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Estimating a small open economy DSGE model with indeterminacy: Evidence from China

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ABSTRACT

Considering that monetary policy instability may cause indeterminacy of the macroeconomic equilibrium, this paper derives the boundary condition between determinacy and indeterminacy in a small open economy DSGE model, and then uses this model to investigate China's monetary policy and macroeconomic fluctuations under indeterminacy during the period from 1992 to 2011. The empirical results show that the nominal interest rate reacts not only to inflation and output gap, but also to the changes in RMB exchange rate. Moreover, the indeterminacy in the macro-dynamics indicates the instability in China's monetary policy, and it stems from two sources, the sunspot shock and the indeterminate propagation of fundamental shocks. In addition, we find that the monetary policy shock affects macroeconomic dynamics significantly in the short run, while in the long run, it only influences nominal variables, such as the inflation and the exchange rate, but not the real output.

1. Introduction

Over the last decades, economists, central bankers and financial market analysts have shown increasing interest in monetary policy analysis.[1] One of the most prominent studies is the well-known *Taylor rule*, which was proposed as a guideline to evaluate and describe central bank policy actions intuitively. As shown in Taylor (1993), the central bank could adjust the interest rate according to inflation deviation (the deviation of inflation rate from its target) and output gap (the deviation of real output from its potential value). From then on, economists extended the original rule to various Taylor-type rules and applied them to examine monetary policy reaction functions in different countries (Clarida et al., 1998, 2000; Taylor, 2001). However, amongst these single-equation models, they fail to establish a clear link between the conduct of monetary policy and the performance of the economy, which makes the model economy far away from the real world, and hence the relevant concluding remarks might be inaccurate and unreliable.

Recently economists are increasingly making use of dynamic stochastic general equilibrium (DSGE) models for macroeconomic analysis and monetary policy evaluation in academic research, especially at central banks. For example, the European Central Bank uses the DSGE model developed by Smets and Wouters (2003) to analyze the economy of the Euro zone as a whole. In fact, compared to other structural models such as vector autoregression (VAR), structural VAR, and simultaneous equation model, the DSGE model has three apparent advantages: Firstly, it provides a theoretical discipline on the structure of the model economy, in which it relates the reduced-form parameters to the structural parameters, and connects the short-run dynamics with the long-run equilibrium; secondly, it shows a more suitable framework for analyzing social welfare and designing an optimal policy, as the agents' utility in the economy can be taken as a measure of welfare explicitly; lastly, it makes use of the micro-founded model for monetary policy analysis more appropriately, i.e. less subject to the *Lucas critique*. Furthermore, as shown in An and Schorfheide (2007) and Chib and Ramamurthy (2010), no matter how complicated the DSGE model is, the standardized Bayesian method can be used to realize the model estimation quickly.

Although a large fraction of DSGE models are assumed to a closed economy (Justiniano et al., 2010; Rabanal, 2007), more and more studies have considered the open economy version of the DSGE model to

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[1] By now there is a large and growing amount of macroeconomist work on DSGE models from theoretical perspective and empirical methodology. This includes Lubik and Schorfheide (2003, 2004, 2007), Fernádez-Villaverde and Rubio-Ramírez (2005, 2007), and Farmer et al. (2010, 2011). Subsequently, DSGE models have been elaborated by many central banks, such as Smets and Wouters (2003) for EMU, SIGMA for the US, BEQM for England, TOTEM for Canada, AINO for Finland, and so on. In addition, Belaygorod and Dueker (2009) is one of the successes in the financial industry.

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examine monetary policy and concern important factors in open economy, such as exchange rate and terms of trade. For example, Galí and Monacelli (2005) extended the DSGE model to a small open economy setting, and analyzed the macroeconomic implications of three alternative rule-based policy regimes from a theoretical point of view. Bergin (2003) was the first one to extend the small open economy model in the empirical direction. Lubik and Schorfheide (2007) examined whether central banks target exchange rates, and found that the central banks of Australia and New Zealand don't, whereas the Bank of Canada and the Bank of England do include the nominal exchange rate in their policy rules. Using the model in Lubik and Schorfheide (2007) and the economic data in Chile, Caputo and Liendo (2005) found that the inflation persistence played an important role for small open economy, and Del Negro and Schorfheide (2009) assessed the robustness of conclusions to the presence of model misspecification. Dib (2010) found that small open economy and closed economy models in Canada lead to qualitatively similar structural parameter estimates and the effects of monetary policy shocks and other domestic shocks.

However, it's worth mentioning that most DSGE models in the existing literature are estimated at the boundary of the determinacy region. As a matter of fact, in order to solve DSGE models and keep them tractable, most economists typically use linear rational expectations (LRE) models as local approximation. Depending on the number of stable eigenvalues in the LRE model, the numerical solution might be non-existent, exhibit unique or multiple equilibria, and the unique and multiple equilibria are often referred to as determinacy and indeterminacy, respectively. More importantly, the dynamic response of the economy under indeterminacy would show some specific characteristics, such as sunspot shock and indeterminate propagation of fundamental shocks, which would not be present under determinacy.

Essentially, indeterminacy can arise if the central bank follows a Taylor-type rule and does not raise interest rates aggressively enough in response to an increase in inflation. For example, Lubik and Schorfheide (2004) firstly applied the standard new Keynesian monetary DSGE model to test the indeterminacy.[2] They found that the US monetary policy before 1979 contributes to the aggregate instability and that the policy becomes more stabilizing during the Volcker–Greenspan period. Treadwell (2009) used the same model to access the role of monetary policy across the G7 countries preceding and during the *Great Inflation*. Belaygorod and Dueker (2009) implemented a change point methodology to extend the model to encompass a sample period that includes both determinacy and indeterminacy. To the best of our knowledge, the indeterminacy is mostly concerned in the prototypical monetary DSGE model, whereas it's not taken seriously in the limited empirical papers within the framework of small open economy DSGE model.

This paper extends the small open economy DSGE model developed by Lubik and Schorfheide (2007) to the parameter space which allows for both determinacy and indeterminacy, and gives the general solution in a standard form. Besides, based on the empirical findings showing the unstable behavior of China's monetary policy,[3] we estimate the small open economy DSGE model with indeterminacy for China, and investigate the monetary policy and macroeconomic fluctuations.[4] Obviously, there are two contributions in this paper. One is to derive the boundary condition between determinacy and indeterminacy, and present the numerical solution for a small open economy DSGE model. The other one is to re-examine the monetary policy reaction function, especially test whether the PBC includes RMB exchange rate in its policy rule, and investigate monetary policy effect and macroeconomic fluctuations in a more accurate way.

The remainder of this paper is organized as follows. In Section 2, we outline a log-linearized small open economy DSGE model, discuss the determinacy and indeterminacy, and then present the numerical solution for a canonical linear rational expectations (LRE) model. Section 3 briefly shows the econometric approach, data description and choice of prior. In Section 4, we report the estimation results and analyze the macroeconomic dynamics using impulse response functions and variance decompositions. Section 5 offers some concluding remarks.

2. Small open economy DSGE model and its numerical solution

2.1. Small open economy DSGE model

Following Lubik and Schorfheide (2007) and Del Negro and Schorfheide (2009), in this paper we consider a small open economy model, which includes two economies, home (China) and rest-of-the-world (world). The consumption Euler equation can be rewritten as an open economy IS curve,

$$y_t = E_t(y_{t+1}) - (\tau+\lambda)(R_t - E_t\pi_{t+1}) - \rho_z z_t - \alpha(\tau+\lambda)E_t(\Delta q_{t+1}) + \frac{\lambda}{\tau}E_t(\Delta y^*_{t+1}) \ , \tag{1}$$

where $0<\alpha<1$ is the import share, and the equation reduces to its closed economy variant when $\alpha=0$. τ is the intertemporal substitution elasticity and $\lambda=\alpha(2-\alpha)(1-\tau)$. R_t, π_t and y_t denote the interest rate, CPI inflation rate and aggregate real output, respectively. z_t is the growth rate of an underlying non-stationary technology process A_t, q_t is the terms of trade, defined as the relative price of exports in terms of imports, and y^*_t is the world output. In order to obtain stationarity of the model, all real variables are expressed in terms of percentage deviations from A_t.

