PHYSICAL REVIEW B, VOLUME 65, 134507

Josephson current in ferromagnet-superconductor tunnel junctions

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(Received 23 August 2001; published 19 March 2002)

We present a phase diagram of a ferromagnetic superconductor (FS) in T-h plane with T the temperature and h the effective exchange field, in which a first-order phase-transition line separates the superconducting region from the normal-state region. The Josephson currents in an FS/I/FS junction (with I denoting thin insulating layers) are calculated as a function of the temperature, exchange field, and insulating barrier strength. It is found that the presence of h always suppresses the Josephson critical current at higher temperatures, or for weak barrier strength, or for a parallel configuration of the magnetic moments of two FS electrodes. The only exception that the critical current increases with h occurs if all three conditions are satisfied: at low temperatures, for strong barrier strength, and in an antiparallel configuration.

DOI: 10.1103/PhysRevB.65.134507 PACS number(s): 74.50.+r, 74.80.Dm, 73.40.Gk, 73.40.Rw

I. INTRODUCTION

The Josephson effect at superconductor/insulator/superconductor ($S/I/S$) junctions has been a most interesting subject. Unlike single-particle tunneling, the tunneling of Cooper pairs can lead to a finite supercurrent (or Josephson current) flowing across the $S/I/S$ junction in the absence of bias voltage. The Josephson current is given by $I=I_c \sin\phi$ where I_c is the critical current and ϕ is the phase difference between the two superconducting electrodes. In recent years the ferromagnet (F) has been introduced to the Josephson tunnel structure, giving rise to some new physical effects. As an example, if very thin insulating layers in a $S/I/S$ junction are replaced by thin ferromagnetic layers, a new type of $S/F/S$ Josephson junction is formed.[1–10] In the $S/F/S$ junctions the tunneling electrons with spin-up and spin-down experience different potentials in F, and the Andreev reflection at F/S interfaces plays an important role in the properties of the $S/F/S$ junction.[5,7,9] With increasing the exchange field in F, the Josephson current displays a damped oscillation, a change of its sign from positive to negative corresponding to a transition from a zero phase difference between two S electrodes to a phase difference of $\phi=\pi$, (the so-called "π" junction). Very recently, Bergeret, Volkov, and Efetov[11] proposed a Josephson tunnel junction of two F/S bilayers separated by a thin insulating film. On the assumption that a thin F/S bilayer is equivalent to a homogeneous ferromagnetic superconductor (FS), the $S/F/I/F/S$ structure may be simplified as a FS/I/FS junction. They found that the presence of an exchange field may increase the critical current in the $S/F/I/F/S$ junction in the case of an antiparallel alignment of the magnetization in the ferromagnets at low temperatures. However, this conclusion was drawn by using the tunneling Hamiltonian approach, in which a strong barrier strength is assumed for the thin insulating layers. It is highly desirable to clarify the effect of the barrier strength on the Josephson current in such a FS/I/FS tunnel junction, as well as its dependence on the temperature and exchange field.

Coexistence of superconductivity and ferromagnetism has attracted much attention recently.[12–14] It was predicted in early 1960's by Fulde and Ferrel[15] and Larkin and Ovchinnikov[16] (FFLO) that pairing still can occur when electron momenta at the Fermi energy are different for two spin directions, for instance as the result of an exchange field in an FS. Unlike the conventional Cooper pair, in which two electrons have opposite spins and momenta $(K\uparrow,-K\downarrow)$, the "FFLO" pairing in the presence of an exchange field has a finite center-of-mass momentum $Q\propto 2h/\hbar v_F$ and consequently leads to a spatially modulated superconducting order parameter, where $2h$ is the exchange energy corresponding to the difference in energy between the spin-up and spin-down bands, and v_F is the Fermi velocity. The "FFLO" state with $[(K+Q/2)\uparrow,(-K+Q/2)\downarrow]$ was never observed in bulk materials. It stems from the fact that in a bulk ferromagnet, h is at least two orders of magnitude larger than the energy gap Δ_0 of a bulk superconductor, while the normal state is recovered as soon as h exceeds $\Delta_0/\sqrt{2}$, which is called Clogston criterion[17] at zero temperature. However, this criterion may be satisfied for a thin F/S bilayer whose effective exchange field h and effective superconducting order parameter Δ may be of the same order of magnitude.[11]

