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# Electrically and magnetically controlled optical spanner based on the transfer of spin angular momentum of light in an optically active medium

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An optical spanner is a light beam that can exert a torque on an object. It is demonstrated in this Rapid Communication that, with the aid of applied electric and magnetic fields, a light beam with initially linear polarization and initially zero total spin angular momentum can interact with an optically active medium, resulting in a change of the ratio of left-handed circularly polarized photons to right-handed ones. Thus the total spin angular momentum of the light is changed, which leads to a torque, creating an electrically and magnetically controlled optical spanner on the medium. For a linearly polarized 632.8 nm laser beam incident on a 100-$\mu$m-long $Ce:Bi_{12}TiO_{20}$ whisker crystal with 5 $\mu$m radius, if the magnetic field is fixed at −1.8 T, both the left- (right-)handed circularly polarized photon number and the total spin angular momentum vary with the applied electric field in a sinusoidal way, which means the torque exerted by the optical spanner on the crystal also varies sinusoidally with the electric field. It is found that at 50 (or −50) kV/cm, 56% right- (left-)handed circularly polarized photons are translated into left- (right-)handed ones, which corresponds to a transfer of $0.56\hbar$ spin angular momentum contributed by each photon.

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The angular momentum of light as well as its transfer to form an optical spanner are of great interest and have attracted a number of theoretical and experimental researchers [1–3]. Poynting first theoretically associated the angular momentum of light with circular polarization [4]. By mechanical detection, Beth verified that, along the propagation direction of the beam, each circularly polarized photon possesses a spin angular momentum of $\hbar$ [5]. Recent interest in optical angular momentum can be traced back to the work of Allen *et al.*, who identified the total angular momentum of a classical light field and discussed the separation of spin and orbital angular momentum components [6,7]. Barnett made a further development in this direction. He derived the classical relation between spin (orbital) angular momentum flux and energy flux carried by light [8]. Van Enk and Nienhuis first considered the quantization of the spin and orbital angular momenta of light. They found that both spin and orbital angular momenta of a photon are well defined and separately measurable; and only the components along the propagation direction can be measured [9]. Simpson *et al.* proposed an experiment using optical tweezers and optical spanners with Laguerre-Gaussian modes [10]. The phenomenon of light-induced rotation of absorbing microscopic particles due to the transfer of angular momentum from light to the material was observed by using elliptically polarized beams [11] or beams with helical phase structure [12,13]. The novel phenomena of optically driven motors and optical torques on transparent birefringent particles of calcite held by optical tweezers have been demonstrated by Friese and co-workers [14,15]. Optically induced angular alignment of trapped birefringent micro-objects using linearly polarized light has also been demonstrated [16]. O'Neil and Padgett observed the analogous phenomenon in which the rotations of metallic particles were induced by the orbital angular momentum of the light beam [17]. Piccirillo *et al.* reported another kind of experiment using an elliptically shaped laser beam to control the molecular alignment in liquid-crystal films by the transfer of orbital angular momentum of light [18]. Recently, Padgett and co-workers exploited deep the intrinsic and extrinsic nature of the orbital angular momentum of a light beam [19] and proposed interferometric methods to measure the orbital and spin or the total angular momentum of a single photon [20]. And recently we presented a way using an applied dc electric field to manipulate the transfer of the spin angular momentum of light in an optically active medium [21]. We found that the manipulation of the transfer of spin angular momentum of light in an optically active medium can be realized much more neatly through the mutual action of the Pockels and Faraday effects. In this Rapid Communication, we will discuss this kind of control and its application as an optical spanner.

To discuss this, we first develop a theory describing the mutual action of Pockels and Faraday effects in an optically active crystal. The authors of *Handbook of Optics* [22] stated that "crystals of quartz are optically active, and this complicates use of the material as an electro-optic modulator." For the case of an optically active medium with electric and magnetic fields applied simultaneously, the theory of the interaction between light and the medium will become considerably more complicated. To avoid the difficulties of the traditional method, the wave coupling idea involved in She and Lee's theory [23] of the Pockels effect was adopted here.

