BER Performance of DPSK Subcarrier Modulated Free Space Optics in Fully Developed Speckle

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***Abstract*--In this paper a DPSK subcarrier intensity modulated (SIM) free space optical (FSO) link is considered in negative exponential atmospheric turbulence environment. To mitigate the scintillation effect the selection combining spatial diversity scheme (SelC) is employed at the receiver. Bit error rate (BER) analysis is presented with and without SelC spatial diversity, and it is shown that at BER of 10^{-6} using four independent PIN-photodetectors, a diversity gain of not less than 38 dB is achievable.**

Key words—Free-space optics, DPSK, turbulence, spatial diversity, Negative exponential distribution.

I. INTRODUCTION

In today's access network, the FSO technology is playing an increasing complementary role to the RF based techniques. This is attributable to its fundamental feature of huge bandwidth that is comparable to that obtainable from optical fibre but with an added advantage of lower deployment cost and time [1]. In recent years we have seen a steadily growing research interest in FSO links, supported by the successful field trials that are now culminating into commercial deployments. [1-4]. The earlier scepticism about FSO's efficacy, its dwindling acceptability by service providers cum slow market penetration are now rapidly fading away judging by the number of service providers, organisation, government and private establishments that now incorporate FSO into their network infrastructure [5, 6].

Terrestrial FSO is not free of challenges though; the atmospheric constituents (gases, aerosol, rain, fog, and smoke) extinguish and scatter photons traversing the atmosphere. The most deleterious being the thick fog, which could result in up to 270 dB/km attenuation coefficient [1]. This potentially limits the achievable link range to less than 500 m during such condition. In clear atmospheric conditions, longer link ranges are palpable. However, due to atmospheric turbulence effects small but random changes occur in the atmospheric temperature. This by extension results in random changes in the atmospheric index of refraction along the path of the optical radiation. This metamorphoses into random phase and irradiance fluctuations (scintillation) of the optical radiation at the photodetector. Detailed study of the atmospheric turbulence can be found in [7-9]. The scintillation effect can be likened to the random fading effect on radio communication though not caused by the multipath propagation and can cause severe degradation in FSO link performance if unmitigated.

Moreover, in terrestrial FSO link; the simple and widely used on-off keying (OOK) [10] signalling technique requires adaptive threshold to perform optimally because of turbulence. This poses a serious design difficulty, which can be overcome by employing SIM. The SIM FSO also outperforms OOK with adaptive threshold [11, 12] and for the full and seamless integration of FSO into existing networks the study of SIM becomes compelling because existing networks already contain subcarrier signals.

