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Modified Scheme of PID Controllers for Robot Manipulators with an Uncertain Jacobian Matrix

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Abstract—**Performance and robustness of the traditional PID controllers are both limited since only three parameters are available to tune in controller design. A new modified scheme of PID controller is proposed in this paper for Cartesian regulation of robot manipulators with uncertain Jacobian matrix, in which the integral action is adapted to error amplitude. The proposed controller improved performance and robustness of the resulting system significantly and it is demonstrated by simulation comparison with the traditional one.**

I. Introduction

Due to simplicity, easy tuning and effectiveness for the majority of the industrial plants, PID controllers are widely used despite the remarkable theoretical progress and breakthroughs in control techniques. It is well known that PID controllers can effectively deal with nonlinearity and uncertainties of dynamics [1~3], and asymptotic stability is achieve accordingly [4~7]. Various schemes and their modification appear in literature [8][9]. For example, PID controller consisted of saturated-P and saturated-D plus a gravity compensation [4][5], and PID-like controller *viz.* linear PD plus an integral action of a nonlinear function of position errors [6] are presented recently. In the presence of Jacobian uncertainty, Cheah et al. derived nonlinear PD plus adaptive or perfect gravity compensation [10][11], an approximate Jacobian matrix PD (PID) control law [12] and PID controllers [13] for task space set-point problem of robot manipulator; they also solves H_∞ tuning problem for PID control of task space [14]. Huang et al. [15] presented a class of transpose Jacobian-based NPID regulators, which includes a nonlinear function of errors in proportional and integral action. The same authors [16] also developed a new nonlinear PID controller; it shows improvement on robustness and transient performance of the resulting system. In consideration of the limit actuator torques, a standard saturated PID compensation is studied in [17], which yields semi-global asymptotic stability if the torque bounds are larger than gravitational torque. It should be noted that a systematic method to select gains of a discrete PID controller is presented in [18] for nonlinear plant in a second-order controller canonical form including robot dynamic. It is to be helpful to tune PID gains applicable to nonlinear plants with incaccurate models.

Most of work concerning PID control is focusing on tuning, self-tuning, auto-tuning and robustness analysis as well [19~23], they can not be directly applied to nonlinear system viz. robot manipulator. And besides, it is not enough to get tighter performance although many modified schemes and tuning techniques are available to design [18]. Furthermore, the limitation of performance, which is imposed by the simple structure of PID controller, has received less attention [20]. This paper is to explore new scheme of PID controller for Cartesian task of robot manipulators with Jacobian matrix uncertainty.

II. Preliminaries

Throughout this paper, $\lambda_M\{\cdot\}$ denotes the maximal eigenvalue of matrix. The norm of vector x is defined by $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}$ and that of matrix $\mathbf{M}$ is defined by the corresponding induced norm $\|\mathbf{M}\| = \sqrt{\lambda_M\{\mathbf{M}^{\mathrm{T}}\mathbf{M}\}}$.

In the absence of friction and other disturbances, the dynamics of a rigid serial n-link robot manipulator is given in joint space as follows,

$$\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is joint angle vector, $\mathbf{M}(\boldsymbol{q})$ is the $n \times n$ inertial matrix; $\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$, $\boldsymbol{g}(\boldsymbol{q})$ and $\boldsymbol{\tau} \in \mathbb{R}^n$ denote the centrifugal Coriolis force, the gravitational force and control inputs, respectively.

In the presence of kinematics uncertainties, we get the following relationship,

$$\boldsymbol{x} = \psi(\boldsymbol{q}),\ \boldsymbol{x}_d = \psi(\boldsymbol{q}_e),\ \boldsymbol{x}_d = \hat{\psi}(\boldsymbol{q}_d) \tag{2}$$

where the function $\psi(\boldsymbol{q})$ ($\hat{\psi}(\boldsymbol{q})$) describes the exact (estimated) kinematic relationship between joint space and Cartesian space; $\boldsymbol{q}_e$ and $\boldsymbol{x}_d$ denote the equilibrium configuration and the desired position in Cartesian space, respectively; $\boldsymbol{q}_d$ is the estimated (desired) configuration that is calculated via the estimated kinematics viz. $\hat{\psi}(\cdot)$.

Therefore, we can define error signals below,

$$\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}_e,\ \tilde{\boldsymbol{x}} = \boldsymbol{x} - \boldsymbol{x}_d\,. \tag{3}$$

and the analytical Jabobian matrix:

$$\dot{\boldsymbol{x}} = \partial\psi(\boldsymbol{q})/\partial\boldsymbol{q}\cdot\dot{\boldsymbol{q}} = \mathbf{J}(\boldsymbol{q})\cdot\dot{\boldsymbol{q}} \tag{4}$$

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$$\dot{\boldsymbol{x}} = \partial\psi(\boldsymbol{q})/\partial\boldsymbol{q}\cdot\dot{\boldsymbol{q}} = \hat{\mathbf{J}}(\boldsymbol{q})\cdot\dot{\boldsymbol{q}} \tag{5}$$

The following Jacobian denotes the analytical Jacobian if no specifications.

