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Nonextensive and extensive thermostatistic properties of Fermi systems trapped in different external potentials*

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The thermostatistic properties of a q-generalized Fermi system trapped in a generic power-law potential are studied, based on the generalized statistic distribution derived from the Tsallis entropy. The total number of particles, the total energy, and the heat capacity at constant volume of the system are derived. The thermostatistic characteristics of the system are discussed in detail. It is found that the thermostatistic properties of such a system depend closely on parameter q, dimensional number of the space, kinetic characteristics of particles and shapes of the external potential, and the external potential has a great influence on the thermostatistic properties of the system. Moreover, it is shown that the results obtained here are very general and can be used to unify the description of the nonextensive and extensive thermostatistic properties of a class of Fermi systems trapped in different external potentials so that the important conclusions of many typical Fermi systems in the literature may be directly derived from the present paper.

Keywords: generalized Fermi system, external potential, thermostatistic property
PACC: 0530, 0520, 0570

1. Introduction

In recent years, it has been considered that the systems with spatial and/or temporal long-range interactions are nonextensive and the conventional Boltzmann–Gibbs (BG) statistical mechanics needs generalizing for the statistical description of the features of the systems. The nonextensive generalization of BG statistical mechanics was carried out first by constructing a new form of entropy with a nonextensive parameter q different from unity.[1] And subsequently it was further developed and widely used to analyse the thermostatistic properties of many such nonextensive systems.[2−8] A representative set of examples are the dynamic linear response theory,[7] the Lévy distributions,[9] the ground-state geometry of silicon cluster,[10] the nonionized hydrogen atom system[11] with $q < 1$, the dark magnetism,[12] the cosmic background radiation,[13] the pure-electron plasma with $q = 1/2$,[14] the quantum scattering of spinless particles,[15] etc. The novel results obtained show that the nonextensive generalization of BG statistical mechanics is a powerful tool in such studies and the nonextensive parameter q can play an important role.

It is well known that the Bose and Fermi systems are two types of the basic quantum systems in the natural world. In the experiment of the Bose–Einstein condensation (BEC), it can be said that the external potential creates favourable conditions for controlling degenerate atomic gases and quantitatively investigating their performance. Unlike the Bose system, the Fermi system does not undergo a phase transition under the constrained conditions of external potentials and very low temperatures. However, under suitable conditions the fermions may form Fermi pairs and become a Bose gas.[16] The BEC may occur in such a system and was recently achieved by Markus Greiner *et al.*[17] In the last few years, the q-generalized statistical mechanics has been used to investigate the q-generalized free Fermi system and some important results have been obtained.[18] However, the properties of a trapped q-Fermi system, which may be more closely related to the experiments, have been seldom studied.

In the present paper, we systemically investigate the thermostatistic properties of a q-generalized Fermi system trapped in a generic power-law potential, so

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that the results obtained here can unify the description of the nonextensive and extensive thermostatistic properties of a class of Fermi systems.

2. Total particle number and energy of the system

With the help of the dilute gas assumption and the approximation method called factorization approach, some researchers[19−21] derived a generalized Fermi–Dirac (FD) distribution function, i.e.

$$n_q = \frac{1}{[1+(q-1)\beta(\varepsilon-\mu)]^{1/(q-1)}+1}, \tag{1}$$

where n_q is the average occupation number in a state with energy ε, $\beta = 1/(kT)$ with T being the absolute temperature, and μ is the chemical potential. When $q = 1$, expression (1) becomes the well-known FD distribution.

It should be pointed out that the deviation of the factorization approximation (FA) can be neglected when the temperature is out of the forbidden zone of the dilute approximation (FZDA).[22] For example, the forbidden zone will appear at 10^{10} K for a system with the particle number $N_q = 10^5$ and 10^{20} K for $N_q = 10^{15}$. As a matter of fact, most of practical systems are far away from the FZDA so the FA can be used to investigate the properties of the quantum nonextensive systems.

