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Sensitivities of two free vibration torsion pendulums

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In this article, for the necessity of measuring the damping of new materials, a detailed comparison was made between two important free vibration torsion pendulums, that is, the inverted torsion pendulum and the Collette pendulum. Special attention was paid to the strong effect of the torsionally weak suspension in both pendulums on the difference between measured damping of the overall system and the real damping exclusively contributed by the investigated specimen. This difference was successfully quantitatively characterized by a parameter: the sensitivity of the pendulum only via which the accurate damping for the specimen can be determined for each case. The comparison showed that the sensitivities of the two systems obviously behave differently. Although it produces smaller performing strain amplitude on the specimen and has a narrower preset vibration frequency range, the sensitivity of the Collette pendulum is always lower than that of the inverted torsion pendulum. © *1996 American Institute of Physics.* [S0034-6748(96)01012-X]

I. INTRODUCTION

The free vibration decay method is one of the major methods for measuring low frequency internal friction in solids. There are three kinds of torsion pendulums developed for low frequency free vibration decay, i.e., the normal torsion, inverted torsion, and Collette pendulums. Among those torsional pendulums, the normal torsion or the Coulomb-type torsion pendulum came into being first. In consisted basically of an inertia member (e.g., a horizontal bar) suspended from a wire specimen (see Fig. 1) and, therefore, seemed somewhat simply to copy the setting arrangements of the galvanometer suspension apparatus (shown in Fig. 2) (for quasistatic measurement of the elastic aftereffect) which date back well over a century in the work of Webber in 1835.[1] This apparatus, although occasionally called the ''Kê pendulum'' in the literature, for some unclear reason, was believed to be invented earlier before Kê's work in 1947 (Ref. 2) because it was employed much earlier by Guye in 1912 (Ref. 3) and Ishimoto in 1919.[4] Since then it became a dominant method for awhile to measure the low frequency internal friction in solids. Subsequently, there were many refined forms[5–10] for this kind of pendulum including Kê's work in 1947 (Ref. 2) in which the effort was made by this pendulum to measure anelastic relaxation internal friction but limited only to metals [the anelastic relaxation internal friction in solids was first highlighted by Zener in 1948 (Ref. 11) but was systematically theorized by Nowick and Berry in 1972 (Ref. 12)]. However, in the normal pendulum, only the specimen itself serves as the suspension wire. Thus, the tensile load exerted on the specimen by the weight of the inertia member (which is necessary to lower or change the system vibration frequency) is rather high and will inevitably lead to the undesired tension or creep deformation of an investigated specimen in addition to the torsion deformation. Another fatal drawback of the normal pendulum is the fact that its vibration frequency is totally determined by the natural frequency or the torsion modulus of the specimen. This will inevitably lead to serious error during an internal friction measurement over a wide temperature range (also a wide range of variation in the modulus of the specimen) which, on the other hand, however, is required for the measurement of activation energy associated with relaxation internal friction or for monitoring any material structure change. Therefore, the normal pendulum greatly limits its applications by its design. Although some laboratories are still employing this kind of torsion pendulum to measure internal friction, we should be aware that this torsion measurement will lead to serious problems as previously discussed, either when any activation energy is measured over a large range of temperature, any obvious modulus change or soft modulus of the investigated specimen is involved during changing temperature measurement, or when any undesired creep deformation in the specimen occurs. For this consideration, most torsion pendulums presently used in materials science are changed to be operated in the ''inverted'' configuration as shown in Fig. 3. The specimen here is located below the inertia member which hangs on a thin suspension wire (called a torsionally weak suspension wire) of low damping. In doing so, only a very small force on the specimen is needed in order to keep the system straight and the pendulum can perform without any inertia weight suspended at the end of specimen. Usually the suspension wire is made so thin that its torsional rigidity can be neglected with respect to that of the specimen. Then the measured damping of the inverted pendulum can be thought to be directly contributed to by the specimen provided that the specimen modulus is less variable (nearly at a constant) and much higher as well, than that of the suspension wire. Also, as this additional suspension wire is soft with respect to the investigated specimen, the vibration frequency of the system becomes less sensitive to the modulus variation of the specimen but is determined by both the specimen and the weak suspension wire. Therefore, it can reduce to a certain degree, the frequency-variation range due to the variation of

