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The dynamics of phase farming: A mathematical model of economic aspects of switching between cropping and land rehabilitation

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USE OF THESIS

THE DYNAMICS OF PHASE FARMING

A Mathematical Model of Economic Aspects of Switching

between Cropping and Land Rehabilitation

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ABSTRACT

In this thesis we consider the following problem: Suppose that a farmer wishes to determine the best course of action to maximise returns from his / her land which has undergone some form of degradation. In order to rehabilitate the land, the farmer may have to change to a different farming practice for some time until the previous practice becomes profitable again. Switching from cropping to rehabilitation or from rehabilitation to cropping incurs costs. From an economical point of view, the question then arises: When is the optimal time to switch from cropping to rehabilitation and when is it optimal to switch back to cropping again in order to maximise profit? In this thesis, we give a mathematical formulation of the farmer's problem and derive necessary conditions for optimality using the calculus of variations. We then apply our model to the specific case of a rotation between wheat farming and oil mallee plantation. We determine optimal switching times for two scenarios - break even and current performance levels- and explore the effects of the rates of change of the water level and the discount rate on the optimal switching times.

DECLARATION

I certify that this thesis does not incorporate without acknowledgment any material previous submitted for a degree or diploma in any institution of higher education; and that to the best of my knowledge and belief, it does not contain any material previously published or written by another person except where due reference is made in the text.

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CHAPTER 1

INTRODUCTION

In this chapter we introduce the problem we will study and discuss hackground material relevant to the mathematical description.

1.1 INTRODUCTION

The situation we wish to model may be described as follows: Suppose that a farmer wishes to determine the best course of action to maximise returns from his / her land and at the same time try to rehabilitate land. The land is assumed to have been under agricultural exploitation for some time and the soil quality has undergone degradation of some form, such as a depletion of nutrients, a rise in acidity or salinity or possibly soil compaction. The reduction in soil quality leads to a decline in the yearly yield and hence in the profit. Unless some action is taken, farming the land will eventually become unprofitable and the farmer may have to give up farming it completely. There are a number of courses of action that can be taken. In the case of depletion of nutrients, addition of fertiliser may counteract the decline in yield. In the case of increased acidity, an addition of lime might be beneficial. This scenario has been studied by Hertzler and Tierney

(1995). In the case of an increase in salinity, these measures will not work. Here a reversal necessitates the land to be allowed to lie fallow or the planting of deep-rooted perennial trees. While this is regarded an environmentally sound solution, the question arises as to how to make the approach economically viable. There must be some incentive for the farmer to invest money for rehabilitating his / her land. The land should also give some benefit during the rehabilitation period, and the cash flow should be encouraging enough for the farmer to make his / her decision (Barbier, 1990).

The problem of salinity is of particular concern in Western Australia, where clearing of the native bush for cropping has destroyed the balance of the ecosystem. Dryland agriculture reduces the water consumption allowing in the soil a build up of saline water which in turn leads to a mobilisation of stored salt and so to waterlogging and increased salinity (Bartle et al, 1996). One proposal to reverse this type of degradation is that of planting salt water resistant plants or trees in order to lower the water level. In this setting, the following questions arise: Firstly, when should the switch from cropping to rehabilitation occur. Secondly, given that the switch from cropping to rehabilitation has taken place, is it profitable to switch back to cropping and if so, when is the optimal time?

These are the questions for which we shall try to determine an answer.

1.2 PROBLEM SPECIFICATION

1.2.1 Background

Problems involving the optimisation of some performance index incorporating a switching time have been discussed amongst others by Hoel (1977) and Dasgupta et al (1982). In both papers, the setting concerned natural resource management where the substitution for an exhaustible resource in future or the introduction of new technology included some uncertainty factor. However, in both papers, the time at which a switch needed to occur was an exogenous variable and so not a decision variable of the associated optimisation problem. A similar problem had already been considered by Nickell (1977) in the case of an investment decision.

Tomiyama (1985) considered the case where the switching time was a decision variable. The performance index to be maximised can be written as :

$$J = \int_{t_0}^{t_1} L_1(t,x,u)dt + \int_{t_1}^{t_2} L_2(t,x,u)dt \tag{1.1}$$

where t_2 can be either finite or infinite, t_1 is the switching time, u **is a control variable,** x **is the state vector** defined by :

$$x' = \begin{cases} f_1(t,x,u) & \text{on } [t_0, t_1) \\ f_2(t,x,u) & \text{on } (t_1, t_2] \end{cases} \tag{1.2}$$

where :

$$x(t_0) = x_0 ; t_0 : fixed$$
$$x(t_2) : free \tag{1.3}$$

Here L_1 and L_2 are two possible profit functions; f_1 and f_2 are the rates of change of the state variable associated with stages 1 and 2. L_1, L_2, f_1 and f_2 are assumed at least continuously differentiable in x, u and t.

Tomiyama used techniques from control theory to find the necessary conditions for maximising (1.1), subject to (1.2) and (1.3).

Tomiyama and Rossana (1989) expanded the formulation to allow the profit function of the second integrand to explicitly depend on the switching time. None of these authors included a cost for making the switch in their formulation. The switching cost was first considered by Amit (1986) in the setting of the exploitation of a petroleum reservoir. In his formulation, the question was when to optimally switch from primary to secondary recovery. Here the problem was to maximise :

$$\int_{t_0}^{t_1} F(t,x,u)dt + \int_{t_1}^{t_1} G(t,x,u)dt - \phi(t_1, x(t_1), u(t_1))$$

subject to

$$x' = \begin{cases} f(t,x,u) & t_0 \le t \le t_1 \\ g(t,x,u) & t_1 \le t \le t_2 \end{cases}$$

where

$$t_0; x(t_0) = x_0 : \textit{fixed}$$
$$t_1; x(t_1); t_2; x(t_2) : \textit{free}$$

where $\boldsymbol{x}$ and $\boldsymbol{u}$ are n-dimensional and m-dimensional vector valued functions respectively. Amit used techniques from the calculus of variations to solve his problem.

Kamien and Schwartz (1991), in addition, described the case of jumps in the state variable.

The above problems are multistage optimisation problems. Each stage consists of a performance index together with conditions on state variables. Therefore these problems may also be regarded as dynamic programming problems.

Babad (1995) recast the formulation of the multistage optimisation problem in the language of multiprocess theory. This approach allows for the weakening of conditions on the functions describing the n-stage process. His approach will not be pursued here.

Multistage optimisation problems associated with farming practices were introduced by Hertzler in 1990. He formulated a model which can be applied to up to **n** farming practices. Assuming that the steady state is reachable, he introduced the discrete choice model:

$$J_0 = \sum_i Max \int_{t_i}^{t_{i+1}} e^{-\delta(t-t_0)} \pi_i (X_t, z_i) dt + e^{-\delta(t_n - t_0)} J_n (X_{t_n})$$

subject to

$$X_t = g_i(X_t, z_i) \ ; \quad i = 0, 1, ..., n-1; \quad t_i \le t \le t_{i+1}$$

and

X_{t_0} is fixed.

Here π_i and g_i are the annual profit and the rate of change of the land resource at stage i; z_i is a continuous control variable; δ is the discount rate. Hertzler and Tierney (1995) applied this model to determine the optimal management of soil acidity by liming. Their model and our model differ in the following aspects. First, the cost for switching from one farming practice to another was ignored. This means that the objective function of the problem is concave. This is no longer the case in our model. Second, it was also assumed that the system is in steady state. This assumption is unrealistic in our case.

In the paper presented to the 41st Annual Conference for the Australian Agricultural and Resource Economics Society, Gold Coast, Queensland, Schilizzi and Mueller formulated an n-stage problem where costs incurred with the switches were included. The generalised formula then was applied to three stages which was called C-R-C (Cropping – Rehabilitation – Cropping). The fonnula was expressed as follows:

$$J = \max\{\sum_{k=0}^{n} \int_{t_{2k}}^{t_{2k+1}} f^1[(t, x(t), u(t)]dt + \int_{t_{2k+1}}^{t_{2k+2}} f^2[t, x(t), u(t)]dt$$
$$- \varphi_{2k}[t, x(t_{2k+1}), u(t_{2k+1})] - \varphi_{2k+1}[t, x(t_{2k+2}), u(t_{2k+2})]\}$$

subject to

$$x'(t) = \begin{cases} g^1[t, x(t), u(t)] & \text{for } t_{2k} \leq t \leq t_{2k+1} \\ g^2[t, x(t), u(t)] & \text{for } t_{2k+1} \leq t \leq t_{2k+2} \end{cases} \quad \text{and } k = 0,1,..n$$

where

$x(t_0) = x_0$ is fixed and t_{2k+1}, $x(t_{2k+1})$, t_{2k+2}, $x(t_{2k+2})$ are free.

We will consider a special case of this model in this thesis.

1.2.2 Problem Specification

Solving the farmer's problem in full generality requires finding the optimal solution of a multistage optimal control problem in which each type of cultivation is one stage. The time at which the farmer decides to change from one type of cultivation to another is called the switching time. The general problem consists of n stages and n-1 switching times. In this thesis, the problem will be considered as a three-stage problem which may be summarised by the following mathematical description:

Maximise

$$J = \int_{t_0}^{t_1} D(t,x(t),u(t))dt + \int_{t_1}^{t_2} F(t,t_1,x(t),u(t))dt + \int_{t_2}^{t_3} G(t,t_2,x(t),u(t))dt$$
$$-\Phi_1(t_1,x(t_1),u(t_1)) - \Phi_2(t_2,x(t_2),u(t_2)) \qquad (1.4)$$

Subject to :

$$x_i' = \begin{cases} d(t,x(t),u(t)) & t_0 \le t < t_1 \\ f(t,x(t),u(t)) & t_1 \le t < t_2 \\ g(t,x(t),u(t)) & t_2 \le t < t_3 \end{cases} \qquad (1.5)$$

where

- t_0, $x(t_0) = x_0$ are fixed, and t_1, $x(t_1)$, t_2, $x(t_2)$, t_3, $x(t_3)$ are free. (1.6)
- *D, F, G* are profit functions associated with stage 1, 2, 3 respectively.
- u is a control variable.
- x is the state variable (soil quality or water level).
- t_1, t_2 are switching points.
- Φ_1, Φ_2 are switching cost functions.

Our task is to give expressions for ***D, F, G***, x', Φ_1, Φ_2 and to find the optimal switching times from cropping to planting and from planting back to cropping, t_1 and t_2 to maximise profit, given ***D, F, G*** and x

1.3 DATA

Data that may be of use in the modeling of this problem have been collected by Schilizzi and White (1997). However, because of the poor representation of the data, they can only be used as suggestions for choosing reasonable parameters and making assumptions for the research.

In this thesis, we make use of the data from Bartle et al (1996) to decide the switching cost from cropping to planting mallee trees, the density of trees planted, the weight of leaves we harvest yearly (to choose suitable parameters for the tree growth function) and the revenue obtained by selling one ton of leaves.

We use the data from the Department of Agriculture, Western Australia (1988) to decide the average rate which ground water lowers and the shape of the function for the depth of the water table in time.

The rate of increase of water table under cropping is also based on data from the Department of Agriculture (1990-1996). All other parameters which we need to decide are chosen by reasonable guessing.

1.4 STRUCTURE OF THE DISSERTATION

Apart from the introductory chapter, this report contains six more chapters. In **Chapter 2**, we will derive the necessary conditions for maximising the general three-phase optimal control problem using the theory of the calculus of variations. In **Chapter 3**, we will formulate a specific model for three-phase farming. **Chapter 4** contains the calculations for the necessary condition for the specific model from chapter 3. **Chapter 5** describes the Excel workbook which we use to implement the model from chapter 3, the necessary conditions from **Chapter 4** and the parameters essential for the implementation, based on the data. These parameters will be used for the analysis in **Chapter 6**. In **Chapter 6,** we will investigate the impacts of the discount rate and the change of the water table on the optimal solution. We also compare the result we obtain by solving the model using the necessary conditions from **Chapter 4** with the results we obtain by using the Solver tool, without the necessary conditions.

CHAPTER 2

DERIVATION OF NECESSARY CONDITIONS

In this chapter, we will derive the necessary conditions for maximising problem (1.4) subject to (1.5) and (1.6) from chapter 1. The derivation of the necessary conditions for the three-phase optimal control problem will be based on the derivation of the necessary conditions for optimality for the two-phase optimisation problem given by Amit (1986). The techniques which will be used are the techniques of the calculus of variations.

An optimal control problem in its simplest form consists of an objective function to be maximised / minimised together with a first order differential equation describing the evolution of the system. When the specific problem is an economics problem, the objective function is often called the performance index. The performance index depends on two classes of variables, both of which are functions of time. They are state variables and control variables. In this setting, it is not necessary that the number of state variables and the number of control variables are the same. A state variable is ruled by a first order differential equation; a control variable affects the objective function both explicitly and implicitly. The problem we consider in this thesis is of the form:

Maximise

$$J = \int_{t_0}^{t_1} D(t,x_1(t),...,x_n(t),u_1(t),...,u_m(t))dt + \int_{t_1}^{t_2} F(t,t_1,x_1(t),...,x_n(t),u_1(t),...,u_m(t))dt +$$
$$\int_{t_2}^{t_3} G(t,t_2,x_1(t),...,x_n(t),u_1(t),...,u_m(t))dt - \Phi_1(t_1,x_1(t_1),...,x_n(t_1),u_1(t_1),...,u_m(t_1))$$
$$-\Phi_2(t_2,x_1(t_2),...,x_n(t_2),u_1(t_2),...,u_m(t_2))$$

(2.1)

Subject to:

$$x'_i = \begin{cases} d_i(t,x_1(t),...,x_n(t),u_1(t),...,u_m(t)) & t_0 \le t \le t_1 \\ f_i(t,t_1,x_1(t),...,x_n(t),u_1(t),...,u_m(t)) & t_1 \le t \le t_2 \\ g_i(t,t_2,x_1(t),...,x_n(t),u_1(t),...,u_m(t)) & t_2 \le t \le t_3 \end{cases} \quad (2.2)$$

where:

$t_0, x_i(t_0)$ are fixed and $t_1, x_i(t_1), t_2, x_i(t_2), t_3, x_i(t_3)$ are free.

We define the vector-valued functions $x : R \dashrightarrow R^n$ and $u : R \dashrightarrow R^m$ by:

$$x(t) = (x_1(t),...,x_n(t)) \in R^n$$

$$u(t) = (u_1(t),...,u_m(t)) \in R^m$$

Here x is **the state vector** and u is **the control vector** of the problem.

Similarly, we abbreviate:

$$d(t,x(t),u(t)) = (d_1(t,x(t),u(t)),..,d_n(t,x(t),u(t)))$$
$$f(t,x(t),u(t)) = (f_1(t,x(t),u(t)),..f_n(t,x(t),u(t)))$$
$$g(t,x(t),u(t)) = (g_1(t,x(t),u(t)),..,g_n(t,x(t),u(t)))$$

and assume d, f, g to be **continuously differentiable** in t, x, u.

Let $\lambda_k = (\lambda_{k1}(t) \ldots \lambda_{kn}(t))$ ($k = 1, 2, 3$) be the continuously differentiable Lagrange multiplier functions associated with the state variables. As constraint (2.2) depends on t in the interval $[t_0, t_3]$, the Lagrange multipliers have to be functions in t. They are also referred to as the costate variables and represent the marginal value of the associated state variable at the time t (see Kamien and Schwartz, 1991). We define the **Hamiltonian functions** H_1, H_2, H_3:

$$H_1 = D + \sum_{i=1}^{n} \lambda_{1i} d_i = D + \lambda_1 d \qquad t_0 \le t \le t_1$$

$$H_2 = F + \sum_{i=1}^{n} \lambda_{2i} f_i = F + \lambda_2 f \qquad t_1 \le t \le t_2$$

$$H_3 = G + \sum_{i=1}^{n} \lambda_{3i} g_i = G + \lambda_3 g \qquad t_2 \le t \le t_3$$

For simplicity, we assume that there is one state variable x and one control variable u. Thus the problem may be rewritten as maximise (1.4), subject to (1.5) and (1.6)

We will modify the proof by Amit (1986) to derive necessary conditions for optimality in our problem. Our proof and his proof differ in two aspects. Firstly, we have to determine optimality conditions for two switching times and secondly, there is an explicit dependence of the integrands on the switching times. As we will see below, this affects the optimality conditions for the switching time through the addition of terms involving the derivation of the Hamiltonians with respect to the switching time.

Theorem

The necessary conditions for maximising (1.4), subject to (1.5) and (1.6) are:

$$\lambda'_1 = -\frac{\partial H_1^*}{\partial x} \qquad t_0 \le t < t_1 \tag{1}$$

$$\lambda'_2 = -\frac{\partial H_2^*}{\partial x} \qquad t_1 \le t < t_2 \tag{2}$$

$$\lambda'_3 = -\frac{\partial H_2^*}{\partial x} \qquad t_2 \le t < t_3 \tag{3}$$

$$\frac{\partial H_1^*}{\partial u} = 0 \qquad t_0 \le t < t_1 \tag{4}$$

$$\frac{\partial H_2^*}{\partial u} = 0 \qquad t_1 \le t < t_2 \tag{5}$$

$$\frac{\partial H_3^*}{\partial u} = 0 \qquad t_2 \le t < t_3 \tag{6}$$

$$G^*(t_3) + \lambda_3(t_3)g^*(t_3) = 0 \tag{7}$$

$$\lambda_1(t_1^-) + \frac{\partial \Phi_1^*}{\partial x} = \lambda_2(t_1^+) \tag{8}$$

$$\lambda_2(t_2^-) + \frac{\partial \Phi_2^*}{\partial x} = \lambda_3(t_2^+) \tag{9}$$

$$\lambda_3(t_3) = 0 \tag{10}$$

$$\frac{\partial \Phi_1^*}{\partial u} = 0 \tag{11}$$

$$\frac{\partial \Phi_2^*}{\partial u} = 0 \tag{12}$$

If $t_0 < t_1 < t_2 < t_3$, then we must have (13.1.a) and (13.2.a):

$$D^* + \lambda_1(t_1^-)d^* + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt = F^* + \lambda_2(t_1^+)f^* + \frac{\partial \Phi_1^*}{\partial t} \quad (13.1.a)$$

$$F^* + \lambda_2(t_2^-)f^* + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = G^* + \lambda_3(t_2^+)g^* + \frac{\partial \Phi_2^*}{\partial t} \quad (13.2.a)$$

If $t_0 = t_1 < t_2 < t_3$, then we must have (13.1.b) and (13.2.b):

$$D^* + \lambda_1(t_1^-)d^* + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \leq F^* + \lambda_2(t_1^+)f^* + \frac{\partial \Phi_1^*}{\partial t} \quad (13.1.b)$$

$$F^* + \lambda_2(t_2^-)f^* + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = G^* + \lambda_3(t_2^+)g^* + \frac{\partial \Phi_2^*}{\partial t} \quad (13.2.b)$$

If $t_0 < t_1 = t_2 < t_3$, then we must have (13.1.c) and (13.2.c):

$$D^* + \lambda_1(t_1^-)d^* + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \geq F^* + \lambda_2(t_1^+)f^* + \frac{\partial \Phi_1^*}{\partial t} \quad (13.1.c)$$

$$F^* + \lambda_2(t_2^-)f^* + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} = G^* + \lambda_3(t_2^+)g^* + \frac{\partial \Phi_2^*}{\partial t} \quad (13.2.c)$$

If $t_0 < t_1 < t_2 = t_3$, then we must have (13.1.d) and (13.2.d):

$$D^* + \lambda_1(t_1^-)d^* + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt = F^* + \lambda_2(t_1^+)f^* + \frac{\partial \Phi_1^*}{\partial t} \quad (13.1.d)$$

$$F^* + \lambda_2(t_2^-)f^* + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \geq G^* + \lambda_3(t_2^+)g^* + \frac{\partial \Phi_2^*}{\partial t} \quad (13.2.d)$$

If $t_0 < t_1 = t_2 = t_3$, then we must have (13.1.e) and (13.2.e):

$$D^* + \lambda_1(t_1^-)d^* + \int_{t_1}^{t_2} \frac{\partial H_2^*}{\partial t_1} dt \geq F^* + \lambda_2(t_1^+)f^* + \frac{\partial \Phi_1^*}{\partial t} \quad (13.1.e)$$

$$F^*+\lambda_2(t_2^-)f^*+\int_{t_2}^{t_3}\frac{\partial H_3^*}{\partial t_2}dt \ge G^*+\lambda_3(t_2^+)g^*+\frac{\partial \Phi_2^*}{\partial t} \qquad (13.2.e)$$

If $t_0 = t_1 = t_2 < t_3$, then we must have (13.1.f) and (13.2.f):

$$D^*+\lambda_1(t_1^-)d^*+\int_{t_1}^{t_2}\frac{\partial H_2^*}{\partial t_1}dt \le F^*+\lambda_2(t_1^+)f^*+\frac{\partial \Phi_1^*}{\partial t} \qquad (13.1.f)$$

$$F^*+\lambda_2(t_2^-)f^*+\int_{t_2}^{t_3}\frac{\partial H_3^*}{\partial t_2}dt \le G^*+\lambda_3(t_2^+)g^*+\frac{\partial \Phi_2^*}{\partial t} \qquad (13.2.f)$$

If $t_0 = t_1 < t_2 = t_3$, then we must have (13.1.g) and (13.2.g):

$$D^*+\lambda_1(t_1^-)d^*+\int_{t_1}^{t_2}\frac{\partial H_2^*}{\partial t_1}dt \le F^*+\lambda_2(t_1^+)f^*+\frac{\partial \Phi_1^*}{\partial t} \qquad (13.1.g)$$

$$F^*+\lambda_2(t_2^-)f^*+\int_{t_2}^{t_3}\frac{\partial H_3^*}{\partial t_2}dt \ge G^*+\lambda_3(t_2^+)g^*+\frac{\partial \Phi_2^*}{\partial t} \qquad (13.2.g)$$

where D^*, F^*, G^*, d^*, f^*,g^*, are values of the corresponding D, F, G, d, f, g evaluated at the optimal t, x, and u.

PROOF

In this proof, we will calculate the first variation of J, and derive the necessity conditions from it. For brevity, the time dependence of functions λ_k, x and u will not be shown. We denote by

u^* : the optimal control function on $[t_0, t_3]$, with the corresponding switching times t_1, t_2.

x^* : the state variable.

J^* : the maximum profit achieved corresponding to the optimal switching points.

δt_i : small changes (positive or negative) (i = 1, 2, 3) in t_1, t_2 and t_3.