The optimal price setting strategy of domestic firms leads to the following Phillips curve,

$$\pi_t = \beta E_t\pi_{t+1} + \alpha\beta E_t\Delta q_{t+1} - \alpha\Delta q_t + \frac{K}{\tau+\lambda}(y_t - \bar{y}_t) \ , \tag{2}$$

[2] After estimating monetary policy reaction functions of reduced form, Clarida et al. (2000) suggested that the monetary policy rule in the US before 1979 is destabilizing and it leaves open the possibility of bursts of inflation and output. Recently Mavroeidis (2010) used identification robust methods to reexamine the empirical findings and confirmed that the policy before Volcker leads to indeterminacy, but the model is not accurately identifiable after 1979.

[3] For example Xie and Luo (2002) employed the historical analysis and reaction function method to conduct an empirical analysis of China's monetary policy in the framework of Taylor rule and draw the conclusion that this rule can accurately measure the operation level of China's monetary policy.

[4] In this paper, we assume a small open economy for China in that it does not have strong market power in the international market until now. In literature a small economy is a country that is a price taker in the international market, and it is not closely related to the total output, market size or territory area. The small open economies include not only small countries, i.e. Chile, Mexico and New Zealand, but also several big countries, i.e. Australia, Canada and England.

where $\bar{y}_t = -\lambda\tau^{-1}y_t^*$ is the potential output in the absence of nominal rigidities. Here the slope coefficient κ is a structural parameter and captures the degree of price stickiness in the economy.[5] The nominal rigidities disappear and the price is flexible as long as $\kappa\to\infty$.

It's worth noting that Taylor (2000) argued that central banks in new emerging market economies should respond to changes in exchange rates to improve the effect of monetary policy, and Xie and Zhang (2002) advised the People's Bank of China (PBC) to bring the exchange rate policy into the framework of monetary policy. Keeping that in mind, in this paper we assume that the PBC adjusts the interest rate in response to changes in CPI inflation and real output, moreover, it also takes nominal exchange rate depreciation into consideration. Furthermore, we allow the central bank to partially adjust the interest rate deviations from targets, which is also called interest rate smoothing, then we have the following monetary reaction function,

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[\varphi_1\pi_t + \varphi_2 y_t + \varphi_3\Delta e_t] + \varepsilon_{Rt} \quad , \tag{3}$$

where the interest rate smoothing parameter $0\le\rho_R<1$, the monetary policy reaction coefficients φ_1, φ_2, $\varphi_3\ge 0$, and the error term ε_{Rt} can be interpreted as an unanticipated monetary policy shock. In order to evaluate the hypothesis whether the PBC includes exchange rate changes in monetary policy reaction function, we will estimate the model separately under the restrictions $\varphi_3\ge 0$ and $\varphi_3=0$ and calculate a posterior odds ratio for the two specifications.

This paper also assumes that purchasing power parity (PPP) holds, then we can express the changes in nominal exchange rate as follows,

$$\Delta e_t = \pi_t - \pi_t^* - (1-\alpha)\Delta q_t \quad , \tag{4}$$

where π_t^* is the world inflation rate.

Here it is worth mentioning that the terms of trade is assumed to be exogenous, and a law of motion for their growth rate is,[6]

$$\Delta q_t = \rho_q \Delta q_{t-1} + \varepsilon_{qt} \quad . \tag{5}$$

And the technology shock is assumed to follow an AR(1) process,

$$z_t = \rho_z z_{t-1} + \varepsilon_{zt} \quad . \tag{6}$$

In addition, we also assume that the world output and world inflation, y_t^* and π_t^*, follow exogenous AR(1) processes,

$$y_t^* = \rho_{y^*} y_{t-1}^* + \varepsilon_{y^*t}, \quad \pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \varepsilon_{\pi^*t} \quad . \tag{7}$$

Finally, to fully structure the DSGE model, we introduce rational expectations forecast errors,

$$\eta_t^\pi = \pi_t - E_{t-1}\pi_t, \quad \eta_t^y = y_t - E_{t-1}y_t \quad . \tag{8}$$

2.2. Determinacy and indeterminacy

Before solving the log-linearized DSGE model with standard techniques, we have to figure out the boundary condition between the determinacy and the indeterminacy region. In general, the numerical solution under indeterminacy is much more complicated compared to the one under determinacy.

Since the interest rate smoothing parameter ρ_R is independent of the boundary condition, in this subsection we consider a special case of the small open economy DSGE model with $\rho_R=0$ for simplicity. That is, the monetary policy rule is given as follows,

$$R_t = \varphi_1\pi_t + \varphi_2 y_t + \varphi_3\Delta e_t + \varepsilon_{Rt} \quad . \tag{3$'$}$$

Then we can rewrite the Phillips curve in Eq. (2) as,[7]

$$E_t\pi_{t+1} = \frac{1}{\beta}\pi_t - \frac{\kappa}{(\tau+\lambda)\beta}y_t + \varepsilon^\pi(\Delta q_t, y_t^*) \quad . \tag{2$'$}$$

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[5] As shown in Del Negro and Schorfheide (2009), the slope coefficient is written as $\kappa=(1-\theta\beta)(1-\theta)/\theta$, where β is the discount rate for the representative household, and θ is the fraction of firms that update their prices by the steady-state inflation rate, while the remaining $1-\theta$ can set prices optimally in each period.

[6] In general, economists use real output, inflation rate, interest rate, exchange rate and terms of trade to estimate small open economy DSGE models. If the terms of trade is assumed to be endogenous, i.e. $\Delta q_t=(\Delta y_t^*-\Delta y_t)/(\tau+\lambda)$, then the number of exogenous shocks is less than the observable variables, which probably results in stochastic singularity in the model estimation. Theoretically, we can introduce a measurement error into any of the structural equations. Unfortunately, if we add a measurement error to a PPP equation, then we cannot identify it from exogenous shock π_t^* without further cross-equation restrictions. If we add a measurement error to the endogenous terms of trade dynamics, then in principle the model provides enough independent restrictions to identify the measurement error since the world output determines the domestic potential output. However, Lubik and Schorfheide (2007) argued that the model with endogenous terms of trade is too tightly restricted, and the tight link between the terms of trade and output growth brings a conflict with output and inflation dynamics as governed by the IS curve and the Phillips curve, and thereby causes difficulty in model estimation. In fact, they found that those model specifications do not converge and the parameter estimates are either implausible or not at local maxima. Therefore, Lubik and Schorfheide (2007) decided to introduce exogenous terms of trade.

[7] For simplicity, we collect all the exogenous variables in the Phillips curve in the polynomial term $\varepsilon^\pi(\Delta q_t, \pi_t^*)$, since they are unrelated to the stability of the economic system.

