Andreev reflection effect on spin-polarized transport in ferromagnet/superconductor/ferromagnet double tunnel junctions

Zhiming Zheng, D. Y. Xing, Guoya Sun, and Jinming Dong

National Laboratory of Solid State Microstructures and Department of Physics, Nanjing University, Nanjing 210093, China

(Received 27 March 2000)

We take the Andreev reflection into account and apply a quantum-mechanical approach to studying spin-polarized transport in a ferromagnet/superconductor/ferromagnet double tunnel junction and its effect on superconductivity in the superconductor. It is found that in the presence of the Andreev reflection the tunneling magnetoresistance in the double tunnel junction depends strongly on barrier strength. In the antiferromagnetic alignment of the magnetizations, an increase in bias voltage will give rise to a first-order transition from the superconducting state to a normal one.

Spin-polarized transport has received considerable theoretical and experimental attention in recent years.[1] A most noticeable effect is the large tunneling magnetoresistance (TMR) observed in a magnetic tunnel junction composed of two ferromagnetic metal (FM) films separated by an insulating barrier film.[2,3] The tunnel resistance is maximal when the magnetizations of two FMs are antiparallel to each other, while it is minimal when the magnetizations are aligned in a magnetic field, resulting in a large TMR in the FM/FM tunnel junction.[3,4] Recently, the study of the TMR has been extended to FM/normal metal (NM)/FM double tunnel junctions.[5,6]

In reality, the first measurement on the spin polarization of the current was made in FM/superconductor (FM/SC) tunnel junctions.[7] Since the Cooper pairs in spin singlet superconductors are formed between up and down spins, the spin-polarized current tunneling from the FM into the SC induces a spin imbalance and so gives rise to a suppression of the superconductivity in SC. In a FM/SC/FM double tunnel junction,[8] there is a strong competition between superconductivity and magnetism induced by the spin polarization in SC. Very recently, Takahashi, Imamura, and Maekawa[9] have studied the spin-imbalance and TMR in FM/SC/FM double tunnel junctions. They showed that the spin-imbalance in SC can strongly suppress the superconductivity and the TMR exhibits unusual voltage dependence below the superconducting transition temperature T_c. Their calculation for the tunneling current is based on a phenomenological model, in which Andreev reflection[10] is not considered. For high barrier strength, the Andreev reflection has little contribution to the tunneling current. With decreasing barrier strength, however, the Andreev reflection becomes more and more important. The Andreev reflection can be regarded as a conversion of normal current to supercurrent at a NM/SC interface.[11] As a spin-up electron is injected from a NM into a SC through the interface between them, it must be a member of a pair. The other electron with spin down required for the formation of the pair is obtained from the NM, thus leaving behind a hole at the interface. The reflected hole has the same energy and quasimomentum as the incident electron, but the velocity changes sign and so the hole propagates away from the interface. Such a hole is the absence of a spin-down electron, corresponding to a spin-up elementary excitation ''hole.''

In this work we extend the quantum-mechanical approach of Blonder, Tinkham, and Klapwijk[12] for NM/SC tunnel junctions to the FM/SC ones and recalculate the spin-imbalance and TMR in the FM/SC/FM double tunnel junctions. This approach has been used to study the TMR in FM/FM (Ref. 13) and FM/NM/FM (Ref. 5) tunnel junctions. Since the Andreev reflection is considered fully, the present approach is very suitable to a barrier of arbitrary strength at the FM/SC interface. It is shown that the barrier strength has a great influence on the TMR, while its effect on the spin imbalance as well as superconductivity in the SC is relatively smaller. Besides, we find that with increasing the spin imbalance, the energy-gap parameter decreases and has a sudden drop from the superconducting to normal state at a critical bias voltage, exhibiting first-order transition behavior.

Let us consider a FM/SC/FM double tunnel junction. The left and right electrodes are made of the same FM; they are separated from the central SC electrode by two thin insulating layers, respectively. The layers are assumed to be the x-y plane and to be stacked along the z direction. The scattering Hamiltonian of two thin insulating layers is described by two δ-type potentials,[12] yielding

$$H_I = U_0[\,\delta(z-a/2)+\delta(z+a/2)\,], \qquad (1)$$

where a is the thickness of the SC layer and U_0 depends on the product of the barrier height and width. In this work a is considered to be long enough so that the electrons tunneling into the SC satisfy the Fermi distribution. At the same time, it is shorter than the spin relaxation length so that the spin flip can be negligible in the SC layer.[9]