Assumption

It is assumed that robot manipulator is operating in the finite workspace $\boldsymbol{\Omega}$, where the Jacobian matrix $\mathbf{J}(\boldsymbol{q})$ and $\hat{\mathbf{J}}(\boldsymbol{q})$ are non-singular. Furthermore, there exists an only unknown equilibrium configuration $\boldsymbol{q}_e$ that corresponds to the desired posture $\boldsymbol{x}_d$ of end- effector in task-space under kinematic uncertainties.

When Assumption is available, the dynamics can be expressed in task space with respect to measurable position $\boldsymbol{x}$ and the estimated Jacobian matrix $\hat{\mathbf{J}}(\boldsymbol{q})$,

$$\hat{\mathbf{M}}_x(\boldsymbol{q})\ddot{\boldsymbol{x}} + \hat{\mathbf{C}}_x(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{x}} + \hat{\boldsymbol{g}}_x(\boldsymbol{q}) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})\boldsymbol{\tau} \tag{6}$$

where

$$\begin{cases} \hat{\mathbf{M}}_x(\boldsymbol{q}) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})\mathbf{M}(\boldsymbol{q})\hat{\mathbf{J}}^{-1}(\boldsymbol{q}) \\ \hat{\mathbf{C}}_x(\boldsymbol{q},\dot{\boldsymbol{q}}) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})[\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) - \mathbf{M}(\boldsymbol{q})\hat{\mathbf{J}}^{-1}(\boldsymbol{q})\dot{\hat{\mathbf{J}}}(\boldsymbol{q})]\hat{\mathbf{J}}^{-1}(\boldsymbol{q}) \\ \hat{\boldsymbol{g}}_x(\boldsymbol{q}) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})\boldsymbol{g}(\boldsymbol{q}) \end{cases} \tag{7}$$

Remark-1 It is worthy to note that $\hat{\boldsymbol{g}}_x(\boldsymbol{q})$ can be rewritten as the following form:

$$\hat{\boldsymbol{g}}_x^*(\boldsymbol{x}) =: \hat{\boldsymbol{g}}_x(\boldsymbol{q}) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})\boldsymbol{g}(\boldsymbol{q}) \tag{8}$$

It is said that the gravity term can also be described as a unique function of end-effector position $\boldsymbol{x}$. ■

A list of properties of the robot dynamic model (1) is recalled as follow [8][16],

P1 It verifies that

$$\boldsymbol{y}^T[\dot{\mathbf{M}}(\boldsymbol{q}) - 2\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})]\boldsymbol{y} = 0,\ \exists \boldsymbol{y}\in\mathbb{R}^n. \tag{9}$$

P2 For some bounded positive constant λ_g, the gravitational force vector $\hat{\boldsymbol{g}}_x^*(\boldsymbol{x})$ satisfy the following inequality,

$$\left\|\frac{\partial\hat{\boldsymbol{g}}_x^*(\boldsymbol{x})}{\partial\boldsymbol{x}}\right\| \le \lambda_g \tag{10}$$

P3 There exist positive constants k_g and $k_{\hat{J}}$ such that

$$\left\|\hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})[\boldsymbol{g}(\boldsymbol{q}) - \boldsymbol{g}(\boldsymbol{q}_e)]\right\| \le k_g\|\tilde{\boldsymbol{x}}\| \tag{11}$$

$$\left\|\hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})\cdot[\hat{\mathbf{J}}(\boldsymbol{q}_e) - \hat{\mathbf{J}}(\boldsymbol{q})]^{\mathrm{T}}\boldsymbol{z}\right\| \le k_{\hat{J}}\|\tilde{\boldsymbol{x}}\| \tag{12}$$

for some constant vector $\boldsymbol{z}\in\mathbb{R}^n$.

According to (11)~(12), we have the following result:

Proposition 1

For some constant vector $\bar{\boldsymbol{w}}\in\mathbb{R}^n$ and function vector $\Upsilon_x(\tilde{\boldsymbol{x}})\in\mathbb{R}^n$,

$$\Upsilon_x(\tilde{\boldsymbol{x}},\boldsymbol{q}) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q})\Upsilon(\tilde{\boldsymbol{q}}) \tag{13}$$

$$\Upsilon(\tilde{\boldsymbol{q}},\boldsymbol{q}) = \boldsymbol{g}(\boldsymbol{q}_e) - \boldsymbol{g}(\boldsymbol{q}) + [\hat{\mathbf{J}}(\boldsymbol{q}_e) - \hat{\mathbf{J}}(\boldsymbol{q})]^{\mathrm{T}}\cdot[\bar{\boldsymbol{w}} + \mathbf{K}_p(\boldsymbol{x}_e - \boldsymbol{x}_d)] \tag{14}$$

there exist constant μ such that

$$\left\|\frac{\partial\Upsilon_x(\tilde{\boldsymbol{x}})}{\partial\tilde{\boldsymbol{x}}}\right\| \le \mu \tag{15}$$