The purpose of this paper is twofold. The first one is to study the coexistence of superconductivity and ferromagnetism in a FS. Although the coexisting condition $h<\Delta_0/\sqrt{2}$ was known at zero temperature, it has not yet been very clear at finite temperatures. By solving self-consistently the superconducting order parameter $\Delta(T,h)$ from the Bogoliubov–de Gennes (BdG) equation[18] and calculating the difference in thermodynamic potential between the superconducting and normal states, we obtain a boundary line in T-h plane, which separates the superconducting region from the normal-state region. The other purpose of this paper is to extend the approach of Blonder, Tinkham, and Klapwijk[19] to study the Josephson current in FS/I/FS junctions. Analytic expressions for the Josephson currents in parallel and antiparallel alignments of the magnetizations in two FS's are obtained as a function of the temperature, exchange field, and barrier strength. In the limit of strong barrier strength, they reduce to the results obtained recently in the $S/F/I/F/S$ junction.[11] If h is taken to be zero, the expression for the Josephson current in a nonmagnetic $S/I/S$ tunnel junction[20] will be reproduced. It is found that the presence of an exchange field usually reduces the Josephson critical current in the FS/I/FS junction. Only on condition of strong barrier strength, low temperatures, and the antiparallel configuration of the magnetizations, the critical current in the FS/I/FS junction may exceed that of the Josephson junction in the absence of the exchange field.

II. FERROMAGNETIC SUPERCONDUCTOR

Consider a FS/I/FS junction structure of two FS films separated by very thin insulating layers. The FS film may consists of a S/F bilayer on the assumption that the thickness of the superconducting layer is smaller than the superconducting coherent length and that of the ferromagnetic layer smaller than the length of the condensate penetration into the ferromagnet.[11] In this case, solutions of the superconducting order parameter may be regarded as being independent of the coordinates and the influence of the ferromagnetic layer on superconductivity is not local. As a result, a S/F bilayer is equivalent to an FS film with a homogeneous superconducting order parameter Δ and an effective exchange field h. As has been given in Ref. 11, h is much smaller than that in an isolated ferromagnetic film and of the same order of magnitude as the effective value of Δ. We adopt the BdG equation approach[18] to study the superconducting order parameter $\Delta(T,h)$ in an FS film. In the absence of spin-flip scattering, the four-component BdG equations are decoupled into two sets of two-component equations: one for the spin-up eletronlike and spin-down holelike quasiparticle wave function $(u_\uparrow, v_\downarrow)$, the other for $(u_\downarrow, v_\uparrow)$.[21] The BdG equation for $(u_\uparrow, v_\downarrow)$ is given by

$$\begin{pmatrix} H_0-h & \Delta(T,h) \\ \Delta^\star(T,h) & -H_0-h \end{pmatrix}\begin{pmatrix} u_\uparrow \\ v_\downarrow \end{pmatrix}=E\begin{pmatrix} u_\uparrow \\ v_\downarrow \end{pmatrix}, \tag{1}$$

where H_0 is the single-particle Hamiltonian and the quasiparticle energy E is measured relative to the Fermi energy E_F. The effective superconducting order parameter $\Delta(T,h)$ is independent of the coordinates, but depends on the effective exchange field and temperature. From the BdG equations, we get

$$u_\sigma^2=\tfrac{1}{2}[1+\sqrt{1-\Delta^2(T,h)/(E+\eta_\sigma h)^2}], \tag{2}$$

$$v_{\bar\sigma}^2=\tfrac{1}{2}[1-\sqrt{1-\Delta^2(T,h)/(E-\eta_{\bar\sigma} h)^2}], \tag{3}$$

where $\eta_\sigma=1$ for $\sigma=\uparrow$ and $\eta_\sigma=-1$ for $\sigma=\downarrow$, and $\bar\sigma$ stands for the spin opposite to σ. The wave vectors of the electronlike and holelike quasiparticles are given by

$$k_\sigma^e=\frac{\sqrt{2m}}{\hbar}[E_F+\sqrt{(E+\eta_\sigma h)^2-\Delta^2(T,h)}]^{1/2}, \tag{4}$$

$$k_{\bar\sigma}^h=\frac{\sqrt{2m}}{\hbar}[E_F-\sqrt{(E-\eta_{\bar\sigma} h)^2-\Delta^2(T,h)}]^{1/2}. \tag{5}$$