Let J be the profit attained for x, u and switching times $t_1+\delta t_1$, $t_2+\delta t_2$, $t_3+\delta t_3$, then:

$$
\begin{aligned}
J-J^* = & \int_{t_0}^{t_1+\delta t_1} D(t,x,u)dt + \int_{t_1+\delta t_1}^{t_2+\delta t_2} F(t,t_1+\delta t_1,x,u)dt + \int_{t_2+\delta t_2}^{t_3+\delta t_3} G(t,t_2+\delta t_2,x,u)dt \\
& -\Phi_1(t_1+\delta t_1,x(t_1+\delta t_1),u(t_1+\delta t_1)) - \Phi_2(t_2+\delta t_2,x(t_2+\delta t_2),u(t_2+\delta t_2)) \\
& -\{\int_{t_0}^{t_1} D(t,x^*,u^*)dt + \int_{t_1}^{t_2} F(t,t_1,x^*,u^*)dt + \int_{t_2}^{t_3} G(t,t_2,x^*,u^*)dt - \Phi_1(t_1,x^*(t_1),u^*(t_1)) \\
& -\Phi_2(t_2,x^*(t_2),u^*(t_2))\}
\end{aligned}
\tag{2.3}
$$

We now rewrite the above integrals as follows:

$$\int_{t_0}^{t_1+\delta t_1} D(t,x,u)dt = \int_{t_0}^{t_1} D(t,x,u)dt + \int_{t_1}^{t_1+\delta t_1} D(t,x,u)dt$$

$$\int_{t_1+\delta t_1}^{t_2+\delta t_2} F(t,x,u)dt = \int_{t_1}^{t_2} F(t,t_1+\delta t_1,x,u)dt - \int_{t_1}^{t_1+\delta t_1} F(t,t_1+\delta t_1,x,u)dt + \int_{t_2}^{t_2+\delta t_2} F(t,t_1+\delta t_1,x,u)dt$$

$$\int_{t_2+\delta t_2}^{t_3+\delta t_3} G(t,x,u)dt = \int_{t_2}^{t_3} G(t,t_2+\delta t_2,x,u)dt - \int_{t_2}^{t_2+\delta t_2} G(t,t_2+\delta t_2,x,u)dt + \int_{t_3}^{t_3+\delta t_3} G(t,t_2+\delta_2,x,u)dt$$

Then (2.3) can be written as:

$$
\begin{aligned}
J-J^* = & \int_{t_1}^{t_1+\delta t_1} D(t,x,u)dt - \int_{t_1}^{t_1+\delta t_1} F(t,t_1+\delta t_1,x,u)dt + \int_{t_2}^{t_2+\delta t_2} F(t,t_1+\delta t_1,x,u)dt \\
& -\int_{t_2}^{t_2+\delta t_2} G(t,t_2+\delta t_2,x,u)dt + \int_{t_3}^{t_3+\delta t_3} G(t,t_2+\delta t_2,x,u)dt
\end{aligned}
$$

$$+\int_{t_0}^{t_1}[D(t,x,u)+\lambda_1 d(t,x,u)-\lambda_1 x'-D(t,x^*,u^*)-\lambda_1 d(t,x^*,u^*)+\lambda_1 x^{*'}]dt$$

$$+\int_{t_1}^{t_2}[F(t,t_1+\delta t_1,x,u)+\lambda_2 f(t,t_1+\delta t_1,x,u)-\lambda_2 x'$$

$$-F(t,t_1,x^*,u^*)-\lambda_2 f(t,t_1,x^*,u^*)+\lambda_2 x^{*'}]dt$$

$$+\int_{t_2}^{t_3}[G(t,t_2+\delta t_2,x,u)+\lambda_3 g(t,t_2+\delta t_2,x,u)$$

$$-\lambda_3 x'-G(t,t_2,x^*,u^*)-\lambda_3 g(t,t_2,x^*,u^*)+\lambda_3 x^{*'}]dt$$

$$-\Phi_1(t_1+\delta t_1,x(t_1+\delta t_1),u(t_1+\delta t_1))-\Phi_2(t_2+\delta t_2,x(t_2+\delta t_2),u(t_2+\delta t_2))$$

$$+\Phi_1(t_1,x^*(t_1),u^*(t_1))+\Phi_2(t_2,x^*(t_2),u^*(t_2)) \qquad (2.4)$$

We denote by h_i the difference between x and x^* in the interval $[t_{i-1}, t_i]$:

$$h_1=x-x^*\Rightarrow h_1'=x'-x^{*'} \qquad t_0\le t<t_1 \qquad (2.5)$$

$$h_2=x-x^*\Rightarrow h_2'=x'-x^{*'} \qquad t_1\le t<t_2 \qquad (2.6)$$

$$h_3=x-x^*\Rightarrow h_3'=x'-x^{*'} \qquad t_2\le t<t_3 \qquad (2.7)$$

Using integration by parts, we have:

$$-\int_{t_0}^{t_1}\lambda_1 h_1'dt=-\lambda_1(t_1^-)h_1(t_1)+\int_{t_0}^{t_1}(x-x^*)\lambda_1'dt \qquad (2.8)$$

$$-\int_{t_1}^{t_2}\lambda_2 h_2'dt=-\lambda_2(t_2^-)h_2(t_2)+\lambda_2(t_1^+)h_2(t_1)+\int_{t_1}^{t_2}(x-x^*)\lambda_2'dt \qquad (2.9)$$

$$-\int_{t_2}^{t_3}\lambda_3 h_3'dt=-\lambda_3(t_3^-)h_3(t_3)+\lambda_3(t_2^+)h_3(t_2)+\int_{t_2}^{t_3}(x-x^*)\lambda_3'dt \qquad (2.10)$$

In (2.8), we have used $h_1(t_0) = x(t_0) - x^*(t_0) = 0$, as $x(t_0) = x^*(t_0) = x_0$

The numbers $\lambda_1(t_1^-)$ and $\lambda_2(t_1^+)$ are given by

$$\lambda_1(t_1^-) = \lim_{t \leftarrow t_1^-} \lambda_1(t) \qquad \text{and } \lambda_2(t_1^+) = \lim_{t \to t_1^+} \lambda_2(t)$$

Similarly

$$\lambda_2(t_2^-) = \lim_{t \to t_2^-} \lambda_2(t) \qquad \text{and } \lambda_3(t_2^+) = \lim_{t \to t_2^+} \lambda_3(t)$$

With (2.5)-(2.9), (2.4) becomes:

$$J - J^* = \int_{t_1}^{t_1+\delta t_1} D(t,x,u)dt - \int_{t_1}^{t_1+\delta t_1} F(t,t_1+\delta t_1,x,u)dt + \int_{t_2}^{t_2+\delta t_2} F(t,t_1+\delta t_1,x,u)dt$$
$$- \int_{t_2}^{t_2+\delta t_2} G(t,t_2+\delta t_2,x,u)dt + \int_{t_3}^{t_3+\delta t_3} G(t,t_2+\delta t_2,x,u)dt$$

$$+ \int_{t_0}^{t_1} [D(t,x,u) + \lambda_1 d(t,x,u) + \lambda_1' x - D(t,x^*,u^*) - \lambda_1 d(t,x^*,u^*) - \lambda_1' x^*]dt$$

$$+ \int_{t_1}^{t_2} [F(t,t_1+\delta t_1,x,u) + \lambda_2 f(t,t_1+\delta t_1,x,u) + \lambda_2' x$$

$$- F(t,t_1,x^*,u^*) - \lambda_2 f(t,t_1,x^*,u^*) + \lambda_2' x^*]dt$$

$$+ \int_{t_2}^{t_3} [G(t,t_2+\delta t_2,x,u) + \lambda_3 g(t,t_2+\delta t_2,x,u) + \lambda_3' x$$

$$- G(t,t_2,x^*,u^*) - \lambda_3 f(t,t_2,x^*,u^*) + \lambda_3' x^*]dt$$

$$- \lambda_1(t_1^-)h_1(t_1) - \lambda_2(t_2^-)h_2(t_2) + \lambda_2(t_1^+)h_2(t_1)$$

$$- \lambda_3(t_3)h_3(t_3) + \lambda_3(t_2^+)h_3(t_2)$$

$$-\Phi_1(t_1+\delta t_1, x(t_1+\delta t_1), u(t_1+\delta t_1))$$

$$-\Phi_2(t_2+\delta t_2, x(t_2+\delta t_2), u(t_2+\delta t_2))$$

$$+\Phi_1(t_1, x^*(t_1), u^*(t_1)) + \Phi_2(t_2, x^*(t_2), u^*(t_2)) \tag{2.11}$$

Assuming δt_1, δt_2, δt_3, $x - x^*$ *and* $u - u^*$ to be close to 0, we may make the following approximations:

$$\int_{t_1}^{t_1+\delta t_1} D(t,x,u)dt \approx D(t_1, x^*(t_1), u^*(t_1))\delta t_1 \equiv D^*(t_1)\delta t_1 \tag{2.12}$$

$$\int_{t_1}^{t_1+\delta t_1} F(t,t_1,x,u)dt \approx F(t_1, x^*(t_1), u^*(t_1))\delta t_1 \equiv F^*(t_1)\delta t_1 \tag{2.13}$$

$$\int_{t_2}^{t_2+\delta t_2} F(t,t_1,x,u)dt \approx F(t_2, t_1, x^*(t_2), u^*(t_2))\delta t_2 \equiv F^*(t_2)\delta t_2 \tag{2.14}$$

$$\int_{t_2}^{t_2+\delta t_2} G(t,t_2,x,u)dt \approx G(t_2, x^*(t_2), u^*(t_2))\delta t_2 \equiv G^*(t_2)\delta t_2 \tag{2.15}$$

$$\int_{t_3}^{t_3+\delta t_3} G(t,t_2,x,u)dt \approx G(t_3, t_2, x^*(t_3), u^*(t_3))\delta t_3 \equiv G^*(t_3)\delta t_3 \tag{2.16}$$

We now approximate D, F, G, d, f and g by the linear part of Taylor expansions about t, t_1, t_2, x^*, u^*. Then

$$\int_{t_0}^{t_1} [D(t,x,u) + \lambda_1 d(t,x,u) + \lambda_1' x - D(t,x^*,u^*) - \lambda_1 d(t,x^*,u^* - \lambda_1' x^*)]dt$$

$$\approx \int_{t_0}^{t_1} [D(t,x,u) + D_x(t,x^*,u^*)(x-x^*) + D_u(t,x^*,u^*)(u-u^*)$$

$$+\lambda_1 d(t,x^*.u^*) + \lambda_1 d_x(t,x^*,u^*)(x-x^*) + \lambda_1 d_u(t,x^*,u^*)(u-u^*)$$

$$+x\lambda_1{}'-D(t,x^*,u^*)-\lambda_1 d(t,x^*,u^*)-x^*\lambda_1{}']dt$$

$$\approx\int_{t_0}^{t_1}\{[D_x^*+\lambda_1 d_x^*+\lambda_1{}']h_1+[D_u^*+\lambda_1 d_u^*]\delta u\}dt \tag{2.17}$$

Similarly:

$$\int_{t_1}^{t_2}[F(t,t_1+\delta t_1,x,u)+\lambda_2 f(t,t_1+\delta t_1,x,u)+\lambda_2{}'x$$

$$-F(t,t_1,x^*,u^*)-\lambda_2,f(t,t_1,x^*,u^*)-\lambda_2{}'x^*]dt$$

$$\approx\int_{t_1}^{t_2}\{[F_x^*+\lambda_2 f_x^*+\lambda_2{}']h_2+[F_u^*+\lambda_2 f_u^*]\delta u+[F_{t_1}^*+\lambda_2 f_{t_1}^*]\delta t_1\}dt \tag{2.18}$$

$$\int_{t_2}^{t_3}[G(t,t_2+\delta t_2,x,u)+\lambda_3 g(t,t_2+\delta t_2,x,u)+\lambda_3'x$$

$$-G(t,t_2,x^*,u^*)-\lambda_3,g(t,t_2,x^*,u^*)-\lambda_3{}'x^*]dt$$

$$\approx\int_{t_2}^{t_3}\{[G_x^*+\lambda_3 g_x^*+\lambda_3{}']h_3+[G_u^*+\lambda_3 g_u^*]\delta u+[G_{t_2}^*+\lambda_3 g_{t_2}^*]\delta t_2\}dt \tag{2.19}$$

and

$$\begin{aligned}&-[\Phi_1(t_1+\delta t_1,x(t_1+\delta t_1),u(t_1+\delta t_1))-\Phi_1(t_1,x^*(t_1),u^*(t_1))]\\&=-[\Phi_{1x}^*\delta x_1+\Phi_{1t}^*\delta t_1+\Phi_{1u}^*\delta u_1]\end{aligned} \tag{2.20}$$

$$\begin{aligned}&-[\Phi_2(t_2+\delta t_2,x(t_2+\delta t_2),u(t_2+\delta t_2))-\Phi_2(t_2,x^*(t_2),u^*(t_2))]\\&=-[\Phi_{2x}^*\delta x_2+\Phi_{2t}^*\delta t_2+\Phi_{2u}^*\delta u_2]\end{aligned} \tag{2.21}$$

where we have used the following abbreviations:

$$\delta u = u - u^*$$
$$\delta x_1 = x(t_1 + \delta t_1) - x^*(t_1)$$
$$\delta x_2 = x(t_2 + \delta t_2) - x^*(t_2)$$
$$\delta u_1 = u(t_1 + \delta t_1) - u^*(t_1)$$
$$\delta u_2 = u(t_2 + \delta t_2) - u^*(t_2)$$

From (2.12) to (2.21), (2.11) can be rewritten as:

$$J - J^* = D^*(t_1)\delta t_1 - F^*(t_1)\delta t_1 + F^*(t_2)\delta t_2 - G^*(t_2)\delta t_2 + G^*(t_3)\delta t_3$$

$$+ \int_{t_0}^{t_1} \{[D_x^* + \lambda_1 d_x^* + \lambda_1']h_1 + [D_u^* + \lambda_1 d_u^*]\delta u\}dt$$

$$+ \int_{t_1}^{t_2} \{[F_x^* + \lambda_1 f_x^* + \lambda_2']h_2 + [F_u^* + \lambda_2 f_u^*]\delta u + [F_{t_1}^* + \lambda_2 f_{t_1}^*]\delta t_1\}dt$$

$$+ \int_{t_2}^{t_3} \{[G_x^* + \lambda_3 g_x^* + \lambda_3']h_3 + [G_u^* + \lambda_3 g_u^*]\delta u + [G_{t_2}^* + \lambda_3 g_{t_2}^*]\delta t_2\}dt$$

$$-[\Phi_{1x}^* \delta x_1 + \Phi_{1t}^* \delta t_1 + \Phi_{1u}^* \delta u_1] - [\Phi_{2x}^* \delta x_2 + \Phi_{2t}^* \delta t_2 + \Phi_{2u}^* \delta u_2]$$

$$- \lambda_1(t_1^-)h_1(t_1) - \lambda_2(t_2^-)h_2(t_2) + \lambda_2(t_1^+)h_2(t_1)$$

$$- \lambda_3(t_3)h_3(t_3) + \lambda_3(t_2^+)h_3(t_2) \qquad (2.22)$$

We next calculate approximations for δx_1, δx_2 and δx_3 approximations for $h_1(t_1)$ and $h_2(t_2)$.

Let $t_1 + \delta t_1$ be the termination time, $x(t_1 + \delta t_1)$ be the value of the state at $t_1 + \delta t_1$ and $x^*(t_1)$ be the value of the optimal state at t_1.

Then

$$\delta x_1 \approx x(t_1+\delta t_1)-x^*(t_1) \Rightarrow \delta x_1 + x^*(t_1) \approx x(t_1+\delta t_1)$$
$$\approx x(t_1)+x'(t_1)\delta t_1$$
$$\approx x(t_1)+x^{*\prime}(t_1)\delta t_1$$

where we have replaced $x'(t_1)$ by $x^{*\prime}(t_1)$ as their values are approximately equal.

Then we find

$$\delta x_1 - x^{*\prime}(t_1)\delta t_1 = x(t_1)-x^*(t_1) = h_1(t_1)$$

and so

$$h_1(t_1) = \delta x_1 - d^*(t_1^-)\delta t_1 \tag{2.23}$$

where we have used

$$d^*(t_1) = x^{*\prime}(t_1)$$

Let $t_1+\delta t_1$ be the initial time for phase 2, $x(t_1+\delta t_1)$ be the value of the state at $t_1+\delta t_1$, and $x^*(t_1)$ be the value of the state at t_1, then with

$$\delta x_1 - x^{*\prime}(t_1)\delta t_1 = x(t_1)-x^*(t_1) = h_2(t_1)$$

we have:

$$h_2(t_1) \approx \delta x_1 - f^*(t_1^+)\delta t_1 \tag{2.24}$$

Similarly

$$h_2(t_2) \approx \delta x_2 - f^*(t_2^-)\delta t_2 \tag{2.25}$$

$$h_3(t_2) \approx \delta x_2 - g^*(t_2^+)\delta t_2 \tag{2.26}$$

$$h_3(t_3) \approx \delta x_3 - g^*(t_3)\delta t_3 \tag{2.27}$$

Substituting (2.23)-(2.27) into (2.22), we have:

$$J - J^* = D^*(t_1)\delta t_1 - F^*(t_1)\delta t_1 + F^*(t_2)\delta t_2 - G^*(t_2)\delta t_2 + G^*(t_3)\delta t_3$$

$$+\int_{t_0}^{t_1}\{[D_x^* + \lambda_1 d_x^* + \lambda_1^{'}]h_1 + [D_u^* + \lambda_1 d_u^*]\delta u\}dt$$

$$+\int_{t_1}^{t_2}\{[F_x^* + \lambda_2 f_x^* + \lambda_2^{'}]h_2 + [F_u^* + \lambda_2 f_u^*]\delta u + [F_{t_1}^* + \lambda_2 f_{t_1}^*]\delta t_1\}dt$$

$$+\int_{t_2}^{t_3}\{[G_x^* + \lambda_3 g_x^* + \lambda_3^{'}]h_3 + [G_u^* + \lambda_3 g_u^*]\delta u + [G_{t_2}^* + \lambda_3 g_{t_2}^*]\delta t_2\}dt$$

$$-[\Phi_{1x}^*\delta x_1 + \Phi_{1t}^*\delta t_1 + \Phi_{1u}^*\delta u_1] - [\Phi_{2x}^*\delta x_2 + \Phi_{2t}^*\delta t_2 + \Phi_{2u}^*\delta u_2]$$

$$-\lambda_1(t_1^-)(\delta x_1 - d^*(t_1)\delta t_1) - \lambda_2(t_2^-)(\delta x_2 - f^*(t_2^-)\delta t_2)$$

$$+\lambda_2(t_1^+)(\delta x_1 - f^*(t_1^+)\delta t_1$$

$$-\lambda_3(t_3)(\delta x_3 - g^*(t_3)\delta t_3) + \lambda_3(t_2^+)(\delta x_2 - g^*(t_2^+)\delta t_2) \qquad (2.28)$$

Rearranging (2.28) and collecting terms yields

$$\partial J = \{[D^* + \lambda_1(t_1^-)d^*] - [F^* + \lambda_2(t_1^+)f^*] - \Phi_{1t}^* + \int_{t_1}^{t_2}\frac{\partial H_2^*}{\partial t_1}dt\}\delta t_1$$

$$+\{[F^* + \lambda_2(t_2^-)f^*] - [G^* + \lambda_3(t_2^+)g^*] - \Phi_{2t}^* + \int_{t_2}^{t_3}\frac{\partial H_3^*}{\partial t_2}dt\}\delta t_2$$

$$+[G^*(t_3) + \lambda_3(t_3)g^*]\delta t_3$$

$$+[\lambda_2(t_1^+) - \lambda_1(t_1^-) - \Phi_{1x}^*]\delta x_1 + [\lambda_3(t_2^+) - \lambda_2(t_2^-) - \Phi_{2x}^*]\delta x_2$$

$$-\lambda_3(t_3)\delta x_3 - \Phi_{1u}^*\delta u_1 - \Phi_{2u}^*\delta u_2$$

$$\int_{t_0}^{t_1}\left(\frac{\partial H_1^*}{\partial x} + \lambda_1\right)h_1 dt + \int_{t_0}^{t_1}\left(\frac{\partial H_1}{\partial u}\right)\delta u dt$$

$$\int_{t_1}^{t_2}\left(\frac{\partial H_2^*}{\partial x}+\lambda_2\right)h_2dt+\int_{t_1}^{t_2}\left(\frac{\partial H_2}{\partial u}\right)\delta udt$$

$$\int_{t_2}^{t_3}\left(\frac{\partial H_3^*}{\partial x}+\lambda_3\right)h_3dt+\int_{t_2}^{t_3}\left(\frac{\partial H_3}{\partial u}\right)\delta udt \tag{2.29}$$

We will use (2.29) to extract the necessary conditions to maximise (1.4), subject to (1.5) and (1.6). If we choose:

$$\lambda_1'=\frac{\partial H_1}{\partial x}=-[D_x+\lambda_1 d_x] \qquad t\in[t_0,t_1) \tag{2.30}$$

$$\lambda_2'=\frac{\partial H_2}{\partial x}=-[F_x+\lambda_2 f_x] \qquad t\in[t_1,t_2) \tag{2.31}$$

$$\lambda_3'=\frac{\partial H_3}{\partial x}=-[G_x+\lambda_3 g_x] \qquad t\in[t_2,t_3) \tag{2.32}$$

then (2.29) simplifies to

$$\begin{aligned}J-J^*=&\int_{t_0}^{t_1}[D_u^*+\lambda_1 d_u^*]\delta udt+\int_{t_1}^{t_2}[F_u^*+\lambda_2 f_u^*]\delta udt+\int_{t_2}^{t_3}[G_u^*+\lambda_3 g_u^*]\delta udt\\
&+\{[D^*+\lambda_1(t_1^-)d^*]-[F^*+\lambda_2(t_1^+)f^*]-\Phi_{1t}^*+\int_{t_1}^{t_2}\frac{\partial H_2^*}{\partial t_1}dt\}\delta t_1\\
&+\{[F^*+\lambda_2(t_2^-)f^*]-[G^*+\lambda_3(t_2^+)g^*]-\Phi_{2t}^*+\int_{t_2}^{t_3}\frac{\partial H_3^*}{\partial t_2}dt\}\delta t_2\\
&+[G^*(t_3)+\lambda_3(t_3)g^*]\delta t_3\\
&+[\lambda_2(t_1^+)-\lambda_1(t_1^-)-\Phi_{1x}^*]\delta x_1+[\lambda_3(t_2^+)-\lambda_2(t_2^-)-\Phi_{2x}^*]\delta x_2\\
&-\lambda_3(t_3)\delta x_3-\Phi_{1u}^*\delta u_1-\Phi_{2u}^*\delta u_2\end{aligned} \tag{2.33}$$

It is possible for the increments δt_1, δt_2, δt_3, δx_1, δx_2, δu_1 and δu_2 to be equal to 0, thus the requirement of optimality leads to:

$$D_u^* + \lambda_1 d_u^* = 0 \qquad t_0 \le t < t_1 \tag{2.34}$$

$$F_u^* + \lambda_2 f_u^* = 0 \qquad t_1 \le t < t_2 \tag{2.35}$$

$$G_u^* + \lambda_3 g_u^* = 0 \qquad t_2 \le t < t_3 \tag{2.36}$$

Therefore we obtain

$$\begin{aligned}\partial J = &\{[D^* + \lambda_1(t_1^-)d^*] - [F^* + \lambda_2(t_1^+)f^*] - \Phi_{1t}^* + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt\}\delta t_1 \\
&+ \{[F^* + \lambda_2(t_2^-)f^*] - [G^* + \lambda_3(t_2^+)g^*] - \Phi_{2t}^* + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt\}\delta t_2 \\
&+ [G^*(t_3) + \lambda_3(t_3)g^*]\delta t_3 \\
&+ [\lambda_2(t_1^+) - \lambda_1(t_1^-) - \Phi_{1x}^*]\delta x_1 + [\lambda(t_2^+) - \lambda(t_2^-) - \Phi_{2x}^*]\delta x_2 \\
&- \lambda_3(t_3)\delta x_3 - \Phi_{1u}^*\delta u_1 - \Phi_{2u}^*\delta u_2 \end{aligned} \tag{2.37}$$

If $\delta x_1, \delta x_2, \delta t_2, \delta x_3, \delta t_3, \delta u_1, \delta u_2$ are both independent and free, then an optimal solution must satisfy:

$$\lambda_1(t_1^-) + \Phi_{1x}^* = \lambda_2(t_1^+) \tag{2.38}$$

$$\lambda_2(t_2^-) + \Phi_{2x}^* = \lambda_3(t_2^+) \tag{2.39}$$

$$\lambda_3(t_3) = 0 \tag{2.40}$$

$$\Phi_{1u}^* = 0 \tag{2.41}$$

$$\Phi_{2u}^* = 0 \tag{2.42}$$

$$G^*(t_3) + \lambda_3(t_3)g^*(t_3) = 0 \tag{2.43}$$

Finally, (2.37) becomes:

$$\partial J = \{[D^* + \lambda_1(t_1^-)d^*] - [F^* + \lambda_2(t_1^+)f^*] - \Phi_{1t}^* + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt\}\delta t_1$$

$$+\{[F^* + \lambda_2(t_2^-)f^*] - [G^* + \lambda_3(t_2^+)g^*] - \Phi_{2t}^* + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt\}\delta t_2 \qquad (2.44)$$

If $t_0 < t_1 < t_2 < t_3$, δt_1 and δt_2 are free, and they can be positive or negative, *J-J** is non-positive if:

$$[D^* + \lambda_1(t_1^-)d^*] + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt = [F^* + \lambda_2(t_1^+)f^*] + \Phi_{1t}^* \qquad (2.45.a)$$

$$[F^* + \lambda_2(t_2^-)f^*] + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = [G^* + \lambda_3(t_2^+)g^*] + \Phi_{2t}^* \qquad (2.45.b)$$

If $t_0 = t_1 < t_2 < t_3$, δt_1 is non-negative and δt_2 is free (positive or negative), *J-J** is non-positive if:

$$[D^* + \lambda_1(t_1^-)d^*] + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt \leq [F^* + \lambda_2(t_1^+)f^*] + \Phi_{1t}^* \qquad (2.45.c)$$

$$[F^* + \lambda_2(t_2^-)f^*] + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = [G^* + \lambda_3(t_2^+)g^*] + \Phi_{2t}^* \qquad (2.45.d)$$

If $t_0 < t_1 = t_2 < t_3$, δt_1 is non-positive and δt_2 is free, *J-J** is non-positive if:

$$[D^* + \lambda_1(t_1^-)d^*] + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt \geq [F^* + \lambda_2(t_1^+)f^*] + \Phi_{1t}^* \qquad (2.45.e)$$

$$[F^* + \lambda_2(t_2^-)f^*] + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt = [G^* + \lambda_3(t_2^+)g^*] + \Phi_{2t}^* \qquad (2.45.f)$$

If $t_0 < t_1 < t_2 = t_3$ δt_1 is free (positive or negative) and δt_2 is non-positive, *J-J** is non-positive if:

$$[D^* + \lambda_1(t_1^-)d^*] + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt = [F^* + \lambda_2(t_1^+)f^*] + \Phi_{1t}^* \qquad (2.45.g)$$

$$[F^* + \lambda_2(t_2^-)f^*] + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \geq [G^* + \lambda_3(t_2^+)g^*] + \Phi_{2t}^* \qquad (2.45.h)$$

If $t_0 < t_1 = t_2 = t_3$, δt_1 and δt_2 are non-positive, *J-J** is non-positive if:

$$[D^* + \lambda_1(t_1^-)d^*] + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt \geq [F^* + \lambda_2(t_1^+)f^*] + \Phi_{1t}^* \qquad (2.45.i)$$

$$[F^* + \lambda_2(t_2^-)f^*] + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \geq [G^* + \lambda_3(t_2^+)g^*] + \Phi_{2t}^* \qquad (2.45.j)$$

If $t_0 = t_1 = t_2 < t_3$, δt_1 and δt_2 are non-negative, *J-J** is non-positive if:

$$[D^* + \lambda_1(t_1^-)d^*] + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt \leq [F^* + \lambda_2(t_1^+)f^*] + \Phi_{1t}^* \qquad (2.45.k)$$

$$[F^* + \lambda_2(t_2^-)f^*] + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \leq [G^* + \lambda_3(t_2^+)g^*] + \Phi_{2t}^* \qquad (2.45.l)$$

If $t_0 = t_1 < t_2 = t_3$, δt_1 is non-negative and δt_2 is non-positive, *J-J** is non-positive if:

$$[D^* + \lambda_1(t_1^-)d^*] + \int_{t_1}^{t_3} \frac{\partial H_2^*}{\partial t_1} dt \geq [F^* + \lambda_2(t_1^+)f^*] + \Phi_{1t}^* \qquad (2.45m)$$

$$[F^* + \lambda_2(t_2^-)f^*] + \int_{t_2}^{t_3} \frac{\partial H_3^*}{\partial t_2} dt \leq [G^* + \lambda_3(t_2^+)g^*] + \Phi_{2t}^* \qquad (2.45.n)$$

This concludes the derivation of necessary conditions for an optimal solution of (1.4) subject to (1.5) and (1.6).

