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Thermostatistic properties of a *q*-deformed ideal Fermi gas with a general energy spectrum

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Abstract

The thermostatistic problems of a q-deformed ideal Fermi gas in any dimensional space and with a general energy spectrum are studied, based on the q-deformed Fermi–Dirac distribution. The effects of the deformation parameter q on the properties of the system are revealed. It is shown that q-deformation results in some novel characteristics different from those of an ordinary system. Besides, it is found that the effects of the q-deformation on the properties of the Fermi systems are very different for different dimensional spaces and different energy spectrums.

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1. Introduction

It is commonly believed that ubiquitous systems can be naturally described within Boltzmann–Gibbs (BG) statistical mechanics. However, it is found that there is a class of physical systems so that the BG scenario may not be appropriate any longer [1–4] and an extension of the statistical mechanics is required.

There are two principal methods in the literature of introducing the intermediate statistical behavior: the nonextensive statistics introduced by Tsallis [5] and the q-deformed theory related to the quantum groups originally introduced by Biedenharn and Macfarlane [6, 7]. Some possible connections between the nonextensive statistics and quantum groups have been investigated by several researchers [8–13]. For example, the Tsallis entropy can be defined within the q-calculus framework [9–11] and the nonextensivity of classical set theory has been proved to relate to the q-oscillator [13].

The theory of the q-deformed statistics has become a topic of great interest in the last few years because of its possible applications in a wide range of areas, such as anyon physics [14, 15], vertex models [16], quantum mechanics in discontinuous spacetime [17], vibration of polyatomic molecules [18–20], vortices in superfluid films [21] and phonon spectrum in

^{4}He [22], etc. In recent years, many researches are devoted to the investigation of q-deformed physical systems [23–35]. For example, in [26], the thermodynamic properties of the q-deformed bosons and fermions are explored and both low- and high-temperature behaviors for the systems confined in a three-dimensional space and with nonrelativistic energy dispersion are discussed.

In this paper, we continue the work of [26] and study the thermostatistic properties of an ideal q-deformed Fermi gas in any dimensional space and with a general energy spectrum. The paper is organized as follows. In section 2, we give a brief review of the previous literature concerning the q-deformed algebra of fermions and the q-deformed Fermi–Dirac distribution. In section 3, we derive the analytical expressions of some important thermodynamic quantities based on the q-deformed Fermi–Dirac distribution. In section 4, the approximations for the thermodynamic quantities are given at the low- and high-temperature limits. The effects of the q-deformation on the properties of a q-deformed Fermi gas are discussed in section 5 and some novel characteristics are revealed. Some important conclusions are given in section 6.

2. *Q*-deformed fermion algebra and the distribution of the *q*-deformed fermions

The symmetric q-deformed fermion algebra is defined in terms of the creation operators $\hat{a}^+$ and annihilation operators $\hat{a}$ which satisfy [6, 7, 36]

$$[\hat{N}, \hat{a}^+] = \hat{a}^+, \qquad [\hat{N}, \hat{a}] = -\hat{a}, \tag{1}$$

and

$$\hat{a}^+\hat{a} = [\hat{N}], \qquad \hat{a}\hat{a}^+ = [1 - \hat{N}], \tag{2}$$

where $\hat{N}$ is the number operator, the q-basic number $[x]$ is defined as

$$[x] \equiv \frac{q^x - q^{-x}}{q - q^{-1}}, \tag{3}$$

and $q \in \mathrm{R}^+$ is the deformation parameter. For the q-deformed fermions, the Hilbert space with basis $|n\rangle$ is constructed such that [37]

$$\begin{aligned} &\hat{N}|n\rangle = n|n\rangle, \qquad \hat{a}|0\rangle = 0, \\ &\hat{a}^+|n\rangle = [1 - n]^{1/2}|n + 1\rangle, \\ &\hat{a}|n\rangle = [n]^{1/2}|n - 1\rangle. \end{aligned} \tag{4}$$

It should be pointed out that the Pauli principle is also applicable for the q-deformed fermions, i.e., the eigenvalues of the number operator $\hat{N}$ can only be taken the values of $n = 0$ and 1.

