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A Cognitive Analysis of Students' Activity: An Example in Mathematics

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Abstract : In this paper, we argue for an engagement of productive connections between research findings and teaching, this since the vocational training of the teachers. We exemplify how analyses of written tests by using a cognitive approach lead to better interpretations and understanding of the learner's knowledge. We show how a teacher can do it and we discuss the possibility of introducing it to the available scientific knowledge in teacher education, in order to include it in the current practices of teachers' methods.

Introduction

The French national curriculum which defines the professional skills of teachers clearly highlights the need for teachers to take into account the findings of research in their practice (MEN, 2013). As Mayer (2008) says: "Educational practice should be guided by research-based principles" (p. 30); and, in recent years, we have seen the emergence of research paradigms that aim to build connections between research and practice (English, 2003; Anderson et al., 2000), as in mathematics for instance (see special issue, number 54, of *Educational Studies in Mathematics*, 2003, for more details).

Historically, in France, the contribution of research to teaching focused, first, on the nature of knowledge to teach (in mathematics, there are mainly the research works of Brousseau and Vergnaud; see Brousseau (1997) and Vergnaud (1991) for example); second, on teaching practices through especially the Vygotsky' works (Roditi, 2011) and activity theory (Engeström, 2001; Rogalski, 2008).

In recent years, in France, an area of research focuses on studying classroom situations, rather oriented student learning, in order to provide the teachers with information that enables them to understand how students get to using their knowledge (Bastien, 1997; Bastien & Bastien-Toniazzo, 2004; Samurçay & Rabardel, 2004; Pastré, 2008). Focused on questions of the effectiveness of the teaching-learning process, this area of research tends to adopt an interdisciplinary approach (cognitive psychology, social psychology, science education, didactics …) and leans on the analysis of students' activity (Lebahar, 2007; Ginestié, 2009). Accordingly, research about learning must be based on what students do really by themselves, on their productions. From this point of view, a cognitive analysis of students' activity can be one possibility (Richard, 2005; Duval, 2006; Musial et al., 2011).

Analysing students' activity with a cognitive approach can be performed during the lesson (Hérold & Ginestié, 2011). In this paper, we try to show that it is also possible with written tests.

This study illustrates how it is possible for teachers to have a cognitive approach of what the students do in an activity. We analysed written work from middle school students following an evaluation by their mathematics teacher on operations of positive and negative numbers (addition and subtraction exercises). Using incorrect answers given by students, we tried to determine what knowledge they put in place to come up with their answers. Here, we will present a few examples from the analysis, the characteristics of the students and ways of looking at them with regard to pedagogical strategies. Then, we try to show that this kind of analysis can be applied to their students by teachers if teacher education provides tools to the pre-service teachers to engage with research. We describe how such research can enrich the understanding of the use of knowledge by students, in order to improve learning and teaching.

Theoretical Framework

In France, it is true that teaching mathematics at middle school is often a teacher standing in front of a relatively large group of students. So, of all teaching methods, lecturing is the most widespread. Such teaching essentially remains a process for passing on knowledge, during which the teacher presents and explains concepts to the students who listen, take notes and possibly ask questions. In this kind of teaching method, the teacher presents information for the students to learn as clearly and precisely as possible; teaching is designed to bring knowledge to those who have to learn it (Lau et al., 2009). In this model, learning knowledge is the responsibility of the student; this provides autonomous learning. With this way of teaching, in order to find out whether the knowledge passed on has been acquired by the student, the teacher will go on to give tests and make evaluations. A large majority of teachers in France thinks that a student's response to an evaluation exercise is indicative of his/her knowledge. Thus, if the student's response in the exercise corresponds with the expected answer, teachers think that the knowledge aimed at has been acquired. On the other hand, if the answer is not the same as expected, the teacher considers that the knowledge has not been taken in (Bastien & Bastien-Toniazzo, 2004).

But this teaching method is not really suitable for all students. Indeed, the results, for France, of international assessments like PISA, show that there are large differences between students who succeed and those who are struggling (OECD, 2013). Even if we can discuss the epistemological and didactic validity of PISA studies (Bodin, 2005), however, other French national assessments highlight this fact (Bodin, 2006). It is therefore necessary to help *all* students learn. For instance, students who reported that their teachers use formative assessments with feedbacks on their strengths and weaknesses also reported particularly high levels of perseverance. But the use of such strategies among teachers is not widespread (OECD, 2013). Thus, even many French teachers seem to be interested (Vantourout & Maury, 2006), they seem they are unable to produce a true didactic analysis. Also, it is necessary to provide to the teachers new tools to better understand how students use their knowledge to perform a task.

Often, teaching is essentially structured around a group idea, the "class", which is incompatible with the individualised nature of learning knowledge (Bastien, 1997). If all students are different, then one has to take into account this aspect of the individual nature of learning. This understanding, based upon activity analysis, allows characterising the difference between what is expected of students and what they do, and find out the difficulties they face or the obstacles they overcome (Ginestié, 2009). So, analysing students' activity through their responses can become an essential clue for the teacher with regard to the manner in which the students have formed their knowledge and the procedures they have come up with (Bastien & Bastien-Toniazzo, 2004; Kazemi & Franke, 2004; Doerr, 2006; Mayer, 2008).

