provided for different learning styles and tried to connect what she was teaching to the lives of her students (for example, by programming her mathematics around a theme with something of real interest to her students such as a class party and the trip to their class camp at Collie).

Michelle's teaching was related to this category of *Interest* largely in the ways in which she connected the language of formal mathematics to what the children already knew. This could be observed particularly at the beginning of lessons and/or the introduction of new concepts.

[3] The discussion which follows will address each component and subcategory description for Lyn and Michelle. The "typical" lesson series graph (Figures 8a & 8b) will be referred to but data outside that lesson will be drawn upon. Other lessons will be referred to either as "March" lesson (ML), "August" lesson (AL), Field Notes (TU) (Since the analysis has been based on the field notes, reference will be made to text units- TU, deriving from these field notes as part of the NUDIST analysis), and lesson transcripts (LT) or Diary Entry (DE). Data will also be drawn from the Initial Teacher Interviews (ITI), Post Teacher Interviews (PTI), Initial Student Interviews (ISI) and Post Student Interviews (PSI).

Connection (2 1)

This component of interest focused on instances when the teacher related the knowledge, skills and/or values of the curriculum to what the learner already knew (Lyn—29%, drawn from 27 incidences, Michelle—50%, drawn from 23, see Table 2 in Appendix 9).

One way in which Lyn would try to connect the mathematical content of the lesson with the student's experiences was the use of everyday artefacts which were familiar to the children (e.g. timetables, newspapers). She regularly tried to relate what her students already knew with the content of what she was intending to teach, as these examples from the field notes suggest.

TU 109 *real situation to genuinely solve* (In order to plan for the class party the students surveyed the class and worked out how to record preferential voting for particular foods.)

TU 111 *use of skills for future citizens* (Once the students worked out the system behind preferential voting, they discussed how it was used during election time)

TU 183 *students already know how much pizza etc will weigh and how many it will feed, also cake and cool drink* (Students in planning for the class party consulted pamphlets and newspaper advertisements in order to calculate the best value per weight for the products they would require for the party)

TL *She discussed the maximum speeds for a bus e.g. 90 kph and a car e.g. 110 kph,* (Students worked out how to calculate the average speed of various forms of transport in order to decide which was the most cost-efficient to take them to their class camp at Collie)

TU 432 *related problems to their Midland or Perth travel* (Students are working out how to read a timetable in order to calculate the most cost efficient form of transport and route for their class trip to Collie. The Midland to Perth route would be a frequently travelled route for the students although not all would be familiar with the related bus or train timetables)

Lyn was keen for children to see the connection between what they were doing in school and their own lives as this comment indicates from her Initial-Teacher Interview:

ITI *I try to make it very relevant and see that maths is part of their, is all around them all day, part of their whole world and that maths is not just that subject that you do in school and you get 9 out of 10 or 2 out of 10 or whatever, but that our life is full of it all ... if they cannot see this real connection between the world in which they live, then it has no bearing other than that's something that you do in a book, or on a sheet of paper for the teacher and she will mark it and it has no bearing, it has no connection for them in the world, it's only just something that you have to do at school to keep people happy.*

It is not always easy at the outset with a new group of children to make the connections between the curriculum and the learner's experiences. Lyn recognises the need to develop ways of getting to know the children, as the following excerpt from an interview shows.

PTI *L: In this particular one it was a camp so I wanted them to have some ownership and previous-prior knowledge before the camp begins.*
J: So when wouldn't you have active involvement?
L: *Perhaps at the beginning of the year, when you really don't know the children. Perhaps you could look at doing something perhaps on self-esteem, and you actually get to know the children, so you really impose that to begin.*
J: I see.
L: *Or, the curriculum, if it's a Science or Social Studies topic, that they probably don't have a lot of knowledge, about, may not. Then it's for you to provide them ...*

When given the question in the Post-Teacher Interview— *"In what ways do you usually use examples from the children's experiences in their mathematics lessons?"*—Lyn responded:

PTI *I incorporate children's examples whenever possible. Sports Day is a great one, because you have got the 100 metres, 25 metres, how long will it take, stop watches, using the Sports Day, that, just the actual participation but there's lots more you could do with it. The actual organisation of the day, planning the timetable for the day, that type of thing. And it's meaningful for the children, because it's part of what they enjoy, and ... when you have your Olympics, and I've done that before and also the Commonwealth Games, good things for understanding high jumps, long jumps, ... Cooking I use a lot because that's where I get the things of weights, kilograms, grams, litres millilitres, that brings in the measuring, the half, the quarter, three quarters, leading on and I always after a cooking activity do a worksheet that might be, if you were feeding um, if the recipe says "Serves 8" and you were feeding 16, then the doubling, or if you had to do half, adjusting it for 4, I do that a lot. I do also the costing out of items, when they are actually cooking, particularly at the beginning of the year on Pancake Day, we always, work out how much each pancake costs, or they might do the cost for the whole school. (unclear) I've done, and that's with the same group an Italian theme, where we went from actually planning to having a good pizza luncheon:- the planning- to the making- to the cooking- to the setting it up as a restuarant.*

PTI *I talk to them about the school fees, how much is that going to cost and because its relevant to them that it does cost a lot of money, even now they can see the relation ... Someone says "my parents could have bought a house in that time." So I try to make the maths we are dealing with relevant all the time.*

Lyn was concerned about making connections which would not only be of interest to the students, but would also have conceptual relevance, as her comments in her Post-Teacher Interview signals:

PTI *Sometimes as a teacher you are not even really aware that some children have lost track of the lesson, because you're on track with a certain understanding you're on track where you're going and* *you misjudge something because they're on a different wavelength to you. So unless the child comes to you or unless you ask the right sort of questions. Sometimes, you just, you actually miss that. You don't pick that up until much later on when you're actually giving perhaps another example and you don't understand why they didn't know it and then you look back and ... or someone else will come and say to you, "I've got to help so and so because they don't understand," or sometimes you do pick it up yourself because you can see them just sitting there and not doing anything or when you walk around and talk to them you can see from what they have done. I think the difficulty there is sometimes you can be actually lulled into thinking they have got it. Particularly when you've done work and they take it home and they come back the next day. Yes they may have understood it at that point in time but only for the time with you, but when they come back after having had time out doing other things, they haven't really internalised it.*

PTI *What do I do? Well then I have to have small group sessions to go over the whole thing again, take steps back, and where I might have been, tried, expected too much, where there are lots of gaps, I've jumped, that was important ...*

Lyn's teaching approach appears to be driven by her attempts to connect, in meaningful ways, with the interests and conceptual needs of the learners.

In the Initial-Teacher Interview with Michelle she expressed concern about the need to connect the mathematical content of her lessons with the needs and interests of her students:

ITI *I think maths must be relevant to the child's world in everything that we do. I think its like any learning we must have a reason for learning and if you can't see the relevance or importance of it then why learn it.*

Michelle noted that she tended to cater in mathematics lessons, for the common problems encountered by students. She found that it was helpful to tutor some girls after school in order to assist them to connect with what was being taught in mathematics lessons:

ITI *I tend to cater for the more common problems I must admit only through time. Having cross-graded the children they tend to have very similar problems anyway, so I am catering to most children, the extremes of my class I tend not to, only through time, however I believe its very important. There are some children that really need a one to one basic and as a teacher of 23 children in a classroom I just physically don't have that time, in my particular method. I have a lot of girls in maths tutoring after school, and they are just blossoming because they tend to be the children that need to talk through what they are doing and just go that step slower than what I do in the classroom.*

Michelle seemed to be more active in the beginning of lessons in her effort to connect with the children's interests. As the mathematical processes developed during the lesson this became less of a priority. This reflective comment was made in the field notes, regarding Michelle's interaction with the students:

TU 426-427 *interesting that teacher has the capacity to be in tune with the students at the start of a lesson but that the more she goes down the track, the less flexible she seems to be. It may be connected with the good methodological practice of "focus" in the beginning of a lesson together with "motivation" as good teaching habits but not tied to an understanding about the learning principles that these practices emanated from.*

Michelle's response during her Initial-Teacher Interview was consistent with these field notes. She commented about her use of the camp at the beginning of the lesson:

ITI *I think I just used the camp because it was relevant, the kids had just been on camp and it was sort of in their very recent experience. I tried to relate the maths to that ... Some people go from the experience and draw the maths out of it but I do it the other way* *round, I take the maths and integrate it with the kids' experiences.*

Her reference to the camp was another example of using it in the beginning of the lesson rather than it being integral to the mathematics they were being required to do, as these examples from the "typical" lesson series show:

TL *She established that the students know what % means. She provided the example: the number of students who are here that went on the camp is 82%. She asks them how many that would mean were not here. (30.3.92)*

TL *Michelle stimulated students' schema by discussing the proportion of students who went to camp and who didn't like chicken. (6.4.92)*

The textbooks Michelle used tended to have examples which would be likely to appeal and connect with students' interests at this age level. The following text units taken from the "typical" lesson series and field notes illustrate this:

TL *The teacher drew students' attention back to the textbook p.42 of Maths bk (Addison Wesley purple)*[4] *and said "Let's look at a word problem and get the maths out of it." She read the text.* "Subtracting Decimals- Using an electric timer, a tennis serve was clocked at 62.483 meters per second (m/s). A baseball pitch had a speed of 41.556 m/s. How much greater was the speed of the tennis ball."

TU 629 *p. 121 Textbooks (Addison Wesley purple)* "Problem-solving: Practice- Solve. 1. Ned can walk 10 km in 95 min. How many minutes will it take him to walk 1km? 2. The winner of a 10- kilometer roller-skating race had a time of 21.87 min. About how many minutes did it take the winner to skate 1 km? 3. Karen ran 100m in 13.2 seconds (s). Her time was 1.3 s slower for the second 100m. What was her total time for running the 200m?"

[4] Addison Wesley complete reference is Eicholz, R., O'Daffer, P., Fleenor, Charles, R., Young, S., & Barnett, C. (1985). *Addison-Wesley Mathematics.* Menlo Park: Addison-Wesley Publishing Company.

At times Michelle would ask students to write their own problems. She outlined quite specific parameters to which the problem had to conform, as the field notes observed:

ML *The teacher stopped the class and gave extension work to the class. She used a new idea with them where she gave the students a task to give their partner a word problem i.e. they give the information/ problem/ sum to their friend to do the working out and then they changed over. The teacher set the condition that she preferred the type of sums to be similar to the subtraction of decimals exercises which they had been doing from the textbook.*

TU 197 *Homework get them to write out another 3 problems (subtraction with decimals) "Make sure the sum is one where you need to borrow from other columns. Check on the calculators. Make them interesting."*

TU 646 *The technique of getting students to set their own similar problem was used frequently by Michelle in an attempt I believe to have them more involved, however it still remains as her agenda—i.e. what is to be learned is unashamedly prescribed by teacher.*

This discussion in this section, indicates how Michelle connected what was being taught in the classroom with the student's background. Instead of starting with their experiences as she said, she started with the content first and then related it to the students, where possible.

Cultural and/or Gender (2 1 2)

This subcategory of interest refers to the teacher's awareness of the cultural and gender issues that a learner may bring with them to the classroom which may influence how and what they are able to learn (Lyn—12%, Michelle—2%, see Figures 8a and 8b).

When interviewing the students at the outset of the study it appeared that the backgrounds of the students from both classes were very similar, with neither class exhibiting spreads of cultural diversity (Initial-Student Interview asked questions—*7. Are your parents good at maths? 8. Did they do maths in High School/ Year 12/ Uni? 9. Do you think your Dad or Mum uses maths around the home much? 10. Do you think they use it at work much? 11. What about your brothers and sisters?).* Most students were from English-speaking background families although Michelle's class included one student from an Asian background.

The students in Lyn's class were mostly from a middle-class family background. Lyn built on the values the students brought to the classroom about getting value for money in this example where students were catering for the class party:

TU 180 ***worksheets- students have to find best value per weight and amount for party products***

Lyn builds on their cultural experiences within each theme. This example from a cooking lesson shows Lyn's assumption about their prior cultural experiences:

TU 42 ***presupposes cooking, refers to different measures*** (Class is taking part in a small business enterprise where they cook products and sell them during recess time)

She also attempted to confirm the status of mathematics purported within the cultural context of the school as commented in the field notes:

TU 346 ***presentation on walls around classroom to validate it as a subject along with social studies, language etc,*** The following comments from her interview shows how she continually used what had specific relevance for their lives and their future.

PTI *I do costing out of items, when they are actually cooking, particularly at the beginning of the year on Pancake Day, we always work out how much each pancake costs, or they might do the cost for the whole school. (unclear) I've done and that's with the same group an Italian theme, where we went from actually planning to having a good pizza luncheon, to the planning, to the making, to the cooking, to the setting it up as a restuarant.*

There were few examples of this subcategory (2%, see Figure 8a) in Michelle's "typical" lesson series. Michelle's programming documents indicated that she valued the function of mathematics lessons to transmit cultural values:

TU 4 *Rationale for Maths included preparing students for rapidly changing and increased technological world, higher order thinking and positive and confident attitude and teacher modelling this genuinely.* (Rationale taken from her program read: *The aim of a mathematics programme is to provide students with the necessary understanding, skills experiences and attitudes which will equip them to function effectively in a rapidly advancing technological society ... It is important that the teacher displays a genuine, positive attitude towards mathematics so that the students might adopt this stance and find mathematics fun, interesting and enjoyable.*)

Michelle also used a textbook which, as already mentioned, was chosen because it was likely to have appeal to her students. The main subjects depicted were teenage children, and the activities were ones in which the students may have, or may have liked to, participate in such as outdoor adventures and many sporting events.

TU 167 *Textbooks p. 40 (Purple Addison Wesley) got Linley to read* "Adding Decimals. Jason visited some European countries. He has 1 British pound note, 1 German mark, and 1 Greek drachma. What is their total value in U.S. dollars?"

teacher asked if converting American dollars who would be richer- an example of applying maths to a real problem, especially if you're a tourist (which I suspect most of these students have travelled)

Michelle recognised the dilemma of using an American-produced text:

PTI *I like WA Maths. I think that's sort of, I think the levels are more reasonable. I think being Australian, or West Australian, too, its better. The Addison Wesley text is very American, the words are American and the examples are American and that's a downfall, as I said I didn't have a say in that choice, it was a school-based decision.*

One of the last students to be interviewed was Kylene, one of the highest-performing mathematics students in the class. The author reflected on this interview in a diary entry:

DE *Michelle's class 26.10.92*

Still getting the same type of responses to the above from the children except for the top maths girl, in my estimate, who was Kylene T ... Her response was very significant. She wrote down the key words before attacking the problem (showed evidence of a metacognitive approach. See the back of her sheet for indicators). She got very flustered as she did the problem because her strategy was to relate this to something else she'd done before. She claimed she had never done anything like this. I stressed that I didn't want her to get it right necessarily [working out the day and year she would turn 21] but was more interested in how her mind thought things through. I recall only too vividly how upset she was from the video interview in the beginning of the year. I helped her to calculate the year she was going to turn 21 and it meant readjusting the initial plan which she crossed out and wrote the new information. She also circled the final answer. All metacognitive place-holders which I would regard as quite advanced strategies being used by this top student ... also I don't know how much critical numeracy there would be in her as she likes to find the pattern and apply it. (quite a successful strategy in our current system as

> *long as you take those types of subjects). Another interesting response which came from her when I asked who does she go to for help in maths (deliberately vague to see if they consider home or school in terms of help) she said her sister, because she knows lots of short cuts and Mrs W... [Michelle] second because "sometimes I get confused." She mentioned a couple more times she was confused by Mrs W... and I wondered how much teaching was going on at home (when I quizzed Mrs W she said she'd noticed often that her sister had shown her another way).*

Kylene was from an Asian background, and her responses (both verbal and emotional) contrasted with the less flustered comments of her peers. Her interview reflected the enormous pressure she appeared to be under to perform well. This, in turn, placed pressure on her teacher, Michelle (Mrs W.), to cater for the child's cultural background (and the help she received from home).

Register (2 3)

This component of interest refers to the way in which the specific jargon of a subject, in this case mathematics, is introduced into the classroom.

Introducing new mathematical terminology was important to Lyn (13%) as she expressed in her Post-Teacher Interview:

PTI *We look at the word, I might often say go to the dictionary, go to the maths dictionary, or the encyclopaedia, look up the word, have a look at the meaning, discuss the word, write it down, perhaps we'll illustrate the word, if it is possible to call on "Where have you seen this word in real life?" If there's any, you can act it out in a new way, or have the children describe it, have wall charts about it, to make it more real to the children.*

Lyn's own use of mathematical language seemed to move between everyday and more mathematical language:

LT 4.3.92 *Lyn:- About 3. Would it be 3 whole sandwiches or just three quarters of a sandwich? So each can have about one round of sandwiches.*

TL *She demonstrated on the easel how to calculate "average" speeds. The students were gathered near the easel ... She asked the students "What would be the middle price for a dishwasher? intending that they add up the list and divide by 5 to find the "average".*

With each new concept Lyn would introduce the associated mathematical terminology. For example, the trip to Collie topic introduced—"ratio," "average," "timetable;" the Class Party topic—"tally," "pie graph," the Mini Olympics—"scale," "perimeter," "area." When calculating the fraction of sandwiches needed to cater for the class party, Lyn did not introduce associated terminology, such as numerator, denominator, improper fraction, mixed fraction, lowest common fraction etc.

Michelle appeared to be conscious of the special language of the mathematics register (9% was the percentage indicated from the combined scores of the raters, see Figure 8b). Usually before any new concept was taught, Michelle would establish what the students already understood by the term, or would get them to research about the topic for homework and then bring their findings into class the next day.

TU 422 *Catherine discovered septagon rather than heptagon as the terminology which caused classroom investigation for homework.*

TU 514-560 *Teacher asks students to think of 2 digit number and arrange largest digit on left and subtract the reverse order number (pallendrome) ... students will get "nine" (9) no matter what digit they select ... teacher reinforces "pallendrome" term*

TU 303 *established what they knew % meant i.e. students from camp -82% and not here.*

TU 306 *stressed what "/" in 82/100 means you do i.e. it's called a divisor*

Both teachers were aware of the connection between the everyday language the child has and the specific mathematical use of language. However, as this selection of examples shows, Michelle focused on different terminology to that being introduced in Lyn's class. In Michelle's Post-Teacher Interview she was asked about how she usually introduced new terminology:

PTI *Generally if it arises in the class discussion or their textbooks I'd pick it up then and introduce the word for them, or try to use the correct terminology as I am talking to them, and if it comes up in their work I'd address it there ... I think that's important because maths can sometimes get a bit—words can be different to the context that they're used in everyday life, and maths can use words quite differently so you need to be aware of it.*

The following excerpt from a lesson transcript demonstrates how Michelle wove the mathematical terms "numerator" and "equivalent fractions" into her questions, and how she introduced the term "lowest term fraction:"

LT 11.8.92

Michelle: We talked about one third and two sixths being equivalent fractions. Can anybody tell me other fractions that are equivalent to one third? ... Right? Rebecca?

Rebecca: Four twelfths

Michelle: Four twelfths is equal to one third

Child:- Eight sixteenths. One times eight is eight, three times eight is sixteen ...

Michelle: that'll do thanks, there are millions of them, we could go on for eternity. Now of those fractions this one's special and it has a special name called? (Pause) Does anybody know?

Child: A lowest term

Michelle: A lowest term fraction. Why do you think it would be called a lowest term fraction?
Child: Because it can't go any lower than that?
Michelle: The denominator can't go any lower than the three.

Regarding the mathematical register, there was quite a difference in the messages students would have received about what it was to express oneself mathematically. Michelle's class continued to have the everyday language they brought with them to the classroom progressively refined to becoming more mathematical.

Lyn's way of signalling mathematical expression was emphasising that mathematics needed rigour. Her way of communicating this to the students was by stressing neatness and accuracy in layout of final presentation and of figures.

TU 44 *stressed layout on sheet- hint on using diagram, no invented spelling* (Students are adding list of figures that need to be considered when setting a price for some products they are selling as a small business)

TU 129 *stresses neatness in recording and need to keep record* (Students are recording their findings from asking their class mates which foods they preferred for the class party)

From this discussion it may be concluded that the register of mathematics was approached differently in each class. It would appear that the students in Michelle's class were exposed to a wide range of quite specific mathematical language. This language was strongly embedded within the mathematics the students were using, regardless of whether the context was a real world one or a mathematical one. In Lyn's class there was a focus on clarifying any mathematical language which may cause ambiguities but by and large the mathematical terms were confined to those related to the real world context in which the mathematics was being used.

Link New Terminology (2 3 1)

This subcategory of *Interest* refers to the way the specific language of mathematics is connected with the everyday language and meanings that the learners bring with them into the classroom.

This subcategory builds upon the previous discussion under *Register (2 3)* but is specifically concerned with the teacher's awareness of the learner's use and understanding of a term, and how she tries to adjust the language to being more mathematical. Both teachers were concerned about the students' comprehension of new concepts and the associated vocabulary (Lyn—14% Michelle—13%, see Figures 8a and 8b). Both teachers tried to assist the students in developing understanding of the new vocabulary.

In Lyn's thematic approach, new vocabulary was likely to come up spontaneously, whereas Michelle usually anticipated the vocabulary needed for that day's lesson and discussed it at the outset. Lyn's instances of giving attention to new vocabulary come from the field notes and occur during the progress of a lesson while Michelle's occur at the outset of the lesson. Examples from text units which refer to Lyn's lessons are as follows:

TU 399-400 ***students seated in groups Lyn asks "What's meant by a timetable?" establishing meaning of a word and possible confusion of "times-table"*** (Students are working with real timetables in an effort to work out the most cost efficient route and form of transport to their class camp)

TU 402-404 *uses their experiences and puts on blackboard (interested in establishing their meaning) e.g. bus, trains, boats, planes, school timetable, hospital, movies, roster, event (sporting like a program), T.V. Guide, clubs, sports (fixture) (accepting their answer, they feel free to contribute and at times are expert) emerged 1 program, 2 diary, 3 fixtüre*

On one occasion there was confusion because of the use of everyday language:

TL *She asked students "What would be the middle price for dishwasher?" intending that they add up the list and divide by 5 to find the "average."*

The confusion occurred with several students who took the word to mean the middle location in a list of five items. This is an example when the accurate mathematical terminology "average" would have indicated more clearly the process required. Use of everyday language was, in this case, misleading.

The field notes taken during Michelle's lessons showed that her introduction of mathematical terminology occurred at the beginning of a lesson:

TU 348 *When reading the problem mentioned in maths, "of" means times* (Students are about to do exercise from their textbook)

TL *She established that the students know what % means. She provided the example:- the number of students who are here that went on the camp is 82%. She asks them how many that would mean were not here.* (Lesson continues with students being given fractions to make equivalent with .../100, and then textbook exercises along the same lines)

The combined totals for *Register* (2 3) and *Link New Terminology* (2 3 1) show Lyn with 27% and Michelle with 22% (see Figures 8a and 8b) indicating minimal difference between the teachers for the "typical" lesson series. However there appears to be a different starting point for each of the teachers. Michelle drew attention to the mathematical terminolgy the children would need to associate with the mathematical task and continued to reinforce this special language during her classroom discourse. For Lyn, however, the starting point was the child and whether s/he had a need at that time to learn the vocabulary in relation to the real world context in which the mathematics was set.

Different Styles (2 4)

This subcategory of interest refers to the valuing of each learner's own style or preferred mode of learning (Lyn—16%, Michelle—11%, see Figures 8a and 8b).

Lyn discussed in her Initial-Teacher Interview how she catered for the individuality of the learner in her classroom:

ITI *I bear in mind all the time very much that they are individuals:- the very able ones who quickly pick up and see things very quickly, to the ones that connections are just made for them. In trying to do that I provide activities that will interest those and challenge those that have got minds, that are wanting more and more. And then on the other hand, to encourage the ones who find it difficult to just keep trying, to keep having a go, use calculators, use one another, use concrete materials to help them find solutions- to relate it to something they would know, something in the real world to use, if its decimals then to relate it back to money ... I think this is very important if we are to improve maths attitudes of our students in our class, that the children who don't see or do not get the answer as quickly as the more able ones, they don't feel that they are inadequate because whilst they are still thinking it out, the brighter ones have given the answers, finished, ready to go onto the next one, so it's important that we cater for each, for all the children at all different levels ...*

Due to the open-ended nature of the tasks in Lyn's class, there were many opportunities for students to approach tasks in their own way. There was a lot of sharing and valuing of the individual approaches, both by the teacher and by the students.

TL *The teacher discussed the investigation tasks students were involved in- their calculations of the most efficient speeds in a car, train etc to get to the campsite at Collie ... (5.5.92) Stephen had a unique way of adding and shared it with the teacher and found others had the same system ... Lyn was quick to recognise his strategy, and asked him to share it with the class. (13.5.92)*

ML *Lyn asked students for suggestions of how they might use the graph paper. As she talked she sought clarification of their ideas and encouraged them to provide more information. Lyn offered a different suggestion of a pie graph. She got students to clarify their definitions of a pie graph. Lyn encouraged freedom of choice in how they may use the previous discussion on graphs in their own recording of choice of savouries.*

It did however seem to be a problem for those students of Lyn's who seemed to require more structure, that is, whose learning style seemed to need more boundaries and guidelines. At the end of topics, it always seemed to be the same few students who had not completed the project. In particular the author noticed this among the weaker students (e.g. Victoria, Brenda, Ben E.) as in the following selected field notes.

TU 201-253 *Students have project of class party- menu and costing, individual discussion with researcher about what they had learned from the process. Victoria - not articulate about how stages relate, aware of quantity and need to stay within cost, not clear on the maths she's learnt except for addition and how it will be changed if she looks back when she is older. Brenda- not quite finished but well presented in categories of drink etc, not clear on how to express what is going on, confident she's learnt something e.g. prices and where to buy the cheapest food, not sure on the maths* *learning but confident that she likes decorating and displaying in project format. Ben E.- incomplete*

It would appear that for some students the open-ended opportunities did not necessarily achieve the desirable goals for which Lyn had hoped. When the author discussed this with Lyn in the Post-Teacher Interview she agreed that some students got lost along the way:

PTI *Sometimes as a teacher you are not even really aware that some children have lost track of the lesson, because you're on track with a certain understanding, you're on track where you're going and you misjudge something because they're on a different wavelength to you ... I think the difficulty is sometimes you can be actually lulled into thinking they have got it ... but when they come back after having had time out doing other things, they haven't really internalised it.*

It would appear that even when a teacher is aware, as Lyn claimed to be, of the different ways in which students in her class learned mathematics, some students inevitably became lost along the way.

Some students prefer to manipulate "concrete" mathematics materials, others prefer to move around and physically interact mathematically with their environment, while still others are able to conceptualise number relations without calling for hands-on equipment. Michelle tried to cater for different learning styles in the way she approached the syllabus (The Western Australian Syllabus has three strands—Space, Number and Measurement). She referred to this in her Post-Teacher Interview:

PTI *I like to use a range, usually a mixture of space and measurement and number mixed in together, I run those three programs together. If I was doing something in* *number, then I would try to pick something from space and measurement. I think that adds the interest to lessons, giving some variety, and appeals to the different interests of the different children who hate to look at percentages and find it boring, and another one might not. So if you vary then you are not going to have different kids turning off.*

Michelle referred to the difficulties she had in catering for different learning styles with an ability-grouped class. A variety of needs exist in any one class in terms of preferred learning styles, for example, some students learn best with concrete materials and a lot of opportunity to practice, others prefer to watch some-one else do it and form their own version from that, and others have an ability to pick up abtract ideas being presented through an oral medium, and so on. Michelle made the comment:

ITI *It is very hard especially in the lower groups to learn from each other and do peer teaching because of their limitations.*

Michelle presented her mathematical concepts to her class within a tight lesson construction, following a coherent logical sequence. She needed the co-operation of the children and tried to restrict the discussion to the task at hand, as this excerpt from the March lesson indicates:

ML *Michelle recapped on their homework and asked when might they round decimals in their everyday life. Linda responded as to how rounding was used in shops. The teacher encouraged her to give an example. Gabrielle gave the example of road signs which indicate how many kilometres to a destination. (An interjection by Angela was ignored by the teacher) ... Elisabeth went back to the road sign discussion. The teacher asked why you might have to round back to a decimal. Carrie P said a judge might judge you e.g. for speeding fine where what you pay is calculated according to the kilometers over the limit you were* *travelling (however her answer stayed within a whole number). Emma said that when you tell the time e.g. 4.23 you round it off to 4.20 (the fact that this was working with 60ths and not 100ths was not discussed). Melissa talked about why she hadn't got her homework done- parents were starting a restaurant. (This response was not developed by the teacher.) The teacher ignored any off task discussion such as this.*

On several occasions children who were likely to take the class off on a tangential discussion, or if they had their hands up, were ignored. These children included Linley, Emily, Carrie P. and Carrie B. The author remarked on this several times in her field notes:

TL *The problem came when students kept coming up with problems that didn't quite "go" e.g. 23/47 Both Carrie B and Tania appeared to be on same wavelength and dominantly shared how they made equivalent fractions, their method was not the same as Michelle's*

ML *Most students worked on their own. Kylene still confused, and said she knew what to do but couldn't explain it. Carla wanted to use the calculator to check her working out. The teacher encouraged the whole class to do this too. After 2 minutes some had completed the work [p.42 q 1 & 2 Textbook- subtraction of decimals] but Carrie P. was still confused and had not done any.*

There were clear examples in both teachers' classrooms where learners' particular styles were catered for. For example in Michelle's class the students were manipulating wooden cubes to to explore the properties of 3-D shapes. At one stage she used the overhead projector to demonstrate visually the rotation of various shapes. Lyn's class had opprtunities to go outside and measure various parts of the building in order to develop the concept of proportion. They were often involved in cooking which enabled the development of volume and capacity concepts. At any time a student felt it may have been useful for their mathematical constructions they would go to the library or computer laboratory.

Generate Own Rules (2 4 1)

This subcategory of interest refers to instances where the teacher acknowledged the value of a learner's own system for working out problems, rather than placed an expectation on the student that the teacher's way was the only acceptable approach (Lyn—6%, Michelle—11%, see Figure 8a and 8b).

In the "typical" lesson series, only minimal incidences were identified for each teacher in this subcategory of interest (in Lyn's "typical" lesson series there were 6 incidences and in Michelle's 5, see Table 2 in Appendix 9). A closer look at the structure of both lessons suggests that there may have been a number of potential opportunities for students to generate their own rules when doing the broad topic of planning the trip to Collie in Lyn's class. In Michelle's lesson on fractions, however, there was less opportunity for students to generate their own rules.

Both teachers commented in interview sessions that students should not feel bound to do a mathematical process in a certain way just because the teacher said so. If the "typical" lesson series is any indicator then the interpretation of these classroom observations is that students did not make use of this expressed freedom within the classroom context. When observed during the Post-Student Interview the students showed highly idiosyncratic ways of processing the mathematics involved (see Appendix 11) when they worked out the day and the year in which they would turn 21.

As mentioned above both teachers stated that they encouraged students to solve problems using their own approaches. This is Lyn's comment:

PTI *Often children don't understand that there are many ways of doing it, we've often just said to them this is the right answer and this is the way you do it and that's why we have that conflict when mum and dad come " I tried to show them my way but they said they'll get into trouble because its not the teacher's way" So I encourage and say very much, very strongly now, that the maths is individual.*

Lyn tended to rely on the students to discover the most effective way to solve the mathematical problems presented in class. Some of the ways she encouraged this is shown in these excerpts from the "typical" lesson series:

TL *Lyn made the point that when students gave their answers they were to share their different strategies for arriving at that answer. She encouraged their responses and doesn't give the correct answer away too early in the discussion ... Students shared at their tables the strategies they used to work out the average and in what ways it was the same or different from what had already been shared ... Stephen had a unique way of adding and shared it with the teacher and found others had the same system ... (13.5.92). Blake shared his discovery of a pattern in the timetable. (28.5.92)*

With reference to Michelle's teaching, and as noted in the section on *Different Styles (2 4),* Michelle adopted a tight lesson construction and followed a coherent logical sequence in her lessons. Rather than focusing on the students generating their own rules, she appeared to be concerned with making sure that they clearly understood the way the concept was unfolding. She used certain students to check whether all of the students understood.

It seemed reasonable to suggest that too much generation of students' own rules might interfere with the coherent plan of Michelle's lessons.

TU 166 *Teacher ignores any off task at hand discussion*

TU 206 *quite high degree of students not 'getting' what teacher was on about or going down their own tangents*

TU 210 *Carrie P seemed to often be one who was not understanding or going down the wrong track and yet although was obvious to me even on my few visits she didn't seem to get attention*

A sample from a lesson transcript on fractions reveals the way in which Michelle presented a fairly direct way of dealing with a concept, even when working on a one-to-one basis with a student:

LT 11.8.92 *Different students are coming to her with their homework on fractions from the previous night:*

Michelle: Carrie could I see you please? What is it Georgia?
Carrie P: I had troubles.
Michelle: You had a bit of trouble? OK let's go through it together. That one's correct ... four thirds its actually an improper fraction.
Carrie P: I thought it was a, I was putting ... I didn't think it was allowed to be an improper fraction.
Michelle: OK If that's an improper fraction do you think it is likely that the equivalent fraction will also be improper?
Carrie P: Yes. If you times it that will be a larger number if you times it by ... like if you times it by the two or whatever you times it by in the middle. You times the number by that and its gotta be more than the bottom number because the number is less ...
Michelle: I think I know what you mean so let's go through this example. Four thirds equals so many eighteenths ... Olivia would you like to go and see Mrs Bickmore-Brand please. Olivia hasn't done her homework but perhaps she can go through just one or two with you. She has been spoken to.
Carrie: Um, six, if you times by six. If you do four times six is twenty four.
Michelle: You've worked out that three sixes are eighteen.
Carrie: Four sixes are twenty four.

Michelle: And so have we got another improper fraction?
Carrie P: Yes.
Michelle: Let's just check these. OK do you understand it now?
Carrie P: There's ... I worked out there's a pattern each time.
Michelle: And what's the pattern in this one here, number twenty three?
Carrie P: Well in the bottom line you are going by sevens, and in the top line you are timesing by four.
Michelle: Multiples of four. What do we call the bottom and top? Do you remember?
Carrie P: The ... one of them, I think its the bottom one is the denominator and ...
Michelle: Starts with an N ...
Carrie P: Numerator.
Michelle: Numerator OK good girl, that's lovely

In this excerpt it appears that Carrie P. had difficulties. During this lesson several children had deduced their own way of working with fractions which were incorrect but resembled one another's. For example, Georgia and Melissa both followed a patterned approach but not the same one as adopted by their teacher.

This was the third lesson on that topic of fractions and some students were still having difficulties in understanding the concepts involved. This excerpt from the lesson transcript shows how Georgia continues to retain her misconstruction of the concept even when Michelle is attempting to lead her through in what Michelle feels is a systematic and logical way. The lines which show Georgia's struggle have been coded (###). Note how Michelle responds to Georgia at these points.

Michelle: If any one would like to see me having marked those eight come over to me now. Georgia, come and see me. Bring your work. Everyone else quite happy to continue along? ... Come and sit down let's have a look Georgia. What is the major problem?

###Georgia: Oh I haven't got one. I wanted to ask you something else. Shall I do it?
Michelle: I'll just mark your homework for you. How did you find this.
Georgia: Oh its easy. I think.
Michelle: Did it take you very long.
Georgia: No. About five minutes.
Michelle: Well tell me your system?
Georgia: Well. I think I do it differently, but if you say twenty four, Three goes into twelve four times, so times four by four and you get sixteen.
Michelle: Right so whatever you have done to the denominator you ...
Georgia: Do to the numerator
Michelle: Have done to the numerator.
###Georgia: Oh gosh.
Michelle: Oh dear what have we done down here? Let's have a look. One half is equivalent to two quarters which is equivalent to three sixths
###Georgia: And they're all halves and I thought. Oh no.
Michelle: Tell me how you came about six eighths and that will identify the problem.
Georgia: I put two, four, six, eight, ten, twelve. And seeings how you times it by two there and timesed by two to get six and then added two and then added two. I did it completely wrong.
Michelle: Why did you add two?
###Georgia: I don't know. Because I was supposed to be timesing two. No. Yeah. No.
Michelle: Can you tell me what each of these are supposed to be equivalent to in the lowest term?
Georgia: A half.
Michelle: A half, good girl, so that's the lowest term fraction. So six eighths we know is not equivalent to a half.
Georgia: It would be six twelfths, eight sixteenths and ten twentieths.
Michelle: Eight sixteenths and ten twentieths OK. But those aren't the next three equivalent fractions, because the next one is going to be ... eighths. You've got halves, quarters and sixths, and so the next one would need to be eighths. The bottom line, the denominators are going up in multiples of?
###Georgia: Two that's how I did there. I thought.
Michelle: So this one then should be tenths. And this one?
Georgia: Twelfths.
Michelle: Twelfths. Let's fill them in. How many eighths is equal to a half?
Georgia: Four
Michelle: How many tenths is equal to a half?

Georgia: Five
Michelle: And? Six twelfths. Can you understand that? So let's go to this one look at the denominator and the pattern that the denominator is making. What are the next three denominators?
###Georgia: It would be twenty.
Michelle: No its going five, ten, fifteen?
Georgia: Twenty,
Michelle: Twenty, twenty-five?
Georgia: Thirty.
Michelle: Mm. The lowest term fraction that we're looking for?
###Georgia: Two, four, six, eight, ten, twelve, fifteen?
Michelle: Let's have a look at it. It's making a pattern, but is eight twentieths equal to two fifths?
Georgia: (pause)
Michelle: Let's work it out. How many fives in twenty?
Georgia: Four
Michelle: Four. Two fours are?
Georgia: Eight.
Michelle: Is your pattern working? Let's check for the next one. How many fives in twenty five?
Georgia: Five.
Michelle: Five tens?
Georgia: Fifteen. Goes. Is ten.
Michelle: Good, so can you tell me the next answer?
Georgia: Um five into fifteen, thirty goes two, two times six is twelve.
Michelle- Good your pattern worked.
###Georgia: It wasn't really what I was thinking.
Michelle: It doesn't always work in a nice pattern;like that. So you need to be careful and always relate these equivalent fractions back to the lowest term fraction. OK. Understand that? I'll leave the next four to do by yourself tonight and show me tomorrow.

This dialogue took place after considerable classroom work on fractions throughout the year using a certain system, which clearly, now in August, after another series of lessons on fractions, students are still struggling with.

Michelle was gently trying to impose her own approach to this fraction topic. Implicit in this tactic was the assumption that she could persuade Georgia to understand the way she was using pattern to develop equivalent fractions. Towards the end of the dialogue she was using expressions like "Good, your pattern worked" (in spite of Georgia's protests that "it wasn't really what I was thinking"). Thus although Michelle appeared to take account of the child's approach early in the dialogue, later, her reference to "your pattern" did little more than reinforce the fact that she was imposing hers.

When asked about how she felt about students having different methods from hers Michelle said:

PTI *I don't have a problem with students adopting a different method to mine, I think that's OK as long as they've got a good understanding of the concept, and their answers are valid. In fact it's good because other children can see that there's different ways of doing it.*

As the discussion of the above lesson transcript (from 11.8.92) suggests, although Michelle wanted the students to develop their own system, invariably they were encouraged to understand the method she was using.

When Michelle did encourage students to develop their own ideas it was usually in relation to a task such as "How many squares are there on a chess board?" (TU 586). Rigid application of a rule in this case was not the purpose for the task. Michelle also encouraged her students to use visual imagery, as demonstrated by the following text unit:

TU 341 *have them think in their own mind that 1/2 way between 41 and more than 1/2 (i.e. 21 1/2)* (Michelle is conducting a lesson on percentage where the students in this example have to find 80% of 41.)

Lyn appeared to encourage a variety of ways of problem-solving whereas for Michelle, it would seem, that there were less occasions when this was the observed. Although both teachers spoke of encouraging students to generate their own approaches, the practice of this in classroom settings was much more complex.

Various Mediums (2 4 2)

This subcategory of interest refers to the opportunities the learner has to use a range of mediums when exploring a concept (Lyn—10%, Michelle—4%, see Figures 8a and 8b).

Lyn had a wide variety of resources around the room which were freely available to the students. She set open-ended tasks which lent themselves to individual expression using personally preferred mediums.

TU 117 *students enter onto computer in wet area* (Students were calculating which foods were the most popular, from voting, in order to purchase for the class party, some students chose to record the information as a table on the computer, others used poster card and textas, others used graph paper)

TU 163 *this spontaneous programming is assisted by having facilities e.g. computers and materials e.g. graph paper on hand.*

The idea of putting the information on computer was initiated by some students and Lyn demonstrated her flexibility in being able to cater for their different ways of wanting to present the information.

Michelle normally used a range of media to explain a concept, and had a clear idea of the best medium to demonstrate most clearly the mathematical idea being discussed:

TL *Michelle demonstrated the fraction idea with magnetic units/ fractions ... Students were still being asked to contribute to making equivalent fractions .../100.*

AL *She set them the task to draw a diagram to show it had been cut into equal pieces ... Michelle demonstrated* *what she meant on the whiteboard using 5/10 and a square of 10 sections with five parts shaded. She discussed that the shaded part is the same as 1/2 ... She demonstrated the idea using grid paper (1cm squares)*

Michelle also encouraged students to express, in a written form, their understanding of the concept they were working on.

TL *She embellished the workbook task by asking them to put each question into a written context e.g. There are 24% vegetarians in class of 25. The book has the following questions e.g.* 37% of 26, 77% of 125 ...

The differences between the two teachers in relation to the use of resources lay mainly with who could choose the most appropriate medium. For Lyn, the students were given the option to use a medium they preferred. For Michelle the students were given a medium which she believed would enhance their understanding of the concept.

Summary

The idea behind this Principle of *Interest* is the connecting of the content of the classroom with what the learner brings to the learning situation. The backgrounds of the learners, not only in terms of their knowledge, language, skills and or values, but their preferred learning style needs to be taken into account during the instruction. While both teachers were quite articulate in their understanding about this Principle, and would view themselves as approaching it quite positively in their own classrooms, it would appear to have been realised in different ways in each classroom.

For Lyn her whole program was driven by content in which she believed the students would not only have an interest, but would have vested interest in being part of. For Lyn, students' construction of mathematical knowledge came about as a consequence of their needing to work through the problems that were part of each topic, for example, planning the class party, the most efficient trip to Collie and the Mini Olympics. No set steps were laid out for these topics. As the students needed the mathematical tools, they were exposed to them, not as rote formulas, but as ways of solving a problem in which all participants in the class had potential solutions, or ways of working.

Michelle's focus was on the curriculum and on the readiness or otherwise of the students to master what they would need for further mathematical development. Michelle's Initial-Teacher Interview revealed her view that mathematics development follows a developmental sequence and children need to be taken through this:

ITI *maths is one of those subjects that is very developmental and we need to build upon existing understandings. Some of them just don't have anything to build upon, and its those type of children that we need to take right back, in fact I have Year 7's this year (year prior to the study) and I have been working, I would say late year 4 work and they have no understanding. Basic fraction concepts, basic decimal work they have no understanding and we can't hope to use those decimals or use those fractions in anything else unless they have that understanding.*

What this statement reveals, is the teacher's perception of the place of the student's own knowledge in the learning process, while she does indicate she values what the learner already knows, she believes that in some circumstances the learner does not have anything to build upon.

It is difficult to assess how successfully each teacher catered for the different backgrounds of her students. However, the choices of content made by each teacher signalled to the students the value she placed on their cultural backgrounds. In this study catering for the idiosyncrasity of each learner seemed to be an issue. Both teachers had varying degrees of success and neither felt fully satisfied about what they had achieved for every child.

In order to gain some insights into how the application of the ideas contained in this Principle of *Interest* may have been experienced by the students in each class, certain questions were asked. Interviews with the students were carried out at the beginning of the observation period and again at the end. In the Initial Interviews, the students were asked *"When your teacher says its time for maths, how do you feel?"* At the end of the study, they were asked *"Do you enjoy maths?"* (see Initial- and Post-Student Interview responses in Tables 1 and 2 Appendix 12).

Little change in their apparent enjoyment of mathematics was observed between the first and final interviews. Approximately two thirds of the class said they enjoyed maths. From this point of view the students had not changed in their attitude. It is interesting to note that the weakest student in Lyn's class did not like mathematics any more at the end of the year than she did at the beginning.

In Michelle's class there was a similar relationship between the beginning of the study and the end, with two-thirds stating that they enjoyed mathematics. In both classes, those who had initially been less keen on mathematics had not changed their views to any extent by the end of the year.

In her efforts to connect with the students at a conceptual level, Lyn provided open-ended tasks or tasks which could be approached from a variety of learning styles. Students were free to generate their own rules for solving a problem rather than refine their approximations towards a predetermined process or protocol.

With Michelle she seemed aware of the connection of the curriculum with students' lives. At intervals Michelle would relate what she was teaching to the students lives or vocabulary. However, as she said, she would start with the curriculum and then connect the child to that rather than the other way around.

In order to triangulate the data the students in both classes were given the PAT and Western Australian Placement Test J assessments. The results of the PAT test were consistent with the norm for both classes. In other words, Michelle's class performed within the stanines consistent with a top ability-grouped class (i.e. stanines 5-7, see Appendix 13). When a T-test was conducted for the independent variable, there was no statistical difference between the two classes (2-Tailed t-test, $p < .05$). If account is taken of age differences between the classes (average student age in Michelle's class was 12.7 and Lyn's average student age was 11.9), it would suggest that the standard deviations shown in Appendix 13 would overlap. The Placement Test J results did not show any great degree of difference between the two classes, even though Lyn's class average was ten months younger than Michelle's students (i.e. Michelle's class was comprised of Year 6 and 7s and Lyn's Year 6 only), and therefore it could be argued that they had in fact performed well. A closer analysis of the test results (see Appendix 14) indicates that where Lyn's class performed poorly was either in areas in which they had had no exposure, such as algebra (questions 27,28,29,35) or in some aspects of geometry (questions 56,57,48b,51), and/or questions which had very little context (see asterisk indicating a decontextualised question). The results have pointed to a slightly better performance from Lyn's class when age level was taken into consideration.

There were some surprising errors from both classes where they had clearly been given exposure to that content during the year. It was of particular interest to note that Lyn's class had little or no exposure to the types of questions which would have simulated the questions or content found in this test. On the other hand, the types of questions with which Michelle's students were familiar were drawn from textbooks which included very similar questions to the ones found in this test. In other words the students in Lyn's class have developed their own ways of knowing the information found in the test from a less explicit and greatly contextualised situation.

This section has explored some of the dilemmas teachers face when connecting new information with the child's conceptual understanding. It cannot be assumed that the links the teachers make will eventually ensure the students will make those links. Ultimately learners must reconstruct for themselves each new mathematical concept they encounter.

Metacognition: Making Explicit the Learning Processes Which are Occurring in the Learning Situation

Introduction

The underlying ideas in this Principle can be linked to Vygotsky's (1962) notion of "verbal thought," where the learner expresses his/her thinking out aloud. Gradually over time the language goes "underground" and becomes "verbal thought." When faced with a difficult problem it is not uncommon to find people directing themselves out aloud or talking it out with others. Teachers can play an important role in encouraging their students to articulate their thinking processes. For example, teachers can share their own language of thinking with the students by making explicit the "why" of what they are doing as well as the "what." In this way students will not only have modelled for them the logic associated with the particular concept, but they will hear the meta-textual vocabulary which accompanies it.

NUDIST Categories

For the purpose of Nudist analysis the *Metacognition* Principle was broken up into 5 major components and 11 subcategories (see Appendix 7). The major components of metacognition are: the *Explicit Modelling (3 1)* that the teacher gives as she thinks aloud in front of the class; the overt way the teacher makes explicit the connections between what the students are learning and real life (This has been coded as—*Maths Transfer to Real Life (3 2)*); the extent to which the teacher is able to use realia and *Materials (3 3)* which are taken from real life; the ability to develop *Critical Literacy (3 4)* about the mathematical and contextual probability of what they both comprehend and compose; and the teacher's ability to provide *Feedback (3 5)* about the student's written and spoken mathematical processing.

The diagram in Appendix 7 shows the hierarchical relationship between the major components and subcategories. The first of the subcategories under the major component *Explicit Modelling (3 1)* relates to the way in which a teacher makes explicit her problem-solving processing to the class. The teacher may also make explicit how the learning *Integrates (3 1 2)* with other subjects in order to facilitate a transfer of skills across learning areas. Further subcategories are defined not only when a teacher assists students to *Link (3 1 2 1)* and locate the new information within a different subject area but also when *Revisiting Concepts (3 1 3)* so that students have several opportunities to grasp a concept. This also provides opportunities for students to practice a particular skill and keep it part of working memory. The teacher may demonstrate the concept in different modes which may signal to students the *Shifts from one Representation (3 1 2 2)* to another. The teacher can also encourage students to translate their newly learned information into another representation as an indication of the degree to which they have grasped the concept. Another subcategory is defined when the teacher moves from *Concrete to Abstract (3 3 1)* representations of concepts. There should be opportunity for *Students to Verbalise (3 1 4)* their own thought processes. In the act of thinking out aloud the students can often *Discover for Themselves (3 1 4 3)* a concept rather than mimic a teacher's approach. The teacher can encourage the *Students to Share (3 1 4 6)* their ideas aloud so that others may benefit not only by the chance to hear modelled other metacognitive language but to hear other ways of processing. The teacher can encourage whole class participation in the construction of mathematical concepts by providing opportunities for *Joint Construction (3 5 4)* between the teacher and the students. During the joint construction, a *Framework (3 5 5)* may be generated around which further processing can be modelled, or alternately the teacher can provide a problem-solving framework which may assist the students to keep track of their thinking. That is not to suggest that following a protocol is the most desirable procedure. The class problem-solving may be better served by *Open-ended (3 3 4)* activities which require the learner to think through the solution rather than apply a solution method to which they may feel no ownership.

Table 3 (see Appendix 9) presents the totals found for each NUDIST component and subcategory for *Metacognition* in the "typical lesson series" for Lyn and Michelle, as located by the four independent raters. Figures 9a and 9b represent the average of the combined totals of the four raters for the different components and subcategories of *Metacognition* for each teacher.

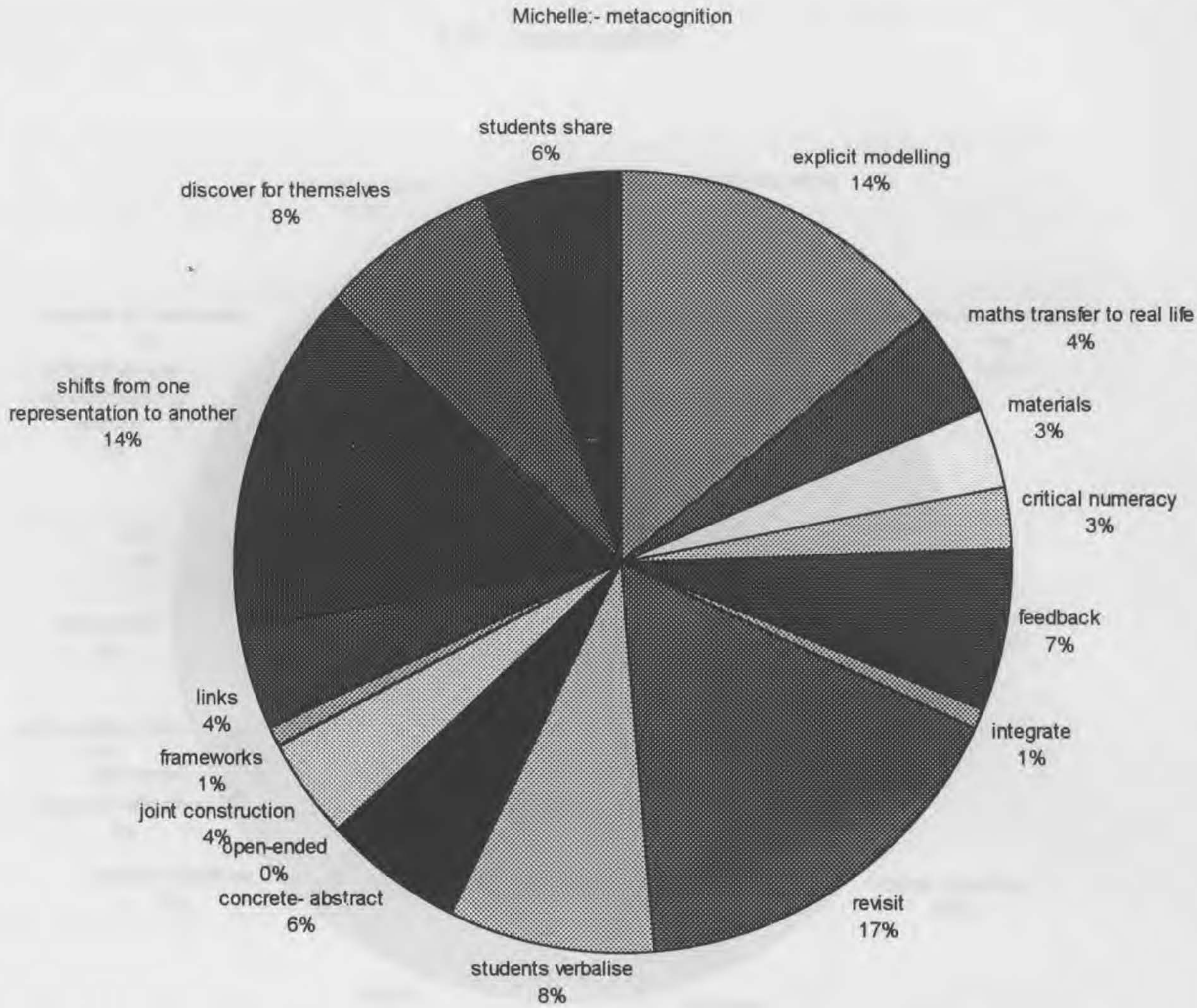

Figure 9b. Summary of NUDIST categories for Metacognition classified by four raters for Michelle.

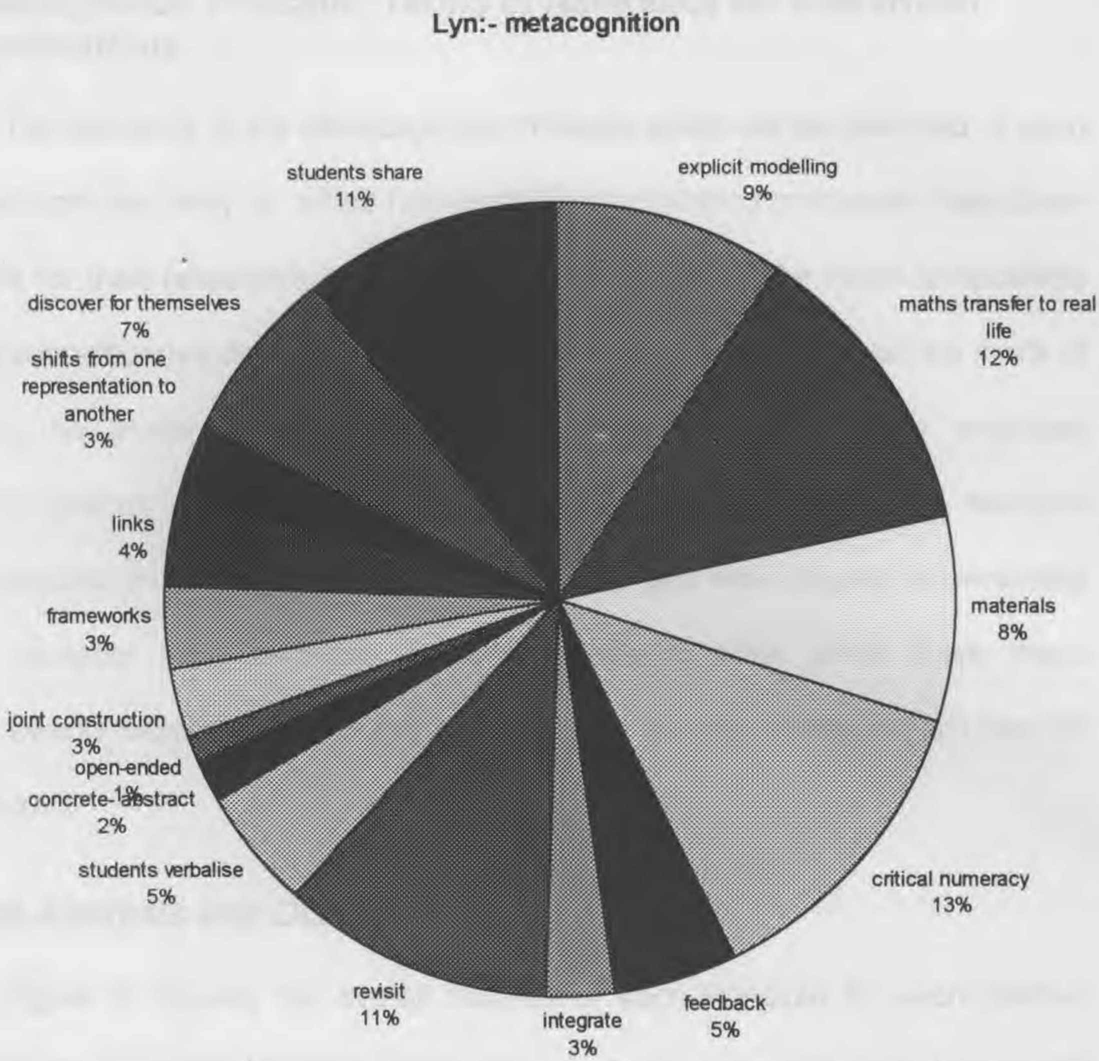

Figure 9b. Summary of NUDIST categories for Metacognition classified by four raters for Lyn.

Metacognition Principle: Terms of Reference for Classroom Observations

The elements of the Metacognition Principle which will be identified in each classroom are likely to reflect how explicitly the thinking processes have been made for their respective students by the two teachers. The major components and subcategories described above will make it possible to analyse the kinds of tasks the students are working on and whether students have employed metacognitive processing. The kinds of frameworks which the teachers constructed to assist students to problem solve and what degree of ownership the students have of these. In other words, to what extent have these frameworks become a part of the students' own thinking strategies, will also be explored.

Data Analysis and Discussion[5]

Figure 6 showing the overall balance of each Principle for each teacher would suggest that Michelle (30%) was much more explicit than Lyn in her teaching of metacognitive strategies and raising students' awareness of the reasonableness of their answers. A major influence on Lyn's coding total of 21%

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[5] The discussion which follows will address each component and subcategory description for Lyn and Michelle. Reference will be made to the "typical lesson series" but other data from the study will also be drawn upon. Other lessons will be referred to either as "March" lesson (ML), "August" lesson (AL), Field Notes (TU) (Since the NUDIST analysis has been based on the field notes, reference will be made to text units—TU, deriving from these field notes as part of the NUDIST analysis), lesson transcripts (LT) or Diary Entry (DE). Data will also be drawn from Initial Teacher Interviews (ITI), Post Teacher Interviews (PTI), Initial Student Interviews (ISI) and Post Student Interviews (PSI).

was the fact that she used students to perform this function with one another (*Students Share* 11% *and Students Verbalise* 5%, see Figures 9a and 9b). The total number of incidences categorised under *Metacognition* in this "typical lesson series" were 119 for Michelle and 180 for Lyn (see Table 3 in Appendix 9).

Explicit Modelling (3 1)

This major component of the *Metacognition* Principle refers to the teacher's overt display of thinking strategies in a whole class forum (Lyn—9%, Michelle—14%, see Figures 9b and 9a).

Although frequent examples of Lyn modelling various metacognitive strategies for the whole class were identified these did not comprise the routine structure of her mathematics lessons. Invariably at the outset of a topic Lyn could be observed using the easel for brainstorming and recording any structures which might be emerging so that the class might have it on display as a guide. Lyn tended to leave specific directions until students signalled that they needed them, as these examples from the "typical lesson series" suggest:

TL ***She demonstrated on the easel how to calculate "average" speeds. The students were gathered near the easel.***

TL ***Lyn ... took Victoria's problem to be one of poor adding and built on that with the class.***

TL ***Lyn stopped them and announced to the class why certain information would need to be on a timetable.***

Lyn appeared to be attempting to develop students' metacognitive awareness when she frequently emphasised doing things in a logical way as she modelled her own way of approaching the topic. This is illustrated in the following field note taken when the children were adding up long shopping lists:

TU 52-53 *Lyn stresses recording in logical way i.e. need for subtotalling as they go, she demonstrates on the blackboard.*

Lyn preferred to have the information the class was working on recorded so that the children had access to the information independently of her. During her Post-Teacher Interview, Lyn explained the specific way in which she used the easel to model metacognitive strategies in explicit ways:

PTI *I use both the blackboard and the easel, perhaps more the easel than the blackboard, because with the easel, it's there the children are closer together and you can actually have more eye contact. Also it can be retained, the big sheets then just stay on there and you can come back to them, reflect on them, go back and check back with them, but once its on the blackboard, it's there and then it's gone.*

From classroom observations and discussions with Lyn it would appear that she wanted the children to realise that thinking strategies were personal and that there was not one set way to approach problems. This can be seen in this discussion in her Initial-Teacher Interview:

ITI *I find that in maths I use it [metacognition] quite a lot. Simply going over and over trying to present the children with a variety of ideas, a variety of strategies or ways of doing a specific process. If it be addition, that there is just not one right way, but a large number, so lots of direct instruction to the whole class or in a small group.*

Lyn's students too were aware of a combined input when problem-solving. This became apparent in their answers to the Post Interview question "*When your class has got problems to solve, how do they go about solving them, as a whole class?*"

OP We usually consult, negotiate things. Have lots of small disputes about it and things like that. (Oliver P.)
PS She'd put up the question on the board, and everybody would sit on the floor and discuss different ways to do it (Paul S.)

Lyn rarely provided a specific protocol or "formula" way of approaching particular problems. In any whole class demonstration there is always the danger that not every child is on the same wavelength, Lyn talks about this in her Post-Teacher Interview:

PTI *I often begin I guess with the whole class looking at what are we doing in the topic and I start with the whole class and invariably you can see with the very mathematically inclined children are with you all the way, but your others you can see dropping by, and so, I guess that one of the things I've learned, that I can see now. You have to just teach to the ones that are getting what you get and for the others you have to just then go back and work with them or get those to come and work with those that already know and can see the direction in which to go*

Explicit modelling of specific thinking strategies would be regarded by Michelle as her strength. She was highly confident of her content and her ability to present this to her students. The author remarked on this in her field notes during a lesson observation where Michelle's class was working on the relationship between 2-D polygons and their interior angles and sides (4.6.92):

TU 611 *because of teacher's comfortable knowledge of subject area she is able to "play" with maths e.g. to see maths patterns and celebrate the discovery of the logic behind these.*

Each lesson had a high proportion of direct delivery of content by Michelle. It was important for the class to follow the lesson's cohesion because the tasks they were sent off to do independently drew on the system they were being shown. When discussing with Michelle (in the Post-Teacher Interview) her demonstration on the blackboard she made it clear that she was using it as a deliberate strategy:

PTI *usually, at the beginning of the lesson before introducing the textbook or the worksheet, I often use the blackboard, sort of a chalk talk session, um talking the problems through and demonstrate what's going on in my head.*

The children too were aware that a systematic approach was being used. When interviewed at the end of the data collection period, the students were asked: *"When your class has got problems to solve, how do they go about solving them, as a whole class?"*

GH Usually Mrs W[6] just calls out a problem, and like she puts it on the board and she calls out every stage. (Gabrielle H.)
MW They just sort of do it in steps. Well they write the major points down, then do whatever, like stuff that needs to be timesed and stuff like that, and then find the answer to the things. (Melissa W.)
CP Well Mrs W would explain it on the board … if no one could solve it Mrs W would go slowly over it. (Carrie P.)
CB Most of the time she puts the question up if we are doing it as a class then she asks people like questions, like what's the first step, and then she asks about the second step … (Carrie B.)
EB Well we'd have to set it out on a piece of paper, say what we, what the problem is, then like, what points are important, and how you would work it out, and then you work it out and then you get the answer. (Elisabeth B.)
AD Well we have problem-solving, and we have, in the back of our book what we do, what's the problem, then we work that out and then write it down and then, how do I solve it, and then we have a conclusion at the end. (Angharad D.)

[6] Michelle

The difference between Lyn and Michelle's explicit modelling of thinking strategies can be demonstrated by the responses from the students in each class. Lyn's class described the problem-solving process as something to be negotiated, and their teacher's model as one of many which are available in the class. In Michelle's class the students are clear about the set way of approaching a problem-solving task and all they have to do is follow the steps.

Revisit Concepts (3 1 3)

This aspect of the *Metacognition* Principle refers to the teacher's awareness of the students' need to backtrack and revise a concept or skill before they progress.

In the "typical lesson series," an average of 11% and 17% (see Figures 9b and 9a) of the coded incidents were rated as *Revisit Concepts* by the four raters for Lyn and Michelle respectively. The incidences identified for Lyn tended to relate to situations in which Lyn responded to a difficulty which the students needed to resolve before they could progress with the activity.

