

The influence of outflows on $1/f$ -like luminosity fluctuations

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ABSTRACT

In accretion systems, outflows may have significant influence on luminosity fluctuations. In this paper, following Lyubarskii's general scheme, we revisit the power spectral density of luminosity fluctuations by taking into account the role of outflows. Our analysis is based on the assumption that the coupling between the local outflow and inflow is weak for accretion-rate fluctuations. We find that, for inflow mass accretion rate $\dot{M} \propto r^s$, the power spectrum of the flicker-noise component will present a power-law distribution $p(f) \propto f^{-(1+4s/3)}$ for advection-dominated flows. We also obtain descriptions of $p(f)$ for both standard thin discs and neutrino-cooled discs, which show that the power-law index of a neutrino-cooled disc is generally larger than that of a photon-cooled disc. Furthermore, the relationship obtained between $p(f)$ and s indicates the possibility of evaluating the strength of outflows through the power spectrum in X-ray binaries and gamma-ray bursts. In addition, we discuss the possible influence of outflow–inflow coupling on our results.

Key words: accretion, accretion discs – ISM: jets and outflows – X-rays: binaries.

1 INTRODUCTION

The emission from Galactic black hole binaries (BHBs) and active galactic nuclei (AGN) displays a significant aperiodic variability over a broad range of time-scales. The power spectral density (PSD) of such variability is generally modelled with a power law, $p(f) \propto f^{-\beta}$, where $p(f)$ is the power at frequency f and the power-law index β stays constant in a certain range of f but changes between different ranges. At high frequencies, the PSDs of both BHBs and AGN present a steep slope with $\beta \sim 2$. In contrast, below a break frequency, typically a few Hz for BHBs, the PSDs flatten to a slope with $\beta \sim 1$, representing flicker noise (see King et al. 2004 and references therein).

Several models have been proposed in order to understand this nearly featureless character of power spectra. The so-called ‘shot-noise models’ (Terrell 1972) attempted to describe the light curves as a series of independent overlapping shots with specific time-scales, amplitudes and occurrence rates. Due to the lack of a physical picture in this scenario, various physically motivated ideas have subsequently been put forward, such as fluctuations of hydrodynamic or magnetohydrodynamic turbulence (Nowak & Wagoner 1995; Hawley & Krolik 2001), magnetic flares or density fluctuations in the corona (Galeev, Rosner & Vaiana 1979; Poutanen & Fabian 1999; Goosmann et al. 2006; Kawanaka, Kato & Mineshige 2008) and Lyubarskii's general scheme (Lyubarskii 1997; King et al. 2004). In Lyubarskii's scheme, it was noted that any variation of accretion rate that is caused by small-amplitude variations in the

viscosity would induce a variation in the accretion rate at the inner radius of the disc, where most of the energy is released. Moreover, observations have shown that the variability is non-linear and the root-mean-square (rms) variability is proportional to the average flux over a wide range of time-scales (e.g. Uttley & McHardy 2001; Gleissner et al. 2004; Uttley, McHardy & Vaughan 2005). This indicates that short time-scale variations are modulated by longer time-scales, which favours Lyubarskii's scheme.

However, the observed power spectra often deviate from the form f^{-1} . For example, the PSD of Cyg X-1 is well described by the form f^{-1} in the soft state, however it exhibits the form $f^{-1.3}$ in the hard state (Gilfanov 2010). In particular, it is shown that the power-law index is around 0.8–1.3 in both the soft state of BHBs and narrow-line Seyfert 1 galaxies (Janiuk & Czerny 2007). Such a dispersion of the power-law index reveals that some other mechanism must exist. A radius-dependent amplitude of α fluctuations may help to alleviate the discrepancy between theories and observations. However, it remains unclear why fluctuations of α should have a strong radius-dependent form.