The order parameter $\Delta(T,h)$ of the FS film is determined by the self-consistent equation $\Delta=g\langle\Psi_\uparrow\Psi_\downarrow\rangle$, where g is the effective attractive potential between electrons, $\Psi_\uparrow=\Sigma_k(\gamma_{k\uparrow}u_{k\uparrow}-\gamma_{k\downarrow}^+v_{k\uparrow}^*)$ and $\Psi_\downarrow=\Sigma_k(\gamma_{k\downarrow}u_{k\downarrow}+\gamma_{k\uparrow}^+v_{k\downarrow}^*)$ with $\gamma_{k\uparrow}$ and $\gamma_{k\downarrow}$ the Bogoliubov transformative operators.[18] With Eqs. (2) and (3) as well as the rules that $\gamma_{k\uparrow}$ and $\gamma_{k\downarrow}$ obey,[18] we obtain

$$1=\frac{g}{2}\sum_k\left(\frac{1-f_{k\uparrow}}{\xi_{k\uparrow}}-\frac{f_{k\downarrow}}{\xi_{k\downarrow}}\right), \tag{6}$$

where

$$f_{k\sigma}=\frac{1}{\exp[\beta(\xi_{k\sigma}-\eta_\sigma h)]+1}, \tag{7}$$

with $\xi_{k\sigma}=\sqrt{(\hbar^2k_\sigma^{e\,2}/2m-E_F)^2+\Delta^2(T,h)}$ and $\beta=1/k_BT$ the inverse temperature. From Eqs. (6) and (7), the self-consistent equation for $\Delta=\Delta(T,h)$ is obtained as

$$\ln\left(\frac{\Delta_0}{\Delta}\right)=\int_0^{\hbar\omega_D}\frac{d\epsilon}{\sqrt{\epsilon^2+\Delta^2}}\left(\frac{1}{\exp[\beta(\sqrt{\epsilon^2+\Delta^2}-h)]+1}+\frac{1}{\exp[\beta(\sqrt{\epsilon^2+\Delta^2}+h)]+1}\right), \tag{8}$$

where $\Delta_0=\Delta(0,0)$ is the BCS gap at zero temperature and in the absence of the exchange field, and ω_D is the Debye frequency. If $h=0$ is taken in Eq. (8), the formula is found to reduce to Eq. (16.27) of Ref. 22. On the other hand, it follows from Eq. (8) that at $T=0$, $\Delta=\Delta_0$ remains unchanged for $h<\Delta_0$; as h is increased to Δ_0, Δ suddenly drops to zero, exhibiting a first-order phase transition from the superconducting state to normal state. This zero-temperature solution that $\Delta=\Delta_0$ for $h<\Delta_0$ and $\Delta=0$ for $h>\Delta_0$ has been obtained previously.[15,16] At finite temperatures, it is found from Eq. (8) that Δ still has a sudden drop from a finite value to zero at a threshold of h. Such a first-order phase transition

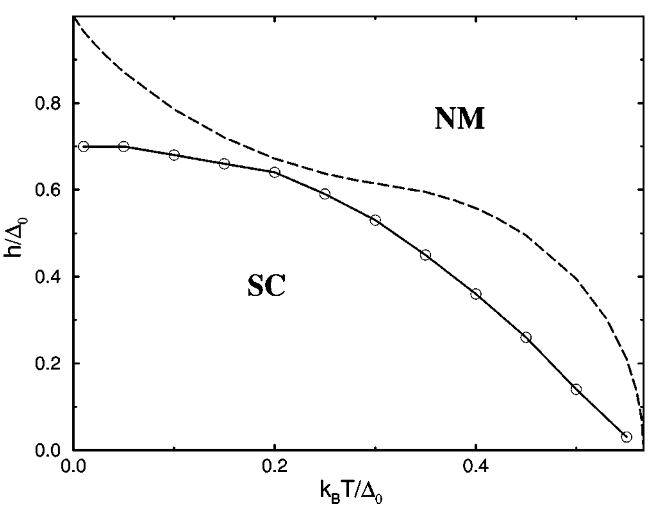

FIG. 1. Phase digram in the h-T plane. The solid line indicates the critical line for a first-order phase transition from superconducting to normal state, and the dashed line stands for a boundary line above which there is no nonzero solution of $\Delta(T,h)$ in Eq. (8).

arises from the presence of an exchange field, similar to the superconducting transition in the presence of an applied magnetic field.

Figure 1 shows the phase diagram in h-T plane, indicating superconducting and normal-state regions. The dashed line stands for the critical line below which there exists nonzero solution for $\Delta(T,h)$. From Eq. (8), one would obtain multivalued solutions for $\Delta(T,h)$. Among them we take only one branch of solutions, corresponding to the lowest thermodynamic potential, in determining each point of the dashed line in Fig. 1. In the region below the dashed line, however, whether there is the superconducting state or the normal state depends on which state has lower thermodynamic potential. For this reason, we must calculate the difference in the thermodynamic potential between the superconducting state $\Omega_s(T,h)$ and the normal state $\Omega_n(T,h)$. This difference may be rewritten as the sum of three differences: $\Omega_s(T,h) - \Omega_n(T,h) = [\Omega_s(T,h) - \Omega_s(T,0)] + [\Omega_s(T,0) - \Omega_n(T,0)] + [\Omega_n(T,0)-\Omega_n(T,h)]$. The first difference can be calculated using the standard integral representation,[23] namely,

