

# Slot Error Rate Performance of DH-PIM with Symbol Retransmission for Optical Wireless Links

S. Rajbhandari, Z. Ghassemlooy, and N. M. Aldibbiat

Optical Communications Research Group, NCRLab,  
School of Computing, Engineering and Information Sciences,

Northumbria University,

Newcastle upon Tyne, NE1 8ST, UK

Email: [fary.ghassemlooy@unn.ac.uk](mailto:fary.ghassemlooy@unn.ac.uk)

## Abstract

In this paper we introduce the dual-harmonic pulse interval modulation (DH-PIM) technique employing a simple retransmission coupled with a majority decision detection scheme at the receiver. We analytically investigate the slot error rate performance and compare results with simulated data for the symbol retransmissions rates of three, four and five, showing a good agreement. We demonstrate that the proposed scheme significantly reduces the slot error rate compared with the standard single symbol transmission system, with retransmission rate of five offering the highest code gain of ~5 dB.

## 1. Introduction

The promise of a high-speed and unregulated bandwidth optical wireless link and the limitations of the available radio bandwidths have encouraged the research interest in optical wireless communication (OWC) systems [1-6]. OWC (both indoor and outdoor) is capable of providing a high data throughput (hundreds of MHz), flexible and secure links in a number of applications such as hospitals, museums, exhibition halls, trains and train stations, aircraft and airports etc. where RF based system may not be the most appropriate scheme to adopt because of security, safety and offered data rates [5-6]. In OWC systems (particularly in indoor application operating at 880-920 nm wavelength range) the allowable transmitter average power is limited mainly by the eye safety requirements, thus affecting the link length and system performance. Also to ensure a longer life span for the optical source in particular

laser diode, it is desirable to adopt a drive signal with high-peak power and low average power for intensity modulation.

To address these issues, a number of modulation techniques have been suggested for OWC links such as on-off keying (OOK), pulse position modulation (PPM), digital pulse interval modulation (DPIM) and DH-PIM, each with its unique power and bandwidth efficiencies [6-11]. The former is the simplest technique but with low power efficiency, whereas PPM, having a fixed symbol length, is the best scheme in terms of the power efficiency and overall performance, but requiring a much higher transmission bandwidth as well as symbol and slot synchronisations [6]. Both DPIM and DH-PIM requires no symbol synchronisation and offer improved bandwidth and power efficiencies compared with the PPM and OOK, respectively [10-12].

The performance of the PPM with/without coding techniques have been evaluated in a number papers such as the Turbo coded PPM [13-14], and the Trellis coded PPM [15-17]. However, in DPIM and DH-PIM schemes, due to the fact that the symbol length is variable, it is not practical to use block coding, as the block codes require a fixed number of input data [18]. In [19-20] the convolutional coding has been applied to both DPIM and DH-PIM. But the properties of the original modulation scheme have not been preserved, thus resulting in a more complex system implementation. In this paper, we propose DH-PIM technique with error detection and correction capabilities, where a DH-PIM symbol is retransmitted a number of times and at the receiver a simple majority decision detection scheme is adopted. We consider symbol retransmission of three, four and five, where errors can be detected and corrected at the cost of reduced data throughput. Symbol retransmission of two is not considered; since it is not possible to determine which symbol is the correct one. We theoretically investigate the slot error rate performance for the symbol retransmission rates of three, four and five and compare it with the simulation data. Results show that the proposed scheme significantly reduces the probability of slot error for a given signal-to-noise ratio (SNR).

The rest of the paper is organised as follow. Section two introduces the DH-PIM focusing on the slot and packet error rates. Details of symbol retransmission are given in Section three and the analytical and simulation results together with discussion are outlined in Section four. Finally, concluding remarks are provided in Section five.

## 2. DH-PIM Modulation Technique

A sequence of  $M$  bit binary symbol  $b_k$  is encoded into its equivalent DH-PIM symbol  $\mathbf{x}_k = (x_1 + x_2, \dots, x_d)$ , where  $x_i = (0,1)$ , and  $d \in \{0, 1, 2, \dots, 2^{M-1} - 1\}$  is the additional slots representing the information. In DH-PIM, depending on the most significant bit (*MSB*) of  $b_k$ , a symbol will have one of two headers as shown in Fig. 1. If *MSB* = 0, then the header 1 ( $H_1$ ) is generated starting with a short pulse of  $\alpha T_s/2$  duration, otherwise header 2 ( $H_2$ ) is generated starting with a wider pulse of  $\alpha T_s$  duration, where  $\alpha$  is a positive integer and  $T_s$  is the slot duration. The remaining part of both headers is filled with a guard band of empty slots to reduced inter-symbol-interference. The number of information slots  $d$  which follow the header is equal to the decimal value of the input word when *MSB* = 0 and decimal value of 1's complement of the input word otherwise. Throughout the paper, DH-PIM will be referred to as  $L$ -DH-PIM $_{\alpha}$ , where  $L = 2^M$ . For example, 16-DH-PIM $_2$  means a DH-PIM symbol with  $M = 4$  and  $\alpha = 2$ .