Combined with Eq. (2′), PPP in Eq. (4) can be rewritten as,

$$E_t\Delta e_{t+1} = E_t\pi_{t+1} + \varepsilon^{\Delta e}(\Delta q_t, \pi_t^*) = \frac{1}{\beta}\pi_t - \frac{\kappa}{(\tau+\lambda)\beta}y_t + \varepsilon^{\Delta e}(\Delta q_t, y_t^*, \pi_t^*) \quad . \tag{4′}$$

Similarly, combined with Eqs. (2′) and (3′), we can represent the IS curve in Eq. (1) as follows,

$$\begin{aligned} E_t y_{t+1} &= y_t + (\tau+\lambda)(R_t - E_t\pi_{t+1}) + \varepsilon^y(\Delta q_t, z_t, y_t^*) \\ &= y_t + (\tau+\lambda)\left[\varphi_1\pi_t + \varphi_2 y_t + \varphi_3\Delta e_t - \left(\frac{1}{\beta}\pi_t - \frac{\kappa}{(\tau+\lambda)\beta}y_t\right)\right] + \varepsilon^y(\Delta q_t, z_t, y_t^*, \pi_t^*, \varepsilon_{Rt}) \\ &= \left(1 + (\tau+\lambda)\varphi_2 + \frac{\kappa}{\beta}\right)y_t + \left((\tau+\lambda)\varphi_1 - \frac{\tau+\lambda}{\beta}\right)\pi_t + (\tau+\lambda)\varphi_3\Delta e_t + \varepsilon^y(\Delta q_t, z_t, y_t^*, \pi_t^*, \varepsilon_{Rt}) \quad . \end{aligned} \tag{1′}$$

Now the dynamics of the macroeconomic system can be represented in the following matrix form,

$$\begin{pmatrix} E_t y_{t+1} \\ E_t\pi_{t+1} \\ E_t\Delta e_{t+1} \end{pmatrix} = \begin{bmatrix} 1 + \frac{\kappa}{\beta} + (\lambda+\tau)\varphi_2 & -\frac{\lambda+\tau}{\beta} + (\lambda+\tau)\varphi_1 & (\lambda+\tau)\varphi_3 \\ -\frac{\kappa}{\beta(\lambda+\tau)} & \frac{1}{\beta} & 0 \\ -\frac{\kappa}{\beta(\lambda+\tau)} & \frac{1}{\beta} & 0 \end{bmatrix} \begin{pmatrix} E_{t-1}y_t \\ E_{t-1}\pi_t \\ E_{t-1}\Delta e_t \end{pmatrix} + \tilde{\boldsymbol{\Psi}} \begin{pmatrix} \Delta q_t \\ z_t \\ y_t^* \\ \pi_t^* \\ \varepsilon_{Rt} \end{pmatrix} + \tilde{\boldsymbol{\Pi}} \begin{pmatrix} \eta_t^y \\ \eta_t^\pi \\ \eta_t^{\Delta e} \end{pmatrix} \quad . \tag{9}$$

Note that the stability properties of the above system depend on the eigenvalues of the auto-covariance matrix in Eq. (9). Therefore, when the DSGE model is at the boundary between the determinacy and indeterminacy regions, the auto-covariance matrix has at least one unit eigenvalue. It follows,

$$0 = \det\begin{vmatrix} \frac{\kappa}{\beta} + (\lambda+\tau)\varphi_2 & -\frac{\lambda+\tau}{\beta} + (\lambda+\tau)\varphi_1 & (\lambda+\tau)\varphi_3 \\ -\frac{\kappa}{\beta(\lambda+\tau)} & \frac{1}{\beta} - 1 & 0 \\ -\frac{\kappa}{\beta(\lambda+\tau)} & \frac{1}{\beta} & -1 \end{vmatrix} \quad .$$

After tedious but straightforward algebra, we can obtain the following boundary condition,

$$\varphi_1 = 1 - \varphi_3 - \frac{(1-\beta)(\lambda+\tau)}{\kappa}\varphi_2 \quad . \tag{10}$$

Notice that Eq. (10) is a necessary, not sufficient boundary condition, since the calculations don't take the size of the other eigenvalues of the auto-covariance matrix.[8]

2.3. Numerical solution

We define $\boldsymbol{\xi}_t = [y_t, \pi_t, R_t, \Delta e_t, E_t y_{t+1}, E_t\pi_{t+1}, \Delta q_t, z_t, y_t^*, \pi_t^*]'$ as the state variable, $\boldsymbol{\varepsilon}_t = \left[\varepsilon_{Rt}, \varepsilon_{qt}, \varepsilon_{zt}, \varepsilon_{y^*t}, \varepsilon_{\pi^*t}\right]'$ as the fundamental shock, and $\boldsymbol{\eta}_t = [\eta_t^\pi, \eta_t^y]'$ as the rational expectations forecast error. Moreover, collect the model parameters in a 17×1 vector $\boldsymbol{\theta} = [\varphi_1, \varphi_2, \varphi_3, \rho_R, \alpha, r, \tau, \kappa, \rho_q, \rho_z, \rho_{\pi^*}, \rho_{y^*}, \sigma_R, \sigma_q, \sigma_z, \sigma_{\pi^*}, \sigma_{y^*}]'$.

Then we can rewrite the log-linearized DSGE model comprised of Eqs. (1)–(8) in "Sims canonical form",

$$\boldsymbol{\Gamma}_0(\boldsymbol{\theta})\boldsymbol{\xi}_t = \Gamma_1(\theta)\xi_{t-1} + \Psi(\theta)\boldsymbol{\varepsilon}_t + \Pi(\theta)\boldsymbol{\eta}_t \quad , \tag{11}$$

[8] Bullard and Mitra (2002) and Llosa and Tuesta (2008) show some formal proof techniques to obtain necessary and sufficient conditions for DSGE models.

where the coefficient matrices $\mathbf{\Gamma_0}(\boldsymbol{\theta})$, $\mathbf{\Gamma}_1(\boldsymbol{\theta})$, $\mathbf{\Psi}(\boldsymbol{\theta})$ and $\mathbf{\Pi}(\boldsymbol{\theta})$ depend on the structural parameters, and to be more specific,

$$\mathbf{\Gamma}_0(\boldsymbol{\theta}) = \begin{bmatrix} 1 & 0 & \tau+\lambda & 0 & -1 & -(\tau+\lambda) & \alpha(\tau+\lambda)\rho_q & \rho_z & -\frac{\lambda}{\tau}\left(\rho_{y^*}-1\right) & 0 \\ -\frac{\kappa}{\tau+\lambda} & 1 & 0 & 0 & 0 & -\beta & \alpha\left(1-\beta\rho_q\right) & 0 & -\frac{\lambda}{\tau}\frac{\kappa}{\tau+\lambda} & 0 \\ -(1-\rho_R)\varphi_2 & -(1-\rho_R)\varphi_1 & 1 & -(1-\rho_R)\varphi_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 1-\alpha & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{\Gamma}_1(\boldsymbol{\theta}) = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&\rho_R&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&\rho_q&0&0&0 \\ 0&0&0&0&0&0&0&\rho_z&0&0 \\ 0&0&0&0&0&0&0&0&\rho_{y^*}&0 \\ 0&0&0&0&0&0&0&0&0&\rho_{\pi^*} \end{bmatrix}, \quad \mathbf{\Psi}(\boldsymbol{\theta}) = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&0&0&0&0 \\ 0&1&0&0&0 \\ 0&0&1&0&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \end{bmatrix}, \quad \mathbf{\Pi}(\boldsymbol{\theta}) = \begin{bmatrix} 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \\ 1&0 \\ 0&1 \\ 0&0 \\ 0&0 \\ 0&0 \\ 0&0 \end{bmatrix}.$$

