

Risk-neutral and Physical Jumps in Option Pricing

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Abstract

When jumps are present in the price dynamics of the underlying asset, the market is no longer complete, and a more general pricing framework than the risk-neutral valuation is needed. Using Monte Carlo simulation, we investigate the important difference between risk-neutral and physical jumps in option pricing, especially for medium- and long-term options.

1 Introduction

In the presence of jumps, the financial market is no longer complete, and option payoffs cannot be replicated by a portfolio of primitive assets. In order to price options in an incomplete market, either strong assumptions are needed for the risk-neutral evaluation approach to be valid, or a more general pricing framework than the risk-neutral method is required.

Early effort to price options with jump risk starts with Merton (1976). He assumes the jump risk is diversifiable and uncorrelated with aggregate market returns, therefore the jump risk is not priced. The argument no longer holds when the underlying asset for the options is the market index itself. Jarrow and Rosenfeld (1984) reject the hypothesis of diversifiable jump risk. Applying the maximum likelihood estimation, they demonstrate that the market portfolio does contain a jump component and the jump risk should be priced.

To take into account of jump risk, various models have been developed. These include Ball and Torous (1983), Bates (1996, 2000), Cox and Ross (1976), Duffie, Pan, and Singleton (2000), Merton (1976), to name a few. Different processes have been assumed to describe the discontinuous jumps, including the variance-gamma jump model of Madan, Carr, and Chang (1998), the log stable model of Carr and Wu (2003), and the popular Lévy process of Carr and Wu (2004), Huang and Wu (2004), and Cont, Tankov, and Voltchkava (2004). By incorporating jumps into the asset price dynamics, these models can develop a multitude of volatility smile patterns observed in the market.

Along the line of risk-neutral evaluation approach, Bates (1991, 2000) and Bakshi, Cao, and Chen (1997) explicitly incorporate both volatility risk and jump risk under the risk-neutral measure. However, they make a simplifying and rather restrictive assumption that the premium for each risk factor is a constant proportion to the amount of risk. Adopting a more mathematical tool, Carr and Wu (2004) and Huang and Wu (2004) try to unify various stochastic volatility and jump-diffusion models via the use of time-changed Lévy process. More recently, empirical evidence from Pan (2002), Eraker, Johannes, and Polson (2003), and Eraker (2004) shows that jumps take place not only in the underlying asset price dynamics but also in the volatility process.

On the other hand, to use a more flexible pricing framework, Naik and Lee (1990) (hereafter NL), Ma and Vetzal (1997) (hereafter MV), and Ma (2006) resort to the general equilibrium framework in which both the diffusion risk and the jump risk are priced, and the risk aversion parameters explicitly incorporated. Built upon an exchange economy of the Lucas (1978), MV assumes recursive utility in the presence of Lévy jumps, and solves for a general pricing formula. The recursive utility allows the separation between intertemporal substitution and risk aversion, while the Lévy jumps are general enough that the jump frequency and the magnitude of the jumps may follow any distribution with finite moments.

In the special case of expected utility and lognormal jump size distribution, the model of Ma and Vetzal (1997) reduces to that of Naik and Lee (1990), who set up a fully stated equilibrium to price diffusion and jump risks in the index options market, and show that the risk premium is equal

to the covariance of option payoff with the change in the marginal utility of equilibrium aggregate wealth¹. The MV model also has a number of other well-known models as special cases. These include the Black-Scholes model when the underlying stock follows a pure diffusion process; the Merton (1976) model when restrictions on the preference parameters lead to a risk-neutral representative agent²; and the Cox and Ross (1976) model when the same restrictions on the preference parameters apply and the underlying price dynamics follows a pure jump process.

In this paper, we use Monte Carlo simulations to demonstrate the important differences that the jump risk in the risk-neutral measure and the jump risk in a fully stated equilibrium have on option prices. In particular, we make comparison in three aspects discussed below and we assume lognormal distribution for the jump size following MV (denoted MV(L)) in the first and second comparisons.

First, we compare option prices from the equilibrium models of NL and MV(L) with those from the risk-neutral measure of Bates (1991) (hereafter BA). We find that, unlike the fully stated equilibrium framework, the risk-neutral measure can only capture mild risk aversion. It consistently overprices options when there is moderate or high level of risk aversion embedded in option prices. As a result, investors are only partially compensated for undertaking the risk.

¹With S&P 500 index options, Santa-Clara and Yan (2004) come to a similar conclusion that the risk premium is a function of both the stochastic volatility and the jump intensity.

