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Procurement Auctions for Differentiated Goods

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We consider two mechanisms to procure differentiated goods: a sealed-bid buyer-determined auction and a dynamic-bid price-based auction with bidding credits. The sealed-bid buyer-determined auction is analogous to the "request for quote" procedure commonly used by procurement agencies, and has each seller submit a price and the inherent quality of his good. Then the buyer selects the seller who offers the greatest difference in quality and price. In the dynamic-bid price-based auction with bidding credits, the buyer assigns a bidding credit to each seller conditional upon the quality of the seller's good. Then the sellers compete in an English auction, with the winner receiving the auction price and his bidding credit. Game-theoretic models predict the sealed-bid buyer-determined auction is socially efficient but the dynamic-bid price-based auction with bidding credits is not. The optimal bidding credit assignment undercompensates for quality advantages, creating a market distortion in which the buyer captures surplus at the expense of the seller's profit and social efficiency. In our experiment, the sealed-bid buyer-determined auction is less efficient than the dynamic-bid price-based auction with bidding credits. Moreover, both the buyer and seller receive more surplus in the dynamic-bid price-based auction with bidding credits.

Key words: procurement; auction; product differentiation; experiment
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1. Introduction

Large enterprises procure many goods for which the alternatives differ in quality. Often in such situations, the sellers find it neither feasible nor profitable to modify the nonprice attributes of their goods. Some examples are the procurement of office furniture, hotel stays, and contract software programming. Traditionally, such goods are bought via a "request for quote" procedure.

As part of their e-procurement agendas, many enterprises search for mechanisms that will provide better combinations of quality and price than the standard "request for quote" mechanism. Most of these searches lead to some form of an English auction.[1] The promise to deliver the lowest-cost seller through cut-throat price competition entices the enterprise. Unfortunately, when goods vary in quality, the use of an English auction can undermine the enterprise's efforts: often the lowest cost seller doesn't provide the best combination of price and quality. To address this issue, we introduce a stage prior to the English auction in which the buyer can assign bidding credits to sellers. Our primary focus in this study is whether an enterprise's interests are better served by our proposed dynamic-bid price-based auction with bidding credits (henceforth "dynamic PBA with credits") or the traditional "request for quote" auction—which we refer to as the sealed-bid buyer-determined auction without bidding credits (henceforth "sealed BDA").[2] The sealed BDA is our baseline for standard industry procurement practice, whereas the dynamic PBA with credits is an alternative we speculate is easy to implement, has good theoretical properties, and has a dominant strategy sellers are likely to adopt.

[1] Within the e-procurement community, the English auction is often called the reverse auction.

[2] This research originated from an evaluation of online auction formats for IBM's procurement division.

The sealed BDA starts with the buyer providing her evaluation criteria for the nonprice attributes, i.e., how she measures quality, to potential sellers. The buyer provides the evaluation criteria, even though the nonprice attributes of a good are fixed. This way a seller knows the quality level assigned to his good.[3] Next, each seller sets a price and provides the fixed description of his product. The buyer then selects the seller who offers the largest margin between quality and price. The winning seller receives his submitted price.

In our formulation of the sealed BDA as a game of incomplete information, we define a seller's type as his realized surplus (quality minus cost) and a seller's bid as a surplus offer (quality minus price). We also assume that the procurer knows the quality of each seller but not his costs, that each seller only knows his own cost and quality, and that each seller assumes the other sellers' qualities and costs are identically and independently distributed.

Under this formulation, the sealed BDA is equivalent to a first-price sealed-bid auction for selling an object to potential buyers with independent private values.[4] A seller's realized surplus in a sealed BDA is equivalent to a buyer's private value in a standard first-price sealed-bid auction; both are the potential gains from exchange that the auction participant provides. A surplus offer in a sealed BDA is equivalent to a buyer's bid in a first-price sealed-bid auction; both are an offer of utility by the bidder to the auctioneer. We exploit this equivalence to derive a symmetric Nash equilibrium for the sealed BDA. The symmetric equilibrium strategy is a surplus offer function that is an increasing function of realized surplus, ensuring the seller with the highest realized surplus wins and the auction is socially efficient.[5]

In the dynamic PBA with credits, as in the sealed BDA, the buyer provides the evaluation criteria to the sellers, who in turn provide their product descriptions. Observing the quality of each good, the buyer assigns a bidding credit to the seller of each good. Then the sellers, each with bidding credit in hand, compete in an English auction. In the auction, sellers offer successively lower prices until the winning seller submits a price that no other seller is willing to improve upon. The winning seller receives the auction price and his bidding credit.

We derive the equilibrium strategies of the dynamic PBA with credits by using a backward induction approach. For any bidding credit assignment, a seller's optimal strategy is to exit the auction when the price falls below his cost less his bidding credit. Anticipating sellers following this strategy, the buyer's optimal bidding credit rule is discriminatory. In the case of two sellers, although the buyer assigns a larger credit to the seller of the higher-quality good, the credit is smaller than his quality advantage. The buyer's optimal rule is reminiscent of the discriminatory policies of an optimal auction when sellers have asymmetric cost distributions.[6] In these optimal auctions and the dynamic PBA with credits, the optimal discriminatory policies promote competitive pressure by subsidizing the disadvantaged sellers, and enrich the buyer at the expense of sellers' profits and social efficiency.

Our experimental results differ from the game-theoretic predictions. In our experiment, the dynamic PBA with credits is more socially efficient, providing a better average outcome to both the seller and buyer than the sealed BDA provides. In the sealed BDA sessions, many subjects' choices do not correspond to the Nash equilibrium strategy. The Nash equilibrium strategy for our sealed BDA is nonlinear, and other studies, such as Chen and Plott (1998), Georee and Offerman (2002), Pezanis-Christou (2002), and Guth et al. (2005), demonstrate that subjects tend not to play nonlinear Nash equilibrium strategies.

[3] Hence, we are not considering the buyer making a strategic choice of evaluation functions for the auction as in studies such as Dasgupta and Spulber (1990) and Che (1993). Furthermore, we do not address the issue of whether a buyer should reveal private information about this demand as in studies such as Tan (1996) and Rezende (2009).

[4] First analyzed in a noncooperative equilibrium framework by Vickrey (1961).

[5] This change of variable approach has been successfully and independently used in two recent papers on procurement with design tournaments: Fullerton et al. (2002) and Che and Gale (2003). In both of the papers, sellers participate in a design contest to determine quality and then the enterprise uses and auction to purchase the innovation. Englebrecht-Wiggans et al. (2007) also use this approach for the same setting as ours.

[6] Myerson (1981), McAfee and McMillan (1989), and Bulow and Roberts (1989) derive the optimal auction for procurement when sellers have asymmetric costs.

Our sessions of dynamic PBA with credits are similar to the sessions of Cornes and Schotter (1999) that consider sealed-bid procurement auctions using price preferences to promote minority representation. Cornes and Schotter (1999) use fixed levels of the price preference as their treatment variable. They find that the level of minority representation increases with the level of the price preference, but procurement costs are minimized at some interior level of the price preference. In our experiment, we allow the buyer to choose the bidding credits. Buyers select discriminatory bidding credits, but not as discriminatory as advocated by the optimal bidding credit rule. The combination of the sellers' nonequilibrium behavior in the sealed BDA and the buyer's overly generous bidding credits in the dynamic PBA with credits gives rise to the outcome of the dynamic PBA with credits Pareto dominating the outcome of the sealed BDA.

Engelbrecht-Wiggans et al. (2007) present a theoretical and experimental study closely related to this one. They also consider procurement for differentiated goods with exogenous quality, and they compare buyer-determined and price-based auctions. Their theoretical analysis shows that, under Nash equilibrium, with a small number of sellers and little correlation between cost and quality, the price-based auction provides more benefit to the procurement organization than the sealed BDA does. This is a result that applies to our setting and shows that even if a buyer assigns zero bidding credits in the dynamic PBA with credits, he should do better than in the sealed BDA. Empirically, Engelbrecht-Wiggns et al. (2007) develop an ingenious experiment in which quality and cost are positively correlated in such a way that equilibrium bid functions are linear. As a result, their sealed BDA auction is slightly more efficient than ours, but surprisingly not by much. In their first-price sealed-bid auction, they observe overaggressive and heterogeneous bidding that is typical in this type of auction. In contrast, we see in our sessions of dynamic PBA with credits that sellers bid very closely to the dominant strategy.

