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Replenishment Modeling for Complex Automatic Picking System

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Abstract— **The replenishment plan of complex automatic picking system(CAPS) determines the efficiency of picking and whether the picking can be operated smoothl[1]y, which is not only highly related to the pallet storage area and picking buffer but also the operation and operating efficiency of Horizontal Dispenser (HD) and Channel Dispenser(CD). The questions here can be summarized as when replenishment should be required and how many cartons of cigarettes should be dedicated to picking buffer. To answer these questions, we first present a comprehensive description for the general multi-tier, multi-model inventory system of CAPS and analyse the stocking and picking activities of HD and CD. Then we consider replenishment in two cases. One is single replenishing request from one picking flue , the other is multiple replenishing requests from more than one picking flues at the same time which is more complex. For the latter case, we solve the problem based-on optimal scheduling policy. Our mathematical model provides a viable solution to optimize replenishment operation for CAPS.**

I. Introduction

Complex Automated Picking System (CAPS) in tobacco distribution center is a kind of automated order picking system for the piece/unit of cigarette that combines the Horizontal Dispensers (HD) and Channel Dispensers (CD) together to increase the picking efficiency, accuracy and reduce the labor intensity of the cigarette order picking. As shown in Fig. 1, the general inventory system of CAPS is a four-tier, six-mode inventory system. The first tier is Back Warehouse (BW) which may be pallet racks or stacks. Its main responsibility is providing restocks to the next tier. The second tier is Pallet Storage Area (PSA) which only receives pallets of SKUs from BW and provides restocks to the next tier. The configuration of PSA could be pallet racks, stacks or pallet flow rack. The third tier is Picking Buffer (PB) which receives restocks from PSA in form of case/carton or pallet and restocks HD or CD. In most cases, the configuration of PB is pallet stack and case flow rack. The fourth tier contains three models: manual picking that is mainly for odd-shape cigarette and whole case, HD and CD.

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Some researches have already been done for the CAPS. Hackman and Rosenblatt (1990) first presented the forward-reserve problem and developed the slotting method under the space constraints. Byung-In KIM (2003) proposed an order-picking sequence algorithm based on Clustering for an automated warehouse. For fully-automated center, the most studied topic is JIT modeling about picking. Shizhen Li (2003) optimized the picking route by dynamic programming. Wuyi Lu (2007) modeled the picking time of each carton. Kai Zhao (2008) studied the same problem as Wuyi Lu did, but his theory is based on the Virtual Queue Containers. Although replenishing planning is one of the most important parts of system operation, there are still few literatures focusing on this topic. This paper utilizes the former research results on CAPS, provides a detailed replenishment analysis of CAPS and optimal replenishment solutions for CAPS aiming to optimize the efficiency of picking operation

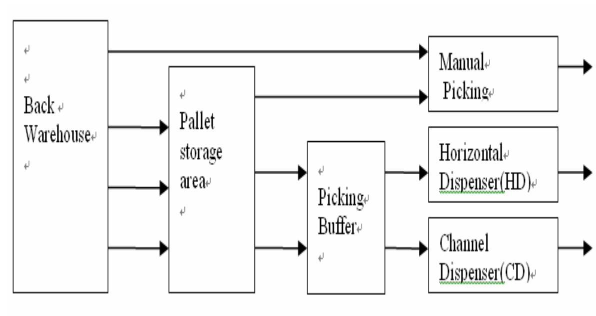

Fig. 1. General Inventory System of CAPS

II. Problem Description

In general, frequent replenishment plan for the HDs and CDs is highly related with the pallet storage area, picking buffer, the capacity of PD, the structure of order and the efficiency of all machines. A general replenishment plan goes as following steps (Figure 2):

Existing picking lines in China are almost half-automatic one whose replenishment relies heavily on handwork. In general, a replenishment point for each PD is first set up, as long as the remaining quantity in PD is less than certain value, a replenish point will be sent out, after then the PD will be fully replenished, however the order details which have an impact on replenishment are totally ignored here, Correspondingly the left cigarettes in PD have to be taken out by handwork if the allocation of brand on picking line has to be reset which increases the workload. Based on CAPS design, this paper proposes a new replenishment planning model.