Following Sims (2002), Lubik and Schorfheide (2003, 2004) showed that, if the state variable $\boldsymbol{\xi}_t$ is non-explosive in an LRE system, then the expectation error $\boldsymbol{\eta}_t$ can be expressed as a linear combination of fundamental shock $\boldsymbol{\varepsilon}_t$ and sunspot shock ς_t. Using singular value decomposition, the numerical solution is given as follows,

$$\begin{aligned} \boldsymbol{\xi}_t &= \mathbf{\Gamma}_1^*(\boldsymbol{\theta})\boldsymbol{\xi}_{t-1} + \mathbf{B}_1(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t + \mathbf{B}_2(\boldsymbol{\theta})\eta_t \\ &= \mathbf{\Gamma}_1^*(\boldsymbol{\theta})\boldsymbol{\xi}_{t-1} + \mathbf{B}_1(\boldsymbol{\theta})\boldsymbol{\varepsilon}_t + \mathbf{B}_2(\boldsymbol{\theta})\left(\tilde{\mathbf{M}}\boldsymbol{\varepsilon}_t + M_\varsigma\varsigma_t\right) , \end{aligned} \tag{12}$$

where M_ς and $\tilde{\mathbf{M}}$ are the arbitrary vector and matrix, respectively. ς_t is a sunspot shock, which represents *animal spirit* or unexplained waves of optimism and pessimism. In order to fully identify the structural parameters, we set $M_\varsigma = 1$ and assume the matrix $\tilde{\mathbf{M}} = \mathbf{M}^*(\boldsymbol{\theta}) + \mathbf{M}$, where $\mathbf{M}^*(\boldsymbol{\theta})$ is to minimize the discrepancy between impulse response functions[9]

$$\frac{\partial\boldsymbol{\xi}_t}{\partial\boldsymbol{\varepsilon}'_t}\left(\boldsymbol{\theta}, \tilde{\mathbf{M}}\right) = \mathbf{B}_1(\boldsymbol{\theta}) + \mathbf{B}_2(\boldsymbol{\theta})\,\tilde{\mathbf{M}} \tag{13}$$

and

$$\frac{\partial\boldsymbol{\xi}_t}{\partial\boldsymbol{\varepsilon}'_t}(g(\boldsymbol{\theta}), \cdot) = \mathbf{B}_1(g(\boldsymbol{\theta})) , \tag{14}$$

where the vector $g(\boldsymbol{\theta})$ is obtained by replacing φ_1 in the vector $\boldsymbol{\theta}$ with the boundary condition between the determinacy and the indeterminacy region in Eq. (10). In our application we use the least square method to obtain the matrix $\mathbf{M}^*(\boldsymbol{\theta})$,

$$\mathbf{M}^*(\boldsymbol{\theta}) = \left[\mathbf{B}_2(\boldsymbol{\theta})'\mathbf{B}_2(\boldsymbol{\theta})\right]^{-1}\mathbf{B}_2(\boldsymbol{\theta})'\cdot\left[\mathbf{B}_1(g(\boldsymbol{\theta}))-\mathbf{B}_1(\boldsymbol{\theta})\right] . \tag{15}$$

Finally, the numerical solution in Eq. (12) can be represented as,

$$\boldsymbol{\xi}_t = \underset{10\times10}{\mathbf{\Gamma}_1^*(\boldsymbol{\theta})}\,\boldsymbol{\xi}_{t-1} + \left[\underset{10\times5}{\mathbf{B}_1(\boldsymbol{\theta})} + \underset{10\times1}{\mathbf{B}_2(\boldsymbol{\theta})}\,\underset{1\times5}{\mathbf{M}^*(\boldsymbol{\theta})} \quad \underset{10\times1}{\mathbf{B}_2(\boldsymbol{\theta})}\right]\begin{bmatrix} \underset{5\times5}{\mathbf{I}} & \underset{5\times1}{\mathbf{0}} \\ \underset{1\times5}{\mathbf{M}} & 1 \end{bmatrix}\begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \varsigma_t \end{bmatrix} , \tag{12'}$$

where $\mathbf{M} = \left[M_R, M_q, M_z, M_{y^*}, M_{\pi^*}\right]$ denotes the indeterminate propagation of fundamental shocks, and the error term $\left[\boldsymbol{\varepsilon}'_t, \varsigma_t\right]'$ is assumed to be multi-normal distributed with mean zero and variance–covariance matrix $\mathbf{\Sigma} = \operatorname{diag}\left(\sigma_R^2, \sigma_q^2, \sigma_z^2, \sigma_{y^*}^2, \sigma_{\pi^*}^2, \sigma_\varsigma^2\right)$.

3. Estimation strategy, data description and choice of prior

3.1. Model estimation

Given the five fundamental shocks in the log-linearized DSGE model, we introduce five measurement equations to prevent stochastic singularity in the model estimation. Considering the data availability in China, we choose the following observable variables: output gap (*Gap*), inflation rate (*Inf*), interest rate (*IR*), RMB appreciation rate (Δ*Ex*) and changes in terms of trade (Δ*ToT*). Following Lubik and Schorfheide (2007), the measurement equation that relates the observable variables and state variables is of the form,[10]

$$\begin{pmatrix} Gap_t \\ Inf_t \\ IR_t \\ \Delta Ex_t \\ \Delta ToT_t \end{pmatrix} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0 \\ 0&4&0&0&0&0&0&0&0&0 \\ 0&0&4&0&0&0&0&0&0&0 \\ 0&0&0&-1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0 \end{bmatrix}\begin{pmatrix} y_t \\ \pi_t \\ R_t \\ \Delta e_t \\ E_t\pi_{t+1} \\ E_t y_{t+1} \\ \Delta q_t \\ z_t \\ y_t^* \\ \pi_t^* \end{pmatrix}. \tag{16}$$

[9] For further details, readers are referred to Lubik and Schorfheide (2004) and Benati (2004).

[10] In small open economy DSGE models, economists generally use direct quotation (i.e. RMB/USD) to express foreign exchange rates, however, the IMF uses indirect quotation (i.e. USD/RMB) to construct nominal effective exchange rate indices in *International Financial Statistics*, hence we have $\Delta Ex_t = -\Delta e_t$.

Considering that the parameters are estimated over the parameter space which allows for both determinacy and indeterminacy, the overall likelihood function of the state space model comprised of the transition equation in Eq. (12') and the measurement equation in Eq. (16), can be evaluated by,

$$\ell\left(\boldsymbol{\theta},\mathbf{M},\sigma_\varsigma\middle|\mathbf{Y}^T\right)=\ell^D\left(\boldsymbol{\theta}\middle|\mathbf{Y}^T\right)\cdot\mathrm{I}\left\{\boldsymbol{\theta}\in\boldsymbol{\Theta}^D\right\}+\ell^I\left(\boldsymbol{\theta},\mathbf{M},\sigma_\varsigma\middle|\mathbf{Y}^T\right)\cdot\mathrm{I}\left\{\boldsymbol{\theta}\in\boldsymbol{\Theta}^I\right\},\tag{17}$$

where $\mathbf{Y}^T=[\mathbf{y}_1,\mathbf{y}_2,\ldots,\mathbf{y}_t]'$, $\mathbf{y}_t=[Gap_t,Inf_t,IR_t,\Delta Ex_t,\Delta ToT_t]'$. $\mathrm{I}\{\cdot\}$ is an indicator function of determinacy and indeterminacy, and $\ell^D\left(\boldsymbol{\theta}\middle|\mathbf{Y}^T\right)$ and $\ell^I\left(\boldsymbol{\theta},\mathbf{M},\sigma_\varsigma\middle|\mathbf{Y}^T\right)$ are likelihood functions over different parameter spaces,

$$\ell^D\left(\boldsymbol{\theta}\middle|\mathbf{Y}^T\right)=\prod_{t=1}^{T}p\left(\mathbf{y}_t\middle|\mathbf{Y}^{t-1},\boldsymbol{\theta}\right),\quad \ell^I\left(\boldsymbol{\theta},\mathbf{M},\sigma_\varsigma\middle|\mathbf{Y}^T\right)=\prod_{t=1}^{T}p\left(\mathbf{y}_t\middle|\mathbf{Y}^{t-1},\boldsymbol{\theta},\mathbf{M},\sigma_\varsigma\right).\tag{18}$$

Notice that the complicate multi-dimension matrix in the likelihood function usually results in many local maxima and very flat surfaces near the optimal solution, which would cause problems in maximum likelihood estimation procedure. Therefore, as to the model estimation, we attempt to use the MCMC method based on the Random-Walk Metropolis–Hastings (RW-MH) algorithm developed by Schorfheide (2000). In general, the RW-MH algorithm takes the following steps,