To derive the mean occupation numbers of each energy level, we choose the Hamiltonian [29]

$$\hat{H} = \sum_k (\varepsilon_k - \mu)\hat{N}_k, \tag{5}$$

where k is a state label, $\hat{N}_k$ and ε_k are, respectively, the number operator and energy associated with state k, μ is the chemical potential of the system. The mean value of the q-deformed occupation number $f_{k,q}$ is defined by [29]

$$[f_{k,q}] = \frac{1}{\Xi}\mathrm{tr}\{\exp(-\beta\hat{H})[\hat{N}_k]\}, \tag{6}$$

where $\beta = 1/(k_B T)$, k_B is the Boltzmann constant, T is the temperature and $\Xi = \mathrm{tr}[\exp(-\beta\hat{H})]$ is the partition function. With the help of the cyclic property of the trace [37, 38], we can get

$$\frac{[f_{k,q}]}{[1 - f_{k,q}]} = \exp[-\beta(\varepsilon_k - \mu)] \tag{7}$$

from equations (2) and (4)–(6). Using equations (3) and (7), one can derive the statistical distribution of the q-deformed fermions as [26]

$$f_{k,q} = \frac{1}{2\ln q} \ln\left[\frac{z^{-1}\exp(\beta\varepsilon_k) + q}{z^{-1}\exp(\beta\varepsilon_k) + q^{-1}}\right], \tag{8}$$

where $z = \exp(\beta\mu)$ is the fugacity of the system.

It is easily proved that when $q = 1$, equation (8) is simplified as

$$f_{k,1} = \frac{1}{z^{-1}\exp(\beta\varepsilon_k) + 1}, \tag{9}$$

which is just the standard Fermi–Dirac distribution. This means that the q-deformed fermions will be the same as the ordinary fermions when $q \to 1$.

Another important property concerned the distribution is that $f_{k,q}$ satisfies the symmetry property, i.e., $f_{k,q} = f_{k,1/q}$. This implies that the q-deformed fermions with the deformation parameter q may possess the same properties as those with the deformation parameter $1/q$, so that we can restrict our discussion to $q \geqslant 1$ in the following discussion.

3. Thermostatistic properties of q-fermions

We consider an ideal gas of q-fermions confined in a D-dimensional box and with the general energy spectrum

$$\varepsilon = ap^s, \tag{10}$$

where p is the momentum of a particle, and a and s are the positive constants.

According to equation (8), the total number of particles and the total energy of the system can be, respectively, expressed as

$$N = \sum_k \frac{1}{2\ln q} \ln\left[\frac{z^{-1}\exp(\beta\varepsilon_k) + q}{z^{-1}\exp(\beta\varepsilon_k) + q^{-1}}\right] \tag{11}$$

and

$$U = \sum_k \frac{\varepsilon_k}{2\ln q} \ln\left[\frac{z^{-1}\exp(\beta\varepsilon_k) + q}{z^{-1}\exp(\beta\varepsilon_k) + q^{-1}}\right]. \tag{12}$$

When the number of particles in the system is large enough, the sum over state k may be replaced by the integral over the phase space, i.e.,

$$N = \frac{g}{h^D} \int \prod_{i=1}^{D} \mathrm{d}p_i\,\mathrm{d}x_i \frac{1}{2\ln q} \ln\left[\frac{z^{-1}\exp(\beta ap^s) + q}{z^{-1}\exp(\beta ap^s) + q^{-1}}\right] = \frac{gV_D}{\lambda^D} h_\eta(z, q) \tag{13}$$

and

$$U = \frac{g}{h^D} \int \prod_{i=1}^{D} \mathrm{d}p_i\,\mathrm{d}x_i \frac{ap^s}{2\ln q} \ln\left[\frac{z^{-1}\exp(\beta ap^s) + q}{z^{-1}\exp(\beta ap^s) + q^{-1}}\right] = \eta k_B T \frac{gV_D}{\lambda^D} h_{\eta+1}(z, q), \tag{14}$$

where x_i and p_i are, respectively, the ith component of coordinate and momentum of a particle, g is the degree of the spin degeneracy, h is the Planck constant, V_D is the D-dimensional volume of the system, $\eta = D/s$,

$$\lambda = \frac{ha^{1/s}}{\pi^{1/2}(k_B T)^{1/s}} \left[\frac{\Gamma(D/2+1)}{\Gamma(D/s+1)}\right]^{1/D} \tag{15}$$

is the generalized thermal wavelength [39],

$$h_n(z, q) = \frac{1}{\Gamma(n)} \int_0^\infty \mathrm{d}x\, x^{n-1} \frac{1}{2\ln q} \ln\left[\frac{z^{-1}\exp(x) + q}{z^{-1}\exp(x) + q^{-1}}\right] \tag{16}$$

may be referred to as the generalized Fermi integral of q-fermions and $\Gamma(x) = \int_0^\infty \exp(-t)t^{x-1}\,\mathrm{d}t$ is the Gamma function. It can be seen from equation (16) that when $q = 1$,

$$h_n(z, 1) = \frac{1}{\Gamma(n)} \int_0^\infty \mathrm{d}x \frac{x^{n-1}}{z^{-1}\exp(x) + 1} \tag{17}$$

is just the standard Fermi integral.

