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THE RELATIONSHIP BETWEEN ESTIMATION SKILL AND COMPUTATIONAL ABILITY OF STUDENTS IN YEARS 5, 7 AND 9 FOR WHOLE AND RATIONAL NUMBERS

By

Phuntsho Dolma
Advanced Diploma in Mathematics (University of Leeds, UK),
B.Ed. Secondary (NIE, Samchi, Bhutan)

A thesis submitted in partial fulfilment of the requirements of
Master of Education

Edith Cowan University
School of Education
Perth, Western Australia
November 2002

ABSTRACT

This study explored the relationship between estimation skill and computational ability for whole and rational numbers. The methods carried out were both quantitative as well as qualitative and data were collected from three primary schools along with their associated high school in the Perth area. The year levels chosen were 5, 7 and 9. There were two classes from each chosen primary school representing Year 5 and Year 7 and three classes of Year 9 from the high school. The total number of students involved was 91, 77 and 73 from the three respective year levels. Instruments used for collecting data were group-administered tests and interview. Two parallel tests with identical items, where one of the pair was estimation and the other written computation were administered to all the students in the chosen year levels. Interviews were conducted for the group of selected students based on the criteria: slightly above the average and slightly below the average. There were eighteen students with nine in each group.

The results of the correlation shows that performance in estimation is positively correlated with written computation in all the year levels. Moreover, the t-test result reveals that there is no significant difference between the two tests except in Year 7. Hence, the findings indicate that a child who is good in estimation skill can also perform well in written computation. As such, the importance of achieving estimation skill in a child would be very helpful in solving computation problems with understanding.

On the other hand, children's performance related to the development of estimation skill and computational ability seems to be in positive direction from Year 5 to Year 7. Whereas the Year 9's performance is lower than Year 7. Among the topics, the children fared better in whole numbers compared to other topics. Performance tends to follow in a descending order from whole number to ratios. The disparities between estimation skill and computational ability are also more towards the difficult topics like division and multiplication of fractions

and decimals. At the same time, the feedback from the interviewees tended to show that, the children from slightly above the average are better at choosing their own sensible strategies for solving the problems, whereas the students from slightly below average are more prone to the rote-learned algorithms.

Although, male students appeared to perform better than the female students, the differences in performances are not that high. Thus, the result depicts that there are no significant gender issues in the selected year levels and topics.

Further research needs to be carried out in order to determine the relationship between estimation skill and computational ability with topics other than whole and rational numbers, especially in measurement topics. Moreover, such studies can be done involving larger samples, and in other countries as well. Doing so can highlight the importance of the integration of estimation skill in teaching and learning mathematics.

DECLARATION

I certify that this project does not, to the best of my knowledge and belief:

i. incorporate without acknowledgment any material previously submitted for a degree or diploma in any institution of higher education;

ii. contain any material previously published or written by another person except where due reference is made in the text; or

iii. contain any defamatory material.

Date: November 1, 2002.

P. Dolma
Perth, Western Australia

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CHAPTER I

INTRODUCTION AND PURPOSE OF STUDY

Background to the Study

To begin with, I would like to discuss in brief, the current situation in learning mathematics in my own country (Bhutan). As in many other countries, Bhutan too faces many problems in the teaching and learning of mathematics. Learning mathematics is considered to be important and essential in every aspect of life, but learning mathematics is still a nightmare for many children in the country (Curriculum and Professional Support Division (CAPSD), 1996). For many years, the country has suffered from a poor performance of school children in mathematics. The deficiency in mathematics becomes very noticeable every year during the admission of college and university students. Very few people opt for study in mathematics. As a result, the country has remained in short supply of skilled people in important fields like education, medicine, industry and technology, which are in high demand in this sophisticated and technological world. As stated, "the advancement and perfection of mathematics are intimately connected with the prosperity of the state" by a famous politician called Napoleon Bonaparte quoted by Usiskin (1986).

According to the current situation, around 60% or more students end up in odd jobs and training, as they are not qualified to seek further studies due to the low percentage attaining the required marks. One of the main factors that pull their overall average marks down is their score in mathematics. Many of them end up getting below the pass mark in the mathematics paper (Kuensel, 2001). It is at this time that we normally see a big gap in children's performance in learning mathematics. As mentioned, if not treated well, mathematics is considered to be the worst curricular villain in driving students to failure in school (National Council Teachers of Mathematics (NCTM), 1989). All the concerned people and the authorities in the country are aware of this situation but they seem to be really trapped in a vicious kind of circle as shown in Figure 1.

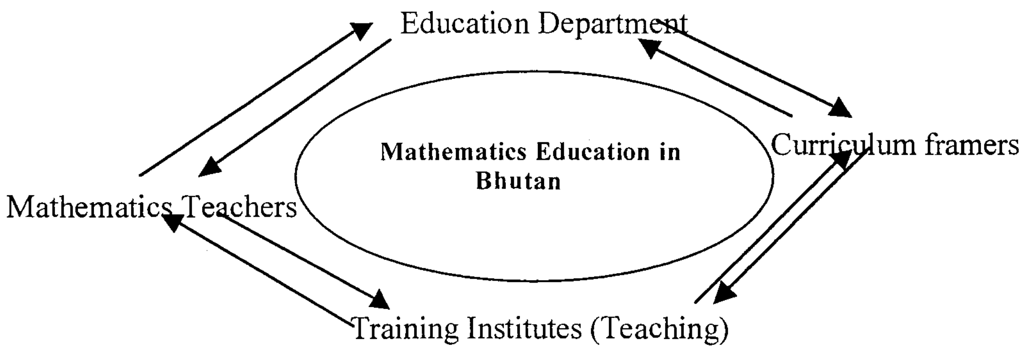

Figure 1: Circle of blame in mathematics education in Bhutan

It has become very difficult to pinpoint the black spot directly to one particular person or body; rather everyone starts blaming each other for the situation. The Education Department points the finger at mathematics teachers for not doing their job properly, whereas the teachers blame curriculum framers for producing a very bulky syllabus. In the same way curriculum framers blame the training colleges for not training the teachers efficiently, whereas training colleges blame the education department for not setting a strong criteria for the selection of mathematics teachers, and so on. Whatever the cause may be, it is high time to break the vicious circle somewhere and start looking for the factors, which contribute to low performance in learning mathematics and try to solve it accordingly.

There could be many reasons for the low performance in mathematics. One of the reasons could be due to lack of a strong foundation in learning the subject. As per my own experience as a mathematics teacher educator and according to research findings, one of the reasons for the weak foundation in mathematics appears to be due to the failure of students in making sense out of computation. May be, we as teachers fail to make sense in teaching and learning mathematics, particularly while dealing with basic topics like whole and rational numbers (fractions, decimals, ratio and percentage). It seems so, because, most of the children seem to face difficulties while computing mathematical problems related to these topics. It appears difficult to many of them, as they often cannot make any sense out of what they do. As such, it affects children's performance tremendously and thus it leads to failure when they reach higher grades.

Similarly, the situation appears to be same in other countries too. For instance, in a study carried out by Leutzinger & Berthean (USA, 1989), it is said that often students' mathematical knowledge is superficial and leads to misconceptions about number. Some of their findings in the classrooms are listed below:

- Seventeen out of twenty students in a fourth grade class responded that $^1/_2$ is the largest fraction less than 1;
- Sixty five percent of class of a sixth grade students selected 0.39 as a decimal that is larger than 0.6; and
- A third grade student adamantly argued that 20 is closer to 90 than to 5. (p. 5)

According to the above findings, many children do not seem to possess any sense of numbers. Lacking that, it hampers them from being able to work flexibly with numbers and give reasons with numerical information. As such, children fail to appreciate mathematics as a tool for solving problems and interpreting events (Ritchhart, 1994). Besides, having a strong sense of numbers can help them to possess a comfortable approach to solving mathematical problems with understanding. One can be in a position to make decisions about what tools and methods to choose for calculating and judge the reasonableness of their results (Jones, Kershaw & Sparrow, 1994). In several findings, it is well argued that, computing mathematical problems without understanding is one of the main concerns of mathematics education today. Many mathematics educators have considered this and come out with several ideas and methods to make learning mathematics more meaningful and useful to learners. Among many key ideas, I strongly feel that estimation skills could be one of the solutions as it has a capacity to ensure whether a computed solution/answer is reasonable or not. As pointed out by Miller (1993):

> For many situations, an estimate is all that is really needed. It appears, however, that little attention is being given to these important skills since national assessments tests continue to show that an alarming number of students are deficient in these areas. (p. 1)

Moreover, estimation is a skill, which involves manipulating quantities in ways that make sense. Having the ability to estimate can help students achieve very important goals (value mathematics, be a confident problem solver, communicate mathematically, and learn to reason) in learning mathematics. For instance, being able to reason and communicate better mathematically improves students' confidence. For that, children come to value mathematics as a distinct way of thinking, instead of viewing it as a collection of unconnected rules and formulas (Micklo, 1999). The same point is well argued by Usiskin, 1986, p. 2) who said:

> Estimating is often more reasonable than avoiding estimates, and estimating is often the only choice one has in a situation. Furthermore, the uses of estimation fit the ideals of mathematics, namely, clarity in thinking and discourse, facility in dealing with problems, and consistency in the application of procedures.

Estimation is also like mental computation, which brings a dynamic quality to learning mathematics and helps students broaden their view of mathematics (Rathmell & Trafton, 1990). Moreover, as suggested by Reys (1992, p. 142), "over 80% of all mathematical applications call for estimation, rather than exact computation". Not only that, in today's society, changes in technology have made estimation skills more important than ever in the development of mathematical power (NCTM, 1993). For instance, use of computing technology (e.g. calculators) now puts a high demand on estimation technique for verifying the reasonableness of computations (Levin, 1981). As such, I strongly agree to what was argued by Carlow (1986, p. 94), "without a well-developed sense of mathematical facts and relationships, students have no way to judge the reasonableness of numerical output from a computer".

To support this point, Poulter and Haylock (1988, p. 27) state, "computational estimation and the ability to judge the reasonableness of results as basic goals for the teaching of mathematics". As such, my main aim in this research project is to find out the place of estimation in improving the quality of learning mathematics. In order to do this, I would like to investigate how the relationship between estimation skill and computation ability can help in solving mathematical problems with understanding. In this study, I will focus only on whole and rational numbers, as they are some of the basic and important topics in learning mathematics.

Significance of the Study

This is an "ice-breaking," exploratory study that is hoped will be seen as the first of many to be undertaken for the benefit of mathematics education in the country of Bhutan. The findings of the study are expected to contribute to the wider sphere of teaching and learning mathematics and particularly to the chosen topics in the following ways, by being able to:

- identify the relationship between estimation skills and computational ability;
- identify the importance of estimation skills in computing mathematical problems related to whole and rational numbers;
- raise awareness of the importance of estimation in computing mathematical problems (numbers) to the mathematics teachers in schools, educators training institutes and curriculum officers in Curriculum and Professional Support Division (CAPSD) in Bhutan; and
- suggest opportunities for all the children to carry out meaningful computation of mathematical problems.

Identify the relationship between estimation skill and computational ability

It is important to find out whether estimation skill can help the children compute the mathematical problems with understanding. For this purpose, answers to two parallel questions can be compared to find out whether there is any relationship between the estimation skill and computation ability.

Identify the importance of estimation skills in computing mathematical problems related to whole and rational numbers

As suggested by Carlow (1986, p. 98), "estimating forms a powerful means of enriching the understanding of number and operations on numbers", and this study is based on the fundamental topics of whole and rational numbers. The main purpose for choosing these topics is that, the children would be in a position to solve mathematical problems with understanding from the very beginning. As such, children would find it easier to compute problems at a later stage (higher level) as well. It is said, "under the right conditions estimating can be an obvious and powerful vehicle for helping children develop the ability to conserve number" (Carlow, 1986, p. 101). The point is very well argued by Showell (1976, p. 25) "the child who has insufficient understanding of the basic concepts is going to find the subject difficult when he gets to his secondary school". In addition to that, it is very important for the children to retrieve simple arithmetic facts so that they would not experience difficulty in other areas of mathematics learning (Ackerman, Anhalt & Dykman, 1986; Geary, 1994 as cited in Hopkins, 2000).

As such, time given to teaching estimation in these topics would help children become more adept at reasoning with numbers, more flexible in thinking, more aware of the relationship between different operations and develop a greater feel for number (Poulter and Haylock, 1988). Besides that, the emphasis on estimation particularly for the chosen topics is designed to help students understand the relationship between whole number and decimal fractions so that they would face less problems while dealing with other topics at a later stage (Reys, Reys, Nohda, Ishida, Yoshikawa & Shimizu, 1991). It is observed that "time spent developing these basic concepts through an estimation approach greatly enhances, and gives meaning to, later work with exact computation" (Reys, 1986, p. 33). It is also believed that computational estimation and general mathematical thinking are highly related in terms of deciding what answer is needed, using mental flexibility, recognising multiple solutions, picking one strategy in favour of another, and checking for reasonableness of results (Reys, 1985).

The research findings suggest that, as skill is developed in estimating rational numbers, it help to improve a child's concept of rational number size. As such, the concept of rational number and skill in estimation can be developed in such a way that they go hand in hand and facilitate each other (Behr, Post & Wachsmuth, 1986).

Raise awareness of the importance of estimation in computing mathematical problems (numbers) to the mathematics teachers in schools

The teachers in school need to be made aware of the importance of estimation before they blindly apply computation skills to the children. Once, they are convinced, they should be able to implement it in their lessons. It should be integrated in every topic rather than taught separately. It is so that estimation can be applied to almost all the mathematical topics. It is suggested by Reys (1986, p. 31) that:

> Estimation, much like problem solving, calls on a variety of skills and is developed and improved over a long period of time... It is not a topic that can be isolated within a single unit of instruction...to be effectively developed, it must be nurtured and encouraged throughout the study of mathematics.

In order to do this, the teachers should be in a position to incorporate estimation activities into all areas of the program on a regular and sustaining basis so that the children can make use of the skills to pose and select alternatives to assess a reasonable answer (NCTM, 1980). It is also pointed out by Clarke, Lovitt and Stephens (1990, p. 175) "estimation tasks, if carefully introduced by teachers, are one way of breaking down students' fear of failure in mathematics".

Raise awareness of the importance of estimation in computing mathematical problems (numbers) to the educators in training institutes

If there are any changes to be made in the field, the teacher educators in teacher training colleges should be aware of that and be able to deal with this at the trainee's level. It should be done so that the trainees will be well aware of its importance and be ready to apply it in the field.

Raise awareness of the importance of estimation in computing mathematical problems (numbers) to the curriculum officers in Bhutan

Any findings from the research that could be applied in the school should be presented to the curriculum officers. They should be convinced as well so that they can include the idea in their curriculum framework for the teachers to apply it in their lessons. As such, findings of my study too will be presented to them, and passed on to be included in the school syllabus also. As pointed out by Trafton, (1986, p. 16):

> Computational estimation is one of the most powerful and useful aspects of estimation, and building a strong computational estimation strand into school mathematics programs must be a top priority for curriculum developers in the near future.

In doing this, one should remember that the main purpose of introducing a systematic estimation program is not to do away with the routines and the analyses of existing mathematics programs. Instead, it is to build a combined linear/analytic and intuitive/holistic approach, which can support the details with a strong informal background of awareness and understanding (Carlow, 1986).

Suggest opportunities for all the children to carry out meaningful computation of mathematical problems

The ultimate but very important expectation of this study is to help the children compute mathematical problems with understanding. For this, they will need to have some skills in estimation. With an idea of estimation, the children are expected to understand the problem and lead to better solutions. Being able to do that is assumes they could perform better in learning mathematics when they reach higher studies. Children with estimation skills could approach problem solving more thoughtfully (Kindig, 1986). It is also predicted that students who are good at estimations are normally confident in their mathematical ability and more likely to attribute success to ability (Sowder, 1992). Besides that, those able students are said to be able to easily link symbols to concepts.

What has always been at the back of the researcher's mind is that the skills and knowledge gained in this research process will be used as a torch light in highlighting the teaching and learning mathematics to all the teacher trainees, school teachers and the curriculum officers in Bhutan.

Aim of Study

The aim of this study is to investigate the relationship between students' computational ability and estimation skill while dealing with whole and rational numbers. More specifically, I would like to explore the following:

- use of estimation in measuring the understanding of mathematical problems;
- estimation skills used in computing whole and rational numbers;
- estimation skills possessed by students in Years 5, 7 and 9;
- development of estimation and computational abilities with age;
- difference in performance among the chosen topics; and
- gender differences in estimation and computational abilities.

Use of estimation in measuring the understanding of mathematical problems

Under this aim, the main purpose is to check the importance of estimation in measuring the understanding of mathematical problems. With the given problem, if the children are able to estimate an appropriate answer, then it is assumed that the child could visualise the content of the problem. That is to say that the child could understand the problem. Likewise, the result is expected to be just the opposite if the child is not able to estimate accurately.

Estimation skills used in computing whole and rational numbers

Since, whole and rational numbers are the foremost and basic topics in learning mathematics, it would be very useful for the children to have a good idea of estimation. Having that idea can help children to understand and learn mathematics better, especially when they move on to higher studies. Therefore, it is always preferable for the children to have strong foundations at the basic level of the learning. With the introduction of estimation skills at this level, it is expected to help children learn mathematics with understanding so that they would have fewer problems learning other more complicated topics at later stages. The major purpose of the study is to investigate how much skill in estimation children posses and make use of while computing the given mathematical problems in estimation and computation tests.

Estimation skills possessed by students in Years 5, 7 and 9

This study investigates estimation skills possessed by students in Year 5, 7 and 9. The main purposes for choosing these year levels is basically for two reasons: Firstly to represent each of the school levels (primary and secondary). Years 5 and 7 represent primary schools, and Year 9 secondary schools. Secondly, the idea is based on the previous research in *Mental Computation in School Mathematics: Preference, Attitude and Performance of Students in Years 3, 5, 7 and 9* by McIntosh, Bana and Farrell (1995), and *Number Sense in School Mathematics: Student Performance in Four Countries* by McIntosh, Reys, Reys, Bana and Farrell (1997) and *Number Sense Performance and Strategies Possessed by Sixth and Eighth Grade Students in Taiwan* by Yang (1995). Around ten items were taken from those studies and are listed below:

a.	9965 + 8972 + 8138 + 8090	b.	18 x 19
c.	96.7 + 147.4 + 62.75 + 36.8	d.	0.72 - 0.009
e.	0.5 x 840	f.	87 x 0.09
g.	54 ÷ 0.09	h.	$^7/_8 + {}^{12}/_{13}$
i.	19.4 x 46.1	j.	563.7 ÷ 2. 93

The actual model of the parallel questions looks like the one given in Table 1 (McIntosh, Reys, Reys, Bana, & Farrell, 1997).

Table 1: Results of two parallel fractions items from the TNST and WCT (TNST = Taiwanese Number Sense Test and WCT = Written Computation Test)

Without calculating an exact answer, Circle the best estimate for $^{12}/_{13} + {}^7/_8$			Calculate: $^{12}/_{13} + {}^7/_8$		
Results (Percentages): TNST			Results (Percentages): WCT		
	Age 12	Age 14		Age 12	Age 14
A. 1	10	20	Correct	61	63
B. 2	25	38	Incorrect	39	37
C. 3	36	14			
D. 21	16	12			
E. I don't know	10	16			
F. No response	3	0			

The above table clearly reveals the actual understanding of children in number sense against the actual performance of computation of the same problem. Keeping that in mind, I wanted to investigate further in the same kind of concepts, using a similar model.