In the present paper, we will take into account another mechanism, outflows, which are a popular phenomenon in accretion systems and have strong observational evidence. One of the best examples comes from Sgr A*, the centre of which harbours a supermassive black hole surrounded by an accretion flow that is likely to be in the form of an advection-dominated accretion flow (ADAF; Narayan & Yi 1994). Radio polarization observations constrain the accretion rate in the innermost region to be nearly two orders of magnitude lower than that measured at the Bondi radius (Marrone et al. 2006), which indicates that intense outflows may be present in this system. Also, the absorption lines from highly ionized

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elements, which have been detected in the X-ray spectrum of some microquasars such as GRO J1655–40 (Ueda et al. 1998; Yamaoka et al. 2001; Miller et al. 2006), GRS 1915+105 (Kotani et al. 2000; Lee et al. 2002) and Atoll sources (see e.g. the review by Díaz Trigo et al. 2006 and references therein), also indicate the existence of outflows. On the other hand, Jiao & Wu (2011) found that outflows generally exist in accretion discs no matter whether the flow is advection-dominated, as in slim discs (Abramowicz et al. 1988) and ADAFs, or radiation-dominated, as in the standard thin disc (Shakura & Sunyaev 1973). In particular, for the three types of advection-dominated flows – ADAFs (gas internal energy dominant), slim discs (trapped photon energy dominant) and hyper-accretion discs (trapped neutrino energy dominant) – outflows may be significantly strong due to positive Bernoulli parameters (e.g. Narayan, Kato & Honma 1997; Liu et al. 2011) or the large radiation pressure (e.g. Gu & Lu 2007). Furthermore, outflows have generally been found in many simulation works (e.g. Ohsuga & Mineshige 2011 and references therein).

In Lyubarskii’s scheme, the power spectrum of luminosity fluctuations is sensitive to the varying mass accretion rate, thus we expect that outflows may have significant effects on the power-law index.

This paper is organized as follows. In Section 2, we investigate the fluctuation power spectrum under a radius-dependent accretion rate following the method of Lyubarskii. The potential application of our results to observations is discussed in Section 3.

2 LUMINOSITY FLUCTUATIONS WITH A RADIUS-DEPENDENT ACCRETION RATE

2.1 Evolution equations of the fluctuations

In the present work, following Lyubarskii’s general scheme, we revisit the power spectral density of luminosity fluctuations by considering a radius-dependent accretion rate. The relevant processes are the conservation of mass and angular momentum. With a radius-dependent accretion rate, these two processes are described as follows:

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{2\pi r} \left(\frac{\partial \dot{M}}{\partial r} - \Phi \right), \quad (1)$$

$$\frac{\partial (\dot{M}\Omega r^2)}{\partial r} = -\frac{\partial}{\partial r} (2\pi r^2 T_{r\phi}) + \Phi l_\phi, \quad (2)$$

where Σ , \dot{M} , Ω and $T_{r\phi}$ are the surface density, accretion rate, angular velocity and $r\phi$ component of the stress tensor at the radius r , respectively. Φ is the change of accretion rate over r in the stationary accretion state, i.e. $\Phi = \partial \dot{M} / \partial r$, and l_ϕ is the angular momentum of Φ .

With $\Omega r^2 = l_{\text{in}}$, $F (= -T_{r\phi} r^2) = 0$ at the inner radius r_{in} , we can obtain

$$\begin{aligned} F &= \frac{1}{2\pi} \left[\dot{M}\Omega r^2 - \dot{M}_{\text{in}} l_{\text{in}} - \int_{r_{\text{in}}}^r \Phi l_\phi dr \right] \\ &= \frac{\dot{M}\Omega r^2}{2\pi} \left[1 - \left(\dot{M}_{\text{in}} l_{\text{in}} + \int_{r_{\text{in}}}^r \Phi l_\phi dr \right) / (\dot{M}\Omega r^2) \right]. \end{aligned}$$

With the following definitions:

$$h = \Omega r^2, \quad (3)$$

$$Q = 1 - \left(\dot{M}_{\text{in}} l_{\text{in}} + \int_{r_{\text{in}}}^r \Phi l_\phi dr \right) / (\dot{M}\Omega r^2), \quad (4)$$

the above description of F can be simplified as

$$F = \frac{\dot{M}h}{2\pi} Q.$$

For standard thin discs and ADAFs, $\Omega \propto \Omega_K = \sqrt{GM/r^3}$, and thus $h \propto r^{1/2}$. We assume $h = b\sqrt{GM}r$ in this paper, where b is a constant. If the accretion rate has a weak dependence on the radius and $l_\phi \propto \Omega r^2 \propto \sqrt{r}$, the value of Q will remain nearly constant. In order to simplify the problem, we assume Q to be constant in our analysis. The validity of this assumption is discussed in Appendix A.

