

# OFDM Transform-domain Channel Estimation Based on MMSE for Underwater Acoustic Channels

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**Abstract**—In this paper, we present a method of transform-domain channel estimation for orthogonal frequency division multiplexing (OFDM). The estimator based on minimum mean squared error (MMSE), and a process of FFT and IFFT is added. The system has been tested in a real underwater acoustic channel – the experimental pool in Xiamen University. The experimental result shows that the transform-domain channel estimation has improved the performance of the OFDM system in underwater acoustic channels.

**Keywords**- Transform-domain, MMSE, OFDM, Channel estimation

## I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) has found application in a number of systems, including wireless local area networks (IEEE 802.11a/g/n)[1], the wire-line Digital Subscriber Loops (DSL), wireless digital audio and video broadcast (DAB, DVB) systems and wireless metropolitan area networks (IEEE 802.16)[2]. It is also considered for the fourth generation cellular systems and ultra-wideband (UWB) wireless communications in general [3].

The underwater acoustic channel (UAC) is possibly nature's most unforgiving wireless communication medium. In this system, fading processes may be fast or slow, frequency selective or frequency nonselective, depending on the direction of propagation and conditions of the water column. The low speed of sound used underwater (nominally  $c=1500$  m/s) leads to severe motion-induced Doppler distortion. The channel is also characterized by severe multi-path propagation and time-varying richly-scattered. However, since the beginning of the 20th century, underwater communication has been used [4] and there is a pressing demand for higher data rate systems that can cope with the highly scattered underwater channel.

OFDM divides the available bandwidth into a large number of sub-bands to extend the duration of the symbol, and it treats the frequency-selective fading as simple amplitude and phase corrections on each narrowband sub-carrier combined with the use of a cyclic extension to mitigate the ISI due to delay spread [5]. So it has been wide studied all over the world not only in wireless communication system but also in underwater acoustic communication system.

The paper describes a new method of channel estimation. In section II, the OFDM system model and channel model are outlined. Section III describes the proposed channel estimation

and the transform-domain method. In section IV, we present simulation result, before finishing with conclusions drawn in section V.

## II. OFDM SYSTEM MODEL AND CHANNEL MODEL

### A. OFDM mode

The basic idea underlying OFDM systems is the division of the available frequency spectrum into several sub-carriers. To obtain a high spectral efficiency, the frequency responses of the sub-carriers are overlapping but orthogonal. This orthogonality can be completely maintained with a small price in a loss in SNR by introducing a cyclic prefix (CP). A block diagram of an OFDM underwater acoustic communication system is shown in Fig.1.