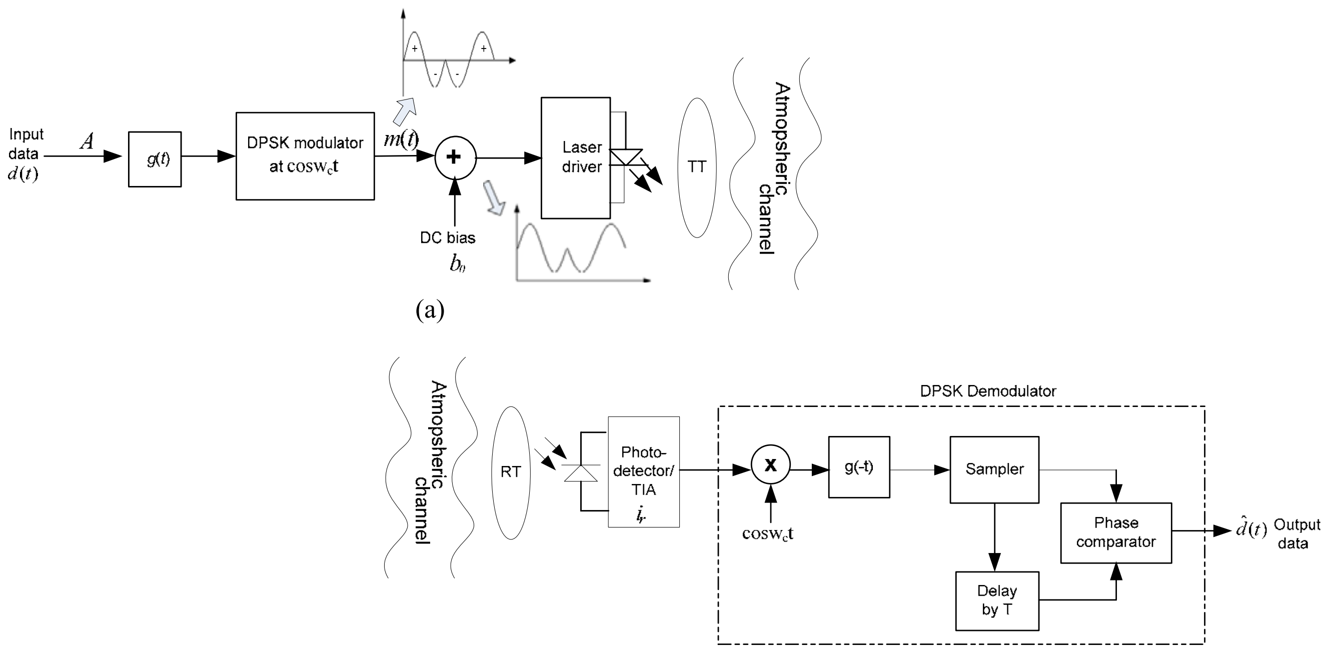
In weak turbulence modelled using the tractable log normal distribution, the spatial diversity has been studied to mitigate turbulence induced irradiance fading [10, 13]. Likewise, FSO employing various forward error control techniques have been reported in literature [14-16] with varying degrees of gain and complexity. In this paper differential phase shift keying (DPSK) pre-modulated SIM terrestrial FSO is presented with the selection combining spatial diversity adopted to ameliorate scintillation effect. In addition to mitigating scintillation without introducing additional latency into the system, spatial diversity also helps to prevent temporary outage/blocking due to birds or other small flying object cross the propagation path; and it's by far simpler/cost effective than using adaptive optics. The SIM is described in section II, the system performance metric analysis with and without selection combining spatial diversity is detailed in section III while the conclusion is presented in section IV.

II. DPSK SUBCARRIER INTENSITY MODULATION

A. System description

To achieve SIM, the data to be transported is pre-modulated on to a radio frequency (RF) signal of frequency ω_c. The modulated subcarrier RF signal is then used to directly modulate the irradiance of an optical carrier which is either an LED or a laser diode. Since the subcarrier signal is sinusoidal with both negative and positive values and since the intensity of an optical carrier can never be negative, a DC bias is usually applied to the subcarrier prior to modulating the optical source to ensure that the driving current of the laser diode is not less than its threshold current.

After traversing the atmospheric channel, the transmit



(a)

(b)

Fig. 1: FSO employing DPSK modulated SIM block diagram (a) transmitter and (b) receiver. TIP-trans-impedance amplifier; TT-Transmit telescope; RT-Receive telescope

telescope focuses the transmitted irradiance onto the direct detection PIN detector with a transimpedance amplifier (TIA). A DSPK demodulator is then used to recover the transmitted data, see Fig. 1.

The SIM leverages on the availability of stable RF oscillators and filters hence any of the evolved digital modulation techniques such as PSK, FSK and QAM can in principle be used. In this work however, binary differential phase shift keying is employed in order to circumvent the absolute phase estimation (and its accompanying ambiguity) requirement of regular PSK techniques [17]. The fact that the atmospheric turbulence channel can be rightly assumed to be ‘frozen’ for more than two consecutive data symbols makes the implementation of DPSK decoder at the receiver feasible. The ‘frozen’ channel assumption is premised on the turbulence correlation time, which is known be on the order of a hundreds of millisecond [10, 18].

The instantaneous photocurrent $i_r(t)$ can now be modelled as [19]:

$$i_r(t) = RI(1+\beta m(t)) + n(t), \qquad (1)$$

where $I = I_{peak}/2$, I_{peak} is the peak received irradiance, R is the photodetector responsivity, β is the modulation index and $m(t) = A(t)g(t)\cos[\omega_c t+\theta]$ is the subcarrier signal; $g(t)$ represents the rectangular pulse shape function and the additive noise $n(t) \sim N(0,\sigma^2)$ is assumed to be dominated by the white Gaussian background radiation.

