



# Dissociation rates of H<sub>2</sub> on a Ni(100) surface: the role of the physisorbed state

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The dissociation and recombination rates of physisorbed H<sub>2</sub>, and the total dissociation rate of gas phase H<sub>2</sub> on the rigid Ni(100) surface, as well as the corresponding kinetic isotope effects, are calculated by using the quantum instanton method, together with path integral Monte Carlo and adaptive umbrella sampling techniques. Both the dissociation and recombination rates of physisorbed H<sub>2</sub> are dramatically enhanced by the quantum motions of H<sub>2</sub> at low temperatures, for instance, the quantum rates are 43 and 7.5 times larger than the classical ones at 200 K, respectively. For the dissociation of gas phase H<sub>2</sub>, at high temperatures, the H<sub>2</sub> can fly over the physisorbed state and dissociate directly, however, at low temperatures, the H<sub>2</sub> is first physisorbed and then dissociates under steady state approximation. The total dissociation rate of gas phase H<sub>2</sub> can be expressed as a combination of the direct and steady state dissociation rates. It has the form of an inverted bell with a minimum value at about 400 K, and detailed analysis shows that the dissociation of gas phase H<sub>2</sub> is dominated by a steady state process below 400 K, however, both the steady state and direct processes are important above 400 K. The calculated kinetic isotope effects reveal that H<sub>2</sub> always has larger rates than D<sub>2</sub> no matter which dissociative process they undergo.

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## 1 Introduction

The dissociative adsorption of hydrogen on nickel surfaces is one of the most extensively investigated subjects.<sup>1–4</sup> This is not only because the understanding of the dynamic of this process is important for elucidating the microscopic mechanisms of metal–adsorbate reactions, but also because nickel catalysts are commonly used in many industrial processes like hydrogenation, methanation and steam reforming of hydrocarbons.

The dissociation and recombination of H<sub>2</sub> on a Ni(100) surface have been investigated both experimentally and theoretically. On the experimental side, the techniques of molecular beam,<sup>5</sup> high-resolution electron energy loss spectra<sup>6</sup> and static secondary ion mass spectroscopy<sup>7,8</sup> have been used, and a great deal of information is available.<sup>9–12</sup> On the theoretical side, several kinds of potential energy surfaces (PESSs)<sup>13–18</sup> have been constructed. Based on PESSs, the classical,<sup>14,19</sup> mixed quantum-classical<sup>20,21</sup> and rather complex quantum mechanical<sup>22–24</sup> simulations have been used to reveal the microscopic mechanisms of these kinds of reactions,<sup>25–45</sup> moreover, the effect of the lattice motions has also been incorporated in several simulations.<sup>46–53</sup> In spite of the great progress that has already been made in our understanding

of the features of the title reaction, some controversies still exist. For instance, the role of the physisorbed state in the dissociation process has not been elucidated.

Experimentally, Hamza and Madix<sup>5</sup> reported a direct mechanism of dissociation for H<sub>2</sub> and D<sub>2</sub> on a Ni(100) surface with nearly mono-energetic beams of hydrogen and deuterium. However, Rendulic *et al.*<sup>10</sup> showed that there exists a mixture of activated and non-activated adsorption paths, and at low temperatures a small amount of hydrogen adsorbs through a precursor, by performing angle resolved measurements of the sticking coefficient for H<sub>2</sub> on Ni(100). Zhu *et al.*<sup>11</sup> also reported the evidence for the coexistence of direct and precursor dynamics in the dissociative chemisorption of H<sub>2</sub> on Ni(100), they found that a molecular precursor mediates dissociative adsorption at low temperatures, and a direct dissociation channel dominates the process for  $T > 200$  K.

Theoretically, Gross *et al.*<sup>54,55</sup> attributed the increase of the sticking probability with decreasing kinetic energy at low kinetic energies to a purely dynamically steering effect for the H<sub>2</sub>–Pd(100) system, in which neither potential barrier nor molecular adsorption well exists. For the H<sub>2</sub>–Ni(100) system, Kresse<sup>19</sup> demonstrated that the high sticking coefficient is related to a combination of steering at low energies and direct activated dissociation at high beam energies. Although no physisorbed molecular hydrogen has been found on clean Ni(100),<sup>56</sup> Andersson and co-workers reported that molecular hydrogen is held on Ni(100) at 150 K in the presence of coadsorbed CO,<sup>57</sup> so it should be reasonable to assume that the physisorbed state exists for the H<sub>2</sub>/Ni(100) dissociative process (indeed, the

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physisorbed state exists in many systems, such as  $\text{H}_2\text{-Cu}(111)^{44}$  and  $\text{H}_2\text{-Pd}(110)^{58}$ , and consider it as a model system, in this case, this dissociative process may be affected by both the steering effect and the physisorbed state. In the present work, we focus on the effects of the physisorbed state.

Lee and DePristo<sup>15</sup> constructed a potential energy surface for the dissociative chemisorption of  $\text{H}_2$  on  $\text{Ni}(100)$ , which is based upon a delocalized effective medium form plus empirical two-body terms, on this PES a potential well exists which corresponds to a physisorbed state. Based on a modified version of Lee and DePristo's PES, Truong *et al.*<sup>26</sup> applied canonical variational transition state theory to examine the dynamics of the dissociative chemisorption of  $\text{H}_2$  and  $\text{D}_2$  on  $\text{Ni}(100)$ . In their calculations, they have applied the steady state approximation which assumes that the concentration of the physisorbed  $\text{H}_2$  molecule remains constant during the time of exposure. However, several years later, Chakravarty and Metiu<sup>59</sup> used the correlation function theory, which provides equations expressing the rate constants in terms of flux-position correlation functions, to distinguish the direct and precursor dynamics through the mean time spent by the  $\text{H}_2$  molecule in the physisorption well. They found that the lifetime of physisorbed  $\text{H}_2$  is extremely short at 300 K, so it is then justified in treating the dynamics as a direct dissociation and ignoring physisorption, but at 100 K physisorbed  $\text{H}_2$  has a long lifetime and one is forced to consider a two-step dynamics:  $\text{H}_2$  is first physisorbed and then dissociated. Though the work of Chakravarty and Metiu has provided reasonable results, it is classical and offering no prescription for intermediate temperatures.

For the present reaction, a remarkable kinetic isotope effect (KIE) has also been reported. Experimentally, Hamza and Madix<sup>5</sup> showed that the initial dissociative sticking probability for  $\text{D}_2$  is lower than that of  $\text{H}_2$ , and reported that the barrier height is  $1.2 \text{ kcal mol}^{-1}$  for  $\text{H}_2$  and  $1.6 \text{ kcal mol}^{-1}$  for  $\text{D}_2$ . Zhu *et al.*<sup>11</sup> reported that the average sticking coefficient of  $\text{H}_2$  is higher than that of  $\text{D}_2$  at 200 K. Theoretically, Mowrey<sup>60</sup> found that the  $\text{D}_2$  needs at least  $0.67 \text{ kcal mol}^{-1}$  additional energy than the  $\text{H}_2$  to pass over the top of the barrier. Truong *et al.*<sup>26</sup> calculated the  $\text{H}_2$  and  $\text{D}_2$  dissociation rate constants, the ratios are 6.6, 4.0, 3.0 and 2.3 at 200, 300, 500 and 800 K, respectively, they also found that the quantum mechanical tunneling enhances the kinetic isotope effect a lot.

In the present paper, we investigate the dissociation and recombination of  $\text{H}_2$  on a rigid  $\text{Ni}(100)$  surface with two specific goals. One is to reveal the quantum effect and kinetic isotope effect during the reaction and examine how they affect rates. The other is to make a systematic analysis of the role of the physisorbed state in the dissociative process over a wide range of temperatures. The absence of lattice motion will limit the accuracy of the calculated rates, in our previous work,<sup>53</sup> we have demonstrated that the classical and quantum motions of the lattice enhance the physisorbed  $\text{H}_2$  dissociation rates by 18% and 49% at 300 K, however, we will not consider it in the present work. To calculate the rates, we use the quantum instanton (QI) approach which is originally proposed by Miller *et al.*<sup>61</sup> for the calculations of reaction rates. The approximation of QI is similar to an earlier semiclassical TST<sup>62</sup> that became known as the instanton model,<sup>63</sup> but it has an advantage that the Boltzmann operator is treated fully quantum

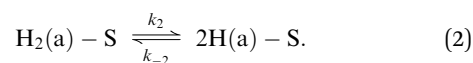
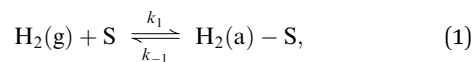
mechanically rather than within the semiclassical approximation. The QI method considers all tunneling paths and automatically gives each path its natural weight by the quantum Boltzmann factor and incorporates the quantum-fluctuation effect correctly. In addition, the QI method can be applied in Cartesian space with a full dimensional potential energy surface, taking into account the effects of the vibrational-rotational coupling and anharmonicity of the reaction system naturally. It has been successfully applied to gas phase reactions,<sup>64-67</sup> proton transfer,<sup>68,69</sup> surface diffusions,<sup>70-72</sup> torsional anharmonicity<sup>73,74</sup> and kinetic isotope effects.<sup>75-78</sup>

The remainder of this paper is as follows: The reaction mechanism is presented in Section 2. In Section 3, we first summarize the working expression of the QI theory, and then apply it to the title reaction. Section 4 describes the computational details. Section 5 gives the numerical results of rates and kinetic isotope effects. Section 6 is the concluding remarks.