In general, a monochromatic plane light wave with frequency $\omega$ in an optically active medium possessing birefringence can be decomposed into two independent polarized components, i.e.,

$$\mathbf{E}(\omega)=\mathbf{E}_1(\omega)+\mathbf{E}_2(\omega)=\mathbf{E}_1(r)\exp(ik_1r)+\mathbf{E}_2(r)\exp(ik_2r), \quad (1)$$

where $\mathbf{E}_1(\omega)$ and $\mathbf{E}_2(\omega)$ denote two cross components of the light field when $k_1=k_2$, while they denote two independent components experiencing different refractive indices when $k_1\neq k_2$. The polarization relative to spatial dispersion, re-

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sponsible for the optical activity, can be written as $\mathbf{P}_1^{(2)}(\omega) = i2\varepsilon_0\kappa_1^{(2)}\colon \mathbf{E}_1(r)\mathbf{k}_1 \exp(ik_1 r) + i2\varepsilon_0\kappa_2^{(2)}\colon \mathbf{E}_2(r)\mathbf{k}_2 \exp(ik_2 r)$ [24,25], where ":" stands for the double scalar product; and $\kappa_1^{(2)}$, $\kappa_2^{(2)}$ denote the second-order susceptibility tensor of natural optical activity, and generally $\kappa_1^{(2)} \neq \kappa_2^{(2)}$. In addition, there exist another two second-order polarizations responsible for the Pockels effect and Faraday effect, when the medium is simultaneously subject to a slowly varying electric $\mathbf{E}(0)$ and magnetic field $\mathbf{B}(0)$. They are, respectively,

$$\mathbf{P}_2^{(2)}(\omega) = 2\varepsilon_0\chi^{(2)}(\omega,0)\colon \mathbf{E}_1(r)\mathbf{E}(0)\exp(ik_1 r) + 2\varepsilon_0\chi^{(2)}(\omega,0)\colon\mathbf{E}_2(r)\mathbf{E}(0)\exp(ik_2 r)$$

and

$$\mathbf{P}_3^{(2)}(\omega) = i2\varepsilon_0\eta^{(2)}\colon \ \mathbf{E}_1(r)\mathbf{B}(0)\exp(ik_1 r) + i2\varepsilon_0\eta^{(2)}\colon \ \mathbf{E}_2(r)\mathbf{B}(0)\exp(ik_2 r)$$

[23–25], where $\chi^{(2)}$ and $\eta^{(2)}$ denote the second-order susceptibility tensors of the Pockels and Faraday effects, respectively. Then the total second-order polarization can be written as

$$\begin{aligned}\mathbf{P}^{(2)}(\omega) &= \mathbf{P}_1^{(2)}(\omega) + \mathbf{P}_2^{(2)}(\omega) + \mathbf{P}_3^{(2)}(\omega)\\ &= i2\varepsilon_0\kappa_1^{(2)}\colon\mathbf{E}_1(r)\mathbf{k}_1 \exp(ik_1 r)\\ &\quad + i2\varepsilon_0\kappa_2^{(2)}\colon\mathbf{E}_2(r)\mathbf{k}_2 \exp(ik_2 r)\\ &\quad + 2\varepsilon_0\chi^{(2)}(\omega,0)\colon\mathbf{E}_1(r)\mathbf{E}(0)\exp(ik_1 r)\\ &\quad + 2\varepsilon_0\chi^{(2)}(\omega,0)\colon\mathbf{E}_2(r)\mathbf{E}(0)\exp(ik_2 r)\\ &\quad + i2\varepsilon_0\eta^{(2)}\colon\mathbf{E}_1(r)\mathbf{B}(0)\exp(ik_1 r)\\ &\quad + i2\varepsilon_0\eta^{(2)}\colon\mathbf{E}_2(r)\mathbf{B}(0)\exp(ik_2 r).\end{aligned} \tag{2}$$

Similarly to Ref. [23], starting from Maxwell's equations, only considering the second-order nonlinearity and neglecting the high-order nonlinearity as well as the linear absorption, we can derive, under the slowly varying amplitude and the no-walk-off approximations, the wave-coupling equations of the mutual action of the Pockels and Faraday effects in an optically active medium possessing birefringence as follows:

$$\frac{dE_1(r)}{dr} = \left(\frac{f_{0N}}{n_1} + \frac{f_{0B}}{n_1} - id_1\right)E_2(r)\exp(i\Delta k r) - id_2E_1(r), \tag{3a}$$

$$\frac{dE_2(r)}{dr} = \left(-\frac{f_{0N}}{n_2} - \frac{f_{0B}}{n_2} - id_3\right)E_1(r)\exp(-i\Delta k r) - id_4E_2(r), \tag{3b}$$