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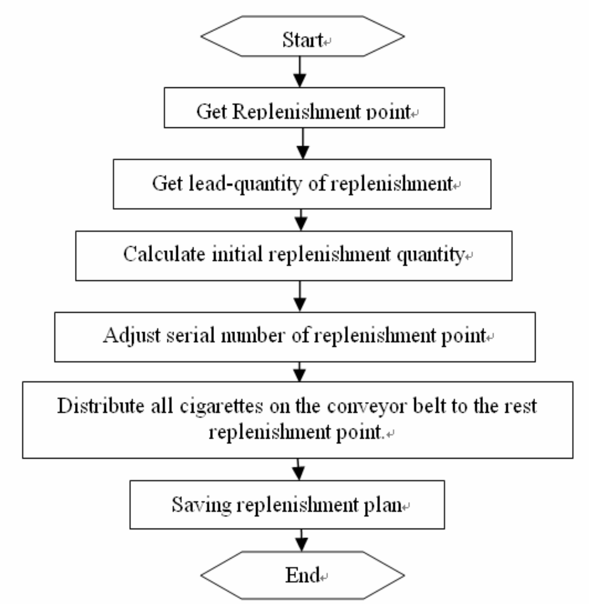

Fig.2. Replenishment Workflow

III. MATHEMATICAL MODELS

A. HYPOTHESIS

In a Tobacco Distribution Center (TDC), the picking operation is processed day by day and orders are picked one after another. The picked orders have been collected one day before. Suppose that there are M orders need to be picked on one day , and N Picking Dispensers (PD) in one picking line, Each PD picks only one brand which means the number of PD equals to the total number of cigarette brands. Before the replenishment operation, order and PD optimization has been done. The replenishment point of PD n is calculated according to the storage capacity of PD and total number of brand n picked on PD n. The Stocking Dispensers (SD) and PD are collected along a conveyor belt. The SD replenishes each PD when receiving a replenishment request and the replenishment quantity should be controlled strictly based on the request quantity from PD . Besides hypothesis discussed above, following hypothesizes are also considered:

H1: Current replenished cigarettes cannot be picked until the replenishment is completed;

H2: Replenishment point is triggered at time of picking order x;

H3: All conveyor belts run at the same speed.

H4 : The movement of Intelligent Replenishment Vehicle(IRV) among PDs can be ignored.

B. VARIABLES

- n : PD n (1≤n≤N)
- m : Order m (1≤m≤M)
- $P[n][m]$: The quantity of brand n in order m
- q_{nml} : The remaining quantity of PD n in order m
- Q_{nc} : Capacity of PD n
- D_{sp} : The distance of conveyor belt from SD TO PD
- V : The speed of conveyor belt
- q_{nmr} : The replenishment quantity of PD n in order m
- p_{nmr} : The replenishment point of PD n($2 \leq p_{nmr} \leq M$)
- t_{nmp} : The picking time of order m at PD n(second)
- t_{nmr} : The total replenishment time of PD n in order m
- t_{nms} : The picking time of SD for PD N in order m
- t_{nmvr} : The replenishment time of IRV for PD n in order m
- D_{oo} : The distance between two orders on picking conveyor belt
- D_{cc} : The distance between two cigarettes on picking conveyor belt
- E_n : The efficiency of PD n ($E_n = 5(CD)$, $E_n = 1(HD)$)
- E_S : The efficiency of SD (cartons per second)
- E_V : The efficiency of IRV (cartons per second)
- T_n : The time interval between dispensing of PD n
- q_x : The lead time of replenishment

C. MODEL ANALYSIS

N PDs and M orders form a N×M matrix. The value of each cell is $P[n][m]$. If brand x is not ordered in order m, then $P[x][m]$ =0. All data should be pre-processed. If $P[x][m] \geq 50$, then $P[x][m] = P[x][m] \bmod 50$ (a case contains fifty cartons). The picking efficiency could be improved greatly by big-unit picking.

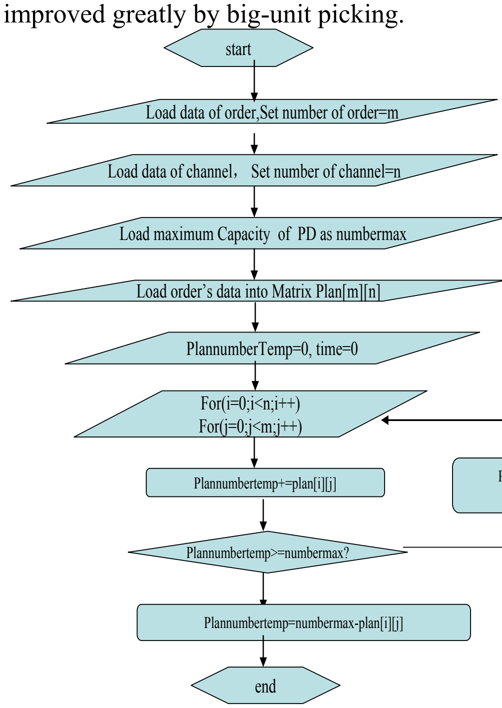

Fig. 3. Algorithm flow chart

While $\sum_{m=1}^{M} P[n][m] \le Q_{nc}$, total quantity for brand n picked on PD n is less than the capacity of PD n and no replenishment should be taken into consideration. While $\sum_{m=1}^{j} P[n][m] > Q_{nc}(j = 1,2\cdots M)$,replenishment happens. An ideal situation is that there is only one PD that needs to be replenished at a specific time. However, because of the complexity and discrete of orders, it's possible that more than one PD need to be replenished simultaneously, that may arise conflict and obstruction on the conveyor belt from SD to PD, and cause the efficiency reducing of the system. So our model deals with these two different cases: One is one-PD replenishment, the other is multi-PD replenishment. The former one is the basic solution to replenishment plan, and the latter focuses on conflict solution. The algorithm flow chart goes as follows (Figure 3).