TL *she discusses what mathematical processes they'd be likely to need such as division and multiplication* (Students are to calculate the distance over time to travel in any way to their campsite)

TL *She demonstrated on the easel how to calculate "average" speeds. The students were gathered near the easel.*

TL *(Lyn) took Victoria's problem to be one of poor adding and built on that with the class.*

In contrast, incidences coded as *Revisit Concepts* for Michelle usually related to Michelle's carefully constructed summary sessions which she conducted at the end of each lesson. Michelle was most aware of the concepts students needed in order to do each day's work. She also timed her lessons sufficiently well to allow for a recapping component at the end of each lesson. This was also important for the students as Michelle regularly used homework to revisit a concept; for example:

TL *Michelle asked them if they remembered doing these last year. As they were streamed last year, most acknowledged they had been exposed to it. Michelle wanted to get on with more complex use of percentage but needed to back pedal until they had this part right.*

TU 62 *11.30- teacher going over problems from last Friday, students were reminded they were subtracting fractions*

TU 263 *recap at end of lessons getting them to see common themes and make use of knowledge of last question to make it easier to solve future problems* (Teacher is revising a three-step process for problem-solving)

Michelle felt that teaching of mathematics should follow a developmental approach, as this comment indicated:

ITI *Maths is one of those subjects that is very developmental and we need to build on existing understandings.*

For this reason Michelle would often revisit concepts when she felt that the next stage of concept development was dependent on certain prior knowledge.

Students Verbalise (3 1 4)

This aspect of metacognition refers to the technique a teacher uses of getting students to record or express their mathematical processing (Lyn—5%, Michelle—8%, see Figures 9b and 9a).

This subcategory has similarities in function to the *Explicit Modelling (3 1)* component of *Metacognition* discussed above. Examples coded as *Students Verbalise* could arise, for example, when a teacher asks a student to demonstrate her approach on the blackboard or to describe it to the whole class. The student's verbalisation in these situations is usually accompanied by teacher prompting.

Lyn frequently used pair and group work which meant students spent much of their time discussing how they were solving their problems. She encouraged the thinking through of problems "out loud" by having students demonstrate on the blackboard how they went about it. At the end of each lesson there was a sharing time where each group would be asked to report back. This was an opportunity for students to verbalise their different approaches and activities. This example has been taken from a lesson early in the year where she was establishing the lesson protocol for what would become routine (4.3.92).

LT *Lyn(4.3.92): (Addressing whole class). Right what I'd like you to do is to report back to us what you have been discussing in your groups. So in the next few minutes I'd like you to decide in your group- who your spokesperson's going to be to report back to the group, or are you going to have 2 or 3. Alright so that's your next 3 minutes and be ready to share with the whole group.*

The following field notes show the ways in which Lyn encouraged students to verbalise:

TU 87 ***Students used blackboard to work out and record most popular drinks and record it in own way*** (Lesson on tallying for class party catering 4.3.92)

TU 155 *Teacher asked students to describe the process they had gone through in long-hand* (Lesson on representation of ranking results from preferential voting 11.3.92)

TU 390-391 *Stephen had a unique way of adding and shared it with teacher, and found others had his same system* (Lesson on finding and comparing the average speeds for different modes of transport 13.5.92)

The students' awareness of using discussion or verbalising, as a way of helping them to solve mathematical problems was noted in the interview when they were asked how they went about solving problems:

ISI *We go into groups and then discuss all the matters. (William B.)*

ISI *Brainstorm it all first and then work it out from there. (Steven H.)*

ISI *Well some people argued with her [Mrs M][7]. (Mark S.)*

Michelle recognised the need to encourage students to verbalise their thinking, and to share ideas with each other. As this comment shows, it was an area in which she felt more could be achieved with her class.

ITI *We need to get children involved in the learning program and actually use their language to learn language ... you need a chance to bounce ideas off each other, you need a chance to talk. I find that, as I said before, that is something that we need to work on.*

An average of 8% of all incidences was identified as *3 1 4* for Michelle's class. The following field notes drew from observations made of lessons other than the "typical lesson series."

TU 65 *Teacher gets students to support their answer by reasoning* (Lesson on subtraction of fractions 28.1.92)

[7] Lyn

TU 195 *Students do a couple of problems on blackboard* (Lesson on subtraction of decimals 12.3.92)

TU 351 *Gets them to elaborate on the logic of their answer* (Lesson revising long multiplication 6.4.92)

When asked to respond to the statement "I use students to share the different ways they solve problems," Michelle replied *always*. Michelle also commented on how she would ask students to verbalise their mathematical processing after she had demonstrated on the blackboard:

PTI *Usually at the beginning before introducing the textbook or worksheet, I often use the blackboard, sort of a chalk and talk session, talking problems through and demonstrate what's going on in my head. I often ask the children to do the same with either the blackboard or easel to the class in the same way.*

However, the author did observe that the students who were used in the whole class verbalising were those who tended to support the teacher's idea with their example. The author reflected upon this in the field notes:

TL *About half the class seemed to be following her with their hands up. Linley and Gabrielle drifted in and out. Sue contributed often.*

TU 645 *This blackboard modelling is deceptive- it can look like joint construction and yet it really can only be directed teaching when the input from the students is only allowed when it suits the direction teacher is heading* (Lesson on multiplication and division of decimals 18.6.92)

The author discussed this with Michelle and she acknowledged that it was expedient for her to direct the whole class interaction towards the middle, rather than select a wider range of students.

ITI *I tend to cater more to the common problems I must admit, only through time. Having cross graded the children they tend to have very similar problems* *anyway, so I am catering to most of the children, the extremes of my class I tend not to, only through time, however I believe it's very important*

Further details regarding the students Michelle chose to give demonstrations to the others in the class can be by found in the lesson transcript in the *Scaffolding* section of this chapter. These transcripts suggest only a few students tended to be chosen to model for the class. Those encouraged were those whose approach seemed to be consistent with the main direction in which the lesson was heading.

Students Share (3 1 4 6)

This aspect of *Metacognition* refers to the opportunity students have to explore informally among themselves their mathematical processing (Lyn—11%, Michelle—6%, see Figures 9b and 9a).

Reference has already been made to the frequent use of groupwork which occurred in Lyn's mathematics lessons. Although she encouraged students to share in order to help one another, they were encouraged to not necessarily change their own opinion. These examples show how the groups operated to share ideas.

TL *Students shared at their tables the strategies they used to work out the average and in what ways it was the same or different from what had already been shared* (Lesson on finding and comparing the average speeds for different modes of transport)

TU 142 *encouraged cross group discussion* (Lesson where each group had decided which foods were their most popular and ultimately a class decision would be made about what to buy for the class party)

LT 4.3.92—*Planning the class party*
Lyn:- Just before we go (to recess) do you have any questions that you'd like to ask different groups? Group 1 would you like to ask anything? Or Reuven? (Lesson transcript 4.3.92- Planning the class party)

LT 3.7.92 *Lyn:- "If you want to keep it to whole groups not just your partner that's OK"*

LT 5.8.92 *Lyn:- "Talk to the person next to you."*

Although the students were seated in groups in Michelle's class the function was not fully maximised. Data pertaining to this form of grouping will be discussed in the *Community* section of this chapter. Students were frequently called upon to share their ideas or working out.

TU 47 *comparisons of answers* (Lesson where children are discussing different ways to solve a set mathematical problem)

TU 185 *new idea–gave students a task to give partner a word problem–and they give the information/ problem/ sum to their friend and then once both done it they do each others sums. teacher set boundary by saying she preferred area with subtraction of decimals.* (Lesson on subtraction of decimals where once the textbook examples had been done students practised on one another)

TU 420-421 *Difficult task for most students. Gabrielle H's table had difficulty. No table able to complete the task. Students not working in groups just at same table- points to a need to share/ cheat off others in problem-solving.* (Lesson on polygons where each group was given a different regular polygon and had to deduce the relationship between the sides and the angles).

Although Michelle valued student sharing and sat the students in grouped tables, she was aware that it was not fully maximised in her mathematics classroom:

ITI *Yes, I am really trying to work with more group learning. I am very aware that children need to talk about these and to become relaxed in their discussions and talk with each other and I am trying to develop that more in my room, it doesn't happen terribly well.*

This discussion suggests that the topics for group discussion were set by Michelle and that this was not negotiable. Possible consequences of restricting the direction of a lesson to the teacher's own agenda will be discussed in the *Community* section under the *Negotiation* subcategory. At this point the author would suggest that, when the focus is on trying to keep up with the teacher's agenda, students appeared to be less relaxed in the classroom.

Discover (3 1 4 3)

The idea behind this aspect of *Metacognition* is that students can come to an understanding about a mathematical idea through their own idiosyncratic approaches rather than through the imitation of the teachers' approach (Lyn—7%, Michelle—8%, see Figures 9b and 9a).

It is not suggested, however, that such learning be achieved independently and without a conducive learning environment. An incident was coded as *Discover* when a student's approach was recognised and/or encouraged. Such a category is the antithesis of teacher-directed approaches where the student tries to discover the teacher's set formula or protocol.

Lyn's main intention was for the students to find which approaches work best for each individual. The author observed very few examples where Lyn offered direct advice, preferring students to work through problems in their own way. The extract below shows Lyn discussing with a visiting class what her class had been working on and the individual discovery her students had made about certain Olympic Sports:

> ***Each of these children were given an olympic sport and they had to write down all the mathematical information that was needed if you were a competitor of that sport, if you were an organiser of that sport, if you were the designer of something, of the equipment or whatever of that sport. What mathematical knowledge did they need to know to be able to do it?***

TU 373 ***last night's homework on blackboard "Find out—where you see these speed limits? When you might use them?- 10 kph, 20 kph ... 110 kph. Any other speed limits?"***

The author also noticed Lyn withholding information from the students about a specific process so that rather than emphasising the solution, a range of ways of tackling the problem was encouraged. This example is taken from a lesson where the students were working out the average weight of the children sitting at their table:

TU 378 ***Teacher made the point that the response they gave was in order to share different strategies for working out an average, the right answer wasn't given too early***

In order to see if Michelle's practice had been influenced by discovery-learning models of education, the author discussed this with her during the Initial-Teacher Interview. *There are things like Rogerian approaches which involves getting kids to be more reflective, more responsible for their own learning and there are other approaches like discovery-learning. Is there any that you are aware of?* Her response was:

ITi *No specific models, but yes I tend to want the children to learn independently rather than rely on me to give them the knowledge. I think they need to* *discover for themselves particularly in maths, and that tends to run through my general program.*

Michelle expressed the difficulty she had in assisting students to discover which strategy to choose when solving a problem:

ITI *I find it incredibly difficult. How do you tell a child how to solve a problem? Although we go through many strategies and solve lots of problems, it's a hard thing to tell the children, to explain to them how to choose a strategy on how to solve a problem. I find that difficult.*

The author's impression was that Michelle's focus was on students' understanding a strategy so well that it could be called upon as a resource when needed. The following examples indicate how Michelle tended to move the whole class through a guided discovery of a particular method. While she did react positively to students who came up with alternatives or discovered the strategy prematurely, her key focus was the whole class understanding of the concept so that other subsequent ideas could be attached to this foundation.

LT (31.8.92) *Michelle: Yes girls do 21 in your books. I know we have done it on the board but it will give you a guide to start off with.*

TU 352 *encourages students to go back to example to look at what class has done* (Lesson on finding the percentage of 2- and 3- digit numbers)

TU 423 *Catherine discovered the term "septagon" rather than "heptagon" as the terminology for a 7-sided polygon. Teacher suggested classroom investigation for homework* (Lesson had been introducing different polygons with their specific terminology)

TU 473 *deduction of patterns being revealed by her controlled progression through each regular polygon, logic is obvious* (This lesson used a worksheet subsequent to that from some homework students had had difficulty with, on polygons progressing from 3- through to 10- sided figures)

When asked in the Initial-Teacher Interview about her use of discovery learning and direct instruction Michelle acknowledged that she did not always give the students the opportunity to discover their own strategies.

ITI *Direct instruction as I see it is exposing the children to the knowledge without giving them a chance to discover it themselves ... I use direct instruction in my case with a group of children with severe problems ... I just give them the information and then let them use it themselves after that.*

Michelle frequently made reference to the importance of adhering to a set curriculum over a certain timeframe. This came out in her Initial-Teacher Interview concerning how much responsibility she felt she could give students for their own learning

ITI *I don't tend to give the children a lot of leeway in that regard I tend to set the problems and I tend to set the curriculum, no there are very few instances where the children get to develop their own learning.*

The decision between a more teacher-directed approach as opposed to a more discovery one will depend on many factors. In Chapter 5 discussions will be directed to the ambivalence Michelle felt about following a syllabus in a timeframe which could not easily accommodate too much side-tracking for students to discover the same content on their own.

Integration (3 1 2)

The idea behind this aspect of *Metacognition* is that the more topics and subjects are integrated by the teacher the easier the student will find the connections between the concepts and are more likely to be able to transfer their knowledge into different contexts.

On the "typical lesson series," Lyn's total average percentage of incidences for integration was 3%. Although this suggests that she paid little attention to the idea of integration, Lyn's program, was in fact, organised around this and as many different subject areas as possible were integrated with them. There were several key themes a year and most lasted over a month or even a term. For example, planning a class party, the most efficient form of transport to get to the class camp site, the Mini Olympics, and so on.

Looking at Michelle's program it was possible to see how she integrated in other subjects especially language and social studies but her mathematics was separate because this was not her full-time regular class. Consequently her total for this sub-category was 1%. She discussed her attitude to integration in the Initial-Teacher Interview:

ITI *I would like to see maths being integrated in the general course program and that doesn't happen ... occasionally my maths would be integrated with my science for instance we would do a lot of measuring and investigative work, but I'm afraid it doesn't happen as much as I would like it to.*

Michelle did, however, have a mobile of mathematical terms in Italian (Text Unit 26), but this was not directly used with the class in this study.

Links Lessons (3 1 2 1)

This aspect of *Metacognition* is connected with the above section in that where a teacher may or may not integrate the mathematical content across subjects, they may link across lessons or topics. It was hoped that such linking between lessons may facilitate the student's ability to transfer the concepts and skills that they have learned in one context to another.

Due to the long timeframe allowed for each mathematical theme in Lyn's class, her students had the opportunity to build on concepts rather than revisit them. Lessons tended to flow onto one another with the students continuing on with their activity. Lyn would reinforce mathematical concepts, when necessary, with a worksheet. At times she would recap by way of focusing what they needed in order to get going with the day's work. Many field notes reflected this flow of lessons:

TL *Lyn then gave the students the task of calculating the distance over time to travel along any route they chose from the school at Guildford, to the campsite at Collie. She explained to them that they would be given school time to do this and that this would be what their maths was about this term.*

TU 204 *task builds onto previous experience* (Students are working out the best value for money after their supermarket visit, having had a previous lesson working out the best value for money from pamphlets and promotional material)

TU 395 *in previous lessons since I last visited students worked on scales with worksheets to reinforce this new concept* (Students need to become good at doing averages in order to work out most efficient form of transport and route to camp site)

TU 431 *recap on yesterday's lesson asking which is central or important information e.g. Hughes St or Lawsonville* (Students need to become adept at using timetables in order to work out most efficient form of transport and route to camp site)

Michelle tended to move from one topic one week to another the next. Linking lessons was apparent within these topics. Within each topic, Michelle would consciously construct a logical progression. If most of the students made an error, she constructed a worksheet to address that problem in specific terms:

TL *11.20a.m. Michelle introduced lesson by saying that they needed to get this work done before the end of term, which was in 2 weeks, and that it would be needed for next semester's work.*

TU 220 *9.30 am going back to party calculation of 8c from worksheet she'd developed* (Students had made errors in calculating the sharing of the $150.00 between the class when working on the class party activity I had set)

Because whole school ability grouping had been adopted, there was also the potential for linking content from the previous year with the next stage of development of that concept:

TL *Michelle asked them if they remembered doing these last year. As they were streamed last year, most acknowledged they had been exposed to it. Michelle wanted to get on with more complex use of percentage but needed to back pedal until they had this part right.*

TU 314 *when students say they've learnt something from last year then it is quite apparent that streaming assists the whole group progression of the concept*

Some students had difficulty when topics were changed frequently.

TU 214 *no obvious linking of new concepts to previous works, topics change without much signalling* (Lesson had begun with rounding decimals then progressed to percentages, then adding decimals, and then subtracting decimals about which students had homework set. Previous lessons had been on fractions)

TU 636 *students confused, especially those who had just had revision of decimal, also unrelated to the preceding part of the lesson* (Students doing problem-solving exercise from textbook)

The "sandwich" example discussed in the *Context* section of this chapter caused difficulties for the students for several reasons. Some of these reasons will be discussed in the *Critical Literacy* subcategory in this section. Several groups of students, however, were trying to calculate the average of the figures presented in this "sandwich" example problem. The difficulty the students were having may have arisen because they were trying to link what they had learnt in the previous week's work on averages to this new problem. This line of thinking was probably cued by the phrase "at this rate." The "sandwich" example problem required the students to suspend the mathematical concepts they had been developing in last week's work on averages because this week there was a new topic. Michelle was wanting them to deduce from the "sandwich" example problem that to draw up a table was a useful metacogitive strategy that could be applied in order to solve this problem. This strategy was in her program for that week. It was not necessary to relate what they had learned the previous week to this week's lesson.

The author shared her observations with Michelle and later in her Post-Teacher Interview she discussed the difficulties children may have had in changing topics:

PTI *Yeah that's common when you're changing topics, and the kids are drawing back on what you said yesterday, and thinking how does all this relate and how do we switch off from there to here. Um I think if there are many experiencing the same problem then you just address the whole class, but if its just a few then you just need to speak to them on their own- "Yesterday we were doing ... this is something totally different, you need to do that in this way, this is* *something that we do differently." It is a problem, but I think most kids get around it.*

Michelle believed that switching from topic to topic is a routine feature of schooling and that students need to adapt to it.

Shift From One Rep (3 1 2 2)

This aspect of *Metacognition* refers to the choice the teacher gives students to change the form in which they represent their information. By encouraging students to do this, they have an opportunity to transfer the knowledge they have gained in one context to a new context (Lyn—3%, Michelle—14%, see Figures 9b and 9a).

In Lyn's class minimal direction was given by the teacher about which representation should be used to display their work. Lyn offered the students latitude both in their mathematical processing and how they chose to present their information. At times she would suggest when to use graphing or a different way of counting but examples were limited.

TU 124 *teacher prompts need for grid/graph paper* (Recording raw data of preferences by each class group)

TU 126 *teacher offers suggestions of pie graph* (Students already know how to do a pie graph from a previous topic)

Lyn valued the students' use of different forms to represent their information, as this comment from her Initial-Teacher Interview shows:

ITI *They'll have different ways of presenting different ideas, and then they discuss things with their peers ... they use the 'dye' material where you actually give them lots of 3-D type things to go and make and then transpose onto pieces of paper.*

The examples from Lyn's classroom show her assisting the students in order to refine the way they present their mathematical information.

Michelle was very conscious of the effect using a particular representation of a concept might have on the student's ability to process that concept (see her comments under the metacognition component of *Materials* in this section). Michelle would select a concrete example when introducing a concept, whenever possible, particularly as she stated, to support the textbook. There was, however, very little opportunity for students to present their information in any alternative representation. Most work was pen-and-paper oriented, unless the topic was based on materials, such as, polygons or tessellations.

TL *Michelle demonstrated the fraction idea with magnetic units.* (Lesson showing percentage represented as a fraction)

AL *She set them the task to draw a diagram to show it had been cut into equal pieces.*

This section from a lesson transcript shows the way Michelle tried to shift the students' representation of their information toward a more mathematically precise form.

LT (31 8 92) *Michelle:- I want somebody to come up to me and demonstrate two and two thirds using those three squares. Think you can do it? Come and show me. (Elisabeth draws on the board) OK stay there. Elisabeth's drawn or coloured in two whole squares and a part of the third square. But how many thirds are there? How many thirds has she coloured in? Elisabeth can you make your diagram more clear so that you can show exactly how many thirds you have coloured in?*

The shifting from one representation to another which occurs in Michelle's classroom is designed to assist the students to grasp certain mathematical protocols.

Maths Transfer (3 2)

This aspect of the *Metacognition* Principle refers to the teacher's overt connection about what is being taught in the classroom to the world in which the student lives or may in the future find him/herself (Lyn—12%, Michelle—4%, see Figures 9b and 9a).

The broad thematic topics dealt with by Lyn had obvious relevance to the real world. These topics included, for example, planning a class party, efficient transport to the class campsite and designs and layouts of Olympic sporting equipment and game rules. Lyn frequently referred to how the students would use what they were doing in real life:

TU 32 *Lyn describes what they're doing like setting up a business* (A cooking lesson producing packets of savoury nuts to sell to the rest of the school)

TU 169 *real life tasks that need mathematical skills to solve* (Recording of preferential voting)

TU 201 *tasks presented as if it was an adult, legitimate responsible use for mathematical skills* (Different groups of students visit three supermarkets and compare value for money on potential items needed for class party)

TU 347 *Students metacognitively encouraged to reflect on the process and validity to real life* (Students had to present class party procedure as a project and write what it was they had learned that would be useful for life)

TU 387 *Lyn discusses where problems could occur for them in the future* (Lyn refers to other situations in life where it would be useful to be able to work out averages)

TU 407 *Lyn makes explicit why they would need to know this info* (Lesson looking at various forms of timetables and how to read them in order to work out a train or bus trip to their camp site)

Michelle's class was also asked to dedicate a lesson for planning a class party. The teacher was most reluctant to develop the class party idea over any greater period of time than a single lesson on the grounds that it would take too much time away from the curriculum and that it was difficult enough to get through the content as it was:

PTI *I did feel very constrained to the syllabus to a very large extent ... it's important to follow and complete the year's work, but also try to integrate it with the children's interest and everyday activities.*

Michelle clearly saw a tension between the constraints of the syllabus and her wish to integrate syllabus content with the children's interests. In most of the lessons observed during this study, covering syllabus content appeared to be the higher priority. This observation was made of a lesson on fractions:

TU 152 *No real world application made explicit except in a token way in textbook showing oranges, even when the reality of real world problem which required this sharing of fractions of oranges was not even referred to in passing.*

Towards the end of a series of lessons on fractions, the author discussed with Michelle that many of the students were still having difficulty, and that suggested that a real world example might be helpful to the students. Prior to the completion of the final lesson in that series, Michelle discussed the real life function of what they had been doing in the class (Lesson transcript 11.8.92):

Michelle: Why on earth are we bothering to have a look at equivalent fractions? Why do we do it? Do you remember when we were talking about decimals we said where in the real world would we ever use decimal numbers, and we talked about accountants and we talked about calculator work and all of that. Where do we use equivalent fractions? Do we ever use it outside the classroom? Where Rebecca?

Rebecca: Well fractions can be changed into decimals so it goes back to when we were talking about decimals.

Michelle:Oh good girl. How can a fraction be turned into decimals?

Rebecca: Forgotten

Michelle:If we looked at the fraction three quarters it can be divided, converted into a decimal number. Because this number means? I'm sorry this line here means a … ? Divided by sign. Three divided by four and if you did that on your calculator you would come up with a decimal number that equals three quarters. Georgia?

Georgia: Like last year I think it was we had a pizza and my brother ate more well, than us. My sister and I could only have one so we wanted to divided it into three fifths and he could get three and we'd get the other two …

Michelle:So you divided the pizza into fifths, he had three and you had two. So you have used a fraction good girl.

Child: If you had half a block of chocolate and you had you and your friend and you had a quarter each … for chocolates and things so you would sort of …

Michelle:So you know that two quarters is equivalent to?

Child: One half.

Michelle:One half good girl. Any other examples of equivalent fractions? Carrie?

Carrie P:When like, when you're in a supermarket and like you said when you change it to decimals, you can sometimes like use approximation in it like, once you've changed it into decimals? Actually as its a fraction its being a fraction?

Michelle:So you're converting your fractions to decimals again. We're actually doing that in a couple of weeks time so you're well ahead. Angharad?

Angharad:Like when you're measuring something when you're cooking. And you might have a cup of something and it doesn't have to be the things that you need so you can change it.

Michelle:Mm I wonder if I said to you, say for instance if you got the recipe off the back of a sugar packet and it said for you to use one quarter of this packet and there were twenty- five grams in that packet, how many grams are you going to use? If it said to you to use a quarter of a packet of sugar and there were ... I'll make it easier there were 24 grams in that packet, which is highly unlikely. How many grams ... what fraction of that packet are you going to use?

In spite of her infrequent reference to mathematics in relation to real life Michelle exercised quite a different role when presenting mathematical challenges in other situations. Once every two weeks, Michelle was responsible for "House Maths" which was a cross-age and non-ability-grouped lesson, which saw the whole school using mathematics for a variety of real-life situations as a fun activity time rather than as formal mathematical instruction. Her comments reveal how her regular classroom operated:

TU 766 *Teacher said tomorrow they'll have to use the opposite materials to what they used today (i.e. she is saying that they have to take from the books and translate it into real life rather than start from the real life)*

Materials (3 3)

This aspect of *Metacognition* refers to the use of materials, either from real life or commercially produced, which will enhance the student's ability to grasp a skill or concept (Lyn—8%, Michelle—3%, see Figures 9b and 9a).

Lyn's materials were predominantly those from real life, for example, timetables, newspapers, voting cards etc. Lyn rarely used worksheets which outlined for students the steps they should take. She tended to list on the blackboard aspects which the students would need to address when presenting their work, or she would put up posters of points they may need to consider. When discussing with Lyn about materials in the Post-Teacher Interview she acknowledged the difficulty of using materials in the upper grades:

PTI *Not as many materials probably as I would like to use, for a couple of reasons, because the children have to be re-educated into "its OK to use materials," and they're not a baby because they are still using materials, because, the two grades which they come through where concrete materials are not a feature, so you have that disadvantage before you. And we probably don't have as much materials on hand readily available, so its up to the teacher then to keep finding those each time, and that's where I would fall down, in not getting as much, but wherever possible I try to use concrete materials.*

The author observed confusion on the part of students on several occasions because of belated directions in how to use the materials. Because Lyn allowed the students to shape where a topic may lead it was not always possible to think through the directions a topic may need ahead of the students' use of the materials. Lyn was clear that the materials used should be hands on and was critical of teachers who taught straight from the textbook as this comment from her Initial-Teacher Interview shows:

ITI *I feel very concerned that we just do- that. Many teachers just do exercises out of books, not following the same sort of themes, but just doing one example after another, after another. Children see success regularly or some may see constant failure all the time.*

Michelle, in her Initial-Teacher Interview, talked about problems associated with the need for textbooks to be supplemented by other materials.

ITI *Yes we like to use a lot of concrete materials, the book doesn't tend to allow for that as much as I would like it to ... I am trying to relate the page in the* *text to a concrete exercise or activity that children can do to reinforce that.*

Michelle often used the textbook for protocol ways to approach a problem.

For example:

TU 171 *p. 42 (Addison Wesley- purple) teacher- "Let's look at a word problem and get the maths out of it"*

At times the maths textbook caused problems. For example:

TU 366 *Textbooks don't always model the way a teacher is recommending, nor exactly matches the way a lesson has evolved after the teacher has planned for its use.* [Michelle has been teaching percentage, and used the following example **In a class of 25 students 24% of the students play a musical instrument** However when the textbook was used to provide examples for students to try and practice the process, the questions were written in a more complex manner e.g. 8.2% of 200]

This became particularly problematic when homework was set from these texts and was clearly unsuitable.

TU 110 *Addison Wesley [p 301 questions 21-24] good examples of poor support for the specific skill the students were having trouble with i.e. example was oranges and not neat squares, also example was halves, a simpler concept than thirds, also exemplar in book showed a rule* **"multiply the whole number by the denominator- add the numerator to the product- write the sum over the denominator"** *which teacher had not emphasised, the questions required 100'ths, 10'ths, 5'ths, i.e. only familiar one was 6'ths*

Michelle mentioned this difficulty with unsuitability of texts when discussing the "typical lesson series" in the Post-Teacher Interview:

PTI *That particular text did become extremely complex, and tended to put the children off because they start off very confident, the first few examples are easy and then suddenly they've got these enormous numbers to tackle.*

Michelle commented in her Initial-Teacher Interview that the textbook would be a weakness if the teacher were too dependent upon it:

ITI *I think a teacher needs to be aware that children have to use concrete materials and have to get in there and do the maths and cannot learn from the textbooks. So I think the ideal maths teacher is someone that goes for the understanding rather than covering the content.*

It would appear that both teachers experienced dilemmas when using materials at the upper primary level. Even though Lyn uses realia there is still a perception by some students that this is not acceptable. Michelle experienced the dilemma of trying to coordinate materials with a textbook which did not always present the information in a form that suited her.

Concrete to Abstract (3 3 1)

This aspect of *Metacognition* is a subcategory of the *Materials* category discussed above. It refers to the conscious introduction of concrete materials by the teacher in order to lead a student towards more abstract understandings (Lyn—2%, Michelle—6%, see Figures 9b and 9a).

As discussed above, Lyn's teaching centred on the use of real world tasks in the mathematics classroom. A high use was therefore made of concrete materials which were built into the real world context (e.g. timetables, pamphlets, brochures). Lyn discussed the role of concrete materials in her Initial-Teacher Interview:

ITI ... *we need to do a lot more with concrete materials right from the beginning and have lots more 'hands on, make, do' type activities.*

In the event that the students were struggling with abstract processing she would intervene and reinforce with a concrete aid.

TU 131 *Teacher prompts that they would need to calculate foods first and then do graph* (Students would need to go back to raw data sheets of the survey showing the tally records for each food before translating the information into graph form)

In Michelle's class each new concept was introduced with a concrete aid where possible. Michelle's knowledge of concept development and her teaching experience meant she had a range to draw upon. Sometimes she used the textbook to illustrate the example before providing exercises on the concept.

TU 150 *use made of concrete materials to reinforce concept* (Equivalent fractions were being taught with students colouring in squares)

TU 175 *Teacher encouraged students to see pattern and used example of a block of chocolate to reveal trading.* (Lesson on subtracting decimals)

TU 168 *Textbook p. 40 (purple Addison Wesley) got Linley to read* **"Adding Decimals. Jason visited some European countries. He has 1 British pound note, 1 German mark, and 1 Greek drachma. What is their total value in U.S. dollars?"** (Lesson on percent)

For both teachers the introduction of concrete materials was made in order to assist students to understand a more abstract form of the concept. The material, whether, it be realia, as in Lyn's class, or a diagram, as in one of Michelle's examples, could be regarded as a crutch for some students helping them to focus their thinking.

Open-Ended (3 3 4)

This aspect of *Metacognition* refers to the kinds of topics or mathematical contexts which were considered to be more likely to encourage students to experiment with various thinking strategies (Lyn—1%, Michelle—0%, see Figures 9b and 9a).

In Lyn's mathematics class the students were generally involved in researching an area which was quite open-ended and was based on their choice of what interested them within a given theme. This excerpt from one of the lessons concerned with planning a class party illustrates the latitude given to the students. The class has already discussed in groups the kinds of games, decorations, food and type of party they wanted. Lyn attempted to raise the students' awareness of budgeting, using the allocated $150.

LT 4.3.92 *Lyn: What I really want to focus on, not so much the games, not so much on the equipment, because I really want to focus on your $150. So that's why I've set you the task where you've collected these pieces of paper with your items of food (Students have pasted on A-3 cartridge paper food prices brought from home, collected from local junk mail and promotional material). Each group has got that. So this will give you an idea of the prices and how much things cost ... (She then puts up posters on the blackboard to guide their selection) So as I send you off to look at your food you need to consider 1. Organising a balanced menu. 2. Calculate what amount is required to feed the class. 3. Use ads to calculate costs of items.*

Other major open-ended topics observed in Lyn's class during the study were calculating the most efficient form of transport to the class camp site and selecting an Olympic sport which could have an aspect of it represented mathematically.

As Michelle discussed in her interviews, she tended to be concerned about fulfilling her responsibility of covering the set syllabus in the given time frame so her lessons rarely incorporated much choice, or encouraged too many opportunities for children to diverge away from the main lesson focus.

PTI *You are getting through the syllabus as well. You're not getting side-tracked and concentrating on one area rather than covering, doing an overall scan of the syllabus.*

Michelle tended to give open-ended tasks to her mathematics class within the mathematical routine of the concept they were learning. For example in these lesson excerpts the students had the freedom to explore different ways of tackling the given task.

LT (31.8.92) *You do circles, boxes, anything you want to show. First five please in diagramatic form. Off you go. Use some colour to demonstrate.* (Lesson on fractions)

LT 12.3.92 *Make sure the sum is one where you need to borrow from other columns. Check on the calculators. make them interesting.* (Students are designing a problem for their partner to solve during a lesson on subtracting decimals)

TU 265 *"How many ways can you use 10c, 5c, 2c and 1c to make 18c?"* (Lesson prompted by students' inability to answer my question how could the class party money of $150 be divided up among the class evenly using notes and coins.)

It would appear that although Michelle knew how to generate open-ended tasks she felt under obligation to keep the choices fairly closed in order to complete the syllabus.

Critical Numeracy (3 4)

This subcategory of *Metacognition* refers to those text units which could be linked to a student's use of thinking strategies to assess the reasonableness of the answer when considering the circumstances of the context (Lyn—13%, Michelle—3%, see Figures 9b and 9a).

Because Lyn used real-life problems there was always a need for the students to reflect on the reasonableness of their actions. This brought about many opportunities to exercise critical numeracy 13%, as can be gauged from Figures 9a and 9b a summary of NUDIST categories for *Metacognition* classified by four raters for Lyn and Michelle, on the "typical lesson series."

An example of Lyn developing critical numeracy skills was identified when students were planning the catering for the party as this field note records:

TU 180 *Students have to find best value per weight and amount for party*

Students were calculating how much more of the ingredients for a recipe would be needed if they were to cater for a staff function with chocolate cake. Lyn reminded them of the need to be more critical of their calculations:

TU 51 *Lyn aware of estimation differences in their answers "I could make a chocolate cake out of all the left-overs!"*

Lyn demonstrated a broad understanding of the value of the students using their mathematical knowledge to challenge ideas or opinions. This example comes from her Post-Teacher Interview in which she discussed how she used the camp to develop the notion of critical numeracy.

PTI *The direction, the understanding of the exact direction of whether we were going north, the exact location of Collie, not just that its down there, but exactly where it is down there. So I felt that they actually, in the planning of the route, gave them a deeper understanding of where they were actually going, how long it takes, and what's involved in going there to the camp... In doing this when we actually left, for the camp the children were aware of* *the camp, of where it was, of the quick‹ ' way of going and the most economical- money-wise for children.*

One skill which can be used in critical literacy is to estimate or visualise the answer to a question before actually doing the calculation. Lyn's answer to the Post-Teacher Interview question *"Do you ask the students to estimate or visualise their answers to mathematics questions?"* revealed a lack of confidence in this function of mathematics pedagogy:

PTI *Um, yes, but probably the estimates and the visualising, not as good as I would say to them when I am actually doing in a language lesson. I would consciously now, thinking about it, I probably don't say to "estimate", because I don't know that children, the children that we get would have perhaps done enough of that estimation work to really understand it, and maybe that's something right at the beginning of the year when you get your children to look at, estimation strategies, and to visualise. But for children who have seen maths as right or wrong you don't visualise much, they just know it. However they are areas that I'm not sure.*

(re-What effects do you see this having?) I can see the effects, the positive effects if you actually did allow them, to have a guess at what they think it's going to be. But in doing that they would still, I wonder if they have the strategies to go about it. Because I don't know that we've actually, coming through the younger grades, particularly the group I had, whether they would have had those skills. So I don't know.

Lyn expressed her concern for the lower ability children. She has concerns that these children don't have the basic operational skills they need to be critically numerate:

ITI ... *to be really honest I don't know that I achieve a lot, I try to help those children as much as I can. What I've done this year [Initial-Teacher Interview prior to class in study] is a lot of work with the calculator with the estimating first what they think the answer's going to be, looking at the problems, is it addition, subtraction, division, multiplication then to use the calculator as a tool that* *they just use all the time, but knowing that they have to have some idea of what the answer's going to be.*

In the Post-Teacher Interview, Michelle had this to say about estimation skills.

PTI *Using calculators, they need estimation skills. You can easily press a wrong button and come out with some sort of outrageous answer, that doesn't make any sense. So they need to have some sort of estimation skills and even when they are not using a calculator, come up with an answer they need to know that its a sensible solution, so if they are not estimating they might not know if its sensible. When children estimate or visualise they are assessing whether they have come to a reasonable or a sensible solution and it requires them to actually draw upon their understanding of the concept, to realise if its a sensible solution.*

Michelle's comments display a confidence in her understanding of the need for students to estimate before they actually calculate. The "sandwich" example discussed in the *Context* section of this chapter, indicates that it was not always possible to apply critical numeracy skills to the textbook or worksheet simulations of real life in Michelle's class. The total of sandwiches that could be made in that problem was 35 by Kathy in the last 15 minute period and 32 by Jill. During the classroom observation where this problem was used, the author noted several groups of students discontented by the improbability of their results (even though they were correct). In other words attempts to apply critical numeracy skills were frustrating because of the problem's unreasonable answers.

This raises issues about the kinds of problems which were used in class. However on another occasion (6.4.92) Michelle complained to the author that the students' answers were incorrect, largely because they hadn't checked on the reasonableness of their answers. Although Michelle valued critical literacy, there were no recorded examples from Michelle's "typical lesson series" sample, however there were other recorded examples taken from the field notes. In one lesson on decimals using a textbook example showing different exchange rates for international currency, Michelle asked the class `Who would be the richer?` (Lesson on percentage—12.3.92.)

On other occasions Michelle stressed the need to be systematic and when getting students to work out the combinations of using a 5c, 2c, and 1c coin she asked `How do you know when you have all the combinations?` (Lesson began with students systematically sharing $150 between each of the 26 class members.)

It would appear from these examples that students may have been receiving mixed messages in Michelle's class about when to apply their critical thinking—not all tasks were given to this form of analysis.

Feedback (3 5)

This aspect of *Metacognition* refers to the type of feedback a teacher gives a student which encourages them to be metacognitive and develop their thinking skills (Lyn—5%, Michelle—7%, see Figures 9b and 9a).

As can be seen from the three lesson samples (TL, ML and AL), Lyn spent a lot of time going from group to group encouraging them to be aware of their processing. Here are two examples, the first is a food bill which needed to be checked when Lyn encouraged students to reflect on their efforts at designing different ways to solve a problem. The second example is where students are looking over the expenses of running their small business and Lyn's questioning provides feedback to the students in an attempt to raise their awareness about matters that they need to take into consideration regarding calculating the costs per item:

TU 63 ***Students rewarded for systematic approach and recording even if not accurate answers***

TU 125 ***Teacher asks for suggestions and builds—seeks clarification, seeks more information*** (Lesson with students using graph paper to record data)

Lyn rarely assigned homework which would then need to be marked. Rather, units of work which involved research projects were set. Quite comprehensive feedback was given at the end of each unit of work responding to the process the students had gone through when doing the task, and their awareness of the experience for themselves as learners. Whole class feedback occurred as a routine part of each lesson where groups shared what they had been doing. Lyn would verbally monitor and respond to their statements, encouraging them to articulate their thinking.

In Michelle's class she would set the students on a task and then move around the room checking on any students having difficulty. Consequently the figure of 7% (see Figure 9a) taken from the "typical lesson series" alone, does not reflect what the author observed to be her practice on other occasions. In terms of whole class feedback Michelle would draw the whole class' attention to it. This example, shows how Michelle drew the class together once

TU 196 *where there was a specific feature teacher talked through the problem* (Some students were having problems with borrowing in subtraction of decimals. The whole class was asked to look at the blackboard)

With regard to their behaviour in her class, Michelle's feedback to the students, both written and verbal, was such that the students clearly knew where they stood:

TU 35 *clear expectations reinforced by behaviour and product and time reminders* (Observation made after reading feedback in workbooks e.g. *"Elizabeth rushes her work to finish rather than complete quality work"*)

TU 37 *reinforces class rules—teacher concern for "chatty" classroom although on task*

Michelle's feedback encouraged students to develop a pattern to the way they thought through a mathematical task rather than merely ad hoc guessing:

TU 273 *being logical has high value and working systematically* (Teacher recapped at the end of the lesson 16.3.92, by getting them to see the common patterns so they would make use of the knowledge they had to make it easier to solve the subsequent ones)

TU 230-232 *Reward has been for system not just hit and miss, comments on the students' work which reflect this:- "You are working to a system. Now continue!", "Can you be more systematic?" "Are you sure that all combinations are being included?" "There are many more than this- can you be more systematic?" "A good try. Can you be more systematic?" "A good start" "Keep going." "How do you know when you have all the combinations?"*

Both teachers gave feedback which not only referred specifically to ways in which the students could improve the way they approached mathematical tasks, but to how they were expected to behave in the classroom.

Frameworks (3 5 5)

This subcategory of *Metacognition* refers to the structure that is made available for students to use when the teacher is not there. These structures can be in the form of a protocol in their books or on display (Lyn—3%, Michelle—1%, see Figures 9b and 9a).

Lyn's charts were sometimes identified as performing the function of a framework. Lyn displayed a range of student work which meant that it was readily accessible by the class as a reference resource. Even after a topic had been completed the information was often laminated and bound. Some of the books and signs noted were:

TU 6 *% posters- definition of % as well as examples of its use*

TU 8 *Big books made by class also maths e.g. book of 14, 100, 1000 etc*

TU 90 *Also on blackboard the following signs:- Facts to consider- Purchasing of food, Party location, Decorations, Games, Utensils*

The above signs were actually generated from ideas the students had brainstormed during discussion in small groups as this transcript indicates:

LT 4.3.92 *Lyn: The headings. First of all [there] was this one- Decorations (holds up poster). This is what some people mentioned. Not many in the decorations, maybe you didn't think of it, I'm not sure. But today you're going to extend it. All I've got are balloons, party poppers, silly string, streamers.) If I didn't include some of these things perhaps I didn't really understand what you really meant- perhaps it was not relevant to a swimming party.*

These frameworks were not in any way mandatory, but were established to assist students to process or present their information.

Michelle quite explicitly set up frameworks and programmed certain strategies in order to teach, and continue to give, students practice in applying these in a variety of circumstances. In her programming file listed the following strategies which she would plan to during the year:-

1. Guess and Check.
2. Draw a Picture.
3. Make an Organised List.
4. Make a Table.
5. Work Backwards.
6. Look for a Pattern.
7. Use Logical Reasoning.

The children had in the back of their books a checklist for problem-solving.

1. What is the problem?—state the problem in your own words.
2. Solving the problem—all the working out.
3. Solution—sentence. (Polya, 1981)

In their text book there was also a similar model (Addison Wesley p.T8)

1. Understand the question,
2. Find the needed data,
3. Plan what to do,
4. Find the answer,
5. Check back.

Her emphasis on these structures or frameworks was also evident in Michelle's classroom discourse as these field notes recorded:

TU 230-232 *Teacher stressed they had actually been doing a double check, which was therefore a system. Reward has been for system not just hit and miss, comments on the students' work which reflect this: "You are working to a system. Now continue!", "Can you be more systematic?" "Are you sure that all combinations are being included?" "There are many more than this- can you be more systematic?" "A good try. Can you be more systematic?" "A good start" "Keep going." "How do you know when you have all the combinations?"*

As the lesson transcripts on the 31.8.92 and 11.8.92 used in this section and the lesson samples (TL, ML, AL, see Appendices 1, 2 & 3) illustrate, Michelle's could be interpreted as a cohesively directed lesson. While these lessons do not in themselves conform to a direct instruction approach, they do involve a high degree of teacher direction. These comments from Michelle's Initial-Teacher Interview indicated a more negative view of a direct instruction approach than her practice would suggest:

ITI *Direct instruction as I see it is exposing the children to the knowledge without giving them the chance to discover it themselves, rather than teacher taught ... I think there is a place for direct instructions depending on the type of children you're teaching. The more capable child probably wouldn't learn as well with direct instructions. They need more investigative work problem-solving work but there are some children that you just need to sit down and say look this is it, this is the way you do it, off you go because they are not going to learn it any other way.*

There were many occasions when the field notes made reference to the logic of the lesson not necessarily being followed by many of the students e.g.

TU 323 *problem of idiosyncracity of how each processes difficult when teacher's cohesion drives that logic* (Comment made because I had observed up to half the class at a time not following (TU-311) Michelle's way for changing a fraction to .../100 for working as a percentage)

TU 355-8 *(book seems to offer a lot of variety but gets complex very quickly) Students don't seem to understand the reason for multiplying or why the number % means two decimal places. This showed in problem—8.5% of 200—where pattern became more complex. They're just learning the pattern of it—no real knowledge of why. Teacher pays token emphasis on where students are coming from without really listening to and building on their concepts, unless it fits with her schema/planned lesson*

TU 477 *problem because of time constraints, pressure to move along and bypass students' pace*

TU 277 *very coherently structured lesson gives the impression of logic and therefore should be easily understood by students which often is only a surface feature.*

Michelle herself on several occasions at the end of a lesson would have an aside with the author about the students who quite clearly had not grasped the particular concept. On one occasion (6.4.92), Michelle gave the author a note attached to some homework the class had done, after she had taught a percent lesson which the author had observed. It read:

Many made errors with decimal places i.e. 20% of 66 = 132 (not 13.2) These girls are not checking the "reasonableness" of their answer.

One interpretation of the observations is that many of the students had to work out Michelle's expectations, and how and when to apply the framework she seemed to be imposing on them. What was clear, however, to observer and teacher alike, was that not all students were able to follow her approach. Although observations suggest that Michelle gave a highly logical presentation, nevertheless some students were not able to approach the problem in the same "logical" way as the teacher. They had used an idiosyncratic approach which they had worked out for themselves.

Joint Construction (3 5 4)

This subcategory of *Metacognition* refers to the teacher using the whole class or a group to jointly negotiate meanings, rather than rely on teacher- or individual student-constructions. (Lyn-3% and Michelle-4%, see Figures 9b and 9a)

Although these percentages of *Joint Constructions* (out of the total components and subcategories identified by the four raters) were similar for the two teachers, the two teachers differed in the ways in which these joint constructions were negotiated in class. For example, Lyn tended to involve students rather than provide a total demonstration herself as these two examples from the field notes show:

TU 85 *Lyn tallied it on blackboard while one student did recording*

TU 360 *Teacher and students together decided 1cm= 7.5 k*

At times Lyn's assistance came too late:

TU 135 *Teacher recapped and put major question on board although by that stage the students had had it confirmed by her group discussion and didn't need the prompt*

What does become obvious is the high number of student-teacher interactions during these joint construction sessions. Table 1a (see end of section in this chapter) illustrates the number and distributions which occurred during 7 lesson observations.

As in the "typical lesson series", the March (ML) and August (AL) lesson samples, Michelle appeared to jointly work through problems quite regularly. This was especially noticeable at the beginning of a new formula or concept with a pattern. As mentioned in the preceeding discussion on *Frameworks*, observations suggested that the framework was one that seemed to be imposed upon the students. Michelle's task seemed to be concerned with the logic of her approach, rather than jointly constructing a shared meaning, built upon the meanings as they arose.

TL *Students were given 14/50= ?/100 and were asked to make equivalent fractions. She asked the students to offer a fraction which the whole class could solve as an equivalent fraction .../100. Michelle selected those problems which demonstrate equivalence easily.*

On one level it could appear very efficient as this field note could be interpreted:

TU 606 *Teacher goes over diagram and sees if they can deduce other patterns e.g. difference between square no's*

In the field notes for several of Michelle's lessons, the author observed that the students seemed poised waiting to guess what was in their teacher's head rather than pursue their own understandings. The 31.8.92 lesson transcript below illustrates this. The following excerpt from this lesson, shows Georgia's confusion over a lesson on fractions. This lesson was an additional revision one on the topic because the class had appeared confused after a similar lesson a week earlier.

LT 31.8.92 *Georgia:- Um I forgot how to do mixed numbers.*
Michelle:- Exactly what we've just done. Two thirds? We've drawn two whole cakes if you like and two thirds of a cake.
Georgia:- Do you divide it then, or?
Michelle:- Sorry?
Georgia:- Divide it then or...?
Michelle:- OK let's just do one more. Let's do the first one together in fact. It says two and three quarters. (Directed to whole class) Could you look at the board rather than your book, otherwise you're going to get very confused. Two and three quarters- equals how many quarters we don't know. We're going to draw a diagram to find out, alright? We'll use um (pause) cakes.

Rectangular cakes. There's one cake. How many cakes will I need to draw for..
Child:- Three
Michelle:- Three OK. Georgia what are we looking for? What do we have to divide our cakes into?

One concern recorded in the field notes was that Michelle's "joint" construction seemed to be more about encouraging the students to adopt her method than trying their own approaches. This extract is taken from the March Lesson (see Appendix 2).

ML *She discussed different methods for doing the problem and settled on the method: 80/100 X 41/1. She used what they already knew e.g. 50% of 21 would be (201/2). She suggested that they approximate and had them think in their own mind that if half way between 41 was 201/2, then 80% would be more than half. Michelle instructed the students on how to work this out on their calculator. When asked to estimate first, the majority of students found it hard.*

The lesson transcripts of both teachers referred to in the *Scaffolding* section in this chapter show the difference between how each teacher attempted to jointly construct meaning in the classroom. Tables 1a and 1b illustrate the observation that, in Lyn's lessons a greater number of students contribute to the class discussions than observed in Michelle's lessons. This is not to say that the others were not engaged in the process, but they did not engage verbally in the interaction. Neither does it indicate the one-to-one visiting each teacher made to individuals and groups, nor the small group discussions where a teacher would bring a group onto the floor at the front.

Table 1a

Student-Teacher Interactions in Whole Class Discussions: Lyn (taken from field notes class maps from 7 lessons)

	Lesson Observations						
Students	4 Mar	11 Mar	5 May	13 May	10 Jun	30 Jul	5 Aug
Ben B	√√	√	√	√	√√√		
Will B	√	√			√	√√	√√
Ben C		√		√	√√	√	√
Olivia D	√		√√	√	√√√		√√
Ben E		√		√	√		√
Louise G			√		√	√	√
Chris G		√	√		√		
Reuven G	√√	√√	√	√√		√√√	√
Stephen H	√√√	√√	√√√	√√	√√	√√√	√√√
Dean J	√√	√	√	√		√	
Matthew J	√√	√	√	√		√	
Melanie K	√√√	√√	√√	√√	√√√	√	√√
Aaron K					√	√√	√
Melissa L	√		√√	√√		√	√
Stuart M	√	√√√	√	√	√√	√	√
Carmen O	√		√	√	√	√√	√
Oliver P	√√		√√			√	√
Victoria P	√√√	√√√	√√	√√√	√	√	√√
Blake S	√√√		√	√	√	√√	√√
Brenda S	√√	√√	√√	√√	√	√√√	√
Mark S			√		√	√	√
Paul S	√√√		√√√	√√√	√	√√√√	√
Paul T	√√√		√	√		√	√
Joe W	√	√			√		√
Tim W	√√√	√		√	√√	√	
Chris W			√	√	√	√	
Student interactions	19	15	20	19	20	21	20
Total interactions	39	23	30	28	30	34	27

Table 1b

Student-Teacher Interactions in Whole Class Discussions: Michelle (taken from field notes class maps from 7 lessons)

Students	28 Jan	16 Mar	6 Apr	3 Jun	9 Jun	18 Jun	10 Jul
	Lesson Observations						
Carrie B		√		√√√	√		
Elizabeth B	√√	√		√		√√√	
Genevieve C	√						
Rebecca C	√√	√				√√√	
Gabrielle C	√√	√√					
Emma C		√√		√			
Angharad D	√	√√		√			
Susan G		√√√√	√				
Gabrielle H				√		√√	
Emily K			√√		√√		√√√
Olivia K	√√			√√	√		
Carla L	√	√√			√√		√
Amy M		√					
Elizabeth M				√√	√	√	√√
Angela M		√			√	√√	√
Georgia M	√√√	√√√	√	√√√√	√√√√	√	√√√
Adeline M							
Naomi M		√√					
Linda N		√	√			√√	
Carrie P			√		√√√√		
Catherine R	√√	√√		√√			
Rebecca S	√√	√			√		
Lynleigh S	√		√	√√	√		
Kylene T		√√√		√√		√√	
Chantelle V	√	√	√	√√√√			
Melissa W	√	√√	√√				
Rebecca W	√√						
Alana W				√	√√	√√	
Leanne T	√						
Student interactions	15	18	8	13	11	9	5
Total interactions	24	32	10	26	20	18	10

Summary

The idea behind this Principle of *Metacognition* is the making explicit the use of thinking strategies and thinking aloud discourse in the classroom. While each teacher stated that she valued the development of metacognitive processing in each of their students their classroom practice reflected different methodologies. Michelle was explicit about the ways in which she encouraged the students to use thinking strategies. Not only did she ask them to record problem-solving steps in their workbooks, but also by consciously including the teaching and practice of each of the following metacognitive strategies into her mathematics classes: Guess and Check, Draw a Picture, Make an Organised List, Make a Table, Work Backwards, Look for a Pattern, Use Logical Reasoning.

Lyn attempted to develop her students metacognitive approaches through the integration of the mathematical ideas with real life situations and tried to raise the students' awareness of being critically literate about their mathematical processing (critically numerate).

In order to discover what aspects about metacognition the students seemed to be absorbing they were asked two questions in the Post-Student Interview. The first was: "*How does your class problem solve?*" Fourteen of Michelle's students referred to an explicit strategy whereas only one example of this was given by Lyn's students, as the excerpt below suggests:

Lyn's class:

MK Like she just set it out on the board, the question out, and then, take bits off and do it bit by bit. Then gradually get everything together and work it out, and then get the answer. (Melanie K.)

Michelle's class:

EB We'd have to set it out on a piece of paper, say what we, what the problem is, then like, what points are important, and how you would work it out, and then you work it out then you get an answer. (Elizabeth B.)

RC So Mrs W. gives us a problem and we've got to solve it...well we sort of talk about the problem, and then we look at it closely and pick out the figures, and sort out what we're going to do about it, and then we decide within ourselves what, how we're going to figure it out, and then we all figure it out together. (Rebecca C.)

AD Well we have problem-solving, and we have, in the back of our book what we do, what's the problem, then we work that out and then write it down and then, how do I solve it, and then we have a conclusion at the end. (Angharad D.)

AM Well, we have to write down in our own words, shortened, then work it out, then write an answer, solution...at the bottom. (Angela M.)

The second question evoked a response by the students that was most revealing, even to both of the teachers. In the final interview, each of the children was given an unfamiliar problem to solve in which it was hoped would be relevant and of interest: "*On what day and in what year will your 21st birthday fall.*" (see Tables 1 and 2 in Appendix 11 for results and analysis). Regardless of whether the children knew what happened in leap year or got the answer correct, all children in the study demonstrated an idiosyncratic way of solving this problem. There were only two students from each class who solved the problem without scaffolding assistance from the author during the interview. There was no evidence to suggest that children from Michelle's class who had been given explicit teaching about problem-solving strategies actually applied them to this situation, or were more systematic than Lyn's class who had experienced more real life opportunities to apply metacognitive strategies. Approximately one-third of the students in Lyn's class approached the task in a systematic way, whereas only three students in Michelle's class seemed to follow any strategy. In general Michelle's students were more willing to accept a prompt.

Once again the issue of lack of transfer occurred with both classes when analysing the Placement Test J concept items. For, example, one-third of Lyn's class got the questions on time and money incorrect even though these concepts had been explored during each of the major topics. One-third of Michelle's class got the question on decimals incorrect which had been a major concept dealt with at regular intervals throughout the year. Over half of Michelle's class made mistakes with the questions relating to the numberline, even though they had had a considerable experience with number lines during the year.

It would appear that the efforts by each teacher to develop thinking strategies among their students was not as powerful a factor as allowing the students to use their own idiosyncratic ways when processing a problem. It may be concluded that each teacher was preparing their students for more of the same of what they had been doing in the classroom. In Michelle's class, for example, students had experienced mainly textbook-oriented activities which had involved the application of a protocol or strategy. In Lyn's class, students had applied thinking strategies to everyday problems. In both classes the students showed evidence that they were aware of the emphasis their teacher was taking toward thinking strategies. In many ways students in Lyn's class had the opportunity for idiosyncratic problem-solving which might be expected to give them an advantage over those in Michelle's class when applying such strategies to an unfamiliar task. The results, however, show no real distinction between the two classes. This raises questions about the practice of instructing students in problem-solving protocols.

Scaffolding: Challenging Learners To Go Beyond Their Current Thinking, Continually Increasing Their Capacities

Introduction

This Principle refers to the action of the "mentor" or "expert" with the learner. Scaffolding builds at the point of the learner's conceptual development, in order to stretch the learner to the next stage of development, which s/he may not otherwise be able to achieve if left to her/his own devices. Vygotsky (1962) used the expression the "zone of proximal development" to describe the slightly higher level in which the "mentor" operates in order to challenge the learner to communicate at a higher level than previously demonstrated. Scaffolding is tailored to meet the needs of the individual, and is provided by the "mentor" at any point during the task when a shared focus might be seen to be beneficial to the learner. There is no expectation that learners will become independent learners independently. Rather, through scaffolding, the teacher provides support while at the same time attempting to challenge and extend the student's thinking. It is a hand-holding process.

Characteristic of scaffolding is the opportunity of changing of roles, between "mentor" and "learner" as the learner develops confidence to direct her/his own thinking. The mentor's discourse style is of particular importance in the way in which each child's communication is valued, and in the way the mentor models reflective language, and uses encouragement when the child is stumbling. Consistent with the notion of scaffolding is the teacher's recasting of a child's statements in a more complex way, for example, by asking the child a clarifying question. In this way the teacher helps the child to develop his/her conceptual understanding through the use of appropriate mathematical language.

NUDIST Categories

For the purpose of NUDIST analysis, the *Scaffolding* Principle was broken into 5 major components and 6 subcategories (see Appendix 7). One of the major components of scaffolding is the *Stretch (4 1)* function which a "mentor" or "expert" might perform with a learner. This describes how the teacher might through dialogue, challenge learners to go beyond their ideas. Another form which scaffolding may take can be as a *Framework (4 2),* such as a chart or a proforma which might act as a prompt, assisting the learner to become less dependent. Another key component referred to in the literature about scaffolding is the process of *Self Destruction (4 3)* where the teacher's assistance is no longer appropriate, as the support becomes redundant for the learner who has achieved the learning.

The teacher can scaffold by taking opportunities to *Generalise (4 4)* about how the learning in which the class is currently engaged might extend to other contexts. Although the notion that the teacher provides a scaffold for the child might suggest an engineering of the child's thinking down a path that has been determined by the teacher, the concept of *Transformational Freedom (4 5)* addresses this. By signalling *Transformational Freedom* through the teacher's discourse there is an acknowledgement to students that they have freedom in how they choose to express their mathematical ideas. In other words there is a tolerance by the teacher as the students experiment with more or less mathematical expression. The teacher, while allowing this flexibility, at the same time encourages the student toward more mathematical expression. The teacher is assisting the students to have control of the language in which they express their understandings. The teacher is making explicit to the learners, through his/her own discourse, what forms of language represent the language of power in certain situations.

The diagram in Appendix 7 shows the hierarchical relationship between the major components and the subcategories used in the NUDIST analysis for *Scaffolding*. The teacher may exhibit a degree of flexibility in how much time she allows students to do their work. This catering for the individual rates of learners is part of how a teacher might scaffold a child and has been coded as *Time Varies (4 1 2)* for the purpose of NUDIST analysis. The teacher may also adjust set tasks to *Regulate Difficulty (4 1 2)* and tailor the learning for different degrees of conceptual understanding among the class. This may also mean providing for the students to have different degrees of independence in approaching the task.

One aspect of a teacher's *Framework (4 2)* may be the use of *Routines (4 2 1)* in the classroom, such as the sequence followed in the lesson or way of approaching a topic. The familiarity and predictability of the routine can build the child's confidence and skill reinforcement. Within the *Framework (4 2)* of the activity the teacher may use *Peers (4 2 2)* to teach other students by getting them to share their ideas with the class or to work alongside other students. In order to assist some children to reinforce particular skills, the teacher may devise situations which may be similar to, or which may involve, new contexts. Examples of this have been categorised as *Situations Constructed (4 2 3)*. Lastly as part of *Transformational Freedom (4 5)* the teacher may allow for a switching of roles to occur where learners can flex their newly learnt skill by taking on the role of the teacher (Categorised as *Switching Roles (4 5 1)*)

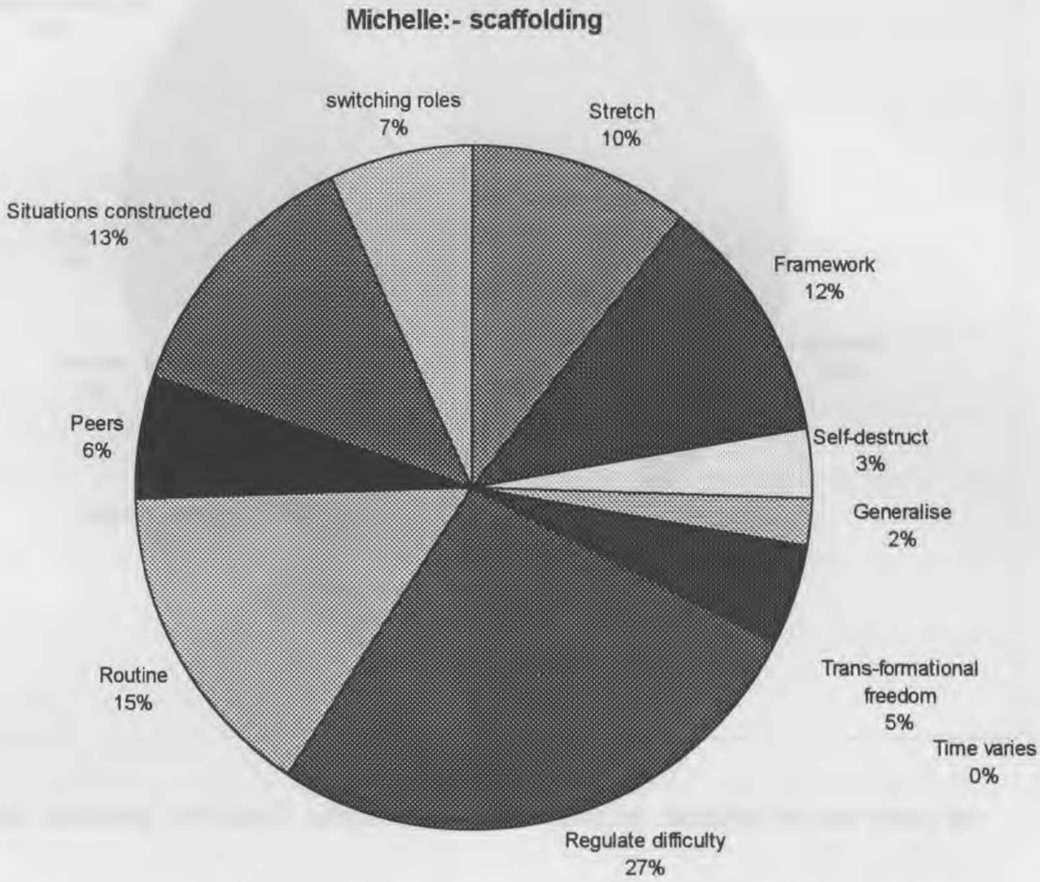

Figure 10a. Summary of NUDIST categories for Scaffolding as classified by four raters for Michelle.

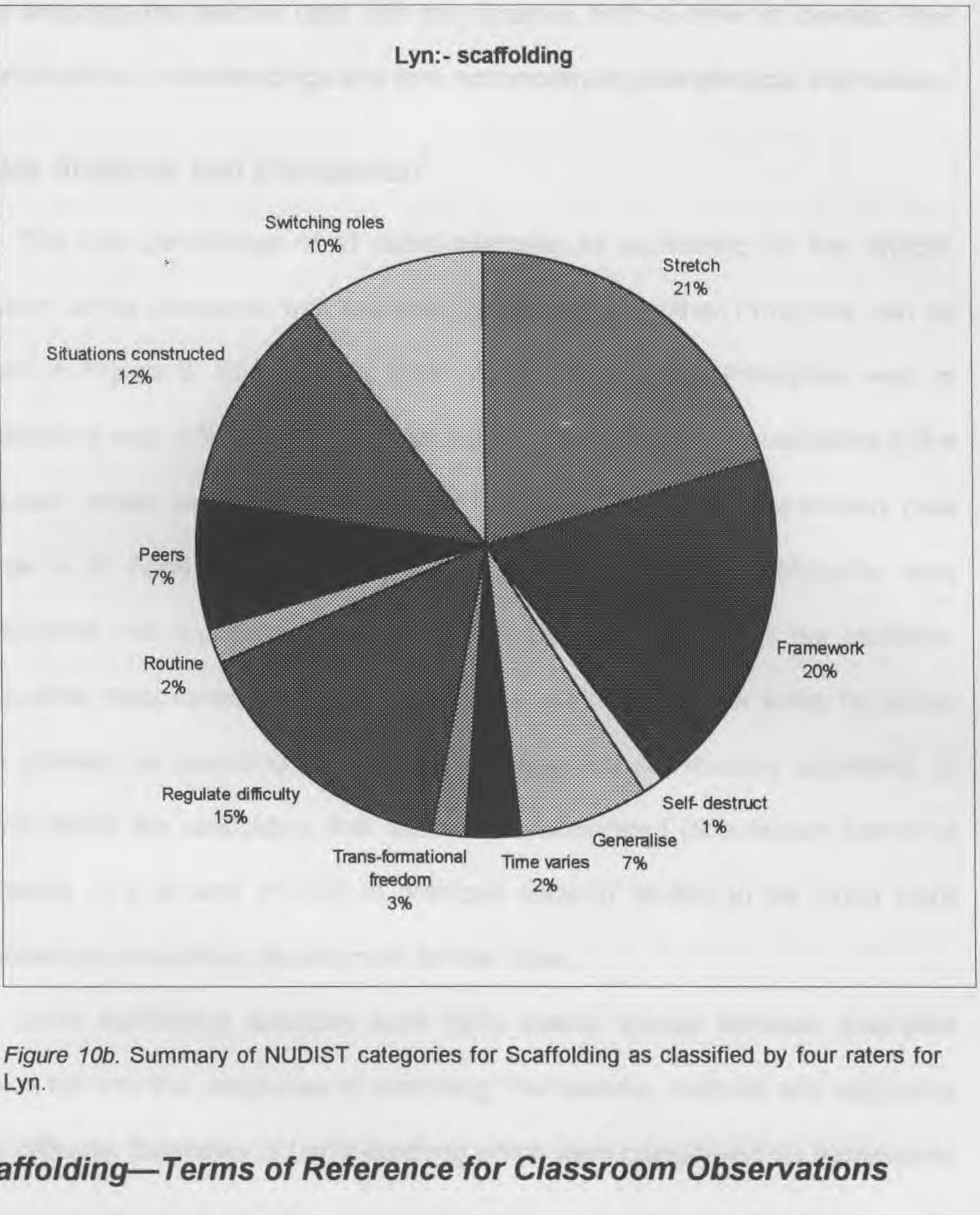

Figure 10b. Summary of NUDIST categories for Scaffolding as classified by four raters for Lyn.