## 2 Reaction mechanism

In this section, we present the mechanism of the  $\text{H}_2/\text{Ni}(100)$  reaction on a rigid surface, thus we neglect the motions of metal atoms. In the modified version<sup>26</sup> of Lee and DePristo's PES,<sup>15</sup> the classical potential energy barrier from the gas phase  $\text{H}_2$  to two chemisorbed H atoms is  $-0.3 \text{ kcal mol}^{-1}$ , which is much lower than the barrier ( $3.46 \text{ kcal mol}^{-1}$ ) reported by Kresse<sup>19</sup> with the density functional theory. This discrepancy is mainly caused by the fact that this modified PES involves a physisorbed state ( $-3.1 \text{ kcal mol}^{-1}$ ), and the classical potential energy barrier from this physisorbed state to chemisorbed state is  $2.8 \text{ kcal mol}^{-1}$ . The zero point energy corrected potential energies at the physisorbed and transition states are  $-1.4$  and  $0.6 \text{ kcal mol}^{-1}$  respectively, relative to the overall zero of energy which is set at the infinite separation of  $\text{H}_2$  from the surface, so the effective barrier to chemisorption from the physisorbed state is  $2.0 \text{ kcal mol}^{-1}$ . Since the potential well at the physisorbed state is deep, it is probably that the hydrogen molecules with low energies will be trapped in the physisorbed state, while that with high energies may pass over it directly.

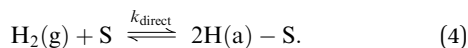
At low temperatures, it is reasonable to assume that physisorbed  $\text{H}_2$  is formed and equilibrated,<sup>26</sup> by applying the steady state approximation, the reaction can be modeled by the following set of reactions



Here, S represents the empty site on the Ni surface.  $k_1$  is the rate constant ( $\text{cm}^3 \text{ site}^{-1} \text{ s}^{-1}$ ) for the molecular adsorption of gas phase  $\text{H}_2$  molecules;  $k_{-1}$  is the rate constant ( $\text{s}^{-1}$ ) of the desorption of adsorbed  $\text{H}_2$  molecules;  $k_2$  is the rate constant ( $\text{s}^{-1}$ ) of the dissociative chemisorption of adsorbed  $\text{H}_2$  molecules; and  $k_{-2}$  is the recombination rate ( $\text{s}^{-1}$ ) of  $\text{H}_2$ . The overall steady state rate constant  $k_{\text{steady}}(\text{cm}^3 \text{ site}^{-1} \text{ s}^{-1})$  can be written as

$$k_{\text{steady}} = \frac{k_1 k_2}{k_{-1} + k_2}. \quad (3)$$

At high temperatures, the H<sub>2</sub> molecules have enough energies to pass over the physisorbed state, so the dissociation reaction should be direct,<sup>59</sup>



However, at intermediate temperatures, the physisorption mediated process and the direct process should coexist. The total rate constant  $k$  could be expressed using the following formula

$$k = k_{\text{direct}} \times f + k_{\text{steady}} \times (1 - f), \quad (5)$$

where  $f$  is the fraction of gas phase H<sub>2</sub> that has an energy greater than or equal to a certain value  $E_1$ , with which the gas phase H<sub>2</sub> can fly over the physisorbed state. The relation  $E_{\text{GS}} \geq E_1$  can be further written as

$$E_{\text{GS}}(\text{H}_2) - E_{\text{PS}}^{\text{G}}(\text{H}_2) \geq V_{\text{GS}}(\text{H}_2) - V_{\text{PS}}(\text{H}_2), \quad (6)$$

with

$$E_{\text{GS}}(\text{H}_2) = E_{\text{GS}}^{\text{tr}}(\text{H}_2) + E_{\text{GS}}^{\text{rot}}(\text{H}_2) + E_{\text{GS}}^{\text{vib}}(\text{H}_2), \quad (7)$$

where  $E_{\text{GS}}$  denotes the summation of the translational ( $E_{\text{GS}}^{\text{tr}}$ ), rotational ( $E_{\text{GS}}^{\text{rot}}$ ) and vibrational ( $E_{\text{GS}}^{\text{vib}}$ ) energies of H<sub>2</sub> in the gas phase.  $E_{\text{PS}}^{\text{G}}$  stands for the minimum vibrational energy that the physisorbed H<sub>2</sub> must hold, since all the rotational and translational modes of physisorbed H<sub>2</sub> are restricted, we treat them as vibrational modes, and the superscript 'G' stands for all six vibrational modes of physisorbed H<sub>2</sub> treated as being in the ground state when calculating the minimum vibrational energy.  $V_{\text{GS}}$  and  $V_{\text{PS}}$  represent the classical potential energies of H<sub>2</sub> in the gas phase and physisorbed state. Overall, eqn (6) and (7) mean that part of the energy of gas phase H<sub>2</sub> will convert to the minimum vibrational energy of physisorbed H<sub>2</sub>, and the rest can be used to overcome the potential well in the physisorbed state.

According to statistical mechanics, the characteristic rotational and vibrational temperatures of gas phase H<sub>2</sub> are 88 and 5987 K, which mean that the rotational degrees of freedom are well excited and the vibrational mode of motion is frozen out in our tested temperature range of 160–800 K. So the vibrational energy of gas phase H<sub>2</sub> is equal to the ground state energy ( $\hbar\omega/2$ ) which can be obtained by vibrational frequency ( $\omega$ ), and the rotational energy ( $k_{\text{B}}T$  per molecule) of gas phase H<sub>2</sub> can be obtained from the equipartition theorem. By substituting the vibrational (using the vibrational frequencies of both gas phase and physisorbed H<sub>2</sub>), rotational and classical potential energies into eqn (6), we get

$$E_{\text{GS}}^{\text{tr}}(\text{H}_2) \geq 4.75(\text{kcal mol}^{-1}) - k_{\text{B}}T. \quad (8)$$

Now, we can use the Maxwell-Boltzmann distribution, the fraction of hydrogen molecules with translational energy equal to or in excess of  $E_2$  ( $E_2 = 4.75(\text{kcal mol}^{-1}) - k_{\text{B}}T$ ) can be expressed as

$$f_{\text{MB}}(E_2) = \frac{2}{\sqrt{\pi}} a e^{-a^2} + \text{erfc}(a), \quad (9)$$

where  $a = (E_2/k_{\text{B}}T)^{1/2}$  and  $\text{erfc}(a)$  is the co-error function.

Finally, the total rate constant  $k$  in eqn (5) could be expressed as follows

$$k = k_{\text{direct}} \times f_{\text{MB}}(E_2) + k_{\text{steady}} \times (1 - f_{\text{MB}}(E_2)). \quad (10)$$

## 3 Rate theory

### 3.1 Summary of the quantum instanton theory

In this section, we summarize the work formulas for the QI approximation. The detailed derivation can be found in ref. 61, 64 and 65. The QI model proposes the following thermal rate constant

$$k_{\text{QI}} Q_{\text{r}} = \frac{\sqrt{\pi\hbar} C_{\text{ff}}(0)}{2 \Delta H(\beta)}. \quad (11)$$

Here,  $Q_{\text{r}}$  is the reactant partition function per unit volume.  $C_{\text{ff}}(0)$  is the zero time value of the flux-flux correlation function

$$C_{\text{ff}}(t) = \text{tr}[e^{-\beta\hat{H}/2} \hat{F}_{\text{a}} e^{-\beta\hat{H}/2} e^{i\hat{H}t/\hbar} \hat{F}_{\text{b}} e^{-i\hat{H}t/\hbar}], \quad (12)$$

where  $\beta$  is the inverse temperature ( $1/(k_{\text{B}}T)$ ),  $\hat{H}$  is the Hamiltonian operator of the reaction system, and  $\hat{F}_{\text{a}}$  and  $\hat{F}_{\text{b}}$  are the flux operators, given by

$$\hat{F}_{\gamma} = \frac{i}{\hbar} [\hat{H}, h(s_{\gamma}(\hat{\mathbf{r}}))], \quad (13)$$

with  $\gamma = \text{a, b}$ . In eqn (13),  $h$  is the Heaviside function,  $r$  represents the Cartesian coordinates of the reaction system, and  $s_{\text{a}}(r)$  and  $s_{\text{b}}(r)$  define two separate dividing surfaces *via* the equations  $s_{\text{a}}(r) = 0$  and  $s_{\text{b}}(r) = 0$ , both  $s_{\text{a}}(r)$  and  $s_{\text{b}}(r)$  being positive (negative) on the product (reactant) side of the dividing surfaces.