(a) *Use* **Csmiwel** *optimization routine to maximize the log likelihood function in Eq. (17) and obtain the posterior mode* $\hat{\theta}$[11];
(b) *Calculate the inverse of the (negative) Hessian matrix* $\hat{\Sigma}$ *at the posterior mode* $\hat{\theta}$ *numerically;*
(c) *Draw* $\theta^0\sim\mathcal{N}\left(\hat{\theta},\hat{\Sigma}\right)$, *where* $\mathcal{N}$ *denotes normal distributions*;
(d) *For* $s=1,2,\cdots,G$, *draw* $\theta^{(s)}\sim\mathcal{N}\left(\theta^{(s-1)},c^2\hat{\Sigma}\right)$,[12] *and the new draw is accepted* $(\theta^{(s)}=\vartheta)$ *with probability* $\min\{1,r(\theta^{(s-1)},\vartheta|\mathbf{Y}^T)\}$ *and rejected* $\theta^{(s)}=\theta^{(s-1)}$ *otherwise. Here,*

$$r\left(\theta^{(s-1)},\vartheta|\mathbf{Y}^T\right)=\frac{\ell\left(\mathbf{Y}^T|\vartheta\right)p(\vartheta)}{\ell\left(\mathbf{Y}^T|\theta^{(s-1)}\right)p\left(\theta^{(s-1)}\right)}.$$

Then the marginal data density (MDD) can be expressed as follows,

$$p^s\left(\mathbf{Y}^T\right)=\int\mathrm{I}\{\boldsymbol{\theta}\in\boldsymbol{\Theta}^s\}\cdot\ell\left(\boldsymbol{\theta}\middle|\mathbf{Y}^T\right)p(\boldsymbol{\theta})d\boldsymbol{\theta},\quad s\in\{D,I\}\tag{19}$$

and the logarithm of the MDD can be interpreted as maximized log-likelihood function panelized for model dimensionality.

Additionally, we can use the posterior probability of indeterminacy

$$\pi^I=\frac{p^I\left(\mathbf{Y}^T\right)}{p^I\left(\mathbf{Y}^T\right)+p^D\left(\mathbf{Y}^T\right)}\tag{20}$$

to infer the characteristic in the model equilibrium: if π^I goes to zero, then the data prefers equilibrium determinacy and suggests that the monetary policy is stabilizing; in contrast, if π^I is close to one, then the sunspot shock or the indeterminate propagation of fundamental shocks is extremely important in the macroeconomic dynamics (Lubik and Schorfheide, 2004).

[11] For simplicity, in the subsequent context, the parameter vector $\boldsymbol{\theta}$ also includes M and σ_ς if necessary.

[12] The tuning parameter c is typically chosen to obtain a rejection rate of about 50% in RW-MH sampling. For more details, please refer to Chib and Greenberg (1994).

As described before, one of the questions that we address empirically is whether the PBC reacts to exchange rate movements. And it motivates us to estimate two versions of the small open economy DSGE model, the first version $\mathcal{M}_1$ with $\varphi_3>0$ and the alternative version $\mathcal{M}_0$ with $\varphi_3=0$. Following Lubik and Schorfheide (2007), a natural way of assessing which model is more plausible is to construct the posterior odd of $\mathcal{M}_0$ versus $\mathcal{M}_1$,

$$\frac{\pi_{0,T}}{\pi_{1,T}}=\frac{\pi_{0,0}}{\pi_{1,0}}\cdot\frac{p\left(\mathbf{Y}^T|\mathcal{M}_0\right)}{p\left(\mathbf{Y}^T|\mathcal{M}_1\right)}\tag{21}$$

where the first factor is the prior odds ratio in favor of $\mathcal{M}_0$, and the second term is called the Bayes factor and summarized the sample evidence in favor of $\varphi_3=0$.

3.2. Data description

This paper uses China's quarterly data covering the period 1992:Q1–2011:Q4 with 80 observations in total. The original data for output gap and inflation rate are available from the China Economic Information (CEI) database (http://db.cei.gov.cn) and *China Monthly Economic Indicators*, while the data for interest rate is published by the *People's Bank of China Quarterly Statistical Bulletin* and the PBC's website (http://www.pbc.gov.cn). In addition, the exchange rate of RMB is obtained from the *International Financial Statistics* (IFS) database (http://www.imfstatistics.org/imf/), and the terms of trade is calculated with the data from the CEI database and the website of China Custom Statistics (http://www.chinacustomsstat.com).

The details of data selection, processing, and description are given as follows.

3.2.1. Output gap

In order to get the output gap, we have to compute the real output and potential output. In this paper, the real output is measured as gross domestic product (GDP), and its HP trend is taken as the potential output. In China, until 1992 the official statistics provide quarterly nominal GDP at current prices and cumulative growth rate of real GDP on a year-over-year basis. Following Zheng et al. (2012), we firstly calculate the real GDP Y_t (suppose the base year is 1992) based on the GDP growth rate, and then compute the seasonally adjusted series via the Tramo/Seats method executed in **Eviews**. After that we apply the HP filter (the smoothing constant is 1600) to obtain the potential output Y_t^*, then the annualized output gap is calculated as percentualized log-deviation of real output with respect to potential output, $Gap_t=100\times\ln(Y_t/Y_t^*)$.

3.2.2. Inflation rate

In this study, we use consumer price index (CPI) to measure the inflation. Notice that we must use the quarter-on-quarter inflation rate given the mathematical derivation and economic interpretation in the DSGE model. Although we can transfer the month-on-month CPI into the quarter-on-quarter data easily, the required data was not available until 2001. Hence, in our study we use the monthly year-on-year CPI to calculate the quarter-on-quarter inflation rate as follows. To begin with, we use the monthly year-on-year CPI to construct a price index (setting year 1991 = 100), and obtain the quarterly price index P_t by calculating the geometric mean. After seasonal adjustment executed by the Tramo/Seats method, we compute the annualized inflation rate as $Inf_t=400\times\ln(P_t/P_{t-1})$.

3.2.3. Nominal interest rate

Following Xie and Luo (2002), this paper selects the 7-day CHIBOR (*China inter-bank offered rate*) as a proxy variable for nominal interest rate, and it's also assumed to be the monetary policy instrument in China. In fact, we calculate the weighted average to obtain the quarterly

Table 1
Prior for distributions.

Parameter	Interpretation	Range	Density function	Para(1)	Para(2)
φ_1	Inflation deviation coefficient	$(0,+\infty)$	Gamma	1.10	0.50
φ_2	Output gap coefficient	$(0,+\infty)$	Gamma	1.00	0.50
φ_3	Depreciation rate coefficient	$(0,+\infty)$	Gamma	0.10	0.05
ρ_R	Interest rate smoothing parameter	[0,1)	Beta	0.50	0.20
α	Degree of openness	[0,1)	Beta	0.25	0.05
r^A	Steady state interest rate	$(-\infty,+\infty)$	Normal	2.00	2.00
κ	Inflation–output trade-off	$(0,+\infty)$	Gamma	0.50	0.25
τ	Intertemporal substitution elasticity	[0,1)	Beta	0.50	0.20
ρ_q	AR (1) for terms of trade shock	[0,1)	Beta	0.50	0.20
ρ_z	AR (1) for technology shock	[0,1)	Beta	0.50	0.20
ρ_{y^*}	AR (1) for world output shock	[0,1)	Beta	0.50	0.20
ρ_{π^*}	AR (1) for world inflation shock	[0,1)	Beta	0.50	0.20
M_R	Propagation of monetary policy shock	$(-\infty,+\infty)$	Normal	0.00	1.00
M_q	Propagation of terms of trade shock	$(-\infty,+\infty)$	Normal	0.00	1.00
M_z	Propagation of technology shock	$(-\infty,+\infty)$	Normal	0.00	1.00
M_{y^*}	Propagation of world output shock	$(-\infty,+\infty)$	Normal	0.00	1.00
M_{π^*}	Propagation of world inflation shock	$(-\infty,+\infty)$	Normal	0.00	1.00
σ_R	Std. dev of monetary policy shock	$(0,+\infty)$	Inverse gamma	0.10	2.00
σ_q	Std. dev of terms of trade shock	$(0,+\infty)$	Inverse gamma	1.50	2.00
σ_z	Std. dev of technology shock	$(0,+\infty)$	Inverse gamma	0.10	2.00
σ_{y^*}	Std. dev of world output shock	$(0,+\infty)$	Inverse gamma	1.50	2.00
σ_{π^*}	Std. dev of world inflation shock	$(0,+\infty)$	Inverse gamma	0.50	2.00
σ_ς	Std. dev of sunspot shock	$(0,+\infty)$	Inverse gamma	0.10	2.00

Note: The inverse gamma $I\mathcal{G}(s,v)$ priors are of the form $f(\sigma|s,v)\propto\sigma^{-v-1}e^{-vs^2/(2\sigma^2)}$, where v can be interpreted as the degree of freedom.