²There is a difference in that in Merton's model the jump risk is not priced because it is assumed to be diversifiable. Here it is not priced because the representative agent is risk-neutral given the restrictions on utility function.

Next, we examine the effect that different utility functions have on option prices. We compare option prices from NL with expected utility and from MV(L) with recursive utility (Epstein and Zin (1989)). The fanning parameter b in MV(L) controls the shape of indifference curve and results from non-expected utility certainty equivalent. When $b < 0$, the indifferent curve displays fanning out, which leads to a resolution to the well-known Allais paradox in experimental economics (see Epstein (1992), Ma and Vetzal (1997), and Machina (1982)). When b moves towards zero the fanning out effect is minimized. As a result, the recursive utility reduces to the expected utility and both NL and MV(L) give very similar option prices.

Finally, we make comparison between the lognormal and the uniform distribution as distribution for the jump size, following the specifications in Ma and Vetzal (1997). The option prices are denoted as MV(L) and MV(U) when the jump size follows the lognormal and the uniform distribution, respectively. We find that when the standard deviation of the jump size distribution is small, the choice between the two makes little difference. However, if the standard deviation of jump sizes is large, employing uniform distribution leads to significantly higher option prices.

The rest of this paper proceeds as follow. Section 2 discusses the theoretical relationship between risk-neutral and physical jumps in the context of pricing European options. Section 3 presents simulation results from four alternative option pricing models across time to maturity, strike prices, and level of risk aversion. Finally, Section 4 concludes.

2 Methodology

In the risk-neutral environment, the underlying non-dividend-paying stock S_t follows the stochastic jump diffusion process

$$\frac{dS_t}{S_t} = \alpha^Q dt + \sigma dZ_t + \int_0^\infty (u - 1) \Pi^Q(dt, du) \quad (1)$$

where α^Q is the expected return conditional on jumps occurring and equal to $r - \lambda^Q \mu_j^Q$; the risk-neutral jump distribution Π^Q is assumed to be lognormal with mean jump size μ_j^Q and jump intensity λ^Q ; σ is the instantaneous variance; and Z_t is a standard Brownian motion. Based on equation (1), the Bates (1991) pricing model for a European call option is given by

$$C_t(S_t, T, X) = e^{-rT} \sum_{n=0}^{\infty} \left[\frac{e^{\lambda^Q T} (\lambda^Q T)^n}{n!} \right] [S_t e^{h(n)T} N(d_{1n}) - X N(d_{2n})] \quad (2)$$

where

$$\begin{aligned} h(n) &= -\lambda^Q \mu_j^Q + \frac{n}{T} \ln(1 + \mu_j^Q) \\ d_{1n} &= \frac{\ln(S_t/X) + h(n)T + \frac{1}{2}(\sigma_T^2 + n(\sigma_j^Q)^2)}{\sqrt{\sigma^2 T + n(\sigma_j^Q)^2}} \\ d_{2n} &= d_{1n} - \sqrt{\sigma^2 T + n(\sigma_j^Q)^2} \end{aligned}$$

where σ_j^Q is the risk-neutral jump volatility.

In contrast, in an exchange economy of Lucas (1978), assume that the stock process S_t follows a jump-diffusion process with constant volatility,

$$\frac{dS_t}{S_t} = \alpha dt + \sigma dZ_t + \int_0^\infty (u - 1) \Pi(dt, du) \quad (3)$$

where α and σ are the drift and volatility of the diffusion process, and the Poisson measure is characterized by jump intensity λ and jump size distribution Π .

In terms of the utility function, the recursive utility function is defined as

$$U_t = W(c_t, \mu(U_{t+1}|\mathcal{F})) \quad (4)$$

where W is a utility aggregator and μ is a certainty equivalent that values the random future utility U_{t+1} conditional on the current information \mathcal{F} (Epstein and Zin (1989) and Ma (1992, 2006)). In continuous time, the utility function becomes (Ma (1992, 2000))

$$U_t = E_t \left[\int_t^T \left(f(c_{s-}, U_{s-}) + \lambda_0 \int_R [M(U_{s+}, U_{s-}) - U_{s+} + U_{s-}] \Pi(du) \right) ds + U_T \right]. \quad (5)$$

The recursive utility function is determined by two primitive functions $f(\cdot, \cdot)$ and $M(\cdot, \cdot)$, representing intertemporal substitution and risk aversion, respectively:

$$f(kc, k^\eta v) = k^\eta f(c, v), \quad M(x, y) = y\phi\left(\frac{x}{y}\right) \quad (6)$$

where η is a measure of *local* risk aversion and $\phi(\cdot)$ is a measure of *global* risk aversion (Ma and Vetzal (1997) and Ma (1992, 2006)). In this way, risk aversion and intertemporal substitution can be separated.