In this way, the analysis of errors done by students constitutes an important information source for teachers (Astolfi, 1997; Bastien & Bastien-Toniazzo, 2004; Deblois, 2006; Ravizza et al., 2008; Kramarski & Zoldan, 2008). Errors indeed reveal the kind of knowledge used by the student when forming an answer to the question that is asked. As Borasi (1996) says: "student errors are seen as a valuable source of information about the learning process, a clue that researchers and teachers should take advantage of for uncover what a student really knows" (p. 40). Therefore, error' analysis is a possibility as an essential indication for the teacher in understanding the knowledge used by students in an activity.

For instance, errors analysed with Newman's error analysis guideline (Newman, 1977) show that many errors are often due to the fact that the students use inappropriate skills in an attempt to find a solution (Clements, 1980). In the same way, Boder (1992) introduced the concept of "familiar knowledge" which is a well-know knowledge but inappropriate for the present task, a procedural knowledge than the role is to help the subject "make sense" of the situation. If a student uses such of knowledge, it is often due to the fact that he is overloaded (Hérold, 2012). In fact, the cognitive load, linked to the situation for the task's achievement and imposed by processing instructional material, depends on levels of learner knowledge (Kalyuga & Sweller, 2004), or/and is a function of the proportion of time during which the task captures attention (Barrouillet et al., 2004). Anderson et al. (2000) use a similar concept, the "strong knowledge", knowledge which can be remembered and called to attention rapidly and with some certainty: strengthened through practice, strong knowledge is more likely to be available when needed (Ritter et al., 2007). Thus, this is knowledge used by default, since the student has yet to acquire sufficient knowledge for the task.

Bastien & Bastien-Toniazzo (2004) show that students may propose some wrong answers even when they use correct knowledge (many times, students adopt an erroneous point of view due to the "structure" of the question). They can produce some incorrect answers but they follow different ways of thinking and, finally, they can propose correct answers that spring from incorrect knowledge. Some wrong answers are knowingly given by students, when there is "no better option". Indeed, the number and types of errors can be an important source of information for teachers (Ayres, 2001; Hersan & Perrin-Glorian, 2005; Duval, 2006; Ravizza et al., 2008).

In order to find out why students make mistakes on written mathematical tasks, Newman (1977) suggest one useful method for solving the error identification (White, 2005), with a procedure which is based on students' interviews. But, it is not always possible for teachers to do that, especially in assessing written tests (in France, most teachers assess students' writing tests at their home). However, we assume that it is possible for a teacher to gather information with a cognitive analysis of the activity of the student to understand why the student is wrong and how the student is wrong. As Mayer (2008) says, "in order to help students learn, it is useful to understand how people learn" (p. 33). But, to ensure a real impact for practice in an efficient way, research has to provide tools and processes for use by practitioners (Burkhard & Schoenfeld, 2003). Teachers must have the possibility of providing feedback to help students learn. Data gathered with a cognitive analysis of students' activity seems to be a possibility.

Aim of the present study

The aim of this paper is to offer teachers some cognitive indicators likely to take a generic status and, so, be reusable in other learning situations.

Previous research results (Hérold, 2012) show that:

- If the student systematically incorrectly reinterprets the situation by using "strong knowledge" to solve a problem, then the student is in difficulty and has not formed the necessary knowledge. Therefore, s/he must be offered remedial help.
- If the student incorrectly reinterprets the situation by using "strong knowledge" on a problem involving several mental operations ("complex tasks"), this can only be due to the cognitive load imposed by the problem. An acceptable teaching strategy can hence be envisaged: signalling techniques, using advance organisers, (see Mayer (2008) for details).

- If the student incorrectly reinterprets the situation only on a few different problems, s/he is probably in the transitional phase of learning. One must therefore analyse how the exercise is worded, in order to underline the elements that have led the student to reinterpret the situation in this way to form his/her answer and set other exercises for him/her, changing the structure (reducing the cognitive load). The parts that "troubled" the student will be re-introduced gradually.

In this study, we worked with a mathematics middle school teacher. His students are from grade 7.

The complexity that characterizes teaching and learning mathematics seems to have yielded a multiplicity of researches and as Ayres (2001) says: "considerable research has been conducted on the nature and cause of mathematical errors" (p. 227), such as, for instance, research about operations involving negative numbers in arithmetic (Hativa & Cohen, 1995; Prather & Alibali, 2008).

So, first of all, in order to understand what is the nature of the main difficulties for students in performing procedures with positive and negative numbers, we established a list of difficulties encountered by students in the 7th grade in learning arithmetic of positive and negative numbers. This list is obviously not exhaustive but is established based on different remarks made by teachers (41 mathematics middle school teachers).

The list shows that:

- Students have difficulties about the magnitude or quantity associated with numbers, when negative numbers are concerned (why -50 is higher than -100 even though 50 is lower than 100 is an example of the kind of questions they ask themselves);
- Although tasks relating to the addition of relative numbers do not pose many problems, subtraction activities are difficult for students;
- Tasks involving the removal of brackets are generally very difficult for students;
- Students have difficulty in interpreting the semantics of the - sign (a difficulty that is particularly noticeable during calculator work, when students confuse the +/ - sign with the - sign).

This list corresponds to the difficulties suggested by several researchers as mentioned by Hativa & Cohen (1995).

Method

Procedure

Twenty-five students (7th grade, Year 7) from a middle to lower socio-economic demographic in the Marseille metropolitan area were involved in this study. Following a test in their mathematics lesson, we analysed their submissions.

