# Application of Residue Number Systems to Bent-Pipe Satellite Communication Systems

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***Abstract*—To reduce the impact on the performance of satellites caused by space radiation environments, redundant structures are introduced into hardware design. Meanwhile, it brings in a new trouble, that is, a great deal of hardware resource is consumed. To overcome the problem, a lot of methods are proposed, one of which is using the residue number system (RNS). The RNS is proved to be suitable to all linear systems in this paper. Since the main modules of Bent-pipe (BP) satellites such as beam forming, finite impulse response (FIR) filters can be approximated to linear systems, RNS becomes an effective way to reduce the area overhead of hardware in BP mode satellite communication systems.**

***Keywords-RNS; linear systems; satellite communication; bent-pipe***

## I. INTRODUCTION

Most of the satellite communication systems use *BP transponders* which is called *transparent transponders* as well. They amplify the uplink signals and transform the uplink frequency to the downlink frequency. Due to the simple structures and reliable performances, BP transponders are widely used in satellite communication systems especially under the circumstance that payloads and electric power on board are limited.

In the traditional BP mode in satellite, it usually uses analog transponder to process the received signals, but with the digitization trends, analog transponders have been replaced by digital transponders, digital signal processing has been the principal duties in bent-pipe systems. So bent-pipe referred in this paper is not the traditional ones but the bent-pipe with the digitized constrained. It includes beam forming for generating the expecting beam according to DOA (direction-of-arrival), FIR filters for signal filtering, FFT to deal with time-frequency transforming of sampling data and so on.

Due to the space radiation environment, SEUs (Single Event Upsets) are common effects on hardware circuits, various kinds of redundant structures are used to overcome SEUs, which increase the area and power overhead significantly, such as TMR (Triple Modular Redundancy) [1] and DMRC (Dual-Modular Redundancy with Comparison) [2]. This two structures lead to triple and double of the original logic, respectively, thus, area overhead is increased greatly, especially under the large computation.



However, the resources on board are extraordinary limited. Moreover, with the miniaturization trend of modern satellites, reducing the circuit scale becomes an essential work.

For the purpose of saving costs caused by redundancy, a lot of methods have been proposed in recent years. One of them is using RNS (Residue Number System). RNS transform the operation of large numbers into that of remainders such that huge computation is divided into several small ones, which simplifies the calculation greatly. Since RNS cannot be utilized to division, sign detection or magnitude comparison [3], it is not as universal as other methods for simplifying hardware circuit. But in special cases, it has unique advantages. For instance, RNS have been applied to Finite Impulse Response (FIR) filters successfully [4] [5].

In this paper, RNS is proved feasible for linear systems. Furthermore, the main functional modules in bent-pipe mode satellites can be regarded as linear systems. Thus, RNS can be applied to bent-pipe mode satellite communication systems in order to solve the problem that area overhead of hardware systems caused by redundant structures for overcoming SEUs is quite considerable.

This paper is organized as follows. In Section II, the RNS is briefly introduced. Section III gives the properties of linear systems, and shows the relationship between RNS and linear systems. Classic linear systems in bent-pipe mode satellite communications are analyzed in Section IV. Section V concludes this paper.

## II. PRELIMINARY

Before the introductions of RNS, two important conceptions are presented. They are CRT (Chinese Remainder Theorem) and Residue Class Ring, their contents will be described in the following subsections.

### *A. Chinese Remainder Theorem*

CRT and its mathematical demonstration can be found in Samkhya easily, so only a brief review is given here to establish the notation.

Consider the following congruence group

$$x \equiv 2(mod\ m_1),\ x \equiv 3(mod\ m_2)\ \dots\ x \equiv 2(mod\ m_k) \quad (1)$$

Let $\{m_1, m_2, \dots, m_k\}$ are a set of relatively prime integers, called moduli set. If $M=m_1m_2...m_k=m_iM_i$, then the group congruence (1) has a unique solution:

$$x \equiv N_1M_1b_1+N_2M_2b_2+\dots+N_kM_kb_k(mod\ M) \quad (2)$$

Where $N_iM_i \equiv 1(mod\ m_i)$ $(i=1,\dots,k)$, $[0, M-1]$ is the dynamic range of $x$. Therefore, $x$ can be calculated as long as the congruence group in the shape of Eq. (1) is defined. When choosing moduli set, a noteworthy factor is the dynamic range $M$. The precondition of Eq. (2) is that $x$ should be in the interval $[0, M-1]$. Once the moduli set we choose is not large enough such that $x > M-1$, then the solution of Eq. (2) is no longer the data we expected, but the expected data module M. So choosing moduli set is a key step for CRT.