D. MODELLING

1) Model A

In model A, only one PD (PD n) put a request for replenishment at a specific time.

Before picking the first order, all PDs have been replenished. If $\sum_{m=1}^{M} P[n][m] \le Q_{nc}$, then $q_{n0r} = \sum_{m=1}^{M} P[n][m]$, else $q_{n0r} = Q_{nc}$.

For the second time replenishment, the replenishment plan relies on $\sum_{m=1}^{j} P[n][m](j = 1,2\cdots M)$.

If $\sum_{m=1}^{j} P[n][m] > Q_{nc}(j = 1,2\cdots M, P[n][j] \ne 0)$ and $\sum_{m=1}^{j-1} P[n][m] < Q_{nc}$, a p_{nmr} is born. Although it's possible that $P[n][1] \ge Q_{nc}$ or $P[n][m] \ge Q_{nc}$, according to the inequality, it doesn't exist at all. The capacity of PD n is far larger than single $P[n][m]$. To eliminate the waiting time of order j, replenishment should be finished before picking order j, so the replenishment point should be triggered at $j - x(x \ge 1)$ and x is decided by the comparison of picking time and replenishment time. The total picking time of order (j-1) includes two parts, one is the picking time of order j-1 and another is time interval of between two pickings on PD n. The picking time of order (j-1) is $t_{n(j-1)p} = P[n][j-1]\backslash E_n \times T_n + (D_{oo} + D_{cc} \times \sum_{x=n+1}^{N} P[x][j-1] + D_{cc} \times \sum_{x=1}^{n-1} P[x][j]) / V$.

The total replenishment time $t_{n(j-1)r} = t_{n(j-1)s} + D_{sp}/V + t_{n(j-1)vr}$.

If $t_{n(j-1)p} \ge t_{n(j-1)r}$, then $p_{n(j-1)r} = j-1$, $q_{n(j-1)r} = \min(\sum_{m=1}^{j-1} P[n][m], \sum_{m=1}^{M} P[n][m] - \sum_{m=1}^{j-1} P[n][m])$.

Else if $t_{n(j-1)p} + t_{n(j-2)p} + \cdots t_{n(j-q-1)p} < t_{n(j-1)r} \le t_{n(j-1)p} + t_{n(j-2)p} + \cdots t_{n(j-q)p}$ and $q \ge 2$, then

$p_{n(j-q)r} = j - q$,

$q_{n(j-q)r} = \min(\sum_{m=1}^{j-1} P[n][m], \sum_{m=1}^{M} P[n][m] - \sum_{m=1}^{j-1} P[n][m])$.

If $\sum_{m=1}^{j} P[n][m] = Q_{nc}(j = 1,2\cdots M, P[n][j] \ne 0)$, a p_{nmr} is generated. It's possible that $P[n][j+1]=0$, but it's also probable that $P[n][j+1] \ne 0$. To make this model feasible, this paper takes the latter assumption into consideration .

If $t_{njp} \ge t_{njr}$, then

$p_{njr} = j$,

$q_{njr} = \min(\sum_{m=1}^{j} P[n][m], \sum_{m=1}^{M} P[n][m] - \sum_{m=1}^{j} P[n][m])$.

Else if $t_{njp} + t_{n(j-1)p} + \cdots + t_{n(j-q-1)p} < t_{njr} \le t_{njp} + t_{n(j-1)p} + \cdots + t_{n(j-q)p}$ and $q \ge 1$, then

$p_{n(j-q)r} = j - q$,

$q_{n(j-q)r} = \min(\sum_{m=1}^{j-1} P[n][m], \sum_{m=1}^{M} P[n][m] - \sum_{m=1}^{j-1} P[n][m])$.

If $t_{nmr} < \sum_{x=m-q}^{m} t_{nxp}(0 \le q \le m)$, it's possible that the quantity of cigarettes after replenishing is more than the capacity of PD, so Q_{nc} needs a buffer.

After replenishing, the existing quantity in PD n is Q_{nc} and the algorithm above could be applied to other replenishments.