According to equations (13) and (14), we can derive the specific heat at constant volume as

$$\begin{aligned} C_V &= \left(\frac{\partial U}{\partial T}\right)_{V_D} = \left(\frac{\partial U}{\partial T}\right)_{V_D,z} + \left(\frac{\partial U}{\partial z}\right)_{V_D,T}\left(\frac{\partial z}{\partial T}\right)_{V_D} \\ &= Nk_B\left[\eta(\eta+1)\frac{h_{\eta+1}(z,q)}{h_\eta(z,q)} - \eta^2\frac{h_\eta(z,q)}{h_{\eta-1}(z,q)}\right]. \end{aligned} \tag{18}$$

Because of the general form of the energy spectrum adopted here, the expressions derived above are valid for a variety of q-deformed fermion and ordinary fermion systems. For example, if $D = 3$, $s = 2$ and $a = 1/(2m)$, equations (13), (14) and (18) may be, respectively, simplified as

$$N = \frac{gV_D}{\lambda^3} h_{3/2}(z, q), \tag{19}$$

$$U = \frac{3}{2}k_B T \frac{gV_D}{\lambda^3} h_{5/2}(z, q), \tag{20}$$

and

$$C_V = Nk_B\left[\frac{15}{4}\frac{h_{5/2}(z,q)}{h_{3/2}(z,q)} - \frac{9}{4}\frac{h_{3/2}(z,q)}{h_{1/2}(z,q)}\right], \tag{21}$$

where $\lambda = \sqrt{h^2/2\pi m k_B T}$ and m is the mass of a particle. Equations (19)–(21) give the properties of a nonrelativistic q-deformed Fermi gas in a three-dimensional space. If $D = 3$, $s = 1$ and $a = c$, equations (13), (14) and (18) become

$$N = \frac{gV_D}{\lambda^3} h_3(z, q), \tag{22}$$

$$U = 3k_B T \frac{gV_D}{\lambda^3} h_4(z, q), \tag{23}$$

and

$$C_V = Nk_B\left[\frac{12h_4(z,q)}{h_3(z,q)} - \frac{9h_3(z,q)}{h_2(z,q)}\right], \tag{24}$$

where $\lambda = hc/(2\pi^{1/3}k_B T)$ and c is the light speed. Equations (22)–(24) present the properties of an ultrarelativistic q-deformed Fermi gas in a three-dimensional space. If $q \to 1$ is set, equations (19)–(21) and (22)–(24) can be further simplified and used to describe the properties of ordinary nonrelativistic and ultrarelativistic Fermi gases in the three-dimensional space, respectively. On the other hand, if D is chosen to be equal to 1 or 2, equations (13), (14) and (18) can be used to describe the characteristics of q-deformed Fermi systems in a low-dimensional space.

4. Low- and high-temperature behaviors of q-fermions

At very low temperatures, the generalized Fermi integral $h_n(z, q)$ can be written as a quickly convergent series:

$$h_n(z,q) = \frac{(\ln z)^n}{\Gamma(n+1)}\left[1 + n(n-1)\frac{\pi^2}{6}\gamma_1(q)\frac{1}{(\ln z)^2} + n(n-1)(n-2)(n-3)\frac{7\pi^4}{360}\gamma_3(q)\frac{1}{(\ln z)^4} + \cdots\right], \tag{25}$$

where

$$\gamma_n(q) = \int_0^\infty \mathrm{d}x \frac{x^n}{2\ln q} \ln\left[\frac{\exp(x)+q}{\exp(x)+q^{-1}}\right] \bigg/ \int_0^\infty \mathrm{d}x \frac{x^n}{\exp(x)+1} \tag{26}$$

is a factor related to the deformation parameter q. It can be proved that $\gamma_n(q) > 1$ for $q \neq 1$ and $\gamma_n(q) = 1$ when $q = 1$.