Given the price dynamics in (3) and the information structure, Ma and Vetzal (1997) shows that an European call option, with strike price X , underlying price S_t at time t , and expiration time T , is valued as follows³,

$$C(S_t, T, X) = X e^{-rT} \mathcal{L}^{-1} \{ \Phi_T(\cdot) \} \left(\ln \frac{X}{S_t} \right) \quad (7)$$

where

$$\Phi_T(s) \equiv \frac{\Theta^T(s)}{s(s+1)}$$

³See also Ma (1992, 2006).

$$\log \Theta(s) = -[\alpha + (\eta - 1.5)\sigma^2]s + \frac{1}{2}\sigma^2 s^2 + \lambda \int_0^\infty (u^{-s} - 1)\phi'(u^\eta)u^{\eta-1}\Pi(du)$$

with \mathcal{L}^{-1} the bilateral Laplace inverse operator and s any real integer less than -1 .

This is a very general closed form pricing model for European call options built on the equilibrium economy. It can be reduced to a number of pricing models in the existing literature as special cases. They include the Black-Scholes model when the jump component is removed by setting $\lambda = 0$, and the NL model when the recursive utility function is reduced to the expected utility function with jump size following the lognormal distribution.

More specifically, we let the recursive utility function (6) be as follows,

$$f(c, v) = \frac{\beta [c^\zeta - (\eta v)^{\frac{\zeta}{\eta}}]}{\zeta (\eta v)^{\frac{\zeta}{\eta}-1}}, \quad \phi(z) = \frac{\eta}{a} [z^{1-\frac{b}{\eta}} - z^{\frac{b}{\eta}}] \quad (8)$$

where $z > 0$, $\eta \equiv a + 2b \leq 1$ measures risk aversion, and $\frac{a}{b} < -1$ to ensure risk averseness. In this specification, the local risk aversion and the elasticity of intertemporal substitution is controlled by $1 - \eta$ and $(1 - \zeta)^{-1}$, respectively, and the subjective rate of time preference is given by β . The level of risk preference is jointly determined by a and b , and the sign of b also determines the shape of indifference curves, which is termed fanning in or fanning out. When $b \neq 0$, preferences are nonlinear in probabilities. If $b < 0$, we have fanning out, which generates predictions that are consistent with Allais experimental results, known as Allais paradox. The restrictions of $b = 0$ and $a = \zeta$ reduce the recursive utility specification of (6) and (8) to the expected utility model. In more detail, further simplification of utility function (8) into

$$f(c, v) = \frac{c^\gamma}{\gamma} - bv, \quad \phi(z) = z - 1 \quad (9)$$

leads to the iso-elastic utility form and hence the model by Naik and Lee (1990). Here $\gamma \leq 1$ reflects the degree of risk aversion and $\gamma = 1$ indicates risk-neutrality.

In the framework of Ma and Vetzal (1997), we also have the additional flexibility that the jump size can follow different distributions, such as the lognormal distribution or the uniform distribution (Ma and Vetzal (1997) and Ma (2006)). Assume the jump size follows lognormal distribution. The moment generating function for the corresponding normal distribution is as follows,

$$g(x) = \exp \left[\left(\ln(1 + \mu_J) - \frac{1}{2}\sigma_J^2 \right) x + \frac{1}{2}\sigma_J^2 x^2 \right] \quad (10)$$

With the utility function (8) and moment generating function (10), formula (7) simplifies to

$$C(S_t, T, X) = X e^{-rT} H \quad (11)$$

where

$$H = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{Re} \left[\frac{X e^{x+iy}}{S_t(x+iy)(x+iy+1)} \Theta^T(x+iy) \right] dy$$

$$\begin{aligned} \ln \Theta^T(s) &= \frac{\lambda T}{a} [(a+b)(g(a+b-s-1) - g(a+b-1)) - b(g(b-s-1) \\ &\quad - g(b-1))] - (r - \frac{1}{2}\sigma^2)sT + \frac{1}{2}\sigma^2 s^2 T \end{aligned}$$

With the lognormal distribution for the jump size, there exists a theoretical relationship between the risk-neutral and physical jumps in terms of jump intensity and jump size as follows,

$$\lambda^Q = \lambda \exp \left((\eta - 1) \left[\ln(1 + \mu_J) - \frac{1}{2}\sigma_J^2 \right] + \frac{(\eta - 1)^2}{2}\sigma_J^2 \right) \quad (12)$$

$$\mu_J^Q = (1 + \mu_J) \exp \left((\eta - 1)\sigma_J^2 \right) - 1. \quad (13)$$

They come from the more general formulae of equations (31) and (32) in Ma (2006). The risk-neutral jump intensity λ^Q and the mean jump size μ_j^Q are proportional to their physical counterpart. The mean jump size has been assumed to be negative for downward jumps and it tends to go up and less negative when moving from the risk-neutral measure to the physical measure indicating smaller downward movement. But it is less clear which direction the jump intensity will follow when moving to the physical measure.