The condition $|\beta m(t) \le 1|$ must always be fulfilled for the continuous wave optical transmitter to operate within its dynamic range. This places an upper bound on the amplitude of the subcarrier for a given value of β. With the subcarrier pre-modulated using DPSK which has non-varying amplitude and normalising β to unity, the peak amplitude becomes $A(t) = A \le 1$. From (1), the electrical signal-to-noise ratio SNR_e can thus be derived as:

$$SNR_e = A^2R^2I^2/2\sigma^2 \,. \qquad (2)$$

The work presented in this paper is for a single subcarrier, there exists however the possibility of aggregating more than one subcarrier and using the resulting composite signal to modulate the optical carrier irradiance.

B. Atmospheric turbulence

In FSO links spanning over 1 km, the turbulence induced multiple scatterings do take place and the log normal turbulence model [8] characterised by single scattering event clearly becomes invalid. The strength of irradiance fading encountered in atmospheric turbulence is often referred to as the scintillation index (*S.I.*). By definition $S.I = \dfrac{E[I^2]}{E[I]^2} - 1$ and it is the normalised log irradiance variance. For links covering over 3 kms, the turbulence effect can easily tend towards saturation, in which the *S.I.* begins to decrease due to multiple scattering and it then settles at a value of unity [7, 8]. This is the fully developed speckle regime. In this regime, the optical radiation field fluctuation obeys the Rayleigh distribution [8], which gives rise to the negative exponential varying irradiance given by (3). The validity of the negative exponential irradiance fluctuation is widely acknowledged in literature and has also been experimentally verified [7]. Other turbulence models notably the log-normal-Rician [20], the *I-K* [21] and the gamma-gamma [9] all reduce to the negative exponential distribution in the limit of strong turbulence.

$$p(I) = I_o^{-1}\exp(-I/I_o), \qquad I_o > 0 \tag{3}$$

where E[I] = I_o is the mean irradiance. For analogue SIM systems such as cable television (CATV) signal over fibre, the average SNR suffices as a performance indicator but not for the digital SIM system. Hence, in the next section the BER performance metrics suitable for the digital SIM under consideration is analysed.

III. PERFORMANCE METRIC ANALYSIS

A. Bit-error-rate (BER)

In a situation where the channel state information is completely known by the receiver, the BER is given by:

$$P_{ec} = 0.5\exp(-0.5SNR_e). \tag{4}$$

However, in the face of scintillation the unconditional BER is obtained by averaging (4) over the irradiance fluctuation statistics given by (3). The result is the following BER expression:

$$\begin{aligned} BER &= \int_0^\infty P_{ec}\,p(I)\,dI \\ &= \frac{1}{2I_o}\int_0^\infty \exp\left(-\frac{I}{I_o} - \frac{A^2R^2I^2}{4\sigma^2}\right)dI \end{aligned} \tag{5}$$

The above expression can be conveniently simplified by invoking equation (3.322.2) in [22] to obtain:

$$BER = \sqrt{\pi/SNR}\,\exp(SNR^{-1})\,Q(\sqrt{2/SNR}), \tag{6}$$

where the subcarrier amplitude A has been normalised to unity and $SNR = R^2E[I]^2/\sigma^2$. The plot of this BER is shown in Fig. 2. It is quite apparent from this plot that to achieve an acceptable level of performance, a huge amount of optical power is required. For instance, over 118 dB of normalised SNR is required to achieve BER of 10^{-6}.

It is therefore imperative to mitigate the atmospheric scintillation that led to this high optical power requirement. Amongst the applicable options (forward error control, adaptive optics spatial and time diversity), only the spatial diversity technique employing an array of *N*-PIN photodetectors is considered here. Because in addition to mitigating scintillation without introducing additional latency into the system, spatial diversity can also help prevent temporary outage/blocking due to birds or other small flying object cross the propagation path; and it's by far simpler/cost effective than using adaptive optics. The following thus give the BER analysis with spatial diversity.