$\Delta H$  in eqn (11) is a specific type of energy variance, given by

$$\Delta H(\beta) = \hbar \sqrt{\frac{-\ddot{C}_{\text{dd}}(0)}{2C_{\text{dd}}(0)}}. \quad (14)$$

In order to get the correct free particle (high temperature) limit (that would be 25% too large otherwise), an ad hoc term is added to  $\Delta H(\beta)$ ,  $\Delta H_{\text{mod}}(\beta) = \Delta H(\beta) + (\sqrt{\pi} - \sqrt{2})/\beta$ , which has very little effect in the low temperature regime.  $C_{\text{dd}}(0)$  and  $\ddot{C}_{\text{dd}}(0)$  are zero time value and its second time derivative, respectively, of the "delta-delta" correlation function

$$C_{\text{dd}}(t) = \text{tr}[e^{-\beta\hat{H}/2} \Delta(s_{\text{a}}(\hat{\mathbf{r}})) e^{-\beta\hat{H}/2} e^{i\hat{H}t/\hbar} \Delta(s_{\text{b}}(\hat{\mathbf{r}})) e^{-i\hat{H}t/\hbar}], \quad (15)$$

where the generalized delta-function operator is

$$\Delta(s_{\gamma}(\hat{\mathbf{r}})) = \delta(s_{\gamma}(\hat{\mathbf{r}})) \sqrt{\sum_{i=1}^N \frac{1}{m_i} (\nabla_i s_{\gamma}(\hat{\mathbf{r}}))^2} \quad (\gamma = \text{a, b}). \quad (16)$$

Here,  $N$  is the total number of atoms,  $\nabla_i = \partial/\partial r_i$ ,  $r_i$  denotes the Cartesian coordinates of the  $i$ th atom and  $m_i$  is its atomic mass.

The dividing surfaces are determined by the stationary condition

$$\frac{\partial}{\partial c_k} C_{\text{dd}}(0; \{c_k\}) = 0, \quad (17)$$

where  $\{c_k\}$  is a collection of parameters that is involved in the location of the dividing surfaces. This condition originates from the SC instanton model, and the resulting dividing surfaces correspond qualitatively to the turning points of the periodic orbit that runs on an upside down PES in imaginary time.

### 3.2 Path integral evaluation of the quantum instanton rate

Since all the relevant quantities in the QI expression (eqn (11)) involve only the quantum Boltzmann operator, they can be readily evaluated using the imaginary time Path Integral Monte Carlo (PIMC)<sup>79</sup> method. In realistic calculations, we rewrite eqn (11) as the product of several ratios

$$k_{\text{QI}} = \frac{\sqrt{\pi\hbar} C_{\text{dd}}(0) C_{\text{ff}}(0)}{2 Q_{\text{r}} C_{\text{dd}}(0) \Delta H(\beta)} \quad (18)$$

The terms of  $C_{\text{ff}}(0)/C_{\text{dd}}(0)$  and  $\Delta H$  are directly calculated as a constrained average over the same ensemble of paths<sup>64,65</sup>

$$C_{\text{ff}}(0)/C_{\text{dd}}(0) = \langle f_{\text{v}} \rangle_{\ddagger}, \quad (19)$$

$$\Delta H^2 = \frac{1}{2} \langle F^2 + G \rangle_{\ddagger}, \quad (20)$$

where

$$f_{\text{v}} = \left( \frac{iP}{2\hbar\beta} \right)^2 \frac{\sum_{i=1}^N \nabla_{i s_{\text{a}}} (r^{(0)}) \cdot (r_i^{(1)} - r_i^{(P-1)})}{\sqrt{\sum_{i=1}^N m_i^{-1} (\nabla_{i s_{\text{a}}} (r^{(0)}))^2}} \times \frac{\sum_{i=1}^N \nabla_{i s_{\text{b}}} (r^{(P/2)}) \cdot (r_i^{(P/2+1)} - r_i^{(P/2-1)})}{\sqrt{\sum_{i=1}^N m_i^{-1} (\nabla_{i s_{\text{b}}} (r^{(P/2)}))^2}}, \quad (21)$$

and

$$F = -\frac{P}{\hbar^2 \beta^2} \left\{ \sum_{s=1}^{P/2} - \sum_{s=P/2+1}^P \right\} \sum_{i=1}^N m_i (r_i^{(s)} - r_i^{(s-1)})^2 + \frac{2}{P} \left\{ \sum_{s=1}^{P/2-1} - \sum_{s=P/2+1}^P \right\} V(r^{(s)}), \quad (22)$$

$$G = \frac{2fP}{\beta^2} - \frac{4P}{\hbar^2 \beta^3} \sum_{s=1}^P \sum_{i=1}^N m_i (r_i^{(s)} - r_i^{(s-1)})^2. \quad (23)$$

Here,  $P$  is the number of imaginary time slices, and  $m_i$  denotes the mass of the  $i$ th H atom.  $r^{(s)}$  and  $r_i^{(s)}$  represent the Cartesian coordinates for the  $s$ th time slice and for the  $s$ th time slice of the  $i$ th H atom, respectively.  $f$  is the degree of freedom for quantized H atoms.  $\Delta\beta$  is defined by  $\Delta\beta = \beta/P$ .

The  $\langle \dots \rangle_{\ddagger}$  in eqn (19) represents the constrained path average

$$\langle \dots \rangle_{\ddagger} = \frac{\int dr^{(1)} \int dr^{(2)} \dots \int dr^{(P)} \Delta(s_{\text{a}}(r^{(0)})) \Delta(s_{\text{b}}(r^{(P/2)})) \exp[-\beta\Phi](\dots)}{\int dr^{(1)} \int dr^{(2)} \dots \int dr^{(P)} \Delta(s_{\text{a}}(r^{(0)})) \Delta(s_{\text{b}}(r^{(P/2)})) \exp[-\beta\Phi]}, \quad (24)$$

with

$$\Phi = \frac{P}{2\hbar^2 \beta^2} \sum_{s=1}^P \sum_{i=1}^N m_i (r_i^{(s)} - r_i^{(s-1)})^2 + \frac{1}{P} \sum_{s=1}^P V(r^{(s)}). \quad (25)$$

The evaluation of  $C_{\text{dd}}(0)/Q_{\text{r}}$ , however, meets a challenge because the  $C_{\text{dd}}(0)$  is the quantity associated with the transition state, while  $Q_{\text{r}}$  with the asymptotic reactant domain. First, we write it to be

$$C_{\text{dd}}(0)/Q_{\text{r}} = \frac{\text{tr} \left[ e^{-\beta\hat{H}/2} \Delta(s_{\text{a}}(\hat{\mathbf{r}})) e^{-\beta\hat{H}/2} \Delta(s_{\text{b}}(\hat{\mathbf{r}})) \right]}{\text{tr} \left[ e^{-\beta\hat{H}/2} \delta(s_{\text{a}}^0(\hat{\mathbf{r}})) e^{-\beta\hat{H}/2} \delta(s_{\text{b}}^0(\hat{\mathbf{r}})) \right]} \times \frac{\text{tr} \left[ e^{-\beta\hat{H}/2} \delta(s_{\text{a}}^0(\hat{\mathbf{r}})) e^{-\beta\hat{H}/2} \delta(s_{\text{b}}^0(\hat{\mathbf{r}})) \right]}{Q_{\text{r}}}, \quad (26)$$

where  $s_{\text{a}}^0$  and  $s_{\text{b}}^0$  denote the reaction coordinates in the reactant region. Then, the first term on the right in eqn (26) can be solved by using eqn (16) and the adaptive umbrella sampling technique.<sup>80</sup> For the second one, it is evaluated directly when calculating the rate  $k_2$  ( $Q_{\text{r}}$  is written as  $\text{tr} [e^{-\beta\hat{H}/2} h(s_{\text{a}}^0(\hat{\mathbf{r}})) e^{-\beta\hat{H}/2} h(s_{\text{b}}^0(\hat{\mathbf{r}}))]$ , where  $h(s_{\text{a}}^0)$  and  $h(s_{\text{b}}^0)$  constrain the hydrogen molecule in the physisorbed state), and it is evaluated analytically when calculating the rate  $k_{\text{direct}}$  (see Appendix).