Development of number sense, estimation and computational abilities with age

This is to investigate whether there is any development in estimation and computational abilities with age. For this, there are some common questions across two or three levels so that the researcher can explore and find out its development with age.

Difference in performance among the chosen topics

Mathematical topics involved in these studies are whole numbers, fractions, decimals, ratios and percentage. Therefore, with the help of the data collected, it is to find out whether there are any differences in performance by the children in each year level in the taught topics.

Gender differences in estimation and computational abilities

The study will involve students in co-educational settings. Thus, it will be possible to find out whether there is any gender difference in estimation and computational abilities.

Research Questions

Related to above aims, the research questions are divided in two parts: Primary part with one main question and the secondary part with five sub-questions. The data collected are expected to answer these questions accordingly.

<u>Primary question</u>

What is the relationship between the estimation skill and computation ability of students in Years 5, 7 and 9 for whole and rational numbers?

<u>Secondary questions</u>

- What is the correlation between computation and estimation skills in Year 5, 7, and 9?
- What development is there in computational ability and estimation skill over Years 5, 7 and 9?
- How are performances in computation and estimation related to one another in each of the topics (whole numbers, fractions, decimals, percentage and ratios)?

- What disparities are there between estimation skill and computational ability?
- Are there any genders differences in performing estimation and computation?

Summary

As a whole, the main intention of my study is not concerned with how fast children can compute a mathematical problem but how they make sense out of computation. Having said that, I would like to find out whether estimation skills could help children compute mathematical problems of the chosen topics effectively and more meaningfully. For instance, the study should be able to depict the capability of students in estimation and how it helps in making sense in their computations.

CHAPTER II

LITERATURE REVIEW

Introduction

In this chapter, some of the main points related to the research topic and their importance in the teaching and learning of mathematics will be discussed. The main point of discussion will be on estimation and its importance in learning mathematics. However, the chapter covers the importance of estimation and computation and their relationship to number sense. The idea behind including number sense is to show its importance in learning mathematics with understanding. So, in order to implant a strong sense of number in children, basic skills of computational estimation are urgently required. As it clearly pointed out by Edwards (1984, p. 60), "the justification for teaching computational estimation lies in the need to develop 'number sense'". Each topic is discussed in brief separately first and then later in paired relationships as follows: estimation and number sense, computation and number sense, and estimation and computation.

What is Estimation?

According to Micklo (1999, p. 142), estimation is nothing more than quickly and reasonably developing an idea about the quantity or size of something without actually counting or measuring it. To be more precise, estimating as per Lang (2001, p. 462) "is the process of thinking about 'how many' or 'how much' problem and possible solutions". As such, Micklo (1999) has concluded that estimation is a method of thinking that is used to solve real problems, rather than a wild guess. He has also pointed out that "to make a guess you do not have to think about how many there are. Any number can be a guess. To make an estimate you have to think" (p. 142).

Importance of Estimation

Estimation is more or less recognised as one of the important parts of learning mathematics. Estimation is used widely in day-to-day life activities. There are not many events where estimation is not implemented. In fact, it is representative of the type of mathematical skill that is widely applied by adults in daily living situations and thus likely to represent a general outcome of school mathematics curricula (Foegen & Deno, 2001). Not only that, according to Reys (1992), it is noted that "estimation is a basic skill, and its growing importance in a technological society is recognised. It is used much more than exact computation" (p. 281). The same point is given so much importance by the National Research Council (1989), which stated that "in today's society, changes in technology have made estimation skills more important than ever in the development of mathematical power", cited in (Gulley, 1998, p. 324). Similarly, Usiskin (1986, p. 9) argues that:

> ...even with calculators and computers taking the work out of computation, estimating may make things a lot easier with no important loss in the quality of the answers. In fact, answers derived using suitable estimates may be more reasonable and more realistic than those that attempt to be exact.

Thus, the greatest reward of an extensive estimating program can be the greatly enriched preparation for meaningful learning (Carlow, 1986). As such, it is very important that the children should be exposed to skills like these so that in devising their estimates, students have gained enough to develop sound problem solving and sense-making skills. As said by Woodcock (1986, p. 115), "it is very important for students to learn to estimate so they will spot careless errors and be able to answer the critical question, 'is my answer reasonable?'" Moreover, according to May (1994, p. 24):

> It is difficult to imagine anyone functioning effectively in the real world without being able to estimate measurements. How high is it? How much does it weigh? How long will it take? Questions like these are asked in all kinds of everyday situations.

Despite that, the teaching of estimation is a relatively recent phenomenon in the long history of mathematics education (Hanson & Hogan, 2000). Until recently, curriculum developers have not given much importance to estimation skills. In fact, it is noted that estimation is one of the most neglected skills in the mathematics curriculum (Carpenter, Coburn, Reys & Wilson, 1976). As such, the topic has not received as much attention as other mathematical skills and abilities, although people make use of it without being an aware of it. Only lately, its importance is gaining recognition in the world of mathematics. As such, many of the mathematics educators have been struggling to bring this topic into the limelight, and thus more research is being undertaken.

As discussed above, the integration of estimation skill in the mathematics curriculum has become very urgent. It is becoming a part of learning mathematics in many countries, including in Australia. There is a strong need for it to be a significant part of the mathematics curriculum of all countries. It should be so in order to help the learners to see meaning in learning mathematics both inside and outside the classroom.

Otherwise, till now, in many schools, learning mathematics has remained a difficult subject for the learners. It is always taken as not something for their life but as a burden for them especially in getting promoted to the next grade, level or whatever. As pointed out by Micklo (1999, p. 142), "estimation, therefore, needs to be integrated into the entire mathematics curriculum, and not be taught as stand-alone concepts". The same point is strongly supported by Harte and Glover (1993, p. 76):

> Estimation can be integrated into any mathematics content and bridged into any curriculum area with a little creative planning. Students quickly become much more aware of mathematical relationships and more sophisticated in their thinking.

Therefore, a need for that is mainly because estimation is crucial to becoming a good problem solver. Being able to solve problems successfully in life is one of the key aims of mathematical education.

Estimation in Learning Mathematics

Many people seem to view estimation as somehow foreign to the mainstream of mathematics and overlook the skills of estimation. They tend to think that there is no reason to estimate when they can work out the answer exactly. However, I would like to differ on this opinion and agree with Usiskin (1986, p. 3) who has listed that estimation is necessary when:

- an exact value is known but for some reason an estimate is used (e.g., the estimate 1.732 for a square root of 3);
- an exact value is possible but is not known and an estimate is used (e.g., the age of an old sequoia tree before it is chopped down); and
- an exact value is impossible (e.g. the estimated life of a bulb).

As indicated above, there are hardly any activities in life, where the concept of estimation is not involved. In fact, our daily lives are filled with situations that require estimation. For instance, in comparing prices at a store, changing the amounts of ingredients used in a recipe, determining the best routes when driving, and verifying calculator computations (Micklo, 1999). Moreover, as McIntosh (1992) has suggested, over 80% of all mathematical applications use estimation instead of exact computation.

Yet, many people are not aware of the fact that estimation provides a framework for judging the reasonableness of answers, whether done with pencil and paper or on a calculator (Ritchhart, 1994). Moreover, being able to estimate and decide the type of answer needed for a problem is an important part of mathematical thinking as argued by Reys (1985, p. 41):

> Every component of estimation-deciding on the type of answer required, choosing and carrying out appropriate strategies, and checking reasonableness of the answer-reflects the kind of high level thinking that is associated with problem solving and mathematical thinking.

Furthermore, as said by Usiskin (1986, p. 2), " the uses of estimation fit the ideals of mathematics, namely, clarity in thinking and discourse, facility in dealing with problems, and consistency in the application of procedures". Hence, without such knowledge, children fail to understand and solve the mathematical problem meaningful and effectively. According to the National Council of Teachers of Mathematics (1989), there are five goals for students in learning mathematics. They are as follows:

- value mathematics;
- become confident in their ability;
- become a math solver;
- learn to communicate mathematically; and
- learn to reason.

Having the ability to estimate can help students reach all these goals in learning mathematics. Knowing when and how to estimate provides students with tools and strategies to solve problems. Being able to reason and communicate mathematically improves students' confidence. Having these qualities would help them to value mathematics as a distinct way of thinking and not as a collection of unconnected rules and formulas (Micklo, 1999). To add to this, Trafton (1978, pp. 199-200) has summarised a version of those goals into three important points of how estimation contributes to the mathematics curriculum as it:

- can bring a new dimension and vitality to the study of computation;
- enhances the development of qualitative thinking; and
- develops problem-solving skill.

Not only that, as per Clarke, Lovitt and Stephens (1990, p. 175) state, "estimation tasks, if carefully introduced by teachers, are one way of breaking down students' fear of failure in mathematics". They also argue that well presented estimation activities can provide teachers with a link from traditional teacher-owned lessons to active mathematics learning owned by the students.

Status of Computation in the Current Curricula

Computation has long been the driving force of the school mathematics curriculum at all levels and is often viewed as the key purpose for learning mathematics (Rathmell & Trafton, 1990). It is thought to be a kind of method/procedure followed in order to solve a particular problem. Hence, many individuals believe that the word computation means using paper and pencil algorithms, a set series of written steps to get the correct answer. As such, people are made to believe that mathematics is being about getting right answers rather than about clear creative thinking (Payne, 1990). In fact, Payne has also stated that, "the rules and procedures of mathematics are too often learned without any real understanding"(p. 2). However, as pointed out by Rathmell et al (1990, p. 171):

> Curricular demands no longer permit teaching with minimal understanding...procedures that children have memorised without understanding do not further the development of number sense, the ability to judge the reasonableness of results, a flexibility in thinking with numbers, or a comprehensive view of computation.

Similarly, Hamrick and William (1978) support the concept that learning the process of computation combined with the skills of estimation and approximation is useful in terms of readiness for future learning. Moreover, according to Coburn (1989), "the role of the computation in the mathematics curriculum is to furnish the individual with useful skills and to facilitate further learning in both mathematics and related disciplines" (pp. 52-53). Jones, Kershaw, and Sparrow (1994) support the same point:

> Children must be allowed to decide what computational methods meet the demands of the tasks in which they are engaged. This means that children must feel confident in using a range of methods (such as the calculator, computer, and paper and pencil). The teacher's responsibility is to provide suitable mathematical experiences, which offer children choice and support personal inventiveness. (p. 56)

As such, one of the primary understandings in computation involves knowing which operation to perform and deciding which calculator button to push. Side by side, a child should be taught to check whether the computed answer is reasonable or not. Having to do this requires more thinking than what is needed for the rote manipulations of a paper and pencil algorithms (Coburn, 1989). As such, effective computation is something in which one requires to decide how accurate the results need to be, what and how operations are to be performed and finally whether a derived answer makes any sense or not (Wills, 1990). Likewise, Swan and Bana (1998, p. 580) argues that:

> When faced with a mathematical problem, a person must at some point determine whether or not a calculation is required. Given that calculation is required, the problem solver must then determine whether an exact or only an approximate answer is needed.

An approximation or estimate is needed here as the part of the process of finding answers, since estimating is a valuable way of checking the computation (Rathmell & Trafton, 1990). Thus, the problem solver should be able to decide accordingly and proceed further in solving the given problem appropriately. As argued by Rathmell et al (1990, p. 171), "decisions about computing encourage reflection on the problem and the computation involved". As such, children should be provided with an opportunity to decide what computational methods meet the demands of the tasks in which they are engaged. This means that children should feel confident in using a range of methods and tools (Jones, Kershaw & Sparrow, 1994). A model of the computation process is given in Figure 2.

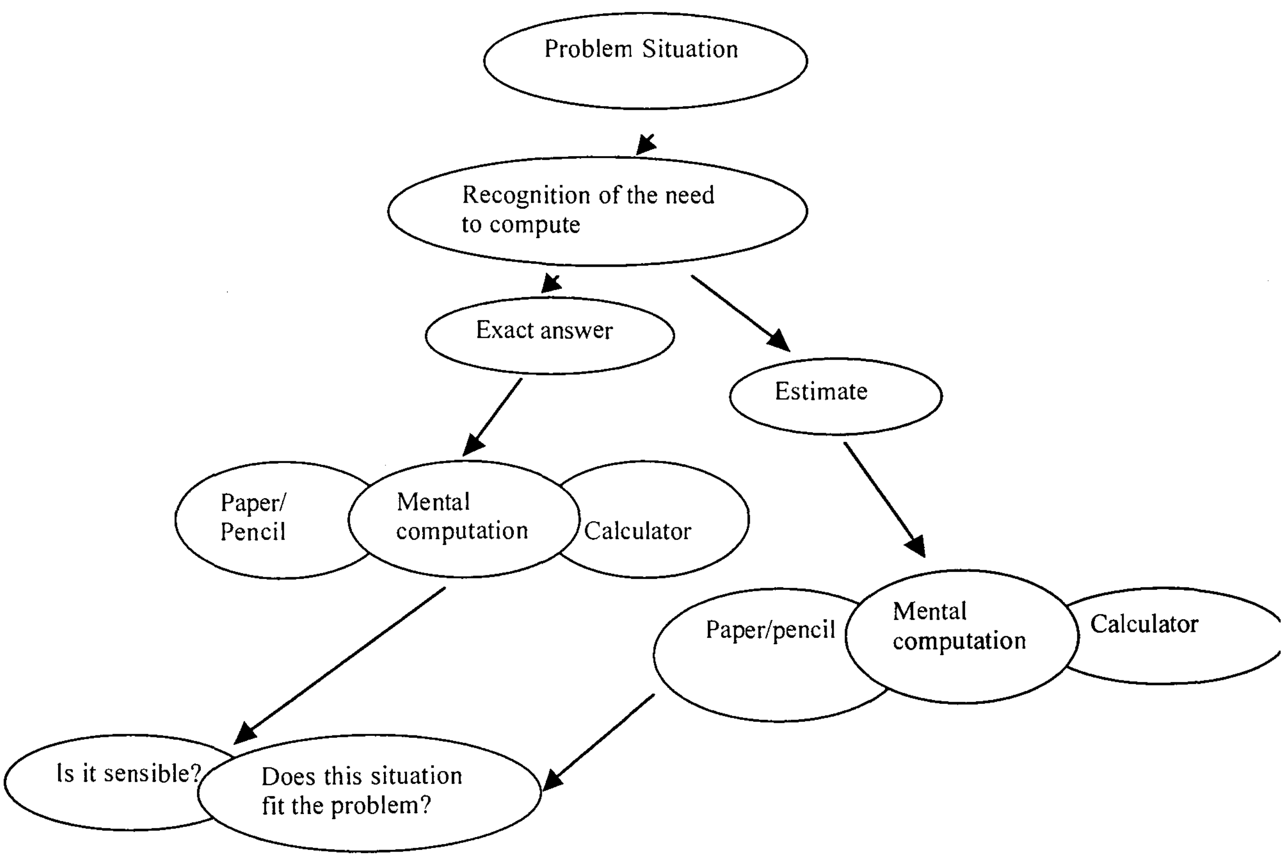

Figure: 2 An overview of computation (NCTM, 1989) cited in Rathmell & Trafton (1990, p. 153)

As given in Figure 2, the study of computation should promote a meaningful and understanding range of learning in the world of mathematics. Hence, the thrust of current curriculum reform should not reduce the importance of computation but rather, should broaden the concept of computation and encourage the importance of problem solving (Coburn, 1989). Thus, it should be able to make children active participants in creating knowledge rather than becoming passive receivers of rules and procedures. Doing that implants in children a belief that learning mathematics is a sense-making experience (National Research Council, 1989).

Number Sense in Learning Mathematics

One of the key objectives of the elementary school mathematics curriculum is to instil in students a basic understanding of the number system (Leutzinger & Berthean, 1989). In simple terms, number sense means sense making of mathematics (McIntosh, Reys, Reys, Bana, & Farrell, 1997). It involves the formation of relationships between numbers and an understanding of their relative magnitudes. Children who have acquired a good number sense should have understood number meanings, developed many relationships among numbers, recognised the relative magnitudes and the relative effect of operations on numbers. It is rather the theme of learning mathematics as a sense-making activity (NCTM, 1989).

Similarly, Reys and Yang (1998, p. 226) support that "number sense refers to a person's general understanding of number and operations". For example, as suggested by Sowder (1992), Greenes, Schulman and Spungin (1993), and Ritchhart (1994), children should be able to recognise that:

- six is simultaneously half a dozen, four less than ten;
- the difference between 5 and 9 is the same as the difference between 635 and 639;
- 1000 marbles wouldn't fit in the jam jar - the reasonableness of the magnitude of the number in relation to the context;
- items costing 85c and $1.05 respectively are each close to $1 and so the total will be about $2.00 using estimation to check reasonableness of result; and

- 73 – 29 will produce the same results as 74 – 30, that is, when a "1" was added to 73, it also had to be added to 29 to maintain the same difference between the number-number relationship.

As such, number sense exhibits itself in various ways as the learner engages in mathematical thinking, including awareness of various levels of accuracy and sensitive for the reasonableness of computations (McIntosh, Reys, Reys, Bana, & Farrell, 1997). Making children understand numbers is very important if they are to make sense of the ways numbers are used in their everyday world (NCTM, 1989). Therefore, one of the most important tasks for mathematics teachers would be to help the learners to achieve a good number sense so that children can have a strong foundation in learning mathematics as a whole.

Estimation and Number Sense

My understanding of estimation is that, one can estimate only if he/she can make sense of the mathematical problem presented. As Schoen, Bean, and Ziebarth (1996, p. vii) point out, "estimating aids in concept development, but at the same time a solid conceptual understanding improves one's ability to make good estimates". An important by-product of learning to estimate is better conceptual understanding (Schoen et al., 1996). Further, as pointed out by Rathmell and Trafton (1990, p. 156), "estimations help children develop confidence in their ability to reason with numbers and provide a base for making judgements about the reasonableness of results". That can be done either with pencil and paper, or on a calculator (Ritchhart, 1994).

Likewise, children rich in number sense can engage in any form of computation successfully as they would be able to understand the problem and carry out the process using the right kind of method accordingly. Many reports and studies (Burton, 1993; Case, 1989; Edwards, 1984; Greenness, Schulman & Spungin, 1993; Greenness, 1991; Greenos, 1989; Hiebert, 1989; Markovits & Sowder, 1994; Macintosh, Reys, & Reys, 1992; National Council of Teachers of Mathematics, 1989; Resnick, 1989; Reys et al., 1991; Sowder, 1989, 1992a; 1992b; Treffers, 1991; Van de Walle & Watkins, 1993) are cited in Yang (1995, p. 6), who states that:

> Computational estimation plays an important role in the development of number sense. Pooring (weakness) in performance of computational estimation may reveal a lack of number sense. However, there is no research evidence, which correlates number sense and ability at paper and pencil computation.