$$\Omega_s(T,h)-\Omega_s(T,0)=\int_0^1 \frac{d\lambda}{\lambda}\langle\lambda\hat{H}_1\rangle_\lambda, \tag{9}$$

where $\Omega_s(T,0)$ is the thermodynamic potential for the BCS Hamiltonian in the absence of the exchange field, and $\langle\cdots\rangle$ denotes the ensemble average with the BCS Hamiltonian and $\lambda\hat{H}_1$. In the present case, $\hat{H}_1$ should be taken as[24]

$$\hat{H}_1=\sum_k h(c^\dagger_{k\uparrow}c_{k\uparrow}-c^\dagger_{k\downarrow}c_{k\downarrow}). \tag{10}$$

Substituting Eq. (10) into Eq. (9), we obtain

$$[\Omega_s(T,h)-\Omega_s(T,0)]/N(0)=2\int_0^h du\int_0^{\hbar\omega_D}d\epsilon\left(\frac{1}{1+\exp[\beta(\sqrt{\Delta^2(T,u)+\epsilon^2}+u)]}-\frac{1}{1+\exp[\beta(\sqrt{\Delta^2(T,u)+\epsilon^2}-u)]}\right), \tag{11}$$

where $N(0)$ is the electronic density of states at E_F. If Δ is taken to be zero in Eq. (11), we obtain the third difference in the thermodynamic potential as

$$[\Omega_n(T,0)-\Omega_n(T,h)]/N(0)=h^2. \tag{12}$$

In the absence of h the thermodynamic potential difference between the superconducting and normal states has been calculated previously, yielding[23]

$$\begin{aligned}[\Omega_s(T,0)-\Omega_n(T,0)]/N(0) &= -\frac{4}{\beta}\int_0^{\hbar\omega_D}d\epsilon\ln[1+\exp(-\beta\sqrt{\Delta^2(T,0)+\epsilon^2})] \\ &\quad -\frac{1}{2}\Delta^2(T,0)\left(1+2\ln\left[\frac{\Delta_0}{\Delta(T.0)}\right]\right)+\frac{\pi^2}{3\beta^2}.\end{aligned} \tag{13}$$

Combining Eqs. (11)–(13) with Eq. (8), we can evaluate $\Omega_s(T,h)-\Omega_n(T,h)$. The calculated result indicates that $\Omega_s(T,h)$ is smaller than $\Omega_n(T,h)$ at small h and at low temperatures; with increasing h or T, the difference between them decreases gradually. As h and/or T are increased to a set of critical values, $h=h_c$ and $T=T_c$, there will be $\Omega_s(T,h) =\Omega_n(T,h)$. These sets of (h_c,T_c) form a critical line, as shown by the solid line in Fig. 1, below which there is a stable superconducting state. In the region between the solid and dashed lines, even though there is nonzero solution for Δ in Eq. (8), the system is still in the normal state due to $\Omega_s(T,h)>\Omega_n(T,h)$ there. It is worth mentioning that we get $[\Omega_s(0,h)-\Omega_n(0,h)]/N(0)=h^2-\Delta_0^2/2$ at $T=0$, yielding $h_c =\Delta_0/\sqrt{2}$, which is just the result obtained previously by Clogston.[17]

III. JOSEPHSON CURRENT IN FS/I/FS JUNCTIONS

In the FS/I/FS junction under consideration, two FS electrodes are assumed to be identical in Δ and h, except for a phase difference, their magnetic moments being parallel or antiparallel to each other. The insulating layer is perpendicular to x axis and located at $x=0$. The insulating barrier may be modeled by a δ-type potential $V\delta(x)$ where V indicates the barrier strength. Consider an electronlike quasiparticle incident on the insulating barrier at $x=0$ from the left FS. With general solutions of the Eq. (1), the wave functions are given by

$$\Psi_{L\sigma}(x)=e^{ik^e_{L\sigma}x}\begin{pmatrix}u_{L\sigma}e^{i\phi_L/2}\\ v_{L\bar{\sigma}}e^{-i\phi_L/2}\end{pmatrix}+a_{\bar{\sigma}}e^{ik^h_{L\bar{\sigma}}x}\begin{pmatrix}v_{L\bar{\sigma}}e^{i\phi_L/2}\\ u_{L\sigma}e^{-i\phi_L/2}\end{pmatrix}+b_\sigma e^{-ik^e_{L\sigma}x}\begin{pmatrix}u_{L\sigma}e^{i\phi_L/2}\\ v_{L\bar{\sigma}}e^{-i\phi_L/2}\end{pmatrix} \tag{14}$$