The test comprised three parts of extremely unequal weight. The first part consisted of an exercise to put a list of seven positive and negative numbers into ascending order (one integer and six real numbers). The second part of the test is the one we worked upon. Students had to make calculations using positive and negative numbers by "detailing the different phases", as the question stated. They were asked to make fifteen calculations. The test comprised a third and final exercise where students were asked to mark out three points on a straight line matching their relative values. Students had fifty minutes to do the test.

With regard to the second part of the test, which complemented our analysis, the exercises were of varying difficulty and not in any particular order. It included simple additions of positive and negative numbers (two operands and an operator), simple subtractions, but also multiple operations with additions and subtractions, operands in brackets, etc. The operands could be integer numbers, but also real numbers (see Table 1).

Figure 1 presents the marks given by the teacher of eight offerings (the marks are given in the Y-axis; the maximum possible mark is 20), randomised from the 25 submitted.

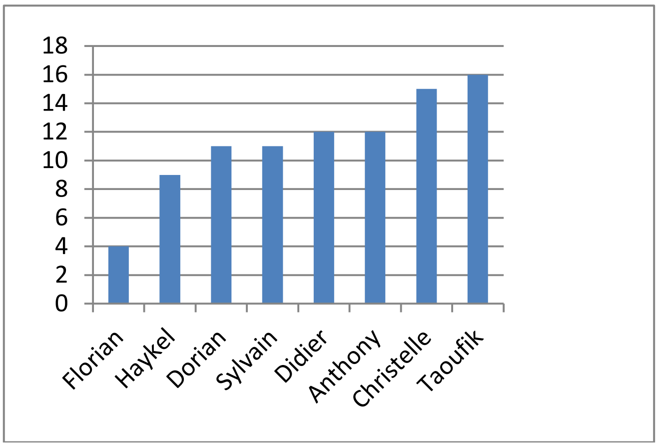

Figure 1: Marks given by the teacher for the 8 offerings used for our analysis

Exercises	Correct answers (N = 8)	Incorrect answers (N = 8)	No answer (N = 8)
A = (-5) + (-9)	5	3	0
B = (-7) - (-4)	6	2	0
C = (+3) – (+7) – (-10)	4	4	0
D = -2 + 5 – 6 – 1 + 4	8	0	0
E = 6 – 8 + 9 – 5 - 2 + 1	7	1	0
F = -2 + 6 – 5 – 1 – 3 + 5.5	7	1	0
G = 14 – 13 + 5 – 7 + 10 – 11	6	2	0
H = -1.3 + 3.6 – 2.4 – 1.1 + 0.3	1	7	0
I = 0.7 + 1.58 – 3 – 0.7 + 3 – 1.58	5	2	1
J = (2 – 3) + (4 – 6)	4	4	0
K = -3 – (-1 + 5)	4	3	1
L = 2 – (4 – 7) – (-2 + 6)	2	5	1
M = 3 – (-1 – 5 + 2) + (-2 + 8)	1	6	1
N = 2.5 – (0.3 – 5.2) – (-1.6 + 10.8)	1	5	2
O = -5 – (-5 – 7) + (-5 + 9) – 1 + (3 – 8) + 1	1	5	2

Table 1: The exercises of our analysis (part 2 of the test)

Global Analysis of the Answers

The students' interpretations of the numerical answers allowed us to observe, firstly, that the correct answers in exercises A to G are due to the fact that, for these exercises, the cognitive process which is used can be executed in the same time and in the same way as the process of the encoding of the calculations (the way of reading, left to right).

For exercise H, there was only one good answer: the fact that there are simultaneously integer numbers and real numbers was too difficult for students.

In exercise I, the higher number of correct answers is probably due to the fact that operands can be treated two by two, so it was easier for the students (so, we decided not to use this exercise for our analysis).

Exercises J to O were mainly unsuccessful. This is probably due to the fact that operations in brackets involve solution planning, a process which cannot be managed in the same way as the process of the encoding of the exercise. In that case, it is necessary to break the problem into sub-goals, to monitor what we are doing, to memorise in the working memory information that is relevant to the solution (Mayer, 2008). So, cognitive load becomes important. As Ayres (2001) says: "it is therefore argued that tasks which combine a knowledge of brackets with the manipulation of algebraic expressions and negative numbers are fairly complex and may exert a heavy load on working memory" (p. 230).

Results

For this analysis, we used a grid elaborated from previous research results (Hérold, 2012) and adapted it to this category of exercises. Figure 2 shows how the grid is elaborated and used.