Remark-2 In (10) and (15), the boundedness estimate of $\partial\hat{\boldsymbol{g}}_x^*(\boldsymbol{x})/\partial\boldsymbol{x}$ and $\partial\Upsilon_x(\tilde{\boldsymbol{x}})/\partial\tilde{\boldsymbol{x}}$ may be much conservative. In fact, these two boundedness estimate both dependent upon the robot configuration $\boldsymbol{q}$. ■

III. DESIGN OF MODEFIED PID CONTROLLER

3.1 The proposed PID control law

The new PID-type control laws are proposed as

$$\boldsymbol{\tau} = \hat{\boldsymbol{g}}(\boldsymbol{q}_d) - \mathbf{K}_v\dot{\boldsymbol{q}} - \hat{\mathbf{J}}^T(\boldsymbol{q})\cdot\mathbf{K}_p\tilde{\boldsymbol{x}} - \hat{\mathbf{J}}^T(\boldsymbol{q})\cdot\mathbf{K}_I\boldsymbol{\tau}_I \tag{16}$$

$$\boldsymbol{\tau}_I = \int_0^t \boldsymbol{\Gamma}(\tilde{\boldsymbol{x}}(\varsigma))\cdot\tilde{\boldsymbol{x}}(\varsigma)\,\mathrm{d}\varsigma \tag{17}$$

where $\mathbf{K}_p$, $\mathbf{K}_v$ and $\mathbf{K}_I$ are all positive definite, and

$$\boldsymbol{\Gamma}(\cdot) = diag\{\boldsymbol{\Gamma}_i(\cdot)\},\ i = 1,\cdots,n \tag{18}$$

Here $\boldsymbol{\Gamma}_i(\cdot)$ is a continuous differentiable decreasing function satisfying

$$\begin{cases} \mathrm{d}\boldsymbol{\Gamma}_i(x)/\mathrm{d}x \le 0 \\ \bar{\varepsilon}_i \ge \boldsymbol{\Gamma}_i(x) \ge \underline{\varepsilon}_i \ge 0 \end{cases},\ i = 1,\cdots,n \tag{19}$$

From (19), it is easy to verify that

$$\begin{cases} \boldsymbol{\Gamma}_i(x) \to \underline{\varepsilon}_i,\ |x| \to +\infty \\ \boldsymbol{\Gamma}_i(x) \to \bar{\varepsilon}_i,\ |x| \to 0 \end{cases},\ i = 1,\cdots,n \tag{20}$$

Remark-3 The difference between this new PID control and other PID-like control is that, in the former, integrator action is error-dependent. When $\boldsymbol{\Gamma}_i(\cdot)$ is chosen as constant, the proposed scheme is identical with the traditional PID controllers. ■

3.2 Stability of PD controller

Suppose that the following PD controller with some constant compensation $\bar{\boldsymbol{\tau}}_I\in\mathbb{R}^n$ is employed in robot manipulator (1),

$$\boldsymbol{\tau} = -\mathbf{K}_v\dot{\boldsymbol{q}} - \hat{\mathbf{J}}^{\mathrm{T}}(\boldsymbol{q})\mathbf{K}_p\tilde{\boldsymbol{x}} - \hat{\mathbf{J}}^{\mathrm{T}}(\boldsymbol{q})\cdot\bar{\boldsymbol{\tau}}_I \tag{21}$$

The closed-loop system dynamics is obtained by substituting the control action $\boldsymbol{\tau}$ into the dynamics of robot manipulator (1),

$$\begin{aligned}&\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\mathbf{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}+\boldsymbol{g}(\boldsymbol{q})\\&=\hat{\boldsymbol{g}}(\boldsymbol{q}_d)-\mathbf{K}_v\dot{\boldsymbol{q}}-\hat{\mathbf{J}}^{\mathrm{T}}(\boldsymbol{q})\mathbf{K}_p\tilde{\boldsymbol{x}}-\hat{\mathbf{J}}^{\mathrm{T}}(\boldsymbol{q})\cdot\bar{\boldsymbol{\tau}}_I\end{aligned}\tag{22}$$

