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Directed transport of coupled Brownian ratchets with time-delayed feedback*

Gao Tian-Fu(高天附)^{a)b)}, Zheng Zhi-Gang(郑志刚)^{c)†}, and Chen Jin-Can(陈金灿)^{b)‡}

^{a)} College of Physical Science and Technology, Shenyang Normal University, Shenyang 110034, China
 ^{b)} Department of Physics, Xiamen University, Xiamen 361005, China

^{c)}Department of Physics and the Beijing–Hong Kong–Singapore Joint Centre for Nonlinear and Complex Studies, Beijing Normal University, Beijing 100875, China

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A time-delayed feedback ratchet consisting of two Brownian particles interacting through the elastic spring is considered. The model describes the directed transport of coupled Brownian particles in an asymmetric two-well ratchet potential which can be calculated theoretically and implemented experimentally. We explore how the centre-of-mass velocity is affected by the time delay, natural length of the spring, amplitude strength, angular frequency, external force, and the structure of the potential. It is found that the enhancement of the current can be obtained by varying the coupling strength of the delayed feedback system. When the thermal fluctuation and the harmonic potential match appropriately, directed current evolves periodically with the natural length of the spring and can achieve a higher transport coherence. Moreover, the external force and the amplitude strength can enhance the directed transport of coupled Brownian particles under certain conditions. It is expected that the polymer of large biological molecules may demonstrate a variety of novel cooperative effects in real propelling devices.

Keywords: delayed feedback ratchet, double-well ratchet potential, centre-of-mass velocity, unbiased timeperiodic force

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1. Introduction

Many microscopic systems each have noisy dynamics governed by thermal fluctuations, making their control and theoretical treatment complicated. The Brownian ratchet is such a system that can take advantage of noise as it converts random fluctuations into directed motion in the absence of external forces.^[1–3] This interesting phenomenon has been a subject of statistical physics and the driving mechanisms of a variety of biological motors.^[1,4]

To observe directed transport the breaking of spatial and/or time-reversal symmetries is a necessary condition. A classical example of this mechanism is the flashing ratchet, which consists of a particle undergoing overdamped diffusion and also being subjected to a space-asymmetric potential.^[5] Whether the potential is switched on or off according to a random/periodic time sequence can be indicated by the current of particles. This current is driven by the energy injected into the system when the potential is switched on. Another way of breaking the time-reversal symmetry is obtained by coupling the system with reservoirs at two different temperatures: an asymmetric intruder in such a multi-temperature environment displays an average drift, acting as a Brownian motor.^[6] For many-body systems this has been extensively studied.^[7]

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The purpose of these models is often practical, i.e., to extract the energy from a highly fluctuating environment, such as a living cell.^[8] Brownian ratchets are also valid probes for the non-equilibrium properties of the fluctuating medium, and the value of the current is sensitive to the time scale as well as diffusion coefficient (temperature).^[9,10] An analogous phenomenon can be observed in this paper.

Inspired by the biological machines, there exist several proposals to utilize the principles of Brownian motors to perfect the nanotechnology.^[11] Many actual Brownian motors are relatively large and complex systems, and therefore models with more comprehensive and controllable designs are needed.^[1,3,4] Besides, a number of artificial Brownian motors and ratchets have been realized, such as cold atom Brownian motors and ratchets^[12] and quantum ratchets.^[13]

Our aim is to demonstrate that the transport of the directed current can be controlled by using a time-delayed feedback control protocol. Although the delayed feedback method was originally proposed by Pyragas^[14] to stabilize unstable states in deterministic systems, it has been utilized in various other contexts.^[15,16] Recently the effect of the time-delayed feedback on the rectification of thermal motion of Brownian particles has been studied in overdamped ratchet systems.^[17-20]

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[†]Corresponding author. E-mail: zgzheng@bnu.edu.cn

[‡]Corresponding author. E-mail: jcchen@xmu.edu.cn

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Such a feedback mechanism for a flashing ratchet has been realized experimentally with an optical line trap recently.^[21] It has been observed that the use of feedback increases the ratchet velocity up to 1 order of magnitude, which is in agreement with the theory.

