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3D NUMERICAL MODEL FOR PEARL RIVER ESTUARY

K. W. Chau

Associate Professor, Department of Civil & Structural Engineering, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

Y. W. Jiang

Research Assistant, Department of Civil & Structural Engineering, Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong

ABSTRACT: A 3D numerical model with orthogonal curvilinear coordinate in the horizontal direction and sigma coordinate in the vertical direction has been developed. This model is based on POM (the Princeton Ocean Model). In this model a second moment turbulence closure sub-model is embedded, and the stratification caused by salinity and temperature is considered. Furthermore, in order to adapt to estuary locations where flow pattern is complex, the horizontal time differencing is implicit with the use of a time splitting method instead of the explicit method in POM. This model is applied to the Pearl River estuary, which is the largest river system in South China with Hong Kong at eastern side of its entrance. The computation is verified and calibrated with field measurement data. The computed results mimic the field data well.

KEY WORDS

Three-dimensional, orthogonal curvilinear coordinate, time-splitting, Pearl River estuary

INTRODUCTION

The Pearl River (Zhujiang) is the largest river system in South China. The Pearl River Delta Region (PRDR), which includes eight cities (Guangzho, Shenzhen, Dongguan, Huizhou, Foshan, Zhongshan, Jiangmen), is one of the most important economic zone in China, even in Asia and the world. With the economic boom of PRDR, the resources of the estuary such as the harbor, channel, reclamation zone etc. have been exploited. Also, the water quality is deteriorating and red tide (algae blooms) has occurred several times in the previous two years.

On July 1st 1997, sovereigns and administration of Hong Kong were restored to the People Republic of China. Since then more projects and concern have focused on the area of PRDR. Some models (two dimensional or quasi-three-dimensional) have been developed to simulate the environmental hydraulics in PRDR (Binnie and Partners 1988; Walker and Jones 1991; Chen and Li 1991; Hu and Kot 1997 etc.).

In this paper, a more complete three-dimensional model is developed, the numerical model is based on POM model (the Princeton Ocean Model, Mellor 1996). The principal attributes of the model that described in this paper are as follows:

1. It contains an embedded second moment turbulence closure sub-model to provide vertical mixing

coefficients.

2. The curvilinear orthogonal coordinate is used in the horizontal direction and sigma coordinate in the vertical direction.
3. The horizontal and vertical time differencing are treated semi-implicitly. A time-splitting method is used for the horizontal time differencing of the external mode.
4. Complete thermodynamics have been implemented and the stratification of salinity and temperature are considered.

The main difference between this model and the POM is in the third attribute. In POM the horizontal time differencing is entirely explicit, but in this model the horizontal time differencing is semi-implicit with the use of a time splitting method. The time step of this model is larger than that in POM. This attribute can be used in applications with complex flow pattern and large current caused by the tide and river discharges such as Pearl River estuary. A semi-implicit version of Blumberg-Mellor model, ECOMsiz, has also been developed recently. (Quamrul Ahsan and Blumberg 1999) In ECOM-siz, the barotropic pressure gradient in the momentum equations and the horizontal velocity divergence in the continuity equation are treated implicitly and an untransformed vertical coordinate (z-levels) system is used instead of sigma coordinate system. The major differences with ECOM-siz are the time-splitting alternating direction implicit scheme in the external mode and sigma coordinate system employed in this model.

The model is applied to the Pearl River estuary, which includes four outlets of Pearl River system and the main part of Hong Kong seawaters (Fig. 1). This is probably the first application of this type of model (3-D and baroclinic) the PRDR. The model certainly has significance from an engineering point of view as a tool for exploring the dynamics and circulation of the PRDR. The computation is calibrated and verified with field measurement data.

DESCRIPTION OF THE GOVERNING EQUATION AND SOLUTION METHOD

The model uses a sigma (σ) coordinate condition in the vertical direction, and an orthogonal curvilinear coordinate in the horizontal direction. In the σ stretching system, σ spans the range from σ=0 at the surface of water to σ=-1 at the bottom. The σ coordinate is more suitable for simulating current flow and salinity transportation than the Z system (Leendertse *et al* 1973) since there it provides the same number of layers regardless of water depth. This handles well domains of large topographic variability.