### *B. Residue Class Ring*

The residue class ring is a kind of finite rings. Addition and multiplication defined on them are given by Eq. (3) and Eq. (4), respectively. Residue class ring module m is a set described as $\{0, 1, 2, \dots, m-1\}$.

$$(a_1 \pm a_2)=((a_1)_m \pm (a_2)_m)_m \quad (3)$$

$$(a_1\ a_2)_m=((a_1)_m*(a_2)_m)_m \quad (4)$$

where $a_1$, $a_2$, $m$ are integers, $b \neq 0$.

It is obvious that results of the operation defined on residue class ring must be in the interval $[0, m-1]$. Different modulus will lead to a group of different results, which can be used to calculate the actual result via Eq. (2) in CRT.

### *C. Characteristics of RNS*

#### *1) Simplify Computation*

In residue number systems, a group of prime number, $m=\{m_1, m_2, \dots m_p\}$ where $m_i < m$, is chosen as the moduli set. By moduling $m_i$, large operands will turn to smaller ones. Take a 10-bit operand $A$ as an example. Assume $m_1=2^{5-1}=31$ and $B=A$ mod $m_1$, then $B$ is a 5-bit operand at most, such that 5 bits are saved at least. Similarly, additions or multiplications of two 10-bit operands can be reduced to those of two 5-bit operands. Further more, it can be found from Eq. (3) and Eq. (4) that only remainders need to be taken care of while carries don't. Thus, RNS divides huge computations into several small ones, or the complex multi-bit computing become simpler.

#### *2) Division*

Division is not arithmetic on the residue class ring because of the existence of element 0. So we can not define division between residue class rings. In other words, RNS is not allowed to be applied to a system in which division is concluded.

#### *3) Overflow detection*

In RNS, overflow is common phenomena due to data accumulation time after time. Unfortunately, overflow detection in RNS is difficult because of the non-weighted of RNS and it requires a determination of the magnitude of the result. However, in a redundant RNS (RRNS), overflow can be detected by a simple mixed radix conversion. The details of RRNS are in [6].

## III. LINEAR SYSTEMS AND RNS

Linear systems are widely used in control systems, signal processing and so on for they are easy to handle. In many cases, systems are regard as linear systems ideally.

Linear system is a mathematical model which is composed of linear mapping who keeps addition and multiplication between two linear spaces. Linear system is also called linear operator or linear transformation. In linear algebra theorem, linear mapping is defined by the follow definition.

Definition: Let $M$ and $N$ be the vector spaces or linear spaces on the field $P$, then the function $f: M->N$ is called the linear mapping if and only if the elements $m_1$ and $m_2$ in $M$ and the elements $p$ in $P$ satisfy the following equations:

$$f(m_1+ m_2)=f(m_1)+f(m_2) \quad (5)$$

$$f(pm_1)=pf(m_1) \quad (6)$$

The vector space or linear space mention above can be simply described as a set who is closed for the addition and multiplication defined on it. In other words, sum or product of any two elements is still the element of this set.

Obviously, the operators in linear systems are addition and multiplication; namely, linear systems are composed of add operations and multiply operations. Notice that, none of the actual physical system is the absolute linear system in the strict sense. But most of them can be regarded as linear systems via approximating and simplifying. In most cases, we regard a system as a linear one if and only if it satisfies both superposition theorem and homogeneity property which are described as follows.

Superposition Theorem: For a multi-input system, the response of system output is the sum of responses caused by each input.

Homogeneity property: For a multi-input system, when all of the inputs are multiplied or divided by $K$, then the output is multiplied or divided by $K$ as well, where $K$ is a constant.

The mathematical abstraction of these two theorems is described in Eq. (7), Eq. (8) and Eq. (9).

$$L(m_1+m_2)= L(m_1) + L(m_2). \quad (7)$$

$$L(Km_1)=KL(m_1). \quad (8)$$

$$L(K_1m_1+K_2m_2)=K_1L(m_1)+K_2L(m_2). \quad (9)$$

In addition, all of the linear systems are composed of addition and multiplication while not all systems composed of additions and multiplications are linear systems. For example, consider the following system

$$y=a_1x_1+a_2x_2+b \quad (10)$$

Where $x_1$, $x_2$ are inputs, $y$ is output, $a_1$, $a_2$ and $b$ are constants. Although only addition and multiplication are defined in this system, it isn't a linear one because it doesn't satisfy to superposition theorem and homogeneity property.

As mentioned in Section II.B, addition and multiplication are suitable for RNS. So RNS is allowed to be utilized by any linear system. It can be concluded that, computations in linear systems can be simplified by RNS in any case.