The best known biological motors are kinesins^[22] which are able to take nanometer steps along protein tracks in the cytoplasm. Kinesin is a two-headed protein linked by a domain (neck) and a tail which attach a cargo or vesicle to be carried. The two heads perform a processive walk over the microtubules inside cells. These motors transport a wide variety of loads, power cell locomotion, and when acting in large ensembles, allow organisms to move. Motivated by experimental results, several authors have introduced diverse models in order to understand the particular walking of kinesins.^[23-26] In the present paper we will demonstrate a model inspired by the walking of motor proteins like kinesin on microtubules, but it is not restricted to the walking of motor proteins. The model consists of two Brownian particles coupled through elastic force and subjected to independent white noise. Meanwhile, the system of two coupled particles is brought into action by an ensemble averaged force, due to the presence of a ratchet potential. Because the kinesin can perform one-dimensional walk and drag effectively the load against the external force and the viscous drag from the environment, we consider an unbiased time-periodic force and a constant external force in our model. In the present delayed feedback system we will adopt the double-well ratchet potential^[27] that can be implemented experimentally and investigate how the time delay, natural length of the spring, amplitude strength, angular frequency, external force, and the structure of the potential affect the current transportation of two coupled Brownian particles in the control of delayed feedback.

2. A coupled delay-feedback ratchet

The delayed feedback ratchet considered here consists of two overdamped Brownian particles at temperature T_0 in a one-dimensional periodic double-well potential U(x). One can choose different periodic potentials according to different motivations. Now, the spatially periodic double-well ratchet potential^[27]

$$U(x) = -U_1 e^{-\sin^2(\pi x)/2\sin^2(\pi R)} - U_2 e^{-\sin^2(\pi(x-d))/2\sin^2(\pi R)}$$
(1)

is used to replace the piecewise linear sawtooth potential that is often used in the literature and its schematic diagram is shown in Fig. 1. Here U_1 and U_2 determine, respectively, the depths of the stronger and weaker wells, which are separated by a distance d and have width R. The ratio $\beta = U_1/U_2$ denotes the asymmetry parameter of the potential, x is the position, and the ratchet period L = 1. The parameters U_1 and U_2 have the energy unit, and d, R, x, and L have the same length unit. The two overdamped Brownian particles interact through an elastic coupling and are subjected to an external, unbiased timeperiodic force $A \cos \omega t$ with angular frequency ω and amplitude strength A. Additionally, a constant external force F acts on the system.



Fig. 1. (color online) Schematic diagram of coupled Brownian particles in the double-well ratchet potential U(x).

The potential force acting on the particles is $F_{\text{pot}}(x) = -dU(x)/dx$. By considering the free boundary condition, the overdamped Langevin equations of the above coupled ratchets can be written as

$$\gamma \dot{x}_{1}(t) = \alpha(t) F_{\text{pot}}(x_{1}(t)) + k(x_{2} - x_{1} - l) + A \cos \omega t + F + \xi_{1}(t), \qquad (2)$$

$$\gamma \dot{x}_{2}(t) = \alpha(t) F_{\text{pot}}(x_{2}(t)) - k(x_{2} - x_{1} - l) + A \cos \omega t + F + \xi_{2}(t).$$
(3)

The average force due to the potential switch being on is

$$f(t) = \frac{1}{2} \sum_{i=1}^{2} F_{\text{pot}}(x_i(t)) = \frac{1}{2} \left[F_{\text{pot}}(x_1(t)) + F_{\text{pot}}(x_2(t)) \right].$$
(4)

In Eqs. (2) and (3), γ is the friction coefficient, which is related to the diffusion coefficient *D* through Einstein's relation $D = k_{\rm B}T_0/\gamma$ with $k_{\rm B}$ being the Boltzmann constant, *k* is the coupling constant, and *l* is the natural length of the spring. Thermal fluctuations due to the coupling of the two particles with the environment are modeled by zero-mean, and $\xi_i(t)$ is Gaussian white noise with an autocorrelation function $\langle \xi_i(t) \xi_j(t') \rangle = 2\gamma k_{\rm B}T_0 \delta_{ij} \delta(t-t')$. We consider a controller $\alpha(t)$,^[17,18,20] which measures the sign of the average force and, after a time τ , switches the potential on ($\alpha = 1$) if the ensemble average of the force is positive or switches the potential off ($\alpha = 0$) if it is negative. This delayed control protocol can be expressed as

$$\alpha(t) = \begin{cases} \Theta(f(t-\tau)), & t \ge \tau, \\ 0, & \text{otherwise,} \end{cases}$$
(5)

where Θ is the Heaviside function.