Substituting equation (25) into equations (13), (14) and (18) and keeping terms up to the second power of k_BT/ε_F only, one can obtain the expressions of μ, U and C_V as the explicit functions of temperature. The results are, respectively, given by

$$\mu = \varepsilon_F\left[1 - \frac{\pi^2}{6}(\eta-1)\gamma_1(q)\left(\frac{k_BT}{\varepsilon_F}\right)^2\right], \tag{27}$$

$$U = \frac{\eta}{\eta+1}N\varepsilon_F\left[1 + \frac{\pi^2}{6}(\eta+1)\gamma_1(q)\left(\frac{k_BT}{\varepsilon_F}\right)^2\right], \tag{28}$$

and

$$C_V = Nk_B\eta\frac{\pi^2}{3}\gamma_1(q)\frac{k_BT}{\varepsilon_F}, \tag{29}$$

where

$$\varepsilon_F = a\left[\frac{h^D\Gamma(D/2+1)}{g\pi^{D/2}}\frac{N}{V_D}\right]^{1/\eta} \tag{30}$$

is the Fermi energy of undeformed Fermi system [40]. It is seen from equations (28) and (29) that the q-deformation increases the total energy and heat capacity at low temperatures, since the factor $\gamma_n(q) > 1$ for $q \neq 1$. The result can be explained by comparing the statistical distribution of the q-deformed fermions with that of the ordinary fermions. According to equation (8), one can find that $f_{k,q} > f_{k,1}$ for $\varepsilon_k > \mu$ and $f_{k,q} < f_{k,1}$ for $\varepsilon_k < \mu$ when $q \neq 1$. This indicates that the q-deformation increases (decreases) the occupation of fermions in the high (low) level at non-zero temperature and hence increases the total energy and heat capacity.

Setting $T = 0$ K in equations (27) and (28), one can obtain the Fermi energy and ground-state energy of the q-deformed Fermi system, which are, respectively, given by equation (30) and

$$U_0 = \frac{\eta}{\eta+1}N\varepsilon_F. \tag{31}$$

It is clearly seen from equations (30) and (31) that both the Fermi energy and ground-state energy are independent of q and the same as those of an original Fermi gas. In fact, it can be further proved that all the properties of the q-deformed fermions are the same as those of the original fermions at $T = 0$ K.

On the other hand, at high temperatures, $k_B T \gg \varepsilon_F$ and hence z is very small, so that $h_n(z, q)$ may be expressed as a series, i.e.,

$$h_n(z, q) = \sum_{i=1}^{\infty} (-1)^i \frac{q^{-i} - q^i}{2 \ln q} \frac{z^i}{i^{n+1}}. \tag{32}$$

Substituting equation (32) into equations (13), (14) and (18) and keeping only the lowest-order correction due to the finite temperature, one can express μ, U and C_V as

$$\mu = \eta k_B T \left(\ln \frac{\varepsilon_F}{k_B T} \right) \left[1 + \ln \left(\frac{1}{\Gamma(\eta + 1)} \frac{2 \ln q}{q - q^{-1}} \right) \Big/ \left(\eta \ln \frac{\varepsilon_F}{k_B T} \right) \right], \tag{33}$$

$$U = \eta N k_B T \left[1 + \frac{1}{2^{\eta+1} \Gamma(\eta + 1)} \frac{q + q^{-1}}{q - q^{-1}} \ln q \left(\frac{\varepsilon_F}{k_B T} \right)^{\eta} \right], \tag{34}$$

and

$$C_V = \eta N k_B \left[1 + \frac{1 - \eta}{2^{\eta+1} \Gamma(\eta + 1)} \frac{q + q^{-1}}{q - q^{-1}} \ln q \left(\frac{\varepsilon_F}{k_B T} \right)^{\eta} \right]. \tag{35}$$

At high temperatures, the second term in the square bracket in equations (33)–(35) can be neglected, so that the expressions for μ, U and C_V are reduced to those of ordinary Boltzmann gases and independent of q.

5. Effects of the *q*-deformation on the properties of *q*-fermions

In order to understand more clearly the effects of the q-deformation on the properties of q-deformed Fermi gases, we can use equations (13) and (18) to plot the characteristic curves of the chemical potential and heat capacity varying with the temperature for different $\eta = D/s$, as shown in figures 1 and 2, respectively.

