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博士后学位论文

可压微极性流体的局部存在性与正则性准则

Local well-posedness and regular criterion of compressible, viscous, micropolar fluid

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摘 要

可压微极性流体是指流体中散布着粒子的悬浮液，比如血液、有添加剂的润滑油和聚合物溶液等。与经典的可压Navier-Stokes方程主要差别是放弃Euler-Cauchy应力原理的假设，考虑流体运动时存在非零应力偶张量的情况，其结果导致非对称的应力张量。由于流体内部的这些粒子被非对称的张力作用，并且这些粒子本身的相互作用能够影响流体本身，流体呈现出非牛顿流体的特性。这是一类很重要的非牛顿流体，被认为是经典可压Navier-Stokes方程的推广而被人们广泛的关注。

本文考虑在初始密度在某些子区域上含有真空并且初始值满足某种相容性条件的情况下，局部强解的存在性。首先，我们采用线性化方法将微极性流体模型线性化，然后运用一致的先验估计，给出线性化模型的整体强解的存在性。然后，我们运用迭代方法给出原模型的近似解的一致先验估计，运用这些一致先验估计，我们给出近似解的一致收敛性。最后，我们运用逼近方法证明了微极性流体的局部强解的存在性。

同时，我们考虑可压微极性流体整体强解存在性的正则性准则。像经典的可压Navier-Stokes方程一样，我们暂时无法证明整体强解的存在性。那么，如果T是强解存在的最大时间，在T时刻是流体的哪一种量产生了奇性，是我们非常关注的事情。在本文中，我们参考可压Navier-Stokes方程，给出了微极性流体在速度的梯度关于空间的无穷模和时间的L1模有界时，整体强解会一直存在下去。

关键词：可压微极性流体；局部存在性；正则性准则

Abstract

A continuous fluid mechanics with randomly oriented particles suspended in the medium is assigned to the rotation or spin of molecular subunits, such as blood, additive lubricant and liquor of polymer etc. Compared with classical compressible Navier-Stokes equation, the main difference is to give up the hypothesis of Euler-Cauchy principle of stress and the movement with non-zero stress of tenser. The molecular structure can affect the fluid flow due to the interaction of internal particles described by asymmetric stress. This is one of the most important generalization of classical compressible Navier-Stokes equation.

In this paper, we consider the existence of local well-posedness of compressible micropolar fluid only if the initial data satisfied some compatibility conditions with the initial vacuum in some subset. Firstly, we consider the linearized model, then use the a priori estimates to construct global strong solutions of the linearized model. Then, we use iterative methods to construct the approximation solutions of the micropolar fluid and deduce the uniform convergence of the approximation solutions. At last, we use iterative methods to prove the local well-posedness of the micropolar fluid.

In the same meaning, we consider the regular criterion of the global strong solutions of compressible micropolar fluids. Like the classical compressible Navier-Stokes equation, we can not prove the existence of global strong solutions to the compressible micropolar fluids. Ones then wonder which quantity of the solution would blowup in finite time. In this paper, motivated by the regular criterion of compressible Navier-Stokes equation, we deduce the regular criterion of micropolar fluids under the condition that the gradient of velocity is uniformly bounded in domain and L1 bounded in time.

Keywords: Compressible micropolar fluid; local well-posedness; regular criterion

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