The curvilinear coordinate has become more popular in recent years because it can eliminate the stagger grid in Cartesian coordinate system and improve the representation of the numerical model. There are two kinds of curvilinear coordinate, orthogonal and non-orthogonal. The orthogonal coordinate has the advantage that the motion equation is simpler but it cannot generate grid for a domain with complex geometry. Consequently, if the model is applied to a domain with a complex boundary, a non-orthogonal coordinate system is highly recommended.

The equations, which govern the dynamics of coastal cycle, contain fast moving external gravity waves and slow moving internal gravity waves. A splitting technique has been used (Simons 1974) to divide the three-dimensional motion equations into two-part sub-mode where one part is an internal mode (vertical structure), and the other part is an external mode (vertically averaged). The advantage of this technique is that it permits the calculation of free surface elevation with little sacrifice in computational time. This is achieved by solving the velocity transport separately from the three-dimensional calculation of velocity and the thermodynamic properties (Mellor 1996). The governing equations of two sub-modes are written below:

Internal Mode

The internal model is a vertical structure mode described by the original three-dimensional equations based on the σ coordinate system

Continuity equation:

$$\frac{\partial UD}{\partial x}+\frac{\partial VD}{\partial y}+\frac{\partial \omega}{\partial \sigma}+\frac{\partial \eta}{\partial t}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial UD}{\partial t}+\frac{\partial U^2 D}{\partial x}+\frac{\partial UVD}{\partial y}+\frac{\partial U\omega}{\partial \sigma}-fVD+gD\frac{\partial \eta}{\partial x}+\frac{gD^2}{\rho_0}\int_\sigma^0\left[\frac{\partial \rho}{\partial x}-\frac{\sigma}{D}\frac{\partial D}{\partial x}\frac{\partial \rho}{\partial \sigma}\right]d\sigma = \frac{\partial}{\partial \sigma}\left[\frac{K_M}{D}\frac{\partial U}{\partial \sigma}\right]+F_x \tag{2}$$

$$\frac{\partial VD}{\partial t}+\frac{\partial UVD}{\partial x}+\frac{\partial V^2 D}{\partial y}+\frac{\partial V\omega}{\partial \sigma}+fUD+gD\frac{\partial \eta}{\partial y}+\frac{gD^2}{\rho_0}\int_\sigma^0\left[\frac{\partial \rho}{\partial y}-\frac{\sigma}{D}\frac{\partial D}{\partial y}\frac{\partial \rho}{\partial \sigma}\right]d\sigma = \frac{\partial}{\partial \sigma}\left[\frac{K_M}{D}\frac{\partial V}{\partial \sigma}\right]+F_y \tag{3}$$

Temperature and salinity transport equation:

$$\frac{\partial TD}{\partial t}+\frac{\partial TUD}{\partial x}+\frac{\partial TVD}{\partial y}+\frac{\partial T\omega}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left[\frac{K_H}{D}\frac{\partial T}{\partial \sigma}\right]+F_T \tag{4}$$

$$\frac{\partial SD}{\partial t}+\frac{\partial SUD}{\partial x}+\frac{\partial SVD}{\partial y}+\frac{\partial S\omega}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left[\frac{K_H}{D}\frac{\partial S}{\partial \sigma}\right]+F_S \tag{5}$$

Turbulence energy equation:

$$\frac{\partial q^2 D}{\partial t}+\frac{\partial Uq^2 D}{\partial x}+\frac{\partial Vq^2 D}{\partial y}+\frac{\partial \omega q^2}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left[\frac{K_q}{D}\frac{\partial q^2}{\partial \sigma}\right]+\frac{2K_M}{D}\left[\left(\frac{\partial U}{\partial \sigma}\right)^2+\left(\frac{\partial V}{\partial \sigma}\right)^2\right] +\frac{2g}{\rho_0}K_H\frac{\partial \tilde{\rho}}{\partial \sigma}-\frac{2Dq^3}{B_1 l}+F_q \tag{6}$$