From the curves in figure 1, one can obtain some important results, which are listed as follows:

(i) When $\eta = 0.5$, which may correspond to the system of nonrelativistic ideal fermions in a one-dimensional space, the chemical potential μ is not a monotonic function of temperature and there exists a maximum $\mu_{\max}$ at a certain temperature T_m for any values of q, as shown in figure 1(a). It is also observed from figure 1(a) that there exists a cross point between the curves with $q > 1$ and with $q = 1$ at a certain temperature T_c, so that $\mu_{q>1} > \mu_{q=1}$ when $T < T_c$ and $\mu_{q>1} < \mu_{q=1}$ when $T > T_c$. Figure 3 further shows the curves of $\mu_{\max}/\varepsilon_F$, $k_B T_m/\varepsilon_F$ and $k_B T_c/\varepsilon_F$ varying with the parameter q. It is seen that $\mu_{\max}/\varepsilon_F$ increases monotonically with q, while $k_B T_m/\varepsilon_F$ and $k_B T_c/\varepsilon_F$ decrease monotonically with q.

(ii) When $\eta = 1.0$, which may correspond to the system of nonrelativistic ideal fermions in a two-dimensional space or the system of ultrarelativistic ideal fermions in a one-dimensional space, the chemical potential μ is a monotonically decreasing function of temperature for any values of q, as shown in figure 1(b). At very low temperatures, μ remains nearly equal to the Fermi energy ε_F, which is independent of q. The result coincides with equation (27), since the coefficient of $(k_B T/\varepsilon_F)^2$ in equation (27) becomes zero when $\eta = 1$. This indicates that at low-temperature region, the difference of the chemical potentials between the q-deformed and ordinary Fermi systems disappears in the case of $\eta = 1.0$. At other temperature regions, $\mu_{q>1}$ is always smaller than $\mu_{q=1}$.

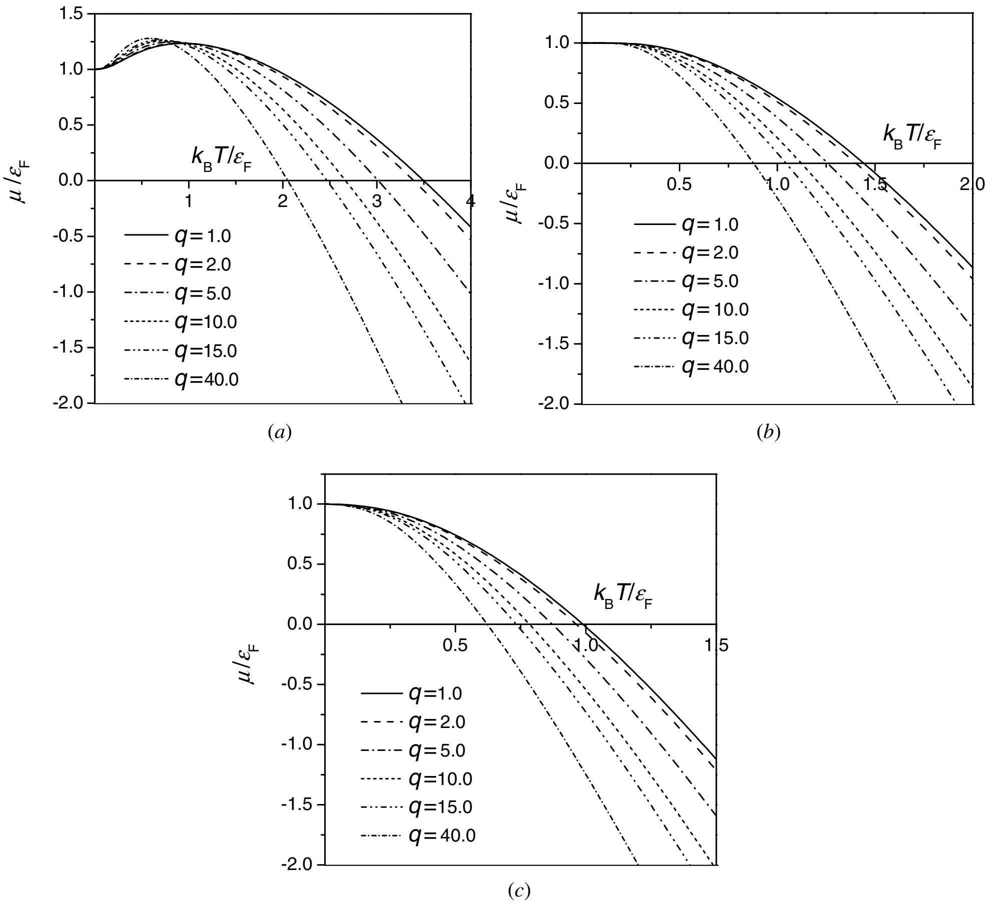

Figure 1. The curves of the scaled chemical potential μ/ε_F varying with the dimensionless temperature k_BT/ε_F for the q-deformed fermions with different parameter q in the cases of (a) $\eta = 0.5$, (b) $\eta = 1.0$ and (c) $\eta = 1.5$, respectively.