In our problem we have one state variable per stage: this is the depth of the water level, and the control variable is the density of the trees grown in the second phase.

CHAPTER 3

MODEL FORMULATION

In this chapter, we will simplify our general model for the purposes of testing and for the development of a spreadsheet. Specifically, we will assume that the only control variable is the density of the trees grown in the second phase. For the first and third phase the problem is uncontrolled. This means that the types of crop to be grown in the cropping phases are predetermined. This will lead to a lack of necessity constraints. Therefore, we will need to make more assumptions in the next chapter in order to be able to solve the problem.

We will make further simplifying assumptions concerning the topography of the land and the behaviour of the water level with time during cropping and rehabilitation phases.

The specific assumptions will be described in section 3.1. In section 3.2, we will state the simplified model, and the specific settings for the variables will be given in section 3.3.

3.1 ASSUMPTIONS

We will assume that the land to which the model applies is flat and homogeneous in composition. The saline water level is site-specific. We will regard the water depth to be positive in the direction from the surface to the centre of the earth. We assume that the rate of change of the water level is dependent on the depth of the water level. In cropping phases, the water level will be raised close to the surface, this is reversed in the rehabilitation phase, where trees decrease the water level.

We will further assume that the rate of decrease of the depth of saline ground water is be directly proportional to the density of the trees. However, there is an upper bound for the number of trees that can be planted per hectare. We will assume that tree density is constant and it must be a positive whole number. That is, the density is a discrete variable and not dependent on time.

3.2 FORMULATION

3.2.1 Water Level

We will assume that in the cropping phases, the rate of change of the water level with respect to time can be described by an exponential decay model. This model has been chosen in preference to a logistic model to take into account the assumption that the water level has already started to rise. We assume that the land has been used for cropping for some time span already. During the rehabilitation phase, the function describing the water level obeys a logistic model. We therefore have the state variable ruled by:

$$x' = \begin{cases} d(t,x(t)) = -\alpha x & 0 \le t < t_1 \\ f(t,x(t),u) = \beta x u\left(1 - \dfrac{x}{m}\right) & t_1 \le t < t_2 \\ g(t,x(t)) = -\gamma x & t_2 \le t < t_3 \end{cases} \tag{3.1}$$

Here, m denotes the maximum depth of the ground water to which the water level can be lowered, while α and γ are intrinsic rates for the increase of the water level with time; β is the intrinsic rate for the decrease of the water level by one tree during the rehabilitation phase (see Schilizzi and Mueller, 1997). The function u is the tree density function. It denotes the number of trees planted in phase 2 and $u \le D_{max}$, where D_{max} is the maximum number of trees planted per hectare. Here u is **a control variable** for phase 2.

We may solve (3.1) to obtain

$$x = \begin{cases} k_1 e^{-\alpha t} & 0 \le t < t_1 \\ \dfrac{k_2 m e^{\beta u t}}{1 + k_2 e^{\beta u t}} & t_1 \le t < t_2 \\ k_3 e^{-\gamma t} & t_2 \le t < t_3 \end{cases} \tag{3.2}$$

where k_1, k_2 and k_3 are constants.

If we require the continuity of the water level at the switching times t_1 and t_2, then we get from the conditions:

$$x(t_1^-) = x(t_1^+)$$

$$x(t_2^-) = x(t_2^+)$$

$$x(0) = x_0$$

the following values for the constants k_1, k_2 and k_3.

$$k_1 = x_0$$

$$k_2 = \frac{k_1 e^{-\beta u t_1}}{m e^{\alpha t_1} - k_1}$$

$$k_3 = \frac{k_2 m e^{\gamma t_2 + \beta u t_2}}{k_2 e^{\beta u t_2} + 1}$$

A graph depicting the behaviour of the water level in time is given in figure 3.1.

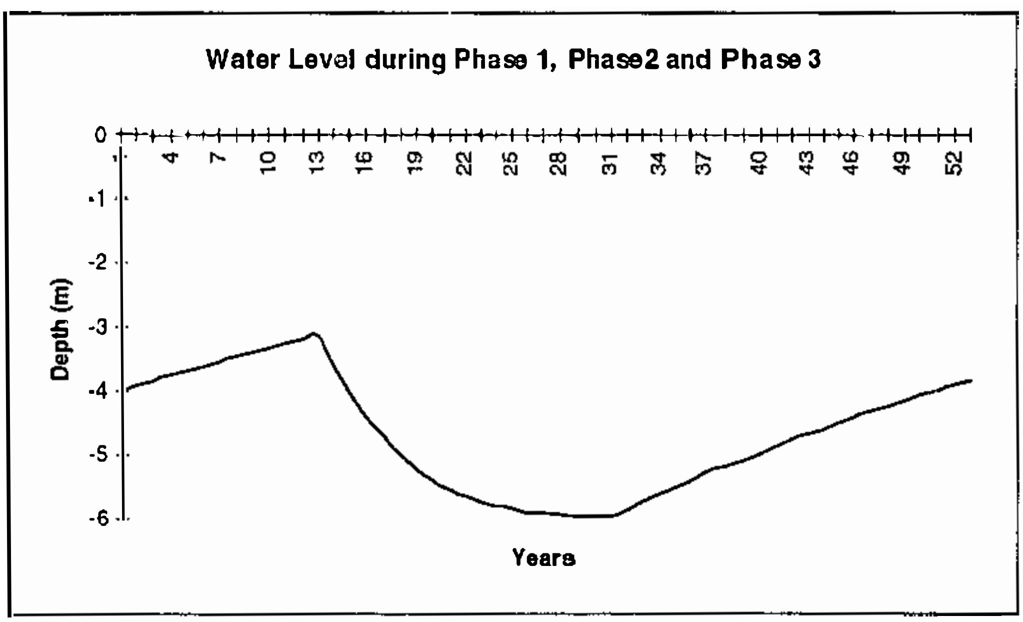

Figure 3.1: *Depth of the saline water level for the case when t_1 = 12, t_2 = 30 and t_3 = 50. For readability, the graph is drawn with the positive direction from the centre to the surface of the earth.*

3.2.2 *Profit Functions*

We assume that in phase 1 and phase 3, wheat is grown and that trees are planted in phase 2 for rehabilitation. The annual profit calculated in today's value is given by:

Yearly profit = (Selling price * Yield – Cost) * Discount factor.

The yearly yield is directly proportional to the maximum yield when there is no salinity; directly proportional to the average depth of ground water in that year; and inversely proportional to the maximum depth of water level. The requirement of direct proportionality of the yield to the depth of the ground water can be explained by the fact that ground water contains salt. When the ground water rises near the surface of the earth, salt will accumulate close to the roots of the crops. This will result in a reduction of the yield.

$$Yield = \frac{Yield\_Max \times x}{m}$$

The inverse proportionality of the yield to the maximum depth of the water level will guarantee that the yield will be a maximum when x is equal to m. This leads to the following yield function in phase 1:

$$Y_1 = \frac{Y_{01} \times x}{m}$$

where $\boldsymbol{m}$ is the maximum depth of water table, $\boldsymbol{Y_{01}}$ is the maximum yield (tons / per hectare) when there is no salinity. The profit function $\boldsymbol{D}$ for cropping in phase 1 then is given by:

$$D(t,x) = (p_1 \times Y_1 - c_1) = (p_1 \times \frac{Y_{01} \times x}{m} - c_1) \times e^{-rt} \tag{3.3}$$

where p_1 is the price of the crops per ton and c_1 is the yearly cropping cost, which is assumed to be constant, for phase 1, and r is the discount rate. Similarly, the profit function for phase 3 is:

$$G(t,x) = (p_3 \times \frac{Y_{03} \times x}{m} - c_3) \times e^{-rt} \tag{3.4}$$

where p_3 is the price of the crops per ton, c_3 is the yearly cropping cost, which is also constant, in phase 3, and Y_{03} is the maximum yield (tons / hectare) when there is no salinity.

We will assume that in phase 2, trees are planted which will contribute to profit via the sales of oil extracted from the leaves. The yield function in this phase is the function describing the tree growth. We assume that trees grow fast in early years and that the tree growth slows down with time. Thus the equation of the tree growth is exponential and is described as:

$$T = \bar{L}(1 - e^{-l(t-t_1)})$$

where $\bar{L}$ is the maximum canopy mass (tons / tree) which can be harvested when the density is less than D_{max} and l is the growth rate of a tree. (See Schilizzi and Mueller, 1997). Here the tree growth rate does not explicitly depend on water level. The profit function for phase 2 is:

$$F(t) = R_2 \times u \times \bar{L}(1 - e^{-l(t-t_1)}) \times e^{-rt} \tag{3.5}$$

where R_2 is the revenue obtained by selling oil extracted from one ton of leaves after subtracting the cost and u is the density of trees per hectare.

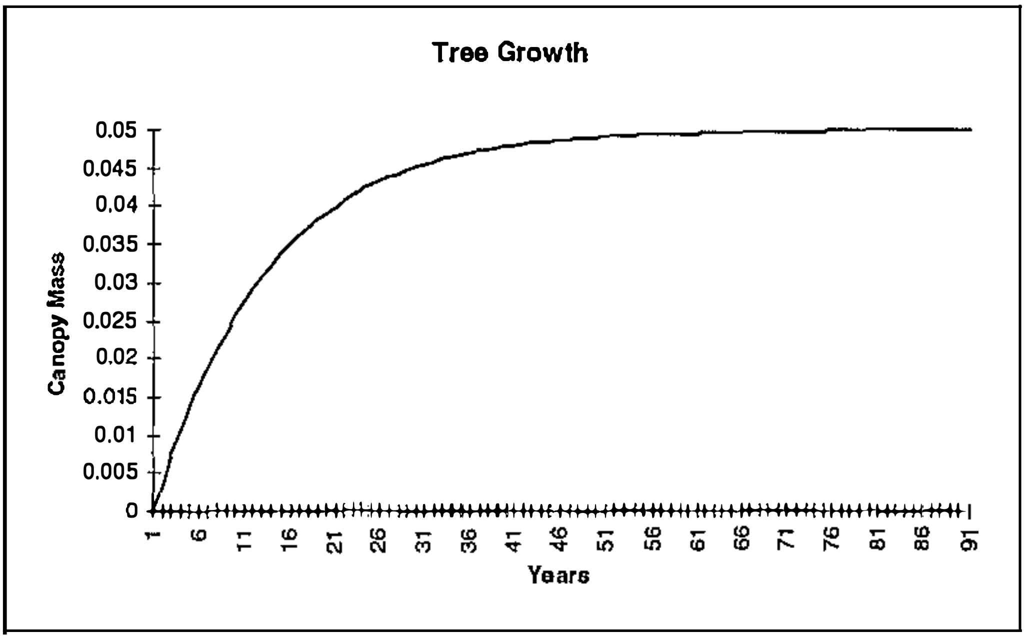

Figure 3.2: The growth of canopy mass of a tree.

It is obvious that D, F and G are all **continuously differentiable** in all arguments.

3.2.3 Formulation of the Switching Costs

The cost for switching from phase 1 to phase 2 consists of the fixed cost per hectare and the cost for planting young trees and caring for them during their initial growth stages. The fixed cost is an establishment cost such as the cost for preparing the land or for fencing. The equation for the switching cost from phase 1 to phase 2 is:

$$\Phi_1(t,u) = (Sw_1 + c_2 \times u) \times e^{-rt} \tag{3.6}$$

where Sw_1 denotes the fixed cost, c_2 is the cost for buying and planting a tree and u is the density of trees per hectare. Here Sw_1 and c_2 are assumed to be constants.

For switching back to cropping, we will assume that the only cost involved is that for clearing the land. This cost may be offset by the sale of the wood resulting from the clearing. Thus

$$\Phi_2(t) = (Sw_2) \times e^{-rt} \tag{3.7}$$

where the establishment cost. Sw_2 may be both negative or positive.

Having formulated the specific model, we will find the necessary conditions for solving t_1, t_2, t_3, and u based on the necessary conditions we obtained in chapter 2.

CHAPTER 4

THE NECESSARY CONDITIONS FOR THE SPECIFIC MODEL

In this chapter, we will find the necessary conditions for solving the specific model which we formulated in chapter 3. Apart from the conditions obtained from chapter 2, it is necessary to make some further assumptions for our particular problem. Firstly, because of the setting proposed in the introductory chapter, our particular model must have at least two phases: the first cropping phase and the rehabilitation phase. Secondly, as the first and third phases are uncontrolled, there will not be enough constraints for obtaining a unique solution as some necessary conditions from chapter 2 cannot be used. The additional assumptions will allow us to narrow our search for the optimal solution and they will be stated and interpreted in sections 4.1 and 4.2

4.1 ASSUMPTIONS

4.1.1 For our particular model, we will assume that the first cropping phase exists, ie., the farmer must be cropping and considering the possibility to switch from cropping to rehabilitation to reverse the degradation. This leads to the requirement that the first switching time t_1 be positive. We will assume that the length of the cultivation we consider for three phases cannot last longer than 100 years, ie. $t_3 < 100$.

4.1.2 We will make the assumption that an unprofitable farming practice will be abandoned. This can be included by requiring the yearly net profit to be non-negative.

4.2 THE NECESSARY CONDITIONS FOR THE SPECIFIC MODEL

In order to find the necessary conditions for solving the specific model, we first will forming the Hamiltonian functions associated with phase 1, phase 2 and phase 3.

For $0 \leq t < t_1$, the Hamiltonian for phase 1 is given by

$$H_1 = D + \lambda_1 d = \left(\frac{p_1 Y_{01} x}{m} - c_1 \right) \times e^{-rt} - \alpha \lambda_1 x \tag{4.1}$$

For $t_1 \leq t < t_2$, we have:

$$H_2 = F + \lambda_2 f = R_2 u \bar{L}(1 - e^{-l(t-t_1)}) e^{-rt} + \beta \lambda_2 u x \left(1 - \frac{x}{m} \right) \tag{4.2}$$

For $t_2 \leq t < t_3$, the Hamiltonian is given by

$$H_3 = G + \lambda_3 g = \left(\frac{p_3 Y_{03} x}{m} - c_3\right) \times e^{-rt} - \gamma\lambda_3 x \tag{4.3}$$

Conditions (1), (2) and (3) of the general case in chapter 2 give the following conditions for λ_1, λ_2, λ_3:

$$\lambda_1' = -\frac{\partial H_1}{\partial x} = \alpha\lambda_1 - \frac{p_1 Y_{01} e^{-rt}}{m} \tag{4.4}$$

$$\lambda_2' = -\frac{\partial H_2}{\partial x} = \beta\lambda_2 u\left(\frac{2x}{m} - 1\right) \tag{4.5}$$

$$\lambda_3' = -\frac{\partial H_3}{\partial x} = \gamma\lambda_3 - \frac{p_3 Y_{03} e^{-rt}}{m} \tag{4.6}$$

Equations (4.4) and (4.6) are first order inhomogeneous linear differential equations and (4.5) is a homogeneous linear differential equation. Solving (4.4), (4.5) and (4.6), we have:

$$\lambda_1(t) = \frac{p_1 Y_{01} e^{-rt}}{m(r+\alpha)} + w_1 e^{\alpha t} \tag{4.7}$$

$$\lambda_2(t) = w_2 (k_2 e^{\beta u t} + 1)^2 e^{-\beta u t} \tag{4.8}$$

$$\lambda_3(t) = \frac{p_3 Y_{03} e^{-rt}}{m(r+\gamma)} + w_3 e^{\gamma t} \tag{4.9}$$

where w_1, w_2 and w_3 are constants. Since phases 1 and phase 3 are uncontrolled, conditions (4) and (6) do not apply, and as the control $\boldsymbol{u}$ in phase 2 does not depend on t, condition (5) is not valid either.

From condition (7), we have:

$$G(t_3) + \lambda_3(t_3)\, g(t_3) = 0 \tag{4.10}$$

and therefore

$$\lambda_3(t_3) = - G(t_3) / g(t_3) \qquad (4.11)$$

We use (4.11) as a boundary condition for solving for w_3. Substituting $t = t_3$ into (4.9), we have:

$$\lambda_3(t_3) = \frac{p_3 Y_{03} e^{-rt}}{m(r+\gamma)} + we^{\gamma t_3} \qquad (4.12)$$

From (4.13) and (4.14):

$$w_3 = \left[-\frac{G(t_3)}{g(t_3)} - \frac{p_3 Y_{03} e^{-rt_3}}{m(r+\gamma)} \right] e^{-\gamma t_3}$$

$$= \frac{p_3 Y_{03} e^{-(r+\gamma)t_3}}{\gamma m} - \frac{c_3 e^{-rt_3}}{\gamma k_3} - \frac{Y_{03} p_3 e^{-(r+\gamma)t_3}}{m(r+\gamma)} \qquad (4.13)$$

where

$$k_3 = \frac{k_2 m e^{(\gamma+\beta u)t_2}}{k_2 e^{(\gamma+\beta u)t_2} + 1}$$

As there is no control in phase 1 and 3, conditions (8) and (10) are no longer valid. Condition (9) gives:

$$\lambda_3(t_2^+) = \lambda_2(t_2^-) \qquad (4.14)$$

We next determine the explicit form for conditions 13.1 and 13.2. The left hand side of condition 13.1 is given by:

$$D(t_1^-) + \lambda_1(t_1^-) d(t_1^-) + \int_{t_1}^{t_3} \frac{\partial H_2}{\partial t_1} dt \qquad (4.15)$$

where

$$D(t_1^-) = \left(\frac{p_1 Y_{01} k_1 e^{-\alpha t_1}}{m} - c_1 \right) e^{-rt_1}$$

and

$$d(t_1^-) = -\alpha k_1 e^{-\alpha t_1}$$

Unfortunately, we cannot calculate $\lambda_1(t_1^-)$ because there is not enough information! Hence, it is not useful to calculate the other elements in condition 13.1. We determine the left hand side of condition 13.2 by:

$$F(t_2^-) + \lambda_2(t_2^-) f(t_2^-) + \int_{t_2}^{t_3} \frac{\partial H_3}{\partial t_2} dt \qquad (4.18)$$

where

$$F(t_2^-) = R_2 u \bar{L}(1 - e^{-l(t_2 - t_1)}) e^{-rt_2}$$

and

$$\lambda_2(t_2^-) f(t_2^-) = \lambda_2(t_2^-) \left(\frac{\beta k_2 m e^{u\beta t_2}}{k_2 e^{u\beta t_2} + 1} \right) \left(1 - \frac{k_2 e^{u\beta t_2}}{k_2 e^{u\beta t_2} + 1} \right)$$

where the value of $\lambda_2(t_2^-)$ is determined by (4.14).

The integral $\int_{t_2}^{t_3} \frac{\partial H_3}{\partial t_2} dt$ is calculated as follows:

$$H_3 = G + \lambda_3 g$$

$$= \left(\frac{p_3 Y_{03} k_3 e^{-\gamma t}}{m} - c_3 \right) e^{-rt} - \gamma k_3 e^{-\gamma t} \times$$

$$\left(\frac{p_3 Y_{03} e^{-rt}}{m(r+\gamma)} + \left(\frac{p_3 Y_{03} k_3 e^{-rt_3} - c_3 m}{m \gamma k_3 e^{-\lambda t_3}} - \frac{p_3 Y_{03} e^{-rt_3}}{m(r+\gamma)} \right) e^{\gamma(t+t_3)} \right)$$

$$= \frac{p_3 Y_{03} k_3 e^{-(\gamma+r)t}}{m} - c_3 e^{-rt} - \frac{\gamma k_3 p_3 Y_{03}}{m(r+\gamma)} \cdot e^{-(\gamma+r)t} + \frac{p_3 \gamma Y_{03} k_3 e^{-(r+\gamma)t_3}}{m(r+\gamma)}$$

$$-\frac{\gamma p_3 Y_{03} k_3 e^{-\gamma t_3} - \gamma c_3 m}{m\gamma e^{-\gamma t_3}} e^{\gamma t_3} \tag{4.19}$$

Differentiating (4. 19) with respect to t_2, we have:

$$\frac{\partial H_3}{\partial t_2} = \left(\frac{p_3 Y_{03}}{m} e^{-(r+\gamma)t} - \frac{\gamma p_3 Y_{03} e^{-(\gamma+r)t}}{m(r+\gamma)} - \frac{p_3 Y_{03} e^{\gamma t_3}}{m} + \frac{p_3 \gamma Y_{03} e^{-(r+\gamma)t_3}}{m(r+\gamma)} \right) \frac{\partial k_3}{\partial t_2}$$

where

$$k_3 = \frac{k_2 m e^{(\gamma+\beta u)t_2}}{k_2 e^{(\gamma+\beta u)t_2} + 1}$$

so

$$\frac{\partial k_3}{\partial t_2} = \frac{k_2 m(\gamma+\beta u) e^{(\gamma+\beta u)t_2}}{(k_2 e^{(\gamma+\beta u)t_2} + 1)^2}$$

Hence

$$\int_{t_2}^{t_3} \frac{\partial H_3}{\partial t_2} dt = \frac{mk_2(\gamma+\beta u)e^{(\gamma+\beta u)t_2}}{(k_2 e^{(\gamma+\beta u)t_2}+1)^2} \left\{ \frac{p_3 Y_{03}}{m(\gamma+r)} \left[e^{-(\gamma+r)t_2} - e^{-(\gamma+r)t_3} \right] \right.$$

$$+ \frac{\gamma p_3 Y_{03}}{m(r+\gamma)^2} \left[e^{-(\gamma+r)t_3} - e^{-(\gamma+r)t_2} \right] - \frac{\gamma p_3 Y_{03} e^{-(r+\gamma)t_3}}{m} (t_3 - t_2)$$

$$\left. + \frac{p_3 \gamma Y_{03} e^{-(r+\gamma)t_3}}{m(r+\gamma)} (t_3 - t_2) \right\} \tag{4.20}$$

Finally, the right hand side of condition 13.2 is given as follows:

$$G(t_2^+) + \lambda_3(t_2^+) g(t_2^+) + \frac{\partial \Phi_2^*}{\partial t}$$

$$= \left(\frac{p_3 Y_{03} k_3 e^{-\gamma t_2}}{m} - c_3 \right) e^{-rt_2} + \lambda_3(t_2^+) k_3 e^{-\gamma t_2} - rSw_2 e^{-rt_2} \tag{4.21}$$

Where $\lambda_3(t_2^+)$ is determined by substituting t_2 into (4.9).

In addition, we have the following constraints from the assumptions 4.1.1 and 4.1.2:

From the assumption 4.1.1, we have

$$t_1 > 0 \tag{4.22}$$

The assumption 4.1.2 gives:

$$D(t_1) \geq 0 \quad \text{and } G(t_3) \geq 0$$

So we have $\dfrac{p_1 Y_{01} x_0 e^{-\alpha t_1}}{m} - c_1 \geq 0$ and $\dfrac{p_3 Y_{03} k_3 e^{-\gamma t_3}}{m} - c_3 \geq 0$

or equivalently

$$t_1 \leq -\frac{1}{\alpha} \ln\left(\frac{c_1 m}{p_1 Y_{01} x_0}\right) \tag{4.23}$$

and

$$t_3 \leq -\frac{1}{\gamma} \ln\left(\frac{c_3 m}{p_3 Y_{03} k_3}\right) \tag{4.24}$$

Having obtained the formulae for d, f, g, D, F, G, Φ_1 and Φ_2 together with the necessary conditions for the specific model, we will find the switching times t_1, t_2; the terminal time t_3 and the density of trees by implementing them in an Excel workbook. The benefit of this is that we can investigate the impacts of the rate of change of the water level and the discount rate on the optimal solution, i.e., the optimal switching times t_1 and t_2 and the total profit. Unfortunately, we could not find condition 13.1 for lack of necessity constraints. Therefore we will use (4.23) and (4.24) as additional constraints in order to solve the problem. However, we are not able to define a unique u, i.e. u will be free. Depending upon on values of chosen $\boldsymbol{u}$, we will obtain different optimal solutions for t_1, t_2, ***and*** t_3. The optimal settings for u, t_1, t_2 and t_3 can then be determined by checking for which value of u the maximum profit occurs.

CHAPTER 5

IMPLEMENTATION OF THE SPECIFIC MODEL

5.1 INTRODUCTION

In this chapter we describe the implementation of the formulae for the water level, yield functions, tree growth function, yearly profit for each phase, switching cost and total profit in an Excel workbook. The necessary conditions are also entered for finding the optimal solution.

5.2 IMPLEMENTATION

Apart from charts showing the evolution of the water depth and total income, the workbook consists of three main worksheets. The three worksheets are **Parameters, Formulation and Optimisation.** If the differential equations describing the water level need to be solved numerically, additional worksheets need to be added.