On the other hand, we can also assume the jump size follows uniform distribution. It turns out that the only change in formula (11) is the definition of moment generating function. If the jump size follows uniform distribution within the interval $[U_L, U_H]$ with $U_H > U_L$, the moment generating function $g(x)$ is given by

$$g(x) = \frac{1}{x+1} \frac{e^{U_H(x+1)} - e^{U_L(x+1)}}{U_H - U_L}. \quad (14)$$

The rest of (11) remains unchanged.

3 Simulations

In this section, we use Monte Carlo simulations to demonstrate the differences that the following pairs have on option prices.

- the risk-neutral and physical jumps;
- the recursive utility function and the expected utility function;
- the lognormal and the uniform distribution for jump size.

We carry out 100,000 simulations in each exercise. We specify that the long-term, medium-term, and short-term options have two years, nine months,

and three months to maturity, respectively.

Following NL, the underlying stock price S_t follows a stochastic differential equation with random jumps which follow the lognormal distribution as in (3),

$$\frac{dS_t}{S_t} = \alpha dt + \sigma dZ_t + \int_0^\infty (u - 1) \Pi(dt, du).$$

The annual expected rate of return and volatility⁴ for the stock are assumed to be 0.06 and 0.17, respectively. The riskless interest rate r is set at 0.03 with zero dividend yield for simplicity, mean jump size μ_J and the standard deviation of jump size σ_J are -0.05 and 0.07, respectively; and jump intensity is 0.5, indicating 1 jump every 2 years.

Figure 1 plots one simulation for the pure diffusion process and the jump-diffusion process over a 2-year period. The stock price starts at 3379 and ends at 3783.5 for the jump-diffusion process, and 4100 for the pure diffusion process due to a crash around time 0.3, roughly on the 110th day. The magnitude of this crash is approximately 263.8 points, 8.35% of the starting value.

3.1 Risk-neutral and Physical Jumps

In Figure 2 we apply the BA model with the risk-neutral jump, and the NL model with the physical jump. In addition to the parameter values mentioned already, we specify the option strike price to be 3200, and risk aversion parameters γ for NL to be 0.42 for moderate risk aversion.

⁴The continuous diffusion volatility is unchanged when moving from the risk-neutral measure to the physical measure. See for example Liu, Shackleton, Taylor, and Xu (2007). For simplicity, the stochastic volatility is replaced by a constant volatility.

Throughout the time series, option values from BA are clearly higher than those from NL, reflecting lower option returns and lower compensation for the jump risk. The price differences mainly come from four sources, all of which result directly from the differences between the risk-neutral and physical measure in pricing options. First, the expected return on the stock is the riskfree interest rate in the risk-neutral measure, but incorporates risk premium in the physical measure; second, investor's risk attitude plays a role. In NL, risk aversion is captured by γ and affects option prices when $\gamma \neq 1$; third, the jump intensity is different as shown in (12); and fourth, the mean jump size is different as shown in (13). However, the divergence decreases as the time to maturity shortens. For three-month options, the difference between BA and NL is negligible. This is intuitive, as we expect the market risk dominates the jump risk when options are close to expiry.

The general equilibrium framework by NL, MV, and Ma (2006) is considered to be more flexible because risk aversion parameter enters into the pricing formulae. This makes it easier to explicitly recover investors' risk preferences from the options market. The effect of investor's risk aversion on option prices is illustrated in Figure 3. All option prices in Figure 3 are generated by NL across time to maturity and risk aversion. The values of 0.18, 0.42 and 0.84 for γ indicate strong, moderate, and mild risk aversion. Not surprisingly, the highest level of risk aversion correspond to the lowest option prices in each graph, and the effect is much less dominant for short-term options.

Calibrated option prices from BA and NL are also presented in Table 1 in the columns headed BA (column 1) and NL (columns 2 to 4). We report

call option prices across a range of moneyness, time to maturity, and risk aversion levels. Under the header NL, the letters I, II, and III correspond to strong, moderate, and mild risk aversion when γ is equal to 0.18, 0.42, and 0.84, respectively. For BA model, the option prices are lower than those from NL when there is mild jump risk aversion. This indicates that the BA model is unable to capture moderate to strong jump risk aversion even if this preference is embedded in the option prices.