The free energy surface is defined by

$$F(s_{\text{a}}, s_{\text{b}}) = -k_{\text{B}} T \log [C_{\text{dd}}(0; s_{\text{a}}, s_{\text{b}})], \quad (27)$$

and the free energy profile along the reaction path is defined by

$$F(s) = -k_{\text{B}} T \log [C_{\text{dd}}(0; s, s)], \quad (28)$$

where  $s = s_{\text{a}} = s_{\text{b}}$ .

### 3.3 Application to the H<sub>2</sub>/Ni(100) reaction

We now apply the QI method to the hydrogen molecule dissociation and recombination reactions on a Ni(100) surface. In the present calculations, the modified version<sup>26</sup> of Lee and DePristo's PES<sup>15</sup> is adopted, and all six degrees of freedom of H<sub>2</sub> are included. In this PES, both the physisorbed state and transition state have asymmetric orientations, this phenomenon is in conflict with the conventional idea that symmetric geometries are more favorable, such as H<sub>2</sub>/Pd(100).<sup>55</sup> However, this PES should be reasonable, and choosing it allows us to compare our results to previous rate constant calculations.<sup>26,59</sup>

To calculate the rates, it is necessary to define a generalized reaction coordinate  $s(r; \xi)$ , where  $\xi$  is an adjustable parameter that shifts the location of the dividing surface (defined by  $s(r; \xi) = 0$ ). The essential strategy for defining  $s(r; \xi)$  is the same as that in the paper,<sup>65</sup> *i.e.*,  $s(r; \xi)$  is defined by a linear interpolation

between two constituent reaction coordinates  $s_0(r)$  and  $s_1(r)$  through the parameter  $\xi$ ,

$$s(r; \xi) = \xi s_1(r) + (1 - \xi) s_0(r). \quad (29)$$

For the dissociation process,  $s_1^{\text{dis}}(r)$  is a reaction coordinate whose dividing surface is designed to pass through the top of the classical potential barrier. It is noted that the position of potential barrier is determined by both the bond length of  $\text{H}_2$  and the distance of the center of mass of  $\text{H}_2$  above the surface. We thus define  $s_1^{\text{dis}}(r)$  as follows

$$s_1^{\text{dis}}(r) = \frac{Z_{\text{TS}}(\text{H}^1) + Z_{\text{TS}}(\text{H}^2)}{2} - \frac{Z(\text{H}^1) + Z(\text{H}^2)}{2} - [R_{\text{TS}}(\text{H}^1 - \text{H}^2) - R(\text{H}^1 - \text{H}^2)], \quad (30)$$

where  $Z(\text{H}^1)$  and  $Z(\text{H}^2)$  are the coordinates of z-axis (vertical to Ni(100) surface) for the two hydrogen atoms ( $\text{H}^1$  and  $\text{H}^2$ ), respectively.  $R(\text{X}-\text{Y})$  denotes the interatomic distance between atoms X and Y. The subscript TS denotes the values at the transition state geometry.  $s_0^{\text{dis}}(r)$  in eqn (29), on the other hand, describes a dividing surface that is located in the reactant region, which is given by

$$s_0^{\text{dis}}(r) = \frac{Z_R(\text{H}^1) + Z_R(\text{H}^2)}{2} - \frac{Z(\text{H}^1) + Z(\text{H}^2)}{2} - [R_R(\text{H}^1 - \text{H}^2) - R(\text{H}^1 - \text{H}^2)], \quad (31)$$

where the subscript R represents the values in the reactant region (it should be mentioned that the physisorbed state is regarded as the reactant when calculating the rate  $k_2$ , while for the rate  $k_{\text{direct}}$ , the gas phase state is the reactant).

Similarly, the reaction coordinates  $s_1^{\text{rec}}(r)$  and  $s_0^{\text{rec}}(r)$  for the recombination process are given by

$$s_1^{\text{rec}}(r) = -s_1^{\text{dis}}(r), \quad (32)$$

$$s_0^{\text{rec}}(r) = \frac{Z(\text{H}^1) + Z(\text{H}^2)}{2} - \frac{Z_P(\text{H}^1) + Z_P(\text{H}^2)}{2} - [R(\text{H}^1 - \text{H}^2) - R_P(\text{H}^1 - \text{H}^2)], \quad (33)$$

where the subscript P stands for the values at the product domain (the chemisorbed state where two separated hydrogen atoms are chemisorbed in two fourfold hollow sites).

The above definition makes the reaction coordinate only related to one parameter  $\xi$  with  $s(r;0) = s_0(r)$  and  $s(r;1) = s_1(r)$ . As  $\xi$  changes from 0 to 1, the dividing surface moves smoothly from the reactant (product) domain to the transition state region.

$C_{\text{dd}}(0)$  (in eqn (15)) now becomes a function of two parameters  $(\xi_a, \xi_b)^{64,65}$

$$C_{\text{dd}}(0; \xi_a, \xi_b) = \text{tr}[e^{-\beta \hat{H}/2} \Delta(s(\hat{\mathbf{r}}, \xi_a)) e^{-\beta \hat{H}/2} \Delta(s(\hat{\mathbf{r}}, \xi_b))]. \quad (34)$$

It is easy to see from eqn (17) that the condition of locating the dividing surfaces becomes

$$\frac{\partial C_{\text{dd}}(0; \xi_a, \xi_b)}{\partial \xi_a} = 0, \quad \frac{\partial C_{\text{dd}}(0; \xi_a, \xi_b)}{\partial \xi_b} = 0. \quad (35)$$

In this case, locating the two optimized dividing surfaces is switched to finding the two optimized parameters  $(\xi_a, \xi_b)$ .

## 4 Computational details

In the present study, the lattice model used consists of 817 Ni atoms and the Ni lattice constant is 3.52 Å. In the simulations, two hydrogen atoms (all six degrees of freedom) are treated quantum mechanically, while all the nickel atoms are rigid. In path integral calculations, the sampling of the discrete paths is performed using the Monte Carlo method. The number of time slices,  $P$  for the degrees of freedom of two H atoms, is set to  $P = (20-160)$  over the temperature range 160–800 K. The number of the Monte Carlo is about  $(4-8) \times 10^6$  for computing a single ensemble average. It converges most of the values within 10% statistical error.

## 5 Results and discussion

### 5.1 Free energy profile along the reaction path

We first calculate the free energy profile from the gas phase state ( $\text{H}_2(\text{g}) + \text{S}$ ) to the chemisorbed state ( $2\text{H}(\text{a}) - \text{S}$ ) at 300 K, with the two hydrogen atoms treated quantum mechanically. The results are displayed in Fig. 1. It is clearly shown that there is an obvious free energy well in the physisorbed state with a depth of  $1.96 \text{ kcal mol}^{-1}$  corresponding to the free energy in the gas phase state, and the dissociation and recombination free energy barriers for physisorbed  $\text{H}_2$  are 2.82 and  $17.46 \text{ kcal mol}^{-1}$ , which demonstrate that the dissociation reaction of physisorbed  $\text{H}_2$  is much easier than the recombination one. In Fig. 1, we also draw the geometries of  $\text{H}_2$  in the gas phase state (GS,  $\text{H}_2$  is located at about 6.0 bohr above the Ni(100) surface), the physisorbed state (PS, one H atom is located 2.07 bohr above a 4-fold site, and the other resides at 2.67 bohr near the neighboring bridge site), the transition state (TS, one H atom is located 1.12 bohr above a 4-fold site, and the other resides at 2.43 bohr near the neighboring bridge site) and the chemisorbed state (CS, both H atoms are adsorbed at 1.01 bohr above two nearest 4-fold sites) with 6 Ni atoms. It is noted that the distance of the center of mass of  $\text{H}_2$  to the surface gradually

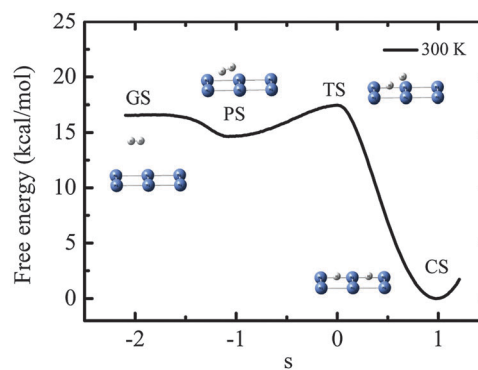


Fig. 1 Free energy profile along the reaction path for  $\text{H}_2/\text{Ni}(100)$  at 300 K. 'GS', 'PS', 'TS' and 'CS' stand for the gas phase, physisorbed, transition and chemisorbed states of  $\text{H}_2$ , respectively.



decreases, while the distance of two H atoms increases monotonically, from GS, PS, TS, to CS.