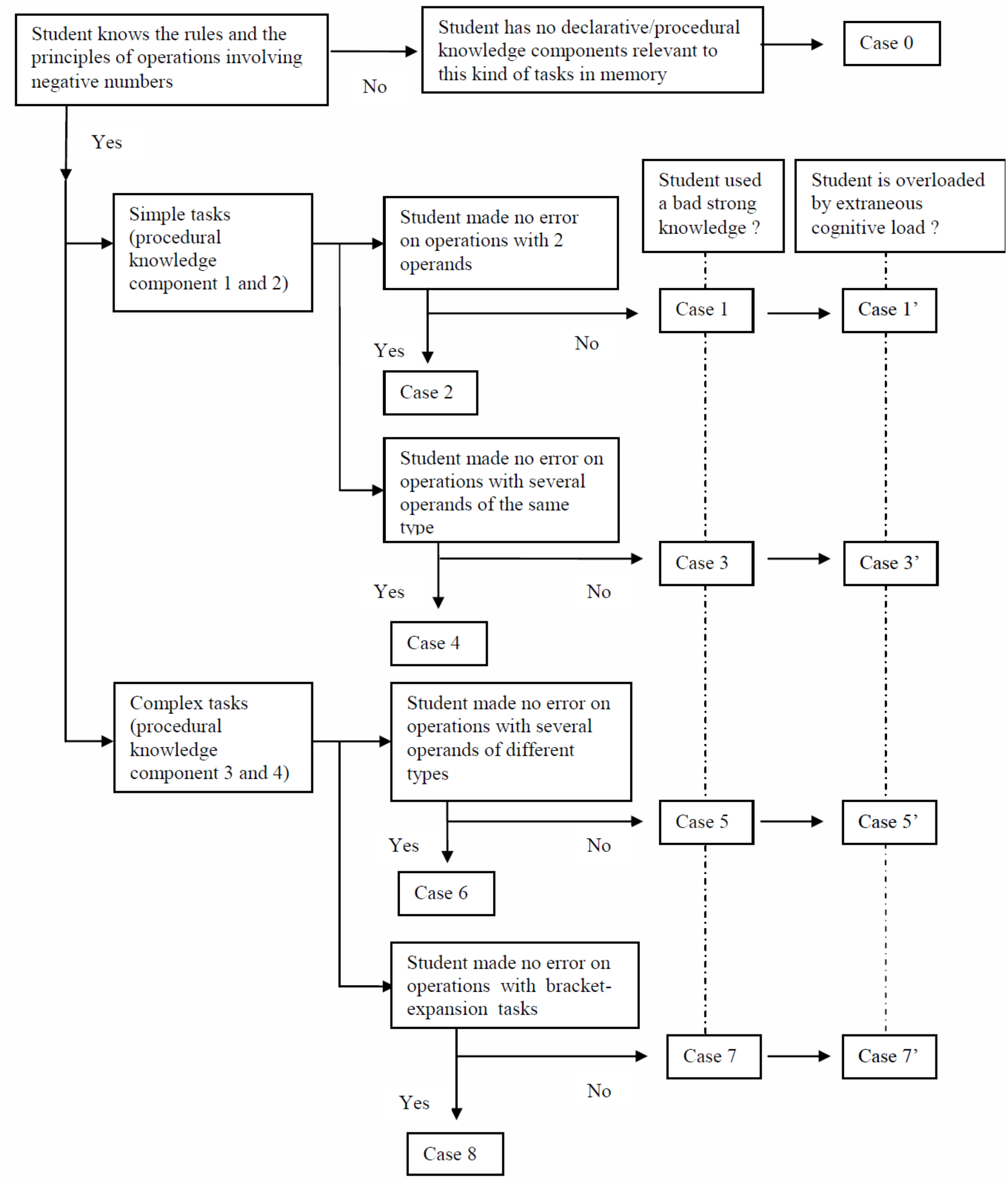

Figure 2: The grid of our analysis

The results of the analysis were put in a table of results like the one shown in Table 2.

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1’	2	3	3’	4	5	5’	6	7	7’	7 ‘’	8
A		X	X											
B				X										
C					X	X								
D							X							
E					X	X								
G							X							
F								X	X					
H									X					
J													X	
K														
L														
M													X	
N													X	
O													X	

Table 2: Results from analysis for Florian (example)

Reading down the columns of the table, one sees, reading across the rows, how results are distributed for the exercises. The grey areas correspond to the correct answers. The other tables of results are showed in the annex. From each table of results, we can establish the following table which shows what procedural knowledge component is learnt and how the profile of the student is determined by the type and the number of odd cases (see Table 3):

Student	Procedural knowledge components learnt	Procedural knowledge components nearly learnt	Number of times the student used a strong knowledge	Number of times the student probably suffered an extraneous cognitive load
Florian		1 – 2	3	5
Haykel	1	2 – 3	3	2
Dorian		1 - 2 - 3	1	5
Sylvain	2	1 – 3	2	1
Didier	1 - 2	3	0	1
Anthony		1 – 2 – 3 - 4	2	3
Christelle	1 - 2 - 3	4	2	2
Taoufik	1 – 2 - 4	3	0	1

Table 3: Results of our analysis

Description of analysis

We next discuss some examples of the way in which this analysis was carried out.

For instance, for exercise C, Florian gave the following response (Figure 3, the answer was crossed out by his teacher in correcting the paper):

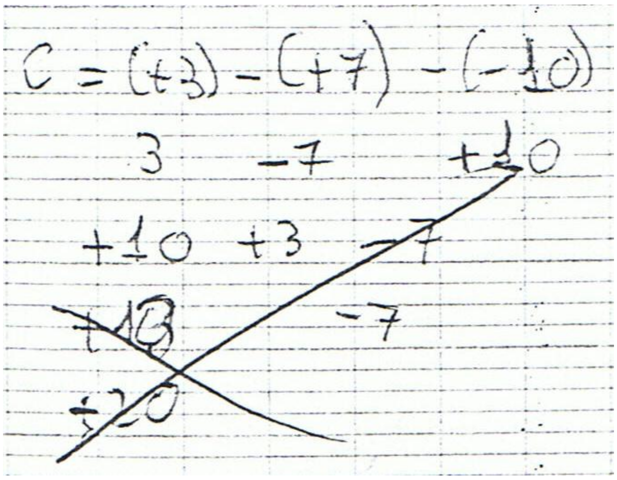

Figure 3: The first example of Florian's answer

We see here that Florian correctly carries out sign transformations of type "- +" gives "- " and "- -" gives "+", but in the last phase of his procedure corresponding to the operation "+13 – 7", he also reinterprets the situation as a simple addition of positive integer numbers and offers "+20" as the result.