3.2 Recursive and Expected Utility Function

Both NL and MV(L) models are special cases of the general pricing model (7), with the former assuming expected utility function while the latter recursive utility function. In the recursive utility function (8), $\eta \equiv a + 2b \leq 1$, a changes with η when b is set to be -0.01 for fanning out. The expected utility function imposes the constraints that $b = 0$ and $a = \zeta$ (intertemporal substitute) and gives $\eta = a = \zeta$.

Subsequently, the difference between the option prices from NL and MV(L) are determined by the value of b . Ma and Vetzal (1997) illustrate that the effect of fanning b on option prices is not significant when b is close to zero, and option values increase sharply when b moves away from zero in either direction⁵.

In Figure 4, we keep η constant at a moderate risk aversion level of 0.42 and set the range for b from -5.5 to 0 and from 0.43 to 5.5 so as to ensure $\frac{a}{b} < -1$ and $a + 2b \leq 1$. We value long-term ATM options when the strike price is

⁵The impact of fanning b shown here is opposite to the conclusion of Ma and Vetzal (1997) because we assume a negative jump while they assume a positive jump.

3400 and the underlying index is 3402.44. With the specified parameter values, option prices from NL is constant at 408.22 regardless of b as b does not enter into NL model. As demonstrated in Figure 4, when b is very close to 0, say at -0.01, the recursive utility function almost reduces to the expected utility function so that the curve of MV(L) prices is tangent to the horizontal line of NL prices. However, the price of MV(L) declines sharply when b moves away from zero and the differences between NL and MV(L) increase. The pattern is the same for medium-term, short-term, ITM, or OTM options.

The pattern has also been tabulated in Table 1. Under the header of MV(L), there are three columns (5 to 7) of call option prices with different risk aversion when b is equal to -0.01 and three columns (8 to 10) of prices when b is equal to -2.50. We can see that regardless of the risk aversion levels, the prices from MV(L) when $b = -0.01$ are the same to those estimated by NL to the second decimal point, reflecting the fact that the recursive utility is very close to the expected utility. When b moves away from zero to -2.50, call prices from MV(L) are lower for the same time to maturity and risk aversion. It is worth noting that in the last six columns of Table 1 (columns 11 to 16), when the jump size follows uniform distribution, the same pattern holds. Option prices from the model when b is equal to -2.50 are lower than those when b is -0.01.

3.3 Lognormal and Uniform Jump Size Distribution

All option prices in Figure 5 come from Ma and Vetzal (1997) where the jump size follows either the lognormal distribution (MV(L)) or the uniform

distribution (MV(U)) across different time to maturity, a range of risk aversion levels, and a range of standard deviation for jump size. The bounds U_H and U_L for the uniform jump size distribution are set such that the uniform distribution has the same first and second moment as the lognormal distribution.

First, regardless of the distributional assumption for the jump size, higher level of risk aversion leads to lower option values, consistent with previous figures. When the standard deviation of the jump size σ_J is small in the region $[0.01, 0.1]$, option prices from the two models are fairly similar across risk aversion levels with the option prices from MV(U) slightly higher than those from MV(L). It implies that to choose either the lognormal or the uniform distribution for the jump size has little impact on option prices. However, when σ_J increases up to 0.4 the differences become very clear, especially for low level of risk aversion. For long-term options when η is equal to 1 and σ_J is equal to 0.4, the price from MV(L) is 824.52 while the price from MV(U) is 1594.71, almost twice as much as that for MV(L). It shows that if there is high variation in the jump size, caution must be exercised to choose an appropriate distribution to model the jump size.

Simulation results are also reported in Table 1. For long-term deep ITM options with moderate risk aversion when $b = -0.01$, option is valued at 779.27 for MV(L) (panel A column 6) but 806.55 for MV(U) (panel A column 12). The difference is greatly reduced for the same option with short-term maturity to 618.12 for MV(L) (panel C column 6) and 622.01 for MV(U) (panel C column 12). For OTM call options, the differences are even smaller.

4 Conclusions

In this study we use Monte Carlo simulations to make a careful distinction between how jump risk enters into the option valuation models and how it affects option prices. In particular, in the risk-neutral framework jump risk premium is assumed to be a constant proportional of the risk in order to simplify modelling. In the more flexible general equilibrium models, a risk aversion parameter explicitly takes into account of jump risk.