In carrying out the analysis a number assumption has been made such as; a distortion free channel, no bandwidth limitations imposed by the transmitter and receiver, the shot noise due to the ambient light is the dominant noise source having white Gaussian characteristics, and an equal occurrence of  $H_1$  and  $H_2$ .

Then the probabilities of the false alarm and erasure errors on the received DH-PIM symbol are given, respectively as [11]:

$$p_{e0} = Q\left(\frac{k\sqrt{SNR_{OOK}}}{\psi}\right), \quad (1)$$

$$p_{e1} = Q\left(\frac{(1-k)\sqrt{SNR_{OOK}}}{\psi}\right), \quad (2)$$

where,  $\psi = \sqrt{16M\bar{L}/9\alpha^2}$ ,  $k$  is the threshold level, the SNR for the input OOK word is given as  $SNR_{OOK} = 2R\bar{P}^2 / \eta R_b$ ,  $\bar{L}$  is the average length of a symbol,  $R_b$  is the OOK bit rate,  $\bar{P}$  is the average transmitted optical power,  $\eta$  is the noise spectral density, and  $R$  is the photodetector responsivity .

Thus, the slot error rate for the DH-PIM is given by:

$$P_{se} = \frac{\bar{L} - 3\alpha}{4\bar{L}} Q \left[ k \sqrt{SNR_{OOK}} \right] + \frac{3\alpha}{4\bar{L}} Q \left[ (1-k) \sqrt{SNR_{OOK}} \right]. \quad (3)$$

And the packet error rate that can be approximated as  $P_{pe} \approx \frac{N_{pkt} \bar{L} P_{se}}{M}$  [8], is given by:

$$P_{pe} \approx \frac{N_{pkt}}{4M} \frac{\bar{L} - 3\alpha}{\bar{L}} Q \left[ k \sqrt{SNR_{OOK}} \right] + \frac{3\alpha N_{pkt}}{4M} Q \left[ (1-k) \sqrt{SNR_{OOK}} \right], \quad (4)$$

where,  $N_{pkt}$  is the number of bits in a packet.

### 3. Symbol Retransmission

In modulation schemes with a non-fixed length symbol such as DPIM and DH-PIM, as well as packet based network systems, a single error in a symbol may affect more than one symbol. When this happens the entire packet composed of a number of symbols is usually discarded and a request for retransmission is made. In this case, a more appropriate quality of service metric would be the packet error rate (PER) or the slot error rate (SER) rather than the bit error rate (BER). Here we consider symbol retransmission rates of three, four and five for DH-PIM scheme. In the case of retransmission rate of two, the receiver compares the two symbols and selects the first symbol rather than requesting a retransmission. However, the downside is that one cannot determine which symbol is the correct one, therefore not recommended for practical applications. In the case of three symbols retransmission, if an error occurs in only one symbol and the remaining two symbols are the same, then one of them is chosen as the valid correct symbol. In this way, a slot error confined to one symbol can be detected and corrected. Slot errors appearing in more than one symbol could be detected and corrected by increasing the symbol retransmission rate, but this is not practical due to the reduced data throughput.

Consider a DH-PIM symbol  $x(t)$  transmitted  $R_{rt}$  times, where  $R_{rt} = 1, 3, 4$  or  $5$ . Since the approach adopted at the receiver selects the correct symbols from a group of received symbols based on the majority decision mechanism, then symbols repeated the most will be selected. For the case where more than one symbol are different but are repeated the same number of times, then the first symbol is always selected. The algorithm used at the receiver to select the correct symbols for  $R_{rt}$  of 4 is shown in Fig. 2. Here, each DH-PIM symbol  $x(t)$  is transmitted four times and the received group of symbols are  $y_r(t) = \{y_1(t), y_2(t), y_3(t), y_4(t)\}$ .

The decoded symbol  $y$  is correct as long as it is equal to  $x$  (we will be using  $x$  and  $y$  instead of  $x(t)$  and  $y(t)$  hereafter for simplicity). As can be seen from Fig. 2,  $P(y_r = y_i)$  depend on the equality of the received symbols. Symbols equality  $y_i = y_j$  for all  $i, i \neq j$  depends two factors: (i) the probability of the received symbol being error-free and/or (ii) the error(s) slot position in different symbols. Thus in mathematical analysis both probability of slot error and the probability of erroneous symbols being equal are taken into consideration. In addition to assumptions made above to calculate the SER for DH-PIM with retransmission rate of one, additional assumptions have been made in determining the SER for higher values of retransmission rates. All the possible symbols are equi-probable, the probabilities of occurrence of errors in all slots and symbols are equal, and the occurrence of error in two or more slots within a same symbol is very low.