Scaffolding—Terms of Reference for Classroom Observations

The scaffolding under discussion here is expressed through the teacher and student discourse. It focuses on the ways in which the teacher may support the students in their becoming independent. The discussion will also be reflected in the language the teacher uses with the students, both in order to develop their mathematical understandings and their accompanying mathematical expression.

Data Analysis and Discussion[8]

The total percentage of all noted examples of scaffolding for the "typical" lesson series compared with the total for each of the other Principles can be seen in Figure 6. For Michelle 21% of her total for the Principles went to scaffolding and 14% for Lyn. The total number of incidences of scaffolding in the "typical" lesson series for Michelle and Lyn were 86 and 98 respectively (see Table 4 in Appendix 9). About 27% of these examples for Michelle were associated with *regulating the level of difficulty* of the lesson for her students. The other major forms of scaffolding Michelle recorded was her ability to *stretch* the children by providing *frameworks*, *routines* and *constructing situations*. In other words the scaffolding that Michelle demonstrated (see lesson transcript excerpts 11.8.92 and 31.8.92 in previous section) tended to be much more focused on conceptual development for her class.

Lyn's scaffolding activities were fairly evenly spread between examples which fell into the categories of *stretching*, *frameworks*, *routines* and *regulating the difficulty*. Examples of Lyn's teaching which were categorised as *frameworks*

[8] The discussion which follows will examine each component and subcategory for Lyn and Michelle. Unless otherwise specified, the data will be derived from the "typical" lesson series (TL), but data from other sources will also be utilised. Other lessons will be referred to either as "March" lesson (ML), "August" lesson (AL), Field Notes (TU) (Since the analysis of the "typical" lesson series has been based on the Field Notes, reference will be made to Text Units (TU) deriving from these Field Notes as part of the NUDIST analysis), lesson transcripts (LT) or Diary Entry (DE). Data will also be drawn from the Initial Teacher Interview (ITI), Post Teacher Interview (PTI), Initial Student Interview (ISI) and Post Student Interview (PSI).

and *regulating the difficulty*, for example, included many instances in which she departed from the main lesson theme and responded to the students' lack of understanding of particular concepts. For example, in the "typical" lesson series, when she saw that the students were having difficulty working out the most efficient route and transport for their school camp, she spent several weeks working on averages and timetables (see "typical" lesson series 5.5.92- 28.5.92, Appendix 1).

Data from other lessons observed in Lyn's classroom suggest that Lyn frequently provided challenging tasks for the children and continually extended the demands of the task (*stretching* and *regulating the difficulty*) so that what the children achieved was never static or bounded in any way (see Appendices 1, 2 & 3).

Stretch (4 1)

This component of *Scaffolding* refers to the discourse the teacher uses to support the students in their next phase of understanding. The teacher's interaction with the student features sequences of focus questions, interspersed with comments, information, suggestions and modelling. Figures 10a and 10b indicate the average combined total of the four raters for the component of *Scaffolding*- 21% for Lyn and 10% for Michelle.

Lyn appeared to take her cue from the students when they signalled some need. She seemed to clarify the need in the first instance, and then suggest a further application which might extend the students' thinking. These field notes demonstrate some of the different ways in which this occurred.

TU 33 *Lyn asked students rhetorical question- will they make a gain or loss?* (in regard to the small business enterprises they were working on)

TU 48-49 *Lyn recommends clustering of amounts in sections to each group this was after deciding that it was a common problem* (students are adding price lists)

TU 125 *Teacher asks for suggestions and builds- seeks clarification, seeks more info*

TU 129 *Teacher encourages even though working in pairs, to use different methods*

TU 131 *Teacher prompts that they will need to calculate foods first and then do graph*

Michelle specifically referred to mathematical processes in her suggestions to assist the students. This became apparent, for example, in extended dialogue, as shown, for example, in the following text units, as well as in the lesson transcripts at the end of this section.

TU 113 *Rebecca already knew how to multiply, teacher built on this*

TU 162 *Teacher asks why you might have to round back to a decimal*

TU 351 *gets them to elaborate on their answer*

TU 514 *Teacher asks students to think of 2 digit number*

Michelle appeared to be aware of when students were not following her procedures during a whole class discussion, and could be regularly observed doing what she remarked upon in the Post Interview:

PTI *I try to pick up on children who aren't understanding whole class discussions, you can often pick them a mile off—they've got their hand in the air looking a bit puzzled. If there's just a few of them I try to get to them individually or in little small groups with a couple who aren't getting it, once the class is working independently. If it's a general- the whole class is looking a bit odd, I guess it's time to just stop and start again and pick up on a new track.*

Time Varies (4 1 1)

This subcategory of *Scaffolding* refers to the degree of flexibility available to children in terms of the teacher's expectations relating to their rate of completion of a task, or of an individual's concept development. Figures10a and 10b indicate a combined total of 2% for Lyn and 0% for Michelle.

In Lyn's classroom the students worked in pairs or groups which allowed them to tackle problems at their own pace. In her Post Interview, Lyn explained that she felt that she was there to intervene when they needed help. Very little pressure was put on the students to complete work except towards the end of term, although by then most had already completed the projects on which they were working. In her Post-Teacher Interview, Lyn made the following comments when asked *"How do you balance the time children have to do particular tasks in the mathematics classroom?"*

PTI *I give them as long as they need. I'll stop them if they are not gaining meaning and you can see that they're frustrated or they've had enough of it. Maths can be more difficult in that aspect than in the language. For as many of them don't have positive aspects to maths ... "Oh do I have to do maths!" You'll have to keep going on that—particularly if they can't see where they're going or it's more advanced than they have done recently. Again I think that's a* *um that balancing of time depends on the children. Those that are really mathematically oriented could just do it all day ...*

Observations of Lyn's lessons over the 12-month period of the study revealed that many mathematics lessons continued beyond any programmed time limit, particularly when Lyn felt that the students were too involved to be interrupted. The following two examples illustrate this:

TU 156 *lesson lasted 80 minutes*

DE *Teacher's lesson on tallying the food choices for the class party and for them representing it in a graph form had many involved for more than the one hour math time! Shows the ability to really get engrossed in math if it is real and relevant to them (11.3.92).*

Lyn discussed the way in which she was flexible in relation to the length of her mathematics lessons in her Post-Teacher Interview:

PTI *They do research in school time and they can work for any length of time on a topic, not just for half an hour, if it takes the whole morning, it takes the whole morning, that's fine. I may not have it [maths lessons] the next day, or it could go for three days, at a time. So I don't have just a set ...*

As suggested by 0% for *Time Varies (4 1 1)* in Figure 10a, Michelle followed the sequence of allocated time recommended in the syllabus. This was consistent with the way in which she followed the curriculum in the planning of her lesson content. One text unit during the observation period, recorded that Michelle had permitted students to work at different rates. Some flexibility was also apparent in the following text unit in which Michelle allowed the students to complete their work at their own pace for homework.

TU 180-184 *individual working by students. Kylene confused—knew what to do but couldn't explain it. Carla wanted to use calculator to check—teacher* *encouraged class to do so after 2 mins some completed. Carrie P was still confused, not done one (2-digit decimal subtraction). Teacher stopped class and gave extension for rest of class*

Topics were changed regularly and children were able to revisit concepts the next time they were encountered. When Michelle was asked in the Post-Teacher Interview *"How do you balance the time children have to do particular tasks in the mathematics classroom?"*, she responded indirectedly:

PTI *There's two aspects, I think you need a mixture of whole class, group and individual work and try to balance that. And also you need a balance of activity-based tasks and also chalk-talk or textbook type activities.*

She did however suggest a time constraint in an earlier answer to the question *"How do you usually plan a unit of work?"*:

PTI *I did feel very constrained to the syllabus to a very large extent. I think maths being very sequential and developmental, you do need to make sure prior knowledge is developed before you work on to the more complex concepts. Therefore its important to follow and complete the year's work.*

It would appear from this discussion that Lyn's class had opportunities for extended time to work on their mathematics whereas in Michelle's class this was less evident.

Regulate Difficulty (4 1 2)

This subcategory of scaffolding refers to the tailoring by a teacher of the workload involved in a particular task.

There was a considerable difference between the two teachers regarding this aspect of teaching. The "typical" lesson series total indicated that Michelle

(27%) adjusted the level of difficulty on more occasions than Lyn (15%). (see Figures 10a and 10b).

Lyn would generally allow the students to develop their own approaches and arrive at a finished product at their own level. When there was a common problem, however, she would often call out advice to the class, as these text units illustrate:

TU 31 *Lyn tells they will need calculators*

TU 43-44 *Lyn recommends students estimate answer then calculator for check stressed layout on sheet- hint on using diagram*

TU 52-53 *Lyn stresses the need for recording i.e. need for subtotalling as they go*

TU 429 *one group with her on mat*

Students worked on their projects at their own pace, and the level of difficulty evolved for each student. In her Post Interview she talked of how aware she was of whether children were keeping up with any whole class discussions or not:

PTI *It's really in that you have to assess as you're doing it, as you're talking you've got the facial expressions that they can give you and they do it to you in maths and also in language you can see that you've lost them, or "I don't know what she is talking about ... I just can't see what she means by that" Some of the kids come back and say what's this. Others just look lost. Now I would say to them you look lost.*

Michelle's class was ability grouped. Her Post-Teacher Interview showed that she did not feel the need to regulate difficulty to any great extent.

PTI *although within ability groups you did have, you know the brighter children and those that aren't quite coping as well,... There weren't any extremes, or maths problems within that group. There were some very bright kids, and I guess the average to upper range* *there, which is why I didn't use it[group work] as much.*

Although this class was ability grouped, there were still students working at different levels. A number of field notes make specific reference to difficulties which some students seemed to be experiencing.

TU 480-483 *it seems apparent that the teacher's logic takes over when the students are having trouble i.e. she has a solution if followed will assist them or so she thinks. On one level this is most impressive and shows a keen awareness for diagnostic pinpointing of trouble with concept development however there are two problems with this—1. it is her logic and not starting where the students are having trouble and 2. she feels under pressure, both within the allocated lesson time slot as well as her awareness of the place of this concept in the overall scheme i.e. low level time priority*

The Post-Teacher Interview with Michelle reflected her awareness of the level of her students' understanding, about each concept, and the degree to which their prior knowledge could be built upon:

PTI *Yes as I said maths is sequential and prior learning is vital to understanding what you're actually teaching. If they haven't got the concepts previous, you're not going to build on them and get an understanding of what they are doing, so I think prior learning must be established, even if you need to back track, back to the previous year even. If you need to do that you need to do that.*

Michelle adjusted the tasks so that they were manageable for the majority of the students. These field notes show some of the ways in which this occurred:

TU 185 *new idea- gave students a task to give partner a word problem- and they give the info/ problem/ sum to their friend and then once both done it they do each other sums. teacher set boundary by saying she preferred area with subtraction of decimals*

TU 251 *tasks broken into manageable pieces (Lesson calculating the various combinations of coins you could make with 8c)*

The "typical" lesson series recorded many instances where Michelle either chose examples which faithfully embodied the concepts, or when concepts were not grasped, provided appropriate remediation.

Frameworks (4 2)

The idea behind this component of scaffolding is that the teacher provide appropriate background information (for example by way of charts or a proforma). These are intended to act as prompts, which assist the students to grasp the relevant concept.

Although Figures 10a and 10b suggest that both teachers felt that it was important to use frameworks (Michelle- 12% and Lyn- 20%), the form adopted for the frameworks was qualitatively different in each classroom.

The "typical" lesson series suggests that Lyn was reluctant to superimpose any frameworks which might direct students to use a train of thought they would not otherwise be using in their own problem-solving. For example, when she did offer a suggestion to the class, it was usually offered in an attempt to resolve a common problem which many of the students were experiencing; such suggestions were usually presented as optional:

TL *She discussed what mathematical processes they'd be likely to need such as division and multiplication [by going through some of them on the blackboard].*

ML *Lyn discussed and built on the students suggestions until they arrived at the idea of a grid and graph [ideas were noted on the board].*

ML *Lyn recapped using whole class discussion and then wrote the major question on the board.*

Examples drawn from other lessons also indicate different forms of framing used by Lyn.

TU 44 *stressed layout on sheet- hint on using diagram*

TU 126 *Teacher offers suggestions of pie graph*

TU 441 *Question E teacher brings class on floor with a common problem for all of them (A couple of tables are working on a worksheet with timetable exercises).*

In the Initial-Teacher Interview Lyn responded to the following question *"If scaffolding is the teacher's capacity to lead the student to the next level in their development through questioning and prompting ... to what extent do you practice it?":*

ITI: *It's very important to provide them with a framework, but in maths it's difficult because you can provide them with the framework but there is not really just one right way in coming to that, there may be for instance in addition, there may be different ways a child sees to calculate, different ways they remember their number facts, different links that they make, so it's important that you model and scaffold a variety of ways.*

This comment indicates that although Lyn exhibited a high incidence of using frameworks in her "typical" lesson series the fact that it was optional made it qualitatively different from an imposed framework.

Michelle provided frameworks for the students, and these were a routine part of each lesson (see TL, ML, and AL). The frameworks used by Michelle were mostly quite explicit from a mathematical perspective. The type of mathematical discourse used by Michelle in providing "frameworks" for her students is illustrated by the following text units:

TL *Michelle wrote on the blackboard as she talked through working out % and stressed that when they see "/" in 82/100 it is called the "divisor."*

TU 232 *reward has been for system not just hit and miss, comments on the students work which reflect this:- "You are working to a system. Now continue!", "Can you be more systematic?" "Are you sure that all combinations are being included?" "There are many more than this- can you be more systematic?" "A good try. Can you be more systematic?" "A good start" "Keep going." "How do you know when you have all the combinations?"*

TU 240 *stresses summary- the process of problem-solving (listed in back of books 1. What is the problem? 2. Solving the problem 3. Solution- summary sentence)*

TU 638 *Teacher reminds students to estimate and check if answer is reasonable*

Specific frameworks for problem-solving were a feature of Michelle's classroom discourse. As mentioned in the previous section on *Metacognition* when the students were given the novel question in the Post Interview to solve out aloud (*"On what day and in what year will your 21st birthday fall?"*), there was no discernable evidence that Michelle's class had been taught any specific strategies, within these problem- solving frameworks.

Routine (4 2 1)

This subcategory of *Scaffolding* can be described as a technique that a teacher might put in place in order to build confidence through the predictability of the exchange and to provide for skill reinforcement.

As Figure 10b suggests, few text units were classified as "routine" in Lyn's classroom (2%). The routines identified could be described in terms of the group structure adopted for the class, and the projects undertaken by the students. Within their group projects the students were given opportunities to practice a skill. For example the calculation of averages in the following excerpts from the "typical" lesson series was classified as "routine":

TL *She demonstrated on the easel how to calculate "average" speeds. The students were gathered near the easel ...*

TL *Once students had entered their information from their investigation, they rotated to another group to enter their information onto the sheet.*

For Michelle, the routines identified (15%, see Figure 10a) were mainly associated with the way in which she developed each new set topic/ concept. At the beginning of every six day cycle, Michelle introduced the students to the new topic/concept. As in the "typical" lesson series, Michelle would begin most lessons with teacher input either in the form of discussing the homework, or mental, or concept introduction. Exercises on the concept would follow and then a summarising time at the end of the lesson when homework was set. The author deliberated on the merits of this as these field notes show:

TU 207 *the type of tasks in textbooks would be good preparation for PAT test type of questions*

TU 211 *a lot of post lesson follow up by teacher with homework making it easier for her ... to be aware of the conceptual problem they were having*

TU 275 *repetition of task builds reassurance, my question is will they necessarily call upon it in their everyday lives–how to make that link*

It would appear that Michelle had a pattern to her lesson plan which was transparent to the students, thereby giving them many opportunities for skill practice.

Peers (4 2 2)

The idea behind this subcategory of *Scaffolding* is the opportunities the teacher provided for peer tutoring. Teachers may have students teach other students by asking them to share their ideas with the class or to work alongside other students. The average of the combined totals of the four raters for this subcategory of *Scaffolding* was found to be Lyn—7% and Michelle—6% (see Figures 10b and 10a).

In Lyn's teaching she used group work and seemed to create many opportunities available for peer teaching. She would ask students to use the blackboard to do the explaining to the rest of the class, for example, these student demonstrations, together with sharing sessions by small groups, provided opportunity for any students who had not quite grasped the idea to hear the language of their peers. These examples from the "typical" lesson series and other lesson observations demonstrate the use Lyn made of peers in her classroom.

TL *She encouraged students who even though they were working in pairs were to use different methods individually and compare.*

TU 87 ***Students used blackboard to work out and record most popular drinks and record it in own way***

TU 390 ***Stephen had a unique way of adding and shared it with teacher***

TU 415 *uses students' answers to explain to other students who don't understand*

TU 439 *once groups back all working, top 2 groups paired with third group to explain how to timetable because it is non-conventional*

Lyn seemed to use peers to reinforce the idea that there may be more than one way of doing a task and that one student's explanation might in some way be clearer than either her own or another student's.

Michelle used pair work so that the students could check one another's work, or to set similar problems to provide students with more practice. Teacher-directed whole-class discussion encouraged students to listen to their peers. As these text units indicate, the author observed that certain students were rarely if ever asked. Over time the author recognised that these students were the ones who were likely to take the class off into a different direction from the one which Michelle had planned.

TU 182 *Carla wanted to use calculator to check. Teacher encouraged class to do so*

TU 352 *encourages students to go back to example to look at what class has done*

TU 210 *Carrie P seemed to often be one who was not understanding or going down the wrong track and yet although was obvious to me even on my few visits she didn't seem to get attention*

TU 612 *situation with Carrie P who invariably wants to not do the mainstream is allowed but not embraced nor used to distract the rest of the class unless its convenient for the teacher's plan*

A similar treatment of Georgia can be located in the lesson transcript at the end of this section. Table 1b in the *Metacognition* section supports the observation that not all students are being included in Michelle's lesson during whole class exchanges.

Situations Constructed (4 2 3)

This subcategory of *Scaffolding* is part of the framework a teacher may devise to allow for skill reinforcement in either similar contexts to that of the task or which may involve new contexts. The average of the combined totals of the four raters for this subcategory of *Scaffolding* was Lyn—12%, and Michelle—13% (see Figures 10b and 10a).

Lyn tended to rely on real-world tasks which provided numerous opportunities to practice a skill. For example, in the class party task, the calculations involved built on similar tasks that the children had worked on during an earlier small business activity.

TU 42 *presupposes cooking, refers to different measures (Each table has a different recipe to make with the ingredients and equipment they need, they rotated groups throughout the morning)*

TU 83 *same idea but with chips (Students have learned how to score preferential voting with drinks and will practice with chips in order to decide on the items to be purchased for the class party)*

TU 183 *Students already know how much pizza etc will weigh and how many it will feed, also cake and cool drink (Students have to find the best value per weight and amount for party catering)*

The situations constructed in these examples from Lyn's class provide for the repetition of a recently learned skill or concept. The requirement to repeat the task comes not as a pedantic exercise, or to prove mastery of a concept or skill, but rather is one which is performed out of a genuine need to solve a problem.

Michelle used the textbook extensively as part of her lesson routine. The exercises were usually presented as sentence problems with accompanying exercises in a similar vein. An initial exercise example on each page was there to act as a proforma or guide with the answer either elaborated on in the book or to be developed by the teacher on the blackboard.

TU 168 Textbook p. 40 (purple Addison Wesley) got Linley to read "Adding Decimals. Jason visited some European countries. He has 1 British pound note, 1 German mark, and 1 Greek drachma. What is their total value in U.S. dollars?" (The textbook went from this prose to demonstrating the algorithm for the addition of decimals. Then followed 35 exercises without text and lastly 4 sentence problems which imitated the original style of the prose example)

Although many of the problems set for the children followed this pattern, the textbook was not always easy for the students to follow.

TU 110 *Addison Wesley p 201 q 21-24 good example of poor support for the specific skills the students were having trouble with i.e. example was oranges and not neat squares, also example was halves, a simpler concept than thirds, also exemplar in book showed a rule "multiply the whole number by the denominator" which teacher had not emphasised, the questions required 100ths, 10ths, 5ths, i.e. only familiar one was 6ths*

The situations constructed gave opportunity for skill reinforcement as did the homework which was taken from a workbook belonging to the same textbook series of publications.

Self- Destruct (4 3)

Another key component of *Scaffolding*, and one which is much harder to detect, is the process of self destruction of the teacher's assistance for the child. Once competence in a skill or concept is achieved the teacher's support is no longer required. The teacher is likely, however, to be "raising the ante" for the next extension of the concept or skill development. The average of the combined totals of the four raters for this component of *Scaffolding* was Lyn—1%, and Michelle—3% (see Figures 10b and 10a).

As Lyn continually worked with students on a one-to-one basis, it was difficult to be aware of how the previous support she had offered had dropped away. She would at times check the students' knowledge by questioning and if they seemed to be on the right track she would no longer provide scaffolding. The following section from the "March" lesson series shows how Lyn withdrew support regarding how the students might graph their information. However, she seemed to recognise that some students were not quite ready for this independence and therefore gave prompts.

ML *Lyn encouraged freedom of choice in how they may choose the previous discussion on graphs in their own recording of choice of savouries.*

ML *Lyn stressed neatness in recording and reiterated the need to keep a record.*

ML *She encouraged students who even though they were working in pairs were to use different methods individually and compare.*

ML *She gave the prompt to calculate foods first and then do graph.*

The shifting of support demonstrated in these text units is the closest illustration of the self-destruct discourse employed by Lyn.

Michelle would tend to move onto the next concept so that the self destruction of scaffolding came by default and, because the pace was being driven by the syllabus. Thus the self-destruction was built into the sequence, regardless of whether the students were ready for it or not. An example of this can be seen in the lesson transcript below. Michelle has set out in her program for that day's lesson that she would start with mixed fractions and demonstrate these by using diagrams. Then she planned to identify the improper fraction, convert mixed fractions to improper fractions without the diagrams, and then do the reverse by converting improper fractions to mixed fractions. The double period (hour and twenty minute lesson) followed a regular pattern of demonstration by teacher on the whiteboard then on to tasks set from the textbook while the teacher roamed around the room assisting individuals. The teacher repeated this pattern six times. Michelle did achieve her set goal from her program but it was clear the next day that many children had had difficulties doing the set homework. Sensing their insecurity Michelle collected their workbooks at the end of the lesson.

LT *Michelle (31.8.92):*

Children are sitting in their desks in grouped tables while teacher is demonstrating on the whiteboard. The lesson has been going for about forty minutes and yet there is a hesitancy by the class to contribute to Michelle's leading them through the concept.

Michelle: Ten quarters. OK back to your question Georgia, you wanted to know did you have to change that to two and one half. Do we have to?

Georgia: Well its lowest terms but since if you have already converted it do you need to convert it again or if I did that I got it wrong, but it seems like the right thing to do.

Michelle: It does and you are not wrong and in fact ten quarters would in fact be equivalent to...?

Georgia: Two and two quarters?

Michelle: You said two and two quarters was the same as two and one half.

Georgia: Are you supposed to do that?

Michelle: Well let's work that out now. Two and one half is how many halves?

Georgia: Oh, um (pause)

Child: Two.

Michelle: Two and one half? Five halves isn't it OK well we said that two and two quarters is equivalent to two and one half so ten quarters must be equivalent to? Five halves.

Georgia: Yeah but what I mean is like you didn't have the right question, what I meant is like, if you do get two and two quarters as your, like your answer from changing it from an improper fraction to a mixed numeral.

Michelle: Yes I know what you mean.

Georgia: Should you do it in lowest terms?

Michelle: It doesn't matter, it's a good habit to get into but it doesn't matter.

Georgia: But you see I did and I got my answer wrong in the book.

Michelle: Where in your chapter review you're talking about? I think you'll find that they're not wrong they're just equivalent fractions. I'll make sure. Alright girls, (turning her attention back to the rest of the class), so when in reverse looking at the first set of questions 1-20, they ask you to go from an improper fraction to a mixed numeral. And this time rather than multiplying we are saying, eleven divided by four is two and three left over, two and three left over, so OK, if I gave you the improper fraction of fifteen quarters, Linleigh could you convert that to a mixed numeral? Fifteen quarters.

(After three more cycles of demonstration and working from books the lesson is about to end. Georgia voiced the hesitancy felt by many of her classmates.)

Michelle: Alright could you have a look in your practice books at page (pause) 76 I think. Yes 76. Look at the page because the numbers aren't (pause) as evident. Girls last time we did page 76 first column. Tonight would you do the last column 4-48, just that last column questions. Be careful after question 24 it changes. The last column. Yes Georgia.

Georgia: Is there a test in our practice book?
Michelle: No there's not.
Georgia: Are we allowed to bring it home, our textbook?
Michelle: (To Georgia) I'm quite happy about that. Questions four and that column down the page. I would like your books though girls, so do that on paper tonight. On my filing cabinet, no I'll stand there. Open up to today's page and I'll collect your workbooks please.

This sequence would indicate a gradual self destruction of support provided by the teacher as the students worked from pictorial representations through the algorithm to workbook exercises, using mathematical symbols.

Generalised Together (4 4)

This component of *Scaffolding* arises when the teacher uses opportunities to try to generalise, with the students, the concepts with which they are currently working, to other contexts. Figures 10b and 10a show the average of the combined totals of the four raters for this component of *Scaffolding* (Lyn—7%, Michelle—2%).

Lyn explained in her Post-Teacher Interview, how she helped students see how a given concept might relate to other situations.

PTI *Lyn: In discussion with the children, when what you have done is actually very simple then saying to them how else could you have tried it in another situation? Do you know any situations where you have seen it happen recently, you've seen it in the paper... and you try to bring it back into their lives so that they can understand it. Days later, go back to "Do you remember when we did this? What was that?" Try and tie it in other subject areas as well. That's difficult.*

Jennie:- How do you think students make this transfer from a particular context to a range of contexts.

Lyn: I believe that if the student really hasn't got it on board then that doesn't occur, its only relevant to the child or when the child has to actually do something, they can do, they can then make those connections. And I think that for many of them they still haven't made the connections even though we have been dealing with it ... they say yes we have done tables and we have done all that but they might have done all that, but they haven't really made it they only know the pattern, or they only know the steps to follow because the teachers taught them not because they have made those connections for themselves.

Thus, although Lyn attempts to make explicit the generalisability of an idea or concept, she argues that the child must make these connections for themselves.

Michelle was asked about whether the students would be able to generalise when the teacher goes from topic to topic. She responded that a broad range of experiences could achieve this:

PTI *I think the only way you can do it is to give them just a wide range of examples, and just give them different uses I guess, for the concept you are using. I think that's the only way you can do it is to give them a vast experience of it, and relate it as much as you can to their little worlds.*

The lesson transcript at the end of this section on *Scaffolding* shows how Michelle called on the students to use the fraction skills they had been developing to apply to another way of working with equivalent fractions. A segment of this transcript is given below:

LT *Michelle 11.8.92*
Michelle: Right for today, we are ging to do the same thing in reverse? OK? We talked about one third and two sixths being equivalent fractions. Can anybody tell me other fractions that are equivalent to one third?

In the "typical" lesson series where Michelle asks the students to estimate their answer before they work through percentage problems, it could be argued that a student would need to generalise from their prior knowledge and what has previously been demonstrated to the whole class. in order to perform the calculations.

Transformational Freedom (4 5)

This component of *Scaffolding* describes situations in which the teacher's discourse makes it possible for a range of approaches to be used as students move towards more mathematical expression.

In Lyn's class the students had a great deal of freedom in how they chose to represent their mathematical ideas. Lyn would provide suggestions for a specific mathematical device which may speed up their processing, but students could choose whether or not to use it. The "typical" lesson series showed how Lyn encouraged students in their efforts to express their mathematical ideas (see Figures 10b and 10a for average of combined totals of the four raters for this subcategory of *Scaffolding*, Lyn—3% and Michelle—5%).

ML *Lyn discussed and built on the students' suggestions until they arrive at the idea of a grid and graph*

ML *Lyn asked the students for suggestions of how they might use the graph paper. As she talked she sought clarification of their ideas and encouraged them to provide more information.*

These examples show how Lyn's use of transformational freedom acted as a scaffold for the students. They are supported in their efforts to develop their mathematical understandings about the use and function of graphs for their immediate purposes.

Michelle's frequent use of workbook/ textbook exercises appeared to provide little opportunity for transformational freedom. However, when she dealt with topics not directly from the textbook e.g. polygons, some options were given.

TU 579 *Teacher encouraged students to use square paper-colour shade it-do as much as is needed (Students are attempting to solve her investigation question "How many squares are there on a chessboard? This was not related to the class work but was a question on the blackboard for her regular class to do in their spare time)*

This excerpt from a "number" lesson on fractions shows how the students had some freedom with regard to the ways in which they might present their information.

LT *Michelle (11.8.92)*
Michelle: So you are showing me in a diagram form what it would be as an improper fraction.
Georgia: OK.
Michelle: Angela?
Angela: Does that mean you have to do eleven boxes... for number 4?
Michelle: You do circles, boxes anything you want to show. First five please in diagramatic form. Off you go. Use some colour to demonstrate.

These examples show that some transformational freedom was possible within the often well-defined boundaries of a specific mathematical exercises.

Switching Roles (4 5 1)

This subcategory of *Scaffolding* refers to any opportunities the teacher provides for the students to switch roles—learners can flex their newly learnt skills taking on the role of the teacher. Figures 10b and 10a indicate the average of the combined totals of the four raters for this subcategory of *Scaffolding* (Lyn—10%, Michelle—7%).

Lyn was observed allowing her students to make their own way through problems with minimal guidance from her when necessary. She actively encouraged students to take on the role of teacher or "expert."

TU 33 *Lyn asked students rhetorical question—will they make a gain or loss? (Students are engaged in setting up a small business where they sell their cooked products to the school)*

TU 390-392 *Stephen had a unique way of adding and shared it with teacher and found others had his system. Lyn quick at cueing in to this. (Students adding speeds to work out the most efficient route to their campsite 5.5.92)*

TU 402 *Uses their experience and puts on blackboard (interested in establishing their meaning) e.g. bus, trains, boats, planes school timetable, hospital, movies, roster, event (sporting like a program), TV guide, clubs, sports (fixtures) (accepting their answer, they feel free to contribute and at times are expert) (Teacher establishing what they know about timetables 28.5.92)*

In Lyn's Post-Teacher Interview she talked about how she could learn from the children or else she might let the children take over some of the teaching role:

PTI *Lyn: And I think somewhere in here there was a way that one of the boys taught me a way that I hadn't even thought of. (see Stephen's reference in TU 390-392 above) ... each child comes to it with a different way of thinking about it and how they see numbers. So if they help each other; I think too the children could actually help each other sometime better. You see the adult can only see one answer and can't understand why the child can't see that.*

These examples show an openness by Lyn to accept the expertise of her students that they might be able to teach both her and/or their classmates something they didn't know.

Michelle's "typical" lesson series was given a total of 7% for this component of *Scaffolding*. When students were asked to demonstrate on the board, it was made clear to the students that the teacher would monitor demonstration carefully. This example from the "typical" lesson series shows how Michelle acted as an adjudicator with respect to the content shared with the class. The lesson transcript extract (31.8.92) which follows, also demonstrates a monitoring of the student's role as a teacher by Michelle.

TL *Students were given 14/50= ?/100 and were asked to make equivalent fractions. She asked the students to offer a fraction which the whole class could solve as an equivalent fraction .../100. Michelle selected those problems which demonstrate equivalence easily.*

LT *Michelle 31.8.92*
Michelle: OK You've come to that conclusion. This was causing quite a bit of problem, so today what I'd like you to do in your books in a second is to draw the diagrams to show me that you understand what we are doing. Ok have a look at the green diagram to start with- two and two thirds and I've drawn three squares on the board. I want somebody to come up to me and demonstrate two and two thirds using those three squares. Think you can do it? Come and show me. (Elisabeth comes to the board) You do what you think you can?... to show me that two and two thirds can be shown as an improper fraction. (pause while Elisabeth draws on board) OK stay there. Elisabeth's drawn or coloured in two whole squares and a part of the third square. But how many thirds are there? How many thirds has she coloured in? Elisabeth can you make your diagram more clear so that you can show exactly how many thirds you have coloured in. (pause) So how many thirds have you coloured in?

Elisabeth: Two.
Michelle: In the last diagram...
Elisabeth: Yes.
Michelle: How many thirds then including the two whole squares? How many thirds have you coloured in?
Elisabeth: Eight. Eight thirds.
Michelle: Can you show the diagram more clearly? To show eight thirds? (pause) Good girl. Sit down ... Elisabeth's now coloured in eight thirds.

To conclude this section on *Scaffolding*, segments from lesson transcripts from each teacher will be used to illustrate the differences in how each teacher scaffolded her students. It became apparent from the discussion in this section on *Scaffolding*, that although each teacher quite clearly scaffolded starting with the child's understanding, differences in how each teacher proceeded as concept development unfolded was apparent in the dialogue.

The transcript from one of Michelle's lessons began with dialogue which related to a difficulty the class had been having with equivalent fractions. Her remediation hints focused on trying to have the children grasp the pattern she had in mind rather than shaping the idea from the child's mis/conception. This is particularly evident in Michelle's responses to Georgia.

LT *Michelle (11.8.92)*
Children are sitting in their desks in grouped tables while teacher is demonstrating on whiteboard as she talks.
Michelle: Alright, are you ready? Right girls eyes this way please. What we will be doing today is a continuation of what we did Monday and yesterday. Yesterday we were looking at equivalent fractions and we were working out how to find equivalent fractions of a simple fraction such as a third without actually having to draw those large graphs everytime you need to work out an equivalent fraction. What is that way that we came up with yesterday? Do you remember we saw a pattern between the different numbers? For instance

we could tell instantly how many sixths is a third. Gabrielle?

Gabrielle: Two.

Michelle: And how did you get two sixths?

Gabrielle: Um two thirds is six and one's two is two?

Michelle: Right you realised that whatever the denominator was multiplied by, the numerator was then multiplied by the same number. One times two is two. What else did we say to prove that one third does actually does equal two sixths? We came up with a way to prove that two sixths was equal to a third. Do you remember what we said yesterday? We said that we know if I had two sixths of a cake I would have just the same amount as someone that had one third... and what was that way Georgia?

Georgia: Because in the top number goes into the bottom number three times and in the second fraction the top number goes into the bottom number three times as well.

Michelle: That's in this instance but it doesn't always happen. I was actually talking about this little thing here that we talked about yesterday, what did we say about that number Angela?

Angela: That you times it by … you can … it's a whole number and you can say two … it's a whole number …

Michelle: Right you're saying two halves of two is a whole number right so you're saying two halves is one. One third multiplied by two halves equals two sixths. And all we are doing is multiplying a third by?

Class: One.

Michelle: And we know whatever number we multiply by one we end up with the same number. Because ten times one is ten. Twenty- five times one is? … Pardon?

Class: Twenty- five.

Michelle: OK So that's what we looked at yesterday. Now I know some of you had a little trouble with that so the plan today is to put you to work at your desk and then I'm going to mark the homework that you did last night and Mrs Bickmore-Brand will also mark your homework last night, individually so that we can see that you really understood what we talked about yesterday.

Child: But I didn't get it.

Michelle: That's right you can talk about that when you talk about your homework. Right for today, we are going to do the same thing in reverse OK? We talked about one third and two sixths being equivalent *fractions. Can anybody tell me other fractions that are equivalent to one third? … Right? Rebecca?*

Rebecca: Four twelfths.

Michelle: Four twelfths is equal to one third

Child: Eight sixteenths.

Michelle: Eight sixteenths. One times eight is eight, three times eight is sixteen … that'll do thanks, there are millions of them, we could go on for eternity.. Now of those fractions this one's special and it has a special name called? (Pause) Does anybody know?

Child: A lowest term.

Michelle: A lowest-term fraction. Why do you think it would be called a lowest term fraction?

Child: Because it can't go any lower than that?

Michelle: The denominator can't go any lower than the three. Now the work that you will be doing today will be asking you, for instance it will give you the fraction eight sixteenths and you will need to say what is the lowest term fraction of eight sixteenths. And whereas yesterday we worked out equivalent fractions by mutliplying by one this time we are going to?

Class & Michelle: Divide by one.

Michelle: So in this case we're going to say eight divided by?(Long pause)

Child: Eight?

Michelle: Eight sixteenths, divided by eight eighths. We are dividing by? (Pause) one. Eight divided by eight is one. Sixteen divided by eight is? … three.

Child: Two.

Others: Its two there.

Michelle:Is this correct then? No it's not. It's not eight sixteenths at all. What should it be?

Child: Eight twelfths.

Michelle: Good girl, I'm glad you picked that up. Lucky you were watching. We will have to start this one again. Eight twenty fourths. Eight divided by eight is one, twenty four divided by eight is?

Class: Three.

Michelle: OK Now we cannot go any further. Let's try one more because we made an error with that one. Something different we will go for...six eighths. What is six eighths, in its lowest form? (Pause) The first thing you will need to do is to find the number that goes into six and eight?

Others: Two.
Michelle: Divided by two. Two halves, one whole. Divided by two. Six divided by two is?
Class & Michelle: Three, eight divided by two?
Class: Four.
Michelle: Now is that our lowest term fraction?
Class: Yes.
Michelle: How do you know Carrie?
Carrie: It means you can't go any lower, than three quarters ... you go one but you would just get the same number.
Michelle: You could divide both numbers by one but you would just get the same—you'd end up with three quarters again. Is there any other number that will go into three and four equally?
Class: No.
Michelle: There's no common factor and so we have to stop there, that is our lowest common fraction. Right. What I want you to do please is to have a look at page 198.

In this transcript Michelle started at a point where many of the students were experiencing difficulties. She used this base of the students' understanding to develop scaffolding to support *their* concept development. As the lesson prog. ed, however, Michelle attempted to ensure that the students' understanding corresponded to *her* understanding.

Michelle overtly altered the children's everyday non-mathematical language to more precise mathematical terminology. She also introduced the more abstract terminology of "lowest term" and reinforced "denominator" and "common factor."

The following transcript has been taken from a small-group discussion in which Lyn tried to develop students' understanding of fractions so that they could solve a problem which a small group was having with working out how many sandwiches would be needed for a class party. Each time a child had not quite grasped the idea she tried to give a "concrete" demonstration of the idea. The absence of abstract mathematical terminology is apparent in this excerpt. The language used emphasises everyday associations, and uses, for example, diagonal, quarters, halves, and so on.

LT *Lyn (4.3.92)*

Lyn stops at one group who are working on sandwiches.

Lyn: So you'll have to write something about bread. How many loaves of bread you will need.

Joe: We're going to have sandwiches and cut them like this (makes 2 diagonal cuts with his hands). How much bread will we need?

Lyn: Right. How many slices in a loaf of bread?

Ben C: 24.

Lyn: I was going to say 24 as well. Right so if you get 24 and you put one on top of the other (demonstrates with her hands) so that's ...?

Joe: Twelve.

Lyn: Twelve Sandwiches. If you do them into ...? (makes a diagonal cut with her hands).

Joe: Halves.

Ben C: No quarters (makes 2 diagonal cuts)

Lyn: Quarters. So there'll be 4 lots of 12...

William: 48.

Lyn: 48 sandwiches.

Ben C: Not everybody would eat it so you'd be able to have 24 quarters.

Lyn: Is that for 2 loaves of bread?

Joe: Mrs?

Lyn: Remember we said there were 12 slices (demonstrates by drawing square with her fingers), and we cut those 12 into 4

William: 24.

Lyn: You've got 24 slices but when you make them into double sides (demonstrates with hands) that reduces it to how many stacks (demonstrates with hands) of bread?

William: 12.

Lyn: Right. Now if you cut that 12 into lots of 4 how many is that?

William: 48.

Lyn: And you've got 2 loaves of bread. (pause) What's 48 plus ...?

Joe: 48.

Ben C: 90, 92

Lyn: Not 92, 48 and 48. What's 8 and 8?

William: 16.

Lyn: 16. So it's ...?

William: 96.
Lyn: And if there's 32 of us approximately, how many will each child have? Approximately?
William: They'll have about half.
Lyn: How many 30's in 96?
Ben C: 3.
Lyn: About 3. Would it be 3 whole sandwiches or just three quarters of a sandwich? So each can have about one round of sandwiches.

Later, the group reported back to the class:
Lyn: So you decided what to do about your sandwiches?
Ben C: Some people don't like sandwiches, but we decided that you could get 3/4 of a sandwich each and we needed 2 loaves of bread.

Lyn has attempted to develop the students' fraction knowledge in a context which is purposeful for the students. In addition, the concept has been tempered with a critical numeracy (see *Metacognition*) understanding of the skills they are using ("Some people don't like sandwiches").

Summary

The ways in which each teacher has chosen to use scaffolding are qualitatively different. Both teachers have relied on constructing situations in which they can provide a framework for mathematical skill and concept development. They both use the students' peers in the construction of the scaffolding, although this has qualitative differences. Both teachers started with a problem or concept which was targetted specifically for the child/class.

The differences lie in how each scaffolding dialogue was constructed. Michelle signalled her preferred pathway for the development of a concept, and although clearly aware of a child's misunderstanding continued to drive on with a predetermined method for approaching that skill or concept. Lyn however continues to shape and develop a skill or concept down a pathway which is jointly constructed between her and the child/class. She works with their ideas as they come up, for example, Joe wants the sandwiches cut diagonally, and Ben suggests quarters. Her explanation of a skill or concept continues to be reworked in an effort to refine the communication of the idea rather than presenting a system for approaching the task.

Michelle continued to "raise the ante" in terms of regulating the difficulty of the concept development. Increased demands were placed on the students to apply their conceptual understanding of fraction concepts. Lyn tended to focus on the immediate conceptual difficulty the children were facing as they tried to solve the problem they were tackling.

The examples drawn from the category of *Scaffolding* demonstrate the degree to which the children are developing their mathematical language and use of mathematical proformas. In Michelle's approach, there is a conscious scaffolding of the children's language. She consistently locates the language which accompanies the mathematical concept for the children (e.g. denominator, lowest common fraction). Lyn, however, tends to focus on mathematical language when it appeared to be responsible for the children's misunderstanding. Sometimes Lyn was able to identify an everyday word for which the children need to adjust their range of meanings. The vocabulary the teacher introduced was readily accessible to the children e.g. diagonal, halves.

Overall, Michelle and Lyn adopted different scaffolding styles, with Michelle diagnosing a child's misunderstanding of a mathematical skill or concept and imposing strict mathematical language. She seemed unable or unwilling to adjust her own language or methodology to embrace that of the child's, or to allow alternative ways of approaching a solution. Lyn tried to use scaffolding to help to shape the skill or concept around the child's understanding. Lyn's own language, however, did not always lead students to a refining of their mathematical language.

Modelling : Providing Opportunities to See the Knowledge, Skills and/or Values in Operation By a Person Who is 'Significant' to the Learner

Introduction

This Principle focuses on the influence that people whom we admire can have over us to a point where we try to emulate them. In all walks of life, increased importance is being given to the use of mentors in which learners are placed alongside "experts" in a supportive way. Authenticity is an important aspect of modelling.

NUDIST Categories

For the purpose of NUDIST analysis this Principle of *Modelling* was broken into its 4 major components and 4 subcategories (see Appendix 7). The major components of modelling are: the *Real Life (5 1)* modelling of mathematics in her own life, displayed by the teacher, the *Charts (5 2)*—either teacher- or child-made—that the teacher makes available; the *Students Modelling (5 3)* and opportunities for the students to see the *Many Uses (5 4)* to which the mathematical concept can be put.

The diagram in the Appendix 7 shows the hierarchical relationship between the major components and the subcategories. The subnode under the node category of teacher's *Real Life (5 1)* modelling refers to the occasions when the teacher is *Overt (5 1 2)* in her efforts to show how she is using a particular mathematical skill in her own life. Under this sub category at another level is the *Informal Explanation (5 1 2 2)* with which the teacher may model her use of mathematics in her own life, as she interacts with individuals in one-to-one or small group situations. The teacher may share her own *Grappling (5 3 1)* with the processing of an idea which may not be coming to her easily. Providing opportunities for the *Students to Compare (5 3 1)* how they derived a solution to a problem with their own approaches has been interpreted in this study as reflecting the importance of valuing different approaches.

Table 5 (see Appendix 9) shows the totals for each NUD!ST component and subcategory of *Modelling* in the "typical" lesson series of Lyn and Michelle. Figures 11a and 11b show the average of the combined totals of the four raters for the different components and subcategories of *Modelling* for each teacher.

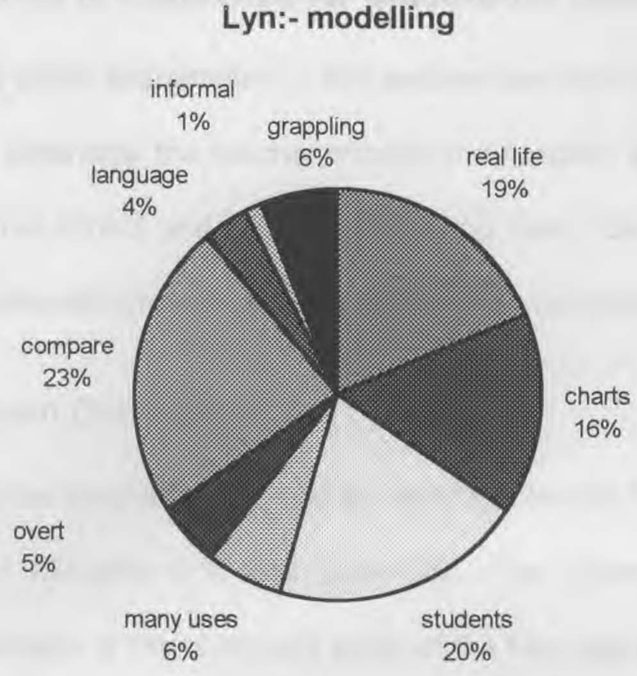

Figure 11a. Summary of NUDIST categories for Modelling as classified by four raters for Lyn.

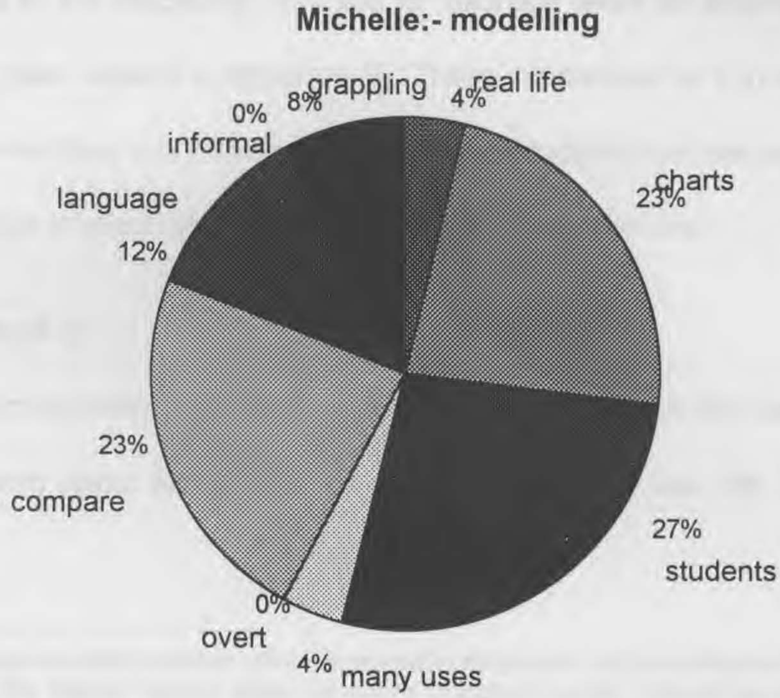

Figure 11b. Summary of NUDIST categories for Modelling as classified by four raters for Michelle.

Modelling—Terms of Reference for Classroom Observations

The modelling under examination in this section has been analysed in terms of how overtly or otherwise the teacher models the function of mathematics in her own life—both at school and at home. Modelling here, does not, in general, refer to teacher demonstration of how to do classroom mathematics exercises.

Data Analysis and Discussion[9]

Figure 6 indicates the difference in totals identified for this Principle between Michelle and Lyn: Michelle—7% and Lyn—12%. The information has been drawn from the average of the combined totals of the four raters for the different components and subcategories of each Principle. The number of incidences recorded in the *Modelling* Principle for Michelle were 26 whereas for Lyn there were 81 (see Table 5 in Appendix 9). These incidences for Lyn were the result of the opportunities she took to share with the students her personal life in regard to the ways in which she used mathematics in her own life.

Real Life (5 1)

This component of *Modelling* refers to the information the teacher displays to the children about the function of mathematics in her own life. Figures 11a and

[9] The discussion which follows will address each component and subcategory for Lyn and Michelle. The "typical" lesson series pie charts (Figures11a and 11b) will be referred to but other data outside that lesson will be drawn upon. Other lessons will be referred to either as Field Notes (TU) (Since the analysis has been based on the field notes, reference will be made to text units- TU deriving from the treatment of these field notes as part of the NUDIST analysis), lesson transcripts (LT) or Diary Entry (DE). Data will also be drawn from the Initial Teacher Interview (ITI), Post Teacher Interview (PTI), Initial Student Interview (ISI) and Post Student Interview (PSI).

11b show the average combined total of the four raters for the component of Modelling—19% for Lyn and 4% for Michelle.

With regard to this component of her teaching, Lyn would often share events or activities from her personal life with the students, for example, they helped her to plan her daughter's party, she coached them in tennis and helped them work out the scoring and the fixtures, she explained how she worked out how to sew her own clothes and showed these to some students. When the Principal went on an overseas trip, Lyn talked with the students as they plotted his journey. These field notes demonstrate some of the occasions in which she revealed how mathematics was related to her own life:

TL *Lyn shared with the class that she wants to buy a new dishwasher, she had been given the following quotes from different companies and/or stores. She wrote the list of five figures on the blackboard.* (Lesson had begun by discussing the previous night's homework to locate certain speed limits on their way to and from school. The class was working out how to derive an average from a set of numbers when Lyn shared her problem concerning what was the average price she could expect to pay for a new dishwasher. She had been given 5 quotes from various companies, the class worked out what would be an average price for a dishwasher 13.5.92)

TL *Lyn provides examples of timetables from my diary and hers* (Students needed to be able to read different timetables, so she showed a sample to give them a real life idea 28.5.92)

Her comments in the Post Interview support her belief that it was important to share with the children how she used mathematics in her own life. Lyn was asked the question *"Do you normally share your own experiences with the students?"*:

PTI *Yes I do, whenever I, as I was at this time, it was the dishwasher I think they worked that out for me. If I'm going overseas or if I'm* *travelling interstate, how long will it take me how much will my fares be, yes I do, because they will have those sorts of experiences somewhere down the track.*

Lyn's students were asked to comment on the question *"Do you think Mrs M uses mathematics in her everyday life?".* The students' responses have been summarised in Tables 2a and 2b later in this section. The information shows how her students perceived Lyn used mathematics as a tool in her everyday life.

Michelle presented herself as a fairly private person and so it was unlikely that the students would know much about how she went about her day-to-day life except for the thoroughness with which she prepared for the mathematics lessons. Her total was 4% (see Figure 11b) which was derived solely from one rater's coding of the following as *Real Life (5 1):*

TL *Students were given 14/50= ?/100 and were asked to make equivalent fractions. She asked the students to offer a fraction which the whole class could solve as an equivalent fraction .../100. Michelle selected those problems which demonstrate equivalence easily.*

When asked the following question during the Post-Teacher Interview, *"Do you normally share your own experiences with the students?"*, Michelle seemed unclear about what aspects of her own experiences she shared with the students:

PTI *Oh whenever I can, if the opportunity arises just to show that we all use maths in some context in one way or another.*

Jennie: Have you got some you can say? What sort of things?

Michelle: I couldn't think of any in particular, I just generally when I'm talking—distance ratio, you can talk about how far you travel in your car, that sort *of thing. Just to pick up on everyday issues, nothing in particular.*

Michelle's students were asked to comment on the question *"Do you think Mrs W uses mathematics in her everyday life?"*. The question was asked at the Post-Student Interview in order to gain information about how Michelle's students perceived she used mathematics as a tool in her everyday life. The student answers are set out in Tables 2a and 2b near the end of this section.

Overt (5 1 2)

This subcategory of *Modelling* provides specific detail about what the teachers model, and refers to the situations when the teacher shared the personal way in which she goes about solving a mathematical problem. The average combined total of the four raters for the subcategory of *Modelling* was Lyn—5%, and Michelle—0% (see Figures 11a and 11b).

Lyn made a point of modelling her own thinking strategies for the children during mathematics lessons:

ITI *... you have to model the stages you go through and take it step by step logically and clearly and explain why it's happening and what you're doing to reach that level, or to reach that answer ... it's important that they see all your working out, as little or as much as it might be, the more working out I believe the better because then that helps them to see the patterns and the connections in your thinking.*

Some examples Lyn shared with the class of the personal way she solved a problem were found in the following field notes. In each case, the students were able to choose their own way of recording and were not directed to use Lyn's approach:

TU 85-87 *Students had to vote for each and then work out preferential scoring. Lyn demonstrates how to tally and do preferential scoring. Students used blackboard to work out and record most popular drinks and record it in their own way.* (Lesson deciding which food should be considered for the class party 4.3.92)

TL *She demonstrated on the easel how to do "average" speeds ... The students were gathered near the easel ... The students then went back to work on finding the average of quantities they would be familiar with (e.g. body weights of their friends ...) ... Lyn made point that when students gave their answers they were to share their different strategies for arriving at that answer. She encouraged their responses and doesn't give the correct answer away too early in the discussion.* (Students needed to know how to calculate averages for determining which route and transport would be the most efficient to get to their school camp 13.5.92)

There were no recorded references made to Michelle's "own" way of processing mathematically or any application to her day-to-day life. The examples taken from the "typical" lesson series, the March and August lessons show Michelle modelling for the students the model which was consistent with the method they would find in their textbook. As students worked from the textbook for homework the example became a reference point which was reinforced. The author made the following observation after conducting classroom observations over eight months:

TU 742 *Students not seeing the teacher model using this skill* (equivalent fractions) *or concept in order to solve a mathematical problem let alone real life*

Michelle's modelling was not included in the terms of reference for this section on *Modelling* because it was in the form of providing an exemplar for the students to follow which is discussed in the *Metacognition* section, rather than providing any personal strategies she might offer from her own experience.

Language (5 1 2 1)

This subcategory of *Modelling* is used to describe the language used by the teacher to communicate mathematical meanings which are associated with the teacher's use of mathematics in her own life. The average combined total of the four raters for the subcategory of *Modelling* was Lyn—4% and Michelle—12% (see Figures 11a and 11b).

Where there may have been possible confusion over certain vocabulary Lyn's discourse would act as a model of the correct use of that language in that context:

TL *She introduced the word "average" and explained why the children would be needing it* (Lesson looking at maximum and average speeds different forms of transport have in order for them to calculate the most efficient route and form of transport to get to class campsite 5.5.92)

TL *Students were seated in groups, and Lyn asked "What's meant by a timetable? She did this to establish the meaning of the word and to avoid any possible confusion with the word "times-table".* (Lesson extrapolating the features of timetables, which they will need to get to the campsite 28.5.92)

Michelle's classroom discourse would act as a model of the correct use of specific vocabulary for school mathematics. Some of the terminology recorded included the words septagon, octogon, heptagon ..., denominator, numerator, and lowest common factor, for example.

Informal Explanations (5 1 2 2)

This subcategory of *Modelling* is a subset of the previous subcategory of *language (5 1 2 1).* It refers to the teacher's spontaneous interactions with small groups or in a one-to-one situation between teacher and student about the use of mathematics in the teacher's everyday life.

Figures 11a and 11b shows the average combined total fo the four raters was 1% for Lyn and 0% for Michelle, which may reflect more on the difficulties in capturing these kinds of interactions in the data collection used in this "typical" lesson series sample. The result reflects very little of the unstructured occasions in the class and the opportunities for informal discussion. The following excerpt indicates the degree of informality the students and Lyn shared in their classroom discourse. One student, Paul, had been listening in on the discussion which was happening at another table and was drawn into the discussion:

LT *Lyn (4.3.92)*

Paul: (overhearing from another group and interrupting) But they may not like ham.

Joe: We could conduct a survey.

Lyn: Right, so you could conduct a survey to see how many people liked sandwiches.

Paul: But you would need to do it with other things like Coke ...

Lyn: Right, so you need to put stars against those things that you need to do surveys with—favourite fillings, favourite chips, favourite lollies ...

Ben: Well so far its costing us ...?

Paul: (to Ben) What about drinks?

Ben: We could just buy squash.

Paul: No you need Coke and not everybody likes squash.

Lyn: Could you get away without doing the drinks by just buying a variety?

Paul: But it costs more. But if you work out exactly what everybody wants you could even it out. But if you buy a variety then you ... Children, might miss out ... on someone.

Lyn: Right. In your drinks are you buying them in cans or bottles?

As this exchange suggests, where there was overt teacher input it tended to be jointly constructed with the active contributions of the students, for example Joe suggests conducting a survey. The informal explanations of Lyn's concerning conducting the survey actually get taken up by the class later in that lesson when the whole class reports back. That group of students was given the responsibility of preparing the survey sheet for the rest of the class to use as a model (see Lessòn Transcript 4.3.92 in *Scaffolding* section).

Michelle's total was 0% for this subcategory because she provided most explanations at a whole class level. Even when talking to individuals at their desks she would often direct her comments to the whole class who were working quietly. The following transcript shows that even in a less formal interaction with a small group Michelle consistently reinforced the specific more formal mathematical language associated with the particular concept being taught.

LT *Michelle (31.8.92)*

Michelle: (To the whole class) Remember if you divide something up into quarters does each quarter need to be the same size? Or is it alright if someone gets less than another person? It must be the same size mustn't it, or else its not a quarter. Make sure that you're diagrams are approximately the same size. (pause)

(Michelle goes up to a group of children working at their table)

Michelle: If I divided up a cake would you rather have a third or a quarter?

Child: Depends which sort of cake it was.

Michelle: Ah. You're most favourite cake and I cut it up. Would you rather have a third or a quarter Melissa?

Melissa: Um a quarter?

Michelle: Why a quarter.

Melissa: A third.

Michelle: Why?

Melissa: Um 'cause a third is bigger?

Michelle: A third is bigger than a quarter?

Melissa: No

Michelle: You don't sound too sure. If I had a cake and I divided it up between three would you have more cake

than if I divided it among four? Yes you would. So would you rather a third or a quarter?
Melissa: A third.
Michelle: Yes you would.

Grappling (5 1 2 3)

This subcategory of *Modelling* is used to define text units in which the teacher shows a genuine example of working her way through an unfamiliar problem. It demonstrates to the children orally the language of thinking and changing directions. The average combined total of the four raters for the subcategory of *Modelling* was Lyn—6% and Michelle—8% (see Figures 11a and11b).

In Lyn's lessons there were occasions where the author was able to observe Lyn openly grappling with what the next step might be in order to assist the students, rather than adhering to a prescribed format. In her Post-Teacher Interview, Lyn described how she planned broad steps for a unit of work, but left the finer details to be shaped as the whole class worked on a topic:

PTI *doing this type of thing you would never actually get to the end of your, where you see the end of the task because you could actually go off on tangents the whole way, and then the children they're thinking that they are actually going to calculate or do something else and then we say "OK we're doing decimals now!"*

In the following field note the author suggests that Lyn's grappling was not always clear enough for many students who may have preferred more guidance.

TU 60 *Lyn's system not obvious but developed better as time progressed* (Section of lesson on clustering as a thinking strategy for adding up long shopping lists 4.3.92)

This subcategory of *grappling* was not a common feature in Michelle's class where she was well planned and her daily program had a thought through lesson procedure of how to transmit the concept development. She responded to the statement *"I share difficulties I may have with solving a mathematical problem"* with a hesitant *often*. The 8% was drawn from two raters coding an incidence each.

Charts (5 2)

This component of *Modelling* is the provision of teacher or student-made display materials which can act as a resource or exemplar for the children. The average combined total of the four raters for the component of *Modelling* was Lyn—16% and Michelle—23% (see Figures 11a and 11b).

Lyn displayed children's work so that it could provide a model for future reference and promote pride in quality work as she suggests in this interview:

ITI *I am very much into displaying the children's work where the children go and read each other's work ... the 6's have been peer tutors for the year 3's and they had to find ways that they were going to teach the year 3's- 10 and 100. They had made their own booklets- 10, 100 and 1000, and the cry came back, "Can I go and get my book and talk to the children about 10mls and 10kgs." So things that they have recorded then, they had to go back and teach someone else . That way then they are clarifying in their minds what they understand by those concepts."*

Around Lyn's classroom the author noted the following in the field notes:

TU 3 *lots of mobiles—mathematics projects e.g. the origins of maths, poetry, percentage definition poster*

TU 8 *Big books made by class also maths e.g. book of 14, 10, 100, 1000 etc*

TU 90 *also on blackboard the posters for the class party e.g. party location, preparation of food, time and duration, serving of food, timetable.*

During an extended topic the posters on display acted as a model for the kinds of prompts the students needed to consider as they worked through the topic. This transcript suggests the flexibility with which the prompts can be used by the students.

LT *Lyn (4.3.92)*
Lyn: ... These are just some suggestions you can discuss this later. I'm going to give you some ideas, give you some clues. (Blu tacs posters around walls). I'll put them up here so you can all see them. With your food—what about considering these kinds of things? Can you read that for me please Christian?
Christian: Food 1. Look at organising a balanced menu. 2. Calculate what amount required to feed the class. 3. Use ads to calculate costs of items.
Lyn: So as I send you off to look at your food you need to consider 1. organising a balanced menu. What do I mean by a balance menu? Reuven?

There was a frequent changeover of both teacher and student-made charts around the room. A storage rack had a collation of all the students' work next to the easel should it be needed for reference during whole class discussion.

The charts and mobiles displayed around the room in Michelle's classroom were for her regular class (referring to reading and written language procedures) although some included mathematical content, for example the numbers from 1-10 in Italian. Michelle did not refer to them with the mathematics class in this study. She provided handouts and activity sheets as a resource or reference:

TU 417 `Hand-out activity sheet of cut out shapes` (Lesson on polygons)

TU 284 *fraction- decimal- % on blackboard* (Before lesson showing relationship between these)

The textbook invariably had an introduction to each concept which included an exemplar for how to do subsequent problems of its type.

Students (5 3)

This component of *Modelling* refers to the use of students for the purpose of displaying to others how they have used their mathematical skills. The average combined totals of the four raters for the component of *Modelling* was Lyn—20% and Michelle—26% (see Figures 11a and 11b).

Lyn frequently used students to demonstrate to their peers in order to encourage a variety of methods and to encourage the use of language to which students felt they could relate. A regular time-slot for students to share their approaches was built into each lesson, and a high level of participation from all class members was expected.

TU 87 *Students used blackboard to work out and record most popular drinks and record it in own way*

TU 120 *Five different sheets, each group will report on findings* (Each group had displayed the results of a brainstorming session to describe what they would like to have at a party 11.3.92)

TU 489 *Paul S idea on angles was shared with class* (Students have been researching trying to locate the mathematics behind the different Olympic sports 5.8.92)

As discussed in the *peers* section of *Scaffolding*, Michelle would use students as a model when she felt confident that their contribution would be consistent with the direction the lesson was heading. She would often ask students to read aloud the question from the textbook.

TU 65 *Teacher gets students to support answer and reasoning* (Lesson was about changing mixed fractions to improper fractions in order to subtract. Students were asked to say how they would do it 28.1.92)

TU 467 *asks students to demonstrate* (Lesson was on polygons 3-10 sided figures. Students were asked to show on blackboard how they drew each 4.6.92)

TU 490 *some students came up with the formula over the weekend* (Lesson was on deducing the formula from the relationship between sides and angles in each polygon 9.6.92)

A comparison of the number of students who are actively contributing in whole class discussion in each class was made in Tables 1a and 1b and presented in the *joint construction* section of *Metacognition.*

Compare (5 3 1)

This subcategory of *Modelling* is a subcategory of the previous section on *students*. It refers to text units which relate to how teachers involve a range of students to demonstrate their approaches to a given problem, so that a variety of strategies can be available for class members to choose from (Lyn—23%, Michelle—23%, see Figures 11a and 11b).

In Lyn's class the frequency of text units categorised as *comparing* was 23% for this "typical" lesson series. This relatively high frequency reflects the opportunities students had in Lyn's class to see how others approached a particular problem and compare it with their own approaches. Lyn asked students to the blackboard often to share their strategies.