Haruvy and Katok (2008) adopt the Engelbrecht-Wiggans et al. (2007) setting to compare the performance of the sealed BDA versus the dynamic-bid buyer-determined auction (henceforth "dynamic BDA"). This study is significant because the dynamic BDA is the most common auction form used in online procurement auctions. Their rather surprising result is that the sealed BDA soundly outperforms the dynamic BDA in their experiment. This result, combined with the results presented in the current paper—namely, that the dynamic PBA with credits results in a better outcome than the sealed BDA for both buyers and sellers—demonstrates strong potential for the dynamic PBA with credits as a powerful procurement tool.

Table 1 **Experimental Treatments Used in This Study and Related Studies**

	Price based	Bidding credits	Buyer determined
Dynamic		Cornes and Schotter (1999), present study	Haruvy and Katok (2008)
Sealed bid	Engelbrecht-Wiggans et al. (2007)		Haruvy and Katok (2008), Engelbrecht-Wiggans et al. (2007), present study

Table 1 organizes these related studies. The rows correspond to whether the auction is dynamic or sealed bid. The columns correspond to whether the auction winner is determined with no consideration for quality difference (price based), full consideration of quality differences (buyer determined), or some partial consideration for quality differences (bidding credits).

2. Game-Theoretic Models and Predictions

In our analysis and experiment, we consider the case of two sellers and a buyer. Levels of cost and quality characterize a seller. Seller i's cost to produce a unit is a random variable, denoted c_i, which is uniformly distributed on the interval $[c_L, c_H]$. A seller incurs this cost only when he makes a sale. The quality of seller i's good is a random variable, denoted v_i, which is uniformly distributed on the interval $[v_L, v_H]$. You can think of this quality as the buyer's maximum willingness to pay for the good. The cost and the quality of each seller's good are independent random variables. We ensure that the quality of a seller's good always exceeds its cost by assuming $v_L \geq c_H$. At the time of the auction, each seller knows his quality and cost, but only the distributions of the quality and cost of the other seller's good. Also, the buyer knows the quality of each seller's good but only the distribution of his costs. This information structure is common knowledge.

Two of our assumptions merit additional comments. First, consider the assumption that both the buyer and seller observe the quality of the seller. In our study, quality differences between sellers are the differences in the buyer's willingness to pay for each seller's good. The dubious aspect of this assumption is that the seller observes private information that is held by the buyer. Whether the seller knows this information has no impact in the dynamic PBA with credits, because a seller's behavior does not depend upon his quality. This assumption does remove influential uncertainty from the sealed BDA, but this should lead to a more positive assessment of the sealed BDA than if this uncertainty was incorporated. The second assumption is the independence of a seller's quality and cost. In some procurement situations, quality differences are likely idiosyncratic and uncorrelated with cost. It is likely in other applications of our results that quality and cost will be correlated. Our theoretical analysis is conceptually easy to extend to incorporate correlation, and we describe how this is done at the appropriate point. We do leave open the important question of how the correlation between quality and cost impacts behavior.

2.1. Sealed-Bid Buyer-Determined Auction

In this mechanism, potential sellers simultaneously submit prices. Seller i's submitted price is denoted p_i. Seller i wins the auction if $v_i - p_i$ is the maximum of $\{v_1 - p_1, v_2 - p_2\}$ and receives the price p_i. In the case of a tie, a seller is selected at random from the set of winning sellers. The winning seller i's profit is $p_i - c_i$, the other seller's profit is zero, and the buyer's achieved surplus is $\Pi = v_i - p_i$.

Although seller i's type is the pair (v_i, c_i), the relevant economic information is simply the difference of the two variables. The potential gains from exchange seller i provides is $s_i = v_i - c_i$. We call s_i seller i's "realized surplus." The random variable s_i has a distribution function, denoted $F(\,)$, which is the convolution of v_i and $-c_i$.[7] Instead of explicitly considering the submitted price, we consider seller i's "surplus offer," $o_i = v_i - p_i$. Under this formulation, the seller who offers the buyer the largest surplus offer wins the sealed BDA auction.

With this change of variables, the sealed BDA has the same formulation as a first-price sealed-bid auction used to sell a single object to buyers with private values. In such a setting, each buyer's value is her realized surplus, or the potential gains from exchange she provides (assuming the seller has a cost of zero). When a buyer makes a bid, she is making a surplus offer to the seller. And the highest bid is simply the greatest surplus offer. The equivalence of the sealed BDA, under the change of variables, and the first-price sealed-bid auction allows us to derive a symmetric Bayes–Nash equilibrium using standard arguments such as those found in McAfee and McMillan (1987).

The symmetric Bayes–Nash equilibrium surplus offer function for the sealed BDA is

$$o_i^* = O(s_i) = s_i - \frac{\int_{V_L - C_H}^{s_i} F(z)\,dz}{F(s_i)}.$$

This expression calculates how much of seller i's realized surplus he offers in equilibrium. From the equilibrium surplus offer function and the definition of realized surplus we get the equilibrium bid function

$$p_i^*(v_i, c_i) = c_i + \frac{\int_{V_L - C_H}^{s_i} F(z)\,dz}{F(s_i)}.$$

This expression provides the margin demanded by the seller as a function of cost and quality. For the n-seller case, the equilibrium strategies are

$$o_i^* = O(s_i) = s_i - \frac{\int_{V_L - C_H}^{s_i} [F(z)]^{n-1}\,dz}{[F(s_i)]^{n-1}} \quad \text{and}$$

$$p_i^*(v_i, c_i) = c_i + \frac{\int_{V_L - C_H}^{s_i} [F(z)]^{n-1}\,dz}{[F(s_i)]^{n-1}}.$$

To better illustrate this derivation, let's consider the following example, which uses the parameters of our experiment. Let c_i and v_i be independent and uniformly distributed with supports of $[c_L, c_H] = [40, 80]$

[7] If quality and cost are correlated, then we would derive the distribution of realized surplus by using the multivariate transformation theorem.

and $[v_L, v_H] = [100, 130]$. Then the distribution of realized surplus, s_i, is

$$F(s) = \begin{cases} \dfrac{(s-20)^2}{2{,}400} & \text{for } 20 \le s < 50, \\ \dfrac{s-35}{40} & \text{for } 50 \le s < 60, \\ 1 - \dfrac{(90-s)^2}{2{,}400} & \text{for } 60 \le s \le 90. \end{cases}$$

The equilibrium surplus offer and bid functions resulting from this distribution are

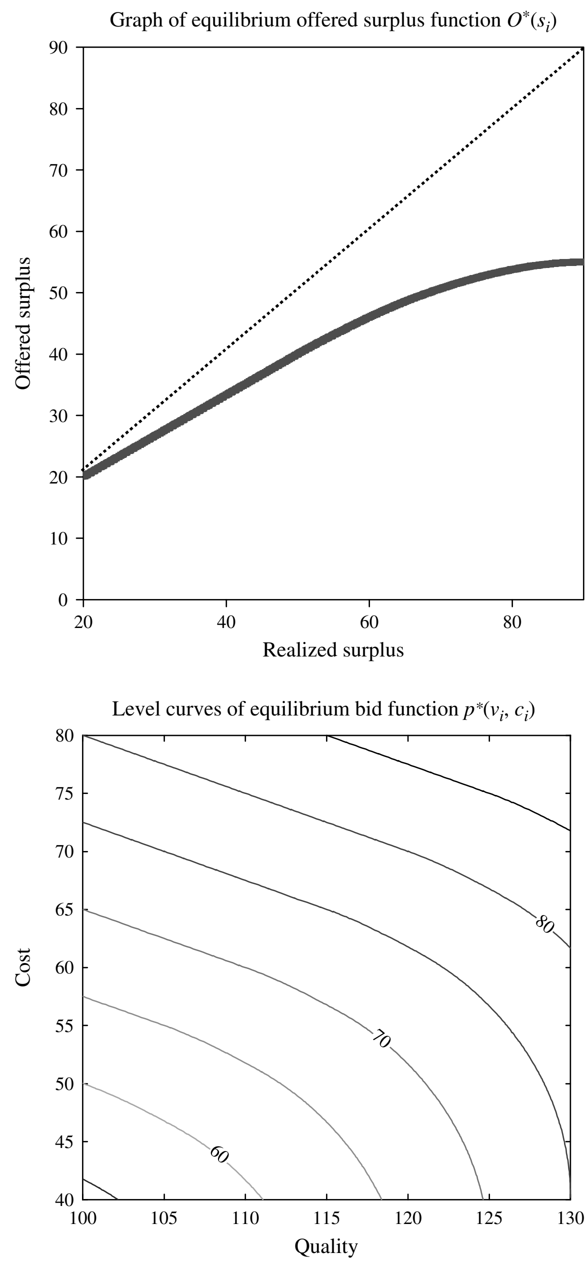
$$o^*(s_i) = \begin{cases} \dfrac{2s_i + 20}{3} & \text{for } 20 \le s < 50, \\ s_i - \dfrac{300 + (s_i - 20)(s_i - 50)}{2(s_i - 35)} & \text{for } 50 \le s < 60, \\ s_i - \dfrac{2{,}400(s_i - 55) + (1/3)(90 - s_i)^3}{2{,}400 - (90 - s_i)^2} & \\ & \text{for } 60 \le s \le 90, \end{cases}$$

and

$$p^*(v_i, c_i) = \begin{cases} \dfrac{2c_i + v_i + 20}{3} & \text{for } 20 \le s < 50, \\ c_i + \dfrac{300 + (v_i - c_i - 20)(v_i - c_i - 50)}{2(v_i - c_i - 35)} & \\ & \text{for } 50 \le s < 60, \\ c_i + \dfrac{2{,}400(v_i - c_i - 55) + (1/3)(90 - v_i + c_i)^3}{2{,}400 - (90 - v_i + c_i)^2} & \\ & \text{for } 60 \le s \le 90. \end{cases}$$