With $\chi = (1/2\pi) \int \Phi (l_\phi - \Omega r^2) dr$, equation (2) becomes

$$\dot{M} = -\frac{\partial [2\pi(F + \chi)]}{\partial h}. \quad (5)$$

Substituting the above equation into equation (1), we have

$$\frac{\partial \Sigma}{\partial t} = \frac{b^4(GM)^2}{2h^3} \frac{\partial^2(F + \chi)}{\partial h^2} - \frac{b^2 GM}{2\pi h^2} \Phi. \quad (6)$$

The relationship between Σ and F can be deduced using the α prescription and the local balance between heating and cooling in the disc. The formula is similar to that with a constant accretion rate (Filipov 1984; Lyubarskii & Shakura 1987; Narayan & Yi 1994), i.e.

$$\Sigma = \frac{b^4(GM)^2 F^{1-m}}{2(1-m)Dh^{3-n}}, \quad (7)$$

where the exponents m and n are determined by the disc model, which is discussed in Section 2.2, and D is a function of α . It should be noted that the analysis presented here is not strict for ADAFs. For a stringent analysis for ADAFs, the reader is referred to Appendix B.

Substituting equation (7) into equation (6), we have

$$\begin{aligned} \frac{\partial F}{\partial t} &= \frac{2DF^m}{b^4(GM)^2 h^{n-3}} \left[\frac{b^4(GM)^2}{2h^3} \frac{\partial^2(F + \chi)}{\partial h^2} \right. \\ &\quad \left. - \frac{b^2 GM}{2\pi h^2} \Phi \right] + \frac{F}{(1-m)D} \frac{\partial D}{\partial t}. \end{aligned} \quad (8)$$

By assuming $\alpha = \alpha_0[1 + \bar{\beta}(t, r)]$, where $\bar{\beta}(t, r) (\ll 1)$ is decorrelated at different radial scales and its correlation time-scale is of the order of the local viscous time-scale, the disturbed quantities D and F are

$$D = D_0(1 + \eta\bar{\beta}), \quad F = \left(\frac{\dot{M}_0 h}{2\pi} \right) Q + \psi,$$

where η is defined as $D \propto \alpha^\eta$ and the subscript 0 denotes unperturbed quantities. Substituting the above equations into equation (8) and including the following stationary condition:

$$\frac{b^4(GM)^2}{2h^3} \frac{\partial^2}{\partial h^2} \left(\frac{\dot{M}_0 h}{2\pi} Q + \chi \right) - \frac{b^2 GM}{2\pi h^2} \Phi = 0,$$

we have

$$\frac{\partial \psi}{\partial t} = \frac{D_0 Q^m}{h^{n-m}} \left(\frac{\dot{M}_0}{2\pi} \right)^m \frac{\partial^2 \psi}{\partial h^2} + \frac{\dot{M}_0 h Q}{2\pi(1-m)} \frac{\partial \eta \bar{\beta}}{\partial t}. \quad (9)$$

Here, we assume that the coupling between the local outflow and inflow is weak for the accretion-rate fluctuations. A simple discussion on this issue is presented in Section 2.4.

2.2 The analysis of luminosity fluctuations

Assuming the radius-dependent accretion rate is

$$\dot{M}_0 = \dot{M}_{0,\text{in}} \left(\frac{r}{r_{\text{in}}} \right)^s, \quad (10)$$

where $\dot{M}_{0,\text{in}}$ is the accretion rate at the inner radius r_{in} , we have

$$\frac{\partial \psi}{\partial t} = \frac{C}{h^{n-m-2sm}} \left(\frac{\dot{M}_{0,\text{in}}}{2\pi} \right)^m \frac{\partial^2 \psi}{\partial h^2} + \frac{\dot{M}_{0,\text{in}} h Q \eta}{2\pi(1-m)} \frac{\partial}{\partial t} \left[\bar{\beta} \left(\frac{h^2}{b^2 G M r_{\text{in}}} \right)^s \right] \quad (11)$$

and $C = D_0 Q^m / (G M r_{\text{in}} b^2)^{sm}$. This is a linear diffusion equation with a radius-dependent accretion rate. For $s = 0$, i.e. a constant accretion rate in the system, we have $Q = 1$ and equation (11) is reduced to

$$\frac{\partial \psi}{\partial t} = \frac{D_0}{h^{n-m}} \left(\frac{\dot{M}_{0,\text{in}}}{2\pi} \right)^m \frac{\partial^2 \psi}{\partial h^2} + \frac{\dot{M}_{0,\text{in}} h \eta}{2\pi(1-m)} \frac{\partial \bar{\beta}}{\partial t}, \quad (12)$$

which is the exact form of equation (9) in Lyubarskii (1997).