According to the Pauli principle and expression (1), one has

$$1+(q-1)\beta(\varepsilon-\mu)\geq 0 \tag{2}$$

for an arbitrary value of q. From inequality (2), we obtain[18]

$$\varepsilon \begin{cases} \leq \mu + \dfrac{1}{(1-q)\beta} & (q\leq 1) \\ \geq \mu - \dfrac{1}{(q-1)\beta} & (q\geq 1) \end{cases}. \tag{3}$$

On the other hand, it is easily seen from the expression $[1+(q-1)\beta(\varepsilon-\mu)]^{\frac{1}{q-1}}$ that for an arbitrary value of q, the expression is smaller than 1 when $\varepsilon-\mu<0$ and larger than 1 when $\varepsilon-\mu>0$. Using the relation $[1+(q-1)\beta(\varepsilon-\mu)]^{1/(q-1)} = Z_q^{-1}[1+(q-1)\beta Z_q^{q-1}\varepsilon]^{1/(q-1)}$ and expression (1), we obtain

$$n_q = \begin{cases} \sum\limits_{j=0}^{\infty}(-1)^j Z_q^{-j}[1+(q-1)\beta Z_q^{q-1}\varepsilon]^{\frac{j}{q-1}} & (\varepsilon<\mu), \\ \dfrac{1}{2} & (\varepsilon=\mu), \\ \sum\limits_{j=1}^{\infty}(-1)^{j-1} Z_q^{j}[1+(q-1)\beta Z_q^{q-1}\varepsilon]^{\frac{j}{1-q}} & (\varepsilon>\mu), \end{cases} \tag{4}$$

where

$$Z_q = [1+(1-q)\beta\mu]^{\frac{1}{1-q}} \tag{5}$$

is called the q-generalized fugacity.[23]

Now, we consider a q-generalized Fermi system trapped in a generic power-law potential. The Hamiltonian of a single particle in the system can be written as[24,25]

$$H = ap^s + \sum_{i=1}^{n} U_i\left|\frac{r_i}{L_i}\right|^{t_i}, \tag{6}$$

where a, s, U_i and L_i are all positive constants, and p and r_i are the momentum and the i-th component of coordinate of a particle, respectively. The energy spectrum given above is very general in which the different values of a and s indicate the different kinematic characteristics of particles and the different parameters U_i, L_i and t_i correspond to different strengths and shapes of the external potentials.

When the number of particles in the system is large and the potential energy of particles in a trap is much smaller than their kinetic energy, the Thomas–Fermi semiclassical approximation is valid,[26] so that the total number of quantum states for $H\leq\varepsilon$ may be expressed as

$$\sum(\varepsilon) = \frac{g}{h^n}\int_{H\leq\varepsilon}\prod_{i=1}^{n}(\mathrm{d}r_i\mathrm{d}p_i), \tag{7}$$

where g is the degree of degeneracy and h is the Planck constant. The derivation of expression (7) with respect to ε yields the expression of the density of states for an ideal system trapped in a generic power-law potential, i.e.

$$\begin{aligned} & D(\varepsilon) \\ = & \frac{\partial \Sigma(\varepsilon)}{\partial \varepsilon} \\ = & \frac{2^n g \pi^{n/2}}{h^n \Gamma(n/2+1) a^{n/s}} \\ & \times \prod_{i=1}^{n} \frac{L_i \Gamma(1/t_i+1)}{U_i^{1/t_i}} \frac{\Gamma(n/s+1)}{\Gamma(\delta)} \varepsilon^{\delta-1}, \end{aligned} \tag{8}$$

where

$$\delta = n/s + \sum_{i=1}^{n} 1/t_i. \tag{9}$$

It is seen from expression (9) that δ is a characteristic parameter of the system. It is dependent only on the space dimensionality n, kinematic characteristics of particles s, and positive parameters t_i which describe the external potential, but it is independent of the nonextensivity parameter q. It is significant to note that the different values of δ represent different Fermi systems which may have different external potentials or different kinematic characteristics of particles, but the different Fermi systems may have the identical values of δ because one value of δ may correspond to the different choices of parameters n, s and t_i $(i = 1, 2, 3, ..., n)$. For example, when $\delta = 3$, the Fermi system may be a relativistic Fermi gas confined in a three-dimensional rigid container ($n = 3, s = 1$, and $t_i \to \infty$), or a nonrelativistic Fermi gas trapped in a three-dimensional harmonic potential ($n = 3,\ s = 2,$ and $t_i = 2$), or a nonrelativistic Fermi system confined in a six-dimensional rigid container ($n = 6,\ s = 2,$ and $t_i \to \infty$). It should be further pointed out that although the different Fermi systems may have identical values of δ, these Fermi systems will have different thermostatistic properties because parameters n,s, t_i $(i = 1, 2, 3, ..., n)$, a, U_i, and L_i are closely dependent on the choice of different systems.

