Arbitrary-to-linear or linear-to-arbitrary polarization controller based on Faraday and Pockels effects in a single BGO crystal

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Abstract: We propose an arbitrary-to-linear or linear-to-arbitrary polarization controller based on the mutual action of Faraday and Pockels effects in a single $Bi_4Ge_3O_{12}$ (BGO) crystal after the wave coupling theory describing these two effects. It is demonstrated that, the expected conversion of arbitrary-to-linear or linear-to-arbitrary polarization state of light can be realized by adjusting the applied electric and magnetic fields.

OCIS codes: (260.5430) Polarization; (190.4360) Nonlinear optics, devices; (230.2090) Electro-optical devices; (160.3820) Magneto-optical materials.

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1. Introduction

The state of polarization (SOP), as a fundamental property of a light wave, has drawn a great deal of attention due to its interesting properties and potential applications. In polarization sensitive systems such as waveguides [1], coherent detections [2] and polarization-based switches [3], SOP effect plays an important role. And in wave-length-division-multiplexing (WDM) systems, polarization-mode dispersion (PMD), polarization-dependent loss (PDL) and unpredicted SOP drift (due to thermal, mechanical or pressure perturbations) can accumulate and become considerable obstacles to the long-haul telecommunications [4-7]. Also, the use of elliptically polarized fields has added a new dimension to the study of multiphoton processes, such as the asymmetry of angular distributions in above-threshold ionization (ATI), in which the laser ellipticity is an important parameter [8,9]. In micromanipulation it was demonstrated that the polarization of laser beam (optical spanner) can smoothly control the rotation of small particles or orientation of molecules [10-11]. The elliptically polarized pump-probe technology has revealed that the electron-spin relaxation in semiconductor quantum well depends evidently on the incident ellipticity of laser beam [12]. Besides, recent interest is also attracted to explore the potential of elliptically polarized light in promoting the asymmetric photolysis of racemic organic substrates and producing measurable enantiomeric excesses [13].

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For the control of the SOP of light, a number of useful polarization controllers, such as the squeezed fiber [14], rotating wave-plates [15], electro-optic wave-plates [16], Faraday rotators [17-20], and rotating magnetic field type [21] have been reported. Other kinds are based on nematic liquid crystals, which possess a superiority of low operating voltage [22,23].Very Recently, a simple waveplate analyzer using magneto-optic (MO) polarization rotators is also proposed [24]. In this paper, we propose another kind of polarization controller, i.e., arbitrary-to-linear or linear-to-arbitrary polarization controller, which consists of only one crystal without mechanically moving parts, completely driven by magnetic and electric fields, so operating fast, accurately and stably.

2. Theory

Here, we would like to use the wave coupling theory of the mutual action of Faraday and Pockels effects to discuss the principle of our polarization controller. Recently, we proposed an electrically and magnetically controlled optical spanner based on the wave coupling theory of Faraday and Pockels effects in an optically active medium [25]. We find from the theory that, if a suitable medium with Faraday and Pockels effects but without natural optical activity (e.g., $Bi_4Ge_3O_{12}$ (BGO) crystal [26]) is used to construct the SOP controller, the conversion of arbitrary-to-linear or linear-to-arbitrary polarization of light can be easily achieved by adjusting the applied magnetic and electric fields. Now we begin to discuss its principle. When a light propagates in a medium, which exhibits Faraday and Pockels effects but without natural optical activity and is simultaneously subject to external magnetic field $\mathbf{B}(0)$ and electric field $\mathbf{E}(0)$, the second-order polarizations induced by the two fields should be

$$\begin{aligned}\mathbf{P}^{(2)}(\omega) &= \mathrm{P}_{MO}^{(2)}(\omega) + \mathrm{P}_{EO}^{(2)}(\omega) \\ &= i2\varepsilon_0\eta_{MO}^{(2)}(\omega,0)\mathbf{:}\mathbf{E}_1(r)\mathbf{B}(0)\exp(ik_1r) + i2\varepsilon_0\eta_{MO}^{(2)}(\omega,0)\mathbf{:}\mathbf{E}_2(r)\mathbf{B}(0)\exp(ik_2r) \\ &\quad + 2\varepsilon_0\chi_{EO}^{(2)}(\omega,0)\mathbf{:}\mathbf{E}_1(r)\mathbf{E}(0)\exp(ik_1r) + 2\varepsilon_0\chi_{EO}^{(2)}(\omega,0)\mathbf{:}\mathbf{E}_2(r)\mathbf{E}(0)\exp(ik_2r).\end{aligned} \tag{1}$$