Otherwise, the lack of number sense tends to present insurmountable barriers to learning mathematics. For instance, if a child fails to understand that 1.50 is a representation of 1.5 and $^3/_4$ is less than 1, then that particular child will have to remember a host of rules in order to deal practically with everyday numerical situations. Owing to the above-mentioned points, for many children, learning of mathematics appears to offer no other way than to learn it by rote. For them, it is something that is needed only to get the required answers or marks to get through the exam.

One of the main problems, which could be foreseen here, is lack of making sense of what they do or learn. For example, in one of my lessons (Year 5), I gave an activity on addition of whole numbers (234 + 456) and asked them to estimate the answer without using pen and paper. To my surprise, the answer given was 6810. I realised immediately that the children could remember vaguely some procedure for solving the problem mechanically using the standard algorithm but lacked number sense, as they were not able to judge whether the number they added was worth a thousand or not. Such an example demonstrates that many children are not used to working with numbers and relationships but with digits (Hope, 1986). As such, Carpenter and his colleagues (1976) cited in Sowder (1992, p. 381) concluded, "before students can estimate well, they must develop a quantitative intuition (number sense), a feel for quantities represented by numbers".

Similarly, findings of Macintosh, Reys, and Reys (1997, p. 73) stated that in estimating 24 x 0.98 from the choices "more than 24", "less than 24", and "impossible to tell without working it out", over sixty percent of grade eight students incorrectly responded to this question. Another example is when 13-year-old children in U.S. were asked to estimate the sum of $^{12}/_{13}$ and $^{7}/_{8}$, given the choices of 1, 2, 19, 21, and "I don't know", over fifty percent incorrectly answered 19 or 21 (Carpenter, Corbitt, Kepner, Lindquist & Reys, 1980; McIntosh, Reys, Reys, Bana & Farrell, 1997).

The findings stated above clearly reveal children's lack of understanding of number sense, operations and computations. As such, a lack of number sense could be one of the main reasons for children lacking a clear visualisation of mathematical problems and the application of estimations. However, as stated by Sowder (1992, p. 382), "estimation and mental computation are not only useful tools in everyday life but they can also lead to better number sense". Thus, although one could solve a mathematical problem correctly, one could not explain how he/she has done it. It is so because students are more often encouraged to follow and memorise the rules and symbols rather than making sense of the numerical situations (Yang, 1995). In order to break this prevailing notion of students, many mathematics educators have undertaken research and have come out with numerous evidence of how estimation plays an important role in the development of number sense (Campbell & Clements, 1990). The point is well supported by Poulter and Haylock (1988, p. 28), who stated that:

> Time given to teaching estimation will pay considerable dividend. Not only do pupils acquire genuinely useful skills particularly if estimation is taught in applied contexts but also in our experience they become more adept at reasoning with numbers, more flexible in their thinking, more aware of the relationship between different operations and generally develop a greater feel for number.

Thus, students' number sense is enhanced when they are encouraged to use numbers in real life situations and is forced to estimate quantities in different mathematical settings (Welchman, 1999). The same point is very well supported by Lang (2001) that by offering rich opportunities in estimating number, varying the contexts, and using appropriate questioning techniques, teachers can help children develop the foundation necessary to build better in number sense. As such, the practice of estimating is a useful subsidiary skill for developing number competence and confidence (Duffin, 1999).

Number Sense and Computation

Ever since I started teaching mathematics in schools, I have always come across children who could solve mathematical problems mechanically using standard algorithms but could not explain why or how if someone asked them. The only possible explanations the children could give were:

- *I checked the answers given at the back of the textbook;*
- *I followed examples given by the teacher; or*
- *My teacher told me to do it this way.*

The above statements reveal that children are always exposed to a mechanical kind of learning where they fail to get the real meaning of what they do in the mathematics classroom. Instead, they seem to be blindly guided by mathematical terms and rules without any understanding about what it actually means. For example, a bus holds 22 children, how many buses are needed to take 121 children for a picnic? A common answer is $5^1/_2$, which is not applicable in a real situation like the one mentioned above. Therefore, computation involves not only applying arithmetic rules but also considering the context in which the numbers are being used (Ritchhart, 1994). Similarly, Sowder (1988, p. 227) has pointed out that "students should not only learn how to calculate an exact answer, but develop a better understanding of number meanings and understanding relationships between numbers and operations".

Unfortunately, in the current practice many children are led to rely solely on procedures and cannot themselves judge whether their answers are reasonable or not. Perhaps, this is because of certain situations as pointed out by Ritchhart, (1994, p. 5):

> In many classrooms, students are not given the opportunity to construct their own meaning based on personal experiences. Much of students' early work in mathematics concentrates on developing computational skills rather than on rich activities that teaches them.

As such, children are left in the dark without knowing where they are heading and what they have done and why they have done it. It is like a person with weak arms trying to climb over a cliff. In other words, 'cliff' represents the world of mathematics and 'weak arms' refers to learning mechanically using standard algorithms. It is so because, at any time there is a chance of misleading them to wrong concepts of mathematics as they lack a strong foundation of number sense. It is like having weak arms and not being able to grasp the cliff firmly.

The same point is also stressed by Swan (1990, p. 70), as "the facts and skills that are taught mechanically using traditional approaches are often quickly forgotten precisely because there is no conceptual foundation." It could be so, as "the knowledge of rote procedures interferes with students' attempts to construct meaningful algorithms" (Mack cited in Yang, 1995, p. 30). As discussed earlier, children are often asked to follow the rules and procedures without conceptual understanding of the same. Doing that, one leads children to learn methods by ways of memorisation and little understanding. Thus, much of the current attention on developing number sense is a reaction to over emphasis on computational procedures that are often algorithmic and lack number sense (Reys & Yang, 1998). The following quotation from Jones, Kershaw, and Sparrow (1994) indicates the difference between computational estimation with and without number sense:

> Consider the problem of finding the difference between 1.9 and 3.6. A child who demonstrated number sense ability said that the solution would be about 1.5. She mentally made the 1.9 up to 2.0, said the difference was now 1.6, added on the 0.1 and gave an answer of 1.7. Another child when presented with the same problem said she had a mental picture of the 1.9 sitting below the 3.6 with the decimal points lined up. She then proceeded to explain how she had used the decomposition method of subtraction to arrive at a solution of 1.7. Both girls provided a correct solution but the second girl did not show as flexible an understanding of numbers and their relationships as evidenced by her method of checking her answer. (pp. 29-30)

The way in which the first girl solved the problem illustrates how number sense and estimation were used together, quickly and successfully. Such a formal and rigid kind of procedure as used by the second girl above, suggests that children are enslaved by a technique and never exposed to any other alternative methods of solution.

Robitaille (in Hope, 1986, p. 50) reported similar conclusions about the apparent inability of students to reason with numbers:

> Although students perform satisfactorily on computational skill items, results are weaker in areas involving what might be termed 'numeracy'. Computation is seen by most children and adults as a way of getting a correct answer, whether the answer makes sense or not is of little concern to the majority of users. (p. 2)

Likewise, there is a lot of evidence which says that students with excellent results on traditional paper and pencil tests can also show surprising weakness in number sense (Ekenstam & Greger, 1982; Sowder, 1992; Yang, 1995). Macintosh (1990, p. 25) stated that:

> Mechanical computation is now an anachronism and is surely worth the struggle to replace it with a more relevant alternative, one in which children are enabled to increase their ability and confidence in selecting and using the most appropriate form of computation.

There could be so many other reasons for doing so, but I support some of the of reasons suggested by Reys (1984), Hope (1986), Jones (1988), and Sowder and Sowder (1989) cited in Jones, et al. (1994, p. 23) against concentrating solely on standard algorithms (mechanical learning) in teaching mathematics. The reasons they highlighted were:

- Children spend time practising the methods rather than developing an understanding of the mathematics needed to solve problems;
- Little understanding of the number system and number properties is gained and number relationships are not used;
- By emphasising standard procedures of written algorithms, ability to create mental strategies may be hampered; and
- Reasonableness of solutions is not checked and children seem to believe that solutions reached in this way are correct.

Hence, it is very important that children be allowed to use their knowledge of number sense and invent algorithms to arrive at a quick and accurate solution (Hope, 1986).

Computation and Estimation

There seems to be much controversy over computation and estimation and their importance in teaching and learning mathematics. To date, most people believe that mathematics means calculation and getting an exact answer to a given problem. As such, it leads them to believe that estimation is a weak sister to exact computation. In fact estimation is quite often considered to be the stronger sister (Usiskin, 1986). He also stressed that obsessions with exact answers lead children to make unnecessary calculations and keep them from gaining experience and confidence in estimation judgements. Such an idea can also kill intuition and reinforce the false notion that exactness is always to be preferred to estimation.

Therefore, the idea of estimations should be adopted formally to enable children to have the opportunity to use their skills in approaching mathematical problem and compute it successfully. Moreover, since the emphasis in teaching mathematics is more towards the understanding of the underlying structure of the operations, the teaching of estimation skills becomes even more important in the process of computation (Poulter & Haylock, 1988). As Trafton in McIntosh, De Nardi and Swan (1994, p. 83) has pointed out:

> Estimation, mental computation, and calculators need to be accepted as legitimate computational methods. Students often feel that the estimations and mental-computation strategies they develop on their own must be kept from teachers because their use would not be considered "proper".

Moreover, some findings say that, being good at estimation can make computation easier as a person would be in a position to change the numbers in some way to make calculations easier. Such research evidence suggests that developing skills in estimation prior to paper and pencil computation is both effective and powerful. Moreover, simple cases of estimation with a particular operation precede related written computational procedure for obtaining exact answers. In the process, there is a chance for the students to acquire more of a number sense prior to the use of formal written computation (Coburn, 1989).

The same point is also argued by Trafton (1978, p. 205) "estimation brings a new dimension and vitality to the study of computation...particularly in upper grades, where students review familiar skills and focus on more complex levels of computation". Otherwise, children would concentrate only on procedures and avoid understanding of the algorithms and how they can be used (Rathmell & Trafton, 1990). They have also suggested that estimation can encourage children to think about computation in a more holistic manner than with paper and pencil algorithms. Having said that, it is a must for the children to be made aware of the importance of estimation so that they are able to decide on their own the methods, tools to choose for calculating, and to judge the reasonableness of their results. As Miller (1993) has mentioned:

> The fact that many everyday situations call for an estimate leaves little doubt that some degree of proficiency in making guesses should be expected of students at all ability levels. Students should be able to make a quick mental estimate to decide whether a written or calculator answer is reasonable. (p. 1)

Summary

As discussed above, the integration of estimation skills in learning mathematics is found to be very useful in developing number sense in children. Having strong number sense in children should help them to understand the mathematics problem better, and thus it can ease the computation. The following chapter will discuss the methodology of the investigation of the relationship between estimation skill and computation ability.

CHAPTER III

METHODOLOGY

Introduction

The purpose of this chapter is to describe the research methodology or system of methods and principles used in this particular study. In research, "methods" means a range of techniques used to gather data for analysis and interpretation with respect to the research questions of the study. This chapter is divided into five parts. The first discusses the design of the study in which the present study is grounded. The sample used for the study is discussed in the second part. The instruments developed for collecting the data are presented in the third part. The fourth part describes the procedure used by the researcher in collecting the required data. The fifth part concludes with a brief summary of the whole chapter.

Design of the Study

Methods adopted for this study were both quantitative and qualitative. Quantitative methods were used for the group-administered tests conducted with four selected schools in the Metropolitan area of Perth. Qualitative methods were used for the interviews conducted with the school children of those selected schools. As such, instruments employed in this study were written tests and interviews. The purpose of using these two instruments was to ensure that the data gathering encompassed more than one technique. Burgess (1996) explains how one method contributes to the other and vice versa on the phases of design, data collection and analysis.

The purpose of conducting written tests was to find out whether the students' skills in estimation were related to their computational performance. As such two parallel forms of test items were developed. The interview was mainly to investigate the feedback from the students in the given tests. It was to find out whether children used their estimation skills while explaining the procedures of their workings. The purpose of interview with the students is seen in allowing the participants to express their views and opinions freely, since there was no opportunity to do so in the written test. As Ritchhart (1994) has stated, interviews are extremely valuable tools for gathering information about students' understanding of mathematical ideas and it is said to be one of the best sources of information. Moreover, this instrument has a function of not only getting the honest view but can triangulate data gathering with other means such as the written tests.

As such, an interview of the students was carried out with a sample selected from those who completed the written tests. The type of interview used in this study was based on non-structured questions, where a researcher may not have a set questionnaire, but only a number of key points around which to build the interview (Appendix E). It was individually administered. The questions were open-ended with specific intent, allowing individual responses. However, the researcher had to exercise a certain degree of intervention at times when the interaction deviated from the topic.

The mathematics curriculum documents were consulted to draw items for those selected levels. The items were based on the current curriculum practised in schools in Perth and also keeping in mind the situation in my country of Bhutan (syllabus for IV to X). This was done so that the test conducted would be based on the topics taught in those selected schools. The students were not allowed to use a calculator, as it would not force children to think and use their number sense in estimation. In the same way, it would not allow the children to use their computation skills in solving problems. The specifics of the schools and number of students involved are discussed below:

Sample

The study being at an exploratory stage was not designed to have a large representative sample. Furthermore given the limited time for data collection, the researcher had to decide to sample from schools to which she had ready access. Hence, the sampling technique adopted was that of a purposeful one. Wiersma (1991) offers the strength of the method:

> ...the researcher essentially selects available units to meet the purpose of the research study. Such sampling goes by a variety of names: judgmental, purposive, or purposeful. The selections of the units must be based on prior, identified criteria for inclusion. Such sampling is not haphazard. Researchers must be knowledgeable about the characteristics of the units, such as variability and the existence of extreme cases. Units, whether sites or individuals, are selected because of the information they can provide relevant to the research problem. (p. 265)

Thus, keeping the above points in mind, data were gathered from four schools in the Perth Metropolitan area. The four schools were comprised of a high school and three primary (feeder) schools. The samples included students from those four selected schools. The three primary schools served as feeders to the selected high school. The reason for selecting a secondary school together of its major "feeder" primary schools (K-7) was to enable more meaningful between-year comparison to be made (Mcintosh, Bana & Farrell, 1995). Moreover, there can be a continuation of the smooth flow of the standard of Years 5 and 7 from the primary school to Year 9 in secondary school, as the year levels selected for this study were 5, 7 and 9.

Thus a child who studied in one of those feeder schools would generally go to that high school later on, so that Year 9 students should have similar backgrounds and ability. It was therefore assumed to be appropriate for the researcher to compare the results in estimation and computation abilities across year level and age. Hence, one can check and investigate the development of the skills and concepts from Year 5 to Year 9.

Within each selected primary school, one class each was randomly selected at each of the year levels 5 and 7. Students in all classes were heterogeneously grouped, as is the custom in most Australian primary schools (Mcintosh, et. al.). In the secondary school where students were streamed on ability, as is case in many Australian secondary schools, stratified sampling was used to select three classes of Year 9 roughly representing students from those associated primary schools selected for the study. The total numbers of subjects involved were 91, 77 and 73 in Years 5, 7 and 9 respectively. More information on this is given in Table 2.

Table 2: Number of students tested in each year level and school

School	Year 5	Year 7	Year 9	Total
High School	NA	NA	73	73
Primary School A	32	25	NA	57
Primary School B	29	24	NA	53
Primary School C	30	28	NA	58
Total	91	77	73	241

Those 241 students were from nine classrooms – three classes in each of Year 5, Year 7 and Year 9. The class sizes are shown in Table 2, except for Year 9, which was comprised of three classes of 16, 28 and 29.

Instruments

Two different instruments developed for the study were the group-administered written tests and interview. The group-administered tests included two parallel sets of items but in different forms. Each of the instruments is discussed in the following section.

Written Tests

As mentioned earlier, the written test is of two forms. These are estimation and computation, so both tests contain the same items but in two different forms. The first test required estimated answers whereas the second part required the exact answer with its working procedures. The number of questions differed in each of the year levels as per the topic coverage and student abilities. Accordingly, the time taken for each question also differs for each year level.

Since the Estimation Test (ET) items, were identical to those in the Written Computation Test (WCT), both the tests includes a total of 45 items each with 10 items for Year 5, 15 for Year 7, and 20 for Year 9. A sample of matching pairs for computational ability and estimation skill test items are given in Table 3:

Table 3: Examples of matching estimation and computation items

Estimation	Computation
Without calculating the exact answer, circle the best estimate for: $^5/_6 + ^8/_9$ a. 1 b. 2 c. 13 d. 15	Calculate: $^5/_6 + ^8/_9$
Without calculating the exact answer, circle the best estimate for: 29 x 0.98 a. A little less than 29 b. A little more than 29 c. A lot less than 29 d. A lot more than 29	Calculate: 29 x 0.98
Without calculating the exact answer, circle the best estimate for: 54 ÷ 0.09 a. A little less than 54 b. A little more than 54 c. A lot less than 54 d. A lot more than 54	Calculate: 54 ÷ 0.09

Each matched pair of items was not presented side by side but in two different tests one following the other. The children were given the estimation test first and then given the questions on computation. The estimation test consisted wholly of multiple-choice items, with four possible answers for each question. The children were expected to estimate and choose the nearest possible answer from those four. As shown in Table 3, the stem of all estimation items was "Without calculating the exact answer, circle the best estimate for:". The questions for computation required children to calculate the correct answer using any method they liked. Enough space was provided with each question for the children to show their chosen procedures. The stem of the computation items was "Calculate:". The test items for the different year levels are given in Appendix C and Appendix D.

Out of twenty-six different test items, ten items were taken from a number sense test used by McIntosh, Reys, Reys, Bana and Farrell (1997). In those previous researches, test items extracted were mainly used to test the number sense in children of ages 12 and 14. The pattern and the style of tests conducted by Yang (1995) were also similar to this study, with two sets of parallel questions but in different forms–Number Sense Test and Computation Tests. The only difference with theirs was investigating number sense with computation, whereas the current one is to investigate the relationship between estimation and computation. Those ten borrowed questions were spread all over the chosen levels based on the commonly taught topics. The spreads of questions are shown in Table 4. This was done to ensure that development of skills through the year levels could be investigated.

Table 4: Items used in previous research

Items from Previous Research	Year 5	Year 7	Year 9
9965 + 8972 + 8138 + 8090	✔	✔	✔
18 x 19	✔	✔	✔
96.7 + 147.4 + 62.75 + 36.8	✔	✔	✔
0.72 – 0.009	✔	✔	✔
0.5 x 840		✔	
87 x 0.09		✔	✔
54 ÷ 0.09		✔	✔
$^{7}/_{8} + ^{12}/_{13}$			✔
19.4 x 46.1			✔
563.7 ÷ 2.93			✔
Total	4	7	9

The first four questions were spread over these three selected levels, whereas the sixth and seventh items were only for Years 7 and 9 as these questions were too difficult for the Year 5 students. The rest of the items were for Year 9, except an item

5, which was only for Year 7. The researcher constructed the remaining items to cover the designated topics and a specialist in a mathematics education checked them.

In general, some of the test items were the same all the way through the levels. Those items were mainly from topics that were covered in all the levels. For instance, questions from topics like whole numbers and the first two operations of decimals as shown in Table 5. As discussed before, the topics covered for the test items were whole and rational numbers (decimals, fractions, percentage and ratios).