CHIBOR using the monthly interest rate and its corresponding trading volume.[13] Due to data availability, we use the weighted average of all term interest rates to compute the quarterly interest rate during the period of 1992–1995, which is also published in Xie and Luo (2002).

3.2.4. Exchange rate

As to the proxy variable for the nominal exchange rate, in this paper we choose the nominal effective exchange rate of RMB provided by the International Monetary Fund (IMF). We take log difference to obtain the percentage changes in exchange rate $\Delta Ex_t = 100\times\ln(Ex_t/Ex_{t-1})$.[14]

3.2.5. Terms of trade

The terms of trade is measured as the relative price index of exports in terms of imports, $ToT_t = EX_t/IM_t$, and it's converted in log differences to obtain percentage changes in the terms of trade $\Delta ToT_t = 100\times\ln(ToT_t/ToT_{t-1})$, which denotes improvement or deterioration in terms of trade. Following Wu and Wang (2006), we use the relative volume of exports in terms of imports to calculate the terms of trade in 1992, because the relative price index was not published until 1993.

Finally, all series are demeaned prior to estimation.

3.3. Choice of prior

In order to obtain accurate and reliable estimation results, we apply the MCMC method to estimate the model with the information in both given samples and prior distributions. However, the choice of prior is not an easy task and requires an important degree of judgment. In general, we choose them on both evidence from previous research and stylized facts in China. Table 1 presents the prior for distributions, in which the restrictions on the parameters, such as non-negativity, are implemented either by truncating the distribution or properly defining the parameters actually to be estimated (Del Negro and Schorfheide, 2009; Lubik and Schorfheide, 2004, 2007). Now we classify the parameter set into two groups, monetary policy parameters and structural parameters for the subsequent analysis.

3.3.1. Monetary policy parameters

First of all, we allow the inflation deviation coefficient to take a wide range of values that leads to both determinacy and indeterminacy, that is, φ_1 is distributed around 1.1 so that the fraction of the parameter space that leads to indeterminacy is about half. Secondly, the priors for φ_2 and φ_3 are centered at relatively small values, 1.0 and 0.1, which are commonly associated with the monetary policy rules in open economy models. Finally, the prior for the interest rate smoothing parameter ρ_R is the beta distribution with mean 0.5 and standard deviation 0.2.

3.3.2. Structural parameters

We choose the beta distribution with mean 0.25 and standard deviation 0.05 as the prior for the import share α, which is in line with the calibration result in Liu (2008). Our prior for annual interest rate r^A is centered at 2.0 with a standard deviation of 2.0.[15] As to the slope coefficient κ in the Phillips curve, its mean is set at 0.5 and the standard deviation is 0.25 in the gamma distribution. The prior for τ in the IS curve is centered at 0.5, which makes the representative agents more risk averse than those with log-utility. Following most literature, the beta distribution with mean 0.5 and standard deviation 0.2 is used as the prior for AR(1) coefficients. The priors for the components of the matrix $\mathbf{M}$ are standard normal distributions, based on the fact that the existing literature typically ignores the indeterminacy by setting $\mathbf{M}=\mathbf{0}$. For the variance of shocks, we use an inverse gamma prior, as is common for exogenous shocks in DSGE models. In fact, the standard deviations of the monetary shock and sunspot shock have the same distribution with $s=0.1$, while the prior mean (s) of σ_q, σ_{y^*} and σ_{π^*} are exactly the same as those in Lubik and Schorfheide (2007), which are centered at 1.5, 1.5 and 0.5, respectively. Additionally, following previous research on Chinese economy

[13] The quarterly weighted average is calculated as follows,

$$\bar{i} = i_1\frac{f_1}{\sum f} + i_2\frac{f_2}{\sum f} + \cdots + i_n\frac{f_n}{\sum f} = \frac{\sum if}{\sum f},$$

where i_k is the monthly interest rate and f_k is the corresponding trading volume.

[14] In this paper we use a linear interpolation to calculate the change in exchange rate in 1994:Q1, because the unification of the *dual exchange rates*, official exchange rate and swap market exchange rate, made the normal effective exchange rate of RMB depreciate a lot in the first quarter of 1994.

[15] In empirical studies, the DSGE model is usually parameterized in terms of the steady state real interest rate r^A, rather than the discount factor β, and the annualized interest rate is given as $\beta\equiv e^{-r^A/400}$.

(Liu, 2008), we set $v=2$ to allow the standard deviations to vary widely, and even to go to infinity.

4. Empirical analysis

In this section, we apply the MCMC method to estimate the open economy model during the period of 1992–2011 in China, and report the model estimation results in detail; afterwards, we present the impulse response dynamics of the open economy to exogenous shocks, and compute variance decompositions to gauge the importance of individual shocks.

4.1. Estimation results

In this subsection, we estimate two log-linearized DSGE models with or without exchange rate in the monetary policy rule, which are labeled models $\mathcal{M}_1$ and $\mathcal{M}_0$, respectively. From the estimation results in Table 2, we can find that the posterior probability of indeterminacy in both models is about 1, which indicates equilibrium indeterminacy in the model and implies the instability in the monetary policy conducted by the PBC. Moreover, when we compare the log marginal data densities calculated by the harmonic mean method described in Geweke (1999), the model $\mathcal{M}_1$ with exchange rate has a value of −819.74 versus −819.83 for the model $\mathcal{M}_0$. And the calculated posterior odd of $\mathcal{M}_0$ versus $\mathcal{M}_1$ is 0.9139. They illustrate the improvement in data fitting when the PBC is allowed to respond to exchange rate in its monetary policy, and lead us to take the exchange rate of RMB into account when studying macroeconomic fluctuations in China.

From economic intuition, the explanation for the monetary policy with exchange rate could be summarized as follows. Firstly, as shown in the law, the aim of the monetary policies shall be to maintain the stability of the value of the currency and thereby promote economic growth. Theoretically, the currency stabilization means not only the price stability, but also the stability in exchange rate dynamics. Secondly, the monetary policies in China are influenced by the exchange rate significantly, which is reflected on two aspects at least: one is the issues and suspension of the central bank bills with Chinese characteristics, and the other is the re-peg of RMB to US dollar during the periods from mid-2008 to June 2010. Last but not least, the export is one of the most important driving forces to promote economic growth in China, and it seems irrational for the PBC to ignore the exchange rate and thereby export growth when implementing monetary policies.

4.1.1. Monetary policy reaction function

In the subsequent analysis we only focus on model $\mathcal{M}_1$. As shown in Table 2, the posterior mean of inflation deviation coefficient φ_1, output gap coefficient φ_2 and depreciation rate coefficient φ_3 are 0.71, 0.39 and 0.09, respectively. If the actual inflation is 1% higher than the target inflation, the nominal interest rate raises by 72 base points but the real interest rate declines; if the real output is 1% higher than its potential value, both nominal and real interest rates increase by 39 base points simultaneously; similarly, if the nominal exchange rate of RMB depreciates by 1%, the central bank would increase the interest rate by 9 basis points. In addition, the interest rate smoothing parameter ρ_R is approximately 0.92, which shows the evidence of partial adjustment and implies a strong willingness of the PBC to smooth the movement of interest rate.