At the receiver when decoding, comparisons are carried out at the symbol level, therefore we first determine the probability of symbol error  $P_{syne}$  which is as given in [12]:

$$P_{syne} = 1 - (1 - P_{se})^{\bar{b}}, \quad (5)$$

where  $P_{se}$  is the probability of slot error. The detail derivative of slot error rate calculation for  $R_{rt}$  of 3, 4 and 5 is described below. In the following derivatives,  $P(x)$  denotes the probability of the event  $x$ ,  $P_{syne}(m, n)$  denotes the probability of occurring errors in exactly  $n$  symbols out of  $m$  symbols,  $P_{neq}$  and  $P_{uneq}$  are the probability of  $n$  symbols being equal and unequal, respectively,  $P_{nm}$  and  $P_n$  are the dummy variables denoting error probability due to  $m$  symbols being equal out of  $n$  erroneous symbols and the total symbol error probability due to error in  $n$  symbols.

### Case 1: three symbol retransmission ( $R_{rt} = 3$ )

The decoded symbol will have an error if one of the following conditions is true.

1.  $P_1 = P_{syne}(3,3) = P_{syne}^3$ . (6)

- 2.

- (a)  $P_{22} = P_{2eq} \cdot P_{syne} \cdot 2 \cdot \frac{1}{L} \left[ \frac{3!}{2!} P_{syne}^2 (1 - P_{syne}) \right]$ , (7a)

- (b)  $P_{21} = \frac{2}{3} P_{2uneq} \cdot P_{syne}(3,2)$ ,

$$= \frac{2}{3} \left( 1 - \frac{1}{L} \right) \left[ \frac{3!}{2!L} P_{syne}^2 \left( -P_{syne} \right) \right], \quad (7b)$$

The total probability of symbol error for  $R_{rt}$  of 3 is the summation of (6), (7a) and (7b) given as:

$$P_{syne3r} = P_{syne}^3 + \frac{3!}{2!L} P_{syne}^2 \left( -P_{syne} \right) + \frac{2}{3} \left( 1 - \frac{1}{L} \right) \left[ \frac{3!}{2!L} P_{syne}^2 \left( -P_{syne} \right) \right], \quad (8)$$

On simplification, (8) is given by:

$$P_{syne3r} = P_{syne}^2 \left[ \frac{1}{L} + 2 \right] - P_{syne}^3 \left( \frac{1}{L} + 1 \right), \quad (9)$$

By substituting (5) into (9) the SER for  $R_{rt} = 3$  is given as:

$$P_{se3r} = 1 - \left[ 1 - \left( P_{syne}^2 \left[ \frac{1}{L} + 2 \right] - P_{syne}^3 \left( \frac{1}{L} + 1 \right) \right) \right]^{\frac{1}{L}}. \quad (10)$$

## Case 2: $R_{rt} = 4$

The conditions for a decoded symbol to be erroneous for  $R_{rt} = 4$  are given below:

$$1. P_4 = P_{syne}(4,4) = P_{syne}^4, \quad (11)$$

2.

$$(a) P_{33} = P_{3eq} \cdot P_{syne} \left( \left( 3, 3 \right) \right) = \frac{1}{L^2} \cdot P_{syne} \left( \left( 3, 3 \right) \right), \quad (12a)$$

$$(b) P_{32} = P_{2eq} \cdot P_{syne} \left( \left( 3, 3 \right) \right) = \frac{3L \left( -1 \right)}{L^3} \cdot P_{syne} \left( \left( 3, 3 \right) \right), \quad (12b)$$

$$(c) P_{31} = \frac{3}{4} \cdot P_{3uneq} \cdot P_{syne} (4,3), \quad (12c)$$

Hence the symbol error probability due to an error occurring in three symbols is given by:

$$P_3 = \left[ \frac{1}{L^2} + \frac{3}{L} - \frac{3}{L^2} + \frac{3}{4} \left\{ 1 - \frac{1}{L^2} + \frac{3}{L} - \frac{3}{L^2} \right\} \right] P_{syne} \left( \left( 3, 3 \right) \right). \quad (13)$$

$$3. P_{22} = \frac{1}{2} \cdot P_{2eq} \cdot P_{syne} (4,2), \quad (14)$$

The symbol error probability is obtained by summation of (11), (13) and (14) given by:

$$P_{syne4r} = \frac{3P_{syne}^2}{L} + P_{syne}^3 \left( 3 - \frac{3}{L} - \frac{2}{L^2} \right) + P_{syne}^4 \left( \frac{2}{L^2} - 2 \right). \quad (15)$$

