

# Implementation of Carrier Interferometry OFDM by Using Pulse Shaping Technique in Frequency Domain

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**ABSTRACT:** This paper presents a novel design of carrier interferometry (CI) spreading codes by Using inverse fast Fourier transform (IFFT) instead of linear transform matrix . Pulse shaping method in frequency domain is used to generate different carrier interferometry orthogonal frequency division multiplexing (CI/OFDM) signals. Peak-to-average power ratio (PAPR) performance has been compared among OFDM, original CI/OFDM and our proposal. Simulation results show that our proposal could simplify the system implementation and improve the signal processing efficiency without PAPR performance degradation.

**KEYWORDS:** Pulse Shaping, IFFT, Carrier Interferometry (CI), CI/OFDM

## I. INTRODUCTION

As shown in fig. 1, in carrier interferometry orthogonal frequency division multiplexing (CI/OFDM), information symbols are modulated onto all of the  $N$  parallel carriers. In order to separate symbols located on the same carrier, a phase offset known as spreading code is used. The CI spreading code for  $k^{\text{th}}$  information symbol is defined as  $\beta^k = (e^{j0}, e^{j\Delta\theta_k}, \dots, e^{j(i-1)\Delta\theta_k}, \dots, e^{j(N-1)\Delta\theta_k})$ , where  $\Delta\theta_k = (2\pi/N)k$  will ensure orthogonality among the  $N$  information transmitted symbols which occupy the same carrier at the same time [1].

For reasons of brevity, we focus only on the case of complex baseband CI/OFDM signals in this paper. Hence, the complex baseband transmitted signals in CI/OFDM is

$$s(t) = \sum_{k=0}^{N-1} s_k(t) = \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} a_k e^{j\frac{2\pi}{N}ki} e^{j2\pi i\Delta f t} \quad (0 \leq t \leq T_s) \quad (1)$$

where  $s_k(t)$  is the transmitted signal for the  $k^{\text{th}}$  information symbol  $a_k$ ,  $\Delta f = 1/T_s$  ( $T_s$  is one CI/OFDM symbol duration) to ensure orthogonality among carriers, and  $(2\pi/N)k \cdot i$  is the phase offset used to generate the spreading code for  $a_k$  which ensures the orthogonality among the  $N$  information symbols.

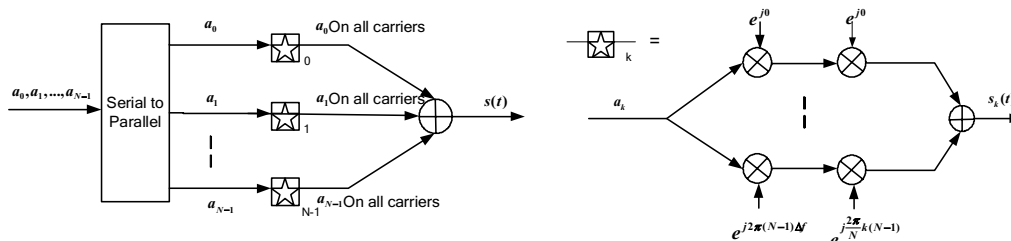


Fig.1 Conceptual CI/OFDM Transmitter

Although CI/OFDM system was shown to possess good peak-to-average ratio (PAPR) [2], [3] and significantly outperform the OFDM systems in terms of BER in multipath fading channel [4], [5], its implementation is complicated, and only concept transmitter and receiver models have been given in the literature. In this paper, the inverse fast Fourier transform (IFFT) is used to perform spreading in transmitter, and fast Fourier transform (FFT) is used to implement de-spreading at the receiver. This proposal significantly simplifies the CI/OFDM system implementation and facilitates CI/OFDM practical application. Another advantage of our design is that it is easy to employ diversity combining technique at receiver, which is necessary in fading channel environment.

The paper is organized as follows: Section II details the theory of our proposal. Section III presents the PAPR performance results of the proposed CI/OFDM system. Conclusions are drawn in Section IV.