$$\frac{\partial q^2 lD}{\partial t}+\frac{\partial Uq^2 lD}{\partial x}+\frac{\partial Vq^2 lD}{\partial y}+\frac{\partial \omega q^2 l}{\partial \sigma}=\frac{\partial}{\partial \sigma}\left[\frac{K_q}{D}\frac{\partial q^2 l}{\partial \sigma}\right]+E_1 l\frac{K_M}{D}\left[\left(\frac{\partial U}{\partial \sigma}\right)^2+\left(\frac{\partial V}{\partial \sigma}\right)^2\right] +E_1E_3 l\frac{g}{\rho_0}K_H\frac{\partial \tilde{\rho}}{\partial \sigma}-\tilde{W}\frac{Dq^3}{B_1}+F_l \tag{7}$$

The horizontal viscosity and diffusion terms are defined according to:

$$F_x=\frac{\partial}{\partial x}(H\tau_{xx})+\frac{\partial}{\partial y}(H\tau_{xy}) \tag{8}$$

$$F_y=\frac{\partial}{\partial x}(H\tau_{xy})+\frac{\partial}{\partial y}(H\tau_{yy}) \tag{9}$$

$$F_\phi=\frac{\partial}{\partial x}(Hq_x)+\frac{\partial}{\partial y}(Hq_y) \tag{10}$$

where,

$$\tau_{xx} = 2A_M \frac{\partial U}{\partial x}, \tau_{xy} = \tau_{yx} = A_M \left[\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right], \tau_{yy} = 2A_M \frac{\partial V}{\partial y}$$

$q_x = A_H \dfrac{\partial \phi}{\partial x}, q_y = A_H \dfrac{\partial \phi}{\partial y}, \phi$ represents $T, S, q^2, q^2 l$.

In the above equations, U, V, ω are mean fluid velocities in the x, y, σ direction respectively; η is the elevation of sea surface above the undisturbed level; f is the Coriolis parameter; $D = \eta + H$, H is the depth of the water, and g is the Earth's gravitational acceleration; ρ_0 is the fluid density; ρ is the fluid density after subtraction of the horizontally averaged density, $\tilde{\rho}$ is the buoyant fluid density; T is temperature; S is salinity; q^2 is the turbulence energy; l is the mixing length; $K_{M,} K_H, K_q$ are vertical turbulent flux coefficients; A_M, A_H are horizontal turbulent coefficients; $\tilde{W}$ is wall proximity function and $B_1, E_{1,} E_3$ are constants determined from laboratory experiments.

From these equations, it can be seen that a $q^2 \sim q^2 l$ turbulence model is considered. It involves two prognostic equations, in essence being identical to that used in the $K \sim \varepsilon$ approach (Davis *et al* 1995).

External Mode

The external mode, which is integrated vertically by the continuity and momentum equations, can be written as two-dimensional dynamic equations:

The Continuity equation:

$$\frac{\partial \bar{U} D}{\partial x} + \frac{\partial \bar{V} D}{\partial y} + \frac{\partial \eta}{\partial t} = 0 \tag{11}$$

Momentum equation:

$$\frac{\partial \bar{U} D}{\partial t} + \frac{\partial \bar{U}^2 D}{\partial x} + \frac{\partial \bar{U}\bar{V} D}{\partial y} - \tilde{F}_x - f\bar{V}D + gD\frac{\partial \eta}{\partial x} = < wu(-1) > - \frac{gD}{\rho_0}\int_{-1}^{0}\int_{\sigma}^{0}\left[D\frac{\partial \rho}{\partial x} - \frac{\sigma \partial D}{\partial x}\frac{\partial \rho}{\partial \sigma}\right] d\sigma d\sigma \tag{12}$$

$$\frac{\partial \bar{V} D}{\partial t} + \frac{\partial \bar{U}\bar{V} D}{\partial x} + \frac{\partial \bar{V}^2 D}{\partial y} - \tilde{F}_y + f\bar{U}D + gD\frac{\partial \eta}{\partial y} = < wv(-1) > - \frac{gD}{\rho_0}\int_{-1}^{0}\int_{\sigma}^{0}\left[D\frac{\partial \rho}{\partial y} - \frac{\sigma \partial D}{\partial y}\frac{\partial \rho}{\partial \sigma}\right] d\sigma d\sigma \tag{13}$$

