

# A Novel Cladding-mode Coupler Formed by Long Period Fiber Gratings and Microsphere Resonator

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## ABSTRACT

There are numerous methods for evanescently coupling energy into the modes of a sphere. In this paper, we provided a new methods, used long period fiber gratings (LPFG) coupling evanescent wave into microsphere resonator. We have illustrated this coupling mechanism in theoretical. As an potentially application, we describe a four-port passive Add/Drop device based on the microsphere-LPFG coupling mechanism.

**Keywords:** long-period fiber grating (LPFG)    microsphere resonator    evanescent wave    whispering gallery mode (WGM)

## 1. INTRODUCTION

A long-period fiber grating (LPFG) written in a single mode fiber is capable of coupling light from the fundamental mode to a specific cladding mode at specific wavelength (the resonance wavelengths) over a range of wavelengths [1]~[5]. If the light energy coupled to the cladding mode cannot be tapped, it will be either absorbed or made to radiate away. In reference [2], [5] they have demonstrated experimentally the phenomenon of evanescent-field coupling between two parallel LPFGs. According to this property, LPFG would be used as an optical add/drop multiplexer (OADM). But from the results of measured output spectra of tapping fiber, the bandwidths of the output spectra were relatively broad. If we can sharp the bandwidth, it will be near to the practical use.

On the other hand, fused-silica microspheres exhibiting whispering gallery modes have been attracting considerable attention because of their unique properties of high Q (up to  $10^{10}$ [7,10]) and small mode volume. Whispering gallery modes (WGM) in fused-silica microspheres have been excited using an evanescent field provided by a tapered optical fiber[8] or a prism[12], a side-polished optical fiber[11].

In this paper, we propose a new method, used a long period fiber to couple evanescent wave into microsphere resonators, to form a novel cladding-mode coupler. The arrangement of a long-period fiber gratings evanescently side-coupled to a sphere is depicted in Fig.2. This method has the advantage that the excitation source is an optical fiber and is thus convenient to use.

## 2 CLADDING-MODE COUPLING BETWEEN LPFG AND MICROSPHERE

### 2.1 Cladding Mode Coupling Configuration

Long period fiber grating (LPFG) with a periodicity of hundreds of microns, which is very long

compared to a Bragg grating with periodicity less than  $1\mu\text{m}$ , can couple the forward fundamental guiding mode to forward cladding leaky modes [4]. In Fig.2, we can depict the coupling figure as follow.

The LPFG #1 converts light from the core mode into a cladding mode of that fiber at a desired wavelength, and other wavelengths are not affected. Then, the cladding mode propagates in the cladding and a coupling region between the transmission fiber and the microsphere. The third, the evanescent wave of the cladding mode excites a whispering gallery mode of the microsphere. The fourth, the whispering gallery mode of the microsphere excites a similar cladding mode in another fiber where an identical LPFG #2 is inscribed. In that fiber the LPFG #2 transforms cladding mode into core mode. As a result, only light at the resonance wavelength of the two LPFGs and the microsphere resonator will be coupled from the core of the LPFG #1's fiber into the core of the LPFG#2's fiber.

## 2.2 Propagation Constant of the Cladding Mode of LPFG.

Long-period fiber gratings with periodicities in the hundreds of microns have been used for coupling the guided fundamental mode in a single-mode fiber to forward propagating cladding modes. These modes decay rapidly as they propagate along the fiber axis owing to scattering losses at the cladding air interface and bends in the fiber.

Consider a single-mode fiber with the propagation constant of the fundamental mode,  $LP_{01}$ , denoted by  $\beta_{01}$ , and the propagation constant of the cladding modes given by  $\beta_{cl}^{(n)}$ , where the superscript denotes the order of the mode, The phase matching condition between the guided mode and the forward propagating cladding modes is given by :

$$\beta_{01} - \beta_{cl}^{(n)} = \frac{2\pi}{\Lambda} \quad (1)$$

Where  $\Lambda$  is the grating periodicity required to couple the fundamental mode to the  $n$ th-cladding mode.

To refer to [4,5], we get Fig.1 (a), theoretical relationship between grating period and resonance wavelengths where guide-to-cladding mode coupling take place.

## 2.3 Propagation Constant of WGM Resonance for Microsphere

We refer to a sphere of size parameter  $x=2\pi a/\lambda$  (where  $a$  is the sphere radius and  $\lambda$  is the wavelength of light in free space) [13,14].