4.1.2. Structural parameters

Firstly, the estimated import share α is about 0.22, which is by and large consistent to the fact that China has been increasing the degree of openness in recent years. The posterior mean for the steady state interest rate r^A is 1.20, but its posterior probability interval is too wide to make sure the significance level. Actually, in order to stimulate consumption, expand investment, and thereby promote high economic growth, the Chinese central government keeps a very low or negative real interest rate in a long time. Secondly, the slope coefficient κ in the Phillips curve is about 0.85, which shows the evidence for inflation–output trade-off, thus the expectation-augmented Phillips curve is suitable to capture the movement of inflation in

Table 2
Model estimation results.

Parameters	Model $\mathcal{M}_1$ ($\varphi_3 \geq 0$)		Model $\mathcal{M}_0$ ($\varphi_3 = 0$)	
	Mean	95% interval	Mean	95% interval
φ_1	0.7098	[0.5291, 0.8825]	0.7379	[0.5503, 0.9453]
φ_2	0.3893	[0.1169, 0.6868]	0.3604	[0.1087, 0.6043]
φ_3	0.0944	[0.0223, 0.1737]	–	–
ρ_R	0.9237	[0.8913, 0.9567]	0.9279	[0.8953, 0.9564]
α	0.2195	[0.1553, 0.3057]	0.2368	[0.1646, 0.3115]
r^A	1.1961	[−2.2129, 4.8386]	2.1655	[−1.5517, 5.5370]
κ	0.8460	[0.6539, 0.9976]	0.8595	[0.6463, 0.9992]
τ	0.4711	[0.3170, 0.6430]	0.4395	[0.3111, 0.5752]
ρ_q	0.4370	[0.2894, 0.5739]	0.4592	[0.3306, 0.5768]
ρ_z	0.7126	[0.5230, 0.8823]	0.7107	[0.5174, 0.8907]
ρ_{y^*}	0.8669	[0.7882, 0.9529]	0.8802	[0.7946, 0.9573]
ρ_{π^*}	0.1952	[0.0382, 0.3486]	0.2015	[0.0566, 0.3663]
M_R	0.4192	[−1.1409, 1.9756]	−0.1367	[−1.4843, 1.1966]
M_q	0.3175	[0.2304, 0.4246]	0.3051	[0.2004, 0.4196]
M_z	−1.7757	[−2.5812, −1.1008]	−1.7619	[−2.5412, −1.1254]
M_{y^*}	−0.0928	[−0.3086, 0.1333]	−0.1171	[−0.3856, 0.1577]
M_{π^*}	0.3585	[0.2730, 0.4315]	0.3510	[0.2484, 0.4373]
σ_R	0.1220	[0.1027, 0.1408]	0.1227	[0.1062, 0.1408]
σ_q	4.2750	[3.5945, 4.9347]	4.2786	[3.6307, 4.9912]
σ_z	0.5359	[0.3055, 0.8121]	0.5510	[0.3246, 0.7821]
σ_{y^*}	1.3548	[0.6123, 2.3260]	1.0611	[0.5384, 1.6585]
σ_{π^*}	3.8928	[3.1382, 4.6159]	3.7201	[3.1591, 4.2603]
σ_ς	0.1313	[0.0355, 0.2302]	0.1347	[0.0487, 0.2659]
π^I	1.0000		1.0000	
Log-MDD	−819.74		−819.83	
Rejection rate	0.5690		0.5818	

Note: we generate MCMC runs of 100,000 iterations and discard the first 10,000 iterations as burn-in. π^I is the posterior probability of indeterminacy, “log-MDD” is short for log marginal data density, and “rejection rate” is the rejection probability in random-walk MetropolisHastings algorithm.

China. The intertemporal substitution elasticity τ is approximately 0.47, which is close to the estimates for the representative household in China (Liu, 2008). Thirdly, the AR(1) coefficients for shocks are significant, and it's also worth noting that the persistence in world output (0.87) is much higher than that in world inflation (0.20), just as it is in Lubik and Schorfheide (2007). Fourthly, the coefficients of the matrix **M** are not all significant, i.e. M_z is negative, M_q and M_{π^*} are positive, whereas M_R and M_{y^*} might be insignificant. Hence, to some extent, the effect of indeterminacy is to change the transmission of structural shocks related to technology, terms of trade and world inflation. Finally, all the estimated variances of fundamental and sunspot shocks are significant. Moreover, in line with Lubik and Schorfheide (2007), the standard deviations of world output (1.35) and world inflation (3.89) are much larger than that of the monetary shock (0.12).

4.2. Impulse response analysis

The system-based estimation approach allows us to investigate the propagation of fundamental and sunspot shocks in the open economy model. Fig. 1 depicts the posterior mean responses and pointwise 95% probability intervals of real output, inflation rate, interest rate and depreciation rate to one unit structural shocks. First of all, in response to an unanticipated tightening of monetary policy, the interest rate rises immediately, and has a negative effect on aggregate demand and

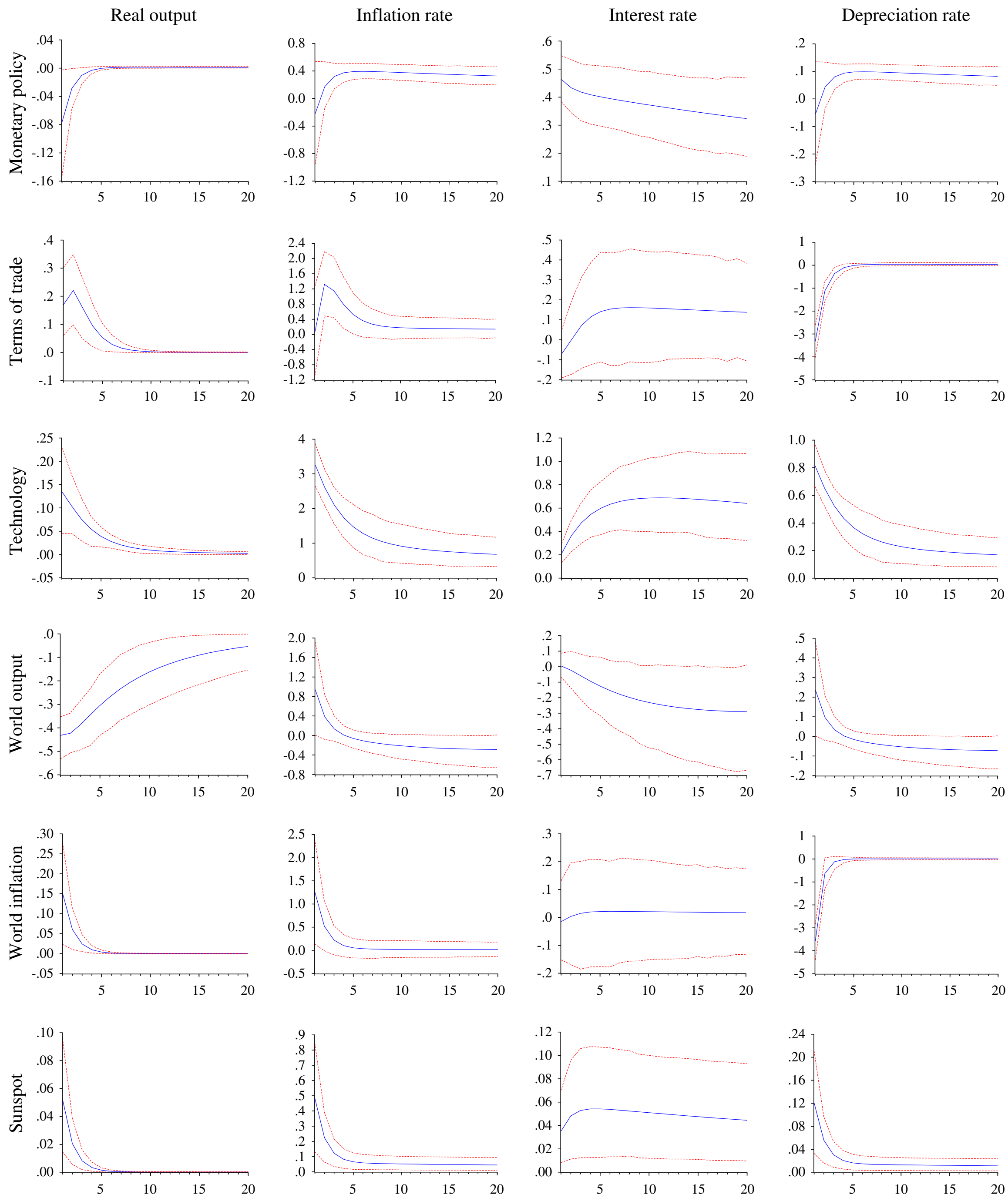

Fig. 1. Impulse responses. Note: The figure depicts posterior means (solid lines) and pointwise 95% posterior probability intervals (dashed lines) for impulse responses of real output, inflation rate, interest rate and depreciation rate to one unit structural shocks.