We can also see that the teacher has incorrectly crossed out the two lines before the last one: these two lines are correct and it is only the very last line that Florian is wrong. It may be indicative of the fact that the teacher must not really use formative assessment with feedbacks on the weakness of his/her students.

Florian repeats the same error for exercise E = 6 - 8 + 9 - 5 - 2 + 1 (Figure 4, again having been corrected and crossed out by the teacher):

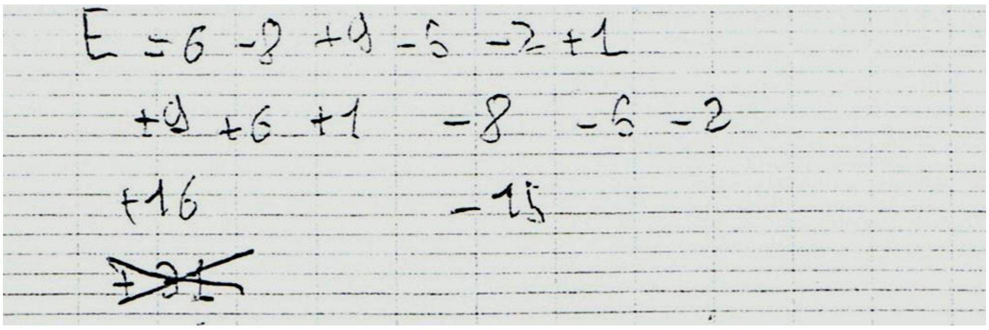

Figure 4: Another of Florian's answers

Here we see that Florian, in the first instance, groups together terms with the same sign; in doing this, he follows the teacher's instructions. He then adds up terms with the same sign: the subgroup of positive whole numbers and negative ones. So, we can say he knows the rules and the principles of operations involving positive and negative integer numbers. But he makes a mistake in the final part of this procedure. For the operation: "+16 – 15", he reinterprets the situation as being a simple addition of positive integer numbers and therefore gives "+31" as his answer.

Whilst he knows the rules about signs, it seems that Florian has not mastered the procedures for arithmetical processing of positive and negative numbers. Faced with a problem for which he does not have the processing procedure, he uses knowledge that works to do the exercise. This is where the addition that he carries out comes from. So, for these two exercises, C and E, Florian is categorised case 3 (using a *strong* knowledge). And, as he does the mistake at the last calculation, he is also marked as case 3' (overloaded by extraneous cognitive load): “For a given period of time, the cognitive cost that a given task involves is a function of the time during which it captures attention” (Barrouillet et al., 2004, p. 86).

Florian reinterprets the situation as a simple addition of positive integer numbers when he makes a mistake and this type of error is quasi-systematic (as the whole of his answer sheet shows) and always occurs in the last step of his calculation. When he makes mistakes, he uses strong knowledge. The analysis of the errors made by Florian shows that he is sensitive to the extraneous cognitive load (see his table of results). That is why he has a number of cases relative to the effect of the extraneous cognitive load in his table of results.

Sylvain, for instance, for the exercise: "A = (-5) + (-9 " offers "-5 – 9" as the first phase of his calculation, which is correct, but "-4" as his final answer. This "-4" can be interpreted in the following way: 4 is the quantity that one must add in order to go from 5 to 9. Here, Sylvain considers the operation "5-9" which gives "-4" (Sylvain has therefore already formed knowledge of the arithmetical processing of relative numbers). However, his cognitive system “forgets” the "-" sign present in front of the operand 5. Now having reached an impasse regarding "-5 -9", Sylvain's cognitive system reinterprets the situation by using more well-known knowledge, a strong knowledge. This accounts for his not bearing the "-" sign in front of the number 5 in mind.

We see the same type of error in the question "J = (2 – 3) + (4 – 6) " where Sylvain suggests " (-1) + (-2) " for the first calculation phase, which is correct, but "+1 " as the final answer. Once again, there is a reinterpreting of the situation through a simple subtraction between 2 and 1, while “forgetting” the "-" sign in front of the number 2.

Anthony makes exactly the same error with "B = (-7) – (-4)", for which he offers "-7 + 4" for the first part of his calculation, which is correct (therefore he knows the rule "- -" gives "+", and is therefore in the transitional learning phase), but he gives "11" as the final answer, which corresponds to the addition of 7 and 4, “forgetting” the "-" sign in front of the number 7.

Other students also make errors of this kind, but less systematically. Thus, Haykel reinterprets the addition scenario where the kind of operand seems to pose a problem, such as the operation "-31 + 29" where he gives the answer "- 60", whereas for the operation "-15 +16" he correctly answers "+1". We can therefore assume that having to process high value numbers induced a cognitive load that penalised the student (calculators were not allowed for this invigilated task). On operands corresponding to real numbers, he makes the same kind of error. For instance, for the exercise H, at the last line of his calculation, "-4 + 3.9", he wrongly answers "-7.9".

