Sturm-Liouville 边值问题的多解结果①②

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摘　要　利用锥映射的不动点指数证明了含参数 $\lambda>0$ 的 Sturm-Liouville 边值问题至少存在两个正解.

主题词　Sturm-Liouville 边值问题; 正解; 多解; 锥; 不动点指数
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Sturm-Liouville 边值问题

$$[p(t)u']' + \lambda f(t,u) = 0,\quad 0<t<1 \tag{1}$$

$$au(0) - b\lim_{t\to 0} p(t)u'(t) = 0,\quad cu(1) + d\lim_{t\to 1} p(t)u'(t) = 0 \tag{2}$$

来源于边界层理论与应用数学领域之中[1, 2], 此处参数 $\lambda>0$, 实常数 $a, b, c, d\geqslant 0$, 并满足 $ad+bc+ac>0$, $f(t,u)$是$[0,1]\times[0,\infty)$上的非负连续函数.

关于边值问题(1), (2)的研究成果, 有变号解的存在性[3,4] 和正解的存在性[1, 2, 5~7]. Erbe 等研究了当 $\lambda=1$ 且 $p(t)=1$ 的一个特殊情形, 给出了多解存在性结果[8]. 文中则对 $\lambda>0$ 且 $p(t)\neq 1$ 的情形, 证明了问题(1), (2)至少存在两个正解. 假设

(H_1)　$p(t)$是$[0,1]$上的非负可测函数, 当 $t\in(0,1)$时, $p(t)>0$ 并满足 $\int_0^1 \frac{dt}{p(t)}<+\infty$;

(H_2)　$\lim\limits_{u\to 0}\max\limits_{t\in[0,1]}\frac{f(t,u)}{u}=0$,　$\lim\limits_{u\to+\infty}\max\limits_{t\in[0,1]}\frac{f(t,u)}{u}=0$;

(H_2)′　$\lim\limits_{u\to 0}\min\limits_{t\in[0,1]}\frac{f(t,u)}{u}=+\infty$,　$\lim\limits_{u\to+\infty}\min\limits_{t\in[0,1]}\frac{f(t,u)}{u}=+\infty$;

(H_3)　存在常数 $l_1>0$ 和 $M_1>0$, 使当 $u\geqslant l_1$ 时, 有 $f(t,u)\geqslant M_1$;

(H_3)′　存在常数 $l_2>0$ 和 $M_2>0$, 使当 $0\leqslant u\leqslant l_2$ 时, 有 $f(t,u)\leqslant M_2$.

称函数 $u(t)$是边值问题(1), (2)的一个正解, 如果它满足: (i) $u(t)\in C[0,1]\cap C^1(0,1)$并在$(0,1)$内 $u(t)>0$; (ii) $p(t)u'(t)$在$[0,1]$上是绝对连续函数; (iii) $u(t)$于$(0,1)$内几乎处处满足方程(1)并满足条件(2).

由边值问题(1), (2)可导出等价的积分方程为

$$u(t) = \lambda\int_0^1 G(t,s)f(s,u(s))ds$$

式中
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$$G(t,s)=\begin{cases}\frac{1}{\rho}\left[b+a\int_0^s\frac{d\tau}{p(\tau)}\right]\left[d+c\int_t^1\frac{d\tau}{p(\tau)}\right], & \text{ye}0\leqslant s\leqslant t\leqslant 1\\ \frac{1}{\rho}\left[b+a\int_0^t\frac{d\tau}{p(\tau)}\right]\left[d+c\int_s^1\frac{d\tau}{p(\tau)}\right], & 0\leqslant t\leqslant s\leqslant 1\end{cases} \tag{3}$$

$$\rho = ad+bc+ac\int_0^1\frac{d\tau}{p(\tau)}>0 \tag{4}$$

引理　(1) 对任何的 $s, t\in[0,1]$ 都有 $G(t,s)\leqslant G(s,s)$;

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(2) 设 $\delta\in(0,\frac{1}{4})$, 则存在 $q>0$, 使对 $t\in[\delta,1-\delta]$ 和 $s\in[0,1]$, 有 $G(t,s)\geqslant qG(s,s)$;

(3) 对上述的 δ 和 q, 当 $s\in[\delta,1-\delta]$ 和 $t\in[\delta,1-\delta]$ 时, 有

ur　　17　$$G(t,s)\geqslant\frac{1}{\rho}\left[b+a\int_{\delta}^{1-\delta}\frac{d\tau}{p(\tau)}\right]\left[d+c\int_{\delta}^{1-\delta}\frac{d\tau}{p(\tau)}\right]$$

