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Level Statistics for the Nilsson Single-Particle Levels *

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We perform level statistics of the Nilsson single-particle levels. The effects of the l^2 and $l \cdot s$ terms are discussed as well as their interplay with the deformations. The results show that when the l^2 term is added to the harmonic oscillator potential, chaotic motion occurs. The strength ranges of the l^2 term in which chaotic motion exists are related to the deformation of the harmonic oscillator potential. The calculations of the localization length in different bases demonstrate that it is the spherical or axial symmetries that govern the chaotic motion. The degree of chaoticity increases significantly with the $l \cdot s$ term included.

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The problem of regular and chaotic motion has been an interesting topic in both classical and quantum mechanics for a long time.[1,2] For a quantum system, signatures of chaos are expected if the dynamical symmetries are broken or, the number of good quantum numbers is less than that of the degrees of freedom. In many-body systems such as nuclei, residue interactions bring correlations among non-interacting basis vectors and are likely to induce chaotic motion. There have been works showing that GOE[3] fluctuations do appear in some kinds of nuclei.[4−6] In single-particle systems such as billiards, the dynamical symmetry is related to the shape of the boundary. In 1984, Bohigas *et al.*[7] found that level fluctuations of the quantum Sinai's billiard are consistent with the GOE predictions. This is the first demonstration that single-particle systems can also show signatures of chaos. However, Sinai's billiard is a model that has no counterpart in the realistic world. In 1994, Heiss *et al.*[8] found that for single-particle levels of an axially symmetric potential with quadrupole and octupole deformations the fluctuations are also consistent with the GOE predictions. The axially symmetric quadrupole deformed potential is often used in nuclear models for deformed nuclei, such as the projected shell model[9] or the cranking shell model,[11] and an octupole term should be considered for nuclei with octupole deformations.[12] Therefore Heiss's work suggests that chaotic motion exists at the mean field level for nuclei. However, the l^2 and $l \cdot s$ terms were not considered in Heiss's work. Gu *et al.* in 1997[13] discussed the fluctuations of single-particle levels of the mean field based on the two-center shell model, where the mean field is also axially symmetric, and the effect of the l^2 and $l \cdot s$ terms, however, was discussed qualitatively. It was found that the $l \cdot s$ term obviously favors the chaotic motion while the l^2 term only has slight effect on it.

The Nilsson potential is the most widely used mean field in the nuclear models for deformed nuclei. It was used to give the single-particle states in the particle-rotor model,[11] the cranking shell model[11] and the projected shell model.[9] It contains a quadrupole deformed harmonic oscillator potential and the l^2 and $l \cdot s$ terms. Usually the potential is assumed to be axially symmetric. However, in recent years potentials with triaxial deformations are considered in the description of transitional nuclei.[10] As a general case, we discuss single-particle levels in the triaxialy deformed potentials in this Letter. The level fluctuations of the harmonic oscillator potential were discussed by Berry *et al.* in Ref. [14]. However, how do the fluctuations change when the l^2 and $l \cdot s$ terms are taken into account? Moreover, how do these terms interplay with the mean field deformation? This Letter is devoted to discuss these problems. We will study the effects of the l^2 and $l \cdot s$ terms on the statistical behavior of the Nilsson levels, with the γ deformation taken into account. First, we give an introduction to the statistical quantities. Then we present our results and discussions.

We unfold the energy levels[11] to obtain the sequence $x(i)$ with a constant mean level density 1. The nearest neighbor spacing distribution $P(s)$ is defined as the probability distribution of $s = x(i+1) - x(i)$. For classically integrable systems, $P(s)$ is given as the Poisson distribution: $P(s) = \exp(-s)$,[3] while in the fully chaotic case the Wigner distribution (predicted by GOE): $P(s) = \frac{\pi}{2} s \cdot \exp\big(-\frac{\pi}{4} s^2\big)$.[3] For realistic systems the distribution can be fitted to the Berry–

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Robnik distribution with a parameter q ranging from 0 to 1,[11] which reads

$$P(s) = \Big[\bar{q}^2 \mathrm{erfc}\Big(\frac{\sqrt{\pi}}{2} q s\Big) + \Big(2q\bar{q} + \frac{\pi}{2} q^3 s\Big) \cdot \exp\Big(-\frac{\pi}{4} q^2 s^2\Big)\Big] \exp(-\bar{q}s), \quad (1)$$

where $\bar{q} = 1 - q$. In the two limited cases of $q = 0$ and $q = 1$, the Poisson and the Wigner distributions are obtained respectively. Thus q measures the degree of chaoticity of the system.