The equilibrium surplus offer and bid functions are depicted in Figure 1. After deriving a sequential Nash equilibrium of the dynamic PBA with credits, we will present some of the economic implications of this symmetric Nash equilibrium.

Figure 1 Graphs of Sealed BDA Nash Equilibrium Strategy



2.2. Dynamic-Bid Price-Based Auction with Bidding Credits

There are two stages in this mechanism. In the first stage, the buyer assigns a bidding credit to each seller, conditioning it upon the seller's quality. Seller i's bidding credit is denoted b_i. In the second stage of the auction, each seller is told his respective bidding credit, and then the sellers participate in an English auction. The winning seller receives a monetary amount equal to the auction price and his assigned bidding credit.

We now derive a sequential Nash equilibrium for this mechanism. Each of a seller's information sets in stage two is defined by his bidding credit and the cost and quality of his good. A seller i's behavioral strategy is to set an exit price, e_i, for the English auction, i.e., the continuation game, at each of his information sets.

Also, we require that the seller update his belief about the other seller's type at each of his information sets via Bayes' Rule. This is only a formality because each seller has a weakly dominant strategy.

Proposition 1. *Seller i has a weakly dominant strategy*: $e_i^*(b_i, c_i, v_i) = c_i - b_i$.

In other words, seller i remains in the auction as long as the standing price is greater than or equal to the seller's cost less his assigned bidding credit.

Proof. Apply one of the standard arguments, such as that of Krishna (2002, p. 15), that establishes the weakly dominant strategy in the second-price sealed-bid or English auctions. Just recalibrate the seller's zero payoff price to his cost less his bidding credit. Q.E.D.

Now we derive the buyer's optimal bidding credit assignment in stage one. At this point, one could be tempted to simply appeal to the famous optimal mechanism literature result, Myerson (1981), which states for this example that you give a bidding credit equal to half the quality advantage in conjunction with a quality-specific reserve price, and then show that the dynamic PBA with credits implements this. Unfortunately, we cannot make that appeal because we are requiring the procurement official to make a purchase. In terms of the Myerson (1981) formulation of the optimal mechanism problem, the constraint that the sum of the allocation probabilities assigned to each bidder must be less than or equal to one is now a constraint that the sum of these probabilities must be one. This precludes the auctioneer placing his own bid (as a reserve price) that could win and preempt the awarding of the contract. In the absence of reserve prices, this simple bidding credit is no longer optimal.[8] So we must proceed to directly calculate the optimal bidding credit rule.

Recall that a buyer's achieved surplus is the difference between the quality and the price paid (auction price plus bidding credit) of the procured object. The buyer's expected achieved surplus[9] for a pair of bidding credits—when sellers adopt their dominant strategies—is

$$\begin{aligned} E[\Pi(b_1, b_2)] = \Pr[c_1 - b_1 \le c_2 - b_2] & \\ \cdot (v_1 - E(c_2 - b_2 \mid c_2 > c_1 - b_1 + b_2) - b_1) & \\ + \Pr[c_1 - b_1 > c_2 - b_2] & \\ \cdot (v_2 - E(c_1 - b_1 \mid c_1 > c_2 + b_1 - b_2) - b_2) & \end{aligned}$$

or

$$\begin{aligned} E[\Pi(b_1, b_2)] = \Pr[c_1 - b_1 \le c_2 - b_2] & \\ \cdot (v_1 - E(c_2 \mid c_2 > c_1 - b_1 + b_2) + b_2 - b_1) & \\ + \Pr[c_1 - b_1 > c_2 - b_2] & \\ \cdot (v_2 - E(c_1 \mid c_1 > c_2 + b_1 - b_2) + b_1 - b_2). & \end{aligned}$$

Inspection of this payoff function reveals that there are payoff-equivalent strategy classes for the buyer: two pairs of bidding credits, (b_1, b_2) and (b_1', b_2'), yield the same expected payoff if $b_1 - b_2 = b_1' - b_2'$. Let K be the set of payoff-equivalent strategies with the typical element k, where $k \in K = \{(b_1, b_2): b_1 - b_2 = k\}$. From this point, when we consider a particular k we will be considering the unique bidding credit pair for which at least one of the sellers receives a bidding credit of zero. With this notation, the buyer's expected achieved surplus is

$$\begin{aligned} &E[\Pi(k)] \\ &\quad = \Pr[c_1 - k \le c_2](v_1 - E(c_2 \mid c_2 > c_1 - k) - k) \\ &\quad\quad + \Pr[c_1 - k > c_2](v_2 - E(c_1 \mid c_1 > c_2 + k) + k). \quad (1) \end{aligned}$$

The term $\Pr[c_1 - k \le c_2]$ is the probability that seller 1 wins the auction. This corresponds to the probability of event $A = \{c_1 - k \le c_2\}$. Figure 2 shows the two shapes this event can take in the support of (c_1, c_2).

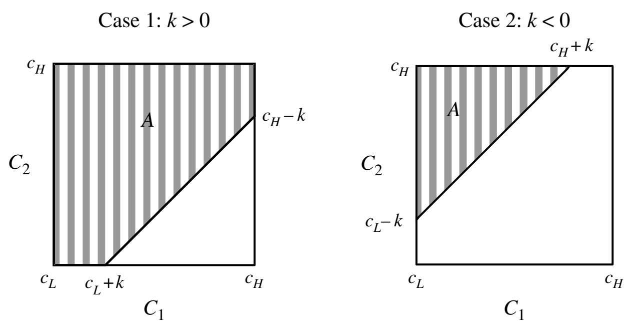
With a rectangular distribution on the support, the probability that seller 1 wins the auction is

$$\Pr[c_1 - k \le c_2] = \begin{cases} \dfrac{2(c_H - c_L)^2 - (c_H - c_L - k)^2}{2(c_H - c_L)^2} & \text{if } k \ge 0, \\[2ex] \dfrac{(c_H - c_L + k)^2}{2(c_H - c_L)^2} & \text{otherwise.} \end{cases} \quad (2)$$

[8] Intuitively, reserve prices aggressively seek surplus from bidders with high types, and the Myerson (1981) bidding credit is relatively generous in its promotion of the higher-quality type. In the absence of a reserve price, the optimal bidding credit is less generous, because it derives value from creating competitive pressure for high-quality types.

[9] If quality and cost were correlated, we would express the expectation of each cost term also to be conditional upon the corresponding quality.

Figure 2 Event Space of Seller 1 Winning the Auction



Start with the case $k \geq 0$. When seller 1 wins the auction the expected auction price is $E(c_2 \mid c_2 \geq c_1 - k,\ k \geq 0)$. To calculate this expectation, we need the probability density function for the auction price. The cumulative distribution function (CDF) for the auction price is

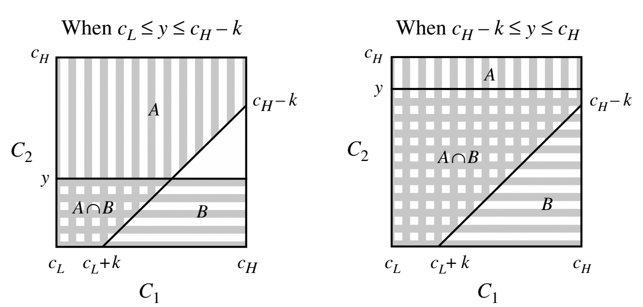
$$\begin{aligned} G(y) &\equiv \Pr(\{c_2 \leq y \mid c_2 \geq c_1 - k\}) \\ &= \frac{\Pr(\{c_2 \leq y\} \cap \{c_2 \geq c_1 - k\})}{\Pr(\{c_2 \geq c_1 - k\})}. \end{aligned} \tag{3}$$

Recall $A = \{c_1 - k \leq c_2\}$, and let $B = \{c_2 \leq y\}$. Figure 3 shows the relevant events in the support of (c_1, c_2) for the case where $k \geq 0$.