By using expression (8), the total particle number and the energy of the system can be, respectively, written as

$$N = \int D(\varepsilon) n_q \mathrm{d}\varepsilon = \frac{2^n g \pi^{n/2}}{h^n \Gamma(n/2+1) a^{n/s}} \prod_{i=1}^{n} \frac{L_i \Gamma(1/t_i+1)}{U_i^{1/t_i}} \frac{\Gamma(n/s+1)}{\Gamma(\delta)} I_{\delta-1} \tag{10}$$

and

$$E = \int \varepsilon D(\varepsilon) n_q \mathrm{d}\varepsilon = \frac{2^n g \pi^{n/2}}{h^n \Gamma(n/2+1) a^{n/s}} \prod_{i=1}^{n} \frac{L_i \Gamma(1/t_i+1)}{U_i^{1/t_i}} \frac{\Gamma(n/s+1)}{\Gamma(\delta)} I_{\delta}, \tag{11}$$

where the parameter I_λ is given by

$$\begin{aligned} I_\lambda = & \int \frac{\varepsilon^\lambda \mathrm{d}\varepsilon}{[1+(q-1)\beta(\varepsilon-\mu)]^{\frac{1}{q-1}}+1} \\ = & \sum_{j=0}^{\infty} (-1)^j Z_q^{-j} \int_{-\infty}^{\mu} [1+(q-1)\beta Z_q^{q-1}\varepsilon]^{\frac{j}{q-1}} \varepsilon^\lambda \mathrm{d}\varepsilon + \sum_{j=1}^{\infty} (-1)^{j-1} Z_q^{j} \int_{\mu}^{\infty} [1+(q-1)\beta Z_q^{q-1}\varepsilon]^{\frac{j}{1-q}} \varepsilon^\lambda \mathrm{d}\varepsilon \end{aligned} \tag{12}$$

and λ is the parameter which may be equal to δ or $\delta - 1$. It should be pointed out that expression (12) is directly derived from expressions (4), (10) and (11), and the lower and upper bounds of the integration in expression (12) depend not only on the value of μ but also on the relation between ε and μ for the different cases of $q \le 1$ and $q \ge 1$. For example, when $\mu < 0$, it is necessary only to calculate the second integral in expression (12) because of the requirement of $\varepsilon \ge 0$. When $\mu \ge 0$, we have to calculate two integrals in expression (12) simultaneously. When $q \le 1$ or $q \ge 1$, we must use the first or the second relation in inequality (3) to determine the lower and upper bounds of the integrations in expression (12), respectively. Detailed discussion is given in Section 3.

3. General properties of the system

It is well known that the energy ε of a single particle in the system is a nonnegative real number, i.e.

$$\varepsilon \ge 0, \tag{13}$$

and it is constrained by different conditions when the values of q are different. Thus, it is necessary to discuss the thermostatistic properties of the system for two different cases of $q \geq 1$ and $q \leq 1$, separately.

3.1. The case of $q \geq 1$

When $\mu \leq 0$ and $0 \leq \varepsilon < \infty$, expression (12) may be expressed as

$$I_\lambda = \sum_{j=1}^{\infty} (-1)^{j-1} Z_q^j \int_0^\infty [1 + (q-1)\beta Z_q^{q-1}\varepsilon]^{\frac{j}{1-q}} \varepsilon^\lambda \mathrm{d}\varepsilon = \sum_{j=1}^{\infty} \frac{(-1)^{j-1} Z_q^j}{[(q-1)\beta Z_q^{q-1}]^{\lambda+1}} \frac{\Gamma\left(\frac{j}{q-1} - \lambda - 1\right)\Gamma(\lambda+1)}{\Gamma\left(\frac{j}{q-1}\right)}. \quad (14)$$

In order to guarantee that the integral value in expression (14) is larger than zero, the parameter q has to be restricted, and consequently, the condition $\frac{j}{q-1} > \lambda + 1$, i.e. $q < \frac{\lambda+2}{\lambda+1}$, must be satisfied. Substituting expression (14) into expressions (10) and (11) yields the total particle number and the total energy of the system, respectively, as

$$N = \frac{2^n g \pi^{n/2} \Gamma(n/s+1)(kT)^\delta}{h^n \Gamma(n/2+1) a^{n/s}} \times \prod_{i=1}^{n} \frac{L_i \Gamma(1/t_i + 1)}{U_i^{1/t_i}} f_{q,\delta}(Z_q) \quad (15)$$

and

$$E = \delta N k T \frac{f_{q,\delta+1}(Z_q)}{f_{q,\delta}(Z_q)}, \quad (16)$$

where

$$f_{q,D}(Z_q) = \sum_{j=1}^{\infty} \frac{(-1)^{j-1} Z_q^{j-(q-1)D}}{(q-1)^D} \times \frac{\Gamma\left(\frac{j}{q-1} - D\right)}{\Gamma\left(\frac{j}{q-1}\right)} \quad (17)$$

is the generalized Fermi integral.