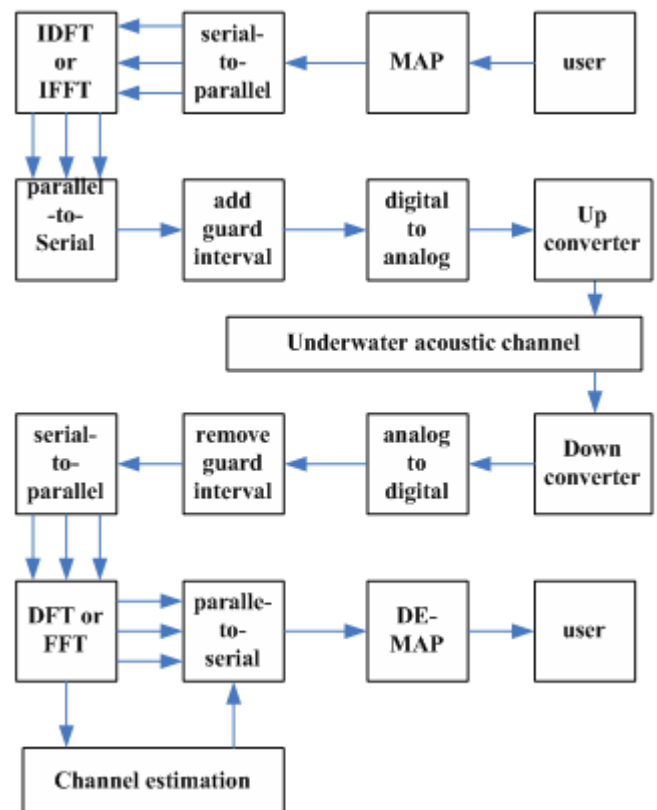


Figure 1. Fig. 1 OFDM system

The binary information is first mapped according to the modulation in a “signal mapper”. Assume the length of the data after being mapped is  $N_c$ , also is the number of the useful sub-carriers, an N-point inverse discrete-time Fourier transform (IDFT/IFFT) block transforms the data sequence into time domain where  $N = 4N_c$  (with zeros appended to full length of  $4N_c$ ). This IFFT size is twice the minimum need, but was chosen to avoid the need to upsample the baseband signal prior to frequency translation.[3] Following the IFFT block, a cyclic extension of time length  $T_{cp}$ , chosen to be larger than the expected delay spread, is inserted to avoid intersymbol and intercarrier interferences. The D/A converter contains low-pass filters with bandwidth  $1/T_{fs}$ , where  $T_{fs}$  is the sampling interval. The useful data duration is  $T_s$ , so the total time for an OFDM symbol is  $T = T_s + T_{cp}$ . Consider the influence of the cyclic, the waveform of the  $k$ th carrier is

$$\Phi_k(t) = \begin{cases} \frac{1}{\sqrt{T_s}} \exp[j2\pi \frac{W}{N} k(t - T_{cp})], t \in [0, T] \\ 0, t \notin [0, T] \end{cases} \quad (1)$$

Note that, when  $t \in [0, T_{cp}]$ ,  $\Phi_k(t) = \Phi_k(t + N/W) = \Phi_k(t + T_s)$ , the  $i$ th OFDM symbol is

$$s_i(t) = \sum_{k=0}^{N-1} X_i(k) \Phi_k(t - iT) \quad (2)$$

In the receiver,

$$y = Xh + n \quad (3)$$

where  $y$  is the received vector,  $X$  is a diagonal matrix containing the transmitted signaling points,  $h$  is a channel attenuation vector, and  $n$  is a vector of independent identically distributed complex zero-mean Gaussian noise with variance  $\sigma_n^2$ . The noise  $n$  is assumed to be uncorrelated with the channel  $h$ .

### B. Channel model

We consider a fading multipath channel model, consisting of  $M_c$  impulses, [6]

$$g(\tau) = \sum_{k=0}^{M_c-1} \alpha_k \delta(\tau - \tau_k T_{fc}) \quad (4)$$

Where  $\alpha_k$  are zero-mean complex Gaussian random variables with a power-delay profile  $\theta(\tau_k)$ . The exponentially decaying power-delay profile  $\theta(\tau_k) = C e^{-\tau_k/\tau_{rms}}$  and the delays  $\tau_k$  are uniformly and independently distributed over the length of the CP.

In the channel model, the attenuation on tone becomes

$$h_k = \sum_{i=0}^{M_c-1} \alpha_i e^{-j2\pi(k/N)\tau_i} \quad (5)$$

The correlation matrix for the attenuation vector  $h$ ,  $R_{hh} = E\{hh^H\} = [r_{m,n}]$

Where

$$r_{m,n} = \frac{1 - e^{-L((1/\tau_{rms}) + 2\pi j(m-n)/N)}}{\tau_{rms} (1 - e^{-L/\tau_{rms}}) (\frac{1}{\tau_{rms}} + 2\pi j \frac{m-n}{N})} \quad (6)$$

A uniform power-delay profile can be obtained by letting  $\tau_{rms} \rightarrow \infty$ , result in

$$r_{m,n} = \frac{1 - e^{-2\pi jL(m-n)/N}}{2\pi jL(m-n)/N} \quad (7)$$

where  $L$  is the length of the CP.

## III. CHANNEL ESTIMATION

The use of differential phase-shift keying (DPSK) in OFDM systems avoids need to track a time varying channel; however, it limits the number of bits per symbol and results in a 3 dB loss in SNR. Coherent modulation allows arbitrary signal constellations, but efficient channel estimation strategies are required for coherent detection and decoding.

To coherently demodulate OFDM signals, channel state information (CSI) must be available at the receiver side. For underwater acoustic applications where the channel is both time-and -frequency-selective dynamic channel estimation is necessary. The literature on OFDM channel estimation is abundant, most of which addresses channel estimation based on pilot tones inserted in the OFDM symbol stream.

In this paper we assume that the underwater acoustic channel is time-variant. Therefore, the transfer function for the present data block should be obtained independently of that for the previous data block. We choose comb-pilot in our system, as show in fig.2. Through the paper, adjacent  $L(L \ll N_c)$  sub-carriers are grouped together, without overlapping between adjacent groups. In each group, the first sub-carrier is used to transmit pilot signal and it is called the pilot sub-carriers. The

rest of the sub-carriers bear information data and thus are called information sub-carriers. Therefore, there are total  $M = Nc/L$  pilot sub-carriers and  $Nc - M$  information sub-carriers. In this paper, we set  $Nc = 4M$ .