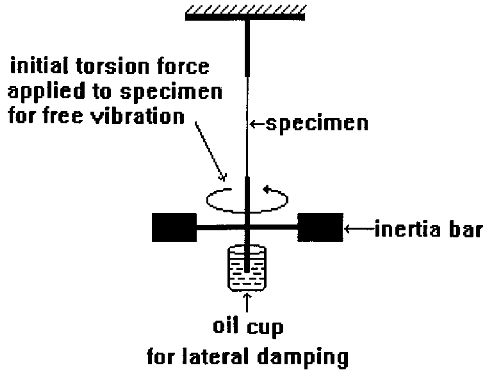

FIG. 1. A scheme of normal torsion pendulum apparatus described by Kê (Ref. 2).

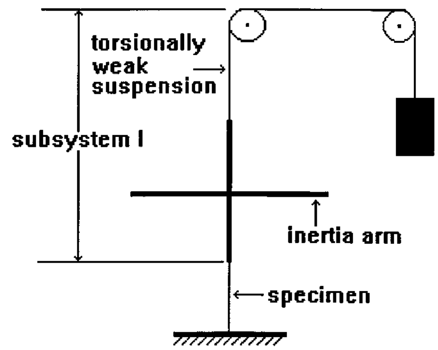

FIG. 3. Scheme of inverted torsion pendulum.

the specimen modulum as shown in the normal pendulum. Because most of the conventional metallic materials specimens can meet the above conditions, this technique has become so much a standard method in materials science that the damping measured with an inverted torsion pendulum is without further consideration commonly taken as the internal friction of the specimen. However, when the torsional modulus of a specimen becomes so variable during measurement that the vibration frequency very obviously changes during measurement or when the modulus of the specimen is so small that it becomes comparable with that of the suspension wire, the measured damping will become insensitive to the internal friction of the specimen but measured that of the suspension wire as analyzed by Sinning,[13] the normal torsion pendulum no longer becomes applicable. This is exactly the case of the measurement of the increasingly concerned damping in new materials like polymer, glass, nanomaterial, composite, etc., or any material undergoing some phase transition in which the soft modulus or the soft mode associated with the transition often appears. Although this kind of internal friction can be more easily measured by the forced vibration method[14–17] (in which the vibration frequency of the specimen is determined by an external signal and not by the natural frequency of the specimen or system itself), a more complicated type of inverted torsion pendulum for free decay measurement has been specially developed as well by Collette *et al.* for this case.[18] The Collette pendulum is formed on the basis of the inverted pendulum by adding a second suspension wire between the inertia member and the specimen as shown in Fig. 4. For such an arrangement, a more narrow frequency range than that in the inverted pendulum can be preset by adding such an appropriate soft suspension wire regardless of the modulus softening of the specimen and in this way the pendulum can also perform at a very small strain amplitude. Therefore, this pendulum has successfully been identified by the work of Sinning *et al.*[13] and some false experimental phenomena in the works[19,20] concerning low-frequency internal friction associated with the metallic glass transition which is measured by the inverted pendulum.

Obviously, the total measured damping in the Collette pendulum is never identical with that of the specimen but is always significantly contributed by the weak suspension wires. Also, for the general consideration as discussed above, the measured internal friction in the inverted pendulum should be considered to be contributed both by the specimen and by the torsionally weak suspension wire. However, how to calculate the internal friction with an emphasis on how to separate the different contributions, respectively, from the specimen and the weak suspension wires both in the inverted pendulum and in the Collette pendulum, there is no accurate formula found so far and there have always been disagree-

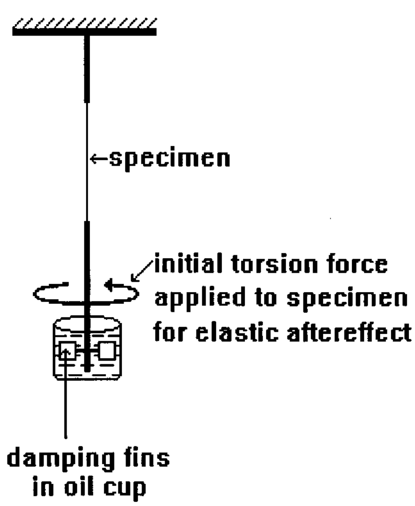

FIG. 2. A scheme of Galvanometer suspension apparatus for quasistatic measurements described by Kê (Ref. 2).