Thus, the SER for  $R_{rt} = 4$ ,  $P_{se4r}$  is given by:

$$P_{se4r} = 1 - \left[ 1 - \frac{3P_{syne}^2}{L} + P_{syne}^3 \left( 3 - \frac{3}{L} - \frac{2}{L^2} \right) + P_{syne}^4 \left( \frac{2}{L^2} - 2 \right) \right]^{\frac{1}{L}}. \quad (16)$$

### Case 3: $R_{rt} = 5$

For  $R_{rt}$  of 5 the decoded symbol will be error-free if at least three symbols are received correctly. However, an error will occur in the decoded symbol if one of the following conditions applies.

$$1. P_5 = P_{syne}(5,5) = P_{syne}^5. \quad (17)$$

2.

$$(a) P_{41} = \frac{4}{5} \cdot P_{4uneq} \cdot P_{syne} \binom{4,5}{-} = \frac{4}{5} \left[ \frac{\bar{L} \binom{4}{-1} \binom{3}{-2} \binom{2}{-3}}{\bar{L}^4} \right] P_{syne} \binom{4,5}{-}, \quad (18a)$$

$$(b) P_{42} = \binom{4}{-} P_{4uneq} \cdot P_{syne} \binom{4,5}{-}, \quad (18b)$$

Hence the probability of selecting erroneous symbols due to an error occurring in any of 4 symbols is given by:

$$P_4 = \left[ \frac{4}{5} \cdot \frac{\bar{L} \binom{4}{-1} \binom{3}{-2} \binom{2}{-3}}{\bar{L}^4} + 1 - \frac{\bar{L} \binom{4}{-1} \binom{3}{-2} \binom{2}{-3}}{\bar{L}^4} \right] \cdot \frac{5!}{3!2!} P_{syne}^3 \binom{4}{-} P_{syne} \binom{4}{-}. \quad (19)$$

$$3. a) P_{33} = P_{3eq} \cdot P_{syne} \binom{3,3}{-} = \frac{1}{2} P_{syne} \binom{3,3}{-}, \quad (20a)$$

$$b) P_{32} = \frac{1}{2} \cdot P_{2eq} \cdot P_{syne} \binom{3,3}{-} = \frac{3\bar{L} \binom{3}{-1}}{2\bar{L}^3} P_{syne} \binom{3,3}{-}, \quad (20b)$$

$$\text{Hence, } P_3 = \left[ \frac{3}{2\bar{L}} - \frac{1}{2\bar{L}^2} \right] \left[ \frac{5!}{3!2!} P_{syne}^3 \binom{4}{-} P_{syne} \binom{4}{-} \right]. \quad (21)$$

Summing (17), (19) and (21) and with further simplification, the symbol error rate for  $R_{rt} = 5$ ,  $P_{syne5r}$  is given by:

$$P_{syne5r} = \left( \frac{15}{L} - \frac{5}{L^2} \right) P_{syne}^3 + \left( 4 - \frac{24}{L} - \frac{1}{L^2} + \frac{6}{L^3} \right) P_{syne}^4 + \left( -3 + \frac{9}{L} + \frac{6}{L^2} - \frac{6}{L^3} \right) P_{syne}^5. \quad (22)$$

Thus, the slot error rate for  $R_{rt} = 5$  is given by:

$$P_{se5r} = 1 - \left[ 1 - \left( \frac{15}{L} - \frac{5}{L^2} \right) P_{syne}^3 + \left( 4 - \frac{24}{L} - \frac{1}{L^2} + \frac{6}{L^3} \right) P_{syne}^4 + \left( -3 + \frac{9}{L} + \frac{6}{L^2} - \frac{6}{L^3} \right) P_{syne}^5 \right]^{\frac{1}{L}}. \quad (23)$$

#### 4. Result and Discussion

The proposed scheme is simulated using Matlab for retransmission rates of 3, 4 and 5 for a direct line-of-sight link configuration. The simulation system block diagram is depicted in Fig. 3 and all the important system parameters adopted for simulation are given in Table 1. The input binary data  $b_k$  is first converted to its equivalent DH-PIM symbol  $x_k$  as outlined in Section 2. The retransmission encoder duplicates  $x_k$  ' $r$ '-times, and its output symbol sequence  $x_{rk}$  is applied to the optical transmitter. Assuming the noise signal  $n(t)$  being white and Gaussian, the received signal is  $z(t) = [x(t)+n(t)]$ . At the receiver the output of photodetector is passed through a matched filter the output of which is sampled at the slot rate  $T_s^{-1}$ , prior to being applied to the threshold detector to regenerate the transmitted DH-PIM symbol stream,  $\bar{z}_k$ . The function of retransmission decoder is reverse of the retransmission encoder, the only difference is that the encoder outputs  $\{x_{1k} \dots x_{rk}\}$  are identical whereas the decoder outputs  $\{y_{1k}, \dots, y_{rk}\}$  are not. The output of the decision circuit  $y_k$  is an approximation of transmitted symbol  $x_k$  based on the received DH-PIM sequence  $\{y_{1k}, \dots, y_{rk}\}$  following the algorithm described in Section 3. To determine SER  $x_k$  and  $y_k$  are compared slot by slot. It is possible to determine the BER by comparing  $b_k$  and  $\hat{b}_k$ , but for variable symbol length modulation scheme such as DH-PIM, PER is the preferred option as was outlined above, which is directly calculated from the SER [10].