where $(wu(-1), wv(-1)) = -C_Z (U^2 + V^2)^{1/2} (U, V), \sigma \to -1$. $\bar{U}, \bar{V}$ are the vertically integrated velocities; $(\bar{U}, \bar{V}) = \int_{-1}^{0} (U, V) d\sigma$; $\tilde{F}_x, \tilde{F}_y$ are horizontal turbulence diffusion term, the <*wu*(-1)> and <*wv*(-1)> are bottom stress components; C_Z is Chezy's coefficient.

The Interaction of Internal and External Modes

The velocity advection (the second and third terms of the above momentum equations 12 and 13), horizontal diffusion (the fourth term) and density gradient (the eighth term) are integrated vertically from the corresponding terms of internal equations. In equations 12 and 13, the bottom stress is derived from the velocity of internal mode. On the other hand when computing the internal mode, the elevation of the water surface is obtained from the external mode directly. The internal and external modes have different truncation errors, so the vertical integrals of the internal mode velocity may differ slightly from $\overline{U}, \overline{V}$. In order to eliminate the current velocity of internal mode (U,V) is adjusted to fit the following condition : $\int_{-1}^{0} U d\sigma = \overline{U}$.

Structure of the Internal Mode Calculation

For a detailed description of internal mode, refer to Blumberg and Mellor (1980 and 1987) and Mellor (1996). The method is semi-implicit and all terms of momentum equations (2) and (3) are treated explicitly except for the vertical flux (the first term in the right hand side) that is treated implicitly. All equations here are based on Cartesian coordinate; readers can refer to Chau and Jin (1995) for equations under the orthogonal curvilinear coordinate.

Structure of the External Mode Calculation

The solution of the external mode in the POM is entirely explicit and use C grid. The time step is based on Courant-Friendrichs-Lewy (CFL) condition, which requires the condition: $dt < dx \Big/ \sqrt{2gh} + U_{MAX} / \sqrt{2}$. For small grid sizes, the corresponding time step will be set small to keep the computation stable, this will require large amount of time.

In order to develop a three-dimensional numerical model that can represent the Pearl River estuary (the smallest size of generated orthogonal curvilinear grid is 50m) and can operate on personal computer with Pentium II, a semi-implicit method is used in the external mode. This method is expressed as a time-splitting alternating direction implicit scheme (ADI) on the "Arakawa C" grids as shown in Fig. 2 (Chau and Jin 1995).

x-direction:

Continuity equation:

$$\frac{\eta^*_{i,j} - \eta^n_{i,j}}{\Delta t} + \frac{\overline{U}^*_{i+1,j}(D^n_{i,j} + D^n_{i+1,j}) - \overline{U}^*_{i,j}(D^n_{i-1,j} + D^n_{i,j})}{2\Delta x} + \frac{\overline{V}^n_{i,j+1}(D^n_{i,j} + D^n_{i,j+1}) - \overline{V}^n_{i,j}(D^n_{i,j-1} + D^n_{i,j})}{2\Delta y} = 0 \tag{14}$$

Momentum equation:

$$\frac{\overline{U}^*_{i,j} - \overline{U}^n_{i,j}}{\Delta t} + g\frac{\eta^*_{i+1,j} - \eta^*_{i,j}}{\Delta x} - f\frac{\overline{V}^n_{i-1,j} + \overline{V}^n_{i-1,j+1} + \overline{V}^n_{i,j} + \overline{V}^n_{i,j+1}}{4} = A^n \tag{15}$$

y-direction:

Continuity equation:

$$\frac{\eta^{n+1}_{i,j} - \eta^*_{i,j}}{\Delta t} + \frac{\overline{V}^{n+1}_{i,j+1}(D^*_{i,j} + D^*_{i,j+1}) - \overline{V}^{n+1}_{i,j}(D^n_{i,j} + D^n_{i,j+1})}{2\Delta y} - \frac{\overline{V}^n_{i,j+1}(D^n_{i,j} + D^n_{i,j+1}) - \overline{V}^n_{i,j}(D^n_{i,j-1} + D^n_{i,j})}{2\Delta y} = 0 \tag{16}$$

Momentum equation:

$$\frac{\overline{V}^{n+1}_{i,j} - \overline{V}^n_{i,j}}{\Delta t} + g\frac{\eta^{n+1}_{i,j+1} - \eta^{n+1}_{i,j}}{\Delta y} + f\frac{\overline{U}^*_{i-1,j} + \overline{U}^*_{i-1,j+1} + \overline{U}^*_{i,j} + \overline{U}^*_{i,j+1}}{4} = B^n \tag{17}$$

At the end of each time step the following equation is assured:

$$\overline{U}_{i,j}^{n+1} = \overline{U}_{i,j}^{*} \quad (18)$$

where η^* and $\overline{U}^*$ are two middle unknowns in first time-splitting step in the x-direction. A^n, B^n are the terms which can be obtained from the internal sub-mode. Equations in each direction can be written in tri-diagonal matrix and solved with the use of double-sweep algorithm method (Leendertse and Crittion 1971). The solution consists of two time-splitting steps which advances the solution first from time level nt to t* in the x-direction and gets $\overline{U}^*, \eta^*$, and the second from t* to (n+1)t in the y-direction and $\overline{V}^{n+1}, \eta^{n+1}$ are obtained. Finally let $\overline{U}^{n+1} = \overline{U}^*$.

Numerical Stability

Since the numerical scheme used here is semi-implicit, the terms of vertical diffusion in the internal mode and the elevation gradation term in the external mode are treated implicitly. The time step of numerical computation cannot exceed the limits associated with the advection terms, the Coriolis term, the baroclinic pressure gradient term and horizontal diffusion term. The semi-implicit numerical algorithm permits the time step many times greater than that based upon the CFL condition. From the computational experiments of this model applied in Pearl River estuary, the maximum time step of the external mode is 100 seconds and the maximum time step of the internal mode is 60 seconds. Simply, this model uses the same time step—60 seconds in both of the two sub-modes. If the POM is applied in the research area with the same grid, the maximum time steps are 6 seconds and 100 second for external and internal modes respectively. The model described here requires a core memory of 14.8 mega words and about 20 seconds per hour run. On the other hand the POM needs 14.2 mega words memory and 28 seconds per hour run based on a PII-450 PC.

The Errors Caused by the Sigma Coordinate Pressure Gradient

Numerical models using sigma coordinates have the capability of dealing with ocean application with large topographic variability. The disadvantage of this model, however, is its hydrostatic inconsistency due to the bottom topography effect. This inconsistency is caused by the horizontal density gradient along the constant σ layer. Mellor *et al* (1997) utilized some strategies to reduce them which have been used in this model. The first is to examine the topography and adjust H (depth of water) so that H can fit the equation given by Haney (1990) $\left|\frac{\sigma}{H}\frac{\partial H}{\partial x}\right|\delta x < \delta\sigma$. The second method adopted in this model is to subtract a climatological density (or temperature and salinity) before computing fluxes and the climatological density (or temperature and salinity) fields can be set to:

$$d\rho_{c\lim} / dt = a(\rho - \rho_{c\lim}) \quad (19)$$

where a is the inverse of decay time. ρ and $\rho_{c\lim}$ are density and climatological density respectively. The value of a adopted in this Pearl River estuary model is (1/t), where t is the period of major tidal constituent. In this model we choose t=0.5175 day, which is the period of M2 (Ip and Wai 1990).

Application of the Model to Pearl River Estuary

The Pearl River estuary has been established as a very important zone in South China. With the economy quickly developing in this area, the environment has been deteriorating and hence more and more attentions put on this research area estuary. As the hydrodynamic numerical model is an efficient tool for environmental impact assessment and feasibility study of projects, the model has been applied to this estuary and calibrated.