We simulate option prices over time along a number of lines. These include different time to maturity, moneyness, level of risk aversion, utility function assumed, and the underlying distribution for jump sizes, in order to highlight the effect of jump risk on option prices. Results show that when jump risk premium is assumed proportional to the risk in the risk-neutral measure, as in Bates (1991) and Bakshi, Cao, and Chen (1997), the model overprices options across board, and it is able to capture only a mild to moderate level of risk aversion embedded in option prices. Therefore investors are not fully rewarded for taking the risk. With a more flexible pricing model as Naik and Lee (1990) and Ma and Vetzal (1997), a risk aversion parameter enters into the model to account for the risk. In addition, using either the expected or the recursive utility function similarly reflect investor's aversion to jump risk when the fanning parameter b is very close to zero. When b moves away from zero, pricing model with expected utility tends to overprices options. Moreover, for jump size distribution the choice between the lognormal or the uniform distribution makes a big difference when the the standard deviation of the jump size is high. However, as time to expiry shortens, all the effects different choices mentioned above on option prices diminish as the aggregate

market risk dominates option prices.

In this study, we only use Monte Carlo simulations and specify the parameter values in the model. A natural extension would be to empirically infer the parameters values from traded option prices to examine further the differences between risk-neutral and physical jumps.

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Figure 1. Stock processes with and without jumps

The drift and volatility of the geometric Brownian motion (GBM) and the jump-diffusion (JD) process are 0.03 and 0.17, respectively. For the jump-diffusion process, the jump intensity is 0.5, and the jump size follows lognormal distribution with mean -0.05 and volatility 0.07.

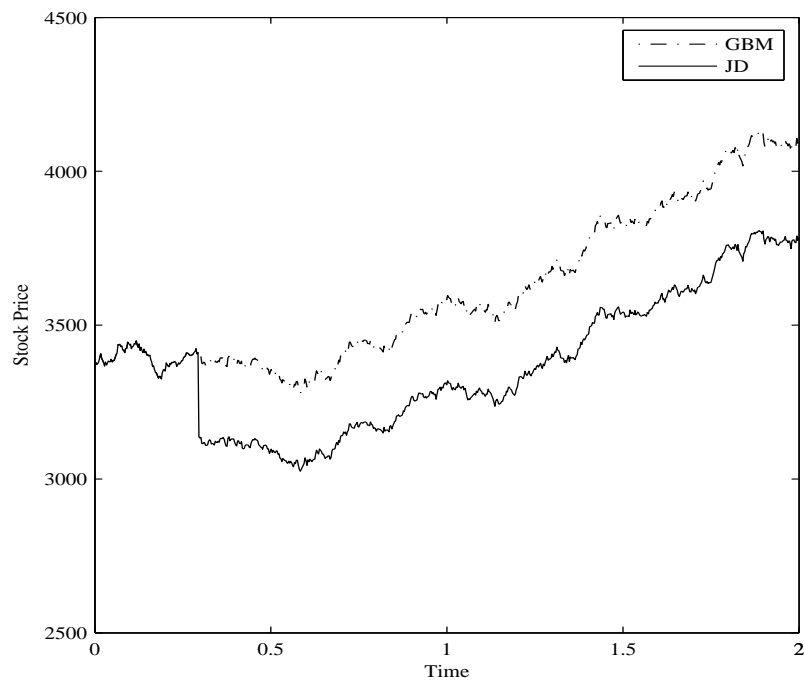


Figure 2. Option prices with risk-neutral or physical jumps across maturity

Option prices come from the BA model with risk-neutral jumps and the NL model with physical jumps. For NL, risk aversion parameter γ is 0.42. The option strike is 3200.