Whispering gallery mode (WGM) resonances are electromagnetic resonances that occur in circularly symmetric dielectric particles. They correspond to light trapped in circling orbits just within the surface of the particle, being continuously totally internally reflected (TIR) from the surface.

The WGM is characterized by the mode numbers  $n$ ,  $l$ , and  $m$ , the radial, angular, and azimuthal mode numbers, respectively. To maximize the advantage of a microcavity resonance, we are interested in exciting modes with low  $n$  and with  $m \approx l$ . These are the modes with the smallest mode volume, as they are most closely confined to the surface of the sphere (lowest radial mode number  $n$ ) and to the sphere equator ( $|m| = l$ ). In this case we can approximate the propagation constant  $\beta_s$  by

$$\beta_s = \frac{kl}{x_{nlm}} \quad (2)$$

where  $x_{nlm}$  is the size parameter that corresponds to the  $n$ ,  $l$ , and  $m$ , resonance and  $k$  is the free-space propagation constant. This formula gives the correct value because  $2l$  is the number of maxima in the angular variation of the resonant field around the microsphere equator. We refer to [13], obtain Fig.1 (b) Calculated values of the propagation constants(the wavelengths) for the first and the second radial mode numbers of WGM resonance for spheres of different sizes

## 2.4 Phase-matched Excitation of Whispering-gallery-mode Resonance

To excite the microsphere efficiently, it is crucial to obtain good matching between the cladding modes of LPFG and whispering gallery modes of the sphere, namely, we need to match the propagation constant ( $\beta_s$ ) of the WGM at the surface of the sphere to the propagation constant ( $\beta_{cl}^{(n)}$ ) of the appropriate mode in the cladding mode of LPFG,

$$\beta_{cl}^{(n)} = \beta_s \quad (3)$$

Assumed the reflective indexes of the microsphere and LPFGs cladding are the same, according to formula (3) and Fig.1, one can obtain the value of the radius of microsphere and the grating periodicity  $\Lambda$  of LPFG mode coupling will be enabled. One can predict the wavelengths at which mode-coupling will be enabled by a particular grating period, and to make use of the formulas of (1), (2), (3), can calculate the sphere radius (R) corresponding to a given  $\Lambda$  of the LPFGs. From Fig.1, one can get the range of R and  $\Lambda$ ,  $R=50\sim 200 \mu m$ ,  $\Lambda=750\sim 850 \mu m$ .

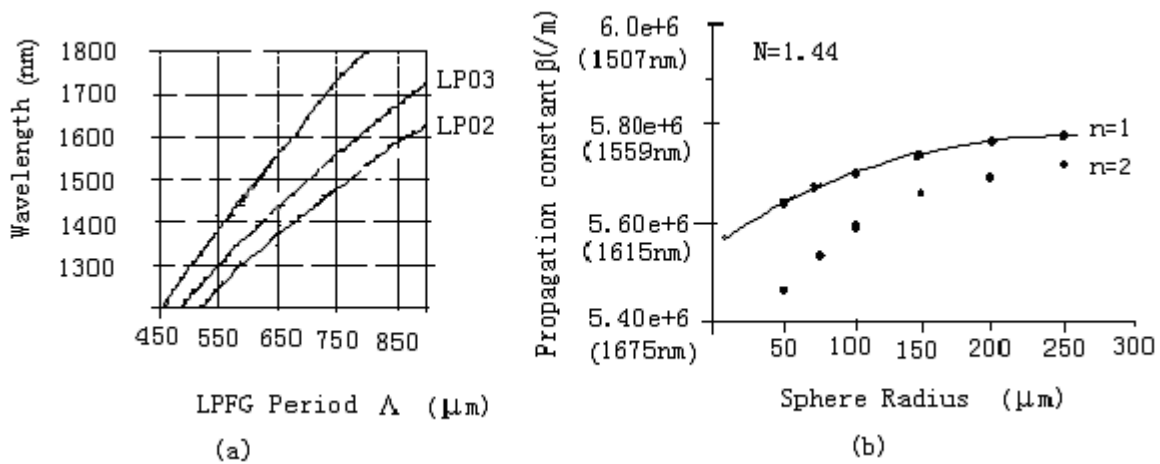


Fig.1 (a) theoretical relationship between grating period and resonance wavelengths where guide-to-cladding mode coupling take place, (b) Calculated values of the propagation constants (the wavelengths) for the first and the second radial mode numbers of WGM resonance for spheres of different sizes

### 3 COUPLING STRENGTHS EVALUATION

#### 3.1 Modes and Fields of a Dielectric Sphere

We refer to Ref. [9]. The electromagnetic fields associated with a sphere are vector fields. If however, the direction of polarization can be assumed to be constant along a fixed set of spherical coordinates throughout all space, then the solution to the Helmholtz equation in spherical coordinates is separable. This solution comprises the well known Spherical Bessel functions for the radial dependence, and Spherical Harmonics for the angular dependencies.