where $\Delta k = k_2 - k_1$ and $d_i$ $(i=1,2,3,4)$ are the same as those in Ref. [23]; and $f_{0N} = -\Sigma_{jkl}(k_0^2)(a_j n_2 \kappa_{2jkl}^{(2)} b_k \hat{k}_l) = \Sigma_{jkl}(k_0^2) \times (b_j n_1 \kappa_{1jkl}^{(2)} a_k \hat{k}_l)$ (the second equation is the requirement of the law of conservation of energy), $f_{0B} = -\Sigma_{jkl}(k_0 B_0) \times (a_j \eta_{jkl}^{(2)} b_k m_l)$, where $a_j$, $b_k$, $m_l$, and $\hat{k}_l$ are the components of four unit vectors **a**, **b**, **m**, and $\hat{\mathbf{k}}$ parallel to $\mathbf{E}_1$, $\mathbf{E}_2$, $\mathbf{B}(0)$, and **k**, respectively. It should be noticed that in derivation of Eqs. (3a) and (3b), the relation $\kappa_{jkl}^{(2)} = -\kappa_{kjl}^{(2)}$ and $\eta_{jkl}^{(2)} = -\eta_{kjl}^{(2)}$ [24] has been used, which leads to $\Sigma_{jkl} a_j \kappa_{1jkl}^{(2)} a_k \hat{k}_l = 0$, $\Sigma_{jkl} b_j \kappa_{2jkl}^{(2)} b_k \hat{k}_l = 0$, $\Sigma_{jkl} a_j \eta_{jkl}^{(2)} a_k m_l = 0$, and $\Sigma_{jkl} b_j \eta_{jkl}^{(2)} b_k m_l = 0$.

Equations (3a) and (3b) can describe the propagation of a light with an arbitrary polarization state traveling along an arbitrary direction in any birefringent optically actively medium with slowly varying electric and magnetic fields applied along arbitrary directions. Under certain conditions, the equations will reduce to those describing pure natural optical activity, the Pockels effect, the Faraday effect, or a cross effect of two of them. For example, supposing a linearly polarized light with initial condition $E_1(0) = E_{in}$ and $E_2(0) = 0$ propagates along the optical axis of a uniaxial crystal or in a cubic crystal without inversion symmetry, we have $\Delta k = 0$. For the case of no applied electric field and magnetic field, $d_i = 0$ $(i=1,2,3,4)$ and $f_{0B} = 0$; Eqs. (3a) and (3b) have the solution $E_1(r) = E_{in}\cos(f_N r)$ and $E_2(r) = -E_{in}\sin(f_N r)$, which are just the equations describing the phenomena of pure natural gyrotropy and $f_N$ $(= f_{0N}/n_o)$ is the optical rotatory power. And, for a medium without optical activity and with only a magnetic field along the optical axis, $f_{0N} = 0$ and $d_i = 0$ $(i=1,2,3,4)$, the solution of Eqs. (3a) and (3b) becomes $E_1(r) = E_{in}\cos(f_B r)$ and $E_2(r) = -E_{in}\sin(f_B r)$. One can see that the ratio $f_B/B_0 = f_{0B}/(n_o B_0)$ is just Verder's constant known in Faraday rotation.

We now use the above theory to discuss the manipulation of the transfer of spin angular momentum of light in an optically active crystal $Ce:Bi_{12}TiO_{20}$ (Ce:BTO) and an electrically and magnetically controlled optical spanner on this crystal. The Ce:BTO crystal exhibits not only optical activity, but also the Pockels and Faraday effects as in crystalline quartz; in addition, its electro-optic coefficients and refractive index are larger than those of crystalline quartz. In the following, a Ce:BTO whisker crystals with 5 $\mu$m radius is considered. For simplicity, suppose that the propagation of light and the magnetic field are along the $[\bar{1}10]$ direction of the whisker crystal, while the electric field is along the [110] direction, i.e., $\mathbf{a} = (0,0,1)$, $\mathbf{b} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$, $\mathbf{c} = (1/\sqrt{2}, 1/\sqrt{2}, 0)$, $\mathbf{m} = (-1/\sqrt{2}, 1/\sqrt{2}, 0)$, and $\Delta k = 0$. According to the nonvanishing elements and symmetry of the second-order tensor of the Ce:BTO crystal (belonging to class 23) [26], we get $d_1 = d_3 = k_0 n_0^3 E_0 \gamma_{63}/2 = d$, $d_2 = d_4 = 0$ and $f_N = f_{0N}/n_0 = -k_0^2 \kappa_{xyz}^{(2)}$, $f_B = -k_0 \eta_{xyz}^{(2)} B_0/n_0$. Subsequently, Eqs. (3a) and (3b) read