For more difficult questions, we can have other types of answers, as shown in the following example (Figure 5):

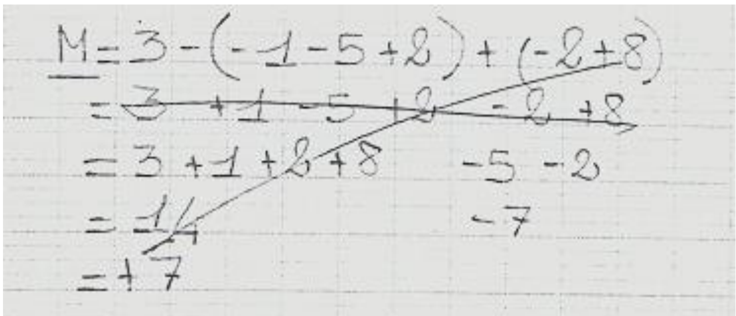

Figure 5: Didier's answer to the M question with parentheses

For this kind of exercise, the presence of brackets to define priority operations seems to induce a heavy load on working memory (Ayres, 2001) and leads him to make errors (A

French national evaluation on about 500 students, 7th grade, Year 7, showed that 68 % of students make errors in this kind of exercise).

Here, two strategies appear to be used by the students. First, there are students like Christelle, Taoufik and Anthony, who first carry out the operations in brackets, and change the sign of the result if necessary. This strategy uses prior knowledge, *strong* knowledge, so this knowledge is well-known. It is then easier to give the correct answers. Second, there are students who suppress the brackets, but in doing that, they change the sign, when it is necessary, only for the first operand in brackets. It seems to be not understanding for all the students. So, it involves students in making the same kind of error. An example of this kind of error, from Haykel's answer sheet, is in Figure 6:

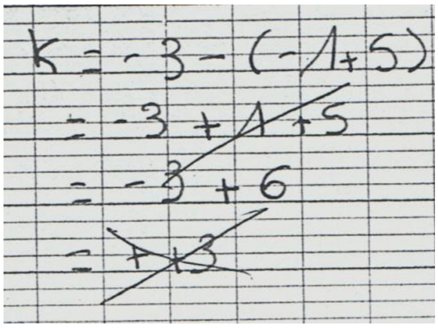

Figure 6: Haykel's answer to the K question

This is an example where the cognitive load seems to cause the errors. In the question "M = 3 - (-1 -5 + 2) + (-2 + 8)", Christelle offers "M = 3 - (-8) + (+6)" as the first part of her solution. Here, the student adds up all the terms in the first set of brackets and keeps the "-"sign, which leads her to suggest "-8" as her result instead of "-4". Christelle goes on to repeat the same error for the following exercise, where the question involved operations with real numbers, and she makes a mistake with a simple calculation ("17 - 11 = 8") on the next question, the final calculation of the fifteen that were set. But overall, this student produces a very good piece of work (her mark was 15/20). We can therefore suppose that errors were due to a lack of concentration on these final three calculations, where the adding up of numbers that was carried out is an operation that corresponds to an automatism that is well anchored in the student's memory.

We have the same effect on error in Taoufik's work (his mark was 16/20). Thus, for the problem "H = -1.3 + 3.6 – 2.4 – 1.1 + 0.3", the student suggests "-4.6 + 3.9" for the intermediate phase. Here, the error really seems to correspond to a problem of cognitive overload, due to the presence of several real numbers. Indeed, Taoufik is not wrong in his calculation for the whole part of the real numbers, but for the decimal part, he appears to carry out the function "3 + 4 – 1", as shown in this extract from his answer sheet (Figure 7):

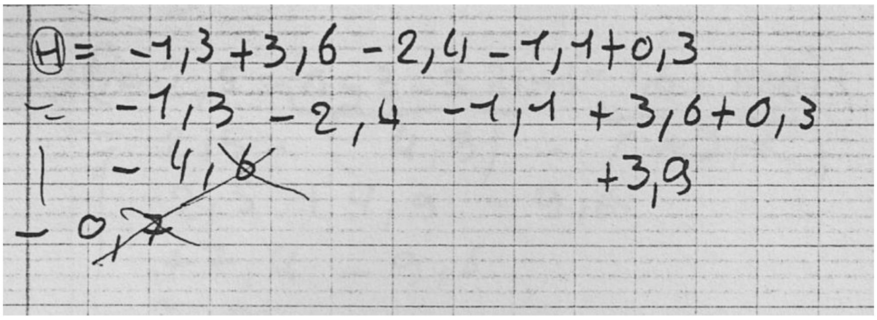

Figure 7: Taoufik's error

Anthony makes essentially the same error, but in a more accentuated way. For the same exercise (question H), he offers the solution "3.9 – 4.8 = 1.1". He actually processes the whole and the decimal parts of these two decimal numbers separately. Firstly, he deals with the decimal part and carries out "9-8 equals 1" (reading from left to right, focusing his attention on the decimal part of the operands). He then deals with the whole parts of the numbers. His attention remained on the second operand, which he hence deals with first: he takes the whole 4 and subtracts 3 (because there is a "-" sign in the function, so a subtraction has to be done), which equals 1.