The information entropy defined in a normalized eigenstate $|\psi\rangle$ is[4]

$$I_H(|\psi\rangle) = \sum_{i=1}^{d} -|\omega_i^{\psi}|^2 \ln(|\omega_i^{\psi}|^2), \quad (2)$$

where ω_i^{ψ} is the expansion coefficient of $|\psi >$ and d is the dimension of the basis space. For the GOE distribution the average value of I_H is $\langle I_H \rangle = \ln(0.48d)$.[4] To eliminate the size effect of the dimension the average localization length is defined[4]

$$\langle L_H \rangle = \frac{\exp(\langle I_H \rangle)}{0.48d}, \quad (3)$$

where the average is taken over all the eigenstates of the Hamiltonian. The information entropy and the localization length are defined to measure the delocalization of the state(s) with regard to the basis, and they are therefore basis dependent.

In the calculations of realistic nuclei the ratio of the l^2 strength D to $\hbar\omega_0$ is around 0.02.[15] For nuclei in different mass regions, the values are slightly different.[16] We take $\hbar\omega_0 = 1$ and change the value of D around 0.02 to study the effect of the l^2 term on the level fluctuations. However, we notice that when $D = 0$ the system comes back to the case of harmonic oscillator in which the parameter q can not measure the chaoticity,[14] and as D increases from 0 the fluctuation pattern should change continuously, thus for very small values of D, q cannot measure the degree of chaoticity either. To avoid the case like this we take D larger than 0.005. Then the histogram of $P(s)$ is obviously different from the harmonic oscillator case.

For small β (for example, $\beta = 0.1$ with various γ values), q stays 0 for all values of D (ranging from 0.005 to 0.03), showing that there is no chaoticity. For small values of γ (say $\gamma = 2°$), q also stays 0 for all values of D even if β is as large as 0.5. This suggests that the breaking of the spherical or axial symmetry is essential for the occurrence of chaotic motion when the l^2 term is included. Note that the l^2 term is rotationally invariant. If the mean field is spherical or axially symmetric, the system as a whole possesses the rotational symmetry, thus no chaotic motion can be expected.

For larger values of β and γ chaos sets in a certain range of D. Figure 1 gives the degree of chaoticity q for different D and β, with γ fixed at 20°. We can see from Fig. 1 that q increases with D when D is relatively small and reaches its maximum value at $D \sim 0.02$. As D becomes larger, q starts to decrease. Finally, when D reaches a certain value, q comes back to 0, and remains unchanged as D increases further. Similar phenomena can be found for other values of γ. The value of D above which q remains 0 is denoted as D_m in the following. The value of D_m depends on β. For $\beta = 0.2$, $\gamma = 20°$, D_m is 0.0285, while for $\beta = 0.5$ and the same γ D_m is 0.0410. Systems with larger β have larger D_m. We also calculate the value of D_m for different values of γ, with β fixed at 0.3, 0.4 or 0.5, respectively. The results are given in Fig. 2. We see in Fig. 2 that D_m increases with γ when γ is less than 30°. When γ is close to 60°, D_m starts to decrease. Thus, combining the relationship of D_m with β and γ, one can conclude that D_m is larger for potentials with a larger deviation from the spherical or axially symmetric shape.

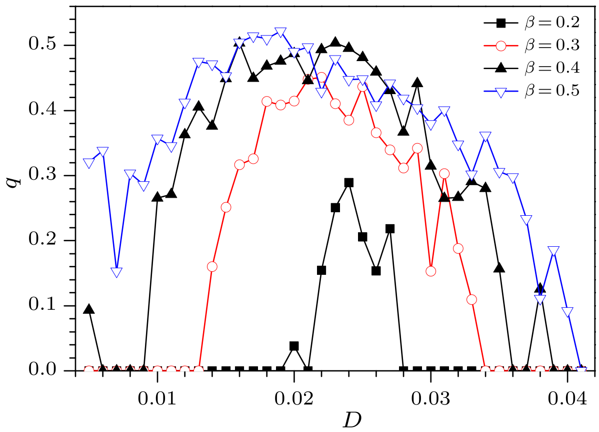

Fig. 1. The Berry–Robnik parameter for different D and β, with γ fixed at 20°.