证明　由 $G(s,s)u=\frac{1}{\rho}\left[b+a\int_0^s\frac{d\tau}{p(\tau)}\right]\left[d+c\int_s^1\frac{d\tau}{p(\tau)}\right]$, 知

$$\frac{G(t,s)}{G(s,s)}=\begin{cases}\left[d+c\int_t^1\frac{d\tau}{p(\tau)}\right]/\left[d+c\int_s^1\frac{d\tau}{p(\tau)}\right], & 0\leqslant s\leqslant t\leqslant 1\\ \left[b+a\int_0^t\frac{d\tau}{p(\tau)}\right]/\left[b+a\int_0^s\frac{d\tau}{p(\tau)}\right], & 0\leqslant t\leqslant s\leqslant 1\end{cases}$$　了当

因此, (1)式成立. 令

$$q=\text{mina}\left\{\left[d+c\int_{1-\delta}^1\frac{d\tau}{p(\tau)}\right]/\left[d+c\int_0^1\frac{d\tau}{p(\tau)}\right],\quad\left[b+a\int_0^{\delta}\frac{d\tau}{p(\tau)}\right]/\left[b+a\int_0^1\frac{d\tau}{p(\tau)}\right]\right\}$$

则知(2)式成立. (3)式显然.

定义映射 $T_\lambda:K\to K$, $(T_\lambda u)(t)=\lambda\int_0^1G(t,s)f(s,u(s))ds$　　(5)

此处　$K:=\{u\in C[0,1];\ u(t)\geqslant 0,\ \int_{\delta}^{1-\delta}u(t)dt\geqslant q(1-2\delta)\|u\|,0<\delta<\frac{1}{4}\}$　　(6)

是 $C[0,1]$ 中的锥. 由引理的(1), (2)知道

$$(T_\lambda u)(t)=\lambda\int_0^1G(t,s)f(s,u(s))ds\leqslant\lambda q\int_0^1G(s,s)f(s,u(s))ds,$$

$$\int_{\delta}^{1-\delta}(T_\lambda u)(t)dt=\int_{\delta}^{1-\delta}\left[\lambda\int_0^1G(t,s)f(s,u(s))ds\right]dt\geqslant\lambda q(1-2\delta)\int_0^1G(s,s)f(s,u(s))ds,$$

再由以上两个不等式, 有

$$\int_{\delta}^{1-\delta}(T_\lambda u)(t)dt\geqslant q(1-2\delta)\|T_\lambda u\|,\qquad\delta\in(0,\frac{1}{4})$$

因此, 有 $T_\lambda(K)\subset K$. 另外知 T_λ 是全连续的.

定理1　假设条件(H_1), (H_2), (H_3)成立, 则存在常数 $l>0$ 和 $\lambda_1>0$, 使当 $\lambda>\lambda_1$ 时, 边值问题(1), (2)都至少存在两个正解 $u_1(t)$ 和 $u_2(t)$, 并满足 $0<\|u_1\|<l<\|u_2\|$.

证明　取 $0<\delta<1/4$, $l=(l_1+1)/q$, 令 $\Omega_l=\{u\in K;\ \|u\|\leqslant l\}$, 再令 $E_\delta=\{t\in[\delta,1-\delta];\ u(t)\geqslant l_1\}$, 则对 $u\in\partial\Omega_l$, 有

$$\int_{\delta}^{1-\delta}u(t)dt\geqslant q(1-2\delta)\|u\|=ql(1-2\delta)$$

$$\int_{\delta}^{1-\delta}u(t)dt=\int_{E_\delta}u(t)dt+\int_{[\delta,1-\delta]\setminus E_\delta}u(t)dt\leqslant l\,\text{mes}E_\delta+l_1(1-2\delta-\text{mes}E_\delta)$$

由以上两个不等式推出　$\text{mes}E_\delta=[(ql-l_1)(1-2\delta)]/(l-l_1)>1/(2l)>0$

再由(H_3), 有　$\int_{E_\delta}f(s,u(s))ds\geqslant M_1\text{mes}E_\delta>M_1/(2l)$　　(7)