where $\mathbf{E}_1(r)$ and $\mathbf{E}_2(r)$ denote two cross or independent components of light in the crystal; and subscripts MO and EO denote Faraday rotation [27] and Pockels effect [28], respectively. Similarly to Refs. [25,28], starting from Maxwell's equations, considering only the second-order nonlinearity described by Eq. (1), we can derive, under slow varying amplitude approximation and no-walk-off approximation, the wave-coupling equations describing the mutual action of Faraday and Pockels effects in the medium, as follows:

$$dE_1(r)/dr = (f_B/n_1 - id_1)E_2(r)\exp(i\Delta k) - id_2E_1(r), \tag{2a}$$

$$dE_2(r)/dr = (-f_B/n_2 - id_3)E_1(r)\exp(-i\Delta k) - id_4E_2(r). \tag{2b}$$

where Δk and $d_i(i=1,2,3,4)$ are the same as those in Ref. [28], while $f_B = -\sum_{jkl}(k_0B_0)a_jb_k\eta_{jkl}^{(2)}m_l$. It should be pointed out that in derivation of Eqs. (2), the relation $\eta_{jkl}^{(2)} = -\eta_{kjl}^{(2)}$ [27] is utilized, which leads to $\sum_{jkl}a_j\eta_{jkl}^{(2)}a_km_l = 0$ and $\sum_{jkl}b_j\eta_{jkl}^{(2)}b_km_l = 0$.

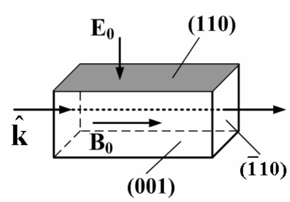

Fig. 1. The configuration of the BGO crystal and the directions of the two external fields.

We now choose one $5\text{mm}\times5\text{mm}\times4\text{cm}$ BGO crystal as the working medium, which is so designed: two (110) faces are coated with electrodes while two $(\bar{1}10)$ faces are well polished;

and the light travels along $[\bar{1}10]$ direction, i.e., $\hat{\mathbf{k}} = (-1/\sqrt{2}, 1/\sqrt{2}, 0)$ and $\Delta k = 0$; the magnetic $\mathbf{B}(0)$ is also applied along $[\bar{1}10]$ direction, then $\mathbf{a} = (0,0,1)$, $\mathbf{b} = \mathbf{c} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$, and $\mathbf{m} = (-1/\sqrt{2}, 1/\sqrt{2}, 0)$, where **a**, **b**, **c**, and **m** are four unit vectors parallel to $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{E}(0)$ and $\mathbf{B}(0)$, respectively (as shown in Fig. 1). Therefore $f = f_B/n_1 = f_B/n_2$, $d_2 = d_4 = 0$, and $d = d_1 = d_3 = k_0 n_0^3 \gamma_{63} E_0 / 2$; Eqs. (2) become

$$dE_1(r)/dr = (f - id)E_2(r), \qquad dE_2(r)/dr = (-f - id)E_1(r). \tag{3}$$

Assume that a laser at 632.8nm is used, then the corresponding Verder's constant $\mathrm{V} = 0.308\,\mathrm{rad/(T \cdot cm)}$ [29], electro-optic coefficient $\gamma_{63} = 1.03\,\mathrm{pm/V}$ and refractive index $n_0 = 2.07$ [30]. Subsequently, we can derive that $f = vB_0$ (in $\mathrm{cm^{-1}}$) (v=0.308) and $d = \tau E_0$ (in $\mathrm{cm^{-1}}$) ($\tau = 0.04536$), where B_0 (in Tesla) and E_0 (in kV/cm) denote the amplitudes of magnetic and electric fields, respectively.

3. Applications

3.1. Arbitrary-to-linear polarization controller

Firstly, we discuss the conversion of arbitrary-to-linear SOP. In this case, the initial conditions of light field are $E_1(0) = \cos\alpha$ and $E_2(0) = \sin\alpha \exp(i\delta)$, where α and δ are two parameters, which can be measured and determined by an Agilent 8509C polarization analyzer [19]. Subsequently, the solutions of Eq. (3) are

$$E_1(L) = \cos\alpha\cos(\sqrt{f^2 + d^2}L) + \sin\alpha\sin(\sqrt{f^2 + d^2}L)\exp[i(\delta - \theta)], \tag{4a}$$

$$E_2(L) = \sin\alpha\cos(\sqrt{f^2 + d^2}L)\exp(i\delta) - \cos\alpha\sin(\sqrt{f^2 + d^2}L)\exp(i\theta), \tag{4b}$$