In the spin-polarized free-electron approximation, the electron Hamiltonian in the FMs is given by

$$H_{FM} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \mathbf{r}^2} - \mathbf{h}(z)\cdot\boldsymbol{\sigma}, \qquad (2)$$

where the first term on the right-hand side is the kinetic energy, and the second one is the internal exchange energy with $\mathbf{h}(z)$ the molecular field and $\boldsymbol{\sigma}$ the conventional Pauli spin operator. In the ferromagnetic (F) alignment of magnetizations of the two FM electrodes, $\mathbf{h}(z<-a/2)=\mathbf{h}(z>a/2)$; while in the antiferromagnetic (A) alignment, $\mathbf{h}(z\leq a/2)=-\mathbf{h}(z>a/2)$, where the magnitude of $\mathbf{h}$ is equal to $\Gamma/2$ with Γ the difference between the bottoms of the

spin-up and spin-down energy subbands. The one-electron energies relative to the chemical potential E_F are given by $E_{k\uparrow} = \hbar^2 k_\uparrow^2/2m - E_F$ and $E_{k\downarrow} = \hbar^2 k_\downarrow^2/2m + \Gamma - E_F$, respectively, for the majority and minority spin directions (spin parallel and antiparallel to the local magnetization).

The bias voltages $-V/2$ and $V/2$ are applied to the left and right electrodes, respectively. In the F alignment, the number of the spin-up (spin-down) electrons tunneling into the SC through the left FM/SC junction is equal to that of the spin-up (spin-down) electrons tunneling out of the SC through the right SC/FM junction, so that there is no non-equilibrium spin density in the SC. In the A alignment, however, the situation is quite different. The difference in number between spin-up (spin-down) electrons tunneling into and out of the SC induces accumulation of electrons with one spin and deficiency with the other spin. Owing to the spin-polarized tunneling, the chemical potentials of the spin-up and spin-down quasiparticles are shifted by $\delta\mu$ oppositely from that in the equilibrium state. The electron Hamiltonian in SC is written as

$$H_{SC} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \sum_k \delta\mu(c_{k\uparrow}^\dagger c_{k\uparrow} - c_{k\downarrow}^\dagger c_{k\downarrow})$$

$$- \frac{g}{2} \sum_{kk'\sigma} c_{k'\sigma}^\dagger c_{-k'\bar\sigma}^\dagger c_{k\bar\sigma} c_{-k\sigma}, \tag{3}$$

where $\epsilon_k = \hbar^2 k^2/2m - E_F$ is the one-electron energy relative to E_F. In the last term on the right-hand side, g is the interaction potential between electrons, and the sums over momenta run only over the intervals in which $-\hbar\omega_D < \epsilon_k, \epsilon_{k'} < \hbar\omega_D$ with ω_D the Debye frequency, By the Bogoliubov transformation: $\gamma_{k\sigma} = u_k c_{k\sigma} - \eta_\sigma v_k c_{-k\bar\sigma}^\dagger$, where $\bar\sigma$ is the spin opposite to σ, $\eta_\sigma = 1$ for $\sigma = \uparrow$, and $\eta_\sigma = -1$ for $\sigma = \downarrow$, Hamiltonian (3) can be diagonalized as

$$H_{SC} = \sum_k E_{k\uparrow} \gamma_{k\uparrow}^\dagger \gamma_{k\uparrow} + \sum_k E_{k\downarrow} \gamma_{k\downarrow}^\dagger \gamma_{k\downarrow}, \tag{4}$$

with

$$E_{k\sigma} = \xi_k + \eta_\sigma \delta\mu. \tag{5}$$

Here $\xi_k = \sqrt{\epsilon_k^2 + \Delta^2}$ is the excitation energy with Δ the gap parameter, and