The study area (Fig. 1) is a delta estuary. The depth of the open side of the estuary ranges from 20-28m, and becomes shallower toward the inner bay. The mean depth of the estuary is 7 meters. There are four river outlets in the northwest of the estuary (Hu men, Jiao men, Heng men, Hongqi men). According to the published data (Pang and Li 1998), the net discharges of the four outlets in different seasons are listed in Table 1. It can be seen that the discharges in different year vary very much. The tide in Pearl River estuary is semi-diurnal and irregular. The mean tidal range is about 1.0 m. At the entrance of the estuary, the mean tide range is 0.85-0.95 m, and the range is higher in the inner estuary. The mean tide range in Hu men is 1.6m (Kot and Hu 1995). In the wet season (May to September) the runoff of the rivers becomes the strongest to become the predominant hydrodynamic forcing in Pearl River estuary, and while in the dry season (December to March) the tidal current is the main force (Lu 1997).

The horizontal grid of the orthogonal curvilinear coordinate system is displayed in Fig.3, and the corresponding transformed grid is shown in Fig.4. There are 3400 grids in each of the 6 vertical layers. Each layer has the same $\delta\sigma$ which value is *1/6*. The number of layer and grid points are chosen such that reasonable accuracy is achieved in both horizontal and vertical discretizations while simultaneously computational efficiency is not deterred.

Open Boundary Condition

At the open boundary (southern boundary and eastern boundary), the tide elevation is the model forcing. It can be interpolated from the observed data at two stations (Macau and North Point) according to tidal wave propagating speed $\sqrt{gh}$ (Huang and Lu 1995). The velocity values of external and internal modes at the open boundary are derived from the radiation condition; for example $\frac{\partial v}{\partial t} - c_i \frac{\partial v}{\partial y} = 0$, $c_i = \sqrt{H/H_{\max}}$ at the southern open boundary. The values of salinity, temperature and turbulence kinetic and turbulence dissipation at the open boundary condition are derived from the following equations:

Ebb time: $\frac{\partial A}{\partial t} + U\frac{\partial A}{\partial x} = 0$ (20)

Flood time: $A = A_{set}(t,\sigma)$ (21)

“A” represents the salinity, temperature, turbulence kinetic and turbulence dissipation. During ebb, at open boundaries, *A* are calculated using the “upwind” differenced advection equation. During flood the *A* is linearly interpolated from its value at end of ebb to a fixed *A* that depends on the depth and observed data. The open boundary conditions of four outlets in the northwest of the research area are governed by water discharges.

Closed Boundary Conditions:

The convection and diffusion terms require the value of velocity at the outer boundary to close the motion equations. Two kinds of closed boundary conditions have been tested in this model. For $\frac{\partial u}{\partial y}$, the first is no-slip condition that assumes $u = 0$, and the second is free slip condition in which $\frac{\partial u}{\partial y} = 0$. The model applied to the Pearl River estuary weighs two methods together and gets the semi-slip boundary condition: $\frac{\partial u}{\partial y} = \frac{\beta u}{\Delta y}, 0 \le \beta \le 1$. Sensitivity tests with different β values have been conducted, with little change in results.

CALIBRATIONS

In June to July 1990, the Hong Kong Civil Engineering Department Port Development Division carried out a hydrological survey. The survey data observed at three tidal elevation stations and three tidal current stations were used for calibration of this model. Fig.1 shows the location of these stations. The observed data from 21st June to 20nd July, 1990, of tidal elevation on two stations, Macau and Northpoint, are available to be used as boundary conditions. The model simulated one month's hydrodynamics of Pearl River estuary in this period.

To calibrate the simulations, the computational results of three components have been verified with the corresponding observed data. The three components are tidal elevation, flow velocity and direction. For lack of detailed salinity data, only the contour of salinity contour is shown.

