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Structural Design of Composite Nonlinear Feedback Control for Nonminimum Phase Linear Systems with Input Saturation

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***Abstract*— The general design procedure of composite nonlinear feedback (CNF) control does not consider the structure information of the system. As a result, the tuning of the nonlinear feedback gain is very difficult, especially for nonminimum phase systems. In this paper, a novel design method is proposed to construct a CNF control law by using the structure information of the system in a special coordinate basis (SCB) form. First, the system is transformed into the SCB form, in which the system is divided into three parts, i.e., stable zero dynamics part, unstable zero dynamics part, and integration part. For a nonminimum phase linear system, a virtual linear feedback gain is designed to stabilize the unstable zero dynamics. With this virtual gain, the system can be transformed to an integration system which is connected to a stable system. Then, the CNF control law is tuned only for the integration part of the system. Since the target system is an integration system, the proposed method simplifies the tuning of the nonlinear function in the CNF design.**

***Index Terms*— nonlinear feedback, tracking, nonminimum phase systems, input saturation.**

I. INTRODUCTION

The composite nonlinear feedback (CNF) control technique is to improve the transient performance of the closed loop system by introducing a nonlinear feedback law. Nonlinear techniques for improving transient performance of servomechanisms can be tracked back to the work of McDonald [13], where an analytical interpretation of the effect of nonlinear elements for nonlinear servo problems is given by using phase plane and space analysis. However, the composite nonlinear feedback control technique is proposed quite later by Lin *et al.* in [12] for a class of second order linear system with input saturation. The CNF control method combines a linear feedback control and a nonlinear feedback control. The linear part is designed to yield a closed-loop system with a small damping ratio for a quick response, and the nonlinear part is introduced to increase the damping ratio of the closed-loop system while the system output approaches the target reference to reduce the overshoot caused by the linear part. Turner *et al.* [16] extended the results of [12] to multivariable systems. Furthermore, Chen *et al.* [2] developed a CNF control to a more general class of systems with measurement feedback. The results of [2] are extended to multivariable system in [8] and [15]. More recently, Lan *et al.* [10] extended the CNF control technique to a class of nonlinear systems. The CNF control technique for discrete-time system can be found in [7] and [9]. The applications of the CNF control technique are also reported in the literature, for examples, the helicopter flight control system [1] and the hard disk driver servo system [2], [3].

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An important and challenging task in the design of the CNF control law is to choice an appropriate nonlinear feedback control law for the system, that is the selection of the nonlinear function in the CNF control law. The tuning methods are investigated in [11] for a second order integration systems. However, the general design method of CNF control does not consider the structure information of the system. The tuning of the nonlinear function are in general based on trial and error. For a nonminimum phase system, the unstable zero dynamics part will dominate the performance of the closed system. Thus, it will be very difficult to tune an appropriate nonlinear function directly for the given system. In this paper, we are tying to simplify the tuning of the nonlinear function in the CNF control by transforming the given system into a special coordinate basis (SCB) form (see, e.g., [4], [14]). In SCB form, the system is divided into two parts for a minimum phase system (stable zero dynamics, and integration part), and three parts for a nonminimum phase system (stable zero dynamics, unstable zero dynamics, and integration part). With a virtual feedback gain that stabilize the unstable zero dynamics, a nonminimum phase system can be further transformed to a new system in which a integration system is connected to a stable system. Thus, to design the CNF control law for the original system, we just need to design a CNF control law for the integration part of the new system. Since the nonlinear function only changes the poles of integration part of the system, the tuning of the nonlinear function in the CNF design is also simplified. As an illustration, we design a CNF control law for the inverted pendulum on a cart system.

II. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider a linear system with input saturation

$$\begin{aligned} \dot{x} &= Ax + B\mathrm{sat}(u), \quad x(0) = x_0 \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}$ control input, $y \in \mathbb{R}$ controlled output. A, B, C are appropriate dimensional constant matrices, and sat: $\mathbb{R} \to \mathbb{R}$ represents the actuator saturation defined as

$$\mathrm{sat}(u) = \mathrm{sgn}(u)\min\{u_{\max}, |u|\} \tag{2}$$

with $u_{\max}$ being the saturation level of the input. The following assumptions on the system matrices are required:

A1 (A, B) is stabilizable;

A2 (A, B, C) is invertible and has no zeros at $s = 0$.