Using the definition of the heat capacity at constant volume

$$C_{V,q} = \left(\frac{\partial E}{\partial T}\right)_V \quad (18)$$

and expression (16), one can calculate the heat capacity at constant volume of the system as

$$C_{V,q} = \delta N k \left[(\delta+1)\frac{f_{q,\delta+1}(Z_q)}{f_{q,\delta}(Z_q)} - \delta \frac{f_{q,\delta}(Z_q)}{f_{q,\delta-1}(Z_q)}\right]. \quad (19)$$

When $0 < \mu \leq \frac{1}{\beta(q-1)}$, $1 \leq q \leq 1 + \frac{1}{\mu\beta}$ and $0 \leq \varepsilon < \infty$, expression (12) may be expressed as

$$\begin{aligned} I_\lambda &= \sum_{j=0}^{\infty} (-1)^j Z_q^{-j} \int_0^\mu [1 + (q-1)\beta Z_q^{q-1}\varepsilon]^{j/(q-1)} \varepsilon^\lambda \mathrm{d}\varepsilon + \sum_{j=1}^{\infty} (-1)^{j-1} Z_q^j \int_\mu^\infty [1 + (q-1)\beta Z_q^{q-1}\varepsilon]^{\frac{j}{1-q}} \varepsilon^\lambda \mathrm{d}\varepsilon \\ &= \sum_{j=0}^{\infty} \frac{(-1)^j Z_q^{-j}}{[(q-1)\beta Z_q^{q-1}]^{\lambda+1}} \frac{(Z_q^{q-1} - 1)^{\lambda+1} \mathrm{H}\left[\frac{j}{1-q}, \lambda+1, \lambda+2, 1 - Z_q^{q-1}\right]}{\lambda+1} \\ &\quad + \sum_{j=1}^{\infty} \frac{(-1)^{j-1} Z_q^j}{[(q-1)\beta Z_q^{q-1}]^{\lambda+1}} \frac{\mathrm{H}\left[\frac{j}{q-1}, \frac{j}{q-1} - \lambda - 1, \frac{j}{q-1} - \lambda, \frac{1}{1 - Z_q^{q-1}}\right]}{\left(\frac{j}{q-1} - \lambda - 1\right)(Z_q^{q-1} - 1)^{\frac{j}{q-1} - \lambda - 1}}, \end{aligned} \quad (20)$$

where $q < (\lambda+2)/(\lambda+1)$ must also be satisfied,

$$\mathrm{H}[a, b, b+1, c] = b \int_0^1 (1 - ct)^{-a} t^{b-1} \mathrm{d}t \quad (21)$$

is the so-called hyper-geometric function with a, b and c being some parameters which are independent of t. By using expressions (10), (11), (18) and (20), the expressions of N, E, and $C_{V,q}$ are, respectively, given by

$$N = \frac{2^n g \pi^{n/2}}{h^n \Gamma(n/2+1) a^{n/s}} \prod_{i=1}^{n} \frac{L_i \Gamma(1/t_i+1)}{U_i^{1/t_i}} \frac{\Gamma(n/s+1)}{\Gamma(\delta)} (kT)^{\delta} f_{q,\delta}(Z_q), \tag{22}$$

$$E = NkT \frac{f_{q,\delta+1}(Z_q)}{f_{q,\delta}(Z_q)}, \tag{23}$$

and

$$\frac{C_{V,q}}{Nk} = (\delta+1)\frac{f_{q,\delta+1}(Z_q)}{f_{q,\delta}(Z_q)} - \delta \frac{\partial f_{q,\delta+1}(Z_q)/\partial Z_q}{\partial f_{q,\delta}(Z_q)/\partial Z_q}, \tag{24}$$

where the generalized Fermi integral

$$f_{q,D}(Z_q) = \sum_{j=0}^{\infty} \frac{(-1)^j Z_q^{-j-(q-1)D}}{(q-1)^D} \frac{(Z_q^{q-1}-1)^D \mathrm{H}\left[\frac{j}{1-q}, D, 1+D, 1-Z_q^{q-1}\right]}{D} + \sum_{j=1}^{\infty} \frac{(-1)^{j-1} Z_q^{j-(q-1)D}}{(q-1)^D} \frac{(Z_q^{q-1}-1)^{D-\frac{j}{q-1}} \mathrm{H}\left[\frac{j}{q-1}, \frac{j}{q-1}-D, \frac{j}{q-1}-D+1, \frac{1}{1-Z_q^{q-1}}\right]}{\left(\frac{j}{q-1}-D\right)}. \tag{25}$$

