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Continuous Modeling for Transient and Steady Cyclic Hardening /Softening Behavior of Metallic Materials under Amplitude Change Loading

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Abstract. This paper is concerned with the performance of the plastic strain amplitude measure, originally motivated and then abandoned by Haupt et al. [1]. Reasons for success and failure in modeling for the amplitude dependence are discussed. It is shown that if the memory parameter in the variable to indicate the center of the plastic strain region is determined suitably, the measure gives satisfactory performance in reflecting the amplitude behavior. By incorporating a hardening function dependent on the measure into the intrinsic time scale used in Valanis' endochronic plasticity, various transient and steady cyclic phenomena of hardening materials can be reproduced.

Introduction

It has been well known that with the variation of plastic strain amplitude, the cyclic hardening/ softening degree of the hardening materials changes continuously and appears memory to the prior maximum hardening history partially or completely [2,3]. In the last two decades, some proposed amplitude parameters are used to simulate these behaviors. Chaboche [2] proposed the concept of maximum plastic strain region. Ohno [4] proposed the cyclic non-hardening region. Improvements for Ohno's measure have been discussed in several works [5]. Tanaka [6] used a new variable to describe the center of the plastic strain region and took the distance between it and the current plastic strain point as an amplitude parameter. On the other hand, Haupt et al. [1] intended to accomplish a continuous describing of the hardening and softening, where an amplitude measure was defined and hoped to proportional to a direct measure of the plastic strain amplitude. As a result, the measure is related to the back stress, which will lose the objectivity.

To describe the amplitude effects in a continuous manner is important so that a constitutive model can easily be used for analysis of engineering components subjected to random loading. This presentation is intended to qualitatively study the initial amplitude measure defined in [1]. The measure, defined by the accumulation of instantaneous plastic strain amplitude with respect to the plastic deformation process, evolves continuously and naturally includes a fading memory to the prior amplitude history. The features of the amplitude measure are discussed in detail. By incorporating it into the hardening variables used in the endochronic theory [5], it is shown that various important amplitude effects on behavior of uniaxial cyclic deformation can be predicted in a satisfactory manner if the memory parameter in the measure is determined suitably.

In the following, stress and strain are expressed in the form of vectors in Ilyushin's five-dimensional deviatoric vector space. Let $|x|$ denote the absolute value of a quantity x.

Measure for cyclic plastic strain amplitude

In uniaxial symmetric tension-compression or torsion cycling, clearly, the instantaneous size of the plastic strain region, written as q_{in}, can be represented by

$$q_{in} = |\varepsilon^p| \tag{1}$$

where ε^p is the uniaxial component of plastic strain vector. To define a suitable amplitude measure, two ways are available. One way is that q_{in} is directly used, but a memory rule on it or on the target value of the internal variable dependent on it has to be introduced, such as that similar to Tanaka's consideration [6]. Another way, by Haupt's idea [1], is that the measure is defined by the accumulation of q_{in} with respect to the plastic deformation process. It evolves continuously and naturally includes the memory to the prior amplitude history. The later way is adopted and discussed here. To begin with, we take the integration of q_{in} as

$$\zeta_A(\zeta) = \int_0^{\zeta} q_{in}(\bar{\zeta})d\bar{\zeta} \tag{2}$$

where $\zeta = \int |d\varepsilon^p|$ is the current plastic strain arc length. Let q_A denote the amplitude measure, it may be assumed as

$$q_A = \frac{\zeta_A}{\zeta} \tag{3}$$

which gives an average mean amplitude value with respect to the plastic deformation process. It can be seen from Eq. 3 that when $\zeta = 0$, q_A becomes indeterminate. In fact, in the case of infinitesimally small ζ, q_A is approximately equal to $|\varepsilon^p|$. Therefore, as $|\varepsilon^p| = 0$, q_A=0.

Noting that q_A in Eq. 3 is assumed to depend on the entire previous plastic deformation process, a permanent memory is spontaneously caused. It means that the integration of q_{in} should be taken in a recent finite plastic deformation interval $[\zeta - \zeta_0, \zeta]$, where ζ_0 is the length of the interval. Thus, Eq. 3 may be replaced by

$$q_A(\zeta) = \frac{1}{\zeta_0}\int_{\zeta-\zeta_0}^{\zeta} q_{in}(\bar{\zeta})d\bar{\zeta}\,. \tag{4}$$

This is the measure initially proposed by Haupt et al. [1]. The constant ζ_0 specifies the memory range to the recent amplitude history. The larger the value of ζ_0, the slower the evolution rate of the amplitude measure.

It is apparent that the measure provides a transient amplitude variation process and a stable oscillatory value for cycling with a fixed amplitude. And the calculated average values at the stable states are approximately proportional to the actual plastic strain amplitudes. If the actual amplitude is changed from one level to another, q_A changes continuously until the new stable level is approached. To a certain extent, the memory degree of q_A to the prior amplitude history can be controlled through changing the value of ζ_0.