记 $\gamma=\frac{1}{\rho}(1-2\delta)\left[b+a\int_0^{\delta}\frac{d\tau}{p(\tau)}\right]m\left[d+c\int_{1-\delta}^1\frac{d\tau}{p(\tau)}\right]$, 利用引理的(3)和(7)式, 对 $u\in\partial\Omega_l$, 有

$$\|T_\lambda u\|=\int_0^1\left[\max_{t\in[0,1]}\lambda\int_0^1G(t,s)f(s,u(s))ds\right]dt\geqslant\lambda\int_{\delta}^{1-\delta}dt\int_{\delta}^{1-\delta}G(t,s)f(s,u(s))ds\geqslant$$

$$\lambda\gamma\int_{\Omega_\delta}f(s,u(s))ds\geqslant\lambda\gamma M_1/(2l^2)\|u\|$$

因此, 对任何 $\lambda>\lambda_1=2l^2/(\gamma M_1)$, 都有 $\|T_\lambda u\|>\|u\|$. 由文献[8]知, $i(T_\lambda,\Omega_l,K)=0$.　　(8)

记 $M=\int_0^1 G(s,s)\mathrm{d}s$，由假设($H_2$)，存在 $0<r<l$，使当 $0\leqslant u\leqslant r$ 时，有 $f(t,u)<\varepsilon u$，此处 $\varepsilon>0$，满足 $\varepsilon\lambda M<1$. 令 $\Omega_r=\{u\in K;\ \|u\|\leqslant r\}$，则对 $u\in\partial\Omega_r$，有 $(T_\lambda u)(t)\leqslant\varepsilon\lambda\int_0^1 G(s,s)u(s)\mathrm{d}s<\|u\|$，即有 $\|T_\lambda u\|<\|u\|$. 再由文献[8]，有 $i(T_\lambda,\Omega_r,K)=1$. (9)

根据(H_2)，对上述 ε，存在 $R_1>0$，使 $f(t,u)\leqslant R_1+\varepsilon u$，对一切 $t\in[0,1]$ 和 $u\geqslant 0$ 成立. 取 $R>\max\{l,\lambda R_1M/(1-\varepsilon\lambda M)\}$，并令 $\Omega_R=\{u\in K;\ \|u\|<R\}$，则对 $u\in\partial\Omega_R$，有

$$(T_\lambda u)(t)\leqslant\lambda\int_0^1 G(t,s)(R_1+\varepsilon u(s))\mathrm{d}s\leqslant\lambda(R_1+\varepsilon R)M<\|u\|$$

即有 $\|T_\lambda u\|<\|u\|$. 由文献[8]，有 $i(T_\lambda,\Omega_R,K)=1$. (10)

根据(8)，(9)，(10)式，得 $i(T_\lambda,\Omega_R\setminus\mathring{\Omega}_l,K)=1$，$i(T_\lambda,\Omega_l\setminus\mathring{\Omega}_r,K)=1$，因此 T_λ 在 $\Omega_l\setminus\mathring{\Omega}_r$ 和 $\Omega_R\setminus\mathring{\Omega}_l$ 中各有一个不动点 $u_1(t)$ 和 $u_2(t)$，并有 $0<\|u_1\|<l<\|u_2\|$.

定理2　假设条件(H_1)，(H_2)$'$，(H_3)$'$成立，则存在常数 $\lambda_2>0$ 和 $l_2>0$，使对任何 $0<\lambda<\lambda_2$，边值问题(1)，(2)都至少存在两个正解 $u_1(t)$ 和 $u_2(t)$，并且有 $0<\|u_1\|<l_2<\|u_2\|$.

证明　令 $\Omega_{l_2}=\{u\in K;\ \|u\|\leqslant l_2\}$. 对 $u\in\partial\Omega_{l_2}$，由(H_3)$'$，有

$$(T_\lambda u)(t)=\lambda\int_0^1 G(t,s)f(s,u(s))\mathrm{d}s\leqslant\lambda MM_2$$