$$dE_1(r)/dr = (f_N + f_B - id)E_2(r), \tag{4a}$$

$$dE_2(r)/dr = (-f_N - f_B - id)E_1(r). \tag{4b}$$

Assuming that $f_N > 0$ and the incident beam is linearly polarized with initial values $E_1(0) = E_{in}$ and $E_2(0) = 0$, then we have solutions of Eqs. (4a) and (4b) such that $E_1(r) = E_{in}\cos[\sqrt{(f_N+f_B)^2 + d^2}\, r]$ and $E_2(r) = E_{in} \times \sin[\sqrt{(f_N+f_B)^2 + d^2}\, r]\exp[i(\theta + \pi)]$, where $\theta = \arg(f_N + f_B + id)$. Generally, this light field can also be expressed as $\mathbf{E}(r) = [E_1(r), E_2(r)]^T = E_{in}(\alpha_L[1/\sqrt{2}, -i/\sqrt{2}]^T + \alpha_R[1/\sqrt{2}, i/\sqrt{2}]^T)$, where $\alpha_L = \{\cos[\sqrt{(f_N+f_B)^2 + d^2}\, r] - i\sin[\sqrt{(f_N+f_B)^2 + d^2}\, r]\exp(i\theta)\}/\sqrt{2}$ and $\alpha_R = \{\cos[\sqrt{(f_N+f_B)^2 + d^2}\, r] + i\sin[\sqrt{(f_N+f_B)^2 + d^2}\, r]\exp(i\theta)\}/\sqrt{2}$.

Let $\mathbf{e}_{-1}=[1/\sqrt{2},-i/\sqrt{2}]^T$, $\mathbf{e}_{+1}=[1/\sqrt{2},i/\sqrt{2}]^T$; then we have $\mathbf{e}_{+1}\cdot\mathbf{e}_{-1}^*=0$ and $\mathbf{e}_\mu\times\mathbf{e}_{\mu'}^*=-i\mu\hat{\mathbf{k}}\delta_{\mu\mu'}$ $(\mu,\mu'=\pm1)$, which show that the field $\mathbf{E}(r)$ is the superposition of left- and right-handed circularly polarized light. According to Ref. [27], in the basis of circular polarization the spin angular momentum operator is $\hat{\mathbf{J}}_s=\Sigma_k\hbar\hat{\mathbf{k}}(\hat{n}_{k,+1}-\hat{n}_{k,-1})$ ($\hat{n}_{k,+1}$ and $\hat{n}_{k,-1}$ are the right- and left-handed circularly polarized photon number operators, respectively), which means that the spin angular momentum of each right- (left-)handed circularly polarized photon is $\hbar\hat{\mathbf{k}}(-\hbar\hat{\mathbf{k}})$ and the total spin angular momentum associated with any one wave vector $\mathbf{k}$ is $\hbar\hat{\mathbf{k}}$ times the difference between the number of right- and left-handed circularly polarized photons. For the present case, there exists only one $\mathbf{k}$ mode, so $\hat{\mathbf{J}}_s=\hbar\hat{\mathbf{k}}(\hat{n}_{k,+1}-\hat{n}_{k,-1})$. On the other hand, it is known that the ratio of spin angular momentum flux to the energy flux of light is $\sigma\hbar/\hbar\omega$ [8], where $\sigma=\pm1$ for right- and left-handed circularly polarized light, respectively. Suppose the input and output surfaces of the Ce:BTO whisker crystal have perfect antireflection coatings; then at the output surface $N_L(-\hbar)/(c\varepsilon_0|\alpha_L E_{in}|^2/2)=-\hbar/\hbar\omega$ and $N_R\hbar/(c\varepsilon_0|\alpha_R E_{in}|^2/2)=\hbar/\hbar\omega$, where $N_L$ and $N_R$ are the numbers of left- and right-handed circularly polarized photons transmitted per unit area per second. Therefore

$$N_L = c\varepsilon_0|\alpha_L E_{in}|^2/2\hbar\omega, \quad N_R = c\varepsilon_0|\alpha_R E_{in}|^2/2\hbar\omega. \tag{5}$$