## 5.2 Dissociation and recombination rates of physisorbed H<sub>2</sub>

Based on the free energy profile, we can calculate the dissociation ( $\text{H}_2(\text{a}) - \text{S} \rightarrow 2\text{H}(\text{a}) - \text{S}$ ) and recombination ( $2\text{H}(\text{a}) - \text{S} \rightarrow \text{H}_2(\text{a}) - \text{S}$ ) rates of physisorbed H<sub>2</sub> on the rigid Ni(100) surface, which correspond to  $k_2$  and  $k_{-2}$  (eqn (2)), respectively. In order to investigate the quantum effect of H<sub>2</sub> motions on the rates, we also calculate the corresponding classical rates, labeled as  $\text{QI}_{\text{cl}}$ , with use of the classical limit of QI. The previous studies<sup>66,81</sup> have demonstrated that the classical limit of the QI formula is the same as the rate constant expression of the classical transition state theory (TST). Essentially, the time slice  $P \rightarrow 1$  in the path integral corresponds to the classical limit of QI. However, according to eqn (21),  $P$  must be even and the minimum  $P$  is 4. Thus,  $P = 4$  is used in the present work. These classical QI rates should have the similar property as the classical TST rates although they are not exactly the same. In Table 1, we summarize the present QI and  $\text{QI}_{\text{cl}}$  rates for both  $k_2$  and  $k_{-2}$ , as well as the results of canonical variational transition state theory with the small-curvature semiclassical adiabatic approximation (CVT/SCSAG).<sup>26</sup> We also tabulate the ratios of quantum and classical rates in Table 1.

Fig. 2 displays the corresponding Arrhenius plots of rates. For the values of  $k_2$ , the QI and  $\text{QI}_{\text{cl}}$  have an obvious difference over the whole tested temperature range. At 800 K, the QI is 1.21 times larger than the  $\text{QI}_{\text{cl}}$ , considering that our classical simulation ( $P = 4$ ) can be enough to capture most of zero point energy at 800 K, it is clear that the difference between QI and  $\text{QI}_{\text{cl}}$  should mainly come from the tunneling effect, which usually enhances the rate by lowering the free energy barrier. Indeed, the classical free energy barrier is higher than the quantum one by 0.2 kcal mol<sup>-1</sup> at 800 K. At 200 K, the ratio of QI and  $\text{QI}_{\text{cl}}$  increases to 42.9, this remarkable difference is due to both the zero point energy and tunneling effect, indeed, the zero point energy correction can decrease the effective potential barrier by 0.8 kcal mol<sup>-1</sup>, and our calculated classical free energy barrier is 1.0 kcal mol<sup>-1</sup> higher than the quantum one. All these situations reveal that the quantum

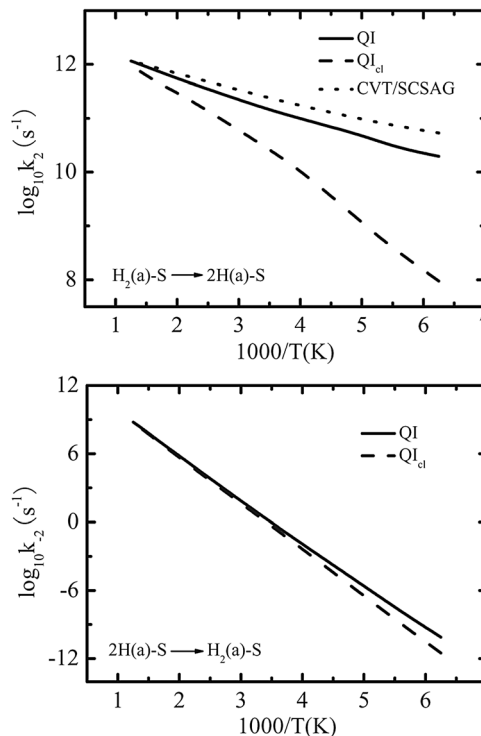


Fig. 2 Arrhenius plots of dissociation and recombination rate constants for physisorbed H<sub>2</sub> on a Ni(100) surface. The solid and dashed lines are the QI results with H atoms treated quantum mechanically and classically, respectively, and the dotted line denotes the rates of CVT/SCSAG.

effect of H<sub>2</sub> is remarkable at both low and high temperatures for this process.

Based on the same PES adopted in the present work, Truong *et al.*<sup>26</sup> have applied the steady state approximation which assumes that the physisorbed H<sub>2</sub> is in equilibrium, and reported the rates ( $k_2$ ) using canonical variational transition state theory with semiclassical tunneling approximation (CVT/SCSAG). Comparing the QI rates with that of CVT/SCSAG, as shown in Fig. 2, we can see they are in good agreement with each other at high temperatures, however, the QI is smaller than the CVT/SCSAG at low temperatures even though they still have the same orders of magnitude, this discrepancy is mainly caused by the fact that these two methods have used different ways to incorporate the tunneling effect, and the tunneling effect affects the rates a lot at low temperatures.

Chakravarty and Metiu<sup>59</sup> have reported a value of  $1.9 \times 10^8$  (s<sup>-1</sup>) for  $k_2$  at 300 K, which is much lower than that of the QI and the CVT/SCSAG, the main reason is that Chakravarty and Metiu's method does not consider the quantum effect (such as zero point energy correction and quantum tunneling), indeed, the zero point energy correction can decrease the effective potential barrier by 0.8 kcal mol<sup>-1</sup>. Another reason is that Chakravarty and Metiu's method has incorporated the recrossing effect, which usually decreases the rates, while the QI and CVT theories do not take it into account, this situation reflects that the recrossing effect may also be remarkable for the present process.

Table 1 Rate constants<sup>a</sup> of the physisorbed H<sub>2</sub> on the Ni(100) surface

T (K)	$k_2$			$k_{-2}$			
	CVT/SCSAG <sup>b</sup>	QI	$\text{QI}_{\text{cl}}^c$	QI/ $\text{QI}_{\text{cl}}$	QI	$\text{QI}_{\text{cl}}$	QI/ $\text{QI}_{\text{cl}}$
160	5.36(10)	1.97(10)	9.50(7)	207	7.68(-11)	3.14(-12)	24.4
180	7.28(10)	2.88(10)	3.52(8)	81.7	2.12(-8)	1.83(-9)	11.6
200	9.63(10)	4.91(10)	1.14(9)	42.9	2.54(-6)	3.39(-7)	7.51
250	1.72(11)	9.83(10)	1.12(10)	8.80	1.16(-2)	4.50(-3)	2.58
300	2.67(11)	1.65(11)	3.47(10)	4.75	3.75(0)	2.08(0)	1.80
400	4.78(11)	3.49(11)	1.39(11)	2.52	6.25(3)	4.26(3)	1.47
500	6.82(11)	5.49(11)	3.00(11)	1.83	6.56(5)	4.63(5)	1.42
600	8.63(11)	7.67(11)	4.66(11)	1.64	1.27(7)	1.06(7)	1.20
800	1.16(12)	1.15(12)	9.53(11)	1.21	6.01(8)	5.56(8)	1.08

<sup>a</sup> Unit: s<sup>-1</sup>, powers of 10 are in parentheses. <sup>b</sup> From ref. 26. <sup>c</sup>  $\text{QI}_{\text{cl}}$  denotes the classical limit of QI rates.

For the recombination process ( $k_{-2}$ ), Fig. 2 shows that QI and  $QI_{cl}$  are close to each other at high temperatures, while the classical rates become much smaller than the quantum ones at low temperatures. At 800 K, the QI is 1.08 times larger than the  $QI_{cl}$ , and the rate ratio increases to 7.51 at 200 K. These phenomena can also be explained by the differences in quantum and classical free energy barriers, indeed, the calculated quantum and classical free energy barriers are nearly the same at 800 K and the former is about  $0.5 \text{ kcal mol}^{-1}$  lower than the latter at 200 K. Experimentally, Zhu and White<sup>8</sup> have reported an activation energy of  $22.7(\pm 0.2) \text{ kcal mol}^{-1}$  and a preexponential factor of  $1.5 \pm 0.6(\times 10^{14}) \text{ ML}^{-1} \text{ s}^{-1}$  in the temperature range of 300–340 K. To compare with it, we calculate the activation energy ( $E_a$ ) and preexponential factor ( $A$ ) by fitting the QI rates to the Arrhenius form

$$k_{QI} = A \exp[-E_a/RT] \quad (36)$$

over the temperature range 250–400 K. The calculated  $E_a$  and  $A$  are  $17.48 \text{ kcal mol}^{-1}$  and  $2.23 \times 10^{13} \text{ s}^{-1}$ , it is clear that our preexponential is consistent with the experimental data but the activation energy is much smaller. This much smaller activation energy reveals that the recombination potential barrier of the present PES may need to be further refined.