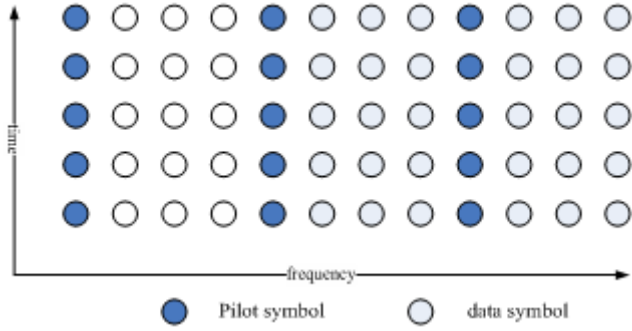


Figure 2. Pilot pattern

Through the whole, in the discrete frequency domain,  $k'$  is further expressed as  $k' = mL + l$ , with integers  $l \in [0, L - 1]$  and  $m \in [0, M - 1]$ . The OFDM signal modulated on the  $k'$ th sub-carriers can be expressed as

$$X(k') = X(mL + l) = \begin{cases} \text{pilot} & \text{data}, l = 0 \\ \text{information} & \text{data}, l = 1, \dots, L - 1 \end{cases} \quad (8)$$

#### A. MMSE channel estimation

The MMSE estimator employs the second-order statistics of the channel conditions to minimize the mean-square error. The MMSE estimate of the channel attenuations  $h$  in (3), given the received data  $y$  and the transmitted symbols  $X$ , is

$$H_{MMSE} = R_{hh} (R_{hh} + \sigma_n^2 (XX^H)^{-1})^{-1} H_{LS} \quad (9)$$

Where  $H_{LS} = X^{-1}y = [\frac{y_0}{x_0}, \frac{y_1}{x_1}, \dots, \frac{y_{M-1}}{x_{M-1}}]$  is the least-

squares (LS) estimate of  $h$  by the pilot data.  $\sigma_n^2$  is the variance of the additive channel noise, and  $R_{hh} = E\{hh^H\}$  is the channel autocorrelation matrix. The superscript  $(\bullet)^H$  denotes Hermitian transpose.

The MMSE estimator yields much better performance than LS estimation, especially under the low SNR scenarios. A major drawback of the MMSE estimator is its high computational complexity.

#### B. Transform-domain MMSE processing

The channel estimation method based on pilot signals, MMSE, and transform-domain processing is depicted in fig.3.

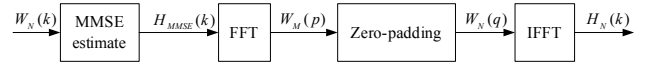


Figure 3. Transform-domain MMSE channel estimation

According to the slow-variation property, the channel transfer function can be viewed as the sum of several sinusoidal functions with respect to  $k$ . However, the number and the “frequencies” of the sinusoids vary due to the changing in the mobile radio channel. To avoid the model mismatch problem, we do not transform  $kM(p)$  back to frequency domain and then perform interpolation. Instead, a high-resolution interpolation approach based on zero-padding and FFT/IFFT is used.

Assume that, the MMSE channel estimation by  $M$ -sample pilot data after transform domain is  $W_M(p) = [B_0, B_1, \dots, B_{M-1}]$ , as we had set  $Nc = 4M$ , we extend  $W_M$  to  $W_{Nc}(q) = [B_0, \dots, B_{M/2-1}, 0, \dots, 0, B_{M/2}, \dots, B_{M-1}]$  by zeros padding. By performing an  $Nc$  point IFFT, the transfer function is obtained as [7]

$$H = a \sum_{q=0}^{N-1} W_{Nc}(q) \exp(-j * 2 * \pi * qk / Nc), 0 \leq k \leq Nc - 1 \quad (10)$$

Noticing that the  $M$ -point FFT and  $Nc$  point IFFT are performed between frequency domain and time domain, a constant  $a$  is need for calibration.