As it can been seen, for any given vector $\bar{\boldsymbol{\tau}}_I\in\mathbb{R}^n$, the equilibrium position $\bar{\boldsymbol{x}}_e$ with the corresponding equilibrium configuration $\bar{\boldsymbol{q}}_e$ are decided by the following equation

$$\hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q}_e)[\hat{\boldsymbol{g}}(\boldsymbol{q}_d)-\boldsymbol{g}(\bar{\boldsymbol{q}}_e)]-\mathbf{K}_p(\bar{\boldsymbol{x}}_e-\boldsymbol{x}_d)-\bar{\boldsymbol{\tau}}_I=0\tag{23}$$

Lemma 1

Suppose that matrix $\mathbf{K}_p=\mathbf{K}_p^{\mathrm{T}}>0$ with

$$\mathbf{K}_p>\lambda_g\,\mathrm{I}\,,\tag{24}$$

then the following statement are available:

i) The equation (23) has a unique equilibrium solution

$$\bar{\boldsymbol{x}}_e=\varphi(\bar{\boldsymbol{\tau}}_I)\ \text{ or }\ \bar{\boldsymbol{q}}_e=\varphi^*(\bar{\boldsymbol{\tau}}_I)\tag{25}$$

for all $\boldsymbol{x}_d,\bar{\boldsymbol{\tau}}_I\in\mathbb{R}^n$, where $\varphi(\cdot)$ and $\varphi^*(\cdot)$ are both globally defined C^1-functions such that

$$\begin{aligned}&\hat{\mathbf{J}}^{-\mathrm{T}}(\varphi^*(\bar{\boldsymbol{\tau}}_I))\cdot[\hat{\boldsymbol{g}}(\boldsymbol{q}_d)-\boldsymbol{g}(\varphi^*(\bar{\boldsymbol{\tau}}_I))]\\&\quad-\mathbf{K}_p(\varphi(\bar{\boldsymbol{\tau}}_I)-\boldsymbol{x}_d)-\bar{\boldsymbol{\tau}}_I=0\end{aligned}\tag{26}$$

ii) The steady-state map $\varphi(\cdot)$ and $\varphi^*(\cdot)$ are invertible.

iii) $\partial\varphi(\bar{\boldsymbol{\tau}}_I)/\partial\bar{\boldsymbol{\tau}}_I$ is negative-definite for all $\bar{\boldsymbol{\tau}}_I\in\mathbb{R}^n$.

Proof. *i*) As we can see, the equilibrium $\bar{\boldsymbol{x}}_e$ with $\bar{\boldsymbol{q}}_e$ is the solution of the following equilibrium equation,

$$\hat{\boldsymbol{g}}(\boldsymbol{q}_d)-\boldsymbol{g}(\bar{\boldsymbol{q}}_e)-\hat{\mathbf{J}}^{\mathrm{T}}(\bar{\boldsymbol{q}}_e)\mathbf{K}_p(\bar{\boldsymbol{x}}_e-\boldsymbol{x}_d)-\hat{\mathbf{J}}^{\mathrm{T}}(\bar{\boldsymbol{q}}_e)\cdot\bar{\boldsymbol{\tau}}_I=0\tag{27}$$

According to the Contraction Mapping Theorem, it follows (24) that equation (27) has a unique equilibrium $\bar{\boldsymbol{x}}_e$ and $\bar{\boldsymbol{q}}_e$.

Since both $\hat{\mathbf{J}}(\boldsymbol{q})$ and $\boldsymbol{g}(\boldsymbol{q})$ are C^1-functions, we always can find some globally defined C^1-function $\varphi(\cdot)$ and $\varphi^*(\cdot)$ so as to make (26) available.

ii) According to the above fact

$$\hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q}_e)[\hat{\boldsymbol{g}}(\boldsymbol{q}_d)-\boldsymbol{g}(\bar{\boldsymbol{q}}_e)]-\mathbf{K}_p(\bar{\boldsymbol{x}}_e-\boldsymbol{x}_d)-\bar{\boldsymbol{\tau}}_I=0\tag{28}$$

it yields

$$\varphi^{-1}(\bar{\boldsymbol{x}}_e)=\hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q}_e)[\hat{\boldsymbol{g}}(\boldsymbol{q}_d)-\boldsymbol{g}(\bar{\boldsymbol{q}}_e)]-\mathbf{K}_p(\bar{\boldsymbol{x}}_e-\boldsymbol{x}_d)\tag{29}$$

iii) From (23), we have

$$\mathbf{I}-\Big(\frac{\partial\hat{\boldsymbol{g}}_x^*(\bar{\boldsymbol{x}}_e)}{\partial\bar{\boldsymbol{x}}_e}+\mathbf{K}_p\Big)\frac{\partial\varphi(\bar{\boldsymbol{\tau}}_I)}{\partial\bar{\boldsymbol{\tau}}_I}=0\tag{30}$$