When $1/[\beta(q-1)] < \mu$, it is seen from inequality (3) that there is a constrained condition $\varepsilon > 0$, while $\varepsilon = 0$ is unallowable. It is in contradiction with inequality (13). On the other hand, it is seen from expression (5) that when $1/[\beta(q-1)] < \mu$, Z_q may become an imaginary number. It implies that the case of $1/[\beta(q-1)] < \mu$ is not allowed for the Fermi systems with $q > 1$, which is a common characteristic of q-generalized Fermi systems.

3.2. The case of $q \leq 1$

From inequalities (3) and (13), we obtain the following relation:

$$0 \leq \varepsilon \leq \mu + 1/[(1-q)\beta]. \tag{26}$$

When $\mu < 0$, then $1 + 1/(\mu\beta) \leq q \leq 1$. By using inequality (26), expression (12) may be written as

$$I_\lambda = \sum_{j=1}^{\infty} (-1)^{j-1} Z_q^j \int_0^{\mu+\frac{1}{(1-q)\beta}} \left[1+(q-1)\beta Z_q^{q-1}\varepsilon\right]^{\frac{j}{1-q}} \varepsilon^{\lambda} \mathrm{d}\varepsilon. \tag{27}$$

Substituting expression (27) into expressions (10), (11) and (18), we find that the forms of total particle number, total energy and heat capacity at constant volume of the system are all the same as those of expressions (15), (16) and (19) respectively, while in this case the q-generalized Fermi integral may be expressed as

$$f_{q,D}(Z_q) = \sum_{j=1}^{\infty} \frac{(-1)^{j-1} Z_q^{j+(1-q)D}}{(1-q)^D} \frac{\Gamma\left(\frac{j}{1-q}+1\right)}{\Gamma\left(\frac{j}{1-q}+D+1\right)}. \tag{28}$$

When $\mu \geq 0$, the expressions of N, E, and $C_{V,q}$ are the same as expressions (22)–(24), while the q-Fermi integral is given by

$$f_{q,D}(Z_q) = \sum_{j=0}^{\infty} \frac{(-1)^j Z_q^{-j+(1-q)D}}{(1-q)^D} \frac{(1-Z_q^{q-1})^D \mathrm{H}\left[\frac{j}{1-q}, D, 1+D, 1-Z_q^{q-1}\right]}{D} + \sum_{j=1}^{\infty} \frac{(-1)^{j-1} Z_q^{(1-q)(D-1)}}{(1-q)^D} \frac{\mathrm{H}\left[1-D, \frac{j}{1-q}+1, \frac{j}{1-q}+2, Z_q^{q-1}\right]}{\frac{j}{1-q}+1}. \tag{29}$$

It can be seen from inequality (26) that when $q < 1$ and $\mu > 0$, the energy of particles is not allowed to be larger than the chemical potential. At zero temperature, all the energy levels below the Fermi energy level E_{F} are filled by particles, while all the energy levels above the Fermi energy level E_{F} are empty. From expression (10), one can derive the Fermi energy of the system as

$$E_{\mathrm{F}} = \left[\frac{N h^n \Gamma(n/2+1) a^{n/s}}{2^n g \pi^{n/2}} \prod_{i=1}^{n} \frac{U_i^{1/t_i}}{L_i \Gamma(1/t_i+1)} \frac{\Gamma(\delta+1)}{\Gamma(n/s+1)} \right]^{1/\delta}, \tag{30}$$

which is independent of q and the same as that of an original extensive Fermi system trapped in a generic power-law potential.

