Multi-focus Image Fusing Based on Non-negative Matrix Factorization

Le Xu, Ji-yang Dong*, Cong-bo Cai, and Zhong Chen

Abstract — *Multi-focus image fusion is a process of obtaining a new all in-focus merged image from two or more partially defocused images of the same scene and same imaging condition. The merged image includes the information of the original images and improves the reliability and intelligibility for object detection and target recognition. The most widespread methods for image fusion are wavelet transform based methods. However, the facts that the original pixel values of input images are not preserved in the fused image and different multi-scale image fusion schemes will lead to different results cause that the wavelet methods present a limited quality performance compared with a cut and pasted fusion reference model. In this paper, a new multi-focus image fusion approach is proposed based on non-negative matrix factorization (NMF). The cut and pasted fusion scheme is adopted in the new fusion approach. Cut the source images into small-size blocks, factorize the corresponding image blocks using NMF, pick out the sharpest blocks according the NMF coefficient, and combine them as an in-focus image. The experiment results show that the proposed approach outperforms the wavelet based fusion methods, both in visual effect and objective evaluation criteria[1].*

Index Terms — Multi-focus image, non-negative matrix factorization (NMF), image fusion.

I. INTRODUCTION

Image fusion can be defined as the process by which a set of images is combined to produce a new image that integrates complementary, multi-temporal or multi-view information from the sources. Due to the limited focus depth of the optical lens, it is often not possible to get an image that contains all relevant objects in focus. In an image captured by those devices, only those objects within the depth of field are focused, while other objects are blurred. To obtain an image with every object in focus, a multi-focus image fusion process is required to fuse the images taken from the same view point under different focal settings. The fused image gives a better view for human or machine perception [1].

Many fusion methods have been proposed in the past decades. The primitive fusion scheme is the weighted mean method which takes the average of the source images pixel by pixel. However, along with simplicity it comes several undesired side effects including reduced contrast. In order to solve this problem, image fusion methods based on pyramid, including Laplacian pyramid, gradient pyramid, ratio-of-low-pass pyramid, etc, were explored. However, using this simple salient features to fuse images obviously still may not produce optimal results [2]. Wavelet transform is one of the most widespread methods in this filed in recent years. Wavelet transform can provide better fusion performance than pyramid decomposition due to its orthogonality, symmetry and compact support [3][4]. But this kind of method replaces wavelet coefficients in the transformed domain and the original pixel values of source images are not preserved in the fused image. In addition, wavelet based methods are shift-variant because of an underlying down-sampling process. This fact causes that the wavelet methods present a limited quality performance when compared with a cut and pasted fusion reference model [5]. Furthermore, some edge information of the original images may lose and ringing effect may appear in the fused image [6].

Recently, Lee and Seung proposed a new technique called Non-negative Matrix Factorization (NMF) to obtain a reduced representation of data [7]. NMF imposes the non-negativity constraints in learning basis images. The pixel values of resulting basis images, as well as coefficients for reconstruction, are all non-negative. By this way, only non-subtractive (or additive) combinations are allowed. This ensures that the components are combined to form a whole in an accumulative means. For this reason, NMF is considered as a procedure for learning a parts-based representation [7]. Together with the non-negativity constraints, NMF has been shown to have various applications to image processing, including image fusion. Zhang presented in their paper a multi-focus image fusion method based on NMF [9], namely Zhang's method at the following part. However, as shown in section III, Zhang's method is in fact a weighted mean fusion method.

A new multi-focus image fusion approach based on NMF is proposed in this paper. The cut and pasted fusion scheme is adopted in the new fusion approach. Firstly, the source images are cut into some small-size blocks. Secondly, the corresponding blocks from different source images are factorized by NMF to distinguish the in-focus image block from de-focus image blocks. Lastly, the sharpest blocks are picked out to reconstruct the all-in-

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focus image according to the coefficients of NMF. In the proposed approach, the part-based representation of NMF, which was discarded in Zhang's fusion method, was utilized to make the construction coefficients different between the clear image block and the blur image blocks. The experiment results show that the proposed approach can avoid the problem of shift-variant, caused by wavelet based fusion method. A better image fusion performance can be obtained compared with wavelet based method.