A dielectric sphere is an open cavity supporting tunneling leaky waves. The eigenvalues of the exact solution would need to be complex in order to satisfy the radiation condition. Such solutions, with fields that grow unbounded in the radial direction far from the sphere, are difficult to normalize. On the other hand, it is only the near field, the bound portion of the field, which contributes to coupling with the external excitations. The field can be written as

$$\psi_{l,m,n}(r, \theta, \Phi) = N_s \psi_r(r) \psi_\theta(\theta) \psi_\Phi(\Phi) \quad (4)$$

where the component contributions take the form :

$$\psi_\Phi(\Phi) = \exp[\pm jm\Phi] \quad (5a)$$

$$\psi_\theta(\theta) = \exp\left[-\frac{m}{2}\theta^2\right] H_N(\sqrt{m}\theta), \quad m \gg 1 \gg \theta \quad (5b)$$

$$\psi_r(r) = \begin{cases} j_\ell(kn_s r), & r \leq R_s \\ j_\ell(kn_s R_s) \exp[-\alpha_s(r - R_s)], & r > R_s \end{cases} \quad (5c)$$

and the coefficients are

$$N_s = \left\{ \sqrt{\frac{\pi}{m}} 2^{N-1} N! R_s^2 \left[ \left(1 + \frac{1}{\alpha_s R_s}\right) j_\ell^2(kn_s R_s) - j_{\ell-1}(kn_s R_s) j_{\ell+1}(kn_s R_s) \right] \right\}^{-1/2} \quad (6a)$$

$$\alpha_s = \sqrt{\beta_\ell^2 - k^2 n_s^2}, \quad \beta_\ell = \frac{\sqrt{\ell(\ell+1)}}{R_s} \quad (6b)$$

$$N = \ell - m, \quad k = \frac{2\pi}{\lambda} \quad (6c)$$

$\psi_{l,m,n}(r, \theta, \phi)$  represents either the  $E_\theta$  or  $H_\theta$  component of the electromagnetic field. The field consists of the following components [9].

1) The azimuthal contribution  $\psi_\phi$  with integer mode  $m \leq l$  number (here  $m$  is defined to range only over positive values).

2) The polar contribution  $\psi_\theta$  which is often expressed in terms of the exact solutions, the Associated Legendre Polynomials  $P_m^l(\cos\theta)$ . We are mainly interested in large  $m$  and  $l$  values, and polar angles near  $\theta \approx 0$ .

3) The radial contribution  $\psi_r$  is comprised of the exact Spherical Bessel functions  $j_l(kn_s r)$  interior to the sphere. Exterior to the sphere, but very close to the surface, the fields decay exponentially,  $\alpha_s$  is the decay constant away from the sphere in the radial direction, while  $\beta_l$  is the propagation constant parallel to the surface of the sphere. The propagation constant parallel the surface of the sphere, but along the equator, is the projection of  $\beta_l$  onto the equator. This propagation constant has the value  $\beta_m = m/R_0$ . Often, the propagation constant is also written as  $\beta = 2\pi N_{\text{eff}}/\lambda$  where  $N_{\text{eff}}$  is the effective index and  $\lambda$  is the wavelength. Then the range of validity  $m \gg 1$  indicated in (5) amounts to  $N_{\text{eff}} R_0 / \lambda \gg 1$  or that the sphere radius must be much larger than the effective wavelength.

$N_s$  is the normalization constant associated with the intensity flow through a plane transverse to the effective direction of propagation. In our coordinate system, the effective direction of propagation is along the  $\hat{\Phi}$  direction, in the equatorial plane, while the transverse plane is the  $\hat{\theta} - \hat{r}$  plane.  $N_s$  is found from the volume integral of  $\psi_{l,m,n}^2$  over all space, divided by the equatorial path length  $2\pi R_0$ .