Table 5: Estimation and computation test item distribution

Number	Operation	Year 5	Year 7	Year 9
Whole Numbers	Addition	2	1	1
	Subtraction	2	1	1
	Multiplication	2	2	2
	Division	1	1	1
Fractions	Addition		1	1
	Subtraction		1	1
	Multiplication		2	2
	Division			1
Decimals	Addition	2	2	1
	Subtraction	1	1	1
	Multiplication		2	2
	Division		1	2
Percentages				2
Ratios				2
Total		10	15	20

Interview

This section discusses the content and purpose of the interview schedule. In this study, the interview was focused on the way the students performed in their test. It was intended to provide additional perspective on the estimation skill possessed by slightly above and slightly below average students. To facilitate this purpose, an interview instrument was created, and interview data were collected from the selected students in Year 5, 7 and 9. Two students from each class were selected from the four schools with the help of the class teachers. Selection for an interview was based on children's performances and the relevant teacher's opinion of that child. As such, criteria were based on those of slightly above average and slightly below average abilities in each class. Therefore, in total, each year had three slightly above average and three slightly below average interviewees for a total of 18.

Procedures

As discussed earlier, one secondary and three primary schools were requested to participate in the research. Permission was sought from the principals, respective teachers and the parents of the concerned children (Appendix A). All the formalities were completed by the end of April 2001. The written test was conducted in the first two weeks of May 2001. Time allotment for the two tests together was about 50 to 60 minutes per year level. The estimation test was conducted first in every year level, immediately followed by the computation test.

Interviews with the students were completed by June 2001. There was a gap of one to two weeks' duration between the test and the interview. The main reason for the short duration was to keep afresh in the children's memory what they did in the test for the follow-up interviews. Otherwise, children might not be able to remember what or how they did in their test and relate it to the questions at interview. Besides, a week's duration was needed to finalise the correction of the test papers, select the interviewees and arrange schedules. The interviews were based on the performance of those selected students in the written tests.

The class teachers were requested to conduct both the tests with their classes. The reason behind that was that, the researcher wanted the test to be conducted in a normal situation, so that it would appear normal for the children and not get them distracted unnecessarily. The detailed procedure on how those tests and interviews were conducted is discussed below. This part is divided into three sections– procedures for the estimation test, the computation tests, and the interview.

The Conduct of the Estimation Test

Each class teacher was provided with a package, which contained both the test papers (Appendices C & D) and a sheet of instructions on how to conduct the tests (Appendix B). Time allotted for each question to estimate the answer was 30 seconds for all the items across the selected year levels (Appendix B). The teacher distributed the paper on estimation to the individual children. The children were provided with a blank paper each to cover their answer to avoid miss-conduct of copying from one another. The teacher read out every question one by one and gave 30 seconds each for the children to estimate the answer from the four given multiple–choice answers. Children were expected to estimate and choose the best within that time. The result sheets were collected as soon as the time was up.

The Conduct of the Computation Test

Immediately following the Estimation Test (ET), the Written Computation Test (WCT) was administered. The class teacher gave each student a copy of the WCT (Appendix D). The instructions on administration (Appendix B) were previously handed over to the teacher who conducted the test. Students were to work independently on the given questions during the allotted time.

Unlike the time allotment in estimation, the time allotted for each items on computation were four minutes each for Year 5, three minutes for Year 7 and two and half minutes for Year 9. The variation of time is basically due to the number of questions and their mental and computational abilities. Moreover, it had to be adjusted to the time limit allotted by the schools. The maximum time provided by the school was one period of 45 to 60 minutes per class depending on year level. Both the tests had to be completed within that given period. Therefore, items were set in such a way that the children were expected to finish the tests within that given time.

In order to help the children finish answering in time, the teacher was instructed to move students to the next question after each time allotment. Doing that the child could move on to the next items accordingly and come back later to the incomplete item if time permitted. The answer sheets were collected immediately after and handed over to the researcher. The correction of test papers was done immediately after the test. It was done that way so that the researcher would have the relevant results ready for the interviews. As mentioned earlier, the questions asked at interview were based on the performance of the children in each of the test questions. Therefore, the researcher had to sort out and select the test items to be covered in the interviews.

The Conduct of the Interview

Eighteen interviews were conducted within a period of almost a month (8th May to 1st June 2001). They took place a week or two later after conducting the tests. The researcher requested the class teacher to decide the time for interview that best suited his/her teaching convenience. In each of the three primary (feeder) schools, four students each were selected for individual interviews. There were two each from Year 5 and Year 7. Where as from the associated high school, there were three students from each of the two bigger classes. Individual interviews were conducted privately in the school interview room. The researcher reviewed each class program and designed an interview schedule that interfered as little as possible with the students' schedules. The order was similar to the test conducted, with estimation first followed by computation.

The researcher presented one item at a time. Items included for interview varied from child to child depending on his/her performance in the given tests. The number of questions asked ranged from nine to eighteen depending on how each student performed. The average of the questions asked was about 15 questions per student. The time taken for each interviewee was 15 to 20 minutes. The children were asked to explain some of their procedures for getting the solutions. They were asked to explain both the ways – why correct and why wrong in both the papers side by side. The type of pattern followed is given in interview schedule (Appendix E):

Such an opportunity was made possible for the children as they were allowed to use any method they liked in the test. Probes and additional follow-up questions were asked to gain a good understanding of the students' thinking. The interviews were audio recorded. The interviewer also recorded the response and explanations made by the students, by making limited field notes. The actual procedures are listed and given in Appendix E. In order to issue consistency in measuring the characteristics of estimation and computation, the researcher listened to each audiotape and recorded each response.

Scoring

Each of the two tests, estimation and computation, were scored according to the following points system. Since the ET was designed to elicit the use of estimation skill and investigate the correlation between estimation and number sense performance, correct answers scored 2 points. No point was scored if the answer was incorrect. Therefore, the total possible score of ET was 20 points for Year 5, 30 points for Year 7 and 40 points for Year 9.

The WCT was developed to find an exact answer, and to explore the correlation between the performance of written computation and estimation. If an item was correct in both answer and procedures, then that item was awarded two points. One point was awarded for the right procedure with the wrong answer. Similarly, one point was awarded for a correct response and wrong procedures. No credit was awarded for the wrong answer with wrong procedures. Therefore, the total possible scores were the same as the ET: 20 points for Year 5, 30 points for Year 7 and 40 points for Year 9.

Summary

The two tests (ET and WCT) were group-administered to a sample of 91, 77 and 73 students from Year 5, 7 and 9 respectively. Data from this administration were scored, coded, and entered into Excel and set for analysis. The interview data was collected from nine students of slightly above average and nine from slightly below average ability. All the interviews were tape-recorded. The results will be discussed in the following chapter.

CHAPTER IV

ANALYSIS OF RESULTS

Introduction

The purpose of this chapter is to analyse the data in relation to the original major research question:

What is the relationship between the estimation skill and computational ability of students in Years 5, 7 and 9 in relation to whole and rational numbers?

Associated with the main research question are the subsidiary questions outlined in the opening chapter of this thesis. Essentially the subsidiary questions focus on five purposes of this study. They are as repeated below:

- What is the correlation between computation and estimation skills in Year 5, 7 and 9?
- What development is there in computational ability and estimation skill in Year 5, 7 and 9?
- How are performances in computation and estimation related to one another in each of the topics (whole numbers, fractions, decimals, percentages and ratios)?
- What disparities are there between estimation and computational skills?
- Are there any genders differences in performing estimation and computation?

In order to answer the above stated questions, this chapter is divided into five sections. The first section presents the results of the correlation between estimation and computational abilities in Year 5, 7 and 9. The main purpose of this section is to summarise the group-administered test results in order to help answer the primary research question: *What is the relationship between the estimation skill and computation ability of students in Years 5, 7 and 9 in relation to whole and rational numbers?* The second section will be a brief presentation on development of estimation and computational abilities over Years 5, 7 and 9. In the third section, how the performances in computation and estimation are related to one another in each of the topics will be discussed. In the fourth section, disparities between estimation and computation skills are presented in brief, and the fifth section will examine gender-related differences on the two parallel tests. A brief closing summary will be presented after every section of the chapter.

The data analysis procedures took the following course. The quantitatively collected data for the two tests were coded, then analysed with the help of SPSS Microsoft. Using the same program, mean scores, t-test and correlation analyses were calculated. The entire group-administered test scores were input into an Excel database for analysis. The interview audiotapes were reviewed and transcribed by the researcher to collect and categorise qualitative data concerning students' computation and estimation strategies.

Correlation between Computation and Estimation Skills

The Pearson correlation coefficients between estimation skill and computational ability reveal that mathematics achievement scores correlate positively in all the Years 5, 7 and 9 and especially for Year 7. The mean score on each item is calculated in the parallel tests and the correlation between them is found accordingly. The details are shown below one by one in Tables 6–9.

Tables 6–9 present the details of correlation coefficients between estimation and computational abilities of Years 5, 7 and 9 along with the overall result. The result is highest in Year 7 with a positive correlation of $r = 0.74$, $p < 0.01$, and the lowest is Year 9 with $r = 0.44$, $p < 0.05$. The correlations are also significant for the overall results. As a whole, the results in the Tables 6–9 indicate that there is significant relationship between estimation skill and computational ability, although it is somewhat low in Year 9. The lower relationship in Year 9 is probably due to the greater reliance on calculator use at this level. As such, the results tend to indicate that a child with good estimation skills is more likely to perform well in written computation. Hence, there is support for the important point that children should have skills in estimation in order to compute mathematical problems with understanding.

Table 6: Correlation coefficient between estimation and computational abilities of Year 5

	Estimation Ability	Computational Ability
Estimation Ability:		
Pearson Correlation	1.00	0.73
Sig. (2-tailed)	0.00	0.02
Computational Ability:		
Pearson Correlation	0.73	1.00
Sig. (2-tailed)	0.02	0.00
N	10	10

*Correlation Coefficient→ ($r = 0.73$, $p < 0.05$)

Table 7: Correlation coefficient between estimation and computational abilities of Year 7

	Estimation Ability	Computational Ability
Estimation Ability:		
Pearson Correlation	1.00	0.74
Sig. (2-tailed)	0.00	0.01
Computational Ability:		
Pearson Correlation	0.74	1.00
Sig. (2-tailed)	0.01	0.00
N	15	15

*Correlation Coefficient → ($r = 0.74$, $p < 0.01$)

Table 8: Correlation coefficient between estimation and computational abilities of Year 9

	Estimation Ability	Computational Ability
Estimation Ability:		
Pearson Correlation	1.00	0.44
Sig. (2-tailed)	0.00	0.05
Computational Ability:		
Pearson Correlation	0.44	1.00
Sig. (2-tailed)	0.05	0.00
N	20	20

* Correlation Coefficient→ ($r = 0.44$, $p < 0.05$)

Table 9: Overall correlation coefficient between estimation and computational abilities on all 45 items for Year 5, 7 and 9

	Estimation Ability	Computational Ability
Estimation Ability:		
Pearson Correlation	1.00	0.56
Sig. (2-tailed)	0.00	0.00
Computational Ability:		
Pearson Correlation	0.56	1.00
N	45	45

* Correlation Coefficient→ ($r = 0.56$, $p < 0.01$)

Mean percentage scores on ET and WCT

Table 10 shows the mean percentage scores of correct responses for each level on both the parallel tests. The overall performance in estimation is highest in Year 5 and lowest in Year 9, whereas in computation Year 7 has scored the highest and the lowest is again in Year 9. Year 5 and Year 9 have done better in estimation than in computation, whereas Year 7's result is just the reverse.

Table 10: Mean percentage scores on estimation and computation

Year	Estimation	Computation
Year 5	41	35
Year 7	40	51
Year 9	36	31
Overall Means	38	39

Looking at the result, overall performance in estimation is almost the same as in computation. The difference in means is not significant as shown in Table 10. Likewise, the difference in means between estimation and computation is rather low in both Year 5 and Year 9. Moreover, performance in estimation is better than computation in those two year levels. However in Year 7, it is the other way round with performance in computation better than in estimation. The one possible reason could be that Year 7 was much more adapt at the rote-learned algorithms compared to Year 5 students. With Year 9, children tend to be more dependent on the calculator and thus failed to score high in written computation. The reason for scoring low in estimation could be that children have problems in understanding concepts like multiplication and division of decimals and fractions. The problem is even more pronounced in the case of ratios and percentages. As such, their performance in estimation is lower than the other two-year levels. The significance of the difference between estimation and computation is discussed in the following section with the help of t-test results.

T-test result of differences between estimation and written computational tests

T-tests were calculated to assess the significant difference between students' mean scores on each of the 15 items on the parallel tests (estimation and written computational skills). The result for Year 7 is shown in Table 11. From the information derived from Table 11, t-test results of Year 7 with t (14) = 2.87, $p < 0.01$, indicate a highly significant difference between the estimation skill and computational ability in that year level. However, the t-test results of Year 5 [t (9) = 1.26, $p < 0.24$] and Year 9 [t (19) = 1.10, $p < 0.29$] show that the differences are not significant.

Table 11: T-test result of Year 7 on estimation and written computation tests

Mean	SD	SE of Mean	t-value	df	2-tail Sig
11.07	14.93	3.86	2.87	14	0.01

* $t(14) = 2.87$, $p < 0.01$

Summary of relationship between estimation and computational skills

From the correlation calculated by year level, the results depict a positive correlation in all the year levels, although the extent varies. In Year 9, the relationship is somewhat weaker compared to the other two year levels. However, the relationship between estimation skill and computation ability is quite strong in Year 5 and Year 7. Moreover, as shown in Table 10, the mean percentage of correct responses on estimation and computation is also quite low in Year 9 compared to Year 7 and Year 5. As such, one possible reason could be that Year 9 students felt handicapped without the privilege of using the calculators. Hence, they have scored only a 31% mean in computation, which is lower than their score in estimation. The case is quite similar in Year 5 with the performance on estimation, which is also higher than in computation. In the case of Year 5, children's low performance in computation could be due to the results of memorizing the rules and formulas without understanding the concepts properly, or those they have had limited experience with algorithms.

The t-t results depict a slightly a different picture for Year 7 compared with the other two year levels. They show that there is highly significant difference between the performance of estimation skill and computational ability. On the other hand, the t-test results for Year 5 and Year 9 indicate that there is no significant difference between the two tests. Overall, being able to perform well in estimation is positively associated with better performance in written computation.

Development of Estimation and Computational Abilities over Years 5, 7 and 9

The development of estimation and computational abilities across the year levels 5, 7 and 9 are discussed mainly to help the researcher to get some information on the development of concepts in the respective topics across the year levels. It will be done with the help of three different points based on common test items. Each of them will be discussed under the following headings:

- descriptions of the differences for all the common test items across the year levels;
- descriptions of the differences on common test items within the topics;
- students' responses to selected items on estimation.

Descriptions of the differences for all the common test items across the year levels

Table 12 contains the results of all the common test items tested across the year levels. Seven test items are common to all the three levels and five in any two of the levels. The differences in performance between estimation and computational abilities with those common test items will be discussed with the help of information given in the Table 12. Each of the common test items will be discussed one by one to investigate the differences in performance across the year levels. The first item results are shown in Table 13.

Table 12: Percentage scores on estimation and computation across year levels for all the common test items

Topics	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
Whole Numbers						
Addition						
9965 + 8972 + 8138 + 8090	46	63	68	84	63	77
Subtraction						
312 – 119	41	47	48	81	53	66
Multiplication						
18 x 19	31	13	52	47	49	41
51 x 48	28	12	44	60	30	56
Division						
598 ÷ 9	46	17	55	46	52	38
Decimals						
Addition						
590. 43 + 312.5	46	62	88	77		
96.7 + 147.4 + 62.75 + 36. 8	40	37	48	74	57	70
Subtraction						
0.72 – 0.009	25	10	26	44	37	47
Multiplication						
87 x 0.09			17	38	41	30
Division						
54 ÷ 0.09			20	27	12	7
Fractions						
Subtraction						
$^{7}/_{8} - ^{3}/_{4}$			17	36	13	12
Multiplication						
$^{5}/_{8}$ of 512			14	21	19	14

Table 13: Item analysis for 9965 + 8972 + 8138 + 8090

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	46	52	2	63	37	0
Year 7	68	32	0	84	15	1
Year 9	63	37	0	77	23	0

The item in Table 13 was tested across all the years 5, 7 and 9. In all the three levels, the performance in computational abilities is much higher than in the estimation abilities. The differences between estimation and computational abilities vary in each of the year levels with 17% in Year 5, 16% in Year 7 and 14% in Year 9. The order of difference in percentage scores in the tests tends to follow from high to low from Year 5 to Year 9, but changes are slight. However, performance in computational ability is better than estimation. The reason could be that children were more aware of the rote–learned algorithms on addition of four digit numbers than making sense of what they did. For instance, an abstract from one interview says:

I: You got it correct in computation but not in estimation, why?
Yr 7: um...that was because um...yeah! I added up properly there...and then so...

Table 14: Item analysis for 312 - 119

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	41	59	0	47	53	0
Year 7	48	52	0	81	18	1
Year 9	53	46	1	66	34	0

Table 14 shows that for the item 312 –119, overall performance is quite low in all the year levels except for Year 7 with 81% in computation. Yet, it does appear that Year 7 children were quite weak in making sense of what they did, seeing their performance in estimation, which is was only 48%. Rather, it indicates that the children relied more on the rote-learned algorithms than on number sense. Where as in Year 5 and Year 9, the difference in performance between ET and WCT is not great. The order of the percentage score in estimation seems to be following the age and year level with the highest in Year 9 with 53% and the lowest in Year 5 with 41%. Thus, it indicates some development of number sense along with the age or year level.

Table 15: Item analysis for 18 x 19

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	31	67	2	13	86	1
Year 7	52	48	0	47	50	3
Year 9	49	50	1	41	59	0

As in the previous item, Table 15 presents performance of item 18 x 19 across Years 5, 7 and 9 levels, which appeared to be very low especially in Year 5. Where as the difference between the ET and WCT score is quite small in Year 7 and 9. In estimation, Year 7 have done better than Year 9 and Year 5. Unlike in previous test items, performance in computation here is lower than in estimation. Thus, result indicates that children had a problem for two possible reasons. It could be either due to lack of knowledge of multiplication of whole numbers especially with Year 5, or that children were used to a hand calculator in the case of Year 7 and Year 9. The extract below supports that children depended on rote-learned times table:

Q: 18 x 19
I: You got it wrong here. What could be your problem?
Yr 5: Oh! I am not really good at times table…

Looking at the result, it indicates that the children have serious problems understanding multiplication of whole numbers, especially in Year 5. The reason could well be that Years 7 and 9 students have had more practice at algorithms than Year 5 children.

Table 16: Analysis for 51 x 48

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	28	61	1	12	87	1
Year 7	44	54	2	60	36	4
Year 9	30	63	7	56	61	3

For this item in Table 16, Year 7 seem to have done 14% and 16% better than Year 9 and Year 5 in estimation. The performance in Year 5 in computations is quite low compared to Year 7 and 9. Looking at the results, it appears that Year 5 children have very little idea of multiplication of whole numbers. One of the reasons is clearly shown with their scores in estimation, which is only 28% and thus depicts their low level of understanding the concept.