We aim to design a state feedback control law for (1) such that the resulting closed-loop system is stable and the output of the closed-loop system will track a step reference input r rapidly without experiencing large overshoot. To improve the transient performance, we will design a CNF control law in the form of

$$u = Fx + Gr + \rho(r, y)B'P(x - xe)$$

The CNF control law consists of a linear feedback control and a nonlinear feedback control. The linear feedback law is designed to stabilize the system with a small closed-loop damping ratio for quick tracking. The nonlinear feedback law is to increase the closed-loop damping ratio as the system output approaches the reference input to reduce the overshoot while it keeps the closed-loop stability.

Remark 2.1: A general design procedure of CNF control is proposed in [2] for a single-input single-output linear system, which gives a CNF control law of the form

$$u = Fx + Gr + \rho(r, y)B'P(x - xe) \tag{3}$$

where F is selected such that $A + BF$ is stable, $\rho(r, y)$ is a non-positive function,

$$\begin{aligned} G &= -[C(A + BF)^{-1}B]^{-1} \\ x_e &:= -(A + BF)^{-1}BGr \end{aligned}$$

and P is a positive definite solution of

$$(A + BF)^{\mathrm{T}}P + P(A + BF) = -W$$

for some given $W > 0$. In general, we can simply let $W = I$. It is shown in [2] that, the closed-loop system comprising the given plant in (1) and the CNF control law of (3) is locally asymptotically stable. Specifically, for any $\delta \in (0, 1)$, let $c_\delta > 0$ be the largest positive scalar satisfying the following condition:

$$|Fx| \le u_{\max}(1 - \delta), \quad \forall x \in \mathbf{X}_\delta := \{x : x^{\mathrm{T}}Px \le c_\delta\} \tag{4}$$

Then, for any nonpositive function $\rho(r, y)$, locally Lipschitz in y, the composite nonlinear feedback law in (3) is capable of driving the system controlled output $h(t)$ to track asymptotically the step command input of amplitude r, provided that the initial state x_0 and r satisfy

$$\tilde{x} := (x_0 - x_e) \in \mathbf{X}_\delta, \quad |Hr| \le \delta u_{\max} \tag{5}$$

where

$$H \; := \; [1 - F(A + BF)^{-1}B]G.$$

■

Remark 2.2: Various forms of the nonlinear function $\rho(r, y)$ in (3) are used in the literature. In [11], a nonlinear function is given in the form of

$$\rho(r, y) = -\beta e^{-\alpha\alpha_0|y-r|} \tag{6}$$

where

$$\alpha_0 = \begin{cases} \frac{1}{|y_0 - r|}, & y_0 \ne r \\ 1, & y_0 = r \end{cases} \tag{7}$$

α and β are the parameters to be tuned. Because α_0 changes with different tracking target r, the closed-loop performance is robust to the variation of tracking targets. ■

To utilize the structure information of the system in the CNF controller design, we transform the system (1) into a so-called special coordinate basis (SCB) form. To this end, consider a linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{8}$$

It follows from [4] and [14] that there exist nonsingular state and input transformations

$$x = \Gamma_s \bar{x}, \quad u = \Gamma_i \bar{u} \tag{9}$$

with

$$\bar{x} = \begin{bmatrix} x_a^- \\ x_a^+ \\ x_d \end{bmatrix}, \; x_a^+ \in \mathbb{R}^{n_a^+}, \; x_a^- \in \mathbb{R}^{n_a^-}, \; x_d = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{n_d} \end{bmatrix}$$

such that

$$\begin{aligned} \dot{x}_a^- &= A_{aa}^- x_a^- + L_{ad}^- y \\ \dot{x}_a^+ &= A_{aa}^+ x_a^+ + L_{ad}^+ y \\ \dot{x}_1 &= x_2, \;\; y = x_1 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n_d - 1} &= x_{n_d} \\ \dot{x}_{n_d} &= E_{da}^- x_a^- + E_{da}^+ x_a^+ + E_1 x_1 + \cdots E_{n_d} x_{n_d} + \bar{u} \end{aligned}$$

where $\lambda(A_{aa}^-)$ contains all the stable system invariant zeros, $\lambda(A_{aa}^+)$ all the unstable system invariant zeros, and n_d is the relative degree of (8).

Remark 2.3: Assumption A1 implies that (A_{aa}^+, L_{ad}^+) is stabilizable. ■

III. Design Procedure

In this section, we present a novel design method to construct a CNF control law for the system (1) by using the SCB technique. We assume that the given system (1) satisfies assumptions A1 and A2, and all the states of the system are available for feedback. The CNF control law can be constructed by the following step-by-step procedure.