The theoretical results for the SER for 16-DH-PIM<sub>2</sub> for different retransmission rates, is displayed in Fig. 4. It is observed that at low values of  $P_{se}$  the code gains for  $R_{rt}$  of 3 and 4 are very close. This can be explained with reference to (14) and (21) where it is shown that  $P_{serr}$  depends on the second and higher powers of  $P_{se}$ , with the latter having a negligible contribution in  $P_{serr}$  as  $P_{se}$  decreases. The marginal improvement in the code gain for  $R_{rt}$  of 4 compared to  $R_{rt}$  of 3 is mainly due to a large coefficient of  $P_{se}^2$  in (16). Since the coefficient of  $P_{se}^3$  is negative in (10) and positive in (16), the difference in the code gain is larger at lower values of SNR compared with the higher values. In case of  $P_{se5r}$ , the dominant term is  $P_{se}^3$ , therefore, the code gain increases as  $P_{se}$  decreases compared with  $R_{rt}$  of 3 and 4.

Figure 5 shows the theoretical and Monte-Carlo simulation results for the SER against the electrical SNR for different retransmission rates at data rate of 1 Mbps for 16-DH-PIM<sub>1&2</sub>. Although in the analysis it is assumed that the error per symbol is limited only to one slot, it is observed that the simulation and theoretical results match closely up to  $P_{se}$  of  $10^{-3}$ . The



divergence between the predicted and simulated result at lower  $P_{se}$  is due to computational power as number of symbols generated for each case is only  $10^4$ . At very high values of  $P_{se}$  there is little or no code gain that improves with decreasing  $P_{se}$ . As expected increasing the rate of retransmission will decrease the SNR requirements to achieve a certain SER at the cost of reduced system data throughput and increased system complexity. At a  $P_{se}$  of  $10^{-5}$ , the SNR code gains for DH-PIM<sub>2</sub> are 3.6, 4.3 and 5.2 dB for  $R_{rt}$  of 3, 4 and 5, respectively compared with  $R_{rt}$  of 1. Whereas for DH-PIM<sub>1</sub> the SNR gain drop to 2.4, 3.3 and 4.2 dB for  $R_{rt}$  of 3, 4 and 5, respectively. This drop in the SNR gain is mainly attributed to the symbol header composed of three slots with a pulse of half slot duration; see Fig.1 (b), where samples are taken at half the slot rate.

Tables 2 illustrates the code gain for 16-DH-PIM retransmission system at  $P_{se}$  of  $10^{-4}$  for  $R_{rt}$  of 3, 4 and 5 and  $M$  equal to 3, 4 and 5. The SNR code gain decreases as  $M$  increases, which is due to the fact that the average length of a symbol increases with  $M$ , thus resulting in a higher probability of symbol error. This can be explained with reference to (14), (21) and (23), in which it is shown that the probability of slot error for the retransmission case  $P_{serr}$  not only depends on the  $P_{se}$  but also on the average symbol length where symbols with a longer length will encounter higher probability of slot error per symbol.

## 5. Conclusions

This paper introduced the dual-har pulse interval modulation with a simple retransmission capability to achieve error detection and correction. The slot error rate performance is investigated theoretically and the results obtained were compared with the simulation data. We demonstrated that the proposed scheme significantly reduced the slot error rate compared with the standard single symbol transmission system, with retransmission rate of five offering the highest code gain of  $\sim 5$  dB at  $P_{se}$  of  $10^{-4}$  for DH-PIM<sub>2</sub>. The code gain depends on the average number of slots per symbol decreasing with increase of the bit resolution.

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**Figure 5(a):** The slot error rate against the electrical SNR for retransmission rate of 3, 4 and 5 at data rate of 1 Mbps for 16-DH-PIM<sub>2</sub>

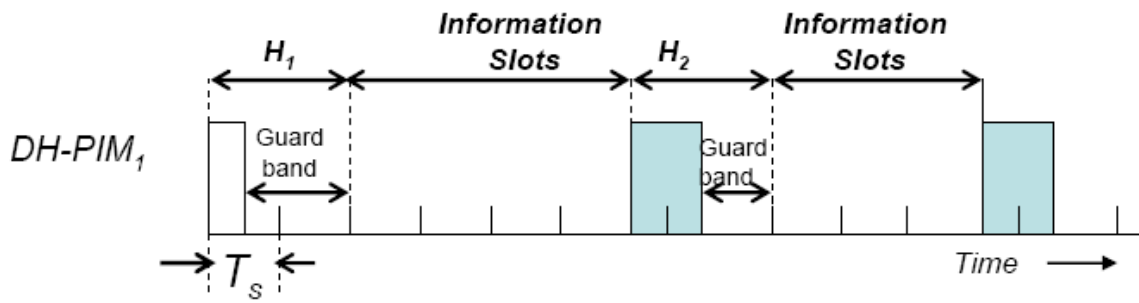
**Figure 5(b):** The slot error rate against the electrical SNR for retransmission rate of 3, 4 and 5 at data rate of 1 Mbps for 16-DH-PIM<sub>1</sub>.