$$u_k^2 = \frac{1}{2}(1 + \epsilon_k/\xi_k),$$

$$v_k^2 = \frac{1}{2}(1 - \epsilon_k/\xi_k). \tag{6}$$

The gap parameter is determined by the self-consistent equation

$$\Delta = g \sum_k u_k v_k (1 - \langle \gamma_{k\uparrow}^\dagger \gamma_{k\uparrow} \rangle - \langle \gamma_{k\downarrow}^\dagger \gamma_{k\downarrow} \rangle). \tag{7}$$
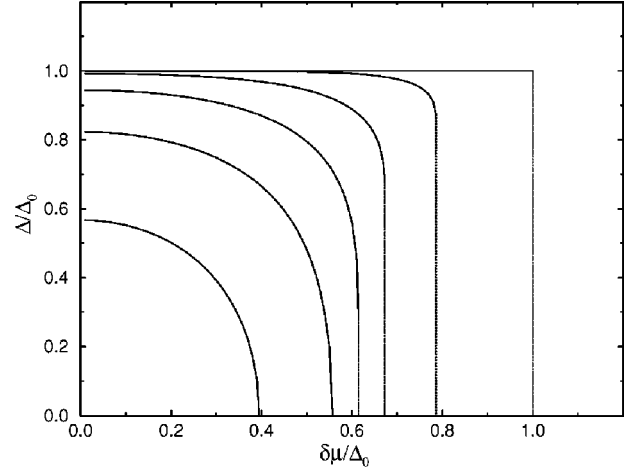
Substituting Eq. (6) into Eq. (7), one gets

FIG. 1. Energy-gap parameter Δ of the SC as a function of $\delta\mu$ (solid lines) in the A alignment for $k_B T/\Delta_0$ $= 0$, 0.1, 0.2, 0.3, 0.4, and 0.5 in order from upper to lower. The dotted line stands for a sudden drop in Δ.

$$\ln\left(\frac{\Delta_0}{\Delta}\right) = \int_0^{\hbar\omega_D} \frac{d\epsilon_k}{\xi_k}\left(\frac{1}{1 + \exp[\beta(\xi_k + \delta\mu)]}\right.$$

$$\left. + \frac{1}{1 + \exp[\beta(\xi_k - \delta\mu)]}\right). \tag{8}$$

Here Δ_0 is the zero-temperature energy gap in the absence of spin density ($\delta\mu = 0$), and $\beta = 1/k_B T$ is the inverse temperature. A derivation of Eq. (8) will be given in Appendix A.

From Eq. (8), Δ is obtained as a function of $\delta\mu$ for several temperatures below T_c, as shown in Fig. 1. At zero temperature, $\Delta = \Delta_0$ remains unchanged for $\delta\mu < \Delta_0$, independent of increasing $\delta\mu$. As $\delta\mu$ is increased to Δ_0, Δ suddenly drops to zero and the superconductivity vanishes due to the presence of nonequilibrium spin density. At finite temperatures, Δ decreases monotonously with increasing $\delta\mu$, but still has a sudden drop from a finite value to zero. Such a drop in Δ occurring at a threshold of $\delta\mu$ shows that there is a first-order phase transition from the superconducting state to the normal state. The drop in Δ is maximal at $T = 0$ and decreases with increasing temperature. For $k_B T/\Delta_0 \geq 0.4$, the drop of Δ has been very small and cannot be distinguished in Fig. 1. However, it is not equal to zero until $T = T_c$. Figure 2 shows the phase diagram in the $\delta\mu$-T plane, indicating superconducting and normal regions, and the critical line $\delta\mu_c(T)$ or $T_c(\delta\mu)$ between them. This line stands for the first-order transition discussed above and culminates at T_c in a second-order critical point. This phase diagram is somewhat similar to that in the H-T plane where H is the applied magnetic field. It can be understood by the fact that a nonequilibrium spin density may induce an internal magnetic field. We wish to point out here that the abscissa of Fig. 1 is $\delta\mu$ rather than the applied bias voltage V. For the normal state, a simple relation in the A alignment was used that $\delta\mu = \frac{1}{2} P e V$ with P the spin polarization.[6,9] In the superconducting state, the relation between $\delta\mu$ and eV is much more complicated, depending not only on temperature but also on barrier strength.

In what follows we study tunneling conductance in the FM/SC/FM double junction, and its effect on the supercon-
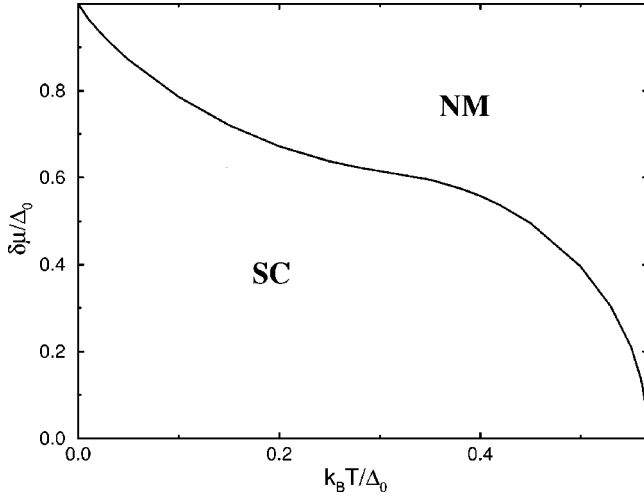
FIG. 2. Curve of the critical $\delta\mu$ as a function of temperature. It indicates a first-order transition from the superconducting to normal state and culminates at T_c in a second-order critical point for $\delta\mu=0$.