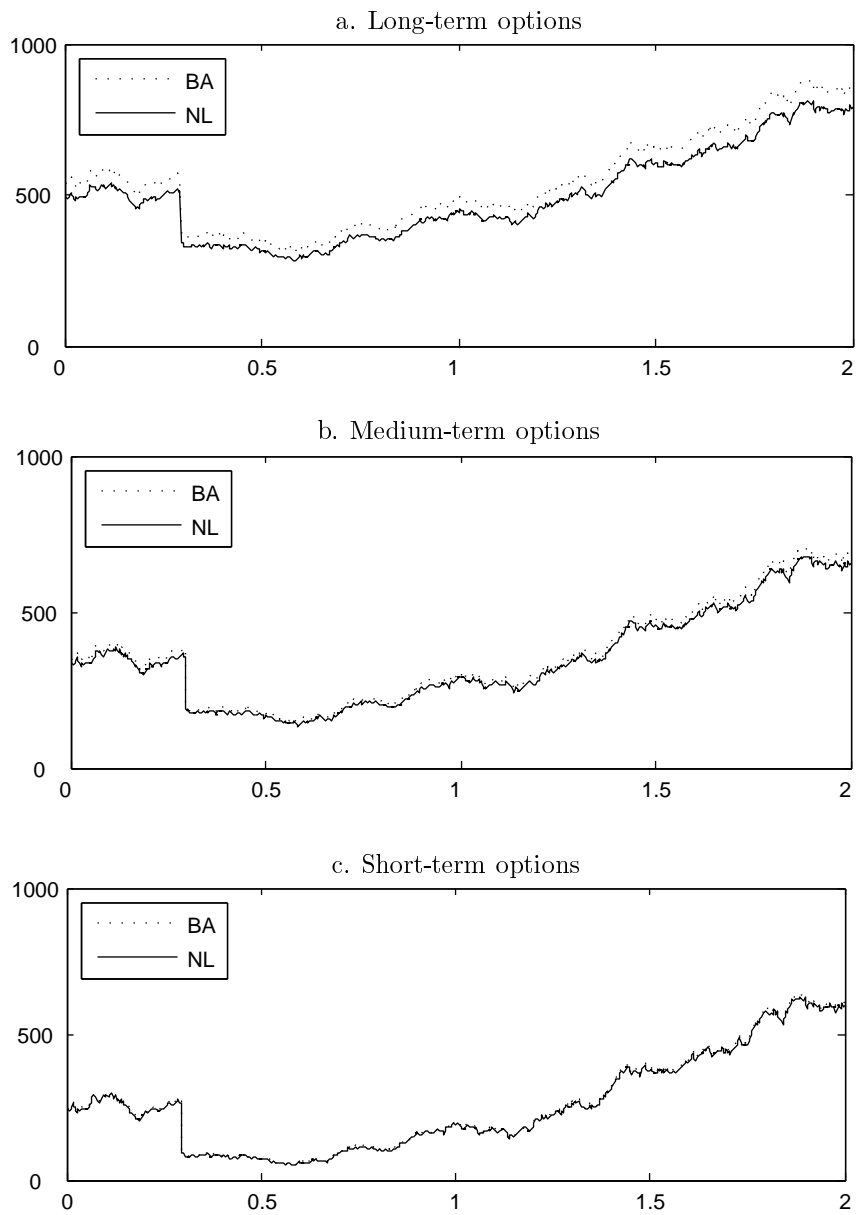


Figure 3. Option prices across different levels of risk aversion

Option prices come from the NL model. The risk aversion parameter γ takes three different values, 0.18, 0.42, and 0.84 for strong, moderate and mild risk aversion.