II. NON-NEGATIVE MATRIX FACTORIZATION

Non-negative matrix factorization is a method to obtain a representation of data using non-negativity constraints. It finds two non-negative factor matrices ***W*** and ***H*** to approximate the observed data matrix ***V*** [8],

$$V \approx V' = WH . \tag{1}$$

Given a set of multivariate ***n***-dimensional data vectors ***V***, the vectors are placed in the columns of an $n \times m$ matrix, where m is the number of examples in the data set. This matrix is then approximately factorized into an $n \times r$ matrix W and an $r \times m$ matrix ***H***. Usually, ***W*** is called basis matrix whereas ***H*** is called weight matrix.

To find an approximate factorization $V \approx WH$, we first need to define cost functions that quantify the quality of the approximation. Such a cost function can be constructed using some measure of distance between two non-negative matrices ***V*** and $V' = WH$. One useful measure is simply the square of the Euclidean distance between ***V*** and V',

$$F = \sum_{i=1}^{n} \sum_{j=1}^{m} \left[V_{ij} - (WH)_{ij} \right]^2 \tag{2}$$

This cost function can be related to the likelihood of generating the images in V from the basis matrix ***W*** and the weight matrix ***H***.

The procedure of iterative updates of ***W*** and ***H*** to reach a local minimum of this objective function can be described as follows [10]:

1. Initialize ***W*** and ***H*** to be two random non-negative matrices;

2. Keep updating ***W*** and ***H*** until the cost function converges. During the updates, we should update ***W*** and ***H*** simultaneously. That means, after updating one row of ***W***, we need to update the corresponding column of ***H***. The multiplicative update rules are as shown in Equ.(3).

$$W_{ia} = W_{ia} \sum_j \frac{V_{ij}}{(WH)_{ij}} H_{aj} \tag{3a}$$

$$W_{ia} = \frac{W_{ia}}{\sum_j W_{ja}} \tag{3b}$$

$$H_{aj} = H_{aj} \sum_i W_{ia} \frac{V_{ij}}{(WH)_{ij}} \tag{3c}$$

In NMF, each data vector in the matrix ***V*** is a linear and non-negative combination of the ***r*** parts, i.e., the basis vector in ***H***. The non-negative constraints which only allow additive, not subtractive combinations of the original data lead to a part-based representation of the observation data. Because of the fact that most of the physics signals such as image, sound, video, and so on, are positively defined, NMF has been shown to have various applications to image processing, and made the experiment results more meaningful and realistic.

III. MULTI-FOCUS IMAGE FUSION ALGORITHM

In the multi-focus image fusion problem, given two partially focused source images ***A*** and ***B***, the observation data ***V*** can be constructed by putting ***A*** and ***B*** pixel by pixel on the two columns of ***V*** respectively. With a reasonable basic number ***r***, the observed matrix ***V*** can be decomposed by NMF into a basis matrix ***W*** and a weight matrix ***H***. ***W*** is an ***r*** columns matrix, and each column of ***W*** represents a basis vector in the data space, while each column in ***H*** corresponds to the weights needed to approximate the corresponding column in ***V*** using the basis from ***W***. And both of the source images ***A*** and ***B*** are linear and non-negative combination of the ***r*** basis vectors in ***W***.

Zhang presented in their paper a multi-focus image fusion method based on NMF. They put all partially focused images in ***V***, and find an approximate factorization $V \approx V' = WH$ using NMF in which the parameter ***r*** is set to be only one. In this way, the basis matrix ***W*** has only one column. Because the only basis vector of ***W*** would represent the substantial feature (global feature) of all the input images in ***V***, they regarded ***W*** as the fused image of the input partially focused images.

However, we can prove that the Zhang's fusion method is in fact a weighted mean method.