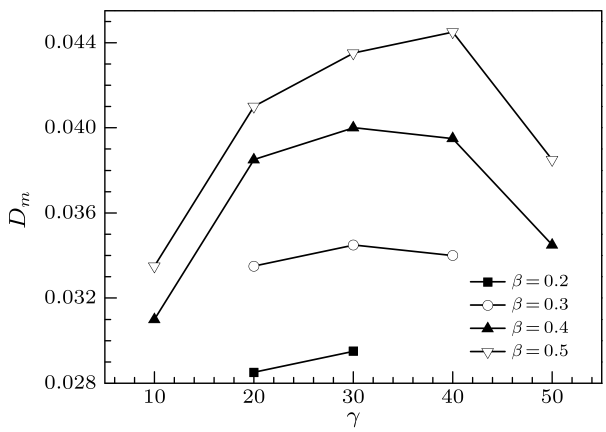

Fig. 2. The values of D_m for different β and γ.

We turn to some explanations to the results presented in Figs. 1 and 2. We know that the occurrence of chaotic motion is related to the breaking of dynamical symmetries, and we use the localization length to measure the degree of the symmetry lost. Figure 3 shows the localization length for different D (in $|nlm\rangle$ basis). Compared with Fig. 1, we can find that in the region of D where q increases, L_H also shows an increase. The maximums of L_H and q corresponds to similar D values (~ 0.02). For larger values of D, L_H decreases as q does. Since there is an obvious similarity between the changing patterns of q and the localization length L_H, q has a close relationship with the localization length, or with the degree of lost of the spherical or axial symmetry. The deformation of the potential breaks these symmetries and the rotationally invariant term l^2 tends to restore them. Thus systems with larger deformations need larger values of D to restore the symmetries, and the values of D_m are larger.

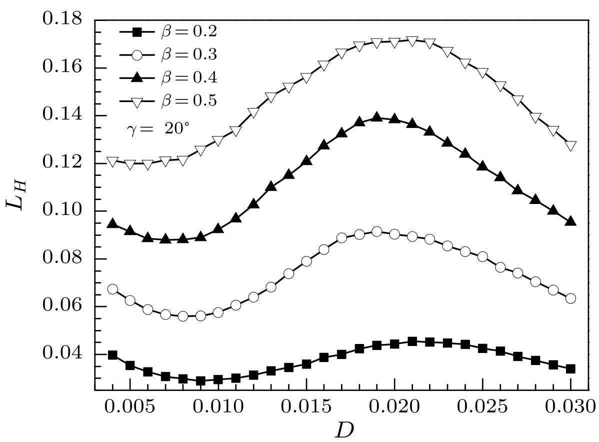

Fig. 3. The localization length in the $|nlm\rangle$ basis for different deformations.

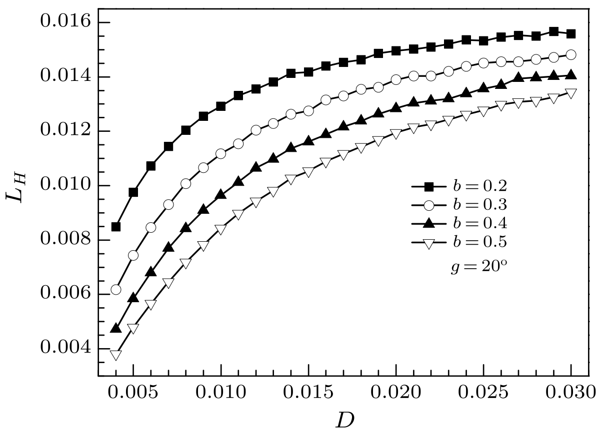

Fig. 4. The localization length in the $|n_xn_yn_z\rangle$ basis.

From Fig. 3 one can find that the localization length calculated in the $|nlm\rangle$ basis increases steadily with β. However, as seen in Fig. 1, the values of q do not have an obvious dependence on β. To understand this it must be revealed that there are other symmetries worth considering besides the spherical or axial ones. With $D=0$ the system has the three quantum numbers n_x, n_y, n_z, while with $D>0$ the eigenstates become linear combinations of basis with different n_x, n_y, n_z values. The breaking of the symmetries possessed by the n_x, n_y, n_z bases can be measured by the localization length calculated in this basis. Figure 4 shows the localization length calculated in the $|n_xn_yn_z\rangle$ bases for different deformations. We find that L_H in this basis decreases as β increases. The value of q stands for the degree of chaoticity of the system, thus should be affected by all kinds of symmetries. Since L_H in the two sets of bases show opposite trends, it is easy to understand that q does not have an obvious dependence on the deformation.