For all $\bar{\boldsymbol{x}}_e\in\mathbb{R}^n$, (24) implies that

$$\Big(\frac{\partial\hat{\boldsymbol{g}}_x^*(\bar{\boldsymbol{x}}_e)}{\partial\bar{\boldsymbol{x}}_e}+\mathbf{K}_p\Big)>0\,,\tag{31}$$

or

$$\frac{\partial\varphi(\bar{\boldsymbol{\tau}}_I)}{\partial\bar{\boldsymbol{\tau}}_I}=-\Big(\frac{\partial\hat{\boldsymbol{g}}_x^*(\bar{\boldsymbol{x}}_e)}{\partial\bar{\boldsymbol{x}}_e}+\mathbf{K}_p\Big)^{-1}<0\,.\tag{32}$$

As for robot manipulator with Jacobian matrix uncertainty, some results regarding PD controller (21) are presented below.

Lemma 2

Consider the dynamics of robot manipulator (1) with uncertain Jacobian matrix that can be described as

$$\left\|\mathbf{I}-\hat{\mathbf{J}}(\boldsymbol{q})\mathbf{J}^{-1}(\boldsymbol{q})\right\|\le p\tag{33}$$

where p is a positive constant; for any given $\bar{\boldsymbol{\tau}}_I\in\mathbb{R}^n$, PD control law (21) make the closed loop system (3):

i) globally asymptotically stable about the equilibrium point $[\dot{\boldsymbol{q}}^{\mathrm{T}}\ \ \tilde{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}}=0$;

ii) locally exponentially stable about $[\dot{\boldsymbol{q}}^{\mathrm{T}}\ \ \tilde{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}}=0$

provided that

$$\mathbf{K}_p>[\lambda_g(1+p)+p\lambda_M\{\mathbf{K}_p\}]\cdot\mathbf{I}\tag{34}$$

Proof: *i*) *Global asymptotic stability*

Consider the Lyapunov function candidate

$$V_1=\tfrac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\mathbf{M}(\boldsymbol{q})\dot{\boldsymbol{q}}+V_0(\tilde{\boldsymbol{x}})\tag{35}$$

where

$$V_0(\tilde{\boldsymbol{x}})=\int\big\{\mathbf{K}_p\tilde{\boldsymbol{x}}+\Upsilon_x(\tilde{\boldsymbol{x}})\big\}^{\mathrm{T}}\hat{\mathbf{J}}(\boldsymbol{q})\mathbf{J}^{-1}(\boldsymbol{q})\,\mathrm{d}\,\tilde{\boldsymbol{x}}\tag{36}$$

With the help of (34) and the following manipulation

$$\begin{aligned}V_0(\tilde{\boldsymbol{x}})&=\int\big\{\mathbf{K}_p\tilde{\boldsymbol{x}}+\Upsilon_x(\tilde{\boldsymbol{x}})\big\}^{\mathrm{T}}[\mathbf{I}+\hat{\mathbf{J}}(\boldsymbol{q})\mathbf{J}^{-1}(\boldsymbol{q})-\mathbf{I}]\,\mathrm{d}\,\tilde{\boldsymbol{x}}\\&\ge\int\tilde{\boldsymbol{x}}^{\mathrm{T}}[\mathbf{K}_p-\lambda_g(1+p)\mathbf{I}-p\lambda_M\{\mathbf{K}_p\}\cdot\mathbf{I}]\,\mathrm{d}\,\tilde{\boldsymbol{x}}\end{aligned}\tag{37}$$

it follows the convex function condition that $V_1\ge 0$.

The time derivative along the trajectory (22) is

$$\begin{aligned}\dot{V}_1&=\dot{\boldsymbol{q}}^{\mathrm{T}}\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}+\tfrac{1}{2}\dot{\boldsymbol{q}}^{\mathrm{T}}\dot{\mathbf{M}}(\boldsymbol{q})\dot{\boldsymbol{q}}+[\mathbf{K}_p\tilde{\boldsymbol{x}}+\Upsilon_x(\tilde{\boldsymbol{x}})]^{\mathrm{T}}\hat{\mathbf{J}}(\boldsymbol{q})\dot{\boldsymbol{q}}\\&=-\dot{\boldsymbol{q}}^{\mathrm{T}}\mathbf{K}_v\dot{\boldsymbol{q}}\end{aligned}\tag{38}$$

Thus, $\dot{V}_1(\dot{\boldsymbol{q}},\tilde{\boldsymbol{x}})$ is global negative semi-definite, where

$$\dot{V}_1(\dot{\boldsymbol{q}},\tilde{\boldsymbol{x}})=0\ \text{ if and only if }\ \dot{\boldsymbol{q}}=0\,.\tag{39}$$

Using LaSalle Invariance Principle, it means that the equilibrium $[\dot{\boldsymbol{q}}^{\mathrm{T}}\ \ \tilde{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}}$ is globally asymptotically stable.