### 5.3 Kinetic isotope effects for physisorbed H<sub>2</sub>

The kinetic isotope effect (KIE) is defined as the ratio  $k^i/k^j$ , where  $k^i$  is the rate for the isotopic reaction with lighter mass, and  $k^j$  is that for the corresponding heavier isotopic reaction. Here, we consider the KIEs of the dissociation ( $k_2$ ) and recombination ( $k_{-2}$ ) processes by examining the values of  $k_2(\text{H}_2)/k_2(\text{D}_2)$  and  $k_{-2}(\text{H}_2)/k_{-2}(\text{D}_2)$ . These KIE values are tabulated in Table 2, in which we also list the CVT/SCSAG results.<sup>26</sup>

From Table 2, we can know that the dissociation rates ( $k_2$ ) of physisorbed H<sub>2</sub> and D<sub>2</sub> are in the same orders of magnitude, and the rates of H<sub>2</sub> are larger than that of D<sub>2</sub>. The recombination rates ( $k_{-2}$ ) of H<sub>2</sub> and D<sub>2</sub> have the same trend as that of the dissociation ones. These differences between the rates of H<sub>2</sub> and D<sub>2</sub> can be explained by the differences in free energy profiles. Fig. 3 displays the dissociation and recombination free energy profiles for physisorbed H<sub>2</sub> and D<sub>2</sub> at 300 K. Explicitly, both the dissociation and recombination free energy

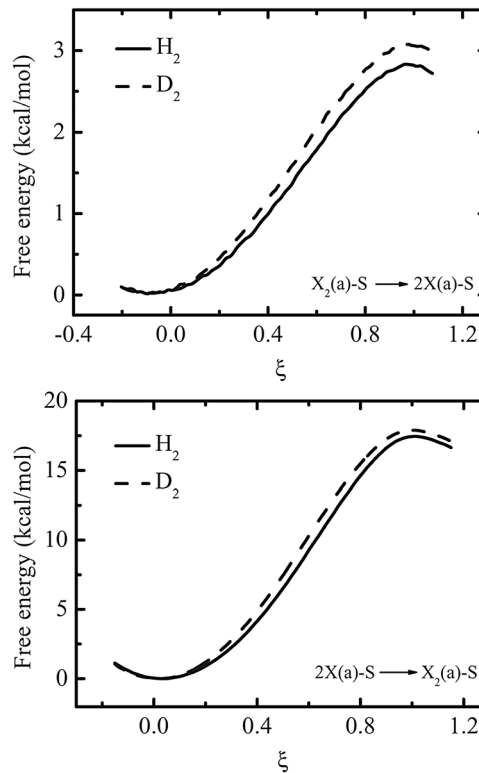


Fig. 3 Free energy profiles of dissociation and recombination for two isotopic reactions at 300 K. The solid and dashed lines are the results corresponding to H<sub>2</sub> and D<sub>2</sub>, respectively.

barriers of physisorbed H<sub>2</sub> are lower than that of D<sub>2</sub> by 0.26 and  $0.44 \text{ kcal mol}^{-1}$  at 300 K.

Table 2 shows that the KIEs are remarkable for both processes. The values of  $k_2(\text{H}_2)/k_2(\text{D}_2)$  are 4.02, 3.18, 2.18, 2.10 and 1.85, and the values of  $k_{-2}(\text{H}_2)/k_{-2}(\text{D}_2)$  are 12.1, 6.20, 2.27, 1.93 and 1.55 at 160, 200, 300, 500 and 800 K, respectively. Both KIEs increase with decreasing temperature, this behavior is due to the fact that the tunneling effect enhances the rates of H<sub>2</sub> much larger than that of D<sub>2</sub> at low temperatures.

Comparing the values of  $QI(\text{H}_2/\text{D}_2)$  with the results of CVT/SCSAG, we can see that they are in good agreement with each other at high temperatures, however, the QI ones are smaller than that of CVT/SCSAG at low temperatures. These discrepancies at low temperatures can be explained by the fact that the QI and CVT/SCSAG have used different ways to incorporate the tunneling effect and the tunneling effect affects the KIE a lot.

### 5.4 The total rates of H<sub>2</sub> on the Ni(100)

In the above sections, we have calculated the dissociation rates ( $k_2$ ) of physisorbed H<sub>2</sub> on the rigid Ni(100) surface, considering that the  $k_1$  and  $k_{-1}$  should not be affected by the tunneling effect, we use the CVT data<sup>26</sup> of  $k_1$  and  $k_{-1}$  directly in our following calculations. With use of these rates, we can obtain the steady state rate  $k_{\text{steady}}$  (as seen in eqn (3)) under steady state approximation. We can also calculate the direct rate ( $k_{\text{direct}}$ ) using a direct reaction mechanism by ignoring the physisorbed state. The total rate ( $k$ ) is a combination of the  $k_{\text{steady}}$  and  $k_{\text{direct}}$  as shown in eqn (10).

Table 2 The kinetic isotope effects (KIEs) of physisorbed H<sub>2</sub>

T (K)	KIE( $k_2$ )		KIE( $k_{-2}$ )
	QI(H <sub>2</sub> /D <sub>2</sub> )	CVT/SCSAG(H <sub>2</sub> /D <sub>2</sub> ) <sup>a</sup>	QI(H <sub>2</sub> /D <sub>2</sub> )
160	4.02	6.37	12.1
180	3.33	5.09	9.25
200	3.18	4.32	6.20
250	2.50	3.26	3.31
300	2.18	2.79	2.27
400	2.11	2.34	2.09
500	2.10	2.12	1.93
600	2.12	1.97	1.66
800	1.85	1.79	1.55

<sup>a</sup> From ref. 26.

Table 3 Rate constants<sup>a</sup> for H<sub>2</sub> on the Ni(100) surface

<i>T</i> (K)	<i>k</i> <sub>1</sub> <sup>b</sup>	<i>k</i> <sub>-1</sub> <sup>b</sup>	<i>k</i> <sub>2</sub> <sup>c</sup>	<i>k</i> <sub>steady</sub>	<i>k</i> <sub>direct</sub>	<i>f</i> <sub>MB</sub>	<i>k</i>
160	4.43(-11)	8.20(11)	1.97(10)	1.04(-12)	4.50(-12)	3.72(-6)	1.04(-12)
180	2.28(-11)	1.03(12)	2.88(10)	6.19(-13)	4.31(-12)	1.84(-5)	6.19(-13)
200	2.56(-11)	1.42(12)	4.91(10)	8.55(-13)	3.26(-12)	6.55(-5)	8.55(-13)
250	1.91(-11)	2.61(12)	9.83(10)	6.93(-13)	2.32(-12)	6.62(-4)	6.94(-13)
300	7.72(-12)	3.70(12)	1.65(11)	3.29(-13)	1.90(-12)	3.00(-3)	3.34(-13)
400	4.17(-12)	4.88(12)	3.49(11)	2.78(-13)	1.68(-12)	1.90(-2)	3.05(-13)
500	3.09(-12)	5.82(12)	5.49(11)	2.66(-13)	1.73(-12)	5.61(-2)	3.49(-13)
600	2.68(-12)	6.56(12)	7.67(11)	2.81(-13)	1.87(-12)	1.13(-1)	4.60(-13)
800	1.49(-12)	4.51(12)	1.15(12)	3.03(-13)	2.38(-12)	2.64(-1)	8.51(-13)

<sup>a</sup> Unit: cm<sup>3</sup> site<sup>-1</sup> s<sup>-1</sup> for *k*<sub>1</sub>, *k*<sub>steady</sub>, *k*<sub>direct</sub> and *k*; s<sup>-1</sup> for *k*<sub>-1</sub> and *k*<sub>2</sub>; powers of 10 are in parentheses. <sup>b</sup> *k*<sub>1</sub> and *k*<sub>-1</sub> are the CVT results from ref. 26. <sup>c</sup> *k*<sub>2</sub> is the QI rates.

Table 3 lists the *k*<sub>1</sub>, *k*<sub>-1</sub>, *k*<sub>2</sub>, *k*<sub>steady</sub>, *k*<sub>direct</sub>, *f*<sub>MB</sub> and *k* for H<sub>2</sub> at temperatures ranging from 160–800 K, the corresponding Arrhenius plots are displayed in Fig. 4. For the H<sub>2</sub> dissociation, both the *k*<sub>steady</sub> and *k*<sub>direct</sub> have nonmonotonic behaviors with larger values at both low and high temperatures. The increase of *k*<sub>direct</sub> at high temperatures is due to the fact that it is an activated process, while the increase of *k*<sub>direct</sub> at low temperatures is most probably caused by the quantum effect, as seen in Fig. 5, the effective free energy barriers from the gas phase to the transition state become very low at low temperatures. The direct process seems like a non-activated process at very low temperatures, under this situation, the steering effect may play an important role.