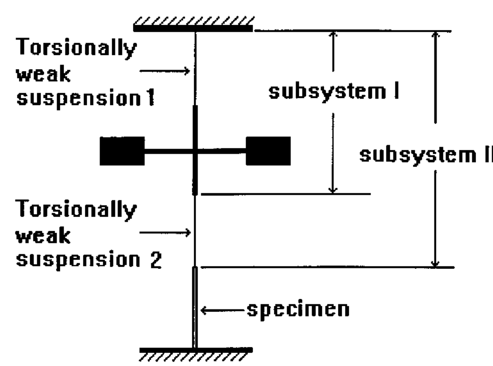

FIG. 4. Scheme of Collette torsion pendulum.

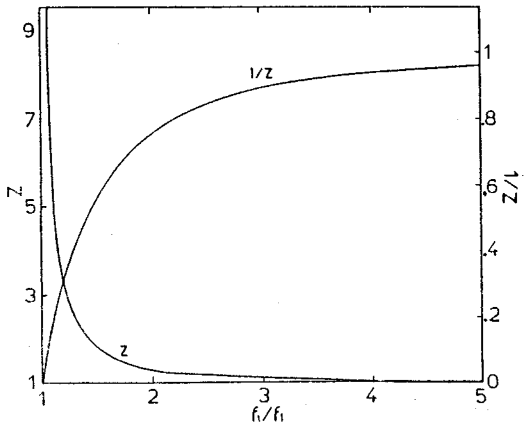

FIG. 5. Parameter Z and sensitivity $S_I(1/Z)$ vs parameter f_T/f_I curves for inverted pendulum.

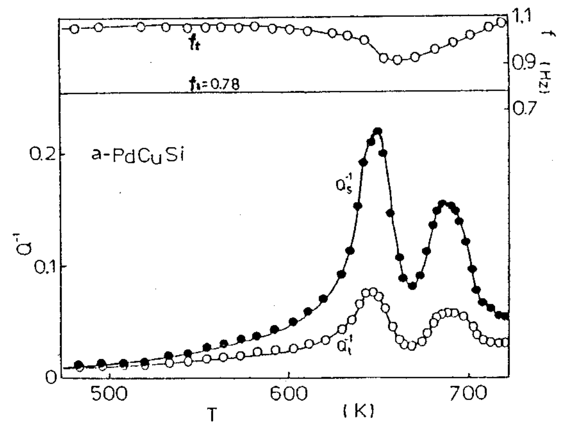

FIG. 6. Internal friction and frequency curves measured in ascending temperature by an inverted pendulum in an amorphous PdCuSi alloy.

ments because the available internal friction calculation formulas are all approximate.[13,18,21,22] This situation greatly limits their potential applications.

For the above considerations, this article on the basis of the previous works, has analyzed and discussed in detail the two different damping contributions from the torsionally weak suspension and the specimen both in the inverted torsion pendulum and in the Collette pendulum and has presented as well the accurate formulas to differentiate and calculate those two kinds of damping contributions in both pendulums.

II. INVERTED TORSION PENDULUM

A. Damping calculation formula

In the inverted torsion pendulum as shown in Fig. 3, if the detailed effect of the torsionally weak suspension is considered on the basis of work[13] according to Ref. 23 in which the effect of the logarithmic decrement on the vibration frequency is particularly accounted for, an accurate relation of the internal friction originated from the specimen, Q_s^{-1} with the measured overall system internal friction, Q_t^{-1} and the internal friction contributed by the weak suspension wire (the subsystem I), Q_{I}^{-1} can be obtained as follows:

$$Q_s^{-1}=ZQ_t^{-1}-(Z-1)Q_{\mathrm{I}}^{-1}, \tag{1}$$

where Z is defined as

$$Z=(f_T/f_{\mathrm{I}})^2/[(f_T/f_{\mathrm{I}})^2-1]. \tag{2}$$

Here, $f_T^2=(4\pi^2+\delta_t^2)f_t^2$ and $f_{\mathrm{I}}^2=(4\pi^2+\delta_1^2)f_1^2$, δ_t, f_t, and δ_1, f_1 are the logarithmic decrement and the vibration frequency, respectively, for the overall system and for the subsystem I.