(iii) When $\eta = 1.5$, which may correspond to the system of nonrelativistic ideal fermions in a three-dimensional space, the curves of the chemical potential varying with the temperature share similar characteristics with the case of $\eta = 1.0$.

From the curves in figure 2, one can find some important characteristics of the heat capacity varying with the temperature for different values of η and q, which are listed as follows:

(i) When $\eta = 0.5$, there exists a maximum of the heat capacity at a certain temperature for any parameter q and the heat capacity at high temperatures approaches $C_{V,B} = 0.5Nk_B$, the value predicted by the Boltzmann distribution, from above, as shown in figure 2(a). It is also observed that the q-deformation increases the heat capacity at any temperatures in the case of $\eta = 0.5$.

(ii) When $\eta = 1.0$, the curves of $C_V/Nk_B \sim k_BT/\varepsilon_F$ display different characteristics for different values of q, as shown in figure 2(b). When q is smaller than a certain value q_0, C_V is a monotonically increasing function of temperature and $\lim_{T\to\infty} C_V = Nk_B - 0$. When $q > q_0$, there is a maximum of C_V and $\lim_{T\to\infty} C_V = Nk_B + 0$. In order to

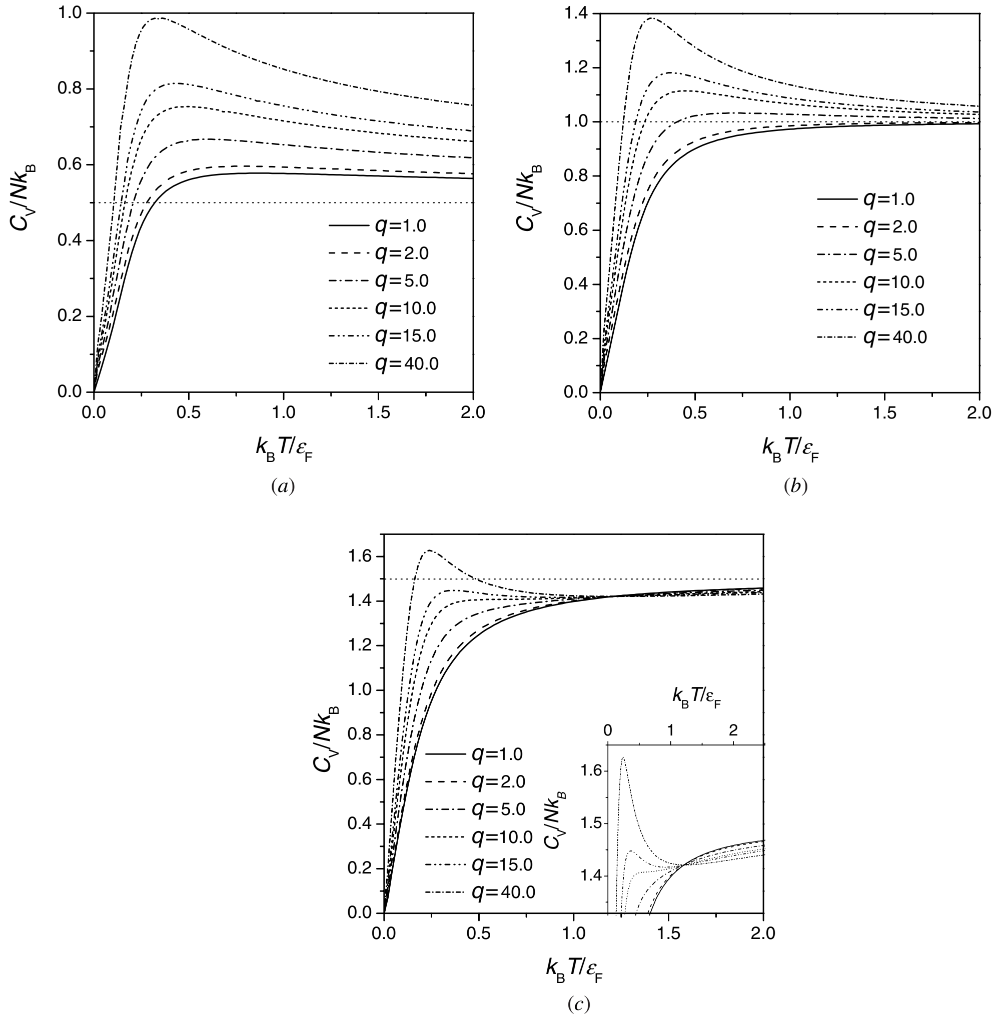

Figure 2. The curves of the scaled heat capacity C_V/Nk_B varying with the dimensionless temperature k_BT/ε_F for the q-deformed fermions with different parameter q in the cases of (*a*) $\eta = 0.5$, (*b*) $\eta = 1.0$ and (*c*) $\eta = 1.5$, respectively. The C_V/Nk_B-axis in the inset is partly stretched in order to show the characteristics of the curves more clearly.