**Table 1:** The simulation parameters

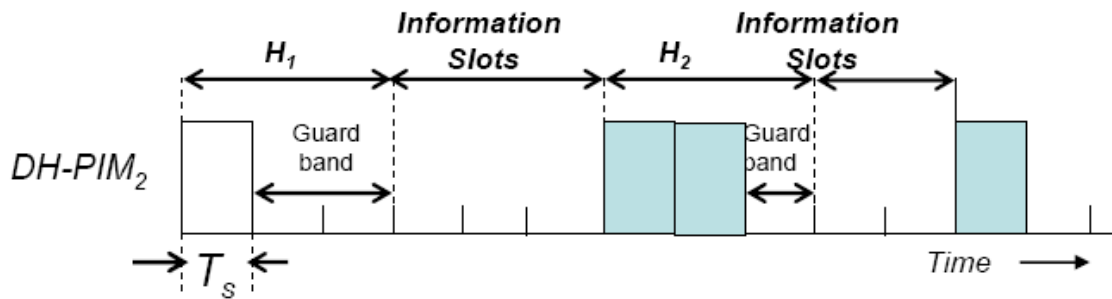
Parameter	Value
$\alpha$	1 and 2
$M$	3, 4 and 5
Detector responsivity $R$	1 A/W
Ambient induced shot noise current $I_b$	200 $\mu$ A
Bit rate $R_b$	1 Mbps
Retransmission rate $R_{rt}$	3, 4 and 5

**Table 2:** Code gain for 16-DH-PIM retransmission system at  $P_{se}$  of  $10^{-4}$

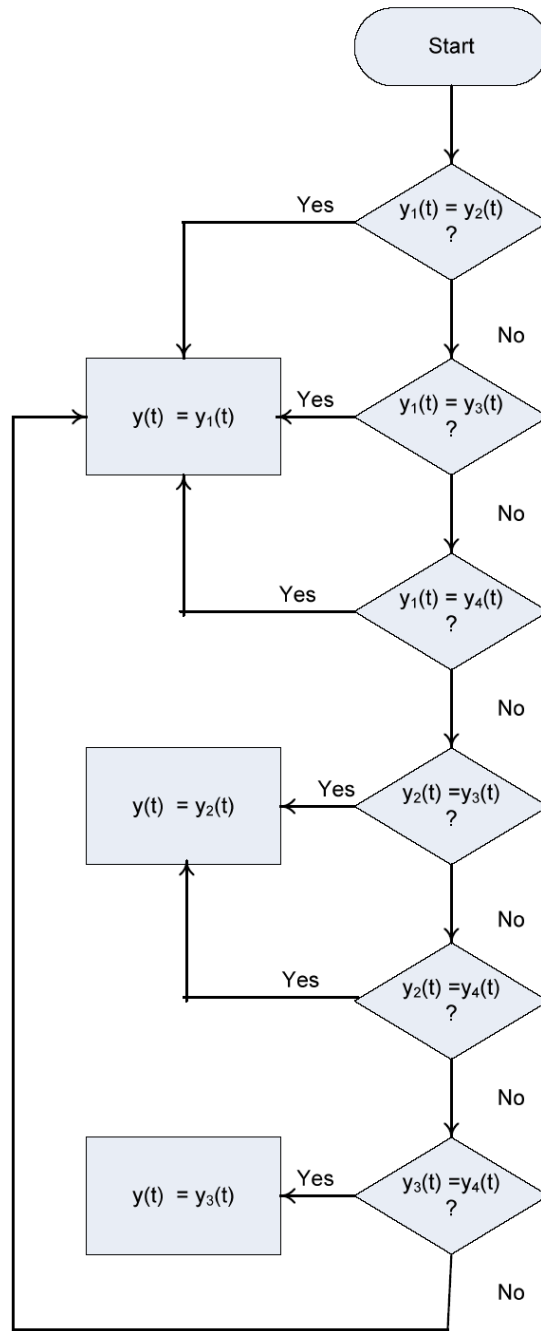
DH-PIM	Code gain (dB)		
	$R_{rt} = 3$	$R_{rt} = 4$	$R_{rt} = 5$
$\alpha = 2, M = 3$	2.72	3.35	4.38
$\alpha = 2, M = 4$	2.57	3.27	4.18
$\alpha = 2, M = 5$	2.33	3.20	3.90
$\alpha = 1, M = 3$	2.48	3.28	4.11
$\alpha = 1, M = 4$	2.26	3.16	3.86
$\alpha = 1, M = 5$	2.07	2.93	3.59