Step 1. By neglecting the input saturation, we can transform the system (1) into SCB form by the state and input transformations (9), which gives a system in the form of,

$$\begin{aligned} \dot{x}_a^- &= A_{aa}^- x_a^- + L_{ad}^- y \\ \dot{x}_a^+ &= A_{aa}^+ x_a^+ + L_{ad}^+ y \\ \dot{x}_d &= A_q x_d + B_d v \\ y &= C_d x_d \end{aligned}$$

with

$$A_q = \begin{bmatrix} 0 & I_{n_d-1} \\ 0 & 0 \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C_d = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and

$$\begin{aligned} v &= E_{da}^- x_a^- + E_{da}^+ x_a^+ + E_1 x_1 + \cdots E_{n_d} x_{n_d} + \bar{u} \\ &=: \bar{E}\bar{x} + \bar{u} \end{aligned} \tag{10}$$

STEP 2. Select a vector $F_a \in \mathbb{R}^{1\times n_a^+}$ such that $A_{aa}^+ + L_{ad}^+ F_a$ is stable. This is possible because of Remark 2.3. Then define a set of state transformations

$$\begin{aligned} \tilde{x}_a^- &= x_a^- \\ \tilde{x}_a^+ &= x_a^+ \\ \tilde{x}_1 &= x_1 - F_a x_a^+ \\ \tilde{x}_2 &= x_2 - F_a A_{aa}^+ x_a^+ - F_a L_{ad}^+ x_1 \\ \tilde{x}_3 &= x_3 - F_a (A_{aa}^+)^2 x_a^+ - F_a A_{aa}^+ L_{ad}^+ x_1 - F_a L_{ad}^+ x_2 \\ &\vdots \\ \tilde{x}_{n_d} &= x_r - F_a (A_{aa}^+)^{n_d-1} x_a^+ - \sum_{i=1}^{n_d-1} F_a (A_{aa}^+)^{n_d-1-i} L_{ad}^+ x_i \end{aligned}$$

which is denoted as

$$\tilde{x} = \tilde{\Gamma}_s^{-1} \bar{x} \tag{11}$$

with

$$\tilde{x} = \begin{bmatrix} \tilde{x}_a^- \\ \tilde{x}_a^+ \\ \tilde{x}_d \end{bmatrix}, \quad \tilde{x}_d = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \cdots \\ \tilde{x}_{n_d} \end{bmatrix}$$

we have

$$\begin{aligned} \dot{\tilde{x}}_a^- &= A_{aa}^- \tilde{x}_a^- + L_{ad}^- \tilde{x}_1 \\ \dot{\tilde{x}}_a^+ &= (A_{aa}^+ + L_{ad}^+ F_a)\tilde{x}_a^+ + L_{ad}^+ \tilde{x}_1 \\ \dot{\tilde{x}}_d &= A_q \tilde{x}_d + B_d \tilde{v} \\ y &= \begin{bmatrix} 0 & F_a & C_d \end{bmatrix} \tilde{x} \end{aligned} \tag{12}$$

where

$$\begin{aligned} \tilde{v} &= v - F_a (A_{aa}^+)^{n_d} x_a^+ - \sum_{i=1}^{n_d} F_a (A_{aa}^+)^{n_d-i} L_{ad}^+ x_i \\ &=: \tilde{E}\bar{x} + v \end{aligned} \tag{13}$$

STEP 3. Select F_d such that $A_q + B_d F_d$ is asymptotically stable. And, define

$$A_{ad} = \begin{bmatrix} A_a^+ + L_{ad}^+ Fa & L_{ad}^+ C_d \\ 0 & A_q \end{bmatrix}, \quad B_{ad} = \begin{bmatrix} 0 \\ B_d \end{bmatrix}$$
$$C_{ad} = \begin{bmatrix} F_a & C_d \end{bmatrix}, \quad F_{ad} = \begin{bmatrix} 0 & Fd \end{bmatrix}$$