determine q_0, we calculate the heat capacity at high temperatures to the second order in ε_F/k_BT from equations (13), (18) and (32). The result is given by

$$C_{V,\text{high}} = Nk_B\left[1 + \frac{5q^2 + 5q^{-2} - 22}{(q - q^{-1})^2}\frac{(\ln q)^2}{108}\left(\frac{\varepsilon_F}{k_BT}\right)^2\right]. \qquad (36)$$

It is seen from equation (36) that if $5q^2 + 5q^{-2} - 22 < 0$, $\lim_{T\to\infty} C_{V,\text{ high}} = Nk_B - 0$, and if $5q^2 + 5q^{-2} - 22 > 0$, $\lim_{T\to\infty} C_{V,\text{ high}} = Nk_B + 0$. It can be determined from the above analysis that $q_0 = \sqrt{(11 + 4\sqrt{6})/5} \approx 2.0$. Similar to the case of $\eta = 0.5$, the heat capacity always increases with the increase of q at any temperature in the case of $\eta = 1.0$.

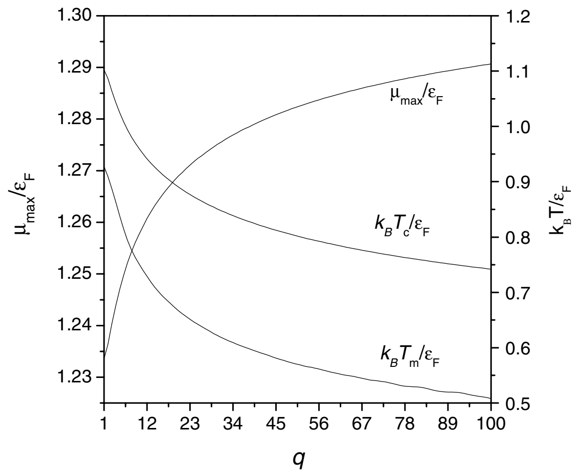

Figure 3. The curves of the maximal scaled chemical potential $\mu_{\max}/\varepsilon_F$ and the corresponding dimensionless temperature $k_B T_m/\varepsilon_F$ along with the dimensionless temperature $k_B T_c/\varepsilon_F$ varying with the deformation parameter in the case of $\eta = 0.5$.

(iii) When $\eta = 1.5$, the curves of $C_V/Nk_B \sim k_B T/\varepsilon_F$ become more complicated, as shown in figure 2(*c*). There exists a cross point between the curves of $q > 1$ and $q = 1$, so that $C_{V,\,q>1} > C_{V,\,q=1}$ when $T < T_d$ and $C_{V,\,q>1} < C_{V,\,q=1}$ when $T > T_d$, where T_d is the temperature at the cross point. The influence of the parameter q on the heat capacity is more obvious in the region of $T < T_d$ than in the region of $T > T_d$. For the small parameters q, such as $q = 1.0$ and 2.0, the heat capacity increases monotonously with the temperature. For the large parameters q, such as $q = 15.0$ and 40.0, however, C_V first increases with the temperature and reaches a maximum, then decreases and reaches a minimum below $C_V = 1.5Nk_B$. Unlike the cases of $\eta = 0.5$ and $\eta = 1.0$, the heat capacity at high temperatures approaches $C_V = 1.5Nk_B$ from below for any parameters q. The result can be seen from equation (35) as well, since the coefficient of $(k_B T/\varepsilon_F)^\eta$ in equation (35) is negative in the case of $\eta = 1.5$.

6. Conclusions

With the help of the q-deformed Fermi–Dirac distribution, we have studied the thermostatistic properties of a q-deformed Fermi gas in any dimensional space and with a general energy spectrum. Some important conclusions are obtained as follows. (i) The effects of the q-deformation on the properties of q-deformed Fermi gases display different characteristics for different dimensional spaces and energy spectrums. (ii) The q-deformation may significantly affect the low-temperature behaviors of a Fermi system but does not alter the ground-state properties of the system. (iii) At high temperatures ($k_B T \gg \varepsilon_F$), the q-deformed statistics reduces to the undeformed statistical mechanics, which implies that the q-deformation is a pure quantum effect.