## IV. BP Mode Satellite Communication Systems

### A. Overview

Transponders are the most important modules in communication satellites. There are two types of transponders. One is BP transponder while the other is OBP (on-board-processing) transponder. We call the communication systems bent-pipe mode satellite communication system if they use BP transponders.

The primary work of BP transponders is transmitting the signals between two earth stations. When receive the uplink signals, BP transponders accomplish the amplifying and frequency conversion of them and then retransmit them as downlink signals. The procedure is transparent for signals, so the BP transponders are also called transparent transponders. Besides of amplifying and frequency conversion, some other technologies such as carrier switching is also transparent for signals, so they can be classified to BP satellite communication systems as well.

### B. Main modules in BP Mode Satellite Communication

In digital signal processing, there are a lot of modules can be regarded as linear systems. Three of which are introduced here. As mentioned before, RNS is suitable to linear systems. So these systems are good candidates for applying RNS.

#### 1) Beam Forming

The signals in mobile communication systems are usually badly ruined due to multipath channel. At the same time, the existence of reflection, diffraction, dispersion interference and Doppler shift are all the important effect factors. Smart antenna is one of the core technologies in 3G system. It adopts increasing the antenna gain, generating null steering in antenna pattern for interference, and diversity techniques to increase signal-to-noise ratio (SNR). A radiation pattern is showed in Fig. 1, it's obvious that main beam is formed on 130 degree and 225 degree.

Beam forming is the core technology of smart antenna, the principle of beam forming is generating the weighted vectors of antenna arrays, steer the phases of antenna according to the direction-of-arrival (DOA), forming main beam in the direction of useful signals while null steering in that of interferences.

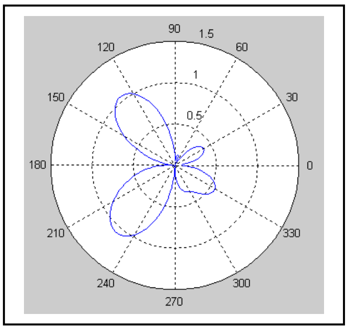


Figure 1. Radiation pattern of beam forming

Assume the receiving signal of the $m$th element is

$$x_m(t)=\sum_{i=1}^{L}s_i(t)e^{-j(i-1)k_i}+n_m(t) \quad (11)$$

Where $s_i(t)$ is the $i$th signal that the $m$th element received, $n(t)$ is the noise of the $m$th element [7].

According to DOA, the generated weights can be described as an $M$ dimension vector

$$W= [w_1\ w_2\ ...\ w_M]^T \quad (12)$$

Then the output of antenna arrays is

$$y(t)=\sum_{i=1}^{M}\omega_i^* x_i(t)=W^H X(t) \quad (13)$$

Beam forming is a system with multi-input, the output $y(t)$ is formed by weighting each $x_i$. It is obvious beam forming is a linear system according to the definition in section III. Thus, it is reasonable to apply RNS to beam forming systems, Eq. (14).

$$(y(t))_{m_i}=(\sum_{i=1}^{M}(\omega_i^*)_{m_i}(x_i(t))_{m_i})_{m_i}=(W^H)_{m_i}(X(t))_{m_i} \quad (14)$$

Weights $w_i$ in the above equation are complex numbers, they are in form of $a+jb$, modular arithmetic for complex numbers is that for real part and imaginary part respectively. That is:

$$(a+jb)_{m_i}=(a)_{m_i}+j(b)_{m_i} \quad (15)$$

A particular case need to be mentioned is modular arithmetic is used for integers in common case, so it needs to be transformed to integers if the operands are decimals. Output will be the same to the original one as long as inverse transformations act on these operands after modular arithmetic.

*2) Finite Impulse Response (FIR)Fliters*

As an elementary unit in digital signal processing systems, FIR filters have been widely used in satellite communication systems. They keeps linear frequency-phase characteristic at an arbitrary frequency-amplitude characteristic, moreover, no pole existing in the system function ensures the stability of FIR.