## II. DESCRIPTION OF THE PROPOSED CI/OFDM SYSTEM

### A. New Realization of CI spreading and de-spreading in Proposed CI/OFDM System

We rewrite the discrete form of (1) with the Nyquist sampling rate of  $N\Delta f$

$$s(n) = \sum_{k=0}^{N-1} s_k \left( \frac{nTs}{N} \right) = \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} a_k e^{j\frac{2\pi}{N}ki} e^{j\frac{2\pi}{N}ni} = \sum_{i=0}^{N-1} \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}ki} e^{j\frac{2\pi}{N}ni} \quad n = 0, 1, \dots, N-1 \quad (2)$$

It is clear that (2) can be performed by  $N$  points weighted inverse discrete Fourier transform (IDFT), where the weights are CI spreading codes for all information symbols. It could be expressed as a vector

$$\mathbf{p} = (\rho_0, \rho_1, \dots, \rho_i, \dots, \rho_{N-1}) = \left( \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}k \cdot 0}, \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}k \cdot 1}, \dots, \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}k \cdot i}, \dots, \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}k \cdot (N-1)} \right) \quad (3)$$

Above vector is same as the IDFT with sample  $N$  except for a constant coefficient. Hence, the CI/OFDM signal model employed in this paper corresponds to

$$\mathbf{s} = N \cdot \text{IDFT}(\mathbf{p}) = N \cdot \text{IDFT}[N \cdot \text{IDFT}(\mathbf{a})] = N^2 \text{IDFT}[\text{IDFT}(\mathbf{a})] \quad (4)$$

where  $\mathbf{s} = (s(0), s(1), \dots, s(N-1))$  and  $\mathbf{a} = (a_0, a_1, \dots, a_{N-1})$ .

Therefore, we can replace the CI spreading transform matrix by an IDFT, and this would simplify CI/OFDM system implementation significantly.

After transmitted over a slow, frequency selective fading channel, the received signal in discrete form at receiver side is

$$r(n) = \sum_{i=0}^{N-1} \sum_{k=0}^{N-1} \alpha_i a_k e^{j\frac{2\pi}{N}ki} e^{j\frac{2\pi}{N}ni} e^{j\phi_i} + n_0(n) \quad n = 0, 1, \dots, N-1 \quad (5)$$

where  $\alpha_i$  and  $\phi_i$  are the amplitude fade and phase offset on the  $i^{\text{th}}$  carrier.  $n_0(n)$  is the discrete form of complex baseband for additive white Gaussian noise (AWGN)  $n_0(t)$ .

Perfect phase tracking and synchronization are assumed. After detection, the output signal of the receiver is

$$\hat{\mathbf{s}} = \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} \mathbf{r} e^{-j\frac{2\pi}{N}ki} e^{j\frac{2\pi}{N}ni} = \text{DFT}[\text{DFT}(\mathbf{r})] \quad (6)$$

where  $\hat{\mathbf{s}} = (\hat{s}(0), \hat{s}(1), \dots, \hat{s}(N-1))$  and  $\mathbf{r} = (r(0), r(1), \dots, r(N-1))$ . We can replace IDFT by IFFT to achieve signal processing efficiency.

## B. Configuration of the Proposed CI/OFDM System

The transmitter and receiver models based on above realization are shown in fig.2. The transmitter incorporates channel coding, symbol mapping. The first IFFT accomplishes the CI spreading and the second IFFT performs the typical OFDM modulation. A cyclic prefix is also appended as a guard interval to the CI/OFDM symbol in order to combat the inter symbol interference (ISI) induced by multipath delay spread in the selective fading channel. In addition, a pilot signal is inserted to estimate channel condition. At receiver, two FFTs are used to demodulate and de-spread the signals respectively. Different diversity combining techniques are easy to implement based on this system structure. Equal gain combining (EGC) is an optimal combining technique in an AWGN channel, while other methods such as maximum ratio combining (MRC) and maximum mean square error combining (MMSEC) are used to eliminate ISI in the selective fading channel environment. By using diversity combination, the output signal of the receiver corresponds to

$$\hat{\mathbf{s}} = \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} w_i r e^{-j\frac{2\pi}{N}ki} e^{j\frac{2\pi}{N}ni} = DFT[\mathbf{w} \cdot DFT(\mathbf{r})] \quad (7)$$

Where  $\mathbf{w} = (w_0, w_1, \dots, w_{N-1})$  is weight used in diversity combining scheme.