For a low damping material like conventional metals, $4\pi^2 \gg \delta^2$, and δ^2 can be neglected. Thus, Eq. (2) can be simplified as

$$Z=f_t^2/(f_t^2-f_1^2). \tag{3}$$

Equation (3) is just the approximate formula given by Sinning[13] and cannot be applicable to high damping measurement.

B. Sensitivity of inverted pendulum

In order to get a more clear physical meaning, we define the sensitivity of a free vibration pendulum as the variation of the overall system internal friction S_{I} with the specimen internal friction Q_s^{-1} as defined by Sinning.[22] Therefore, we have

$$S_{\mathrm{I}}=\partial Q_t^{-1}/\partial Q_s^{-1}. \tag{4}$$

Using Eqs. (1) and (4), we get

$$S_{\mathrm{I}}=1/Z. \tag{5}$$

In terms of Eq. (5), we can see that when $Z\rightarrow 1$, $S\rightarrow 1$, the larger Z, the smaller S_{I}. The detailed relations among the parameters: Z, $1/Z(=S_{\mathrm{I}})$ and f_T/f_{I} can be easily obtained from Eqs. (1), (2), (4), and (5) and have been shown in Fig. 5 as well. From Fig. 5, it is obvious that the sensitivity S_{I} increases very rapidly to its final value 1 when f_T/f_{I} increases and when f_T/f_{I} approaches 5, and S_{I} approaches 1. It means that in the inverted pendulum, if f_T/f_{I} is close to 5, Q_t^{-1} can be used to replace Q_s^{-1} when Q_t^{-1} is not very large. This is the case of the internal friction measurement of common metallic materials in the inverted pendulum, therefore

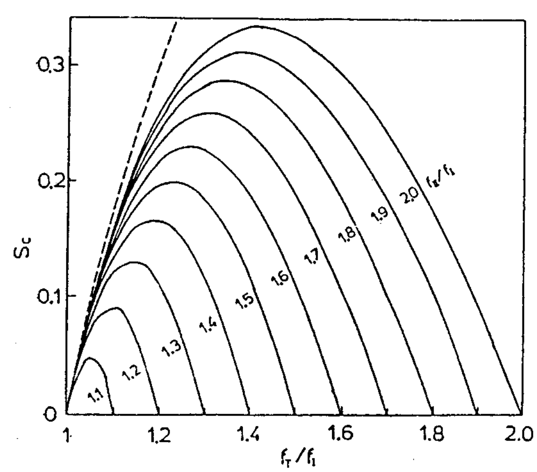

FIG. 7. Sensitivity S_C vs f_T/f_{I} curves at different value of parameter $f_{\mathrm{II}}/f_{\mathrm{I}}$.

TABLE I. Essential features of two pendulums.

	Inverted pendulum	Collette pendulum
Number of suspension and setting	One suspension above specimen	Two suspensions above and below specimen
Sensitivity expression and feature	$S_I = 1/\{(f_T/f_I)^2/[(f_T/f_I)^2-1]\}$ $S_I<1$	$S_C = (f_{II}^2-f_T^2)(f_T^2-f_I^2)/[f_T^2(f_{II}^2-f_I^2)]$ $S_C<1/3$
Specimen internal friction	(i) $Q_s^{-1}=Q_t^{-1}$ if $f_T/f_I>5$ and Q_I^{-1} not large (ii) $Q_s^{-1}=$ Eq. (2) if $f_T/f_I<5$	$Q_s^{-1}=$ Eq. (6)
Merit and drawback	(i) higher sensitivity (ii) cannot be at small amplitude (iii) frequency depending more on specimen	(i) lower sensitivity (ii) can be at small amplitude (iii) frequency depending less on specimen

the effect of the torsionally weak suspension can be ignored. However, for the specimen of a material whereby f_T/f_I is small so that S_I is substantially less than 1, Eq. (1) should be used to calculate internal friction, especially when f_T/f_I approaches 1, S_I approaches 0, and the calculation error of the internal friction value obtained by Eq. (1) will be reduced. The experimental results in this case should be treated more carefully.