ii) *Local exponential stability*

Consider the local linearization of the closed loop system (21),

$$\hat{\mathbf{M}}(\boldsymbol{q}_e)\ddot{\tilde{\boldsymbol{x}}}+\hat{\mathbf{K}}_v(\boldsymbol{q}_e)\dot{\tilde{\boldsymbol{x}}}+\bar{\mathbf{K}}_p\tilde{\boldsymbol{x}}=0\tag{40}$$

where

$$\hat{\mathbf{M}}(\boldsymbol{q}_e) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q}_e)\mathbf{M}(\boldsymbol{q}_e)\hat{\mathbf{J}}^{-1}(\boldsymbol{q}_e)$$
$$\hat{\mathbf{K}}_v(\boldsymbol{q}_e) = \hat{\mathbf{J}}^{-\mathrm{T}}(\boldsymbol{q}_e)\mathbf{K}_v\hat{\mathbf{J}}^{-1}(\boldsymbol{q}_e) \tag{41}$$

Since $\dot{\hat{\boldsymbol{x}}} = \hat{\mathbf{J}}(\boldsymbol{q}_e)\dot{\boldsymbol{q}},\ \dot{\boldsymbol{x}} = \mathbf{J}(\boldsymbol{q}_e)\dot{\boldsymbol{q}}$, the Lyapunov function

$$V(\dot{\hat{\boldsymbol{x}}}, \tilde{\boldsymbol{x}}) = \dot{\hat{\boldsymbol{x}}}^{\mathrm{T}}\mathbf{M}(\boldsymbol{q}_e)\dot{\hat{\boldsymbol{x}}} + \int \tilde{\boldsymbol{x}}^{\mathrm{T}}\overline{\mathbf{K}}_p\hat{\mathbf{J}}(\boldsymbol{q}_e)\mathbf{J}^{-1}(\boldsymbol{q}_e)\,\mathrm{d}\,\tilde{\boldsymbol{x}} \tag{42}$$

is positive-definite when (35) is valid, where

$$\overline{\mathbf{K}}_p = \frac{\partial\, \hat{\boldsymbol{g}}_x^*(\boldsymbol{x}_e)}{\partial \boldsymbol{x}_e} + \mathbf{K}_p \tag{43}$$

It verifies that the above system (40) is asymptotically stable with respect to the equilibrium $[\dot{\hat{\boldsymbol{x}}}^{\mathrm{T}}\ \tilde{\boldsymbol{x}}^{\mathrm{T}}] = 0$. Using the LaSalle Invariance Principle, one concludes that the origin is the global asymptotic attractor for the linearized system (40). This implies that all the eigenvalues of the Jacobian matrix $\mathbf{J}_L$ of the system (40) is within the left-hand side of the complex plane, where

$$\mathbf{J}_L = \begin{bmatrix} 0 & \mathbf{I}_{n\times n} \\ -\hat{\mathbf{M}}^{-1}(\boldsymbol{q}_e)\overline{\mathbf{K}}_p & -\hat{\mathbf{M}}^{-1}(\boldsymbol{q}_e)\hat{\mathbf{K}}_v(\boldsymbol{q}_e) \end{bmatrix} \tag{44}$$

Thus, the system (40) is exponentially stable with respect to the origin. It follows *Lemma 3.* in *Appendix* that the original system (20) is locally exponentially stable with respect to the equilibrium $[\dot{\boldsymbol{q}}^{\mathrm{T}}\ \ \tilde{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}} = 0$. ▲

3.3 Stability of the Proposed PID controlled system

In the presence of Jacobian matrix uncertainty, only the estimated gravitational force as well as the estimated Jacobian matrix is available to compensate. As a result, asymptotic stability can not be guaranteed under PD controller (26). Hence, PID controller like (16-17) should be employed, in which the integrator is used to eliminate such steady-state position error.

Theorem

Consider the dynamics of robot manipulator (1) with uncertain Jacobian matrix that can be described as

where p is a positive constant; PID control law (16-17) make the closed-loop system globally asymptotically stable with respect to equilibrium point $[\dot{\boldsymbol{q}}^{\mathrm{T}}\ \ \tilde{\boldsymbol{x}}^{\mathrm{T}}]^{\mathrm{T}} = 0$ provided that (35) is satisfied.

Proof. The proof is omitted due to limited space.