Didier makes a similar error in the same exercise (Figure 8):

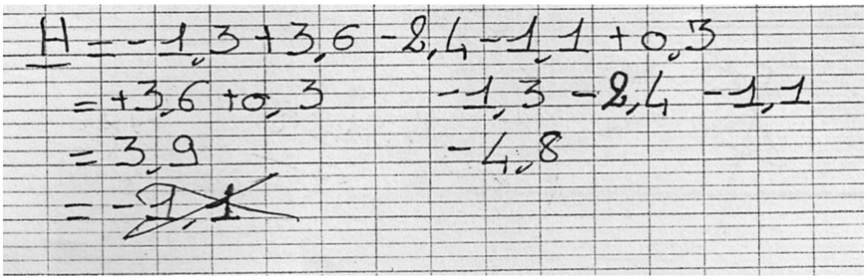

Figure 8: Didier's error

However, Didier gives as an answer a negative number. This may be due to a better mastery of real numbers on his part: he deals with the whole and the decimal parts together and not separately like Anthony, but keeps the distinction between the whole and the decimal. He carries out the operation "3-4" and the operation "9-8", allowing him to give "-1.1" as his answer.

General Discussion

Overall, the results of the cognitive analysis show that there does not seem to be any student which is in great difficulty (the maximum number of use of a strong knowledge by a student is 3). However, while Florian's mark was only 04 out of 20, he also seems to be in cognitive overload five times. So, we can suppose that Florian (like Haykel and Dorian) has a much higher level of knowledge than that suggested by his mark. As far as Anthony and Sylvain are concerned, they seem to have the same level of knowledge as Christelle.

So, we can establish the following analysis' results (Table 4):

Students	*Final result*
Florian, Haykel and Sylvain	Reinterpret the situation mainly on tasks involving several mental operations (“complex tasks”): this can only be due to the cognitive load imposed by the tasks.
Dorian and Anthony	Probably in the transitional phase of learning. Needs more investigation (from results of other exercises, recall interviews, etc) to improve our analysis.
Didier	Strongly penalised by the lack of understanding of how to remove the brackets.
Christelle	Seems to have learnt the four procedural knowledge components tested. Needs more investigation about the last task.
Taoufik	Seems to have learnt the four procedural knowledge components tested.

Table 4: Results of the Cognitive Analysis

The results of the cognitive analysis of the students’ activity show that, for most of the students, errors seems to be caused by limitations in working memory (Kintsch & Greeno, 1985), due to the fact that their procedural knowledge has not yet been strengthened through use (Ritter et al., 2007). The limitations could also be due to the nature of the material used in the tasks (Bastien & Bastien-Toniazzo, 2004) and the time during which the task captures attention (Barrouillet et al, 2004).

For most of the students, it seems that they know more than is suggested by their marks on the test (such as Florian, for instance). Thus, we can say that this kind of analysis gives the teacher richer information about the students’ knowledge than the marks. This analysis also shows that the nature of the material for the test is very important and teachers must take care to provide exercises aimed at helping students use their knowledge by relating it to what is presented. In the end, the analysis showed the fact that the teacher has probably failed in teaching the principle of brackets removal.

Through the examples of activity analysis that we have shown, we see that the kind of exercises given to students heavily determines the way in which knowledge is used by students. In analysing incorrect answers, we have seen that they revealed the type of knowledge called upon (Bastien & Bastien-Toniazzo, 2004; Mayer, 2008). Based on the manner in which the situation was reinterpreted by the student's cognitive system, we can determine whether the student is in great difficulty or in a transitional learning phase, which can be normal. The remedial assistance by the teacher will of course be different in both cases. For a student in considerable difficulty, one must begin by finding a different way of explaining the knowledge that is to be acquired (Bastien & Bastien-Toniazzo, 2004).

Indeed, we have seen that when the student completed an exercise incorrectly, it was essentially a *strong* knowledge that was used. The function of the "-" sign is not perceived as it should be by the student, in terms of looking at one number compared to another. During their previous schooling, students learned that the "-" sign was the one used for subtraction, and now they are taught that it also means that the number following it is negative, which does not necessarily mean anything to them. Thus, relevant knowledge has to be used, meaning knowledge that will allow the student to have a point of view that is adapted to the situation. Using the positioning of values on a graduate axis might be one possibility: students will be able to visualise it and remember it easily (Mayer, 2008; Hérold, 2012).

Hence, this strengthens learning about number order and the role played by the sign in front of a number.

The student must then be helped to form calculation procedures, using simple examples and then increasingly difficult ones in order to comply with the cognitive load limitations in the questions. The teacher will be able to explain problems that have been solved, in order to provide immediate feedback for students with regard to interpretation of the problem and the constraints in procedures for solving it (Tricot, 2003; Kazemi & Franke, 2004). At this level of learning, the use of similar situations can of course be beneficial to students (Scott et al, 1991; Merrill, 2002). Here, calling upon interdisciplinary approaches provides a wealth of possibilities (Beswick, 2011). By using themes broached in other disciplines, it is indeed possible to render mathematical knowledge functional: by giving meaning to what the student learns, the integration of new knowledge into his memory is facilitated (Bastien, 1997; Anderson, 2000). He will then be in a position to use this knowledge to complete or understand a specific task. Furthermore, the integration of this new knowledge will be even longer-lasting, given that its functional characteristics will be evident, making its re-use and then its automation even easier (Merrill, 2002; Mayer, 2008).