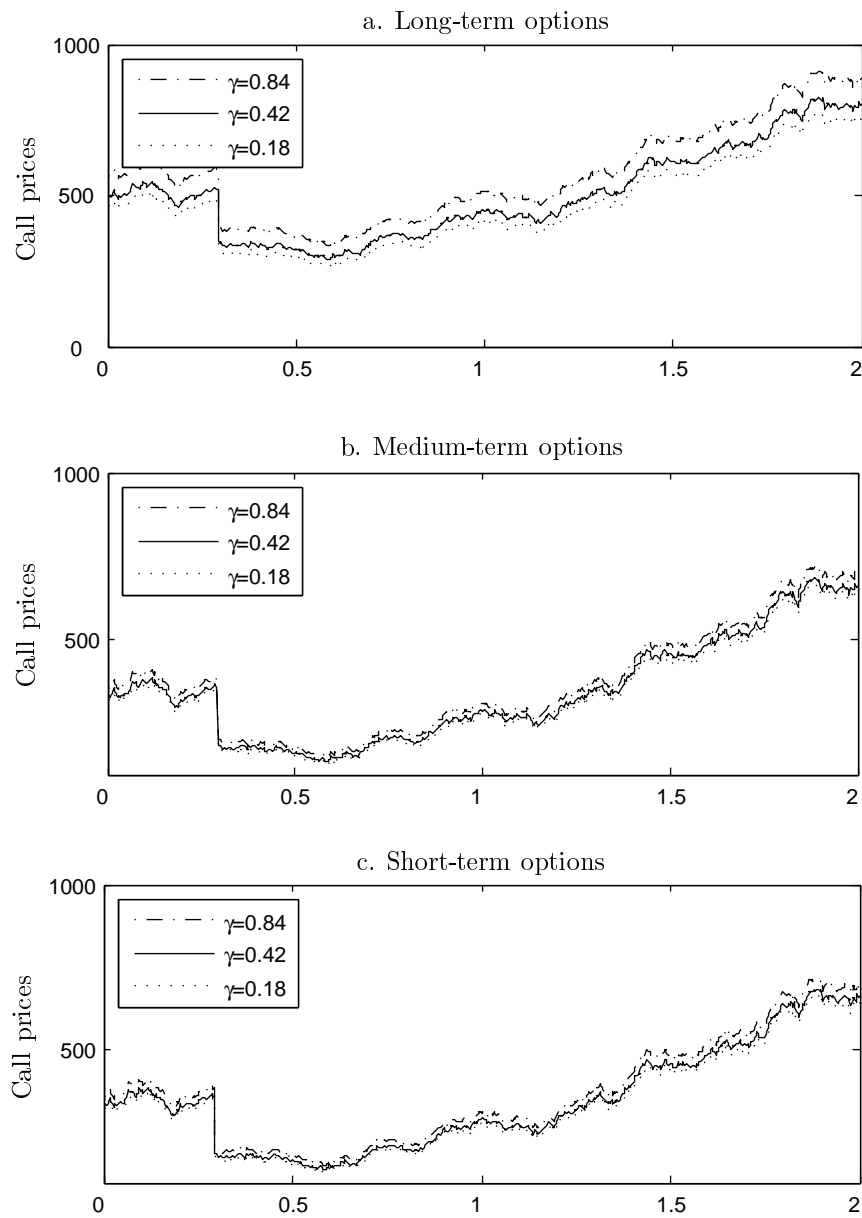


Figure 4. The effect of fanning on option prices

The prices of long-term ATM option with strike price 3400 and underlying index level at 3402.44 are plotted when the fanning parameter b is between -5.5 to 0, and between 0.43 to 5.5 using MV(L). For each value of b , a is chosen to fix η at 0.42. The prices of the same option from NL are constant at 408.22 with changes of b .

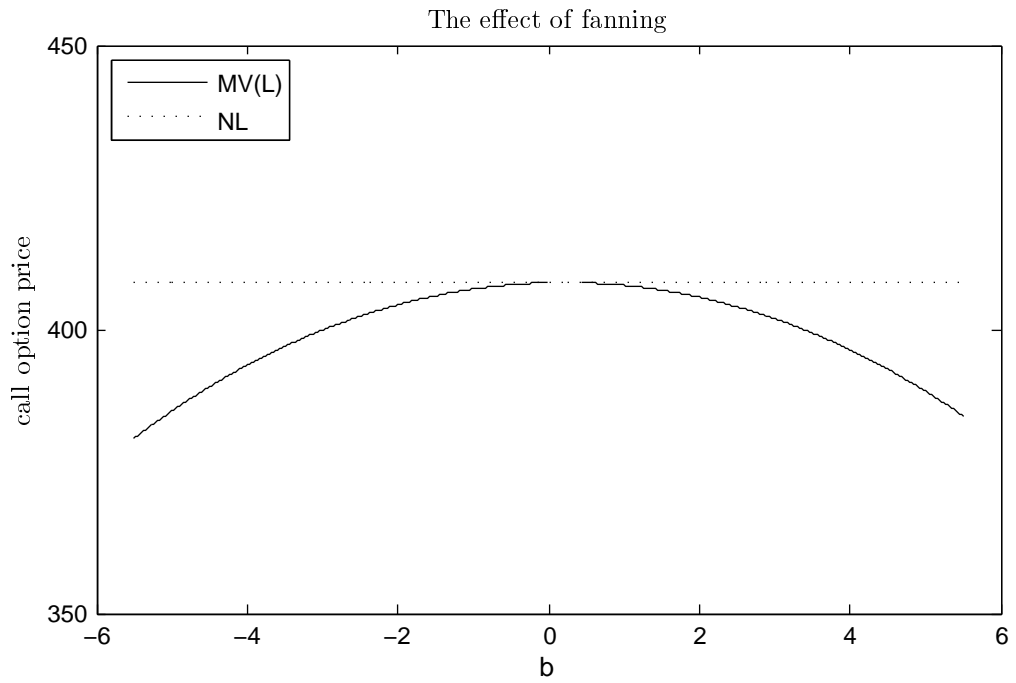


Figure 5. Option prices with different jump size distribution

The option prices come from either MV(L) with lognormal jump size distribution or MV(U) with uniform jump size distribution, with η and σ_J denoting risk aversion and the volatility of the jump size distribution.