The worksheet **Parameters** is used to enter all parameters related to the problem. They are:

Price1 (p_1):	Price of crops planted in phase 1, dollars/ton.
Price2 (or Revenue2) (R_2):	Revenue obtained by selling leaves after subtracting harvest cost, dollars/ton.
Price3 (p_3):	Price of crops planted in phase 3, dollars/ton.
Cost1 (c_1):	Cropping cost per hectare per year in phase 1, dollars/ha.
SwCost1(Sw_1):	Fixed cost per hectare for switching from phase 1 to phase2, dollars/ha.
Cost2 (c_2):	Planting cost per tree in phase2, dollars/ tree.
SwCost2 (Sw_2):	Fixed cost per hectare for switching from phase 1 to phase 2, dollars/ha.
Cost3 (c_3):	Cropping cost per hectare per year in phase 3, dollars/ha.
Discount_Rate(r):	Farmer Discount rate.
Alpha(α):	Intrinsic rate of increase of water level in phase1.
Beta (β):	Intrinsic rate of decrease of water level thanks to planting trees in phase 2.
Gamma(γ):	Intrinsic rate of increase of water in phase 3.

m (m):	Maximum water depth that trees can lower.
Y_01:	Maximum crops yield in phase 1 when the water table is at the maximum depth.
Y_03:	Maximum crops yield in phase 3 when the water table is at the maximum depth.
L_Bar ($\bar{L}$):	Maximum canopy mass given by one tree per year, tons.
l (l):	Growth rate of tree leaves.
Den (u):	Density of trees per hectare, the control variable for phase 2.
D_{max} :	Maximum number of trees grown per hectare.
X1_0 (x_0):	Initial depth of water level at the beginning of phase 1, metres.
K1, K2, K3:	The constants of the equations of water depth when solving the differential equation.

PARAMETERS

Name	Cell Referred	Unit	Comment
Price1	800	Dollars/ton	Price of crop planted in phase 1
Price2 (Revenue2)	20	Dollars/ton	Revenue obtained by growing trees in phase 2
Price3	800	Dollars/ton	Price of crop planted in phase 3
Cost1	200	Dollars/ha	Cropping cost for phase 1
SwCost1	1000	Dollars	Fixed cost for switching from phase 1 to phase 2
Cost2	0.5	Dollars/tree	Cost planting 1 tree/plant in phase 2
SwCost2	80	Dollars	Fixed cost for switching from phase 2 to phase 3
Cost3	200	Dollars/ha	Cropping cost for phase 3
Discount Rate	0.1		Farmer discount rate
Alpha	0.03	m/year	Intrinsic rate of change of water level in phase 1
Beta	0.001	m/year	Intrinsic rate of change of water level in phase 2
Gamme	0.03	m/year	Intrinsic rate of change of water level in phase 3
m	6	m	Maximum water depth under tree stand
Y_01	1.5	ton/ha	Max crop yield with no salinity for phase 1
Y_03	1.5	ton/ha	Max crop yield with no salinity for phase 3
L_bar	0.05		Maximum canopy mass haversted per tree
l	0.08		growth rate of tree
Dmax	160		Maximum number of trees per hectare
u	120	trees/ha	Density of tree /ha
X1_0	4	metre	initial depth of saline water (the first phase)
K_1	4.000000		
K_2	0.003321		
K_3	16.672979		

The Parameters worksheet.

The worksheet **Formulation** is used for calculating water depth, profit for three phases, switching costs, yearly profit and cumulative profit. It consists of 18 columns containing formulae for the particular model to be optimised.

Column A:	Time in years.
Column B:	Discount rate in year *t*.
Columns C - E:	Water level in phase1, phase 2, phase 3 of the model.
Column F:	Expected yield in phase 1.
Column G:	Revenue in year *t* for phase 1, before discount.
Column H:	Net revenue in year *t* for phase 1.
Column I:	Expected tree growth in year *t* for phase 2.
Column J:	Revenue obtained from harvesting leaves in year *t*, in phase 2, before discount.
Column K:	Net revenue in year *t* in phase 2.
Column L:	Expected yield in phase 3.
Column M:	Revenue in year *t* for phase 3, before discount.
Column N:	Net revenue in year *t* for phase 3.
Column O:	Switching cost from phase 1 to phase 2.
ColumnP:	Switching cost from phase 2 to phase 3.
Column Q:	Yearly profit.
Column R:	Cumulative profit for the model.

Time Phase1 t	Discount	Water Level 1.0000 X(t)	Water Level 2.0000 X(t)	Water Level 3.0000 X(t)	Yield 1 Y1(t)	Revenue 1 "Price* Yield1" "-Cost1"
0.00	1.0000	4.0000	0.0000	0.0000	1.6667	1166.6667
1.00	0.9139	3.8432	0.0000	0.0000	1.6013	1101.3157
2.00	0.8353	3.6925	0.0000	0.0000	1.5385	1038.5272
3.00	0.7634	3.5477	0.0000	0.0000	1.4782	978.2007
4.00	0.6977	3.4086	0.0000	0.0000	1.4202	920.2396
5.00	0.6376	3.2749	0.0000	0.0000	1.3646	864.5513
6.00	0.5827	3.1465	0.0000	0.0000	1.3110	811.0464
7.00	0.5326	3.0231	0.0000	0.0000	1.2596	759.6396
8.00	0.4868	2.9046	0.0000	0.0000	1.2102	710.2484
9.00	0.4449	2.7907	0.0000	0.0000	1.1628	662.7939
10.00	0.4066	2.6813	2.6813	0.0000	1.1172	617.2001
11.00	0.3716	2.5761	3.0401	0.0000	1.0734	573.3940
12.00	0.3396	2.4751	3.3977	0.0000	1.0313	531.3057
13.00	0.3104	2.3781	3.7442	0.0000	0.9909	490.8676
14.00	0.2837	2.2848	4.0708	0.0000	0.9520	452.0151
15.00	0.2592	2.1952	4.3706	0.0000	0.9147	414.6861
16.00	0.2369	2.1092	4.6395	0.0000	0.8788	378.8207
17.00	0.2165	2.0265	4.8753	0.0000	0.8444	344.3617
18.00	0.1979	1.9470	5.0785	0.0000	0.8113	311.2538
19.00	0.1809	1.8707	5.2505	0.0000	0.7794	279.4440
20.00	0.1653	1.7973	5.3943	0.0000	0.7489	248.8816
21.00	0.1511	1.7268	5.5131	0.0000	0.7195	219.5175
22.00	0.1381	1.6591	5.6102	0.0000	0.6913	191.3049
23.00	0.1262	1.5941	5.6891	0.0000	0.6642	164.1984
24.00	0.1153	1.5316	5.7527	0.0000	0.6382	138.1548
25.00	0.1054	1.4715	5.8037	0.0000	0.6131	113.1324
26.00	0.0963	1.4138	5.8445	0.0000	0.5891	89.0911
27.00	0.0880	1.3584	5.8770	0.0000	0.5660	65.9925
28.00	0.0805	1.3051	5.9028	5.9028	0.5438	43.7997
29.00	0.0735	1.2539	5.9233	5.6714	0.5225	22.4770

A part of the Formulation worksheet.

The third worksheet is the **Optimisation** worksheet. It is used to perform the optimisation. This worksheet contains the first optimal switching time, Time1, from phase 1 to phase 2, the second switching time, Time2, from phase 2 to phase 3; the terminal time, Time3, the density of trees per hectare D. The above variables then are rounded to give OpTime1, OpTime2, OpTime3 and u respectively. The time constraints such as:

$$0 < t_1 < t_2 \tag{5.1}$$

$$t_1 < t_2 \leq t_3 \tag{5.2}$$

$$t_3 \leq 100 \tag{5.3}$$

are entered directly into Solver tool box. The necessary condition 13.2 and the additional constraints (4.23) and (4.24) are used to find the optimal solution. As we could not caiculate condition 13.1, its value is not entered into the spreadsheet. The maximum profit is displayed in the last row of the optimal part of the Optimisation worksheet. Apart from the fact that condition 13.1 cannot be used and constraints (4.23) and (4.24) are inequality expressions, our objective function is not strictly concave. Therefore a solution which satisfies all of the constraints and condition may not be the optimal solution. It only one possible candidates of the optimum (see the explanation in chapter 7). So in order determine the optimal switching times, we may have to solve our problem more than one time and select the solution which gives the best profit. As we limit the length of the three phases in no more than 100 years, if we cannot find t_1, t_2 and t_3 such that the left and right hand sides of condition 13.2 are equal, then there will not be a switch back from rehabilitation to cropping.

Optimisation Switching Time t1, t2 and Profit

Parameters :		**Comments :**
Time0	0	Initial time
Time1	32	Switching time 1
Time2	48	Switching time2
Time3	72	Terminal time
Den	120	Tree density (smaller than 160)
u	120	Tree density (rounded)
OpTime1	**32**	Switching t1 (rounded)
OpTime2	**48**	Switching t2 (rounded)
OpTime3	**72**	Optimal terminal time (rounded)
Cond 13.1.1	**0**	Left side cond 13.1
Cond 13.1.2	**0**	Rightside cond 13.1
Cond 13.2.1	**3**	Left side cond 13.2
Cond 13.2.2	**3**	Right side cond 13.2
Cons. (4.23)	**35**	
Cons. (4.24)	**80**	
Max Profit =	**4036.0**	

The Optimisation worksheet..

5.3 PARAMETERS FOR THE SPECIFIC MODEL

In this section, we determine the parameters that we defined in the previous section in order to solve for a concrete solution. The values of these parameters are based on the data collected by Schilizzi and White (1997). As mentioned in the introductory chapter, because of the poor presentation of the data, we just use them as suggestions for reasonable choices of our parameters.

5.3.1 The Intrinsic Rate of Change of the Water Level.

We assume that wheat is grown in phase 1 and phase 3. Research by Vincent-Llewellyn (1985) indicates that the rate with which the water level rises during cropping wheat is 47mm for an annual rainfall of 162mm and 139mm for an annual rainfall of 258mm, respectively. Based on that information, the intrinsic rates α and γ in phase 1 and phase 3 can be assumed to lie between 0.02 and 0.05. This will make the rate with which the water level rises lie between 0.04m and 0.14m per year. Figure 5.1 depicts the ground water level after 18 years cropping wheat associated with $\alpha = \gamma = 0.02$ and $\alpha = \gamma = 0.05$.

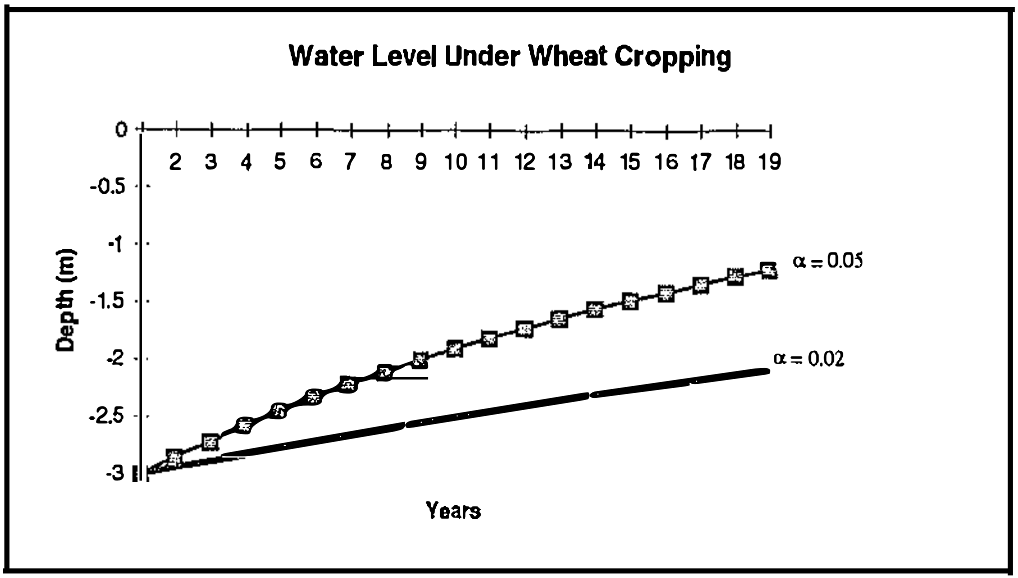

Figure 5.1: *The depth of ground water under cropping with* $\alpha = 0.02$ *and 0.05.*

We assume that mallee trees are planted in phase 2. We do not have data indicating how mallee trees change the depth of the water table, but research carried out at Alex Campbell Plantation (unpublished data, see Schilizzi and White (1997), Tag SA2) indicates that under bluegums, the ground water drops in average of 0.4m per year. There is no information about tree density. Research by Engel and Negus (1988), shows that a density of 80 trees per hectare planted near Narrogin from 1981 to 1986 lowered the water level from about 1.2m to 2.2m below the surface. The rate was higher for higher density. For a density of 160 trees per hectare, the water level was lowered from 1.2m to about 2.6m below the surface. The experiment is valid only on sand. Therefore, based on the above information, the rate of decrease of the water level is considered to be between 0.2m to 0.4m per year. Hence the associated β to be chosen will vary between 0.0010 and 0.0025. Figure 5.2 depicts the ground water level under bluegums after five years of planting when $\beta = 0.0010$ and $\beta = 0.0025$.

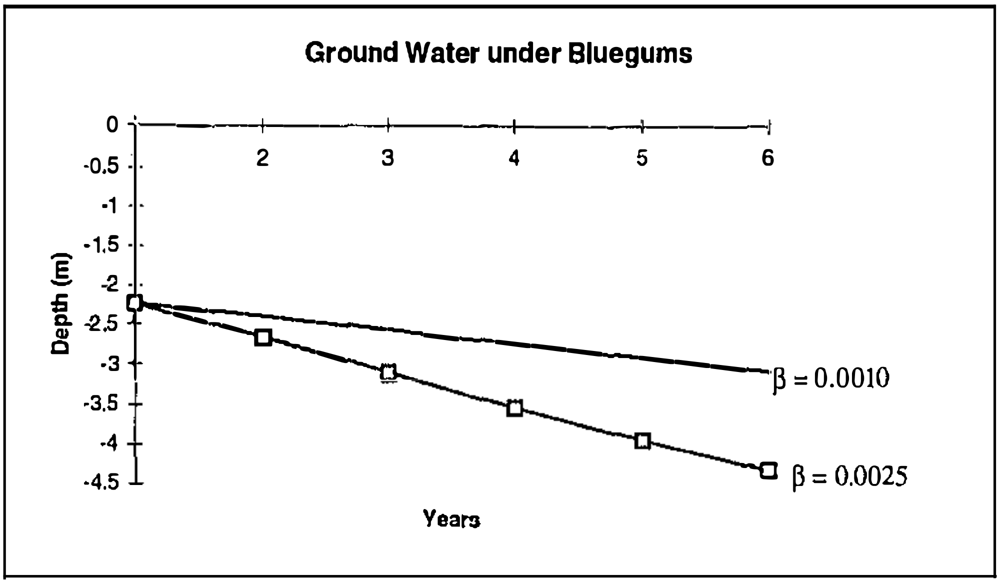

Figure 5.2: The depth of ground water under bluegums planting when β = 0.0010 and 0.0025

5.3.2 Other Parameters

The discount rate r: In this thesis, we will investigate the impact of the discount rate on the optimal switching time, therefore we assume r will be between 0.01 and 0.1.

D_{max}: The maximum tree density per hectare is assumed to be equal to 160 trees / ha.

x_0: The initial depth of the water level in this thesis is 4m.

Y_{01} and Y_{03} Assuming that wheat is grown in the first and the third

	phase, in WA because of the poor quality of the land, the maximum yield of wheat per hectare is about 1.5 tonnes.
p_1 and p_3:	Prices of crop in phase 1 and phase 3 are assumed to be constant and equal to $800 / tonne.
c_1 and c_3	Cropping costs in phase 1 and phase 3 are assumed to be constant and equal to $200 / ha.
m	The maximum depth of the water table is assume to be 6m.
Sw_1:	Fixed cost for switching from phase 1 to phase 2 is $1000 / ha.
Sw_2:	Fixed cost for switching from phase 2 to phase 3 is $80 / ha.
c_3 :	Planting cost is $0.50 / tree.
l:	Growth rate of tree biomass is 0.08

In order to determine the revenue of selling one tonne of leaves (R_2), the maximum canopy mass ($\bar{L}$) given by a tree per year and the control u, we refer to the data given by Bartle et al (1996). Mallee trees were planted together with wheat and occupied 10% of the land in two rows with the length of 100m and width of 10m. In general, two rows with length of 1km and width of 10m is considered one hectare of trees. The yield, cost and profit obtained by the planting are listed in table 5.1.

	Scenario 1	Scenario 2
Parameters	Break even	Assumed level of performance
Leaf Yield	11.5 tonnes / ha / yr	2.5 tonnes / km / yr or 5 tonnes / ha / yr
Oil content	53kg / tonne	40kg / tonne of leaf (freshweight)
Harvest / extraction cost	$34 / tonne	$60 / tonne of leaf
Establishment cost	no break even	$500 / km hedge or $1000 / ha
Oil price	$2.65 kg	$2 / kg

Table 5.1: *Break even production levels for oil mallee, Bartle et al(1996).*

Approximately, if the distance between two trees in a row is 15m, **the density of trees per hectare is 120 trees**. Hence, we assume the following values for maximum canopy mass ($\bar{L}$) and the revenues (R_2):

	Scenario 1	Scenario 2
Parameters	Break even	Assumed level of performance
$\bar{L}$	0.1 tonne / tree. This will give approximately 12 tonnes of leaves per ha per year.	0.05 tonne / tree. This will give approximately 5 tonnes of leaves per ha per year
R_2	$106 / tonne	$20 / tonne

Table 5.2: *Values of the maximum canopy mass per tree $\bar{L}$ and the revenue obtained from one tonne of leaves R_2 for the two scenario.*

In the analysis in chapter 6, we will make use the above parameters and will look at the impacts of the rate of change of the water level and the discount rate on the optimal switching times for these two scenarios.

CHAPTER 6

ANALYSIS

In this chapter, we solve the particular model formulated in **chapter 3** using the necessary conditions and constraints from **chapter 4** and the parameters defined in **chapter 5**. We will explore the impact of the discount rate on the optimal switching times for given rate of change of the water level. We will also investigate how the rate of change of the water level influences the optimisation for some given discount rate.

6.1 THE IMPACT OF THE DISCOUNT RATE ON THE SWITCHING TIMES AND OPTIMAL PROFIT.

In this section we look at the effect of the farmer discount rate on the switching times and on the optimum profit. To do so, we keep the intrinsic rates of change for the water level in the three phases fixed and allow the discount rate to vary from 0.01 to 0.1. We give solutions for the two scenarios summarised in table 5.1. Recall that scenario 1 concerns the break even case and scenario 2 gives figures at which an oil mallee production would operate currently. The results for the two scenarios are listed in tables 6.1 and 6.2. Here the intrinsic rates of change of the water level are $\alpha = 0.02$, $\beta = 0.0010$ and $\gamma = 0.02$.

α	β	γ	r	t_1	t_2	t_3	Profit
0.02	0.0010	0.02	0.01	1	100	100	64428
0.02	0.0010	0.02	0.02	1	100	100	40721
0.02	0.0010	0.02	0.03	1	100	100	27342
0.02	0.0010	0.02	0.04	1	100	100	19347
0.02	0.0010	0.02	0.05	1	100	100	14299
0.02	0.0010	0.02	0.06	1	100	100	10948
0.02	0.0010	0.02	0.07	2	100	100	8624
0.02	0.0010	0.02	0.08	5	100	100	7057
0.02	0.0010	0.02	0.09	9	100	100	6018
0.02	0.0010	0.02	0.10	12	100	100	5310

Table 6.1: *Optimal solutions for scenario 1 when r varies from 0.01 to 0.1 with* $\alpha = \gamma = 0.02$ *and* $\beta = 0.0010$.

For scenario 1, there is no switch back to cropping within the assumed time span of 100 years. However, it is clear that **the higher the discount rate is, the longer duration of the first phase will be**. It means, when the discount rate increases, t_1 also increases. The impact of the discount rate on the optimal profit is obvious. **When the rate goes up, the total profit goes down**. Figure 6.1 depicts the difference of the optimal solution when r = 0.01 and r = 0.1 for scenario 1.

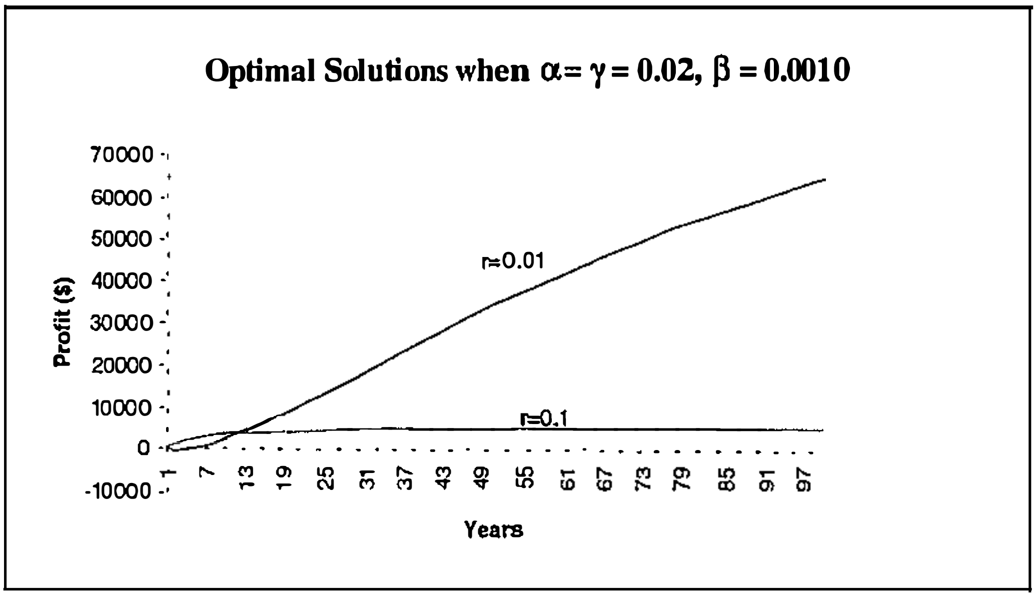

Figure 6.1: *Optimal switching times and the corresponding profit for scenario 1. For $r = 0.01$: $t_1 = 1$, $t_2 = t_3 = 100$; for $r = 0.1$: $t_1 = 12$, $t_2 = t_3 = 100$.*

Table 6.2 lists the optimal switching times and the corresponding profit for scenario 2 obtained by using the necessary conditions from chapter 5 (normal font) and the optimal solutions obtained using the SOLVER tool in Excel by trial and error (in italics). The differences of the solutions by the two methods will be explained in the next chapter. We can see that the optimal solutions obtained by both methods are consistent in terms of how the discount rate affects the switching times and the maximum profit. For $r \leq 0.03$, the duration of the third phase goes beyond our limit of 100 years so the solutions corresponding to those values of r are not analysed. For $r \geq 0.04$, it is obvious that **the higher the discount rate is the longer the first phase becomes and the shorter the second and third phases are (see table 6.2.a).** Figure 6.2 depicts the optimal solutions when $r = 0.04$ and $r = 0.1$ for scenario 2.

				Necessary Conditions				Solver Only			
α	β	γ	r	t_1	t_2	t_3	Profit1	t_1	t_2	t_3	*Profit2*
0.02	0.0010	0.02	0.01	22	54	100	22582	*22*	*47*	*100*	*23128*
0.02	0.0010	0.02	0.02	22	45	100	15804	*22*	*43*	*100*	*15850*
0.02	0.0010	0.02	0.03	23	41	100	11874	*23*	*40*	*100*	*11877*
0.02	0.0010	0.02	0.04	25	40	100	9542	*25*	*40*	*100*	*9542*
0.02	0.0010	0.02	0.05	27	40	100	8070	*27*	*40*	*100*	*8070*
0.02	0.0010	0.02	0.06	29	41	100	7073	*29*	*41*	*100*	*7073*
0.02	0.0010	0.02	0.07	34	46	100	6362	*34*	*46*	*100*	*6362*
0.02	0.0010	0.02	0.08	37	48	100	5804	*37*	*48*	*100*	*5804*
0.02	0.0010	0.02	0.09	55	65	100	5358	*55*	*65*	*100*	*5358*
0.02	0.0010	0.02	0.10	64	73	100	4973	*64*	*73*	*100*	*4973*

Table 6.2: *Optimal solutions for scenario 2 when r varies from 0.01 to 0.1 and* $\alpha = \gamma = 0.02$ *and* $\beta = 0.0010$.

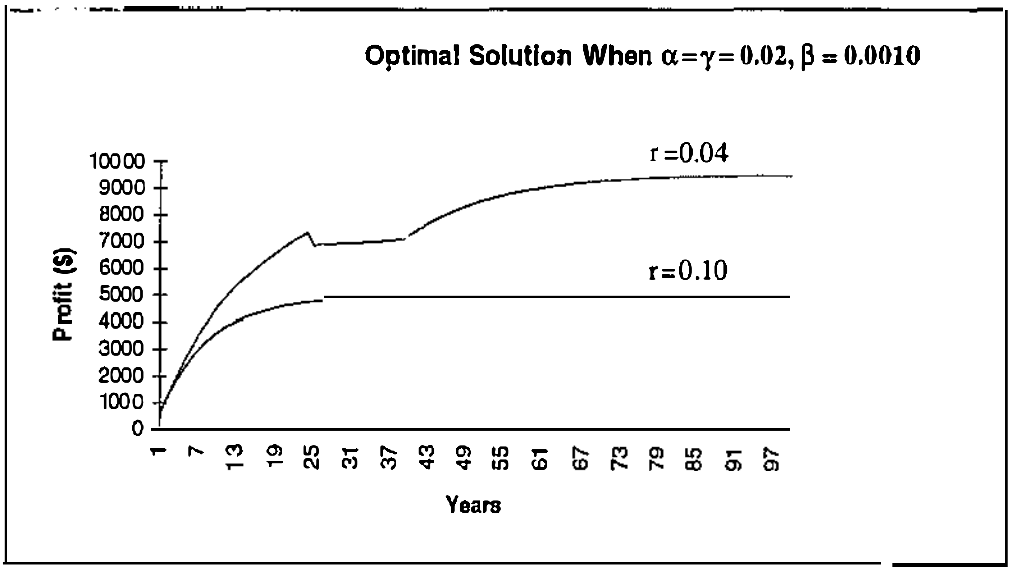

Figure 6.2: *Optimal switching times and the corresponding profit for scenario 2.* $r = 0.04$: $t_1 = 24$, $t_2 = 39$; $r = 0.1$: $t_1 = 64$, $t_2 = 73$.