Fir filter is composed of adders, multipliers and delay units, structure of an N-tap FIR filter is showed in Fig. 2. An N-tap FIR filter is expressed by Eq. (16) in time-domain while Eq. (17) in frequent-domain.

$$y(n)=\sum_{i=0}^{N-1}h(i)x(n-1) \quad (16)$$

$$Y(K)=X(K)H(K) \quad (17)$$

Where *{x(n)}* is infinite sampling data, *{h(n)}* is the set of coefficients, *X(K)* and *H(K)* are the Fourier transform of *x(n)* and *h(n)*. The same to beam forming system, FIR filter is also a linear system composed of additions and multiplications. Applying RNS to FIR filter has been proposed for decades of years. In residue number system, FIR filter is expressed by Eq. (18).

$$(y(n))_{m_i}=(\sum_{i=0}^{N-1}(h(i))_{m_i}(x(n-i))_{m_i})_{m_i} \quad (18)$$

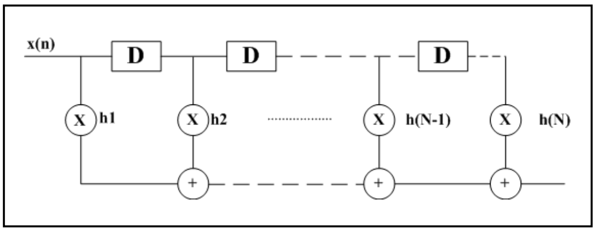


Figure.2 Structure of FIR filter

*3) Fast Fourier Tranform (FFT)*

FFT is the fast algorithm of DFT, used as the transforming of signals between time and frequency domain. FFT is another frequently-used technique in satellite communication systems. A lot of works are based on FFT, including spectral estimation, convolution and interference detection. Generally speaking, FFT runs through the whole procedure of satellite communication.

The main idea of FFT is dividing the N-point sequence *x(n)* in to $2^i$ groups (*i* is a nonnegative integer). DFT is applied to each group separately, then an N-point DFT turn to be $2^i$ $N/2^i$-point DFTs. When *i*=0, FFT reduced to DFT.

Assume *x(n)* is a finite sequence of 8 points, DFT of *x(n)* is showed in Eq. (19). In Fig. 5, we describe an 8-point algorithm graph of FFT with time divide two without discussing details about algorithm. According to Fig. 3, the FFT with time divided two can be described as Eq. (20), $x_1(r)$ and $x_2(r)$ are the odd sequence and even sequence of *x(n)* according to the parity of *n*.

$$X(k)=\sum_{n=0}^{7}x(n)W_8^{nk} \quad (19)$$

$$X(k)=\sum_{r=0}^{3}x_1(r)(W_8^2)^{rk}+W_8^k\sum_{r=0}^{3}x_2(r)(W_8^2)^{rk} \quad (20)$$

Undoubtedly, the same as beam forming and FIR filter, FFT is also a linear system. It can be expressed by Eq. (21) in RNS.

$$(X(k))_{m_i}=(\sum_{r=0}^{3}(x_1(r))_{m_i}(W_8^2)^{rk}+W_8^k\sum_{r=0}^{3}(x_2(r)_{m_i}(W_8^2)^{rk})_{m_i} \quad (21)$$

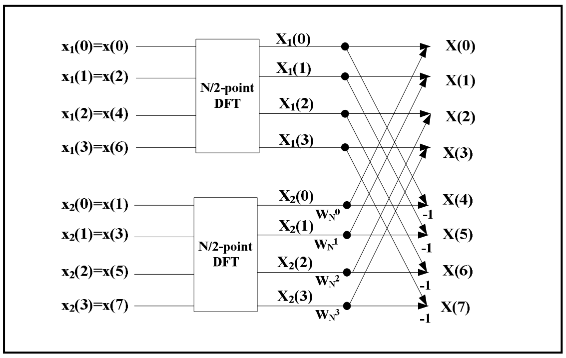


Figure.3 Algorithm graph FFT with time divided two

Thus, we can come to a conclusion that most prime modules in bent-pipe mode satellite communication systems should be satisfy the precondition of utilizing RNS. So applying RNS to bent-pipe mode satellite communication systems can reduce the size of hardware on a large scale. Take FIR filter as an example. When applying RNS to an 8-tap FIR filter which is protected by TMR, overhead of the LUTs is 2.025 times of the original logic, compared to the area of traditional TMR which triples the area of original filter, LUTs are saved approx. 32.5%. We choose {7, 11, 13} as the moduli set here.

Compared to the FIR filter, beam forming and FFT have the similar structures, but we can not give the precise data of the saved costs of area, because it is decided by the dimension of operands and the moduli set we choose. Even though for FIR filters, different sampling data, coefficients and moduli sets will lead to different results. Moreover, when we need to transform decimals to integers or converser, the overhead will be increased to some extent. But what can be confirmed is that, the area saved in all of the three systems will be roughly the same.

## V. CONCLUSIONS

In this paper, it is proved that RNS can be applied to linear systems. Since the prime modules in BP satellite communication systems such as beam forming, FIR filter and FFT can be regarded as linear systems, we proposed a standpoint that RNS can be applied to TMR protected BP satellite communication systems for reducing the area overhead of hardware. It is easy to find out that RNS reduces the area overhead of hardware significantly according to the data.

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