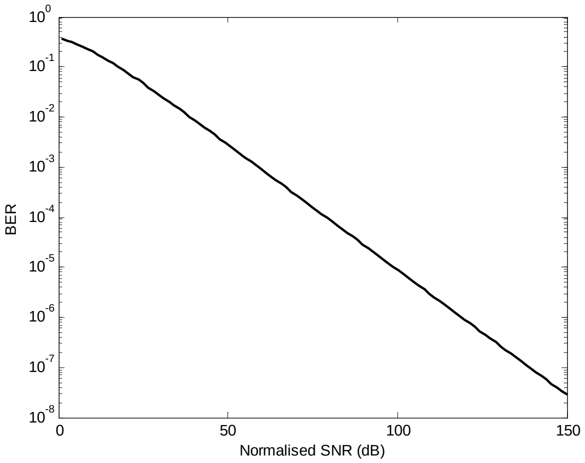


Fig. 2: The graph of BER against normalised $SNR = R^2E[I]^2/\sigma^2$ without spatial diversity

With RF based subcarrier systems employing the coherent demodulation, there is an issue associated with the absolute phase estimation [17], and this has informed our choice of the DPSK. The implication of this choice is that the spatial diversity techniques adopted must also not require absolute subcarrier phase estimation. This requirement led us to the selection combining spatial diversity (SelC) technique which requires no phase synchronisation of all the *N*-photodetectors outputs. In selection combining diversity, only the PIN photodetector with the highest SNR is selected out of all the *N* possible photodetectors. The selected link is also the link with the highest received irradiance provided all the *N*-links have the same noise level.

For fair comparison with single photodetector system, each receiver aperture is made equal to A_p/N, where A_p is the receiver aperture for a single receiver system. Without any loss of generality, A_p is normalised to unity. For the *N*-photodetector to be uncorrelated, the spacing, *s* between any two photodetectors needs to be greater than the correlation distance, ρ_o of the atmospheric turbulence which is only a few centimetres [13]. For a shot noise limited link as is the case in this work, the noise level as well as the photocurrent is proportional to the receiver aperture. The electrical SNR can therefore be easily derived as:

$$SNR_{e(SelC)} = \frac{R^2A^2I^2}{2N\sigma^2}. \tag{7}$$

In the absence of the complete channel state information at the receiver, the following gives the unconditional BER:

$$BER = \int_0^\infty P_{ec}\,p(I_{\max})\,dI \tag{8}$$

where $P_{ec} = 0.5\exp(-0.5SNR_{e(SelC)})$ and $I_{max} = max\{I_i\}_{i=1}^{N}$ is the strongest of all received irradiance from all the *N*-PIN photodetectors. The probability density function (pdf) of I_{max}, given by $p(I_{max}) = p(max\{I_i\}_{i=1}^{N})$ is obtained first by finding the cumulative distribution function of I_{max} at an arbitrary point and later differentiating. The resulting pdf is given below with the detailed proof presented in the Appendix I.

$$p(I_{max}) = \tfrac{N}{I_o}\exp(-I/I_o)(1-\exp(-I/I_o))^{N-1}. \quad (9)$$

The plot of the pdf of I_{max} is shown in Fig. 3 for different values of N and the resulting BER from the combination of the preceding equations (7), (8) and (9) is depicted in Fig. 4.

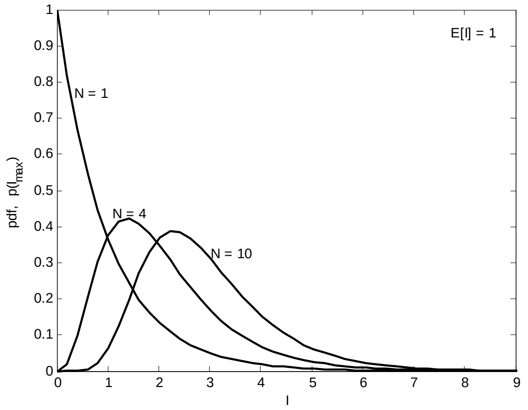


Fig. 3: The pdf $p(I_{max}) = p(max\{I_i\}_{i=1}^{N})$ for N = 1, 4 and 10, and $E[I] = I_o = 1$