for $x<0$, and

$$\Psi_{R\sigma}(x)=c_\sigma e^{ik^e_{R\sigma}x}\begin{pmatrix}u_{R\sigma}e^{i\phi_R/2}\\ v_{R\bar{\sigma}}e^{-i\phi_R/2}\end{pmatrix}+d_{\bar{\sigma}}e^{-ik^h_{R\bar{\sigma}}x}\begin{pmatrix}v_{R\bar{\sigma}}e^{i\phi_R/2}\\ u_{R\sigma}e^{-i\phi_R/2}\end{pmatrix} \tag{15}$$

for $x>0$. Here a_σ, b_σ, c_σ and d_σ correspond, respectively, to the coefficients, for the Andreev reflection,[25] normal reflection, transmission to the right FS as electronlike quasiparticles and transmission as holelike quasiparticles. ϕ_L and ϕ_R stand for the macroscopic phase of the left and right FS, respectively. Subscript L (R) is the index for the left (right) FS, and the spin index $\sigma=\uparrow$ or $\downarrow$.

The wave functions must satisfy the boundary conditions

$$\Psi_{R\sigma}(x=0^+)=\Psi_{L\sigma}(x=0^-), \tag{16}$$

$$\left(\frac{d\Psi_{R\sigma}}{dx}\right)_{x=0^+}-\left(\frac{d\Psi_{L\sigma}}{dx}\right)_{x=0^-}=\frac{2mV}{\hbar^2}\Psi_{R\sigma}(x=0^+). \tag{17}$$

From Eqs. (14)–(17), we obtain

$$a_\sigma(\phi,E)=\frac{(E+h\eta_\sigma)(\cos\phi-1)-i\Omega_\sigma\sin\phi}{(E+h\eta_\sigma)^2-\Delta^2(T,h)\cos\phi+Z\Omega_\sigma^2}\Delta(T,h) \tag{18}$$

for the parallel configuration, and

$$a_\sigma(\phi,E)=\frac{(E+h\eta_\sigma)\cos\phi-(E-h\eta_\sigma)-i\Omega_\sigma\sin\phi}{(E^2-h^2)-\cos\phi\Delta^2(T,h)+Z\Omega_\uparrow\Omega_\downarrow}\Delta(T,h) \tag{19}$$

for the antiparallel configuration. Here $\phi=\phi_R-\phi_L$, $Z=1+mV/\hbar^2k_F^2$, and $\Omega_\sigma=\sqrt{(E+h\eta_\sigma)^2-\Delta^2(T,h)}$. Since h is much smaller than E_F, we have made the approximation of $k_\uparrow^e=k_\downarrow^e=k_\uparrow^h=k_\downarrow^h=k_F$ with k_F the Fermi wave vector of the FS.

Having obtained $a_\sigma(\phi,E)$, the Josephson current can be calculated using the generalized coefficient of the Andreev reflection $a_\sigma(\phi,i\omega_n)$, which is obtained by analytic continuation of E to $i\omega_n$, yielding[5,20]

$$I=\frac{ek_BT\Delta(T,h)}{2\hbar}\sum_{\omega_n}\left(\frac{a_\uparrow(\phi,i\omega_n)-a_\uparrow(-\phi,i\omega_n)}{\Omega_{n\uparrow}}+\frac{a_\downarrow(\phi,i\omega_n)-a_\downarrow(-\phi,i\omega_n)}{\Omega_{n\downarrow}}\right), \tag{20}$$

with $\Omega_{n\sigma}=\sqrt{(\omega_n+ih\eta_\sigma)^2+\Delta(T,h)^2}$, and the Matsubara frequency $\omega_n=2\pi k_BT(n+1/2)$.