Combining the results of the localization length in the two sets of bases, we conclude that the spherical or axial symmetry (represented by $|nlm\rangle$ basis) governs the degree of chaoticity of the system, considering the similarity between the changing patterns of q and l_H in the bases. However, the symmetry possessed by $|n_xn_yn_z\rangle$ bases also plays a role.

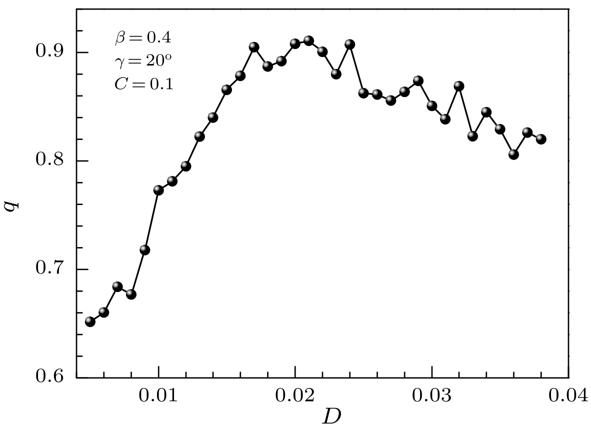

Fig. 5. The Berry–Robnik parameters for different D, with and without the $l\cdot s$ term.

In the calculations of realistic nuclei the ratio of the $l\cdot s$ strength C to $\hbar\omega_0$ is around 0.1.[15] The actual values are also different for different mass regions.[16] For $\beta=0.4, \gamma=20^\circ$ and $C=0.1$ we show the parameter q changing with D in Fig. 5. We notice in Fig. 5 that q ranges from 0.6 to 0.9 for all values of D. Compared with Fig. 1 we find the degree of chaoticity is enhanced significantly by the $l\cdot s$ term. The reason for the increasing of chaoticity can be understood as the breaking of the $SU(3)$ symmetry possessed by the deformed harmonic oscillator potential. This coincides with the result of the two-center shell model.[13]

For D values larger than D_m, q stays 0 if there is no $l\cdot s$ term. However, with the $l\cdot s$ term taken into account q can be positive when $D>D_m$. Moreover, positive q values need only a very small value of the $l\cdot s$ strength C. Calculations show that the 'critical' value of C (denoted as C^*) above which q becomes positive is of the order of 10^{-4}. The values of q for different C are shown in Fig. 6 with a D larger than D_m at the corresponding deformation. In Fig. 6 we find that when $C > C^*$ q shows a steady increase with increasing C.

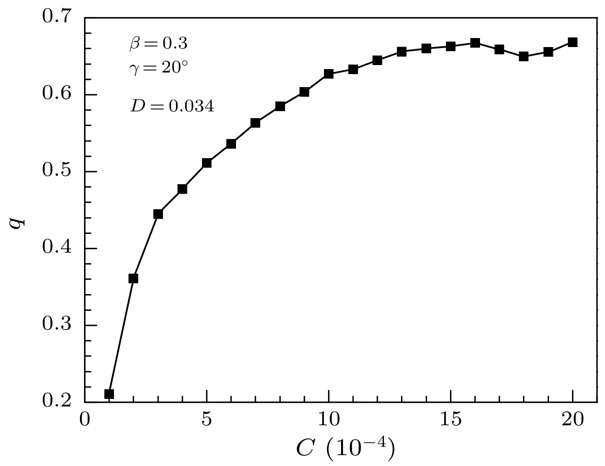

Fig. 6. The Berry–Robnik parameters for different C.

In summary, with the β and γ deformation, chaotic motion exists for a certain range of the l^2 strength D. For each deformation there is an upper limit of D, above which the chaoticity disappears. The value of the upper limit depends on the deformation. The chaotic behavior of the system with the l^2 term is dominated by the spherical or axial symmetry represented by the $|nlm\rangle$ basis. The symmetries represented by the $|n_x n_y n_z\rangle$ basis also plays some role.

When both of the l^2 and the $l \cdot s$ terms are included, the chaoticity is significantly larger than that of the case with the l^2 term only. Particularly, when D is larger than D_m, chaoticity will show up if the $l \cdot s$ term is considered. The strength of the $l \cdot s$ term that is needed to produce chaotic motion is of the order of 10^{-4}.

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