Then, the linear feed back gain is given by

$$\tilde{v}_L = F_d \tilde{x}_d + G_{ad} r \tag{14}$$

where

$$G_{ad} = -[C_{ad}(A_{ad} + B_{ad}F_{ad})^{-1} B_{ad}]^{-1}$$

STEP 4. Given a positive-define matrix $W \in \mathbb{R}^{n_d \times n_d}$, solve the Lyapunov equation

$$(A_q + B_d F_d)' P_d + P_d (A_d + B_d F_d) = -W \tag{15}$$

for $P > 0$. Note that such a P_d exists since $A_q + B_d F_d$ is asymptotically stable. We also let

$$\begin{aligned} H_{ad} &:= [1 - F_{ad}(A_{ad} + B_{ad}F_{ad})^{-1} B_{ad}] G_{ad} \\ x_e &:= -r(A_{ad} + B_{ad}F_{ad})^{-1} B_{ad} G_{ad} \\ &:= \begin{bmatrix} x_{ea} \\ x_{ed} \end{bmatrix}, \quad x_{ed} \in \mathbb{R}^{n_d} \end{aligned}$$

Then, the nonlinear feedback control law $\tilde{v}_N(t)$ is given by

$$\tilde{v}_N = \rho_d(r, y) B_d' P_d (\tilde{x}_d - x_{ed}) \tag{16}$$

where $\rho_d(r, y)$ is any non-positive function locally Lipschitz in y.

STEP 5. The CNF control law for (12) is given by combining the linear and nonlinear feedback law derived in the previous steps,

$$\tilde{v} = \tilde{v}_L + \tilde{v}_N = F_d \tilde{x}_d + G_{ad} r + \rho_d(r, y) B_d' P_d (\tilde{x}_d - x_{ed}) \tag{17}$$

STEP 6. Using the state and input transformations (9), (11), (10) and (13), it is not difficult to transform the CNF control law (17) for (12) to the CNF control law for (1) in the form of

$$u = Fx + Gr + \rho(r, y) B_d' * P_d (T_d x - x_{ed}) \tag{18}$$

where

$$\begin{aligned} T_d &= \begin{bmatrix} 0 & I_{n_d} \end{bmatrix} (\Gamma_s \tilde{\Gamma}_s)^{-1} \\ F &= \Gamma_i (F_d T_d - (\bar{E} + \tilde{E}) \Gamma_s^{-1}) \\ G &= \Gamma_i G_{ad} \\ \rho(r, y) &= \Gamma_i \rho_d(r, y) \end{aligned}$$

Remark 3.1: From the design procedure, we can see that the key step to design a CNF control law for the system (1) is to design a CNF control law (17) for the system (12). Since the system (12) is in a SCB form, its performance is more clear than that of the original system (1). Thus, utilizing the structure information, we can simplify the work of tuning the parameters of the nonlinear function $\rho(r, y)$ in the CNF design. ■

IV. INVERTED PENDULUM SYSTEM

To demonstrate the design procedure of the proposed design method, we will design a CNF control law for the tracking control problem of the inverted pendulum on a cart system.

The inverted pendulum on a cart system, shown in Figure 1, is a well known unstable nonlinear system that can be found in many universities' control labs. Let M be the mass of the cart, m the mass of the block on the pendulum, l the length of the pendulum, y the position of the cart, θ the angle of the pendulum makes with vertical, g the acceleration due to gravity, b the coefficient of viscous friction for motion of

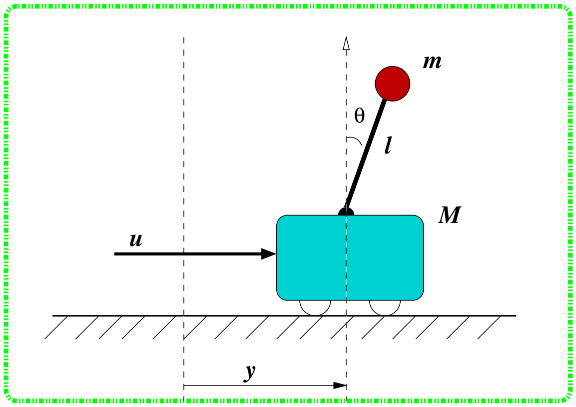


Fig. 1. Inverted pendulum on a cart system.

the cart, and u the applied force, the state space model of the inverted pendulum on a cart system is given by [6],