Substituting Eq. (18) into Eq. (20), we obtain the Josephson current for the parallel configuration as

$$I_P=\frac{\pi k_BT\Delta^2(T,h)\sin\phi}{eR_N}\times\sum_{\omega_n}\frac{\omega_n^2-h^2+\alpha\Delta^2(T,h)}{(\omega_n^2-h^2+\alpha\Delta^2(T,h))^2+4\omega_n^2h^2} \tag{21}$$

with $\alpha=(\cos\phi+Z)/(1+Z)$ and R_N the normal-state resistance.[20] Similarly, we obtain the Josephson current for the antiparallel configuration as

$$I_{AP}=\frac{\pi k_BT\Delta^2(T,h)}{eR_N}\sum_{\omega_n}\frac{(1+Z)\sin\phi}{\omega_n^2+h^2+\cos\phi\Delta^2(T,h)+Z\sqrt{(\omega_n^2-h^2+\Delta^2(T,h))^2+4\omega^2h^2}}. \tag{22}$$

In Eqs. (21) and (22), the superconducting gap $\Delta(T,h)$ depends on both the temperature T and exchange field h, its value being determined by Eq. (8). For given T and h, a substitution of the solution for $\Delta(T,h)$ into Eqs. (21) and (22) yields the Josephson currents I_P and I_{AP}.

IV. RESULTS AND DISCUSSIONS

If h is taken to be zero in Eqs. (21) and (22), we have $I=I_P=I_{AP}$, yielding

$$I=\frac{\pi k_BT\Delta^2(T,0)\sin\phi}{eR_N}\sum_{\omega_n}\frac{1}{\omega_n^2+\alpha\Delta^2(T,0)}, \tag{23}$$

which is in agreement with Eq. (20) of Ref. 20. However, in the presence of h, the Josephson currents of the junction are different from each other for the parallel and antiparallel configurations. Figures 2 and 3 show numerical results for I_P and I_{AP}, respectively, as functions of the exchange field at different temperatures in the superconducting region. It is found that for the parallel configuration, the exchange field always suppresses the critical current I_p, regardless of whether the barrier strength is weak [$Z=1$ in Fig. 2(a)] or strong [$Z=5$ in Fig. 2(b)]. For the antiparallel configuration, the situation is somewhat different. With increasing h, the critical current I_{AP} decreases for weak barrier strength [$Z=1$ in Fig. 3(a)] or at higher temperature [dashed lines in Figs. 3(a) and 3(b)], but increases for strong barrier strength ($Z=5$) and at low temperatures, as shown by the upper two lines in Fig. 3(b). The interesting effect that I_{AP} increases with h has been recently reported in Ref. 11, in which Fig. 2 is very similar to the present Fig. 3(b). It is worth pointing out here that this effect holds only for strong barrier strength, low temperatures, and the antiparallel configuration. It is easily seen that, in the large Z limit ($z\to\infty$ and $\alpha=1$), Eqs. (21) and (22) in this paper are reduced just to Eqs. (7) and (8) of Ref. 11, respectively. With decreased Z, the Andreev reflection coefficients become large gradually and the exchange field always suppresses I_P and I_{AP}. The temperature is another important factor of changing the critical currents and an increase in T always giving rise to a decrease in I_P and I_{AP}, as shown in Figs. 2 and 3.

In Figs. 4 and 5 the temperature dependences of the Josephson critical currents I_P and I_{AP} are plotted for different h. With increasing temperature, I_P and I_{AP} decrease and drop to zero at $T_c(h)$, which is lower than $T_c(h=0)$. Such a sudden drop in I_P or I_{AP} in the presence of h results due to a drop in $\Delta(T,h)$ from a finite value to zero at $T_c(h)$, at which there is a first-order phase transition from a superconducting state with finite $\Delta(T,h)$ to a normal state of $\Delta=0$. $T_c(h)$, at which the Josephson current drops to zero, shown by Fig. 1, depending neither on the magnitude of Z nor on the magnetic

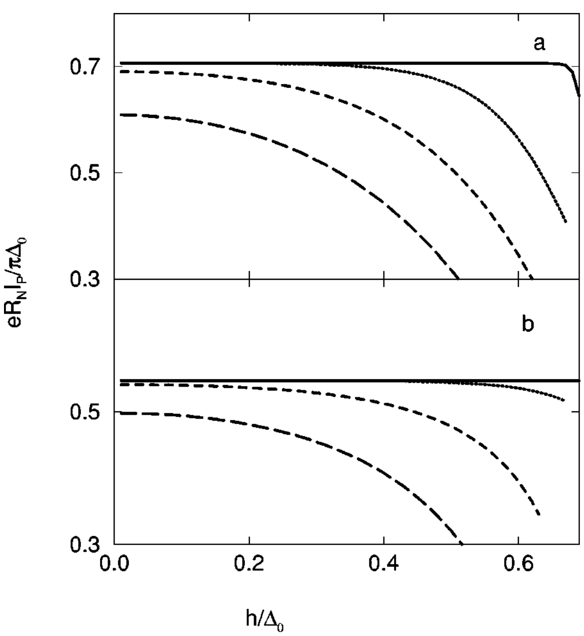

FIG. 2. Dependence of the normalized critical current on h for different temperatures: $k_BT/\Delta_0 = 0.01$ (solid line), 0.1 (dotted line), 0.2 (short-dashed line), and 0.3 (long-dashed line), in the case of a parallel orientation. Here $\phi = \pi/2$, $Z = 1$ (a) and $Z = 5$ (b).