hence on total output. According to the Phillips curve and PPP, inflation falls below steady state and the RMB exchange rate decreases. As the interest rate falls towards steady state, real output returns to steady state after six quarters. It should be noted that an increase in nominal interest rate may have a slight inflation effect in the long run, just as mentioned in Lubik and Schorfheide (2004).

An improvement in the terms of trade (a positive shock) decreases the exchange rate, and then increases the real output and inflation rate. Responding to positive inflation and overheated economy, the central bank increases the interest rate by implementing a tight monetary policy. A positive technology shock reduces the marginal costs of production and thus increases the real output. In line with Lubik and Schorfheide (2004), the presence of indeterminacy creates an inflationary effect of technology shock ε_z since M_z is negative. Consequently, RMB depreciates as inflation increases, and then the central bank increases the interest rate.

Now it comes to the dynamic responses of the economy with respect to shocks from the rest of the world. Under a positive world output shock, domestic output declines along with an increase in inflation rate and exchange rate. It deserves special mention that the world output shock lowers the domestic potential output under the estimate of $\tau<1$, which essentially suggests a substitution effect between domestic and foreign products, and implies countercyclicality of domestic and world output (Lubik and Schorfheide, 2007). The negative inflation and RMB appreciation lead the central bank to decrease the interest rate. As to the world inflation shock, it directly appreciates RMB, but the interest rate increases slightly since the central bank reacts to imported inflation and overheated economy.

Lastly, under an inflationary sunspot belief (a negative sunspot shock), the expected real interest rate declines and the expected output growth is negative. The fall in real interest rate stimulates current consumption and thereby output. As is consistent with the Phillips curve, the inflation rate is positive which validates the assumption of sunspot-driven inflation expectations. In other words, the economic system in China suffers from the inflation that can be generated by self-fulfilling expectations. In turn, the central bank raises the interest rate to deal with the overheated economy with high inflation, as well as RMB depreciation.

4.3. Variance decomposition

In order to gauge the importance of individual shocks, we compute variance decompositions in this subsection. Table 3 presents the variance decompositions for real output (deviations from trend), inflation rate, interest rate and depreciation rate. According to posterior estimates, the variance decompositions can be summarized as follows. First, world output shock contributes most fluctuations in output (83%), and world inflation shock plays an important role to explain RMB exchange rate swings (42%), which is by and large consistent with Lubik and Schorfheide (2007). Second, the interest and inflation rates are largely driven by technology shock, and also shocks in monetary policy and terms of trade. Last but not least, similar to Lubik and Schorfheide (2004), sunspot shock does not have a notable impact on macroeconomic fluctuations.

5. Summary and concluding remarks

In recent years, under the guideline of *bring in and go global* economic strategies, China has gradually increased its degree of economic openness and integrated into the global economy; in the meanwhile, the international society puts more and more pressure on RMB appreciation given the current account surplus. The profound changes in international environment call for monetary and exchange rate policies under international perspective, thus the PBC should not just concentrate on domestic economic situations any longer. Furthermore, the theory and practice of monetary policy should pay attention to important international factors, such as terms of trade and exchange rate of RMB. Therefore, this paper aims to re-examine China's monetary policy and its link with the macroeconomic fluctuations within the framework of open economy DSGE model. Since the preferred open economy DSGE models in the existing literature are estimated at the determinacy region, we extend the DSGE model considering the parameter space which allows for both determinacy and indeterminacy.

In this paper, we firstly derive the boundary condition between determinacy and indeterminacy in a small open economy DSGE model, which provides a theoretical foundation for the empirical study, and then apply this model to investigate the monetary policy and macroeconomic fluctuations in China. Using the quarterly data from 1992 to 2011, we obtain some important conclusions as follows. Firstly, there is significant evidence of interest rate smoothing; the interest rate reacts not only to inflation deviations and output gaps, but also to changes in exchange rate of RMB. Furthermore, the monetary policy rule includes an exchange rate that fits the real data better than the one that does not. These empirical results support the view that central banks in emerging economies should take exchange rates into consideration when making monetary policies (Taylor, 2000; Xie and Zhang, 2002). Secondly, the open economy empirical DSGE model prefers equilibrium indeterminacy, which indicates the instability in the conduct of monetary policy by the PBC. Actually, the indeterminacy stems from two sources, the sunspot shock and the indeterminate propagation of the fundamental shocks, such as technology shock, world output shock and world inflation shock. Hence the impulse responses and variance decompositions under indeterminacy are essential to improve the dynamic macroeconomic analysis and monetary policy evaluation in China. Thirdly, the monetary policy shock has a significant influence on the dynamics of economy in the short run, while in the long run, it just affects nominal economic variables, i.e. inflation and exchange rates, but not real output. To some extent, though the central government could stimulate short-run economic growth by loose monetary policies, they have to rely

Table 3
Variance decompositions.

	Real output	Inflation rate	Interest rate	Depreciation rate
Monetary policy	0.0075	0.0978	0.1977	0.0241
	[0.0000, 0.0234]	[0.0160, 0.2193]	[0.0191, 0.3889]	[0.0032, 0.0634]
Terms of trade	0.0994	0.0816	0.0450	0.3981
	[0.0113, 0.2194]	[0.0034, 0.2078]	[0.0000, 0.1506]	[0.2730, 0.5367]
Technology	0.0362	0.6781	0.5877	0.1358
	[0.0047, 0.0744]	[0.4424, 0.9046]	[0.2735, 0.8854]	[0.0453, 0.2846]
World output	0.8280	0.1026	0.1557	0.0225
	[0.6624, 0.9637]	[0.0027, 0.3064]	[0.0001, 0.4793]	[0.0011, 0.0834]
World inflation	0.0253	0.0328	0.0093	0.4209
	[0.0000, 0.0629]	[0.0008, 0.0866]	[0.0000, 0.0354]	[0.2759, 0.5754]
Sunspot	0.0035	0.0070	0.0045	0.0014
	[0.0001, 0.0099]	[0.0003, 0.0177]	[0.0002, 0.0107]	[0.0000, 0.0035]

Note: The table reports posterior means and 95% probability intervals (in brackets).

on strategic emerging industries and structural adjustment to maintain sustained, rapid and sound development in the long run.

This paper estimates a small open economy DSGE model with indeterminacy, and the empirical results in China appear to be fairly good. However, it should be recognized that the monetary policy and macroeconomic fluctuations are contingent upon the small open economy DSGE model settings. Moreover, as discussed in Schorfheide (2011), there are several shortcomings of the DSGE model itself, such as the fragility of parameter estimates, the concern that whether exogenous shocks capture aggregate uncertainty or misspecification, the difficulty in reconciling the estimated DSGE model with low frequency behavior in time series, as well as measuring uncertainty in monetary policy as the predictions of the effects of rare policy changes often rely exclusively on extrapolation by theory, and so on.

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