Teaching requires one to be sufficiently aware not only of the status and type of knowledge that is to be passed on (Giordan & Guichard, 2004), but also of learning processes, in order to correctly identify the cognitive resources that are available to the student and hence to develop help and guidance which will be useful for learning the targeted knowledge (Weil-Barais & Lemeignan, 1993).

If pre-service teachers do not necessarily ask themselves the same questions as researchers, the vocational training of teachers must build bridges between these two groups, researchers and teachers, in order to develop a really reflexive position among teachers. This reflexive position, which is possible for a teacher in the teaching situation with written tests as we have shown in this paper, is necessary for teachers, in order that they abandon the basic “right or wrong” for a better understanding of students' learning problems. But developing a reflexive position in teaching situations requires learning: learning how to interpret the results of the research, learning to use them, and learning how to implement them in practice (Gitlin et al., 1999). That is why it is necessary to introduce it into the vocational training of teachers: “an alternative model [which] would provide time within the student teaching experience to engage in the study of teaching and the reading of research” (Gitlin et al., 1999, p. 767).

It is therefore necessary to develop instructional strategies in ways that integrate data from research during the teacher preparation classrooms. In France, for instance, at the University of Aix-Marseille, pre-service teaching students must perform a dissertation that is both a critical reflection on the teaching-learning process but also and above all an integration of research results. For that, students have to read research articles, analyse research results and use an analysis grid and the methods and tools of researchers. So, as Gitlin et al. (1999) say, research linking theory with experience “is seen as having a value” (p. 766) by the students. That is why it is necessary to make research accessible by showing its application to real learning situations. We have tried to show in this paper that it is really possible to make research practical, even accessibility must also be discussed including issues like time, point of view, physical and temporal space (Gitlin et al., 1999).

There is not just one effective teaching method. Nevertheless, in order to really promote learning, one needs to establish connections between research and teaching in teacher education.

Conclusion

In this article, we have considered the need to use a cognitive approach in the teaching-learning process to allow teachers to improve their practices which are all too often rooted in their experiences of their own further education (Tunks & Weller, 2009). This could be possible with various forms of teacher-researcher collaboration, with a focus on teachers' interpretations of their students' learning (English, 2003).

We have highlighted the importance of greater awareness of the implicit information contained within a student's wrong answer. By analysing a number of examples of answers given by middle school students in mathematics exercises, we highlighted the benefit of identifying what knowledge a student called upon to find his/her answer, the effects of cognitive load (nature of the tasks, time to do the task). This was done in order to determine how much difficulty the student was experiencing - whether s/he was in great difficulty or in a transitional learning phase, for instance.

Teachers have to promote meaningful learning with different tools to respond to students' needs. Teachers have to help students to learn in ways that are truly effective. It is therefore necessary to provide teachers, as clearly as possible, with the tools that they need in order to reflect upon and improve the way in which they do their instruction. Activity's analysis with a cognitive approach can be one of these elements.

Research provides many results useful for teaching and "the development of educational interventions should be informed by the growing bodies of research in cognitive and social science" (Anderson et al., 2000, p. 13). That is why it is necessary to include the use of research findings in the practice of pre-service teachers.

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Appendix One

Student Results

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1'	2	3	3'	4	5	5'	6	7	7'	7 ''	8
A				X										
B			X											
C						X								
D							X							
E							X							
G							X							
F										X				
H									X					
J											X			
K														X
L													X	
M													X	
N												X	X	
O												X		

Table 5: Results from analysis for Dorian.

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1'	2	3	3'	4	5	5'	6	7	7'	7 ''	8
A				X										
B				X										
C						X								
D							X							
E							X							
G					X	X								
F										X				
H								X	X					
J														X
K													X	
L													X	
M													X	
N											X			
O													X	

Table 6: Results from analysis for Haykel

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1’	2	3	3’	4	5	5’	6	7	7’	7 ‘’	8
A				X										
B				X										
C							X							
D							X							
E							X							
G							X							
F										X				
H									X					
J														X
K													X	
L													X	
M														
N														
O														

Table 7: Results from analysis for Didier

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1’	2	3	3’	4	5	5’	6	7	7’	7 ‘’	8
A				X										
B				X										
C							X							
D							X							
E							X							
G							X							
F										X				
H										X				
J												X		
K														X
L														X
M											X	X		
N											X	X		
O												X		

Table 8: Results from analysis for Christelle

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1’	2	3	3’	4	5	5’	6	7	7’	7 ‘’	8
A				X										
B				X										
C							X							
D							X							
E							X							
G							X							
F										X				
H									X					
J														X
K														X
L														X
M														X
N														X
O														X

Table 9: Results from analysis for Taoufik

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1’	2	3	3’	4	5	5’	6	7	7’	7 ‘’	8
A		X												
B				X										
C							X							
D							X							
E							X							
G							X							
F										X				
H									X					
J											X			
K													X	
L													X	
M													X	
N													X	
O													X	

Table 10: Results from analysis for Sylvain

		Procedural knowledge component 1			Procedural knowledge component 2			Procedural knowledge component 3			Procedural knowledge component 4			
	0	1	1’	2	3	3’	4	5	5’	6	7	7’	7 ‘’	8
A				X										
B		X												
C						X								
D							X							
E							X							
G						X								
F										X				
H									X					
J														X
K														X
L											X			
M												X		
N														X
O														

Table 11: Results from analysis for Anthony