ductivity in the SC. As one considers a spin-up electron incident on the interface from the left FM with energy E, there will be two sets of reflected quasiparticle waves in the left FM: normal reflection as an electron with spin up and Andreev reflection as a hole with spin up. The wave function in the left FM is given by

$$\Psi_{FM}(z)=e^{iq_{e\uparrow}z}\begin{pmatrix}1\\0\end{pmatrix}+a_\uparrow e^{iq_{h\uparrow}z}\begin{pmatrix}0\\1\end{pmatrix}+b_\uparrow e^{-iq_{e\uparrow}z}\begin{pmatrix}1\\0\end{pmatrix}. \quad (9)$$

Here the first term on the right-hand side is the incident wave, the second one is the Andreev reflection, and the third one is the normal reflection. $\hbar q_{e\uparrow}=\sqrt{2m(E_F+E)}$ is the momentum of the spin-up electron with energy E, and $\hbar q_{h\uparrow}=\sqrt{2m(E_F-E-\Gamma)}$ is the momentum of the spin-up hole, corresponding to that of the spin-down electron. Owing to the asymmetry of the spin-up and spin-down subbands in the FM, the magnitude of $q_{e\uparrow}$ is always not equal to that of $q_{h\uparrow}$ while they are identical to each other in a nonmagnetic metal where $\Gamma=0$. In the SC, the transmitted wave, including electronlike and holelike parts, is given by

$$\Psi_{SC}(z)=c_\uparrow e^{ik_{e\uparrow}z}\begin{pmatrix}u_k\\v_k\end{pmatrix}+d_\uparrow e^{-ik_{h\uparrow}z}\begin{pmatrix}v_k\\u_k\end{pmatrix}, \quad (10)$$

where $\hbar k_{e\uparrow}=\sqrt{2m[E_F+\sqrt{(E-\delta\mu)^2-\Delta^2}]}$ and $\hbar k_{h\uparrow}=\sqrt{2m[E_F-\sqrt{(E+\delta\mu)^2-\Delta^2}]}$. Since each of E, $\delta\mu$, and Δ is much smaller than E_F, $k_{e\uparrow}$ and $k_{h\uparrow}$ can be approximately replaced by the Fermi wave vector k_F. Applying matching conditions of the wave functions,

$$\Psi_{SC}(z=-a/2)=\Psi_{FM}(z=-a/2),$$

$$\left(\frac{\partial\Psi_{SC}}{\partial z}\right)_{z=-a/2}=\left(\frac{\partial\Psi_{FM}}{\partial z}\right)_{z=-a/2}+\frac{2mU}{\hbar^2}\Psi_{FM}(z=-a/2), \quad (11)$$

we obtain the Andreev and normal reflection amplitudes as

$$a_\uparrow=4q_{e\uparrow}k_F u_k v_k/D,$$

$$b_\uparrow=-1+2[q_{e\uparrow}k_F+(q_{e\uparrow}q_{h\uparrow}-2iZq_{e\uparrow}k_F)(u_k^2-v_k^2)]/D, \quad (12)$$

where

$$D=[(1+4Z^2)k_F^2-2iZ(q_{e\uparrow}-q_{h\uparrow})k_F+q_{e\uparrow}q_{h\uparrow}](u_k^2-v_k^2)$$
$$+(q_{e\uparrow}+q_{h\uparrow})k_F,$$

and $Z=U_0/\hbar v_F$ with v_F the Fermi velocity. The dimensionless parameter Z is introduced to describe barrier strength.[12] The transmission ratio is given by $T_{\uparrow\uparrow}(E)=1-|b_\uparrow(E)|^2$ and $T_{\downarrow\uparrow}(E)=|a_\uparrow(E)|^2 q_{h\uparrow}/q_{e\uparrow}$, from which the tunneling current for spin-up electrons incident on the FM/SC interface can be obtained.