We can also see that the *k*<sub>direct</sub> is always much larger than *k*<sub>steady</sub>, and the ratio of *k*<sub>direct</sub> and *k*<sub>steady</sub> is 5.78 at 300 K, this discrepancy is caused by the fact that the zero point energy corrected potential barrier for the direct process is only 0.6 kcal mol<sup>-1</sup>, which is much lower than the effective potential barrier (2.0 kcal mol<sup>-1</sup>) of physisorbed H<sub>2</sub>.

In order to obtain the total rate *k*, we calculate the fraction (*f*<sub>MB</sub>) of gas phase hydrogen molecules with translational energy equal to or in excess of a certain energy (4.75(kcal mol<sup>-1</sup>) - *k*<sub>B</sub>*T*) with the Maxwell–Boltzmann distribution. In Table 3, we can see that the values of *f*<sub>MB</sub> are very small at low temperatures, which means only the steady state process is allowed for the H<sub>2</sub>, and the *f*<sub>MB</sub> increases with increasing temperature, which demonstrates that more and more hydrogen molecules are undergo a direct process of dissociation. However, even at

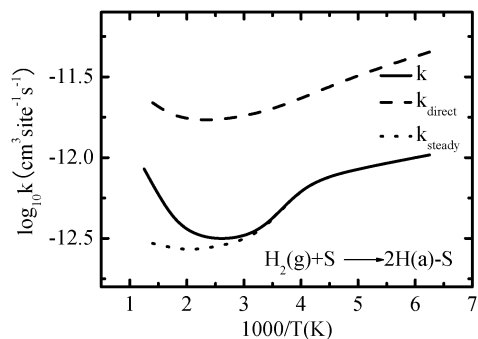


Fig. 4 Arrhenius plots of rate constants for H<sub>2</sub> on Ni(100). The solid, dashed and dotted lines are the total rate *k*, *k*<sub>direct</sub> and *k*<sub>steady</sub>, respectively.

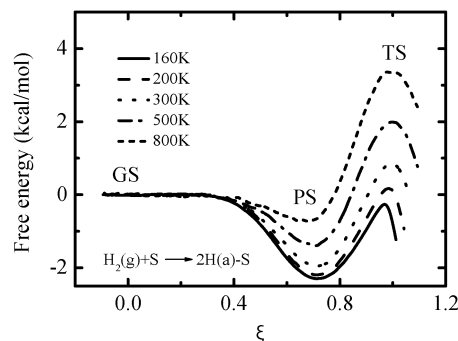


Fig. 5 Free energy profiles from the gas phase state to the transition state for H<sub>2</sub> on Ni(100). The solid, dashed, dotted, dash-dotted and short dashed lines are the results corresponding to 160, 200, 300, 500 and 800 K, respectively.

800 K, only 26% of gas phase H<sub>2</sub> are able to dissociate directly. With use of the *f*<sub>MB</sub>, the total rate *k* is calculated. In Fig. 4, we can see the total rate *k* has the form of an inverted bell, and its minimum value appears at about 400 K, at which the contributions of *k*<sub>steady</sub> and *k*<sub>direct</sub> to *k* are 89% and 11%, respectively. At low temperatures, *k* is equal to *k*<sub>steady</sub> because the *f*<sub>MB</sub> is very close to zero, while it becomes close to *k*<sub>direct</sub> at high temperatures with the increase of *f*<sub>MB</sub>, for instance, the contributions of *k*<sub>steady</sub> and *k*<sub>direct</sub> to *k* are 26% and 74% at 800 K. We have plotted the free energy profiles from the gas phase to the transition state at different temperatures in Fig. 5. It is clear that the free energy well at the physisorbed state is very deep at low temperatures, however, it becomes shallow with the increase of temperature, which demonstrates that the hydrogen molecules become less favorable to reside at the physisorption site at high temperatures. Finally, we can conclude that the dissociation of H<sub>2</sub> mainly proceeds through a physisorbed state below 400 K, but both the direct and physisorbed dynamics are important above 400 K.

Compared with others, Chakravarty and Metiu<sup>59</sup> used the classical correlation function theory and reported that H<sub>2</sub> is first physisorbed and then dissociated at 100 K, which is in good agreement with our conclusion at low temperatures, however, they treated the dynamics as a direct dissociation and ignored physisorption at 300 K, which conflicts with ours that the physisorbed dynamics still play an important role even above 300 K.



Table 4 Rate constants<sup>a</sup> for D<sub>2</sub> on the Ni(100) surface

T (K)	$k_1^b$	$k_{-1}^b$	$k_2^c$	$k_{\text{steady}}$	$k_{\text{direct}}$	$f_{\text{MB}}$	$k$
160	1.63(-11)	3.22(11)	4.90(9)	2.44(-13)	1.09(-12)	1.64(-5)	2.44(-13)
180	1.10(-11)	5.16(11)	8.64(9)	1.81(-13)	8.23(-13)	6.81(-5)	1.81(-13)
200	8.07(-12)	7.59(11)	1.54(10)	1.61(-13)	7.06(-13)	2.22(-4)	1.61(-13)
250	4.79(-12)	1.54(12)	3.94(10)	1.19(-13)	6.71(-13)	1.69(-3)	1.20(-13)
300	3.50(-12)	2.47(12)	7.57(10)	1.04(-13)	6.05(-13)	6.44(-3)	1.07(-13)
400	2.61(-12)	4.53(12)	1.65(11)	9.18(-14)	6.11(-13)	3.32(-2)	1.09(-13)
500	1.22(-12)	3.34(12)	2.61(11)	8.85(-14)	6.80(-13)	8.69(-2)	1.40(-13)
600	8.99(-13)	3.08(12)	3.63(11)	9.47(-14)	8.76(-13)	1.62(-1)	2.21(-13)
800	6.61(-13)	2.63(12)	6.23(11)	1.27(-13)	1.18(-12)	3.39(-1)	4.83(-13)

<sup>a</sup> Unit: cm<sup>3</sup> site<sup>-1</sup> s<sup>-1</sup> for  $k_1$ ,  $k_{\text{steady}}$ ,  $k_{\text{direct}}$  and  $k$ ; s<sup>-1</sup> for  $k_{-1}$  and  $k_2$ ; powers of 10 are in parentheses. <sup>b</sup>  $k_1$  and  $k_{-1}$  are the CVT results from ref. 26. <sup>c</sup>  $k_2$  is the QI rates.

### 5.5 The total rates of D<sub>2</sub> on the Ni(100)

We also calculate the  $k_{\text{steady}}$  and  $k_{\text{direct}}$  for D<sub>2</sub> on the Ni(100) surface, as well as the fraction ( $f_{\text{MB}}$ ) of D<sub>2</sub> with translational energy equal to or in excess of a certain energy ( $4.26(\text{kcal mol}^{-1}) - k_{\text{B}}T$ ). All the values are tabulated in Table 4, and the corresponding Arrhenius plots are displayed in Fig. 6.

In Table 4, we can see the  $k_{\text{direct}}$  is always much larger than  $k_{\text{steady}}$  over the whole tested temperature range 160–800 K, for instance, the ratio of  $k_{\text{direct}}$  and  $k_{\text{steady}}$  is 5.82 at 300 K, this phenomenon is because the effective potential barriers for the direct dynamics and physisorbed D<sub>2</sub> are 0.36 and 2.26 kcal mol<sup>-1</sup>, respectively. Fig. 6 shows that both  $k_{\text{direct}}$  and  $k_{\text{steady}}$  decrease firstly, then they reach minimum values at about 300 and 500 K, respectively, finally they increase with increasing temperature. The total rate  $k$  is calculated with  $k_{\text{direct}}$  and  $k_{\text{steady}}$  according to eqn (10), it is noted that the total rate  $k$  is exactly the same as  $k_{\text{steady}}$  at low temperatures, while it gets close to  $k_{\text{direct}}$  with the increase of temperature. The total rate  $k$  of D<sub>2</sub> also has the form of an inverted bell, with a minimum at about 300 K, below which the steady state dynamics dominates the dissociation process.