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= \frac{1}{g(x)}\left(f(x,u) - mg\cos x_3 \sin x_3\right)\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= \frac{1}{lg(x)}\left((M+m)g\sin x_3 - f(x,u)\cos x_3\right)\end{aligned}$$

where

$$x_1 = y,\quad x_2 = \dot{y},\quad x_3 = \theta,\quad x_4 = \dot{\theta}$$

and

$$\begin{aligned} f(x,u) &= u + mlx_4^2 \sin x_3 - bx_2\\ g(x) &= M + m(\sin x_3)^2\end{aligned}$$

with

$$x = \left[\begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array}\right]^T$$

Assume the maximum control input is ± 10N, the linearization model with input saturation is given by

$$\begin{aligned}\dot{x} &= Ax + B\text{sat}(u)\\ y &= Cx\end{aligned} \tag{19}$$

with $u_{\max} = 10$,

$$A = \left[\begin{array}{cccc} 0 & 1 & 0 & 0\\ 0 & -\frac{b}{M} & -\frac{mg}{M} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{b}{lM} & \frac{(M+m)g}{lM} & 0\end{array}\right], B = \left[\begin{array}{c} 0\\ \frac{1}{M}\\ 0\\ -\frac{1}{lM}\end{array}\right]$$

and

$$C = \left[\begin{array}{cccc} 1 & 0 & 0 & 0\end{array}\right]$$

The system (19) has two invariant zeros at

$$\pm\sqrt{\frac{g}{l}}$$

Thus, the inverted pendulum on a cart system is a non-minimum phase system. The objective is to design a CNF control law such that the closed-loop system is stable, and the output of (19) will track a step reference r as quick as possible with a very small overshoot or without any overshoot.

Step 1. Assume the parameters of the system are given by

$$M = 1.278\text{kg},\ m = 0.051\text{kg},\ l = 0.325\text{m},$$
$$g = 9.8\text{m/sec}^2,\ b = 12.98\text{kg/sec}$$

then, the state and input transformations

$$x = \Gamma_s \bar{x},\quad u = \Gamma_i \bar{u}$$

with

$$\begin{aligned}\Gamma_s &= \left[\begin{array}{cccc} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0.1792 & -0.1792 & -3.0769 & 0\\ -0.9838 & -0.9838 & 0 & -3.0769\end{array}\right]\\ \Gamma_i &= 1.3780\end{aligned}$$

will transfer the system (19) into the SCB form,

$$\begin{aligned}\dot{x}_a^- &= A_{aa}^- x_a^- + L_{ad}^- y\\ \dot{x}_a^+ &= A_{aa}^+ x_a^+ + L_{ad}^+ y\\ \dot{x}_d &= A_q x_d + B_d v\\ y &= C_d x_d\end{aligned} \tag{20}$$

where

$$A_{aa}^- = -5.4913,\ L_{ad}^- = 47.1535$$
$$A_{aa}^+ = 5.4913,\ L_{ad}^+ = 47.1535$$
$$A_q = \left[\begin{array}{cc} 0 & 1\\ 0 & 0\end{array}\right],\ B_d = \left[\begin{array}{c} 0\\ 1\end{array}\right],\ C_d = \left[\begin{array}{cc} 1 & 0\end{array}\right]$$

and

$$v = \bar{E}\bar{x} + \bar{u}$$

with

$$\bar{E} = \left[\begin{array}{cccc} -0.0650 & 0.0650 & 1.1160 & -9.4194\end{array}\right]$$

Step 2. Let

$$F_a = -0.2$$

then

$$A_{aa}^+ + L_{ad}^+ F_a = -3.9394$$

is stable. Then by a state transformation

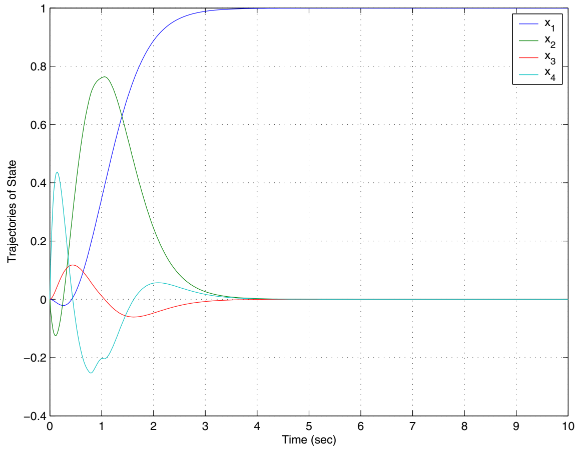
$$\bar{x} = \tilde{\Gamma}_s \tilde{x}$$