TU 87 *Students used blackboard to work out and record most popular drinks and record it in own way* (Students given choices of cans, bottles, boxes and asked how they would buy drinks 4.3.92)

TU 390-391 *Stephen had a unique way of adding and shared it with teacher and found others had his same system* (Students working on averages 13.5.92)

TU 415 *uses students' answers to explain to other students who don't understand* (Students working on reading various forms of timetables 28.5.92)

TU 417 *Blake shares his discovery of pattern in timetable* (Students working on defining components of timetables 28.5.92)

Michelle found it useful to ask students to work in pairs at times which would have given them the opportunity to compare their answers. She routinely compared their answers after the class had been given mental exercises.

TL *The textbook presented the question in a complex manner 25 X .24, which recalled the need for long division (which needed revising), Michelle demonstrated long division on the blackboard. Michelle got them to estimate and elaborate on the logic of their answer. She set them to work and encouraged them to look back at the example the class had done on the board.* (6.4.92)

TL *Michelle discussed different methods for doing problem--80% of 41 and settled on the method 80/100 X 41/1* (Percentage work using estimation first and then process and then check with the calculator 6.4.92)

TU 418-420 *Teacher model first, Rebecca first to show class, Gabrielle C has a rectangle and is resistant in showing how it can be made into a square* (Teacher has cut out shapes which students have to change these polygons into another regular polygon shape 3.6.92

Many Uses (5 4)

This component of *Modelling* refers to exposing the students to the variety of locations in which a skill, concept or particular mathematical feature may appear. The average combined total of the four raters for the component of *Modelling* was 6% for Lyn and 4% for Michelle (see Figures 11a and 11b).

Lyn constructed contexts which by their nature provided follow up and practice for a skill in a variety of situations. For example, tallying needed to be repeated for every food group in the party costings; in the lesson series on the most cost-efficient route and form of transport to Collie, students had been working on averages for speed limits and Lyn then asked them to find averages of several weights by way of practice. This interview comment from Lyn describes how she uses different contexts to help reinforce particular number contexts:

ITI *Cooking I use a lot because that's where I get the things of weights, kilograms, grams, litres, millilitres, that brings in the measuring, the half, the quarter, three quarters, ... I always after a cooking activity do a worksheet that might be if you were feeding, if the recipe says served 8 and you were feeding 16, then the doubling, or you had to do half adjusting it for 4, I do that a lot.*

Comments made by Michelle in her Post-Teacher Interview reflect her familiarity with the content area.

PTI *I like to use a range, usually a mixture of space and measurement and number mixed together, I run those three programs together. If I was doing something in number, then I would try to pick something from space and measurement ... it exposes them to more maths ideas and concepts.*

The two text units reproduced here also reflect how Michelle's "comfortable knowledge" of mathematics means that she can "celebrate the discovery" in her mathematics lessons.

TU 263 *recap at end of lessons getting them to see common themes and make use of knowledge of last one to make it easier to solve future* (Lesson has used three-step problem-solving strategy which she revised at the end 16.3.92)

TU 611 *because of teacher's comfortable knowledge of subject area she is able to 'play' with maths e.g. to see maths patterns and celebrate the discovery of the logic behind these.* (Students have been set an investigation to find out how many squares there are on a chessboard 9.6.92)

The average percentage of incidences of text units which were classified as *many uses* were relatively low for both Lyn (6%) and Michelle (4%). It is possible that the raters found this a category which was quite difficult to recognise.

Post-Student Interview

In their Post-Student Interview, all students were asked *"Do you think your teacher uses mathematics in her everyday life?"*. The question was included in an attempt to learn what message students had received about the role of mathematics in their teacher's own everyday lives. The students' responses are included in Table 2a for Lyn's class, and Table 2b for Michelle's class.

Table 2a

Lyn's class response to Post Interview Question "Do you think your teacher uses mathematics in her everyday life?"

Students	Shopping-related answers	School-related answers	Mixed answers
Ben, B.		"In the mornings she gets out her books and like looks back on some of the things she does and they have maths problems of what she had to do."	
Will, B.		teach people	
Ben, C.		in teaching- doing posters, orders and stuff	
Ben, E.		"in the classroom, how many kids can go at once to somewhere or do something"	"... so she knows how much money she has got, how much she can spend..."
Louise, L.			(Not interviewed)
Chris (male), G.			"She does sewing and she needs to know how long the material needs to be"
Reuven, G.	"One time there was this book sale and you had to work out the prices of some of the stuff for it."		
Stephen, H.			"Tennis, because she is pretty good at tennis, just working out all the bills and stuff like how much her trip was going to cost"
Dean, J.		"teaches us"	"Don't know"
Matthew, J.			"Adding up money and score of games like tennis."
Melanie, K.		"school work"	"cooking, maybe talking about something"
Melissa, L.	"shopping"		"Just around the house, with the angles in her house and all those sorts of things."
Stuart, M.	"shopping"	"school"	
Carmen, O.	"shopping"	"teaches us"	"everyday needs"
Oliver, P.	"shopping, clothes"		
Victoria, P.			"Everything"
Blake, S.			"To work out how much money she has saved for holidays, and to buy food, and to bank money and how much money she gets a month, or every two weeks. And how much money she'd get spare as well to spend on luxury."
Brenda, S.	"shopping, buying things, dresses"		"dresses and things, making how much material she'll need."
Mark, S.	"shopping"		"If the person gets it wrong on the calculator... um the machine, whatever it is, she can try (and correct them)."
Paul, S.	"shopping"	"teaching"	

Table 2a

Lyn's class response to Post Interview Question "Do you think your teacher uses mathematics in her everyday life?" (ctd.)

Paul, T.			"When she was moving she used it and when she does handwriting and everything so she goes nice and straight."
Joe, W.	"shopping, buying something that costs her quite a bit of money"		
Tim, W.			"Like when she's going to America now she had to work out how much it will cost and how much money she's got."
Chris, W.			"If she has milk in the morning, how much she pours into a cup, how much butter to put on her toast."

Table 2b

Michelle's class response to Post Interview Question "Do you think your teacher uses mathematics in her everyday life?"

Student	**Shopping-related answers**	**School-related answers**	**Mixed answers**
Carrie, B.	"shopping"	"adding subtracting"	
Elizabeth, B.			"I don't know. But she's very mathematical person!" She probably just like, say I wonder how, what the area of that thing is (carpet floor tile)."
Genevieve, C.			"I don't know."
Rebecca, C.			"same as everybody"
Gabrielle, C.	"shopping, buying petrol"		
Angharad, D.	"shop"	'teach us'	
Gabrielle, H.			"Maths is basically part of everyday life"
Olivia, K.	"shopping"		medicine
Carla, L.	"shopping"	"percentage in test scores"	"dividing something evenly between anyone"
Elizabeth, M.	"shopping"		taxes
Angela, M.	"shopping"	"teaching"	
Georgia, M.			"She says she does, but I'm not sure she does as much as she says she does"
Adeline, M.			(not interviewed)
Natalie, M.			"No"
Linda, N.	"shopping"	"teaching"	
Emily, P.			"She works out, I don't know, something like out how many students are missing or something I don't know."
Carrie, P.		"add test scores"	"tax returns, cooking"
Catherine, R.	"pay bills"	"teach people"	
Rebecca, S.		"know how many sheets to photocopy"	
Lynleigh, S.	"shopping"	"homework"	
Kylene, T.	"shopping"		"I know that you use maths in just about everything."
Leanne, T.	"shopping"	"timesing"	
Chantelle, V.			"I'm not quite sure.... I don't think so."
Melissa, W.	"shopping"	"teaching"	
Rebecca, W.	"shop- rounding off"		"Not sure."
Alana, W.		"homework"	"lotto"

For both classes their responses indicated a general impression by the students, that their teachers used mathematics for "shopping" and in order to "teach" them. Responses relating to these categories have been placed in columns 2 and 3. The fourth column reveals a qualitative difference between the students' reponses from Lyn's class and Michelle's. More students in Michelle's class seemed less sure of how Michelle would use mathematics in her everyday life than in Lyn's class. The responses of those who were more sure did not reflect a great deal of personal information about Michelle—the responses appeared to be worded in more general terms—medicine, tax returns, cooking. One student (Elizabeth) made an interesting observation. She said:

> *I don't know. But she's very mathematical person!" She probably just like, say- I wonder how, what the area of that thing is (carpet floor tile).*

Students in Lyn's class gave a wide range of responses to this question. It would appear that the students had access to different kinds of information about Lyn's personal use of mathematics, with examples including sewing, playing tennis, book sales, cooking, trip to America, and so on. Because of the informal structure in Lyn's classroom, each student may have had different occasions to access information about how Lyn used mathematics in her everyday life. It appears that the students in the two classes received different messages from their mathematics lessons about the role of mathematics in their teacher's every day life.

Summary

The idea behind the Principle of *Modelling* is the importance of having a significant person authentically modelling the desired skills for learners. Each teacher valued this notion of modelling, whether as modelling in their own approaches or through modelling by their students.

The opportunities for students to model their approaches to mathematics differed for Lyn and Michelle. Lyn would frequently create opportunities for students to share their ideas—her emphasis being that there are many ways to approach a problem. In Michelle's class, the examples which Michelle encouraged her students to present would normally be those which were were consistent with her own approaches.

The kind of mathematical language each teacher modelled for her students also differed. Michelle overtly introduced and reinforced mathematical terminolgy, as discussed in the *Interest* and *Scaffolding* sections. Lyn also introduced specific mathematical terminology, but tended to draw students' attention to it only when there was likely to be some confusion with how it was being used in that context.

The students' views about the role of mathematics in their teachers' everyday lives reflect differences between the two classrooms.

Responsibility: Developing in the Learner the Capacity to Accept Increasingly More Responsibility for Their Learning

Introduction

The premise behind this Principle is that the "expert" is not the one who is generating the learning. Ultimately it is the learner who chooses what is taken from the knowledge, skills and/or values operating in the classroom. In the light of the other preceeding Principles, the notion of learners being responsible for their learning has a more comprehensive meaning. The *Scaffolding* Principle introduced the idea that rather than leaving the learner to survive or else, the learner could be supported to gradually become more independent.

NUDIST Categories

For the purpose of NUDIST analysis, the *Responsibility* Principle was broken up into 2 major components and 1 subcategory (see Appendix 7). The major components of *Responsibility* are: the *Gradual Release (6 1)* that a teacher exercises in order to allow the child to experience some degree of independent control; this will mean that the learner will have to *Accept (6 2)* responsibility for their actions.

The diagram in Appendix 7 shows the hierarchical relationship between the major components and the subcategory of *Responsibility*. The next level under *Gradual Release (6 1)* includes the notion that there should be a *Hand-holding (6 1 1)* function performed by the teacher. This reinforces the idea that the learner is not expected to become independent without support.

Table 6 (see Appendix 9) indicates the combined totals for each NUDIST component and subcategory for *Responsibility* in the "typical" lesson series of Lyn and Michelle, as located by the four independent raters. Figures 12a and 12b represent the average of the combined totals of the four raters for the different components and subcategories of *Responsibility* for each teacher.

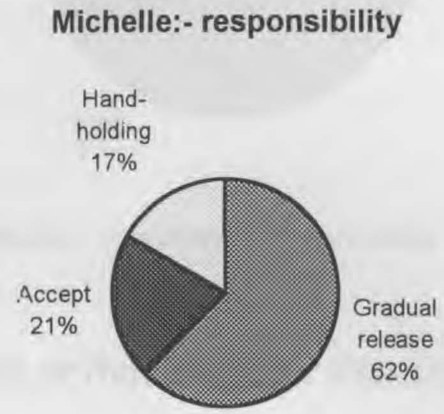

Figure 12a. Summary of NUDIST categories for Responsibility classified by four raters for Michelle.

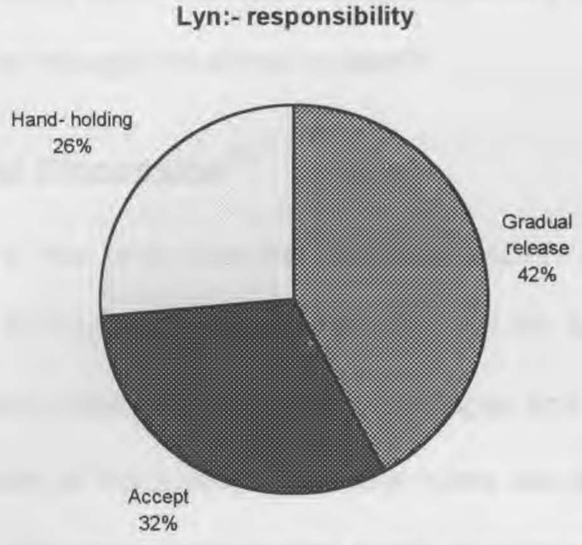

Figure 12b. Summary of NUDIST categories for Responsibility classified by four raters for Lyn.

Responsibility—Terms of Reference for Classroom Observations

The relationship between the teacher and the learner can be one in which there is a negotiation of roles. A misinterpretation of this Principle would be to say that learners do not need teachers and can learn on their own. This may be one interpretation of constructivism but such an interpretation undersells the function that the teacher can have in creating a learning environment which enhances that learning. It is true that teachers cannot force learners to learn. Ultimately it is the learners who choose what meaning to take from what they have been exposed to; it is the learners who decide what they find useful and preferably meaningful. The focus of this section is the degree to which learners are encouraged to take increasingly more responsibility for their learning. The notion of responsibility raises questions about the kinds of tasks with which students are confronted in schools. Do these tasks have any real consequences, other than progression through the school system?

Data Analysis and Discussion[10]

The percentage of text units classified as *Responsibility* compared with the overall total of each Principle was 6% for Michelle and 8% for Lyn (see Figure 6). The small difference between these two percentages and its position as the lowest recorded number of incidences by the four raters out of all the Principles, perhaps reflects the difficulty in capturing this quality in classroom observation.

Gradual Release (6 1)

This component of *Responsibility* is concerned with the teacher's ability to hand over responsibility in a gradual manner, to the learners rather than leaving them to their own discovery of the world, or alternately, expecting too much too soon.

This sub category of *Gradual Release* received the highest number of totals recorded by the interraters for Lyn (42%), compared with the other two subcategories of *Responsibility* (Accept-32%, Hand-holding-26%, see Figures 12a and 12b). In Lyn's class, although the tasks were always real life tasks for which the consequences for their problem-solving was significant, the early

[10] The discussion which follows will address each component and the subcategory description for Lyn and Michelle. The "typical" lesson series graphs (Figures 12a and 12b) will be referred to, but data outside that lesson will be drawn upon. Other lessons will be referred to either as "March" lesson (ML), "August" lesson (AL), Field Notes (TU) (Since the analysis has been based on the field notes, reference will be made to text units- TU, deriving from these field notes as part of the NUDIST analysis), and lesson transcripts (LT) or Diary Entry (DE). Data will also be drawn from the Initial Teacher Interview (ITI), Post Teacher Interview (PTI), Initial Student Interview (ISI) and Post Student Interview (PSI).

tasks for the year had less grave consequences than the latter ones. For example, the tasks progressed from: spending $150 on a class party, to planning the most efficient way to get to their school camp and in the last part of the year the tasks involved the students in marking to scale the school oval for each sports event in the school Mini Olympics. The earlier tasks were more corporate and therefore shared the responsibility among the group members.

TU 119 *the students were recording food preferences of their classmates on sheets and were given the freedom to record or tally the information in whatever way they found suited them.*

One of the later tasks in the year was individual and therefore there was more chance for an error to slip through, for example, to the groundsman who was trying to use the students' drawings to mark out the oval.

In the August Lesson field notes, the author documented the way in which Lyn would start a topic which would demand quite a high degree of responsibility for decisions from the students and how she gradually adjusted the task as it became apparent that they needed more guidelines. These excerpts came from the August lesson (see Appendix 3)

AL *8.30 Lyn began the lesson by reminding them about their latest maths project which was for them to choose a sport related to the forthcoming Olympic Games and demonstrate the mathematics in connection with that sport. They could do it as individuals, pairs or in small groups. They could also treat the project like a secret where the class has to guess which sport it was- different parts of the information would gradually be revealed during the presentation...*

Lyn then adjusted the parameters so that they can now choose any sport, not necessarily an Olympic Games related sport ...

Lyn adjusted the task so that the students now have to draw to scale a piece of equipment or the field etc used in the sport they have chosen...

She encouraged the class to share where they got their ideas from e.g. school, reading, home etc, so that they consider a broader range of resources for their project. Several students then went to the library and encyclopaedias in the back of the room ...

This last comment showed how the students had freedom to follow their own ideas and get their own resources within what was acceptable school behaviour.

This subcategory of *Responsibility* as rated by the four raters was 62% for Michelle (see Figures 12a and 12b). In Michelle's class the students' opportunities for responsibility although they were frequent, tended to be defined within the constraints of a specific task:

ML *The teacher stopped the class and gave extension work to the class. She used a new idea with them where she gave the students a task to give their partner a word problem i.e. they give the information/ problem/ sum to their friend to do the working out and then they change over. The teacher set the conditions that she preferred the type of sums to be similar to the subtraction of decimals exercises which they had been doing from the textbook.*

This limited freedom may have been due to the fact that Michelle did not have this class of students full time. The field notes suggest a gradual release of Michelle's work expectations as she monitored the students' ability to handle the work. The following excerpts taken from the "typical" lesson series demonstrate this:

TL *Michelle introduced lesson by saying that they needed to get this work done before the end of term, which was in 2 weeks, and that it would be needed for next semester's work.*

students were given 14/50 = ?/100 and were asked to make equivalent fractions. She asked the students to offer a fraction which the whole class could solve as an equivalent fraction.

About half the class seemed to be following her with their hands up...

Michelle asked them if they remembered doing these last year. As they were streamed last year, most acknowledged they had been exposed to it. Michelle wanted to get on with more complex use of percentage but needed to back pedal until they had this part right.

Qualitative differences between the classes, in the kinds of tasks for which the students were being given responsibility, were observed. For example, the parameters for the students provided by Lyn were broad, but were modified according to how she perceived the students were grasping the idea. Michelle demonstrated that she gave the students quite narrow parameters for tasks and focused on making the curriculum manageable for them.

Hand-holding (6 1 1)

This subcategory of *Responsibility* has been influenced by the ideas found in the *Scaffolding* section. The assumption here is that learners will not be expected to become independent without some support. Figures 12a and 12b show the average combined total of the four raters for the component of *Responsibility* was 26% for Lyn and 17% for Michelle.

Due to the independent nature of the students' work, Lyn was almost always in a position to move around the class assisting students who were having difficulty. When a problem appeared to be a common one for several students, Lyn would stop the whole class and guide them through. Alternately she would take a small group aside and work with them.

TU 56 *Lyn decided to stop and form a class answer so the whole class can move on* (Students grappling with how to add their food bills which have been generated at each table from the different products they have made for their small business)

TU 429 *one group with her on the mat* (Students working on the most efficient route to their class campsite, Lyn has any students who are not sure how to read timetables out the front)

This example from a lesson transcript taken from a series of lessons on the Olympic Games shows one way in which Lyn developed responsibility—by giving the students options:

LT *Lyn:*
This morning we have about three different options that you can do. First of all some of you haven't finished and would like to finish your diagrams and your um of your grid, not of your grids, of your plans. That is an option ...

The options acted as "hand holding," in which the students could experience a degree of responsibility for their own learning.

Michelle also moved around the class in every lesson providing assistance, (hence the total of 17%-*Hand-holding*). As the following example shows the assistance for the students was in order to complete the task.

ML *Carla wanted to use the calculator to check her working out. The teacher encouraged the whole class to do this too.* (Students are doing subtraction of decimal exercises from a textbook)

Michelle encouraged the students to use the calculator as way of increasing their independence as learners (see also "typical" lesson series). She would also encourage them to make up more exercises like the ones she was doing with them in order to develop some responsibility for their learning:

AL *She then set the task for the class to make up other equivalent fractions following this last example.*

The form of hand-holding that Michelle demonstrated was allowing the students some freedom in doing their own tasks in line with a pattern that she, as their teacher, had already established.

Accept (6 2)

This component of *Responsibility* is concerned with the opportunity students are given to accept responsibility for the decisions they make. Figures 12a and 12b show the average combined total of the four raters for the component of *Responsibility* was 32% for Lyn and 21% for Michelle.

In Lyn's class, due to the nature of the real life problems the class worked on, there was usually vested interest, from all concerned, in seeing the problem solved. Students were given group decision-making tasks, the results of which were likely to have an impact on the whole class.

TU 28 *recipe written by Lyn and children had to decide in 6 groups the utensils they'd need*

TU 136 *Find ways of discovering which foods are the most popular so we can have a menu suitable to all of us*

It is also clear from the Post-Teacher Interview with Lyn that the children were entrusted with a considerable degree of independence in relation to their study habits:

PTI *They do research in school time, and they can work for any length of time on a topic, not just for half and hour, if it takes the whole morning, it takes the whole morning, that's fine. I may not have it the next day, or it could go for 3 days, at a time*

Lyn's mathematics lessons would usually be the first lesson for the day. Then, if the students were heavily involved in their tasks, it would go on until morning recess and sometimes even until lunch time. Encouraging students to accept responsibility for their learning was Lyn's main goal in her classroom:

ITI *That's a very strong feature, just part and parcel of all the activities in the classroom- that they take responsibility for their learning and have that same ownership, have that desire to want to look for different patterns, look to improve their mathematical understanding, the same way as with learning whatever subject it be.*

It was apparent that as well as giving the students tasks which would potentially make an impact on the whole class, Lyn allowed students the opportunity to develop responsibility for their own approach to learning.

In Michelle's class she would encourage students to take responsibility, especially in regard to tests.

TU 608 *Homework "which ever area needs most improvement in your tests"* (Students have recently had some test results returned and although homework is expected, none has been specifically set)

In Michelle's classroom, one responsibility given to the students was that of proving to their teacher that they knew what they were doing, as captured in this extract:

LT *31.8.92 Michelle: A lot of you have realised that girls there is an easy way to do it. For instance in this case, two times four is eight add three is eleven, eleven quarters. But I want you to show me what it means in real terms. If I did have two and* *three quarter cakes, how many quarters would I have I want you to show me the picture of it. Just to prove to me you really understand what you are doing.*

It was encouraging to see one student, Georgia, accept responsibility for doing some extra homework after having completed a lesson on fractions and still being unsure.

LT *Michelle 31.8.92*
End of a forty-five minute lesson.
Michelle: Alright could you have a look in your practice books at page (pause) 76 I think. Yes 76. Look at the page because the numbers aren't (pause) as evident. Girls last time we did page 76 first column. Tonight would you do the last column 4-48, just that last column questions. Be careful after question 24 it changes. The last column. Yes Georgia?
Georgia: Is there a test in our practice book?
Michelle: No there's not.
Georgia: Are we allowed to bring it home—our textbook?
Michelle: (To Georgia) I'm quite happy about that. Questions four and that column down the page. I would like your books though girls, so do that on paper tonight. On my filing cabinet, no I'll stand there. Open up to today's page and I'll collect your workbooks please.

When asked in the Post-Teacher Interview to respond to the statement *"Students have to accept responsibility,"* Michelle replied "*often.*" However, she explained her interpretation of students taking responsibility in her Initial-Teacher Interview:

ITI *I don't tend to give the children a lot of leeway in that regard I tend to set the problems and I tend to set the curriculum, no there are very few instances where the children get to develop their own learning. As far as taking responsibility, yes I tend to give them responsibility in that, for instance they are in group and they were talking about a particular concept, now if they want to extend on that or if they want to go on a different tangent then yes that's* *fine, but I set the original problem and if they wanted to take it in different ways then yes they can … although I tend to think that the teacher should know where they should be going and I think that's more important than giving the children opportunities.*

This suggests a tentativeness on Michelle's part regarding children's perception of their ability. It would appear that Michelle perceived that she would know what was best for her students.

In contrast, Lyn provided wide parameters for the tasks she gave her students, and freedom to be able to work out what was best for themselves. In many cases in Lyn's classroom, the field notes suggest that this responsibility was given prematurely.

TU 108 *Students left to own devices often prematurely* (Students had been given a range of newspaper advertisements and had to work out the most economical buys. The wide range of different weights needing to be converted, and the actual calculation of what quantities would be needed, and what proportion of the $150 could be used on various products, had not been discussed)

TU 141 *Teacher seemed unconcerned with actual rate of individuals except those who prompted her for questions* (Students were recording preferential voting information in various forms, many of them graphs and some using pie graphs which involved percentage conversion they had had no experience with.)

In Lyn's classroom, there were also occasions when the students seemed to find the tasks so wide that they were uncertain what was being asked of them. The following excerpt from the "typical" lesson series demonstrates this:

TL *Lyn provided examples of timetables from my diary and hers, although these were not exactly like the timetable she was to discuss later (Train timetable). Lyn conducted a whole class brainstorming session in which she summarised their experiences. She established the meanings the children had for the word "timetable" e.g. bus, train, boats, planes, school timetable, hospital,* *movies, roster, event (like a program), TV guide, clubs, sports (fixtures). They all felt free to contribute and at times were more expert than their teacher. Lyn suggested that they cluster the information. Three categories emerged e.g. 1- program, 2- diary, 3- fixture. Lyn set the task for them to list the features they would expect to see on a timetable. This was attempted by the students although it has not been made explicit how this would help them to calculate the most efficient route to the campsite. About six students misunderstood and actually started to design their own timetable. Lyn stopped them and announced to the class why certain information would need to be on a timetable.*

Lyn had high expectations of the students which had to be adjusted as they learned to take on more responsibility for their learning.

Post-Student Interview- The Textbook Problem Analysis

In order to gain some insights into how well students had accepted responsibility for learning the content, the students were asked to indicate on a five-point scale what they perceived their capacity was in performing certain mathematical processes or types of problems. The problems were representative of the type of content covered in each class over the 9-month observation period. A different textbook was used to cover the range of content covered in each classroom. The textbook, which had been approved by the class teacher as suitable for the year level, had not been previously used with the students of either class. The students were told that they would be shown a range of problems which, although they did not have to calculate the problem, they understood, that if called upon during the interview, they could be asked to do the problem. The students were not told whether their teacher would be shown their comments.

The mathematical processes and/or problems for Michelle's class were as follows: addition up to 8 digits including decimals, subtraction up to 8 digits, problem-solving in sentence format, conversions both time and distance, pronumerals, percentage as fractions not decimals, ratio, maps, tables and graphs, multiplication including 2-digit decimals, powers, division including 4-digit division, averages, factors, perimeter, geometry in relation to solids and angles, fractions each of the 4 processes, area of rectangles and parallelograms, volume including prisms.

The 5 point scale was as follows:

? = never learnt
X = don't know
O = O.K.
√ = easy
√√ = confident

Each student's class mathematics teacher was then given the results of the oral interview of the student's perceptions and asked to enter onto the following table their perception of each student's assessment according to the following three headings: Fairly good estimate, over confident, underconfident. The results are shown for Michelle in Table 3a and for Lyn in Table 3b.

Table 3a

Michelle's responses to students' assessment of their ability to perform textbook tasks

Student	Fairly good estimate	Over confident	Underconfident
Kylene	√		
Rebecca			√
Emily		√	
Melissa W	√		
Linda			√
Gabrielle H	√		
Catherine	√		
Carrie P	√		
Georgia	√		
Rebecca C	√		
Elizabeth M	√		
Carla		√	
Olivia	√		
Genevieve	√		
Rebecca W	√		
Adeline	√		
Alana		√	
Chantelle	√		
Lynleigh	√		
Leanne	√		
Angela M	√		
Angharad	√		
Gabrielle C		√	
Elizabeth B	√		
Carrie B	√		

The mathematical processes and/or problems for Lyn's class were as follows: problem-solving in sentences, distance, timelines, fractions using all 4 processes, area of rectangles and parallelograms, circles, place value, division up to 2-digit, weights, graphs and tables, multiplication up to 3-digit, angles, perimeter, directions, solids, conversion, percentage.

Table 3b

Lyn's responses to students' assessment of their ability to perform textbook tasks

Student	Fairly good estimate	Over confident	Under confident
Melanie		√	
Oliver	√		
Mark S	√		
Brenda	√		
Blake	√		
Victoria		√	
Carmen	√		
Stuart	√		
Melissa	√		
Matthew	√		
Stephen	√		
Dean		√	
Tim W	√		
Chris			√
Ben B	√		
William	√		
Ben C	√		
Ben E	√		
Christian	√		
Reuven			√
Joe	√		
Paul T	√		
Paul S	√		

The concordance of tho students' rating of their own ability with the teachers' perceptions of those students' abilities suggests that most students have been realistic in their self-assessments. Given the different teaching styles in operation in the two classrooms, this is an important finding.

Summary

It is important to consider the function of this Principle of *Responsibility* within the operation of the other Principles. The *Scaffolding* section, in particular, discussed the idea that responsibility is developed through the relationship between the teacher and the learner. The teacher can allow children ownership by gradual degrees, letting them increasingly experience the consequences of their actions. This could be observed in Lyn's classroom in her final topic for the year, where the children's mathematics was integrated into the curriculum in such a way that the children were responsible for designing to scale, the Mini Olympics courses for each event. Their diagrams were then passed onto the gardener, to use for marking up the oval. The potential lesson to be learned was more than using scales to represent the oval—there were obvious consequences to the child's mis/calculations. The activity provided a situation which demonstrated that loose approximations would be inadequate.

In Michelle's class the consequence for students not accepting responsibility for the learning was a slowing in their exposure to the more advanced features of the concept. It would appear that the teacher's concern was that the student's immature attempts at working with a particular concept could become a permanent fixture of the child's repertoire unless the teacher intervened.

The results of the textbook study (see Tables 3a and 3b) suggest that students have a fairly accurate idea of what they know. Regardless of the teaching approaches used, ultimately it is the learners who choose what meaning to take from what they have been exposed to; it is the learners who decide what they find useful and meaningful.

Community: Creating a Supportive Classroom Environment Where Children Feel Free to Take Risks and be Part of a Shared Context

Introduction

The preceeding Principles presuppose a certain classroom climate in order for the ideas for enhancing learning for the students to be maximised. A supportive classroom environment could be described as a community of learners in which the participants can share their successes, their problems and their failures. The learners in such an environment can assume that they will not only be understood by those with whom they communicate, but also that they will be valued regardless of their level of understanding or skill level.

NUDIST Categories

For NUDIST analysis this Principle of *Community* was broken into 5 major components and 9 subcategories (see Appendix 7). The component of *Support (7 1)* refers to how the teacher and students relate to one another. The teacher in this definition of *Community* provides for and acknowledges different *Learning Styles (7 2)*. The teacher encourages the creation of a community where learning is seen as a social, rather than an individual accomplishment and one where it is possible for all participants to become a *Resource for one Another (7 3)*.

The community culture being described here is one where the tasks the students are involved in are an *Empowering (7 4)* experience—not one that finishes at the door of the classroom or at the end of the year. It describes a classroom culture which accepts the different aspirations that each might have, and conveys an *Expectation of Success (7 6)* for the outcomes of each student.

The hierarchical relationship between the major components and subcategories is shown in Appendix 7. The next level under *Support (7 1)* is the necessary ingredient of *Risk-taking (7 1 5)*. One way a teacher can encourage students to take risks is where the *Teacher Admits faults (7 1 5 1)* from her own experiences. The students may recognise from this behaviour of their teacher that they are not the only ones on a learning journey. Where participants are open about their misunderstandings it enables the teacher to *Start with Errors (7 1 5 2)* the students are making and assist the students in building their own constructs.

Within the component of *Learning Styles (7 2)* is the subcategory which describes the designing of classroom activities which cater for *Different Levels (7 2 1)* of ability. Part of the students being a *Resource (7 3)* for one another is their *Collaborative (7 3 1)* role with one another. This releases the teacher as the main focus of expertise and maximises the contribution that each participant can make to the learning community. In order to participate in *Empowering (7 4)* another not only in order to take control of their learning, but also to be in a position to accept responsibility for the consequences of their decision-making. In order to share with the students her *Expectation of Success (7 6)* the teacher will need to be quite explicit about both her *Work Expectations (7 6 3)* of how the class will operate and the *Quality of the Products (7 6 3 2)* the students produce. These components and subcategories are seen not so much as a structure imposed on the classrooms and students, but rather as environments created in a spirit of cooperation and negotiation

Table 7 (see Appendix 9) indicates the totals by the four independent raters for each component and subcategory for *Community* in the "typical" lesson series of Lyn and Michelle.

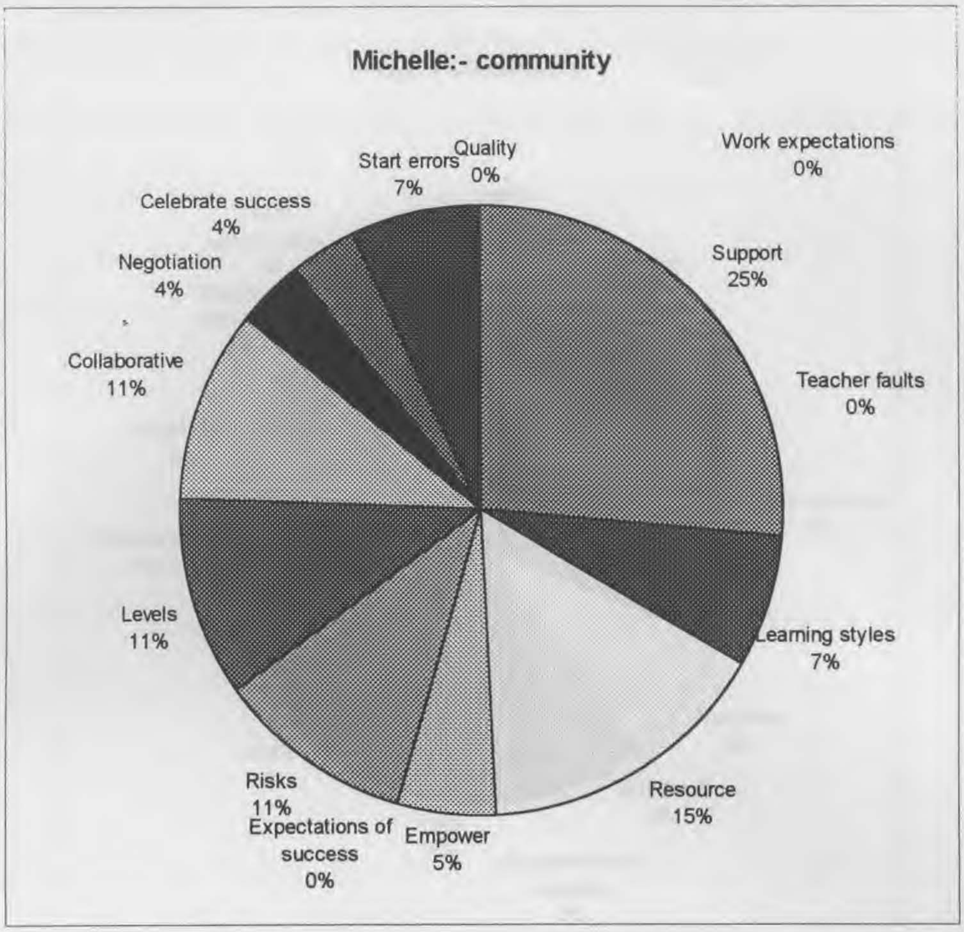

Figure 13a. Summary of NUDIST categories for Community as classified by four raters of Michelle.

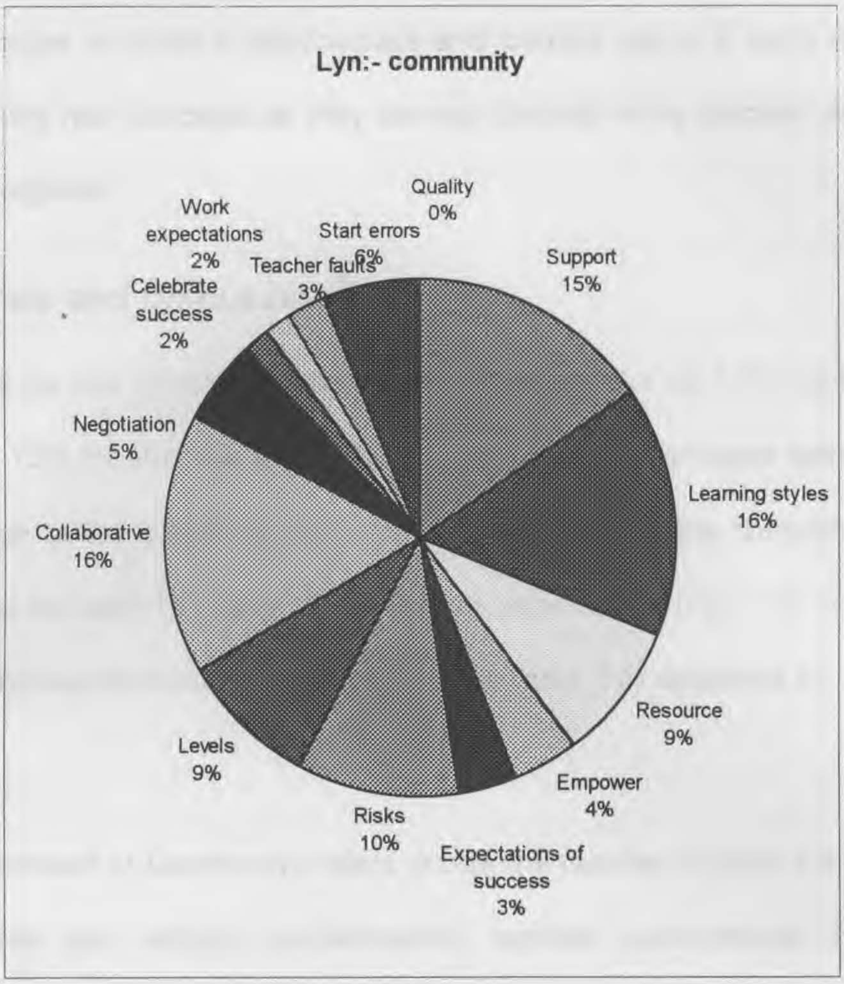

Figure 13b. Summary of NUDIST categories for Community as classified by four raters of Lyn.

Figures 13a and 13b show the average of the combined totals of the raters for each of the components and subcategories.

Community—Terms of Reference for Classroom Observations

The notion of *Community* in this study will be examined to check the extent to which a culture of cooperation and negotiation has been created by the teacher. Such a culture is one in which students feel free to take risks and are supported in their efforts to take control of their own learning. It is a culture which encourages students to approximate and tolerate learners' early attempts at understanding new concepts as they develop towards more precise use of the mathematics register.

Data Analysis and Discussion[11]

The totals for this Principle compared with the totals for all 7 Principles (see Figure 6) are 15% for Michelle and 16% for Lyn. These percentages were based on the average of the combined totals by the four raters of the component and subcategories for each Principle. Lyn's figures were drawn from 116 combined incidences whereas Michelle's was from 57 (see Table 7 in Appendix 9).

Support (7 1)

This component of *Community* refers to how the teacher creates a climate in which students can, without condemnation, explore mathematical concepts about which they are unclear, both in whole class situations and on a one-to-one basis.

This component pertains specifically to incidences when the teacher built upon an individual's or the class's problem. For both teachers this was a high priority. Figures 13a and 13b show that the average combined total of the four raters for the component of Community was 15% for Lyn and 25% for Michelle.

[11] The discussion will look at each component and subcategory for Lyn and Michelle. The "typical" lesson series graph will be referred to, but other data outside that lesson will be drawn upon. Other lessons will be referred to either as "March" lesson (ML), "August" lesson (AL), Field Notes (TU), lesson transcripts (LT) or Diary Entry (DE). Data will also be drawn from Initial Teacher Interviews (ITI), Post Teacher Interviews (PTI), Initial Student Interviews (ISI) and Post Student Interviews (PSI).

For example, the following field notes taken while observing Lyn's lessons, suggest that she showed a high level of responsiveness to the students in her class. In both examples, not being able to understand a mathematical concept was likely to prevent the students from fully participating in and contributing to, decisions the students in the class were making. The first example refers to the class party, and the second example is an extract from the lesson dealing with finding the most efficient trip to the school campsite.

TU 135 *Teacher recapped and put major question on board although by that stage the students had had it confirmed by her group discussion and didn't need the prompt.* (Question: Find ways of discovering which foods are the most popular so we can have a menu suitable to all of us)

TU 429 *one group with her on the mat* (Lyn was working with a group who were still not sure about how to read a timetable after yesterday's discussion)

In a similar way Michelle's field notes suggest her sensitivity toward individual students:

TU 266 *Emily (new to class from top group) got flustered by presenting the combinations, liquid papered and then was relieved when teacher stopped her.*

Michelle admitted that she found it difficult to provide enough support to her students during class and said in her Initial-Teacher Interview, that she was holding tutoring sessions with some of the girls after school:

ITI *There are some children that need a one-to-one basic and as a teacher of 23 children in a classroom I just physically don't have the time in my particular method. I have a lot of girls in maths tutoring after school and they are just blossoming because they have got that one-to-one attention ...*

It is interesting to note that Michelle stated her "particular method" made it difficult for her to service all the children in the class. By many standards, Michelle's class with 23 students was not a large class size.

Risks (7 1 5)

This subcategory of *Community* refers to the messages that teachers give students that it is alright to "have a go" and make mistakes without condemnation.

As will be illustrated, both teachers would argue that they encouraged their students to take risks. The incidence of this Principle varies according to the topic. This Principle was less apparent in the "typical" lesson series for both teachers, and the percentages for both Lyn (10%) and Michelle (11%) should be regarded as low.

Lyn's dialogue was such that students felt free to contribute and have a go at an idea rather than trying to guess what it was the teacher wanted to hear, as this field note suggests.

TU 34 *lots of alternative questions from students* (Teacher and students are calling out questions they would need to ask of the business they are setting up e.g. "Will they make a gain or loss?")

Lyn discussed what she was striving to achieve in her mathematics teaching in her Initial-Teacher Interview. She felt that it was pivotal for students to feel free to take risks:

ITI *I respect the difficulties that children experience in learning maths in coming to terms with all sorts of skills and concepts that they need. Taking the risk, allowing them to have a go at it, it doesn't matter if its wrong, have a go at it even if the answer is not right,* *look at all the steps along the way. Providing an environment that is mathematically rich ...*

Michelle encouraged risk taking when the topic was from the "Space" strand rather than a topic from the "Number" strand which she considered to be more "core" in nature.

TU 423 *Catherine discovered septagon rather than heptagon as the terminology which caused classroom investigation for homework* (Students had a discovery task to find a pattern in regular polygons)

TU 238 *Teacher encourages students to be 'brave'* (Students had been working on calculating the different combinations of coins (1c, 2c, 5c) which would make up 8c. This activity had not been planned as part of the main lesson but was included, at Michelle's initiative, in response to students' difficulties in performing such tasks. These difficulties were revealed in my Initial Interview with each student, and I had shared these details with Michelle.)

Michelle discussed the difference in classroom interaction when working with different strands from the syllabus in her Post-Teacher Interview:

PTI *I think number lessons tend to be very formal and very, um your typical chalk-talk, sit at the seat and do the exercises. I think that's the only way to teach certain number concepts ... When I am teaching more space, measurement, the children are more actively involved and they are the ones recording data, listing numbers, looking for patterns, and coming up with some sort of solutions and I tend to take a bit of a back seat and let them work it out.*

It would appear from this discussion that Michelle considered the "Space" and "Measurement" strands to be more appropriate than the "Number" strand for encouraging students to take risks.

Teacher Faults (7 1 5 1)

This subcategory of *Community* refers to the classroom climate which is created when a teacher openly acknowledges when a mistake has been made. It serves as an example to the students as a natural part of problem-solving and not anything to lose face over. Figures 13a and 13b show that the average combined total of the four raters for the subcategory of *Community* was 3% for Lyn and 0% for Michelle.

In this "typical" lesson series, field notes taken in Lyn's class noted one incidence of Lyn acknowledging a flaw in her knowledge; there were no incidences in the field notes taken in Michelle's classroom.

TL *She discussed the maximum speeds for a bus e.g. 90kph and a car e.g. 110 kph, and she admitted that she did not know the speed for either the train or plane.*

Lyn appeared to be quite comfortable admitting this lack of knowledge. The following text unit is taken from field notes made when the class was working out the costings for producing and selling class-made food as a small business.

TU 35 *Lyn admits faults she's made in explaining:* (Lyn wanted the students to cost in the expenses of electricity and wages etc but had not made it clear how they might work these out)

Lyn discussed in her Initial-Teacher Interview that she did not always feel confident in what she is teaching.

ITI *When I go and do the [in service] workshops and they ask you to do puzzles, and I do some of those things, it has not been something as a strategy I've learned. I have a seven-year-old who could do it better than I could do. When I used to do those things with the children, I guess I overcome it by saying I am not going to be expert, they are going to be!*

Lyn seemed to be quite comfortable with not being the "expert" in all knowledge in her classroom.

Few examples were noted in the field notes for Michelle's lessons to indicate that she admitted her faults or deficiencies in front of the class. One example was noted in a lesson transcript on fractions, which indicated how Michelle might handle such a situation:

LT *31.8.92*

Michelle: Eight sixteenths, divided by eight eighths. We are dividing by? (Pause) one. Eight divided by eight is one. Sixteen divided by eight is? ... three.

Child: Two.

Others: It's two there.

Michelle: Is this correct then? No it's not. It's not eight sixteenths at all. What should it be?

Child: Eight twelfths.

Michelle: Good girl, I'm glad you picked that up. Lucky you were watching. We will have to start this one again. Eight twenty fourths. Eight divided by eight is one, twenty four divided by eight is?

Michelle made no direct apology for the error, and appeared to deal with it in a matter of fact manner so that the lesson could proceed unhindered.

Start Errors (7 1 5 2)

This subcategory of *Community* reflects the teacher's ability to identify how closely the students are following the activities in which they are engaged in the classroom, and whether any back-tracking or adjustment may need to be made by the teacher. For example, the teacher may need to start with the errors the students have made in order to proceed with a concept the whole class is developing.

Both teachers were aware of student errors, and in Lyn's class, because of the use of themes, students were not always doing the same type of mathematics calculations as a whole class, at any one time. The low number of incidences of *Start Errors* identified in the field notes of the "typical" lesson series (Lyn 6%, Michelle 7%) suggests that this was not a strategy highly used by either teacher. It would also be fair to acknowledge that both teachers were being observed and when students failed to understand it was likely to be produce a degree of stress.

These observations from the "typical" lesson series and other field notes demonstrate how Lyn handled a situation in which she felt she needed to back-track.

TL *Lyn demonstrated on the easel how to do average again.*

AL *Lyn took the problem of one student and built on that and encouraged the whole class to share in solving the problem.*

TU 49 *Lyn decided to stop class and form a class answer so that whole class can move on*

TU 441 *Question E [teacher] brings class on floor with a common problem for all of them* (Students had been working from a work sheet using a timetable when they came across a question **"Look at the times of departure and try to explain why the trains leave at different intervals?"** which involved prior knowledge of train routines and the answer could not be immediately drawn from the timetable itself).

In these examples, Lyn drew the attention of the whole class to a problem for which the solution would be applicable to all the students. She chose this approach so that the class, as a whole could make progress.

In Michelle's lessons, the topics changed weekly or fortnightly. Very little backtracking was done. This segment of a lesson transcript shows Michelle backtracking when the majority of students misunderstood:

LT *Michelle:*
Michelle: Alright girls eyes this way. Thankyou. There was a couple of things on Friday that we were looking at, and a few of you had a lot of problems so I thought we'd do a bit of a back track today and have a look at a smaller part of Friday's lesson where everyone had a few problems.

Errors or misunderstandings did present a problem for Michelle who had clearly mapped out the term's work and did not have a lot of space built in for diversions as my diary entry tries to capture:

Michelle 18.06.92 - lesson which was once again driven by the content of the curriculum and pressure perceived by the teacher to fulfil the school's requirement to complete the text book in that year. I feel this agenda is getting in the way and stressing Michelle out because once something goes wrong and she has to back track it becomes visibly stressful for her.
The lesson showed her demonstrating division by decimals ... which was simple division ... Then left them to get on with a problem-solving page from their workbook ... Most of the class was struggling because the problems involved two digit division and division by 10's and 100's which was a weakness for the group although she had been doing multiplication and touched on division last week, as she reminded them. Because of the logical cohesion of her lessons she is most reluctant to let things get in the way of her planning. What it further revealed to me was the weakness of textbooks and the need for teachers to be selective and not just give out whole pages but pick and choose ...
Even once she had drawn the class back together to work on the division issue she was under stress because of running out of time. Reteaching and Georgia's divergent working out couldn't be embraced because of time and stress.

> *Lots were confused and yet the homework given out was totally different, much easier ...*
> *how content driven her teaching is and how inflexible it makes her ... to exclude the children's own working out ...*

Michelle clearly knew when the students were having difficulty, the problem was having enough time allocated in the schedule to do much about it. As discussed above, Michelle was tutoring students after school, which was an indication of the problem she had building in time for their specific problems.

Learning Styles (7 2)

This component of *Community* refers to the teacher's ability to design activities which will enable a range of learning styles to be catered for.

The difference between the two percentages for each teacher (Lyn 16% Michelle 7%) suggests that students in Lyn's class had more opportunities for adopting a range of approaches to accomplish the set tasks. In the overall codings within this component of the Principle *Community* this percentage of instances for *Learning Styles* was the same as that for the subcategory *Collaborative (7 3 1)*—both showed 16%—the highest for this Principle. The following examples from the "typical" lesson series illustrate the instances noted for *Learning Styles* in Lyn's classroom.

TL *Students shared at their tables the strategies they used to work out the average and in what ways it was the same or different from what had already been shared.*

TU 41 *could produce lots of alternative measurements by students* (Students were in groups and working out what aspects they would need to take into account to make their small business cost-efficient)

TU 59 *lots of alternative ways of adding- clustering* (Students were working out the cost per item from a recipe they had just made and intended to sell)

TU 128 *encourages freedom of choice* (Students were recording on grid paper or a graph in any way they chose how the results of their tally could be represented)

TU 143 *encouraged originality* (Students were recording on the grid paper or a graph the results of their tally, representing them in any way they chose)

In speaking with Lyn at the Post-Teacher Interview, it was clearly important to her that the students were given the freedom and support to tackle problems in different ways:

PTI *I like to use the students' ideas because the way I learned to do maths, and the way I see maths may not be the way that the child sees it and I think somewhere in here there was a way that one of the boys taught me a way that I hadn't even thought of. I didn't need that because in my life I could do maths, I just saw the answers and off I went. And even when I help my daughter, I try and show her different sorts of ways and ask her what she's done, because each child comes to it with a different way of thinking about it and how they see numbers. So if they help each other. I think the children could actually help each other better sometimes. You see the adult can only see one answer and can't understand why the child can't see that. I think maths is difficult in that area because we have moved on so far and the children are, many of the children are still a long way behind.*

In Michelle's class, patterning on a methodical way of solving a problem, was the focus. There were occasions when students were encouraged to share their ideas, but this usually took place when there seemed to be little likelihood that such sharing would distract from the thrust already given by the teacher. This is illustrated by the following two field-note entries.

TU 234 *Teacher interested in Carlene and Melissa's system for variable by using 10c and 8c variations* (Students were working on calculating the different combinations for small change for 8c. The activity was an aside from the main lesson as a response to a weakness revealed from my initial interviewing with each of the students which I had shared with Michelle)

TU 346 *Melissa had an interesting way for 77% of 125 where she did 8 X 12 by rounding off*

In terms of different learning styles it was more apparent in Michelle's class that there was a preferred way of approaching tasks. When Michelle's students were asked a question about how their class went about problem-solving, the difference in answers were quite marked (see Appendix 11). The majority of the students in Michelle's class (14 out of the 25 respondents) referred to a specific strategy or approach which implied a systematic way of solving a problem. These strategies were usually teacher guided, as these examples from the Post-Student Interview demonstrate:

CB well she like puts a problem on the board ... most of the time she puts a question up if we're doing it as a class then she asks people like questions, like what's the first step, and then she asks about the second step. (Carrie B.)

AD well, we have a problem-solving, and we have, in the back of our book what we do, what's the problem, then we work that out and then write it down and then, how do I solve it, and then we have a conclusion at the end. (Angharad D.)

In Lyn's class there was minimal reference to specific strategies (3 out of the 22 respondents). The strongest response (14 out of the 22 respondents), was that problems were solved as a class, sharing their ideas co-ordinated by the teacher, as these student responses show:

SH Brainstorm it all first and then work it out from there. (Steven H.)

ML Normally we get someone to work it out and then she asks different people and then asks everybody if they think it is correct. (Melissa L.)

SM (As a whole class) ... On the blackboard and then we get the best idea in. (Stuart McG.)

OP We usually consult, negotiate things. Have lots of small disputes about it and things like that. (Oliver P.)

Catering for different learning styles seemed to be handled quite differently by each teacher. As a consequence, a different community was created in each class. In Lyn's class, as the students indicated, there was room for a range of opinions. In Michelle's class, the students were clear about the main problem-solving strategy that was being quite explicitly recommended—it was handwritten by the students in the back of their workbooks.

Levels (7 2 1)

This subcategory of *Learning Styles* refers to the teacher's ability to design activities which will enable a range of learning styles to be catered for at a variety of different levels.

Although Michelle's class was already streamed for students of a higher ability level, there was still a spread of abilities in the class. The percentage of text units identified in this "typical" lesson series for *Learning Styles* was 9% for Lyn and 11% for Michelle.

Lyn's class included students with a wide range of abilities. Her broad themes catered for this, as these examples from the "typical" lesson series, and the field notes, illustrate.

TL *Lyn made the point that when students gave their answers they were to share their different strategies for arriving at that answer.*

TU 5 *Birthday party posters given broad parameters of task* (Class was to organise the entertainment side of Lyn's daughter's birthday party. Posters gave major points that they would need to consider e.g. wet weather alternatives, number of guests, equipment...)

TU 344 *due to the broad scope in the topics there is opportunity for different students to learn different things at their level* (Students have planned a class party and shared with me what they had learned from the experience).

As a consequence of ability grouping, Michelle had to cater for less levels. At times she would let students design their own problems for the rest of the class to follow as this example from the "typical" lesson series shows:

TL *Students were given 14/50 = ?/100 and were asked to make equivalent fractions. She asked the students to offer a fraction which the whole class could solve as an equivalent fraction .../100. Michelle selected those problems which demonstrate equivalence easily.*

TU 556-9 *Students deduce the patterns learning that maths has regularity and consistency, reliability. Whole class attend to her [Michelle] to correct common problem. Encourages students (Olivia) to make a general assumption that they can deduce from the pattern* (Section of lesson on addition of 2-digit numbers to discover a palindrome)

The second example (TU 556-9), shows a form of catering for different levels, within the confines of a discrete task in which the students have some freedom to discover the pattern at their own rate.

Resource (7 3)

This component of *Community* refers to the creation of a community where learning is seen as a social, rather than an individual accomplishment. As all participants become a resource for one another, a culture is created where students take on different roles and responsibilities for the learning environment.

It was clear in the "typical" series lesson alone, that both teachers used their students, during lessons, as a resource (Lyn—9%, Michelle—15%, see Figures 13a and 13b). The discussion in the preceeding subcategory (*Levels*) illustrates how each teacher had a qualitatively different view of how to use their students. The tasks each teacher gave her class had different outcomes for the kind of sharing that was available to the students. Lyn actively encouraged an interdependency between the class as my field notes illustrated:

TL *She [Lyn] set an investigation task which she wrote on the blackboard:- "Find out- Where you see these speed limits? When might you use them? 10 kph, 20 kph ... 110kph. Any other speed limits?"*

TL *Once students had entered their information from their investigation, they rotated to another group to enter their information onto the sheet.*

TU 76 *peer support in problem-solving a common strategy* (Comment made after reflecting on two months of classroom observations)

TU 173 *building a community that is not solely reliant on her to accept responsibility for the knowledge—evident from the outset* (Comment made after reflecting on three months of classroom observations)

It was clear from the following segment of her Post-Teacher Interview, when Lyn was discussing what might happen if some children did not understand a whole class discussion, that she used different students as a resource for one another:

PTI *You have to just teach to the ones that are getting what you get and for the others you have to just then go back and work with them or get those to come and work with those that already know and can see the direction in which to go.*

At another point in the Post-Teacher Interview she had this to say when asked the extent to which she used pair and group work in the mathematics classroom. This confirmed the value which Lyn placed on the contributions made by all students:

PTI *Lots really: Group work, pair work, if I can't give them an understanding of a concept which is being stressed I just say go and talk to some people or go and help someone. They use each other, I use the children very much. I think this is always beneficial to the children because as long as the children can choose, and not choose one who would put the other children down, and that they don't have that sort of- "You're dumb" approach ... Just recently a very slow child, a child who has difficulty in learning could actually make a rectangular and triangular prism, and the very bright ones in maths, in number, could not get it to work, and so the less able student normally who was seen by the group as the less able student became the peer tutor and so that was wonderful- and this child doesn't speak very much at all ... "Go and see James ...," so from that point of view. So whenever possible that's what I'd do.*

Michelle tended to use students as a resource when their ideas were likely to be consistent with hers:

TL *Most students grasped it in the first 10 minutes, then there was whole class discussion. Michelle seemed to use Carrie P to check whole class understanding as she often took a while to catch on.* (Students working out fractions from %)

TL *Students were still being asked to contribute to making equivalent fractions .../100.*

TU 158 *Linda responded as to how rounding was used at shops, teacher encourages her to give an example.* (Opening of a lesson on continuation of decimals where Michelle was asking when the class would use decimals in everyday life)

TU 490 *some students came up with formula over the weekend* (Investigation task set for homework to find out how to deduce sides of polygons from the sum of the internal angles)

In the Post-Teacher Interview, Michelle was asked if she used students as a resource to explain to one another. Michelle replied—"*often*." She did make the statement in the Post-Teacher Interview that it was an area where she would like to develop:

PTI *Yes, I am really trying to work with more group learning, I am very aware that children need to talk about these and to become relaxed in their discussions and talk with each other and I am trying to develop that more in my room, it doesn't tend to happen terribly well.*

This qualitative difference was mentioned at the outset of this discussion on *Resource (7 3)*. The learning climate in Michelle's class was one in which the input by the students was quite explicit and discrete in nature. In Lyn's class the topics were broad and students' contributions would often alter the course of the lesson or direction of the topic. As Lyn said herself ***"They use each other. I use the children very much."***

Tables 1a and 1b drawn up in *Metacognition* (see section in this chapter) when discussing how the teacher jointly constructed the ideas in the classroom shows the difference in the number of students involved, and therefore available, to be a resource for one another during whole class exchanges. Similarly the sociograms (see Figures 14a and 14b) at the end of this section provide a compelling illustration of the ways the students used one another in each classroom.

Collaborative (7 3 1)

This aspect of *Community* is a subcategory of students being a *Resource (7 3)* for one another where the students have more than just an opportunity to share ideas, but tasks which involve a *Collaborative (7 3 1)* role with one another. This releases the teacher as the main focus of expertise, and maximises the contribution that each participant can make to the learning community.

Although the "typical" lesson series indicates Michelle had less of this feature recorded than Lyn (Lyn 16% Michelle 11%, see Figures 13a and 13b), it does not indicate the times when Michelle would get students to work in pairs and groups. By and large, this did not involve collaborative skills, where students have to negotiate and work through differences, but rather a sharing or tabling of ideas.

Each of Lyn's topics involved calculations which were likely to be of direct benefit for the class. Collaboration was therefore integral to the success of the outcome, for example, planning the most efficient way to get to their campsite. The following examples are taken from two topics which involved collaborative sharing of information.

TL ***Once students had entered their information from their investigation they rotated to another group and entered their information onto the sheet*** (Class was sharing the locations in which various speed signs were located so that they could calculate the most efficient route to their class campsite)

TU 136 *[blackboard] Find ways of discovering which foods are the most popular so we can have a menu suitable to all of us* (Class were planning a class party)

Lyn created a classroom climate in which collaboration was not only valued and encouraged, but was integral to the success of the class decision-making which was a feature of the major topics undertaken by the class (Planning a class party; Planning the most efficient route to class campsite; Marking out the Oval for a Mini Olympics).

The examples of *Collaboration (7 3 1)* identified in Michelle's "typical" lesson series tended to be ones which illustrate sharing rather than collaborate decision-making between peers. Michelle, however, did appear to value such collaboration, as this extract from her Policy Statement (recorded in the field notes), demonstrates:

TU 3 *Policy Statement—shows valuing co-operation and caring atmosphere, problem-solving, responsibility for own items* (Policy reads- *General Aim:- To encourage a co-operative and caring atmosphere in the class so that children will share ideas in a friendly manner. To provide a stimulating and enquiring learning environment where children are encouraged to work alone and in groups situations to solve problems of an academic, or emotional, nature.*)

The types of tasks which were identified from the text units as collaborative, were in fairly well-defined areas, as suggested by this transcript from a lesson series on fractions:

LT *Michelle:*
Michelle: I think girls what we'll do, if I haven't marked yours I would have marked someone on your table. Compare your answers with those people and mark

them off that book please. I will be with you in a second.

In her Post-Teacher Interview, Michelle acknowledged that she had not fully capitalised on the use of group work with her class:

PTI *I try to use group work. I probably didn't use it as much as I should have. Looking back I think I could probably have made more use of it. It's a good way to get through to children that are struggling, if they are not understanding me, then there's a good chance they'll understand their peers-for the fact that they can talk on their own level and their own terms. I think I could have used it a lot more than I did, looking back.*

Later in the Post-Teacher Interview, Michelle made reference to the importance of having a cooperative atmosphere, and stated again that this was an area that "we need to work on":

PTI *I think they must be relaxed, you have to have an atmosphere that's co-operative and you need a chance to bounce ideas off each other, you need a chance to talk. I find that as I said before that is something that we need to work on.*

When asked to comment on my observation that *"Students are grouped but mostly work as individuals, or share in pairs,"* Michelle replied "*often*," confirming the interaction patterns established in her classroom. This was in contrast to the collaboration observed taking place in Michelle's class at the beginning of the study, when both classes were given the task of planning a class party. A high level of groupwork was observed among different small groups of Michelle's students. Here is a sample conversation which took place in one group:

Emily: Let's just have Catherine. She hasn't said anything. What do you think we should have?
Catherine: I don't know.

Sarah: Do you think we should buy the food or we should...
Elisabeth: I think we should buy the food.
Georgia: Let's do it by vote. Everybody put their hand in if you think we should buy the food.
Catherine: Monolity jule [sic] *(Laughs)*
Georgia: Majority Rules.

The collaborative skills displayed by this group at the beginning of the year, (in response to my request for them to plan a class party, using the $150 allowed) did not manifest themselves in the same way later in the year during the lesson observation period.

Empowering (7 4)

This component of *Community* is used to describe a classroom environment in which students have the opportunity to experience different roles and responsibilities. If such opportunities are to be empowering experiences for the students, then they must extend beyond the discrete confines defined by the classroom.

The percentages of instances of this component (Lyn 4% Michelle 5%, see Figures 13a and 13b) were relatively low; total incidences of empowerment identified were 3 for Michelle and 5 for Lyn, respectively (see Table 7 in Appendix 9). Decisions to code an incidence as *empowerment* were related to whether the raters felt the students were being allowed to take risks, and the degree of responsibility they were given in these situations. Clearly, such instances will be difficult to assess, and the instances identified are likely to stand out clearly.

Lyn provided broad parameters within which the students could work. When some students were grappling with ideas, other students appeared to feel empowered to explain their own ideas to them as this example from the "typical" lesson series shows:

TL *Students continued to contribute to the brainstorming session and Lyn encouraged them to explain to others who didn't seem to understand or who weren't able to locate the information from their timetable.*

The task the students were working on was locating information from a timetable—an empowering skill for successful functioning in their culture. Other tasks in which the students engaged during the observation period provided similar opportunites for empowering them to function in their society.

Both Lyn and Michelle had a low percentage of incidences of the *Empowering (7 4)* component. The number of incidences for Michelle was 3 and 5 for Lyn (see Table 7 in Appendix 9). The field notes reflected the fact that Michelle interpreted empowerment in ways which were consistent with fulfilling the syllabus requirements by working through the topics over the year's work. In other words, she would have felt that students were in a strong position to cope with the following year's work, and because they were competitive on any standardised tests. Only one rater scored Michelle's "typical" lesson series as including an example of empowerment. The following example was coded:

TL *She asked children to check it with the calculator.*

The development of appropriate calculator skills could be regarded as empowering for the students as such skills are important for them to function successfully in their society.

Negotiation (7 4 1)

This subcategory of *Community* refers to the opportunity students have for negotiating with the teacher and with each other in order that they not only take control of their learning, but also that they are in a position to accept responsibility for the consequences of their decision-making.

In this "typical" lesson series, the percentage of instances of *Negotiation (7 4 1)* identified by the raters for both teachers was relatively low (Lyn 5% drawn from 6 instances and Michelle 4% drawn from 2 instances, see Table 7 in Appendix 9).

As the preceding discussions have indicated, Lyn set broad topics within which students could negotiate their contribution, choosing to work individually or with peers. Class discussion tended to be led by the more able students whose ideas would often take the class into new areas. This was quite acceptable to their teacher:

Lyn:- So as I send you off to look at your food you need to consider 1, organising a balanced menu. What do I mean by a balanced menu? Reuven?
Reuven:- A bit of everything.
Paul:- So we all get the same thing.
Lyn:- So we all get the same amount? Yes.
Olivia:- You could have healthy food and junk food.
Lyn:- Yes a combination of healthy food and junk food.
Mark S:- Snack food.
Lyn:- Yes, snack food, junk food. Yes?