			Necessary Conditions			Solver Only		
α=γ	β	r	Duration of p1	Duration of p2	Duration of p3	*Duration of p1*	*Duration of p2*	*Duration of p3*
0.02	0.0010	0.01	21	33	44	*22*	*25*	*53*
0.02	0.0010	0.02	22	23	55	*22*	*21*	*57*
.02	0.0010	0.03	23	18	59	*23*	*17*	*60*
0.02	0.0010	0.04	25	15	60	*25*	*15*	*60*
0.02	0.0010	0.05	27	13	60	*27*	*13*	*60*
0.02	0.0010	0.06	29	12	59	*29*	*12*	*59*
0.02	0.0010	0.07	34	12	54	*34*	*12*	*54*
0.02	0.0010	0.08	37	11	52	*37*	*11*	*52*
0.02	0.0010	0.09	55	10	35	*55*	*10*	*35*
0.02	0.0010	0.10	64	9	27	*64*	*9*	*27*

Table 6.2.a: *Duration of phases 1, 2 and 3 obtained by 2 methods.*

6.2 THE IMPACT OF THE RATE OF CHANGE OF THE WATER LEVEL ON THE SWITCHING TIMES AND THE OPTIMAL PROFIT

In this section, we will investigate the impact of the rate of change of the water level on the switching times and the corresponding maximum profit. We first consider how the rate with which the ground water rises in phase 1 and phase 3 affects the optimal solutions. We then consider the impact of the rate of decrease of the water depth in phase 2 on the switching times and profit.

6.2.1 The Impact of the Intrinsic Rate of Change of the Water Level in Phase 1 and Phase 3 on the Problem

As mentioned in section 5.3.1, the rate with which the water table rises in cropping phases lies between 0.04m and 0.14m per year and the values of the intrinsic rates α and γ associated with these values lie between 0.02 and 0.05. This variation is caused by the differences of annual rainfall and soil type. In this part, we investigate the effect of the change of α and γ while the discount rate *(r)* and the rate with which the water level decreases *(* β *)* in phase 2 are fixed.

We consider the break even scenario with β equal to 0.0010 and let α and γ vary from 0.02 to 0.05. In order to see the pattern clearly, we investigate the optimal solutions for r = 0.08 to r = 0.1 listed in tables 6.3, 6.4 and 6.5. (The solutions corresponding to other values of r are tabulated in Appendix A).

α, γ	t_1	t_2	t_3	Profit
0.02	5	100	100	7057
0.03	4	100	100	7013
0.04	3	100	100	6991
0.05	3	100	100	6972

Table 6.3: *Optimal solutions when* β = 0.0010 and r = 0.08.

α, γ	t_1	t_2	t_3	Profit
0.02	9	100	100	6018
0.03	6	100	100	5913
0.04	5	100	100	5852
0.05	4	100	100	5816

Table 6.4: *Optimal solutions when $\beta = 0.0010$ and $r = 0.09$.*

α, γ	t_1	t_2	t_3	Profit
0.02	12	100	100	5310
0.03	8	100	100	5145
0.04	6	100	100	5047
0.05	6	100	100	4976

Table 6.5: *Optimal solutions when* $\beta = 0.0010$ and $r = 0.1$.

As can be seen in these tables, an increase in the rate with which the water table rises in the cropping phases leads to a shortening of the first cropping phase and an increase in the duration of the rehabilitation phase. Figure 6.3 depicts the optimal switching time t_1 and the maximum profit when $\alpha = \gamma$ vary from 0.02 to 0.05 and $r = 0.1$.

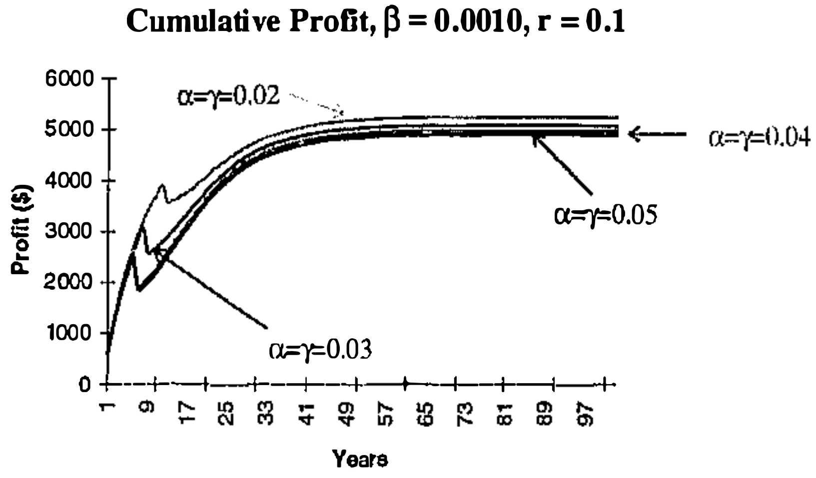

Figure 6.3: *Optimal solutions for scenario 1, with $r = 0.1$, $\beta = 0.0010$; α and γ change from 0.02 to 0.05.*

Since there is no switch back to cropping in this case, we cannot obtain any further conclusion about the duration of phase 3 for this scenario. For scenario 2, the situation is different. As we will see below, there is a switch back from rehabilitation to cropping. To be consistent with the previous analysis, we will consider the case where β is equal to 0.0010 and α and γ vary from 0.02 to 0.05. We also investigate the optimal solutions for $r = 0.08$ to $r = 0.1$ listed in tables 6.6, 6.7 and 6.8. (The solutions corresponding to other values of r can be seen in Appendix B).

α, γ	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.02	37	48	11	100	52	$5804
0.03	31	45	14	79	34	$5115
0.04	26	42	16	75	33	$4559
0.05	21	38	17	59	21	$4100

Table 6.6. *Optimal solutions when* $\beta = 0.0010$ and $r = 0.08$.

α, γ	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.02	55	65	10	100	35	$5358
0.03	33	46	13	75	29	$4760
0.04	29	44	15	72	28	$4275
0.05	22	38	16	59	21	$3862

Table 6.7: *Optimal solutions when $\beta = 0.0010$ and $r = 0.09$.*

α, γ	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.02	64	73	9	100	27	$4973
0.03	38	51	13	72	21	$4463
0.04	32	48	16	68	20	$4036
0.05	29	39	19	58	19	$3655

Table 6.8: *Optimal solutions when* $\beta = 0.0010$ and $r = 0.1$.

The optimal solutions for this scenario listed in tables 6.6, 6.7 and 6.8 indicate that **an increase in the intrinsic rate of change of the water level in phase 1 (α) and phase 3 (γ) will decrease the duration of the cropping in phases and increase the duration of the rehabilitation phase.**

α, γ	Average Profit r = 0.08	Average Profit r = 0.09	Average Profit r = 0.1
0.02	$58.04	$53.58	$49.73
0.03	$64.75	$63.47	$61.99
0.04	$60.79	$59.38	$59.35
0.05	$69.49	$65.46	$63.02

Table 6.9: *The average profit per year when $r = 0.08$, 0.09 and 0.1.*

As we can see from table 6.9, even though an increase of rate with which the water depth rises decreases the total profit, there is no pattern for the average profit. Figure 6.4 depicts the impacts of the rate of change of the water depth in phase 1 and phase 3 on the optimal solution. **When α and γ increase, the total profit drops and the terminal time decreases.**

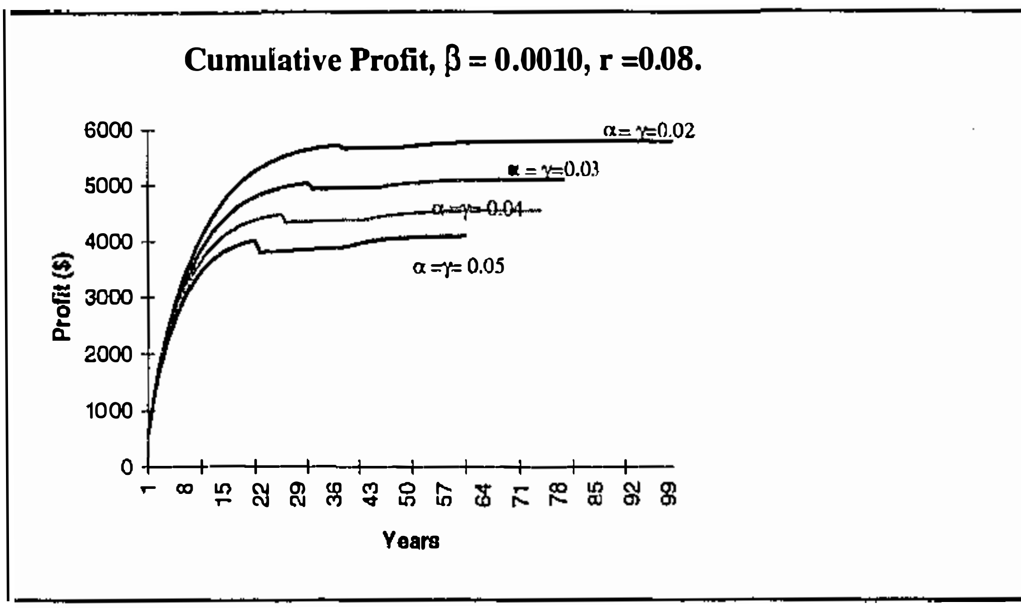

Figure 6.4: *Total profit and the optimal switching times for scenario 2 change when α and γ increase from 0.02 to 0.05 with r = 0.008. $\alpha = \gamma = 0.02$, $t_1 = 37$, $t_2 = 48$; $\alpha = \gamma = 0.02$, $t_1 = 31$, $t_2 = 45$; $\alpha = \gamma = 0.04$, $t_1 = 26$, $t_2 = 42$; $\alpha = \gamma = 0.05$, $t_1 = 21$, $t_2 = 38$.*

6.2.2 The Impacts of the Rate of Change of the Water Level in Phase 2 on the Optimisation

In this part, we explore how the intrinsic rate of change of the water level in the rehabilitation phase affects the optimal solution. As explained in 5.3.1, the intrinsic rate in phase 2 **(β)** varies from 0.0010 to 0.0025. We now investigate how these values influence the switching times and the total profit. In order to do so, we fix the value of α and γ to 0.02 and let β change. Tables 6.10 to 6.19 show the optimal solutions for scenario 2 with different values for the discount rate. The optimal solutions for the break even scenario are the same for all values of β therefore they are not useful for analysis.

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	22	47	25	100	53	$23128
0.0020	22	38	17	100	62	$25223
0.0025	15	28	13	100	72	$25274

Table 6.10: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with r = 0.01.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	22	45	23	100	55	$15850
0.0020	23	36	13	100	64	$17545
0.0025	15	26	9	100	74	$18004

Table 6.11: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with r = 0.02.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	23	41	18	100	59	$11877
0.0020	24	36	12	100	64	$13094
0.0025	16	25	9	100	75	$13367

Table 6.12: *Optimal solutions for scenario2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with $r = 0.03$.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	25	40	15	100	60	$9542
0.0020	25	36	11	100	64	$10361
0.0025	16	24	8	100	76	$10833

Table 6.13: *Optimal solutions when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with $r = 0.04$.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	27	40	13	100	60	$8070
0.0020	26	36	10	100	64	$8598
0.0025	17	24	7	100	76	$8944

Table 6.14: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with $r = 0.05$.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	29	41	12	100	59	$7073
0.0020	27	36	9	100	64	$7378
0.0025	17	24	7	100	76	$7629

Table 6.15: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with $r = 0.06$.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	34	46	12	100	54	$6362
0.0020	27	35	8	100	65	$6539
0.0025	18	24	6	100	76	$6681

Table 6.16: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with r = 0.07.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	37	48	11	100	52	$5804
0.0020	27	35	8	100	65	$5896
0.0025	18	23	5	100	77	$5965

Table 6.17: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with r = 0.08.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	55	65	10	100	35	$5358
0.0020	27	35	8	100	65	$5384
0.0025	18	23	5	100	77	$5413

Table 6.18: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with r = 0.09.*

β	t_1	t_2	Duration of phase 2	t_3	Duration of phase 3	Profit
0.0010	64	73	9	100	27	$4973
0.0020	28	35	7	100	65	$4978
0.0025	19	24	5	100	76	$4977

Table 6.19: *Optimal solutions for scenario 2 when $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 with r = 0.10*

From tables 6.10 to 6.19, we can conclude that an increase in the intrinsic rate of change of the water depth will:

- decrease the duration of the first cropping phase.
- decrease the duration of the rehabilitation phase.
- increase the duration of the third cropping phase.
- increase the total profit.

Figure 6.5 depicts the impacts of the increase of β in phase 3, when $r = 0.06$ and α and $\gamma = 0.02$.

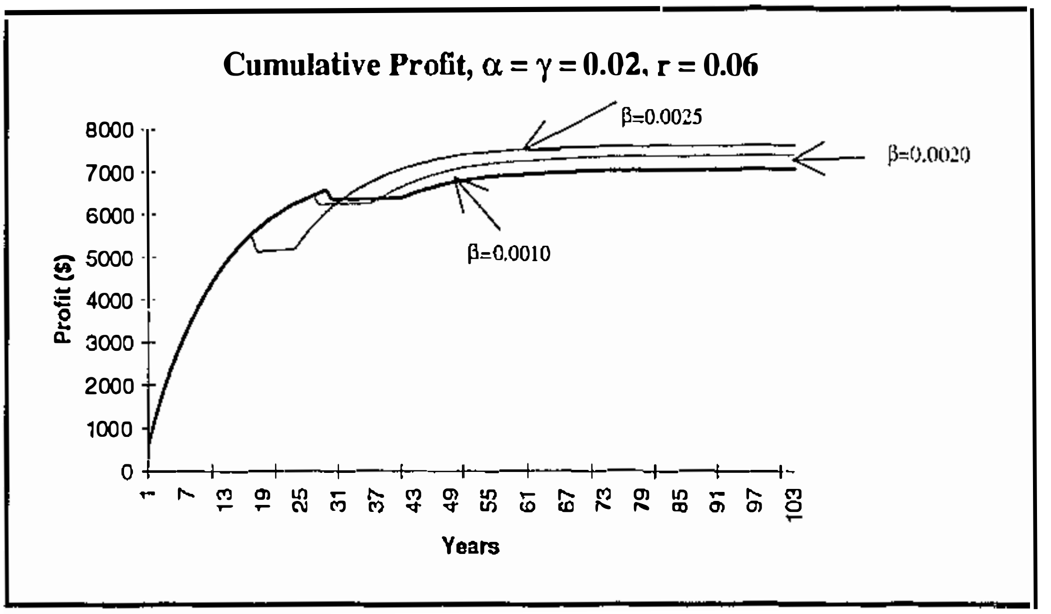

Figure 6.5: *Optimal solutions of the specific model for scenario 2 with $\alpha = \gamma = 0.02$ and β changes from 0.0010 to 0.0025 when $r = 0.06$. Before the first switching time occurs, the cumulative profit is the same for all values of β. After that, when β increases the total profit also increases.*

CHAPTER 7

CONCLUSION

7.1 SUMMARY

From chapter 3 to chapter 6, we formulated a particular model for farming practice under land degradation and solved it for two scenarios determined by the performance level for a mallee plantation. In the case of the break even scenario, most solutions indicate that planting trees is more profitable than cropping. When the discount rate is relatively small ($r \leq 0.06$), the first switching time is 1, and we do not switch back to cropping for any values of r, α, β and γ. As we assume that the land has been used for cropping and t_1 has to be positive, the result indicates that the first and third phases should be omitted and that we should switch to rehabilitation immediately. Unless for some other reasons apart from the income, land in that situation should only be used for planting trees. In the scenario corresponding to the current performance level, we have two switches for the cultivation: cropping - rehabilitation - cropping, and the optimal solutions under this scenario obey the following pattern:

When the farmer discount rate increases:

- The duration of the first cropping phase increases.

- The duration of the rehabilitation phase and the third phase decreases.
- The total profit decreases.

When the rates with which the water level rises in phase 1 and phase 3 increase:

- The duration of the cropping phases decreases.
- The duration of the rehabilitation phase increases.
- The total profit decreases.
- The terminal time decreases.

When the intrinsic rate of change of the water level in phase 2 increases:

- The duration of phase 1 and phase 2 decreases.
- The duration of phase 3 and the cumulative profit increase.

In section 6.1, we noted that the solutions obtained by using the SOLVER tool only and by applying the necessary conditions were not consistent for some cases. As the cumulative profit obtained by the SOLVER tool was higher than that obtained by using the necessary conditions, those solutions are the true optima and the solutions obtained by the necessary conditions were not optimal solutions for those cases. This can be explained by two reasons:

1) In the derivation of the necessary conditions in chapter 2, we assumed t_3 to be free. Therefore, δt_3 in equation (2.27) is also free. In chapter 4, we used this result to find the boundary value for solving the differential equation to determine $\lambda_3(t)$. This value then was used to calculate the left and right hand sides of

condition (13.2). When we limit our terminal time to up to 100 years, for some values of r, the terminal time t_3 becomes fixed; δt_3 in equation (2.29) is no longer free. Hence the boundary value we used to determine $\lambda_3(t)$ is no longer valid. This means that condition (13.2) does not apply. Consequently, the solutions are not valid either.

The purpose of this thesis is to explore the impacts of the intrinsic rate of change of the water level and the farmer discount rate on the optimal switching times, so we let t_3 be free. However, when we have concrete values of the rate of change and the farmer discount rate, it is possible that we modify our assumptions to find the necessary conditions in the case where t_3 is fixed.

2) The necessary conditions found in chapter 2 are also sufficient if the graph of the objective function is strictly concave (convex) for the case of maximisation (minimisation). In our case, due to the switching costs, the profit function is not strictly concave. Therefore the solutions we obtained by applying the necessary conditions are sometimes not the optimal ones but only the possible candidates (see figure 7.1). We may sometimes obtain many solutions satisfying (13.1) and (13.2), and we have to choose the best one.

One may wonder, so why we need the necessary conditions while we can solve our special problem using the SOLVER tool? The answer is that the SOLVER tool in Excel only gives local optima. In order to obtain the global optimum, we have to try many initial combinations of t_1, t_2 *and* t_3. The necessary conditions will limit our search for those values of t_1, t_2 *and* t_3 which satisfy (13.2).

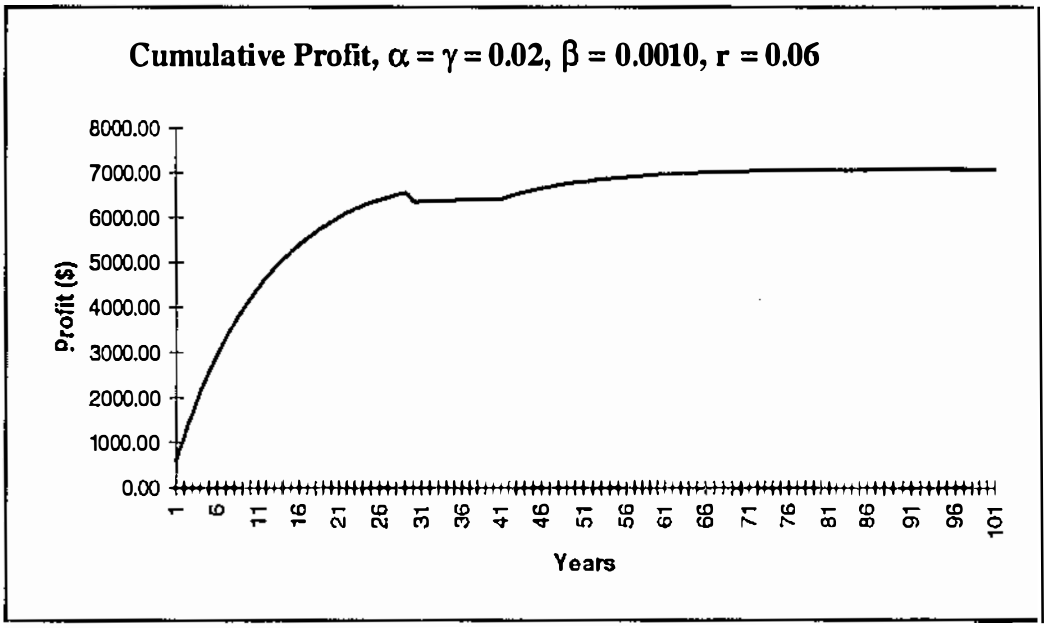

Figure 7.1: *The graph of the cumulative profit is not strictly concave, therefore the necessary conditions are not sufficient.*

7.2 FURTHER RESEARCH

The specific model in this thesis is formulated based on the assumptions that the land is flat; soil is homogenous; the type of crop is predefined; the prices of crop, the cropping costs and the farmer discount rate are constants. These assumptions lead to a model which is oversimplified. A more general model should take into account the spatial variability of the land; the soil concentration or the variation of the discount rate. It would also meet the prices as stochastic variables. As the type of crop is predefined, we do not have any control for phase 1 and phase 3. The only control variable in the model - the tree density in phase 2 - is also a constant during the rehabilitation. In reality, one should consider a model which has some control in the first and third phases such as the type of crop for cropping phases. In the rehabilitation phase, one possible control is the kind of trees which makes the cultivation more profitable or we can let the density of trees be a function in time in stead of constant. The problem we model is cyclic: cropping - rehabilitation - cropping. In practice, it may be more beneficial to grow crop and to plant trees together. This will allow farmers to crop and to conserve their land at the same time. This approach will ensure avoiding the shortage of food which may lead to famine crisis in some countries where only industrial trees are planted. In conclusion, phase farming under land degradation / rehabilitation is a fanstastic issue to be explored.

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APPENDIX A

The Solutions for the Optimal Switching Times and Profit when the Yield and Revenue of Mallee Oil is of Break Even Scenario.

α	β	γ	r	u	t_1	t_2	t_3	Profit
0.02	0.0010	0.02	0.01	*120*	1	100	100	64428
0.02	0.0010	0.02	0.02	*120*	1	100	100	40721
0.02	0.0010	0.02	0.03	*120*	1	100	100	27342
0.02	0.0010	0.02	0.04	*120*	1	100	100	19347
0.02	0.0010	0.02	0.05	*120*	1	100	100	14299
0.02	0.0010	0.02	0.06	*120*	1	100	100	10948
0.02	0.0010	0.02	0.07	*120*	2	100	100	8624
0.02	0.0010	0.02	0.08	*120*	5	100	100	7057
0.02	0.0010	0.02	0.09	*120*	9	100	100	6018
0.02	0.0010	0.02	0.10	*120*	12	100	100	5310

Table A.1: Solutions for break even scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.02$ and $\beta = 0.0010$.

α	β	γ	r	u	t_1	t_2	t_3	Profit
0.03	0.0010	0.03	0.01	*120*	1	100	100	64428
0.03	0.0010	0.03	0.02	*120*	1	100	100	40721
0.03	0.0010	0.03	0.03	*120*	1	100	100	27342
0.03	0.0010	0.03	0.04	*120*	1	100	100	19347
0.03	0.0010	0.03	0.05	*120*	1	100	100	14299
0.03	0.0010	0.03	0.06	*120*	1	100	100	10948
0.03	0.0010	0.03	0.07	*120*	1	100	100	8623
0.03	0.0010	0.03	0.08	*120*	4	100	100	7013
0.03	0.0010	0.03	0.09	*120*	6	100	100	5913
0.03	0.0010	0.03	0.10	*120*	8	100	100	5145

Table A.2: Solutions for break even scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.03$ and $\beta = 0.0010$.

α	β	γ	r	u	t_1	t_2	t_3	Profit
0.04	0.0010	0.04	0.01	*120*	1	100	100	64428
0.04	0.0010	0.04	0.02	*120*	1	100	100	40721
0.04	0.0010	0.04	0.03	*120*	1	100	100	27342
0.04	0.0010	0.04	0.04	*120*	1	100	100	19347
0.04	0.0010	0.04	0.05	*120*	1	100	100	14299
0.04	0.0010	0.04	0.06	*120*	1	100	100	10948
0.04	0.0010	0.04	0.07	*120*	1	100	100	8623
0.04	0.0010	0.04	0.08	*120*	3	100	100	6991
0.04	0.0010	0.04	0.09	*120*	5	100	100	5852
0.04	0.0010	0.04	0.10	*120*	6	100	100	5047

Table A.3: Solutions for break even scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.04$ and $\beta = 0.0010$.

α	β	γ	r	u	t_1	t_2	t_3	Profit
0.05	0.0010	0.05	0.01	*120*	1	100	100	64228
0.05	0.0010	0.05	0.02	*120*	1	100	100	40721
0.05	0.0010	0.05	0.03	*120*	1	100	100	27342
0.05	0.0010	0.05	0.04	*120*	1	100	100	19347
0.05	0.0010	0.05	0.05	*120*	1	100	100	14299
0.05	0.0010	0.05	0.06	*120*	1	100	100	10948
0.05	0.0010	0.05	0.07	*120*	1	100	100	8623
0.05	0.0010	0.05	0.08	*120*	3	100	100	6972
0.05	0.0010	0.05	0.09	*120*	4	100	100	5816
0.05	0.0010	0.05	0.10	*120*	6	100	100	4976

Table A.4: Solutions for break even scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.05$ and $\beta = 0.0010$.

α	β	γ	r	u	t_1	t_2	t_3	Profit
0.02	0.0020	0.02	0.01	*120*	1	100	100	64428
0.02	0.0020	0.02	0.02	*120*	1	100	100	40721
0.02	0.0020	0.02	0.03	*120*	1	100	100	27342
0.02	0.0020	0.02	0.04	*120*	1	100	100	19347
0.02	0.0020	0.02	0.05	*120*	1	100	100	14299
0.02	0.0020	0.02	0.06	*120*	1	100	100	10948
0.02	0.0020	0.02	0.07	*120*	2	100	100	8624
0.02	0.0020	0.02	0.08	*120*	5	100	100	7057
0.02	0.0020	0.02	0.09	*120*	9	100	100	6018
0.02	0.0020	0.02	0.10	*120*	12	93	100	5310

Table A.5: Solutions for break even scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.02$ and $\beta = 0.0020$.