Direct calculation yields

$$\Pr(A \cap B) = \begin{cases} \dfrac{(y-c_L)(2k+y-c_L)}{2(c_H-c_L)^2} \\ \quad \text{if } c_L \leq y \leq c_H - k, \\ \dfrac{2(y-c_L)(c_H-c_L)-(c_H-c_L-k)^2}{2(c_H-c_L)^2} \\ \quad \text{if } c_H - k \leq y \leq c_H. \end{cases} \tag{4}$$

Figure 3 Two Possible Configurations of the Events That Seller 1 Wins the Auction and Seller 2's Cost Is Below the Value y



Substitution of (2) and (4) into (3) gives us

$$G(y) = \begin{cases} \dfrac{(y-c_L)(2k+y-c_L)}{2(c_H-c_L)^2-(c_H-c_L-k)^2} \\ \quad \text{if } c_L \leq y \leq c_H - k, \\ \dfrac{2(y-c_L)(c_H-c_L)-(c_H-c_L-k)^2}{2(c_H-c_L)^2-(c_H-c_L-k)^2} \\ \quad \text{if } c_H - k \leq y \leq c_H. \end{cases}$$

Differentiate this expression to obtain the density function of y:

$$g(y) = \begin{cases} \dfrac{2(k+y-c_L)}{2(c_H-c_L)^2-(c_H-c_L-k)^2} & \text{if } c_L \leq y \leq c_H - k, \\ \dfrac{2(c_H-c_L)}{2(c_H-c_L)^2-(c_H-c_L-k)^2} & \text{if } c_H - k \leq y \leq c_H. \end{cases}$$

The expected value of c_2 conditional upon k and seller 1 winning the auction is

$$\begin{aligned} E(y) &= \int_{c_L}^{c_H} y g(y)\, dy \\ &= \frac{\int_{c_L}^{c_H-k} (2y^2 + 2ky - 2c_L y)\, dy + \int_{c_H-k}^{c_H} 2(c_H-c_L) y\, dy}{2(c_H-c_L)^2-(c_H-c_L-k)^2} \end{aligned}$$

or

$$E(y) = \frac{2}{3}c_H + \frac{1}{3}c_L - \frac{k(c_H-c_L)^2 + k^2(c_H-c_L-k)}{6(c_H-c_L)^2 - 3(c_H-c_L-k)^2}. \tag{5}$$

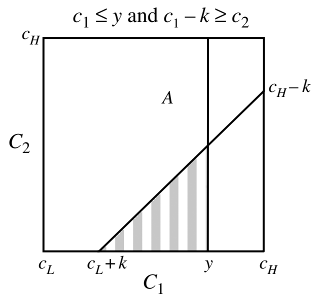
The first term in this expression is the expectation of the first-order statistic of the draw of two costs, and the second term is the deviation from this expectation as k changes.

To better see how these results relate to our experiment, we include our experiment parameters and see that

$$\begin{aligned} E(y) &\equiv E(c_2 \mid c_2 \geq c_1 - k,\ k \geq 0) \\ &= 66\tfrac{2}{3} - \frac{k}{3}\left(1 - \frac{40k}{1600 + 80k - k^2}\right). \end{aligned}$$

One consistency check on this expression is to set $k = 0$, i.e., the special case of a simple English auction. When $k = 0$, the expected value of the auction price conditional upon seller 1 winning is the expected value of the maximum cost statistic, or $66\frac{2}{3}$. A second consistency check is to set $k = 40$ and guarantee that seller 1 wins the auction. Here the expected value of y is 60, the unconditional expectation of seller 2's cost.

Figure 4 **Union of the Set of Events in Which Seller 2 Wins the Auction and Seller 1's Cost Is Less Than the Value of y**



Now we calculate the expected price conditional upon seller 2 winning the auction, i.e., $E(c_1 \mid c_1 - k \ge c_2, k \ge 0)$. In this instance,

$$\Pr(\{c_1 - k \ge c_2\}) = \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2} \quad \text{and}$$

$$\Pr(\{c_1 \le y\} \cap \{c_1 - k \ge c_2\}) = \begin{cases} 0 & \text{if } y \le c_L + k, \\ \dfrac{(y - c_L - k)^2}{2(c_H - c_L)^2} & \text{if } c_L + k \le y \le c_H. \end{cases}$$

One can verify these probabilities with Figure 4.

The cumulative distribution function of the auction price when seller 2 wins is

$$G(y) = \begin{cases} 0 & \text{if } y \le c_L + k, \\ \dfrac{(y - c_L - k)^2}{(c_H - c_L - k)^2} & \text{if } c_L + k \le y \le c_H. \end{cases}$$

This is the CDF of the maximum statistic for two independent draws from a uniform distribution on the interval $[c_L + k, c_H]$, permitting us to state

$$E(c_1 \mid c_1 - k \ge c_2, k \ge 0) = \tfrac{2}{3}c_H + \tfrac{1}{3}c_L + \tfrac{1}{3}k. \tag{6}$$

After substituting (2), (5), and (6) into (1) and simplifying, the buyer's expected achieved surplus for $k \ge 0$ is

$$\begin{aligned} &E[\Pi(k \mid k \ge 0)] \\ &= \left[\frac{2(c_H - c_L)^2 - (c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] \\ &\quad \cdot \left(v_1 - \frac{2}{3}c_H - \frac{1}{3}c_L + \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2} - k\right) \\ &\quad + \left[\frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right]\left(v_2 - \frac{2}{3}c_H - \frac{1}{3}c_L + \frac{2}{3}k\right). \end{aligned}$$

Let's consider the case where $k < 0$. The symmetry of the probability and conditional expectation calculations allows us to immediately state the expected achieved surplus in this case:

$$\begin{aligned} &E[\Pi(k \mid k < 0)] \\ &= \left[\frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right]\left(v_1 - \frac{2}{3}c_H - \frac{1}{3}c_L - \frac{2}{3}k\right) \\ &\quad + \left[\frac{2(c_H - c_L)^2 - (c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] \\ &\quad \cdot \left(v_2 - \frac{2}{3}c_H - \frac{1}{3}c_L + \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2} + k\right). \end{aligned}$$

Without loss of generality, assume seller 1 is the seller with the higher-quality good. The following proposition indicates that it is never in the buyer's interest to give a larger bidding credit to the lower-quality seller, i.e., choose $k < 0$.

Proposition 2. $E[\Pi(0)] > E[\Pi(k \mid k < 0)]$.

Proof.

$$\begin{aligned} &E[\Pi(0)] - E[\Pi(k \mid k < 0)] \\ &= \frac{1}{2}(v_1 - v_2) - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2} v_1 - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] v_2 \\ &\quad + \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\frac{2k}{3} - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] k \\ &\quad - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2}. \end{aligned}$$

The term

$$\frac{1}{2}(v_1 - v_2) - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2} v_1 - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] v_2$$

is strictly positive because a negative k reduces the probability of seller 1 winning below one-half. Also, the term

$$\frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\frac{2k}{3} - \left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] k$$

is strictly positive. Finally, the term

$$-\left[1 - \frac{(c_H - c_L + k)^2}{2(c_H - c_L)^2}\right] \frac{k(c_H - c_L)^2 + k^2(c_H - c_L - k)}{6(c_H - c_L)^2 - 3(c_H - c_L - k)^2}$$

is also strictly positive. Therefore,

$$E[\Pi(0)] - E[\Pi(k \mid k > 0)] > 0. \quad \text{Q.E.D.}$$

With this proposition, we examine the case of $k \geq 0$ for the optimal bidding credit assignment k^*. The first-order condition for the maximization of the buyer's achieved surplus is

$$\frac{dE[\Pi(k^*)]}{dk} = \frac{2k^{*2} - [3(c_H - c_L) + (v_1 - v_2)]k^* + (c_H - c_L)(v_1 - v_2)}{4} = 0.$$

The second-order condition is

$$\frac{d^2E[\Pi(k^*)]}{dk^2} = \frac{4k^* - 3(c_H - c_L) - (v_1 - v_2)}{4} \leq 0.$$