Ben C:- What about if we just want junk food?
Lyn:- Will your money go sufficiently far for you to eat that you'll feel as if you're satisfied by just eating junk food? That's another thing you'll need to discuss in your group and it will depend on what time the party will be.
Steven H:- It's not the best to swim after you've just eaten either.
Lyn:- No so maybe you'll have to have your swim first. Yes Olivia?
Olivia: - Maybe we can have lunch and then a quiet time of games and then have a swim.

Lyn agreed that the way in which she worked with the students meant that *"You would never actually get to the end of your, where you see the end of the task because you could actually go off on tangeants the whole way"* (PTI).

Negotiation with some students became less flexible when work deadlines were approaching and they needed to be called to task (i.e. when the end of term was imminent or they were about to go on camp. The following excerpt from the class party transcript provides some idea of the negotiating discourse Lyn engaged in with her class.

LT *Now did you actually address that issue or did you put this issue in. (shows another poster- Equipment—Utensils, plates, cups, games, presents). You didn't perhaps put a heading. I got these off your work—plates and cups, so I gave it the heading Utensils. I've got games over here, perhaps I should have put presents for the party. Are you going to have prizes? Can you indicate by putting your hand up if I left something out perhaps from your webs or your brainstorming- that I haven't got under decorations, food, games or equipment? Yes?*
Carmen:- You didn't put music down.
Lyn:- I didn't have music—do I need to? I was thinking more of the $150. Do I need to pay for that?
Carmen:- No some people will probably bring their own tapes and that.
Lyn:- So I haven't looked on that side OK? Music could that not go under this section in Equipment? Perhaps I should have put it here

as games. Right, come up with a better title to include music and tape recorder.

Later in that same lesson the students presented their group ideas back to the whole class. The discussion became heated over whether people are able to sleep over in the school classroom after the party. As the recess bell rang Lyn made this comment:

LT *Lyn: Don't forget we have to come to a consensus, and Mrs Bickmore-Brand and I would like to be consulted. We have to think about what we really want and reach a consensus.*

Michelle dedicated one lesson for me to observe her planning the class party. The following excerpt from the transcript demonstrates the negotiation process which took place in Michelle's classroom.

Michelle:- Now this is what you've come up with (Points to three A-3 sheets from each group blutacked to whiteboard). We need to come up with one sheet. Let's put these three together with one sheet. Let's use those ideas now and see if we can come up with a list of things that are the same in every case? What things are the same in every case? What things do we need?
Georgia:- I think we definitely need food.
Michelle:- OK So food will be one category that we'll need to look at.
Child:- Drinks.
Michelle:- Definitely we'll need drinks don't you think?
[later in lesson]
Child 1:- Cake.
Michelle:- The cake?
Child 2:- You don't need a cake.
Child 3:- That comes under...
Kylene:- It's not a party.
Child 1:- Every party needs a cake.
Kylene:- Its not a birthday party.
Michelle:- In your little groups you came up with a cake. Let's have a majority rule. How many think we do need a cake (Class votes) I think majority rules.
Michelle:- Let's keep this a group discussion rather than little groups for now. What do you think?

Child:-$10.
Michelle:- Why 10?
Child:- Because if we're going to have decorations, we know where we're having it, it's really big ...
Michelle:- Such as what, what type of decorations are we looking at?
Child:- Balloons and streamers.
Michelle:- Balloons, streamers, anything else?
Child:- How about party-poppers?
Michelle:- I think they're a bit dangerous in a school situation so we won't do that.

The negotiation although including the ideas of the class members, comes across as decisions which in the end will need to have Michelle's approval.

Expectation of Success (7 6)

This component of *Community* is to do with how explicit the teacher makes her expectations about standards for class behaviour. In the context of this study, a positive learning environment is one in which the teacher conveys an expectation of success with respect to the students' contributions and achievements.

Even though in the "typical" lesson series both teachers had set tasks which they anticipated their students would be able to handle, the average percentage of the number of incidences identified for *Expectation of Success (7 6)* was only 3% for Lyn and 0% for Michelle (see Figures 13a and 13b).

The members of Lyn's class were given quite ambitious tasks which she believed they would be able to solve. When Lyn was asked whether she experienced any difficulty designing her lessons around broad topics to teaching mathematics—like planning the most efficient route to their campsite—she explained:

PTI *Yes, because it brings in so many different aspects of maths, and the children don't always have those skills. So it could be that you talk about proportion and they haven't really learnt proportion, so you often have to stop and teach them about it. In fact, sometimes you actually have to do it for them.*

Some of the tasks Lyn set were ambitious and she anticipated that some students would need help. This appeared, at first sight, to contradict the idea of having an expectation of success in the classroom. However, it was Lyn's goal that all students should participate to the best of their ability, and there was no penalty if some did not "keep up" with the rest of the class.

Michelle included, in the *Rationale for Mathematics* section of her year's program the goal that her students would be able to have *"higher order thinking and positive and confident attitudes"* (TU 4). This signals the importance Michelle placed on the students feeling positive about their successful contribution to the class. She explained how she would try to achieve this by relating the concepts students already knew to the new ideas she was introducing.

PTI *If they haven't got the concepts previous to, you're not going to build on them and get an understanding of what they are doing. So I think prior learning must be established even if you need to back track, back to the previous year even.*

In order to gain some indication as to whether the students perceived they could manage the work expected of them in their class, each student was asked *"Do you need much help in maths lessons?"* in the Post-Student Interview. There seemed to be an overall confidence in both classes about their capacity to do the work set in class. There were certain students in both classes who recognised that they needed help (see Tables 1 and 2 in Appendix 19).

Celebrate (7 6 1)

This subcategory of *Community* is concerned with the instances in which the teacher periodically celebrated the achievement of learning events with her class.

Although relatively few examples of *Celebrate (7 6 1)* were identified in the "typical" lesson series for either Lyn or Michelle (Lyn 2% drawn from 2 incidences and Michelle 4% drawn from 2 incidences, see Table 7 Appendix 9), other data suggests the value placed on *Celebrate* by the two teachers. For example, Lyn's class celebrated in several ways. One of Lyn's practices was to keep a published record of the students' work as part of the class resource, as the field note below indicates. This book was not only read during silent reading time, but was also taken to the lower grades and shared with students in these classes:

TU 8 *Big books made by class also maths eg. Book of 14, 100, 1000 etc*

ITI *I am very much into displaying the children's work where the children go and read each other's work ...*

The 6's have been peer tutors for the year 3's and they had to teach the year 3's what 100 means and what 10 means and so they had to find ways that they were going to teach the year 3's, 10 and 100. They had made their own booklets—ten, hundred, thousand, and the cry was can I go and get my book and talk to the children about 10mls, 10kgs ...

Because of the thematic nature of her mathematics program, Lyn would inevitably complete a topic with a climax of a celebration, as in the class party, camp, and Mini Olympics.

Michelle tended to use a *games* approach, for example, working out how many squares there are on a chessboard for *problem-solving* (TU 27), and celebrating successes was a natural part of such an approach. Praise for students who had had success in test results was also a regular feature during the topic cycle.

Although very little celebrating was apparent in Michelle's mathematics class there was frequent evidence of having fun with mathematics in Michelle's regular class (home class). Each week an investigation was set for them on the blackboard (for example *"How many minutes have you been alive?"*). Michelle also designed and conducted a cross-age faction or "House Maths" on a fortnightly basis for the upper grades. Students were actively involved in solving problems such as "*using field markers work out how many students could fit on a basketball court.*" Michelle was also involved in planning and implementing "Mad Maths" days which had similar range of activities on a whole school basis once a year.

The following diary entry was made when the author returned from an unscheduled observation of Michelle teaching "House Maths."

Michelle's 9.9.92

> *A great revelation today with Michelle teaching "House Maths"; which was with non ability grouping. I saw her energised about maths, relaxed and not content driven. I really saw her bloom. I heard her use enabling language with her students. I saw her set practical concrete examples. I saw her give time for kids to complete it at their own pace. I saw her modelling and stretching them forward and I wish I had been able to tape her language. Her lesson was out in the assembly area which she asked the children to calculate how many people would be able to fit on the assembly area. She then asked them to use a hoop or a dome and in groups work out how to calculate the number of people. She then took them inside and gave them an activity in which they had 4 cubes which had to be arranged in such a way that there would always be a right angle somewhere and that sides joined and not edges and to draw these on isometric paper. Wonderful! I challenged her as to why she didn't teach this way every time and she said she was constrained by the school text books and the streaming exercise ...*

NOTE: In a later diary entry made after the author had debriefed Michelle about the observation data, and after she had listened to the tapes of her students in the Post-Student Interviews, she concluded that:

Michelle:- 2.11.92

> *she would continue to teach in the same way i.e. more content driven than process oriented because she believed at this age they needed it before high school, and that just like language short-sold punctuation and grammar skills with this whole language approach she wasn't going to let the skills go by in maths. I asked her why she couldn't teach like she did for "House Maths" and she didn't believe that the content would be covered.*

These observations and diary entries raise questions about the tension faced by Michelle between her beliefs and her role as the teacher. These constraints will be discussed in the summary at the end of this section.

Work Expectations (7 6 3)

This subcategory of *Community* refers to the instances of the teacher making explicit her expectations about the standards of work she would like the students to produce.

In the "typical" lesson series, the number of instances where the teacher gave a clear indication of work expectations was low (Lyn 2% drawn from 2 instances and Michelle 0%, see Figures 13a and 13b). In other lessons observed, however, both Lyn and Michelle made routine comments regarding the expected standards of work. Lyn was often less overt during the stages of each project undertaken by her students, but her standards were made more explicit as the project neared completion.

TU 22 *presentation and layout continued to be emphasised* (Field notes made after reading Lyn's comments on students' projects displayed on classroom walls.)

TU 63 *Students rewarded for systematic approach and recording even if not accurate answers* (Students recorded their ideas—so that these can be shared with the rest of the class—about what items will need to be costed in to determine the cost per item of food the class had produced as they made plans for setting up as a business.)

The feedback Lyn gave her students indicated they should be methodical in their working out and careful in their final presentation. Each topic concluded with a poster displaying what each child believed they had learned from the exercise.

NOTE: In order to complete the interviewing of Lyn's class the author continued to work in her class after Lyn had gone on Long Service Leave. The class had a relief teacher for the remainder of the year. This Diary entry was made after returning home from a session of interviewing the students:

> *DE: I started my final interviews with Lyn's class although Lyn has gone on leave. S ... O ... is the teacher and the class was in rows and her desk was up the front and well organised. The children, although she said they had been asking for me, were subdued when I came and visited. (15.10.92)*

In actual fact the author had been quite surprised by the behaviour of the class. They were not their usual animated selves. Sitting at the back of the room with a student the author managed to overhear S ... giving them a mathematics lesson. The author was quite surprised at how the students were reacting. She felt like saying to the students who did not have their hands up or who weren't responding- "Hey you know that, remember when you did ... !" What was also surprising was certain students who had been less confident in Lyn's class, contributing more. One student in particular was William B who seemed to be keen to answer anything the teacher raised and was highly participatory. The author commented later to S ... about what seemed a bit of a character change in William, and she remarked that he was "sweet" on her. It would appear that S ... had an ingredient in her teaching which reached William in a way which Lyn's teaching had not.

Needless to say this raises questions about what students perceive their teacher wants and how well they are able to, or choose to fulfil those expectations.

Michelle's comments to her students were qualitatively different from Lyn's in that they related specifically to details of the mathematics she expected to see in their work, with less emphasis on presentation.

TU 198 *"Make sure the sum is one where you need to borrow from other columns. Check on the calculators. Make them interesting."*

TU 185 *new idea—gave students a task to give partner a word problem—and they give the info/problem/sum to their friend and then once both done it they do each others sums. Teacher set boundary by saying she preferred area with subtraction of decimals.*

In Michelle's class the students were given specific information about what mathematical approach they should take in each particular activity.

Quality Products (7 6 3 2)

Incidents classified as *Quality Products* belong to a subcategory of *Work Expectations (7 6 3)*. Such incidences can be described in terms of the specific attention a teacher may make to excellence in what the students have produced and how they conducted themselves in the classroom.

The "typical" lesson series did not contain any examples of incidences of *Quality Products* from either Lyn or Michelle (Lyn—0%, Michelle—0%). Both teachers, however, have drawn attention to their expectation of high standard work produced by their students.

Lyn, for example, frequently displayed student work and was adamant about the importance of the accuracy of these "public" documents which were open to scrutiny by parents, and which she used to establish and build her assessment records. Examples of Lyn's attention to the quality of students' work can be seen from these field notes.

TU 44 *stressed layout on sheet—hint on using diagram, no invented spelling*

TU 165 *Students work samples become key artefacts of record keeping*

TU 345 *presentation used as variety to show maths at work without an exercise book*

Michelle also had high expectations of not only her students' finished products, but how they conducted themselves. These were made explicit at the outset but as the year progressed became a "given" and unstated part of classroom practice as these lesson excerpts indicated.

ML *She said "Make sure the sum is one where you need to borrow from other columns. Check on the calculator. Make them interesting."*

LT- *11.8.92 Olivia would you like to go and see Mrs Bickmore-Brand please. Olivia hasn't done her homework but perhaps she can go through just one or two with you. She has been spoken to.*

Michelle envisaged quite well-defined standards for her students and was reluctant to accept anything less.

Further Analysis

In order to gain some insight into the class dynamics from the student's point of view each individual was asked the following questions, during the Post-Student interview: *"Who is the best person at mathematics in your class?"* (see Figures 1 and 2 in Appendix 16) and *"Who do you go to for help in mathematics lessons?"* The responses of this latter question was set out as a sociogram (see Figure 14a for Michelle's class and Figure 14b for Lyn's class).

In Michelle's class, the responses to the first of these questions suggest that about five students were regarded by 15 out of the 25 respondents as "best at maths" in their class (see Figure 2 Appendix 16). The remaining 10 respondents in this class responded by saying that they didn't know who was the best:

LS: Oh, I couldn't really say. Coz I don't really, I usually work by myself or the person sitting next to me, and they're all just as good as each other really. (Lynleigh S.)

RC: Um.. I'm not sure, because there are quite a few people that … um Gabrielle probably is quite good, Carla is good, Kylene's good, there's quite a few people, it's not just like one person (Rebecca C.)

Georgia's comment also reveals the effect ability grouping might have on certain students:

GM Um well I think that's a bit unfair because at the beginning of the year some people were moved up but now that the other group's on the 8 book there's people like Kylene T ..., Rebecca C ... and Carla L ... and Carrie P ... who just get all in the 90's on tests but they haven't moved up, and I don't think that's very fair; I'm really happy though I wouldn't like to, move up, because I just want to stay where I

> am but I think it's unfair to a couple of people. (Georgia M.)

In the Post-Teacher Interview with Michelle, she maintained that most children felt very positive about ability grouping because it was not competitive. Michelle also reacted to the differing sociograms (see Figures 14a and 14b) for both her class and Lyn's which had also been shown to Lyn when she was interviewed. She felt it was difficult to get to know the students and for them to get to know her in 40 minute lessons or even double periods, because she felt committed to get through the syllabus. However, Michelle's own "home" class had a similar sociogram (see Appendix 15) constructed from students' responses to the question *"Who do you go to for help during class?"* The original question asked of the other two classes was tailored to take into account the fact that they did not have Michelle for mathematics lessons.

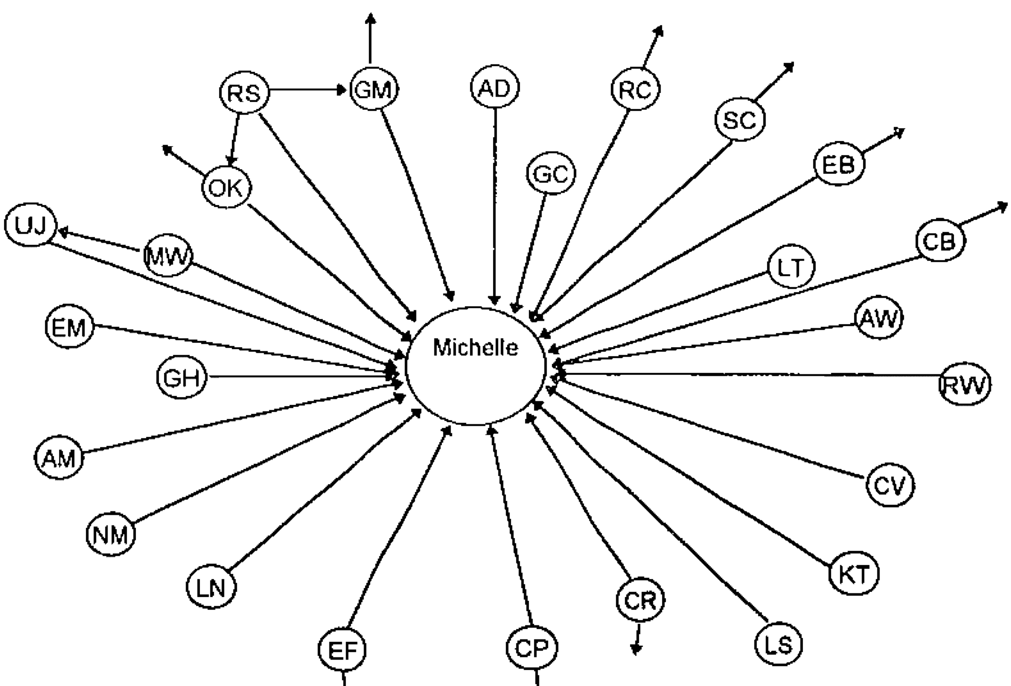

Figure 14a. Michelle's class response to the question "Who do you go to for help in mathematics lessons?"

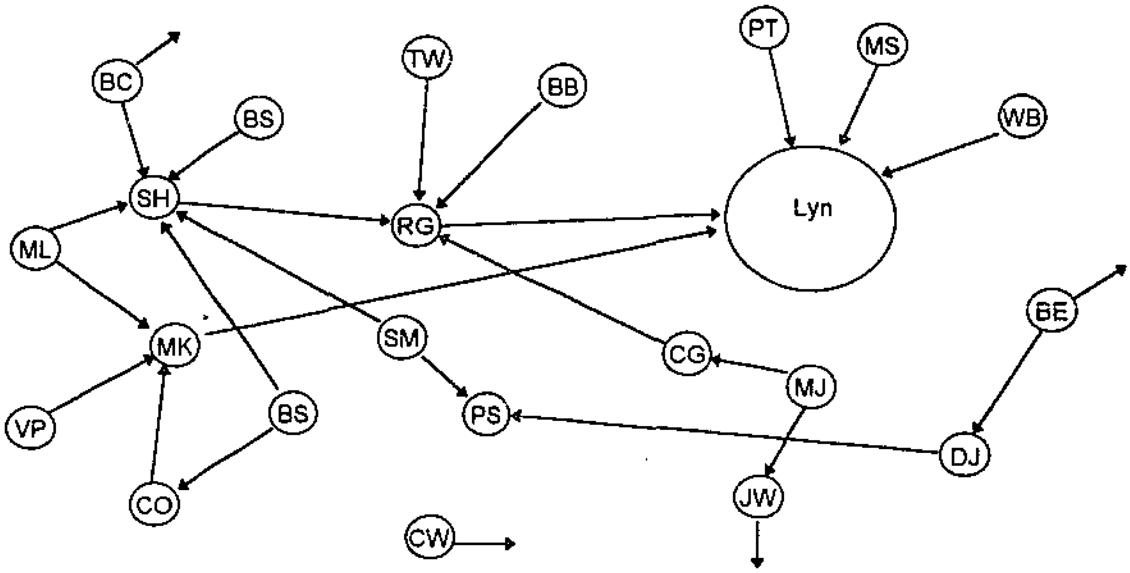

Figure 14b. Lyn's class response to the question "Who do you go to for help in mathematics lessons?"

In sharp contrast was the key boy identified in Lyn's class as the top student—Steven H—who also received the most choices for the question *"Who do you go to for help?"* (see Figure 14b). This class knew who was best and that they could use one another as a resource as well as the teacher. The graph in Figure 1, Appendix 16, suggests that, in Lyn's class, a range of students were identified by their peers as having expertise, and that the students could be used as a resource.

PS I'm one of the best people, and then there's H..., Stephen H and we're about pretty much the same, but in different areas, and then there's Melanie, she's pretty good. (Paul S.)

DJ Paul S who sits opposite me. He knows a lot about maths. (Dean J.)

BS Usually somebody who's good at maths in class like maybe Steven H if I've got real trouble or just the teacher, Mrs M Some people even come for help to me. (Blake S.)

BB I'm sort of a different, I do different things that I am good at than maths. (Ben B.)

Lyn describes in her Initial Interview the kind of learning community she attempts to create:

ITI *By being in a meaningful context ... visually, orally ... making approximations, taking risks, setting an environment where children do not feel threatened that there is going to be any sort of failure that whatever their opinions, they are going to be valued and that they all have worthy contributions to make, and that the teacher is not the overall, the teacher is really the facilitator and not the person who says "thou shalt do" type thing. And the children take responsibility for their own learning and are independent as well as the very strong notion of—you cannot do it in isolation, that co-operative, that group support both from your peers, your teachers and all those sort of factors together then.*

This comment of Lyn's reveals how it is possible for her to create a classroom climate which achieves the effect of the sociogram. It shows a web of interactions where the teacher, as Lyn admitted is not viewed as the only "expert".

Michelle was aware that the sense of "community" in her class was not as ideal as she would have preferred it to be. These comments from her Initial-Teacher Interview raise some of her issues.

ITI *Down the other end [below year 6 and 7] they are basically allowed to teach the way they want because they have their own class.*

I don't feel as though I am creating a maths community as I would like to only because I don't have my class. I have my specific maths class and I have time constraints and within that time I have to get the maths done.

When asked in the Initial-Teacher Interview *"To what extent do you create a feeling of a maths community in your class?"* Michelle marked about half way along a scale from 0-100%. It would appear that for Michelle there are two impediments to her creating a learning climate in the way she would prefer. One is the fact that her mathematics class is ability-grouped and she does not have the whole day contact with all of the students. Second, she is aware of the obligation to deliver the syllabus to the class within a limited timeframe.

Summary

The ways in which each teacher developed a classroom climate in this study were quite different. Probably one of the most noticeable differences suggested by the sociograms was the building of a community in Lyn's classroom in which the students became a resource for one another. The sociogram from Michelle's class suggests that the students regarded their teacher as the expert, and that the students preferred to use their teacher as a resource rather than their peers.

Another difference between Lyn and Michelle was the degree to which students could negotiate their learning—both the content and the process. Both teachers selected the content, Michelle's being closely related to the Syllabus set for her class year level. Lyn selected broad themes which she believed would have student appeal and provide potential for the mathematical skills and concept development recommended in the Syllabus. The effect created in Lyn's class was one where the direction of the topic was loosely set by Lyn but could be taken off on other directions as the students negotiated related areas of interest.

Michelle's class climate gave students the confidence that what they were learning would be needed for high school and further studies. Both the teacher and the students shared this agenda concerning the content and processes of the classroom. The support Michelle offered was quite specifically mathematical and focused on school success.

If one of the goals of education is that students should enjoy mathematics and have an expectation that they will succeed in the subject then both teachers achieved this, as the students' comments indicated. It may be concluded that regardless of differences in the ways in which each teacher created a classroom community, a common goal was achieved by teacher and learners alike in each classroom.

TEACHERS OF MATHEMATICS TEACH MATHEMATICS DIFFERENTLY: A CASE STUDY OF TWO TEACHERS

VOLUME 2

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This thesis is presented for the Degree of Doctor of Philosophy

at

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CONTENTS

VOLUME 1

Chapter Five

MAJOR RESEARCH FINDINGS

This chapter summarises the results obtained using each Principle as a framework for discussion. The key findings are drawn together in Chapter Six "Emerging Themes."

Context

Context as used in this paper is informed by the work of whole language exponents who believe that most learning occurs naturally within a context which makes the value of the learning obvious to the learner and motivates the learner to acquire the skill (Cambourne, 1988; Goodman, 1983; Holdaway, 1986; Smith, 1988). It has also been informed by writers in the field of mathematics education, for example, Shweder (1983), D'Ambrosio (1984), and de Lange (1987). The studies of Goodman (1983), Cambourne (1988), Holdaway (1986) and Smith (1988), observed the way a learner grasped the complex rules of language from the cultural context into which children were born and raised. The language was acquired naturally because the rules were an integral part of the culture in which the child was immersed. The learning was not separated into pieces but presented as a whole, and in a context which made sense or from which sense could be construed by the learner. The children's learning was not an imitation but an approximation which gradually became refined. Children's learning develops in such a way that they have an ability to cope with less contextualised situations or contexts which were different from that in which the orginal learning took place. Bruner (1986) described a form of instruction which presents concepts in a spiral where the learner can revisit concepts and gradually progress.

The teaching approach used by Lyn in this study closely resembled the teaching methodology in this whole and contextual way. In Lyn's classroom the knowledge, skills and values were integral to the larger topics or themes around which she programmed. For example, the trip to the school campsite at Collie involved students working out scales, reading maps, finding averages, reading timetables, investigating available and preferred forms of transport. Students needed to use a range of mathematical and literacy processes as they worked on each of the larger topics or themes.

Michelle, who also valued the use of context in her classroom, did not use it as a base around which knowledge, skills and or values were interwoven. Instead she invariably used context to introduce a topic. In most instances however, the context introduced did not relate to the rest of the lesson which tended to become decontextualised after the initial connections had been made.

The messages the students received from the teaching approaches adopted by their teacher, about the relationship between school mathematics and their own everyday lives, varied widely. The interviews with the students in Michelle's class indicated that there was a greater distance between what they were learning in school and how they perceived it related to their own everyday lives than the responses would indicate from the students in Lyn's class.

Clements and Del Campo (1990), however, have concerns about the degree of transference from a concept in one context when learners need to work with that concept in another context. Similarly Clements and Lean (1988) and Ellerton and Clements (1994) indicated that, for example, there is no guarantee that having "fraction-related knowledge" in one real world context e.g., being able to share a drink equally between three glasses will be linked conceptually with other real world contexts (Clements & Lean, 1994), or that it could be considered as indicative of "fraction knowledge." This section of the study does not discuss the issue of transference but quite significant results have been dealt with later in the *Metacognition* section of this chapter. Without preempting the discussion it too raised questions about the transferability.

The notion of presenting skills in context does not only have support from the language acquisition field, but also from psychologists (Bruner, 1986; Kelly, 1955; Piaget & Inhelder, 1969). These authors have emphasised the importance of involving the learners wholly in the process of acquiring knowledge rather than attempting to break the skills into distinct units. In 1987, de Lange developed *The Realistic Mathematics Education Model* which emphasised the need to provide a context so that the meanings and purposes of classroom mathematics are made more explicit, and to draw connections between the students' world and the mathematical world (see also Clements, 1990; Lowe, 1990). Similarly, Mousely (1990a) recommended that problems used in classroom mathematics are drawn from the students' daily lives, so that they themselves can make the links.

Many writers (for example, Applebee & Langer, 1983; Freudenthal, 1973; Sadler & Whimbey, 1985) are critical of the artificial boundaries of subject areas, curriculum and textbooks which present information in chunks or in a disjointed way. Such lack of continuity makes it difficult for learners to link isolated pieces of information or to generalise across to other contexts. In Michelle's class, observations were made of students having difficulty on more than one occasion when topics were changed and they found themselves trying to work out the purpose of the activity as well as identify which mathematical skills they were required to bring to bear on the problem. When questioned about this issue in the Post Interview, Michelle's response suggested that such dilemmas were just part of school life and "most kids get around it."

Textbook Contexts

Both teachers were asked about their use (or otherwise) of a textbook when teaching mathematics. Lyn said that she had not found one which covered an integrated approach to mathematics rather than focusing on "crunching of number" activities. Michelle, on the other hand, was quite ardent about her use of a textbook, even though the one she used was not always well linked with the work she was doing in class. She explained that she relied on the textbook to cover a lot of content, and to help keep her "on track."

The Classroom Context

Language is the most social of human activities and is currently a necessary state for human society. It is developed in a social setting and therefore when a particular linguistic item does not embody communal meaning then its survival is unlikely (Holdaway, 1988). Lemke (1987) states that meaning making is interdependent on the social practices of the community in which it is embedded. Similarly, those participating in language exchanges attend to what makes sense and ignore what is useless.

It seems reasonable to assume that this also applied to students in both classes. However, what makes sense in Lyn's class for Lyn's students is likely to be quite different from what makes sense in Michelle's class for Michelle's students. In particular, because Lyn's students usually worked on projects, meanings were built up over time. In Michelle's class, meaning making tended to occur over shorter periods of time since the focus was often on specific and separate mathematical concepts.

Mathematics for a Purpose

Boomer (1988) believed in the social negotiation of the learning context in order to involve the learner rather than encourage rote learning. For this reason he and others (Applebee & Langer, 1983, Barnes, 1976; Skemp, 1977) advocated the importance of students recognising the intention behind a particular classroom activity, so that learners could see how this activity fits into the general area being studied, and the purpose for doing it at all.

It was revealing when early in this study with the two classes the author set up a class party investigation, where the students were to be given $150 to spend. The common reaction was disbelief that there would actually be a class party or that $150 would be available for them to spend. The students were, however, willing to go through the motion of planning the class party with their classmates and teacher, even though they held the belief that it was not going to happen. The author believes this says a lot about the kinds of experiences we put students through in the name of education. Here was an activity which was highly contextualised and likely to be relevant to most students, and yet they were unconvinced that this could be part of their school mathematics lesson.

Fitzsimons (1990) provided evidence of research to indicate the growing trend towards real applications for the mathematics which is being studied in classrooms, even at the upper secondary level (see Day, 1995; Gnanadesikan, Schaeffer & Swift, 1987; Landwehr, 1990; Landwehr, Swift & Watkins, 1987; Schwa..z & Curcio, 1995). In this study Michelle was quite explicit about the importance she placed on preparing her students for high school which would involve having a basic understanding of a range of mathematical processes.

Praeger (1993) discussed the decontextualised nature of mathematics education which she believed has led to poor attitudes towards mathematics in the community. Anderson (1990) had the same opinion, "Do we intentionally mislead or confuse our students by failing to include meaningful context or by presenting mathematical models as reality?" (p. 327). Lovitt and Clarke (1988) suggested that an over-emphasis on the skills of mathematical manipulations in schools may have been the cause "for so many pupils, not being exposed to the holistic process, failing to see and appreciate the applicability of mathematics" (p. 538).

Four areas of discontinuity between learning done in school and mathematical activities done out of school were identified by Resnick (1987a). First, school tasks are predominantly individual, whereas in society calculations are often socially shared. Second, schools discourage the use of aids, whereas in society people generally access these for efficiency. Third, schools tend to value abstract and symbolic processing, whereas in society, mathematics is situational and directly connected with objects. Lastly, schools aim to give a student a broad knowledge base, whereas society tends to require a competence which is location specific.

Upon analysing the students' responses to the question *"Do you think what you have learned will be useful for everyday life?,"* and *"Do you think there is anything you have learned that will not be useful for everyday life?,"* the students in Lyn's and Michelle's classes had different perceptions about the value of school mathematics. It would appear from their answers that Lyn's teaching methodology positively addressed the area that school mathematics should be relevant to her students, and the responses from Michelle's students seemed to reflect the discontinuity that Resnick described. The students in Lyn's class were not only able to extract the school learning from the activities in which they were involved, but they could also see how the mathematics they were learning might relate to their own lives. For Michelle's class the real life activity the students mainly referred to was how mathematics could assist them in shopping.

Nesher (1988) is strongly critical of the lack of transferability of school mathematics into the real worlds of the students. She decries the proportion of school time in which a student spends working on "exercises that do not teach him anything" (p. 72). Clements and Lean (1988) pursue the idea that teachers need to make explicit the links between the familiar real world concepts, the formal mathematical language, and accompanying symbolic manipulation (see diagram in Clements & Lean, 1988).

What becomes problematic for students is when mathematical meaning being developed in a classroom context does not have a context outside the one being created for the purpose of developing a particular mathematical practice. That is, when attempts are made to develop mathematical meaning in the classroom, where the sole emphasis is on encouraging students to practice formal mathematical skills, then students find themselves faced with a dilemma, for example—knowing how to "do" percentages really means knowing how to work out percentages in a particular "correct" way. In fact, a particular sequence for working through a strategy (for example, "Make a Table") is likely to be repeated many times in the mathematics classroom. Knowing the method is as important as the result (Chapman, 1992). This learning of set sequences, all too familiar to mathematics classrooms, has been questioned by constructivist Cobb (1990a). Cobb (1990a) questioned the practice of teaching the rules before giving students the opportunity to learn the rules during the process of working through real life situations.

At one stage in this study Michelle responded quite overtly to an aside the author had made to her about how the fractions being worked on in the lesson could be related to the real world (see lesson transcript 11.8.92, Appendix 17). As can be seen from the transcript excerpt she started to talk to the class about this:

> *Why on earth are we bothering to have a look at equivalent fractions? Why do we do it? Do you remember when we were talking about decimals we said where in the real world would we ever use decimal numbers, and we talked about accountants and we talked about calculator work and all of that. Where do we use equivalent fractions? Do we ever use it outside the classroom?*

The rather abrupt ending of the whole class discussion, switching over to the homework, possibly signalled where Michelle's real focus was. Certainly her students received a limited message about the applicability of mathematics to their everyday life. When asked *"What do you think you have learned that will be useful for everyday life?"* her students' responses suggested that, other than potential applications in shopping, mathematics was something you did for school or to get you to University. As previously discussed in Chapter 4 within the data analysis section on *Context, Big Picture (1 2) and Purpose (1 3)* the study showed direct transcripts of Michelle believing and transmitting the message that the purpose for doing the mathematics lessons was in order to be able to do further mathematics.

The students in Lyn's class were involved in activities which were integral to their own community and culture. The positioning of this choice of content for their mathematics lessons had the potential to reinforce for them the links between the mathematics learned in school with the real world. Although the students from Lyn's class recognised the mathematical concepts with which they were working (e.g., decimals, percentages, multiplication, fractions) their responses to interview questions revealed considerable depth about the applicability of these concepts to their own lives.

Overall, the responses from the students in Michelle's class suggested that they had negative opinions about the usefulness of what they were learning for everyday life. Approximately one-third of the students, however, referred to specific examples of work that they could "remember sitting in class and thinking when are we going to use this?"

It would appear from Michelle's perspective, that, in order to prepare students for higher mathematics, some experience with decontexualised texts is important. Ellerton and Clements (1991) support the idea that it is possible to move back and forth between the more everyday experiences that the student recognises and formal mathematical abstraction. They refer to Clements and Lean's diagram (1988, p. 222, see Figure 2) which symbolises the dynamic interrelationship between these elements. Teaching in a contextualised way and shifting to the more decontextualised language of mathematics acknowledges that, as teachers of mathematics, we are not only assisting our students to cope with the world, but to expand the horizons of their thinking mathematically.

Interest

Interest as defined in this paper refers to more than catering for what might appeal to the learner, or even for what the learner might have vested interests in. Although these are certainly important elements, the main idea behind the *Interest* Principle is the connecting of the content being discussed in the classroom with what the learner brings to the learning situation. The background of learners, not only in terms of their knowledge, language, skills and or values, but also their preferred learning styles need to be taken into account during instruction. This Principle has been well established in the pertinent literature of Ausubel (1968), Bruner (1983), Cazden (1983), Cobb, Yackel and Wood (1989), Goodman (1983), Halliday (1975), Kelly (1955), Lampert (1989), Novak and Gowin (1993), Piaget and Inhelder (1969), Vygotsky (1962), and Wells (1981).

Mapping New Information to Old

The most important aspect of this Principle is that the outside source or information needs to be connected to the learner's prior knowledge. Many teachers, aware of the need to make connections with what students already know, provide student-centred activities which are either concrete or analogies which help learners frame new information. Ausubel (1968) regards the most important single factor influencing learning to be what the learner already knows. The Berlin-White Integrated Science and Mathematics (BWISM) model identifies this as one of six general propositions upon which the model is developed. This model would appear to encorporate key tenets of the constructivist movement from early views about the construction of knowledge for the learner (Driver,1988) to more recent social semiotic perspectives (Chapman, 1992) and incorporates the interpretation given to the "interest" Principle in this thesis.

- Knowledge is built upon previous knowledge;
- Knowledge is organised around big ideas, concepts or themes;
- Knowledge involves the internalisationship of concepts and processes;
- Knowledge is situation- or concept-specific;
- Knowledge is advanced through social discourse; and
- Knowledge is socially constructed over time. (1995, p. 23)

Both of the teachers in this study believed that it was important to connect the content of the curriculum to the conceptual understandings of their learners. However, the starting point for each teacher was quite different. The teaching methodology used by Lyn in this study was not only driven by what she believed the students would have an interest in, but by what she believed they would be likely to be committed to participate in. For Lyn, learning would occur as a consequence of the students working through problems within each of the major topics. For example, in order to plan their trip to Collie the students needed to learn to read scales on maps, locate information on timetables, calculate the average speed over certain distances for various forms of transport, and so on. Lyn's students were encouraged to develop their own repertoire of mathematical tools as they needed them. Although Lyn demonstrated some mathematical processes for the students—for example how to work out an average—these processes were presented in specific problem contexts. Furthermore, all participants in the class had potential solutions or approaches to find solutions to the problems being considered. In other words, Lyn's methodology was to start with what the learners knew and what skills the learners brought with them to the problem context and link the mathematics curriculum to these skills and understandings as and when appropriate.

Michelle's teaching methodology focused on the content of the curriculum and the readiness or otherwise of the students to master what they would need for further mathematical development, as described by the syllabus. Both the teacher and the textbook made only cursory associations with other mathematical or scientific concepts and/or the students' own experiences. The repetition of particular procedures, and experiences with certain types of problems, was planned as part of the need to lay foundations on which to build further concept development. In other words, Michelle's methodology was to start with the curriculum and draw connections, where applicable, to the student's own experiences.

From one point of view, the results of the Placement Test J indicated that there was no significant advantage or disadvantage in either teacher's methodology. From another perspective, the results show a surprising competence on the part of the students in Lyn's class for whom this form of mathematical problem was not part of the normal class routine.

Generating Own Rules

This discussion on students' idiosyncratic approaches to problem-solving raises the issue of students generating their own rules. Kamii (1989) recommends that children be given the opportunity to generate their own rules and procedures. Carraher and Schlieman (1985) have also noted that students display a wide variety of self-generated strategies and competencies, particularly outside the school setting (see also Ginsburg, 1982; Saxe & Posner 1983; Scribner, 1984). Constructivists believe that learners will feel a sense of "ownership" of mathematical knowledge when it is actively linked to their own world. "When someone actively links aspects of his or her physical and social environments with certain numerical, spatial, and logical concepts, a feeling of 'ownership' is often generated" (Ellerton & Clements, 1992, p. 4).

The constructivist Cobb (1987a) has a concern about basing pedagogy on an information processing model, when it continues to perpetuate the metaphysical realist assumptions that signify the objectified existence of mathematical knowledge. Teachers working within the framework of such assumptions, place themselves in the role of translating the "public information" of the culture for the students. In turn, the students' role is to decode the information and then encode it into their own schemas or "private knowledge." This section of dialogue with Georgie shows Michelle operating in this way:

(Lesson transcript 11.8.92)

Michelle: Right you realised that whatever the denominator was multiplied by, the numerator was then multiplied by the same number. One times two is two. What else did we say to prove that one third *does actually equal two sixths? We came up with a way to prove that two sixths was equal to a third. Do you remember what we said yesterday? We said that we know if I had two sixths of a cake I would have just the same amount as someone that had one third ...and what was that way Georgia?*

Georgie:- Because in the top number goes into the bottom number three times as well.

Michelle:- That's in this instance but it doesn't always happen. I was actually talking about this little thing here that we talked about yesterday, what did we say about that number, Angela?

The content is clearly driving this interaction and students are being encouraged to recall one specific way of going about the problem.

The relationship between students and constructivist teachers was described by Steffe and Weigel (1992) in the following terms:

> The most basic responsibility of constructivist teachers is to learn the mathematical knowledge of their students and how to harmonize their teaching methods with the nature of that mathematical knowledge. (p. 445)

Michelle saw the issue of providing opportunities for children to verbalise their thinking processes as quite problematic. To Michelle, allowing children to decide on their own approaches meant a considerable reduction in time available to cover the prescribed syllabus. Lyn, however, welcomed the children's divergences, acknowledging that the "track" that one child chooses may also be useful to another, even if some clarification is needed. Lyn often found herself being driven down unanticipated paths to which many of the children had a greater commitment than the one she had in mind.

If teachers are to help learners make links which connect the knowledge, skills and values of the curriculum with the current knowledge, skills and values of the learners, at times it may be necessary to make compromises between the need to allow for a learner's conceptual development, and the need for completion of a set syllabus over time.

Register

The linguistic term "register" has been used by Halliday (1978) to describe a "set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p. 195). Language is not neutral, and words have different meanings for each person depending on their experiences. Vygotsky (1962) concluded that language is so closely linked to concepts that it is difficult to separate a concept from its name. Pimm (1987) introduces the term "register" in relation to how language is modified through attempts to communicate mathematical ideas (p. 196). There is increasing support for the practice of using the learners' own language to clarify their thinking, to make linkages and to understand new concepts (Del Campo & Clements, 1987a; Hersh, 1990; Hughes, 1986; McIntosh, 1988; Reeves, 1986; Robinson, 1986; Seeger, 1994; Waters & Montgomery, 1992, 1993).

In this study, each teacher valued both the introduction of mathematical terminology and the linking of the terminology with what the students may already know. However, there appeared to be a different starting point for each of the teachers. Michelle seemed to be aware of the mathematical terminolgy the children would need to associate with the particular mathematical task and continued to reinforce this special language during her classroom discourse. For

Lyn, however, the starting point was the learner's own language and meanings and whether s/he had a need at that time to learn new vocabulary in relation to the real world in which the mathematics was being used.

It could be argued that Lyn's class may be disadvantaged because they have had insuficient exposure to formal mathematical discourse. Certainly Michelle used formal mathematical terminology more frequently than Lyn. In the Placement Test J the language of the test items did present a problem for Lyn's class, particularly in questions which involved highly specialised symbolic representations (see Appendix 14). One test item which asked students to find the "product" of two fractions caused problems for students from both classes.

Boomer (1988) recommended that "the more of their own language that children can weave around the new idea, and the more variously they can represent the new to their own experiences (through talking, writing, drawing, modelling, etc) then the more likely it will be that they will come to understand" (p. 4). Although it may be desirable that natural language forms can be used as much as possible in mathematics classrooms, a fundamental goal is for students to gain an appreciation and understanding of appropriate mathematical concepts (Clements, 1987; Pengelly, 1987). There is increasing evidence that students shift between more mathematical and less mathematical language/ representations on a continuum in their attempts to operate and control the register of school mathematics and its complex systems of meaning relations (Chapman, 1992; Clements & Lean, 1988).

Child-Centred Curriculum

The notion of a child-centred curriculum has been well established in the literature. Lehr (1985), for example, gives a comprehensive analysis of the works of many researchers where children's interest successfully drove the curriculum content (Bruner, 1978; Cazden, 1983; Clark, 1976; Halliday, 1978; McKenzie, 1985; Vygotsky, 1962; Wells, 1981). However, as already pointed out, the elements behind this *Interest* Principle are more than the literal name might suggest. "Interest" is a label which tries to locate the learner at the centre of classroom pedagogy. In other words, what needs to be considered is what the learner brings with him/her to the classroom in terms of language, learning style and all other facets of the learner's background. This should not be taken to imply that the teacher should not have a clear sense of the cognitive and affective goals for her classroom. The intent of this Principle is to help guide the proactive role the teacher can take in designing learning activities which attempt to relate the content to the learners' experiences.

Metacognition

Metacognition as used in this paper refers to the awareness learners have of their own cognitive processing, and the degree to which they can regulate and make use of this in different situations. It has been used to refer to individuals' declarative knowledge about their own cognitive processes (Schoenfeld 1992). The notion of *Metacognition* is not associated with what you know. Rather, it refers to how you use what you know, when you use it and ultimately whether you choose to use it.

Transfer of Learning

The ability to transfer what has been learned into a new context has been widely recognised (Baker & Brown, 1984, as cited in Weaver, 1987; Barrett, 1985; Boomer, 1988). Furthermore, Weinstein (1987) believes that the ability to apply metacognitive strategies learned in one context to a different context creates the distinction between experts and novices. Schoenfeld (1989) conducted a study of high school students who failed to monitor their thinking strategies when faced with an unfamiliar problem. The conclusion that he drew from this was that this was due to the predictability of the school test situations where students could assume that the formulae or approach to be followed would be consistent with those used in their most recent instruction experiences.

Burghes (1980) notes that many concepts in mathematics often appear rather abstract and not really accessible. He argues that they can come to life when explored through practical "modelling" situations. However, Schoenfeld (1989), like Steffe and D'Ambrosio (1995), recommended the development of thinking strategies within the context of a "genuine" need to use those skills. However, the question whether such approaches assist students in developing skills which enable them to apply thinking strategies to novel situations needs to be addressed. Lovitt and Clarke (1988) suggest that "mathematical modelling" is a process where learners can "predict what would happen if changes occur. These can be then tested against that reality" (p. 537).

This study has presented data to show how one teacher, Lyn, helped her students to develop their own metacognitive strategies in the classroom activities she designed for them. The question that is unanswered at this stage is why the development of metacognitive strategies did not seem to provide any significant advantage for Lyn's students when they were confronted with an unfamiliar problem. In fact, the results from this study also suggest that there is no particular advantage for the students in Michelle's class who had been taught specific thinking strategies in predictable "classroom" problems (see Appendix 13 and discussion in Chapter 4 in the summary section of the data analysis in the *Interest* section concerning the Placement Test J test results for both classes, Appendix 11 and discussion in Chapter 4 in the summary section of the data analysis in *Metacognition* on how students approached the novel problem *"On what day and in what year did your 21st birthday fall?"*). It is possible that the distance between the skills they had learned and the skills required for the new task was too great for the students to bridge. It is also possible that the interview situation in which the students were presented with an unfamiliar problem created an additional hurdle which prevented or inhibited appropriate responses.

Idiosyncratic Processing shown by the Learner

When analysing the approaches used by Michelle's students when solving the novel problem (see Appendix 11) they appeared to exhibit less systematic problem-solving behaviours than Lyn's students, even though they had been exposed regularly to a systematic problem-solving approach. They were also more willing to accept a prompt given by the author, in order get the correct answer. Lyn's students were less inclined to accept the author's prompts to assist them to solve the problem and overall, exhibited more variety in their approaches. Dawe (1993) and Callingham (1993) argue strongly for visual imagery in classrooms and draw the conclusion that visualising is highly personal and idiosyncratic in nature. The author would argue that the idiosyncratic way in which the students processed the unfamilar question in this study, regardless of whose classroom they were from, signals the importance of recognising these highly personal approaches in all aspects of students' work.

The suggestion that the focus during classroom discourse should be shifted away from teacher dominated talk comes from many studies (see, for example, Carpenter, Fennema, Peterson, Chiang & Loef, 1989; Forman & Cazden, 1983; Steffe & D'Ambrosio, 1995). These writers emphasise the need to give students the opportunity to talk about how they worked through their informal knowledge of solving particular problems. An important part of this process is that of learning to respect the ideas presented by others, and to compare and contrast these with one's own ideas. This was developed further in Schoenfeld's (1992) notion that students' informal strategies are readily accessible in classroom activities, and from there the teacher can help build students' mathematical understandings. Tables 1a and 1b in the *Metacognition* section of Chapter 4, show students' involvement in problem-solving activities in the two classrooms, and reflects the different ways in which the two teachers supported students' construction of mathematical knowledge. In Michelle's class the patterns of interaction indicate the limited use she made of the class members. It would appear that the students selected were those whose responses were more closely aligned with the direction she wanted the lesson to take.

Teacher Constraints

An analysis of the Post Interview with Michelle suggests that she felt under pressure, not only to stay within the allocated lesson time, but to follow the syllabus and to ensure that the concept is appropriately placed in the overall scheme. Phrases she used included: *I did feel very constrained to the syllabus ... its important to follow and complete the years work ... getting through the syllabus ... you're not getting side-tracked ... as soon as they get into the door I had to get down to it in order to cover the syllabus ...* This may be a dilemma many teachers face that influences their interactions with students.

Michelle was able to generate open-ended tasks when she ran the school "House Maths" for the whole school every two weeks, (see Chapter 4 for a discussion under the heading *Community- Celebrate*, which describes a Diary entry for 9.9.92, where the author inadvertently witnessed Michelle teaching "House Maths"). The constraints felt by Michelle, therefore, should not be taken as reflecting whether or not she could generate more open-ended approaches. Rather, the data presented in this study suggest that these constraints can become all pervasive, and can dictate the nature of the classroom approach adopted.

Integration Across the Subject Areas

One of the aspects which Mousely (1990a) argues would assist the development of metacognitive strategies in the classroom is the integration of concepts across subject areas. The 1995 Year Book of the NCTM "Connecting Mathematics Across the Curriculum" attests to the significance of this trend in mathematics (see House & Coxford, 1995). Clements (1990), too has provided examples of teachers exploring the ideas of integration across the curriculum. This study provides several examples of how this might be possible. In Lyn's program, for example, the students were involved in real world tasks which drew upon a range of their skills and understandings. For example, the topic on the Mini Olympics used not only mathematical skills and concepts but relied on the skills the students were developing in science, language arts and art. When discussing the lack of integration opportunities that ability grouping a school for mathematics might present, Michelle expressed her disappointed that this was a by-product of ability grouping in her situation, *I would like to see maths* *being integrated in the general course program and that doesn't happen.*

Whose Thinking Strategies Have Priority in the Classroom?

Figure 6 showed that both teachers gave a high priority to helping students develop metacognitive skills. As shown in Table 3 in Appendix 9, the number of classroom incidences which showed *Metacognition* were far greater in Lyn's class because of the ways in which she encouraged students to participate in her class. The students were just as much involved in explicit instruction as she was. Lyn was trying to help her students to realise that thinking strategies were personal, and that there were many ways in which problems could be worked out. In Michelle's class the students were taught specific thinking strategies which were included in her lesson plans so that regular experience with these might build up student's thinking abilities. The question that remains is *whose* thinking strategies should be given priority in the classroom and for *what* purposes?

The issue with school mathematics is that students are rarely called upon to use their metacognitive skills for authentic real world tasks, and feel as though they are causal agents of their own cognitive processing (Kameenui & Griffin, 1989). The kinds of tasks that Lyn gave her students tended to encourage students to use their metacognitive skills in situations which resembled real-life scenarios. In Michelle's class, the students' mathematical processing efforts focused on mastering the set curriculum.

Scaffolding

The literature review suggests that a strong case can be made for adopting the notion of scaffolding in the classroom. Vygotsky's (1962, 1978) term "zone of proximal development" is echoed in the work of Bruner (1978), Palinscar and Brown (1984) ahd Seeger (1994). It refers to the support given by a mentor or "expert" to learners in order to assist them to achieve more than may have otherwise been possible. The scaffold enables the learner to develop skills with the support of a mentor.

Turn-Taking During the Teaching and Learning Dialogue

This study showed both teachers supporting their students when they were having trouble doing a task independently. Lyn's way of providing support was more widely characterised by a mentor providing support which included using students in the "expert" role. Dansereau (1987), Singer (1978), Snow (1976), Stern (1975) and Wells (1981) all refer to this turn-taking aspect of scaffolding. The transcripts in this study showed that when turn-taking occurred in Lyn's class she, as the teacher, became less directive and more passive, while the students undertook the teaching role. This flexibility in turn-taking between the mentor or "expert" and the learner has been referred to as role reversal (Tizzard & Hughes, 1984). Michelle gave her class the opportunity for turn-taking by letting them come up to the blackboard and explain their working out. During these interactions, however, Michelle was not passive and maintained the control position.

Taylor's (1992) study of a Year 12 mathematics teacher noted how difficult it was to adopt a "teacher as learner" role. It would appear that there were personally constraining beliefs ("technical curriculum rationality") about his function which maintained him in the role of "teacher as controller" (see also the findings of Ball, 1988; 1989; Schuck, 1996; Wilcox, Lanier, Schram & Lappan, 1992). The frequent references by Michelle in this study to "getting" through the Syllabus may have placed her in a position which made it less plausible for her to modify her role with the Year 7 class she was preparing for high school. Mayers (1994) notes that primary school teachers in mathematics in general seem to teach their other subjects differently to their teaching of mathematics.

Frameworks Being Used as Scaffolds

The component of scaffolding most commonly adopted in the literature is that of providing a scaffold in the form of a framework or protocol. Polya's (1981) heuristics would fall into this category. The genrists (Christie, 1987; Kress, 1985; Painter, 1986) have encouraged the adoption of scaffolds in the form of writing frameworks which can be applied across the curriculum where, for example, report writing of a proof is required in mathematics (see, for example, Bickmore-Brand & Chapman, 1996).

Characteristic of this framework component of scaffolding is the predictability of the routines accompanying the use of the frameworks. In this study Michelle regularly demonstrated a combination of such frameworks as part of her classroom routine. However, when a novel question was put to her students, they indicated little evidence of transfer (see Chapter 4 data analysis discussion on *Metacognition*). The "typical" lesson series for the two teachers demonstrated differences in the way in which a framework might be used by each teacher. In Michelle's transcript (11.8.92, see Appendix 17) the discourse continually returns to the students grasping a highly recognisable "rule" for converting fractions to equivalent fractions. In Lyn's transcript (4.3.92, see Appendix 18), also on fractions, the framework adopted requires the computation of equivalent fractions where the emphasis is not on the "rule" but on the accuracy of the calculation to cater in a reasonable way for a class party. The teacher gave students the freedom to adopt her suggestions or to continue to negotiate about the amount of sandwiches required, now that they had the calculation correct. The students displayed this freedom to accept or reject the prompt of the mentor during their solution to the novel question posed to them in the post interview *"On what day and in what year will your 21st birthday fall?"* (see discussion in Chapter 4 analysis in the *Metacognition* section).

Who is in Control of the Scaffolding?

The question of who is in control of scaffolds is debated in Cambourne and Turbill (1986) who perceive that control over learning should be exercised by the learner and not by the adult (see also Graves, 1983; Harste, Woodward & Burke, 1984). Bruner (1986), Lehr (1985), Painter (1986), and Wells (1981), all focus on the proactivity of the adult or mentor in this role, and see an active removal of the scaffolding. Although this study has contributed no new information concerning whose role it should be to remove scaffolding, the use of a proforma or framework as scaffolding was clearly more possible to see being removed than the self destruct nature of scaffolding which can only be detected by analysing the shifting of the discourse onto a different level.

Shifts in Mathematical Expression

The patterns of discourse in the two classrooms in this study were different. Michelle, for example, made greater use than Lyn of mathematical language during her classroom exchanges, and showed a conscious introduction of vocabulary to support the mathematical concept under discussion. The students' mathematical understanding, as well as their mathematical vocabulary, were continually checked by Michelle.

On the other hand, Lyn was less overt in her use of mathematical vocabulary. There were times, however, when everyday words may have caused confusion and she was keen to establish the mathematical context for that word. During discussion the students shifted between everyday and mathematical language, although this was not necessarily in response to any cues Lyn was giving the students by her discourse. Chapman (1992) argues that successful students rely on transformational language shifts as they grow towards mathematical understandings. Chapman (1992), and Austin and Howson (1979) have noted that it is the language of the teacher which takes the learner's language and develops it, through interaction, to be increasingly more conversant with the mathematical register.

In this study Lyn, while encouraging discourse in which students could shift between everyday and mathematical language, did not consciously establish scaffolding of mathematically appropriate language. Michelle encouraged the students to use mathematical language and presented it as a layer above the students' language (zone of proximal development). Concern was expressed in the discussion earlier in this chapter on the analysis related to *Interest,* that rather than the teacher generating a joint construction of mathematical meanings with the students, through her discourse, the mathematical language was presented by the teacher as that of an "expert" who controls the language being used by the students.

The Role of Scaffolding in Current Pedagogy

If scaffolding is to have a place in mathematics instruction it needs to assist teachers who are currently unsure whether a constructivist approach to learning is likely to be more facilitative than a traditional didactic approach. Many teachers find this dilemma very real as they do not feel comfortable with the idea that students develop strategies and mathematical concepts for themselves. At the same time, they do not wish to resort to teaching concepts and strategies by direct, "correct" instruction followed by sustained practice for the children. Scaffolding as described here locates a starting point for the development of a learner's own language, and involves the teacher in a proactive role, assisting students to refine their mathematical language and the accompanying conceptual development.

Modelling

The literature from which the Principle of *Modelling* was drawn was directed at the relationship between the teacher and the learner. Most of the self-concept research carried out by psychologists such as Combs (1962), Jourard (1964) and Purkey (1970) acknowledge that feedback provided by parents, siblings and "significant others" assists in shaping an individual's self-concept—that is, what is worth valuing and not valuing about oneself. Clearly, teachers are in a powerful position to influence the children they teach. The *Modelling* Principle embraces the notion that children should be able to participate in realistic contexts where they can observe their teacher and peers modelling authentic engagement with the task. The students are then able to observe how the skills of their teacher or peers, within that context, are being used for a meaningful purpose. Holdaway (1986) writes about authenticity and the way it affects the relationship of the teacher and learner: "in terms of the spectator/performer relationship, the learner at this stage acts as a fascinated 'spectator' while the skilled person is active, not as a 'performer,' but as a genuine user of the skill" (p. 59). The *Modelling* Principle is therefore not only about the teacher providing an example; it is about providing opportunities for the learner to be able to observe the teacher's skills being used within a genuine situation.

Modelling Within the Classroom Context

The teaching methodology of Lyn, in this study, showed each mathematical topic providing a context for which the purpose of learning and practising the skills could be construed as meaningful. The students in Lyn's class were engaged in activities in which they were given opportunities to model to their peers, their own ways of approaching a task.

Mason and Davies (1991) argued that one of the core concepts of "mathematical modelling" is a process of knowing when and where a particular mathematical technique might be useful. Burkhardt (1981), classified the kinds of questions students are given in mathematics classrooms;

> Action problems—where the results affect the pupils' lives;
> Believable problems—which could plausibly be Action problems at some time;
> Curious problems—which intrigue and stimulate;
> Dubious problems—which are really covers for exercising mathematical techniques; and
> Educational problems—which are constructed to make some important point but which are not directly related to pupil experience.

He claimed that textbooks occasionally set "curious problems," but were usually "dubious." Mason and Davis (1991) included this study of Burkhardt in their discussion on modelling and note that "After a while it becomes difficult to distinguish the boundaries between problem-solving (with real problem-solving at one end of the spectrum, and abstract exploration at the other), and mathematical modelling" (p. 5). The problems set by Lyn for the students in her mathematics classroom tended to reflect the "Action problems."

Michelle's teaching methodology reflected the interpretation of modelling that the *National Statement on Mathematics for Australian Schools* (AEC, 1990) and the *VCE Mathematics Study Design* (1989) use. For example, the term "modelling" is used within the problem-solving process to imply that a teacher may wish to set up a model or situation, even a simulation in order to formulate a mathematical description of the problem: "Problem-solving and modelling activities provide a vehicle for the development of mathematical concepts and skills" (p. 77).

Srivastava (1984) also designed a process for mathematical modelling which entailed the following phases (a) formulation, (b) construction, (c) solution, (d) validation, (e) interpretation, (f) implementation, and (g) reformulation. Michelle valued the place of problem-solving in her teaching. This was reflected not only in her program but by the 3- and 5-step problem-solving strategies the students had written in the back of their workbooks and found in their textbooks.

Student Interpretation of their Teacher's Modelling

Both teachers were also explicitly and implicitly modelling the place of mathematics in their everyday lives. What is consistent with these understandings of modelling, is that users gain increasing control over their environment because the outlet for their mathematical processing is being tested against reality itself.

Students responses to the question *"Do you think that your teacher uses maths in her everyday life?"* were quite different in the two classes. The students in Lyn's class referred to a wide range of ways in which they interpreted their teacher to use mathematics in her everyday life (see Tables 2a and 2b and discussion in Chapter 4 within the analysis concerning *Modelling*). Although students in both classes noted that their teacher would use mathematics for "shopping," and for "teaching," the wide variety of responses from Lyn's class seemed to reflect a more personal knowledge about Lyn's everyday life. This can be interpreted as reflecting the degree of authenticity with which Lyn modelled, through action or discussion, how much she valued the place of her mathematical skills in her own life. It appears that this application of mathematics to the real world was a feature that Lyn's class seemed more articulate about than Michelle's students.

Modelling the Language of Mathematics

Chapman (1992) discusses the way teachers "introduce and model 'mathematical' words and language structures which are privileged over other language forms. Learning mathematics involves learning its register" (p. 39). The data from the current study showed that the range and type of mathematical language used by Michelle and Lyn were different. Lyn would draw the students' attention to how she used the mathematical terminology when there was a chance that the word may have been misinterpreted, (for example "average" and "timetable.") Michelle's language was generally more dense in her use of specific mathematical terminology in each lesson (see, for example the terminology of denominator, numerator and lowest common factor in Appendix

18). In relation to Chapman's idea that learning mathematics involves learning the mathematics register, it would appear from this study that Lyn's class did not have as much exposure as Michelle's to formal mathematical language.

Smith (1988) raises the issue that, although children are immersed in an environment rich with different adult models, they do not reproduce language in the same way. For example, children do not end up as replicas of their school teachers. The relationship with a particular model, possibly that of their peers, continues to be an important ingredient in the teaching/ learning process. Smith discusses the richness of this modelling:

> Learning is vicarious ... But the only way I can account for the enormous amounts of unwitting learning that children accomplish, much of which is apparently error-free on the first trial, is that children actually learn from what other people do—provided they are the kind of people the children see themselves as being. (p. 8)

It would seem that learning from modelling can occur on two planes. On one plane it is the functional level which achieves meaning and the desired communication for those purposes. On another level students also learn through the "hidden curriculum"—the values the teacher, or the person modelling, places on the skill being demonstrated.

This study has raised the issue of the development of a mathematical register through the modelling of the teacher (see also *Interest* and *Scaffolding* sections in Chapter 4). The teacher as the adult or expert, is there to model and facilitate students' attempts, to construct mathematical meaning, and to learn about the appropriate use of certain language constructions. It would appear from this study that there can be differences in the emphasis and interpretation the teacher places on this aspect of education. Learning the mathematical register would be facilitated by greater exposure to appropriate mathematical language.

In concluding this section it must be reiterated that this study has highlighted that students may be receiving different messages about mathematics, both in the emphasis their teacher places on aspects of their teaching and the ways in which they perceive their teacher puts these emphases into practice. It is, perhaps, disturbing that more than two thirds of the students in the study, perceived "shopping" and "teaching" as the *main* application for their teacher's mathematical skills. Clearly, such a finding must challenge mathematics educators to explore ways of broadening such perceptions.

Responsibility

The terms of reference for the label of this Principle of *Responsibility* deals with who is responsible for the learning which occurs in the classroom. Ultimately it is the learners who choose from what they have seen, heard and experienced in the mathematics classroom, and learn what they perceive to be useful, important and meaningful.

Independence

The responsibility that is being recommended in this Principle, however, is regulated and structured in such a way that the learners take increasingly more responsibility for doing the task as the "expert" (see *Scaffolding* section in this chapter) gradually lets go. It is not a case of the learner being left to their own devices. Davis (1989) reinforces the premise that mathematical concepts are socially constructed, and that there is a danger that an individual's construction of reality may not be challenged when children are left to do the learning entirely to their own devices. Cobb (1991) is critical of controlled mathematical teaching which supposedly makes "discovery" of a concept easier for the child "because children must necessarily mentally construct mathematics for themselves" (p. 8). Ellerton and Clements (1991) ask the question "But do they have the background to choose wisely, and how and when should the teacher guide?" (p. 57). This suggests that some tensions may exist in how much and under what circumstances responsibility will be encouraged by teachers.

The two teachers in this study showed different approaches towards responsibility, reflecting their different attitudes. Lyn provided a social context in which the students could undertake their own mathematical activities with the support of their teacher and peers. Michelle was concerned that the students may not be able to make appropriate choices in more open situations and therefore limited the opportunities the students had for decision making. The results of the Placement Test J assessment showed no disadvantage for Lyn's students when given more responsibility for the learning tasks in the classroom. The textbook study (see Tables 3a and 3b referred to in Chapter 4 concerning the data analysis related to *Responsibility*) not only indicated a wide range of learning was occurring in both classrooms, but also that students had a fairly accurate assessment of their ability without necessarily needing to resort to information from external testing devices. This may suggest that learners could be trusted more frequently than commonly occurs, to accept responsibility for their own learning.

Ownership

The issue is one of the "ownership" of knowledge, which Steffe (1990) believes learners construct through actively seeking out and making connections rather than by listening to teachers and absorbing what is written in textbooks. He notes that "A particular modification of a mathematical concept cannot be caused by a teacher any more than nutriments can cause plants to grow" (p. 392). Rice and Mousley (1990) go further and suggest that a striving toward ownership of mathematical problems and a generating of problem posing rather than merely problem-solving can lead to a greater capacity to solve problems both in and out of school.

Those who opt for a negotiated curriculum (Boomer, 1988; Friere, 1994) acknowledge the need for negotiation not only with respect to the roles of the teacher and the learners but also with regard to the content and processes which are valued in the classroom (see *Community* section in Literature Review, Chapter 1). The reality for children in schools is that they have very little to say in determining what counts as knowledge, and decisions in schools are rarely made as a direct response to children's initiatives (Searle, 1984). As Rogers (1961) observes, personal growth occurs when students commit themselves to risk-taking experiences in their pursuit of unknown solutions to their problems.

