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Generalized Polynomial Chaos Decomposition for the Stokes Equations with Stochastic Inputs

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Abstract: Thanks to the great progress in the development of numerical methods and computer resources, many classical partial differential equations can now be solved very efficiently with high accuracy. However, in many cases, the coefficients, boundary conditions as well as the geometry of the considered partial differential equation may contain uncertainties. In order to provide meaningful predictions to problems involving uncertain data, there is a need to investigate efficient numerical methods for handling general stochastic partial differential equations. In fact there exist several families of numerical techniques to treat such problems. The first method, so-called Monte Carlo method, consists in sampling the random data of the equations, and using standard numerical methods to generate identically distributed approximations of the solution. Then the Monte Carlo method is used to compute corresponding sample averages of these approximations. The accuracy of the Monte Carlo method depends heavily on the number of sampling points, and the computational cost increases quickly with increasing accuracy requirement. An alternative method is based on finite dimensional approximations of the random data, turning the original stochastic problem into a deterministic parametric problem. A standard numerical method is then used to approximate the corresponding deterministic solution, yielding approximations of the desired statistics. Recently, the stochastic Galerkin method, using Karhunen-Loéve decomposition for the stochastic input, has received more attentions for the reason that the type of polynomial chaos can be chosen according to the type of underlying random variables in the stochastic inputs.

In this paper we propose and analyze a high order numerical method to solve the Stokes equations with random coefficients and forcing terms. A stochastic Galerkin approach, based on a finite dimensional Karhunen-Loève decomposition technique for the stochastic inputs, is used to reduce the original stochastic Stokes equations into a set of deterministic equations for the expansion coefficients. Then a $P_N \times P_{N-2}$ spectral method, together with a block Jacobi iteration is applied to solve the resulting problem. This work can be seen as a generalization of the stochastic Galerkin method proposed in [Babuska et al. 2004] for the linear elliptic equation. It is worthwhile to emphasize that the generalization of such a method to the Stokes problem is far from direct due to the coupling feature of the velocity and pressure in the Stokes equations. We establish the well-posedness of the weak formulation and its discrete counterpart. Moreover, we provide a rigorous convergence analysis and demonstrate exponential convergence with respect to the degrees of the polynomials chaos expansion used for the approximation in the direction of the random variable. Finally, a series of numerical tests is presented to support the theoretical analysis.

Keywords: *Stochastic Stokes equations, Generalized polynomial chaos, Spectral methods, Error estimate.*

Abstract only