故对任何 $0<\lambda<\lambda_2=l_2/(MM_2)$，都有 $\|T_\lambda u\|<\|u\|$，T_λ 见式(5). 由文献[8]知

$$i(T_\lambda,\Omega_{l_2},K)=1. \tag{11}$$

由(H_2)$'$，存在 $0<r<l_2$ 和 $R_2>l_2$，使当 $0\leqslant u\leqslant r$ 和 $u\geqslant R_2$ 时都有 $f(t,u)\geqslant Nu$，此处 $N>0$ 使 $\lambda Nq\gamma(1-2\delta)>1$，$0<\delta<(1/4)$. 选择 $R>R_2$，则对 $u\in\partial\Omega_r$ 和 $u\in\partial\Omega_R$，都有

$$\|T_\lambda u\|=\int_0^1[\max_{t\in[0,1]}\lambda\int_0^1 G(t,s)f(s,u(s))\mathrm{d}s]\,\mathrm{d}t\geqslant\lambda\int_\delta^{1-\delta}\mathrm{d}t\int_\delta^{1-\delta}G(t,s)f(s,u(s))\mathrm{d}s\geqslant$$

$$\lambda\gamma N\int_\delta^{1-\delta}u(s)\mathrm{d}s\geqslant\lambda\gamma qN(1-2\delta)\|u\|>\|u\|$$

即有 $\|T_\lambda u\|>\|u\|$. 于是 $i(T_\lambda,\Omega_r,K)=0$，　$i(T_\lambda,\Omega_R,K)=0$ (12)

从(11)，(12)式知，$i(T_\lambda,\Omega_{l_2}\setminus\mathring{\Omega}_r,K)=1$，$i(T_\lambda,\Omega_R\setminus\mathring{\Omega}_{l_2},K)=-1$. 因此，$T_\lambda$ 在 $\Omega_{l_2}\setminus\mathring{\Omega}_r$ 和 $\Omega_R\setminus\mathring{\Omega}_{l_2}$ 中各有一个不动点 $u_1(t)$ 和 $u_2(t)$，满足 $0<\|u_1\|<l_2<\|u_2\|$.

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Measurement of Temperature Coefficient of Thermistor

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Abstract In order to keep the measuring system in isolated and balanced state, isothermal field radiation method was applied to measure the temperature factor of thermistor. The characteristic and classification of thermistor and the design idea of the measuring instrument were demonstrated. This instrument guarantees the precision of temperature measuring and decreases the system error caused by the non-thermal balance. In order to ensure the synchronization of the resistance and temperature of thermistor, two electric-bridge time synchronizous automatic recording instruments were used. Finally, the measuring results were analyzed and discussed.

Subject terms thermistor, temperature factor, isothermal field radiation method, thermal balance, least square method

Multiple Positive Solutions to Sturm-Liouville Boundary Value Problem

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Abstract It is proved that Sturm-Liouville boundary value problem with $\lambda > 0$ has at least two positive solutions by using the fixed point index for the mapping in cones.

Subject terms Sturm-Liouville boundary value problem, positive solution, multiple solution, cone, fixed point index

Sedimentary Features of Dujiatai Reservoir of Shu 22 Blocks

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Abstract To gain a clear idea of the sedimentary environment and its sedimentary feature of a sand body is the basis and key to understanding the heterogeneity, evaluating the exploitation effect and making further adjustment. Dujiatai reservoir of Shu 22 blocks is a set of fan delta sediment which developed on the base of complex paleostructure and paleotopography. It is necessary to dissect the Du $\text{II}_8 \sim \text{II}_{11}$ and Du $\text{III}_3 \sim \text{III}_5$ layer of the research area and make further study about its sedimentary microfacies by applying electrical assemblage, combining the development extent of the sand body with the condition of its plane and space distribution. The two major reservoirs of the research area are both the frontal subfacies of fan delta and the subfacies of frontal fan delta. And the frontal subfacies of fan delta can also be subdivided into distributary microfacies, interdistributary microfacies channel mouth bar microfacies and frontal thin bedded sand microfacies. Each microfacies developed from different small layers, and each microfacies has its own sedimentary features.

Subject terms Shu 22 blocks, sedimentary feature, sedimentary microfacies, frontal subfacies of fan delta, subfacies of frontal fan delta

Micro Heterogeneous Characteristics of Dujiatai Reservoir of Shu 22 Blocks

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Abstract According to the data of core analysis as the major information, we have studied the micro heterogeneous of Dujiatai reservoir of Shu 22 blocks through the analysis of mercury-injection, SEM, thin section of cast mold, oil bearing thin section and pore image. The clay mineral content of the reservoir is high and its main components are mixed-layer of illite and montmorillonite. Intergranular pore and vugular solution pore are the major pore type, and the large or middle pore with moderate permeability and fine throat is the important type of the pore structure.

Subject terms Shu 22 blocks, micro heterogeneity, clay mineral, type of pore, pore structure

Macro Heterogeneous Characteristics of Dujiatai Reservoir of Shu 22 Blocks

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