IV. NUMERICAL EXAMPLE

A two-link manipulator shown in *Fig.1* is considered,

$$\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \tau \tag{46}$$

whose parameters are given as follows,

$$m_1 = 5\,kg,\ m_2 = 5\,kg$$

Actual length $l_1 = 0.8\ (m),\ l_2 = 0.8\ (m)$,

estimation $\hat{l}_1 = 0.9\ (m),\ \hat{l}_2 = 0.7\ (m)$

The maximal torque of joint actuator is

$$\bar{\boldsymbol{\tau}}_i = 50\,(N \bullet m)\ ,\ i{=}1{,}2$$

The initial conditions are

$$x(0) = [0.2364\ 1.5643]^T\,(m),\ \dot{q}(0) = [0\ \ 0]^T\,(rad/s)$$

The desired position is $x_d = [1.2\ 0.9]^T\,(m)$.

The traditional transpose Jacobian-based PID control law is available in literature as following,

$$\boldsymbol{\tau} = \hat{\boldsymbol{g}}(\boldsymbol{q}_d) - \mathbf{K}_v\dot{\boldsymbol{q}} - \hat{\mathbf{J}}^T(\boldsymbol{q})\cdot\mathbf{K}_p\tilde{\boldsymbol{x}} - \hat{\mathbf{J}}^T(\boldsymbol{q})\cdot\mathbf{K}_I\int_0^t \tilde{\boldsymbol{x}}(\varsigma)\cdot\mathrm{d}\varsigma \tag{47}$$

As for the proposed controller (16-17), the following function is chosen as $\boldsymbol{\Gamma}_i(\cdot)$,

$$\boldsymbol{\Gamma}_i(\cdot) = \frac{1}{1+15\tilde{\boldsymbol{x}}_i^2}\ ,\ i{=}1{,}2 \tag{48}$$

Parameters of the traditional controller (47) and the proposed scheme (16~17)(48) are tuned as follows,

$$\mathbf{K}_p = 60\mathbf{I},\ \mathbf{K}_v = \begin{bmatrix} 46.8 & 0 \\ 0 & 31.2 \end{bmatrix},\ \mathbf{K}_I = 10\mathbf{I} \tag{49}$$

Simulation results with the tuning (49) in Fig.2 shows that: 1) compare with the traditional PID (47), the pro- posed scheme (16-17)(48) provides better performance; 2) both the two controllers make the resulting system asymptotically stable at the expense of sluggish response, which is not expected in practice. Therefore, the integral action is strengthened and parameters is tuning as follows

Case-1

$$\mathbf{K}_p = 50\ \mathbf{I},\ \mathbf{K}_v = \begin{bmatrix} 46.8 & 0 \\ 0 & 31.2 \end{bmatrix},\ \mathbf{K}_I = 20\mathbf{I};$$

Case-2

$$\mathbf{K}_p = 60\mathbf{I},\ \mathbf{K}_v = \begin{bmatrix} 46.8 & 0 \\ 0 & 31.2 \end{bmatrix},\ \mathbf{K}_I = \begin{bmatrix} 40 & 0 \\ 0 & 46 \end{bmatrix};$$

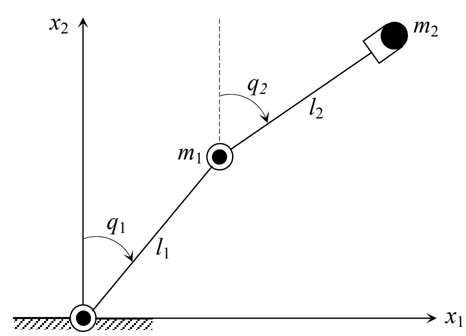
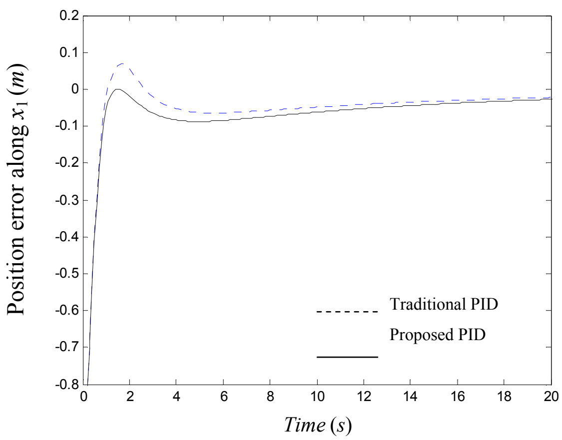
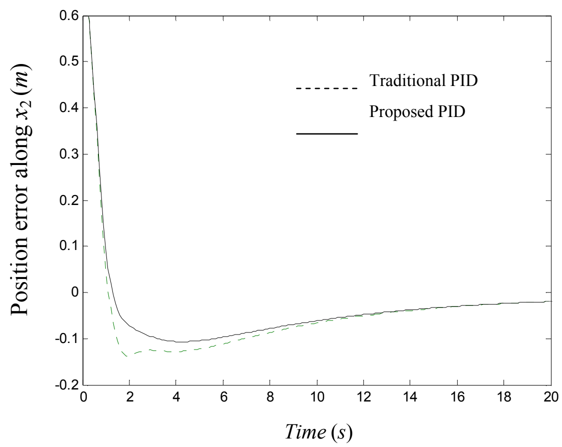