where $\theta = \arg(f + id)$ and $L = 4\mathrm{cm}$. It is found that when $\theta = \delta$, i.e., $d/f = \tan\delta$, Eqs. (4) can be simplified to $E_1(L) = \cos(\alpha - \sqrt{f^2 + d^2}L)$ and $E_2(L) = \sin(\alpha - \sqrt{f^2 + d^2}L)\exp(i\delta)$, from which one can conclude that, when $2(\alpha - \sqrt{f^2 + d^2}L) = s\pi$ $(s = 0, \pm1, \pm2 \cdots)$, the output is linearly polarized, where $s = \pm1, \pm3, \pm5, \cdots$ corresponds to the horizontal linear polarization output (H), while $s = 0, \pm2, \pm4, \cdots$ to the vertical one (V). The corresponding mapping algorithm on Poincare sphere for the arbitrary-to-linear SOP conversion is shown in Fig. 2, marked by the blue lines. And, the required magnetic and electric fields can be easily calculated by using the following expressions:

$$B_0 = \frac{(s\pi/2 - \alpha)}{vL\sqrt{1 + \tan^2\delta}}, \qquad E_0 = \frac{(s\pi/2 - \alpha)\tan\delta}{\tau L\sqrt{1 + \tan^2\delta}}. \tag{5}$$

For example, with the input SOP $[E_1(0), E_2(0)]^T = [\cos 50°, \sin 50° \exp(i20°)]^T$, we can immediately calculate out that, $B_0 = 0.5326\mathrm{T}$ and $E_0 = 3.616\mathrm{kV/cm}$, for a horizontally linear polarization output, while $B_0 = 0.6658\mathrm{T}$ and $E_0 = 4.521\mathrm{kV/cm}$ for a vertically linear one. For an arbitrary input SOP, $0 \le \alpha \le \pi/2$. But $\alpha = 0$ and $\alpha = \pi/2$ are trivial since in these cases the input lights are linearly polarized already. So what we are concerned about is the case of $0 < \alpha < \pi/2$. If we wish only linearly polarized output, we can simply set $s = 0$. Then we can deduce the maximum fields required, $B_0 = 1.275\mathrm{T}$ and $E_0 = 8.657\mathrm{kV/cm}$, respectively, where $8.657\mathrm{kV/cm}$ is far smaller than the breakdown field of the material considered [31]. The required magnetic field can be supplied by an adjustable electromagnet [32], or for some special uses, by superconducting Helmholtz coil [33].

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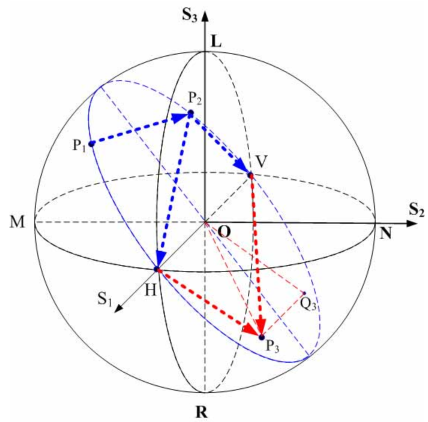

Fig. 2. Arbitrary-to-linear or linear-to-arbitrary SOP mapping algorithms on the Poincare sphere: V, vertical linear; H, horizontal linear; M, +45° linear; N, -45° linear; L, left-handed circular; R, right-handed circular; P_1, an arbitrary input SOP; P_3, an arbitrary output SOP; Q_3, projection of P_3 in S_2OS_3 plane. Conversion of arbitrary-to-linear polarization (blue lines): $P_1 \rightarrow P_2$ mapping $d/f = \tan\delta$; $P_2 \rightarrow V(\text{or H})$ to $2(\alpha - \sqrt{f^2+d^2}L) = s\pi$, where s is an odd (even) integer. Conversion of linear to arbitrary polarization (red lines): $H\,(\text{or V}) \rightarrow P_3$ mapping $\angle S_2OQ_3 = 2\sqrt{f^2+d^2}L$ and $\angle S_1OP_3 = \pi + \theta$ (or $\angle S_2OQ_3 = \pi - 2\sqrt{f^2+d^2}L$ and $\angle S_1OP_3 = \theta$).

3.2. Linear-to-arbitrary polarization controller

Secondly, we discuss the conversion of linear-to-arbitrary polarization. With the initial condition of horizontally linear polarization, i.e., $E_1(0) = 1$ and $E_2(0) = 0$, the solution of Eq. (3) can be easily gotten. It is

$$E_1(L) = \cos(\sqrt{f^2 + d^2}L)\,,\quad E_2(L) = \sin(\sqrt{f^2 + d^2}L)\exp[i(\pi + \theta)]\,, \tag{6}$$

where $\theta = \arg(f + id)$. The output light filed can be equivalently described by the following Stokes vector [34]