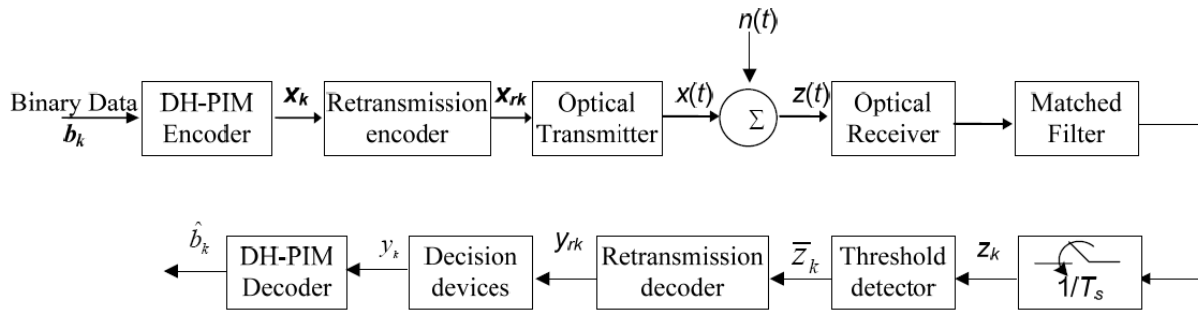
**Figure 1(a):** DH-PIM<sub>1</sub> symbol structure



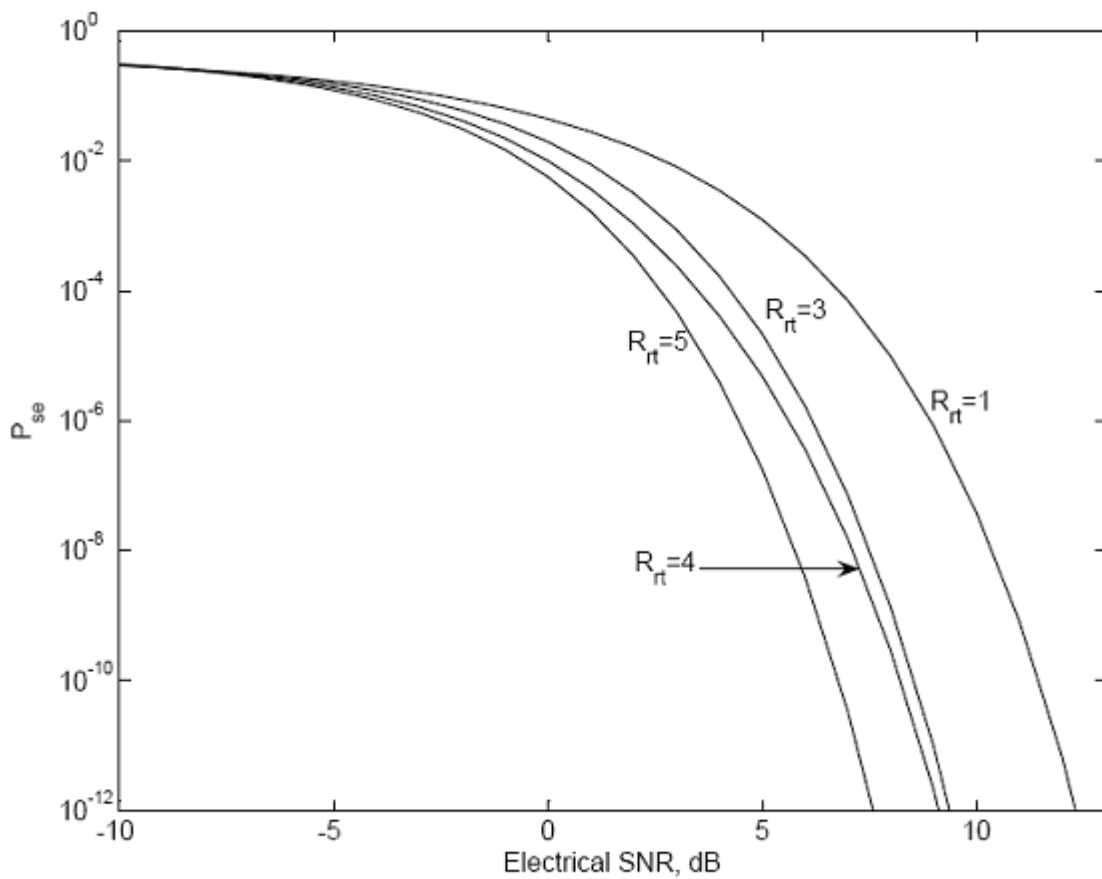
**Figure 1(b):** DH-PIM<sub>2</sub> symbol structure