Table 17: Analysis for 598 ÷ 9

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	46	51	3	17	79	4
Year 7	55	44	1	46	50	4
Year 9	52	41	7	38	61	1

For the item 598 ÷ 9 in Table 17, the performance in estimation is much better than in computational abilities in all the three levels. It also shows that, there is not much difference in estimation among the year levels compared to the computational abilities with so much disparity between Year 5 and Year 7. However, overall results indicate that the children had some problem with the division of whole numbers.

Table 18: Item analysis for 590.43 + 312.5

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	46	54	0	62	38	0
Year 7	88	12	0	77	20	3
Year 9	NA	NA	NA	NA	NA	NA

Item 590.43 + 312.5 was tested only in Year 5 and 7 as shown in Table 18. As per the result presented, the performance is quite high for Year 7 in both the tests. As such, results in Year 7 indicate sound knowledge of the addition of decimals. The following extract supports this statement.

Q: 590.43 + 312.5
I: Great! You were correct in both the tests. Would you mind explaining how you carried out the estimation?
Yr 7: Um...I added up...oh...I rounded up to 600 and then 300 and added that up and then thought...which stand closer to ...then I look for more than 900 because a bit of extra with 12 makes it a little more than 900

Looking at the procedures carried out above, this child showed a strong number sense. As such, he could also perform correctly with the written computation to achieve positive results. More of such interview abstracts showing strong number sense are given in Appendix F. Year 5 performed reasonably well in computation but showed limited understanding of addition of decimals as evidenced by a much lower estimation score.

Table 19: Item analysis for 96.7 + 147.4 + 62.75 + 36.8

Year level	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	40	58	2	37	63	8
Year 7	46	54	0	74	24	2
Year 9	57	39	4	57	40	3

The item in Table 19 was tested all across the year levels. The results show that, for Year 7, performance in computation is far better than in estimation. Thus, one of the reasons may be that children did not have a good understanding of the given problem but faired well in written computation. Another possible reason could be that the children were good at remembering rote-learned methods rather than making sense of what they did. . In the case of Year 9, the performance is quite good in both the tests with a score of 57% each. The result is quite low for Year 5 in both estimation (40%) and in computation (37%). Children in Year 5 appear to have some problem remembering formal rules in addition of decimals where there are many addends, as indicated in the extract of interview below:

Q: 96.7 + 147.4 + 62.75 + 36.8
I: Unlike in the previous question (590.43 + 312.5), you were wrong in this question. What could be your problem?
Yr 5: Oh...that was a bit more numbers...I got confused!

Table 20: Item analysis for 0.72 – 0.009

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	25	71	4	10	88	2
Year 7	26	74	0	44	52	4
Year 9	37	62	1	47	49	4

Table 20 shows those performances in the item 0.72 – 0.009, which are low in both the tests across all the year levels, and particularly in Year 5. It appears that the children in all the three-year levels had a serious problem with understanding subtraction of decimals. Thus, there is apparently a lack of understanding of decimal numeration, and this hindered both estimation and computation as illustrated below:

Q: 0.72 – 0.009
I: What went wrong with this estimation?
Yr 7: Um...I got struck with...two zeros after the decimal point...like what this...like hundredths or tens...something like that...
I: So, that was your problem...
Yr 7: Yeah! Because I am not really good at decimals and fractions...but when I get through my head...I had to work out....

Table 21: Item analysis for 87 x 0.09

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	NA	NA	NA	NA	NA	NA
Year 7	17	82	1	38	56	6
Year 9	41	58	1	30	65	5

The item 87 x 0.09 was tested only in Year 7 and Year 9 with results shown in Table 21. Looking at the result, it clearly shows that children posses a serious problem in understanding the concept of multiplication of decimals. The main problem seems to be in understanding the concept of decimals per se, as in the extracts given below:

Q: 87 x 0.09
I: Something went wrong here? What could be your problem?
Yr 7a: Um...huh...I probably didn't see the other zero so... it is a lot less than 87...yeah! I thought it was just zero point 9...so, I put a little less than 87
Yr 7b: Oh! That one, I have no idea...I thought decimals and it is not good thing...I wasn't sure...

The information from the above statements depicts that the children were not that clear about the concept. As such, the performance is very low in both the tests across the year levels.

Table 22: Item analysis for 54 ÷ 0.09

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	NA	NA	NA	NA	NA	NA
Year 7	20	80	0	27	55	18
Year 9	12	87	1	7	80	13

Table 22 shows the results for the item 54 ÷ 0.09, which was tested only in Year 7 and Year 9. Overall results of the two-year levels are extremely low in both the tests. As such, the result indicates that majority of the children were quite weak in division of decimals. It appeared that children got mixed up with the idea of division of whole numbers as indicated in the extracts below:

Q: $54 \div 0.09$
I: What happened to these questions you have got wrong on both the test papers?
Yr 7a: Um...that would be a lot less than 54...because it is not a whole number....
I: Is it? Are you sure?
Yr 7a: Because...it is 0.09, which is of one hundredth...
Yr 7b: I am afraid, I got totally confused here ...

It is also evident that while 27% of children in Year 7 could manage the algorithm, Year 9 children were lost without a calculator.

Table 23: Item analysis for $^7/_8 - ^3/_4$

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	NA	NA	NA	NA	NA	NA
Year 7	17	83	0	36	61	3
Year 9	13	86	1	12	88	0

Table 23 shows results for the item $^7/_8$ – $^3/_4$ In both the year levels, overall performance is very low. It appears that the children do not have much idea of the concept of fractions. Rather, most of the children were not able to understand that both $^7/_8$ and $^3/_4$ are numbers close to one. Instead, children were more concerned with the rote-learned algorithms and thus failed to make sense of the item to get an estimate as shown below:

Q: $^7/_8 - ^3/_4$
I: What about this question? You got it wrong here, why?
Yr 7: Um...Yeah...that was a bit harder because we haven't done much...um taking away with fractions in class...so I did through guessing...

Table 24: Item analysis for $^5/_8$ of 512

Year	Estimation Ability			Computational Ability		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Year 5	NA	NA	NA	NA	NA	NA
Year 7	14	86	0	21	74	5
Year 9	19	78	3	14	86	0

Table 24 presents the result of the item $^5/_8$ of 512. As in the previous item on fractions, the results for this item are also very low in both the year levels. In both the tests and for both the year levels, percentage scores are below 25%. The children seem to have found it very difficult to solve it, as they tended to be quite weak in overall concepts of fractions.

Descriptions of the differences on common test items within the topics

The test items were compiled in given topics. Comparisons of different topics (whole numbers, decimals and fractions) were made between each of the year levels 5, 7 and 9. Accordingly, percentage scores on topics are given in Table 25.

Table 25, indicates the overall performance of the common test items based on selected year levels in the tested topics. Year 7 has done better than the other two levels in both the tests for most topics. Among topics tested, children scored highest in whole numbers and the lowest in fractions. Performance for both the Years 7 and 9 were very low in fractions. Surprisingly Year 9 is even lower than Year 7 in both the tests. The results indicate that the children were really weak in understanding the concept of fractions. The results are quite low in the topics of percentages and ratios as well.

Table 25: Overall comparison of percentage scores on common items for estimation and computation within topics across year levels

Topic	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
Whole Numbers	44	33	55	66	52	58
Decimals	34	30	34	49	43	37
Fractions			22	30	15	24
Percentages					26	25
Ratios					33	8

Students' responses to selected items on estimation

Some selected items are discussed below to show students' choices of responses in each of the year levels. Estimation items were given in a multiple-choice format and the full results are given in Appendix G.

Table 26: Percentage of estimation choices for 9965 + 8972 + 8138 + 8090

Estimates	Year 5	Year 7	Year 9
a. 24 000	8	6	7
b. 30 000	9	10	10
c. 36 000*	46	68	63
d. 42 000	36	16	20
e. No response	1	0	0

* Correct response

Table 26 shows the correct and incorrect estimates for the item 9965 + 8972 + 8138 + 8090. Other than the correct choice number 'c' the children mostly chose 'd', especially Year 5. It was likely that they tended to round off all the addends to ten thousands to get around 40 000 and so be closest to 42 000. Doing that they have ignored some strategies, such as that 8090 is closer to 8000 than 10 000.

Q: 9965 + 8972 + 8138 + 8090
I: It is very good that you were correct in computation but what happened to estimation?
Yr 5: Um...that was because um... yeah! I added up properly there...and then ...I rounded up all those four numbers to 10 000 and estimated to be closer to 42 000.

Table 27: Percentages of estimation choices for 312 – 119

Estimates	Year 5	Year 7	Year 9
a. A little less than 100	8	9	14
b. A little more than 100	46	40	32
c. A lot less than 100	5	3	0
d. A lot more than 100*	41	48	53
e. No response	0	0	1

* Correct response

Table 27 presents the correct and incorrect estimates of item 312 – 119. As per the result shown, the children have chosen 'b' almost equally with the correct estimate 'd'. The one possible reason could be that children have rounded 312 to 300 and 119 to 200, and expected the estimates to be a little more than 100 as shown below:

Q: 312 – 119
I: You were correct in computation but not in estimation. What could be your problem?
Yr 7: Yeah...um...I just thought...oh...300 take 100 would be just a little more than a 100...so now I see that...it would be 'd' (a lot more than 100).

Table 28: Percentages of estimation choices for 18 x 19

Estimates	Year 5	Year 7	Year 9
a. 190	33	24	15
b. 390*	31	52	49
c. 400	12	14	21
d. 490	22	10	14
e. No response	2	0	1

* Correct response

The results presented in Table 28 shows the correct and incorrect estimates for the item 18 x 19. As per the information given, other than the correct choice 'b', the children have diverted their attention to other choices as well, especially with 'a' in Year 5 and Year 7 and 'c' in year 9. The possible reason for choosing 'a' could be that the children rounded 18 to 10 and left 19, as it is to get 190. Whereas for 'c' the possibility is that children could have rounded both the numbers to 20 but forgot to adjust the numbers as both 18 and 19 were less than 20, so got 400. Thus, children seem to have some problems understanding the concept of multiplication of two-digit whole numbers.

For the item 51 x 48, results are shown in Table 29, the children have selected almost equally choice 'b' and the correct estimate 'a'. In fact Years 5 and 9 have a higher percentage choosing 'b' than 'a'. The reason for choosing 'b' was probably, that the children rounded both 51 and 48 to 50 but forgot to adjust for 48 being 2 points less than 50, and 51 a point more than 50. As such they choose 'b' thinking that best estimate to be a little more than 2500. Hence, the result indicates that the children had difficulty in making sense of this product as precisely as required.

Table 29: Percentages of estimation choices for 51 x 48

Estimates	Year 5	Year 7	Year 9
a. A little less than 2500*	28	44	30
b. A little more than 2500	36	30	47
c. A lot less than 2500	16	13	8
d. A lot more than 2500	18	13	14
e. No response	2	0	1

* Correct response

Table 30 indicates the correct and incorrect estimates for the item 598 ÷ 9. The results tell us that, other than the correct choice 'b', many children have also attempted other choices such as 'a' in Year 9 and 'c' in Year 5. The possible reason for choosing 'a' could be that the children rounded the numbers to 600 and 10; and for 'c' it could be mainly because children rounded 598 to 500 and 9 to 10. Thus, those children seem to have problems with number sense in the way they have rounded 598 to 500.

Table 30: Percentages of estimation choice for 598 ÷ 9

Estimates	Year 5	Year 7	Year 9
a. 600 ÷ 10	9	13	34
b. 600 ÷ 9*	46	55	52
c. 500 ÷ 10	24	19	6
d. 500 ÷ 9	18	12	1
e. No response	3	1	7

* Correct response

Table 31: Percentages of estimation choices for 0.72 – 0.009

Estimates	Year 5	Year 7	Year 9
a. 0.06	12	19	22
b. 0.6	11	21	18
c. 0.07	48	34	22
d. 0.7*	25	26	37
e. No response	4	0	1

* Correct response

The estimates for the item 0.72 – 0.009 are given in Table 31. Children seem to have opted more for choice 'c' rather than the correct one in 'd', especially for Year 5 and Year 7. The main problem faced by the majority of the children in this item was again with the decimal concepts. They seem to have difficulty in understanding the difference between 0.009 and 0.09. For them, these numbers tend to look the same and thus they estimated incorrectly. The choices of 0.06 and 0.6 support this.

Table 32: Percentages of estimation choices for 87 x 0.09

Estimates	Year 7	Year 9
a. A little less than 87	18	14
b. A little more than 87	44	22
c. A lot less than 87*	17	41
d. A lot more than 87	20	22
e. No response	1	1

* Correct response

The item 87 x 0.09 presented in the Table 32 was tested only in Year 7 and Year 9. As such, the results show that the children have chosen choice 'b' and 'd' in Year 7 more than a correct choice 'c'. A probable reason may be that they miss-understood the concept of multiplication with a decimal number less than one. They possibly understood that multiplication means increase the quantity. As in previous items on multiplication of decimals, children do have serious problems, especially in understanding the value of digits after the decimal point. As such, the performance is very low in both the year levels.

Table 33: Percentages of estimation choices for 54 ÷ 0.09

Estimates	Year 7	Year 9
a. A little less than 54	27	22
b. A little more than 54	32	41
c. A lot less than 54	21	24
d. A lot more than 54*	20	12
e. No response	0	1

* Correct response

Table 33 indicates the correct and incorrect estimates of item 54 ÷ 0.09 tested in Year 7 and Year 9 only. As shown, the children in both classes have chosen each of 'b', 'a' and 'c' more than the correct estimate in choice 'd'. The possible reason for choosing 'a' and 'c' could be confusion between the division of whole numbers and division of decimals. As such, it indicates that the children have very little idea of the concept of division involving decimals as shown below.

Q: 54 ÷ 0.09
I: What happened? Somehow, you have missed the correct estimate here.
Yr 7: Um:..that would be a lot less than 54...because it is not a whole number...
Yr 9: I am afraid, I got totally confused here with whole numbers and so...

Table 34: Percentages of estimation choices for $^7/_8 - {}^3/_4$

Estimates	Year 7	Year 9
a. 0*	17	13
b. 1	31	23
c. 3	9	23
d. 4	43	40
e. No response	0	1

* Correct response

Table 34 presents the item $^7/_8 - {}^3/_4$, which was tested only in Year 7 and Year 9. As per the result shown, the number of attempts is more in 'd', 'b' and 'c' than the correct choice 'a'. Looking at the various choices made, they indicate that children had a serious problem with this concept. Moreover, the fact is that so many children chose ‘d’ underlines their weakness in number sense about fractions. Here, children seem to have subtracted 3 from 7 or 4 from 8 and thought the correct estimate was 4, instead of 0. Whereas for choice ‘b’ children possibly subtracted numerator from denominator to get 1. As such, students have failed to understand that both the fractions are closer to the whole number one, and therefore the best estimate is zero.

Table 35: Percentage of estimation choice for $^5/_8$ of 512

Estimates	Year 7	Year 9
a. A little less than 240	22	26
b. A little more than 240	38	40
c. A lot less than 240	26	12
d. A lot more than 240*	14	19
e. No response	0	3

* Correct response

The results of the item $^5/_8$ of 512 are presented in Table 35. The choice ‘b’ is more popular with the children compared to the correct choice ‘d’. The way so many children opted for choice ‘b’ suggests that they had some idea of $^5/_8$ as being close to half, but then they seem to have forgotten that it is also slightly more than a half, so the correct estimates is a lot more than 240. As such, it still indicates that children had problems with the concept of fraction.

Table 36: Percentages of estimation choices for $^2/_3 \times {}^3/_4$

Estimates	Year 9
a. 1*	15
b. 2	34
c. 6	33
d. 12	17
e. No response	1

* Correct response

Table 36 presents the result of the item $^2/_3$ x $^3/_4$ that was tested only in Year 9 students. The Year 9 students have chosen 'b' and 'c' more than the correct choice 'a'. The reason could be that the children have taken those fractions as 1 each and correctly added them to get 2 instead of multiplying them. Another problem could be that children had just multiplied the two numerators to get 6 or the two denominators to get 12 without meaning. As such, Year 9 children have failed to understand that both the fractions are more close to 1 and the result cannot be more than 1 x 1 = 1.

Table 37: Percentages of estimation choices for $^5/_6 \div {}^2/_3$

Estimates	Year 9
a. 1*	29
b. 2	44
c. 3	20
d. 5	4
e. No response	3

* Correct response

The correct and incorrect estimates of $^5/_6 \div {}^2/_3$ are shown in Table 37. The results tell us that children have miss-understood the concept of division with fractions. Year 9 students have failed to understand that the division approximates 1 ÷ 1. Moreover, it is supported by their performance on the percentages item in Table 38.

Table 38: Percentages of estimation choices for 20% of 198

Estimates	Year 9
a. A little less than 40*	23
b. A little more than 40	48
c. A lot less than 40	4
d. A lot more than 40	25
e. No response	0

* Correct response

Table 38 indicates the correct and incorrect estimates of 20% of 198. According to the result presented, the choice of attempt is more on 'b' rather than on 'a'. One possible reason could be that children forgot to adjust that 198 is less than 200 and estimated the answer out of 200 directly. As such, they have missed choice 'a' and went for choice 'b' instead. The next popular choice was 'd', which indicates those children were quite weak in the concept of percentage, as the estimates indicated are well away from the correct estimate.

Summary of development of estimation and computational abilities over Years 5, 7 and 9

From the information and the discussion provided earlier, there tends to be some development of computational estimation from Year 5 to Year 7. However, Year 7 had scored higher than Year 9 in many of the test items across the topics. This applied to both the tests. The explanation for scoring lower in written computation could be that the Year 9 children were more used to calculator than pen and paper to help them solve mathematical problems.

On the other hand, it is very difficult to justify the reason for the low performance on estimation. As a matter of fact, Year 9 was expected to perform better than what they did, especially in terms of estimation, with a belief that more experience results in more number sense. Feedback from the interviews support that the children from a slightly above average group were better in number sense as they could depict a variety of strategies to get the solution; whereas the children of slightly below average abilities were more concerned with the rote-learned steps than in seeking their own strategies. As such, there arises the question of how effective it would be for a child to use a calculator if he/she lacks number sense to estimate the answer before pressing the buttons.

Relation between Estimation and Computational Abilities within Topics

In this section, discussion will focus on the relation between estimation and computation within the selected topics used for this study. Table 39 presents the result of the three-year levels across the selected topics.

Table 39: Percentage scores for all year levels by topic

Topics	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
Whole Numbers						
Addition	55	63	68	84	63	77
Subtraction	45	37	48	81	53	66
Multiplication	30	13	48	54	40	49
Division	46	17	55	46	52	38
Overall	**44**	**33**	**55**	**66**	**52**	**58**
Decimals						
Addition	43	50	68	76	57	70
Subtraction	25	10	26	44	37	47
Multiplication			21	47	42	22
Division			20	27	36	4
Overall	**34**	**30**	**34**	**49**	**43**	**37**
Fractions						
Addition			41	74	78	26
Subtraction			17	36	13	12
Multiplication			27	24	17	35
Division					29	11
Overall			**28**	**45**	**34**	**21**
Percentages					**26**	**25**
Ratios					**33**	**8**

Estimation and Written Computation with Whole Numbers

Table 39 shows that there are not many differences in performance between estimation and written computation in whole numbers across all the year levels. Compared to estimation, the performance in computation is 11% better in Year 7 and 6% better in Year 9. Whereas in Year 5, it is other way around; performance in estimation is 11% higher than computation. The range of performances in estimation and computation all across the year level are 11% and 33% respectively.