The total current passing through the left FM/SC tunnel junction is the sum of the spin-up and spin-down currents, $I_1=I_{1\uparrow}+I_{1\downarrow}$. Define $N_\sigma(0)$ as the density of states at E_F for the spin-σ electrons in the left FM, $v_{F\sigma}$ the corresponding Fermi velocity, and A an effective-neck cross-section area. The tunneling current is given by

$$I_1=N_\uparrow(0)ev_{F\uparrow}A\int_{-\infty}^{\infty}dE[T_{\uparrow\uparrow}(E)+T_{\uparrow\downarrow}(E)][f(E-eV/2)$$

$$-f(E-\delta\mu)]+N_\downarrow(0)ev_{F\downarrow}A\int_{-\infty}^{\infty}dE[T_{\downarrow\downarrow}(E)+T_{\downarrow\uparrow}(E)]$$

$$\times[f(E-eV/2)-f(E+\delta\mu)]. \quad (13)$$

By using the same procedure, the tunneling current passing through the right SC/FM tunnel junction can be obtained. Owing to symmetry I_2 must have an expression similar to Eq. (13). Since there is no spin-flip scattering in the present model, the steady-state current for either spin-up or spin-down electrons should be continuous, i.e., $I_{1s}=I_{2s}$ where s is the absolute spin direction. In the F alignment, from $I_{1\uparrow}=I_{2\uparrow}$ or $I_{1\downarrow}=I_{2\downarrow}$, it follows

$$\int_{-\infty}^{\infty}dE[T_{\uparrow\uparrow}(E)+T_{\uparrow\downarrow}(E)][f(E-\delta\mu)-f(E+\delta\mu)]=0. \quad (14)$$

As a result, a conclusion that $\delta\mu=0$ is naturally drawn. In the A alignment, a majority (minority) spin in the left FM/SC junction will be regarded as a minority (majority) spin in the right SC/FM junction, so that the current-continuity condition for each spin channel is given by $I_{1\uparrow}=I_{2\downarrow}$ or $I_{1\downarrow}=I_{2\uparrow}$, yielding

$$\int_{-\infty}^{\infty}dE[T_{\uparrow\uparrow}(E)-T_{\uparrow\downarrow}(E)]\left[f(E-\delta\mu)-\frac{1+P}{2}f(E-eV/2)\right.$$

$$\left.-\frac{1-P}{2}f(E+eV/2)\right]=0, \quad (15)$$

where

$$P=\frac{N_\uparrow(0)v_{F\uparrow}-N_\downarrow(0)v_{F\downarrow}}{N_\uparrow(0)v_{F\uparrow}+N_\downarrow(0)v_{F\downarrow}} \quad (16)$$
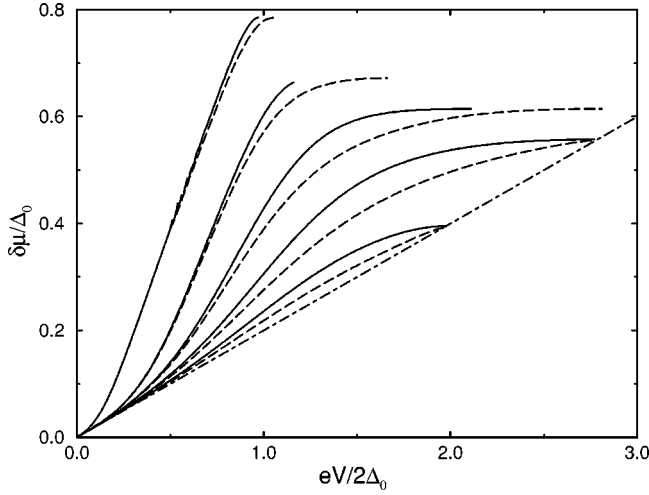
FIG. 3. $\delta\mu$ as a function of bias voltage in the A alignment for $k_B T/\Delta_0 = 0.1$, 0.2, 0.3, 0.4, and 0.5 in order from upper to lower. The parameters used are $P=0.2$, $Z=0$ (solid lines), and $Z=10$ (dashed lines). The dash-dotted line stands for $\delta\mu = \frac{1}{2}PeV$ in the normal state.

FIG. 4. Differential conductance as a function of bias voltage in the F (solid line) and A (dashed line) alignments with $P=0.2$ and $k_B T = 0.2\Delta_0$.

is the spin polarization. In the normal state, $T_{\uparrow\uparrow}$ and $T_{\uparrow\downarrow}$ have only weak E dependence in the effective integral range so that they can be regarded as being constant. It then follows from Eq. (15) that $\delta\mu = \frac{1}{2}PeV$ in the normal state. In the superconducting state, however, this linear relation is valid only in the zero bias-voltage limit; the general relation between eV and $\delta\mu$ must be determined self-consistently by Eqs. (8) and (15). Figure 3 shows calculated results for $\delta\mu$ vs eV. They not only have a big departure from the linear relationship, but also exhibit strong temperature dependence. With lowering temperature, the increase rate of $\delta\mu$ with voltage becomes greater and greater. Besides, they are found to depend on the barrier strength.