with

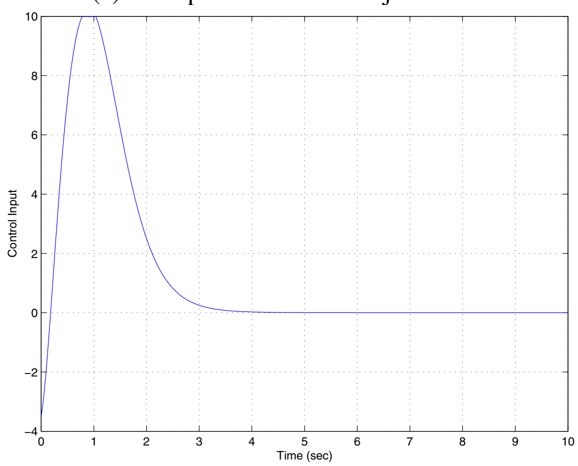
$$\tilde{\Gamma}_s = \left[\begin{array}{cccc} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & -0.2000 & 1 & 0\\ 0 & 0.7879 & -9.4307 & 1\end{array}\right]$$

we have

$$\begin{aligned}\dot{\tilde{x}}_a^- &= A_{aa}^- \tilde{x}_a^- + L_{ad}^- \tilde{x}_1\\ \dot{\tilde{x}}_a^+ &= (A_{aa}^+ + L_{ad}^+ F_a)\tilde{x}_a^+ + L_{ad}^+ \tilde{x}_1\\ \dot{\tilde{x}}_d &= A_q \tilde{x}_d + B_d \tilde{v}\\ y &= \left[\begin{array}{ccc} 0 & F_a & C_d\end{array}\right]\tilde{x}\end{aligned}$$

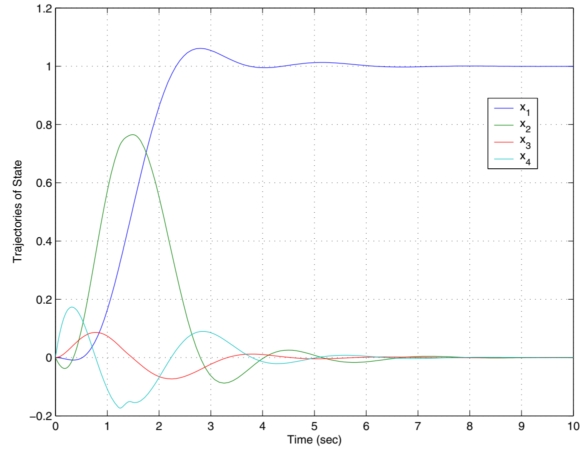


(a). The profile of state trajectories.

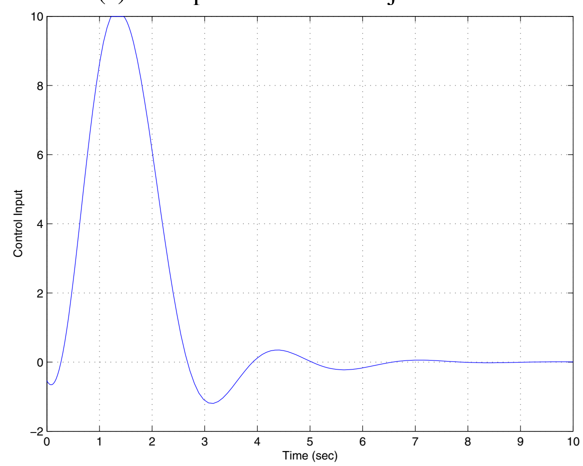


(b). The profile of control inputs.

Fig. 2. Simulation result for the inverted pendulum on a cart system under CNF control.



(a). The profile of state trajectories.



(b). The profile of control inputs.

Fig. 3. Simulation result for the inverted pendulum on a cart system under linear control.

where

$$\tilde{v} = \tilde{E}\bar{x} + v$$

with

$$\tilde{E} = \begin{bmatrix} 0 & 6.0308 & 51.7863 & 9.4307 \end{bmatrix}$$

Step 3. Select

$$F_d = \begin{bmatrix} -3.185 & -0.7 \end{bmatrix}$$

which places the eigenvalues of $A_q+B_dF_d$ at $-0.35\pm1.75i$. Thus,

$$A_{ad} = \begin{bmatrix} -3.9394 & 47.1535 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \; B_{ad} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$C_{ad} = \begin{bmatrix} -0.2 & 1 & 0 \end{bmatrix}$$
$$F_{ad} = \begin{bmatrix} 0 & -3.185 & -0.7 \end{bmatrix}$$