$$S_1 = \cos(2\sqrt{f^2 + d^2}L)\,, \tag{7a}$$

$$S_2 = \sin(2\sqrt{f^2 + d^2}L)\cos(\pi + \theta)\,, \tag{7b}$$

$$S_3 = \sin(2\sqrt{f^2 + d^2}L)\sin(\pi + \theta)\,. \tag{7c}$$

Equations (6) and (7) indicate that the retardation and Stokes vector are the functions of the two applied fields since $f = vB_0$ and $d = \tau E_0$ are the functions of two static fields. And Eqs. (7) have the same forms as those of unit-sphere coordinates with azimuth angle $(2\sqrt{f^2 + d^2}L)$ and polar angle $(\pi + \theta)$. The Stokes vectors described by Eqs. (7) will cover the whole surface of Poincare sphere when $(2\sqrt{f^2 + d^2}L)$ and $(\pi + \theta)$ are tuned in the range from 0 to 2π , which means that the conversion of linear-to-arbitrary polarization can be realized since the Poincare sphere corresponds to all of SOP. The mapping algorithm on the Poincare sphere for linear-to-arbitrary SOP conversion is also shown in Fig. 2, by red lines. To reach point (S_1, S_2, S_3) on the Poincare sphere, the applied fields should be set as

$$B_0 = \frac{S_2 \cos^{-1} S_1}{2vL\sqrt{1 - S_1^2}}\,,\qquad E_0 = \frac{S_3 \cos^{-1} S_1}{2\tau L\sqrt{1 - S_1^2}}\,. \tag{8}$$

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On the other hand, a Stokes vector can be represented by two parameters of the polarization ellipse, ψ and e, i.e.,

$$S_1 = \sqrt{\frac{1-\sin^2(2\tan^{-1}e)}{1+\tan^2 2\psi}}\ , \tag{9a}$$

$$S_2 = \tan 2\psi\sqrt{\frac{1-\sin^2(2\tan^{-1}e)}{1+\tan^2 2\psi}}\ , \tag{9b}$$

$$S_3 = \sin(2\tan^{-1}e)\ , \tag{9c}$$

where $\psi \in [-90°,90°]$ and $e \in [-1,1]$, representing azimuth angle and ellipticity of polarization ellipse, respectively; and the positive and negative for e correspond to right- and left-handed elliptical polarizations, respectively. Therefore in practice, for given ψ and e, we can calculate immediately that the required magnetic and electric fields according to Eqs. (8) and (9). For example, for the designated output SOP $\psi = 25°$ and $e = -0.5$, or $S = (0.3856, 0.4596, -0.8)$, we get $B_0 = 0.24\text{T}$ and $E_0 = -2.8\text{kV/cm}$, respectively. Fig. 3 shows the general correspondence between applied fields and designated output SOP. The color bars in Fig. 3 illustrates that, the maximum amplitudes of two external fields needed are also about 1.25T and 8.6kV/cm, respectively. Similarly, for the case of vertically linear polarization input, i.e., $E_1(0) = 0$ and $E_2(0) = 1$, the required fields can be derived as

$$B_0 = \frac{S_2(\pi - \cos^{-1}S_1)}{2vL\sqrt{1-S_1^2}}, \qquad E_0 = \frac{S_3(\pi - \cos^{-1}S_1)}{2\tau L\sqrt{1-S_1^2}}, \tag{10}$$

while Eqs. (9) are still valid.

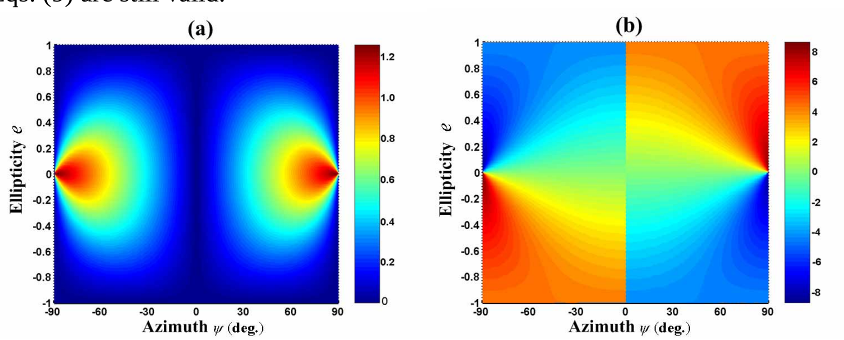

Fig. 3. Correspondence between applied fields and designated output SOP for horizontally linear polarization input: (a) Magnetic field (in Tesla) vs. designated output SOP; (b) Electric field (in kV/cm) vs. designated output SOP.

4. Summary

In summary, we have theoretically demonstrated both arbitrary-to-linear and linear-to-arbitrary polarization controllers based on the mutual action of Faraday rotation and Pockels effect in a single BGO crystal. And, one can find that with such two controllers combined together, the arbitrary-to-arbitrary SOP conversion could also be realized.

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