It is worthwhile to point out that the model adopted here is more general. For the case of $A \cos \omega t = 0$ and F = 0, the model in the present paper may retrogress to that studied in Ref. [28]. It is interesting to note that for the case of k = 0, although there are no explicit mechanical interactions between the particles, the use of information in the control of the system introduces a coupling between the particles.^[17,18] For this reason, for closed-loop strategies^[29] the average velocity is dependent on the number of particles in the ensemble, in contrast to open-loop policies^[30] which, for non-interacting particles, output the same velocity regardless of the ensemble size.^[31]

The first basic quantity of interest in the system is the center-of-mass velocity of two Brownian particles, given by

$$V_{\rm cm} = \lim_{T \to \infty} \frac{1}{2T} \sum_{i=1}^{2} \int_{0}^{T} \dot{x}_{i}(t) \, \mathrm{d}t.$$
 (6)

In order to quantify the transport coherence of the coupled Brownian particles, we introduce the Pe number,^[32] i.e.,

$$Pe = \frac{|V_{\rm cm}|L}{D_{\rm eff}},\tag{7}$$

where D_{eff} is the average effective diffusion coefficient determined by

$$D_{\text{eff}} = \lim_{T \to \infty} \sum_{i=1}^{2} \frac{\left\langle x_i(t)^2 \right\rangle - \left\langle x_i(t) \right\rangle^2}{2 \cdot 2T}.$$
(8)

The Pe number describes the competition between the directional drift and the stochastic diffusion of the particle. The directional drift will increase with the Pe number increasing. Thus, the larger Pe number means that the drift predominates over the diffusion and there is high transport coherence.

Equations (2) and (3) are numerically solved by using the stochastic Runge–Kutta algorithm. The transport processes of two coupled particles are simulated and each trajectory consists of 10^5 steps of integral with a small time step of $h = 10^{-3}$. In the following calculations, we restrict the discussion to a set of driving parameters,^[27,33] which are set to be L = 1, d = 0.36, R = 0.15, $U_1 = 3.2k_BT_0$, $\beta = 1.0$, $\gamma = 1.0$, D = 1.0, A = 4.2, $\omega = 4.9$, l = 0.01, and $k_BT_0 = 1$ where the energy is in units of Joule, unless otherwise mentioned.

3. Results and discussion

3.1. Influence of the delay time

Figure 2(a) shows the curves of center-of-mass velocity $V_{\rm cm}$ varying with delay time τ for different coupling strengths. It can be found that the center-of-mass velocity of the system is a monotonically decreasing function of delay time τ . The correlation between the present sign of the average force and the measured sign actually used in the controller decreases with the increase of delay time. It can be understood that the decrease in center-of-mass velocity is a consequence of the loss of information about the present sign of the average force. Thus, the action of the controller begins to be uncorrelated to the present state of the system and it effectively begins to act as an open-loop ratchet.^[20] The result is compatible with

those obtained in Refs. [18] and [20]. On the other hand, it can be clearly seen that the directed current is improved for the coupled case. This indicates the effect of coupling-enhanced transport.^[34,35]



Fig. 2. (color online) Curves of (a) center-of-mass velocity $V_{\rm cm}$, (b) the values of average effective diffusion coefficient $D_{\rm eff}$, and (c) the *Pe* number varying with time delay τ for the different values of coupling strength *k*, where F = 0.1.

In order to understand the phenomenon in depth, we calculate the values of average effective diffusion coefficient D_{eff} as a function of delay time τ for different coupling strengths, which are shown in Fig. 2(b). For the uncoupled case, particles are easier to diffuse. But for the coupling case, it is difficult for the diffusion of coupled particles and the average effective diffusion coefficient D_{eff} to decrease drastically, implying that the fluctuation of displacement of coupled particles decreases with the increase of the coupling strength. However, while the feedback time is larger, the effective diffusion coefficient D_{eff} does not vary obviously with the increase of delay time.