α	β	γ	r	u	t_1	t_2	t_3	Profit
0.02	0.0025	0.02	0.01	*120*	1	100	100	64428
0.02	0.0025	0.02	0.02	*120*	1	100	100	40721
0.02	0.0025	0.02	0.03	*120*	1	100	100	27342
0.02	0.0025	0.02	0.04	*120*	1	100	100	19347
0.02	0.0025	0.02	0.05	*120*	1	100	100	14299
0.02	0.0025	0.02	0.06	*120*	1	100	100	10948
0.02	0.0025	0.02	0.07	*120*	2	100	100	8624
0.02	0.0025	0.02	0.08	*120*	5	100	100	7057
0.02	0.0025	0.02	0.09	*120*	9	100	100	6018
0.02	0.0025	0.02	0.10	*120*	12	100	100	5310

Table A.6: Solutions for break even scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.02$ and $\beta = 0.0025$.

APPENDIX B

The Solutions for the Optimal Switching Times and Profit when the Yield and Revenue of Mallee Oil is of Current Performance Level Scenario.

					Necessary Conditions				Solver Only			
α	β	γ	r	*u*	t_1	t_2	t_3	Profit1	*t_1*	*t_2*	*t_3*	*Profit2*
0.02	0.0010	0.02	0.01	*120*	21	54	100	22582	*22*	*47*	*100*	*23128*
0.02	0.0010	0.02	0.02	*120*	22	45	100	15804	*22*	*43*	*100*	*15850*
0.02	0.0010	0.02	0.03	*120*	23	41	100	11874	*23*	*40*	*100*	*11877*
0.02	0.0010	0.02	0.04	*120*	25	40	100	9542	*25*	*40*	*100*	*9542*
0.02	0.0010	0.02	0.05	*120*	27	40	100	8070	*27*	*40*	*100*	*8070*
0.02	0.0010	0.02	0.06	*120*	29	41	100	7073	*29*	*41*	*100*	*7073*
0.02	0.0010	0.02	0.07	*120*	34	46	100	6362	*34*	*46*	*100*	*6362*
0.02	0.0010	0.02	0.08	*120*	37	48	100	5804	*37*	*48*	*100*	*5804*
0.02	0.0010	0.02	0.09	*120*	55	65	100	5358	*55*	*65*	*100*	*5358*
0.02	0.0010	0.02	0.10	*120*	64	73	100	4973	*64*	*73*	*100*	*4973*

Table B.1: Solutions for current performance scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.02$ and $\beta = 0.0010$.

					Necessary Conditions				Solver Only			
α	β	γ	r	*u*	t_1	t_2	t_3	Profit1	*t_1*	*t_2*	*t_3*	*Profit2*
0.03	0.0010	0.03	0.01	*120*	22	61	100	18496	*22*	*54*	*100*	*18763*
0.03	0.0010	0.03	0.02	*120*	24	52	100	12975	*24*	*50*	*100*	*12986*
0.03	0.0010	0.03	0.03	*120*	25	48	100	9872	*25*	*48*	*100*	*9872*
0.03	0.0010	0.03	0.04	*120*	26	46	100	8053	*26*	*46*	*100*	*8053*
0.03	0.0010	0.03	0.05	*120*	27	45	100	6904	*27*	*45*	*100*	*6904*
0.03	0.0010	0.03	0.06	*120*	28	44	90	6122	*28*	*44*	*90*	*6122*
0.03	0.0010	0.03	0.07	*120*	29	44	83	5553	*29*	*44*	*83*	*5553*
0.03	0.0010	0.03	0.08	*120*	31	45	79	5115	*31*	*45*	*79*	*5115*
0.03	0.0010	0.03	0.09	*120*	33	46	75	4760	*33*	*46*	*75*	*4760*
0.03	0.0010	0.03	0.10	*120*	38	51	72	4463	*38*	*51*	*72*	*4463*

Table B.2: Solutions for current performance scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.03$ and $\beta = 0.0010$.

					Necessary Conditions				Solver Only			
α	β	γ	r	*u*	t_1	t_2	t_3	Profit1	t_1	t_2	t_3	*Profit2*
0.04	0.0010	0.04	0.01	*120*	21	69	100	15527	*21*	*60*	*100*	*15791*
0.04	0.0010	0.04	0.02	*120*	21	52	95	10992	*21*	*52*	*95*	*10992*
0.04	0.0010	0.04	0.03	*120*	22	48	90	8457	*22*	*48*	*90*	*8457*
0.04	0.0010	0.04	0.04	*120*	23	46	85	6964	*23*	*46*	*85*	*6964*
0.04	0.0010	0.04	0.05	*120*	24	44	83	6018	*24*	*44*	*83*	*6018*
0.04	0.0010	0.04	0.06	*120*	24	43	82	5382	*24*	*43*	*82*	*5382*
0.04	0.0010	0.04	0.07	*120*	25	42	78	4916	*25*	*42*	*78*	*[illegible]16*
0.04	0.0010	0.04	0.08	*120*	26	42	75	4559	*26*	*42*	*75*	*4559*
0.04	0.0010	0.04	0.09	*120*	29	45	72	4275	*29*	*45*	*72*	*4275*
0.04	0.0010	0.04	0.10	*120*	32	48	68	4036	*32*	*48*	*68*	*4036*

Table B.3: Solutions for current performance scenario when r varies from 0.01 to 0.1, with α = γ = 0.04 and β = 0.0010.

					Necessary Conditions				Solver Only			
α	β	γ	r	*u*	t_1	t_2	t_3	Profit1	t_1	t_2	t_3	*Profit2*
0.05	0.0010	0.05	0.01	*120*	5	65	100	11729	*17*	*66*	*100*	*13692*
0.05	0.0010	0.05	0.02	*120*	18	51	84	9575	*18*	*47*	*82*	*9549*
0.05	0.0010	0.05	0.03	*120*	18	45	80	7452	*16*	*41*	*74*	*7463*
0.05	0.0010	0.05	0.04	*120*	18	41	72	6184	*18*	*41*	*72*	*6184*
0.05	0.0010	0.05	0.05	*120*	19	40	70	5362	*17*	*37*	*67*	*5366*
0.05	0.0010	0.05	0.06	*120*	19	38	68	4806	*19*	*38*	*68*	*4806*
0.05	0.0010	0.05	0.07	*120*	20	38	67	4404	*20*	*38*	*67*	*4404*
0.05	0.0010	0.05	0.08	*120*	21	38	61	4100	*21*	*38*	*64*	*4100*
0.05	0.0010	0.05	0.09	*120*	22	38	59	3862	*22*	*38*	*59*	*3862*
0.05	0.0010	0.05	0.10	*120*	30	48	68	3673	*30*	*48*	*68*	*3673*

Table B.4: Solutions for current performance scenario when r varies from 0.01 to 0.1, with α = γ = 0.05 and β = 0.0010.

α	β	γ	r	u	Necessary Conditions t_1	t_2	t_3	Profit1	Solver Only t_1	t_2	t_3	Profit2
0.02	0.0020	0.02	0.01	*120*	22	38	100	25223	*22*	*38*	*100*	*25223*
0.02	0.0020	0.02	0.02	*120*	23	36	100	17545	*23*	*36*	*100*	*17545*
0.02	0.0020	0.02	0.03	*120*	24	36	100	13094	*24*	*36*	*100*	*13094*
0.02	0.0020	0.02	0.04	*120*	25	36	100	10361	*25*	*36*	*100*	*10361*
0.02	0.0020	0.02	0.05	*120*	26	36	100	8598	*26*	*36*	*100*	*8598*
0.02	0.0020	0.02	0.06	*120*	27	36	100	7398	*27*	*36*	*100*	*7398*
0.02	0.0020	0.02	0.07	*120*	27	35	100	6539	*27*	*35*	*100*	*6539*
0.02	0.0020	0.02	0.08	*120*	27	35	100	5896	*27*	*35*	*100*	*5896*
0.02	0.0020	0.02	0.09	*120*	27	35	100	5384	*27*	*35*	*100*	*5394*
0.02	0.0020	0.02	0.10	*120*	28	35	100	4978	*28*	*35*	*100*	*4978*

Table B.5: Solutions for current perfonnance scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.02$ and $\beta = 0.0020$.

α	β	γ	r	u	Necessary Conditions t_1	t_2	t_3	Profit1	Solver Only t_1	t_2	t_3	Profit2
0.02	0.0025	0.02	0.01	*120*	15	29	100	25254	*15*	*28*	*100*	*25274*
0.02	0.0025	0.02	0.02	*120*	15	26	100	18004	*15*	*26*	*100*	*18004*
0.02	0.0025	0.02	0.03	*120*	16	25	100	13637	*16*	*25*	*100*	*13637*
0.02	0.0025	0.02	0.04	*120*	16	24	100	10833	*16*	*24*	*100*	*10833*
0.02	0.0025	0.02	0.05	*120*	17	24	100	8944	*17*	*24*	*100*	*8944*
0.02	0.0025	0.02	0.06	*120*	17	24	100	7629	*17*	*24*	*100*	*7629*
0.02	0.0025	0.02	0.07	*120*	18	24	100	6681	*18*	*24*	*100*	*6681*
0.02	0.0025	0.02	0.08	*120*	18	23	100	5965	*18*	*23*	*100*	*5965*
0.02	0.0025	0.02	0.09	*120*	18	23	100	5413	*18*	*23*	*100*	*5413*
0.02	0.0025	0.02	0.10	*120*	19	24	100	4977	*19*	*24*	*100*	*4977*

Table B.6: Solutions for current perfonnance scenario when r varies from 0.01 to 0.1, with $\alpha = \gamma = 0.02$ and $\beta = 0.0025$.

APPENDIX C

The Excel Spreadsheet for Implementing and Solving the Specific Model

The PARAMETER Worksheet.

PARAMETERS

Name	Cell Referred	Unit	Comment
Price1	800	Dollars/ton	Price of crop planted in phase 1
Prica2 (Revanue2)	20	Dollars/ton	Revenue obtained by growing trees in phase 2
Price3	800	Dollars/ton	Price of crop planted in phase 3
Cost1	200	Dollars/ha	Cropping cost for phase 1
SwCost1	1000	Dollars	Fixed cost for switching from phase 1 to phase 2
Cost2	0.5	Dollars/tree	Cost planting 1 tree/plant in phase 2
SwCost2	80	Dollars	Fixed cost for switching from phase 2 to phase 3
Cost3	200	Dollars/ha	Cropping cost for phase 3
Discount Rate	0.08		Farmer discount rate
Alpha	0.02	m/year	Intrinsic rate of change of water level in phase 1
Beta	0.001	m/year	Intrinsic rate of change of water level in phase 2
Gamma	0.02	m/year	Intrinsic rate of change of water level in phase 3
m	6	m	Maximum water depth under tree stand
Y_01	1.5	ton/ha	Max crop yield with no salinity for phase 1
Y_03	1.5	ton/ha	Max crop yield with no salinity for phase 3
L_bar	0.05		Maximum canopy mass haversted per tree
l	0.08		growth rate of tree
Dmax	160		Maximum number of trees per hectare
u	120	trees/ha	Density of tree /ha
X1_0	4	metre	initial depth of saline water (the first phase)
K_1	4.000000		
K_2	0.005502		
K_3	9.963793		

The OPTIMISATION Worksheet

Optimisation Switching Time t1, t2 and Profit		
Parameters :		**Comments :**
Time0	0	Initial time
Time1	37	Switching time 1
Time2	48	Switching time2
Time3	100	Terminal time
Den	120	Tree density (smaller than 160)
u	120	Tree density (rounded)
OpTime1	**37**	Switching t1 (rounded)
OpTime2	**48**	Switching t2 (rounded)
OpTime3	**100**	Optimal terminal time (rounded)
Cond 13.1.1	**0**	Left side cond 13.1
Cond 13.1.2	**0**	Right side cond 13.1
Cond 13.2.1	**9**	Left side cond 13.2
Cond 13.2.2	**9**	Right side cond 13.2
Cons. (4.23)	**69**	
Cons. (4.24)	**115**	
Max Profit =	**5804**	

The FORMULATION Worksheet

Time Phase t	Discount	Water Level 1 X(t)	Water Level 2 X(t)	Water Level 3 X(t)	Yield 1 Y1(t)	Revenue 1 "Price * Yield1" "-Cost1"
0.00	1.0000	4.0000	0.0000	0.0000	1.0000	600.0000
1.00	0.9231	3.9208	0.0000	0.0000	0.9802	584.1589
2.00	0.8521	3.8432	0.0000	0.0000	0.9608	568.6316
3.00	0.7866	3.7671	0.0000	0.0000	0.9418	553.4116
4.00	0.7261	3.6925	0.0000	0.0000	0.9231	538.4931
5.00	0.6703	3.6193	0.0000	0.0000	0.9048	523.8699
6.00	0.6188	3.5477	0.0000	0.0000	0.8669	509.5363
7.00	0.5712	3.4774	0.0000	0.0000	0.8694	495.4866
8.00	0.5273	3.4086	0.0000	0.0000	0.8521	481.7150
9.00	0.4868	3.3411	0.0000	0.0000	0.8353	468.2162
10.00	0.4493	3.2749	0.0000	0.0000	0.8187	454.9846
11.00	0.4148	3.2101	0.0000	0.0000	0.8025	442.0150
12.00	0.3829	3.1465	0.0000	0.0000	0.7866	429.3023
13.00	0.3535	3.0842	0.0000	0.0000	0.7711	416.8413
14.00	0.3263	3.0231	0.0000	0.0000	0.7558	404.6270
15.00	0.3012	2.9633	0.0000	0.0000	0.7408	392.6546
16.00	0.2780	2.9046	0.0000	0.0000	0.7261	380.9192
17.00	0.2567	2.8471	0.0000	0.0000	0.7118	369.4163
18.00	0.2369	2.7907	0.0000	0.0000	0.6977	358.1411
19.00	0.2187	2.7354	0.0000	0.0000	0.6839	347.0891
20.00	0.2019	2.6813	0.0000	0.0000	0.6703	336.2560
21.00	0.1864	2.6282	0.0000	0.0000	0.6570	325.6375
22.00	0.1720	2.5761	0.0000	0.0000	0.6440	315.2291
23.00	0.1588	2.5251	0.0000	0.0000	0.6313	305.0269
24.00	0.1466	2.4751	0.0000	0.0000	0.6188	295.0267
25.00	0.1353	2.4261	0.0000	0.0000	0.6065	285.2245
26.00	0.1249	2.3781	0.0000	0.0000	0.5945	275.6164
27.00	0.1153	2.3310	0.0000	0.0000	0.5827	266.1986
28.00	0.1065	2.2848	0.0000	0.0000	0.5712	256.9673
29.00	0.0983	2.2396	0.0000	0.0000	0.5599	247.9187
30.00	0.0907	2.1952	0.0000	0.0000	0.5488	239.0493
31.00	0.0837	2.1518	0.0000	0.0000	0.5379	230.3556
32.00	0.0773	2.1092	0.0000	0.0000	0.5273	221.8339
33.00	0.0714	2.0674	0.0000	0.0000	0.5169	213.4811
34.00	0.0659	2.0265	0.0000	0.0000	0.5066	205.2936
35.00	0.0608	1.9863	0.0000	0.0000	0.4966	197.2682
36.00	0.0561	1.9470	0.0000	0.0000	0.4868	189.4018
37.00	0.0518	1.9085	1.9085	0.0000	0.4771	181.6911
38.00	0.0478	1.8707	2.0679	0.0000	0.4677	174.1331
39.00	0.0442	1.8336	2.2334	0.0000	0.4584	166.7248
40.00	0.0408	1.7973	2.4041	0.0000	0.4493	159.4632
41.00	0.0376	1.7617	2.5789	0.0000	0.4404	152.3453
42.00	0.0347	1.7268	2.7566	0.0000	0.4317	145.3684
43.00	0.0321	1.6926	2.9361	0.0000	0.4232	138.5297
44.00	0.0296	1.6591	3.1160	0.0000	0.4148	131.8263
45.00	0.0273	1.6263	3.2951	0.0000	0.4066	125.2557
46.00	0.0252	1.5941	3.4721	0.0000	0.3985	118.8152
47.00	0.0233	1.5625	3.6458	0.0000	0.3906	112.5023
48.00	0.0215	1.5316	3.8151	3.8151	0.3829	106.3143
49.00	0.0198	1.5012	3.9789	3.7395	0.3753	100.2489

50.00	0.0183	1.4715	4.1365	3.6655	0.3679	94.3036
51.00	0.0169	1.4424	4.2870	3.5929	0.3606	88.4760
52.00	0.0156	1.4138	4.4301	3.5217	0.3535	82.7637
53.00	0.0144	1.3858	4.5651	3.4520	0.3465	77.1646
54.00	0.0133	1.3584	4.6920	3.3837	0.3396	71.6764
55.00	0.0123	1.3315	4.8106	3.3167	0.3329	66.2969
56.00	0.0113	1.3051	4.9209	3.2510	0.3263	61.0238
57.00	0.0105	1.2793	5.0231	3.1866	0.3198	55.8552
58.00	0.0097	1.2539	5.1173	3.1235	0.3135	50.7889
59.00	0.0089	1.2291	5.2039	3.0617	0.3073	45.8230
60.00	0.0082	1.2048	5.2831	3.0010	0.3012	40.9554
61.00	0.0076	1.1809	5.3555	2.9416	0.2952	36.1841
62.00	0.0070	1.1575	5.4213	2.8834	0.2894	31.5074
63.00	0.0065	1.1346	5.4811	2.8263	0.2837	26.9232
64.00	0.0060	1.1121	5.5352	2.7703	0.2780	22.4298
65.00	0.0055	1.0901	5.5842	2.7155	0.2725	18.0254
66.00	0.0051	1.0685	5.6283	2.6617	0.2671	13.7082
67.00	0.0047	1.0474	5.6680	2.6090	0.2618	9.4765
68.00	0.0043	1.0266	5.7037	2.5573	0.2567	5.3286
69.00	0.0040	1.0063	5.7357	2.5067	0.2516	1.2628
70.00	0.0037	0.9864	5.7644	2.4570	0.2466	-2.7224
71.00	0.0034	0.9669	5.7901	2.4084	0.2417	-6.6288
72.00	0.0032	0.9477	5.8131	2.3607	0.2369	-10.4578
73.00	0.0029	0.9289	5.8337	2.3140	0.2322	-14.2110
74.00	0.0027	0.9106	5.8520	2.2681	0.2276	-17.8898
75.00	0.0025	0.8925	5.8684	2.2232	0.2231	-21.4959
76.00	0.0023	0.8748	5.8830	2.1792	0.2187	-25.0305
77.00	0.0021	0.8575	5.8960	2.1360	0.2144	-28.4951
78.00	0.0019	0.8405	5.9076	2.0938	0.2101	-31.8911
79.00	0.0018	0.8239	5.9179	2.0523	0.2060	-35.2199
80.00	0.0017	0.8076	5.9270	2.0117	0.2019	-38.4828
81.00	0.0015	0.7916	5.9352	1.9718	0.1979	-41.6810
82.00	0.0014	0.7759	5.9425	1.9328	0.1940	-44.8160
83.00	0.0013	0.7606	5.9489	1.6945	0.1901	-47.8888
84.00	0.0012	0.7455	5.9546	1.6570	0.1864	-50.9008
85.00	0.0011	0.7307	5.9597	1.8202	0.1827	-53.8532
66.00	0.0010	0.7163	5.9643	1.7842	0.1791	-56.7471
87.00	0.0009	0.7021	5.9683	1.7468	0.1755	-59.5837
88.00	0.0009	0.6882	5.9719	1.7142	0.1720	-62.3641
89.00	0.0008	0.6746	5.9750	1.6803	0.1686	-65.0895
90.00	0.0007	0.6612	5.9778	1.6470	0.1653	-67.7609
91.00	0.0007	0.6481	5.9803	1.6144	0.1620	-70.3794
92.00	0.0006	0.6353	5.9826	1.5824	0.1588	-72.9461
93.00	0.0006	0.6227	5.9845	1.5511	0.1557	-75.4619
94.00	0.0005	0.6104	5.9863	1.5204	0.1526	-77.9279
95.00	0.0005	0.5983	5.9878	1.4903	0.1496	-80.3451
96.00	0.0005	0.5864	5.9892	1.4608	0.1466	-82.7144
97.00	0.0004	0.5748	5.9904	1.4318	0.1437	-85.0368
98.00	0.0004	0.5634	5.9915	1.4035	0.1409	-67.3133
99.00	0.0004	0.5523	5.9925	1.3757	0.1381	-69.5446
100.00	0.0003	0.5413	5.9933	1.3485	0.1353	-91.7318
101.00	0.0003	0.5306	5.9941	1.3218	0.1327	-93.8756
102.00	0.0003	0.5201	5.9947	1.2956	0.1300	-95.9770

Net Profit 1 Revenue1*Discount D(t)	Tree Growth T(t)	Revenue2 Tree Revenue	Profit 2 Revenue2 * Discount F(t)	Yield 3 Y3(t)
600.0000	0.0000	0.0000	0.0000	0.0000
539.2467	0.0000	0.0000	0.0000	0.0000
484.5558	0.0000	0.0000	0.0000	0.0000
435.3290	0.0000	0.0000	0.0000	0.0000
391.0262	0.0000	0.0000	0.0000	0.0000
351.1605	0.0000	0.0000	0.0000	0.0000
315.2926	0.0000	0.0000	0.0000	0.0000
283.0264	0.0000	0.0000	0.0000	0.0000
254.0047	0.0000	0.0000	0.0000	0.0000
227.9053	0.0000	0.0000	0.0000	0.0000
204.4378	0.0000	0.0000	0.0000	0.0000
183.3403	0.0000	0.0000	0.0000	0.0000
164.3768	0.0000	0.0000	0.0000	0.0000
147.3345	0.0000	0.0000	0.0000	0.0000
132.0216	0.0000	0.0000	0.0000	0.0000
118.2653	0.0000	0.0000	0.0000	0.0000
105.9098	0.0000	0.0000	0.0000	0.0000
94.8147	0.0000	0.0000	0.0000	0.0000
84.8536	0.0000	0.0000	0.0000	0.0000
75.9125	0.0000	0.0000	0.0000	0.0000
67.8889	0.0000	0.0000	0.0000	0.0000
60.6903	0.0000	0.0000	0.0000	0.0000
54.2336	0.0000	0.0000	0.0000	0.0000
48.4436	0.0000	0.0000	0.0000	0.0000
43.2530	0.0000	0.0000	0.0000	0.0000
38.6009	0.0000	0.0000	0.0000	0.0000
34.4328	0.0000	0.0000	0.0000	0.0000
30.6994	0.0000	0.0000	0.0000	0.0000
27.3563	0.0000	0.0000	0.0000	0.0000
24.3639	0.0000	0.0000	0.0000	0.0000
21.6861	0.0000	0.0000	0.0000	0.0000
19.2907	0.0000	0.0000	0.0000	0.0000
17.1488	0.0000	0.0000	0.0000	0.0000
15.2343	0.0000	0.0000	0.0000	0.0000
13.5237	0.0000	0.0000	0.0000	0.0000
11.9959	0.0000	0.0000	0.0000	0.0000
10.6320	0.0000	0.0000	0.0000	0.0000
9.4150	0.0000	0.0000	0.0000	0.0000
8.3296	0.4613	9.2260	0.4413	0.0000
7.3621	0.8871	17.7427	0.7835	0.0000
6.5001	1.2802	25.6047	1.0437	0.0000
5.7325	1.6431	32.8621	1.2365	0.0000
5.0494	1.9781	39.5616	1.3742	0.0000
4.4419	2.2873	45.7460	1.4668	0.0000
3.9020	2.5727	51.4549	1.5230	0.0000
3.4225	2.8362	56.7249	1.5499	0.0000
2.9969	3.0795	61.5897	1.5535	0.0000
2.6195	3.3040	66.0805	1.5386	0.0000
2.2851	3.5113	70.2261	1.5094	0.9538
1.9890	3.7026	74.0529	1.4693	0.9349