Without loss of generality, let ***A*** and ***B*** be the two source images to be fused, both of ***A*** and ***B*** are with ***N*** pixels. The observation matrix ***V*** would be

$$V = \begin{pmatrix} A & B \end{pmatrix} = \begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \\ \vdots & \vdots \\ a_N & b_N \end{pmatrix}$$

NMF is to find an approximate factorization $V' = WH$, which is as close to ***V*** as possible. When the parameter ***r*** of NMF is set to 1, we have

$$V' = \begin{pmatrix} a'_1 & b'_1 \\ a'_2 & b'_2 \\ \vdots & \vdots \\ a'_N & b'_N \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix} \begin{pmatrix} h_1 & h_2 \end{pmatrix} \tag{4}$$

Rewrite Equ.(4) in vector form, we have

$$A' = W h_1$$

$$B' = W h_2$$

Then,

$$W = \frac{A' + B'}{h_1 + h_2}$$

If NMF is convergence, $A' \approx A$, $B' \approx B$. So

$$W \approx \frac{A+B}{h_1 + h_2} \tag{5}$$

Equ.(5) indicates that $\boldsymbol{W}$ is exactly a weighted mean of the source images $\boldsymbol{A}$ and $\boldsymbol{B}$. So Zhang's fusion method actually is a weighted mean one.

Via the analysis above, we can see that the most important feature of NMF, *i.e.*, the local or part-based representation feature of the input images, would be discarded when set the parameter $\boldsymbol{r}$ be 1. To obtain a better fused image, the parameter $\boldsymbol{r}$ must be bigger than 1.

Now we introduce a new fusion method based on NMF.

In order to make the basis component be more meaningful and interpretable, we construct the observation matrix $\boldsymbol{V}$ putting the source image $\boldsymbol{A}$ and $\boldsymbol{B}$ pixel by pixel on the two rows of $\boldsymbol{V}$ respectively, *i.e.*,

$$V = \begin{pmatrix} A^T \\ B^T \end{pmatrix} = \begin{pmatrix} a_1 & a_2 & \cdots & a_N \\ b_1 & b_2 & \cdots & b_N \end{pmatrix}$$

When the parameter $\boldsymbol{r}$=2, Equ.(1) can be written as

$$\begin{aligned} V \approx V' &= WH \\ &= \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{pmatrix} \begin{pmatrix} h'_{11} & h'_{12} & \cdots & h'_{1N} \\ h'_{21} & h'_{22} & \cdots & h'_{2N} \end{pmatrix} \end{aligned}$$

Here the matrix $\boldsymbol{W}$ is the weight matrix and $\boldsymbol{H}$ is the basis matrix. $\boldsymbol{H}$ is an $\boldsymbol{r}$ rows matrix, and each row of $\boldsymbol{H}$ represents a basis vector in the data space, while each row in $\boldsymbol{W}$ corresponds to the weights needed to approximate the corresponding column in $\boldsymbol{V}$ using the basis from $\boldsymbol{H}$. Because of the normalization procedure of $\boldsymbol{W}$ in NMF algorithm (see Equ.(3b)), the basis vector in each row of $\boldsymbol{H}$ is close to the source images.

Via theoretical and experimental analysis, we found a significant rule that the first row of $\boldsymbol{V}$ would possess more information than the second row when the sum of the first row of $\boldsymbol{W}$ is less than the second row, which implies that image $\boldsymbol{A}$ is sharper than image $\boldsymbol{B}$, and vice verse. This rule can be used to find out the better focused image.

Furthermore, to enhance the local representations ability of NMF, we introduce the cut and paste scheme to the multi-focus image fusion problem. The basic idea underlying the fusion scheme is to choose the clearer image blocks from source images to construct the fused image. The NMF selection rule is used to distinguish the focused image blocks from the defocused image blocks. Then the new fusion algorithm for multi-focus images is:

Step 0: Initialization. Construct a matrix $\boldsymbol{S}$ of the same size as the source images ($\boldsymbol{A}$ and $\boldsymbol{B}$). $\boldsymbol{S}$ is called Label Matrix which records the selection results of clear image blocks. Initialize $\boldsymbol{S}$ with zero. Choose an appropriate iteration number $\boldsymbol{M}$ for the algorithm.

Step 1: **Source Images Division.** Divide the two images to be fused ($\boldsymbol{A}$ and $\boldsymbol{B}$) into $\boldsymbol{K}$ image blocks of appropriate scale, i.e., $\boldsymbol{A}[\boldsymbol{i}]$, $\boldsymbol{B}[\boldsymbol{i}]$, $\boldsymbol{i}$=1,2,….,$\boldsymbol{K}$.