2) Model B

Based on model A, assume that there are $X(X \ge 2)$ PDs that should be replenished at $p_{n(j-1)r}$. Correspondingly a priority strategy is needed. The most basic principle is first dispensing first replenishing. The queue of PD is fixed and the order is picked from the entrance to the exit on the picking line, so the PD near the entrance should be replenished first , otherwise the picking operation would be interrupted. All PDs that should be replenished form an array $PD[\,]$ with length X . $PD[X-1]$ locates near the entrance and $PD[0]$ locates near the exit.

Assume that $PD[x](0 \le x \le X-1)$ is replenished $q_{x(j-1)r}{}^{B}$

$$t_{x(j-1)s} = q_{x(j-1)r}{}^{B} / E_S \text{ , } t_{x(j-1)vr} = q_{x(j-1)r}{}^{B} / E_V \text{ .}$$

Assume that $\frac{q_{x(j-1)r}{}^{B}}{E_S} \le \frac{q_{(x+1)(j-1)r}{}^{B}}{E_V} (0 \le x \le X+2)$.

$$t_{x(j-1)r}{}^{B} = \sum_{y=x}^{X-1} q_{y(j-1)r}{}^{B} / E_V + q_{(X-1)(j-1)r}{}^{B} / E_S + D_{sp} / V \text{ ,}$$

$$t_{x(j-1)p}{}^{B} = \sum_{y=j-1}^{j+q_x-2} t_{xyp}$$

$t_{x(j-1)r}{}^{B} \le t_{x(j-1)p}{}^{B} (0 \le x \le X-1)$. This inequality influences the replenishment quantity of $PD[x]$.

If $q_{x(j-1)r}{}^{B} = q_{x(j-1)r}{}^{A} (0 \le x \le X-1)$ meet that condition, then all PDs should be replenished with the condition mentioned above for one time.

If all $q_{x(j-1)r}{}^{B} (0 \le x \le X-1)$ are equal, then $t_{x(j-1)r}{}^{B} = (X-x)\frac{q_{x(j-1)r}{}^{B}}{E_V} + \frac{D_{sp}}{V} + \frac{q_{x(j-1)r}{}^{B}}{E_S}$,

$$q_{x(j-1)r}{}^{B} = \min\left(\frac{\sum_{y=j-1}^{j+q_x-2} t_{xyp} \times E_S E_V}{(X-x)E_S + E_V} - \frac{E_S E_V D_{sp}}{(X-x)VE_S + VE_V} \right) (0 \le x \le X-1) \text{ .}$$

Each PD should be replenished $q_{x(j-1)r}{}^{A} \setminus q_{x(j-1)r}{}^{B}$ times.

If all $q_{x(j-1)r}{}^{B} (0 \le x \le X-1)$ are not equal, then this model can be recognized as a dynamic programming problem. Under conditions that replenishment time is less than picking time for PDs, the replenishment quantity could be planned with $P[n][m] \left(m \ge j-1+q_x \right)$.

If $P[n][j-1+q_x] = 0$, the picking time for PD n could be extended. Then we can infer that the brand with low-ordered frequency should be placed near the exit of picking line, so the replenishment efficiency could be improved and operation interruption caused by replenishment could be eliminated.

IV. APPLICATION

There are 1092 orders on april 13th,2009 ranging from EES000424942 to EES00042 6295.The total picking quantity is 159406 (carton) with 92 brands. The picking quantity of each brand ranges from 1 to 16171. Suppose that the capacity of PD is 150, then there are only 49 PDs that should be replenished ranging from 163 to 16171. According to the model, after all data have been processed, the picking quantity on PD declines ranging from 73 to 11307, then the number of PDs that should be replenished declines to 48 with picking quantity from 163 to 11307. Using the model, 915 replenishment points are generated. The frist replenishment point is generates at EES000424949 and the last one is EES000426294. There are 223 replenishment request conflicts ranging from 2 to 7. For the conflict 7, all should be replenished at EES000424992. The quantity of order EES000424992 is 70, the total replenishment quantity of 7 PDs is 986. The replenishment quantity relies on E_S 、 E_V 、 E_V 、 D_{sp} .

V. SUMMARY

Replenishment is the "midfielder" of stocking and picking, decomposing cases of cigarettes to cartons. Its purpose is to fill all picking machines to guarantee the operation of picking line. Different from common replenishment method, this paper propose a completely new idea and describe it with mathematical model. This model calculates the replenishment point and replenishment quantity according to practical situation. Also it's not difficulty to change this model into computer programming so that this model is practical. Considering that the logistics technology in many fields including tobacco is still developing in China, this paper will contribute to improve the automation level of Tobacco Distribute Center. For limitation of paper length, this paper doesn't use stimulation tools to test the model which should be included and that will be discussed in the next research.

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