Now, let us consider cycling with non-zero mean plastic strain. Following the works by Haupt et al., a center variable of the plastic strain region, written as y here, is assumed to evolve as follows

$$y'(\zeta) = \frac{1}{\zeta_1}[\varepsilon^p(\zeta) - y(\zeta)] \tag{5}$$

where the constant ζ_1 determines how fast the center y adjusts itself to the actual prescribed mean plastic strain position. Thus, $q_{in}(\zeta)$ in Eq. 4 is replaced by

$$q_{in}(\zeta) = |\varepsilon^p(\zeta) - y(\zeta)|. \tag{6}$$

In [1], the capability of the measure defined by Eq. 4 was examined by incorporating it into Chaboche type kinematic hardening rules to simulate cycling with non-zero mean plastic strain. It was found that nonmonotonous softening occurs just after change of mean plastic strain since y adjusts itself to the new position with a certain delay. For this reason, the measure was abandoned by the proposers.

In fact, nonmonotonous softening is caused by the unsuitable value of ζ_1. According to the definition of Eq. 6, the smaller the value of ζ_1, the larger the instantaneous value of y. And there exists two limit cases that y varies with ζ_1 during mean plastic strain changing. One case is $\zeta_1 \to 0$, $y \to |\varepsilon^p|$ and another $\zeta_1 \to \infty$, $y \to 0$. Thus there should exist a suitable value of ζ_1 to be used in Eq. 6 to give the actural y which is some value between zero and $|\varepsilon^p|$. To make this clear, the material hardening function used to define the intrinsic time scale in Valanis' endochronic plasticity will be related to the measure for simulating the transient and steady amplitude effects. For simplicity of discussions, we briefly review the uniaxial endochronic constitutive equation in the following.

The endochronic theory and the amplitude-dependent hardening function

Based on the concept of the internal variable theory of thermodynamics, Valanis proporsed the endochronic theory along with the notion of intrinsic time. In uniaxial loading, the constitutive equation is written as

$$s = \int_0^z \rho(z - z') \frac{d\varepsilon^p}{dz'} dz' \tag{7}$$

in which s denotes deviatoric stress, $\rho(z)$ is called the hereditary function and may be represented by the Dirichlet series

$$\rho(z) = \sum_{i=1}^{\infty} c_i \mathrm{e}^{-b_i z}; \qquad \sum_{i=1}^{\infty} c_i \to \infty \tag{8}$$

c_i, b_i are positive material constants. In application, two or three terms in the series is sufficient to represent $\rho(z)$ quite accurately. And z is the intrinsic time defined by

$$dz = \frac{d\zeta}{f} \tag{9}$$

in which $d\zeta$ is the accumulated plastic strain increment. *f* is called the hardening function. Usually, *f* is assumed to depend on ζ or *z*.

In order to represent the amplitude dependence in cyclic plasticity, it is assumed $f = f(q_A)$ herein since the transient and steady feature of the amplitude measure, and taking it as a simple form

$$f(q_A) = 1 + \gamma q_A \tag{10}$$

where γ is a nonnegative material constant. It should be pointed out that the special choice of $f(q_A)$ is not necessary. For a certain type of material, other forms can be used, as long as they meet some general considerations, such as $f(0) = 1$ and $f(q_A)$>0, for hardening materials.

Numerical tests

As cited before, the aim of this paper is to qualitatively study the performance of the amplitude measure. The material constants identification procedures are not discussed here. They are chosen from the references or chosen according to the general behavior of the hardening rules.

Fig. 1(a) shows the present result for the plastic strain process used in [1]. We take the constants the same values as in [1], except for $\zeta_1 = 0.05$. It can be seen that nonmonotonous softening does not occur. The stress decreases monotonously after the mean plastic strain changing until the new steady state is approached.

By taking $i = 2$ in Eq. 8, as shown in Fig. 1(b), the predicted mean stress in the direction of changing mean plastic strain relaxes gradually to a steady value gives a stable, symmetric hysteresis loop, which is consistent with the experiments.

Fig. 2 shows the stress-plastic strain hysteresis loops under various cyclic plastic strain amplitudes with different mean plastic strain respectively. The constants take the values chosen in Fig. 1(b). Obviously, the variable y does well give the actual mean plastic strain position even for a moderate mean strain. The simulation for the cyclic hardening after changing a mean plastic strain is also qualitatively consistent with the experiments (see Fig. 7(b) in [7]; Fig.. 4 in [8] for comparison)

Fig. 3 gives the result for plastic strain process similar to the experimental strain process conducted in [4]. Again, values of the material constants in Fig. 1(b) are used, but 0.5 is chosen for ζ_0 here. It is found that the tendencies of peak stress with respect to the cycling numbers agree qualitatively with that in [4].

At the last, the present numerical result illustrated in Fig. 4 shows that the transient softening after reduction of amplitudes and the distinct shakedown phenomena, observed in the experiment of Bruhns et al. [9], can be fairly simulated.

Conclusions

(1) By determining a suitable value of the memory parameter appeared in the variable to indicate the center of the plastic strain region, the plastic strain amplitude measure initially defined by Haupt et al. gives a rational estimation to the actual amplitude.