## IV. SIMULATE AND EXPERIMENTAL RESULT

#### A. Simulate result

The simulation of the transform-domain MMSE channel estimation performance and the LS channel estimation in an AWNG and multipath channel using MATLAB, is shown in Fig.4

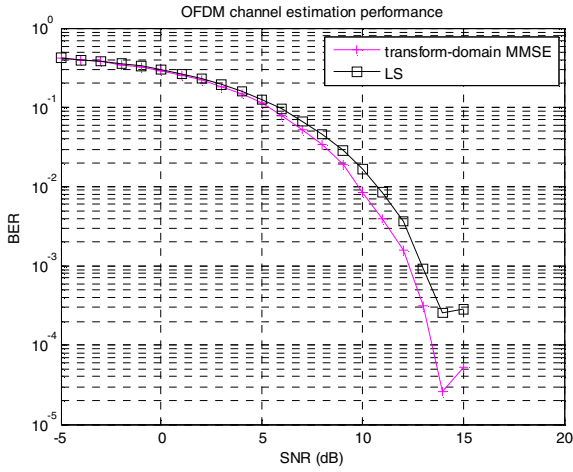


Figure 4. The performance of transform-domain MMSE channel processing simulate use MATLAB

### B. Experimental result

The experiment was carried out at the experimental pool in Xiamen University. The size of the pool is  $4.3m \times 3m \times 2m$ . Fig.5 depicts the location of transmitter and receiver transducers. Both of them kept still during the whole experiment. The distance between the transmitter and the receiver is 3.04m.

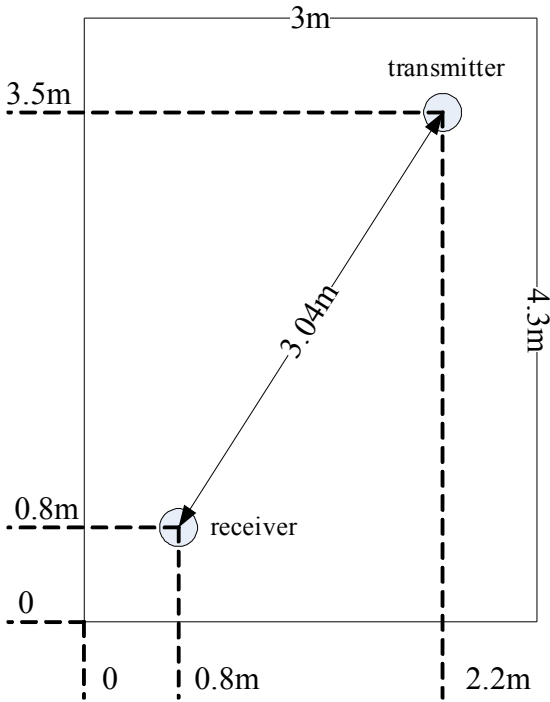


Figure 5. The experimental condition

There are 25 OFDM signals in one frame. A cyclic prefix is used as guard intervals in order to compete with the inter-carrier interference (ICI). In order to know the frame boundary, LFM signals are appended before the data sequence before transmitted into the channel. At receiver side, time

synchronization is done via correlating the received samples with the known LFM sequence. After that, the received data are divided into OFDM symbols.

We set 1024 sub-carriers in this experiment. System specification for the experiment is shown in Table I.

TABLE I. SYSTEM SPECIFICATION

Mapping mode	QPSK
Bandwidth	5000Hz
Carrier frequency	15000Hz
Transmission frequency band	12500-17500Hz
Maximum delay spread	25ms
Sampling frequency	60KHz
Number of sub-carriers	1024
Symbol duration	204.8ms
Symbol rate	3.906symbols/sec
Guard interval	51.2ms

Table II is the result of the experiment. From the table, we can see that the average BER of the LS channel estimation is 0.00991 nearly 10 times of the result of the transform-domain MMSE channel estimation of which the average BER is 0.00108. In the OFDM underwater acoustic communication system, if the channel is not used, the BER is 0.51457, so we can conclude that the channel estimation has improved the performance of the OFDM system. The performance of the latter is much better than the former because the latter method include the information of the channel,  $R_{hh}$  and  $\sigma_n^2$ , and a process of FFT and IFFT is added.

TABLE II. EXPERIMENTAL RESULT

Number of the test	LS	transform-domain MMSE	without channel estimation
1	0.00958	0.00081	0.51227
2	0.00867	0.00076	0.49961
3	0.00898	0.00065	0.48617
4	0.00846	0.00052	0.49943
5	0.01036	0.00146	0.53917
6	0.01076	0.00148	0.50638
7	0.01115	0.00182	0.54852
8	0.01091	0.00133	0.53128
9	0.01120	0.00135	0.50987
10	0.00898	0.00063	0.51305
average	0.00991	0.00108	0.51457

## V. CONCLUSION

As can be shown in the experiment result, the method of transform-domain MMSE has greatly improved the performance of OFDM channel estimation than LS method. The overhead of the transform-domain MMSE is higher than LS because it needs two inverses of matrix and an FFT and IFFT process.

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