Step 2: Blocks Factorization. Initialize the two rows of $\boldsymbol{V}[\boldsymbol{i}]$ with the pixels of $\boldsymbol{A}[\boldsymbol{i}]$ and $\boldsymbol{B}[\boldsymbol{i}]$ respectively, then factorize $\boldsymbol{V}[\boldsymbol{i}]$ into two non-negative matrixes $\boldsymbol{W}[\boldsymbol{i}]$ and $\boldsymbol{H}[\boldsymbol{i}]$ using NMF, for $\boldsymbol{i}$=1,2,…, $\boldsymbol{K}$.

Step 3: Selection Result Recording. Detect the focused blocks with the ***selection rule***, and record the selection results on the label matrix $\boldsymbol{S}[\boldsymbol{i}]$. This can be done by increasing all elements of $\boldsymbol{S}[\boldsymbol{i}]$ by 1 if $\boldsymbol{A}[\boldsymbol{i}]$ is in-focused, for $\boldsymbol{i}$=1,2,…$\boldsymbol{K}$.

Step 4: Stop Condition. Change the division scale and repeated Step 1 ~ Step 3 for $\boldsymbol{M}$ times. Then update the label matrix $\boldsymbol{S}$ with its mean.

Step 5: Fused Image construction. Set the pixels value of the fused image $\boldsymbol{F}$ by the following rules: Let $\boldsymbol{F}(\boldsymbol{m},\boldsymbol{n})$=$\boldsymbol{A}(\boldsymbol{m},\boldsymbol{n})$ if $\boldsymbol{S}(\boldsymbol{m},\boldsymbol{n})$>0.5, otherwise, $\boldsymbol{F}(\boldsymbol{m},\boldsymbol{n})$=$\boldsymbol{B}(\boldsymbol{m},\boldsymbol{n})$, for every pixel ($\boldsymbol{m},\boldsymbol{n}$).

IV. EXPERIMENTS AND RESULTS

A lot of multi-focus image fusion experiments have been done by the proposed fusion algorithm, and the results showed the efficiency of the NMF based fusion algorithm. For example, Fig.1(a) is an original image. Figs. 1(b) is the image with left part self-generated defocusing through adding Gaussian noises and Fig.1(c) is the image with right part self-generated defocusing through adding Gaussian noises. The wavelet transform method is realized using sym4 wavelet basis, maximum coefficients fusion scheme and 3 levels decomposition. Fig.1(d) and Fig.1(e) are the fused images using wavelet based method and the proposed method respectively. Three typical measures, i.e., root mean square error (RMSE), entropy and root cross entropy (RCE), are used for performance evaluation of fused images.

Table 1 shows the evolution for comparison. It indicates that the three measures of the proposed method are better than that of wavelet transform method. For example, RMSE value of the proposed method is 0.4770, which is lower than 1.3932 of the wavelet based method, which indicates that more information are retained on the fused image by the new method.

(a) Original image

(b) Image de-focused on left

(c) Image de-focused on right

(d) Fused image using wavelet

(e) Fused image using NMF

Fig.1 Self-generating partly defocused image and fusion result

Table 1 Comparison of fused images with different image fusion methods

Methods	RMSE	Entropy	RCE
Wavelet	1.3932	5.3265	0.0098
NMF	0.4770	5.3701	0.0093

V. CONCLUSIONS

A feasible and effective multi-focus image fusion method based on NMF is proposed in this paper. The experimental results show that the proposed method can keep the edge information of two spatially registered images to utmost extent and achieve better fusion performance than wavelet transform based method. Furthermore, the proposed method has two advantages: (1) No parameter is needed to be optimized in the new method, while other fusion methods, such as wavelet transform, have to set some parameters according to the given source images; (2) The fusion performance can be improved by repeating the fusion algorithm with more different division scales.

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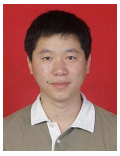

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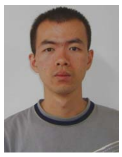

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