The first-order condition is a quadratic. Of the two roots, only the negative one satisfies the second-order conditions for the maximum. The negative root and optimal bidding credit rule is

$$k^* = \tfrac{1}{4}\Big(3(c_H - c_L) + (v_1 - v_2) - \sqrt{[3(c_H - c_L) + (v_1 - v_2)]^2 - 8(c_H - c_L)(v_1 - v_2)}\,\Big).$$

Proposition 3. *The optimal bidding credit assignment is less than the differences in quality, i.e., $k^* < v_1 - v_2$.*

Proof.

$$\begin{aligned} &k* < (v_1 - v_2) \\ &\Rightarrow\ 3(c_H - c_L) - 3(v_1 - v_2) \\ &\qquad - \sqrt{[3(c_H - c_L) + (v_1 - v_2)]^2 - 8(c_H - c_L)(v_1 - v_2)} < 0 \\ &\Rightarrow\ 9(c_H - c_L)^2 - 18(c_H - c_L)(v_1 - v_2) - 9(v_1 - v_2)^2 \\ &\qquad < 9(c_H - c_L)^2 - 2(c_H - c_L)(v_1 - v_2) + (v_1 - v_2)^2 \\ &\Rightarrow\ -16(c_H - c_L)(v_1 - v_2) - 10(v_1 - v_2)^2 < 0. \quad \text{Q.E.D.} \end{aligned}$$

According to Proposition 3, the buyer's best strategy in the dynamic PBA with credits is to use a discriminatory rule that assigns a bidding credit to the high-quality seller that is less than his quality advantage. The impact of this rule bolsters the low-quality seller's competitiveness and leads the high-quality seller to receive lower expected surplus than in the sealed BDA.

Again we consider the parameters of our experiment to form an example: Two sellers independently and uniformly draw costs from the interval [40, 80] and qualities from the interval [100, 130]. After the buyer observes each seller's quality, she assigns to the higher-quality seller the bidding credit

$$k^* = \frac{(120 + v_1 - v_2) - \sqrt{(40 + v_1 - v_2)^2 + 12{,}800}}{4}.$$

An inefficient outcome occurs when the high-quality seller's cost is in the interval $[c_2 + k^*, c_2 + (v_1 - v_2)]$. For example, if $(v_1, c_1) = (120, 60)$ and $(v_2, c_2) = (110, 55)$, seller 1 has the greatest realized surplus but seller 2 wins the dynamic PBA with credits. Specifically, the buyer assigns the optimal bidding credit of 1.58 to seller 1, and seller 2 wins the auction at a price of 58.42.

2.3. Economic Performance

Using the Nash equilibrium strategies for the sealed BDA and dynamic PBA with credits, we can generate theoretical predictions of economic variables such as efficiency, market price, the average quality and cost of the procured good, and the buyer's achieved surplus. Table 2 presents the expected values of various economic variables for the economic environment of our experiment. Each of the following statistics is associated with one of the columns in Table 2: *percentage of efficient outcomes* (or the percentage of auctions that select the seller with the greatest difference between quality and cost), *average realized social surplus* (or the sum of the buyer's achieved surplus and the winning seller's profit), *average winning seller's quality*, *average auction price* (for the dynamic PBA with credits, this is the auction price and bidding credit paid), *average winning seller's cost*, *average buyer's surplus* (or the winning seller's quality less total price paid), and *average winning seller's profit*. We obtained all of the expected values, except *percentage of efficient outcomes* for the sealed BDA, by simulating each auction 10 million times.

The predicted outcomes of the two mechanisms differ for many variables. The sealed BDA always generates a socially efficient outcome because the symmetric Nash equilibrium strategy is strictly increasing in realized surplus. In contrast, the dynamic PBA with credits selects the inefficient seller over 16% of the time. Also, the buyer's discriminatory bidding credit assignments reduce the average winning seller quality. Of course, the dynamic PBA with credits more than makes up

Table 2 Nash Equilibrium Predictions for the Sealed BDA and Dynamic PBA with Credits

Auction	*% of efficient outcomes*	*Avg. winning seller quality*	*Avg. auction price*	*Avg. winning seller cost*	*Avg. buyer surplus*	*Avg. winning seller profit*	*Avg. realized social surplus*
Sealed BDA	100.00	117.90	71.17	54.65	46.74	16.52	63.26
Dynamic PBA with credits	83.41	115.55	66.79	53.37	48.76	13.41	62.17

for this lower quality with a reduced price and a bias toward the lower-cost seller. Also, the dynamic PBA with credits leads to a gain in buyer welfare and a reduction in seller profit.

From a theoretical perspective, the dynamic PBA with credits better serves the buyer's interest than the standard sealed BDA. But does human behavior conform to the models used to derive our predictions? Past experimental studies show that human choice often differs from game-theoretic predictions, and we will see this occur again in our experiment.

3. Experimental Design

All of our experimental sessions, except one, were conducted via a computer software application at the University of California, San Diego Department of Economics EEXCL facility. We conducted the other session at the IBM Thomas J. Watson Research facility. Every session was either a sealed BDA or dynamic PBA with credits treatment. All sessions were conducted in the fall of 2000.

Each subject received a show-up payment of $5 prior to participating in a session (except for the IBM session, for which a $20 payment was given). Before the decision-making portion of a session, each subject read a paper copy of the instructions and then had to successfully complete a simple written test of how the auction worked and how earnings were calculated. After the experiment, each subject was privately paid his or her earnings in U.S. currency.

In a sealed BDA session, all subjects were designated as sellers. The subjects participated in five practice periods with no payments, and then 50 additional periods with cash payments proportional to their experimental earnings. Prior to each period, subjects were randomly paired to participate in different auctions. At the start of each period, each subject was informed of his or her quality and cost. Then, each seller privately submitted a price, and a winner was determined. Subjects were informed of whether they won the auction, the winning auction price, and their period earnings. A complete history of this information was always available to each subject.

In the sessions of dynamic PBA with credits, two-thirds of the subjects were randomly assigned as sellers, and one-third as buyers. After two practice periods, subjects participated in 12 to 16 periods in which cash earnings accumulated.[10] Before each period, a collection of trios was formed by randomly matching two sellers and one buyer. Each trio participated in their own auction. At the beginning of an auction, the buyer was informed of the quality of each seller's good, and each seller was informed of the quality and unit cost of his or her good. Then the buyer had the opportunity to assign a credit to each of the sellers. Once these credits were assigned, they were revealed to the respective sellers.

Next, an iterative English auction commenced with sellers making opening offers. In subsequent iterations, the seller who did not have the lowest current offer could either exit the auction or make an offer lower than the current lowest offer. The seller with the current lowest offer could either maintain his current offer or improve it. When one of the sellers exited, the auction concluded. The current lowest price at the conclusion determined the auction price.

The winning seller received the auction price and his assigned credit less his unit cost. The buyer received the difference between the quality of winning seller's good and the total payment to the winning seller. The losing seller received zero earnings. Over the course of a session, subjects could see a complete history of the information that had been revealed to them. We conducted four sealed BDA sessions with 44 total subjects:

[10] A dynamic PBA with credits period lasted significantly longer than a sealed BDA period, and consequently, we used a 20 experimental dollars to one U.S. dollar exchange rate in the dynamic PBA with credits sessions and a four-to-one exchange rate in the sealed BDA sessions.

Table 3 **Number of Subjects and Periods in Each Dynamic PBA Session**

Dynamic PBA session	Total periods	Practice periods	Sellers	Buyers
1	14	2	10	5
2	18	2	6	3
3	16	2	10	5
4	16	2	8	4

two with 12 subjects each and two 10 subjects each. Each sealed BDA session was completed in less than 90 minutes. We conducted four sessions of dynamic PBA with credits. The number of participants and periods in each session is given in Table 3. All sessions of dynamic PBA with credits were completed in 105 minutes.

4. Results on Empirical Economic Performance

Under Nash equilibrium play, the sealed BDA is socially efficient, whereas the dynamic PBA with credits better serves the buyer's interest at the expense of efficiency and seller profit. Contrary to these predictions, in experiments, the dynamic PBA with credits is more efficient and nominally makes both buyer and seller better off than in the sealed BDA. The difference between the comparative theoretical and empirical economic performances mostly results from play that diverges from the theory in the sealed BDA experiments.

In Table 4, we provide statistics for various performance variables from the sealed BDA and dynamic PBA with credits sessions. For both the sealed BDA and dynamic PBA with credits sessions, we provide the sample mean and standard deviation for each variable. Also, for each variable, we provide the z-statistic and its p-value for the hypothesis test that the means are the same under both auction types. A bold-faced p-value indicates the hypothesis is rejected at a 5% level of significance.