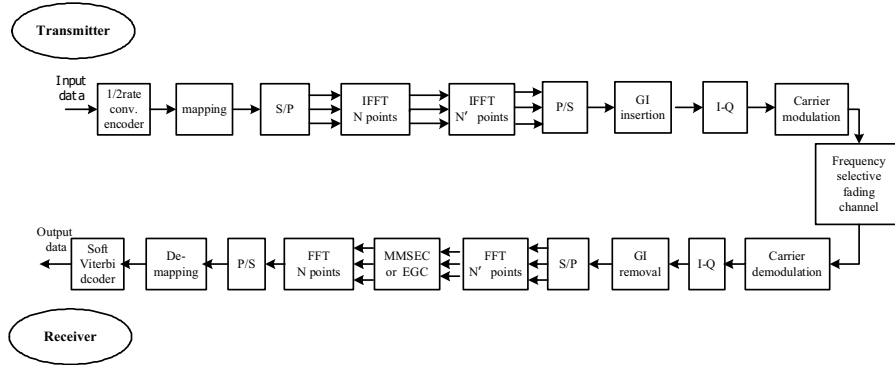


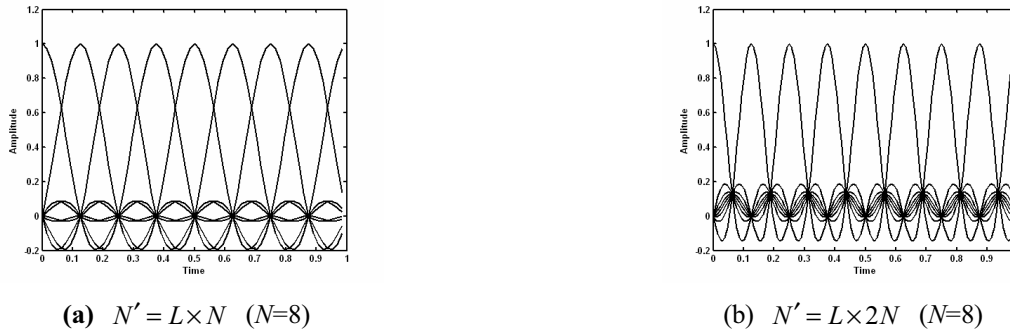
Fig. 2 CI/OFDM Transmitter and Receiver Structures

## C. Pulse Shaping Methods in Frequency Domain

We propose pulse shaping methods to generate CI/OFDM signal in frequency domain based on above system structure. The input vector of the second IFFT in fig. 2 could correspond to  $\mathbf{\rho}' = (\underbrace{\rho_0, \rho_1, \dots, \rho_{N/2-1}}_{N/2}, \underbrace{0, 0, \dots, 0}_{(L-1) \times N}, \underbrace{\rho_{N/2}, \rho_{N/2+1}, \dots, \rho_{N-1}}_{N/2})$  or

$\mathbf{\rho}' = (\underbrace{\rho_0, \rho_1, \dots, \rho_{N-1}}_N, \underbrace{0, 0, \dots, 0}_{(L-1) \times 2N}, \underbrace{0, 0, \dots, 0}_N)$ , where  $L$  is oversampling factor of the second IFFT. In fig. 3 (a), when the length of

the second IFFT is  $N' = L \times N$  points, CI signals are pseudo-orthogonal with each other and the durations of the mainlobe and multiple sidelobes are  $4/N\Delta f$  and  $2/N\Delta f$  respectively. Since both positive and negative frequency inputs of the IFFT are used, its frequency efficiency is higher. While in fig.3 (b), when the second IFFT length is  $N' = L \times 2N$  points, which means all the input signals are only located in positive frequency of the second IFFT, CI signals are orthogonal with each other, and the durations of the mainlobe and multiple sidelobes are  $2/N\Delta f$  and  $1/N\Delta f$  respectively.