We now study the TMR effect in the present double tunneling junction. In either the A or F alignment, the tunneling current can be calculated by Eq. (13). The main difference is that $\delta\mu$ needs to be determined from Eqs. (8) and (15) in the A alignment, while $\delta\mu = 0$ in the F alignment. Besides, in the A and F alignments, $T_{\uparrow\uparrow} + T_{\uparrow\downarrow}$ has different energy dependence because of the difference in the gap parameter. Figure 4 shows the voltage dependence of the differential conductance G_F and G_A in the F and A alignments. For strong barrier strength (large Z), either G_F or G_A has a high peak near $eV=2\Delta_0$, while it is relatively smaller for low voltage. This is because as Z is large enough, the Andreev reflection has little contribution to dI/dV for $eV/2<\Delta_0$. Furthermore, G_A increases with voltage more rapidly than G_F in the greatest range of voltage, leading to an inverse TMR effect ($I_A > I_F$ where $I = \int_0^V [dI/dV]dV$). In the metallic limit of $Z=0$, the contribution of the Andreev reflection makes G_F and G_A decrease with voltage, and G_F is always greater than G_A, resulting in a normal TMR ($I_F > I_A$). The tunneling magnetoresistance is calculated by $\text{TMR}=(I_F-I_A)/I_A$ with the same voltage. The calculated result in Fig. 5 indicates that for low voltages, the TMR is always positive and has relatively weaker dependence on Z, while for $eV/2\Delta>0.75$, the TMR depends strongly on barrier strength and shifts rapidly towards lower with increasing Z, making the sign of
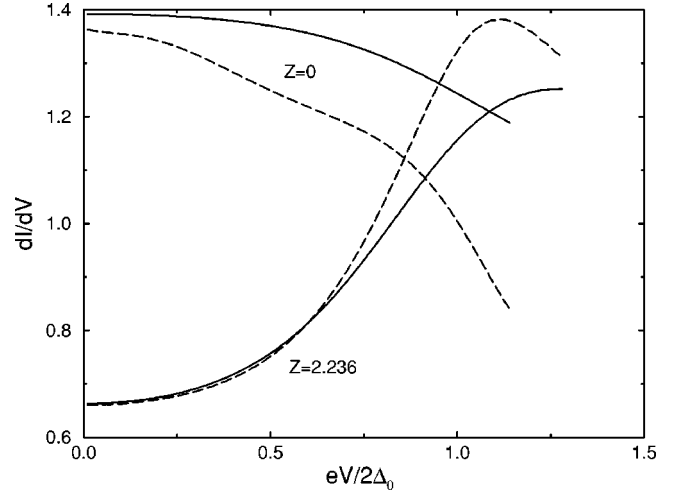
TMR change from positive to negative. The unusual variance of TMR with Z can be understood by the fact that the transmission coefficient, $T_{\uparrow\uparrow}(E)+T_{\uparrow\downarrow}(E)$, has quite a different energy dependence for $Z=0$ and large Z.[12] In the metallic limit of $Z=0$, where the Andreev reflection plays an important role, the transmission coefficient for $E<\Delta$ is about two times greater than that for $E>\Delta$, having a rapid decrease as soon as E is beyond Δ, as shown by the dashed line in Fig. 6. Owing to nonequilibrium spin density, Δ in the A alignment is always smaller than Δ_0 in the F alignment, leading to $I_F > I_A$. With increasing voltage, the increase of $\delta\mu$ enlarges the difference in Δ between the F and A alignments, and so the normal TMR increases with voltage. On the other hand, in the tunneling limit of large Z, the situation is quite different. As shown by the solid line in Fig. 6, the transmission coefficient is very small for $E<\Delta$, exhibits a sharp peak at $E=\Delta$, and tends to a constant value for $E>\Delta$. In this case, the energy range of $E<\Delta$ has little contribution to the tunneling current. As a result, the tunneling current in the A alignment, where Δ is smaller than Δ_0 and the sharp peak in
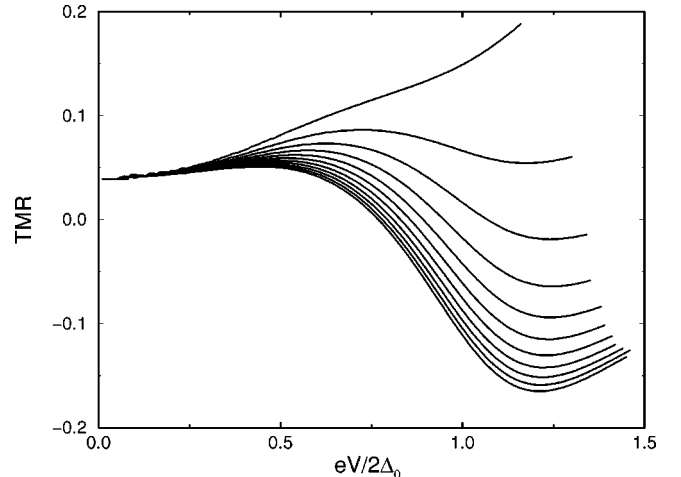