C. Experimental results and analysis

Figure 6 shows the internal friction and vibration frequency results with increasing temperature measured by the inverted pendulum for an amorphous PdCuSi solid. The two internal friction peaks are associated with the glass transition and the crystallization in the amorphous alloy which were once extensively studied and have shown that those processes internal frictions are good candidates to demonstrate the different effects of the suspension wire. The overall system internal friction Q_t^{-1} measurement result is shown as the open circles which is identical with the earlier published data without considering the suspension wire effect.[19] Q_s^{-1}, the true specimen internal friction shown as the full circles are derived by the accurate expression (1) using measured values: f_t, f_1, Q_t^{-1}, and Q_I^{-1} (Q_I^{-1} is constant and not shown in Fig. 6). It is obvious that the true specimen internal friction Q_s^{-1} is much higher than the overall system internal friction Q_t^{-1}, and both positions and shapes of the two peaks in Q_s^{-1} are different from those in Q_t^{-1}. Therefore, the effect of the torsionally weak suspension is very pronounced where any soft modulus of an investigated specimen occurs in the inverted pendulum and the accurate determination of the specimen contribution is critically important in this case.

III. COLLETTE PENDULUM

A. Internal friction calculation formula

Sinning[13,22] also once presented an internal friction calculation formula for the Collette pendulum but he neglected the effect of δ^2 in the term $(4\pi^2+\delta^2)$. The strict equation for the specimen internal friction should be

$$\begin{aligned} Q_s^{-1} = {} & f_T^2(f_{II}^2-f_I^2)Q_t^{-1}/[(f_{II}^2-f_T^2)(f_T^2-f_I^2)] \\ & - f_I^2(f_{II}^2-2f_I^2+f_T^2)Q_I^{-1}/[(f_T^2-f_I^2)(f_{II}^2-f_I^2)] \\ & - f_{II}^2(f_T^2-f_I^2)Q_{II}^{-1}/[(f_{II}^2-f_T^2)(f_{II}^2-f_I^2)], \end{aligned} \tag{6}$$

where $f_I^2=(4\pi^2+\delta_1^2)f_1^2$, $f_{II}^2=(4\pi^2+\delta_2^2)f_2^2$, and $f_T^2=(4\pi^2+\delta_t^2)f_t^2$, f_1, f_2, f_t, and δ_1, δ_2, δ_t are the vibration frequencies and the logarithmic decrements, respectively, for the subsystem I, the subsystem II, and the overall Collette pendulum as shown in Fig. 4. Meanwhile, Q_I^{-1} and Q_{II}^{-1} are the internal frictions, respectively, from the subsystems I and II at the corresponding frequencies.

B. Sensitivity of Collette pendulum

In terms of our definition of the sensitivity of Eq. (4), we can obtain from Eq. (6) the sensitivity in Collette pendulum

$$S_C=(f_{II}^2-f_T^2)(f_T^2-f_I^2)/[f_T^2(f_{II}^2-f_I^2)]. \tag{7}$$

From Eq. (7), we can obtain plots of S_C vs f_T/f_I and f_{II}/f_I and those plots have been shown in Fig. 7. It can be found that for a given value of f_{II}/f_I, there exists a maximum value for S_C, i.e., $S_{C\max}=(f_{II}-f_I)/(f_{II}+f_I)$ when $f_t^2=f_If_{II}$. Therefore, when f_{II}/f_I increases, $S_{C\max}$ also increases in a way as shown in Fig. 7. Generally, the value f_{II}/f_I in a Collette pendulum is less than 2, so S_C should be less than or equal to 1/3. Thus, the Collette pendulum often works with a lower sensitivity compared with the sensitivity in a usual inverted pendulum as shown in Fig. 5. Therefore, the effect of the torsionally weak suspension in the Collette pendulum should be taken into account.

On the other hand, it can be easily proven that when f_{II}/f_I approaches the indefinite, we have

$$S_C=1/Z=S_I. \tag{8}$$

It means that when $f_{II}/f_I\to\infty$, the sensitivity of the Collette pendulum approaches the sensitivity of the inverted torsion pendulum. In other words, for a certain value of f_T/f_I, the sensitivity of the Collette pendulum is always lower than that of the inverted torsion pendulum. This fact is illustrated in Fig. 7 in which the dotted line represents the sensitivity of the inverted torsion pendulum. As previously discussed, it can be concluded that the advantage of the narrow variable range of the vibration frequency and the smaller vibration amplitude of the Collette pendulum are gained at the price of lowering its sensitivity.

According to the above discussion, we can get Table I to summarize the essential features of two pendulums.

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