Estimation and written computation with decimals

Unlike in whole numbers, performance in decimals is quite low in both the tests, although the scores are similar in both skills. Moreover, there is not much difference in overall performance in decimals all across the year levels. The range is 9% in estimation and 19% in written computation.

Estimation and written computation with fractions

As shown in Table 39, fractions are tested only in Year 7 and Year 9. The performance is very low with 21% in computations in Year 9 and with 28% in estimation in Year 7. As a whole, Year 7 scores the highest percentage in computation with 45%, which is 17% more than in estimation. Where as in Year 9, the performance is better in estimation with 13% more than in computation. The range of performance between estimation and written computation is 6% and 24% respectively.

Percentages and ratios for Year 9

The result in the Table 39 indicates that performance on percentages and ratios for Year 9 is very low compared to the other topics. The score is especially low in written computation. From this, we can assume that they were slightly better at making sense of the item than they could compute using pen and paper.

Summary of relation between estimation and computational abilities within topics

As in research conducted on 'computational estimation skill of college students' by Hanson and Hogan (2000), this study also indicates similar kinds of results. Students scored the highest on the estimation tests of addition and subtraction of whole numbers. Their performance is quite low on division and subtraction of fractions. Side by side, on the computational test, student scored the highest on items involving addition, subtraction, multiplication and division of whole numbers. Items with fractions and decimals were more difficult for them. Percentages and ratios proved very difficult for Year 9 children.

Amongst the three selected levels, Year 7's overall performance is the best in all the topics. As such, Year 9's performance is comparatively low considering its year level. This was especially so with written computation, probably because the children relied more on the calculator than on pen and paper. Children seemed handicapped without calculators, as they were not allowed to use it during the test time.

Disparities between Estimation and Computational Skills

The main discussion in this section will be on disparities between estimation and computational skills. For instance, some items will be selected based on either having very high percentage score in computation and a low score in estimation or vice versa. As such, possible reasons will be discussed from the information collected through tests in Table 40 and the interviews. There are several items, where the children have performed better in written computation than in estimation. Among

Table 40: Percentages scores across the year levels by items

Topics	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
Whole Numbers						
Addition						
9965 + 8972 + 8138 + 8090	46	63	68	84	63	77
2333 + 435 + 23 + 9	63	63				
Subtraction						
312 – 119	41	47	48	81	53	66
4012 – 998	48	26				
Multiplication						
18 x 19	31	13	52	47	49	41
51 x 48	28	12	44	60	30	56
Division						
598 ÷ 9	46	17	55	46	52	38
Decimals						
Addition						
590. 43 + 312.5	46	62	88	77		
96.7 + 147.4 + 62.75 + 36. 8	40	37	48	74	57	70
Subtraction						
0.72 – 0.009	25	10	26	44	37	47
Multiplication						
0.5 x 840			25	56		
87 x 0.09			17	38	41	30
19.4 x 46.1					43	14
Division						
54 ÷ 0.09			20	27	12	7
563.7 ÷ 2.93					60	1
Fractions						
Addition						
$^{3}/_{4} + {}^{1}/_{2}$			33	43		
$^{7}/_{8} + {}^{12}/_{13}$					34	11
Subtraction						
$^{7}/_{8} - {}^{3}/_{4}$			17	36	13	12
Multiplication						
$^{1}/_{4}$ of 798			40	27		
$^{5}/_{8}$ of 512			14	21	19	14
$^{2}/_{3} \times {}^{3}/_{4}$					15	55
Division						
$^{5}/_{6} \div {}^{2}/_{3}$					29	11
Percentages						
Percentage for $^{7}/_{12}$					29	27
20% of 198					23	22
Ratios					43	8
3 : 1 = 7 : n, n = ?						
1 : 9 = 1.5 : n, n =?					22	8

those, the item 312 - 119 shows a big difference between estimation and computational scores. The difference between estimation and computation is very high in Year 7 compared to the other two year levels

Subtraction and multiplication of whole numbers

Children in Year 7 have done much better in written computation than in estimation The way the children performed in these two tests suggests that they were more oriented to the formal rules of computation, thus depicting their weakness in number sense. That is, the reason for not performing well in estimation could be due to their weakness in making sense of the given problem.

Other than that there are only few extreme cases with selected items, like 4012 – 998 and 18 x 19, where the performance is much better in estimation than in computation. The reason for the better estimation performance could be that there was too much dependence on algorithms that were not well established, especially in year 5. For instance, a sample from Year 5 is given below from the interviews on one particular question, 18 x 19:

Q: 18 x 19:
I: You got it correct in estimation but wrong in computation, why did it happen that way?
Yr 5: Um...forgot! I know tables only up to 12 but...so, that's how I got it wrong...

As per the information provided above, interviewee from Year 5 tells us that children were more or less appeared to be dependent on times tables and rote-learned steps. As such, they were not able to make sense of the question provided and failed to get the correct answer in written computation.

Division of whole numbers

In the item 598 ÷ 9, the children's performance is better in estimation than in written computation. This difference is very large in Year 5. From the information gathered, children were better at estimation, depicting number sense. On the other hand, their result in computation reflects their low performance there. The reason could be that

Year 5 children could relate the problem based on their daily practice of sharing using their non-formal algorithms and their common sense. But, they failed to compute using formal steps of computation. Moreover, they were frightened off with the size of the number. Many of them found it very large for them to compute effectively. Or, they ended up guessing rather than computing the item as shown below:

Q: $598 \div 9$
I: You got it wrong in computation but right in estimation, what could be the reason?
Yr 5: Wow! Um...it was a kind of a wild guess...

Multiplication of decimals

This particular item 0.5 x 840 was tested only in Year 7, so there is no comparison possible with other year levels. Unlike in many other items, children's performance in this item is quite low, especially in estimation. As such, the reason for scoring very low in estimation was that the children could not make any sense out of the item, as they seemed to lack understanding of the decimal concepts as mentioned earlier. But their performance was better in computation where rote-learned steps could be used without making any sense out of it.

Division of decimals

Year 9 children did not perform well in the item $563.7 \div 2.93$, especially in computation with only 1% correct. But on the other hand, the estimation result of 60% indicates that children had quite good number sense, as they could perform much better in estimation. Therefore, as discussed earlier, the main problem with Year 9 in computation was most likely due to too much dependence on calculators.

Multiplication of fractions

The performance of Year 9 with the item $^2/_3 \times {}^3/_4$ is quite unusual compared to other items. With this, the majority of the children have performed well in computation but not in estimation. The reason could be most probably because of those children who found easier to remember the steps for computation on multiplication of fractions.

As such, they were thorough with the rote-learned steps without making any sense of it while computing. As a result, their score was very low in estimation. Therefore, children's understanding on this item was very weak and confused, as the extract below clearly indicates:

Q: $^2/_3 \times {}^3/_4$

I: You got this correct in computation but got it wrong in estimation, what could be the reason you think?

Yr 9: Um...because it needs calculator...I can't do things in my head...I need to process for them and everything...and moreover not enough time and could not do that...there is no way...it confused me because...I thought answers would be in fractions as well...and it wasn't in whole numbers and that's why I wasn't sure with that one even though I knew the process on how to do it...I thought, it would be one over something or two over something... and never had a clue that it would be closer to any whole number...

Summary of disparities between estimation and computational abilities

There were disparities between estimation and computation performance in many items and, as indicated above, some of these were very great indeed. In some of these cases students could make an estimate but were unable to complete the calculation. In others the reverse was true, in that students knew the algorithm but did not understand the problem.

Gender Related Differences

Another point for investigation in this study was to find out whether there is any difference in performance on computational or estimation skills by gender. The issue will be discussed with the help of data presented in Table 40, which contains the performance of boys and girls in computation and estimation across the year levels, by topics. The item-by-item comparison is given in Appendix G.

Table 41: Percentage scores comparison of performance between male and female students by topics across Years 5, 7 and 9 levels

	Year 5				Year 7				Year 9			
	ET		WCT		ET		WCT		ET		WCT	
Topic	M	F	M	F	M	F	M	F	M	F	M	F
Whole Numbers	50	42	43	38	49	46	59	67	50	51	58	71
Decimals	39	34	52	30	37	37	54	52	39	31	39	36
Fractions					33	34	47	64	27	19	22	32
Percentages									55	56	41	25
Ratios									46	44	19	11
Total	45	38	48	34	40	39	50	61	43	38	36	35

Whole Numbers

From the result collected, there are no large differences in overall performance between boys and girls. At the same time, consistency differs from one level to another and from topic to topic. For instance, in whole number, there is not much difference in Year 5 and Year 7, but in Year 9 females scored much higher than males in written computation. Similarly, the performance of boys is higher than girls in computation of decimals in Year 5. In the same way, in Year 7, girls scored higher than boys did in written computation for fractions. In percentages, Year 9 boys performed better than the girls. Looking at the patterns of the performance, the result shows more disparities in performance with written computation compared to the performance on estimation.

As a whole, the difference in performance favours a little more on boys than girls in both the estimation as well as computational abilities in Year 5. The difference in percentage scores being 8% more in estimation and 5% more in computation. In Year 7, males are better by only 3% in estimation, but girls scored 8% higher in computation. Unlike in Year 5 and Year 7, female performance is better in Year 9 in both the tests with a difference of only 1% in estimation, but 13% in written computation.

Decimals

In Year 5, as in whole numbers, boys are still better than girls in both the tests, especially in computation with the difference of 22% in score. There is almost no difference in performance in decimals between boys and girls in Year 7. But in Year 9, males are slightly better than females in both the tests, although the overall performance is quite low for both in decimals.

Fractions

Girls in Year 7 have performed better than boys in fractions, especially in computational abilities. There is only a 1% difference in estimation between boys and girls but 17 % differences in computational abilities. Whereas in Year 9, males are better than females by 8% at estimation, but it is other way round in computational abilities, in which females have scored 10% more than males.

Percentages and Ratios

Since the percentage and ratio topics are tested only in Year 9, the result is shown only for that year level. Performance in estimation is almost the same for both genders, but girls are better than boys in computational abilities with a difference of 16% for percentages and 8% for ratios. In both the tests, male performance is slightly better than for females.

Summary of gender related difference

The results show some marked differences for particular topics. However, the totals for all topics, as detailed in Appendix H show that overall differences are not great, except in written computation at the Year 7 and Year 9 levels where girls' performances were better than the boys', especially in the topics of whole numbers and fractions.

CHAPTER V

CONCLUSION AND RECOMMENDATIONS

The chapter is divided into five main parts. The first part discusses the findings by summarising the major aspects of the study. Then, it moves on to its limitations as the second part. The third part presents some implications of the study proposed for the curriculum and teaching. Finally, recommendations and then suggestions for further research constitute the fourth and fifth parts of the chapter.

Summary of the Study

In this section, a brief summary of the study will be presented with reference to points discussed in earlier chapters.

A problem in teaching and learning mathematics

As discussed earlier, one of the main problems in learning mathematics is solving problems with understanding. For too long many children had been computing mathematics problems without any in-depth conceptual knowledge. In fact, in many mathematics curricula, the importance was placed more on speed and accuracy of computation than on meaning (Bana & Bourgeois, 1976). Computation was mostly carried out with a few rote-learned algorithmic steps. Young children were being presented with a mathematics problem in which the arithmetic computation is given more importance than conceptualisation of mathematics. Moreover, learning mathematics tends to be geared more to the arithmetic than mathematics as pointed out by Wolfinger (1988, p. 4):

> A sound program dealing with quantitative aspect of the school program for young children should emphasise mathematics rather than arithmetic, should develop understanding rather than answers, and should generate concepts rather than folders of completed worksheets.

Owing to that, the children end up getting the correct answer but often without it meaning anything to them. As such, learning mathematics often remained as something meaningless and not concerned with the development of concepts. Rather, it became more of selling the information and not at all understanding it (Cole, 1987 as cited in Ritchhart, 1994). Therefore, the issue on learning mathematics tended to be not with being able to solve the mathematical problems but not being able to think mathematically. This was due to the type of curriculum that lacks in providing students with skills to solve problems encountered in the real world (Swan, 1991). Hence, it results in students' failure to recognise when answers are not sensible.

Purpose of the study

According to what had been discussed earlier, there was a great need for a kind of study, which would help improve teaching and learning mathematics with understanding. As suggested by Swan (1991), one of the widely accepted purposes of mathematics education is that of preparing students to solve problems that they will encounter in the real world. Swan raised an important question, (1991, p. 1): "Are students being provided with the skills they will use in the real world?" To answer such a question leads mathematics educators to the main concerns in teaching and learning mathematics, which is to help the children in exploring meaningful ways to compute, rather than memorising algorithms (Sowder & Schappelle, 1994). As such, estimation is stressed to be one of the skills, which involves comprehending the problem, judging and verifying reasonableness and thus helping learn mathematics meaningful (Harte and Glover, 1993).

Hence, it is a concern for all the mathematics educators and leads to one of the major reasons for teaching estimations (Trafton, 1986). As it is, knowing how to estimate is one of the important skills that can help children solve problems with understanding as supported by Van de Walle (1988, p. 15), who stated that:

> An important by product of learning to estimate is better conceptual understanding, and conversely – concepts must be understood in order to provide the flexible set of processes and decision-making rules needed by the proficient estimator.

Not only that, as Reys (1988, p. 29) has pointed out, "one of the exciting benefits of teaching estimation is the opportunities it provides for individual thinking to occur".

Therefore, I strongly support what Reys had to say on estimation that "estimation skills are essential and must be given high priority within every school program...only few mathematical topics provide the wealth of benefits both immediate as well as long term as does estimation" (1988, p. 41). In spite of that, estimation is a crucial mathematical strategy that can be woven throughout the entire mathematics curriculum (Whiten, 1994).

Further more, I also support Whiten (1994) who points out that a focus on the use of estimation also gives learner a more balanced perspective about the nature of mathematics. He argues that children grow in their confidence about themselves as mathematicians when they see mathematics as a way of thinking.

Moreover, as argued by Edwards (1984, p. 61), "you cannot use the calculator to find answers until you have some idea what answers you are looking for". As such, estimation skill is not only useful to perform computation without any external aids but also useful for checking the results of the calculation (Levin, 1981). Besides that, it is one topic that has usefulness for both as a situation for developing number sense, as well as a skill in and of itself (Sowder & Shappelle, 1994). As such, in real life, problems and situations more often involve estimation than precise measurement or calculation (Harte and Glover, 1993).

Despite all the importance of estimation as discussed above, the reality that very little attention is actually given by mathematics teachers to the development of this skill in their pupils (Cockcroft Report in Poulter & Haylock, 1988). So, in order to highlight it and find out its effectiveness, this current study had been carried out to investigate the relationship between the estimation skill and computational ability of students in Years 5, 7 and 9 in relation to whole and rational numbers. The reason for choosing those topics was as per the argument made by Poulter and Haylock (1988, p. 28) that "to be a good estimator the student will need to have developed confidence and flexibility in handling numbers and number relationships". Moreover, there is evidence found by Yang (1995, p. 38), that "skill in computational estimation is associated with the flexibility of using and understanding the structure of number system and operations". As such, there is a need for study of estimation integrated with the study of concepts underlying whole and rational numbers so that these concepts can be constructed meaningful by the learners (Reys & Reys, 1990).

Research Methods

Three primary K-7 schools and a secondary school in the same region in Perth suburbs were chosen for the study. The subjects were the students in nine classrooms. There were two classes of Years 5 and 7 from each of the three primary schools; and three Year 9 classes from the secondary school. The total number of students participated in the three respective year levels were 91, 77 and 73.

Two instruments were developed for the study: a set of two parallel tests on estimation and written computational abilities and an interview to triangulate the result derived from those two tests conducted. Both the tests consisted of identical items, with 10, 15 and 20 items for Years 5, 7 and 9 respectively. The only difference was that one test required computation and the other estimation. Several items were repeated for two or three year levels to measure skill development. The administration of the test followed the same pattern in all the classes, with the estimation test first, followed by the written computation test.

Both quantitative and qualitative methods were used to carry out this study. As mentioned in Chapter III, the first stage was used to evaluate the result derived from those tests on estimation and computational abilities. The second stage was involved interviews for the selected group of students. Eighteen students were interviewed with nine students of slightly above average and the other nine of slightly below the average abilities. The class teachers helped the researcher to select those students. Tools used for analysing the collected data were SPSS and Microsoft Excel. The first tool was used to work out the test results in details, where as the SPSS software was used to find the correlation, standard deviation and t-test of those two tests.

Summary of the Results

As suggested in number of studies, a child's number sense and computational estimation is closely allied, and the result from this study also supports the point strongly. The findings in the two tests and interviews through correlation and t-test show that there is a close relationship between estimation and computational abilities in all three-year levels. The result also indicates that, a child who is good at estimation could explain the problem with understanding. Moreover, according to the information given in the Appendix G, results show that being able in estimation generally leads to correct mathematical computations.

On the other hand, many students who were weak in estimation or number sense could still perform computations correctly. From this, one could conclude that it is not necessary to be good at number sense to perform computation. But it is very important to possess good number sense if one is to estimate and make sense of the given computations. Thus, having knowledge of estimation is very important in solving mathematical problems with understandings.

However, for some individual items there were few extreme cases with very high scores in written computation and very low ones in estimation and vice versa. The reason could be that the child was weak in number sense and scored very low in estimation but was good at rote-learned methods and scored higher in computation.

As pointed out by Sowder (1988), justification for teaching computational estimation is that it develops number sense. Likewise, the result of this study also supports the notion that estimation can play significant role in raising the general level of quantitative literacy and mathematical understanding among students and adults (Buchman, 1978; cited in Edwards, 1984). The role of number sense was most apparent when children estimated a solution for mathematical problems. Moreover, estimation activities are valuable in developing and assisting the student's grasp of numbers (Ritchhart, 1994). For instance, a child with good number sense could predict roughly what the solution would be before actually computing the problem. Where as, a child who is weak in number sense may jump directly to the rote-learned steps and tries to get the answer without understanding the problem. For him or her, getting answers seems to be more important than understanding the problem. Likewise, the findings by Yang (1995, p. 180), who states that:

> Interviews with students revealed that high ability students demonstrated a wider range of characteristics of number sense than middle ability students. Middle ability students tended to use the written computation algorithms more often than high ability students.

At the same time, this study also shows that, many of the students did not seem to grasp the values of the number being computed. As such, results from several investigations on estimation depict that good estimators are flexible in their thinking, use a variety of estimation strategies, and demonstrate a deep understanding of number and its operations (Dowker, 1988; in Sowder, 1992). This research also supports that "correct answers are not a safe indicator of good thinking...teachers must examine more than answers and must demand from students more than answers" (Sowder, 1988, p. 227).