In general, the solution of equation (11) is (Lyubarskii 1997; Lynden-Bell & Pringle 1974)

$$\psi(t, x) - \psi(0, x) = \frac{\dot{M}_{0,\text{in}} \eta Q \kappa^2 x^l}{4\pi(1-m)} \int_0^t dt' \int_0^\infty dx_1 \frac{x_1^{l+1}}{t-t'} \times \exp \left[-\frac{(x^2 + x_1^2) \kappa^2}{4(t-t')} \right] I_l \left[\frac{\kappa^2 x x_1}{2(t-t')} \right] \frac{\partial}{\partial t'} \left[\bar{\beta}(t', x_1) \frac{x_1^{4s}}{(b^2 G M r_{\text{in}})^s} \right],$$

where

$$l = \frac{1}{2+n-m-2sm}, \quad x = h^{1/2l}, \quad 4 \left(\frac{l}{\kappa} \right)^2 = C \left(\frac{\dot{M}_{0,\text{in}}}{2\pi} \right)^m. \quad (13)$$

From equation (2), the accretion rate $\dot{M}(t, x)$ is

$$\dot{M}(t, x) - \dot{M}(0, x) = \int_0^t \int_0^\infty G(t, x; t', x_1) \frac{\partial}{\partial t'} \left[\bar{\beta}(t', x_1) \frac{x_1^{4s}}{(b^2 G M r_{\text{in}})^s} \right] dt' dx_1,$$

where

$$G(t, x; t', x_1) = \frac{\eta Q \kappa^4 x^{1-l} x_1^{l+1}}{8l(1-m)(t-t')^2} \exp \left[-\frac{(x^2 + x_1^2) \kappa^2}{4(t-t')} \right] \times \left\{ x_1 I_{l-1} \left[\frac{\kappa^2 x x_1}{2(t-t')} \right] - x I_l \left[\frac{\kappa^2 x x_1}{2(t-t')} \right] \right\}$$

and $I_\nu(z)$ is the Bessel function of the imaginary argument. Based on the above equation, the power spectrum of $\dot{M}(t, x)$ can be obtained following a complex calculation, as presented in section 4 of Lyubarskii (1997). Here, we directly present the result of the power spectrum and focus on the effects of outflows on the luminosity fluctuations. If $\sqrt{\langle \bar{\beta}^2 \rangle} \propto r^\xi$, the power spectrum $p(f)$ of $\dot{M}(t, x)$ is

$$p(f) \propto f^{-[1+4l(\xi+s)]}, \quad (14)$$

which indicates that the power-law index $\beta = 1 + 4l(\xi + s)$. In this work we ignore α fluctuations, i.e. $\xi = 0$, and then the expression of β will be reduced to

$$\beta = 1 + 4ls. \quad (15)$$

For an advection-dominated flow, we have $m = 0$, $n = 1$ and $D \propto \alpha$ (Narayan & Yi 1994) for equation (7), thus equation (13) indicates $l = 1/3$ and therefore β can be simplified as

$$\beta = 1 + \frac{4}{3}s. \quad (16)$$

We would like to stress that the above formula should be valid for all three types of advection-dominated flows mentioned in the first section.

For a standard thin disc it is well known that there exist three regions, according to the different dominant mechanisms for opacity and pressure. We have $m = 0.3$, $n = 0.8$ and $D \propto \alpha^{0.8}$ for the outer region ($p \sim p_{\text{gas}}$, $\kappa \sim \kappa_{\text{ff}}$), $m = 0.4$, $n = 1.2$ and $D \propto \alpha^{0.8}$ for the middle region ($p \sim p_{\text{gas}}$, $\kappa \sim \kappa_{\text{es}}$) and $m = 2$, $n = 7$ and $D \propto \alpha$ for the inner region ($p \sim p_{\text{rad}}$, $\kappa \sim \kappa_{\text{es}}$), thus we easily obtain the following from equations (13) and (15):

$$\beta = 1 + \frac{40}{25-6s}s \quad (17)$$

for the outer region,

$$\beta = 1 + \frac{10}{7-2s}s \quad (18)$$

for the middle region and

$$\beta = 1 + \frac{4}{7-4s}s \quad (19)$$

for the inner region. We point out that outflows may be negligible in standard thin discs, except for the inner region, which is radiation-pressure-dominated and may suffer from thermal instability. Outflows in the inner region may be significantly stronger than in the other two regions, due to thermal instability. However, it remains a matter of controversy as to whether the inner region is indeed thermally unstable or not (e.g. Hirose, Krolik & Blaes 2009) and many mechanisms have recently been proposed to suppress the instability (e.g. Ciesielski et al. 2011; Lin, Gu & Lu 2011; Zheng et al. 2011).