According to $\hat{\mathbf{J}}_s=\hbar\hat{\mathbf{k}}(\hat{n}_{k,+1}-\hat{n}_{k,-1})$ the total spin angular momentum of light transmitted per unit area per second is

$$M_S = \hbar(N_R - N_L) = (c\varepsilon_0 E_{in}^2/2\omega)(|\alpha_R|^2 - |\alpha_L|^2)$$
$$= -(c\varepsilon_0 E_{in}^2/2\omega)\sin\theta\sin[2\sqrt{(f_N+f_B)^2+d^2}r]. \tag{6}$$

Assuming a He-Ne laser beam at 632.8 nm is used, then we know the corresponding refractive index $n_0=2.54$, optical rotatory power $f_N=0.1103$ rad/mm [26], Verder's constant $V=0.0611$ rad/(T mm) [28], and the nonvanishing electro-optic coefficients $\gamma_{41}=\gamma_{52}=\gamma_{63}=7.37$ (in pm/V) [26]. From these parameters, we can get $f_B=0.0611B_0$ (in $\text{mm}^{-1}$) and $d=5.9\times10^{-2}E_0$ (in $\text{mm}^{-1}$), where $E_0$ and $B_0$ are in units of kV/cm and T, respectively. We now use two whisker crystals with lengths of 200 and 100 $\mu$m for calculating the photon numbers $N_L$ and $N_R$ and the total transmitted spin angular momentum $M_S$ using Eqs. (5) and (6). The numerical results are shown in Figs. 1 and 2, respectively.

From Figs. 1 and 2, we find that both the left- (right-) handed circularly polarized photon numbers and the total spin angular momentum are functions of the applied electric and magnetic fields, and they vary with the applied electric field in a sinusoidal way if the magnetic field is fixed at −1.8 T. For a 200 $\mu$m whisker crystal [Figs. 1(c), 1(d), and 2(b)], when the electric field changes from −66 to 66 kV/cm, the transfer of spin angular momentum per photon can change from $-\hbar$ to $\hbar$. However, the field may damage the crystalline structure of the medium when $E_0$ $>50$ kV/cm [29]. Calculations show that, at 50 kV/cm, the transfer of angular momentum can reach about $0.93\hbar$ per photon. For a 100 $\mu$m whisker crystal [Figs. 1(a), 1(b), and 2(a)], a 50 (−50) kV/cm electric field will cause 56% of

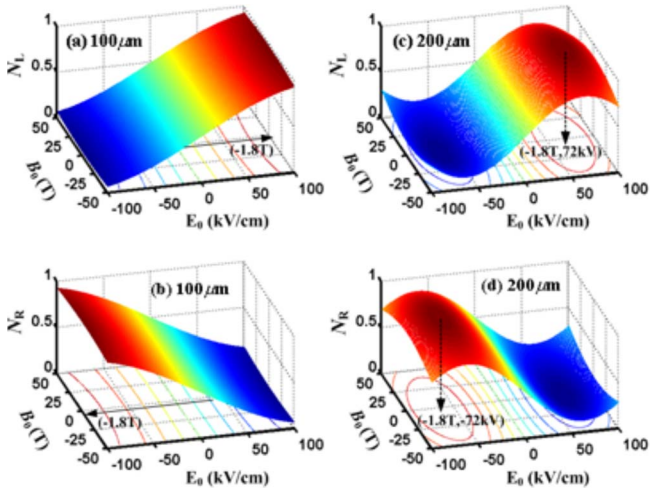


FIG. 1. (Color online) Normalized photon numbers of left- or right-handed circularly polarized light controlled by applied electric and magnetic fields for whiskers with lengths of 100 and 200 $\mu$m.

right- (left-)handed circularly polarized photons translate to left- (right-)handed ones, which corresponds to transfer of $0.56\hbar$ angular momentum per photon. When the electric field is fixed at 50 kV/cm, the transfer of spin angular momentum can be controlled by the magnetic field with the maximum sensitivity in practice. Without the aid of an electric field, the magnetic field cannot change the polarization state of the light any more. Obviously, the present method has much superiority over that using only an electric field to manipulate the transfer of spin angular momentum of light reported in our recent work [21]. Figure 1 demonstrates clearly that the total photon number is conserved although the left- or right-handed circularly polarized number varies with the applied electric and magnetic fields. Unlike the total photon number, the total spin angular momentum of light is not a constant and varies with the applied electric and magnetic fields (see Fig. 2). This indicates that there should be a transfer of angular momentum from the medium to the light induced by the applied fields, since the total spin angular momentum of the incident light is zero. According to the law of conserva-