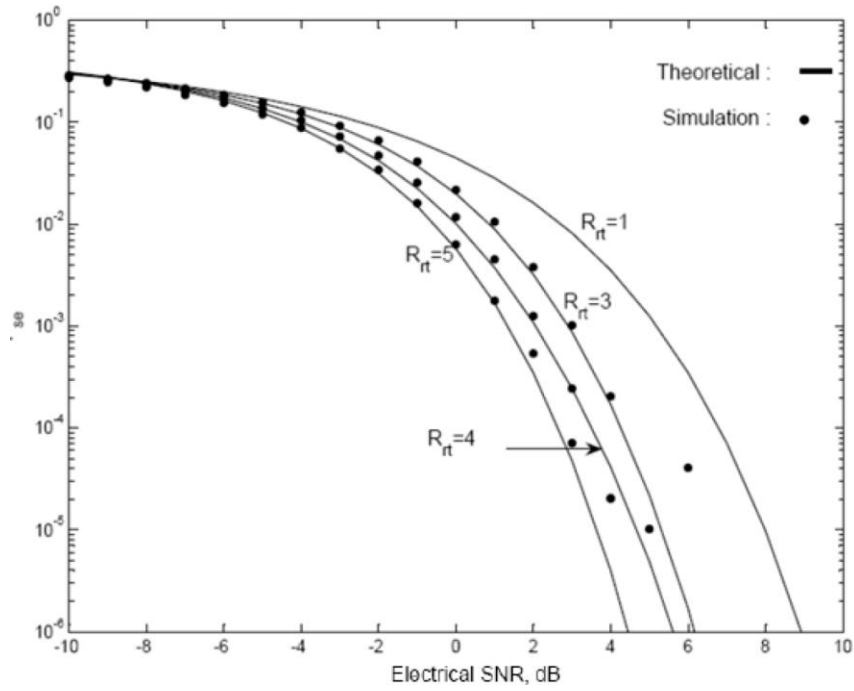
**Figure 2:** Flow chart showing the majority decision process for  $R_{rt} = 4$ .



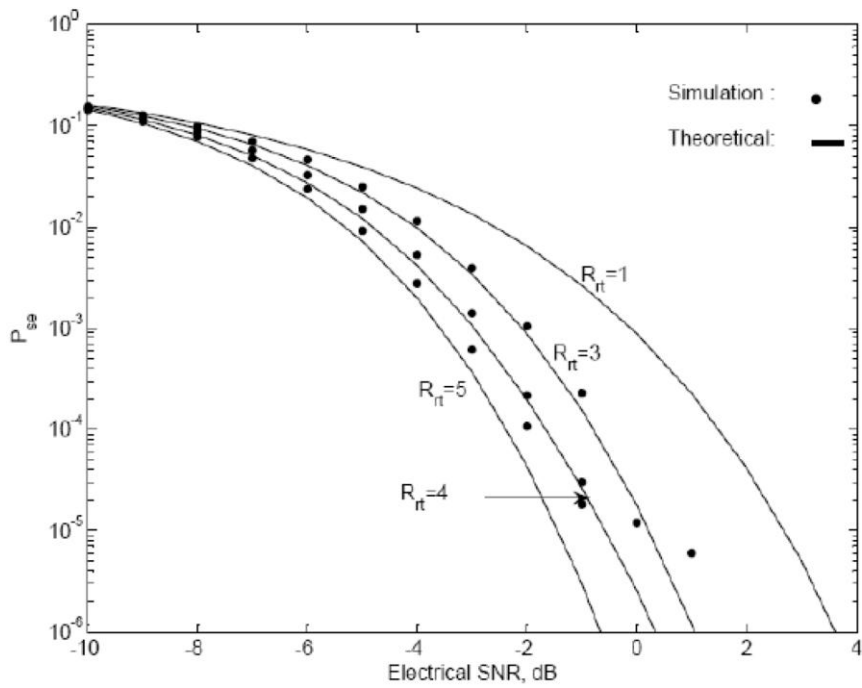
**Figure 3:** The system block diagram of the proposed DH-PIM with symbol retransmission.



**Figure 4:** The predicted slot error rate against the electrical SNR for retransmission rate of 3, 4 and 5 at data rate of 1 Mbps for 16-DH-PIM<sub>2</sub>.



(a)



(b)

**Figure 5:** (a) The slot error rate against the electrical SNR for retransmission rate of 3, 4 and 5 at data rate of 1 Mbps for 16-DH-PIM<sub>2</sub> (b): The slot error rate against the electrical SNR for retransmission rate of 3, 4 and 5 at data rate of 1 Mbps for 16-DH-PIM<sub>1</sub>.