The dynamic PBA with credits, not the sealed BDA, provides a more socially optimal outcome in the experiments. In over 84% of the dynamic PBA with credits auctions, the higher-surplus seller wins and the average total realized social surplus is 63.25, whereas the higher-surplus seller wins only 79% of sealed BDA auctions and the average total realized social surplus is 61.53. Dividing realized social surplus into its two components, we see that the average buyer surplus is about 1.7% greater and the average seller profit is about 5.2% greater in the dynamic PBA with credits than in the sealed BDA.[11]

How buyers and sellers in the dynamic PBA with credits benefit from the advantage in average social surplus is found by examining the average realized qualities, prices, and costs. The sealed BDA generates a higher quality level than the dynamic PBA with credits (117.69 versus 116.26), but also an increase in costs (56.15 versus 53.01). The net effect of these two differences is the 1.71 advantage in total social surplus enjoyed by the dynamic PBA with credits. Also, the average dynamic PBA with credits price is 2.52 lower than the sealed BDA price. From the seller's perspective, the net effect of the price and cost reductions is a 0.62 increase in profit in the dynamic PBA with credits. From the buyer's perspective, the reduction in quality is more than offset by the reduction in cost, and results in a 1.09 increase in buyer surplus in the dynamic PBA with credits.

The differences between the relative empirical performances and the game-theoretic predictions must mean at least one of the auctions is performing differently than its Nash equilibrium predictions. Table 5 presents the observed and theoretical values of the reported performance variables and hypothesis tests that the observed and theoretical values are the same. The theoretical predictions of the sealed BDA are rejected at the 5% level of significance for all variables except *avg. winning seller quality*. On the other hand, for the dynamic PBA with credits, the theoretical prediction is only rejected for a single variable. The observed buyer surplus is significantly less than predicted. Clearly the Nash equilibrium predictions fare worse for the sealed BDA than the dynamic PBA with credits. Subjects not using Nash equilibrium strategies must be the source of the theoretical prediction's failures.

5. Analysis of Individual Behavior

To understand how the equilibrium predictions fail we must identify how subjects' behavior is deviating from the equilibrium strategies. First, we consider

[11] However, the improvement in surplus for the buyer and seller in the dynamic PBA with credits is not significant according to the z-test.

Table 4 Empirical Auction Performance: Sealed BDA vs. Dynamic PBA with Credits

Auction	*% of efficient outcomes*	*Avg. realized social surplus*	*Avg. winning seller quality*	*Avg. auction price*	*Avg. winning seller cost*	*Avg. buyer surplus*	*Avg. winning seller profit*
Sealed BDA	79%	61.53	117.69	68.37	56.16	49.32	12.21
Std. dev.	0.407	12.67	8.18	9.39	10.64	9.86	8.33
Dynamic PBA with credits	84.27%	63.25	116.26	65.85	53.01	50.41	12.83
Std. dev.	0.64	3.56	2.86	3.06	3.26	3.14	2.89
$\mu_{\text{sealed}} - \mu_{\text{dynamic}}$	−5.27%	−1.71	1.43	2.52	3.14	−1.09	−0.62
z-stat.	−2.01	−2.00	2.37	3.45	4.74	−1.25	−0.94
p-value	**0.022**	**0.023**	**0.991**	**1.000**	**1.000**	0.106	0.173

Note. A boldfaced p-value indicates the hypothesis of a difference in means is rejected at a 5% level of significance.

Table 5 Auction Performance: Theoretic Predictions and Empirical Measurements

Auction	*% of efficient outcomes*	*Avg. realized social surplus*	*Avg. winning seller quality*	*Avg. auction price*	*Avg. winning seller cost*	*Avg. buyer surplus*	*Avg. winning seller profit*
Sealed BDA, $n = 1{,}100$							
Theoretical	100%	63.26	117.90	71.17	54.65	46.74	16.52
Observed	79%	61.53	117.69	68.37	56.16	49.32	12.21
Std. dev.	0.407	12.67	8.18	9.39	10.64	9.86	8.33
z-stat.	−17.092	−4.51	−0.85	−9.91	4.69	8.69	−17.16
p-value	**0.000**	**0.000**	0.198	**0.000**	**1.000**	**1.000**	**0.000**
Dynamic PBA with credits, $n = 248$							
Theoretical	83.41%	62.17	115.55	66.79	53.37	48.76	13.41
Observed	84.27%	63.25	116.26	65.85	53.01	50.41	12.83
Std. dev.	0.638	3.56	2.86	3.06	3.26	3.14	2.89
z-stat.	0.374	1.40	1.29	−1.40	−0.61	2.01	−0.94
p-value	0.646	0.919	0.902	0.081	0.270	**0.978**	0.173

Note. A boldfaced p-value indicates the hypothesis of a difference in means is rejected at a 5% level of significance.

subject behavior in the sealed BDA experiments. Here we show that sellers offer too much surplus when they receive high-surplus types and too little surplus when they receive low-surplus types. Also, subjects offer different levels of surplus to the buyer for distinct quality–cost pairs that provide the same realized surplus. Regression analysis shows that there are two distinct types of bidders: those who make nonlinear bids that are correlated with the Nash bids, and those who submit bids linear in cost and quality. This mixture model explains why sellers are too "generous" or too "stingy" depending upon their realized surplus types. In the dynamic PBA with credits, we see that buyers are too generous with their bidding credit assignments and that sellers follow their dominant strategy with one caveat: the losing sellers on average exit the auction about three dollars before their zero profit price. This bias takes away some of the buyer's surplus in the dynamic PBA with credits.

5.1. Behavior in the Sealed-Bid Buyer-Determined Auction

To what extent do subjects' surplus offers correspond to the Nash equilibrium surplus offer function? When we plot surplus offers versus realized surplus in Figure 5 we do not see evidence that subjects follow the Nash surplus offer function. We do see at low levels of realized surplus that subjects will offer less than Nash levels of surplus, whereas at middle levels of realized surplus the surplus offers exceed the Nash levels, and at high levels of realized surplus there is tremendous variation in the level of surplus offers. The scatter plot of surplus offers also has several linear bands. Each of the bands represents a focal amount of profit demanded by a seller such as \$10, \$20, or \$30. These bands could be indicative of subjects who only ask for a fixed absolute margin independent of their quality. The presence of these bands and the large variation in the surplus offers raises a question: Is a subject's surplus offer determined by the difference in quality and

Figure 5 Surplus Offered

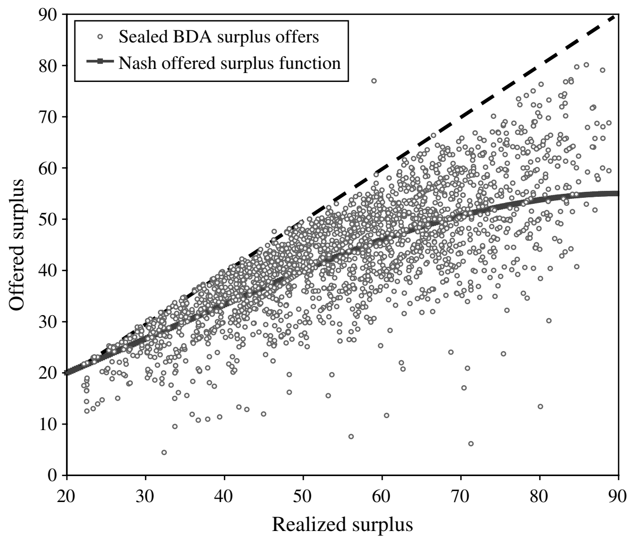
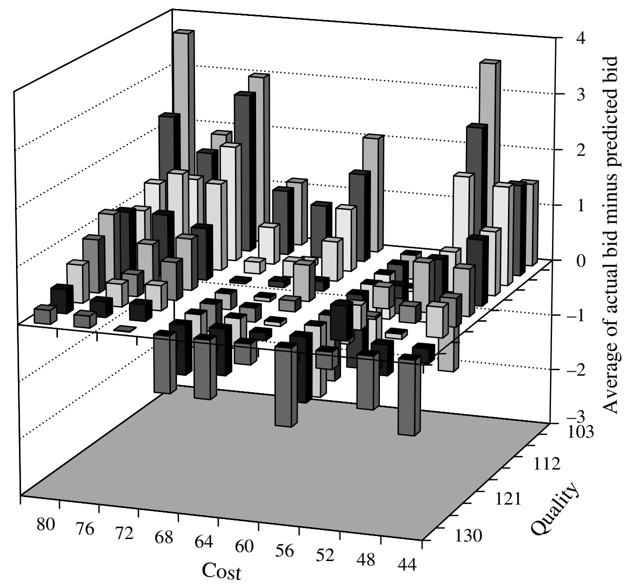


Figure 6 Average Difference Between Actual Bid and Predicted Bid for Different Cost–Quality Types



Note. The shading of a bar is quality specific.

cost, or more generally by the absolute values of quality and costs?