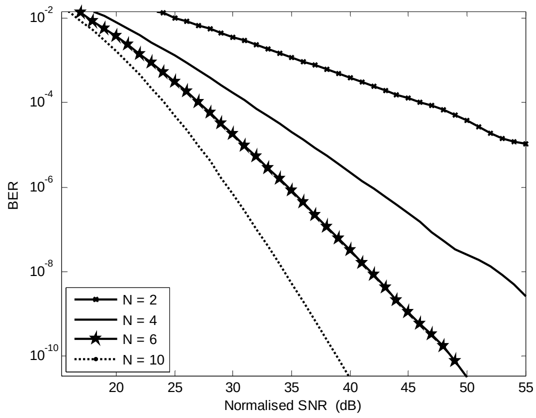


Fig. 4: The graph of BER against the normalised $SNR = R^2E[I]^2/\sigma^2$ with SelC spatial diversity for N = 2, 4, 6, 10

A comparison of Figs. 2 and 4 clearly reveals the resultant gain from the use spatial diversity. It should be noted that the plot for N = 10 in the Fig. 4 is for illustration only; the use of such a large number of detectors pose serious implementation difficulty. Another interesting point of note from these results is that unlike in weak turbulence regime where our previous study [23] and an independent study by Lee and Chan [13] revealed that SelC should not be used during weak irradiance fading/short FSO links, here SelC is highly recommended as it results in a significant reduction in required SNR (diversity gain) of about ~77 dB ((~38.5dB gain in received mean optical power) at BER of 10^{-6} with four independent PIN photodetectors. An explanation for this is that the irradiance fading in fully developed speckle regime is dominant over the reduction in received irradiance due to a reduction in the area of the receiver aperture when SelC spatial diversity is employed. Any scheme that thus mitigates this dominant irradiance fading will clearly result in reduced SNR for a given BER. The results presented here are only valid when the N-photodetectors receive independent irradiances, that is $\rho_o < s \le \vartheta_D L$ where ϑ_D is the divergence angle of the optical source in milliradian and L is the link length in kilometre. For $s < \rho_o$, the received irradiances are correlated resulting in a reduced diversity gain.

IV. CONCLUSION

The performance of the SIM FSO link has been presented in fully developed speckle environment. Expression for the BER has been presented with and without the SelC spatial diversity. Results show that with four PIN photodetectors, a theoretical gain of about 38 dB in received optical power is attainable at a BER of 10^{-6}. This implies that SelC spatial diversity is a potent technique for mitigating scintillation in fully developed speckle regime.

APPENDIX I

Assumption 1: Let the number of independent photodetectors be N.

$$\therefore I_{max} = max\{I_i\}_{i=1}^{N}. \quad (10)$$

And $p(I_{max}) = p(max\{I_i\}_{i=1}^{N})$ Considering an arbitrary received irradiance I, it then follows that $p(I_{max} < I) = p(I_1 < I, I_2 < I, \ldots I_N < I)$ since none of the $\{I_i\}_{i=1}^{N}$ is greater than I_{max}.

The following therefore gives the cumulative distribution function (CDF) of I_{max} for N-independent received irradiances:

$$p(I_{max} < I) = \int_0^I \ldots \int_0^I \int_0^I p(I_1, I_2 \ldots I_N) dI_1 dI_2 \ldots dI_N. \quad (11)$$

Assumption 2: The received irradiances are identically distributed as negative exponential distribution.

$$p(I_{max} < I) = \prod_{i=1}^{N}\int_{0}^{I} p(I_i)dI_i = \left[\int_{0}^{I}\frac{1}{I_o}\exp\left(-\frac{I}{I_o}\right)dI\right]^N$$
$$= \left(1-\exp\left(-\frac{I}{I_o}\right)\right)^N \qquad (12)$$

The required pdf, $p(I_{max})$ is now obtained by differentiating (12) once with respect to irradiance *I*.

$$p(I_{max}) = \frac{d}{dI}\left(1-\exp\left(-\frac{I}{I_o}\right)\right)^N$$
$$= \frac{N}{I_o}\exp\left(-\frac{I}{I_o}\right)\left(1-\exp\left(-\frac{I}{I_o}\right)\right)^{N-1}. \qquad (13)$$

With $N = 1$, (13) gives the negative exponential distribution as will be expected.

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