In comparison among the selected topics, students' performances were much better for addition and multiplication of whole numbers. A majority of the students were quite weak in division, particularly for decimals and fractions. For example, less than a half of the Year 5 students correctly computed 598 ÷ 9, showing lack of understanding of the concept of division of whole numbers. Similarly, Year 7 and Year 9 had problems computing $^5/_8$ of 512 and 54 ÷ 0.09, indicating a poor concept in multiplication of fractions and division of decimals. It appears that not many students were aware of number relationships, and neither could they make any connections between related expressions (Macintosh, Bana & Farrell, 1995). The results also revealed other conceptual difficulties. Besides that, performances of Year

9 students were very low in the other two topics of percentages and ratios. A similar kind of study was also carried out by Sowder and Wheeler (1987) and found out that most students before Year 10 were not able to correctly compare $^5/_6$ and $^5/_9$. Likewise, in another study by Peck and Jencks (1981), a poor performance for comparing fractions such as $^2/_3$ and $^3/_4$ was demonstrated. Both the findings are cited in Yang (1995).

Regarding the performance with the age or year level, the result indicates that there is some development or progress from Year 5 to Year 7, but not to Year 9. However, compared to Year 9, Year 7 have done better in both estimation and in written computation. Thus, results from the current study suggest that it is not always true that the children's development of number sense improves with age or year level.

There is a smooth development of performance across the year levels for the concepts of whole numbers particularly in addition and subtraction. The reason could be that the children have a firm understanding in these operations, as is clearly indicated by the performance in both the tests.

Generally speaking, the problem with Year 5 is mainly with the understanding of the concept and making sense of what they compute. They seemed to have less problems with the written computation, probably through the rote learned formal algorithms. On the other hand, the case is slightly different with Year 9 children, as they were found to be reasonable at estimation but not that sound in written computation. The most likely reason is that the students in Perth (Western Australia) at that level mainly compute with the use of calculators, where as the students in the researchers' home country are not permitted to use the calculator inside the classroom while solving numerical problems. As such, children in the sample tested seem to have lost skills in computation with pen and paper.

The Year 7 case is mixture of two above problems. Children in this level happened to be quite good at both the skills (estimation and computation). They tended to be far better in making sense of the problem than Year 5 and better performers in written computation than Year 9. Overall, performance in Year 7 is more balanced than in the other two levels.

Regarding the performance level in computation and estimation in each of the topics, there is a decreasing order from whole numbers to ratios. The relationship between estimation and computation remains fairly constant but the performance becomes weaker as it moves towards the higher year level as shown in Table 38.

The gender issue was also explored in the study. The main purpose was to find out whether results supported what other studies had found. That is, boys are better than girls in computational estimation (Reys et al:, 1980). Unlike their findings, the result in Table 40 shows little gender difference in performance. According to the results shown, there are some marked differences in particular topics for both girls and boys. For instance, the performances of girls in computation were far better than those of boys in Year 7 and Year 9, especially in the topics of whole numbers and fractions. The difference of percentage scores of Year 7 and Year 9 in whole numbers is 8% and 13% more than the boys. Likewise, in fractions, girls in Year 7 and Year 9 scored 17% and 10% more than what boys obtained. Thus, the result indicates that, although boys are ahead of girls in most of the items (Appendix H), the difference in performance as a whole is very low.

Finally, not many problems were faced while conducting the study. The school authorities and the class teachers were very co-operative, helpful and supportive throughout. The children were very co-operative and frank with their opinions. They tried their best to respond to what the researcher had to ask them. Some of them had a very clear idea about estimation strategies. They could explain so clearly how they had done their computation. At the same time, there were some whose idea of estimation was no better than just narrating the steps of formal algorithms verbally.

Limitations

Several limitations were met in the process of study. The main limitations are discussed as follows.

Sample

The sample sizes were neither large enough nor representative enough for the true generalisation of the study. For instance the relatively small sample of 241 children drawn from a sample of Years 5, 7 and 9 students from a few of schools in the Perth Metropolitan Area makes it difficult to generalise the results to any large extent. However, the trends indicated from the results can add to the growing body of research in this area and in most cases concurs with what other researchers have found.

Situation

This study had to be conducted outside the researchers' home country of Bhutan where the situation and the cultural background are markedly different. Some of the problems faced in the home country may not be reflected in the situation where the research had been carried out. As such, some results derived from the study may not apply in the home country. For instance, the children studying in Perth do have some idea of estimation strategies and could use the process while computing the mathematical problems. The same may or may not be applied in the home country, where the children generally have little exposure to those strategies.

Mathematics Curriculum

There could be some differences in mathematics curriculum too. Some of the topics introduced in certain levels may not be the same in both the countries. As such, there would be some miss-match between these levels. For instance, topics like fractions and decimals are introduced towards late primary in Perth Metropolitan schools, where as those topics are introduced towards early primary education in the researcher's home country.

Time Duration

The researcher had to fit in the time duration as per the time allotted by the class teachers. As such, the number of items prepared had to be adjusted according to the given time. Owing to that, the freedom for the researcher to include more items was limited.

Syllabus Coverage

There was a slight problem in syllabus coverage particularly in Year 5. The term plan set by the class teacher did not tally with some of the items prepared by the researcher. It happened in particular with topics like addition and subtraction of decimals in one of the schools. The teacher in that school has kept those topics for the latter part of the year. As such, the topics were not covered adequately before the children sat for the tests.

Implications

As suggested by Macintosh, Bana and Farrell (1995), this study also leads to a number of implications for curriculum development and teaching practice in the mathematics classroom as follows.

- The curriculum needs to be much more flexible to serve for the wide range of ability, especially in computational estimation.
- Teachers should introduce estimation skills by encouraging strategies that are suited to the individual student. The study of children's estimation strategies is said to serve as a window into their mathematical thinking and problem solving (Ainley, 1991; Dowker, 1992) with the intention that the strategies used may exhibit varying degrees of insight into the nature of a problem or mathematical

domain (Gardiner & Klebanov, 1995). Both these points are cited in Forrester and Pike (1998).

- Students need to develop a sound understanding of the number sense, and need to be made aware of relationships between number facts.
- Teachers should integrate computational and estimation skills in mathematical topics where these apply, so that no computations are undertaken without estimation.
- Lastly, it should be stressed that real-life computation involves much estimation, so classroom teaching should emphasize computational estimation rather than concentrate on the paper and pencil algorithms.

Recommendations to the Bhutanese Schools

Since the study was based on the problem faced by the children in the home country of Bhutan, the researcher's recommendations are to be stressed here for that country. A study of this nature is new in the Bhutanese context. Consequently, the researcher would like to recommend to the Department of Education in Bhutan for the following points, which the researcher considers manageable and, more importantly, useful:

- To set up a committee to look into the national curriculum and national teaching syllabus of mathematics with a view to reviewing the methods of teaching. This review should be done in the context of updating the methods and introducing an approach to estimation to help children compute mathematical problems with understanding;
- To provide an appropriate in-service training for the teachers so that they will be able to implement the objectives of such an approach (computational estimation);
- To equip both the schools and the training institutions with requirements and resources that will help implement an idea of computational estimation in children;

- To increase the understanding and make sense out of their computation;
- To integrate computational estimation in the national teaching syllabus, which should be based on the children's environment so that they can easily relate what they have learnt. In other words, the children should be able to see practical aspects of the concepts they come across in real life;
- The place of computational estimation skills in teaching mathematics should be seen as something which can make one's ability to use mathematics in real situations, faced in everyday life; and
- Lastly, since the introduction of computational estimation requires extra time for the children to get used to it, teachers may need extra time for the coverage of the syllabus. Hence, it is recommended that fewer topics in mathematics to be introduced, especially in early primary education.

Suggestion for Further Research

Throughout this chapter, a number of questions relating to possible further research has been raised. These questions are detailed below:

- What effect does the relationship between number facts and computational estimation have on teaching and learning mathematics?
- What type of estimation items do students prefer to be presented visually/orally?
- What differences are there between strategies used in oral versus visual presentation?
- If time currently spent on written algorithms in classrooms were devoted to computational estimation, what differences would this make in teaching and learning mathematics?
- If the computational estimation items were contextually based what difference would this make to performance?

- What is the relationship between children's computational estimation and their overall number sense?

Lastly, more such studies need to be carried out in many other countries to see whether or not the same results can be applied. Besides, the issue of whether children should be taught to use certain strategies or simply be made aware of them is one that requires more research. Given that a body of knowledge is beginning to be built up about a number of strategies, the question of what is the best way to impart this knowledge to children demands attention. Further research also needs to be carried out to determine the relationship between estimation and computational abilities related to mathematical topics other than whole and rational numbers, such as measurement topics. As pointed out by many mathematics educators, too much time is spent dealing with written arithmetic. As such, the time previously spent on written algorithms might well be used to develop estimation skills. Such a study could be used to determine whether overall computation performance changes as a result of increasing time spent on developing skills in computational estimations.

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APPENDIX A: LETTERS

A Letter to Principals

Dear Principal,

I am writing this letter to provide you with some information about a research project in which I am engaged and to ask if you would be willing for your school to be involved in the project.

The project is for a Masters' thesis that I am working on as part of my studies with the school of education in Edith Cowan University. The main purpose of the project is to gain more detailed information about the development of estimation and computational abilities of Years 5, 7 and 9 students. It is hoped that this information will aid in the development of materials to improve the estimation and computational abilities of children.

Dr. Jack Bana, who is a senior lecturer in mathematics education at Edith Cowan University, is supervising this project.

In the preliminary phase a short written test of about 40 minutes duration will be given to Years 5, 7 and 9 students from a number of schools. Later, a few students will be selected for a follow up interview of approximately 20 minutes duration in which the students will be asked to explain how they go about solving some computation and estimation questions.

All interviews will be audio taped for further analysis. The identity of individual students and individual schools will not be used again once the data is collected. Thus complete confidentiality is assured.

Having taught in schools I realise that the demands placed on teachers are great. The data collection phase has therefore been designed to cause as little disruption as possible to the school and should not involve the relevant staff in any extra workload.

I will be very happy to discuss any matters with yourself and/or your staff prior to you making a decision if you wish.

Thanking you in advance.

Yours sincerely,

Phuntsho Dolma
M.Ed. Student

A Letter to Class Teachers

Dear Sir or Madam,

My name is Phuntsho Dolma and I am a master's student at Edith Cowan University. I am presently undertaking a study of mathematics education by course work and thesis. As a part of my thesis, I would like to develop an understanding of whether estimation skills can help the students in computing mathematics problems. I plan to develop this understanding by conducting a written test with the students of Years 5, 7 and 9 in several schools in Perth.

I would be grateful if you would agree to participate in this study. This would involve administering a written maths test to students in your class (approx. 40 minutes), plus interviewing several students

Thank you.

Yours sincerely,

Phuntsho Dolma
(M.Ed. Student)

A Letter to Parents

Dear Parent,

I am writing this letter to provide you with some information about a research project in which I am engaged and to ask if you would be willing to allow your child to take part.

I am a Postgraduate student in Edith Cowan University who is doing a Master of Education by course work and thesis. The topic of my thesis is 'Investigating the relationship between computational ability and estimation skill'. For this, I need to conduct a written maths test as well as an interview with your child. The main purpose of doing these is to gain more detailed information about the development of computation and estimation abilities of Years 5, 7 and 9 students. It is hoped that this information will aid in the development of more appropriate learning materials.

Dr Jack Bana, who is a senior lecturer in mathematics education at Edith Cowan University, is supervising this project.

The interview will be audio taped for further analysis. The identity of individual students and individual schools will not be used again once the data is collected. Thus complete confidentiality is assured.

If you have any concerns please feel free to contact me through your school.

Thanking you.

Yours sincerely,

Phuntsho Dolma
M.Ed. student
Ph:
Fax:
Email:

I (__________________________) have read the information above and any questions I have asked have been answered to my satisfaction. I agree that the research data gathered for this study may be published provided that my child is not identifiable.

__________________________ ____________
Parent Date

__________________________ ____________
Researcher Date

APPENDIX B: PROTOCOLS FOR ESTIMATION AND COMPUTATIONAL TESTS

Introduction to the teacher concerned:

My name is Phuntsho Dolma. I am doing my master by course work and thesis in mathematical education at Edith Cowan University. I will be studying how students of the chosen levels perform computational estimation. For this, I need your help and appreciate your co-operation today. I would like to request you to administer two different tests to your students, with the first on estimation and the second on computation. They should be conducted separately in one sitting. The computation test should only be administered after collecting the answer sheet for the estimation test. The result will be kept confidential and will be returned to you. Thank you for your co-operation.

Directions for conducting Estimation Test

Test lengths are as follows for both estimation and computation.
Year 5: 10 items; Year 7: 15 items; Year 9: 20 items
Ask class to have an A4 size book or sheet to cover the handout, and a pen or pencil.
Hand out the Estimation test and ask students to cover it as they receive it. Tell students that they must:

- estimate each answer;
- not copy any numbers down; and
- make no marks on the sheet except for their answer

Ask students to uncover the paper and print their name on it.
Read them the instructions at the top of the test paper.
Read the first item aloud and ask them to proceed.
After 30 seconds, say: "30 seconds is up. Move to Item 2", but do not read any further items. Continue in this way for each item to the end of the test, and then please collect papers immediately.

Directions for conducting Computation Test

Tell students that they will now be given a set of computation items where they are not permitted to use a calculator, but can use any other method they wish.
Hand out the papers and ask students to print their names on them.
Read the instructions at the top of the paper to the students.
Read the first item and ask students to proceed. After 4 minutes (Yr 5) or 3 minutes (Yr 7) or 2.5 minutes (Yr 9), say:" Time to move to Item 2". Continue in this way for each item to the end of the test, and then please collect papers immediately.

Conclusion

Please bundle up the two sets of papers with a class list for my marking.

APPENDIX C: ESTIMATION TESTS

Year 5: Estimation Name:________________________

Directions: ***You have 30 seconds for each estimate. Do not calculate exact answers. Do not write anything, except to ring the letter for your choice.***

1. Without calculating an exact answer, circle the best estimate for:
   9965 + 8972 + 8138 + 8090

   a. 24 000 b. 30 000 c. 36 00 d. 42 000

2. Without calculating the exact answer, circle the best estimate for:
   2333 + 435 + 23 + 9

   a A little less than 2 000 b. A little more than 2 000
   c. A lot less than 2 000 d. A lot more than 2 000

3. Without calculating the exact answer, circle the best estimate for: 312 – 119

   a. A little less than 100 b. A little more than 100
   c. A lot less than 100 d. A lot more than 100

4. Without calculating the exact answer, circle the best estimate for: 4012 – 998

   a. A little less than 3 000 b. A little more than 3 000
   c. A lot less than 3 000 d. A lot more than 3 000

5. Without calculating the exact answer, circle the best estimate for: 18 x 19

   a. 190 b. 390 c. 400 d. 490

6. Without calculating the exact answer, circle the best estimate for: 51 x 48

   a. A little less than 2 500 b. A little more than 2 500
   c. A lot less than 2 500 d. A lot more than 2 500

7. Without calculating an exact answer, circle the best estimate for: 598 ÷ 9

   a. 600 ÷ 10 b. 600 ÷ 9 c. 500 ÷ 10 d. 500 ÷ 9

PLEASE TURN OVER

8. Without calculating an exact answer, circle the best estimate for:
   590.43 + 312.5

a.	A little less than 900	b.	A little more than 900
c.	A lot less than 900	d.	A lot more than 900

9. Without calculating an exact answer, circle the best estimate for:
   96.7 + 147.4 + 62.75 + 36.8

   a. 250 b. 300 c. 350 d. 400

10. Without calculating an exact answer, circle the best estimate for: 0.72 - 0.009

   a. 0. 06 b. 0.6 c. 0.07 d. 0.7

Year 7: Estimation Name: ____________________

Directions: ***You have 30 seconds for each estimate. Do not calculate exact answers. Do not write anything, except to ring the letter for your choice.***

1. Without calculating an exact answer, circle the best estimate for:
   $9965 + 8972 + 8138 + 8090$

   a. 24 000 b. 30 000 c. 36 000 d. 42 000

2. Without calculating the exact answer, circle the best estimate for: $312 - 119$

   a. A little less than 100 b. A little more than 100
   c. A lot less than 100 d. A lot more than 100

3. Without calculating the exact answer, circle the best estimate for: 18×19

   a. 190 b. 390 c. 400 d. 490

4. Without calculating the exact answer, circle the best estimate for: 51×48
4

   a. A little less than 2 500 b. A little more than 2 500
   c. A lot less than 2 500 d. A lot more than 2 500

5. Without calculating an exact answer, circle the best estimate for: $598 \div 9$

   a. $600 \div 10$ b. $600 \div 9$ c. $500 \div 10$ d. $500 \div 9$

6. Without calculating an exact answer, circle the best estimate for $^3/_4 + {}^1/_2$

   a. 1 b. 3 c. 4 d. 6

7. Without calculating an exact answer, circle the best estimate for: $^7/_8 - {}^3/_4$

   a. 0 b. 1 c. 3 d. 4

8. Without calculating an exact answer, circle the best estimate for: $^1/_4$ of 796

   a. A little less than 200 b. A little more than 200
   c. A lot less than 200 d. A lot more than 200

PLEASE TURN OVER

9. Without calculating an exact answer, circle the best estimate for: $^5/_8$ of 512

a. A little less than 300 b. A little more than 300
c. A lot less than 300 d. A lot more than 300

10. Without calculating an exact answer, circle the best estimate for:
590.43 + 312.5

a. A little less than 900 b. A little more than 900
b. A lot less than 900 d. A lot more than 900

11. Without calculating an exact answer, circle the best estimate for:
96.7 + 147.4 + 62.75 + 36.8

a. 250 b. 300 c. 350 d. 400

12. Without calculating an exact answer, circle the best estimate for: 0.72 - 0.009

a. 0. 06 b. 0.6 c. 0.07 d. 0.7

13. Without calculating an exact answer, circle the best estimate for: 0.5 x 840

a. 840 ÷ 2 b. 5 x 840 c. 5 x 8400 d. 0.50 x 84

14. Without calculating the exact answer, circle the best estimate for: 87 x 0.09

a. A little less than 87 b. A little more than 87
c. A lot less than 87 d. A lot more than 87

14. Without calculating the exact answer, circle the best estimate for: 54 ÷ 0.09

a. A little less than 54 b. A little more than 54
c. A lot less than 54 d. A lot more than 54

Year 9: Estimation

Name:____________________

Directions: You have 30 seconds for each estimate. Do not calculate exact answers. Do not write anything, except to ring the letter for your choice.