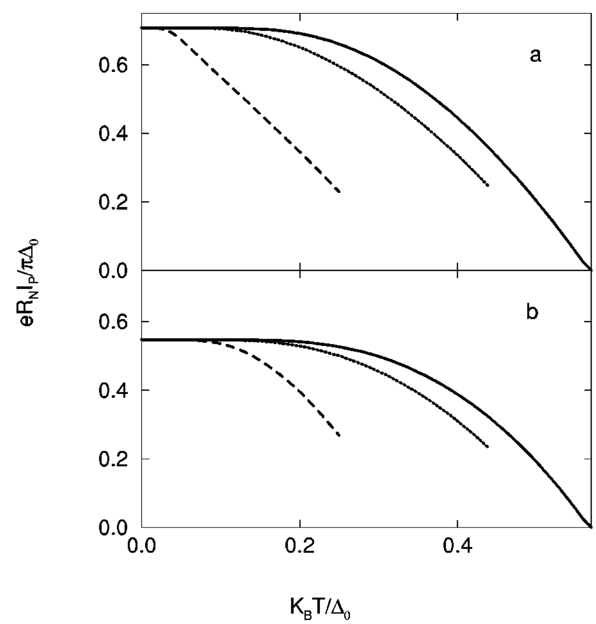

FIG. 4. Dependence of the normalized critical current on T for different exchange fields: $h/\Delta_0 = 0$ (solid line), 0.3 (dotted line), and 0.6 (dashed line), in the case of a parallel orientation. Here $\phi = \pi/2$, $Z = 1$ (a) and $Z = 5$ (b).

configuration of the two FS's, as seen in Figs. 4 and 5. Figure 3(b) shows a feature that for strong barrier strength ($Z = 5$), I_{AP} increases with h at low temperatures but decreases at higher temperatures.

In Figs. 6 and 7, the phase dependences of the Josephson critical currents I_p and I_a are, respectively, plotted for different h. It is found that the relation of I_P (I_{AP}) proportional to $\sin\phi$ holds only for strong barrier strength ($Z = 5$), while for small Z there is a big departure from this relation, as shown in Figs. 6 and 7. This is because $\alpha = (\cos\phi + 1)/(1 + Z)$ appearing in Eqs. (21)–(23) depends strongly on $\cos\phi$ for small Z, while α is very weakly dependent on $\cos\phi$ for large Z and tends towards $\alpha = 1$. Besides, since the exchange field is much smaller than E_F, the Josephson junctions do not change from zero junction to π junction.

In summary we have calculated the phase diagram in the h-T plane for a ferromagnetic superconductor. Two necessary conditions are required for the coexistence of superconductivity and ferromagnetism; one is that there is a nonzero solution for the superconducting order parameter in the en-

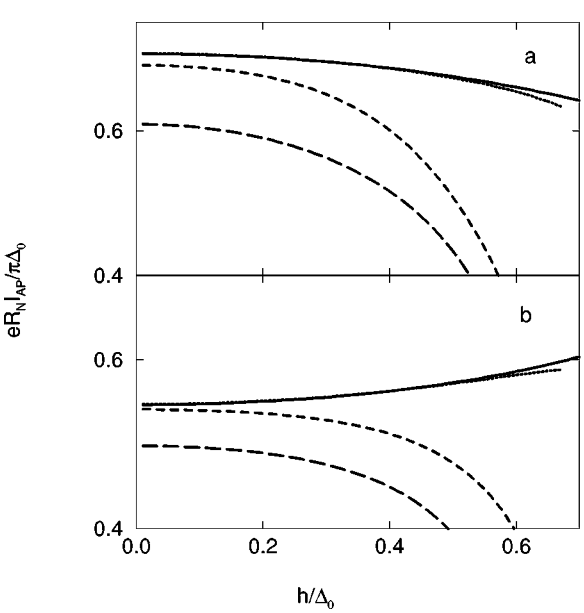

FIG. 3. The same dependence as in Fig. 2 in the case of an antiparallel orientation.