FIG. 5. TMR as a function of bias voltage with $P=0.2$ and $k_B T = 0.2\Delta_0$. The curve shifts gradually lower with Z^2 increasing from zero at a regular interval of 0.5.
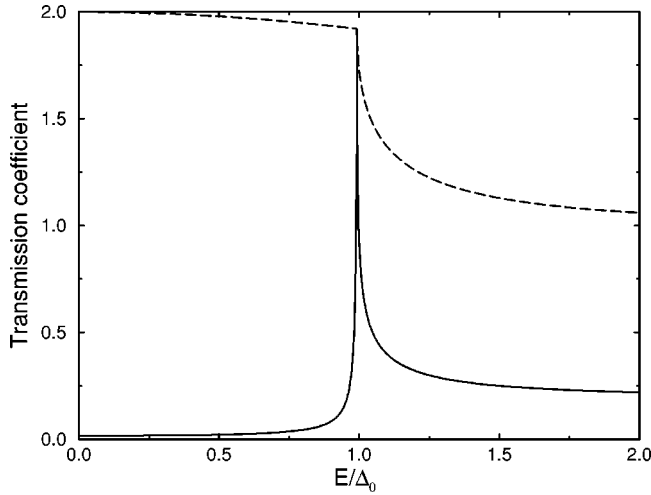
FIG. 6. Energy dependence of the transmission coefficient in the F alignment for $Z=0$ (dashed line) and $Z^2=5$ (solid line) with $P=0.2$ and $k_B T=0.2\Delta_0$.

Fig. 6 will shift toward the left, is greater than that in the F alignment, leading to an inverse TMR.

Finally, we wish to briefly discuss three-dimensional (3D) effects because in this paper a one-dimensional (1D) approach has been approximately applied to a 3D system. This approximation corresponds to only a perpendicular incidence used to replace various possible angles of incidence. If the 3D approach is used, we find that the tunneling current can be expressed in the same form as Eq. (13), but $T_{\sigma\sigma'}(E)$ in it should be replaced by

$$\bar{T}_{\sigma\sigma'}(E) = \frac{1}{2}\int_0^1 u\,du\,\mathrm{Re}[T_{\sigma\sigma'}(E,u)]. \qquad (17)$$

Here $T_{\sigma\sigma'}(E,u)$ has the same expressions as Eq. (12) provided that $q_{e\uparrow}$, $q_{h\uparrow}$, and k_F are replaced by $\sqrt{q_{e\uparrow}^2-k_\parallel^2}$, $\sqrt{q_{h\uparrow}^2-k_\parallel^2}$, and $\sqrt{k_F^2-k_\parallel^2}$, respectively, with $k_\parallel = k_F u$. The 3D formulas have been used to perform numerical calculations; it is found that there is no significant difference in calculated results between the 1D and 3D approaches.

In summary we have shown that the Andreev reflection plays an important role in the tunneling magnetoresistance (TMR) in the FM/SC/FM double tunnel junctions. In the A alignment of the magnetizations, the spin-polarized tunneling current induces spin imbalance in the SC and so gives rise to a first-order transition at a critical voltage where the superconducting gap parameter has a sudden drop from a finite value to zero. With increasing barrier strength, the TMR near $k_B T=2\Delta_0$ changes from positive ($I_F > I_A$) to negative ($I_F < I_A$), which is attributed to a different energy dependence of the transmission coefficient in the metallic and tunneling limits.