1. Without calculating an exact answer, circle the best estimate for:
   $9965 + 8972 + 8138 + 8090$

   a. 24 000 b. 30 000 c. 36 000 d. 42 000

2. Without calculating the exact answer, circle the best estimate for: $312 - 119$

   a. A little less than 100 b. A little more than 100
   c. A lot less than 100 d. A lot more than 100

3. Without calculating the exact answer, circle the best estimate for: 18×19

   a. 190 b. 390 c. 400 d. 490

4. Without calculating the exact answer, circle the best estimate for: 51×48

   a. A little less than 2 500 b. A little more than 2 500
   c. A lot less than 2 500 d. A lot more than 2 500

5. Without calculating an exact answer, circle the best estimate for: $598 \div 9$

   a. $600 \div 10$ b. $600 \div 9$ c. $500 \div 10$ d. $500 \div 9$

6. Without calculating an exact answer, circle the best estimate for $^{7}/_{8} + {}^{12}/_{13}$

   a. 1 b. 2 c. 19 d. 21

7. Without calculating an exact answer, circle the best estimate for $^{7}/_{8} - {}^{3}/_{4}$

   a. 0 b. 1 c. 3 d. 4

8. Without calculating an exact answer, circle the best estimate for: $^{5}/_{8}$ of 512

   a. A little less than 300 b. A little more than 300
   c. A lot less than 300 d. A lot more than 300

9. Without calculating an exact answer, circle the best estimate for: $^{2}/_{3} \times {}^{3}/_{4}$

   a. 1 b. 2 c. 6 d. 12

10. Without calculating an exact answer, circle the best estimate for: $^{5}/_{6} \div {}^{2}/_{3}$

   a. 1 b. 2 c. 3 d. 5

PLEASE TURN OVER

11. Without calculating an exact answer, circle the best estimate for:
$96.7 + 147.4 + 62.75 + 36.8$

a. 250 b. 300 c. 350 d. 400

12. Without calculating an exact answer, circle the best estimate for: 0. 72 - 0.009

a. 0. 07 b. 0.7 c. 0.6 d. 0.06

13. Without calculating the exact answer, circle the best estimate for: 87 x 0.09

a. A little less than 87
b. A little more than 87
c. A lot less than 87
d. A lot more than 87

14. Without calculating an exact answer, circle the best estimate for: 19.4 x 46.1

a. 20 x 50 b. 20 x 45 c. 20 x 40 d. 10 x 50

15. Without calculating the exact answer, circle the best estimate for: 54 ÷ 0.09

a. A little less than 54
b. A little more than 54
c. A lot less than 54
d. A lot more than 54

16. Without calculating an exact answer, circle the best estimate for: 563.7 ÷ 2.93

a. 20 b. 130 c. 190 d. 280

17. Without calculating an exact answer, circle the best estimate as a percentage for: $^{7}/_{12}$

a. 7% b. 12% c. 60% d. 70%

18. Without calculating an exact answer, circle the best estimate for: 20% of 198

a. A little less than 40
b. A little more than 40
c. A lot less than 40
d. A lot more than 40

19. Without calculating an exact answer, circle the best estimate for n:
3 : 1 = 7 : n

a. A little less than 2
b. A little more than 2
b. A lot less than 2
d. A lot more than 2

20. Without calculating an exact answer, circle the best estimate for n:
1 : 9 = 1.5 : n

a. A little less than 14
b. A little more than 14
c. A lot less than 14
d. A lot more than 14

APPENDIX D: COMPUTATION TESTS

Year 5: Computation Name:______________

Directions: *You have about 4 minutes for each question. Find an exact answer using any method you like, except with a calculator. Show your working in the second column and write your answer in the first column.*

Questions & Answers	**Work Space**
1. Calculate: 9965 + 8972 + 8138 + 8090 __________________	
2. Calculate: 2333 + 435 + 23 + 9 __________________	
3. Calculate: 312 – 119 __________________	
4. Calculate: 4012 – 998 __________________	
5. Calculate: 18 x 19 __________________	

6. Calculate: 51 x 48 ____________	
7. Calculate: 598 ÷ 9 ____________________	
8. Calculate: 590.43 + 312.5 ____________________	
9. Calculate: 96.7 + 147.4 + 62.75 + 36.8 ____________________	
10. Calculate: 0.72 - 0.009 ____________________	

Year 7: Computation Name: ____________________

Directions: *You have about 3 minutes for each question. Find an exact answer using any method you like, except with a calculator. Show your working in the second column and write your answer in the first column.*

Questions & Answers	**Work Space**
1. Calculate: $9965 + 8972 + 8138 + 8090$ ____________	
2. Calculate: $312 - 119$ ____________	
3. Calculate: 18×19 ____________	
4. Calculate: 51×48 ____________	
5. Calculate: $598 \div 9$ ____________	
6. Calculate: $^{3}/_{4} + {}^{1}/_{2}$ ____________	
7. Calculate: $^{7}/_{8} - {}^{3}/_{4}$ ____________	

8. Calculate: $^{1}/_{4}$ of 796 ____________________	
9. Calculate: $^{5}/_{8}$ of 512 ____________________	
10. Calculate: 590.43 + 312.5 ____________________	
11. Calculate: 96.7 + 147.4 + 62.75 + 36.8 ____________________	
12. Calculate: 0.72 - 0.009 ____________________	
13. Calculate: 0.5 x 840 ____________________	
14. Calculate: 87 x 0.09 ____________________	
15. Calculate: 54 ÷ 0.09 ____________________	

Year 9: Computation Name: ____________________

Directions: *You have about 2.5 minutes for each question. Find an exact answer using any method you like, except with a calculator. Show your working in the second column and write your answer in the first column.*

Questions & Answers	Work Space
1. Calculate: 9965 + 8972 + 8138 + 8090 ____________________	
2. Calculate: 312 – 119 ____________________	
3. Calculate: 18 x 19 ____________________	
4. Calculate: 51 x 48 ____________________	
5. Calculate: 598 ÷ 9 ____________________	
6. Calculate: $^{7}/_{8} + {}^{12}/_{13}$ ____________________	

7. Calculate: $^{7}/_{8}$ - $^{3}/_{4}$ ________________	
8. Calculate: $^{5}/_{8}$ of 512 ________________	
9. Calculate: $^{2}/_{3}$ x $^{3}/_{4}$ ________________	
10. Calculate: $^{5}/_{6} \div ^{2}/_{3}$ ________________	
11. Calculate: 96.7 + 147.4 + 62.75 + 36.8 ________________	
12. Calculate: 0.72 - 0.009 ________________	
13. Calculate: 87 x 0.09 ________________	
14. Calculate: 19.4 x 46.1 ________________	

PLEASE TURN OVER

15. Calculate: $54 \div 0.09$ ________________	
16. Calculate: $563.7 \div 2.93$ ________________	
17. Calculate: $^{7}/_{12}$ as a percentage ________________	
18. Calculate: 20% of 198 ________________	
19. Calculate n: $3 : 1 = 7 : n$ ________________	
20. Calculate n: $1 : 9 = 1.5 : n$ ________________	

APPENDIX E: INTERVIEWS

Procedures for the Given Interviews

Estimation

- handed over the answer sheet on estimation to the child to go through it;
- picked one question with correct response and asked the child to explain and in the same way moved on to the next item, with a wrong answer;
- repeated that pattern for a few rounds depending on the performances and keeping in mind the topic coverage.

Computation

- handed over the answer sheet on computation to the child to look at;
- picked one question with a correct answer in both the papers and asked the child to explain the procedures/steps;
- next item, with a correct answer in computation but wrong in estimation followed by the other way round, and lastly to both incorrect responses;
- Repeated the pattern for a few more rounds as per their performance.

Interview Questions for the Students

1. Here is the test that you did where you were asked to make estimates. Have a look at it again and see what your results were.
2. Question # is the first one you got correct. Explain how you tried that. [Ask follow-up questions as appropriate]
3. [Repeat (2) with a selection of items both correct & incorrect items as appropriate from each of the topics]
4. Here is the written test that you did where you were asked to calculate. Have a look at it again and see what your results were.

5. Tell me how you worked Question # [with correct answer and a correct matching estimate]. Did you estimate what the answer would be before you did the working? How often do you estimate the answer before calculating? [and appropriate follow-up questions to check for understanding]

6. [Repeat (5) above with another correct item having an incorrect matching estimate]

7. [Repeat (5) above with incorrect item having a correct matching estimate]

8. [Repeat (5) above with incorrect item having an incorrect matching estimate]

9. [Repeat (5), (6), (7), and (8) with another set of items if possible]

APPENDIX F: INTERVIEW ABSTRACTS

The following abstracts represent the type of mathematical thinking and learning taking place in children. Some of them could do it correctly using a very beautiful strategy of their own as shown below:

Q. **9965 + 8972 + 8138 + 8090**

I: Was there any problem solving this question?

Yr. 5: Yes, I had to think about that one…I knew…it has to bit because 8 and 8 are ahum…16, 9 and 9 is 18 so that could be huh, 17…so had to sort of estimate around here…so, when we calculate and think about that twice…it roughly come to 36…and that would be 36000.

I: You got a correct estimate here, that was very good, so, I just would like you to explain how you went about getting that?

Yr. 7: um…I somewhat rounded up to 10 000, 9 000, 8 000 and 8 000…and I thought, it would be more than 24 or 30 000 and it would be less than 42 000.

I: You got it right in both the papers, I just would like to know that…do you remember estimating while calculating this one?

Yr 7: Yeah! I remember estimation about those four 8000's and thought 8 x 4 = 32 and 36, 000 would be closure to it…so, I might get it right!

Q: **4012 – 998**

I: Explain to me how you got this correct?

Yr 5: Well, that …998 equals roughly to 1000 … take away from 4012 equals 3012 which is roughly equals to little more than 3000.

I: How did you go about getting that best estimate?

Yr 7: Wow…take 998 from 4012 would be around 3000 or something or over it…because if 998 takes off all the twelve's and it would be back in 3000…so…it would be little more than 3000.

Q: 590.43 + 312.5

I: What was your problem here?
Yr 5: Ohh…I think…actually the decimal…I could not think well…

I: What was that which confused you most?
Yr 5: Oh yeah! I think, it is the decimal point that confused me…

I: What could that be? Arrangement?
Yr 5: Oh yeah!

I: Did you study this topic before?
Yr 5: Yeah! We did…but can't remember the answer…

Q: $^1/_4$ of 798

I: How did you get it correct in estimation and wrong in computation?
Yr 7: Got rounded to 800 and divided that by 4, which is more closely to 200…so, I picked on a little less than 200.

Q: $^2/_3$ x $^3/_4$

I: You got it correct in computation but wrong in estimation, what could be the reason you think?

Yr 9: Um…because it needs calculator…I can't do things in my head…I need to process for them and everything…and moreover not enough time and could not do that…there is no way…it confused me because…I thought answers would be in fractions as well…and it wasn't in whole numbers and that's why I wasn't sure with that one even though I knew the process on how to do it…I thought, it would be one over something or two over something… and never had a clue that it would be closer to any whole number…

Q: $^7/_8 - {}^3/_4$

I: How did you get this correct estimate?

Yr. 7: Um…I am not really good at that…I just double the both top and bottom number of $^3/_4$ as $^6/_8$, which then subtracted from $^7/_8$ gives $^1/_8$ which is more closer to 0 than any other numbers here.

I: Do you apply such a method to other similar kinds of problem?

Yr. 7: Yeah!

APPENDIX G: RESULTS OF ESTIMATION AND COMPUTATION

Percentages correct on all estimation and computation items across year levels

Topics	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
9965 + 8972 + 8138 + 8090						
a. 24, 000	8		6		7	
b. 30, 000	9		10		10	
c. 36, 000	46	63	68	84	63	77
d. 42, 000	36		16		20	
e. No response	1		0		0	
2333 + 435 + 23 + 9						
a. A little less than 2000	7					
b. A little more than 2000	24					
c. A lot less than 2000	4					
d. A lot more than 2000	63	63				
e. No response	2					
312 – 119						
a. A little less than 100	8		9		14	
b. A little more than 100	46		40		32	
c. A lot less than 100	5		3		0	
d. A lot more than 100	41	47	48	81	53	66
e. No response	0		0		1	
4012 – 998						
a. A little less than 3000	13					
b. A little more than 3000	48	26				
c. A lot less than 3000	13					
d. A lot more than 3000	24					
e. No response	2					
18 x 19						
a. 190	33		24		15	
b. 390	31	13	52	47	49	41
c. 400	12		14		21	
d. 490	22		10		14	
e. No response	2		0		1	
51 x 48						
a. A little less than 2500	28	12	44	60	30	56
b. A little more than 2500	36		30		47	
c. A lot less than 2500	16		13		8	
d. A lot more than 2500	18		13		14	
e. No response	2		0		1	

Topics	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
598 ÷ 9						
a. 600 ÷ 10	9		13		34	
b. 600 ÷ 9	46	17	55	46	52	38
c. 500 ÷ 10	24		19		6	
d. 500 ÷ 9	18		12		1	
e. No response	3		1		7	
590. 43 + 312.5						
a. A little less than 900	22		3			
b. A little more than 900	46	62	88	77		
c. A lot less than 900	3		0			
d. A lot more than 900	29		9			
e. No response	0		0			
96.7 + 147.4 + 62.75 + 36. 8						
a. 250	9		10		11	
b. 300	15		24		18	
c. 350	40	37	48	74	57	70
d. 400	34		18		10	
e. No response	2		0		4	
0.72 - 0.009						
a. 0.06	12		19		22	
b. 0.6	11		21		18	
c. 0.07	48		34		22	
d. 0.7	25	10	26	44	37	47
e. No response	4		0		1	
0.5 x 840						
a. 840 ÷ 2			25	56		
b. 5 x 840			35			
c. 5 x 8400			18			
d. 0.50 x 84			22			
e. No response			0			
87 x 0.09						
a. A little less than 87			18		14	
b. A little more than 87			44		22	
c. A lot less than 87			17	38	41	30
d. A lot more than 87			20		22	
e. No response			1		1	
19.4 x 46.1						
a. 20 x 50					19	
b. 20 x 45					43	14
c. 20 x 40					12	
d. 10 x 50					23	
e. No response					3	

Topics	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
54 ÷ 0.09						
a. A little less than 54			27		22	
b. A little more than 54			32		41	
c. A lot less than 54			21		24	
d. A lot more than 54			20	27	12	7
e. No response			0		1	
563.7 ÷ 2.93						
a. 20					6	
b. 130					4	
c. 190					60	1
d. 280					30	
e. No response					0	
$^{3}/_{4} + ^{1}/_{2}$						
a. 1			33	43		
b. 3			18			
c. 4			27			
d. 6			22			
e. Response			0			
$^{7}/_{8} + ^{12}/_{13}$						
a. 1					28	
b. 2					34	11
c. 19					30	
d. 21					8	
e. No response					0	
$^{7}/_{8} - ^{3}/_{4}$						
a. 0			17	36	13	12
b. 1			31		23	
c. 3			9		23	
d. 4			43		40	
e. No response			0		1	
$^{1}/_{4}$ *of 798*						
a. A little less than 200			40	27		
b. A little more than 200			34			
c. A lot less than 200			8			
d. A lot more than 200			17			
e. No response			1			
$^{5}/_{8}$ *of 512*						
a. A little less than 240			22		26	
b. A little more than 240			38		40	
c. A lot less than 240			26		12	
d. A lot more than 240			14	21	19	14
e. No response			0		3	

Topics	Year 5		Year 7		Year 9	
	ET	WCT	ET	WCT	ET	WCT
$^{2}/_{3} \times {}^{3}/_{4}$						
a. 1					15	55
b. 2					34	
c. 6					33	
d. 12					17	
e. No response					1	
$^{5}/_{6} \div {}^{2}/_{3}$						
a. 1					29	11
b. 2					44	
c. 3					20	
d. 5					4	
e. No response					3	
Percentage for $^{7}/_{12}$						
a. 7%					19	
b. 12%					35	
c. 60%					29	27
d. 70%					14	
e. No response					3	
20% of 198						
a. A little less than 40					23	22
b. A little more than 40					48	
c. A lot less than 40					4	
d. A lot more than 40					25	
e. No response					0	
$3 : 1 = 7 : n, n = ?$						
a. A little less than 2					27	
b. A little more than 2					43	8
c. A lot less than 2					15	
d. A lot more than 2					14	
e. No response					1	
$1 : 9 = 1.5 : n, n = ?$						
a. A little less than 14					22	8
b. A little more than 14					37	
c. A lot less than 14					27	
d. A lot more than 14					11	
e. No response					3	

APPENDIX H: GENDER DIFFERENCES

Gender differences in performance on estimation and computation

	Year 5				Year 7				Year 9			
Topic	ET		WCT		ET		WCT		ET		WCT	
	M	F	M	F	M	F	M	F	M	F	M	F
Whole Numbers												
Addition	65	46	82	69	72	68	90	100	67	61	86	93
Subtraction	49	44	56	41	28	21	42	44	45	47	50	81
Multiplication	35	25	12	19	49	46	55	61	36	47	52	64
Division	49	53	21	24	48	49	48	61	52	49	43	44
Total	198	168	171	153	197	184	235	266	200	204	231	282
Mean	**50**	**42**	**43**	**38**	**49**	**46**	**59**	**67**	**50**	**51**	**58**	**71**
Decimals												
Addition	44	46	80	55	66	60	81	92	48	49	81	44
Subtraction	33	22	23	4	45	20	48	44	43	35	43	44
Multiplication					28	24	52	42	46	23	24	27
Division					10	42	35	29	17	16	7	27
Total	77	68	103	59	149	146	216	207	154	123	155	142
Mean	**39**	**34**	**52**	**30**	**37**	**37**	**54**	**52**	**39**	**31**	**39**	**36**
Fractions												
Addition					38	67	88	71	63	39	31	54
Subtraction					31	20	35	76	14	10	14	10
Multiplication					31	16	19	45	8	12	33	42
Division									24	14	10	21
Total					100	103	142	192	109	75	88	127
Mean					**33**	**34**	**47**	**64**	**27**	**19**	**22**	**32**
Percentages									**55**	**56**	**41**	**25**
Ratios									**46**	**44**	**19**	**11**

Comparison of performance (ET & WCT) between male and female students on items across the year levels

Topics	Year 5				Year 7				Year 9			
	ET		WCT		ET		WCT		ET		WCT	
	M	F	M	F	M	F	M	F	M	F	M	F
9965 + 8972 + 8138 + 8090	49	43	87	80	72	68	90	100	67	61	86	93
2333+ 435 + 23 +9	80	49	77	57								
312 – 119	44	45	56	49	55	42	83	88		47		81
4012 – 998	54	43	56	33								
18 x 19	44	22	10	22	62	46	48	59	43	47	52	81
51 x 48	26	28	13	16	35	46	62	63	29	47	52	47
598 ÷ 9	49	53	21	24	48	49	48	61	52	33	43	65
590. 43 + 312.5	39	51	82	71	79	71	83	90				
96.7 + 147.4 + 62.75 + 36. 8	49	41	77	39	52	49	79	93	48	49	81	44
0.72 – 0.009	33	22	23	4	45	20	48	44	43	65	43	77
0.5 x 840					31	27	62	49				
87 x 0.09					24	20	41	34	29	35	29	44
19.4 x 46.1									62	10	19	10
54 ÷ 0.09					10	42	35	29	10	12	14	12
563.7 ÷ 2.93									24	19	0	42
$^{3}/_{4} + ^{1}/_{2}$					45	10	38	20				
$^{7}/_{8} + ^{12}/_{13}$									24	0	24	2
$^{7}/_{8} - ^{3}/_{4}$					31	20	35	76	14	33	14	7
$^{1}/_{4}$ of 798					41	17	31	51				
$^{5}/_{8}$ of 512					21	15	28	39	10	10	14	63
$^{2}/_{3} \times ^{3}/_{4}$									5	14	52	21
$^{5}/_{6} \div ^{2}/_{3}$									24	53	10	19
Percentage for $^{7}/_{12}$									67	53	48	5
20% of 198									43	58	33	44
3 : 1 = 7 : n, n = ?									62	30	24	19
1: 9 = 1.5 : n, n = ?									29	58	14	2