Figure 2(c) shows the variations of the *Pe* number with delay time τ for different coupling strengths. It can be found from Fig. 2(c) that the *Pe* number is directly proportional to the value of $V_{\rm cm}$ since $D_{\rm eff}$ is a nearly constant for a fixed delay time. Then the *Pe* number of the delayed feedback system is also a monotonically decreasing function of delay time τ . The results show that the feedback ratchet exhibits a lower transport efficiency for the uncoupled case, and the *Pe* number increases with coupling strength increasing. It can be understood that the coupling between the two particles can cause high transport coherence.

3.2. Influence of the amplitude strength

Figure 3(a) shows the plots of the center-of-mass velocity versus amplitude strength A for some different values of external force F. The results show that the velocities are always non-monotonic functions of the amplitude strength. If A is too small, the two coupled particles move under the action of pulling force F. If the amplitude strength is too large, the



Fig. 3. (color online) Curves of the center-of-mass velocity $V_{\rm cm}$ varying with amplitude strength A for the different values of external force F, where $\tau = 0.01$, k = 1.0, (a) D = 0.01 and (b) D = 1.0.

coupled particles can only feel a fixed unbiased time-periodic external force, so it is difficult to facilitate directed transport in the delayed feedback system. Therefore, only for an appropriate match between A and F, can the higher directed transport appear. It is interesting to note that the maximum of the center-of-mass velocity is shifted toward the right when the external force F increases. With the increase of the diffusion coefficient D the corresponding center-of-mass velocities and the heights of the peaks decrease, but the peaks are still shifted toward the right as shown in Fig. 3(b). This indicates once more that an optimal amplitude strength may lead to the most efficient directed transport for the different diffusion coefficient/temperature cases. It can be clearly obtained that directed current depends also on F. This phenomenon will be discussed in the next section.

3.3. Influence of the angular frequency

In order to investigate the effect of the time-periodic force, the curves of center-of-mass velocity $V_{\rm cm}$ versus angular frequency ω for different values of delay time are plotted in Fig. 4. In the limit $\omega \to 0$, it is easier for particles to move forward and the center-of-mass velocity can reach its maximum. When the external driving force changes very quickly, i.e., $\omega \to \infty$, the time averaged periodic force $A' = \int_0^{2\pi/\omega} A \cos \omega t \, dt \to 0$, and the two coupled particles move only under the pulling force F, so the velocity $V_{\rm cm}$ approaches to a constant. Interestingly, at some intermediate values of ω , the center-of-mass velocity evolves "periodically" and the "amplitude" decreases with angular frequency increasing.



Fig. 4. (color online) Curves of the center-of-mass velocity $V_{\rm cm}$ varying with angular frequency ω for the different values of delay time τ , where k = 1.0 and F = 0.1.

3.4. Influence of the external force

Figure 5(a) shows the variations of center-of-mass velocity with external force F for different values of asymmetry parameter of potential β . The larger the external force F, the faster the particles move. Since the delay time τ is fixed, the center-of-mass velocity of two coupled particles increases as the external force is increased. It can be easily found that when the external opposite force is applied, the current will be reversed. In other words, there is a current reversal when the sign of *F* is changed. It is interesting to find that the directed current is roughly proportional to the external force when the asymmetry parameter $\beta \ge 0.5$ as shown in Figs. 5(a) and 5(b). But the directed current is not a monotonic function of the asymmetry parameter of the potential. We will discuss the phenomenon in detail below.



Fig. 5. (color online) Curves of the center-of-mass velocity $V_{\rm cm}$ varying with external force *F* for the different values of asymmetry parameter of potential β , where $\tau = 0.01$, k = 1.0, (a) D = 0.01 and (b) D = 1.0.