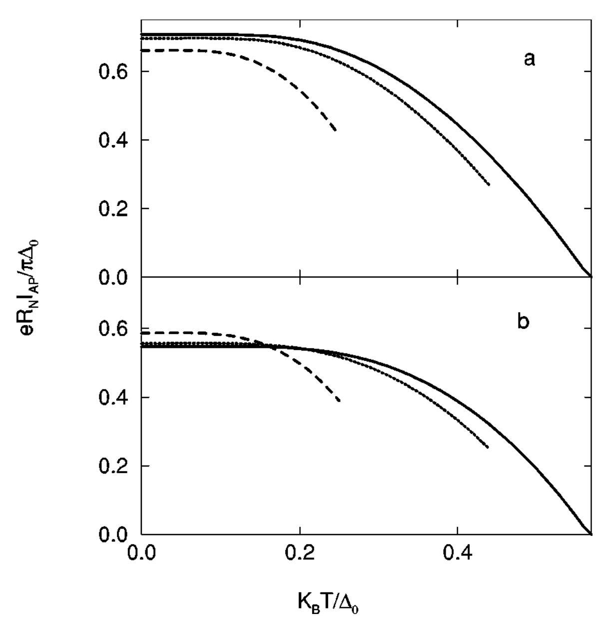

FIG. 5. The same dependence as in Fig. 4 in the case of an antiparallel orientation.

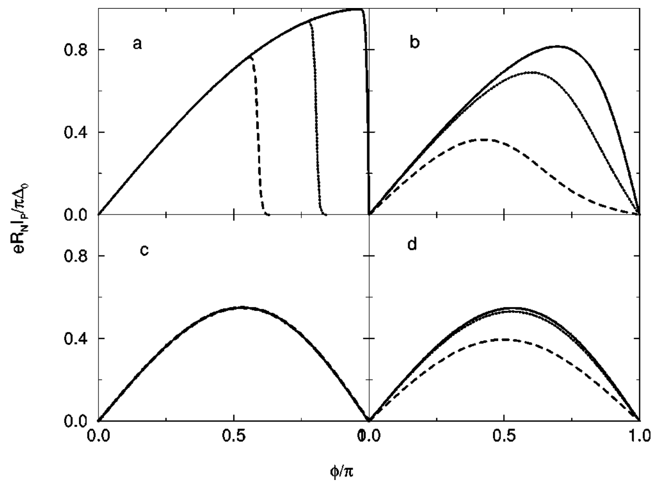

FIG. 6. Dependence of the normalized Josephson current on the phase difference for different exchange fields: $h/\Delta_0=0$ (solid line), 0.3 (dotted line), and 0.6 (dashed line), in the case of a parallel orientation. Here $Z=1$ and $k_BT/\Delta_0=0.01$ (a), $Z=1$ and $T/T_c=0.2$ (b), $Z=5$ and $k_BT/\Delta_0=0.01$ (c), and $Z=5$ and $T/T_c=0.2$.

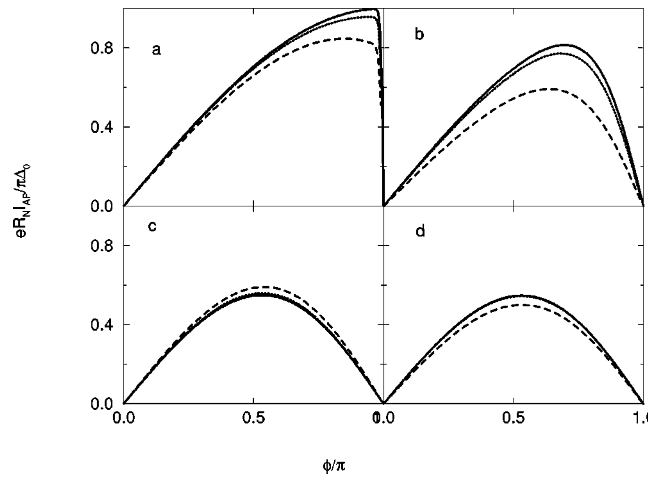

FIG. 7. The same dependence as in Fig. 6 in the case of an antiparallel orientation.

ergy gap equation, the other is that the thermodynamic potential in the superconducting state should be lower than that in the normal state. The Josephson current in a FS/I/FS tunnel junction has been analytically obtained as a function of the temperature, exchange field, and the insulating barrier strength. It is found that an increase in temperature always gives rise to a suppression of the critical current for either parallel (I_P) or antiparallel (I_{AP}) configuration. As the exchange field is increased, I_P always decreases, but I_{AP} may increases at low temperatures and for the strong barrier strength. For weak barrier strength, the Josephson currents (both I_P and I_{AP}) have a big departure from the sin ϕ dependence. With the increased barrier strength, the sin ϕ dependence of the Josephson current is gradually recovered.

ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant No. 10174011 and by the State Key Programs for Basic Research of China under Grant No. G19980614.

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