Apart from the above-mentioned photon-radiation-dominated flows, neutrino-dominated accretion flows (NDAFs) have also been widely studied (e.g. Popham, Woosley & Fryer 1999; Narayan, Piran & Kumar 2001; Gu, Liu & Lu 2006; Chen & Beloborodov 2007; Liu et al. 2007) and may account for the central engines of gamma-ray bursts (GRBs; e.g. Narayan, Paczyński & Piran 1992). The outer region of a NDAF may be advection-dominated, since neutrino cooling cannot balance viscous heating due to low temperature and density. This region can be regarded as an extension of slim discs (e.g. Liu et al. 2008). On the other hand, for high mass accretion rates such as $\dot{m} \gtrsim 1 M_\odot \text{ s}^{-1}$ the inner region may also become advection-dominant, due to the large optical depth for neutrinos (e.g. Di Matteo, Perna & Narayan 2002; Gu et al. 2006). Outflows may occur in NDAFs in particular for these two advection-dominated regions, owing to positive Bernoulli parameters (e.g. Liu et al. 2011).

For NDAFs, with the balance between the cooling rate by pair capture and the energy dissipation rate per unit volume (Popham et al. 1999, equations 5.4–5.5), we have $m = 0$, $n = -0.4$ and $D \propto \alpha^{1.2}$, thus equations (13) and (15) directly give the form of β :

$$\beta = 1 + \frac{5}{2}s. \quad (20)$$

The relationship between β and s for all the mentioned accretion models is presented in Fig. 1, where the short-dashed, solid and long-dashed lines correspond to the inner regions of standard thin discs, ADAFs and NDAFs, respectively. The region with cross symbols represents photon-cooled discs (cooling dominated either by photon radiation or by advection of photon or gas energy), whereas the region with dot symbols represents neutrino-cooled discs (cooling dominated by either neutrino radiation or advection of neutrino energy). The figure clearly indicates that, for comparable s , a neutrino-cooled disc generally has a larger β than a photon-cooled disc. In other words, β is generally larger in a GRB system than in a BHB system.

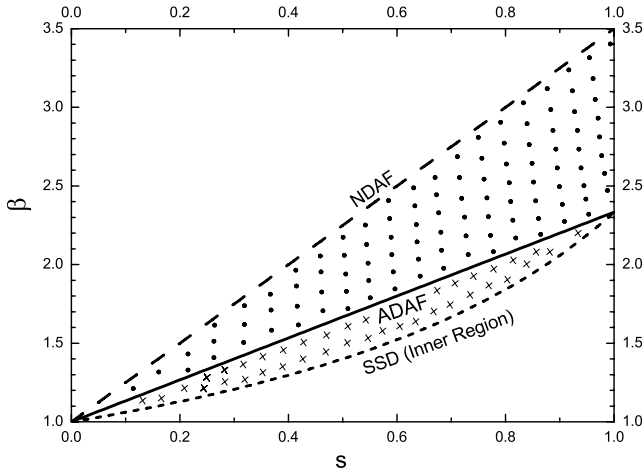


Figure 1. Variation of β with s for three types of accretion models: radiation-pressure-supported standard thin discs (short-dashed line), ADAFs (solid line) and NDAFs (long-dashed line). The regions of photon-cooled discs and neutrino-cooled discs are marked by the cross symbol and the dot symbol, respectively.

2.3 A simple analysis

The dependence of β on s can be well understood using equations (5) and (12). Following these two equations, the amplitude of $\delta\dot{M}(r)$, which is the accretion-rate variation produced at radius r , is proportional to the accretion rate at that radius, i.e. $\delta\dot{M}(r) \propto \dot{M}(r)$. Since variations of the accretion rate in the inner region may be represented as a sum of independent accretion-rate variations produced at different radii (Lyubarskii 1997), the power spectrum for the flow with $\dot{M} \propto r^s$ can be written as

$$p(f) \propto \dot{M}^2 f^{-1} \propto r^{2s} f^{-1}, \quad (21)$$

where \dot{M} is the accretion rate at radius r , which contributes to the fluctuation at frequency f , and the term f^{-1} is taken from Lyubarskii (1997) for the constant accretion-rate case.