In this study a high degree of negotiation was evident in Lyn's classroom, and risk-taking opportunities were built into the nature of the tasks. In Michelle's class the students had very little control over the content. The agenda was established openly that the purpose of the mathematics lessons was to prepare the students for studying mathematics at the secondary school level. Both students and teacher would therefore have considered negotiation over specific content inappropriate. As discussed in Chapter 4 when analysing the data related to the Principle of *Responsibility*, students in both classes experienced periods of confusion and difficulty associated with the ownership of the learning. In Lyn's class, at times, the parameters of how to execute the topic were so broad that the students had difficulty making choices. In Michelle's class the students were sometimes confused when they could not grasp the particular approach their teacher was wanting them to learn.

Hand-Holding

Responsibility needs to be the concern of both the student and the teacher. Hersh (1986) comments that "the student must be given the responsibility to decide what role she wishes mathematics to play in her life, whereas the teacher must be sure that the decision is an informed one ... ultimately they (the students) must decide" (pp. 186-187). Herber and Nelson-Herber (1987) offer the following explanation about students being responsible for their own learning: "Independence does not mean isolation ... it has to do with who's in charge rather than who one's with" (p. 586). A great deal of independent learning can take place within a supportive environment in which the adult can assist the learner's personal constructions of knowledge by extending these meanings (*Scaffolding*) and taking account of the learner's point of view (*Interest*) (Cambourne, 1988; Gray, 1987; Hall, 1986; Smith, 1988; Wells, 1981).

The Teacher's Perception of her Role

The traditional role of the teacher is necessarily challenged if not changed when students are given a high degree of responsibility for their own learning. This can present a dilemma for teachers. The following is an excerpt from Michelle's Initial Interview when asked the question *"To what extent to do you provide opportunities for children to accept responsibility for their learning i.e. owning and developing the curriculum, and accepting responsibility for their decisions? To what extent do you believe children should be given opportunities in the maths classroom?"* It captures Michelle's ambivalance about giving students too much responsibility.

> *I don't tend to give the children a lot of leeway in that regard. I tend to set the problems and I tend to set the curriculum. No there are very few instances where the children get to develop their own learning. As far as taking responsibility, yes I tend to give the responsibility in that, say for instance they are in a group and they were talking about a particular concept, now if they want to extend on that or if they wanted to go down a different tangent then yes that's fine, but I set the original problem and if they wanted to take it in different ways then yes they can. Although I tend to think that the teacher should know where they should be going and I think that's more important than giving the children opportunities.*

The differences between Lyn and Michelle in regard to how comfortable they were with providing opportunitues for students to be responsible for their own learning was evidenced in the kinds of tasks they set for their students. When the self-esteem of teachers is closely linked to their perception of their own roles as "authority" figures, then this clearly represents a difficulty in the degree to which students will be given responsibility in these teachers' mathematics classrooms.

Community

From the literature review (see Chapter 1) it became clear that the kind of learning environment the teacher establishes will have an impact upon the classroom dynamics and the kinds of learning that take place. By creating a community atmosphere in a classroom which allows for sharing of problems and successes (Boomer, 1988; Cairney, 1987), a shared context is created where there is a sense of belonging to a club (Smith, 1988) in which values and intent are shared between teachers and students, and between students. The teacher has a large part in determining what values and intent will be foregrounded.

Teacher and Student Expectations

Cobb (1987b, 1988) observed that students adjusted their goals and activities in line with their teacher's expectations, and adopted a view of mathematics which fitted their teacher's expectations. In this study Michelle was observed teaching in two quite different ways. With the ability-grouped class whom she was preparing for high school, she kept to a structured program which would assure her of getting through the set syllabus. The students shared this goal and when asked about how much help they needed in mathematics they responded in a way which would suggest they were able to keep up with the content (see Appendix 19 Post Interview question *"Do you need much help in mathematics?"*).

Michelle used quite a different approach with the fortnightly "House Maths" which she coordinated across the upper school. The sense of community and collaboration around the open-ended tasks was consistent with those described in the Literature Review (see discussions concerning Boomer, 1988; Holdaway, 1979; Wells, 1981). In Chapter 4 (section on *Community- Work Expectations*) the difference in student behaviour observed in Lyn's class when the relieving teacher took over during Lyn's Long Service Leave was discussed. Weaver (1980) pointed out how a teacher's beliefs about how a subject is taught (reading) impacts on the classroom dynamics. It would appear that both Michelle with her ability-grouped class and the relief teacher, with Lyn's class, held beliefs about mathematics and how it should be taught, which resulted in creating a competitive place which was qualitatively different from other learning environments (Clarke & Clarke, 1990; Mousley & Clements, 1990; Stigler & Baranes, 1988). Gattuso (1994) describes some of the many factors which effect teaching styles, for example, mood, personal relationships and a set of beliefs about how she herself taught. When monitoring her practice, demonstrated differences between her beliefs and practice appeared.

Co-operative Group Learning and Performance

Debate continues to surround the relationship between co-operative group learning and performance (Davidson, 1990). A number of research studies have produced results which reject the idea that co-operative group learning necessarily leads to greater individual gains (see, for example, Gooding, 1990; Healy, Hoyles & Sutherland 1990; Pirie & Schwarzenberger, 1988). It would appear from the Placement Test J results (see Appendix 14 and the discussion in the summary section of Chapter 4 on *Interest)* for the two classes in this study that student achievement was not affected by the different teaching styles of the two teachers. Michelle's class scored slightly higher in the PAT assessment, with less spread which would be consistent with an ability-grouped class. The major concern often expressed about adopting less traditional teaching styles is that the teacher may not have enough time to cover the set curriculum, thereby disadvantaging the students. If performance on a standardised mathematics test is the criterion, then this study suggests that there is little difference between the two approaches investigated here.

Classroom Discourse

Closely related to the notion of cooperation is the way in which classroom discourse sets the stage for the social construction of meaning (Bishop, 1985). Classroom discourse, according to Bishop, is responsive to contextual and cultural aspects of classroom interaction. Lerman (1994a, 1994b) describes the notion of the mind being "fully social" (1994a, p.135) and through social intercourse humans are inculturated from infancy. Personal construct pedagogies fall short when they fail to take into account the dynamics of the classroom interaction which shape the personal construction of meaning. Cobb (1990b) discusses classroom practices which value the learner's voice, encouraging differing opinions which may take students off into alternative interpretations. Constructivist classrooms tolerate dissonance (Pateman & Johnson, 1990). The effect in shifting a student's thinking through some form of social co-operation is not to be underestimated (Steffe, 1990; Wheatley, 1991; Yackel, Cobb, Wood, Wheatley & Merkel, 1990). Mathematics education researchers Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989) argue that it is through discourse that students can be made to see the inconsistencies between their misconceptions.

The transcripts throughout this study show that quite different classroom interactions take place in each classroom. In Lyn's class each opinion is taken to be valid and contributes to the collective knowledge of the class. Similarly, when Lyn does not know something, this is freely expressed. The message that students might receive from this is that the teacher is also on a learning journey and that her way of explaining something may not make sense to some of the participants. Lyn mentions on several occasions during her Post Interview that she uses students to communicate an idea *"I think too the children could actually help each other sometime better. You see the adult can only see one answer and can't understand why the child can't see that. I think* *maths is difficult in that area because we have moved on so far and the children are, many of the children are still a long way behind."*

In Michelle's classroom her errors are portrayed as a slip of the pen, when she says "*I'm glad you picked that up. Lucky you were watching*." The teacher's discourse has a logical coherence and when students have divergent ideas it is not always easy to fold them into the sequence of the explanation: "*That's in this instance but it doesn't always happen. I was actually talking about this little thing here that we talked about yesterday* ... ."

The students adjust to the style of their teacher, as revealed by their answers to the question *"How does your class go about problem-solving"* discussed in the *Metacognition* section of Chapter 4. The students in Lyn's class argued about things, negotiated and tried to include everyone's ideas. In Michelle's class, a set way to approach problem-solving was established as the preferred way. Students who have other ways of approaching the problem took second place to the "expert" way. Habermas (1970, 1972, 1978, 1984) develops this notion of binding the "knower" with their culture through social interactions. In this way it may be argued that, in Lyn's class, the students may be operating outside mainstream culture if they embrace such a diversity of ideas and that Michelle's class is more likely to perpetuate the traditional rituals associated with school mathematics. On the other hand, there is much evidence to suggest that where students felt free to take risks they were in a position to construct more mathematical ideas and to own the ideas rather than imitate someone else's ideas (Del Campo & Clements, 1990; Pateman, 1989). Foucault (1970) discusses the idea of "institutionalised" ways of talking and doing in a community. In their own way both teachers in the study have cultivated their own approaches, and in effect, have transferred these into well-rehearsed routines which the children come to know, to understand and to respect.

Being a Resource for One Another

The degree to which each teacher in this study created a co-operative community which fostered risk taking and collaborative group learning was quite different for both classes. Lyn's class had tasks which were dependent on successful collaboration with their peers. To a large exent the tasks set in Michelle's class required very little collaboration. The negotiation skills which her students displayed during a small group interview which recorded their planning of the class party were not utilised, in general, in the mainstream practice of Michelle's classroom.

Clements (1990) suggested that students would become a resource for each other in classrooms in which students felt free to take risks. The extent to which each teacher in this study created a classroom where participants became a resource for one another was quite different, as the sociograms in Figures 14a and 14b indicate. The sociograms go some way toward demonstrating that the messages the students get from each of their teachers about what counts as knowledge (Chapman, 1992) and about who is expert, are different. In this way this study moves beyond constructivist approach towards a social semiotic perspective.

Where the focus has the teacher as the pivot of information and communication, then a class of more than a small group of students becomes a challenge to service with any effect. Michelle discussed the problem she had in supporting students using her "particular method" and had found one solution which was to provide after-school-hours tutoring. She also mentioned on several occasions in her post interview that she would like to have had group work operating more effectively in her classroom. Where the students act as resources for one another, and where the information and discourse takes place freely between all participants including the teacher, then it is possible to conceive of the successful operation of classes of combinations of group sizes.

In conclusion it may be stated that if one of the by products of education is that students should enjoy mathematics, and have an expectation that they will succeed in the subject, then both teachers achieved this. This was reflected by the students' comments in the Post Interview. From this study it may be concluded that regardless of the difference in how each teacher created their classroom community, a goal was being achieved by each teacher and one which was shared by their students.

Chapter Six

EMERGING THEMES

The discussion in each of the preceding sections in Chapter 5, pertaining to each Principle, has been presented in an attempt to maintain a micro focus. Using each Principle as a lens through which the data was interpreted was often difficult due to the same data at times being used to emphasise different Principles. The Principles provided a framework where each was treated separately even though elements of several Principles could have been discussed concurrently.

This chapter will look at the macro picture and attempt to draw out the major themes which have emerged from the research as a whole. It will integrate the results of the study across all sections. The major themes which have emerged from the study parallel the common issues which emerged from research in the fields of language and mathematics education (see Figure 4 in Chapter 2 The Study and its Conceptual Framework). The nature of qualitative research is such that it is difficult to control the outcomes to match the major research questions that are set at the outset of the study. The major themes which have emerged from the study however resemble closely those established in the early stages of the study (see Figure 4 Chapter 2) and their relationship will be discussed later in this chapter (see section The Relationship Between the Major Research

Questions and the Emerging Themes). The main themes which will be discussed are:

1. The place of context in teaching and learning.
2. The place of explicit instruction in problem-solving strategies.
3. The role of the teacher as scaffolder in developing the mathematical register.
4. Learner-centred curriculum versus content-centred curriculum.

1. The Place of Context in Teaching and Learning

The study demonstrated how the different teaching methodologies of the two teachers were manifest in the teaching of mathematical skills in a given context. Lyn constructed her teaching program around the mathematical knowledge, skills and values she felt could be associated with the larger topics or themes. One example was the broad context of planning a trip to the school campsite at Collie, where the students had to plan the most cost-efficient route and form of transport. Within this topic students had the opportunity to learn about reading scales, maps and timetables, finding averages, investigating available and preferred forms of transport, as well as developing their skills in a wide range of mathematical and literacy processes in order to perform these tasks.

The interviews with students, concerning the potential usefulness of what they were learning suggest that most students believed that what they were doing would be useful for everyday life. The students made comments on two levels. First, their responses revealed a depth about the applicability, for them, of the mathematics to their own lives. Second, they could identify the mathematical concepts they had been working on in the broader task context (for example, decimals, percentages, multiplication, fractions).

Associated with this discussion on context, are the kinds of messages their teacher may have been communicating to these students which might influence their responses. Lyn selected activities which she believed were integral to the society and culture of which the class was a part. The positioning of this choice of content rather than, for example using textbook activities, had the potential to reinforce the links between the mathematics learned in school with their everyday lives. The degree of authenticity with which Lyn modelled through action or discussion how much she valued the role of her mathematical skills in her own life came through clearly from the wide range of perceptive responses from the students to the question *"Do you think your teacher uses maths in her everyday life?"*

Lyn's students were engaged in "Action problems" in contexts for which the purpose of doing the skills was construed by them to be meaningful. At the same time they were in a position to see modelled, their teacher's and other children's engagement in using mathematics for authentic tasks.

Michelle also valued the use of context in her classroom. She would invariably use a real world context to introduce a topic. For example in the "typical" lesson series when Michelle is developing the idea of percentage, she discusses how many students went on a recent camp they had attended. This context, however, was not sustained, and became disconnected from the rest of the lesson in which most activities became decontextualised. Michelle's choice of textbook was one which included mathematical activities in context at the outset, often reinforced with a photograph or other visual material. For example, a picture of a student playing a flute next to the sentence describing the number of students playing musical instruments in a class. The problem posed was to find the percentage of students, from the data given, who played a musical instrument in the class. This was the first of a series of questions converting fractions to percentages, but none of the subsequent questions related to the picture of the flute or musical instrument scenario and the exercises became decontextualised.

Whether the students in Michelle's class could see the relationship between what they were learning and their everyday lives from these contexts or not, was revealed in their answers to the question *"Do you think you have learned anything that will be useful in your everyday life?"*. Their responses suggested that other than for shopping, mathematics was something you did for school or to get you to University. This was consistent with what Michelle had been quite explicit about—preparing them for high school. A third of the class responded along the lines that not a lot of what they were learning was relevant, for example this student's response, *I can remember sitting in class and thinking when are we going to use this?*

The degree to which the students in Michelle's class came to appreciate the value Michelle placed on mathematics in her everyday life, was revealed in their interviews. Students were articulate about their teacher needing to use mathematics in order to shop and in order to teach them. The details from their answers, indicated that they knew very little about any values Michelle held about the importance of mathematics to her everyday life.

Having established that the teachers in this study valued context in their teaching of mathematics but translated it differently in their teaching methodology, the question may be asked "Did the different teaching styles have any effect on academic performance?" It was clear from the Placement Test J results that Michelle's class performed slightly better than average which was consistent with an ability-grouped class. This test reflected more closely the kinds of questions and layout with which Michelle's class would have been familiar. Lyn's class performed on the same test in a manner consistent with an average spread of abilities. The test items did not reflect the kinds of questions and layout that were used in Lyn's lessons. These findings suggest that no major differences in performance could be associated with the different teaching styles. What is important to note is that students who have been taught in a contextualised way—such as the students in Lyn's class—were still able perform adequately overall on the decontextualised standardised test—Placement Test J. Students in both classes performed poorly on the more decontextualised questions regardless of whether the students had had experience with that question type before (see Appendix 14).

One of the most interesting pieces of data to emerge from this study was the choice Michelle made not to teach in a wholly contextualised manner to the class used in this study. It was clear from the way she conducted fortnightly, "House

Maths" (integration of all grades) and her reputation from her school and University lecturers, that she could offer a stimulating program for the teaching of mathematics in a highly contextualised way. After the "House Maths" lesson Michelle made the comment to me "*Tomorrow they'll have to use the opposite materials to what they used today*" (i.e. she was saying that, the next day, the students would have to use their books and "translate" what they learned into real life rather than start from real life).

In the Post Interview with Michelle, the author asked her about her choice to teach in this way. Her response is summarised in my field notes.

> *Michelle would continue to teach in the same way i.e. more content driven than process oriented because she believed at this age they needed it before high school, and that just like language short sold punctuation and grammar skills with this whole language approach she wasn't going to let the skills go by in maths. I asked her why she couldn't teach like she did for "House Maths" and her response was she didn't believe that the content would be covered.* (text units 765-766)

The issue of how constrained teachers feel to cover the content of the syllabus and its subsequent effect on how they chose to teach is important. Michelle agreed to use a lesson on planning a class party for the purposes of this study. She was, however, most reluctant to develop the class party idea over any greater period of time, on the grounds that it would take too much time away from the curriculum and because it was difficult enough to get through the content as it was. Michelle commented in her Post Interview, *I did feel very constrained to the syllabus to a very large extent ... it's important to follow and complete the year's work.*

In her Post Interview, Lyn discussed how she drew her ideas for topics and skills from a range of curriculum documents, and through talking with her colleagues and with the students in her class. When asked if she felt constrained by the syllabus she replied:

In my situation when I teach I really don't have those constraints. That's totally up to me if I don't cover it all, or if I choose to go in a different direction ... provided I immerse them in the skills, that they should need rather than the topic.

The issue of teaching in a contextualised way, as it emerges from this study, is best presented through two examples. The students in Michelle's class *talked* about cutting a cake into two sixths (Michelle, lesson transcript 11.8.92, see Appendix 17), while the students in Lyn's class needed to cut sandwiches into fractions in order to cater for a class party (Lyn, lesson transcript 4.3.92, see Appendix 18). The difference in each teacher's approach to the issue of context was revealed in the effect each approach had on the students' perceptions of the usefulness of the work they were doing in school mathematics for their everyday lives. If one of the goals of education is to help prepare people to take their place in the world, then it would appear from this study that there is more to be gained in teaching in a contextualised way.

2. The Place of Explicit Instruction in Problem-Solving Strategies

The literature review explored ideas around the place of direct instruction such as the teaching of heuristics within a constructivist classroom climate. It was quite apparent that Lyn's class was more exposed to a variety of strategies for processing mathematical ideas than Michelle's class. Students in Lyn's class were encouraged to share their own ways of processing. Their ways of processing were equally as valued as their teacher's:

> *I like to use the students' ideas because the way I learned to do maths and the way I see maths may not be the way that the child sees it and I think somewhere in here was a way that one of the boys taught me a way that I hadn't even thought of. I didn't need that because in my life I could do maths, I just saw the answers and off I went. And even when I help my daughter, I try and show her different sorts of ways because and ask her what she's done, because each child comes to it with a different way of thinking about it and how they see numbers. So if they help each other. I think too the children could actually help each other sometimes better. You see, the adult can only see one answer and can't understand why the child can't see that.*

The difference in how each teacher jointly constructed mathematical knowledge with their students, in the classrooms in this study, was revealed in Tables 1a and 1b, which shows student involvement during whole class discussions. Michelle's whole class interaction patterns seemed to indicate that she made minimal use of the class members. The lesson transcripts also supported this, as also does the observation that Michelle appeared to select those whose thinking would be more consistent with the direction she wanted the lesson to take. As a consequence, the students construed that Michelle would be the person to whom they would defer for assistance in mathematics problems (see sociogram in Figure 14a). The students were aware of a preferred way of approaching a problem which they did not necessarily "own," as these students reported in the Post-Student Interview:

E. B.: We'd have to set it out on a piece of paper, say what we, what the problem is, then, like, what points are important, and how you would work it out, and then you work it out then you get an answer. (Elizabeth B.)

A. D.: Well we have problem-solving, and we have, in the back of our book what we do, what's the problem, then we work that out and then write it down and then, how to solve it, and then we have a conclusion at the end. (Angharad D.)

In the lesson transcripts provided in the study, it seemed apparent that Michelle's logic took over when the students were having trouble. The following excerpt shows how she has a clear view of the algorithm she believes the children should understand and use.

Michelle: What else did we say to prove that two sixths was equal to a third. Do you remember what we said yesterday? We said that we know if I had two sixths of a cake I would have just the same amount as someone that had one third ... and what was that way Georgia?

Georgie: Because the top number goes into the bottom number three times and in the second fraction the top number goes into the bottom number three times as well.

Michelle: That's in this instance but it doesn't always happen. I was actually talking about this little thing here that we talked about yesterday, what did we say about that number Angela?

Angela: That you times it by ... you can ... its a whole number and you can say two ... its a whole number.

Michelle: Right you're saying two halves of two is a whole number right so you're saying two halves is one. One third multiplied by two halves equals two sixths. And all we are doing is multiplying a third by?

The students were having trouble understanding Michelle's approach to the algorithm. Although the dialogue started with an equivalent fraction problem which the students found difficult to solve, it did not fully explore how the students had already thought about the problem and attempted some solutions. Michelle appeared under pressure to move them on to the solution she had in mind. This may also be deduced from her interview where she explained that she was aware, not only of lesson time-frame restrictions, but also of the place of the particular concept in the overall mathematics program to be covered that year.

It was interesting to note the performance of both classes on the novel question used in the Post-Student Interview *"On what day and in what year will your 21st birthday fall?"* From the descriptions of each teacher's approach it may have been assumed, that due to the problem-solving emphasis in Michelle's class, the students were better placed to do solve this problem. It may also be assumed that the students in Lyn's class, where everyday questions were constantly being considered, would be better placed to do the task. However, only two students from each class successfully answered the question without assistance. What was surprising was that Michelle's students exhibited very little evidence of using any systematic procedure. They were generally more reluctant to do the task, and readily accepted the assistance they were given when they struggled with certain parts of the problem (nearly three quarters of the students accepted a prompt during the interview discussion when they appeared to be having difficulty trying to solve the problem).

In Lyn's class nearly half the class ignored the offer of assistance. This behaviour—as well as the range of ways Lyn's class went about solving the problem, with about one-third displaying unusual ways of approaching the task—seemed consistent with a class in which the students had been encouraged to decide upon their own problem-solving strategies. What was, perhaps, unexpected was that another third of the class were quite systematic in how they approached the task and, in general, were less reluctant than Michelle's students, to attempt the problem.

One of the conclusions which may be drawn from this study is the lack of transference of problem-solving strategies to a novel situation, even after explicit instruction in heuristics, as Michelle's students had experienced. It may also be construed from the problem-solving strategies exhibited by Lyn's students, that, if left to their own devices, some students may develop appropriate strategies, where other students may not. What does however seem to be an important observation concerns the idiosyncratic way in which students processed information during the problem-solving task, regardless of the teaching methodology to which they had been exposed.

As discussed previously, the results from the PAT and Placement Test J assessments, indicated that there was no significant performance advantage or disadvantage for the students, from either class. However a detailed analysis of how each class performed on the different test items shows that the students in Lyn's class, who were not normally given problems in this format, exhibited a surprising level of competence. This may suggest that the deliberate teaching of specific mathematical methods, and a progression through a developmental sequence of instruction may not be necessary to help students acquire appropriate mathematical understanding (assuming, that the PAT test and Placement Test J assess mathematical understanding).

Once again the issue of lack of transfer occurred with both classes when analysing the Placement Test J concept items. For example, one-third of Lyn's students gave incorrect responses to the questions on time and money even though these concepts had been explored during each of the major topics. One-third of the students in Michelle's class gave incorrect responses to the question on decimals which had been a major concept dealt with at regular intervals throughout the year. Over half of Michelle's class made mistakes when they used the number line, which had been a resource they had had a lot of experience with.

This evidence, of a lack of transfer, points to the reality that students construe their own meanings from the classroom activities in which they are involved. They make their own connections with what they already know, and how this knowledge relates to any new tasks they are being called to perform, regardless of how routine these might appear to be to others.

3. The Role of Teacher as Scaffolder in Developing the Mathematical Register

The importance of teacher dialogue in assisting students to develop conceptual understandings has been drawn out in different parts of this study (see, for example discussions in Chapter 4, in the *Interest, Scaffolding* and *Metacognition* sections). This study showed each teacher supporting their students when they were having trouble doing tasks independently. Lyn's way of providing support was characterised by the use of mentor support, which may have been her own or that provided by other students. Michelle provided scaffolds which were predominantly in the form of routines or frameworks. The discourse in the two classrooms in this study has provided insights into the different ways in which each teacher developed mathematical language to support the mathematical concept under discussion.

Michelle made greater use of mathematical terminology during her exchanges with students. She was observed not only introducing new vocabulary, and explicitly using this mathematical vocabulary in her own discourse, but she monitored the mathematical vocabulary students were using, as can be seen in the following interaction,

Michelle: Now of those fractions this one's special and it has a special name called? (Pause) Does anybody know?
Child: A lowest term?
Michelle: A lowest term fraction. Why do you think it would be called a lowest term fraction?
Child: Because it can't go any lower than that?
Michelle: The denominator can't go any lower than the three.

Rather than the teacher generating a joint construction of mathematical meaning with the students through her discourse (as discussed in Chapter 5 when looking at the *Interest* conclusions), the mathematical language is presented by the Michelle as an "expert" who controls the language being used by the students. This, in fact, had a positive result when the students were asked to perform the PAT and Placement Test J assessments. Michelle's students seemed less hindered by the language being used in the test items than Lyn's students.

The students in Lyn's class did have difficulty with some of the language on Placement Test J. For example, the majority of her class had difficulty with items involving the solving of algebraic-type problems containing subtraction, multiplication and division ([ ] -10 = 9; 10 X 4 = [ ] X 10; [ ] ÷ 2 = 9), and with the question "What is the product of 1/4 and 1/3?" The lack of emphasis upon the language of mathematics was quite obvious in Lyn's classroom discourse. Although she was keen to clarify any confusion students may have had in everyday words that have different interpretations in different contexts (for example, timetable) Lyn was less overt in her use of mathematical vocabulary. During her interaction with the students, her words shifted between everyday and mathematical language, as this lesson transcript excerpt shows (4.3.92)

Lyn: And if there's 32 of us approximately, how many will each child have? Approximately?
William: They'll have about half.
Lyn: How many 30's in 96?
Ben C: 3.
Lyn: About 3. Would it be 3 whole sandwiches or just three quarters of a sandwich? So each can have about one round of sandwiches.

In the whole class discussion which followed the above interaction, Lyn could be seen giving the students the freedom to adopt her suggestions or those from her classmates, or to continue to negotiate with them (about the amount of sandwiches required). In general, Lyn's classroom discourse showed her to be less directive and more passive, allowing students to take on the lead.

The study therefore identified a dilemma concerning each teacher's approach. If scaffolding takes the form of the language of the teacher taking on the learner's language and developing it, through interaction, so that the learner becomes increasingly more conversant with the mathematical register, then neither teacher in this study appeared to be doing this. Michelle, certainly used the mathematical register, but this was imposed onto the learner's own language. Lyn was sensitive to the learner's own vocabulary but rarely helped students to develop this into a mathematical register. Clearly the mathematical language was not signalled to the students as being more valued than their own language. This was not the case in Michelle's class, where the mathematical register was valued as the preferred discourse.

If scaffolding is to have a place in the mathematics classroom it needs to assist students to deduce strategies and mathematical concepts for themselves, while at the same time enabling their own language to be refined toward developing appropriate communication skills.

4. Learner-centred curriculum versus content-centred curriculum

A dilemma which can face teachers as they try to connect what they are teaching with the background of the learner is whether to start with what the learner knows or whether to present the information and then draw the connections with what the learner knows. Both teachers in this study believed that it was important to make the curriculum relevant to their learners. However, the starting point for each teacher was quite different.

Lyn's methodology was to use content that she believed would relate to the students' interests. In choosing "Action problems" Lyn hoped that the students would be able to appreciate how the results of their mathematical problem-solving was actually affecting their own lives. For Lyn, the learning would evolve as a consequence of the student's needing to work through problems inherent in the topics themselves. For example, in order to set up the oval for the Mini Olympics the students needed to research a sport of their own choosing, interpret tables of records and statistics, read diagrams, draw models to scale-often translating information presented in a different form, consider layout and desirable locations etc. The students developed their repertoire of mathematical tools as they, as learners, needed them. Lyn shares how this occurred:

> *I incorporate children's examples whenever possible. Sports Day is a great one, because you have got the 100 metres, 25 metres, how long will it take, stop watches, using the Sports Day, just the actual participation but there's lots more you could do with it. The actual organisation of the day, planning the timetable for the day, that type of thing. And its meaningful for the children, because it is part of what they enjoy ... It brings in so many different aspects of maths, and the children don't always have those skills, so it could be that you talk about proportion and they haven't really learnt proportion, so you often have to stop and teach them about it ... the children can then see a purpose for doing maths, and that maths is not just numbers and numbers of sums, that are right or wrong.*

Although Lyn did expose the students to certain mathematical processes, for example how to work out scale, they were not presented simply as formulas to be applied. Instead the class explored ways of solving the problem in which all participants had potential solutions or ways of working. In other words, Lyn's methodology was to start with the learner and connect the content of the curriculum wherever she felt it was appropriate.

Michelle's approach to teaching focused on the mathematical content and what she perceived the students would need in order to cover the syllabus. In terms of connecting with the learner's own experiences, both Michelle and the textbook, which formed the basis of the content handled in the class, gave only minor reference to the students' own experiences. The repetition of certain procedures and experiences with particular types of problems was done with the intent of laying down foundations on which to build further concept development. Michelle discussed her orientation in this way in her Post Interview:

> *I think maths being very sequential and developmental, you do need to make sure prior knowledge is developed before you work on to the more complex concepts. Therefore its important to follow and complete the years work, but also try to integrate it with the children's interest and everyday activities ... I tend to choose the maths content first rather than go from their experience. So I'd choose the mathematical concepts that we are going to cover and then use the children's experience to demonstrate the maths. I found that's two things, one that the maths is relevant for the kids because you're using their experiences but also you are getting through the syllabus as well. You're not getting side-tracked and concentrating on one area rather than covering, doing an overall scan of the syllabus. Some people go from the experience and draw the maths out of it but I do it the other way round, I take the maths and integrate it with the kids experiences. And its good for me because I get through the syllabus and its good for them because its using them.*

In other words, Michelle's methodology was to start with the curriculum and draw connections where applicable, to the student's own experiences.

One of the issues this raises from observations in both classrooms was how the students' construction of ideas was handled in either classroom. The author was interested, in particular, with the place of students generating their own rules for working out mathematical problems. Michelle seemed to have difficulty with allowing the students time to go off on their own tangents and "getting side-tracked." As she commented, this meant a considerable reduction in time available for "getting through the syllabus." Hearing Michelle discuss how her lesson would proceed from a blackboard demonstration, it was clear that she valued the students' contributions about their own ways of approaching particular problems. Her priority was that students should achieve the kind of understanding needed so that the next section of content could be built on this understanding:

> *Yes, usually, at the beginning of the lesson before introducing the textbook or the worksheet, I often use the blackboard, sort of a chalk and talk session, um talking the problems through and demonstrate what's going on in my head ... I often ask the children to do the same with either the blackboard or easel to the class in the same way ... I don't have problems with students adopting a different method to mine. I think that's OK as long as they've got a good understanding of the concept, and their answers are valid. In fact its good because other children can see that there's different ways of doing it.*

However on analysing the lesson transcripts (see, for example, Appendix 17 and discussion in the *Scaffolding* section of Chapter 4), her own dialogue, even when providing for students to articulate their processing, directed the discussion towards the meanings and emphases she felt were important.

Lyn's topic choice was consistently one with which the students would be likely to relate. For example, they planned a class party, a trip to their campsite, a Mini Olympics, and they ran small businesses in their class. There could be no doubt from the responses in the Post-Student Interviews that the students could relate to what they were doing in class. How Lyn handled the personal construction of meaning by each class member, including her own, exemplified her valuing of individuality. Each person's construction was not only regarded as significant, but opportunities for frequent sharing of these characterised each lesson. Like Michelle, Lyn also acknowledged that the "track" that one child may be on may also be that of another, and therefore may need clarification or confirmation. The difference, however between the two classrooms lay in how each teacher utilised the classroom discourse. Lyn accepted that she, like the students, could learn from the class members. The results of the sociogram (see Figure 14b) showed the extent to which students felt they were a resource for one another. As the field notes and "typical" lesson series showed, Lyn welcomed the divergencies, and her lessons would often be driven down unanticipated paths. As a direct consequence of this form of lesson development, the students had a greater commitment for the mathematical content they were studying in Lyn's classroom than Michelle's students had for the planned lesson content in their classroom.

The question that this study raises is "What prevents a teacher from jointly constructing the mathematical content in the classroom by starting with a child's understanding?" In this study it was interesting to observe that Michelle was able to generate open-ended tasks with her own home class but felt the restrictions of getting through the syllabus with her mathematics class.

If teachers are to assist learners to make connections between the knowledge, skills and/or values of the curriculum to the learner's own knowledge skills and values, then consideration must be given to the tension between allowing time for a learner's construction of a mathematical idea and the completion of a set syllabus within a certain time-frame.

The Relationship Between the Major Research Questions and the Emerging Themes

The seven teaching/learning Principles have been used as a lens to provide a focus during the data analysis phase of the research. Through this analysis, several key themes have emerged which relate to the original five major research questions (see Figure 4 Chapter 2).

1. *What is the role of context in learning? Related to this is the issue of the extent to which "real world" situations need to be incorporated into the curriculum.* As previously discussed this emerged as a major theme from the data analysis. The study showed that when real world situations are not used in the classroom there is a risk that the students will exit school with an attitude that what they are doing in school lacks usefulness for their everyday lives. Moreover when teaching in a way that involves introducing real world problems using appropriately introduced mathematical processes, students are not disadvantaged on achievement tests.

2. *What is the role of modelling in the teaching and learning of mathematics?* As has been made clear throughout this thesis the term "modelling" does not refer to the exemplar a teacher may provide of a mathematical protocol for approaching a problem, but to the display of the genuine use of mathematical skills in the teacher's everyday life to the students. This study showed two teachers clearly communicating quite different messages about their own engagement in using mathematics for authentic tasks in their everyday lives. The students in both classes recognised that their teacher would need to use

mathematics for "shopping" and for "teaching." The wide range of comments about the various ways in which one teacher communicated her use of mathematics in her daily life signalled that her students had been in a position that the other class had not been in to see modelled the valuing by their teacher of the application of mathematical skills in her own life.

3. What is the role of direct instruction in the teaching of problem-solving strategies? Related to this are two issues: First, the teacher's knowledge, skills, and perceptions about the subject area, and how they communicate with the students. Second, the metacognitive skills of learners and their ability to translate them to different situations. The study showed that explicit problem-solving instruction did not necessarily lead to a transfer of those strategies to a novel problem-solving situation. The students who had been in a class where problem-solving followed certain routines were more responsive to prompts offered by the author during the novel problem-solving situation. These students were familiar with working through the syllabus in a developmental way and were being exposed to certain ways of conducting mathematical processes. These students, however, failed to apply knowledge that had been a significant part of classroom practice to some of the questions in the standardised test they were given.

The study also showed that students who were involved in solving a range of real world problems as part of their regular classroom lessons did not show any obvious transfer of these skills to the novel situation. These same students did not perform well on certain test items in the standardised test even though there had been significant exposure to, and use of, these mathematical concepts during lessons.

The strongest result from the novel question was that students approached the task very idiosyncratically. This in itself raises questions about the deliberate teaching of specific mathematical methods, and a progression through a developmental sequence of instruction set down in the syllabus.

4. How driven should the content of the curriculum be by the learner's interests? Related to this is the issue of learner-centred curriculum versus content-centred curriculum. The study showed a difference in approaches between the two teachers. One teacher's methodology was more consistent with that of a learner-centred approach and the other teacher could be described as having a content-centred approach.

In spite of the suggestion that a content-centred approach would not be concerned with the learner's interests, it was clear that this teacher believed it was important to make the curriculum relevant to her learners. What seemed to get in the way of operating in a learner-centred way was the responsibility she felt to cover the content of the syllabus. As a Year 7 teacher, she was concerned about adequately preparing her students for the work they would be required to do in the following year—their first year in high school. This proved a constraining factor on how much time she allowed for divergencies down pathways in which the students were interested. It became expedient to streamline the delivery of the content. It was therefore problematic in whole class discussions, when progress was being impeded when some students struggled with their own ways of solving a problem. Although academic achievement was an indicator of success for these students, only a few students demonstrated systematic strategies (or unusual strategies) in solving a novel problem. Students appeared reluctant to attempt unfamiliar problems.

The third theme to emerge from the study is related to this discussion about a learner-centred versus a content-centred classroom. It highlights the role the teacher can play in scaffolding the learner's concept development. In the learner centred-classroom the students were supported in their mathematical development largely through the way their teacher encouraged the students to develop their own ways of thinking and communicate these to others. In the content-centred classroom the students were supported through the use of routines and frameworks which assisted them to perform certain mathematical processes. The classroom discourse was quite different in each approach. The students in the learner-centred class had less exposure to the mathematical register. This was due to the teacher's major priority that the students make sense of the experiences in which they are taking part. The students in the content-centred classroom had a high degree of exposure to the mathematical language that was part of their concept development. This can be attributed to the teacher's belief that they would need to know the terminology for their secondary school mathematics studies.

It would appear that some middle ground is warranted. In the first instance when the language of the teacher scaffolds *from* the learner's conceptual understanding towards the development of further mathematical concepts, value is attached to the learner's own meaning. Second, the teacher needs to be conscious of the appropriate language that is used for communication in that field. She is then in a better position to scaffold the students as they experiment with shifting their own everyday language towards more mathematical expression. The teacher can assist the students to communicate their mathematical understandings with others through the use of appropriate formal mathematical language.

5. To what extent and by what means can the learner take responsibility for his/her own learning? Related to this are the issues of ownership and/or empowerment within the classroom climate. As summarised in Chapter 4, when discussing the results of the Principle *Responsibility*, the data did not provide sufficient evidence to draw any strong conclusions about this issue. Consequently this issue did not emerge as a major theme from the study.

Issues concerning responsibility for learning were at the centre of each teacher's approach. It was clear that in the learner-centred class, where the students were working on real world problems, the consequences for the students' mis/calculations were tangible. The topics were such that the students had a commitment to accepting responsibility for working through solutions (for example planning the most cost-efficient route and form of transport to their campsite). In the content-centred class the consequence for students' not accepting responsibility for their learning meant that they were given less exposure to the more advanced features of the concept. The teacher expressed that she felt under pressure and therefore responsible for ensuring that the students had covered the curriculum *I don't tend to give the children a lot of leeway in that regard. I tend to set the problems and I tend to set the curriculum, no there are very few instances where the children get to develop their own learning. As far as taking responsibility, yes I tend to give them responsibility ... although I tend to think that the teacher should know where they should be going and I think that's more important*

What has been suggested from this study, is that classroom mathematics can have the kinds of tasks which will encourage the child's commitment to solving and therefore the learning has intrinsic consequences. On the other hand the tasks can be more focused on the teacher progressing to the next level of development in the curriculum.

Both teachers faced the dilemma that the student's immature attempts may become a permanent fixture of that person's repertoire unless the teacher intervened. As discussed earlier, ultimately it is the learner who chooses to take from what they have been exposed, what they find useful and preferably meaningful.

Conclusion

Through an analysis of the teaching methodologies of two teachers, the topic "Teachers of mathematics teach mathematics differently" has been investigated. The teachers in this study had different attitudes towards the task of teaching, and different teaching foci. They also adopted different teaching methodologies: one could be described as more learner-centred and the other as more content-centred.

The research used a qualitative case study methodology which analysed classroom observations, and interviews with the teachers and students, which were collected over a twelve month period. The research utilised data sources from a range of perspectives. The results suggested that there were differences between the perception, held by the students from the two classes, about the nature of mathematics they were being taught, and its function in their own lives. It appeared from this study that the students from the class which could be described as learner-centred, believed mathematics to be integral to their everyday lives. On the other hand, students from the classroom which could be described as content-centred, believed mathematics to be a necessary skill for further study.

A feature of the analysis and discussion has been the use of seven Teaching/Learning Principles as a framework. An attempt was made in the analysis to draw out positive features of the teaching approaches rather than focus on negative features. The results, when data was triangulated, indicated benefits for the students when their teacher adopted a contextualised approach to mathematics teaching, not only for academic goals but for everyday relevance. The analysis also showed that students were positively influenced by the way in which their teacher and peers modelled the value of mathematics in their own lives.

The study showed that the instruction of mathematical skills need not only be the domain of teachers. Students develop and learn through various social and pedagogical influences. For example, students increasingly can be given opportunities to accept more responsibility for developing their own strategies for tackling mathematical tasks. At the same time, both teachers regarded themselves as accountable for what their students learned. Responsibility is likely to be developed when the learning activities are tasks in which the students have some commitment. The construction of mathematical knowledge, skills and values can be negotiated within a supportive, co-operative community in which individual learning styles are valued and all members operate as resources for one another. The study also showed that the teacher can scaffold the students' development of thinking strategies and enhance their familiarity with and ability to use the mathematical register. However, regardless of the teaching methodologies used, there was evidence that learners constructed their own meanings, and developed their own thinking strategies from their experiences.

In this study two different foci for the roles of teachers was demonstrated. One teacher felt constrained to deliver set mathematical curriculum. In fulfilling this obligation, the teacher made certain choices which were not consistent with approaches she used under other circumstances. In other words the teacher used the methodology which was consistent with the goals she had for the subject and her beliefs about how that subject should be taught.

The other teacher believed that the mathematics curriculum and the teaching methodology she used would develop in her students their capacity to use mathematics in their everyday lives. For her, the subject needed to have relevance for the students beyond the school environment.

A learner-centred approach has been supported in the Literature Review. This study has shown ways which are reflected in current research for delivering the mathematics curriculum to enhance learning for all participants. This study demonstrated approaches that achieve goals for mathematics education that foster in students an appreciation that mathematics has relevance both for school progress and for everyday life.

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LIST OF APPENDICES

Appendix 1

Typical Lesson Series (TL)

LYN M.

Lesson Observation 5.5.92

Lyn has decided that she will use the forthcoming camp at Collie as the context for the new term's work in mathematics. Students will be travelling from Guildford where the school is, to Collie, and Lyn will be focussing on the mathematics involved in travelling to the campsite (approximately 200kms away). Students are to plan the most cost-efficient route and to make decisions on the mode of transport so that plans can be made for the camp itself

9.00 a.m. Lyn outlined the above task and then distributed a map of the area to each of the students (i.e. south western region of Western Australia).

Lyn directed the students to look for the scale. As the scale was not explicitly given the students were then asked to calculate how many cm represent 1 kilometer. The teacher provides the example 1.5cm=10km

After some discussion, the teacher and students decided that the scale for this map was 1cm= 7.5km

Lyn then gave the students the task of calculating the distance over time to travel along any route they chose from the school at Guildford, to the campsite at Collie. She explained to them that they would be given school time to do this and that this would be what their maths was about this term.

She discussed the maximum speeds for a bus eg. 90kph and a car e.g. 110 kph, and she admitted that she did not know the speed for either the train or plane.

She introduced the word "average" and explained why the children would be needing it.

She discussed what mathematical processes they'd be likely to need such as division and multiplication

She set an investigation task which she wrote on the blackboard:- "Find out- Where you see these speed limits? When might you use them?- 10 kph, 20 kph...... 110 kph. Any other speed limits?"

9.50 a.m. lesson ended

Next observation a week later.**13.5.92.** (during the week, the students had been given tasks to work on scales with various maps and worksheets to reinforce this new concept)

9.00 a.m. Lyn checked out whether students knew how to work out "average" speeds. She said they would stay with whole numbers for now explained that they would be taught how to calculate averages with decimals later in the year.

She demonstrated on the easel how to calculate "average" speeds. The students were gathered near the easel.

9.15 a.m Students then went back to their seats to work on finding the average of quantities which they would be familiar with (e.g. body weights of their friends...).

There was the usual spread of children completing this task but Melissa, Brenda and Victoria the first finished. This was perhaps surprising, given that they are fairly weak students.

Lyn made the point that when students gave their answers they were to share their different strategies for arriving at that answer. She encouraged their responses and doesn't give the correct answer away too early in the discussion.

Lyn then demonstrated on the easel how to do average again, although not all students were placed in a position to see.

Students shared at their tables the strategies they used to work out the average and in what ways it was the same or different from what had already been shared.

Lyn shared with the class that she wants to buy a new dishwasher, she had been given the following quotes from different companies and/or stores. She wrote the list of five figures on the blackboard:- $659, $789, $599, $839, $1256

She asked the students "What would be the middle price for a dishwasher?" intending that they add up the list and divide by 5 to find the "average".

Melissa, quite a weak student, divided by 2 thinking that that would be half way or the "middle". Victoria, one of the weakest students in the class answered $599, thinking that because it was third in a list of five it would have to be the "middle".

Lyn failed to pick this up but took Victoria's problem to be one of poor adding and built on that with the class.

Lyn discussed where problems might occur for the students in the future when doing these types of sums. She implied as she talked the students might want to write down some details- Blake didn't catch on and just watched her.

The teacher discussed the investigation tasks students were involved in -their calculations of the most efficient speeds in a car, train etc to get to the campsite at Collie.

Stephen had a unique way of adding and shared it with the teacher and found others had the same system.

Lyn was quick to recognise his strategy, and asked him to share it with the class.

Students choose stations to sit at and record all the things in their environment they had discovered which had the sign:- 10 kph or 20 kph (station 1), 30 kph or 40 kph (station 2) 110 kph or other (station 3) 50 kph or 60 kph (station 4) 70 kph or 80 kph (station 5) 90 kph or 100 kph (station 6)

Once students had entered their information from their investigation, they rotated to another group to enter their information onto the sheet.

10.30 lesson ended

Next observation **28.5.92** two weeks later

9.00 a.m. Students were seated in groups, and Lyn asked "What's meant by a timetable?"
She did this to establish the meaning of the word and to avoid any possible confusion with the word "times-table".

Lyn provided examples of timetables from my diary and hers, although these were not exactly like the timetable she was to discuss later (train timetable).

Lyn conducted a whole class brainstorming session in which she summarised their experiences. She established the meanings the children had for the word "timetable" e.g. bus, train, boats, planes, school timetable, hospital, movies, roster, event (like a program), TV guide, clubs, sports (fixtures). They all felt free to contribute and at times were more expert than their teacher.

Lyn suggested that they cluster the information. Three categories emerged eg. 1 program, 2- diary, 3 fixture.

Lyn set the task for them to list the features they would expect to see on a timetable.

This was attempted by the students although it has not been made explicit how this would help them to calculate the most efficient route to the campsite.

About six students misunderstood and actually started to design their timetable.

Lyn stopped them and announced to the class why certain information would need to be on a timetable.

It was still not clear to the students why they needed to extract this information.

Lyn asked a student to read from the dictionary the definition of "timetable".

The whole class brainstormed what she asked earlier i.e. list the features you would expect to find on a timetable. (2-3 tables were not contributing or once they had contributed, opted out of the discussion.)

Students shared ideas as a class. Suggestions included dates, days, grid, shapes, numerals, a.m., p.m., 24hr time, abreviations e.g. P. E ...

Lyn then handed out a timetable for them to add to their ideas. It was the Bunbury to Perth line and will assist in planning their trip.

Students continued to contribute to the brainstorming session and Lyn encouraged them to explain to others who didn't seem to understand or who weren't able to locate the information from their timetable.

Lyn relied on questioning students, often directing questions to a specific child who she thought was not listening, or who she considered representative of others in terms of understanding the discussion. eg. William, Ben C, Ben B, Ben E, Dean (lower level students)

Blake shared his discovery of a pattern in the timetable.

The lesson ended at 10.30 a.m.

MICHELLE W.

Lesson observation 30.3.92

11.20 a.m.Michelle introduced lesson by saying that they needed to get this work done before the end of term, which was in 2 weeks, and that it would be needed for next semester's work.

She established that the students know what % means. She provided the example:- the number of students who are here that went on the camp is 82%. She asks them how many that would mean were not here...

She reinforced the notion of:- out of 100.

She asked the children to check it with their calculators.

Michelle wrote on the blackboard as she talked through working out % and stressed that when they see / in 82/100 it is called the divisor.

Most students grasped it in the first 10 minutes, then there was whole class discussion. Michelle seemed to use Carrie P to check whole class understanding as she often took a while to catch on.

Students were given 14/50= ?/100 and were asked to make equivalent fractions. She asked the students to offer a fraction which the whole class could solve as an equivalent fraction .../100. Michelle selected those problems which demonstrate equivalence easily.

About half of the class seemed to be following her with their hands up. Linley and Gabrielle H drifted in and out. Sue contributed often.

Michelle asked them if they remembered doing these last year. As they were streamed last year, most acknowledged they had been exposed to it. Michelle wanted to get on with more complex use of percentage but needed to back pedal until they had this part right.

Michelle demonstrated the fraction idea with magnetic units.

Students were still being asked to contribute to making equivalent fractions .../100.

The problem came when students kept coming up with problems that didn't quite "go" e.g. 23/47.

Both Carrie B and Tania appeared to be on the same wavelength and dominantly shared how they made equivalent fractions, their method was not the same system as Michelle's.

There was strong cohesion in the lesson after Michelle presented her logical method for finding percentage.

Grade 7 Addison Wesley (purple) maths books were handed out. Gabrielle and Emma completed their tasks quickly. In fact percentage concepts tested by this workbook were easier than the tasks the children had done together on the board, e.g. p. 296 "Give the percent for each fraction 1/10, 3/4, 3/20, 4/5, 1/100, 5/20, 99/100, 7/1, 8/10, 3/2."

Both Angela and Carrie P converted their answers to decimals when asked to express their answer as a fraction (this was because that was what they had been doing in their most recent lessons).

Tania continued to solve problems in her own method and didn't want to let go and adopt Michelle's system.

About one quarter of the students were having trouble- more through lack of confidence e.g. Emma, Melissa, Linley.

Michelle completed checking answers for theworkbook problems in a whole class forum, reinforcing the concept with magnetic fractions when difficulties arose.

Lesson ends 12.30 p.m.

Lesson observation **6.4.92**
Continuation of work on %. (other mathematics lessons on different content had been conducted in the period between observations)

Michelle stimulated the students schemas by discussing the proportion of students who went to camp and who didn't like chicken.

She discussed different methods for doing the problem of 80% of 41, and settled on the method—80/100 X 41/1.

She used what they already knew e.g. 50% of 41 would be (201/2). She suggested that they approximate and had them think in their own mind that if half way between 41 was 211/2 then 80% would be more than half

Michelle instructed the students on how to work this out on their calculator.

When asked to estimate first, the majority of students found it hard.

Michelle wrote the following examples on the whiteboard for them to do:-
50% of 80, 50% of 38, 75% of 28, 77% of 125

Most students showed a lack of confidence but once they used the calculator they seemed to develop confidence.

Melissa had away of estimating 77% of 125 - she rounded off and made the sum 8 X 12.

Students were set examples from p.302 of Maths Book Addison Wesley (purple) - "Finding a Percent of a Number- In a class of 25 students, 24% of the students play a musical instrument".

Michelle told class that when reading a problem in maths- "of" means "times".

The textbook presented the question in a complex manner 25 X .24, which recalled the need for long division (which needed revising). Michelle demonstrated long division on the board.

Michelle got them to estimate and elaborate on the logic of their answer.

She set them to work and encouraged them to look back at the example the class had done on the board.

She embellished the workbook task by asking them to put each question into a written context eg. There are 24% vegetarians in a class of 25. The book has the following questions e.g. 37% of 26, 77% of 125, 67% of 130, 8.5% of 200, 123% of 94, .5% of 700. The book seemed to offer a lot of variety but quickly became complex.

Students did not seem to understand the reason for multiplying or why the number % means two decimal places. This showed in the problem .5% of 200, where simply applying their formula way of solving it became more complex.

Lesson ended 11.00 a.m.

Appendix 2

March Lesson (ML)

LYN M.
11.3.92 lesson observation (text unit 115-157)

Two lessons have transpired continuing work on the party theme.

Lyn recapped yesterday's work where children were trying to determine the most popular choice of 8 savouries.

Students are allowed to enter their results onto the computer in the wet area.

Lyn emphasised that they are to avoid typing errors.

Lyn chose pairs, students were given the task to record a particular food on a sheet ie how to record/tally easily.

There were 5 different sheets, each group will report on their findings once pairs have done their task.

Lyn asked students for their suggestions for ways of recording.

Most students suggest a tally or sticks.

Lyn discussed and built on the students suggestions until they arrive at the idea of a grid and graph.

Lyn offered the prompt of the need for graph paper.

Lyn asked the students for suggestions of how they might use the graph paper. As she talked she sought clarification of their ideas and encouraged them to provide more information.

Lyn offered a different suggestion of a pie graph.

She got students to clarify their definitions of a pie graph.

Lyn encouraged freedom of choice in how they may use the previous discussion on graphs in their own recording of choice of savouries.

Lyn stressed neatness in recording and reiterated the need to keep a record.

She encouraged students who even though they were working in pairs were to use different methods individually and compare.

She gave the students the prompt to calculate foods first and then do graph.

Some of the girls seemed to be preoccupied with their textas and colours, while the boys got on with tallying.

The greatest productivity came from Melanie and Stewart.

Some 15 minutes later when the students seemed to still be unclear of the task, Lyn recapped using whole class discussion and then wrote the major question on the board:- "Find ways of discovering which foods are the most popular so we can have a menu suitable to all of us." (although by this stage in her discussion the students didn't need the blackboard prompt as they were quite clear).

Lyn then discouraged any off task discussion because time was becoming a constraint.

Even though the task was to be done in pairs, only one couple operated in that way- Tim and Ruben, Tim let Reuven do the recording while he read out the information.

Victoria was noticed to be quite slow at doing it because she was concerned with her spelling.

Lyn seemed to be unconcerned with the actual rate of individuals except those who prompted her for questions.

Lyn moved around the groups and encouraged cross group discussion and originality.

Lyn would let the dilemma of one child or group be public so that it became an open class problem solving discussion.

The students stayed focussed and on task for 40 minutes.

Lyn then gave the students a 5 minute deadline for which they had to be ready to report back in order from groups 1-5.

During this feedback time it was readily obvious to the students which food was the highest in popularity.

One group did have trouble with their records and Lyn facilitated this group during their report back to establish which food was the highest in popularity.

Few groups operated cohesively- breakaway sub groups occurred. There was a common drive to complete their own work. Poor groups skills was very obvious.

The students returned to their groups to complete any unfinished work. By the time the lesson had continued for an hour students had broken away from their group and gone back to their couples for advice.

Lots of levels of completion were evident in the class, with one third of the class unable to sustain the task- Brenda, Victoria, Jo and Matthew (weaker students).

Lyn closed the lesson by asking students to describe the process they had gone through in a long handed way (writing it down), about 4 could do this.

The lesson lasted 80 minutes and it seemed to be driven by the momentum of the more able students (about 6 students).

Michelle W.
12.3.92 lesson observation (text unit 154- 198)

As students arrived they were told to ignore a live dove fluttering in the rafters, this had to be reinforced on more than this occasion when the students were distracted.

Michelle recapped on their homework and asked when might they round decimals in their everyday life.

Linda responded as to how rounding was used in shops. The teacher encouraged her to give an example.

Gabrielle gave the example of road signs which indicate how many kilometres to a destination.

Angela stated that it wasn't likely for us to use them and that Christian Scientists would use them. (This response was ignored by the teacher).

Elisabeth went back to the road sign discussion.

The teacher asked why you might have to round back to a decimal.

Carrie P said a judge might judge you e.g. for a speeding fine where what you pay is calculated according to the kilometres over the limit you were travelling(however her answer stayed with a whole number).

Emma said that when you tell the time e.g. 4.23 you round it off to 4.20 (the fact that this was working with 60ths and not 100ths was not discussed).

Melissa talked about rounding up for test marks e.g. 87%- 90%.

Tania talked about why she hadn't got her homework done- her parents were starting a restaurant. (This response was not developed by the teacher.)

The teacher ignored any off task discussion such as this.

The teacher asked Susan to calculate one of the homework examples out loud.

The students were asked to look up p.40 in their textbooks (Purple Addison Wesley).

The teacher got Linley to read "Adding Decimals. Jason visited some European countries. He has 1 British pound note, 1 German mark, and 1 Greek drachma. What is their total value in U.S. dollars?"

The teacher asked when converting to American dollars who would be richer. (This was an example of applying maths to a real problem, especially if you are a tourist, which the majority of these students would be.)

The teacher asked students who tend to know what she is talking about rather than seeking clarification of understanding from less confident students.

The teacher drew students' attention back to the textbook p. 42 (Addison Wesley purple) and said "Let's look at a word problem and get the maths out of it."

She read the text "Subtracting Decimals- Using an electronic timer, a tennis serve was clocked at 62.483 meters per second (m/s). A baseball pitch had a speed of 41.556 m/s. How much greater was the speed of the tennis ball."

Angela responded and frequently seemed to be on the teacher's wavelength.

Most students were involved as they attended to the teacher's demonstration of calculating subtraction of decimals.

The teacher encouraged students to see the pattern in the method and talked about a block of chocolate to explain the "trading" idea.

Kylene was called upon but struggled to get the answer.

Naomi was a confident contributor.

The teacher set p. 42 Q 1 and 2:-"Warm ups- Subtract 1. 58.42- 26.65 =... 2. 9.27- 5.969=..."

The teacher roved around the room and stopped at Chantelle and checked if she was O.K.

Most students worked on their own.

Kylene was still confused, and said she knew what to do but couldn't explain it.

Carla wanted to use the calculator to check her working out. The teacher encouraged the whole class to do this too.

After 2 minutes some had completed the work but Carrie P was still confused and had not done any.

The teacher stopped the class and gave extension work to the class. She used a new idea with them where she gave the students a task to give their partner a word problem i.e. they give the information/ problem/ sum to their friend to do the working out and then they changed over. The teacher set the condition that she preferred the type of sums to be similar to the subtraction of decimals exercises which they had been doing from the textbook.

Leanne and Angharad were slow to get on with the task.

Georgia got the first problem wrong but it was not immediately noticed. (Once the sums were handed in, the teacher corrected this later.)

The teacher was able to get to tables/groups 2, 3 and 1 (16 students in a class of 26).

Most students stayed on the task of setting problems for one another.

Table 2 announced that they had never worked in pairs like this before for maths in Ms W's class.

The teacher continued to roam around the room checking on those who didn't understand.

Table 4 (6 students) got visited less and table 5 (3 students) not at all.

Students were called up by the teacher to do a couple of problems on the board.

The teacher encouraged the students to talk through their processing out aloud, and asked the class to check at the same time.

Where there were specific features the teacher talked through the problem.

The teacher gave students the task of writing and solving another three problems for homework, using decimals and subtraction.

She said "Make sure the sum is one where you need to borrow from other columns. Check on the calculator. Make them interesting."

Appendix 3

August Lesson (AL)

Lyn M.
Lesson Observation 5.8.92 (22 text units 476-510 less 12)

8.30 Lyn began the lesson by reminding them about their latest maths project which was for them to choose a sport related to the forthcoming Olympic Games and demonstrate the mathematics in connection with that sport. They could do it as individuals, pairs or in small groups. They could also treat the project like a secret where the class has to guess which sport it was- different parts of the information would gradually be revealed during the presentation.

Lyn discussed previous maths lessons they had had and mentioned how they might use what they had learned e.g. decimals.

Lyn gave them three minutes to think about it, but interupted them before the time was up.

Lyn then adjusted the perameters so that they can now choose any sport, not necessarily an Olympic Games related sport.

Lyn continued to reinforce the idea of all the maths calculations that might be involved in a sport, but at this stage the idea is still a little obscure for the students.

Those students who could see how maths fitted into their sport shared their ideas with the class.

The students who were able to do this were Tim W, Dean J, Mark, Melanie and Reuven.

Lyn adjusted the task so that the students now have to draw to scale a piece of equipment or the field etc used in the sport they have chosen.

The students have been exposed to a fairly dense rate of information delivery. It would appear that the idea from the previous week of keeping their selected sport a secret has been ignored.

Lyn put the following onto the blackboard 1. field 2. bars 3. court... cross country, arena...

Lyn recalled for them the scale work they had done last term on size and shape.

She encouraged the class to share where they got their ideas from e.g. school, reading, home etc, so that they consider a broader range of resources for their project. Several students then went to the library and encyclopaedia's in the back of the room.

Lyn took the problem of one student and built on that and encouraged the whole class to share in solving the problem.

Paul S shared his idea on angles for his sport.

Lyn encouraged the students to "talk to the person next to you"

The whole class went back to the task with confidence.

The teacher walked around the groups and took up one student's idea by suggesting the class might like to do a sketch first.

The whole class was engrossed on their tasks.

8.55 as the teacher walked around the groups she questioned, encouraged and scaffolded their ideas.

She got a student to explain her dressage arena to the class(Melissa).

Lyn developed this example as a model for the other students to learn from.

9.05 Lyn called the class to join her at the blackboard. She recapped the explanation of the dressage arena and related it to other student's work, building on the idea by increasing the complexity of what Melissa was trying to do.

Lyn encouraged students to solve the problems as she talked through them using a form of oral cloze.

There was a slight diversion as she revised their "tables" to take advantage of the teachable moment.

The students then went back to jointly problem-solving Melissa's dressage arena.

9.12 the students were sent back to their seats.

The teacher encouraged the students to join together and share with each other, acknowledging to the students that she was not the only expert in the class especially as they know their own sports in more detail.

Lyn reinforced the boundaries for their project and her expectations which have become a lot more specific after quite a loose initial structure.

Students continued to work in groups and pairs with only a couple of students off task. (Observation ended 9.30)

Michelle W. 10.8.92 lesson observation (27text units 652-736 less 57)

Michelle began the lesson at 10.14 by recapping fractions which they would be needing for that lesson. They were familiar with the layout in their textbooks which Michelle used to focus the lesson.

Michelle put the first sum from their textbook (Addison Wesley purple p. 196 q. 1- 3) onto the whiteboard:- 1. 5X3/ 8X3= .../24

She used the analogy of a break in an arm in order to explain the word "fraction". She discussed breaking a bone into many pieces or quantities.

Michelle said that in maths the pieces have to be equal ie. equal pieces.

She set them the task to draw a diagram to show it had been cut into equal pieces.

Michelle demonstrated what she meant on the whiteboard using 5/10 and a square of 10 sections with five parts shaded. She discussed that the shaded part is the same as 1/2.

She demonstrated the same idea using grid paper (1cm squares)

She challenged students by asking them to discuss which was the bigger fraction between the 1/2 and 5/10.

She then set the task for the class to make up other equivalent fractions following this last example.

Emily asked "Is it like you have 10 parts and 5 coloured or ...?"

The teacher went on to a discussion about both the numerator and denominator in a fraction.

She set the task for one person in each group to draw a particular fraction, using grid paper, and then each member of the group had an equivalent fraction to colour in (the students were not told that their fraction would be equivalent to one another).

Elizabeth's square was 12 X 12 and she was told to colour in 2/3
The second student in her group was told to colour in 4/6 of that same square.
The third student was told to colour in 6/9 of that same square.
The fourth student was told to colour in 8/12.

Georgia's square was 16 X 16 and she was told to colour in 1/2
The second student in her group was told to colour in 2/4 of that same square.

The third student was told to colour in 4/8 of that same square.
The fourth student was told to colour in 8/16.

Georgia asked " But isn't 4/8 the same as 2/4?!" To which the teacher replied "That's what may be found out."

Angela's square was 24 X 24 and she was told to colour in 2/3
The second student in her group was told to colour in 4/6 of that same square.
The third student was told to colour in 8/12 of that same square.
The fourth student was told to colour in 12/18.

Chantelle's square was 36 x 36 and she was told to colour in 1/3
Her partner was asked to colour in 5/15.

The teacher's idea behind this activity was to assist students to understand the denominator by colouring them in first before doing the fraction sums.

Students doing division involving 1/3's found it more difficult than those doing 1/2's.

At 10.37 students were asked to report back and show the class their grid paper.

Emily (from Georgia's group) said "You can 'see' that they're exactly half".

Most of the students were still struggling with the concept of identifying each unless they had the grid paper to colour in with.

Lesson ended 11.00 with homework set from Addison Wesley workbook p. 196 1. 5X3/ 8X3= .../24 2. 2X4/ 3X4= 8/...3. 3X10/ 5X10= .../50

Appendix 4

Bickmore-Brand's Pedagogical Principles Explained

Bickmore-Brand, a lecturer in Education, was concerned by the diverse messages being given to University students on their journey to becoming teachers from the various disciplines aligned to or establishing pedagogical principles. She undertook an extensive interpretive literature search (Bickmore-Brand 1989) to see if there were any common underlying principles. The search traversed across many disciplines:- language arts theory, education theory, socio-psycholinguistic theory, language and learning theories, metacognition theory, literacy learning theory, educational psychology theories and early childhood learning theories.

Seven pedagogical principles were synthesised from the massive range of material available. She separated out those principles which have application across the board. The resultant common principles will be readily recognisable to teachers because they are embedded in good practice. Teachers have applied these principles to their classrooms across the curriculum from early childhood through to tertiary education, including areas of adult literacy and post compulsory schooling. It was significant to note synonymous principles in the 'organisational change' literature being produced in the business community. The basis of these teaching/learning principles selected by Bickmore-Brand is their multidisciplinary application to enhance all learning situations.

CONTEXT- creating a meaningful and relevant context for the transmission of knowledge, skills and values
INTEREST- realising the starting point for learning must be from the knowledge, skills and or values base of the learner
MODELLING- providing opportunities to see the knowledge, skills and or values in operation by a 'significant' person
SCAFFOLDING- challenging learners to go beyond their current thinking, continually increasing their capacities
METACOGNITION- making explicit the learning processes which are occurring in the learning environment
RESPONSIBILITY- developing in learners the capacity to accept increasingly more responsibility for their learning
COMMUNITY- creating a supportive learning environment where learners feel free to take risks and be part of a shared context.