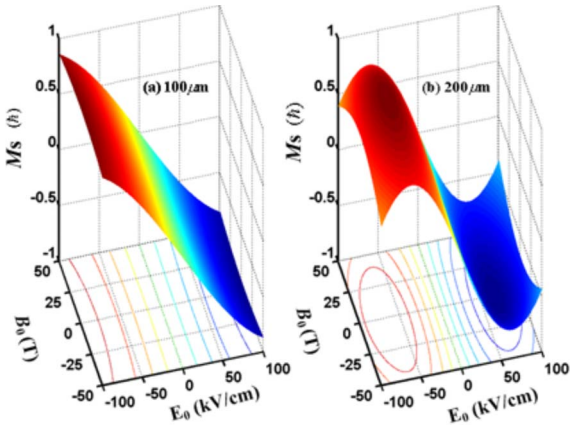


FIG. 2. (Color online). Total spin angular momentum (normalized) of light controlled by applied electric and magnetic fields for whiskers with lengths of 100 and 200 $\mu$m

tion of angular momentum, we know that light should transfer angular momentum to the medium in return and exert a torque on it when light passes through the whisker crystal. According to Eq. (6), a larger torque can be achieved by using a high-intensity light beam since the torque is proportional to intensity. It is known that the transfer of spin or orbital angular momentum from a light beam to particles held within optical tweezers can from an optical spanner [1–3]. We have shown in this Rapid Communication that, with the aid of applied electric and magnetic fields, a linearly polarized light can also form an optical spanner. In addition, the transfer of spin angular momentum and the torque on the medium can be continuously and neatly manipulated by tuning the two applied fields. For a crystal without any resistance, the torque induced by the transfer of angular momentum will drive it to rotate. Padgett *et al.* [30] derived, based on the Einstein box treatment [31], an expression for the medium's angular displacement driven by a single photon, $\alpha=-(l+\sigma)\hbar(n_g-1/n_\phi)s/Ic$, where $I$ is the moment of inertia of the disk, $n_g$ and $n_\phi$ are the phase refractive index and the group refractive index, respectively, and $l$ and $\sigma$ are the orbital and spin quantum numbers, respectively. For our conditions, $l=0$ and $n_g=n_\phi=n_0$. From Eq. (6), we can obtain, near the zero point of time, the rotation velocity of the microdisk. It is

$$\Omega = \frac{s\varepsilon_0 E_{in}^2}{2I\omega}\left(n_0 - \frac{1}{n_0}\right)\sin\theta \sin[2\sqrt{(f_N + f_B)^2 + d^2}s]. \quad (7)$$

Equation (7) indicates that the rotation of the disk, resulting from the torque exerted by light, can be driven and manipulated by tuning the electric and magnetic fields, which is similar to the manipulation of the transfer of spin angular momentum.

The magnetic field considered here is easily obtained from a Nd-Fe-B permanent magnet, whose magnetic field can be over 2 T [32]. On the other hand, since the electric field is applied transversely, we can use a pair of electrodes well polished with a diastema of 50 $\mu$m and then put the whisker crystal in it. The maximal voltage needed is about 250 V. The electrodes are put in a transparent vacuum chamber. This will effectively avoid the difficulty from high voltage. Further calculation shows that, if a crystal with much larger electro-optic coefficients is used, the length of the sample can be shortened remarkably and the maximal electric field needed can be further lowered. As a special case, for example, if a $Sr_{0.75}Ba_{0.25}Nb_2O_3$(SBN) crystal (whose $\gamma_{33}$ is 1340 pm/V) is used, we need only a 24-$\mu$m-long crystal and an 8.4 kV/cm electric field to result in a change of $2\hbar$ per photon when $B=0$ and, for a pair of electrodes with 50 $\mu$m diastema, the voltage needed is only 42 V. Additionally, spin-controlled orbital angular momentum change in inhomogeneous anisotropic $q$ plates has recently been reported [33,34]. We have now shown that the change of spin angular momentum of light can be controlled by electric and magnetic fields. This raises the possibility of using external electric and magnetic fields to control the transfer of orbital angular momentum in a medium. This is an interesting subject, worthy of further study.

In conclusion, we have shown theoretically that applied electric and magnetic fields can control the transfer of spin angular momentum of light in an optically active medium with the Pockels and Faraday effects and make linearly polarized light form an optical spanner, which can exert a torque on the medium. If the medium is without any resistance, the torque induced by the transfer of angular momentum will drive it to rotate, depending on the incident intensity of light and the applied electric and magnetic fields.

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