1.7272	3.8793	77.5854	1.4210	0.9164
1.4959	4.0423	80.8464	1.3669	0.8982
1.2917	4.1928	83.8567	1.3088	0.8804
1.1118	4.3318	86.6355	1.2482	0.8630
0.9533	4.4600	89.2007	1.1864	0.8459
0.8139	4.5784	91.5687	1.1242	0.8292
0.6916	4.6877	93.7546	1.0626	0.8127
0.5844	4.7886	95.7724	1.0020	0.7967
0.4905	4.8818	97.6351	0.9429	0.7809
0.4085	4.9677	99.3546	0.8858	0.7654
0.3371	5.0471	100.9419	0.8307	0.7503
0.2749	5.1204	102.4072	0.7780	0.7354
0.2210	5.1880	103.7598	0.7277	0.7208
0.1743	5.2504	105.0084	0.6798	0.7066
0.1340	5.3080	106.1610	0.6344	0.6926
0.0994	5.3612	107.2250	0.5915	0.6789
0.0698	5.4104	108.2072	0.5510	0.6654
0.0445	5.4557	109.1138	0.5129	0.6522
0.0231	5.4975	109.9508	0.4771	0.6393
0.0051	5.5362	110.7234	0.4435	0.6267
-0.0101	5.5718	111.4366	0.4121	0.6143
-0.0226	5.6048	112.0950	0.3826	0.6021
-0.0330	5.6351	112.7028	0.3551	0.5902
-0.0413	5.6632	113.2638	0.3295	0.5785
-0.0480	5.6891	113.7817	0.3055	0.5670
-0.0533	5.7130	114.2598	0.2832	0.5558
-0.0573	5.7351	114.7011	0.2625	0.5448
-0.0602	5.7554	115.1085	0.2431	0.5340
-0.0622	5.7742	115.4846	0.2252	0.5234
-0.0634	5.7916	115.8318	0.2085	0.5131
-0.0639	5.8076	116.1522	0.1930	0.5029
-0.0639	5.8224	116.4481	0.1786	0.4930
-0.0635	5.8361	116.7212	0.1653	0.4832
-0.0626	5.8487	116.9732	0.1529	0.4736
-0.0614	5.8603	117.2060	0.1414	0.4642
-0.0600	5.8710	117.4208	0.1308	0.4551
-0.0583	5.8810	117.6191	0.1209	0.4460
-0.0566	5.8901	117.8021	0.1118	0.4372
-0.0546	5.8986	117.9711	0.1034	0.4286
-0.0526	5.9064	118.1271	0.0955	0.4201
-0.0506	5.9136	118.2711	0.0883	0.4118
-0.0485	5.9202	118.4040	0.0816	0.4036
-0.0464	5.9263	118.5267	0.0754	0.3956
-0.0443	5.9320	118.6400	0.0697	0.3878
-0.0422	5.9372	118.7446	0.0644	0.3801
-0.0402	5.9421	118.8411	0.0595	0.3726
-0.0382	5.9465	118.9302	0.0549	0.3652
-0.0363	5.9506	119.0124	0.0508	0.3580
-0.0344	5.9544	119.0884	0.0469	0.3509
-0.0325	5.9579	119.1584	0.0433	0.3439
-0.0308	5.9612	119.2232	0.0400	0.3371
-0.0291	5.9641	119.2829	0.0369	0.3304
-0.0274	5.9669	119.3380	0.0341	0.3239

Revenue 3 "Price * Yield3" "-Cost3"	Net Profit 3 Revenue3*Discount G(t)	Switching Cost1	Switching Cost2	Yearly Profit	Total Profit J
0.0000	0.0000	0.0000	0.0000	600.0000	600.00
0.0000	0.0000	0.0000	0.0000	539.2467	1139.25
0.0000	0.0000	0.0000	0.0000	484.5558	1623.80
0.0000	0.0000	0.0000	0.0000	435.3290	2059.13
0.0000	0.0000	0.0000	0.0000	391.0262	2450.16
0.0000	0.0000	0.0000	0.0000	351.1605	2801.32
0.0000	0.0000	0.0000	0.0000	315.2926	3116.61
0.0000	0.0000	0.0000	0.0000	283.0264	3399.64
0.0000	0.0000	0.0000	0.0000	254.0047	3653.64
0.0000	0.0000	0.0000	0.0000	227.9053	3881.55
0.0000	0.0000	0.0000	0.0000	204.4378	4085.99
0.0000	0.0000	0.0000	0.0000	183.3403	4269.33
0.0000	0.0000	0.0000	0.0000	164.3768	4433.70
0.0000	0.0000	0.0000	0.0000	147.3345	4581.04
0.0000	0.0000	0.0000	0.0000	132.0216	4713.06
0.0000	0.0000	0.0000	0.0000	118.2653	4831.32
0.0000	0.0000	0.0000	0.0000	105.9098	4937.23
0.0000	0.0000	0.0000	0.0000	94.8147	5032.05
0.0000	0.0000	0.0000	0.0000	84.8536	5116.90
0.0000	0.0000	0.0000	0.0000	75.9125	5192.81
0.0000	0.0000	0.0000	0.0000	67.8889	5260.70
0.0000	0.0000	0.0000	0.0000	60.6903	5321.39
0.0000	0.0000	0.0000	0.0000	54.2336	5375.63
0.0000	0.0000	0.0000	0.0000	48.4436	5424.07
0.0000	0.0000	0.0000	0.0000	43.2530	5467.32
0.0000	0.0000	0.0000	0.0000	38.6009	5505.92
0.0000	0.0000	0.0000	0.0000	34.4328	5540.36
0.0000	0.0000	0.0000	0.0000	30.6994	5571.06
0.0000	0.0000	0.0000	0.0000	27.3563	5598.41
0.0000	0.0000	0.0000	0.0000	24.3639	5622.78
0.0000	0.0000	0.0000	0.0000	21.6861	5644.46
0.0000	0.0000	0.0000	0.0000	19.2907	5663.75
0.0000	0.0000	0.0000	0.0000	17.1488	5680.90
0.0000	0.0000	0.0000	0.0000	15.2343	5696.14
0.0000	0.0000	0.0000	0.0000	13.5237	5709.66
0.0000	0.0000	0.0000	0.0000	11.9959	5721.66
0.0000	0.0000	0.0000	0.0000	10.6320	5732.29
0.0000	0.0000	54.9281	0.0000	-54.9281	5677.36
0.0000	0.0000	0.0000	0.0000	0.4413	5677.80
0.0000	0.0000	0.0000	0.0000	0.7835	5678.58
0.0000	0.0000	0.0000	0.0000	1.0437	5679.63
0.0000	0.0000	0.0000	0.0000	1.2365	5680.87
0.0000	0.0000	0.0000	0.0000	1.3742	5682.24
0.0000	0.0000	0.0000	0.0000	1.4668	5683.71
0.0000	0.0000	0.0000	0.0000	1.5230	5685.23
0.0000	0.0000	0.0000	0.0000	1.5499	5686.78
0.0000	0.0000	0.0000	0.0000	1.5535	5688.33
0.0000	0.0000	0.0000	0.0000	1.5386	5689.87
563.0131	12.1012	0.0000	1.7195	10.3817	5700.25
547.9044	10.8710	0.0000	0.0000	10.8710	5711.12

533.0949	9.7640	0.0000	0.0000	9.7640	5720.89
518.5786	8.7679	0.0000	0.0000	8.7679	5729.66
504.3498	7.8717	0.0000	0.0000	7.8717	5737.53
490.4028	7.0655	0.0000	0.0000	7.0655	5744.59
476.7319	6.3405	0.0000	0.0000	6.3405	5750.93
463.3317	5.6885	0.0000	0.0000	5.6885	5756.62
450.1968	5.1023	0.0000	0.0000	5.1023	5761.72
437.3221	4.5753	0.0000	0.0000	4.5753	5766.30
424.7023	4.1016	0.0000	0.0000	4.1016	5770.40
412.3323	3.6760	0.0000	0.0000	3.6760	5774.08
400.2073	3.2936	0.0000	0.0000	3.2936	5777.37
388.3224	2.9501	0.0000	0.0000	2.9501	5780.32
376.6729	2.6416	0.0000	0.0000	2.6416	5782.96
365.2540	2.3646	0.0000	0.0000	2.3646	5785.33
354.0612	2.1159	0.0000	0.0000	2.1159	5787.44
343.0901	1.8927	0.0000	0.0000	1.8927	5789.34
332.3362	1.6924	0.0000	0.0000	1.6924	5791.03
321.7952	1.5127	0.0000	0.0000	1.5127	5792.54
311.4630	1.3516	0.0000	0.0000	1.3516	5793.89
301.3353	1.2071	0.0000	0.0000	1.2071	5795.10
291.4082	1.0776	0.0000	0.0000	1.0776	5796.18
281.6777	0.9615	0.0000	0.0000	0.9615	5797.14
272.1398	0.8575	0.0000	0.0000	0.8575	5798.00
262.7908	0.7644	0.0000	0.0000	0.7644	5798.76
253.6269	0.6810	0.0000	0.0000	0.6810	5799.44
244.6445	0.6064	0.0000	0.0000	0.6064	5800.05
235.8400	0.5396	0.0000	0.0000	0.5396	5800.59
227.2098	0.4799	0.0000	0.0000	0.4799	5801.07
218.7505	0.4265	0.0000	0.0000	0.4265	5801.49
210.4586	0.3788	0.0000	0.0000	0.3788	5801.87
202.3310	0.3362	0.0000	0.0000	0.3362	5802.21
194.3643	0.2981	0.0000	0.0000	0.2981	5802.51
186.5554	0.2641	0.0000	0.0000	0.2641	5802.77
178.9011	0.2338	0.0000	0.0000	0.2338	5803.01
171.3983	0.2068	0.0000	0.0000	0.2068	5803.21
164.0442	0.1827	0.0000	0.0000	0.1827	5803.39
156.8356	0.1612	0.0000	0.0000	0.1612	5803.56
149.7698	0.1421	0.0000	0.0000	0.1421	5803.70
142.8439	0.1251	0.0000	0.0000	0.1251	5803.82
136.0551	0.1100	0.0000	0.0000	0.1100	5803.93
129.4008	0.0966	0.0000	0.0000	0.0966	5804.03
122.8782	0.0847	0.0000	0.0000	0.0847	5804.11
116.4848	0.0741	0.0000	0.0000	0.0741	5804.19
110.2180	0.0647	0.0000	0.0000	0.0647	5804.25
104.0752	0.0564	0.0000	0.0000	0.0564	5804.31
98.0541	0.0491	0.0000	0.0000	0.0491	5804.36
92.1523	0.0426	0.0000	0.0000	0.0426	5804.40
86.3673	0.0368	0.0000	0.0000	0.0368	5804.44
80.6968	0.0318	0.0000	0.0000	0.0318	5804.47
75.1387	0.0273	0.0000	0.0000	0.0273	5804.50
69.6905	0.0234	0.0000	0.0000	0.0000	5804.50
64.3503	0.0199	0.0000	0.0000	0.0000	5804.50
59.1158	0.0169	0.0000	0.0000	0.0000	5804.50

Formulae Entered in the FORMULATION Worksheet

Time Phase t	Discount	Water Level 1 X(t)
0	=EXP(-DisRate*A7)	=K_1*EXP(-Alpha*A7)
1	=EXP(-DisRate*A8)	=K_1*EXP(-Alpha*A8)
2	=EXP(-DisRate*A9)	=K_1*EXP(-Alpha*A9)
3	=EXP(-DisRate*A10)	=K_1*EXP(-Alpha*A10)
4	=EXP(-DisRate*A11)	=K_1*EXP(-Alpha*A11)
5	=EXP(-DisRate*A12)	=K_1*EXP(-Alpha*A12)
6	=EXP(-DisRate*A13)	=K_1*EXP(-Alpha*A13)
7	=EXP(-DisRate*A14)	=K_1*EXP(-Alpha*A14)
8	=EXP(-DisRate*A15)	=K_1*EXP(-Alpha*A15)
9	=EXP(-DisRate*A16)	=K_1*EXP(-Alpha*A16)
10	=EXP(-DisRate*A17)	=K_1*EXP(-Alpha*A17)
11	=EXP(-DisRate*A18)	=K_1*EXP(-Alpha*A18)
12	=EXP(-DisRate*A19)	=K_1*EXP(-Alpha*A19)
13	=EXP(-DisRate*A20)	=K_1*EXP(-Alpha*A20)
14	=EXP(-DisRate*A21)	=K_1*EXP(-Alpha*A21)
15	=EXP(-DisRate*A22)	=K_1*EXP(-Alpha*A22)
16	=EXP(-DisRate*A23)	=K_1*EXP(-Alpha*A23)
17	=EXP(-DisRate*A24)	=K_1*EXP(-Alpha*A24)
18	=EXP(-DisRate*A25)	=K_1*EXP(-Alpha*A25)
19	=EXP(-DisRate*A26)	=K_1*EXP(-Alpha*A26)
20	=EXP(-DisRate*A27)	=K_1*EXP(-Alpha*A27)
21	=EXP(-DisRate*A28)	=K_1*EXP(-Alpha*A28)
22	=EXP(-DisRate*A29)	=K_1*EXP(-Alpha*A29)
23	=EXP(-DisRate*A30)	=K_1*EXP(-Alpha*A30)
24	=EXP(-DisRate*A31)	=K_1*EXP(-Alpha*A31)
25	=EXP(-DisRate*A32)	=K_1*EXP(-Alpha*A32)
26	=EXP(-DisRate*A33)	=K_1*EXP(-Alpha*A33)
27	=EXP(-DisRate*A34)	=K_1*EXP(-Alpha*A34)
28	=EXP(-DisRate*A35)	=K_1*EXP(-Alpha*A35)
29	=EXP(-DisRate*A36)	=K_1*EXP(-Alpha*A36)
30	=EXP(-DisRate*A37)	=K_1*EXP(-Alpha*A37)
31	=EXP(-DisRate*A38)	=K_1*EXP(-Alpha*A38)
32	=EXP(-DisRate*A39)	=K_1*EXP(-Alpha*A39)
33	=EXP(-DisRate*A40)	=K_1*EXP(-Alpha*A40)
34	=EXP(-DisRate*A41)	=K_1*EXP(-Alpha*A41)
35	=EXP(-DisRate*A42)	=K_1*EXP(-Alpha*A42)
36	=EXP(-DisRate*A43)	=K_1*EXP(-Alpha*A43)
37	=EXP(-DisRate*A44)	=K_1*EXP(-Alpha*A44)
38	=EXP(-DisRate*A45)	=K_1*EXP(-Alpha*A45)
39	=EXP(-DisRate*A46)	=K_1*EXP(-Alpha*A46)
40	=EXP(-DisRate*A47)	=K_1*EXP(-Alpha*A47)
41	=EXP(-DisRate*A48)	=K_1*EXP(-Alpha*A48)
42	=EXP(-DisRate*A49)	=K_1*EXP(-Alpha*A49)
43	=EXP(-DisRate*A50)	=K_1*EXP(-Alpha*A50)

Water Level

2

X(t)

```
=IF(A7>=OpTime1,K_2*m*EXP(Beta*u*A7)/(1+K_2*EXP(Beta*u*A7)),0)
=IF(A8>=OpTime1,K_2*m*EXP(Beta*u*A8)/(1+K_2*EXP(Beta*u*A8)),0)
=IF(A9>=OpTime1,K_2*m*EXP(Beta*u*A9)/(1+K_2*EXP(Beta*u*A9)),0)
=IF(A10>=OpTime1,K_2*m*EXP(Beta*u*A10)/(1+K_2*EXP(Beta*u*A10)),0)
=IF(A11>=OpTime1,K_2*m*EXP(Beta*u*A11)/(1+K_2*EXP(Beta*u*A11)),0)
=IF(A12>=OpTime1,K_2*m*EXP(Beta*u*A12)/(1+K_2*EXP(Beta*u*A12)),0)
=IF(A13>=OpTime1,K_2*m*EXP(Beta*u*A13)/(1+K_2*EXP(Beta*u*A13)),0)
=IF(A14>=OpTime1,K_2*m*EXP(Beta*u*A14)/(1+K_2*EXP(Beta*u*A14)),0)
=IF(A15>=OpTime1,K_2*m*EXP(Beta*u*A15)/(1+K_2*EXP(Beta*u*A15)),0)
=IF(A16>=OpTime1,K_2*m*EXP(Beta*u*A16)/(1+K_2*EXP(Beta*u*A16)),0)
=IF(A17>=OpTime1,K_2*m*EXP(Beta*u*A17)/(1+K_2*EXP(Beta*u*A17)),0)
=IF(A18>=OpTime1,K_2*m*EXP(Beta*u*A18)/(1+K_2*EXP(Beta*u*A18)),0)
=IF(A19>=OpTime1,K_2*m*EXP(Beta*u*A19)/(1+K_2*EXP(Beta*u*A19)),0)
=IF(A20>=OpTime1,K_2*m*EXP(Beta*u*A20)/(1+K_2*EXP(Beta*u*A20)),0)
=IF(A21>=OpTime1,K_2*m*EXP(Beta*u*A21)/(1+K_2*EXP(Beta*u*A21)),0)
=IF(A22>=OpTime1,K_2*m*EXP(Beta*u*A22)/(1+K_2*EXP(Beta*u*A22)),0)
=IF(A23>=OpTime1,K_2*m*EXP(Beta*u*A23)/(1+K_2*EXP(Beta*u*A23)),0)
=IF(A24>=OpTime1,K_2*m*EXP(Beta*u*A24)/(1+K_2*EXP(Beta*u*A24)),0)
=IF(A25>=OpTime1,K_2*m*EXP(Beta*u*A25)/(1+K_2*EXP(Beta*u*A25)),0)
=IF(A26>=OpTime1,K_2*m*EXP(Beta*u*A26)/(1+K_2*EXP(Beta*u*A26)),0)
=IF(A27>=OpTime1,K_2*m*EXP(Beta*u*A27)/(1+K_2*EXP(Beta*u*A27)),0)
=IF(A28>=OpTime1,K_2*m*EXP(Beta*u*A28)/(1+K_2*EXP(Beta*u*A28)),0)
=IF(A29>=OpTime1,K_2*m*EXP(Beta*u*A29)/(1+K_2*EXP(Beta*u*A29)),0)
=IF(A30>=OpTime1,K_2*m*EXP(Beta*u*A30)/(1+K_2*EXP(Beta*u*A30)),0)
=IF(A31>=OpTime1,K_2*m*EXP(Beta*u*A31)/(1+K_2*EXP(Beta*u*A31)),0)
=IF(A32>=OpTime1,K_2*m*EXP(Beta*u*A32)/(1+K_2*EXP(Beta*u*A32)),0)
=IF(A33>=OpTime1,K_2*m*EXP(Beta*u*A33)/(1+K_2*EXP(Beta*u*A33)),0)
=IF(A34>=OpTime1,K_2*m*EXP(Beta*u*A34)/(1+K_2*EXP(Beta*u*A34)),0)
=IF(A35>=OpTime1,K_2*m*EXP(Beta*u*A35)/(1+K_2*EXP(Beta*u*A35)),0)
=IF(A36>=OpTime1,K_2*m*EXP(Beta*u*A36)/(1+K_2*EXP(Beta*u*A36)),0)
=IF(A37>=OpTime1,K_2*m*EXP(Beta*u*A37)/(1+K_2*EXP(Beta*u*A37)),0)
=IF(A38>=OpTime1,K_2*m*EXP(Beta*u*A38)/(1+K_2*EXP(Beta*u*A38)),0)
=IF(A39>=OpTime1,K_2*m*EXP(Beta*u*A39)/(1+K_2*EXP(Beta*u*A39)),0)
=IF(A40>=OpTime1,K_2*m*EXP(Beta*u*A40)/(1+K_2*EXP(Beta*u*A40)),0)
=IF(A41>=OpTime1,K_2*m*EXP(Beta*u*A41)/(1+K_2*EXP(Beta*u*A41)),0)
=IF(A42>=OpTime1,K_2*m*EXP(Beta*u*A42)/(1+K_2*EXP(Beta*u*A42)),0)
=IF(A43>=OpTime1,K_2*m*EXP(Beta*u*A43)/(1+K_2*EXP(Beta*u*A43)),0)
=IF(A44>=OpTime1,K_2*m*EXP(Beta*u*A44)/(1+K_2*EXP(Beta*u*A44)),0)
=IF(A45>=OpTime1,K_2*m*EXP(Beta*u*A45)/(1+K_2*EXP(Beta*u*A45)),0)
=IF(A46>=OpTime1,K_2*m*EXP(Beta*u*A46)/(1+K_2*EXP(Beta*u*A46)),0)
=IF(A47>=OpTime1,K_2*m*EXP(Beta*u*A47)/(1+K_2*EXP(Beta*u*A47)),0)
=IF(A48>=OpTime1,K_2*m*EXP(Beta*u*A48)/(1+K_2*EXP(Beta*u*A48)),0)
=IF(A49>=OpTime1,K_2*m*EXP(Beta*u*A49)/(1+K_2*EXP(Beta*u*A49)),0)
=IF(A50>=OpTime1,K_2*m*EXP(Beta*u*A50)/(1+K_2*EXP(Beta*u*A50)),0)
=IF(A51>=OpTime1,K_2*m*EXP(Beta*u*A51)/(1+K_2*EXP(Beta*u*A51)),0)
=IF(A52>=OpTime1,K_2*m*EXP(Beta*u*A52)/(1+K_2*EXP(Beta*u*A52)),0)
```

Water Level 3 X(t)	Yield 1 Y1(t)	Revenue 1 "Price * Yield1" "-Cost1"
=IF(A7>=OpTime2,K_3*EXP(-Gamma*A7),0)	=Y_01*C7/m	=F7*Price1-Cost1
=IF(A8>=OpTime2,K_3*EXP(-Gamma*A8),0)	=Y_01*C8/m	=F8*Price1-Cost1
=IF(A9>=OpTime2,K_3*EXP(-Gamma*A9),0)	=Y_01*C9/m	=F9*Price1-Cost1
=IF(A10>=OpTime2,K_3*EXP(-Gamma*A10),0)	=Y_01*C10/m	=F10*Price1-Cost1
=IF(A11>=OpTime2,K_3*EXP(-Gamma*A11),0)	=Y_01*C11/m	=F11*Price1-Cost1
=IF(A12>=OpTime2,K_3*EXP(-Gamma*A12),0)	=Y_01*C12/m	=F12*Price1-Cost1
=IF(A13>=OpTime2,K_3*EXP(-Gamma*A13),0)	=Y_01*C13/m	=F13*Price1-Cost1
=IF(A14>=OpTime2,K_3*EXP(-Gamma*A14),0)	=Y_01*C14/m	=F14*Price1-Cost1
=IF(A15>=OpTime2,K_3*EXP(-Gamma*A15),0)	=Y_01*C15/m	=F15*Price1-Cost1
=IF(A16>=OpTime2,K_3*EXP(-Gamma*A16),0)	=Y_01*C16/m	=F16*Price1-Cost1
=IF(A17>=OpTime2,K_3*EXP(-Gamma*A17),0)	=Y_01*C17/m	=F17*Price1-Cost1
=IF(A18>=OpTime2,K_3*EXP(-Gamma*A18),0)	=Y_01*C18/m	=F18*Price1-Cost1
=IF(A19>=OpTime2,K_3*EXP(-Gamma*A19),0)	=Y_01*C19/m	=F19*Price1-Cost1
=IF(A20>=OpTime2,K_3*EXP(-Gamma*A20),0)	=Y_01*C20/m	=F20*Price1-Cost1
=IF(A21>=OpTime2,K_3*EXP(-Gamma*A21),0)	=Y_01*C21/m	=F21*Price1-Cost1
=IF(A22>=OpTime2,K_3*EXP(-Gamma*A22),0)	=Y_01*C22/m	=F22*Price1-Cost1
=IF(A23>=OpTime2,K_3*EXP(-Gamma*A23),0)	=Y_01*C23/m	=F23*Price1-Cost1
=IF(A24>=OpTime2,K_3*EXP(-Gamma*A24),0)	=Y_01*C24/m	=F24*Price1-Cost1
=IF(A25>=OpTime2,K_3*EXP(-Gamma*A25),0)	=Y_01*C25/m	=F25*Price1-Cost1
=IF(A26>=OpTime2,K_3*EXP(-Gamma*A26),0)	=Y_01*C26/m	=F26*Price1-Cost1
=IF(A27>=OpTime2,K_3*EXP(-Gamma*A27),0)	=Y_01*C27/m	=F27*Price1-Cost1
=IF(A28>=OpTime2,K_3*EXP(-Gamma*A28),0)	=Y_01*C28/m	=F28*Price1-Cost1
=IF(A29>=OpTime2,K_3*EXP(-Gamma*A29),0)	=Y_01*C29/m	=F29*Price1-Cost1
=IF(A30>=OpTime2,K_3*EXP(-Gamma*A30),0)	=Y_01*C30/m	=F30*Price1-Cost1
=IF(A31>=OpTime2,K_3*EXP(-Gamma*A31),0)	=Y_01*C31/m	=F31*Price1-Cost1
=IF(A32>=OpTime2,K_3*EXP(-Gamma*A32),0)	=Y_01*C32/m	=F32*Price1-Cost1
=IF(A33>=OpTime2,K_3*EXP(-Gamma*A33),0)	=Y_01*C33/m	=F33*Price1-Cost1
=IF(A34>=OpTime2,K_3*EXP(-Gamma*A34),0)	=Y_01*C34/m	=F34*Price1-Cost1
=IF(A35>=OpTime2,K_3*EXP(-Gamma*A35),0)	=Y_01*C35/m	=F35*Price1-Cost1
=IF(A36>=OpTime2,K_3*EXP(-Gamma*A36),0)	=Y_01*C36/m	=F36*Price1-Cost1
=IF(A37>=OpTime2,K_3*EXP(-Gamma*A37),0)	=Y_01*C37/m	=F37*Price1-Cost1
=IF(A38>=OpTime2,K_3*EXP(-Gamma*A38),0)	=Y_01*C38/m	=F38*Price1-Cost1
=IF(A39>=OpTime2,K_3*EXP(-Gamma*A39),0)	=Y_01*C39/m	=F39*Price1-Cost1
=IF(A40>=OpTime2,K_3*EXP(-Gamma*A40),0)	=Y_01*C40/m	=F40*Price1-Cost1
=IF(A41>=OpTime2,K_3*EXP(-Gamma*A41),0)	=Y_01*C41/m	=F41*Price1-Cost1
=IF(A42>=OpTime2,K_3*EXP(-Gamma*A42),0)	=Y_01*C42/m	=F42*Price1-Cost1
=IF(A43>=OpTime2,K_3*EXP(-Gamma*A43),0)	=Y_01*C43/m	=F43*Price1-Cost1
=IF(A44>=OpTime2,K_3*EXP(-Gamma*A44),0)	=Y_01*C44/m	=F44*Price1-Cost1
=IF(A45>=OpTime2,K_3*EXP(-Gamma*A45),0)	=Y_01*C45/m	=F45*Price1-Cost1
=IF(A46>=OpTime2,K_3*EXP(-Gamma*A46),0)	=Y_01*C46/m	=F46*Price1-Cost1
=IF(A47>=OpTime2,K_3*EXP(-Gamma*A47),0)	=Y_01*C47/m	=F47*Price1-Cost1
=IF(A48>=OpTime2,K_3*EXP(-Gamma*A48),0)	=Y_01*C48/m	=F48*Price1-Cost1
=IF(A49>=OpTime2,K_3*EXP(-Gamma*A49),0)	=Y_01*C49/m	=F49*Price1-Cost1
=IF(A50>=OpTime2,K_3*EXP(-Gamma*A50),0)	=Y_01*C50/m	=F50*Price1-Cost1
=IF(A51>=OpTime2,K_3*EXP(-Gamma*A51),0)	=Y_01*C51/m	=F51*Price1-Cost1
=IF(A52>=OpTime2,K_3*EXP(-Gamma*A52),0)	=Y_01*C52/m	=F52*Price1-Cost1