Using expressions (15), (22) and (30), one can generate the curves for the chemical potentials of a class of Fermi systems trapped in different external potentials versus dimensionless temperature kT/E_{F} for different values of q and δ as shown in Fig.1. It is seen from the curves in Fig.1 that the chemical potentials of the systems with $q \neq 1$ are different from that of the system with $q = 1$ and closely dependent not only on temperature but also on the parameters q and δ. When $q > 1$, the chemical potential of a nonextensive Fermi system is always less than the Fermi energy and there is a cut-off of the chemical potential. The straight lines in Fig.1 represent the critical condition of the cut-off. The slope of the straight line increases with the decrease of q, and the cut-off temperature decreases with the increase of the slope of straight line. It implies that the curves for the chemical potential of a nonextensive Fermi system against temperature are only allowed to be situated on the right side of the straight line with a slope of $kT/(q-1)$. When the value of q approaches to 1, the slope of the straight line becomes infinite, the cut-off temperature approaches to zero, and the chemical potential at the cut-off temperature is equal to the Fermi energy. When $q < 1$, the chemical potential of a nonextensive Fermi system at any non-zero temperature is always larger than that of an original Fermi system. The more the q value is away from 1, the larger the difference between them will be. When $T \to 0\,\mathrm{K}$, the difference will disappear and all the chemical potentials for the various systems with different values of $q < 1$ are equal to the Fermi energy. It is worth while to point out that the chemical potential of a nonextensive Fermi system with $q < 1$ is not a monotonic function of temperature and may be larger than the Fermi energy in a certain region of temperature.

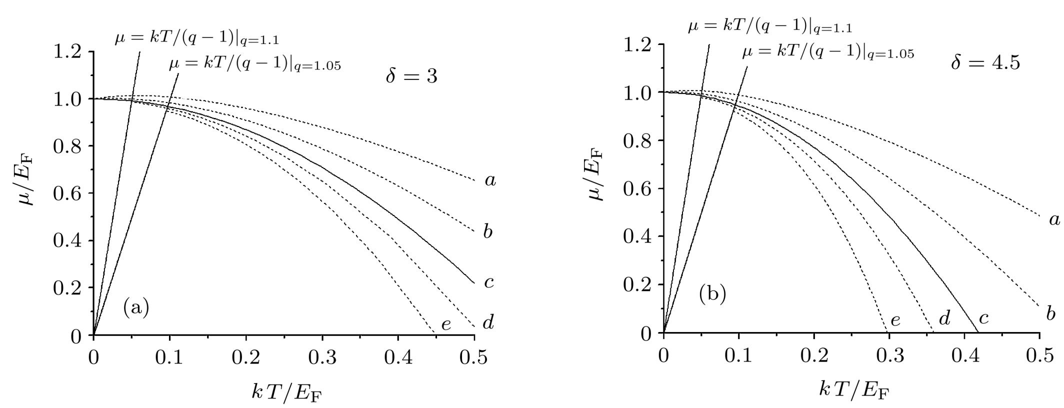

Fig.1. Curves for chemical potentials of a class of Fermi systems versus temperature, where curves a, b, c, d and e correspond to the cases of $q = 0.7$, 0.9, 1, 1.05 and 1.1, respectively.

Similarly, expressions (19), (24) and (30) can be used to plot the curves for heat capacities at constant volume of a class of Fermi systems at low and high temperatures against dimensionless temperature kT/E_{F} for different values of q and δ as shown in Figs.2 and 3, respectively. The dot curve in Fig.2 represents the different cut-off points for the different values of q. It is seen from the curves in Figs.2 and 3 that the heat capacities at constant volume of the Fermi systems increase with the increase of q for the same value of δ, but they are not a monotonic function of temperature in some cases. When $q > 1$, there is a cut-off of the heat capacity for a q-generalized Fermi system. The heat capacities of the systems with $q > 1$ are larger than that of the system with $q = 1$ and increase monotonically with temperature. When $q < 1$, the heat capacity at constant volume of a Fermi system will approach to zero when temperature is close to absolute zero no matter what the q value is, and consequently, the third law of thermodynamics still holds. At any temperature, the heat capacities of the systems with $q < 1$ are always less than that of the system with $q = 1$. When temperature is very low, the difference in heat capacity between the systems with $q < 1$ and $q = 1$ will not be obvious. The heat capacity of the system with $q = 1$ is a monotonically increasing function of temperature, while the heat capacities of the systems with $q < 1$ are not monotonic functions of temperature. However, unlike the nonextensive Bose system[23,27,28] that has a phase transition point and a heat capacity that may be discontinuous at the critical temperature of BEC, the q-generalized Fermi system has a heat capacity that continuously varies with temperature. It first increases and then decreases as temperature increases so that there is a maximum heat capacity at a certain value of temperature, which is due mainly to the fact that the fermions must be constrained by the Pauli principle. In addition, it is significant to note that at high temperatures, the heat capacities of the generalized Fermi systems do not approach to a constant. This is very unusual but coincides with the case of the generalized Bose system.[23,27–31] It implies that at high temperatures, the quantum effects of the generalized Bose and Fermi systems are negligible and consequently their thermostatistic properties tend to be accordant with each other.