The characteristic equation which describes the relationship between the wave vector  $k$  and the eigenvalues  $l$  and  $n$  is determined by matching tangential electric and magnetic fields across the surface  $r = R_0$ . Two independent cases are identified as a consequence of separable solutions: 1) transverse electric (TE) modes, where the electric field is parallel to the surface. The vector components are  $\vec{E} = \hat{\theta}E_\theta \equiv \hat{\theta}\psi_{l,m,n}$   $E_\phi = E_r = 0$ . 2) Transverse magnetic (TM) modes, where the magnetic field is parallel to the surface. The vector components are  $\vec{H} = \hat{\theta}H_\theta \equiv \hat{\theta}\psi_{l,m,n}$ ,  $H_\phi = H_r = 0$ . The remaining  $\vec{H}$  fields of the TE modes, or the  $\vec{E}$  fields of the TM modes, are determined by Maxwell's equations, Matching tangential fields leads to the simple characteristic equation

$$\left(\eta_s \alpha_s + \frac{l}{R_0}\right) j_l(kn_s R_0) = kn_s j_{l+1}(kn_s R_0) \quad (7)$$

$$\eta_s = \begin{cases} 1 & , \text{ TE modes} \\ \frac{n_s^2}{n_0^2} & , \text{ TM modes.} \end{cases}$$

Values for the Spherical Bessel functions are easily computed by using the following recursion formulas.

$$j_0(x) = \frac{\sin x}{x} \quad (8a)$$

$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x} \quad (8b)$$

$$j_{l+1}(x) = \frac{2l+1}{x} j_l(x) - j_{l-1}(x) \quad (8c)$$

### 3.2 Fields of Cladding Mode in a Circular Step-index Fiber

The fields of an optical fiber have well known analytical solutions Because the core radius  $a$  is much smaller than the cladding radius  $\rho$ , the core can be ignored in the calculation of the mode field and the propagation constant of the cladding mode. In that case, the field of the cladding mode, the  $LP_{lm}$  mode, can be approximated by

$$\begin{aligned} \psi_l(R) &= \frac{J_l(UR)}{J_l(U)} & , \quad R \leq 1 \\ &= \frac{K_l(WR)}{K_l(W)} & , \quad R > 1 \end{aligned} \quad (9)$$

where  $R = r/\rho$  is the normalized radial coordinate, and  $J_l$  and  $K_l$  are, respectively, the Bessel function and modified Hankel function of the first kind of order  $l$ . In Eq.(7), the normalized parameters are defined as

$$U = \rho k \left[ n_1^2 - \left( \frac{\beta}{k} \right)^2 \right]^{1/2} \quad (10)$$

$$W = \rho k \left[ \left( \frac{\beta}{k} \right)^2 - n_2^2 \right]^{1/2} \quad (11)$$

with  $V^2 = U^2 + W^2$

The eigenvalue equation that specifies the allowed values of  $U$  (and hence,  $W$  and  $\beta$ ) for a given  $V$  and  $l$  is given by

$$\frac{U J_{l+1}(U)}{J_l(U)} = \frac{W K_{l+1}(W)}{K_l(W)} \quad (12)$$

For the cladding mode, we usually have  $V \gg 1$ . When this condition is satisfied, we can use an asymptotic expression to solve the propagation constant  $\beta_{cl}^{(n)}$  [1].

### 3.3. Coupling Strength

The arrangement of a long-period fiber grating evanescently side-coupled to a sphere is depicted in Fig.2. The surface of the sphere lies a minimum distance  $S_0$  away from the fiber axis. The mode numbers of the sphere and the resonant wave vector are evaluated from the solution to the characteristic equation (7). The propagation constant  $\beta_{cl}^{(n)}$  of the fiber cladding is found from the characteristic equation (12). With the normalized modes determined, coupled mode theory is invoked to evaluate the interaction strengths [10]. The interaction strength between the fiber cladding mode  $\Psi_l$  of (9), and the sphere mode of (4), at the minimum separation is given by the overlap integral

$$\kappa(s_0) = \frac{k^2}{2\beta_{cl}^{(n)}} \int_x \int_y (n_s^2 - n_0^2) \Psi_l(R) \Psi_{l,m,x} dx dy \quad (13)$$

## 4 A NOVEL CLADDING-MODE COUPLER: AN OADM CONFIGURATION

We can conceive a four-port passive add/drop device based on the microsphere-LPFGs coupling mechanism. The four-port device is sketched in Fig.3. Two nearly identical single-mode fibers that contain two identical LPFGs designed for the same spectral region are positioned to tangentially touch opposite sides of an equator of the microsphere. The microsphere sustains WGM resonances at a LPFG-microsphere contact points enables coupling to these resonant modes. The four coupling ports are labeled as shown.