APPENDIX A: DERIVATION OF EQ. (8)

Substituting Eq. (6) into Eq. (7) and replacing the summation over **k** by an integral over ϵ_k, we get

$$1 = N(0)g\int_0^{\hbar\omega_D}\frac{d\epsilon_k}{\xi_k}(1-\langle\gamma_{k\uparrow}^\dagger\gamma_{k\uparrow}\rangle-\langle\gamma_{k\downarrow}^\dagger\gamma_{k\downarrow}\rangle), \quad (A1)$$

where $N(0)$ is the density of states at E_F in the SC. Since $\int_0^{\hbar\omega_D}d\epsilon_k/\xi_k=\sinh^{-1}(\hbar\omega_D/\Delta)\simeq\ln(2\hbar\omega_D/\Delta)$, Eq. (A1) can be approximately written as

$$\frac{1}{N(0)g} = \ln\left(\frac{2\hbar\omega_D}{\Delta}\right) - \int_0^{\hbar\omega_D}\frac{d\epsilon_k}{\xi_k}(\langle\gamma_{k\uparrow}^\dagger\gamma_{k\uparrow}\rangle+\langle\gamma_{k\downarrow}^\dagger\gamma_{k\downarrow}\rangle).$$
$$(A2)$$

At zero temperature and in the absence of $\delta\mu$, $\langle\gamma_{k\uparrow}^\dagger\gamma_{k\uparrow}\rangle=0$ and $\langle\gamma_{k\downarrow}^\dagger\gamma_{k\downarrow}\rangle=0$, so that Eq. (A2) is reduced to

$$\frac{1}{N(0)g} = \ln\left(\frac{2\hbar\omega_D}{\Delta_0}\right), \qquad (A3)$$

where Δ_0 is the zero-temperature energy gap in the absence of spin density ($\delta\mu=0$), as indicated in the text. Then, Eq. (8) can be readily obtained by comparing Eq. (A2) with Eq. (A3) and taking into account $\langle\gamma_{k\uparrow}^\dagger\gamma_{k\uparrow}\rangle=1/\{1+\exp[\beta(\xi_k+\delta\mu)]\}$ and $\langle\gamma_{k\downarrow}^\dagger\gamma_{k\downarrow}\rangle=1/\{1+\exp[\beta(\xi_k-\delta\mu)]\}$. If taking $\delta\mu=0$ in Eq. (8), the formula is found to reduce to (16.27) of Ref. 14.

[1] G. A. Prinz, Phys. Today **48**, 58 (1995).

[2] M. Julliere, Phys. Lett. **54A**, 225 (1975).

[3] J. S. Moodera, L. R. Kinder, T. Wong, and R. Meservey, Phys. Rev. Lett. **74**, 3273 (1995).

[4] R. Y. Gu, D. Y. Xing, and J. Dong, J. Appl. Phys. **80**, 7163 (1996).

[5] X. Zhang, B-Z. Li, G. Sun, and F-C Pu, Phys. Rev. B **56**, 5484 (1998); Z. M. Zheng, Y. N. Qi, D. Y. Xing, and J. Dong, *ibid.* **59**, 14 505 (1999).

[6] J. Barnas and A. Fert, Europhys. Lett. **44**, 85 (1998); A. Brataas *et al.*, Phys. Rev. B **59**, 93 (1999); H. Imamura *et al.*, *ibid.* **59**, 6017 (1999).

[7] P. M. Tedrow and R. Meservey, Phys. Rev. Lett. **26**, 192 (1971); Phys. Rev. B **7**, 318 (1973).

[8] M. Johnson, Appl. Phys. Lett. **65**, 1460 (1994).

[9] S. Takahashi, H. Imamura, and S. Maekawa, Phys. Rev. Lett. **82**, 3911 (1999).

[10] A. F. Andreev, Zh. Éksp. Teor. Fiz. **46**, 1823 (1964) [Sov. Phys. JETP **19**, 1228 (1964)].

[11] R. Kummel, Phys. Rev. B **16**, 1979 (1977); R. J. Soulen, Jr., J. M. Byers, M. S. Osofsky, B. Nadgorny, T. Ambrose, S. F. Cheng, P. R. Broussard, C. T. Tanaka, J. Nowak, J. S. Moodera, A. Barry, and J. M. D. Coey, Science **282**, 85 (1998).

[12] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **25**, 4515 (1982).

[13] X. Yang, R. Y. Gu, D. Y. Xing, Z. D. Wang, and J. Dong, Int. J. Mod. Phys. B **11**, 3375 (1997); Y. Qi, D. Y. Xing, and J. Dong, Phys. Rev. B **58**, 2783 (1998).

[14] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).