Calibration of Tidal Elevation

The entire month's comparison of elevation result versus the observation (from 21st June to 20th July 1990) at West Lamma is shown in Fig. 5. However, a typical 25 hours of comparison can illustrate the slight differences between the computed results and the observations more effectively. Fig. 6 illustrates the computational and measured results of tidal elevation at 3 different elevation stations from 15:30 21st to 22nd June 1990. It appears that the computational results concur well with the measured data. From this figure it is clear that the amplitude of inner station (such as Tap Shek Kok) is larger than the outer stations (such as West Lamma).

Calibration of Tidal Flow

Fig 7 shows the horizontal tidal current pattern during an ebb tide in mean season (April, October and November) at the surface, middle, and bottom layers, respectively, in the Pearl River estuary.

In this figure, the current velocities at the surface are slightly larger than that in the bottom. The flow directions at the surface layer and bottom layer may be slightly nonuniform, especially when the current is not too fast. The maximum flow velocity occurs at narrow channels in Ma Wan Channel and Kap Shui Mun where the water depth is larger than 20m with a maximum speed of 2.5m/s. The current speed in northwest Lantau Island water areas, Urmston Road and West East Channel are higher than other locations, since the bathymetry of these locations cause lateral contraction in the flow channel. Hence faster flow are also expected there. In general, the simulated current pattern agrees well with the observed data (Kot and Hu 1995).

The depth-averaged velocities at three tidal stations are compared with the observed data. Fig. 8 shows the results of comparison of direction and magnitude velocity between computed and observed data from 15:30 21st to 22nd June 1990. The root-mean-squares errors (rms) of the computed tidal level, flow direction and velocity for the one month comparisons are 0.14m, 17 degree, and 0.07m/s respectively. The computed flow direction and velocity coincide well with the observed data, which indicate that the model can simulate well the observed data.

As can be seen in Figure 6 and Figure 8, start up of the simulation requires about 2 to 3 hours before it converges to the accurate solution and matches with the observations well.

The Distribution of Salinity

The distribution of salinity in the wet and dry season is simulated. The measured data such as tidal level and salinity at open boundary from 21st June to 20nd July 1990 are used as boundary condition in wet season. Due to lack of data in winter, same tidal levels are used to simulate the salinity distribution in dry season. The salinity at open boundary is derived from the dry season salinity horizontal and vertical patterns given by Kot and Hu (1995); Broom and Ng (1995). The discharges of the four rivers are used as the data for wet and dry season in 1990 (see Table 1)

As seen from the distribution of salinity during ebb tide in three layers during the wet season (Fig. 9), sharp change of salinity is a common phenomenon in Pearl River estuary during the wet season. This figure indicates that the stratification is only discernable in outer bay and breaks down in the inner bay because the water depth becomes shallower and the turbulence becomes stronger. In the dry season (Fig. 10), the sharp gradient of salinity or stratification is not so notable and the seawater intrusion is farther into the inner delta area. Therefore in the inner delta the gradient of salinity is smaller but the averaged salinity is higher than that in summer. The pattern of the distribution of salinity shows no discrepancy with the pattern described by Kot and Hu (1995).

CONCLUSIONS

A three-dimensional curvilinear hydrodynamic model has been formulated, verified and applied to the Pearl River estuary. This region is the most quickly developing in China with Hong Kong and Macau at its entrance.

In this model, the external and internal gravity waves have been split in the analysis, and the former becomes a two-dimensional mode (external mode), the latter forms a three-dimensional mode (internal mode). In the external mode, the term of tidal elevation gradient is treated implicitly and the other terms are integrated or obtained from the internal mode. In the internal mode, the vertical flux term of momentum equation is treated implicitly and tidal elevation values comes from the external mode directly. The same time step is adopted in both internal and external mode. With the use of these methods, the time step is larger than the POM model and hence less computational time is needed to keep the computation stable.

The model is applied to the Pearl River estuary with the stratification, caused by the salinity and temperature, considered. The computational results of tidal elevation, direction and magnitude of current velocity are compared with the measured data in July 1990. The result shows fairly good agreement between the simulated and survey measured data. The distribution of salinity in different layers during the wet and dry seasons has been illustrated.

Above all, a complicated and efficient three-dimensional model has been developed. It works well when applied to the Pearl River estuary, which is a typical estuary domain.

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