The linear feed back gain is given by

$$\tilde{v}_L = F_d\tilde{x}_d + G_{ad}r$$

where

$$G_{ad} = -[C_{ad}(A_{ad}+B_{ad}F_{ad})^{-1}B_{ad}]^{-1} = -2.2849$$

Step 4. Solving the Lyapunov equation

$$(A_q + B_dF_d)'P_d + P_d(A_d + B_dF_d) = -I$$

yields

$$P_d = \begin{bmatrix} 3.0992 & 0.1570 \\ 0.1570 & 0.9386 \end{bmatrix}$$

Calculate

$$x_e = -r(A_{ad}+B_{ad}F_{ad})^{-1}B_{ad}G_{ad} = \begin{bmatrix} -8.5870 \\ -0.7174 \\ 0 \end{bmatrix}$$

Then,

$$x_{ed} = \begin{bmatrix} -0.7174 \\ 0 \end{bmatrix}$$

The nonlinear feedback control law $\tilde{v}_N(t)$ is given by

$$\tilde{v}_N = \rho_d(r,y)B_d'P_d(\tilde{x}_d - x_{ed})$$

where $\rho_d(r,y)$ is given by (6).

Step 5. The CNF control law is given by combining the linear and nonlinear feedback laws, that is,

$$\tilde{v} = \tilde{v}_L + \tilde{v}_N = F_d\tilde{x}_d + G_{ad}r + \rho_d(r,y)B_d'P_d(\tilde{x}_d - x_{ed})$$

Step 6. Finally, we can transform the CNF control into the original coordinate,

$$u = Fx + Gr + \rho(r,y)B_d' P_d(T_d x - x_{ed}) \tag{21}$$

where

$$\begin{aligned} T_d &= \begin{bmatrix} 0 & I_{n_d} \end{bmatrix} (\Gamma_s \tilde{\Gamma}_s)^{-1} \\ &= \begin{bmatrix} -0.7174 & -0.3128 & -0.5582 & -0.1016 \\ 0 & -0.7174 & -3.0650 & -0.5582 \end{bmatrix} \\ F &= \Gamma_i (F_d T_d - (\bar{E} + \tilde{E})\Gamma_s^{-1}) \\ &= \begin{bmatrix} 3.1486 & 15.0447 & 29.0985 & 5.2080 \end{bmatrix} \\ G &= \Gamma_i G_{ad} = -3.1486 \\ \rho(r,y) &= -1.378\beta e^{-\alpha\alpha_0 |y-r|} \end{aligned}$$

Using the result of [11], we can easily tune the parameter of the $\rho(r,y)$ to get the desired transient response, which yields

$$\alpha = 0.5, \quad \beta = 3.7$$

The simulation result is shown in Figure 2.

For comparison, we also design a linear control law by the ITAE (the integral of the time multiplied by the absolute values of the error) method [5]. The ITAE method obtains the desired transient response by placing the closed-loop pole location to minimize

$$\int_0^\infty t|e|dt$$

By letting the nominal cutoff frequency

$$\omega_0 = 1.85 \text{rad/sec}$$

we get the linear control law

$$u = Fx + Gr \tag{22}$$

where

$$\begin{aligned} F &= \begin{bmatrix} 0.5353 & 13.7612 & 19.3896 & 1.9938 \end{bmatrix} \\ G &= -\left(C(A+BF)^{-1}B\right)^{-1} = -0.5353 \end{aligned}$$

The simulation result under the linear control law (22) is shown in Figure 3. It is clear that the closed-loop system under the linear control has an overshoot 6.16%. But under the CNF control law (21), there is no overshoot in the transient response of the closed-loop system. Also, the settling time under the CNF control law is much smaller than that under the linear control law.

V. Conclusions

The composite nonlinear feedback technique is an efficient tool to improve the transient performance of the closed-loop system. In general, the parameters of the nonlinear function are tuned by trial and error. Thus, it is time consuming to obtain a desired parameter, especially for nonminimum phase systems. Using the special coordinate basis technique, a novel CNF design method is proposed for the nonminimum phase linear systems with input saturation in this paper. The feature of proposed method is that the CNF control law is designed for the systems in special coordinate basis form. The structure information is helpful for the designer to tune the nonlinear function in the CNF control law.

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