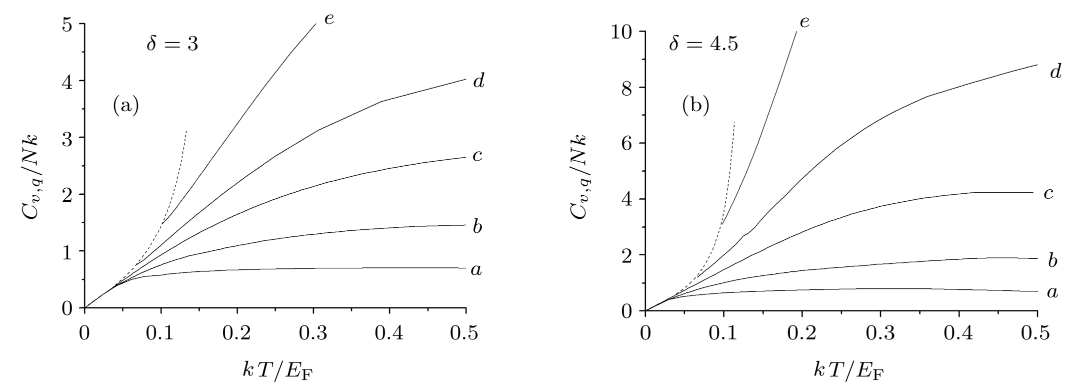

Fig.2. Curves for heat capacities at constant volume of a class of Fermi systems at low temperatures versus temperature, where curves a, b, c, d and e correspond to the cases of $q = 0.7$, 0.9, 1, 1.05 and 1.1, respectively.

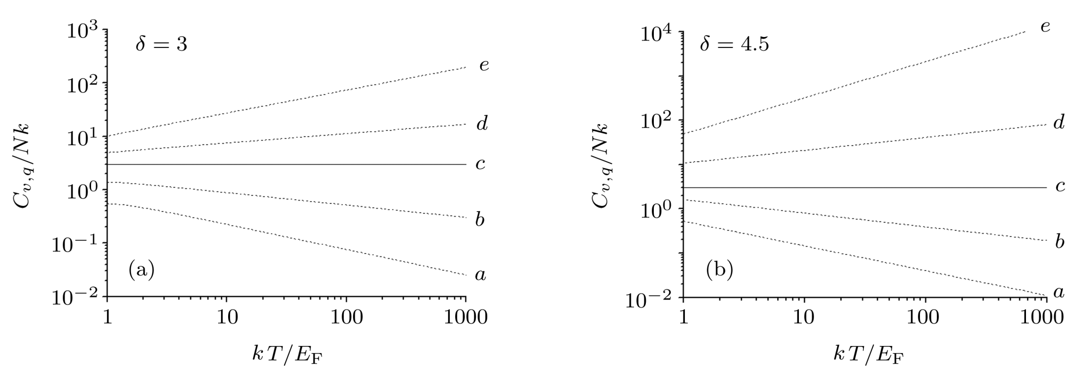

Fig.3. Curves for heat capacities at constant volume of a class of Fermi systems at high temperatures versus temperature, where curves a, b, c, d and e correspond to the cases of $q = 0.7$, 0.9, 1, 1.05 and 1.1, respectively.

4. Discussion

It is interesting to note that the results obtained above are very general and can be used to derive the thermostatistic properties of ordinary and q-generalized Fermi systems trapped in different external potentials by appropriately choosing different values of parameters a, s, U_i, L_i and n. Several typical examples are given as follows.