Defining realized surplus as a seller's type is key to solving for the symmetric Nash equilibrium strategy, but assuming a subject's behavior is solely characterized by his realized surplus, or the difference in quality and cost, may be inappropriate. To understand how subjects condition their choices on the absolute levels of cost and quality, we consider the difference between submitted and Nash bids for different quality–cost pairs. In Figure 6, we present the average difference between submitted and Nash bids for different ranges of cost–quality pairs. We start by defining 100 equal-sized bins that cover the supports of the cost and quality variables. For each bin, we select all the instances when a subject drew a cost–quality pair in the range of the bin. For each of these instances, we calculate the deviation of the submitted bid from the Nash bid. We calculate the average of all the deviations in the bin, and the average is the reported as the height of the bar of the bin in Figure 6.

The graphs of these averages reveal systematic patterns. First, for each level of cost, the difference between the submitted and Nash bids falls as quality increases. Evidently, subjects do not fully appreciate the competitive advantage associated with higher quality levels. Second, when costs are high and quality is low—i.e., there is a low level of realized surplus—submitted bids are greater than Nash bids. This is counter to the Nash equilibrium feature that the lowest type demands zero profit. Third, for low-cost/high-quality bins, the bids are below the Nash levels. Finally, if the subjects condition their behavior only on the level of realized surplus, then we would expect the average bid deviation to be the same for a constant level of realized surplus. Bins corresponding to the same level of realized surplus lie on off-diagonal lines of the cost–quality range. Inspection of the bar graphs does not suggest equal bid biases for bins lying on these off-diagonals. Hence, subjects' behavior is not invariant to the absolute levels of cost and quality.

Given the significant variation observed when we pool subject behavior, we now ask whether subjects' decisions are noisy or whether there is systematic heterogeneity in the subjects' bidding rules. We proceed by allowing for two possibilities: a subject's bids could either be a linear function of cost and quality or correlated with the nonlinear Nash bid function. A linear bid function for subject i has the following form:

$$p_i(c_i, v_i) = \beta_0 + \beta_1 c_i + \beta_2 v_i,$$

where the betas are unknown coefficients. We formulate the nonlinear Nash bid model as

$$p_i(c_i, v_i) = \gamma_0 + \gamma_1 p^*(c_i, v_i),$$

where $p^*()$ is the Nash bid function. If a bidder exactly follows the Nash bidding rule, then $\gamma_0 = 0$ and $\gamma_1 = 1$. The two models allow us to characterize linear bidders and bidders who follow nonlinear rules that are close to the Nash equilibrium strategy. We want to ascertain, for each subject, whether either of these models is appropriate.

We use the J-test of Davidson and MacKinnon (1981) to determine the selection from the two nonnested models. First we imbed the two models into one specification:

$$p_i(c_i, v_i) = (1-\alpha)[\beta_0 + \beta_1 c_i + \beta_2 v_i] + \alpha[\gamma_0 + \gamma_1 p^*(c_i, v_i)].$$

We use ordinary least squares (OLS) to estimate this model. Then we run two hypothesis tests: $\alpha = 0$ and $\alpha = 1$. There are four possible outcomes to this exercise. First, we could reject both hypotheses, and we would then select the larger nesting model. Second, we could not reject either hypothesis. This would indicate that both models are adequate and that the models are highly colinear. Third, we could reject $\alpha = 0$ but not $\alpha = 1$. In this case, we would select the Nash model. Finally, we could reject $\alpha = 1$ but not $\alpha = 0$. In this case, we would select the linear bid model. Recall that the Nash bid function is linear over part of the cost–quality range, and in this range the two models can correspond. This can confound the identification of which bidding function a subject follows. Nevertheless, our results allow us to make a definitive model assignment for half of the subjects.

We apply the J-test to each subject's data in the sealed BDA and find substantial heterogeneity in the bidding strategies. In Table 6, we report for each subject the model selected, the estimated parameters of the selected model, and the r-squared statistic. For 22 of the 44 subjects, we are able to select a single model. Six subjects follow the Nash bidding model, and 16 follow the linear bidding model. The J-test selects both models for 10 subjects. For these subjects we report the coefficient estimates for the model with the cost, quality, and Nash bid parameters. Inspection of the regression results reveals some classic signs of multicollinearity: a high R^2, insignificant coefficients, and coefficients with the wrong sign. For these 10 subjects, the two models are too similar to differentiate. For the remaining 12 subjects, we reject both the linear and Nash models in favor of the nested model. Here we report the regression results for the nested regression and again observe the signs of multicollinearity.

Table 6 **Estimated Bidding Models with J-Test Selection Criteria for Each Subject**

Subject	Intercept	Cost	Quality	NE price	R-squared	Model selected
1	**−4.10**	—	—	1.04	0.85	Nash price
2	**−8.11**	—	—	1.08	0.64	Nash price
3	14.10	—	—	0.78	0.62	Nash Price
4	**9.10**	—	—	0.91	0.54	Nash price
5	**13.80**	—	—	0.79	0.54	Nash price
6	24.19	—	—	0.66	0.53	Nash Price
7	**5.29**	0.74	0.15	—	0.94	Linear cost and bid
8	**−1.69**	0.92	0.12	—	0.93	Linear cost and bid
9	**−2.25**	0.68	0.27	—	0.88	Linear cost and bid
10	**0.05**	0.89	**0.10**	—	0.85	Linear cost and bid
11	**10.20**	0.66	0.19	—	0.82	Linear cost and bid
12	**8.28**	0.68	0.21	—	0.81	Linear cost and bid
13	**9.54**	0.76	**0.11**	—	0.81	Linear cost and bid
14	**12.44**	0.81	**0.08**	—	0.77	Linear cost and bid
15	**2.20**	0.65	0.26	—	0.75	Linear cost and bid
16	**12.88**	0.86	**0.02**	—	0.67	Linear cost and bid
17	**13.82**	0.61	0.22	—	0.65	Linear cost and bid
18	**10.80**	0.63	**0.18**	—	0.64	Linear cost and bid
19	**6.93**	0.61	0.23	—	0.62	Linear cost and bid
20	**18.35**	0.91	**0.05**	—	0.55	Linear cost and bid
21	**18.45**	0.67	**0.14**	—	0.47	Linear cost and bid
22	**38.65**	0.59	**0.05**	—	0.33	Linear cost and bid
23	**−4.05**	0.40	**−0.06**	0.75	0.93	Both models selected
24	**4.43**	**−0.01**	**−0.15**	1.12	0.87	Both models selected
25	**12.30**	−1.20	−0.71	2.89	0.86	Both models selected
26	**−5.12**	−0.63	**−0.26**	2.03	0.85	Both models selected
27	**1.68**	**0.20**	**−0.07**	0.88	0.84	Both models selected
28	**3.19**	−1.48	**−0.60**	3.15	0.83	Both models selected
29	35.84	**−0.45**	−0.66	1.86	0.83	Both models selected
30	**12.41**	−0.89	**−0.36**	2.12	0.70	Both models selected
31	40.41	**−0.31**	**−0.45**	1.39	0.60	Both models selected
32	57.49	−0.75	−0.60	1.76	0.53	Both models selected
33	**−11.02**	**0.08**	**0.01**	1.00	0.82	Neither model selected
34	**15.39**	**0.33**	**0.01**	**0.54**	0.79	Neither model selected
35	**−3.13**	**0.41**	**0.49**	**0.06**	0.71	Neither model selected
36	**−2.15**	**0.30**	**−0.02**	**0.77**	0.70	Neither model selected
37	**−7.04**	**0.19**	**0.30**	**0.53**	0.67	Neither model selected
38	**24.90**	**−0.78**	**−0.49**	2.14	0.58	Neither model selected
39	**8.10**	**−0.17**	**−0.02**	**1.04**	0.57	Neither model selected
40	36.39	**0.23**	**0.06**	**0.25**	0.41	Neither model selected
41	**19.83**	**−0.71**	**−0.36**	**1.95**	0.33	Neither model selected
42	43.29	**0.16**	**0.03**	**0.33**	0.26	Neither model selected
43	**37.00**	**0.43**	**−0.05**	**0.31**	0.21	Neither model selected
44	**27.52**	**0.64**	**0.48**	**−0.63**	0.12	Neither model selected

Notes. A boldfaced p-value indicates the hypothesis coefficient is equal to zero and is rejected at a 5% level of significance; there are 50 observations for each subject. NE, Nash equilibrium.