BICKMORE-BRAND 1990

Bickmore-Brand's teaching/learning principles reflect the dynamic nature of learning environments and therefore each needs to be considered integral with one another. The following is a synthesis of the main ideas behind each principle, for more detailed reading see Bickmore-Brand (1993) in Stephens, Waywood, Clarke and Izard, ***Communicating Mathematics: Perspectives From Classroom Practice and Current Research*** ACER/ AAMT, Melbourne.

CONTEXT- creating a meaningful and relevant context for the transmission of knowledge, skills and values

The premise behind this principle is that most learning occurs naturally embedded within a context which is obvious/ explicit to the learner. It is much easier to learn and practice the subskills when you have an idea of the big picture or can see the relevance of where the learning fits. Its a bit like cooking a dish when you already know what its meant to look and taste like and when it should be served. Learning that has a 'real world' application is gaining popularity in our schools. No longer is it seen appropriate to deliver isolated skills lessons devoid of any obvious context. Textbooks have taken this innovation on board with increased examples of simulated or real life exercises.

Teachers can apply this principle by:

- making explicit the purpose behind the learning about to be undertaken and how that chunk of learning fits into the social context from which it was drawn.
- provide finished products or examples of the skills.
- set tasks which use 'real world' situations.
- when working with abstract concepts which don't have an obvious 'real world' relationship, show learners how the part you are working on fits into the larger picture, or where you're heading conceptually.

INTEREST- realising the starting point for learning must be from the knowledge, skills and or values base of the learner.

The premise behind this strand is that in order for learning to take place the learner has to connect the new information to what they already know. The difficulty for us as teachers is the idiosyncratic way each learner hooks the new information into their schema. If you've ever played 'Pictionary' you'll be well aware of how individual people's thinking can be! Consequently the learning environment needs to be tolerant to a range of learning styles, allowing students to use a variety of mediums and generate their own 'rules' for working things out.

It is fundamental that we get to know the knowledge, skills, values and in many cases the cultural base of the learner so we design our teaching to connect most closely to where they are coming from. In many cases we are transmitting concepts embedded within quite distinctive terminology which needs to become part of the learner's repertoire. Bickmore- Brand's (!993) example illustrates this well:- a seven year old child was questioned after an exemplary lesson teaching him the concept of 'volume' using concrete materials. When asked what 'volume' meant he replied "It's the knob on your radio which makes the noise louder." A teacher using the students own language initially and building the new vocabulary onto that base will enable

that learner to take on board the new ideas as part of their personal knowledge.
Teachers can apply this principle by:

- assisting learners to verbalise during their processing in order to give insights into how their schema is developing.
- start with content likely to create the least distance between the knowledge, skills, values, and cultural base of the learner.
- use and encôurage different styles of learning.
- use and encourage a range of mediums and learning experiences
- encourage individual generation of 'rules' or strategies as alternatives to going about a task in a single, 'right' way
- link new terminology to the learner's own perceptions of the concept

MODELLING- providing opportunities to see the knowledge, skills and or values in operation by a 'significant' person

The modelling principle refers to the influence people whom we admire have over us when we try to take on board their knowledge, skills values and or culture. Observing children's efforts to imitate their peers, sporting or media personalities clearly reinforces what a powerful learning tool this can be. Peers can be used as mentors or the teachers themselves can authentically model the concepts they are teaching. Bickmore-Brand asked children from a maths class, "Did their teacher use maths in her everyday life?" The rich variety of answers from the children showed how authentic the teacher's modelling had been e.g. some knew she played tennis and would use maths for that, others knew she sewed, others knew she was planning a trip overseas, buying a dishwasher etc. They knew a lot more than the standard response- she uses maths to teach them and to go shopping! The message these children got was that the content of the curriculum their teacher was teaching them was fundamental to her everyday life.

A teacher can model for the learners their own thinking strategies (see Metacognition principle) e.g. solving a problem they have a need to solve, either orally or on the blackboard or overhead projector etc. The students see their teacher grappling with ideas rather than presenting everything in a highly structure logical way. Learners are often left with the impression that they should be able to master the efficient short-cuts demonstrated by teachers from the outset rather than appreciate the reality of a struggle through the meaning making process. Similarly getting students to model how they went about a problem and comparing different methods can free up a learner to develop a learning process which suits their own style best.

A model can also be in the form of a finished product. It was suggested in the first principle- 'context', that it is a good idea if the learner can see an example of the finished product. Where possible the learner needs to see, feel or experience the content. These materials can be made available to the students for reference or to model their work on. Often the one off demonstration is not adequate for the learner especially when they're being asked to emulate that model. Similarly seeing the same information being applied in a different context can increase the transferability of the idea. The increased use of video and computers across all subjects makes this experiencing, although often vicarious, more possible. The advent of multi-interactive media will make this learning principle of modelling a virtual reality in our schools of the future!

Teachers can apply this principle by:
- showing students their own genuine use of the knowledge, skills and or values they are trying to teach.
- showing students their own struggle to process an idea or a new skill
- using students to demonstrate to peers.
- providing exemplars for students.
- providing opportunities for students to go back to reinvestigate models or examples.
- providing different contexts where the same knowledge, skill and or value is being used.

SCAFFOLDING- challenging learners to go beyond their current thinking, continually increasing their capacities

The premise behind the scaffolding principle is to provide enough support to the learner to enable a progression to the next stage of development. There is no expectation that the learners will become learners independent of the environment or of other learners. Scaffolding is essentially a hand-holding strategy, tailored to the needs of the individual and is provided at any stage when joint assistance might be beneficial for the learner's development. It will vary in difficulty and length of time in accordance with individual learner's needs.

What is being suggested in the light of the preceding principles, is that the learner's own communication be valued as the starting point. The teacher, it may be a peer 'mentor', uses his/her own language to provide the scaffold. They may model the reflective language of processing, or encourage when the learner is grappling, or reshape the learner's expressions to clarify at times and extend in more complex ways at others.

As learners grow in confidence, they are in the position to take on more responsibility. This might result in a role reversal for a while. As each new stage of development is reached the previous scaffolding is no longer needed and self- destructs ready to have the new scaffolding put in place.

Teachers can apply this principle by:
- recognising the parts along the way that make up the mastery of a concept, idea or skill.
- using enabling language to facilitate the learner's growth.
- providing support for as long as the learner needs and with as much difficulty as that learner is comfortable with.

- providing routines which allow the learner to practice by focussing on the new learning rather than continually creating novel situations until a reasonably level of mastery is attained.
- providing frameworks where the learner can clearly see the steps or components of the task.
- using peers and role- reversals to provide the learner with opportunities to reinforce their learning.

METACOGNITION- making explicit the learning processes which are occurring in the learning environment

This principle is concerned with the ability to be aware of one's thinking processes. Have you ever noticed when you're faced with a difficult problem that you resort to directing yourself out aloud, or you talk it out with others, so that you're in a position to assess the reasonableness of your decisions.

As teachers we need to model what it sounds like to think aloud, at the same time we can make explicit the 'why' of what we're doing and not just the 'what.' Learners will then be in a position to hear not only the logic of the concept but hear the accompanying language.

If students are to practice their reasoning abilities we need to use materials which foster making sense. Making the links explicit from one learning task to another, where possible across the curriculum areas, will make it easier for the learner to generalise about concepts and store them efficiently in their schemas e.g. in an English class, use the Geography assignment materials to teach Report Writing. Assist students to transfer what you're teaching to the 'real life' location for the knowledge skill or value.

Teachers can apply this principle by:

- consciously modelling out aloud their processing.
- integrating with previous learning both within the subject area and across the curriculum.
- encouraging students to verbalise and share their thinking strategies.
- selecting materials which lend themselves to metacognitive strategies eg. open-ended, concrete before more abstract, repetition for practice purposes ...
- teaching students to be critical (both positive and negative) of what they read, see and hear.
- providing them with feedback about how they have processed rather than just the finished product.
- jointly working alongside students to provide metacognitive language or frameworks when needed.

RESPONSIBILITY- developing in learners the capacity to accept increasingly more responsibility for their learning

In order for this principle to be effective it will depend on the integration of the other principles. The idea behind giving learners responsibility for their learning is not that they are left stranded in order to survive or alternately to run amok in our classrooms, but that they will gradually be in a position to accept increasingly more responsibility for the curriculum. Rarely in schools is there any real consequence for the decisions that students make on a daily basis. A class where their scale drawings of Commonwealth Games event layouts were presented to the groundsman for marking up the school oval for a sporting Carnival provided very tangible consequences for their maths calculations.

The reality is that it is the learners themselves who take on board the knowledge skills and or values they choose,consequently we need to connect more closely the curriculum to their starting point (see the interest principle).

Teachers can apply this principle by:
- allowing students to accept responsibility for classroom decisions.
- holding student's 'hands' until they have the skills to take control.
- providing situations which have tangible consequences for students processing.
- diminish the teachers degree of control by allowing a change of roles.

COMMUNITY- creating a supportive learning environment where learners feel free to take risks and be part of a shared context.

The classroom environment implied in the other principles seems to require a fairly special climate. A culture where the underlying values are open-ness, trust and honesty:- Open, in that learners feel supported in this classroom to take risks and to learn from their mistakes without any loss of dignity- and that includes the teacher;

Trust, where learners in this community feel empowered to negotiate the tasks. One where their learning styles and different levels of mastery are not only tolerated but are valued, thereby creating a scenario where the whole class can become a resource for one another. A class was asked ' who do you go to for help in maths?'. The reply was not the common response of- 'the teacher' but "Well it depends ... if you needed help with graphs you'd go to Blake, if you wanted some quick adding up done, you'd use Olivia..." etc. The learners in this community expect to succeed, jointly where necessary. Co-operation is encouraged to achieve excellence rather than a cop out or a time waster in this culture;

Honesty is the underlying ingredient for the other two values to operate successfully. You can't have openness and support if it laced with dishonesty or suspicion at least. Trust will only be achieved if the feedback people are given is honest in a positive and supportive way. In this community there is no need for dishonesty because the students are intrinsically motivated by the content. They, with 'significant people' are using, sharing and admiring what is being taught in a context which is not merely one in which they may have vested interest, but knowledge skills and or values which are valued and sort after by them.

Teachers can apply this principle by:

- supporting students to take risks and not denigrate their errors.
- providing tasks which allow flexible approaches to cater for different learning styles.
- negotiating control with students over some of the content and tasks.
- valuing each member of the class as a resource rather than the teacher as the 'fount of all wisdom'
- recognising and acknowledging your own limitations with students

CONCLUSION

The overall picture these principles paint is not intended to be overwhelmingly utopian, nor clouded with an undercurrent suggesting rising militant students, but rather it is to create a democratic relationship and restore the partnership between learner and teacher. The application of these principles is designed to strategically heighten and value the contribution that each can play in the teaching/learning environment. Ultimately these principles are designed to restore the arbitrary divisions between disciplines and especially between school and the 'real world'. It is the learner who is at the centre of the learning environment needing to learn how to function not only within the immediate context of what is being taught but as part of the fabric of a changing community.

Appendix 5

Initial Student Interview Schedule

Hello, I'm studying teachers and how they teach maths and because you're in Mrs ... class I'd like to see how you feel about maths. I'm asking all of your class separately and going to other schools to do the same. Do you mind?

Do you mind the video? It saves me taking notes.

1. How do you feel when the teacher says its time for maths lesson? You can use this scale (edge of the desk, where position represents happy ... sad).
2. How do you think you compare with the rest of the class in maths ability? You can use this scale (edge of desk, where position represents above average ... below average).
3. Who is the best person at maths in your class?
4. About where would you put yourself compared with him/her (edge of desk used as a scale indicating above that person located at the centre point or below).
5. What do you think kids who are good at maths seem to be able to do?
6. Why do you think they're good at maths? e.g. Do you think it runs in their family? Do you think it is because they work hard? Do you think they're just brainy? Do you think the teacher helps them? Do you think other kids help them?
7. Are your parents good at maths?
8. Did they do maths in High School? Year 12? Uni?
9. Do you think your Dad or Mum use maths around the home much?
10. Do you think they use it at work much?
11. What about your brothers or sisters? Do you think they use maths around the home?
12. What about you? Do you use maths except in maths lessons?
13. What kind of maths homework do you get?
14. Do you like doing maths homework?
15. How far do you think you'll do maths for? High School? Year 12? Uni?
16. Make up a sum that you think a Year 6/7 (matching their year level) student would be able to solve.

17. Can you do it? (if 'yes') Show me.

18. Which things in this book would you be able to do? (Using a textbook the teacher has designated would be appropriate for their year level.)

19. If you were to plan a class party what things would you need to consider? (prompt with ideas about who you would invite, how much you'd spend, etc, if necessary).

20. I'm going to get you to solve this problem on the white/blackboard. (The problem has been handwritten on a piece of paper which is read out to the student following it with my finger as I read).

    If there are 26 (actual number of their class) children in your class and am going to donate $150 to the class to spend on a class party. "Yes I am actually going to give your teacher that money." But the class has too many hassles deciding how to spend it and give up and decide to split the money between them. About how much will each child get? (Just estimate for me). What will be left over?

Can you tell me how you would go about solving this problem?
You can use a calculator if you wish.

How many coins of different types would the teacher need to have in order to give each child the exact money?

That's it thank you. Do you have any questions of me?

Appendix 6

Post Student Interview Schedule

1. I am going to walk you through this maths book for your year level I am going to use a code, you are to respond as if I was to ask you to do it in front of me now. The code is:

x = don't know how to do it
? = I've never learnt it before
O = I'D be O.K. at doing it
√= I'd be good at it
√√ = I'd be very confident to do it

2. Who do you go to for help in maths?

3. Do you need much help in maths?

4. What do you think you have learnt from Ms...........this year?

5. Do you enjoy maths?

6. What do you think you have learned that you can use in your every day life or in the future?

7. Do you think you have learned anything that you doubt will be useful in your daily life or the future?

8. Do you think Ms........... uses maths in her every day life?

9. Who do you think is the best person at maths in your class?

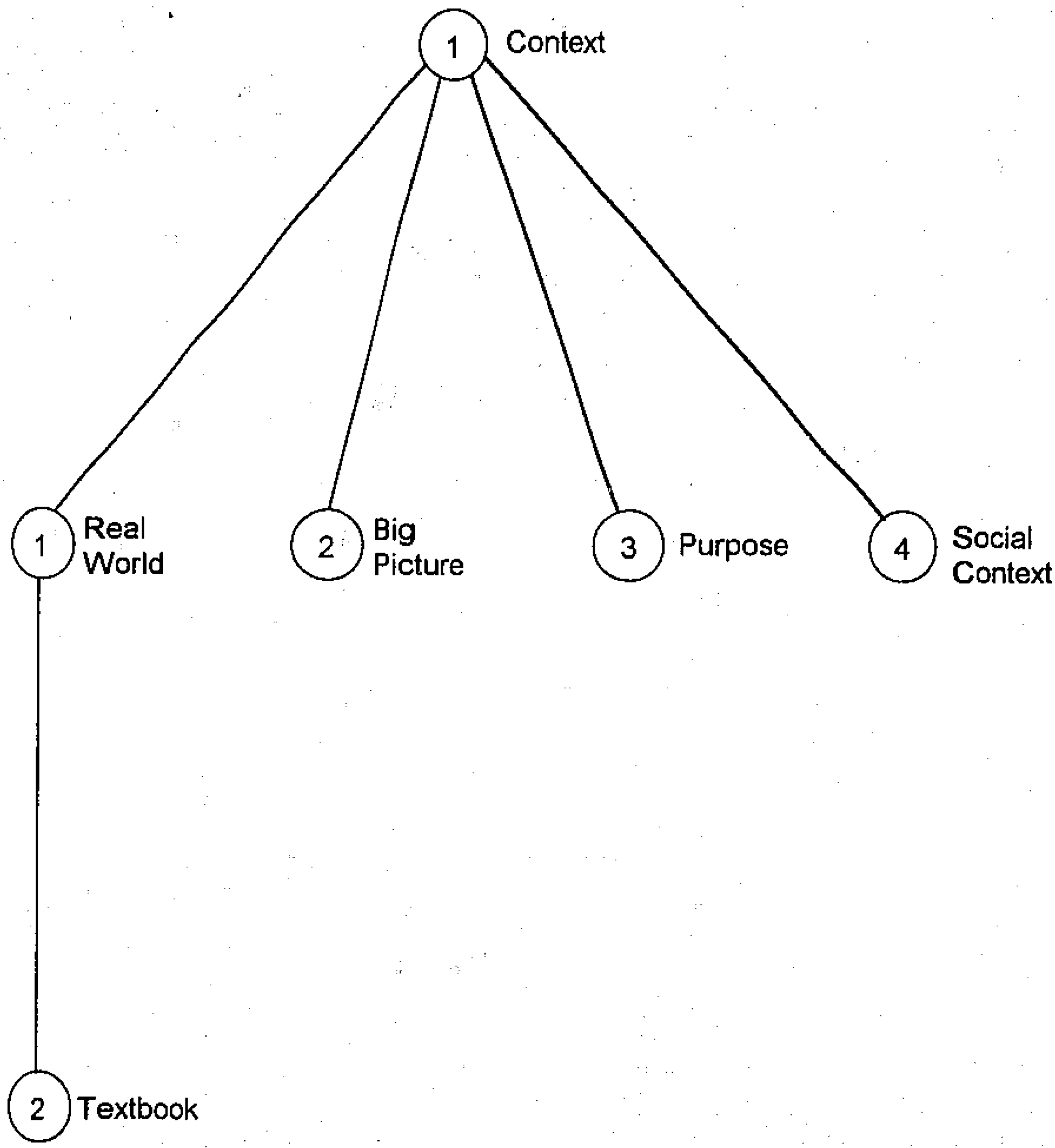
10. When the class has a problem to solve how do they go about solving it as a class.

11. What day of the week will your 21st birthday fall on and in what year?

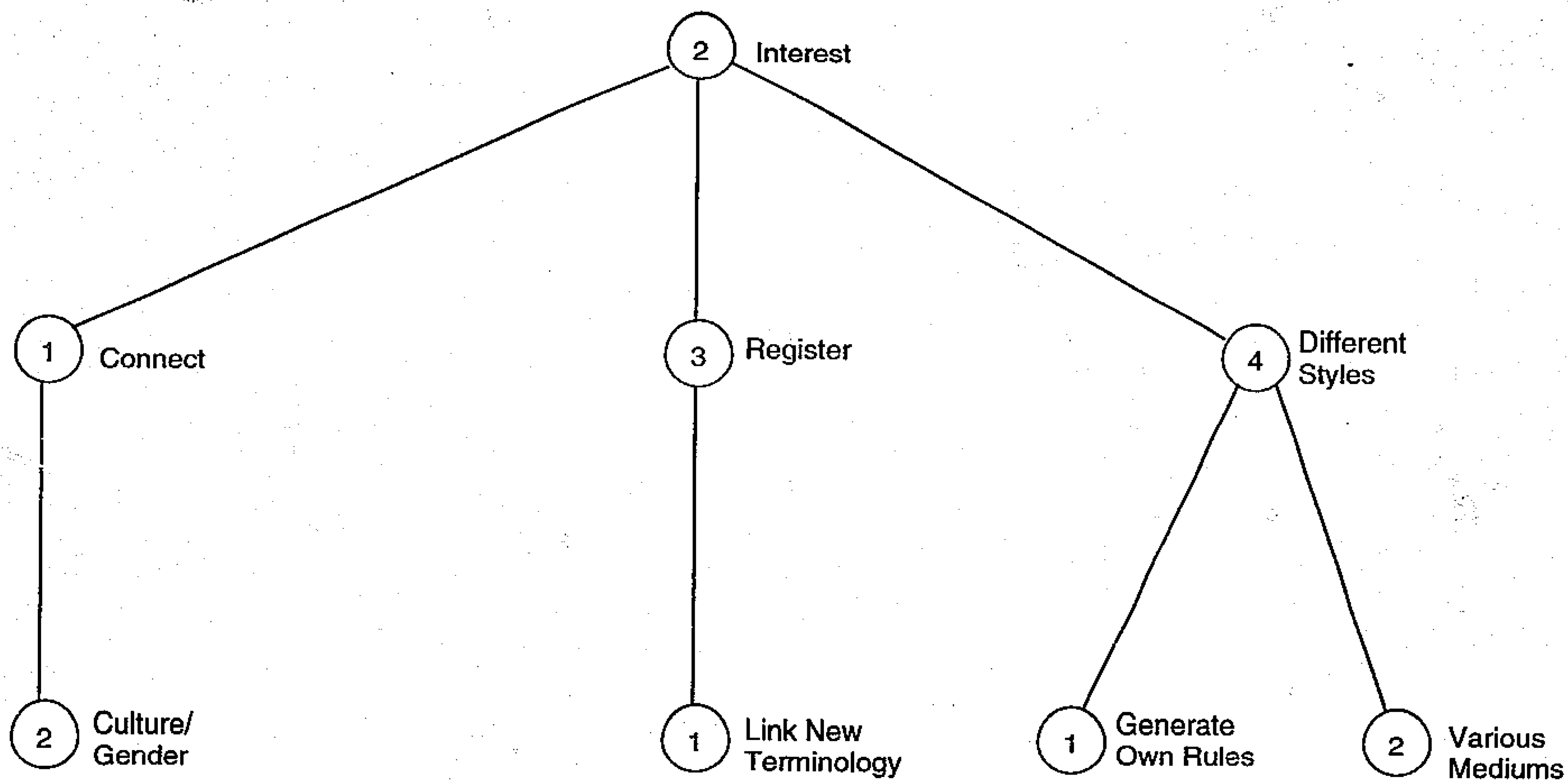
Appendix 7

Hierarchical Representation of the Ideas in Each Principle

CONTEXT

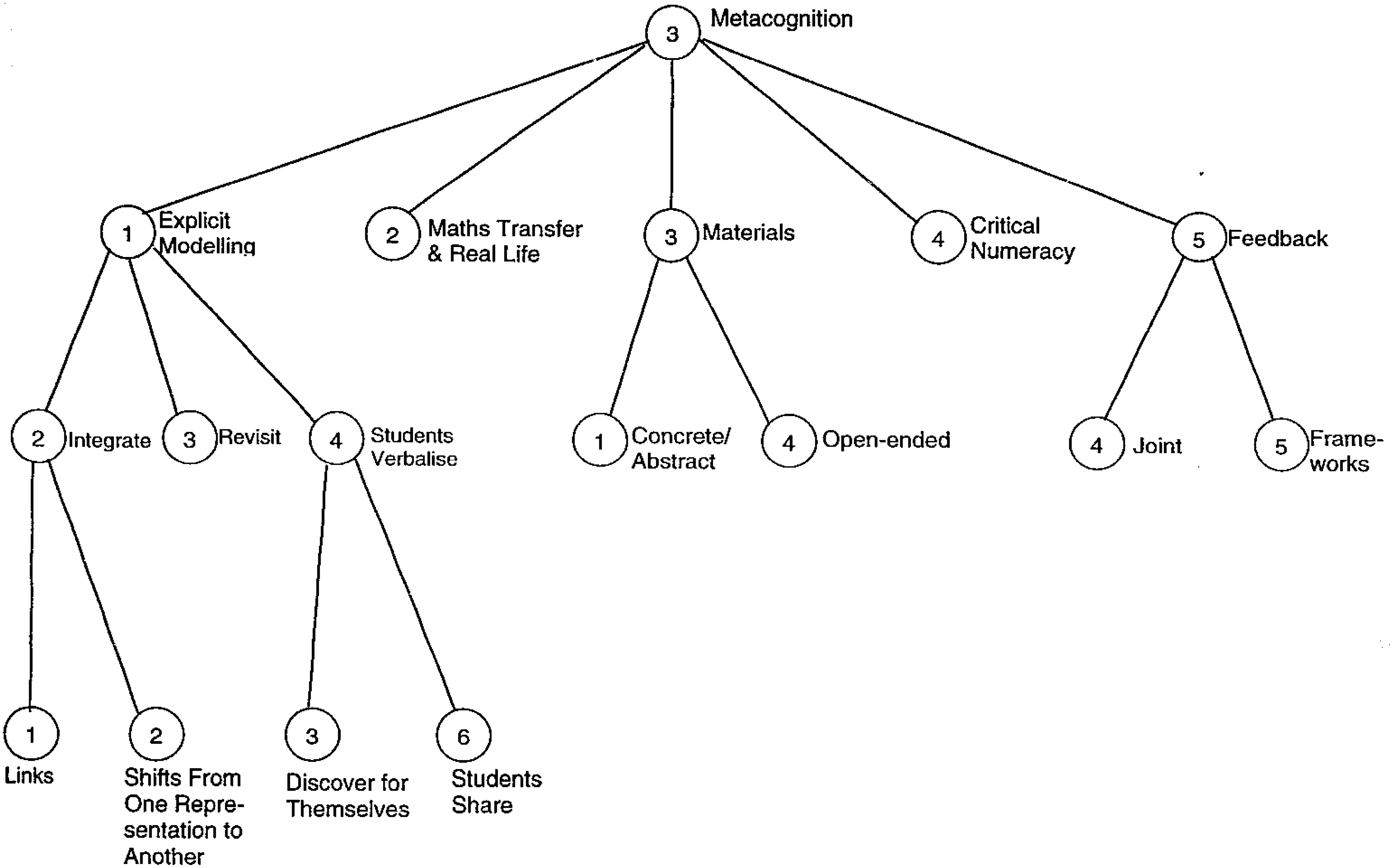


INTEREST

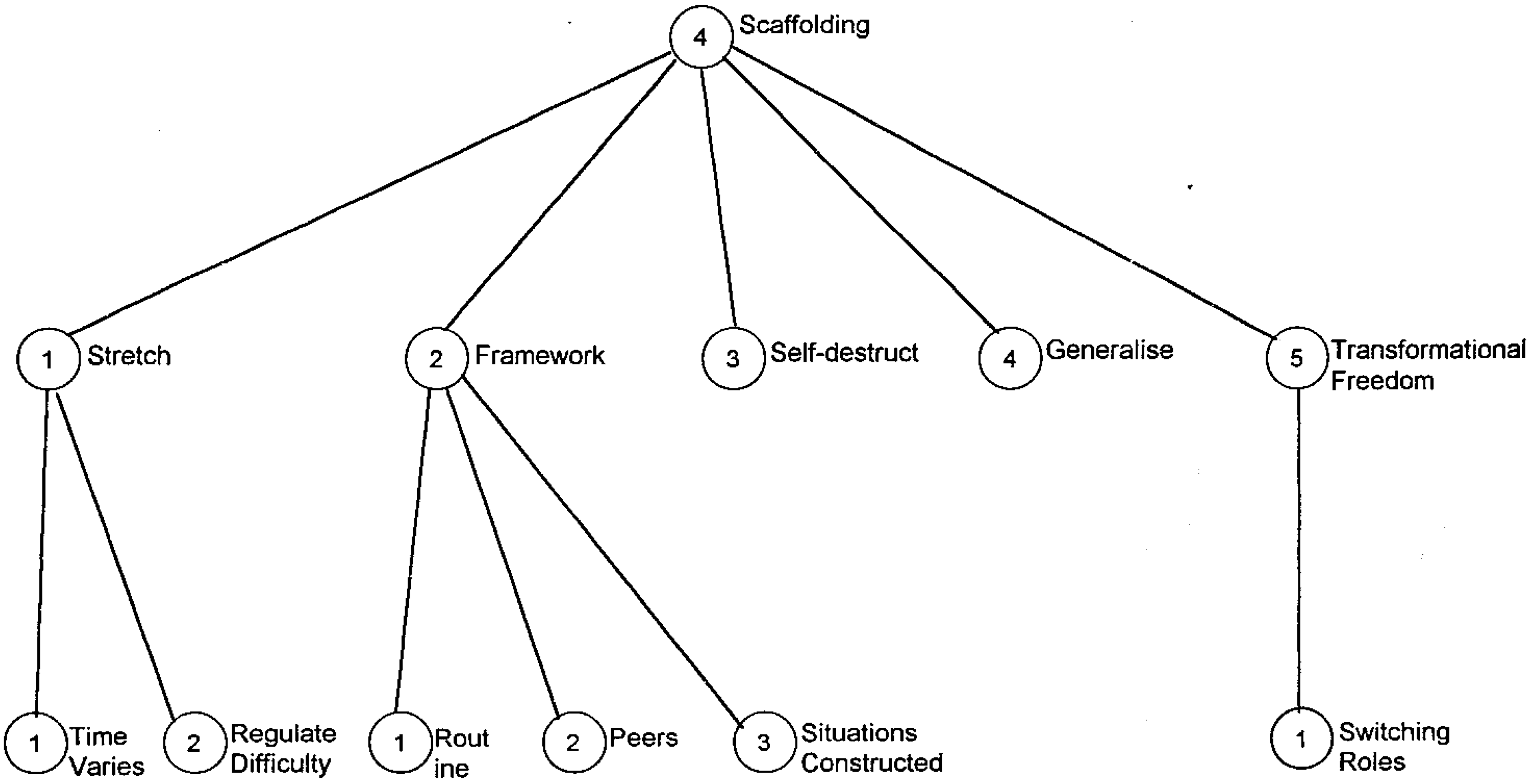


2 Interest
1 Connect
3 Register
4 Different Styles
2 Culture/ Gender
1 Link New Terminology
1 Generate Own Rules
2 Various Mediums

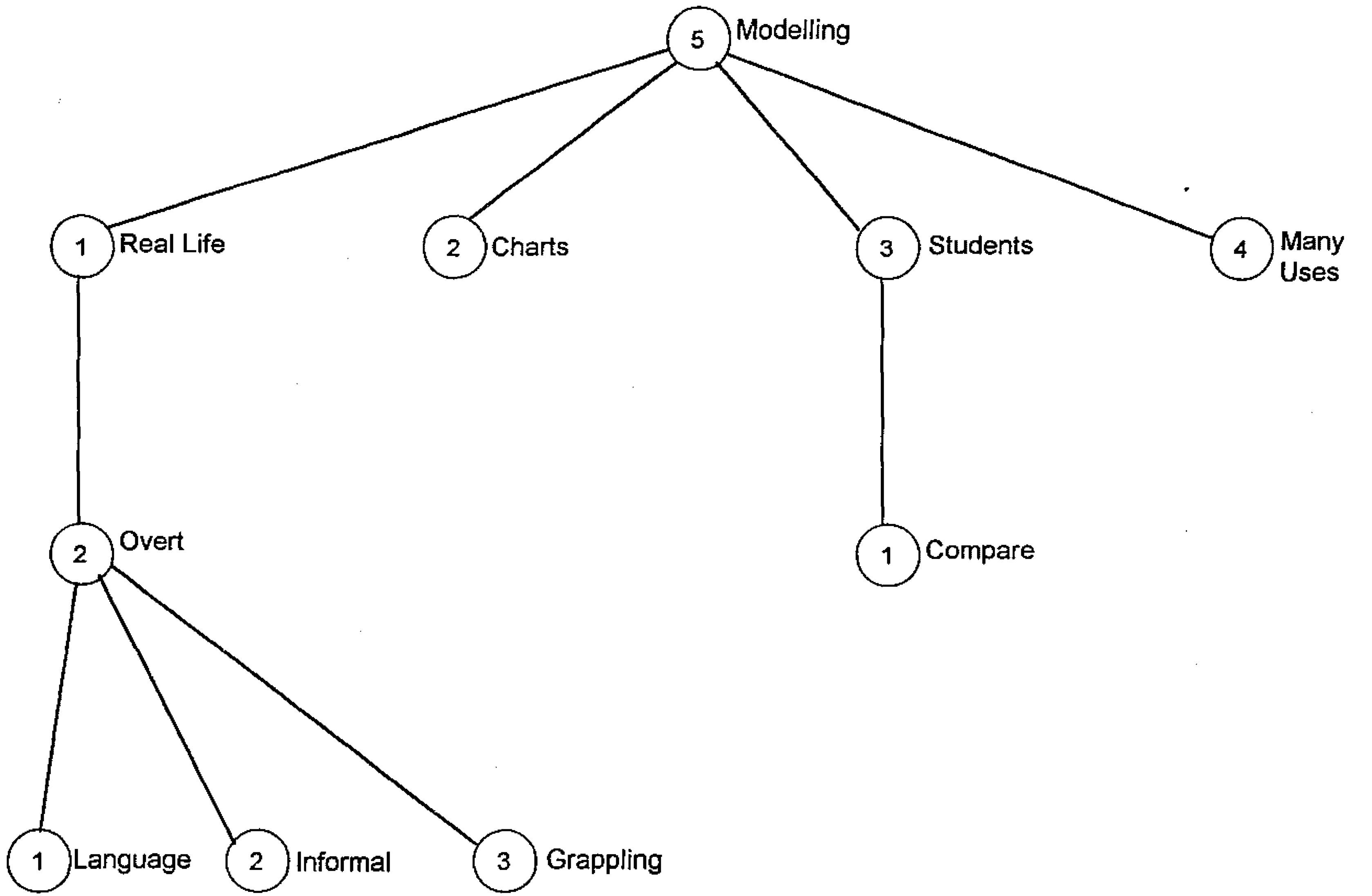
METACOGNITION



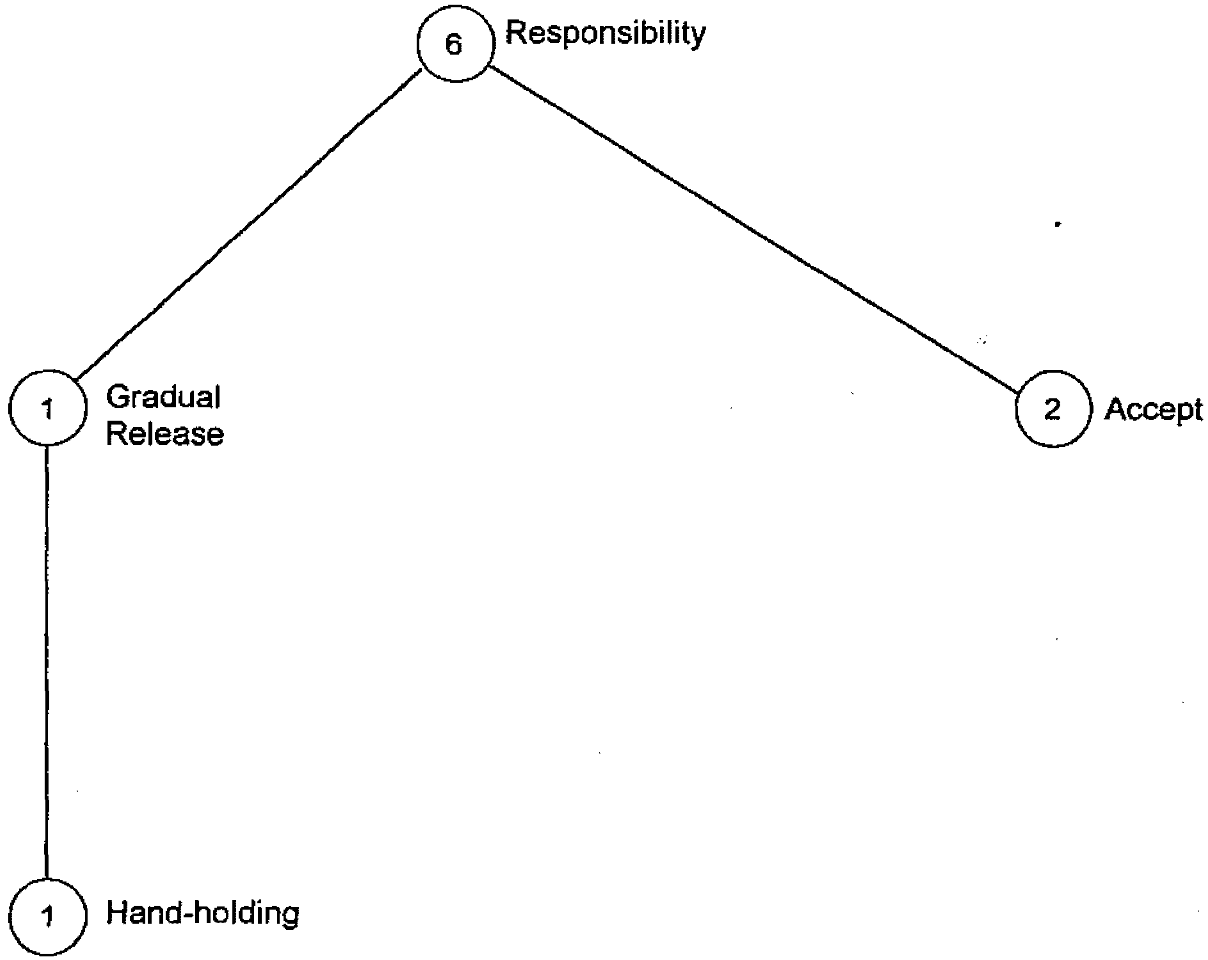
SCAFFOLDING



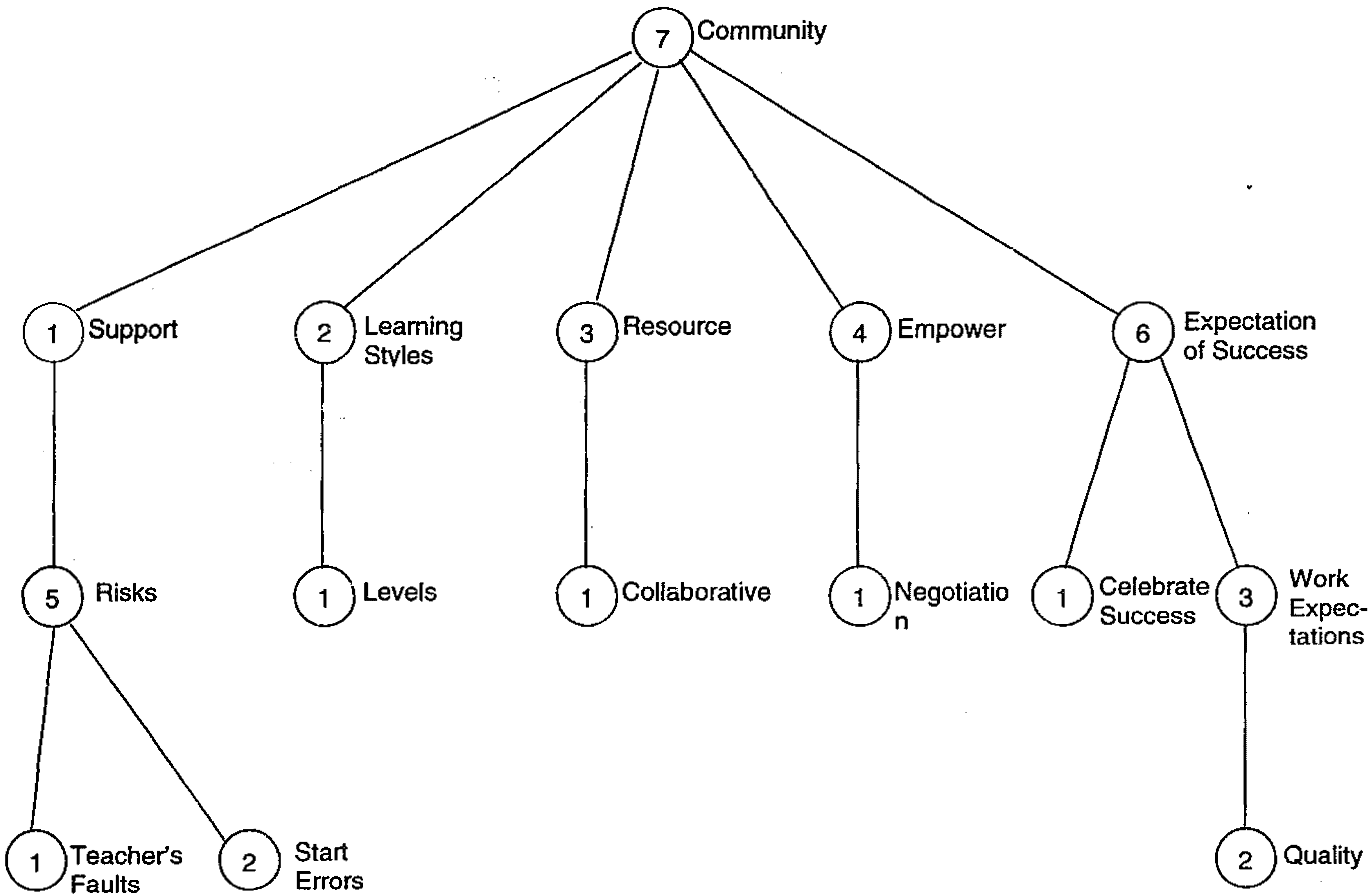
MODELLING



5 Modelling
1 Real Life
2 Charts
3 Students
4 Many Uses
2 Overt
1 Compare
1 Language
2 Informal
3 Grappling


6
Responsibility
1
Gradual Release
2
Accept
1
Hand-holding

COMMUNITY



7 Community
1 Support
2 Learning Styles
3 Resource
4 Empower
6 Expectation of Success
5 Risks
1 Levels
1 Collaborative
1 Negotiatio n
1 Celebrate Success
3 Work Expec- tations
1 Teacher's Faults
2 Start Errors
2 Quality

Appendix 8

Principles Defined

PRINCIPLES DEFINED (with Text Unit examples)

1. Context- creating a meaningful and relevant context for the transmission of knowledge, skills and values.

Teachers can apply this principle by:-

- making explicit the purpose behind the learning about to be undertaken and how that chunk of learning fits into the social context from which it was drawn
- provide finished products or examples of the skills
- set tasks which use 'real world' situations
- when working with abstract concepts which don't have an obvious 'real world' relationship, show learners how the part you are working on fits into the larger picture, or where you're heading conceptually

Examples of Text units to be coded under 1- Context might be:-

Lyn adjusted the task so that the students now have to draw to scale a piece of equipment or the field used in the sport they have chosen

Michelle recapped on their Homework and asked when might they round decimals in their everyday life

2. Interest- realising the starting point for the learning must be from the knowledge, skills and or values base of the learner.

Teachers can apply this principle by:

- assisting learners to verbalise their processing in order to give insights into how their schema is developing
- start with content which is likely to create the least distance between the knowledge, skills, values and cultural base of the learner
- use and encourage different styles of learning
- use and encourage a range of mediums and learning experiences
- encourage individual generation of 'rules' or strategies as alternatives to going about a task in one way
- link new terminology to the learner's own perceptions of the concept

Examples of Text units to be coded under 2 Interest might be

Lyn encouraged freedom of choice in how they might use the previous discussion on graphs in their own recording of choice of savouries.

Linda responded as to how rounding was used in shops. The teacher encouraged her to give an example

3. Metacognition- making explicit the learning processes which are occurring in the learning environment.

Teachers can apply this principle by:

- consciously modelling out aloud their processing
- integrating with previous learning both within the subject area and across the curriculum
- encouraging students to verbalise and share their thinking strategies
- selecting materials which lend themselves to metacognitive strategies e.g. open-ended, concrete before abstract, repetition for practice purposes...
- teaching students to be critical of what they read, see and hear
- providing them with feedback about how they have processed rather than just the finished product
- jointly working alongside students to provide metacognitive language or frameworks when needed.

Examples of Text Units to be coded under 3 Metacognition might be:

Michelle demonstrated what she meant on the whiteboard using 5/10 and a square of 10 sections with five parts shaded. She discussed that the shaded part is the same as 1/2.

Students shared at their tables the strategies they used to work out the average and in what ways it was the same or different from what had already been shared.

4. Scaffolding- challenging learners to go beyond their current thinking, continually increasing their capacities.

Teachers can apply this principle by:

- recognising the signs along the way that make up the mastery of a concept, idea or skill
- using enabling language to facilitate the learner's growth
- providing support for as long as the learner needs and with as much difficulty as that learner is comfortable with
- providing routines which allow the learner to focus on the new learning rather than have to take on board too many variables
- providing frameworks where the learner can see the steps or components of the task
- use peers and role-reversals to enable the learner opportunity to reinforce their learning

Examples of Text Units to be coded under 4 Scaffolding might be:

Lyn directed the students to look for the scale. As the scale was not explicitly given the students were then asked to calculate how many cm represent 1 kilometer. The teacher provides the example 1.5cm=10km.

She challenged students by asking them to dicuss which was the bigger fraction between 1/2 and 5/10.

5. Modelling- providing opportunities to see the knowledge, skills and or values in operation by a 'significant' person.

Teachers can apply this principle by:

- showing students their own genuine use of knowledge, skills and or values they are trying to teach
- showing students their own struggle to process an idea or a new skill
- using students to demonstrate to peers
- providing exemplars for students
- providing opportunities for students to go back to reinvestigate models of examples
- providing different contexts where the same knowledge, skill and or value is being used.

Examples of Text Units to be coded under 5 Modelling might be:

Teacher asked Susan to calculate one of the homework examples out loud

She demonstrated on the easel how to calculate "average" speeds. The students were gathered near the easel.

6. Responsibility- developing in learners the capacity to accept increasingly more responsibility for their learning.

Teachers can apply this principle by:

- allowing students to accept responsibility for classroom decisions
- holding student's 'hands' until they have the skills to take control
- providing situations which have tangible consequences for students' processing

Examples of Text Units to be coded under 6 Responsibility might be:

Students are allowed to enter their results onto the computer in the wet area

Students were given 14/50=?/100 and were asked to make equivalent fractions. She asked the students to offer a fraction which the whole class could solve as an equivalent fraction../100. Michelle selected those problems which demonstrate equivalence easily.

7. Community- creating a supportive learning environment where learners feel free to take risks and be part of a shared context.

Teachers can apply this principle by:

- supporting students to take risks and not denigrate their errors
- providing tasks which allow flexible approaches to cater for different learning styles
- negotiating with students over some of the content and tasks
- valuing each member of the class as a resource rather than the teacher as the 'fount of all wisdom'
- acknowledging your own faults with the students

Examples of Text Units to be coded under 7. Community might be

The teacher encouraged the students to join together and share with each other, acknowledging to the students that she was not the only expert in the class especially as they know their own sports in more detail.

Carla wanted to use the calculator to check her working out. The teacher encouraged the whole class to do this too.

The following Text Units show how multiple coding will inevitably occur.

She encouraged the class to share where they got their ideas from e.g. school, reading, home etc, so that they consider a broader range of resources for their project. Several students then went to the library and encyclopaedia's in the back of the room. (Coded under:-1,2,4,6 and 7)

She used the analogy of a break in an arm in order to explain the word "fraction". She discussed breaking a bone in many pieces or quantities. (Coded under:- 1,2 and 3)

She then set the task for the class to make up other equivalent fractions. (Coded under:- 4,6 and 7)

Lyn discussed and built on the students suggestions until they arrive at the idea of a grid and graph (Coded under:- 2,3,4,5 and 7)

Appendix 9

Tables 1-7

Table 1

Combined Totals of Raters for Context Principle

MICHELLE

	1.1	1.2	1.3	1.4	1.1.2	
Linda	2	0	1	1	1	
Les	1	1	1	0	2	
Anne	3	3	2	3	0	
Jennie	2	1	1	1	2	
Total Incidences	8	5	5	5	5	28
%	28.6	17.9	17.9	17.9	17.9	100

LYN:

	1.1	1.2	1.3	1.4	1.1.2	
Linda	12	0	4	0	0	
Les	4	8	5	0	0	
Anne	21	2	6	18	0	
Jennie	8	2	4	8	1	
Total Incidences	45	12	19	26	1	103
%	43.7	11.7	18.4	25.2	0.97	100

Lyn and Michelle % compared

Category	**Lyn**	**Michelle**
Real World 1.1	43.7	28.6
Big Picture 1.2	11.7	17.9
Purpose 1.3	18.4	17.9
Social Context 1.4	25.2	17.9
Textbook 1.1.2	.9	17.9

Table 2

Combined Totals of Raters for Interest Principle

MICHELLE:

	2.1	2.3	2.4	2.1.2	2.3.1	2.4.1	2.4.2	
Linda	10	1	0	1	2	2	0	
Les	3	0	3	0	2	0	1	
Anne	5	1	0	0	0	1	1	
Jennie	5	2	2	0	2	2	0	
Total Incidences	23	4	5	1	6	5	2	46
%	50	8.6	10.8	2.1	13.0	10.8	4.3	100

LYN:

	2.1	2.3	2.4	2.1.2	2.3.1	2.4.1	2.4.2	
Linda	5	2	1	3	1	1	4	
Les	5	1	2	0	5	0	2	
Anne	10	3	6	1	0	1	2	
Jennie	7	7	7	8	8	4	2	
Total Incidences	27	13	16	12	14	6	10	98
%	27.5	13.2	16.3	12.2	14.2	6.1	10.2	100

Lyn and Michelle % compared

Category	**Lyn**	**Michelle**
Connect 2.1	27.5	50
Register 2.3	13.2	8.6
Different Styles 2.4	16.3	10.8
Cultural/Gender 2.1.2	12.2	2.1
Link New Terminology 2.3.1	14.2	13.0
Generate Own Rules 2.4.1	6.1	10.8
Various Mediums 2.4.2	10.2	4.3

Table 3

Combined Totals of Raters for Metacognition Principle

MICHELLE:

	3.1	3.2	3.3	3.4	3.5	3.1.2	3.1.3	3.1.4	3.3.1	3.3.4	3.5.4	3.5.5	3.1.2.1	3.1.2.2	3.1.4.3	3.1.4.6	
Linda	8	0	0	0	2	1	5	4	0	0	0	0	0	5	2	3	
Les	1	2	0	0	0	0	3	1	0	0	0	0	1	7	3	2	
Anne	2	0	3	1	4	0	5	1	4	0	2	0	2	4	1	1	
Jennie	6	3	1	2	2	0	7	4	3	0	3	1	2	1	3	1	
Total*	17	5	4	3	8	1	20	10	7	0	5	1	5	17	9	7	119
%	14	4.2	3.4	2.5	6.7	0.8	16.8	8.4	5.8	0	4.2	0.8	4.2	14.2	7.5	5.8	100

* Total Incidences

LYN:

	3.1	3.2	3.3	3.4	3.5	3.1.2	3.1.3	3.1.4	3.3.1	3.3.4	3.5.4	3.5.5	3.1.2.1	3.1.2.2	3.1.4.3	3.1.4.6	
Linda	4	2	1	1	3	2	8	3	0	1	2	1	3	0	4	5	
Les	2	5	3	0	0	1	2	0	2	0	0	1	1	3	2	7	
Anne	4	6	5	15	4	0	2	0	0	0	0	1	0	0	0	0	
Jennie	7	9	6	7	2	2	8	6	1	1	3	3	3	2	7	7	
Total*	17	22	15	23	9	5	20	9	3	2	5	6	7	5	13	19	180
%	9.4	12	8.3	13	5	2.7	11.1	5	1.6	1.1	2.7	3.3	3.8	2.7	7.2	10.5	100

* Total Incidences

Lyn and Michelle % compared

Category	**Lyn**	**Michelle**
Explicit Modelling 3.1	9.4	14.2
Maths Transfer to Real Life 3.2	12.2	4.2
Materials 3.3	8.3	3.3
Critical Numeracy 3.4	12.7	2.5
Feedback 3.5	5	6.7
Integrate 3.1.2	2.7	.8
Revisit 3.1.3	11.1	16.8
Students Verbalise 3 1 4	5	8.4
Concrete-Abstract 3.3.1	1.6	5.8
Open-Ended 3.3.4	1.1	0
Joint Construction 3.5.4	2.7	4.2
Frameworks 3.5.5	3.3	.8
Links 3.1.2.1	3.8	4.2
Shifts From One Rep to the Other 3.1.2.2	2.7	4.2
Discover for Themselves 3.1.4.3	7.2	7.5
Students Share 3.1.4.6	10.5	5.8

Table 4

Combined Totals of Raters for Scaffolding Principle

MICHELLE:

	4.1	4.2	4.3	4.4	4.5	4.1.1	4.1.2	4.2.1	4.2.2	4.2.3	4.5.1	
Linda	6	1	1	0	2	0	8	1	0	7	2	
Les	0	3	1	0	0	0	3	2	1	0	1	
Anne	3	1	1	2	2	0	8	3	3	3	3	
Jennie	0	5	0	0	0	0	4	7	1	1	0	
Total *	9	10	3	2	4	0	23	13	5	11	6	86
%	10.4	11.6	3.4	2.3	4.6	0	26.7	15.1	5.8	12.7	6.9	100

* Total Incidences

LYN:

	4.1	4.2	4.3	4.4	4.5	4.1.1	4.1.2	4.2.1	4.2.2	4.2.3	4.5.1	
Linda	7	5	0	5	0	1	2	0	2	6	5	
Les	0	5	0	1	1	0	7	2	2	3	2	
Anne	11	5	0	0	2	0	1	0	3	0	2	
Jennie	2	4	1	1	0	1	5	0	0	3	1	
Total *	20	19	1	7	3	2	15	2	7	12	10	98
%	20.4	19.4	1.0	7.1	3.0	2.0	15.3	2.0	7.1	12.2	10.2	100

* Total Incidences

Lyn and Michelle % compared

Category	Lyn	Michelle
Stretch 4.1	20.4	10.4
Framework 4.2	19.3	11.6
Self-destruct 4.3	1.0	3.4
Generalise 4.4	7.1	2.3
Transformational Freedom 4.5	3.0	4.6
Time Varies 4.1.1	2.0	0
Regulate Difficulty 4.1.2	15.3	26.7
Routine 4.2.1	2.0	15.1
Peers 4.2.2	7.1	5.8
Situations Constructed 4.2.3	12.2	12.7
Switching Roles 4.5.1	10.2	6.9

Table 5

Combined Totals of Raters for Modelling Principle

MICHELLE:

	5.1	5.2	5.3	5.4	5.1.2	5.3.1	5.1.2.1	5.1.2.2	5.1.2.3	
Linda	0	2	0	0	1	2	0	0	1	
Les	1	4	6	0	0	0	2	0	1	
Anne	0	0	0	0	0	2	1	0	0	
Jennie	0	0	1	0	0	2	0	0	0	
Total*	1	6	7	0	1	6	3	0	2	26
%	3.8	23.0	26.9	0	3.8	23.0	11.5	0	7.6	100

* Total Incidences

LYN:

	5.1	5.2	5.3	5.4	5.1.2	5.3.1	5.1.2.1	5.1.2.2	5.1.2.3	
Linda	5	3	3	0	1	10	1	0	2	
Les	2	2	8	1	0	0	2	1	1	
Anne	2	1	3	3	0	4	0	0	0	
Jennie	6	7	2	1	3	5	0	0	2	
Total*	15	13	16	5	4	19	3	1	5	81
%	18.5	16.0	19.7	6.1	4.9	23.4	3.7	1.2	6.1	100

* Total Incidences

Lyn and Michelle % compared

Category	**Lyn**	**Michelle**
Real Life 5.1	18.5	3.8
Charts 5.2	16.0	23
Students 5.3	19.7	26.9
Many Uses 5.4	6.1	3.8
Overt 5.1.2	4.9	0
Compare 5.3.1	23.4	23
Language 5.1.2.1	3.7	11.5
Informal 5.1.2.2	1.2	0
Grappling 5.1.2.3	6.1	7.6

Table 6

Combined Totals of Raters for Responsibility Principle

MICHELLE:

	6.1	6.2	6.1.1	
Linda	8	0	0	
Les	1	2	3	
Anne	3	2	0	
Jennie	3	1	1	
Total*	15	5	4	24
%	62.5	20.8	16.6	100

*Total Incidences

LYN:

	6.1	6.2	6.1.1	
Linda	11	9	2	
Les	7	0	7	
Anne	3	7	0	
Jennie	3	2	6	
Total*	24	18	15	57
%	42.1	31.5	26.3	100

*Total Incidences

Lyn and Michelle % compared

Category	**Lyn**	**Michelle**
Gradual Release 6.1	42.1	62.5
Accept 6.2	31.5	20.8
Hand-holding 6.1.1	26.3	16.6

Table 7

Combined Totals of Raters for Community Principle

MICHELLE:

	7.1	7.2	7.3	7.4	7.6	7.1.5	7.2.1	7.3.1	7.4.1	7.6.1	7.6.3	7.1.5.1	7.1.5.2	7.6.3.2	
Linda	10	1	1	2	0	0	3	2	2	0	0	0	3	0	
Les	2	2	5	0	0	4	2	0	0	0	0	0	0	0	
Anne	1	1	0	1	0	2	0	3	0	2	0	0	0	0	
Jennie	2	0	3	0	0	0	1	1	0	0	0	0	1	0	
Total*	15	4	9	3	0	6	6	6	2	2	0	0	4	0	57
%	26.3	7.0	15.7	5.2	0	10.5	10.5	10.5	3.5	3.5	0	0	7.0	0	100

* Total Incidences

LYN:

	7.1	7.2	7.3	7.4	7.6	7.1.5	7.2.1	7.3.1	7.4.1	7.6.1	7.6.3	7.1.5.1	7.1.5.2	7.6.3.2	
Linda	7	5	2	2	3	8	6	8	3	1	1	0	4	0	
Les	2	1	3	1	0	2	0	3	1	0	0	1	0	0	
Anne	6	7	3	1	0	1	0	4	1	0	1	0	0	0	
Jennie	3	5	2	1	1	1	4	4	1	1	0	2	3	0	
Total*	18	18	10	5	4	12	10	19	6	2	2	3	7	0	116
%	15.5	15.5	8.6	4.3	3.4	10.3	8.6	16.3	5.1	1.7	1.7	2.5	6.0	0	100

* Total Incidences

Lyn and Michelle % compared

Category	Lyn	Michelle
Support 7.1	15.5	26.3
Learning Styles 7.2	15.5	7
Resource 7.3	8.6	15.7
Empower 7.4	4.3	5.2
Expectations of Success 7.6	3.4	0
Risks 7.1.5	10.3	10.5
Levels 7.2.1	8.6	10.5
Collaborative 7.3.1	16.3	10.5
Negotiation 7.4.1	5.1	3.5
Celebrate Success 7.6.1	1.7	3.5
Work Expectations 7.6.3	1.7	0
Teacher Faults 7.1.5.1	2.5	0
Start Errors 7.1.5.2	6.0	7
Quality 7.6.3.2	0	0

Tables 8-13

Table 8

Individual Raters % for March Lesson %: Lyn

Lyn	March			
	Anne %	Les %	Linda %	Jennie %
Context	6.0	2.5	1.9	4.3
Interest	7.6	7.5	9.7	13.8
Metacognition	15.1	20.0	22.1	23.9
Scaffolding	30.0	17.5	13.6	18.1
Modelling	10.6	5.0	7.1	10.1
Responsibility	16.7	17.5	12.3	8.7
Community	13.6	30.0	33.1	21.0

Table 9

Individual Raters % for "typical" Lesson Series %: Lyn

Lyn	Typical			
	Anne %	Les %	Linda %	Jennie %
Context	25.2	12.2	8.1	11.9
Interest	13.6	12.2	8.1	21.9
Metacognition	19.2	20.9	19.7	22.9
Scaffolding	14.1	18.7	16.7	9.0
Modelling	7.1	12.9	13.6	13.9
Responsibility	6.6	10.8	10.6	5.5
Community	14.1	12.2	23.2	14.9

Table 10

Individual Raters % for August Lesson %: Lyn

Lyn	August			
	Anne %	Les %	Linda %	Jennie %
Context	20.5	5.4	6.0	4.8
Interest	18.8	8.9	14.3	11.9
Metacognition	12.8	19.6	23.3	32.5
Scaffolding	17.1	19.6	21.1	8.7
Modelling	7.7	7.1	5.3	12.7
Responsibility	5.1	8.9	9.0	0.0
Community	17.9	30.4	21.0	29.4

Table 11

Individual Raters % for March Lesson %: Michelle

Michelle	March			
	Anne %	Les %	Linda %	Jennie %
Context	27.0	6.5	8.8	7.4
Interest	4.8	9.1	5.3	19.1
Metacognition	34.9	10.4	21.9	29.8
Scaffolding	14.3	18.2	13.2	11.7
Modelling	4.8	13.0	14.9	9.6
Responsibility	7.9	15.6	10.5	7.4
Community	6.3	27.3	25.4	14.9

Table 12

Individual Raters % for "typical" Lesson Series %: Michelle

Michelle	Typical			
	Anne %	Les %	Linda %	Jennie %
Context	11.1	6.2	4.5	7.4
Interest	8.1	12.3	14.5	15.8
Metacognition	30.3	23.5	26.4	38.9
Scaffolding	28.3	14.8	20.0	21.1
Modelling	4.0	17.3	5.5	3.2
Responsibility	6.1	7.4	7.3	5.3
Community	12.1	18.5	21.8	8.4

Table 13

Individual Raters % for August Lesson %: Michelle

Michelle	August			
	Anne %	Les %	Linda %	Jennie %
Context	5.8	6.7	4.5	4.4
Interest	12.8	3.3	11.9	19.1
Metacognition	20.9	23.3	25.4	29.4
Scaffolding	44.2	23.3	16.4	26.5
Modelling	1.2	20.0	16.4	5.9
Responsibility	5.8	20.0	3.0	4.4
Community	9.3	3.3	22.4	10.3

Appendix 10

List of Abbreviations Used in this Thesis

NUDIST	Non-nUmerical Data Indexing, Searching and Theorising
TU	Text Units, Field Notes
ML	March lesson sample
AL	August lesson sample
TL	Typical lesson series sample
LT	Lesson transcripts
ITI	Initial Teacher Interviews
PTI	Post Teacher Interviews
ISI	Initial Student Interviews
PSI	Post Student Interviews
DE	Diary Entry

Appendix 11

Michelle and Lyn's class responses to the Post Interview Question: "What day of the week will your 21st birthday fall, and in what year?"

Table 1

Michelle's class answers to Post Student Interview Q 11- On what day and in what year will your 21st birthday fall?