Fig.1 The two-link robot manipulator

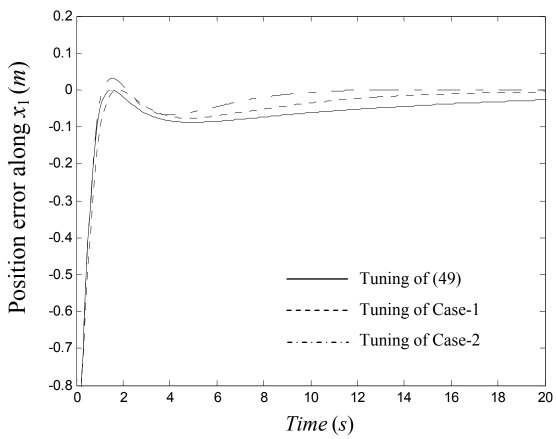


(a) Position error $\tilde{\boldsymbol{x}}_1$

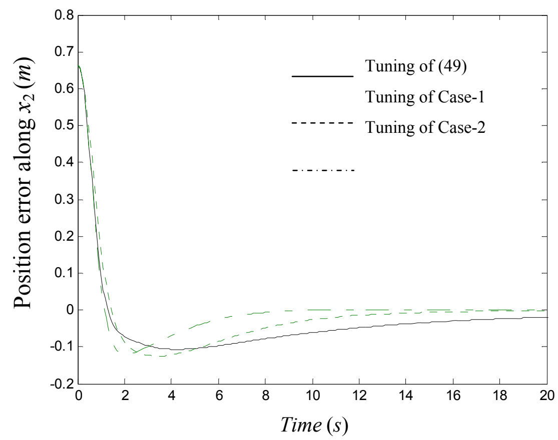


(b) Position error $\tilde{\boldsymbol{x}}_2$

Fig.2 Comparison of simulation result for PID parameter tuning (49)

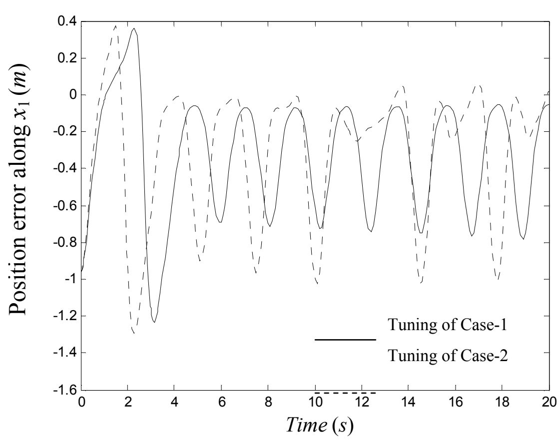


(a) Position error $\tilde{\boldsymbol{x}}_1$

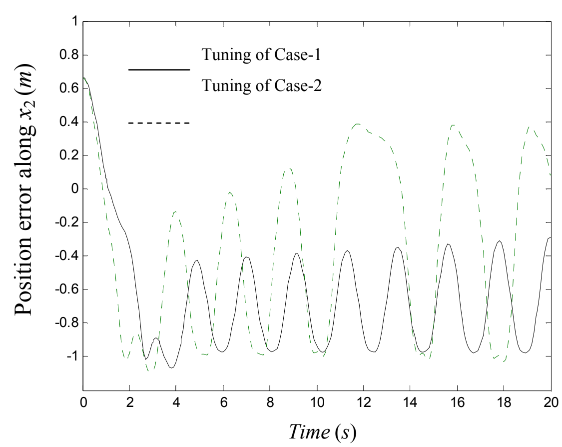


(b) Position error $\tilde{\boldsymbol{x}}_2$

Fig.3 Simulation result of the proposed PID controllers for tuning of (49), Case-1 and Case-2



(a) Position error $\tilde{\boldsymbol{x}}_1$



(b) Position error $\tilde{\boldsymbol{x}}_2$

Fig.4 Simulation result of the traditional PID controllers for tuning of Case-1 and Case-2

Simulation results are presented in Fig.3 and Fig.4 respectively. As for the proposed PID controller, the comparison with the above different tuning is demonstrated in Fig.3, it shows the better transient response when the integral action is strengthened. On the contrary, the traditional PID with the tuning in case-1 and case-2 destabilizes the closed- loop system and the performance deteriorates seriously.

V. Conclusion

In this paper, a new PID-type controller with error-dependent integration is developed, in which the integral action is adapted to amplitude of position error. As a result, the performance improvement is expected due to more parameters that are available to tune and better transient performance is deserved in comparison with the traditional PID controllers.

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