### 5.6 Kinetic isotope effects for the total rate

The kinetic isotope effects of H<sub>2</sub>/D<sub>2</sub> for  $k_{\text{steady}}$ ,  $k_{\text{direct}}$  and  $k$  are calculated and tabulated in Table 5, in this table we also include the CVT/SCSAG results<sup>26</sup> under steady state approximation. We can see that all the KIEs are much larger than 1, this situation reveals that the rates of H<sub>2</sub> are always larger than that of D<sub>2</sub>, which qualitatively agrees with the experimental data<sup>5</sup> that the

Table 5 The kinetic isotope effects for the total reaction

T (K)	KIEs(H <sub>2</sub> /D <sub>2</sub> )			
	$k_{\text{steady}}$	$k_{\text{steady}}(\text{CVT/SCSAG})^a$	$k_{\text{direct}}$	$k$
160	4.25	6.6	4.13	4.25
200	5.32	6.6	4.63	5.32
250	5.81	4.5	3.46	5.77
300	3.17	4.0	3.14	3.11
400	3.03	3.3	2.76	2.79
500	3.01	3.0	2.55	2.49
600	2.96	2.8	2.14	2.08
800	2.39	2.3	2.02	1.76

<sup>a</sup> From ref. 26.

D<sub>2</sub> has a larger dissociation barrier. It is interesting to notice that the KIEs of  $k_{\text{steady}}$ ,  $k_{\text{direct}}$  and  $k$  are all nonmonotonic with respect to temperature, they first increase to maxima, then decrease, and the maximum values are 5.81, 4.63 and 5.77 at 250, 200 and 250 K, respectively. The values in Table 5 also show that the KIEs of  $k_{\text{steady}}$  are larger than that of  $k_{\text{direct}}$  over the whole tested temperature range. The KIEs of  $k$  are close to that of  $k_{\text{steady}}$  at low temperatures, while they become smaller than both  $k_{\text{steady}}$  and  $k_{\text{direct}}$  at high temperatures.

Comparing our QI KIEs of  $k_{\text{steady}}$  to that of CVT/SCSAG, we find that the QI is in good agreement with CVT/SCSAG at high temperatures, however, it is smaller at low temperatures. Truong *et al.* have attributed the large KIEs at low temperatures to the tunneling effect, so the difference between the QI and CVT/SCSAG is mainly caused by the fact that these two methods have used different ways to incorporate the tunneling effect.

## 6 Concluding remarks

We have calculated the dissociation and recombination rates of physisorbed H<sub>2</sub>, the total dissociation rate of gas phase H<sub>2</sub>, and the corresponding kinetic isotope effects on the rigid Ni(100) surface, using the quantum instanton approximation with the modified version of Lee and DePristo's potential energy surface.

For the dissociation and recombination reactions of physisorbed H<sub>2</sub>, we have calculated the quantum and classical rates by considering H atoms as quantum and classical particles, respectively. Both the dissociation and recombination rates show that the quantum rates are obviously larger than the classical

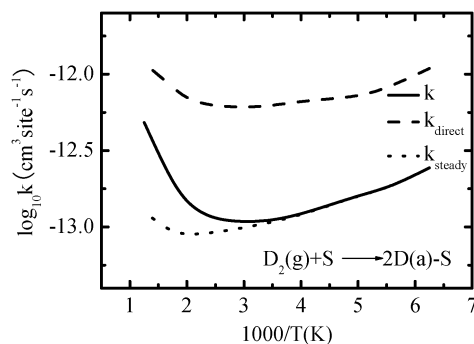


Fig. 6 Arrhenius plots of rate constants for D<sub>2</sub> on Ni(100). The solid, dashed and dotted lines are the total rate  $k$ ,  $k_{\text{direct}}$  and  $k_{\text{steady}}$ , respectively.

ones, especially at low temperatures. This behavior has been explained by the quantum effect that lowers the free energy barriers and enhances the rates. Our fitted activation energy (17.48 kcal mol<sup>-1</sup>) for the recombination process is much lower than the experimental one (22.7 kcal mol<sup>-1</sup>), this shows that the recombination barrier of the present PES should be refined. The obtained kinetic isotope effects of H<sub>2</sub>/D<sub>2</sub> are consistent with that of CVT/SCSAG, and the increase of the KIEs with decreasing temperature demonstrates that the tunneling effect is remarkable.

We have also calculated the steady state, direct and total dissociation rates of gas phase H<sub>2</sub> on the Ni(100) surface, and found that the steady state and direct dissociation rates have minimum values at about 400–500 K, while the total one has the form of inverted bell with respect to temperature. We also demonstrate that, under the conditions that the physisorbed state exists, the gas phase H<sub>2</sub> has two ways to dissociate, one is the steady state process that physisorbed H<sub>2</sub> is formed, equilibrated and dissociated, the other is the direct process that H<sub>2</sub> passes over the physisorbed state directly. Detailed analysis shows that the dissociation of gas phase H<sub>2</sub> is dominated by the steady state process below 400 K, however, both the steady state and direct processes are important above 400 K. The calculated KIEs for the rates reveal that H<sub>2</sub> always has larger rates than D<sub>2</sub> no matter which dissociation process they undergo, and the direct process has a smaller KIE than the steady state process.

## Appendix: the analytical result of C<sub>dd</sub>(0)/Q<sub>r</sub>

We now consider how to obtain C<sub>dd</sub>(0)/Q<sub>r</sub> analytically for the direct process (eqn (4)). For the rigid surface, the Hamiltonian of the system can be written as

$$\hat{H} = \frac{\hat{P}_R^2}{2m_c} + \frac{\hat{P}_r^2}{2\mu_r} + V(\hat{R}, \hat{r}), \quad (\text{A.1})$$

where the scattering vector  $R$  connects the Ni(100) surface and the center of mass of H<sub>2</sub>, the interatomic vector  $r$  connects two hydrogen atoms,  $P_R$  and  $P_r$  are the momenta conjugate to  $R$  and  $r$ , respectively, while the  $m_c$  and  $\mu_r$  are the mass of the center of mass and the reduced mass of H<sub>2</sub>, respectively.

In the gas phase state, H<sub>2</sub> is far away from the Ni(100) surface, and the interaction potential energy between H<sub>2</sub> and the surface is negligible, so the Hamiltonian can be further written as

$$\hat{H} \approx \frac{\hat{P}_R^2}{2m_c} + \frac{\hat{P}_r^2}{2\mu_r} + V(\hat{r}) = \hat{H}_R + \hat{H}_r. \quad (\text{A.2})$$

The term C<sub>dd</sub>(0)/Q<sub>r</sub> has the form that

$$C_{dd}(0)/Q_r = \frac{\text{tr} \left[ e^{-\beta\hat{H}/2} \Delta(s_a) e^{-\beta\hat{H}/2} \Delta(s_b) \right]}{\text{tr} \left[ e^{-\beta\hat{H}/2} \delta(s_a^0) e^{-\beta\hat{H}/2} \delta(s_b^0) \right]} \times \frac{\text{tr} \left[ e^{-\beta\hat{H}/2} \delta(s_a^0) e^{-\beta\hat{H}/2} \delta(s_b^0) \right]}{Q_r}, \quad (\text{A.3})$$

where the first term on the right has been discussed in Section 3.2, and the second term can be evaluated analytically. Following the previous studies,<sup>66,81</sup> and considering that  $s_a^0$  and  $s_b^0$  are only related to the scattering vector  $R$  in the gas phase state, we can get

$$\begin{aligned} & \text{tr} \left[ e^{-\beta\hat{H}/2} \delta(s_a^0) e^{-\beta\hat{H}/2} \delta(s_b^0) \right] \\ &= \text{tr} \left[ e^{-\beta\hat{H}_r} \right] \times \text{tr} \left[ e^{-\beta\hat{H}_R/2} \delta(s_a^0) e^{-\beta\hat{H}_R/2} \delta(s_b^0) \right] \\ &= \text{tr} \left[ e^{-\beta\hat{H}_r} \right] \times 2 \left( \frac{m_c}{2\pi\hbar^2\beta} \right)^2, \end{aligned} \quad (\text{A.4})$$

and

$$\begin{aligned} Q_r &= \text{tr} \left[ e^{-\beta\hat{H}_r} \right] \times \text{tr} \left[ e^{-\beta\hat{H}_R} \right] \\ &= \text{tr} \left[ e^{-\beta\hat{H}_r} \right] \times \left( \frac{m_c}{2\pi\hbar^2\beta} \right)^{3/2}. \end{aligned} \quad (\text{A.5})$$

Finally, we obtain

$$\frac{\text{tr} \left[ e^{-\beta\hat{H}/2} \delta(s_a^0) e^{-\beta\hat{H}/2} \delta(s_b^0) \right]}{Q_r} = \left( \frac{2m_c}{\pi\hbar^2\beta} \right)^{1/2}. \quad (\text{A.6})$$

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