Net Profit 1 Revenue1*Discount D(t)	Tree Growth T(t)
=G7*B7	=IF(A7>=OpTime1,L_bar*u*(1-EXP(-l*(A7-OpTime1))),0)
=G8*B8	=IF(A8>=OpTime1,L_bar*u*(1-EXP(-l*(A8-OpTime1))),0)
=G9*B9	=IF(A9>=OpTime1,L_bar*u*(1-EXP(-l*(A9-OpTime1))),0)
=G10*B10	=IF(A10>=OpTime1,L_bar*u*(1-EXP(-l*(A10-OpTime1))),0)
=G11*B11	=IF(A11>=OpTime1,L_bar*u*(1-EXP(-l*(A11-OpTime1))),0)
=G12*B12	=IF(A12>=OpTime1,L_bar*u*(1-EXP(-l*(A12-OpTime1))),0)
=G13*B13	=IF(A13>=OpTime1,L_bar*u*(1-EXP(-l*(A13-OpTime1))),0)
=G14*B14	=IF(A14>=OpTime1,L_bar*u*(1-EXP(-l*(A14-OpTime1))),0)
=G15*B15	=IF(A15>=OpTime1,L_bar*u*(1-EXP(-l*(A15-OpTime1))),0)
=G16*B16	=IF(A16>=OpTime1,L_bar*u*(1-EXP(-l*(A16-OpTime1))),0)
=G17*B17	=IF(A17>=OpTime1,L_bar*u*(1-EXP(-l*(A17-OpTime1))),0)
=G18*B18	=IF(A18>=OpTime1,L_bar*u*(1-EXP(-l*(A18-OpTime1))),0)
=G19*B19	=IF(A19>=OpTime1,L_bar*u*(1-EXP(-l*(A19-OpTime1))),0)
=G20*B20	=IF(A20>=OpTime1,L_bar*u*(1-EXP(-l*(A20-OpTime1))),0)
=G21*B21	=IF(A21>=OpTime1,L_bar*u*(1-EXP(-l*(A21-OpTime1))),0)
=G22*B22	=IF(A22>=OpTime1,L_bar*u*(1-EXP(-l*(A22-OpTime1))),0)
=G23*B23	=IF(A23>=OpTime1,L_bar*u*(1-EXP(-l*(A23-OpTime1))),0)
=G24*B24	=IF(A24>=OpTime1,L_bar*u*(1-EXP(-l*(A24-OpTime1))),0)
=G25*B25	=IF(A25>=OpTime1,L_bar*u*(1-EXP(-l*(A25-OpTime1))),0)
=G26*B26	=IF(A26>=OpTime1,L_bar*u*(1-EXP(-l*(A26-OpTime1))),0)
=G27*B27	=IF(A27>=OpTime1,L_bar*u*(1-EXP(-l*(A27-OpTime1))),0)
=G28*B28	=IF(A28>=OpTime1,L_bar*u*(1-EXP(-l*(A28-OpTime1))),0)
=G29*B29	=IF(A29>=OpTime1,L_bar*u*(1-EXP(-l*(A29-OpTime1))),0)
=G30*B30	=IF(A30>=OpTime1,L_bar*u*(1-EXP(-l*(A30-OpTime1))),0)
=G31*B31	=IF(A31>=OpTime1,L_bar*u*(1-EXP(-l*(A31-OpTime1))),0)
=G32*B32	=IF(A32>=OpTime1,L_bar*u*(1-EXP(-l*(A32-OpTime1))),0)
=G33*B33	=IF(A33>=OpTime1,L_bar*u*(1-EXP(-l*(A33-OpTime1))),0)
=G34*B34	=IF(A34>=OpTime1,L_bar*u*(1-EXP(-l*(A34-OpTime1))),0)
=G35*B35	=IF(A35>=OpTime1,L_bar*u*(1-EXP(-l*(A35-OpTime1))),0)
=G36*B36	=IF(A36>=OpTime1,L_bar*u*(1-EXP(-l*(A36-OpTime1))),0)
=G37*B37	=IF(A37>=OpTime1,L_bar*u*(1-EXP(-l*(A37-OpTime1))),0)
=G38*B38	=IF(A38>=OpTime1,L_bar*u*(1-EXP(-l*(A38-OpTime1))),0)
=G39*B39	=IF(A39>=OpTime1,L_bar*u*(1-EXP(-l*(A39-OpTime1))),0)
=G40*B40	=IF(A40>=OpTime1,L_bar*u*(1-EXP(-l*(A40-OpTime1))),0)
=G41*B41	=IF(A41>=OpTime1,L_bar*u*(1-EXP(-l*(A41-OpTime1))),0)
=G42*B42	=IF(A42>=OpTime1,L_bar*u*(1-EXP(-l*(A42-OpTime1))),0)
=G43*B43	=IF(A43>=OpTime1,L_bar*u*(1-EXP(-l*(A43-OpTime1))),0)
=G44*B44	=IF(A44>=OpTime1,L_bar*u*(1-EXP(-l*(A44-OpTime1))),0)
=G45*B45	=IF(A45>=OpTime1,L_bar*u*(1-EXP(-l*(A45-OpTime1))),0)
=G46*B46	=IF(A46>=OpTime1,L_bar*u*(1-EXP(-l*(A46-OpTime1))),0)
=G47*B47	=IF(A47>=OpTime1,L_bar*u*(1-EXP(-l*(A47-OpTime1))),0)
=G48*B48	=IF(A48>=OpTime1,L_bar*u*(1-EXP(-l*(A48-OpTime1))),0)
=G49*B49	=IF(A49>=OpTime1,L_bar*u*(1-EXP(-l*(A49-OpTime1))),0)
=G50*B50	=IF(A50>=OpTime1,L_bar*u*(1-EXP(-l*(A50-OpTime1))),0)
=G51*B51	=IF(A51>=OpTime1,L_bar*u*(1-EXP(-l*(A51-OpTime1))),0)
=G52*B52	=IF(A52>=OpTime1,L_bar*u*(1-EXP(-l*(A52-OpTime1))),0)

Revenue2 Tree Revenue	Profit 2 evenue2 * Discou F(t)	Yield 3 Y3(t)
=IF(A7>=OpTime1,I7*Price2,0)	=J7*B7	=IF(A7>=OpTime2,Y_03*E7/m,0)
=IF(A8>=OpTime1,I8*Price2,0)	=J8*B8	=IF(A8>=OpTime2,Y_03*E8/m,0)
=IF(A9>=OpTime1,I9*Price2,0)	=J9*B9	=IF(A9>=OpTime2,Y_03*E9/m,0)
=IF(A10>=OpTime1,I10*Price2,0)	=J10*B10	=IF(A10>=OpTime2,Y_03*E10/m,0)
=IF(A11>=OpTime1,I11*Price2,0)	=J11*B11	=IF(A11>=OpTime2,Y_03*E11/m,0)
=IF(A12>=OpTime1,I12*Price2,0)	=J12*B12	=IF(A12>=OpTime2,Y_03*E12/m,0)
=IF(A13>=OpTime1,I13*Price2,0)	=J13*B13	=IF(A13>=OpTime2,Y_03*E13/m,0)
=IF(A14>=OpTime1,I14*Price2,0)	=J14*B14	=IF(A14>=OpTime2,Y_03*E14/m,0)
=IF(A15>=OpTime1,I15*Price2,0)	=J15*B15	=IF(A15>=OpTime2,Y_03*E15/m,0)
=IF(A16>=OpTime1,I16*Price2,0)	=J16*B16	=IF(A16>=OpTime2,Y_03*E16/m,0)
=IF(A17>=OpTime1,I17*Price2,0)	=J17*B17	=IF(A17>=OpTime2,Y_03*E17/m,0)
=IF(A18>=OpTime1,I18*Price2,0)	=J18*B18	=IF(A18>=OpTime2,Y_03*E18/m,0)
=IF(A19>=OpTime1,I19*Price2,0)	=J19*B19	=IF(A19>=OpTime2,Y_03*E19/m,0)
=IF(A20>=OpTime1,I20*Price2,0)	=J20*B20	=IF(A20>=OpTime2,Y_03*E20/m,0)
=IF(A21>=OpTime1,I21*Price2,0)	=J21*B21	=IF(A21>=OpTime2,Y_03*E21/m,0)
=IF(A22>=OpTime1,I22*Price2,0)	=J22*B22	=IF(A22>=OpTime2,Y_03*E22/m,0)
=IF(A23>=OpTime1,I23*Price2,0)	=J23*B23	=IF(A23>=OpTime2,Y_03*E23/m,0)
=IF(A24>=OpTime1,I24*Price2,0)	=J24*B24	=IF(A24>=OpTime2,Y_03*E24/m,0)
=IF(A25>=OpTime1,I25*Price2,0)	=J25*B25	=IF(A25>=OpTime2,Y_03*E25/m,0)
=IF(A26>=OpTime1,I26*Price2,0)	=J26*B26	=IF(A26>=OpTime2,Y_03*E26/m,0)
=IF(A27>=OpTime1,I27*Price2,0)	=J27*B27	=IF(A27>=OpTime2,Y_03*E27/m,0)
=IF(A28>=OpTime1,I28*Price2,0)	=J28*B28	=IF(A28>=OpTime2,Y_03*E28/m,0)
=IF(A29>=OpTime1,I29*Price2,0)	=J29*B29	=IF(A29>=OpTime2,Y_03*E29/m,0)
=IF(A30>=OpTime1,I30*Price2,0)	=J30*B30	=IF(A30>=OpTime2,Y_03*E30/m,0)
=IF(A31>=OpTime1,I31*Price2,0)	=J31*B31	=IF(A31>=OpTime2,Y_03*E31/m,0)
=IF(A32>=OpTime1,I32*Price2,0)	=J32*B32	=IF(A32>=OpTime2,Y_03*E32/m,0)
=IF(A33>=OpTime1,I33*Price2,0)	=J33*B33	=IF(A33>=OpTime2,Y_03*E33/m,0)
=IF(A34>=OpTime1,I34*Price2,0)	=J34*B34	=IF(A34>=OpTime2,Y_03*E34/m,0)
=IF(A35>=OpTime1,I35*Price2,0)	=J35*B35	=IF(A35>=OpTime2,Y_03*E35/m,0)
=IF(A36>=OpTime1,I36*Price2,0)	=J36*B36	=IF(A36>=OpTime2,Y_03*E36/m,0)
=IF(A37>=OpTime1,I37*Price2,0)	=J37*B37	=IF(A37>=OpTime2,Y_03*E37/m,0)
=IF(A38>=OpTime1,I38*Price2,0)	=J38*B38	=IF(A38>=OpTime2,Y_03*E38/m,0)
=IF(A39>=OpTime1,I39*Price2,0)	=J39*B39	=IF(A39>=OpTime2,Y_03*E39/m,0)
=IF(A40>=OpTime1,I40*Price2,0)	=J40*B40	=IF(A40>=OpTime2,Y_03*E40/m,0)
=IF(A41>=OpTime1,I41*Price2,0)	=J41*B41	=IF(A41>=OpTime2,Y_03*E41/m,0)
=IF(A42>=OpTime1,I42*Price2,0)	=J42*B42	=IF(A42>=OpTime2,Y_03*E42/m,0)
=IF(A43>=OpTime1,I43*Price2,0)	=J43*B43	=IF(A43>=OpTime2,Y_03*E43/m,0)
=IF(A44>=OpTime1,I44*Price2,0)	=J44*B44	=IF(A44>=OpTime2,Y_03*E44/m,0)
=IF(A45>=OpTime1,I45*Price2,0)	=J45*B45	=IF(A45>=OpTime2,Y_03*E45/m,0)
=IF(A46>=OpTime1,I46*Price2,0)	=J46*B46	=IF(A46>=OpTime2,Y_03*E46/m,0)
=IF(A47>=OpTime1,I47*Price2,0)	=J47*B47	=IF(A47>=OpTime2,Y_03*E47/m,0)
=IF(A48>=OpTime1,I48*Price2,0)	=J48*B48	=IF(A48>=OpTime2,Y_03*E48/m,0)
=IF(A49>=OpTime1,I49*Price2,0)	=J49*B49	=IF(A49>=OpTime2,Y_03*E49/m,0)
=IF(A50>=OpTime1,I50*Price2,0)	=J50*B50	=IF(A50>=OpTime2,Y_03*E50/m,0)
=IF(A51>=OpTime1,I51*Price2,0)	=J51*B51	=IF(A51>=OpTime2,Y_03*E51/m,0)
=IF(A52>=OpTime1,I52*Price2,0)	=J52*B52	=IF(A52>=OpTime2,Y_03*E52/m,0)

Revenue 3 "Price * Yield3" "-Cost3"	Net Profit 3 Revenue3*Discount G(t)
=IF(A7>=OpTime2,L7*Price3-Cost3,0)	=M7*B7
=IF(A8>=OpTime2,L8*Price3-Cost3,0)	=M8*B8
=IF(A9>=OpTime2,L9*Price3-Cost3,0)	=M9*B9
=IF(A10>=OpTime2,L10*Price3-Cost3,0)	=M10*B10
=IF(A11>=OpTime2,L11*Price3-Cost3,0)	=M11*B11
=IF(A12>=OpTime2,L12*Price3-Cost3,0)	=M12*B12
=IF(A13>=OpTime2,L13*Price3-Cost3,0)	=M13*B13
=IF(A14>=OpTime2,L14*Price3-Cost3,0)	=M14*B14
=IF(A15>=OpTime2,L15*Price3-Cost3,0)	=M15*B15
=IF(A16>=OpTime2,L16*Price3-Cost3,0)	=M16*B16
=IF(A17>=OpTime2,L17*Price3-Cost3,0)	=M17*B17
=IF(A18>=OpTime2,L18*Price3-Cost3,0)	=M18*B18
=IF(A19>=OpTime2,L19*Price3-Cost3,0)	=M19*B19
=IF(A20>=OpTime2,L20*Price3-Cost3,0)	=M20*B20
=IF(A21>=OpTime2,L21*Price3-Cost3,0)	=M21*B21
=IF(A22>=OpTime2,L22*Price3-Cost3,0)	=M22*B22
=IF(A23>=OpTime2,L23*Price3-Cost3,0)	=M23*B23
=IF(A24>=OpTime2,L24*Price3-Cost3,0)	=M24*B24
=IF(A25>=OpTime2,L25*Price3-Cost3,0)	=M25*B25
=IF(A26>=OpTime2,L26*Price3-Cost3,0)	=M26*B26
=IF(A27>=OpTime2,L27*Price3-Cost3,0)	=M27*B27
=IF(A28>=OpTime2,L28*Price3-Cost3,0)	=M28*B28
=IF(A29>=OpTime2,L29*Price3-Cost3,0)	=M29*B29
=IF(A30>=OpTime2,L30*Price3-Cost3,0)	=M30*B30
=IF(A31>=OpTime2,L31*Price3-Cost3,0)	=M31*B31
=IF(A32>=OpTime2,L32*Price3-Cost3,0)	=M32*B32
=IF(A33>=OpTime2,L33*Price3-Cost3,0)	=M33*B33
=IF(A34>=OpTime2,L34*Price3-Cost3,0)	=M34*B34
=IF(A35>=OpTime2,L35*Price3-Cost3,0)	=M35*B35
=IF(A36>=OpTime2,L36*Price3-Cost3,0)	=M36*B36
=IF(A37>=OpTime2,L37*Price3-Cost3,0)	=M37*B37
=IF(A38>=OpTime2,L38*Price3-Cost3,0)	=M38*B38
=IF(A39>=OpTime2,L39*Price3-Cost3,0)	=M39*B39
=IF(A40>=OpTime2,L40*Price3-Cost3,0)	=M40*B40
=IF(A41>=OpTime2,L41*Price3-Cost3,0)	=M41*B41
=IF(A42>=OpTime2,L42*Price3-Cost3,0)	=M42*B42
=IF(A43>=OpTime2,L43*Price3-Cost3,0)	=M43*B43
=IF(A44>=OpTime2,L44*Price3-Cost3,0)	=M44*B44
=IF(A45>=OpTime2,L45*Price3-Cost3,0)	=M45*B45
=IF(A46>=OpTime2,L46*Price3-Cost3,0)	=M46*B46
=IF(A47>=OpTime2,L47*Price3-Cost3,0)	=M47*B47
=IF(A48>=OpTime2,L48*Price3-Cost3,0)	=M48*B48
=IF(A49>=OpTime2,L49*Price3-Cost3,0)	=M49*B49
=IF(A50>=OpTime2,L50*Price3-Cost3,0)	=M50*B50
=IF(A51>=OpTime2,L51*Price3-Cost3,0)	=M51*B51
=IF(A52>=OpTime2,L52*Price3-Cost3,0)	=M52*B52

Switching Cost1
=IF(A7=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A8=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A9=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A10=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A11=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A12=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A13=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A14=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A15=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A16=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A17=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A18=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A19=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A20=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A21=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A22=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A23=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A24=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A25=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A26=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A27=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A28=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A29=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A30=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A31=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A32=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A33=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A34=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A35=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A36=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A37=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A38=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A39=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A40=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A41=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A42=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A43=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A44=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A45=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A46=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A47=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A48=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A49=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A50=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A51=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)
=IF(A52=OpTime1,(SwCost1+Cost2*u)*EXP(-DisRate*OpTime1),0)

Switching Cost2
=IF(A7=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A8=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A9=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A10=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A11=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A12=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A13=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A14=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A15=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A16=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A17=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A18=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A19=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A20=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A21=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A22=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A23=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A24=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A25=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A26=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A27=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A28=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A29=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A30=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A31=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A32=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A33=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A34=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A35=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A36=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A37=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A38=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A39=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A40=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A41=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A42=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A43=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A44=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A45=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A46=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A47=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A48=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A49=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A50=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A51=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)
=IF(A52=OpTime2,SwCost2*EXP(-DisRate*OpTime2),0)

Yearly Profit	Total Profit J
=IF(A7<OpTime1,H7,IF(A7<OpTime2,K7,IF(A7<OpTime3,N7,0)))-O7-P7	=Q7+R6
=IF(A8<OpTime1,H8,IF(A8<OpTime2,K8,IF(A8<OpTime3,N8,0)))-O8-P8	=Q8+R7
=IF(A9<OpTime1,H9,IF(A9<OpTime2,K9,IF(A9<OpTime3,N9,0)))-O9-P9	=Q9+R8
=IF(A10<OpTime1,H10,IF(A10<OpTime2,K10,IF(A10<OpTime3,N10,0)))-O10-P10	=Q10+R9
=IF(A11<OpTime1,H11,IF(A11<OpTime2,K11,IF(A11<OpTime3,N11,0)))-O11-P11	=Q11+R10
=IF(A12<OpTime1,H12,IF(A12<OpTime2,K12,IF(A12<OpTime3,N12,0)))-O12-P12	=Q12+R11
=IF(A13<OpTime1,H13,IF(A13<OpTime2,K13,IF(A13<OpTime3,N13,0)))-O13-P13	=Q13+R12
=IF(A14<OpTime1,H14,IF(A14<OpTime2,K14,IF(A14<OpTime3,N14,0)))-O14-P14	=Q14+R13
=IF(A15<OpTime1,H15,IF(A15<OpTime2,K15,IF(A15<OpTime3,N15,0)))-O15-P15	=Q15+R14
=IF(A16<OpTime1,H16,IF(A16<OpTime2,K16,IF(A16<OpTime3,N16,0)))-O16-P16	=Q16+R15
=IF(A17<OpTime1,H17,IF(A17<OpTime2,K17,IF(A17<OpTime3,N17,0)))-O17-P17	=Q17+R16
=IF(A18<OpTime1,H18,IF(A18<OpTime2,K18,IF(A18<OpTime3,N18,0)))-O18-P18	=Q18+R17
=IF(A19<OpTime1,H19,IF(A19<OpTime2,K19,IF(A19<OpTime3,N19,0)))-O19-P19	=Q19+R18
=IF(A20<OpTime1,H20,IF(A20<OpTime2,K20,IF(A20<OpTime3,N20,0)))-O20-P20	=Q20+R19
=IF(A21<OpTime1,H21,IF(A21<OpTime2,K21,IF(A21<OpTime3,N21,0)))-O21-P21	=Q21+R20
=IF(A22<OpTime1,H22,IF(A22<OpTime2,K22,IF(A22<OpTime3,N22,0)))-O22-P22	=Q22+R21
=IF(A23<OpTime1,H23,IF(A23<OpTime2,K23,IF(A23<OpTime3,N23,0)))-O23-P23	=Q23+R22
=IF(A24<OpTime1,H24,IF(A24<OpTime2,K24,IF(A24<OpTime3,N24,0)))-O24-P24	=Q24+R23
=IF(A25<OpTime1,H25,IF(A25<OpTime2,K25,IF(A25<OpTime3,N25,0)))-O25-P25	=Q25+R24
=IF(A26<OpTime1,H26,IF(A26<OpTime2,K26,IF(A26<OpTime3,N26,0)))-O26-P26	=Q26+R25
=IF(A27<OpTime1,H27,IF(A27<OpTime2,K27,IF(A27<OpTime3,N27,0)))-O27-P27	=Q27+R26
=IF(A28<OpTime1,H28,IF(A28<OpTime2,K28,IF(A28<OpTime3,N28,0)))-O28-P28	=Q28+R27
=IF(A29<OpTime1,H29,IF(A29<OpTime2,K29,IF(A29<OpTime3,N29,0)))-O29-P29	=Q29+R28
=IF(A30<OpTime1,H30,IF(A30<OpTime2,K30,IF(A30<OpTime3,N30,0)))-O30-P30	=Q30+R29
=IF(A31<OpTime1,H31,IF(A31<OpTime2,K31,IF(A31<OpTime3,N31,0)))-O31-P31	=Q31+R30
=IF(A32<OpTime1,H32,IF(A32<OpTime2,K32,IF(A32<OpTime3,N32,0)))-O32-P32	=Q32+R31
=IF(A33<OpTime1,H33,IF(A33<OpTime2,K33,IF(A33<OpTime3,N33,0)))-O33-P33	=Q33+R32
=IF(A34<OpTime1,H34,IF(A34<OpTime2,K34,IF(A34<OpTime3,N34,0)))-O34-P34	=Q34+R33
=IF(A35<OpTime1,H35,IF(A35<OpTime2,K35,IF(A35<OpTime3,N35,0)))-O35-P35	=Q35+R34
=IF(A36<OpTime1,H36,IF(A36<OpTime2,K36,IF(A36<OpTime3,N36,0)))-O36-P36	=Q36+R35
=IF(A37<OpTime1,H37,IF(A37<OpTime2,K37,IF(A37<OpTime3,N37,0)))-O37-P37	=Q37+R36
=IF(A38<OpTime1,H38,IF(A38<OpTime2,K38,IF(A38<OpTime3,N38,0)))-O38-P38	=Q38+R37
=IF(A39<OpTime1,H39,IF(A39<OpTime2,K39,IF(A39<OpTime3,N39,0)))-O39-P39	=Q39+R38
=IF(A40<OpTime1,H40,IF(A40<OpTime2,K40,IF(A40<OpTime3,N40,0)))-O40-P40	=Q40+R39
=IF(A41<OpTime1,H41,IF(A41<OpTime2,K41,IF(A41<OpTime3,N41,0)))-O41-P41	=Q41+R40
=IF(A42<OpTime1,H42,IF(A42<OpTime2,K42,IF(A42<OpTime3,N42,0)))-O42-P42	=Q42+R41
=IF(A43<OpTime1,H43,IF(A43<OpTime2,K43,IF(A43<OpTime3,N43,0)))-O43-P43	=Q43+R42
=IF(A44<OpTime1,H44,IF(A44<OpTime2,K44,IF(A44<OpTime3,N44,0)))-O44-P44	=Q44+R43
=IF(A45<OpTime1,H45,IF(A45<OpTime2,K45,IF(A45<OpTime3,N45,0)))-O45-P45	=Q45+R44
=IF(A46<OpTime1,H46,IF(A46<OpTime2,K46,IF(A46<OpTime3,N46,0)))-O46-P46	=Q46+R45
=IF(A47<OpTime1,H47,IF(A47<OpTime2,K47,IF(A47<OpTime3,N47,0)))-O47-P47	=Q47+R46
=IF(A48<OpTime1,H48,IF(A48<OpTime2,K48,IF(A48<OpTime3,N48,0)))-O48-P48	=Q48+R47
=IF(A49<OpTime1,H49,IF(A49<OpTime2,K49,IF(A49<OpTime3,N49,0)))-O49-P49	=Q49+R48
=IF(A50<OpTime1,H50,IF(A50<OpTime2,K50,IF(A50<OpTime3,N50,0)))-O50-P50	=Q50+R49
=IF(A51<OpTime1,H51,IF(A51<OpTime2,K51,IF(A51<OpTime3,N51,0)))-O51-P51	=Q51+R50
=IF(A52<OpTime1,H52,IF(A52<OpTime2,K52,IF(A52<OpTime3,N52,0)))-O52-P52	=Q52+R51