4.1. Nonextensive Fermi system trapped in a harmonic oscillator potential

When $a = 1/(2m)$, $t_i = 2$ and $U_i/L_i^2 = m\omega_i^2/2$ $(i = 1, 2, \ldots, n)$, the above results can be used to derive the thermostatistic properties of a q-generalized Fermi system trapped in an n-dimensional harmonic potential with frequency ω_i. In such a case, expression (9) is simplified to

$$\delta = n/s + n/2. \tag{31}$$

When $q \geq 1$, the total number of particles of the system in the case of $\mu \leq 0$

$$N = \frac{g\pi^n \Gamma(n/s+1)(kT)^\delta}{h^n \prod_i \omega_i \Gamma(n/2+1)a^{n/s}(m/2)^{n/2}} f_{q,\delta}(Z_q), \tag{32}$$

and that in the case of $0 < \mu \leq \dfrac{1}{\beta(q-1)}$

$$N = \frac{g\pi^n}{h^n \prod_i \omega_i \Gamma(n/2+1)a^{n/s}} \frac{\Gamma(n/s+1)(kT)^\delta}{(m/2)^{n/2}\Gamma(\delta)} f_{q,\delta}(Z_q) \tag{33}$$

can be, respectively, derived from expressions (15) and (22), while the total energy and heat capacity at constant volume of the system are still given by expressions (16) and (19) when $\mu \leq 0$ and by expressions (23) and (24) when $0 < \mu \leq \dfrac{1}{\beta(q-1)}$. Similarly, one can derive the total particle number, the total energy and the heat capacity at constant volume of the system from the above results when $q \leq 1$.

4.2. Nonextensive Fermi system confined in a rigid container

When $t_i \to \infty$ $(i = 1, 2, \ldots n)$, expression (9) is simplified into

$$\delta = n/s \tag{34}$$

and the above equations can be used to derive the thermostatistic properties of a q-generalized nonrelativistic or relativistic Fermi system confined in a rigid container with an n-dimensional volume. The results obtained are just the same as those derived in Ref.[18].

4.3. Ordinary Fermi system

When $q \to 1$, expression (1) becomes the well-known FD distribution function. It is easily seen from the above equations that the thermostatistic properties of an ordinary Fermi system trapped in a generic power-law potential are given by[25]

$$N = \frac{2^n g\pi^{n/2}\Gamma(n/s+1)(kT)^\delta}{h^n\Gamma(n/2+1)a^{n/s}} \times \prod_{i=1}^{n} \frac{L_i\Gamma(1/t_i+1)}{U_i^{1/t_i}} f_\delta(Z), \tag{35}$$

$$E = \delta NkT\frac{f_{\delta+1}(Z)}{f_\delta(Z)}, \tag{36}$$

and

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \delta Nk\left[(\delta+1)\frac{f_{\delta+1}(Z)}{f_\delta(Z)} - \delta\frac{f_\delta(Z)}{f_{\delta-1}(Z)}\right], \tag{37}$$

where $Z = e^{\beta\mu}$ is the fugacity and $f_D(Z) = \dfrac{1}{\Gamma(D)}\displaystyle\int_0^\infty \frac{x^{D-1}dx}{Z^{-1}e^x+1}$ is the original Fermi integral. Expressions (35)–(37) may be used to discuss the thermostatistic properties of an ordinary Fermi gas system trapped in different external potentials such as a harmonic oscillator potential[32] and a rigid container. For example, when $n = 3$, $s = 2$, $a = 1/(2m)$, and $t_i \to \infty$ are chosen, the thermostatistic properties of a three-dimensional nonrelativistic Fermi system mentioned often in textbooks[33] can be easily derived from expressions (35)–(37).

5. Conclusions

With the help of the q-generalized FD distribution function and the density of states of a Fermi system trapped in a generic power-law potential, we have successfully derived the analytic expressions for the total particle number, the total energy, and the heat capacity at constant volume of a q-generalized Fermi system trapped in a generic power-law potential by introducing some significant physical parameters such as the q-generalized fugacity, generalized

Fermi integral, and so on. The effects of the kinetic characteristics of particles, shape of external potential, dimensional number of the space and nonextensive parameter on the properties of the system are discussed in detail. It is found that when $q > 1$, the chemical potential and the heat capacity at constant volume of a q-generalized Fermi system at low temperatures must be cut off and the chemical potential is always smaller than the Fermi energy; when $q < 1$, the chemical potential of a q-generalized Fermi system in a certain region of temperature may be larger than the Fermi energy and there exists a maximum of the chemical potential which is a common characteristic of q-generalized Fermi systems. It is also found that at low temperatures, the thermodynamic properties of a generalized Fermi system are very different from those of a generalized Bose system. However, at high temperatures, the quantum effects of generalized Fermi and Bose systems are negligible and consequently they have the same unusual behaviour of the heat capacity at constant volume. Moreover, it is expounded that the thermodynamic properties of typical Fermi systems trapped in different external potentials can be derived from the present paper so that the results obtained here can play an important role in unifying the description of thermodynamic properties of many nonextensive and extensive Fermi systems.

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