Code:

*	worth looking at the transcript
1	system of adding 7 days and then a further 2 for 9 years
2	system of adding 9/10 years
3	system of counting on until you get to 21 from the current year
4	system of taking 11/12 (current age) from 21
5	system of starting with day born on
√	Correct answer unscaffolded.
R	Reluctant to attempt or continue once started.
C	Correct answer assisted with prompts from interviewer.
Ch	Spontaneously changed strategy without any prompting to do so.
D	Did accept prompt.
dn	Did not accept prompt.
U	Unusual strategy.
S	Used a systematic approach

Student	Recount	√	C	R	ch	D	dn	U	S
Carrie B	The first thing she thought about was leap year which she said would take off a day every 4 years, which I corrected. Started to add consecutive days from 12 to get to 21 but then decided take 12 from 21 and said 9. Then proceeded to add in leap years and count on each of the next 11 days .(Incorrect answer) 4				ch				S

Elizabeth B	Unsure how the days change, so needed help right from the start. Unsure of her birthday day. Needed to look up calendar. Established leap year, unclear of its impact- "would it be that every leap year it would jump a day, well every year would it jump?... so it would usually be on the same day" got the hang of it, went ahead in clusters- "So it'll be Thursday in 1993, um... then it'll be Friday, Saturday, Sunday, so that's 93,94,95,96, I'm going to be 21 in 2001, so that'll be, 92,93,...,99,2000,2001, so that'll be Saturday" (unsure where that day came from) "Sunday, Monday, hang on, 1, 2, 3, (working out leap years??) so that'll be a Monday there, and it'll be Tuesday, Wednesday,... so it'll be on a Saturday" (Correct answer).*		C	R					
Genevieve C	Reluctant to begin to work it out. Said she was 12 in 1980 (birth) I started the progression of each day increasing and she took over but went too far and realised she need to stop at 23. Abandoned working out days and worked on the year, but calculated she would be 21 in 10 years time instead of 9. Lost confidence and seemed reluctant to want to know about leap year or receive any more help. (Incorrect answer).			R	ch		dn		

Rebecca C	Momentarily forgot when her birthday was. Worked out first what she would need to do before she started to calculate:- said she needed to work out how many years there were between "now and your 21st" Calculated it was 9 years and then said you would have to add on the days taking into account leap year which would make it 11 days- "And so you'd add 11 days on to Wednesday, and there's 7 in a week, so that would be Wednesday again and then there's four so its Wednesday, Thursday, Friday, Saturday, Sunday... um in the year 2001" (Correct answer) *	√							
Gabrielle C	Knew to "start off with the days I was born on". Then said she would need to add 21 years onto that, which she calculated correctly. Unsure what to do next, but knew about the progression of days and leap year, but starting working out the day by jumping two ahead each year. (Incorrect answer) 5					D			S
Angharad D	Started with last year's birthday and said about leap years and deduced that "last year, my 21st birthday would have been on a Saturday", then tried to work out how many leap years there were so she would move the day ahead one day for every four only. (Incorrect answer) *					D			

Gabrielle H	Started with what day her birthday was on this year. Knew that she would have to think how the "days are going to move. Like, if um, in February, like, um a day is added every year" Seemed unsure what a leap year did, abandoned that and went to the year she would turn 21 which she did by adding 10. Went back to the days but thought "February goes back to 28 so it loses a day" Corrected by me, once she got the hang of it she went from each calendar year and each day, building in the leap years as she went. (Correct answer)		C		ch	D			
Emily K	Started off wanting to know the current day's date and then made unclear connections to arrive at the correct years- "My birthday's in April on the 16th. So I... just times...times, well you times 1980 (birth year), well you just find out from the what... It'd be 2001" Thought she would need to start with the day she was born on, then went through the days saying incorrectly that it would be 7 years until her 21st birthday, corrected with scaffolding. Thought leap year missed a day and didn't know how often they occurred. (Correct answer)		C			D			

Olivia K	Started to think of the date of her birthday and the year and how old she was now, counted in her head 8 years until she was 21 although it would have been 9, which I corrected for her. Counted on the days of the week but ignored leap year and thought it was every 3 years. Added 21 years to the current year and worked out incorrectly that she would be 21 in 2012 but adjusted it incorrectly to 2010, Scaffolded her to rework her answer. (Correct answer).		C			D			
Carla L	Reluctant to make a start, then said she would need to think about what year and what age she was then. Said "I'm 12 now, and in another 9 years I'll be 21... Well, 12 plus 9 is 21". Went back to 1991, but self corrected to 1992- "1992 and in 9 years it'll be the year 2001. So, plus 9 will be the year 2001. " But didn't know what to do after that. Knew about the progression of days and knew to work out when the leap years would fall before she started. Progressed through each day and each calendar year taking into account leap years as she went. Seemed self conscious about her answer "But there would have been a more simpler, would there have been much more simpler way of doing that? I don't know but I'm sure there would have been, I'm sure you wouldn't have to go through all that. Is there a more simpler way?... There's not, you've got to do that?" (Incorrect answer) 2			R	ch				S

Amy M	Initially said she would be 21 next year and wrongly decided that she would be 21 in 11 years instead of 9, and so went through the 11 days of the week starting with Tuesday and added on extra two days for the leap years, correct method but incorrect for her actual 21st birthday, which she then realised was in 10 years but she stayed with the same answer. (Incorrect answer)								
Elizabeth M	Started with her birthday this year and knew to build in 2 leap years straight away. Continued to go up by each day and calendar year but stopped at '99, and then regressed and went through her birthday years stopping at 18 and reverting back to the calendar year because it was a leap year. (Correct answer)		C		ch	D			
Angela M	Her birthday was the day before, then went ahead three years counting on the days only until 1995 and said it would be on a Friday, but also said it would be a leap year, and went two days ahead for the next year (a leap year) Tried to go ahead another four years but got mixed up as to which year she was up to and in the end forgot to add on the last leap year. (Incorrect answer)*	√			ch	D			

Georgia M	Said confidently when her birthday was this year and then changed it three times, and then did a subtraction sum out aloud "If I work out my 21st birthday, take my age now, and change that, I've got to borrow one, 2 there, 21 take 2 is 19...no I get confused, oh no carry the ten,... that's 11 take 1, no, 1 take 1 add 1, no it's just 9, yeah. So now I add 1992 add 9 equals 2, 11,... so I'm going to be 21 in the year 2001" Then wrote down the calendar years and then matched the days against them. Needed to be prompted about leap year. Kept working her way counting off 1,2,3,4, but got mixed up with what day she was up to, but continued to self correct. (Correct answer) *		C			D			
Adeline M	Not interviewed								
Linda N	Said she would have to think about "Times tables, the year you were born and the date" Worked out the year she would turn 21 first, when she went to the day she wanted to know what to do with the leap years. Heavily scaffolded through each calendar year and day building in leap years. (Correct answer)		C			D			

Carrie P	Started with her birthday day and the calendar year, and didn't mention the month, added on 10 years to the year 2002, but failed to see that she need only have added on 9 years. "Im 12... I'll write that down, 12. Every seven years, I think I'll do this, so that the next seven years it'll be 1999." OK What made you go seven years? "Because.. there's seven days... oh but there's a leap year." Thought that leap years missed out one day, but started to get mixed up as to which day she was up to and I suggested she write it down. Instead of writing 1993 she just put "3" proceeded to go through each calendar year and the day but went back to 16 and then jumped to 19 before she continued the pattern taking into account leap year. Quite an unweildy way of working through it. (Correct answer) *		C			D		U	

Catherine R	Took her age away from 21 "21, take the age I am now, which is 12, so I take 2 you can't do that so you take away from 21, that becomes 1, then you carry it, 11 take 2 is 9, 1 take 1 is 0. So 9 years." Said that because its was 9 years the day would change 9 times- "so it's Saturday, Sunday which is 1, Monday, which is 2, Tuesday, which is 3...". Prompt for leap year, corrected for days. Then added on the 9 to find out what year it would be- "it's 1992 now, we add nine years, this is going to be in 9 years, so that adds up to 11, 2 and 9 is 11, and carry that, onto the next number, so that's 10 and we carry the one, is another 10, and carry that one, so it's 2001. (incorrect)*					D			
Rebecca S	Started by adding 9 to 12, but then got mixed up- "I need to add 9 to 12 so that's 21, so nine years more than 1992 and that would be 19, no, 9, 2 add 9 would be 11,9,10,0, so it would be the year 2001." counted on 6 more days from her day this year and then changed it to adding on 9 more day. Said leap year had a day less and didn't know thought it may have been every 2 or 3 years. Once she was told she quickly deduced there would be 2 more before her 21st birthday, but changed it to 3 until scaffolded. (Correct answer)		C			D			

Lynleigh S	Knew to start with her birthday year but wanted to times it by 21, when questioned she continued in this unusual way (she would have had 9 years to go before she was 21) "times 9... which is 4 times 9, 36, add 14, 50, and 50 days, and you times 50 by 21,... 50 days so would be" when quizzed she said "trying to think how many days or months or years it would be 'til my birthday, and I'm 12 now... take 3" when quizzed " well 12 add 12 is 24, take 3 is 21, so that's about 9 years until my 21st birthday. Without resolving which year is would be she went on to the day with more scaffolding. Knew to ask about leap years, quite laboured in working through each day and the said "Every time I get asked a certain question I go blank", but continued to go year by year working out her age and the day with great assistance each day even with the final amount (Correct answer)*		C			D		U	

Kylene T	Started unsure which day her birthday was on this year and then said 21 take 12 was 9, but knew that the date would not be the same, but went back to working out which year it would be first. Knew she had to think about a leap year but was unsure what to do and also how each year changes one day. Started to work on a hypothetical day and then said - "Well we've been talking about them and stuff but I'm really confused about it" Seemed reluctant to be told the information she needed, wanted to work it out herself. Once she was told she quickly added on a week i.e. 7 days and added on 2 more to make it the 9, but then got flustered "can't think about it, coz even though I've been doing it I'm really confused about it anyway" needed to be helped about the leap year. (Correct answer).		C		ch		dn		
Leanne T	Seemed stressed about what I might be going to get her to do, but was clear about how to go ahead each day for her next birthday. Started by saying she'd need to think about how many days were in the month, and how many days were in a year until her birthday, and how many months "... 1995 would be Friday, and I'll be 15. 96 would be a Saturday and I'll be 16,.. Sunday, 17, oops Monday, 18..." Confused about leap year and said it went back but knew it was every fourth year and was scaffolded fairly heavily. (Correct answer)		C			D			

Chantelle V	Said she would need to think about how many months and what date her birthday was. Said "Well, if its a leap year this year, when I'm 16 it'll be a leap year, and then when I'm 20 it'll also be another leap year", but thought it would change it to a day less, but unable to say how she would actually do it. Wrote it down as she went putting each age with the year and then the day. Continued by each year until 16 then she jumped ahead 3 years, then added in the leap year. (Correct answer)		C			D			
Melissa W	Said she would need to start with the day she was born which she didn't really know, but would need to add on 21 years. She thought she would need to know "how many days it changes from your birthday each year." Most reluctant to know what to do- "I don't usually do that..." Once prompted she zoomed along to 1997 and then quickly said her final answer. (Correct answer).		C	R		D			
Rebecca W	Started with how old she was now and worked out it would be 9 years before she was 21. Unsure what to do next, needed scaffolding through each of the days, knew about leap years but needed help to build them into her calculating which was written down for her so she could keep track. (Correct answer)		C			D			

Alana W	Knew to start with what day of the week her birthday was on this year, and knew she would have to work out what day it would be on the next year and the year after seemed to stop then and needed prompting about leap year which she said "don't have as many days in each of the years". Needed heavy assistance to continue (Correct answer)		C			D			

Table 2

Lyn's class answers to Post Student Interview Q 11- On what day and in what year will your 21st birthday fall?

Code:

- * worth looking at the transcript
- 1 system of adding 7 days and then a further 2 for 9 years
- 2 system of adding 9/10 years
- 3 system of counting on until you get to 21 from the current year
- 4 system of taking 11/12 (current age) from 21
- 5 system of starting with day born on
- √ Correct answer unscaffolded.
- R Reluctant to attempt or continue once started.
- C Correct answer assisted with prompts from interviewer.
- Ch Spontaneously changed strategy without any prompting to do so.
- D Did accept prompt.
- dn Did not accept prompt.
- U Unusual strategy.
- S **Used a systematic approach**

Student	Recount	√	C	R	ch	D	dn	U	S
Ben B	Wanted to count 21 more years, got mixed up and kept increasing his birthday day by another day "On the 7th December will be a Monday and that will be my 12th birthday and on the 8th of Tuesday it would be my 13th and on the 9th of Wednesday it would be.....". Talked about Boxing Day! Then tried to add on 13 days instead, had trouble building in leap year which he didn't know much about. "It could probably be on the 3rd quarter on the 16th on the Wednesday" (Incorrect answer)					D		U	

Will B	Said straight away how to do it which included allowing for leap years. Said "Just add every four years when its leap year and miss out a day". Followed calendar by saying "12 will be a Tuesday, and when I am 13 it will be a Wednesday...." Stopped after 15th birthday as if he had finished. (Incorrect answer) 3					D			S
Ben C	Knew to add on 10 but calculated the ten days as if they were a seven day week. Thought leap year deducted one year, but later changed his mind. Worked through each year calculating his age and pointed to the day on the calendar. (Correct answer)		C			D			
Olivia D	Not interviewed								
Ben E	Knew that he had to think about how many more years he had to go, but didn't know whether to go forward of backwards on a diary. Needed a lot of scaffolding to know what to do. Knew that a leap year "goes around and around... a leap year has an extra day in it... every four years", but didn't build it into his calcuations. (Incorrect answer).						dn		
Louise G	Not interviewed								
Chris G	Calculated that going from 12 to 21 would be 7. Knew about leap years but thought you deducted three days. (Correct answer)		C			D			
Reuven G	Knew to ask straight away about the leap years. Counted on four days and then matched his age for each of those days- "12th, 13th, 14th, that will be another four days, Friday, Saturday, Sunday and then Tuesday will be my 15th" and then went to 19th birthday and suddenly added on leap years. (Incorrect answer). *								

Stephen H	Started off by trying to find how many weeks there would be:- "You times 52 by 7". Seemed to have a whole lot of unnecessary calculations:-"... then you work out how many there is till 265 and then every four years you need to put on another day. So 14, 36 so a normal day there is 52 weeks and one day in the year so if you work that out its Sunday this year and then how many what was it last year?" (Correct answer)	√						U	
Dean J	He had just turned 11 the previous week. Had system of adding 7 then another three to make it add up to 10 days. Ignorant of real function of leap years and any impact it might have. (Incorrect answer) 1.						dn	U	S
Matthew J	Seemed to have a plan, but most unusual, quite a cumbersome way of adding up days. Even with considering leap years (his idea was incorrect- "at the end of the month like if its a leap year there won't be as many days in that month"), he was reluctant to change his answer having worked it out (good example of 'interest' where learner is unwilling to shift their preconceived ideas). (Incorrect answer)						dn	U	S
Melanie K	Quite clear about how she would go about working it out and quickly worked out the year. Knew her next question needed to be " what day of the month is June 1st" But when she got to the diary she started at 24th June, actually her birthday, also clear about what to do for leap year, but counting the current leap year and skipped a day throwing her answer out although she had been quite methodical, listing each day for each year. (Incorrect answer) 3.								S
Aaron K	Not interviewed								

Melissa L	Counted the days on "because the days go around in a circular". Seemed to be confident to attack problem but failed to see that in 10 years time she would be 22 not 21 if she is 12 now. Most unusual way of counting the days and inadequate knowledge (false) of leap year (1 day shorter). (Incorrect answer) 3						dn	U	S
Stuart M	Quite articulate about what he would need to think about in order to do that type of problem, even before he actually started working it out. Did first part methodically, added 10 days by saying 7 days would make it the same day and add on another 3, but needed the scaffold of what a leap year does. (Correct answer) * 1		C			D			S

Carmen O	Quite quick to answer, very confident, almost adamant that all she needed to do was "go in 10", unable to accept the clue of reconsidering her error of the day saying that her birthday was Monday this year "so I think it will be a Monday", not even begin to talk about leap years. (Incorrect answer)						dn		
Oliver P	Said straight out how he would go about working it out, "How many years it is, how old you are, how many years to go..." but wanted to add on 11 and not 10 years even though he was 11. Worked the year out. But started skipping days when he was working out the actual day, confused himself with leap year. Needed reassurance (Correct answer)		C			D			
Victoria P	Most reluctant to even have a go. Needed prompting about adding 10 years in order to get to 21. Gave up before actually completing the calculation. (Incorrect answer)			R		D			
Blake S	Began confidently by working out what day and date it was that day, and how many days there were left in the month, as well as adding the 11 days to his birthday (11th January). Then tried to work out the days " in ten years time, ten times by 365 take away two" (deducting for leap years). Tried to make hard work of it. He then tried to add two days on for every year, he finally got the right year but had amazing logic for the days" adding on two days on every year. So two times by ten would be twenty, twenty into would be two, plus six over, um six into the Monday would be Sunday, I think" (Incorrect).*					D		U	

Brenda S	Seemed quite confident saying "That'd be easy because mine goes like Monday, Tuesday... like that." She knew she needed a diary or "I could get lots of different calendars and see what days the... how they do it..." After encouraging her to actually work it out she just went through the days of the week counting on ten more but didnot accept prompt that a leap year might alter it. (Incorrect). 2						dn		S
Mark S	Said outright that he would need help to work it out. He knew that it was in nine years from his next birthday and readily calculated the year, needed prompting to change each day for each year but eventually added nine days. Knew about leap years but wanted to say there were going to be four in the next nine years, but after scaffolding worked out correct answer (Correct) 2		C	R		D			S
Paul S	Said the first thing he'd need to think about was that his first birthday was on a Tuesday, then changed it to when he was born being the Tuesday. Quite confident to work it out from there "Well, I was just thinking, well, 21, divided by 3 is 7, so I'd..." Jennie:- "Why are you dividing it by 3?" Paul:-"Divided by 3 to see how many times 7 goes into it, and therefore I'll be able to work out, plus leap years, which is every four years...Saturday...I got it by, by, putting the... adding the number of leap years in 21 years to the final answer which was Tuesday, and that's 5 so it gave me Saturday" Error made in adding on the leap years, but efficient system otherwise. (Correct) *	√			ch			U	

Paul T	Started to work out what year he was born in then said "times 81 plus 20" but didn't actually times it added on 20 to get 2001 but said that would be his 21st birthday, hesitated and said "So probably two thousand .....and one, or two thousand." but stayed with 2001. Took a stab at the day, but then realised I wanted him to work it out. "Well, probably times each day, each month, each year it goes up by one day" but then continued to take stabs at days. Went as far as his 15th birthday and needed to be encouraged on. Needed prompting about leap years which he said "It adds an extra day in the year". (Correct answer)*		C			D		U	
Jo W	Knew to start with this year's day and " count on how many years, 'cause for every year it goes up a day, and for leap years it goes up two". Worked out each year and day concurrently- "Friday, so that was 1992, so 1993 will be Saturday, 1994 will be Sunday, 1995 will be Monday, 1996 will be Tues... Wednesday, it'll be Wednesday, it'll be Wednesday 'cause it was a leap year...." continued to calculate until 18 and then added another 3 more years to take into account leap year. (Correct answer) *		C						
Tim W	Knew he'd have to start with when his birthday was "now". Quickly answered that because it was on a Friday this year it would be on a Friday for his 21st. He reasoned that adding another 10 days would end up with the same day. Needed detailed scaffolding to get him to progress through each age and then the day, and then to work out leap years. When he came to the end he wasn't confident and then reworked it out on paper starting with 1992 and adding on 21 years! Needed scaffolding to change this idea, finally got year and day sorted out. (Correct answer). (Good example of scaffolding) *		C		ch	D			

Chris W	Knew to add on until he got to 21 and deduced correct year, but then decided to work out the correct day he would have to add on 10 more days and got 27, it was then revealed that his birthday was on the 17th and he was adding on ten to make it the 27, he then realised the error and went to working out the days but failed to worry about leap years saying it "misses a day and leaps on to the next month" but that it wouldn't effect his answer. (Incorrect) 3.				ch		dn		S

Appendix 12

Michelle and Lyn's Class Responses to the Initial Interview Question "When the teacher says it's time for maths, how do you feel?" and the Post Interview Question "Do you enjoy doing maths?"

Table 1

Lyn's student responses to questions: Initial Student- Interview: When the teacher says it's time for maths, how do you feel? and Post Student-Interview: Do you enjoy doing maths?

	INITIAL INTERVIEW	POST INTERVIEW
Ben B	Yeah I really enjoy it, it's really good fun.	I do different things that I am good at than maths... Mrs M she is different like she is a maths person sort of than anything else. She gets into the different kinds of maths that we haven't learnt and we don't know much about and that is going to be really important longer in our life.
Will B	I don't really care.	Yeah... she does harder stuff than Mrs O.
Ben C	Not special about it.	Yeah, she uses the board more... Mrs O gives out sheets and says it on like she is holding it up and pointing to it and stuff.
Olivia D	I feel disappointed.	
Ben E	Depends what sort of things it is, say if it's times, I can't really do times, addition, subtraction, multiplication..?	Yeah I enjoy maths.
Louise G	I like maths a lot.	
Chris G	Well I don't really feel all that bad because I know quite a bit about maths.	Yes
Reuven G	I feel happy, it's my best subject.	Yes
Stephen H	I like maths.	Yeah

Dean J	Like it's boring some of the times, Sometimes it's good.	No
Matthew J	Alright	Sometimes
Melanie K	Depending on what he's... like multiplication, or really hard stuff... but if we're doing things like just normal multiplication or division or whatever, then I feel OK.	Yep
Aaron K	I like maths. I really try to hurry up so I can get on with maths.	
Melissa L	Not very happy.	Sometimes, depends on what type of maths we do.
Stuart M	I like maths , it's OK, get bored after a while.	Yes, it's OK.
Carmen O	Sometimes I feel good because maths is my favourite subject.	Sort of.
Oliver P	I enjoy maths	Yes
Victoria P	Bad	No.
Blake S	Really good because I like maths.	Yes quite a bit
Brenda S	It doesn't bother me.	Mmmm, Yes. I'd rather do reading, but it's alright.
Mark S	Good, I like maths.	It's OK.
Paul S	I feel better than when she says it's time for English... it's OK.	Yeah
Paul T	Happy coz I like maths, it's a lot of fun.	Yes
Jo W	Well I feel different ways cos if she says its maths like sums and we sit down and do sums, well I don't feel anything, but if it's maths like drawing up graphs and describing what maths is, things like that I might enjoy.	Um some types.
Tim W	OK It's quite good	Yep
Chris W	Pleased, sort of thing.	Yep

Table 2
Michelle's student responses to questions: Initial Student Interview: When the teacher says it's time for maths, how do you feel? and Post Student Interview: Do you enjoy doing maths?

	INITIAL INTERVIEW	POST INTERVIEW
Carrie B	Well when, if I know we're going to say have a fun maths class.	Yes.
Elizabeth B	not interviewed	(not asked)
Genevieve C	not interviewed	Yeah
Rebecca C	Well I sort of like maths but I don't like packing up all my books and moving to the other classroom! I prefer staying.	Sometimes, and sometimes not very much.
Gabrielle C	Well I'm not that crazy about maths.	(not asked)
Angharad D	Well I feel a bit of a rush to get my books so I can come in so I don't forget anything.	Yes
Susan G	Depends what we're doing in maths, but not that... [much].	
Gabrielle H	I like problem solving.	Well I used to hate it but it's OK now.
Emily K	Well it depends what we're doing, if we're doing something like, I don't like long division or something and I think, oh no, I don't want to go to maths, I don't like long division...	
Olivia K	not interviewed	Yeah
Carla L	not interviewed	I do.
Amy M	Well, I don't really feel anything I just stay there and just do the work.	
Elizabeth M	I just feel like, like....?	(not asked)

Angela M	Quite at home	It depends what we're doing.
Georgia M	Well, I feel I've got to get organised (not clear) my classroom and I feel good because I'm having a good time doing maths this year.	I didn't in the first two terms, but then, I could never do well in maths tests, and then one maths test I got like 90 and it gave me new hope, and now I really like going.
Natalie M	not interviewed	A bit.
Naomi M	Well I like doing maths that. That's because it's quite (not clear)	
Linda N	A bit, it depends on what they do.	Yes
Emily P	not interviewed	Yeah I do much more than I did at my old school.
Carrie P	Excited and a bit nervous.	Yes.
Catherine R	Well, I like maths. (not clear) I'm alright doing maths (not clear)	Yes
Alia S	I don't know.	
Rebecca S	All depends on whether you like, if its a funny feeling (not clear)	Sometimes yeah, but it gets a bit boring and things, and we do things that I'm not very good at.
Tania S	It depends if we are just doing really hard stuff which I am having trouble I don't really look forward to it but if it is something that I enjoy, I feel like...	
Lynleigh S	Well if they (not clear) before... I don't feel good (not clear)	I don't enjoy going to it when I'm tired and things like that but it can be fun.

Kylene T	OK (Unclear)	Sort of. Yeah, sort of, because sometimes we do stuff that's not that good, and sometimes we do stuff that's quite fun.
Chantelle V	Well it's not really, because it's not really exciting, it's just, it's OK but it's not really exciting like Science you get to study things and Health you get to learn about things.	I'm not that good at maths but its OK. But it's not that enjoyable.
Melissa W	not interviewed	Yeah, it's my favourite subject!
Rebecca W	not interviewed	Yeah
Alana W	Um I don't really like maths that much.	Yes
Leanne T	I like maths. Its my favourite subject.	Yes

Appendix 13

Progressive Achievement Test Results for Lyn and Michelle's Class

stanine	1	2	3	4	5	6	7	8	9
LM class	8.3	16.6	16.6	12.5	16.6	12.5	8.3	4.1	4.1
MW class	4.76			9.52	14.28	42.85	23.80	4.76	
normal curve	4	7	12	17	19	17	12	7	4

It can be seen by Lyn's results that while following the normal curve cluster in the 4-5-6 stanines she has a bulge in the 2 stanine

In Michelle's class the bulge is dominantly around the 4-5-6-7 stanines consistent with a class which has been ability grouped

Appendix 14

Analysis of Placement Test J

PLACE-MENT TEST J Questions	LYN 23 students tested	Total	MICHELLE 25 students tested	Total
NUMBER				
1				
3				
4				
5				
6				
7				
10				
11 *	x	1		
12 *				
13				
14 *			x	1
15 *	x	1		
17 *	xx	2		
22	xx	2	xx	2
23 money addition	xxxxxxx	7 !	xxxx	4
24	xxx	3	xx	2
25	xxxxxxx	7	xx	2
26 * pronumerals, algebra addition			xx	2
27 * pronumerals, algebra subtraction	xxxxxxxxxxxxxxxxx x	18	xxxxxxxxxxxxxx	14
28 * pronumerals, algebra multiplication	xxxxxxxxxxxxxxxxx	17	xxxxxxxxxxxxx	13
29 * pronumerals, algebra division	xxxxxxxxxxxx	12	xxxxx	5
30	x	1	x	1
31 fraction shaded	xxxxxxxxxx	10	xx	2
32 * fraction comparison	xxxxxxxxxxxxxxx	15	xxx	3
33 *	x	1	x	1
34 *			xx	2
35 * algebra	xxxxxxxx	8	xxxx	4

39	xxxx	4	xx	2
40 subtraction of 3 digit	xxxxxxx	7	xxx	3
41 write digit numeral	x	7	xxxxxxx	7
42	xxx	3	xx	2
43 Venn diagram	xxxxxxxxx	9	xxxxxxxxx	9
44 weight 4 digit	xxxxxxxxxxx	11	xxxxxxx	7
45 cooking	xxxxx	5	xxx	3
46 money	xxxxxxxxx	9 !	xxxxx	5
47 * "product" in fractions	xxxxxxxxxx	10	xxxxxxxxxxxx	12
54 graphs	xxxxxxxxx	9 !	xxxxxxxx	8
55 * numberline	xxxxxxxxx	9	xxxxxxxxxxxxxx	14 !
58 * decimal place	xxxxxxxxxx	10	xxxxxxxx	8 !
SPACE				
2				
9	xxxx	4	xxxxx	5
19	x	1	x	1
20 poor question design (invalid)	xxxxxxxxxxxxxx	14	xxxxxxxxxxxxxx	14
37				
49			xxx	3
56 parallel lines	xxxxx	5		
57 geometry	xxxxxxxxx	9	xxxx	4
60			xx	2
MEASURE MENT				
7				
16	x	1		
18				
21	x	1	x	1
36	x	1	x	1
38	x			
44				
48a			x	1
48b 2-D rep in measurement	xxxxxxxxxx	10	xxxxx	5
50 clock	xxxxxxxxxx	10 !	xxxxxxxxxx	10
51 geometry-shape	xxxxx	5	x	1
52	xxxxx	5	xxxxxxx	7
53 volume	xxxxxxxxx	9	xxxxxxxxxxxxxxx	15
59	xxxx	4	xxxx	4

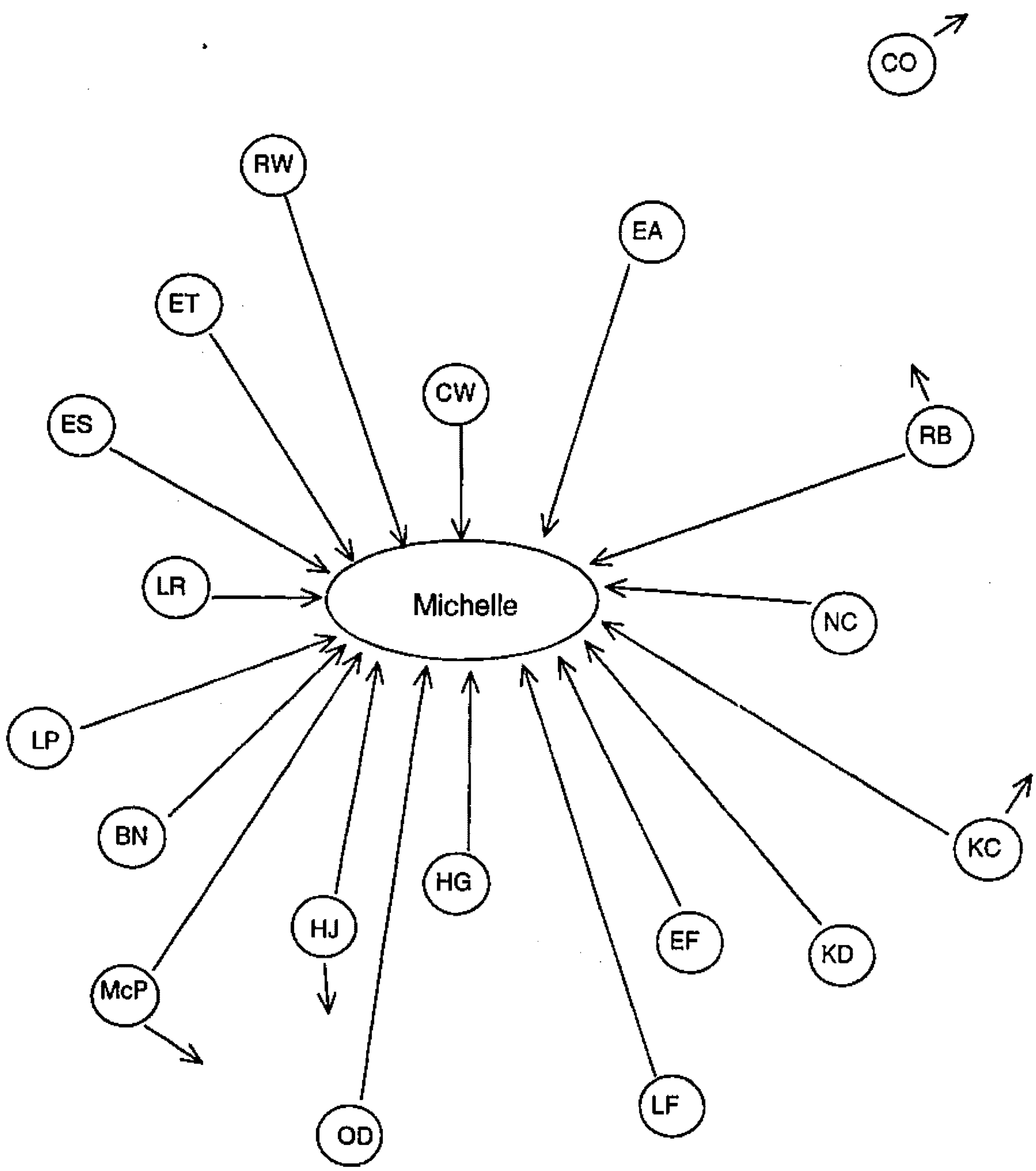
* = decontextualised question or without pictorial support.

! = surprising error due to experiences students had been given in class.

x = an incorrect response.

Appendix 15

Michelle's 'home' class response to the question "Who do you go to for help in class?"



Appendix 16

Students Who Received Choices for the Question "Who is the best person at maths in your class?"

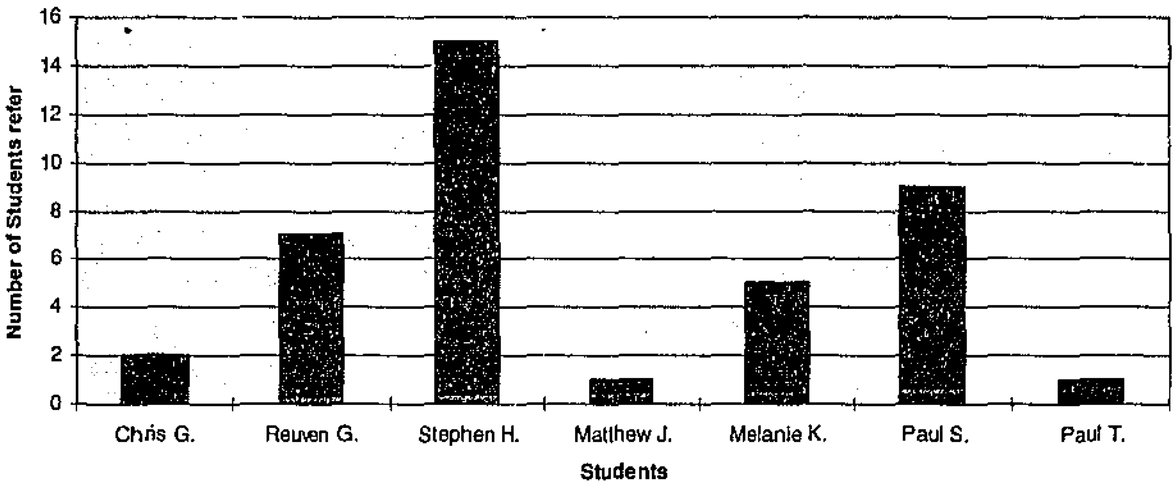


Figure 1.

Lyn's students who received choices for the question "Who is the best person at maths in your class?"

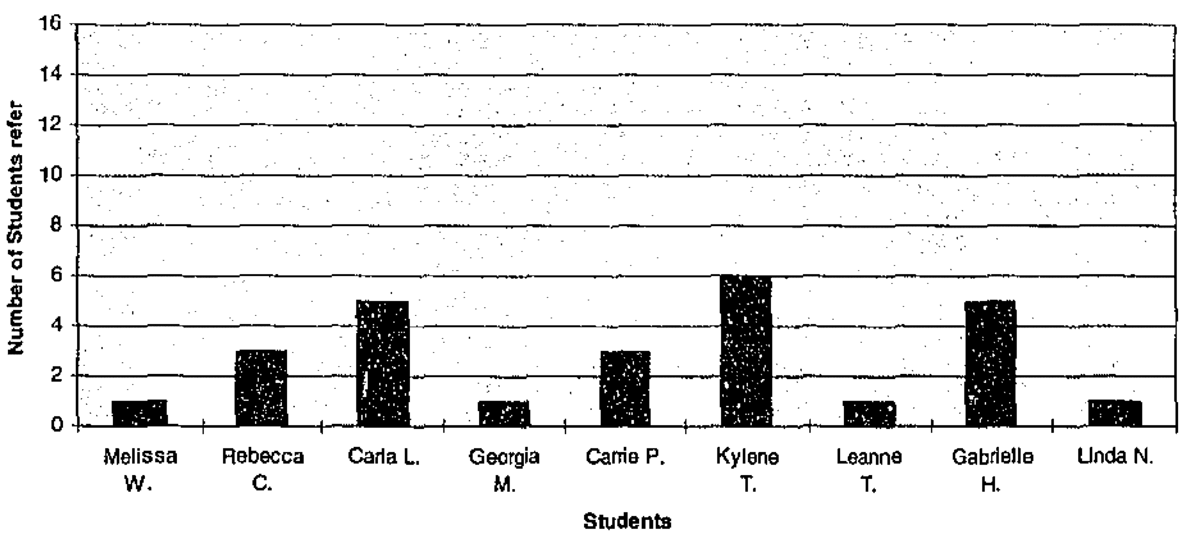


Figure 2.

Michelle's students who received choices for the question "Who is the best person at maths in your class?"

Appendix 17

Michelle Lesson Transcript 11.8.92

LT Michelle (11.8.92)

Children are sitting in their desks in grouped tables while teacher is demonstrating on whiteboard as she talks.

Michelle:- Alright, are you ready? Right girls eyes this way please. What we will be doing today is a continuation of what we did Monday and yesterday. Yesterday we were looking at equivalent fractions and we were working out how to find equivalent fractions of a simple fraction such as a third without actually having to draw those large graphs everytime you need to work out an equivalent fraction. What is that way that we came up with yesterday? Do you remember we saw a pattern between the different numbers? For instance we could tell instantly how many sixths is a third. Gabrielle?

Gabrielle:- Two

Michelle:- And how did you get two sixths?

Gabrielle:- Um two thirds is six and one's two is two?

Michelle:- Right you realised that whatever the denominator was multiplied by, the numerator was then multiplied by the same number. One times two is two. What else did we say to prove that one third does actually does equal two sixths? We came up with a way to prove that two sixths was equal to a third. Do you remember what we said yesterday? We said that we know if I had two sixths of a cake I would have just the same amount as someone that had one third ... and what was that way Georgia?

Georgia:- Because in the top number goes into the bottom number three times and in the second fraction the top number goes into the bottom number three times as well.

Michelle:- That's in this instance but it doesn't always happen. I was actually talking about this little thing here that we talked about yesterday, what did we say about that number Angela?

Angela:- That you times it by ... you can ... its a whole number and you can say two ... its a whole number ...

Michelle:-Right you're saying two halves of two is a whole number right so you're saying two halves is one. One third multiplied by two halves equals two sixths. And all we are doing is multiplying a third by?

Class:- One

Michelle:- And we know whatever number we multiply by one we end up with the same number. Because ten times one is ten. Twenty- five times one is? Pardon?

Class:- Twenty- five

Michelle:- OK So that's what we looked at yesterday. Now I know some of you had a little trouble with that so the plan today is to put you to work at your desk and then I'm going to mark the homework that you did last night and Mrs Bickmore-Brand will also mark your homework last night, individually so that we can see that you really understood what we talked about yesterday.

Child: But I didn't get it.

Michelle:- That's right you can talk about that when you talk about your homework. Right for today, we are going to do the same thing in reverse OK? We talked about one third and two sixths being equivalent fractions. Can anybody tell me other fractions that are equivalent to one third?... Right? Rebecca?

Rebecca:- Four twelfths

Michelle:- Four twelfths is equal to one third

Child:- Eight sixteenths

Michelle:- Eight sixteenths. One times eight is eight, three times eight is sixteen ... that'll do thanks, there are millions of them, we could go on for eternity. Now of those fractions this one's special and it has a special name called? (Pause) Does anybody know?

Child:- A lowest term

Michelle:- A lowest term fraction. Why do you think it would be called a lowest term fraction?

Child:- Because it can't go any lower than that?

Michelle:- The denominator can't go any lower than the three. Now the work that you will be doing today will be asking you, for instance it will give you the fraction eight sixteenths and you will need to say what is the lowest term fraction of eight sixteenths. And whereas yesterday we worked out equivalent fractions by mutliplying by one this time we are going to?

Class & Michelle:- divide by one

Michelle:- So in this case we're going to say eight divided by? ... (Long pause)

Child:- Eight?

Michelle:- Eight sixteenths, divided by eight eighths. We are dividing by? (Pause) one. Eight divided by eight is one. Sixteen divided by eight is? ... three.

Child:- Two

Others:- Its two there.

Michelle:-Is this correct then? No its not. Its not eight sixteenths at all. What should it be?

Child:- Eight twelfths

Michelle:- Good girl, I'm glad you picked that up. Lucky you were watching. We will have to start this one *again. Eight twenty fourths. Eight divided by eight is one, twenty four divided by eight is?*

Class:- Three

Michelle:- OK Now we cannot go any further. Lets try one more because we made an error with that one. Something different we will go for ... six eighths. What is six eighths, in its lowest form? (Pause) The first thing you will need to do is to find the number that goes into six and eight?

Others:- Two

Michelle:- Divided by two. Two halves, one whole. Divided by two. Six divided by two is?

Class & Michelle:- Three, eight divided by two?

Class:- Four.

Michelle:- Now is that our lowest term fraction?

Class:- Yes

Michelle:- How do you know Carrie?

Carrie:- It means you can't go any lower, than three quarters... you go one but you would just get the same number

Michelle:- You could divide both numbers by one but you would just get the same- you'd end up with three quarters again. Is there any other number that will go into three and four equally?

Class:- No

Michelle:- There's no common factor and so we have to stop there, that is our lowest common fraction. Right. What I want you to do please is to have a look at page 198 ...

Different students are coming to her with their homework on fractions from the previous night:-

Michelle:- Carrie could I see you please? What is it Georgia?

Carrie P:- I had troubles.

Michelle:- You had a bit of trouble? OK let's go through it together. That one's correct ... four thirds its actually an improper fraction.

Carrie P:- I thought it was a, I was putting... I didn't think it was aloud to be an improper fraction.

Michelle:- OK If that's an improper fraction do you think it is likely that the equivalent fraction will also be improper?

Carrie P:- Yes. If you times it that will be a larger number if you times it by ... like if you times it by the two or whatever you times it by in the middle. You times the number by that and its gotta be more than the bottom number because the number is less...

Michelle:- I think I know what you mean so let's go through this example. Four thirds equals so many eighteenths ... Olivia would you like to go and see Mrs Bickmore-Brand please. Olivia hasn't done her *homework but perhaps she can go through just one or two with you. She has been spoken to.*

Carrie:- Um, six, if you times by six. If you do four times six is twenty four.

Michelle:- You've worked out that three sixes are eighteen.

Carrie:- Four sixes are twenty four.

Michelle:- And so have we got another improper fraction?

Carrie P:- Yes.

Michelle:- Let's just check these. OK do you understand it now?

Carrie P:- There's ... I worked out there's a pattern each time.

Michelle:- And what's the pattern in this one here, number twenty three?

Carrie P:- Well in the bottom line you are going by sevens, and in the top line you are timesing by four.

Michelle:- Multiples of four. What do we call the bottom and top? Do you remember?

Carrie P:- The ... one of them, I think its the bottom one is the denominator and ...

Michelle:- Starts with an N ...

Carrie P:- Numerator.

Michelle:- Numerator OK good girl, that's lovely.

Untranscribed section

Michelle:- If any one would like to see me having marked those eight come over me now. Georgia, come and see me. Bring you work. Everyone else quite happy to continue along? ... Come and sit down let's have a look Georgia. What is the major problem?

Georgia: Oh I haven't got one. I wanted to ask you something else. Shall I do it?

Michelle:- I'll just mark your homework for you. How did you find this.

Georgia:- Oh its easy. I think.

Michelle:- Did it take you very long.

Georgia:- No. About five minutes.

Michelle:- Well tell me your system?

Georgia:- Well. I think I do it differently, but if you say twenty four, Three goes into twelve four times, so times four by four and you get sixteen.

Michelle:- Right so whatever you have done to the denominator you ...

Georgia:- Do to the numerator

Michelle:- Have done to the numerator.

Georgia:- Oh gosh.

Michelle:- Oh dear what have we done down here? Let's have a look. One half is equivalent to two quarters which is equivalent to three sixths

Georgia:- And they're all halves and I thought. Oh no.

Michelle:- Tell me how you came about six eighths and that will identify the problem.

Georgia:- I put two, four, six, eight, ten, twelve And seeings how you times it by two there and timesed by two to get six and then added two and then added two. I did it completely wrong.

Michelle:- Why did you add two?

Georgia:- I don't know. Because I was supposed to be timesing two. No. Yeah. No.

Michelle:- Can you tell me what each of these are supposed to be equivalent to in the lowest term?

Georgia:- A half.

Michelle:- A half, good girl, so that's the lowest term fraction. So six eighths we know is not equivalent to a half.

Georgia:- It would be six twelfths, eight sixteenths and ten twentieths.

Michelle:- Eight sixteenths and ten twentieths OK. But those aren't the next three equivalent fractions, because the next one is going to be... eighths. You've got halves, quarters and sixths, and so the next one would need to be eighths. The bottom line, the denominators are going up in multiples of?

Georgia:- Two that's how I did there. I thought.

Michelle:- So this one then should be tenths. And this one?

Georgia:- Twelfths.

Michelle:- Twelfths. Let's fill them in How many eighths is equal to a half?

Georgia:- Four

Michelle:- How many tenths is equal to a half?

Georgia:- Five

Michelle:- And? Six twelfths. Can you understand that? So let's go to this one look at the denominator and the pattern that the denominator is making. What are the next three denominators?

Georgia:- It would be twenty.

Michelle:- No its going five, ten, fifteen?

Georgia:- Twenty,

Michelle:- Twenty, twenty-five?

Georgia:- Thirty.

Michelle:- Mm. The lowest term fraction that we're looking for?

Georgia:- Two, four, six, eight, ten, twelve, fifteen?

Michelle:- Let's have a look at it. It's making a pattern, but is eight twentieths equal to two fifths?

Georgia:- (pause)

Michelle:- Let's work it out. How many fives in twenty?

Georgia:- Four

Michelle:- Four. Two fours are?

Georgia:- Eight.

Michelle:- Is you pattern working? Let's check for the next one. How many fives in twenty five?

Georgia:- Five.

Michelle:- Five tens?

Georgia:- Fifteen. Goes. Is ten.

Michelle:- Good, so can you tell me the next answer?

Georgia:- Um five into fifteen, thirty goes two, two times six is twelve.

Michelle:- Good your pattern worked.

Georgia:- It wasn't really what I was thinking.

Michelle:- It doesn't always work in a nice pattern ;like that. So you need to be careful and always relate these equivalent fraction back to the lowest term fraction. OK. Understand that? I'll leave the next four to do by yourself tonight and show me tomorrow.

Michelle:- Girls I think at this stage what I'll do if I haven't marked yours I would have marked someone on your table. Compare your answers with those people and mark them off that book please. I'll help you in a second.

Michelle:- Right Melissa you've got all these incorrect can you tell me your system?

Melissa:- I was halving them.

Unscribed marking of homework continues.

Michelle:- Put up you hand if you are finding these relatively easy? Well done. And when I started fractions you all said "Oh I can't do fractions" And look at you now.

Georgia:- What do you do if you're finsihed.

Michelle:- When you're finished, just... In two minutes. So just check your work carefully

Michelle:- Why on earth are we bothering to have a look at equivalent fractions? Why do we do it? Do you remember when we were talking about decimals we said where in the real world would we ever use decimal numbers, and we talked about accountants and we talked about calculator work and all of that. Where do we use equivalent fractions? Do we ever use it outside the classroom? Where Rebecca?

Rebecca:- Well fractions can be changed into decimals so its goes back to when we were talking about decimals.

Michelle:- Oh good girl. How can a fraction be turned into decimals?

Rebecca:- Forgotten

Michelle:- If we looked at the fraction three quarters it can be divided, converted into a decimals number. Because this number means? I'm sorry this line here means a...? Divided by sign. Three divided by four and if you did that on your calculator you would come up with a decimal number that equals three quarters. Georgia?

Georgia:- Like last year I think it was we had a pizza and my brother are more well, than us. My sister and I could only have one so we wanted to divided it into *three fifths and he could get three and we'd get the other two ...*

Michelle:- So you divided the pizza into fifths, he had three and you had two. So you have used a fraction good girl.

Child:- If you had half a block of chocolate and you had you and your friend and you had a quarter each... for chocolates and things so you would sort of ...

Michelle:- So you know that two quarters is equivalent to?

Child:- One half.

Michelle:- One half good girl. Any other examples of equivalent fractions? Carrie?

Carrie P:- When like, when you're in a supermarket and like you said when you change it to decimals, you can sometimes like use approximation in it like, once you've changed it into decimals? Actually as its a fraction its being a fraction?

Michelle:- So you're converting your fractions to decimals again. We're actually doing that in a couple of weeks time so you're well ahead. Angharad?

Angharad:- Like when you're measuring something when you're cooking. And you might have a cup of something and it doesn't have to be the things that you need so you can change it.

Michelle:- Mm I wonder if you said to you, say for instance if you got the recipe off the back of a sugar packet and it said for you to use one quarter of this packet and there were twenty- five grams in that packet, how many grams are you going to use? If it said to you to use a quarter of a packet of sugar and there were ... I'll make it easier there were 24 grams in that packet, which is highly unlikely. How many grams, er so, what fraction of that packet are you going to use?

Child:- Six grams

Michelle:- Six twenty fourths or six grams good girl. Homework tonight is p. 75 questions 1-10.

Appendix 18

Lyn Lesson Transcript 4.3.92

This is the follow up lesson that Lyn knew would be taped of her planning the class party with the class. They have individually done this and in small groups, all of which have been videoed.

Lyn:- Right you've done the first stage where all of you were in a group. You had you brainstorming and you had your webs- I've taken one (holds up a group's poster of a schematic map of ideas associated with the planning of a class party). I've looked through all of your groups and from your groups it will tell me the types of party you are going to have. I guess the consensus would be some sort of swimming party of all the groups that's what came out. Taking it as a swimming party I then looked at the next sort of headings (holds up her poster) there were- yours and yours and yours (pointing to each group), and put them together sometimes I didn't exactly put all these things down and I want you to extend that.
The headings first of all was this one- Decorations (holds up poster). This is what some people mentioned. Not many in the decorations, maybe you didn't think of it, I'm not sure. But today you're going to extend it. All I've got are balloons, party poppers, silly string, streamers.) If I didn't include some of these things perhaps I didn't really understand what you really meant- or perhaps it was not relevant to a swimming party.
I came to the next heading which seemed to come out strongly- Food, they were basically all the headings that all the groups said. Sometimes the groups went into more specific things and put snakes (under lollies) jellybeans or whatever. Some were more specific and put Samboy under chips. (Food poster has chips, cheezels, lollies, popcorn, soft drinks, pizza, frankfurts, party pies, sausage rolls, sausages, cordial). Some of you had soft drinks and you went into Coke or Fanta, but these were the overall headings.
Then as I looked the sorts of Games you are going to play (Shows another poster- Games- volleyball, murderball, pass the parcel, treasure hunt, races, poison ball, fashion show.) Again they were headings, they were basically summarizing what you just said.
Now did you actually address that issue or did you put this issue in. .(shows another poster- Equipment- Utensils, plates, cups, Games, presents). You didn't perhaps put a heading. I got these off your work- plates and cups, so I gave it the heading Utensils. I've got to games over here, perhaps I should have put presents for the party. Are you going to have prizes. Can you indicate by putting your hand up if I

left something out perhaps from your webs or your brainstorming- that I haven't got under decorations, food, games or equipment. Yes?
Carmen:- You didn't put music down.
Lyn:- I didn't have music- do I need to? I was thinking more of the $150. Do I need to pay for that.
Carmen:- no some people will probably bring their own tapes and that.
Lyn:- So I haven't looked on it on that side, so OK? Music could that not go under then this section in Equipment. Perhaps I should have put it here as games. Right, come up with a better title to include music and tape recorder, right?
Steven H:- I thought games, we have the balls and everything for murderball.
Lyn:- Right so beside all of these things, you might have to say if you're going to collect... what equipment you need for murderball. So you really need to put these together (i.e. Games and Equipment poster). Because in order to achieve volleyball- you've got what equipment do you need. So they could go side by side, perhaps in a ...
Brenda:- You can have Entertainment.
Lyn:- Yes instead of games I could have chosen that and used another heading. Yes?
Paul T:- Drink competition.
Lyn:- I see that belonging to that (games).
What I really want to focus on, not so much the games, not so much on the equipment, because I really want to focus on your $150. So that's why I've set you the task where you've collected these pieces of paper with your items of food (A-3 cartridge paper pasted with newspaper prices cut out from local papers and promotional material). Each group has got that. I've also got different junk mail items and I've got enough for each table. So this will give you an idea of the prices and how much things cost. So I'll give you one of those on your desk as well.
Your task today is looking at specific areas, food is you first specific area you've got to look at. If I look at that (Food poster on easel) and there's 20... how many are there in the class?
Olivia:- 26 students, and Mrs Merry.
Lyn:- Right Mrs Merry. Is there anyone else?
Brenda:- Mrs Bickmore- Brand.
Lyn:- Mrs Bickmore-Brand so that's 28 or would you round it off to ...?
Class:- 30
Lyn:- Right so we're interested in a party that has to cater for how many people?
Class:- 30.
Lyn:- Write that number on your sheet somewhere so you know how many you were catering for. Write on your lined paper there- "The party must cater for.... or ..." How you put that I don't mind. That number we've always got to keep that number 30 in our mind. (pause while children write on their sheets).

Alright ? Do you know how to spell ca-ter-ing? Anyone have a go at ca-ter-ing? What do you think its going to start with.
Victoria:- C ... a ...
Lyn:- C ... a ...
Victoria:- o ...
Lyn:- Not ...o ... c ... a ...
Olivia:- C ... a ... t ... e ... r ... i ... n ... g.
Lyn:- Yes, c...a....t...e...r...i...n...g... So perhaps put it above correctly if you didn't- "Catering for 30 people." Because if you were actually planning that's what you would have to do ... (pause).
So I'm going to send you off in a moment to work with your group. When you look at your Food at you party are you going to think about anything specific in your party in the line of food?
Aaron:- Yeah, um, the names of the plates.
Lyn:- Right you're going to list the foods at you party. What are your foods going to be? Do you know? Have you got any ideas?
Mark:- Chips and
Christian:- Lollies
Lyn:- Chips and lollies and things, so I should have said what time is your party going to be because that's going to perhaps change your food? Having your party first thing when you get here at 8.30 am or 9.00, that's one option, right?
Brenda:- After lunch
Lyn:- After you've eaten your lunch you're going to have a party? Perhaps you're first question on your piece of paper is when is it best to have the party? So perhaps you need to put time down on your paper. Is it going to be a brunch? Between breakfast and lunch? Is it going to be an afternoon tea party or a morning tea party?
Olivia:- Shall we put place as well?
Lyn:- Somebody said shall we put place as well. These are just some suggestions you can discuss this later. I'm now going to give you some ideas, give you some clues. (Blu tacs posters around walls). I'll put them up here so you can all see them. With your food- what about considering these kinds of things? Can you read that for me please Christian?
Christian:- Food, 1. Look at organising a balanced menu. 2. Calculate what amount required to feed the class. 3. Use ads to calculate costs of items.
Lyn:- So as I send you off to look at your food you need to consider 1. organising a balanced menu. What do I mean by a balanced menu? Reuven?
Reuven:- A bit of everything.
Paul :- So we all get the same thing.
Lyn:- So we all get the same amount? Yes.
Olivia:- You could have healthy food and junk food.
Lyn:- Yes a combination of healthy food and junk food.
Mark S:-Snack food.

Lyn:- Yes, snack food, junk food. Yes?
Ben C:- What about if we just want junk food?
Lyn:- Will your money go sufficiently for you to eat that you'll feel as if you're satisfied by just eating junk food? That's another thing you'll need to discuss in your group and it will depend on what time the party will be.
Steven H:- Its not the best to swim after you've just eaten either.
Lyn:- No so maybe you'll have to have your swim first. Yes Olivia?
Olivia:- Maybè we can have lunch and then a quiet time of games and then have a swim.
Lyn:- Yes. Yes?
Ben C. We could start after recess. We could swim then after lunch and then you could have a playtime and then go back to the pool again afterwards.
Lyn:- Mmm. Its up to your group to discuss what options you have. Number 2. I want you to calculate what amount? What do I mean by that question?
Stuart:- Well we have to see how many drinks and we have to get how many party pies and everything and then we have to get the price and add them all up and divide them into 20's, no 30's.
Lyn:- Yes, is everybody going to have the 30 party pies, or are you only going to buy 15 party pies and cut them into half? Of buy large pies and cut them into quarters? Are you going to have frankfurts as you indicated (pointing to poster). If you're going to work out 30 how many will be in a packet?
Brenda:- About 30.
Lyn:- Sausage rolls. Are you going to have the long ones and cut them into smaller or are you going to have little ones? Have a look at the ads, they'll help you with that. What will give you the best value for your money? $150, what will give you the best value? The sausages. Were you going to have a b-b-q? I don't know whether that group thought that through?
3. Use you ads to calculate the cost of items- you've got them there. If you've got to have 30 party pies and they sell packs of 24 what are you going to do?
Steven:- Get more.
Lyn:- So you have to work those things out. So that's when it comes to looking at your food.
Then I wish you to go onto the next section because it leads on too... and you can put it on another piece of paper ...(pause as she blu tacks Shopping List on B/B)
Lyn:- Shopping List
Class:- Shopping List
Lyn:-Would you like to read it please Paul?
Paul:- Prepare your list Include "quality" (reads quality instead of quantity), weight/ capacity, prices.

Lyn:- Let's look back at this one (points to quantity). Do you think that says quality or quantity? Its your shopping list so you must include the quantity, its no good just putting chips, because I don't know when I look at it for 30 people will I need 30, or will I need more? For weight, do you want them in 250 gram packet or 500 gram packet. Or when I put capacity there, what would come under capacity?
Blake:- Drinks
Lyn:- The drinks. And then the prices. If you keep a running list like the shopping list we did the other day (still on B/B) and the amount we want and how much and end that with a tally of your ... ?
Class:- Price.
Lyn:- So I think perhaps I'LL give you time to look at that. I've got some other ... I'll put these up because as you're doing all this you've got to consider these facts. Perhaps I'm telling you too much? (blu tacks poster to b/b) Would you like to read these Melissa?
Melissa:- Facts to Consider- 1. Purchasing of food, 2. preparation of food, 3. serving food, 4. party location, 5. time and duration 6. timetable.
Lyn:- Right let's go back (points to poster) Facts to consider as you're going to do it. The purchasing of food. What do I mean by that um Mark?
Mark:- Who's going to purchase it and when.
Lyn:- Who's going to purchase it and when are you going to purchase it and ... ?
Class:- Where.
Lyn:- Where are you going to purchase it. the next one please Blake, what's that one?
Blake:- Preparation of food.
Lyn:- Preparation of food, what do I mean by preparation of food? Dean?
Dean:- When and how you are going to eat it?
Lyn:- This is sort of before you actually get to eat it.

Joe, Brenda, Tim and Stuart continue to contribute ideas on serving food.

15 minutes group work where children discuss how and what they'll need to organise. Teacher goes from group to group with minimal input unless it is specifically sought by students.

Lyn stops at one group who are working on sandwiches.

Lyn:- So you'll have to write something about bread. How many loaves of bread you will need.
Joe:- We're going to have sandwiches and cut them like this (makes 2 diagonal cuts with his hands). How much bread will we need?
Lyn:- Right. How many slices in a loaf of bread?
Ben C:- 24

Lyn:- I was going to say 24 as well. Right so if you get 24 and you put one on top of the other (demonstrates with her hands) so that's...?
Joe: Twelve.
Lyn:- Twelve. Sandwiches. If you do them into ... ? (makes a diagonal cut with her hands).
Joe:- Halves.
Ben C:- No quarters (makes 2 diagonal cuts)
Lyn:- Quarters. So there'll be 4 lots of 12 ...
Paul:- 48
Lyn:- 48 sandwiches.
William:- Not everybody would eat it so you'd be able to have 24 quarters.
Lyn:- Is that for 2 loaves of bread?
Joe:- Mrs Merry?
Lyn:- Remember we said there were 12 slices (demonstrates by drawing square with her fingers), and we cut those 12 into 4
William:- 24
Lyn:- You've got 24 slices but when you make them into double sides (demonstrates with hands) that reduces it to how many stacks (demonstrates with hands) of bread?
William:- 12.
Lyn:- Right. Now if you cut that 12 into lots of 4 how many is that?
William:- 48.
Lyn:- And you've got 2 loaves of bread. (pause) What's 48 plus ... ?
Joe:- 48.
Ben C:- 90, 92
Lyn:- Not 92, 48 and 48. What's 8 and 8?
William:- 16.
Lyn:- 16. So its ... ?
William:- 96.
Lyn:- And if there's 32 of us approximately, how many will each child have? Approximately?
William:- They'll have about half.
Lyn:- How many 30's in 96?
Ben C:- 3
Lyn:- About 3. Would it be 3 whole sandwiches or just three quarters of a sandwich? So each can have about one round of sandwiches.

Discussion continues to revolve around ingredients for fillings ...

Paul:- But they may not like ham.
Joe:- We could conduct a survey.
Lyn:- Right, so you could conduct a survey to see how many people liked sandwiches.
Paul:- But you would need to do it with other things like Coke ...

Lyn:- Right, so you need to put an asterisk by all the types of things that you need to do surveys with ... favourite fillings, favourite chips, favourite ...
Paul:- (to Ben) What about drinks?
Ben:- We could just buy squash.
Paul:- No you need Coke and not everybody likes squash.
Lyn:- Could you get away with just buying a variety?
Paul:- But it costs more. But if you work out exactly what everybody wants. If you buy- variety then you might miss out on someone.
Lyn:- Right. In your drinks are you buying them in cans or bottles?

(Teacher leaves the group)

Groups work on for another 5 minutes.
Lyn:- (Addressing whole class) Right what I'd like you to do is to report back to us what you have been discussing in your groups. So in the nest few minutes I'd like you to decide in your group- who are your spokesperson's going to be to report back to the group, or are you going to have 2 or 3. Alright so that's your next 3 minutes and be ready to share with the whole group.

Class discuss among themselves, she helps out where necessary.

Lyn:- Right. Stop thankyou. I will ask you to sit at you groups and speak it out very loudly.
Stuart reports back (inaudible on video).
Lyn:- Do you have any idea what you've spent so far?
Stuart:- We estimate between $50 and $90.

When the sandwich group reports back Lyn responds by:-
Lyn:- So I want you to prepare a survey sheet for us the next time. What are the items?
Joe:- Pies, lollies, ... (inaudible).
Lyn:- Sorry?
Joe:- Pizza.
Lyn:- Types of Pizza. Right how could we design one?
Paul:- You could put like headings and then put ...
Joe:- Like for each group and then put their favourite like chips and that and you just see who's got the biggest amount.
Lyn:- So you decided what to do about your sandwiches?
Ben C:- Some people don't like sandwiches, but we decided that you could get 3/4 of a sandwich each and we needed 2 loaves of bread.

Other tables continue to report back.
Lyn:- If you said it was $1.99 if for example you said the pizza was $1.99 because I've gone around and got all of the prices for you. What is going to happen when you actually go shopping?

Paul T:- It will say, well $1.99 but it will be $2.00
Lyn:- It will be higher and so ... ?
Tim:- You should save about $20, just in case.
Lyn:- Right you need to be sure you've got ... you've allowed for ... Actually this group over here worked out on 2 sheets. One piece what was it? $2.99? You explain what you did to us.
Victoria:- Some were $2.99
Tim:- And some were 2 for $4.99.
Victoria:- And they were the same pizzas.
Lyn:- What had happened, one had had a bigger reduction and so ... ?
Ben E:- And we would save $1.79.
Lyn:- Just before we go (to recess) do you have any questions that you'd like to ask the different groups. Group 1 would you like to ask anything or Reuven?
Reuven:- (From Group 1 to Group 5) Um What would happen if some people can't sleep over.
Matt:- Well they could get picked up at um, whenever it suits their parents to go home.
Dean:- They don't have to sleep over.
Olivia:- They can just go home.
Oliver:- Well you'll have lots of food left over and stuff.
Dean:- You can give it back to the staff.
Oliver:- They won't be here.
Olivia:- There'll be Mrs M ... and Mrs Bickmore- Brand.
Paul S:- What about sleeping bags and everything?
Group 5:- You can bring your own.
Stuart:- Well on Saturday Mrs Bickmore-Brand and Mrs M ... don't want to be with us. They want to go and ... Mrs M. might want to go and play tennis and Mrs Bickmore-Brand might want to do something ...
Paul S:- Another thing about food, I mean roast! Who likes roast? Most people hate vegetables.
A few children:- I like vegetables.
Paul S:- Yeah well, a lot of people don't like vegetables so they just wont bother at all, they'll be wasting their money.
Olivia:- They'll all be able to eat. You've got sandwiches and everything (pointing to other groups). You've got all these people.
Lyn:- No, no, can I just interupt, she's saying that they've got all this food. We've got to reach a consensus because you haven't got $150 here and $150 here and $150 here and $150 here and $150 here. We've got $150 in toto.
Olivia:- Well they can go on a 48 hour famine.
Paul S:- Exactly:- And that's the thing- because they don't like what you've done and it may not be convenient for the parents on Saturday. I've got athletics on Saturday.
Chorus:- So have I, I've go ... (classroom noise)
Lyn:- Sh. Melissa?

Melissa:- They said over there (pointing to Group 3), 24 lots of 250 grams of chocolate and that's a bit too much.
Joe:- He was reading the wrong thing.
Lyn:- Right 250 grams. Oliver?
Oliver:- Its a bit unfair if its unconvenient (sic) for other people to sleep over and they have to go home and to um and they miss out on all the other fun.

Discussion continues with small groups presenting their ideas.

Lyn:- Don't forget we have to come to a consensus, and Mrs Bickmore-Brand and I would like to be consulted, we have to think about what we really want and reach a consensus.

Lesson ends.

Appendix 19

Lyn & Michelle's Students Responses to the Post Interview

Question "Do you need much help in mathematics?"

Table 1
Lyn's students' answers to the Post Interview question:- "How much help do you need in maths?"

Ben, B	A fair bit because I'm sort of different, I do different things that I am good at than maths.
Will, B	Just as much as the other people.
Ben, C	Not too much help for maths.
Ben, E	Not much.
Chris, G	Not much I'm really good.
Reuven, G	No, not usually.
Stephen, H	No.
Dean, J	A little.
Matthew, J	Not that much.
Melanie, K	Not a lot.
Melissa, L	Depends on what needs doing, I'm pretty good at maths I think, I don't need a lot of help.
Stuart, M	Not very much.
Carmen, O	Not much.
Oliver, P	Um, not really, no.
Victoria, P	A little bit of help.
Blake, S	Oh not much at all. Some people even come for help to me.
Brenda, S	...fifty-fifty.
Mark, S	Quite a bit.
Paul, S	I don't really need help.
Paul, T	Not really.
Jo, W	Well it depends what type we're doing, like if I'm doing things like really hard problems I'll need... a bit, quite a bit, and like if I'm doing sums or graphs or things like that, none.
Tim, W	No, not that much.
Chris, W	Not much.

Table 2
Michelle's students' answers to the Post Interview question:- "How much help do you need in maths?"

Carrie, B	Not much. Once she's shown us I don't need much.
Elizabeth, B	Um, not very much, but I do get stuck on some things.
Genevieve, C	Sometimes, sometimes, not in every maths lesson, but sometimes I do.
Rebecca, C	Um... it sort of depends, because if it's something that's new to me I'd probably need quite a lot if it's something we're revising over, not very much.
Gabrielle, C	A bit, with most things,... I know lots of other maths questions
Angharad, D	Not very much.
Gabrielle, H	Not a lot, only things, like, that I haven't really learnt before, but most of the stuff we're doing I've already learnt before so it's pretty easy.
Olivia, K	Sometimes if it's hard, quite a bit, but other times I can just go off and work.
Carla, L	Fairly little.
Elizabeth, M	Um... no not really.
Angela, M	It depends on what it is but not usually that much.
Georgia, M	I need... depends, Mrs Wright, like sometimes she really explains it and I can do all my work, but sometimes if it's hard I have to go back to her and ask for another example, and then I'm OK.
Adeline, M	(not interviewed)
Natalie, M	Uh... a bit.
Linda, N	Not really. If I understand it I wouldn't need much help but if it's a bit confusing I go for help.
Emily, P	Well it depends if I don't understand it, or, well, if I don't understand it I just ask them to tell me how to, what to do and things.
Carrie, P	Well, once I take a bit of time to understand things at first, but then once I understand them I can do them, I don't need any help.

Catherine, R	Um, sometimes, I'm alright with fractions, but some things I'm not sure of.
Rebecca, S	Oh, in some things, like the, say we've learnt it and I can't remember, I just have to, they have to refresh me and say, and then I'd remember how to do it.
Lynleigh, S	If it's doing fractions and things that I don't really understand I'm not the best but if it's something OK I don't need much help.
Kylene, T	Not much, as soon as I actually get it, it's quite easy.
Leanne, T	A Little bit.
Chantelle, V	Not that much. Some things I can catch on really quickly and some things I need a bit of time to sink in.
Melissa, W	In different areas I need a lot, in some areas not so much
Rebecca, W	It depends sometimes if there's something I don't understand, I'd normally go to Mrs Wright.
Alana, W	Not really much at all.