Taking ADAFs as an example, we have $f \propto 1/t_{\text{vis}}$ and $t_{\text{vis}} = (r/H)^2/(\alpha\Omega) \propto r^{3/2}$, i.e. $r \propto f^{-2/3}$. Equation (21) is therefore simplified as

$$p(f) \propto f^{-[1+(4/3)s]},$$

which is exactly the same form as equation (16). For other disc models, such as standard thin discs and NDAFs, the results can be understood in the same way.

2.4 Possible influence of the outflow–inflow coupling

As mentioned in Section 2.1, our results are based on the assumption that the coupling between the local outflow and inflow is weak for the accretion-rate fluctuations, i.e. the response of outflows to inflow fluctuations is weak. In principle, the outflow should have fluctuations related to the variations of the local inflow accretion rate. However, the mechanism for outflows may be complicated, and therefore the fluctuations of the outflow may not be determined simply by those of the local inflow. For example, radiation from the inner region of a disc can heat up the materials in the outer region and thus outflows can be produced (e.g. Begelman, McKee & Shields 1983; Metzger, Piro & Quataert 2008). In such cases an outflow may be more relevant to the inner disc than to the outer region in which it occurs.

Here, we offer a simple discussion on the possible influence of local outflow–inflow coupling. If such a coupling exists, the amplitude

of an accretion-rate fluctuation may vary while it propagates into the inner region. With the assumption that $\delta\dot{M}(r)|_{r_1}$ is the accretion-rate variation at the radius r_1 induced by $\delta\dot{M}(r)$, we introduce a factor $D(r, r_1)$ to describe generally the change of the above amplitude owing to coupling effects, i.e., $\delta\dot{M}(r)|_{r_1} = D(r, r_1)\delta\dot{M}(r)$. It is easy to find the form of $D(r, r_1)$ in the following two situations: (1) if the coupling is negligible, the amplitude of $\delta\dot{M}(r)$ will stay unchanged during its propagation, i.e. $\delta\dot{M}(r)|_{r_1} = \delta\dot{M}(r)$ and thus $D(r, r_1) = 1$; (2) if the local outflow is in strong coupling with the local inflow, the relative amplitude $\delta\dot{M}(r)|_{r_1}/\dot{M}(r_1)$ will remain unchanged for varying r_1 , i.e. $D(r, r_1) = \dot{M}(r_1)/\dot{M}(r)$. Obviously, our results come under the former situation. For the latter one, in contrast, the power spectrum of accretion-rate variations at the inner radius r_{in} can be expressed as

$$p(f) \propto D(r, r_{\text{in}})^2 \dot{M}^2 f^{-1} = \dot{M}_{\text{in}}^2 f^{-1} \propto f^{-1},$$

which is the same form as the result with a constant accretion rate (e.g. Lyubarskii 1997). The detailed prescription of $D(r, r_1)$ is, however, beyond the scope of the present work due to the complexity of outflows. Nevertheless, we expect that a real flow may exist between situations (1) and (2) and therefore the value of β may be located between unity and the results presented in Section 2.2.

3 SUMMARY AND DISCUSSION

In this paper, we evaluate the effects of outflows on luminosity fluctuations with Lyubarskii’s general scheme. With a radius-dependent accretion rate $\dot{M} \propto r^s$, the power spectrum of the luminosity fluctuations is $p(f) \propto f^{-\beta}$, where the value of β varies with the disc structure. By assuming that the coupling between the local outflow and inflow is weak for the accretion-rate fluctuations, we obtain the following explicit expressions for β for different disc models: $\beta = 1 + 4s/3$ for advection-dominated discs, $\beta = 1 + 40s/(25 - 6s)$ for the outer region of standard thin discs, $\beta = 1 + 10s/(7 - 2s)$ for the middle region, $\beta = 1 + 4s/(7 - 4s)$ for the inner region and $\beta = 1 + 5s/2$ for NDAFs. The above expressions imply that β in a GRB is generally larger than that in a BHB for comparable s . The expressions for β indicate the possibility of evaluating the strength of outflows using the power spectrum in X-ray binaries and GRBs. In addition, if the coupling is not negligible, the value of β will probably be located between unity and the value presented in the above expressions.