The J-test exercise demonstrates there are large contingencies of both nonlinear Nash bidders and linear bidders. The presence of linear bidders leads to inefficient auction outcomes; linear bidding rules lead to low prices for high-quality goods. Consequently, the buyer's surplus is significantly higher and the seller's profit is significantly lower than under the Nash equilibrium.

5.2. Behavior in the Dynamic-Bid Price-Based Auction with Credits

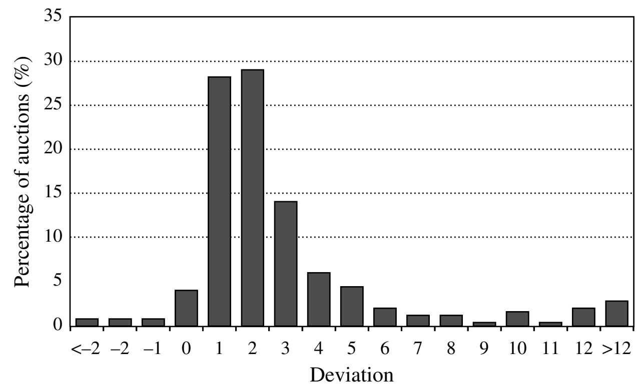
Subject behavior in the dynamic PBA with credits adheres more closely to the game-theoretic predictions than does behavior in the sealed BDA. Again, a seller has a weakly dominant strategy in the auction: exit only when the price falls below cost minus bidding credit. Subjects do follow this prescription with a caveat. The losing seller exits the auction, on average, three dollars above his threshold price. The buyer's Nash strategy is not as apparent as the seller's. Most of the time the buyers do assign nonzero bidding credits, but their assignments are, on average, too generous. The combination of sellers exiting the auction slightly early and the buyer's assigning overly generous bidding credits leads to greater efficiency and seller profit than predicted.

How closely do sellers adhere to the dominant strategy? In Figure 7, we provide a histogram of the difference between the losing seller's exit price and his dominant strategy exit price. Most losing sellers exit slightly above their zero profit prices. Specifically, over 81% of the deviations are between zero and four—the average price was close to 64. In contrast, only 2.4% of the losing sellers exit after the zero profit price, and only three out of 248 auction winners lose money. Sellers clearly understand the dominant strategy and exit close to, but not below, their zero profit prices. We conjecture that the tediousness of the English auction is responsible for the early exit behavior.[12]

The buyers' bidding credit assignments greatly vary and, on average, are more generous than the optimal bidding credits. First, approximately 25% of the time the buyer does not utilize the bidding credits to give

[12] This raises a potential issue with the Nash equilibrium analysis: if the subjects were actually discounting their auction payoffs, then our open outcry implementation of the auction allows alternative equilibrium involving jump bidding as discussed in Isaac et al. (2007).

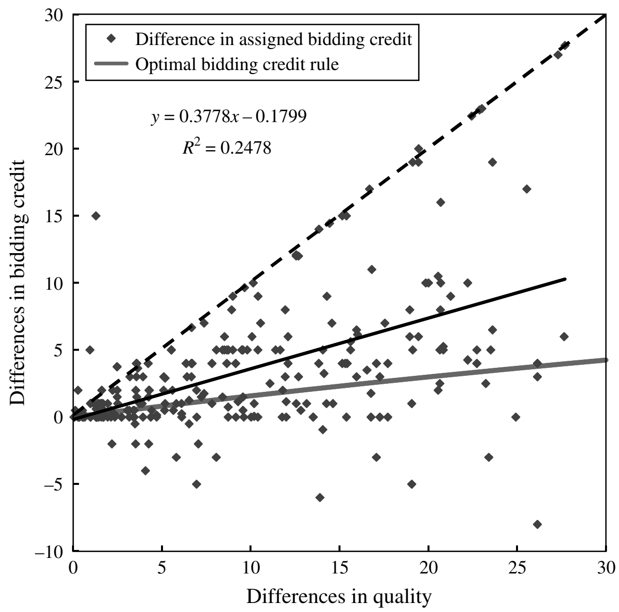
Figure 7 **Deviation of Losing Seller's Exit Price from Dominant Strategy Exit Price**



an advantage to the high-quality seller. Specifically, in over 18% of the auctions, the buyer assigns the same bidding credit to both sellers, and in almost 7% of the auctions, the buyer assigns a larger bidding credit to the lower quality. In these cases, the buyer is certainly not using the bidding credits to manage quality differences. At the other end of the spectrum, in almost 6% of the auctions the difference in the assigned bidding credits is equal to the difference in quality. Here, although the buyer is ensuring the best seller is selected, he is not capturing any additional surplus over the Nash equilibrium outcome of the sealed BDA. In Figure 8, we provide a scatter plot of the difference in bidding credit versus the difference in quality, a graph of the optimal bidding credit rule, and a graph of the OLS trend line. The trend line is above the optimal assignment line, but the OLS trend also has a low R^2 that reflects highly variable buyer behavior. Although the dynamic PBA with credits has a transparent dominant strategy for sellers, the optimal bidding assignment rule proves elusive to the buyers. However, as seen in Figure 8, the majority of the time the buyers do use discriminatory assignments.

The experimental results provide an assessment of the impact the choice of auction has on the effectiveness of procurement activities. The empirical performance of the sealed BDA leaves opportunities for other mechanisms to improve upon the status quo. As our experiment shows, the dynamic PBA with credits produces greater total surplus and better serves the buyer's interest, depending upon the buyer's objective and rule for assigning bidding credits.

Figure 8 Differences in Assigned Bidding Credits vs. Differences in Quality



6. Concluding Remarks

In this study we assess two alternative mechanisms that enterprises may use to procure differentiated goods. The first alternative is the traditional "request for quote" method, characterized as a sealed-bid buyer-determined auction. We show that, under a symmetric Nash equilibrium, the sealed BDA is an efficient mechanism. The second alternative is a dynamic-bid price-based auction with the buyers assigning credits to sellers based upon the differing qualities of the goods for sale. This mechanism has a Nash equilibrium in which a seller has a transparent weakly dominant strategy, and the buyer has an optimal bidding credit assignment, which undercompensates the high-quality seller for his quality advantage. This discriminatory policy improves the buyer's welfare over what she receives in the sealed BDA at the expense of social efficiency and seller profit. However, in our experiments, we find that the dynamic PBA with credits outperforms the sealed BDA for both buyers and sellers because (1) the sellers do not follow the symmetric Nash strategy in the sealed BDA, and (2) the buyers assign overly generous bidding credits in the dynamic PBA with credits.

The transformation of how enterprises procure goods and services is one promise the emergence of e-commerce has actually fulfilled. Part of this transformation is an increase in the use of buyer-determined auction variations. In practice, these auctions for procurement of differentiated goods are not equivalent to the auctions typically studied by economists. Specifically, the buyer-determined auction mechanism does not determine the seller; the auction only sets each participating seller's price. After the auction, the buyer selects the winning seller and pays that seller his exit price from the auction. This is how currently used procurement auctions manage product differentiation (Kinney 2000).

The dynamic PBA with credits is a potentially attractive alternative to current procurement auction practices. In the current business use of English auctions, a seller no longer has a transparent dominant strategy and, more importantly, a buyer cannot credibly commit to a discriminatory policy when they select a seller after the auction. Evaluating the nonprice attributes of goods after the auction, the buyer is less likely to use a discriminatory policy. This would require sometimes selecting a seller who does not provide the best combination of price and quality. With the dynamic PBA with credits, the evaluation of quality prior to the auction is an opportunity to precommit to a discriminatory policy that doesn't suffer from the credibility problem of exercising the policy after the auction.

Another promising application of the dynamic PBA with credits and its strategic transparency to the seller is its use in managing the longer-term procurer–seller relationship. As noted by Jap (2007), current applications of the reverse auction alienate this relationship and can lead to worse procurement performance. This study demonstrates that smart use of the dynamic PBA with credits can lead to making both the sellers and buyer better off. This comes from the fact that in the various forms of procurement auctions without transparent dominant strategies, bidders fail to adopt Nash equilibrium bidding strategies and lead to inefficient market outcomes. The dynamic PBA with credits may allow one to realize additional gains from exchange that would otherwise be lost when using other forms of procurement auctions.

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