(2) The amplitude measure shows both transient process and steady state and evolves continuously

during changing of the actual amplitude and mean plastic strain. Thus, it can be directly incorporated into the hardening variables to reproduce the gradual evolution of the cyclic hardening and softening, especially the downward convex softening and the forward relaxing in the case of loading with non-zero mean strain.

Acknowledgments

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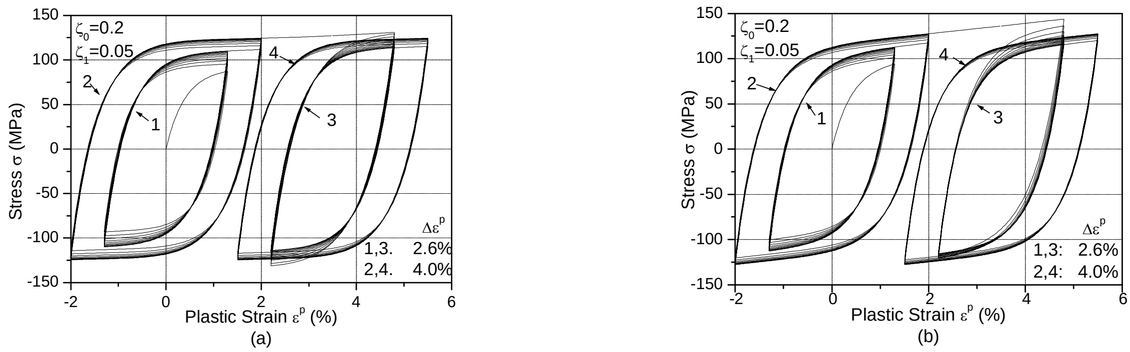
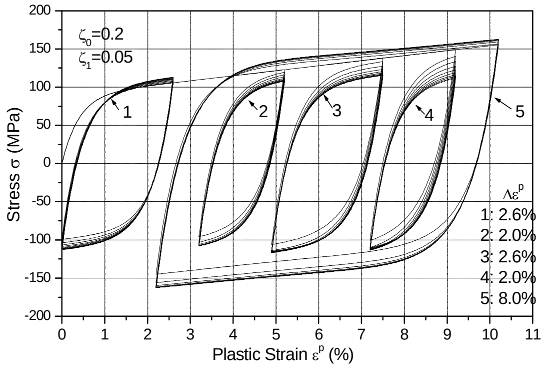


Fig. 1. Stress-plastic strain hysteresis loops for cycles with non-zero mean plastic strain $\varepsilon_m = 5.5\%$ subsequent to cycles with ε_m=0: (a) simulation using one component in Eq. 8 with $c_1 = 22500$, $b_1 = 250$, $\gamma = 40$; (b) simulation using two components in Eq. 8 with $c_1 = 22500$, $c_2 = 580$, $b_1 = 250$, $b_2 = 15$, $\gamma = 30$.



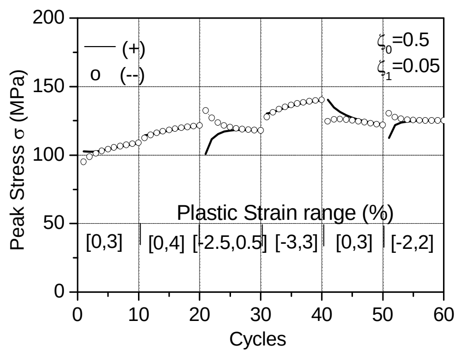


Fig. 2. Stress-plastic strain hysteresis loops for cycles with given plastic strain amplitude and non-zero mean plastic strain.

Fig. 3. Cycling with variable strain range. (+) ---positive peak stress per cycle; (-)---negative peak stress per cycle.

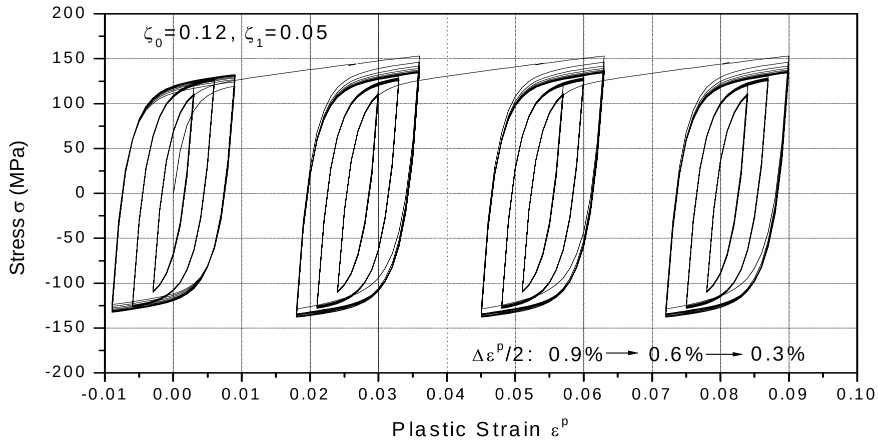


Fig. 4. Cycling with decreasing plastic strain amplitude and non-zero mean plastic strain, with $c_1 = 60000$, $c_2 = 550$, $b_1 = 1400$, $b_2 = 35$, $\gamma = 25$.