If a signal containing several channels at wavelengths  $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \dots$  is injected into the input port 1 where the LPFG #1 is located in the core of the fiber. The LPFG #1 converts light from the core mode into a cladding mode of that fiber at a desired wavelength, and other wavelengths are not affected. Then, the cladding mode propagates in the cladding. There is a coupling region between the two fibers and the microsphere, The cladding mode of the first fiber excites WGM in high-Q microsphere. The microsphere WDM excites a similar cladding mode in the second fiber where an identical LPFG #2 is inscribed. In that fiber, the LPFG #2 transforms cladding mode into core mode. As a result, only light at the resonance wavelength of the two LPFGs and the microsphere will be coupled from the core of LPFG #1's fiber through the microsphere into the core of the LPFG #2's fiber (power transfer to the microsphere and to port 3 will occur for any input channel that is resonant with a WGM ( $\lambda_3$ ) in the Figure). Other nonresonant channels travel through the upper fiber past the microsphere with minimal power. Similarly, for adding a channel, another cladding-mode coupler should be arranged in reverse order. A channel input at port 4 that is resonant with the resonance wavelength of the two LPFGs and the microsphere would be transferred to the upper fiber, and join the other channels at port 2. Channel adding and dropping therefore can be realized simultaneously.

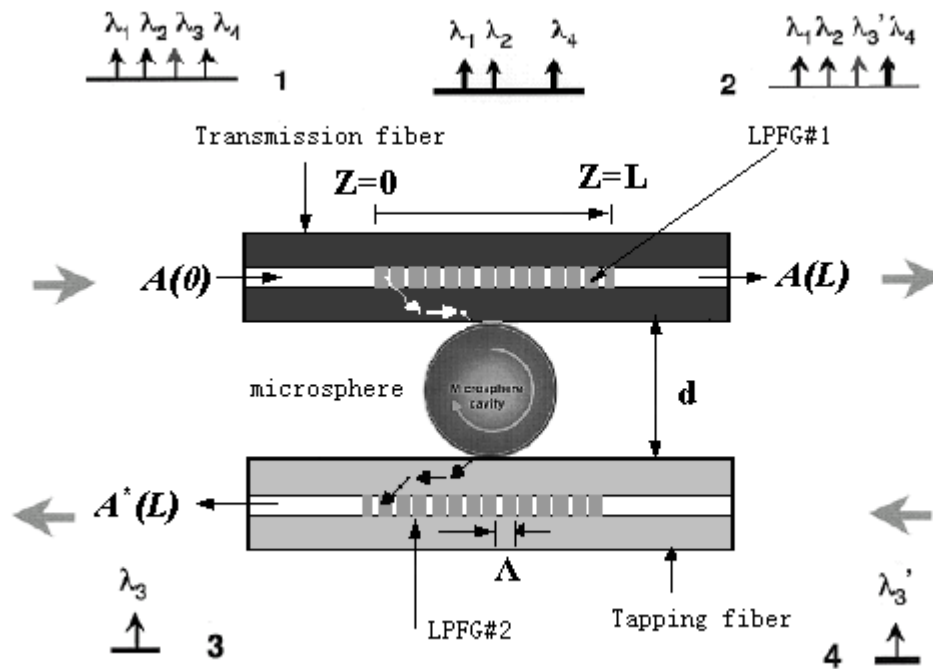


Fig.2. Diagram of coupling mechanism between two parallel identical LPFGs and a microsphere. (Two nearly identical single-mode fibers that contain two identical LPFGs designed for the same spectral region)

The coupling strength will be relatively weak, in order to enhance the coupling efficiency, we can improve the configuration as follows:

- 1) The coupling efficiency could be improved by etching the claddings of the LPFGs .
- 2) It may be desirable to immerse the sphere and coupling fiber into index matching liquids in order to increase their mutual interaction strength. The coupling efficiency could be further increased by index-matching external medium.
- 3) Adjust the position of the microsphere along LPFGs.

## 5 CONCLUSION

We propose a new coupling configuration, used two parallel fibers that contain LPFGs designed for the same spectral region and a microsphere. Our method is based on evanescent-field of LPFG exciting the WGM of microsphere resonator. The key features of the method are illustrated with theoretical, but expecting experimental result.

Considering this type of structure ,there may be several advantages as follow: (i)The diameter of the LPFG is relatively larger than that of taper fiber, there will be easier to operate and manufacture (ii) Owing to the microsphere is a high-Q value resonator , coupled with the very small resonant mode volume, the bandwidth of the tapping fiber output spectrum will be narrower than that of the two parallel long-period gratings evanescent-field coupling.(iii)We can conceive many applications, used as an optical add/drop multiplexer(OADM), refractive-index and chemical sensors.



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