In both BHBs and AGN, ADAFs are usually adopted to describe the quiescent state, the low/hard state and the corona that lies above a cold disc. ADAFs may produce significant outflows and therefore the power spectrum can deviate from f^{-1} based on the present analysis. The exact value of s is, however, difficult to estimate from the theoretical point of view, except for the general constraint $0 < s < 1$ (Narayan & McClintock 2008). On the other hand, some observations indicate $s \sim 0.3$ (Yuan, Quataert & Narayan 2003; Zhang, Yuan & Chaty 2010). Taking this value, we obtain $\beta = 1.4$ for ADAFs, which is close to 1.3, the power-law index of PSDs presented in low-mass X-ray binary systems (Gilfanov & Arefiev 2005). The quantitative difference may be relevant to the coupling between the outflow and inflow, as discussed in Section 2.4.

If only outflow operates in the accreting system, s should be positive. However s can also be negative, due to the evaporation mechanism of a cold disc. For the accreting black holes in BHBs, the observed power-law components in the X-ray spectra are generally attributed to hot, tenuous plasmas, namely accretion-disc coronae. Due to the high temperature in the corona, interaction between the disc and the corona will lead to mass evaporating from the disc to

the corona (Meyer, Liu & Meyer-Hofmeister 2000; Spruit & Deufel 2002). In this case, the value of s for the corona should be negative if outflows are not strong, and therefore it is quite possible for β to be less than unity. Consequently, in this scenario s for the cold disc underneath should be positive.

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APPENDIX A: THE VALIDITY OF THE ASSUMPTION OF CONSTANT Q

In this section, we analyse the validity of the assumption of constant Q .

With the radius-dependent accretion rate described in equation (10), we have

$$\Phi = s \frac{\dot{M}_{0,\text{in}}}{r_{\text{in}}} \left(\frac{r}{r_{\text{in}}} \right)^{(s-1)}.$$

Then equation (4) can be expressed as

$$Q = 1 - \frac{\dot{M}_{\text{in}} l_{\text{in}} + s \int_{r_{\text{in}}}^r \frac{\dot{M}_{0,\text{in}}}{r_{\text{in}}} \left(\frac{r}{r_{\text{in}}} \right)^{(s-1)} l_{\Phi} dr}{\dot{M} \Omega r^2}.$$

We assume that the specific angular momentum corresponding to Φ is proportional to that of the gas in the disc, i.e.

$$l_{\Phi} \propto \Omega r^2 \propto r^{1/2}.$$

This is the case for thermal-energy-driving outflows, magnetic-field centrifugal accelerating winds (Mayer & Pringle 2006) and the disc evaporation model.

For the general situation ($s \neq -1/2$), we have

$$\begin{aligned} Q &= 1 - \frac{\dot{M}_{\text{in}} l_{\text{in}} + \frac{s}{s+0.5} [\dot{M} l_{\Phi}]_{r_{\text{in}}}}{\dot{M} \Omega r^2} \\ &= 1 - \frac{s}{s+0.5} \frac{l_{\Phi}}{\Omega r^2} - \left(l_{\text{in}} - \frac{s}{s+0.5} l_{\Phi,\text{in}} \right) \left(\frac{r_{\text{in}}}{r} \right)^s \frac{1}{\Omega r^2}, \end{aligned}$$

and therefore

$$Q \rightarrow \text{constant} \quad \text{for } s > -1/2,$$

where $l_{\Phi,\text{in}}$ is the angular momentum of Φ at the inner radius r_{in} .

Thus, the analysis presented in this paper holds for $s > -1/2$. For $s < -1/2$, our analysis may present a qualitative result.

APPENDIX B: FLICKER NOISE IN ADAF

The dynamic equations of ADAFs read as follows (Narayan & Yi 1994; Kato, Fukue & Mineshige 2008; Li & Cao 2009):

$$\frac{\partial}{\partial t}(2H\rho) = \frac{1}{2\pi r} \left(\frac{\partial \dot{M}}{\partial r} - \Phi \right),$$

$$\rho r \Omega^2 - \rho r \Omega_K^2 - \frac{\partial p}{\partial r} = 0,$$

$$2H\rho \left[\frac{\partial v_\varphi}{\partial t} + \frac{v_r}{r} \frac{\partial}{\partial r} (r v_\varphi) \right] + \frac{\Phi}{2\pi r^2} \Delta l = \frac{1}{r^2} \frac{\partial}{\partial r} (2Hr^2 \tau_{r\varphi}),$$