Because of the general forms of the energy spectrum adopted, the results obtained here may be used to study the properties of a variety of q-deformed Fermi systems, such as nonrelativistic or ultrarelativistic q-deformed Fermi systems in any dimensional space.

If $q \to 1$ is set, the results obtained here are as well suitable for the systems of the ordinary fermions.

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References

[1] Borges E P, Tsallis C, Ananos G F J and Oliveira P M C 2002 *Phys. Rev. Lett.* **89** 254103
[2] Ananos G F J and Tsallis C 2004 *Phys. Rev. Lett.* **93** 020601
[3] Sotolongo-Costa O and Posadas A 2004 *Phys. Rev. Lett.* **92** 048501
[4] Jang S, Shin S and Pak Y 2003 *Phys. Rev. Lett.* **91** 058305
[5] Tsallis C 1988 *J. Stat. Phys.* **52** 479
[6] Biedenharn L 1989 *J. Phys. A: Math. Gen.* **22** L873
[7] Macfarlane A 1989 *J. Phys. A: Math. Gen.* **22** 4581
[8] Tsallis C 1994 *Phys. Lett.* A **195** 539
[9] Abe S 1997 *Phys. Lett.* **224** 326
[10] Abe S 1998 *Phys. Lett.* A **244** 229
[11] Johal R S 1998 *Phys. Rev.* E **58** 4147
[12] Ubriaco M R 1999 *Phys. Rev.* E **60** 165
[13] Arik M, Kornfilt J and Yildiz A 1997 *Phys. Lett.* A **235** 318
[14] Caracciolo R and Monteiro M R 1993 *Phys. Lett.* B **308** 58
[15] Lerda A and Sciuto S 1993 *Nucl. Phys.* B **401** 613
[16] Witten E 1990 *Nucl. Phys.* B **330** 285
[17] Dimakis A and Müller-Hoissen F 1992 *Phys. Lett.* B **295** 242
[18] Bonatsos D, Daskaloyannis C and Kolokotronis P 1992 *Phys. Rev.* A **46** 75
[19] Bonatsos D, Daskaloyannis C and Kolokotronis P 1993 *Phys. Rev.* A **48** 3611
[20] Bonatsos D, Daskaloyannis C and Kolokotronis P 1997 *J. Chem. Phys.* **106** 605
[21] Bonatsos D and Daskaloyannis C 1996 *Mod. Phys. Lett.* B **21** 1011
[22] Monteiro M R, Rodrigues L M C S and Wulck S 1996 *Phys. Rev. Lett.* **76** 1098
[23] Lavagno A and Swamy P N 2000 *Phys. Rev.* E **61** 1218
[24] Swamy P N 2005 *Physica* A **353** 119
[25] Swamy P N 2006 *Int. J. Mod. Phys.* B **20** 2537
[26] Lavagno A and Swamy P N 2002 *Physica* A **305** 310
[27] Shu Y, Chen J and Chen L 2002 *Phys. Lett.* A **292** 309
[28] Su G, Chen J and Chen L 2003 *J. Phys. A: Math. Gen.* **36** 10141
[29] Tuszynski J A, Rubin J L, Meyer J and Kibler M 1993 *Phys. Lett.* A **175** 173
[30] Algin A and Arik M 2003 *Physica* A **330** 442
[31] Bortz M and Sergeev S 2006 *Eur. Phys. J.* B **51** 395
[32] Naderi M H, Soltanolkotabi M and Roknizadeh R 2004 *J. Phys. A: Math. Gen.* **37** 3225
[33] Algin A 2002 *Phys. Lett.* A **292** 251
[34] Chang Z and Chen S 2002 *J. Phys. A: Math. Gen.* **35** 9731
[35] Ubriaco M R 1997 *Phys. Rev.* E **55** 291
[36] Sun C P and Fu H C 1989 *J. Phys. A: Math. Gen.* **22** L983
[37] Lee C R and Yu J P 1990 *Phys. Lett.* A **150** 63
[38] Lee C R and Yu J P 1992 *Phys. Lett.* A **164** 164
[39] Yan Z 2000 *Phys. Rev.* A **61** 063607
[40] Li M, Yan Z, Chen J, Chen L and Chen C 1998 *Phys. Rev.* A **58** 1445