3.5. Influence of the natural length of the spring

The natural length of the spring l has been set to be 0.01 in the above discussions. Now we investigate the influences of the spring length l on directed transport for different values of diffusion coefficient D. In Fig. 6(a), we plot the center-ofmass velocity against the natural length of the spring. It is interesting to find that V_{cm} changes periodically with l varying and the complicated dynamics depends on the natural length of the spring. It can be clearly observed that the V_{cm} is nonzero even when l/L is an integer, where the two coupled particles trapped in the minimum of the potential can diffuse thermally enough to surpass the potential barrier against the force of spring in the presence of external force. The result is analogous to the case of a time-periodic modulation ratchet obtained in Ref. [36]. Moreover, the change of the diffusion coefficient D (related to the temperature) may significantly influence the current. For a higher temperature, the change of the current is affected slightly by the natural length of the spring. In order to obtain a higher transport coherence, there should be a cooperation between the thermal fluctuation and the harmonic potential.



Fig. 6. (color online) Curves of (a) the centre-of-mass velocity $V_{\rm cm}$, (b) the average effective diffusion coefficient $D_{\rm eff}$, and (c) the *Pe* number varying with the natural length of the spring *l* for the different values of diffusion coefficient *D*, where $\tau = 0.01$, k = 1.0, and F = 0.1.

Figure 6(b) shows that for the fixed parameters, the average effective diffusion coefficient D_{eff} increases quickly when the temperature is increased. While the temperature is higher, the effective diffusion coefficient D_{eff} does not vary obviously with the increase of the natural length of the spring. It is in-

teresting to find that when the diffusion coefficient is small $(D \le 0.05)$ two peaks can be observed in one period of the natural length. The two peaks show that the natural length of the spring can facilitate the diffusion in two different regimes. However, with the increase of D, the two peaks turn into one peak and the average effective diffusion coefficient does not vary obviously for a larger value of diffusion coefficient (D = 0.1).

To understand the transport coherence in depth, we calculate the values of number Pe as a function of l for different diffusion coefficients, which are shown in Fig. 6(c). It can be found from Figs. 6(c) and 6(a) that the value of *Pe* number is roughly proportional to the value of $V_{\rm cm}$ for a fixed delay time. The results show that the curve of Pe number versus lcan obtain two maximal values during one period of the natural length for D < 0.05, which means that the natural length of the spring can lead to a high transport coherence. It is also found that the maximal value of the Pe number depends on the maximal velocity of the coupled particles in one period of the natural length. When the value of diffusion coefficient is increased, the Pe number is decreased and affected slightly by l because in this case $V_{\rm cm}$ is small and $D_{\rm eff}$ is large. It is indicated once again that the higher the temperature, the more difficult the directed transport of particles is, the larger the effective diffusion coefficient becomes, and the more slowly the speed of the effective diffusion coefficient decreases.

3.6. Influence of the asymmetry parameter

Figure 7 shows the dependencies of the center-of-mass velocity on the asymmetry parameter of potential β for some given values of the coupling strength. It is clearly seen that $V_{\rm cm}$ is not a monotonic function of the asymmetry parameter so that there is an optimal value of β at which the center-of-mass velocity attains its extremum. The center-of-mass velocity decreases with the value of β increasing. It is observed that the current enhancement may also be obtained by changing the structure of the two-well ratchet potential.



Fig. 7. (color online) Variations of center-of-mass velocity $V_{\rm cm}$ with asymmetry parameter of potential β for the different values of coupling strength *k*, where $\tau = 0.01$ and F = 0.1.

4. Conclusions

In this paper, we have identified and characterized the present mechanism to control the thermally driven transport for two coupled Brownian particles evolving in a two-well ratchet potential under the action of external force and timedelayed feedback. Through analytical arguments we have calculated the center-of-mass velocity of the coupled ratchet in certain cases. It is found that the coupling can enhance the transport and the directed current is very sensitive to the natural length of the spring and evolves periodically for small diffusion coefficients. Some further issues, such as the influences of the time delay, the external force, the diffusion effect, the structure of the potential, etc., are still under consideration.

Our investigation of the delayed feedback ratchet transport composed of two elastically coupled particles is also relevant to the directed intracellular transport of molecular motors such as kinesin. The complex motor protein can be considered to be the polymer of large biological molecules. It has been observed that structurally similar molecular motors can move in opposite directions.^[37,38] The current study deals with two elastically coupled Brownian particles which are shorter than *N* coupled motor proteins. However, we believe that the collective behavior may demonstrate a variety of novel cooperative effects in real one-, two- or three-dimensional propelling devices.

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