$$\frac{1}{\gamma_2 - 1} \left(\frac{\partial p}{\partial t} + v_r \frac{\partial p}{\partial r} - \gamma_1 \frac{p}{\rho} \frac{\partial \rho}{\partial t} - \gamma_1 \frac{p}{\rho} v_r \frac{\partial \rho}{\partial r} \right) = -\tau_{r\varphi} \left(r \frac{\partial \Omega}{\partial r} \right),$$

where γ_1 and γ_2 are the usual generalized ratios of the specific heat, $\tau_{r\varphi} = -\alpha p$, $H/r = \text{constant}$ and Φ , $\Delta l = l_\Phi - r v_\varphi$ maintain the value of the stationary state. The self-similar solution of the above equations is

$$v_{r,0} \propto r^{-1/2}, \quad \Omega_0 \propto r^{-3/2},$$

$$p_0 \propto r^{-5/2+s}, \quad \rho_0 \propto r^{-3/2+s}.$$

We introduce small deviations of the disc parameters from the stationary parameters as follows:

$$\alpha = \alpha_0(1 + \bar{\beta}), \quad \bar{\beta}' = r^s \bar{\beta},$$

$$v_r = -a_1 \alpha_0 r \Omega_K (1 + r^{-s} v), \quad \Omega = a_2 \Omega_K (1 + r^{-s} \omega),$$

$$p = a_3 \sqrt{r} \Omega_K^2 r^s (1 + r^{-s} \delta), \quad \rho = a_4 r^{-3/2+s} (1 + r^{-s} \sigma),$$

where a_1 , a_2 , a_3 , and a_4 are constant. With the above equations, the evolution equations of the perturbed variables are

$$\frac{1}{\alpha_0 \Omega_K} \frac{\partial \sigma}{\partial t} = -a_1 r \frac{\partial (v + \sigma)}{\partial r},$$

$$a_3 r \frac{d\delta}{dr} - \frac{5}{2} a_3 \delta - 2a_4 a_2^2 \omega - a_4 (a_2^2 - 1) \sigma = 0,$$

$$\frac{1}{\alpha \Omega_K} \frac{\partial \omega}{\partial t} - a_1 r \frac{\partial \omega}{\partial r} + \frac{a_3}{a_4 a_2} r \frac{\partial \delta}{\partial r} - \frac{1}{2} a_1 (1 - 2s) \omega$$

$$- \frac{1}{2} a_1 v - \frac{1}{2} a_1 \sigma + \frac{a_3}{2a_4 a_2} \delta = \frac{a_3}{a_4 a_2} r \frac{\partial \bar{\beta}'}{\partial r} + \frac{a_3}{2a_4 a_2} \bar{\beta}',$$

$$\frac{1}{\alpha \Omega_K} \frac{\partial \delta}{\partial t} - \gamma_1 \frac{1}{\alpha \Omega_K} \frac{\partial \sigma}{\partial t} - a_1 r \frac{\partial \delta}{\partial r} + \gamma_1 a_1 r \frac{\partial \sigma}{\partial r} - (\gamma_2 - 1)$$

$$\times a_2 r \frac{\partial \omega}{\partial r} + \left[\frac{5}{2} a_1 + \left(s - \frac{3}{2} \right) \gamma_1 a_1 + \frac{3}{2} (\gamma_2 - 1) a_2 \right] \delta$$

$$+ \left(\frac{5}{2} - s - \frac{3}{2} \gamma_1 + s \gamma_1 \right) a_1 v - s \gamma_1 a_1 \sigma + \left(s + \frac{3}{2} \right)$$

$$\times (\gamma_2 - 1) a_2 \omega = -\frac{3}{2} (\gamma_2 - 1) a_2 \beta'.$$

The accretion rate is

$$\dot{M} = -4\pi r v_r \rho H$$

and its fluctuating component is

$$\dot{m}(r, f) = 4\pi a_1 a_4 \alpha_0 \left(\frac{H}{r} \right) [v + \sigma].$$

For $\sqrt{\langle \bar{\beta} \rangle^2} \propto r^0$, the power spectrum is (see Lyubarskii 1997, Section 5)

$$p(f) \propto f^{-[1+(4/3)s]}.$$

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