**Fig. 3  $N$  pseudo-orthogonal and  $N$  orthogonal CI signals**

### III. SIMULATION RESULTS OF PAPR

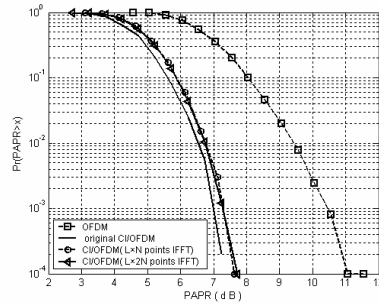
Since a linearly increasing phase offset in the frequency domain corresponds to an offset in the time domain, we note that (1) could be characterized by the following equation

$$s(t) = \sum_{k=0}^{N-1} s_k(t) = \sum_{k=0}^{N-1} a_k c(t + k\tau) = \sum_{k=0}^{N-1} a_k c\left(t + \frac{k}{N\Delta f}\right) \quad (0 \leq t \leq T_s) \quad (8)$$

where  $c(t) = \sum_{j=0}^{N-1} e^{j2\pi i \Delta f t}$  is a CI signal with a mainlobe centered at time 0.  $\tau = 1/(N\Delta f)$  is the minimum time shift to ensure orthogonality among CI signals with a mainlobe positioned at time  $k\tau$  ( $k = 0, 1, \dots, N-1$ ).

According to (8), CI/OFDM can be viewed as a single carrier modulation technique in which the carrier is wideband other than a typical narrowband carrier. The receiver can be a typical OFDM receiver with a CI de-spreading process. It is well explained that CI/OFDM system has a good PAPR characteristic.

Computer simulation results are shown in fig.4. The total number of the transmitted CI/OFDM symbols is 10,000. The number of the carriers  $N$  is 32 and the oversampling factor  $L$  is 4. QDPSK mapping is used. Complementary cumulative distribution functions (CCDF) of the two proposed CI/OFDM signals presented in this paper are compared with the original CI/OFDM signal and typical OFDM signal. As shown in fig.4, two proposed CI/OFDM signals have almost the same PAPR performances which are similar to that of the original CI/OFDM. They all outperform the OFDM signal significantly. Simulation results show that only about 0.3% of the PAPR of proposed CI/OFDM signals and about 0.1% of the PAPR of original CI/OFDM signal are higher than 7dB, whereas almost 40% of the PAPR of the OFDM signal are higher than 7dB. Hence, we could draw a conclusion that our new design has almost the same PAPR characteristic as that of the original CI/OFDM whereas simplifies the system implementation considerably.



**Fig.4 PAPR performance**

#### IV. CONCLUSION

In this paper, we have proposed a new design of CI/OFDM system based on pulse shaping technique in frequency domain. Spreading process is implemented by the IFFT instead of a linear transform matrix. Simulation results show that the PAPR performance of our proposal is similar to that of the original CI/OFDM. In addition, this proposal noticeably simplifies the system implementation and improves the signal efficiency in practice. We also point out that this structure is easy to employ diversity combining scheme at the receiver side.

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#### REFERENCES

- [1] Balasubramaniam Natarajan, Carl R. Nassar, Steve Shattil, Marco Michelini, and Zhiqiang Wu, "High-performance MC-CDMA Via Carrier Interferometry Codes," *IEEE Transactions on Vehicular Technology*, Vol.50, No.6, pp. 1344-1353, 2001.
- [2] C. R. Nassar, B. Natarajan, Z. Wu, D. Wiegandt, S. A. Zekavat, and S. Shattil, *Multi-carrier technologies for wireless communications*, kluwer Academic Press, pp.129, November 2001.
- [3] David A. Wiegandt, C. R. Nassar and Z. Wu, "The Elimination of Peak-to-Average Power Ratio Concerns in OFDM via Carrier Interferometry Spreading Codes: A Multiple Constellation Analysis," *The Thirty-Sixth Southeastern Symposium on System Theory, Conference Proceedings*, pp. 323- 327, 2004.
- [4] Zhiqiang Wu, Carl R. Nassar, "Narrowband Interference Rejection in OFDM via Carrier Interferometry Spreading Codes", *IEEE Transactions on Wireless Communications*, Vol. 4, NO.4, pp. 1491-1505, July 2005.
- [5] Andrew J. Best, Balasubramaniam Natarajan, "The effect of Jamming on the performance of Carrier Interferometry/OFDM," *Wireless and Mobile Computing, Networking and Communications, Conference Proceedings*, pp.66-70, Aug. 2005.